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THE HYDRAULIC JUMP

IN A NON-RECTANGULAR OPEN CHANNEL

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A THESIS

Presented to

the Faculty of the Graduate Division

Georgia Institute of Technology

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Civil Engineering

By

Alberto J. Rodriguez-Diaz

September 1954

THE HYDRAULIC JUMP

IN A NON-RECTANGULAR OPEN CHANNEL

Approved:

Date Approved by Chairman: Sept. 10, 1954

ACKNOWLEDGMENTS

I wish to express my thanks to Dr. Carstens, upon whose suggestion and under whose supervision and guidance this investigation has been conducted.

I also wish to thank Mr. H. J. Bates for his help in the construction of the equipment, Professor Kindsvater and Professor Goglia for their helpful suggestions as members of the thesis reading committee.

Most of all I would like to recognize the love and encouragement given to me by my parents in the pursuit of my education.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

ABSTRACT

The hydraulic jump was studied in a trapezoidal channel in which the sidewalls were vertical and the water surface and the bottom channel were not parallel. The purpose of the study was twofold.

a. The length and depth characteristics of the hydraulic jump were determined for this particular non-rectangular section.

b. It was found that the hydraulic jump did not occur in a triangular channel. Instead, the transition from supercritical flow to subcritical flow was accompanied by a series of large eddies on the shallow side. This phenomenon is far different than the hydraulic jump in which the transition is accomplished through a single breaking wave in which the downstream water surface is comparatively smooth. However, if a wall is placed on the shallow side and is moved closer and closer to the wall on the deep side, a point is reached at which the hydraulic jump will form. This limit for the hydraulic jump was determined for this particular section. This limit was determined so that it can be applied to channels of different cross section and even with non-uniform velocity distribution.

A transition from supercritical flow to subcritical flow was observed in the trapezoidal channel. Observations were made in order to determine the depth and length ratios of the hydraulic jump. Also, the type of wave occurring at the transition was observed. A total of 49 such transitions wer observed with the variables being the Froude Number ($1.4 < 1F < 2.8$) and the channel geometry. The channel geometry was varied

in five increments by changing only the distance between the parallel vertical sidewalls.

The significant results of the experiments are as follows:

a. The length and depth characteristics of the hydraulic jump in this particular channel were similar to those of the much-studied hydraulic jump in a rectangular channel.

b. In order for a hydraulic jump to form the lateral transfer of momentum through the jump is limited. A generalized parameter for measuring this lateral momentum transfer is the ratio of the pressure force plus momentum flux downstream in a unit wide strip to that upstream from the transition. The value of this parameter did not exceed 2.8 when a hydraulic jump was formed.

CHAPTER I

INTRODUCTION

The characteristics of the hydraulic jump in a rectangular channel have been well defined by a great number of experimental studies. These experiments have confirmed the validity of the momentum analysis for the determination of the depth ratio. The other characteristics involving the length of the jump were of necessity determined by experiment. The most complete determination of the characteristics of the hydraulic jump in a rectangular channel were made by Bakhmeteff and Matzke(1).

However, the characteristics of the hydraulic jump in non-rectangular channels have been studied only in a few isolated cases. Lane and Kindsvater(2) studied the hydraulic jump in a circular enclosed conduit. In these experiments, supercritical open-channel flow was changed to enclosed flow by means of the hydraulic jump. This study again demonstrated the validity of the momentum analysis. Posey and Hsing(3) experimented with the hydraulic jump in a trapezoidal channel and found the experimental results and the results of the momentum analysis to be in agreement. They determined the length and depth characteristics and made pertinent observations about the jump appearance.

> In appearance the jump in a trapezoidal channel differs from the jump in a rectangular channel in that wedge-shaped wings form on each side. The wings may meet or be joined by a straight portion. In many of the tests alternate ridges and troughs formed in the downstream portion before the full height was reached.
>
> The length of the jump in a trapezoidal channel is less definite than in a rectangular channel, and for flat side slopes is considerably greater.

This study was initiated to further investigate the characteristics of the hydraulic jump in a non-rectangular channel. The particular non-rectangular channel was chosen so that the flow cross section was the shape of a trapezoid for which the water surface and bottom were not parallel. The purpose of the study was twofold.

1. The length and depth characteristics of the hydraulic jump were to be determined for this particular non-rectangular shape.
2. It was found that the hydraulic jump did not occur in a triangular channel. Instead, the transition from supercritical flow to subcritical flow was accompanied by a series of large eddies on the shallow side. This phenomenon is far different than the hydraulic jump in which the transition is accomplished through a single breaking wave and in which the downstream water surface is comparatively smooth. However, if a wall is placed on the shallow side and is moved closer and closer to the wall on the deep side, a point is reached at which the hydraulic jump will form. This limit for the hydraulic jump was to be determined for this particular section. This limit was to be defined so that it could be applied to channels of different cross section and even with non-uniform velocity distribution.

CHAPTER II

THEORY

Nomenclature.--The particular non-rectangular section chosen for study is shown in cross section in Figure 1.

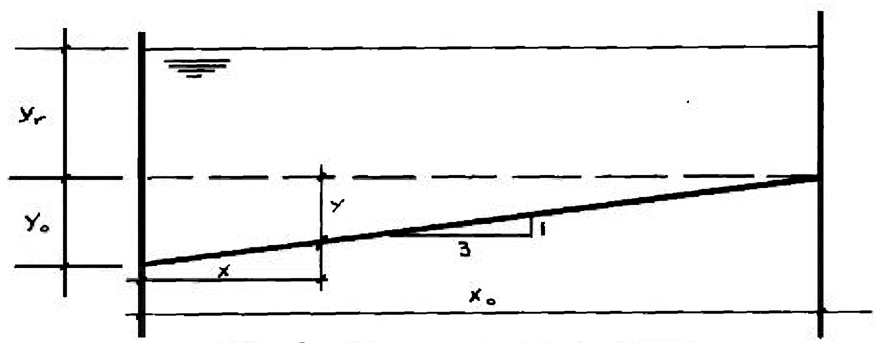

FIG. 1 CROSS SECTION OF FLUME

x_o = width of channel cross section.

y_o = difference between the depths of water at the deep and the shallow sides.

y_r = depth of water at the shallow side.

y_{ave} = average depth.

x_s = length of reverse flow at the shallow side of the channel.

x_d = length of reverse flow at the deep side of the channel.

V = mean velocity.

Q = discharge.

P+M = pressure force plus momentum flux.

G = ratio of pressure force plus momentum flux downstream to that upstream.

ψ = $P + M/A_t \frac{y_o}{2}$.

A_t = $x_o y_o/2$.

ϕ = y_r/y_o.

θ = $(Q/A_t)^2/gy_o$.

$\mathbb{F}_1$ = Froude Number.

γ = unit weight of water.

ρ = mass density.

g = acceleration of gravity.

The subscript 1 is used to denote a section upstream from the wave front, and 2 for a section downstream from the wave front. Subscript s means strip.

Total-Section Analysis.--As in the rectangular channel, the sequent depth relationship is determined by a momentum analysis. The following assumptions are employed.

1. A hydraulic jump which is normal to the wall is possible between sections in which y_r is constant perpendicular to the flow.
2. The velocity distribution is uniform.
3. The boundary shear is negligible.

The expression for the pressure force plus the momentum flux in a section (Figure 1) is as follows:

$$P + M = \tfrac{1}{2}x_o y_r^2\gamma + \tfrac{1}{2}x_o y_o\gamma(y_r + y_o/3) + \rho Q^2/x_o y_r + \tfrac{1}{2}(x_o y_o)$$

In dimensionless form,

$$\psi = \phi^2 + \phi + 1/3 + \theta/(1 + 2\phi)$$

From the equation of momentum, ψ_1 (before jump) = ψ_2 (after jump.) Therefore,

$$\phi_1^2 + \phi_1 + 1/3 + \theta/(1 + 2\phi_1) = \phi_2^2 + \phi_2 + 1/3 + \theta/(1 + 2\phi_2)$$

from an inspection of the expression for θ, it can be seen from the equation of continuity, that θ is the same before and after the jump.

The solution of ϕ_2 for given values of ϕ_1 and θ was performed graphically by plotting ψ versus ϕ for the given value of θ.

A family of curves of ψ versus ϕ were constructed with each curve representing a different value of θ. From these curves the correct value of ϕ_2 could be determined for given values of ϕ_1 and θ, since the value of ψ is the same for both the upstream and downstream sections.

Longitudinal Strip Analysis.--The pressure force plus momentum flux in the strip at a section upstream from the wave is not necessarily the same at a section downstream from the wave, because of the possibility of a lateral transfer of momentum from the deep side to the shallow side of the channel.

If in the strip analysis the ratio of pressure force plus momentum flux at a section downstream from the wave to that for a section upstream from the wave is unity, we have balance for the strip. If the value of that ratio is greater than unity, it means that there is a momentum deficiency upstream; and if the ratio is less than unity, there is a momentum surplus upstream.

It will now be shown that there is a region of momentum surplus on the deep side of the channel (Figure 1) and a deficiency on the shallow side. This condition requires that momentum must be transferred laterally across the channel. If the lateral transfer of momentum becomes too large it is conceivable that the transfer would be physically impossible. For this case the hydraulic jump would not be possible. The criteria for the existence of the hydraulic jump is based upon the magnitude of this lateral transfer of momentum.

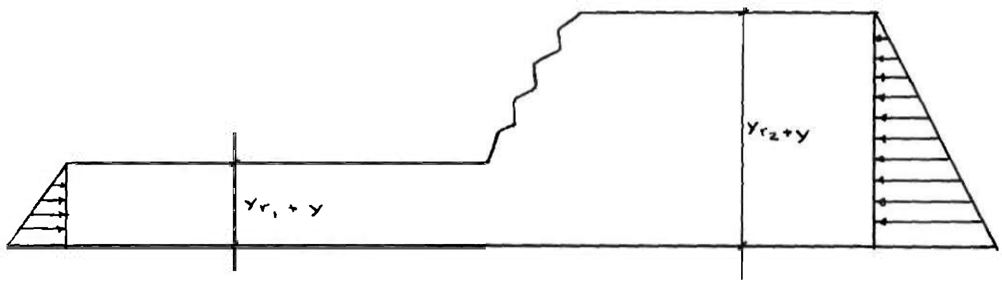

FIG. 2 LONGITUDINAL STRIP SECTION

The momentum equation for a strip can be presented in the following form

$$\tfrac{1}{2}\gamma(y_{r1} + y)^2 + V_1^2(y + y_{r1}) = \tfrac{1}{2}\gamma(y_{r2} + y)^2 + V_2^2(y + y_{r2})$$

Letting $V = Q/A_t(1 + 2\phi)$ and dividing by γy_o^2, the following expression is obtained for the strip

$$P + M/\gamma y_o^2 = \tfrac{1}{2}(y/y_o + \phi_1)^2 + \theta(y/y_o + \phi_1)/(1 + 2\phi_1)^2 =$$
$$\tfrac{1}{2}(y/y_o + \phi_2)^2 + \theta(y/y_o + \phi_2)/(1 + 2\phi_2)^2$$

For balance

$$(P + M)_{s1} = (P + M)_{s2}$$

Letting

$$G = (P + M)_{s2}/(P + M)_{s1}$$

$M = 1$ For balance

$M > 1$ For P + M deficiency upstream

$M < 1$ For P + M surplus upstream

From geometry

$$y/y_o = 1 - x/x_o$$

Substituting:

$$\tfrac{1}{2}(1 - x/x_o + \phi_1)^2 + \theta(1 - x/x_o + \phi_1)/(1 + 2\phi_1)^2 =$$

$$\tfrac{1}{2}(1 - x/x_o + \phi_2)^2 + \theta(1 - x/x_o + \phi_2)/(1 + 2\phi_2)^2$$

$$G = \frac{\tfrac{1}{2}(1 - x/x_o + \phi_2)^2 + \theta(1 - x/x_o + \phi_2)/(1 + 2\phi_2)^2}{\tfrac{1}{2}(1 - x/x_o + \phi_2)^2 + \theta(1 - x/x_o + \phi_1)/(1 + 2\phi_1)^2}.$$

The parameter G resembles the shape parameter H used in boundary layer studies. The value of G is a measure of the momentum defect at any longitudinal strip, and the value of H is a measure of the momentum defect within the boundary layer. However, recent studies(4) indicate that the criterion for boundary layer separation may be better expressed by the value of the rate of change of H rather than the commonly used value of H. Extending the resemblance further, it would seem that either the value of G or the value of $\partial G/\partial(x/x_o)$ might be valid measures as to whether a hydraulic jump is possible.

The maximum value of G occurs at the shallow wall, where $x/x_o = 1$.

$$G_m = \frac{\tfrac{1}{2}\phi_2^2 + \theta\phi_2/(1 + 2\phi_2)^2}{\tfrac{1}{2}\phi_1^2 + \theta\phi_1/(1 + 2\phi_1)^2}$$

The value of G versus ϕ for different values of θ are shown in Figure 3.

In a similar fashion

$$\partial G/\partial(x/x_o)\Big|_{x/x_o = 1} = G(\phi_2 - \phi_1)/\phi_1\phi_2$$

The values of $\partial G/\partial(x/x_o)\Big|_{x/x_o = 1}$ are plotted versus values of ϕ_1 for different values of θ on Figure 4.

In both the diagram of Gm and $\partial G/\partial(x/x_o)\Big|_{x/x_o = 1}$ are shown the values corresponding to the test runs. The test runs were accomplished at five different widths of channel or at five different values of $y_{r1}/y_{r1} + y_o$.

The value of this parameter $y_{r1}/y_{r1} + y_o$ for a rectangular cross section is unity. In this case, the jump is always possible and the ratio G is 1, that is, there is a pressure force plus momentum flux balance in each longitudinal strip. For a triangular section the value of $y_{r1}/y_{r1} + y_o$ is zero, no jump is possible, and undulating waves and eddies on the shallow side are present; the corresponding maximum value of G is infinite which means that there is a pressure force plus momentum flux deficiency upstream and which cannot be supplied, making impossible the formation of the hydraulic jump.

The theoretical values computed for the ratio Gm and plotted in Figure 3 are always greater than unity which means that there is a momentum surplus on the deep side of the channel and a deficiency on the shallow side.

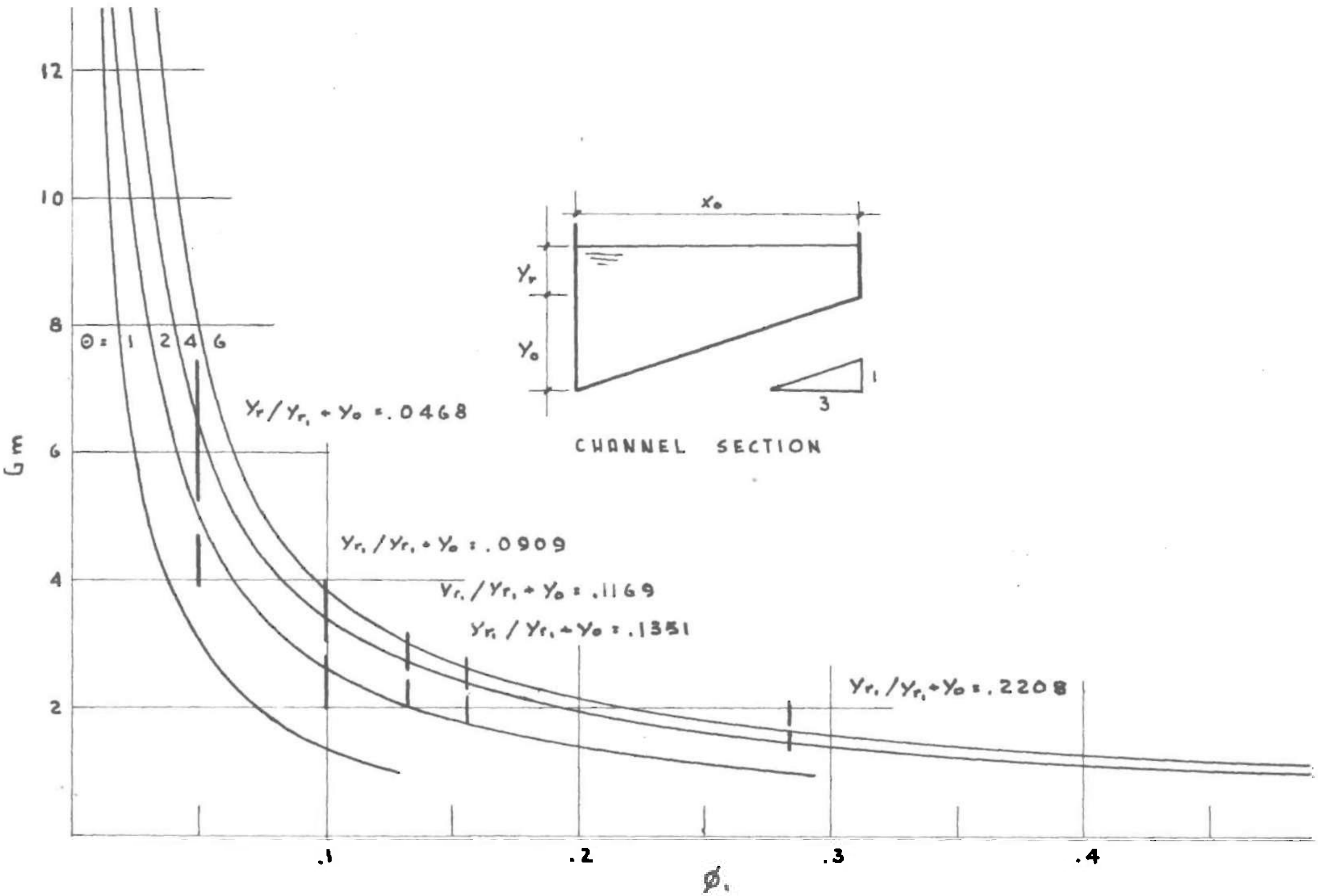

FIG. 3 - DIMENSIONLESS DIAGRAM OF PRESSURE FORCE PLUS MOMENTUM FLUX RATIO (DOWNSTREAM/UPSTREAM) AT THE SHALLOW SIDE.

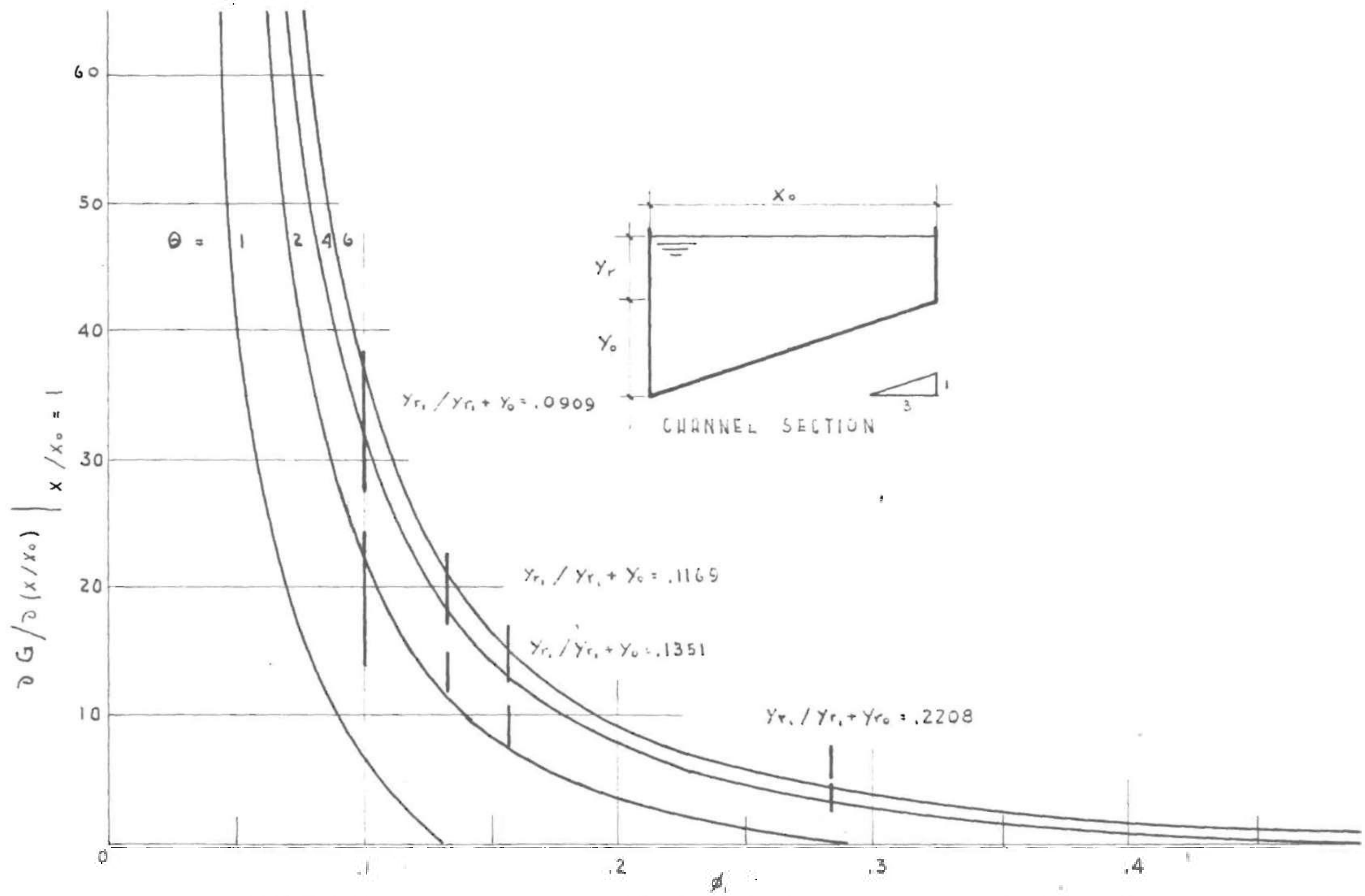

FIG. 4 - DIMENSIONLESS DIAGRAM OF THE RATE OF CHANGE OF M AT THE SHALLOW SIDE

CHAPTER III

EQUIPMENT AND INSTRUMENTATION

Waves were induced into a channel of varying depth in the cross section, the shape of which is shown on Figure 1.

The experimental investigation was performed in a flume eighteen feet long and three feet wide with vertical side walls and sloping bottom in the cross section (three horizontal to one vertical.) The flume was resting on ten legs, each one of which has adjustment screws for use in leveling the flume. The flume was leveled to a horizontal position with aid of a surveyor's transit. Figure 5 is a photograph of this flume.

Water for the test runs was supplied from a constant head tank.

The inlet was always the shape of the channel cross section. This was accomplished by fitting streamlined, hand-carved, wooden blocks into the pointed edge of the originally triangular inlet. The maximum depth of the opening was 0.40 feet and the maximum width was 1.20 feet.

In order to obtain nearly uniform velocity distribution, the forebay side of the opening had one-quarter rounds with one and half inch radius. Also, for the same purpose, two vertical walls in the forebay at both sides of the inlet, a cylindrical screen and four vanes were used.

In order to secure uniform depths upstream from the wave, a cover plate with three suction lines was used. This cover plate which provided an enclosed inlet was six inches wide and had a sharp edge at its downstream end.

The wall on the deep side of the flume was fixed in position. The wall in the shallow side was eight feet long and was movable so that the

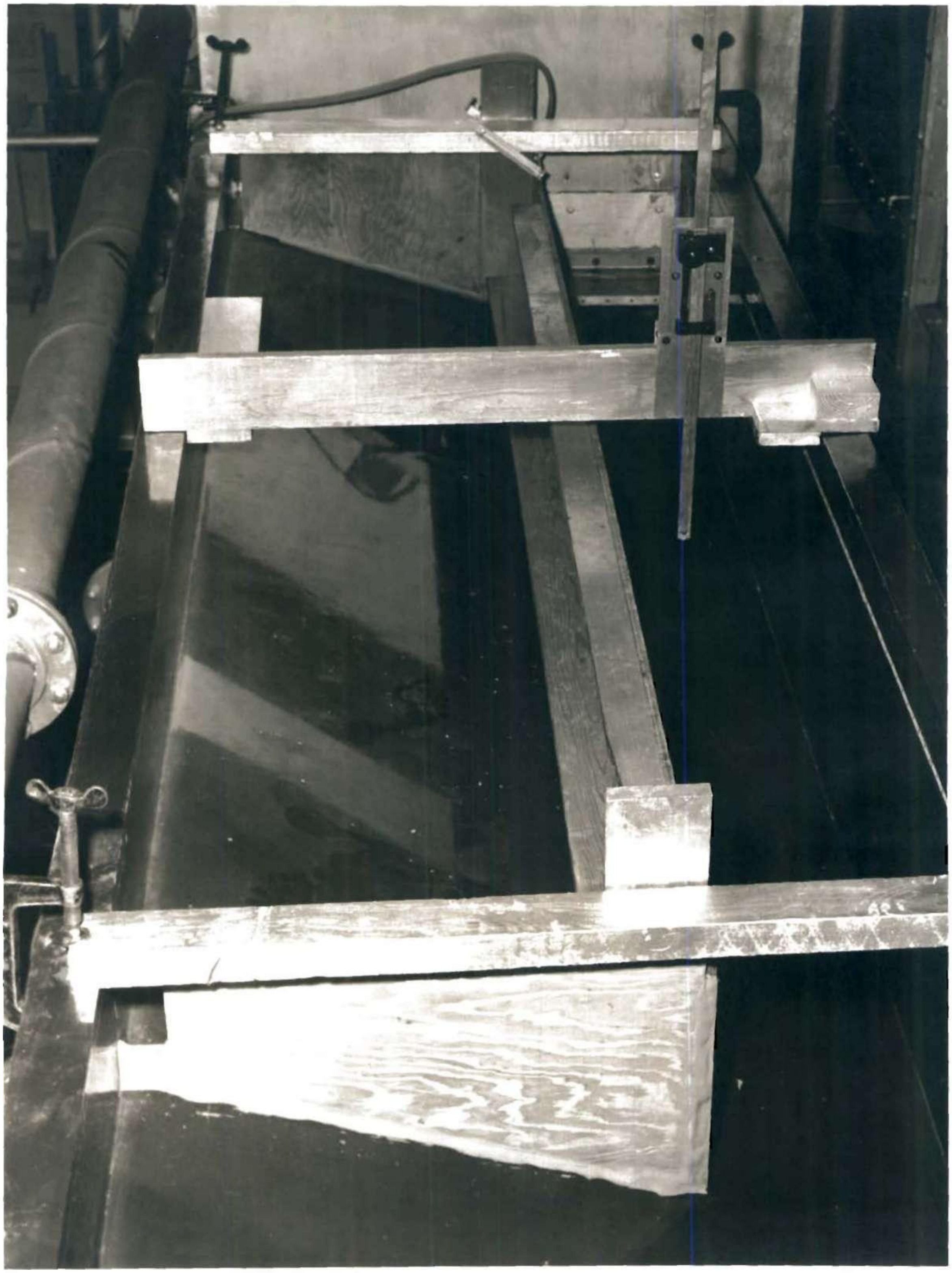

FIG. 5 PHOTOGRAPH OF APPARATUS

channel width could be changed.

A weighing tank was used to measure the discharge.

Water depths were measured with a point gage mounted in a movable bridge.

CHAPTER IV

PROCEDURE

Experimental Procedure.--A wave was induced in the flume as closely as possible to the cover plate at the inlet by changing downstream conditions.

The discharge was measured by means of a weighing tank and an electric stop clock.

Water depths to the nearest 0.001 foot were measured with a point gage mounted on a movable bridge. This bridge was carefully leveled at preset stations both upstream and downstream from the wave.

Waves and pulsations continued sometimes for a considerable distance below the jump with the result that the measurements of the downstream depth were difficult. In order to overcome the inaccuracies due to these conditions, ten to eleven depth measurements were made in each cross section. A good average was expected from these measurements. The upstream depths were also measured at ten to eleven points in the cross section.

Lengths of the reverse flow at both sides of the channel were measured. The length of the reverse flow zone was measured as the distance from the wave front to the downstream point where all entrained air bubbles are moved downstream. The lower end of the jump cannot be precisely determined.

In running the tests, five different channel widths were used; 1.10, 1.05, 1.02, 1.00, and 0.90 feet. Each of these channel widths was tested at nine or ten different discharges.

Analytical Procedure.--The water surface profile upstream from the wave was irregular. This profile was adjusted by plotting the measured depths at ten or eleven different points in the cross section and drawing a horizontal line which compensated for the irregularities of the profile.

The average depth at both sections upstream and downstream from the wave were computed as the average between the depths at the shallow and the deep sides at the corresponding sections. The same average depth upstream from the wave was used in all cases for a given channel width.

The velocities were computed by dividing the discharge by the cross sectional area of the channel. The momentum correction coefficient for the velocity was assumed to be unity. Only slight errors are to be expected from this assumption, because of the provisions made at the inlet in order to get a nearly uniform velocity distribution. The Froude Number at the upstream section was computed by using the mean velocity and average depth of the section.

CHAPTER V

DISCUSSION OF RESULTS

The purpose of the test runs was twofold. First, the characteristics of a breaking wave, hydraulic jump, in a particular non-rectangular channel were to be determined. Second, a criterion was to be determined for the existence of a hydraulic jump in a channel of any shape or velocity distribution.

Two different types of flow were observed. The first type consists of undulating waves across the channel and eddy currents on the shallow side. A well defined second wave was always observed with this type of flow. A sketch of this type of flow is shown in Figure 6a and 6b. The second type consists in a wave perpendicular to the channel cross section which can be called a hydraulic jump; no second wave was observed. This type of flow is also sketched in Figure 6c and 6d.

The criteria used in analyzing the two types of flow observed was that a certain lateral transfer of momentum is required in order to obtain a perpendicular wave; in case this lateral transfer of momentum cannot be supplied, no hydraulic jump is possible.

Five runs were made. Runs number 1 and 4 were of the first type of flow; runs number 2 and 3 were of the second type; run number 5 was in the limiting condition.

The channel width for runs number 1 and 4 was 1.10 feet and 1.05 feet respectively. For Froude Numbers smaller than about 1.83, the wave did not extend across the entire cross section of the channel and non-breaking waves were observed in the deep side. Since the non-breaking

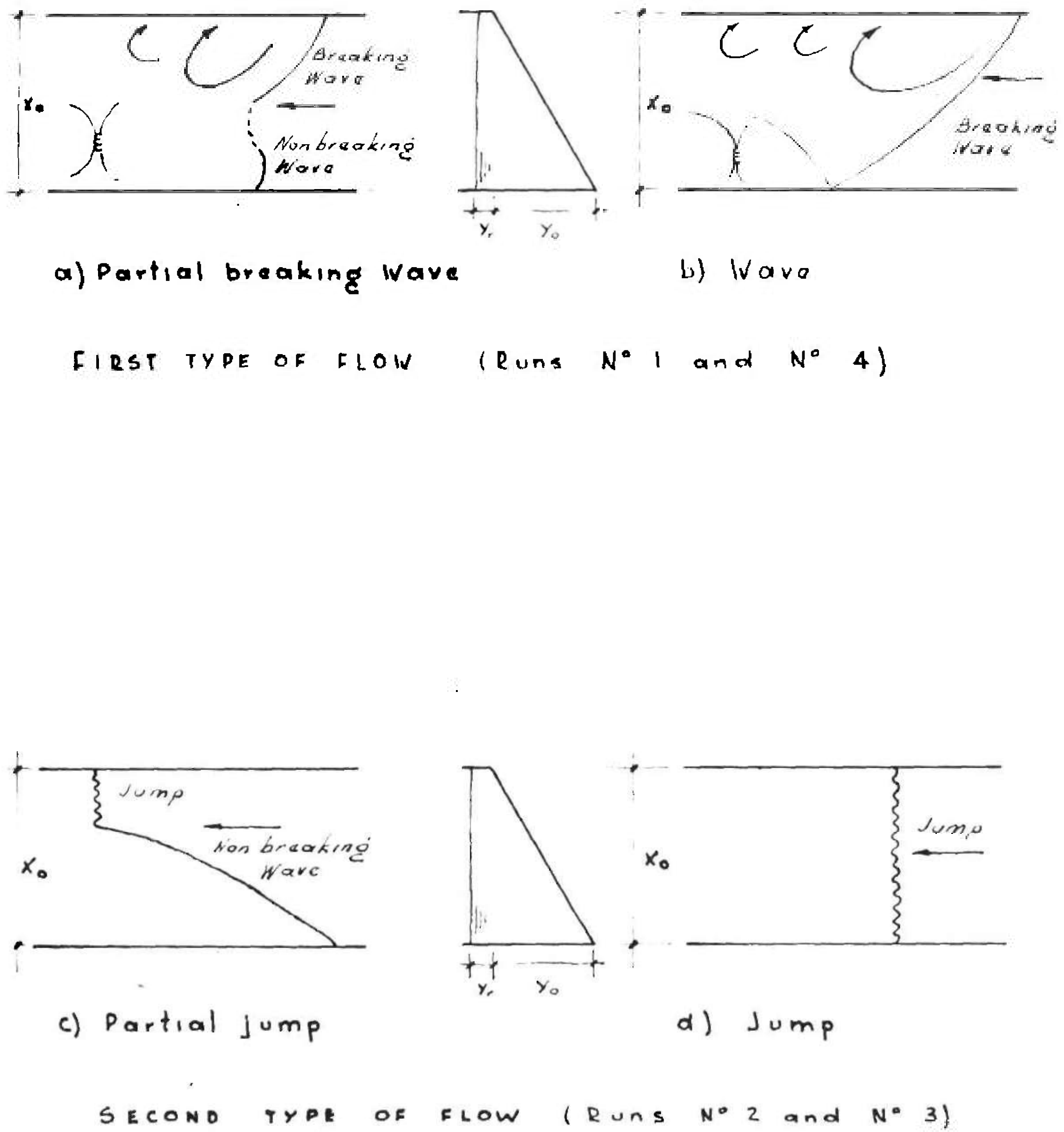

FIG. 6 - GENERAL FLOW SKETCH

wave is very long, this wave influenced the upstream depths on the deep side. There was no reverse flow in the deep side. A sketch of this flow condition is shown in Figure 6a. For greater Froude Numbers, the wave went across the entire section of the channel. The depths upstream from the wave were the same for all tests with Froude Numbers greater than about 1.83. This flow condition is shown in Figure 6b.

No jump was possible for these runs. This means that a greater lateral transfer of momentum was necessary in order to get a perpendicular wave or what can be called a jump; in other words, the lateral transfer of momentum required is greater than that which can be supplied, hence the jump is not possible.

The values of Gm and $\partial G/\partial(x/x_o)\Big|_{x/x_o = 1}$ for test number 1 and 4 can be seen in Table 1 and Figures 3 and 4.

The channel width for runs number 2 and 3 was 1.00 feet and 0.90 feet respectively. For Froude Numbers smaller than about 1.83, a well defined jump in the shallow side and undulating waves in the deep side were observed. Here again the depths upstream from the wave at the deep side were altered. There was no reverse flow on the deep side. This type of flow is shown in Figure 6c. For greater Froude Numbers, a perpendicular wave across the entire section of the channel was obtained. (See Figure 6d.)

As a perpendicular wave was possible, the conclusion is that the lateral transfer of momentum could be supplied for these cases. The values of Gm and $\partial G/\partial(x/x_o)\Big|_{x/x_o = 1}$ are also shown in Table 1 and Figures 3 and 4.

By means of Figure 3, it was found that the channel width should be 1.02 feet in order to cover the range of values of Gm between those

TABLE 1

Type of Wave	IF_1	Gm	$\partial G/\partial(x/x_o)$ at $x/x_o = 1$	$y_{r1}/y_{r1} + y_{ro}$	Run No.
a	1.39	2.0	14	.0909	4A
a	1.58	2.6	21	.0909	4B
a	1.75	2.8	24	.0909	4C
a	1.54	4.0	>60	.0468	1A
a	1.73	4.7	>60	.0468	1B
b	2.07	2.8	18	.1169	5E
b	2.15	2.9	19	.1169	5F
b	2.27	3.0	20	.1169	5G
b	2.37	3.0	20	.1169	5H
b	2.00	3.1	28	.0909	4D
b	2.59	3.2	22	.1169	5I
b	2.72	3.2	23	.1169	5J
b	2.19	3.5	31	.0909	4E
b	2.34	3.5	32	.0909	4F
b	2.49	3.7	34	.0909	4G
b	2.67	3.8	36	.0909	4H
b	2.75	3.9	36	.0909	4I
b	2.84	4.0	37	.0909	4J
b	1.88	5.3	>60	.0468	1C
b	2.08	5.5	>60	.0468	1D
b	2.25	5.9	>60	.0468	1E
b	2.44	6.3	>60	.0468	1F
b	2.67	6.7	>60	.0468	1G
b	2.72	6.9	>60	.0468	1H
b	2.85	7.2	>60	.0468	1I
b	2.97	7.5	>60	.0468	1J
c	1.39	1.4	2.7	.2208	3A
c	1.59	1.4	3.6	.2208	3B
c	1.78	1.6	4.3	.2208	3C
c	1.34	1.8	7.5	.1351	2A
c	2.59	2.0	9.2	.1351	2B
c	1.44	2.1	12	.1169	5A
c	1.70	2.2	11	.1351	2C
c	1.58	2.4	13	.1169	5B
d	1.98	1.8	5.0	.2208	3D
d	2.10	1.9	5.7	.2208	3E
d	2.21	2.0	6.3	.2208	3F
d	2.32	2.0	7.0	.2208	3G
d	2.41	2.0	7.5	.2208	3H
d	2.49	2.1	7.7	.2208	3I

TABLE 1 (Continued)

Type of Wave	$\mathbb{F}_1$	Gm	$\partial G/\partial(x/x_o)$ at $x/x_o = 1$	$y_{rl}/y_{rl} + y_{ro}$	Run No.
d	1.89	2.4	10	.1351	2D
d	1.73	2.4	15	.1169	5C
d	2.09	2.5	14	.1351	2E
d	2.23	2.6	14	.1351	2F
d	2.31	2.6	15	.1351	2G
d	1.91	2.7	17	.1169	5D
d	2.46	2.7	16	.1351	2H
d	2.58	2.8	16	.1351	2I
d	2.65	2.8	17	.1351	2J

corresponding to runs number 4 and 2.

The channel width for run number 5 was 1.02 feet. The tests made with this channel width were very sensitive.

The corresponding values of Gm and $\partial G/\partial(x/x_o)\Big|_{x/x_o = 1}$ are also shown in Table 1 and Figures 3 and 4.

As the condition for the formation of the jump is approached, the values of Gm and $\partial G/\partial(x/x_o)\Big|_{x/x_o = 1}$ decrease. Because this point is not well defined, it is not possible to give an exact value of Gm or $\partial G/\partial(x/x_o)\Big|_{x/x_o = 1}$ corresponding to the limit for the formation of the jump.

From the observations made and with the aid of the corresponding diagrams, the limiting value of Gm for the formation of the jump (Figure 6d) for Froude Numbers above 1.83 can be given as 2.8, and that for $\partial G/\partial(x/x_o)\Big|_{x/x_o = 1}$ between 17 and 18.4, and the values for Froude Numbers smaller than 1.83; the limiting value of Gm is between 2 and 2.4 and of $\partial G/\partial(x/x_o)\Big|_{x/x_o = 1}$ is between 14 and 14.9 (Figure 6c.)

Within the range of the experiments, no noticeable change was found between the theoretical curve of $\mathbb{F}_1$ versus y_{2ave}/y_{1ave} for a rectangular channel and those for the different channel width tested; the values of versus y_{2ave}/y_{1ave} for the test runs were compared with the theoretical curve for a rectangular channel. These values are shown in Figure 7. All values are practically on the same straight line, even for the cases where no jump was obtained. For Froude Numbers smaller than about 1.83, non-breaking waves on the deep side were obtained. This value of the Froude Number is very close to the theoretical limiting value of $\mathbb{F}_1 = 1.73$ for the undular and direct jump conditions in a rectangular channel.

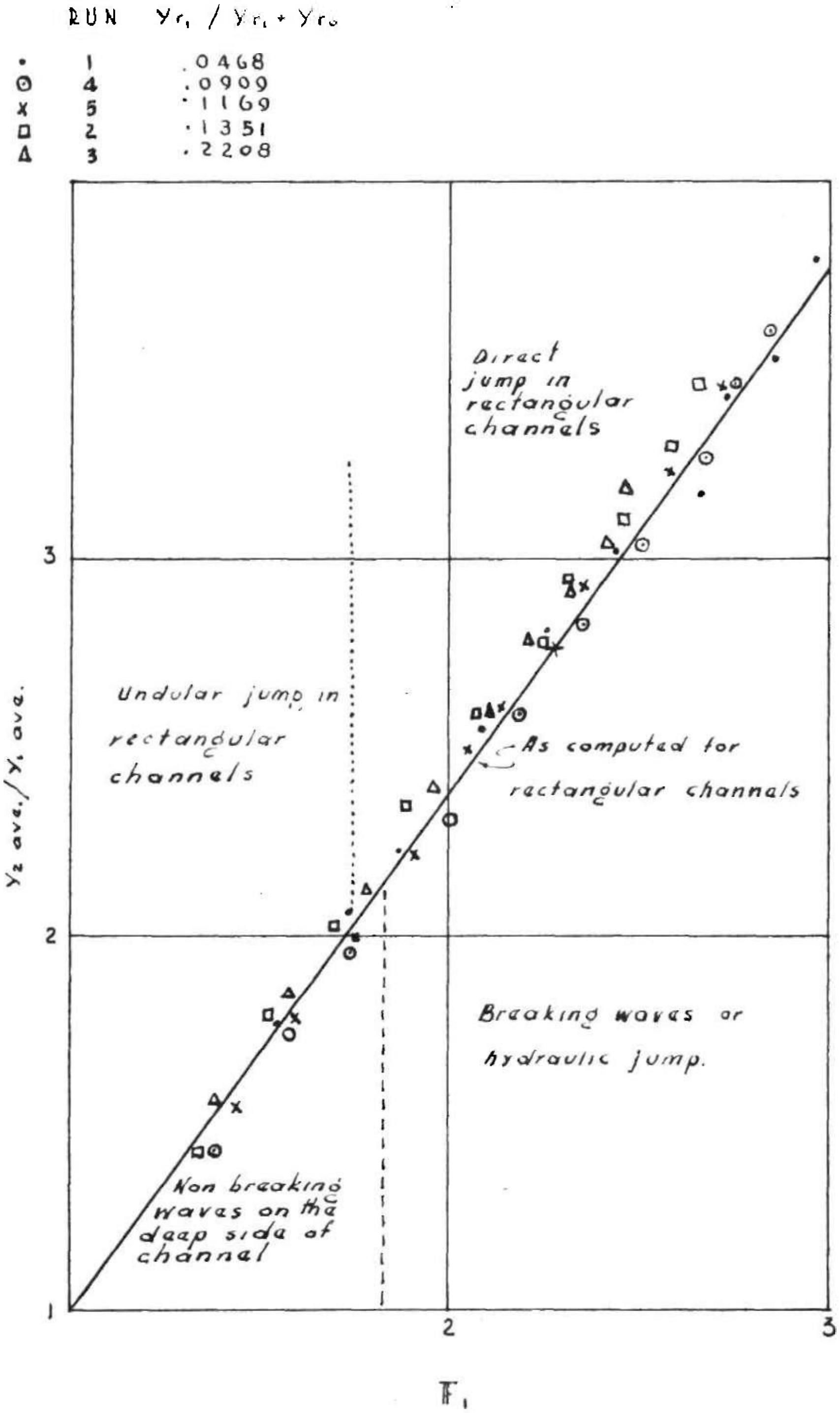

FIG. 7 - AVERAGE DEPTH RATIOS AS FUNCTION OF FROUDE NUMBER.

Thus, it appears that the value of the Froude Number slightly less than 2.0 can be taken as the limit between the breaking and non-breaking wave regardless of the shape of the channel.

In Figure 8, the two types of flow (undulating waves or hydraulic jump) are well differentiated. Values of $x_s/y_{r2} - y_{r1}$ above eight correspond to undulating waves and eddies on the shallow side. When a jump was obtained, the value of $x_s/y_{r2} - y_{r1}$ was never greater than eight. The maximum values obtained by Mr. Bakhmeteff and Mr. Matzke(1) for the length over the height of a jump in a rectangular channel are shown in dotted line in Figure 8. The value of this ratio is about eight in a rectangular channel.

When no jump was obtained, the length of reverse flow on the shallow side increases as the channel width increases; the limit will be when the channel cross section is triangular, the length of reverse flow on the shallow side being infinite; a fact which was proved experimentally. For the jump condition, the length of reverse flow on the shallow side was practically the same for all runs (Figures 8 and 9.)

The maximum values of the ratio of the length of the jump over the depth upstream from the jump for a rectangular channel obtained by Mr. Bakhmeteff and Mr. Matzke(1) are shown in dashed lines on Figure 9. The average value of this ratio is about five.

The length of reverse flow on the shallow side for a non-rectangular channel is smaller than the length of the jump in a rectangular channel for Froude Numbers smaller than 2.8, as can be observed in Figures 8 and 9. Hence, for Froude Numbers smaller than 2.8, a hydraulic jump in a non-rectangular section seems to be more effective as an energy

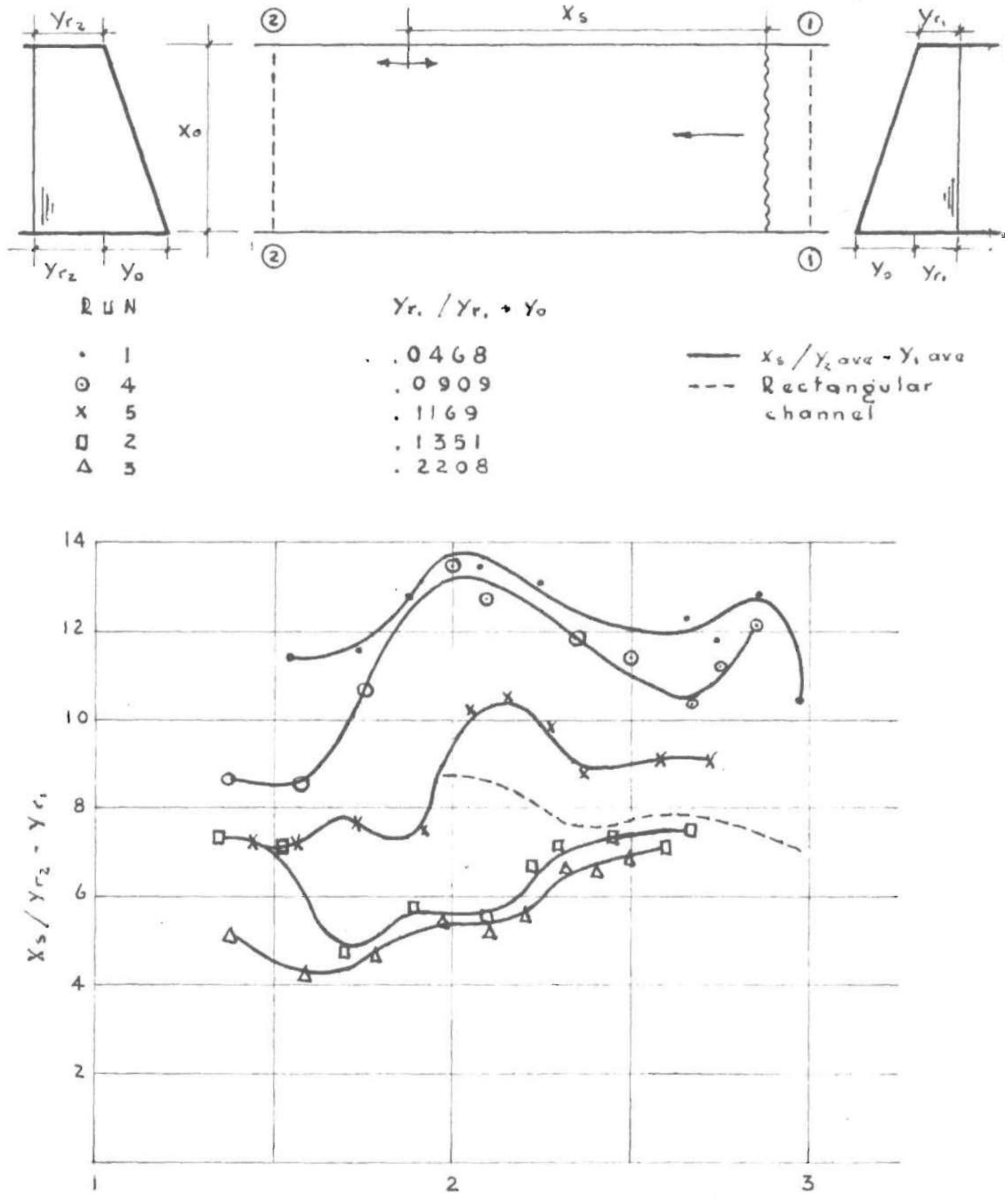

FIG. 8 - DIMENSIONLESS DIAGRAM FOR THE LENGTH OF REVERSE FLOW ON THE SHALLOW SIDE.

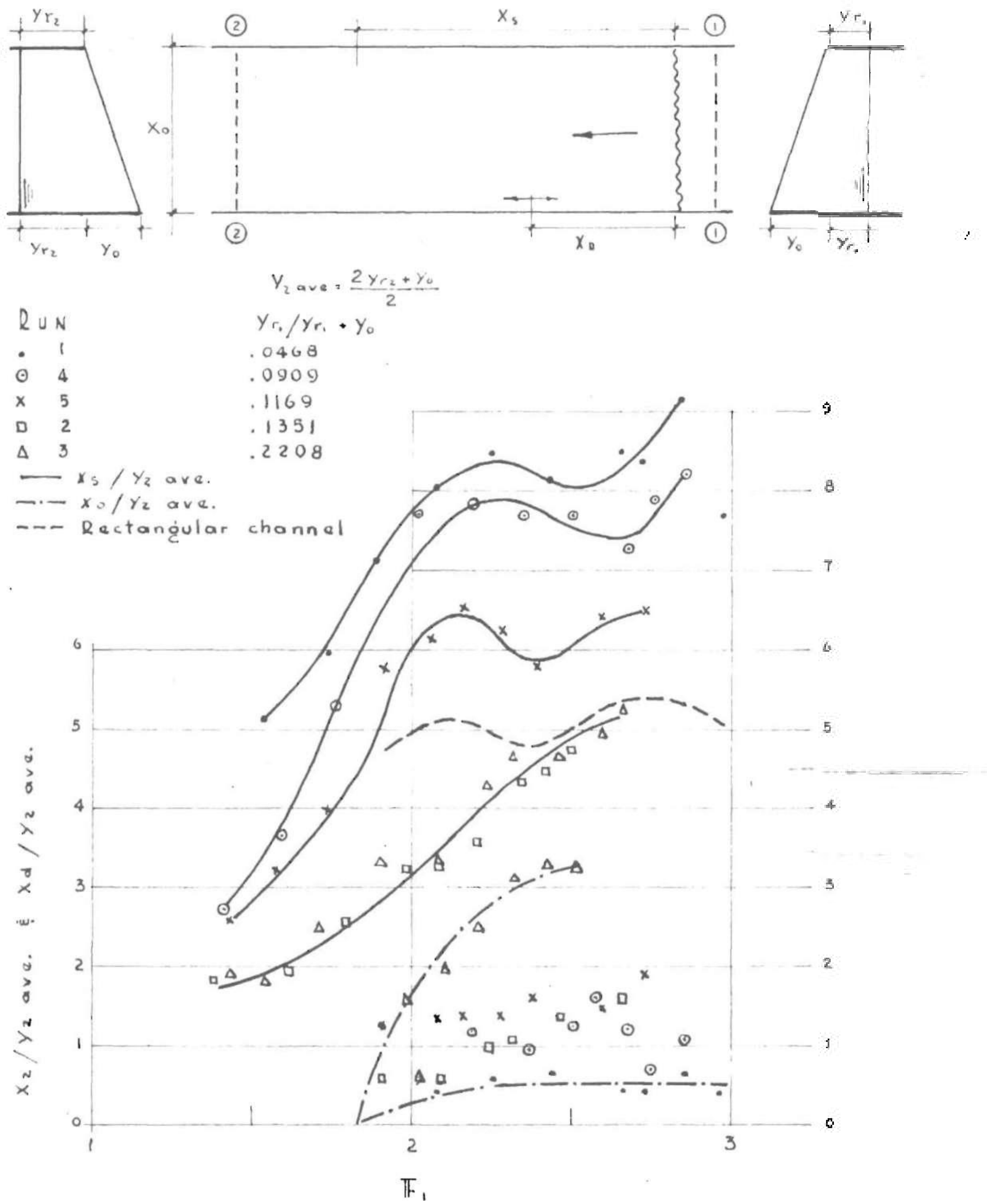

FIG. 9 - DIMENSIONLESS DIAGRAM FOR LENGTH OF REVERSE FLOW AT BOTH SIDES OF CHANNEL.

dissipator than that in a rectangular section.

It can also be observed in Figure 9 that the length of reverse flow in the deep side increases as the width of the channel decreases. This is due to the fact that the influence of the sloping channel bottom becomes less as the width of the channel decreases; the limit will be when the length of reverse flow becomes equal in length at both sides of the channel, which is the case in a rectangular channel.

CHAPTER VI

CONCLUSIONS

1. Depending on the value of the Froude Number, the flow in a non-rectangular channel can be divided into two types. When the Froude Number is smaller than about 1.83, partial breaking waves are obtained; and when it is greater, breaking waves across the entire section of the channel are observed.

2. The theoretical limiting value of $\mathbb{F}_1 = 1.73$ for the undular and direct jump conditions in a rectangular channel is very close to the value $\mathbb{F}_1 = 1.83$ obtained for this particular channel. Thus, it appears that the value of the Froude Number slightly less than 2.0 can be taken as the limit between the breaking and non-breaking wave regardless of the shape of the channel.

3. In a rectangular channel the Froude Number is sufficient to describe the type of wave, since only two types are possible; the undular and the direct jump. In a non-rectangular channel an additional division is necessary, since two completely different phenomena are possible; the type of flow consisting of undulating waves and eddy currents in the shallow side, and that which can be called a hydraulic jump. The line of demarcation between these two types of flow is given by either the value of G or the value of $\partial G/\partial(x/x_o)$ at the shallow side. The condition for the formation of the jump was found to be:

$$Gm < 2.8 \qquad \text{or} \qquad \partial G/\partial(x/x_o)\Big|_{x/x_o = 1} < 18.4$$

4. When Gm is greater than 2.8, the length of reverse flow in the shallow side increases rapidly as the deviation from the rectangular shape becomes marked; being infinite when the channel is triangular.

5. The length of reverse flow in the deep side is always smaller than that in the shallow side and increases as the channel width decreases. The limiting condition is obtained in a rectangular channel where the length at both sides of the channel is the same.

6. The length of reverse flow on the shallow side for a hydraulic jump in a non-rectangular channel ($F_1 > 1.83$ and $Gm < 2.8$) is smaller than the length of the jump in a rectangular channel.

7. The conclusions Number 5 and 6 indicate that a hydraulic jump in a non-rectangular section is more effective as an energy dissipator than one in a rectangular section.

BIBLIOGRAPHY

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LITERATURE CITED

1. Bakhmeteff, B. A., and Matzke, A. E., "The Hydraulic Jump in Terms of Dynamic Similarity," Transactions A.S.C.E., Vol. 101, 1936, p. 630.

2. Lane, E. W., and Kindsvater, C. E., "Hydraulic Jump in Enclosed Conduits," Engineering News-Record, Vol. 121, Dec. 29, 1938, pp. 815-7.

3. Posey, C. J., and Hsing, P. S., "Hydraulic Jump in Trapezoidal Channels," Engineering News-Record, Vol. 121, Dec. 22, 1938, pp. 797-8.

4. Von Doenhoff, A. E., and Tetervin, N., "Determinations of General Relations for the Behavior of Turbulent Boundary Layers," National Advisory Committee for Aeronautics, Report 772, 1943.

OTHER REFERENCES

5. Bakhmeteff, B. A., Hydraulics of Open Channels, McGraw-Hill Book Company, New York, 1932.

6. Rouse, H., Editor, Engineering Hydraulics, John Wiley and Sons, New York, 1950.

7. Rouse, H., Fluid Mechanics for Hydraulic Engineers, First Edition, McGraw-Hill Book Company, New York, 1938.

8. Woodward, S. M., and Posey, C. J., Hydraulics of Steady Flow in Open Channels, John Wiley and Sons, New York, 1951.