

SPECIFIC HEAT LAG IN GASES

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SPECIFIC HEAT LAG IN GASES

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SYMBOLS

a	displacement from central position
a_k	amplitude of vibration
b	momentum
b_k	momentum of vibration
$C_1, C_2, \text{ etc.}$	constants
c_v	specific heat at constant volume
c_p	specific heat at constant pressure
c_{vib}	specific heat of vibrational freedom
c_p'	specific heat of nonvibrational freedom
d	diameter
E, ϵ	unit of energy
ϵ'	nondimensional energy term
f	force
h	enthalpy
I	momentum of inertia
k	gas constant of single molecule
K.E.	kinetic energy
K, K_n	nondimensional parameter
l	length
m	mass
m_k	reduced mass
p	pressure
P.E.	potential energy
q	transferred energy

q	flow per unit thickness
R	universal gas constant
s'	nondimensional entropy of total head defect
t	time
t'	nondimensional time
T	absolute temperature
u	internal energy
u_t	rotational internal energy
u_v	vibrational internal energy
v	volume
w, W	velocity
w'	nondimensional velocity
x, y, z	coordinates
ψ	stream function
ω	angular acceleration
ϕ	frequency
ν	kinematic viscosity
δ	boundary layer thickness

SPECIFIC HEAT LAG IN GASES

SUMMARY

Specific heat lag means a departure from equilibrium states in a gas undergoing rapid changes of states. The existence of a lag in the specific heat in gases has been demonstrated experimentally and analytically by investigators interested in the phenomena concerning the absorption of sound and dispersion of sound velocity with frequency in gases. In order to fully understand the subject, the basic theory of specific heat is presented. The energy content of a molecule is primarily translational, rotational, and vibrational energy. The vibrational energy is absorbed exponentially with a measurable relaxation time. A brief review of the sound absorption investigations is given.

Kantrowitz has developed analytical expressions for determining the total head defect during and after a compression of a gas at the nose of an impact tube due to the specific heat lag. The measured defect is compared to the theoretical value. The relaxation time may be determined from the resulting expression. His work has been carefully studied and is explained. A design of apparatus similar to that which Kantrowitz used is presented to be used to check the present data and for future investigations.

The energy dissipations which have been demonstrated to accompany the lag will decrease the efficiency of any high temperature process and may possibly explain some unexplained, observed phenomena. There

may be a connection with engine knock, increased radiation, and loss of power in an internal combustion engine.

ORIGIN OF THE PROBLEM

In determining explosion pressures for mixtures of hydrogen and oxygen, Lewis and von Elbe (Reference 18) used thermodynamic functions derived from band spectra of the gases. These theoretical values were compared with the pressures obtained by other experimenters. In mixtures containing excess oxygen or nitrogen, experimental pressures obtained by Lewis and von Elbe were in great excess of the theoretical values. To explain this irregularity, the investigators used a theory advanced by Wohl and Magot. According to this theory, there exists a lag in the excitation of the vibrational energy levels. Intense, loud oscillations of pressure and knocks accompanied the increased pressures produced when excess oxygen was introduced in the explosion chamber. The oxygen had far more effect than did the nitrogen indicating a difference in the relaxation time, or the time to take on full energy content, of the two elements. The oxygen and nitrogen may also effect the relaxation time of the other gases in the combustion.

The explanation proposed for the oscillations and knocks concerns the specific heat. When a small fraction of gas burns, the oxygen or nitrogen molecules do not take up their full heat capacity immediately. If a sudden adiabatic compression due to burning of the remainder of the gas is applied, the temperature will increase. There is a contraction in volume, along with the rapid rise of pressure. The rapid shrinkage in volume causes a fast mass movement inward. The result could well be a pressure wave that continues for some time due to little damping of the oscillations.

Lewis and von Elbe suggest that the measurement of explosion pressures might be an approach to the problem of time variable specific heats.

In his study of energy transformations in flowing gases due to shocks, Bailey (Reference 1) observed thermocouple readings in excess of the total gas temperature. These observations were made in connection with high temperature and high pressure shocks. Careful check on methods and data taking eliminated those possibilities of error. The thermocouple readings in the high temperature products of combustion were greater than the total temperature of the gas before it was in equilibrium. As shown, temperature readings close to the nozzle exit were less than those farther from the nozzle. No satisfactory explanation was given. Bailey did point out that the temperature concept assumes a gas in equilibrium and admits only of translational kinetic energy of the molecules; and that any time that one of the assumptions is not satisfied, temperature as an energy concept is in trouble.

In the light of these two observed phenomena, both of which indicate a connection with the time required for a substance to assume its full heat content, the problem of specific heat lag is deemed worthy of study. As the work progresses, the lag may be used to explain other phenomena.

TECHNIQUE OF SPECIFIC HEAT

Frequently specific heat is defined as a quantity of heat necessary to change the temperature of a unit mass of a substance one unit on a given temperature scale. This definition must be used with discretion as it can be applied only to a process in which the quantity of heat has the same value as the change of internal energy or the change of enthalpy. The property which changes depends on the process involved.

In a constant volume process the first partial derivative of the internal energy with respect to the temperature is the "specific heat at constant volume." The term "specific" is misleading in that it indicates a quantity of heat. To conform to accepted practice, "specific" is used; but the definition previously given is considered. By definition

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v \quad (1)$$

For a constant pressure process the first partial derivative of the enthalpy with respect to the temperature is the "specific heat at constant pressure." By definition

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p \quad (2)$$

Since for a perfect gas h and u are independent of pressure and are functions of temperature only, the partial notations may be dropped. Also by definition $h = u + pv$. Then

$$c_p = \frac{dh}{dT} = \frac{du}{dT} + \frac{d(pv)}{dT} = c_v + R \quad (3)$$

Because of this relationship the basic concern can be the change of internal energy u .

Although the classical or kinetic energy theory of specific heat does not furnish accurate values, it gives an extremely simple method for determining the average value of the specific heat. The theory is also useful in explaining the forms of molecular energy. In order to use the theory, the energy must be expressible as definite functions of coordinates since it is based on the principle of equipartition of energy. The principle of equipartition states that "if any part of the energy of a system is simply proportional to the square of the coordinate or of a component of velocity or momentum, then, when the system is in thermal equilibrium at temperature T , the mean value of this part of the energy is $\frac{1}{2}kT$, k being the gas constant for one molecule." (Reference 13)

The energy of a gas may be divided into several parts. (Reference 13)

- a. mutual potential energy
- b. translatory kinetic energy
- c. rotational kinetic energy
- d. vibrational energy
- e. internal atomic energy

The mutual potential energy is the energy of the molecules as a whole due to forces between the molecules. Since the density of a gas is practically negligible, this form of energy is not considered. The vibrational energy is partly potential and partly kinetic energy. The

internal atomic energy, which is due to electronic excitation, is not considered as it occurs in appreciable quantity only at excessively high temperatures. The other forms of energy are explained by their classifications.

Consider a molecule in the shape of a smooth sphere with the center of gravity at the geometric center and with only translatory motion possible. There are three possible directions of motion. In each of these directions the molecule possesses a definite velocity and consequently a momentum. If b_x represents the momentum of the molecule in a particular direction x , the kinetic energy is $\frac{b_x^2}{2m}$. The total kinetic energy is

$$\text{K.E.} = \frac{b^2}{2m} = \frac{b_x^2 + b_y^2 + b_z^2}{2m}$$

The energy is now expressed as a definite function of the squares of coordinates. When thermal equilibrium is attained, the molecule will have a mean value of energy equal to $3/2 kT$. When multiplied by the number of molecules in a unit mass, the energy is $3/2 RT$. Thus for this gas

$$c_v = 3/2 R$$

Into this category are placed monatomic gases such as helium, argon, neon, krypton, and mercury. At high temperature lithium and potassium are also included. (Reference 20)

The next simplest molecule would be two spherical atoms united rigidly and with one axis of symmetry. As with the previous molecule, this one may move in three translatory directions and will possess kinetic energy of translation equal to $3/2 RT$.

There will also be two perpendicular axis about which the molecule may rotate both of which are perpendicular to the axis of symmetry. About each of these axes the molecule will possess kinetic energy equal to $1/2 I \omega^2$. By choosing the proper coordinate system, this energy will satisfy the equipartition principle. The rotational energy may therefore be expressed as $1/2 kT$ for one molecule or $1/2 RT$ for a unit mass of gas. The total rotational energy will then be $2(1/2 RT)$ or RT .

The total energy of this type of molecule must be the sum of the rotational and translational energy.

$$u = u_t + u_r = 3/2 RT + RT = 5/2 RT$$

Then

$$c_v = 5/2 R$$

In addition to the rotations and translation of the molecule, it is possible for the atoms to vibrate along the axis of symmetry. Vibration is expected as the binding force is not expected to be completely inelastic. The energy of the vibrations will be both potential and kinetic. Part of the time it is potential energy; and part of the time, kinetic energy. If the amplitude of the vibration is

small enough, the motion may be considered harmonic. The period will then be given as

$$t = 2\pi \sqrt{\frac{m_k}{f/a}}$$

The frequency ϕ is the reciprocal of the period. The potential energy can be shown to be expressed by the formula

$$P.E. = \frac{4\pi^2}{2} m_k \phi_k^2 a_k^2$$

The kinetic energy is represented by

$$K.E. = \frac{b_k^2}{2m}$$

as for the translational energy. These energy functions satisfy the equipartition principle. As a result, the total vibrational energy for a unit mass is RT .

Adding this energy to u_t and u_r , the result is the total energy of the two atoms rigidly united into a molecule.

$$u = u_t + u_r + u_v = 7/2 RT$$

Then

$$c_v = 7/2 R$$

Among the gases which fall into this category are hydrogen, nitrogen, carbon monoxide, nitrous oxide, chlorine, and bromine. (Reference 20)

With these discussions, the general case may now be considered. The translational energy is always $3/2 RT$. The rotational energy is RT for linear combination of atoms as discussed previously. For any other combination of atoms to form a molecule, rotation is possible about three mutually perpendicular axes. Since the energy of rotation about one axis has been shown to be $1/2 RT$, the rotational energy for nonlinear molecules must necessarily be $3/2 RT$. In a molecule of n atoms, there are $3n$ degrees of freedom of motion. The number of degrees of vibrational freedom is the difference in this total number and the sum of the translational and vibrational degrees of freedom. For the system of linear molecules u_v is $(3n-5) RT$ and for systems of nonlinear molecules u_v is $(3n-6) RT$.

The total internal energy is the sum of all the parts. Thus

$$u_v = 3(n-1)RT \text{ or } (3n-5/2)RT$$

Then

$$c_v = 3(n-1)R \text{ or } (3n-5/2)R$$

All of the degrees of freedom for vibrational motion are not always present and cause the specific heat to vary from this theoretical mean value indicated by the classical theory. In using this theory to apply to vibrational freedoms, the restriction that elasticity of impacts of molecules is preserved and no changes in kinetic energy of gas as a whole occur. By elastic impact is meant an impact in which the shape of the molecule is not destroyed. Further explanation of

this limitation follows from the quantum theory of specific heat.

The classical theory cannot allow a limitation such as must be imposed on it. For this reason the field of quantum mechanics must be entered for an accurate study of specific heat. Specific heat values for monatomic gases are very closely approximated by the classical theory. Gases which exist as such at low temperatures have measured specific heat values practically the same as indicated by the classical theory for monatomic gases. The classical theory is unable to explain this fact. Apparently the number of degrees of freedom for both vibration and rotation tend to decrease with temperature.

The necessary limitation of elasticity of impacts in the classical theory presents the question as to why the impacts should be elastic at all. The impacts might possibly be inelastic and cause a distortion of the molecules which would lead to an increase in potential energy and cause a dissipation in the form of radiant energy. Because of this possibility a generalization from other fields is necessary. Einstein has applied Planck's assumptions from which the radiation laws were derived to the field of specific heats.

Planck was able to show that the energy was absorbed or emitted by particles in whole units or quanta. This unit of energy was shown to be the product of the frequency of vibration and a universal constant, known as the Planck constant. The mechanism of the process remains obscure, but the assumptions are accepted because of their success in correlating experimental data.

Assuming that the various quantum states are numbered in a simple series, Kennard (Reference 12) has shown that the internal mole-

cular energy has the value

$$u_i = RT^2 \frac{d(\ln Z)}{dT}$$

where

$$Z = \sum_j \exp \left[\frac{-\epsilon_j}{KT} \right]$$

Knowing ϵ_j , the energies of the molecular quantum states Z and u in terms of T may be calculated. By the term "quantum state" is meant one of the complete fundamental series of stationary states for a molecule. To each state there corresponds a certain value of energy.

Since the internal energy has been shown to consist of distinct parts, the quantum states must also fall into groups. Very little electronic excitation can be expected at normal temperatures. In order to obtain this form of energy in a molecule, there must be an impact which will transmit an energy of the order of 50 times the average energy of impact at normal temperatures. Such an impact is hardly expected until temperatures are reached where the energy of impacts becomes of order of the energy required for light emission by electrons. To produce electronic excitation, the electrons of the atom must be moved from their orbits to higher orbits. In order to disturb the electrons, the theories of Bohr and Einstein, which have been proved experimentally by Millikan and others, require that the atom must be energized to the point that, on dropping back to next lowest quantum state, light is emitted. At high temperatures such a process does occur as evidenced by

spectroscopic data.

Elastic impacts are sufficient to produce vibration. At normal temperatures practically all impacts are elastic. Some of the impacts should necessarily produce vibrations. As the temperature increases these vibrations should increase and the value of specific heat should show an increase. Rotational energy requires only a small change in energy to jump from one quantum state to another. Sufficient impact energy is available at normal temperature to cause the molecules to show a full capacity of rotation.

On decreasing the temperature, the number of collisions available for impact energy decreases and causes a corresponding decrease in the rotational energy. Most gases liquefy before this decrease becomes appreciable. Hydrogen is an exception and considerable study of this gas has been made. At extremely low temperatures, about 50° absolute, a constant value is reached. This constant value is the same as that given by classical theory for a monatomic gas. This fact is also upheld by the quantum theory. (Reference 20)

There have been several unsuccessful attempts at making models of molecules which would produce theoretical values of specific heat to compare with experimental data. The better method of studying specific heats has been to make use of spectroscopic data concerning energy levels. This method of studying is laborious and theoretical interpretations are left to the theoretical spectroscopist.

Figure 1 indicates the relative values and importance of the three

phases of the internal energy most often encountered. The values are those which have been obtained from hydrogen experiments. At 20° K the value is 2.98 which is the theoretical value for translational energy only. Around 50° K the sharp increase is caused by the rotational freedom beginning to assume its value of energy. The second change in the curve occurs at normal conditions of temperature. The rotational freedom has assumed its full value of energy and the vibrational freedom begins to assume its value. Data were not available to show the effects of electronic excitation. The reasonable change to expect would be one similar to the previous changes. The temperature of this change would be of the order of $20,000^{\circ}$.

REVIEW OF LITERATURE

Lord Rayleigh (Reference 21) appears to have been the first to advance the point that a molecular absorption mechanism might be involved in abnormally large sound absorption in free, still air. He discussed the possibility of a dissipation of energy in a rapid compression process due to a portion of the energy passing from translational form to rotational form. In doing so, the pressure would decrease. He saw no experimental evidence in the behavior of gases which would prevent such a delay occurring.

In Knudsen's investigation of the effect of humidity and temperature upon the rate of absorption of sound in a room, (Reference 16) the results indicated that the absorption was partly due to molecular absorption of some kind. As the investigation was not concerned with this effect, Knudsen left the problem for a later investigation. Later experiments (Reference 18) proved conclusively that some absorption resulted from the transfer of energy between translational and either rotational or vibrational forms of energy during molecular collisions. Knudsen attributed the abnormally high absorption in air to interaction of oxygen and water molecules. He was able to demonstrate his statement by experiments with pure oxygen and water vapor and with pure nitrogen and water vapor. In the latter, he found very little change in the absorption over the classical value. With the oxygen he discovered absorption coefficients of the order of five times as great as for air under the same conditions.

Herzfeld and Rice (Reference 4) had previously introduced the slow rate of exchange of energy between the translational movement and internal degree of freedom to explain the absorption of sound waves. These investigators postulated that the slow exchange kept the internal degrees from taking up the whole amount of heat, and therefore caused a gas to act as if the effective specific heat were decreased. Formulas for the calculation of the absorption and dispersion of sound waves were advanced and checked with previous experimental data. Some agreement was found, but their belief was that sufficient accurate data had not yet been determined due to influencing factors not controlled in previous experiments.

Kneser (Reference 14) developed a theoretical explanation of the anomalous sound absorption in moist air discovered by Knudsen along with the experiments with which Knudsen proved the existence of absorption due to collisions. When a gas is compressed adiabatically, a certain amount of the energy applied is turned into internal energy. In the reverse process the energy may be regained provided the process takes place slow enough so that the gas is in equilibrium states at all times. If the compression and expansion takes place within a time interval of the order of time necessary to establish equilibrium between the normal and vibration degrees of freedom, then some of the vibrational energy will not have been recovered. This unreversed energy will go to produce an irreversible process by raising the temperature. By elimination using observed data and facts, the vibrational energy can be shown to be that which causes the sound absorption irregularities in

these gases. The equilibrium of translational and rotational energy is reached in less than .000001 second. Consequently sound waves of periods longer than .0001 second could not be affected by this adjustment. Experiments have shown these waves to be affected. The vibrational energy of nitrogen is small at room temperature. By elimination and confirmation by Knudsen's experiments with pure oxygen and pure nitrogen, the only possible explanation for the irregular absorption lies with the vibrational oxygen molecules. Kneser also developed a formula based on Einstein's theory of sound dispersion for determining the sound absorption. The absorption coefficient was shown to be directly proportional to the vibrational internal specific heat.

While Kneser and Herzfeld and Rice were developing their theories from fluid flow considerations, Bourgin was preparing a statistical mechanics development. His work, (Reference 3) prepared over a number of years, contained derivations and formulas for the absorption coefficients and velocities of sound in pure gases, vapors, and in mixtures of gases. These discussions are so exact that they are recognized as required reading for investigators of sound absorption. The developments take into account all possible influences on the absorption including the lag in the vibrational molecules assuming their full value of energy and viscosity effects.

The early work has proved beyond a doubt the existence of a lag in the vibrational freedoms assuming their full value of energy. As indicated, the problem was first encountered in attempts to explain abnormal and irregular sound absorption especially in moist air.

Richards (Reference 22) prepared an excellent bibliography for reference

work on the study of this absorption.

IMPACT TUBE MEASUREMENT OF RELAXATION TIME

The following material is a discussion of Kantrowitz's work in the field of specific heat lag. (References 7, 8, 10) The apparatus discussed is similar to that presented in the assembly drawing contained in this thesis.

Consider the flow of a fluid about a very small impact tube. If the compression process is slow enough that the molecules may take on their normal heat capacity and the gas is at all times essentially in equilibrium, the gas will return to its original state after expanding through a nozzle isentropically and being compressed at the nose of an impact tube.

On the other hand, if the compression time is very short as compared to the relaxation time of the gas, some of the vibrational energy will not be regained. The result will be an increase in entropy of the gas. The non-vibrational forms undergo an adiabatic compression, but are located off the thermodynamic surface at stable equilibrium states at some temperature higher than the temperature before expansion. The vibrational forms remain at the temperature immediately after the expansion. On adjustment and transferring of the energies between the various forms, the gas returns to the thermodynamic surface with an increase in entropy. The increase in entropy occurs due to the irreversible process of transferring energy across a finite temperature difference. Assuming these conditions, then the energy involved satisfies the following equality.

$$c_{vib}T_{vib} + c_p'T = c_pT_0 \quad (5)$$

The subscripts refer to the initial conditions in the chamber, 0, the conditions after expansion, 1, and the conditions after compression, 2', and the conditions after adjustment, 2. T represents the temperature of the non-vibrating forms of energy before adjustment.

The increase in entropy when an element of energy transfers during the adjustment is given by

$$ds = \frac{dq}{T_{vib}} - \frac{dq}{T}$$

But

$$dq = c_{vib}dT_{vib} = c_p'dT$$

Therefore

$$ds = c_{vib}dT_{vib} \left(\frac{1}{T_{vib}} - \frac{1}{T} \right) \quad (6)$$

From (5)

$$T = \frac{c_p T_0 - c_{vib} T_{vib}}{c_p'}$$

Substituting in (6) and integrating

$$\Delta s = \int ds = \int_{T_1}^{T_0} c_{vib} dT_{vib} \left(\frac{1}{T_{vib}} - \frac{c_p'}{c_p T_0 - c_{vib} T_{vib}} \right)$$

$$\Delta s = \left[c_{vib} \ln T_{vib} + c_p' \ln \left(\frac{c_p}{c_{vib}} T_0 - T_{vib} \right) \right]_{T_1}^{T_0}$$

From which

$$\Delta s_{max.} = \ln \left(\frac{\frac{c_p}{c_{vib}} - 1}{\frac{c_p}{c_{vib}} - \frac{T_1}{T_0}} \right)^{c_p'} \left(\frac{T_0}{T_1} \right)^{c_{vib}} \quad (7)$$

Since the expansion through the nozzle is isentropic, the Δs from equation (7) will be the difference in entropy of the gas inside the chamber and the gas at equilibrium at the nose of the impact tube. The only energy concerned in this process is the enthalpy of the gas. The enthalpy at the end of the compression will be the same as that inside the chamber. The temperature at the end of adjustment must necessarily be the same as that inside the chamber. The final equilibrium state will then be located on the thermodynamic surface along an isotherm from the initial conditions with increased entropy and consequently a reduced pressure.

From the perfect gas relations

$$\Delta s = c_p \ln \frac{T_2}{T_0} - R \ln \frac{P_2}{P_0}$$

But $T_2 = T_0$. Therefore

$$\Delta s = R \ln \frac{P_0}{P_2}$$

Substitution in (7) yields

$$\frac{P_0}{P_2} = \left(\frac{\frac{c_p}{c_{vib}} - 1}{\frac{c_p}{c_{vib}} - \frac{T_1}{T_0}} \right)^{\frac{c_p}{R}} \left(\frac{T_0}{T_1} \right)^{\frac{c_{vib}}{R}} \quad (8)$$

The ratio of the difference in the pressure of the gas in the chamber and the pressure at the impact tube to the pressure drop through the nozzle is defined as the total head defect. This ratio is expressed as a percent. Thus the percentage total head defect is

$$100 \frac{P_0 - P_2}{P_0 - P_1}$$

The general method of determining the theoretical total head defect is outlined in Appendix A.

The previous discussion has been made with regard to a very small impact tube, an instantaneous compression of the gas and temperature changes in the gas which were either very fast or slow as compared to the relaxation time. In the general case the temperature changes may be neither slow nor fast as compared to the relaxation time. The temperature history of the gas particle must be considered.

Some limitations must be imposed on the problem in order to simplify the solution. The temperature changes must be considered small enough so that the specific heats remain constant. This limitation was assumed in the specific case discussed previously. The velocity distribution is assumed unchanged by the specific heat lag.

The variable ϵ is introduced to represent the excess energy of the lagging specific heat over the energy of equilibrium partition at the translational temperature T . The general energy equation then becomes

$$c_p T + 1/2 w^2 + \epsilon = \text{constant}$$

If the assumption is made that only the vibrational form of energy exhibits the lagging characteristic and that its time rate of adjustment is proportional to the departure from equilibrium, then

$$\frac{dE_{\text{vib}}}{dt} = -m\epsilon$$

Kantrowitz states that this assumption is in agreement with sonic theo-

ries. However, in recent years at least two people (References 23 and 24) have shown that there is a dispersion of sound in hydrogen that is attributed to the failure of the rotational degrees of freedom to follow the temperature associated with the translational degrees of freedom and here the assumption is insufficient. By definition of ϵ

$$\epsilon = c_{vib} T_{vib} - c_{vib} T = E_{vib} - c_{vib} T$$

By differentiating this equation with respect to time and substituting the value of $\frac{dE_{vib}}{dt}$, the result is

$$\frac{d\epsilon}{dt} = -c_{vib} \frac{dT}{dt} - m\epsilon \quad (10)$$

Differentiating equation (9)

$$\frac{dT}{dt} = \frac{-\frac{d\epsilon}{dt} - w \frac{dw}{dt}}{c_p}$$

Equation (10) now becomes

$$\frac{d\epsilon}{dt} \left(\frac{c_p - c_{vib}}{c_p} \right) + m\epsilon = \frac{c_{vib}}{c_p} w \frac{dw}{dt}$$

or

$$\frac{c_p'}{c_{vib}} \frac{d\epsilon}{dt} + \frac{c_{p'm}}{c_{vib}} \epsilon = w \frac{dw}{dt} \quad (11)$$

To find the meaning of m , examine the statement that the total enthalpy remains constant.

$$c_p T + \epsilon = \text{constant}$$

Equation (10) becomes

$$\frac{d\epsilon}{dt} = \frac{c_{vib}}{c_p} \frac{d\epsilon}{dt} - m \epsilon$$

$$\frac{d\epsilon}{dt} = -m \frac{c_p}{c_p'} \epsilon = -\frac{\epsilon}{c_p'/mc_p}$$

The term $\frac{c_p'}{mc_p}$ must be the relaxation time \mathcal{T} of the gas.

Multiplying equation (11) by a term $\frac{2d}{W^3}$, where d is a typical length in the flow and W is a typical velocity, for convenience to be shown later and substituting the value of m in terms of \mathcal{T} , the result is

$$\frac{2d}{W^3} \frac{c_p'}{c_{vib}} \frac{d\epsilon}{dt} + \frac{2d}{W^3} \frac{c_p'}{c_{vib}\mathcal{T}} \epsilon = \frac{2d}{W^3} \frac{WdW}{dt}$$

This equation may be written as

$$\frac{d \left(\frac{\epsilon}{\frac{c_p'}{c_{vib}} \frac{W^2}{2}} \right)}{d \left(\frac{W}{d} \right)} + \mathcal{T} \frac{d \epsilon}{W \frac{c_p'}{c_{vib}} \frac{W^2}{2}} = \frac{d \left(\frac{W}{W} \right)^2}{d \left(\frac{W}{d} \right)} \quad (12)$$

Dimensionless variables are introduced at this point in order to simplify the solutions. The variable ϵ'

$$\epsilon' = \frac{\epsilon}{\frac{c_p'}{c_{vib}} \frac{W^2}{2}} \quad (13)$$

is defined to have the value unity for an instantaneous compression of a gas. The following dimensionless variables are also defined.

$$w' = w/W \quad (14)$$

$$t' = \frac{t}{d/l} \quad (15)$$

$$K = \frac{d}{WJ} \quad (16)$$

Equation (12) now reduces to

$$\frac{d\epsilon'}{dt'} + K\epsilon' = \frac{dw'^2}{dt'} \quad (17)$$

This linear differential equation has the solution

$$\epsilon' = e^{-\int K dt'} \left[\int e^{\int K dt'} \frac{dw'^2}{dt'} dt' + \text{constant} \right] \quad (18)$$

if $w'(t')$ is known and is a decent function.

As previously defined, the rate of energy flow from the temperature T_{vib} to the temperature T is $m\epsilon$. Then

$$\frac{ds}{dt} = m\epsilon \left(\frac{1}{T} - \frac{1}{T_{vib}} \right) \quad (19)$$

But

$$T_{vib} = T + \frac{\epsilon}{c_{vib}}$$

Equation (19) can now be expressed as

$$\frac{ds}{dt} = m\epsilon \left(\frac{1}{T} - \frac{1}{T + \epsilon/c_{vib}} \right)$$

or

$$\Delta s = \int_{t_0}^t m\epsilon \left(\frac{1}{T} - \frac{1}{T + \epsilon/c_{vib}} \right) dt \quad (20)$$

provided the integration is made over all the streamlines of flow.

If only small changes of temperature and pressure as compared to ambient conditions are maintained, constant density may be assumed and incompressible fluid flow laws used. In this case n is a constant. Equation (18) then becomes

$$\epsilon' = e^{-Kt'} \left[\int \frac{dw'}{dt'} e^{Kt'} dt' + \text{constant} \right] \quad (21)$$

Rewriting and expanding equation (20)

$$\Delta s = \int_{t_0}^t \frac{m\epsilon}{T} \left(1 - 1 + \frac{\epsilon}{T c_{vib}} \right) dt$$

Therefore

$$\Delta s = \frac{m}{c_{vib} T^2} \int_{t_0}^t \epsilon^2 dt \quad (22)$$

where T is the average of the temperature over the range t_0 to t .

The dropping of the second order and higher terms is possible because

$\frac{\epsilon}{c_{vib}}$ is small compared with T .

Introducing the non-dimensional quantities into equation (22), it becomes

$$\Delta s = \frac{c_p K}{c_p c_{vib} T^2} \left(\frac{W^2}{2} \frac{c_{vib}}{c_p} \right)^2 \int_{t_0}^t \epsilon'^2 dt' \quad (23)$$

In order to introduce a non-dimensional entropy increase, equation (23) is divided by the entropy increase following an instantaneous compression. This entropy increase is obtained by expansion of equation

(7). (Reference 9) In the expansion, the second order terms must be retained. The division is simplified by expressing \bar{T} as $\frac{T_0 + T_1}{2}$ and W^2 as $2 c_p (T_0 - T_1)$. The resulting equation is

$$\Delta s' = 2K \int_{t_0}^{t'} \epsilon'^2 dt' \quad (24)$$

From equations (21) and (24) K is shown to be the determining factor for ϵ' and $\Delta s'$. Since K is defined as a typical length in the flow divided by the product of the relaxation time and a typical velocity, a determination of the relaxation time should be possible upon the determination of $\Delta s'$. Curves of $\Delta s'$ versus K will be necessary for the particular obstruction in the flow causing the compression. With the aid of such a curve the problem is now reduced to an experimental one. The ratio of actual total head defect to the maximum total head defect as obtained from the instantaneous compression theory is $\Delta s'$.

In order to facilitate the calculation of the anticipated $\Delta s'$ for a given value of K , an impact tube with a source shape is used. The general method of obtaining such a shape is to determine the shape of a surface across which no fluid flows. In a uniform flow with a point source there will be located a surface whose stream function is zero. If a solid body of this shape is inserted in the flow, there is no change in the flow outside the body. Such a body for a single source has been plotted as Figure 4 with the values listed in Table 2.

The velocity along the central streamline of such a flow is given by (Reference 26)

$$w(x) = W \left(1 - \frac{d^2}{16x^2} \right)$$

where x is the distance along the central streamline from the source, W is the velocity far from the body, and d is the diameter of the impact tube.

In non-dimensional form this equation is

$$w'(x') = 1 - \frac{1}{16x'^2} \quad (25)$$

which is obtained by using W as the typical velocity and d as the typical length. By differentiation of (25)

$$dx' = 8x'^3 dw'(x')$$

But

$$x' = -\frac{1}{4} \left[\frac{1}{1 - w'(x')} \right]^{1/2}$$

Therefore

$$dx' = -\frac{1}{8} \left[\frac{1}{1 - w'(x')} \right]^{3/2} dw'(x')$$

$$dt' = \frac{dx'}{w'(x')} = -\frac{dw'(x')}{8w'(x') \left[1 - w'(x') \right]^{3/2}}$$

If t' is chosen as zero when $w' = .99$,

$$t' = -\frac{1}{8} \int_{.99}^{w'} \frac{dw'}{w'(1-w')^{3/2}}$$

$$t' = -\frac{1}{8} \int_{.99}^{w'} dw' \left[\frac{1}{w'(1-w')^{1/2}} + \frac{1}{(1-w')^{3/2}} \right]$$

$$t' = -\frac{1}{8} \left[\ln \left(\frac{11}{9} \frac{1-\sqrt{1-w'}}{1+\sqrt{1-w'}} \right) + \frac{2}{\sqrt{1-w'}} - 20 \right] \quad (26)$$

To find $\epsilon'(t')$, refer to equation (21). Substitute the value of $\frac{dw'^2}{dt'}$ from equation (25)

$$\frac{dw'^2}{dt'} = 2w' \frac{dw'}{dt'} = 2w' \frac{dw'}{dx'} \frac{dx'}{dt'} = 2w'^2 \frac{dw'}{dx'}$$

$$\frac{dw'^2}{dt'} = -16w'^2 (1-w')^{3/2}$$

Then

$$\epsilon'(t') = -e^{-Kt'} \int_0^{t'} 16w'^2 (1-w')^{3/2} e^{Kt'} dt' \quad (27)$$

The actual calculation of ϵ' and $\Delta s'$ becomes laborious. Figure 2 represents the method by which $\Delta s'$ has been determined for $K = 3$. From equation (26), t' may be determined for any value of w' . Careful plotting of a few points will yield the value of w' at any t' . The integral part of equation (27) may now be determined by Simpson's rule for the given K . In Figure 2 the integrand is plotted as curve f and the integration done by planimeter. By careful integration the curve of ϵ'^2 may be determined. The integral of this curve then yields the value of $\Delta s'$. Although more laborious the integration of equation (27) by Simpson's rule will produce more accurate results for the scale involved in this plotting. The results of Figure 2 yielded a value of 0.402 for $\Delta s'$ to compare with 0.405 obtained by Kantrowitz for $K = 3$.

By calculation of a number of values of $\Delta s'$ for different values of K , a curve may be plotted. Due to an entropy increase in the nozzle the values of $\Delta s'$ must be adjusted as will be explained later.

This adjusted curve will serve to determine the relaxation time. From experimental data the value of $\Delta s'$ is determined by dividing the actual total head defect by the maximum total head defect. With this $\Delta s'$ there is only one value of K . From the definition of K , the only unknown is the relaxation time as the typical length is the diameter of the impact tube and the typical velocity is the maximum velocity. Adiabatic expansion is assumed in the nozzle to determine the maximum velocity.

For large and small values of K approximate solutions for $\Delta s'$ can be made which are sufficiently accurate and which reduce the labor of calculations.

When the relaxation time is short, K will be large since $\mathcal{J} = \frac{d}{Kw}$. Examining equation (21) at a particular time t_a'

$$\epsilon'(t_a') = e^{-Kt_a'} \left[\int_{t_0'}^{t_a'} \frac{dw'^2}{dt'} e^{Kt'} dt' \right]$$

or

$$\epsilon'(t_a') = \int_{t_0'}^{t_a'} \frac{dw'^2}{dt'} e^{-K(t_a' - t')} dt' \quad (28)$$

This integral will reduce to very nearly zero for any large value of $t_a' - t'$. Consequently only values of t' very close to t_a' will contribute substantially to the integral. Since t' is close to t_a' , the following approximations may be made.

$$\frac{dw'}{dt'} \approx \left(\frac{dw'}{dt'} \right)_a$$

and

$$t' - t_a' = \frac{w'^2 - w_a'^2}{\left(\frac{dw'^2}{dt'}\right)_a}$$

Substituting in equation (28)

$$\epsilon'(t_a') = \int_{t_0'}^{t_a'} \frac{dw'^2}{dt'} e^{\left[K \frac{w'^2 - w_a'^2}{\left(\frac{dw'^2}{dt'}\right)_a} \right]} dt'$$

or

$$\epsilon'(t_a') = \int_{-\infty}^0 e^{\left[\frac{K}{\left(\frac{dw'^2}{dt'}\right)_a} (w'^2 - w_a'^2) \right]} d(w'^2 - w_a'^2) \quad (29)$$

The lower limit is replaced by $-\infty$ because $w'^2 - w_a'^2$ becomes so large that the exponential becomes negligible. The sign of this limit must be opposite that of $\left(\frac{dw'^2}{dt'}\right)_a$ in order for this substitution to be made.

Integrating equation (29) and applying the limits

$$\epsilon'(t_a') = \frac{1}{K} \frac{dw'^2}{dt'} \quad (30)$$

This equation will develop from both lower limits. Substitution in (24)

$$\Delta s' = \frac{2}{K} \int \left(\frac{dw'^2}{dt'}\right)^2 dt' \quad (31)$$

The integral of equation (31) may be evaluated once and used for all values of K when K is large.

When the relaxation time is long, K is small compared with 1. The problem is divided into two parts as in the usual flow. In the initial period the gas velocity changes appreciably. Finally it settles to a steady state value.

Examining equation (17) when K is small compared to 1, the $K \epsilon'$ term becomes insignificant and reduces the equation to

$$\frac{d\epsilon'}{dt'} = \frac{dw'^2}{dt'}$$

Integrating and applying limits

$$\epsilon' = w'^2 - w_0'^2 \quad (32)$$

From equation (24) $\Delta s'$ for the first period becomes

$$\Delta s'_1 = 2K \int_0^{t_1'} (w'^2 - w_0'^2)^2 dt' \quad (33)$$

In the second period equation (17) reduces to

$$\frac{d\epsilon'}{\epsilon'} = -K dt'$$

or

$$\epsilon' = \epsilon'_1 e^{-K(t'-t_1')}$$

Equation (24) yields

$$\Delta s'_2 = 2K \epsilon_1'^2 \int_{t_1'}^{\infty} e^{-2K(t'-t_1')} dt'$$

Integration and application of limits gives

$$\Delta s_2' = \epsilon_1'^2 \quad (35)$$

To find the value of ϵ_1' examine equation (17) again.

$$\frac{d\epsilon'}{dt'} = \frac{dw'^2}{dt'} - K\epsilon'$$

$$\epsilon_1' = \Delta w'^2 - K \int_0^{t_1'} (w'^2 - w_0'^2) dt' \quad (36)$$

by substituting the value of ϵ' and integrating from zero to end of period 1.

The total entropy increase will be the sum of the parts since the value could not be obtained by a single integration.

$$\Delta s_T' = \Delta s_1' + \Delta s_2'$$

$$\Delta s_T' = 2K \int_0^{t_1'} (w_0'^2 - w'^2)^2 dt' + \left[K \int_0^{t_1'} (w_0'^2 - w_1'^2) dt' + \Delta w'^2 \right]^2 \quad (37)$$

As for the case of large K as compared to 1, these integrals may be evaluated only once for any small value of K. The order of magnitude for K small is less than 0.12; and for K large, greater than 12.

The entropy increase in the nozzle must be calculated and added to the anticipated $\Delta s'$ for the impact tube. The nozzle design is such that

$$\frac{dw'^2}{dt} = \text{constant} = \lambda \quad (38)$$

This equation may be expressed as

$$w dw^2 = \mathcal{K} dx$$

$$\frac{2}{3} w^3 = \mathcal{K} x + c$$

But $c = 0$, since at $x = 0$, $w = 0$.

At $x = \ell$, $w = W$. Hence

$$\frac{2}{3} W^3 = \mathcal{K} \ell$$

If nondimensional terms are introduced by dividing by the typical velocity W and typical length ℓ , \mathcal{K} will have the value $2/3$.

From equation (21)

$$\epsilon' = e^{-K_n t'} \left(\int_0^{t'} \frac{2}{3} e^{K_n t'} dt' + \text{constant} \right)$$

where n refers to the nozzle. Integration yields

$$\epsilon' = \frac{2}{3K_n} (1 - e^{-K_n t'}) \quad (39)$$

where $\epsilon' = 0$ at $t' = 0$. From equation (24)

$$\begin{aligned} \Delta s'_n &= 2K_n \int_0^{t'_{\max}} \epsilon'^2 dt' \\ \Delta s'_n &= \frac{8}{9K_n} \int_0^{t'_{\max}} (1 - e^{-K_n t'})^2 dt' \end{aligned} \quad (40)$$

Again introducing nondimensional terms in equation (38), t'_{\max} is found to be $3/2$. Hence

$$\Delta s_n' = \frac{4}{3K_n} - \frac{16}{9K_n^2} (1 - e^{-3/2 K_n}) + \frac{4}{9K_n^2} (1 - e^{-3K_n}) \quad (41)$$

In this derivation K_n must have the same form as K . The value of K_n is $\frac{2}{wJ}$. Hence

$$K_n = \frac{2}{d} K \quad (42)$$

The value of $\Delta s_n'$ must be added to the previously determined $\Delta s'$ for the tube in order to have the total anticipated $\Delta s'$.

In developing equation (27) for $\epsilon'(t')$, the assumption was made that ϵ' was zero when t' was zero. In other words ϵ' had no value before compression. For a gas with a long relaxation time, there will be a change in ϵ' through the nozzle which will give a value to ϵ' initially. In order to correct (27) for the gases with long relaxation times, the value of equation (39) must be added algebraically. Hence

$$\epsilon'(t') = -e^{-Kt'} \left[\int_{t_0}^{t_1} 16w'^2 (1-w')^{3/2} e^{Kt'} dt' - \frac{2}{3K_n} (1 - e^{-3/2 K_n}) \right] \quad (43)$$

DESIGN OF APPARATUS

The apparatus which has been designed and for which detail drawings are given in the back of this report is based on the principles advanced by Kantrowitz. The design is quite similar to the sketch which he has presented.

The essential features of the apparatus are a chamber in which the gas collects at equilibrium conditions, a nozzle of particular design and an impact or pitot tube of special shape.

The nozzle design is such as to give a constant time rate of temperature drop. This particular design is used to facilitate calculations. From the energy equation

$$c_p dT = d \frac{W^2}{2}$$

Then

$$c_p \frac{dT}{dt} = \frac{dW^2}{2dt}$$

But $\frac{dT}{dt}$ is chosen constant. Therefore

$$\frac{dW^2}{dt} = \text{constant} \quad (44)$$

If low velocity is assumed, then the gas density may be taken as approximately constant. By the continuity equation

$$W = \frac{\text{constant}}{r^2} = \frac{C_1}{r^2} \quad (45)$$

and by definition

$$Wdt = dx \quad (46)$$

By squaring and differentiating (45) and substituting in (44)

$$dt = \frac{C_2 dr}{r^5} \quad (47)$$

Substituting (45) and (47) in (46)

$$Wdt = C_3 r^{-7} dr = dx$$

Integrating

$$x = \frac{C_4}{r^6} + C_5$$

At $x = 0$, $r = \infty$, therefore $C_5 = 0$.

The equation, $r^6 x$ constant, when rotated about the axis will produce the theoretical shape of the nozzle. The last 0.10 inch of the nozzle is made straight in order to straighten the streamlines and allow the static pressure at exit to be atmospheric.

The nozzle should be such that the center streamlines are not disturbed. A correction to reduce the effect of boundary layer must be added to the value of the radius. Assuming laminar flow through the nozzle, which seems possible due to the low velocity, the thickness of boundary layer is given by (Reference 5)

$$\delta = \frac{5.2x}{\sqrt{\frac{Wx}{\rho}}} \quad (48)$$

This formula is for a flat plate but is assumed sufficiently accurate for this problem.

In order to be on the safe side, a maximum velocity of 10,000 inches per second is assumed. No velocity of this order is expected. By (45) an approximate value of velocity along the nozzle may be obtained.

To arrive at a value of kinematic viscosity, carbon dioxide has been used as a guide. The value used is 4.15×10^{-3} in.² / sec which is the condition in the neighborhood of 32°F and 23 psia. These conditions will produce a greater boundary layer thickness than expected conditions.

Substituting values in (43) gives the values of δ , and the actual radius is found by adding δ to the theoretical radius. The results are in Table 1.

The shape of the impact tube is that formed by a single source in a uniform flow of fluid.

The stream function for a steady flow and source is represented by

$$\psi = -Wy - \frac{Q}{2} \tan^{-1} \frac{y}{x}$$

At the boundary between the two flows, this function has the value zero. A solid body inserted in the flow having the shape of the surface where $\psi = 0$ will produce the same effect on the flow as the source. Hence

$$\tan \frac{2\pi Wy}{Q} = -\frac{y}{x}$$

where Q is the flow from the source. At some far removed point where

the source velocity and low velocity are the same

$$Q = 2rW$$

where r is height of source stream at that point. Then

$$x = - \frac{y}{\tan \frac{y}{r}} \quad (49)$$

The graph of this equation is given in Figure 4 with the data listed in Table 2. When this curve is rotated about the x axis, the source shaped body is produced. The value of r is assumed to be unity, for these calculations since it is a parameter for a family of curves.

The impact tube is to be made of glass. Kantrowitz has suggested that glass tubing be drawn out to produce the tube. The inside diameter of the tube should be kept larger than .004 inch. The tube should not be longer than 1/4 inch. The source shape must be ground on the end of the tube. In order to check the shape, the suggestion is made that a silhouette be thrown on a screen during the grinding process.

The heaters are designed from data taken from reference 15. The gas coming into the chamber should be dried and heated to the desired temperature. In this case the inside heaters will only have to supply the heat that is lost to conduction through the walls and radiation to the surroundings. Approximate calculations were made and a safety margin allowed. Approximately 35 feet of #18 nichrome V wire are required. Porcelain beads on the wire provide the necessary insulation. Small clamps on the liner will hold the wire in position. External leads are provided through spark plug type insulators.

One hundred mesh screen is provided in the inner chamber to destroy the streams of gas before the nozzle.

Motion of the clamp holding the impact tube is provided in two perpendicular directions as well as rotational motion about two horizontal perpendicular axes through the end of the nozzle.

Two manometers are needed for the measurement of the pressures. One manometer filled with mercury should have a range of approximately twenty inches of mercury as it is to measure the pressure inside the chamber. To measure the drop in pressure due to the irreversible process at the nose of the impact tube, an alcohol manometer is used. Kantrowitz's work indicated that this manometer should have a range of about six inches.

To measure the temperature of the gas in the chamber, an aluminum-chromel or similar thermocouple should be peened into the end of the thermocouple tube. The junction may be satisfactorily made by soldering a copper plug which the thermocouple has been fastened in the end of the tube.

CONCLUSION

This thesis is intended as the beginning of a long range project. Basic fundamentals of specific heat have been presented. Past research has shown that the vibrational freedom lag behind the other freedoms in taking on their heat contents. Kantrowitz has developed a theory and an apparatus for measuring the relaxation time. This apparatus is simple and uses principles known for years. No new measuring apparatus is needed for the work. A design for the apparatus is presented for the check and continuation of the experimentation with specific heat lag.

There are some limitations to the theory which has been developed.

1. Incompressible fluid flow has been assumed.
2. The entropy change in the nozzle due to friction and turbulence has been neglected.
3. Perfect gas relationships have been used.
4. Small temperature and pressure changes have been assumed.
5. In most instances all except zero and first order terms have been dropped.

These limitations must be recognized in the application of the theory.

BIBLIOGRAPHY

1. Bailey, N.P., "Abrupt Energy Transformation in Flowing Gases," Transactions of the American Society of Mechanical Engineers, Vol. 69, 1947. pp. 749-763.
2. Bloch, Eugene, Kinetic Theory of Gases, Great Britain: E.P. Dulton and Company, 1921. pp. 50-63.
3. Bourgin, D.G., Nature 122:123; Philosophical Magazine 7:821; Physical Review 34:521; Journal of Accoustical Society of America 5: 108; Physical Review 42:721; Physical Review 50:355.
4. Herzfeld, K.F. and Rice, F.C., "Dispersion and Absorption of High Frequency Sound Waves," Physical Review, Vol. 31, 1928. pp. 691-695.
5. Hunsaker, J.C. and Rightaire, B.G., Engineering Applications of Fluid Mechanics, New York: McGraw-Hill Book Company, Inc., 1947. p. 188.
6. Jeans, J.H., Dynamical Theory of Gases, London: Cambridge University, 1925. pp. 391-695.
7. Kantrowitz, A., "Effects of Heat Capacity Lag in Gas Dynamics," Journal of Chemical Physics, Vol. 10, 1942. p. 145.
8. _____, Effects of Heat Capacity Lag in Gas Dynamics, NACA ARR No. 4A22, 1944.
9. _____, Letter to Charley Scott, 1949.
10. Kantrowitz, A. and Huber, P.W., Heat Capacity Lag in Turbine Working Fluids, NACA Bulletin L4E29, 1941.
11. Kennan, J.H., Thermodynamics, New York: John Wiley and Sons, Inc., 1941. pp. 20-22.
12. Kennard, E.H., Kinetic Theory of Gases, New York: First Edition. McGraw-Hill Book Company, Inc., 1927. pp. 358-386.
13. _____, Kinetic Theory of Gases, New York: Second Edition. McGraw-Hill Book Company, Inc., 1938. pp. 238-266.
14. Kneser, H.C., "The Interpretation of the Anomalous Sound-Absorption in Air and Oxygen in Terms of Molecular Collisions," Journal of the Accoustical Society of America, Vol. 5, 1933. pp. 122-126.
15. Knowlton, A.C., Standard Handbook for Electrical Engineers, New York: McGraw-Hill Book Company, Inc., 1941. pp. 18: 19-18:42.

16. Knudsen, V.O., "The Effect of Humidity Upon the Absorption of Sound in a Room, and a Determination of the Coefficients of Absorption of Sound in Air," Journal of the Acoustical Society of America, Vol. 5, 1931. pp. 126-138.
17. _____, "Absorption of Sound in Gases," Science, Vol. 81, 1935. pp. 126-138.
18. _____, "The Absorption of Sound in Air, in Oxygen, and in Nitrogen---Effects of Humidity and Temperature," Journal of Acoustical Society of America, Vol. 5, 1933. pp. 112-121.
19. Lewis, B. and von Elbe, G., "Anomalous Pressures and Vibrations in Gas Explosions," Journal of Chemical Physics, Vol. 3, 1935. pp. 63-71.
20. Loeb, L.B., Kinetic Theory of Gases, New York: First Edition. McGraw-Hill Book Company, Inc., 1927. pp. 358-386.
21. Lord Rayleigh, "On the Cooling of Air by Radiation and Conduction, and on the Propagation of Sound," Philosophical Magazine, Vol. 47, 1899. pp. 308-314.
22. Richards, W.T., "Supersonic Phenomena," Bibliography, Review of Modern Physics, Vol. 11, 1939. pp. 55-64.
23. Rhodes, J.E., "The Velocity of Sound in Hydrogen When Rotational Degrees of Freedom Fail to Be Excited," Physical Review, Vol. 70, 1946. pp. 932-938.
24. Stewart, E.S., "Dispersion of the Velocity and Anomalous Absorption of Sound in Hydrogen," Physical Review, Vol. 69, 1946. p. 632.
25. Sweigert, R.L. and Beardsley, H.W., Empirical Specific Heat Equations Based Upon Spectroscopic Data. Georgia State Experiment Station, Bulletin No. 2. Atlanta: Georgia State Experiment Station, Georgia School of Technology, June, 1938. 14 pp.
26. Tietjens, O.G., Fundamentals of Hydro- and Aeromechanics, New York: McGraw-Hill Book Company, Inc., 1934. pp. 146-149.

FIGURE 1
CHANGE IN SPECIFIC HEAT WITH TEMPERATURE

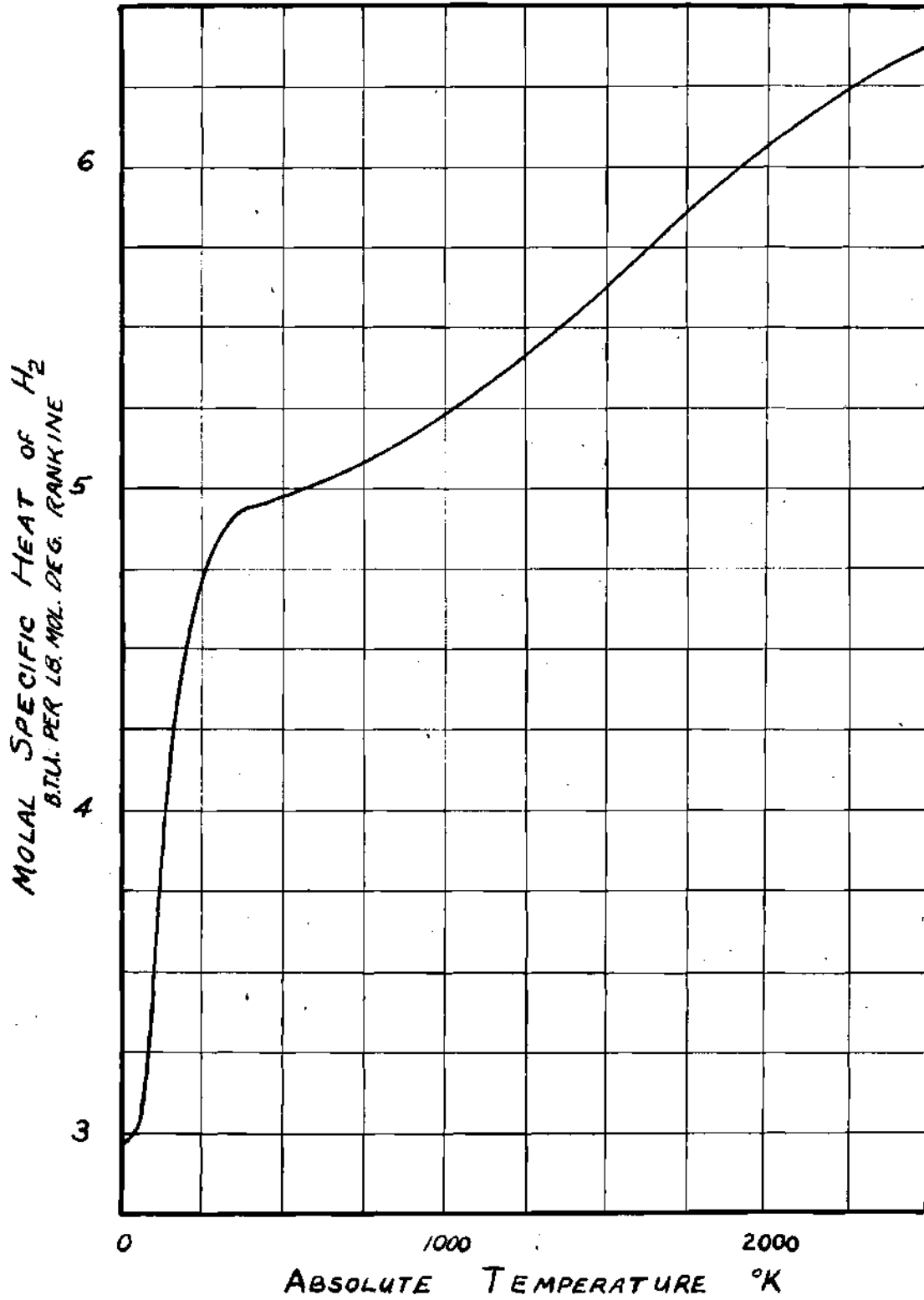
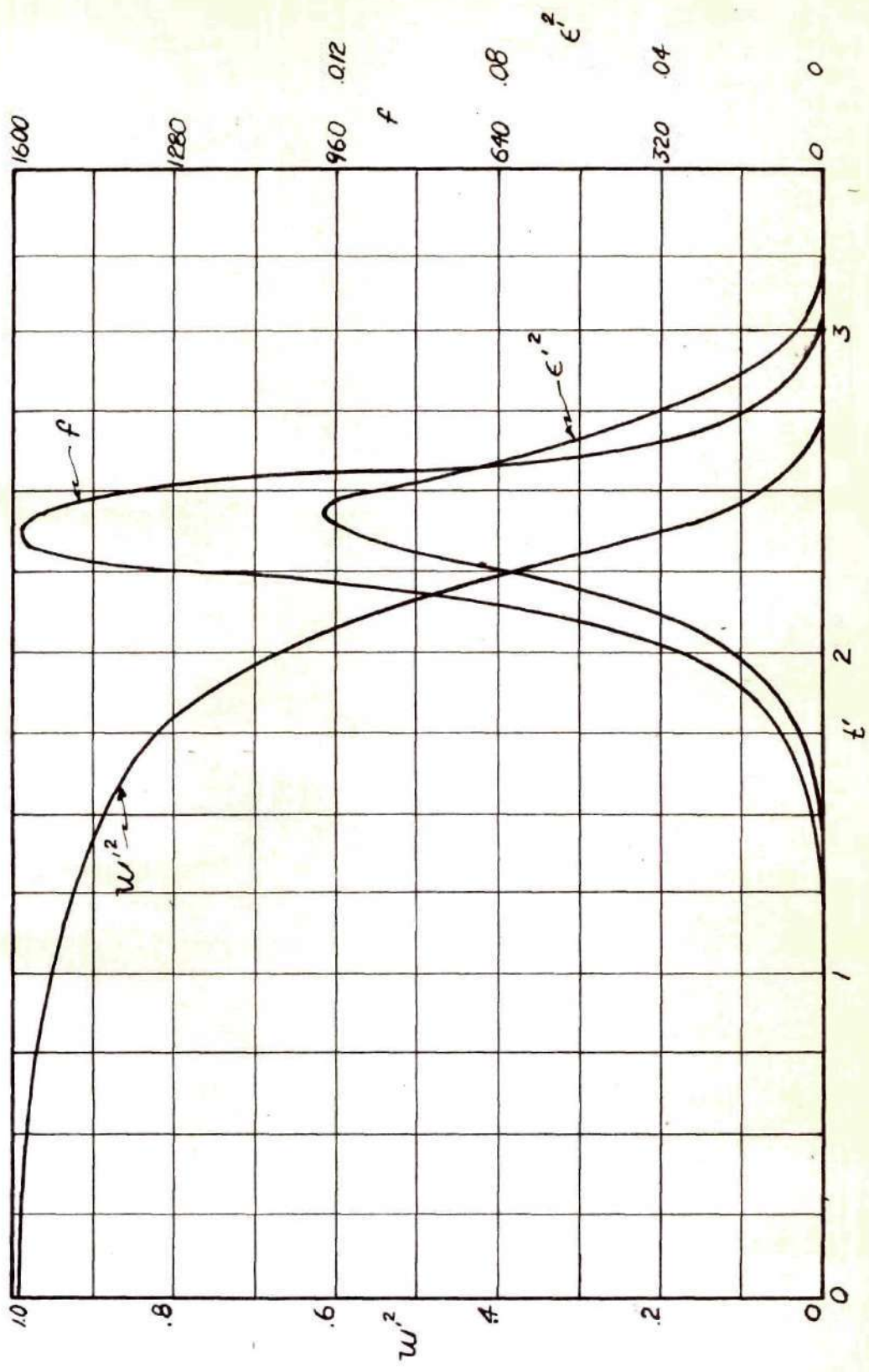


FIGURE 2
DETERMINATION OF ϵ'^2



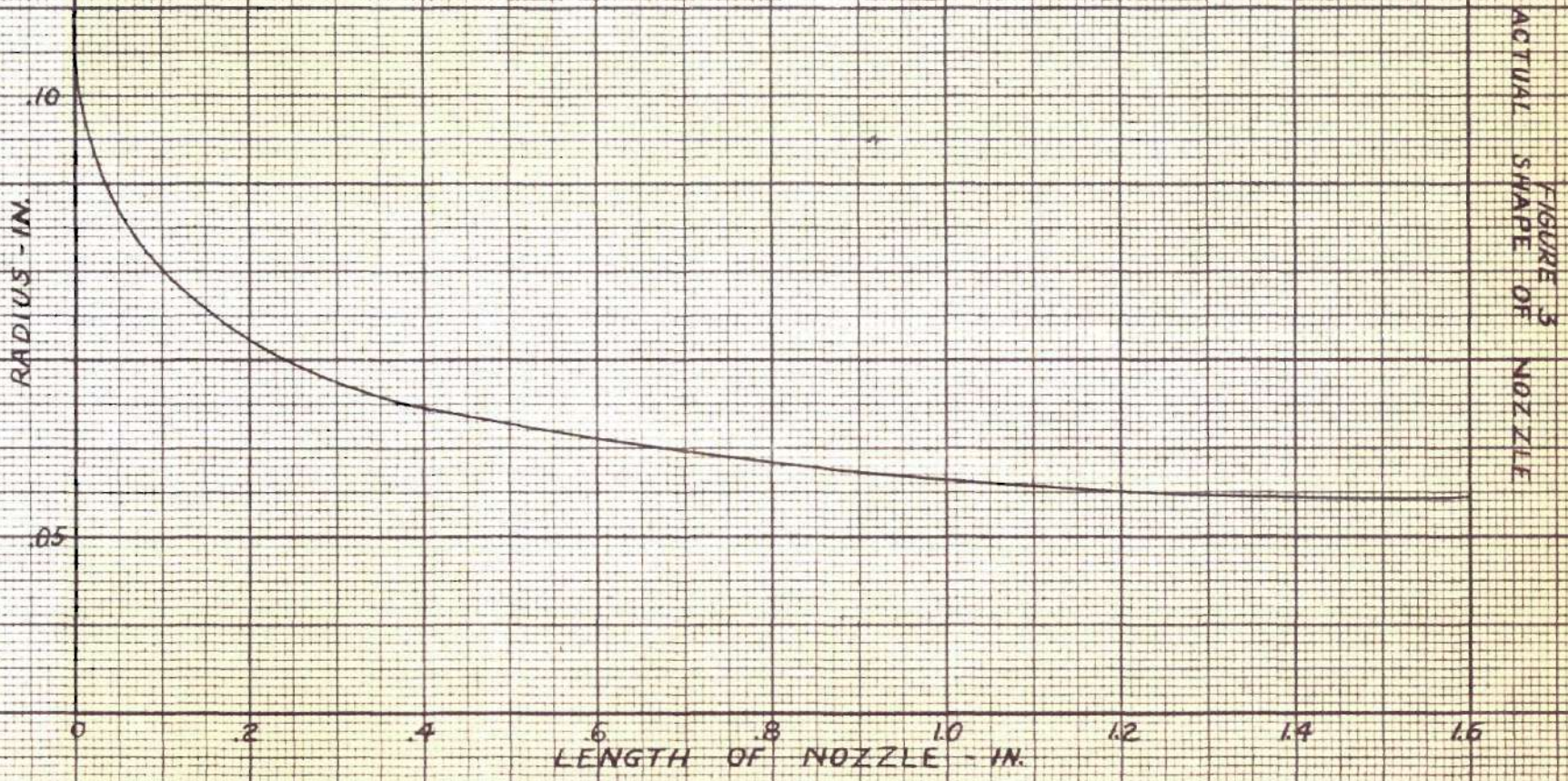
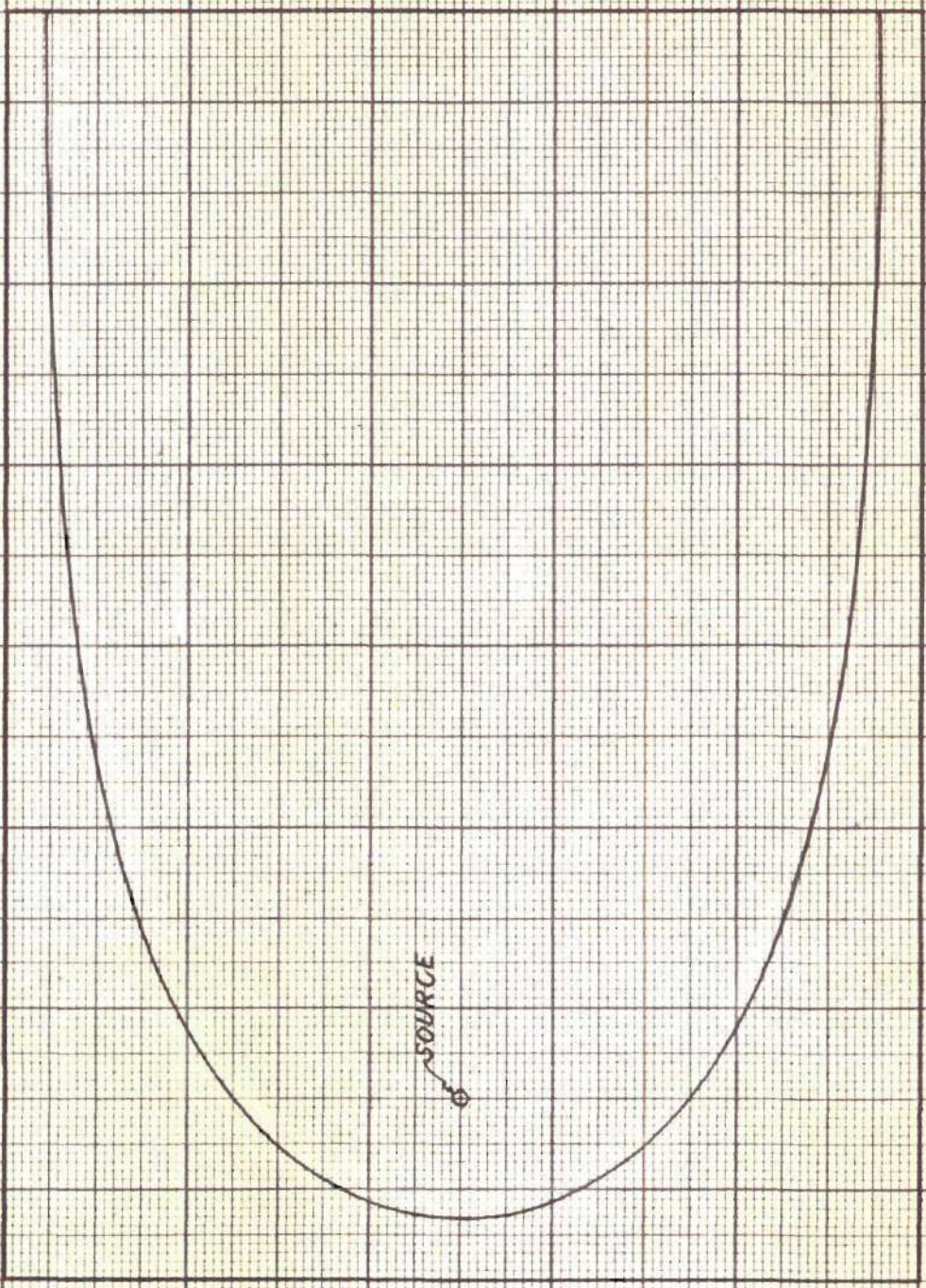


FIGURE 4
SOURCE SHAPE



DIMENSIONS ARE ARBITRARY

TABLE I

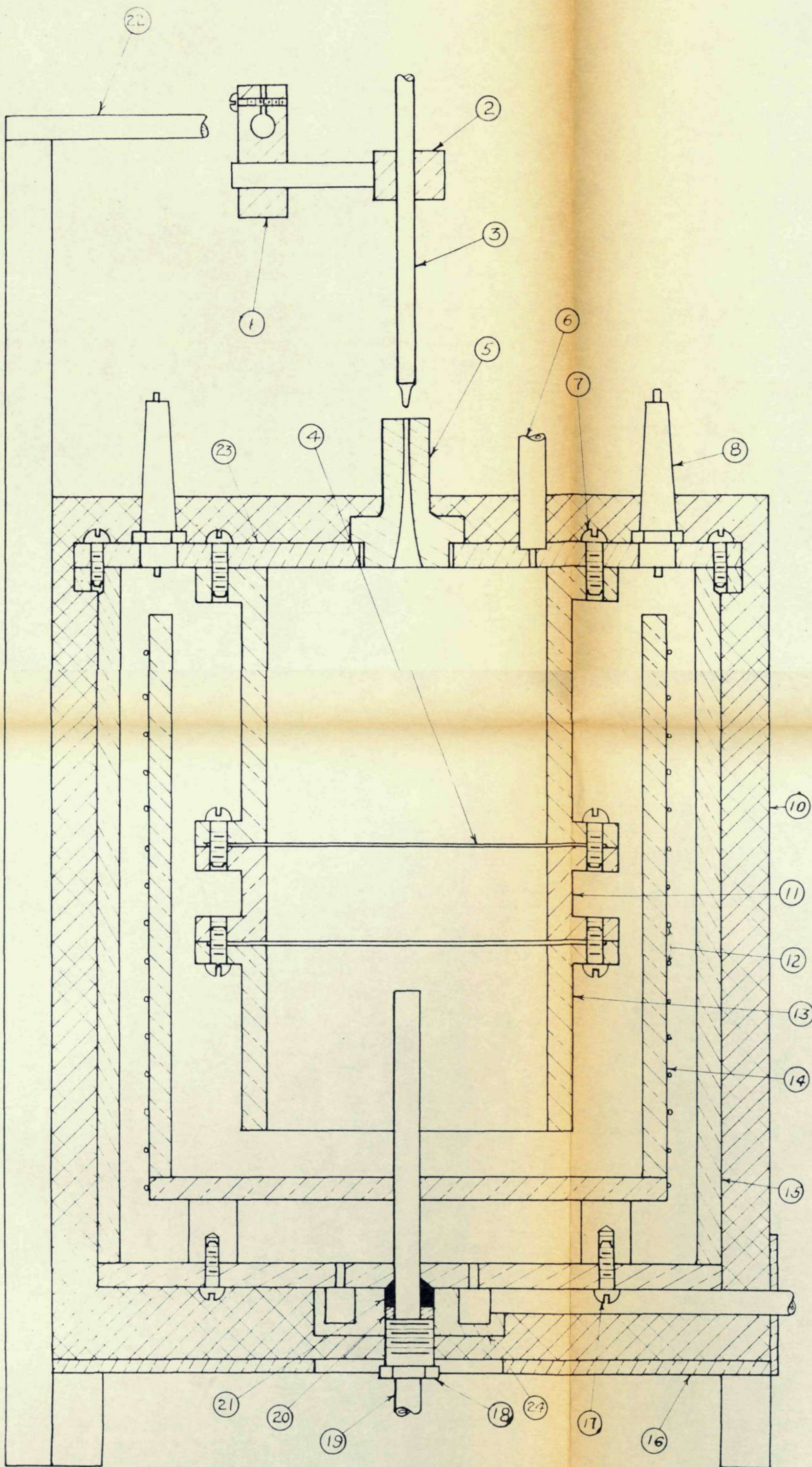
DIMENSIONS OF THE NOZZLE

Pos. inch	r_t inch	W in./sec	v/W in. x 10^6	δ inch	r_a inch
0	∞	0	∞	0	∞
.1	.07852	4050	1.025	.00166	.08018
.2	.06995	5100	.814	.00210	.07205
.3	.06538	5850	.710	.00240	.06778
.4	.06232	6440	.645	.00264	.06496
.5	.06005	6940	.598	.00284	.06289
.6	.58250	7360	.564	.00303	.06128
.7	.05667	7760	.535	.00318	.05995
.8	.05552	8100	.513	.00333	.05885
.9	.05442	8450	.491	.00345	.05787
1.0	.05350	8730	.476	.00358	.05708
1.1	.05265	9000	.461	.00370	.05635
1.2	.05189	9290	.447	.00381	.05570
1.3	.05121	9530	.435	.00392	.05515
1.4	.05058	9760	.425	.00401	.05459
1.5	.05000	10000	.415	.00424	.05424
1.6	.0500	10000	.415	.00424	.05424

TABLE II

VALUES FOR SOURCE SHAPE

y	x
0	-.33333
.1	-.30777
.2	-.27528
.3	-.21796
.4	-.12997
.5	0
.6	.19496
.7	.50854
.8	1.10112
.9	2.76993
1.0	∞



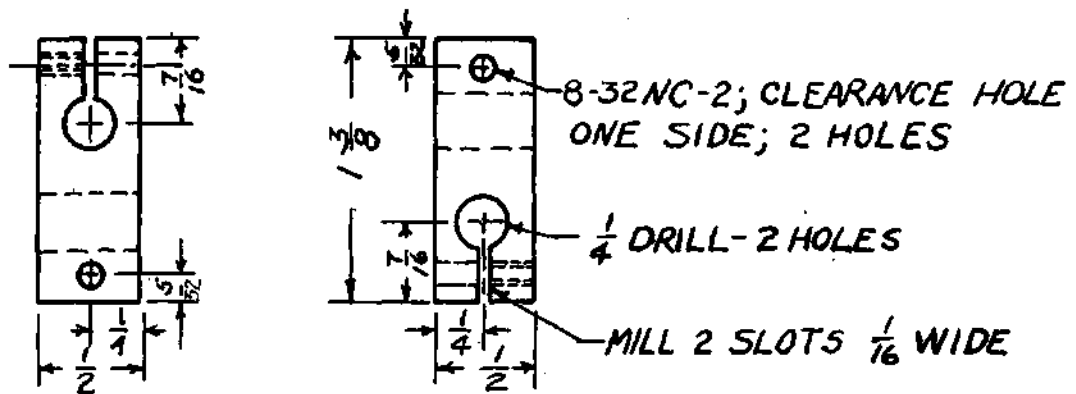
24	DELIVERY CHAMBER	BRASS	1
23	COVER	BRASS	1
22	SUPPORT	BRASS	1
21	ASBESTOS PACKING	ASB.	1
20	PACKING WASHER	BRASS	1
19	THERMOCOUPLE TUBE	COP.	1
18	PACKING SCREW	BRASS	1
17	8-32 x 1/2 RD. HD. SCREW	STEEL	
16	BASE	BRASS	1
15	BUCKET	BRASS	1
14	LINER	BRASS	1
13	LOWER INNER CHAMBER	BRASS	1
12	HEATING COILS	NICH.	1
11	CENTER INNER CHAMBER	BRASS	1
10	ASBESTOS INSULATION	ASB.	1
9	TOP INNER CHAMBER	BRASS	1
8	SPARK PLUG INSULATOR		2
7	10-32 x 1/2 RD. HD. SCREW	STEEL	
6	PRESSURE TUBE	COP.	1
5	NOZZLE	BRASS	1
4	100 MESH SCREEN	COP.	2
3	DEFECT TUBE	GLASS	1
2	CLAMP	CRS	1
1	SUPPORT BRACKET	CRS	1
MK	NAME		MATL. NO.

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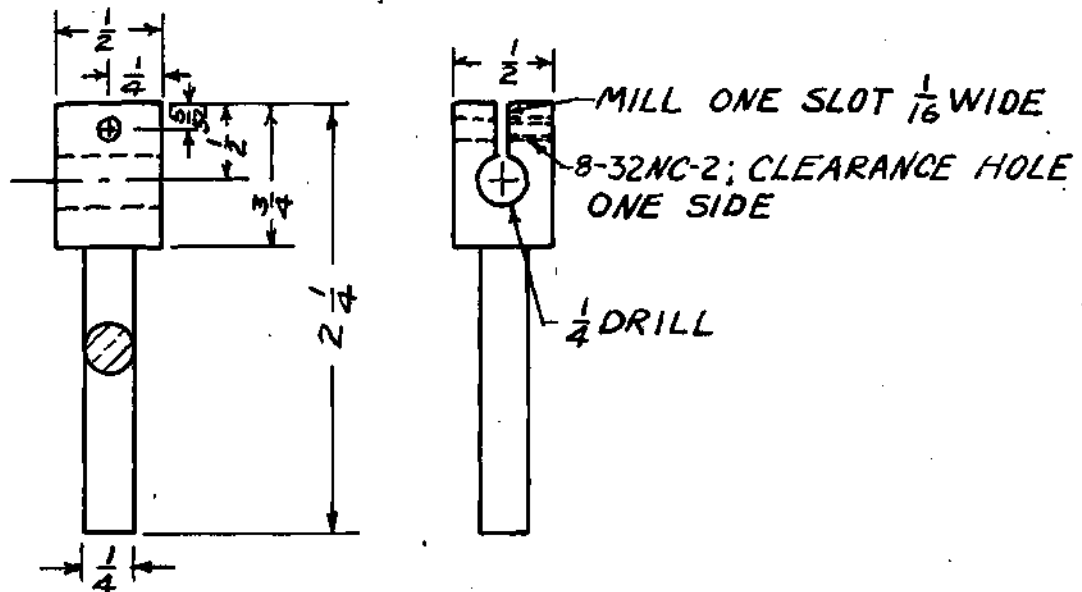
HEAD DEFECT APPARATUS ASSEMBLY

DRAWN BY C. SCOTT 8-49
TRACED BY C. SCOTT 8-49
CHECKED BY

FULL
A-200



① SUPPORT BRACKET; ONE REQ'D; C.R.S.

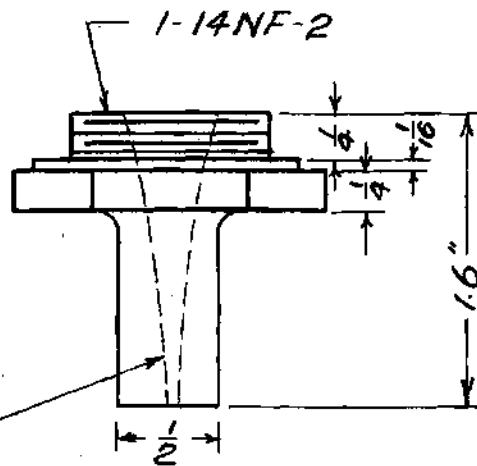
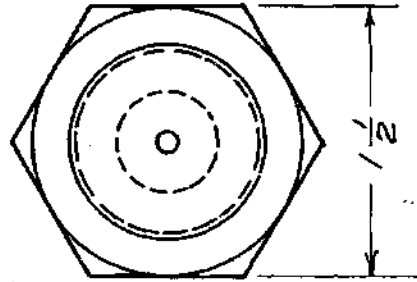


② CLAMP ; ONE REQD; C.R.S.

BREAK ALL CORNERS $\frac{1}{64}R$

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PARTS 1 AND 2		
DRAWN BY	C. SCOTT	B-49
TRACED BY	C. SCOTT	B-49
CHECKED BY		
		FULL
		D-201

⑤ NOZZLE, 1 REQD; BRASS



← SHAPE MUST BE IN ACCORDANCE WITH FIG. 3

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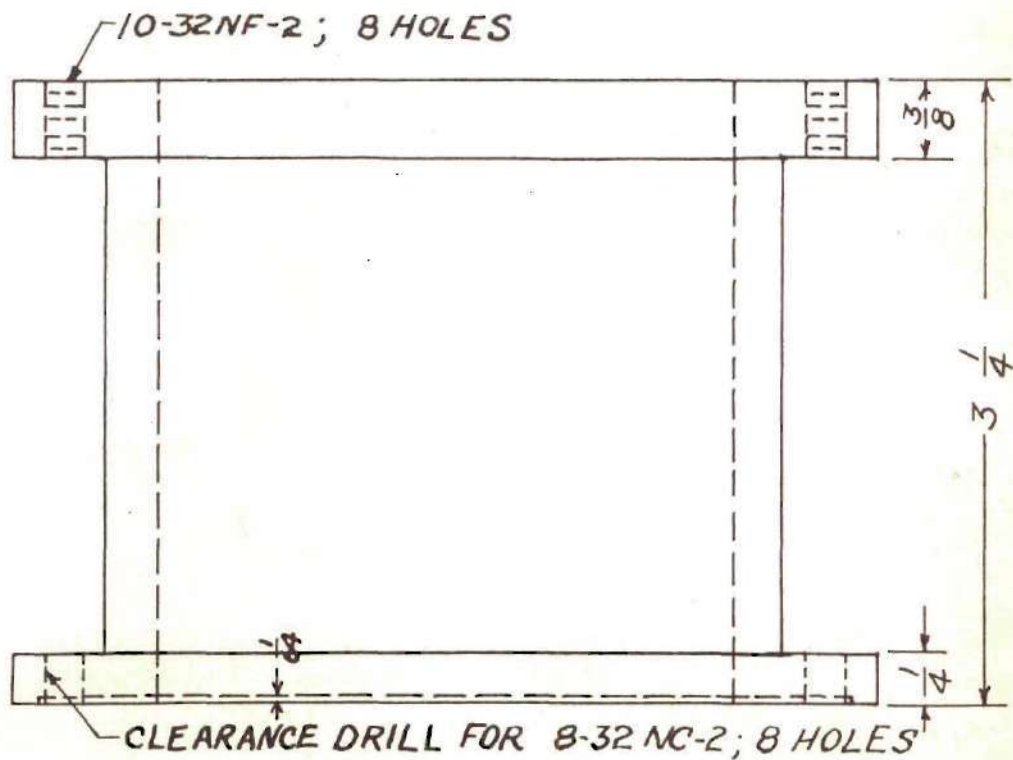
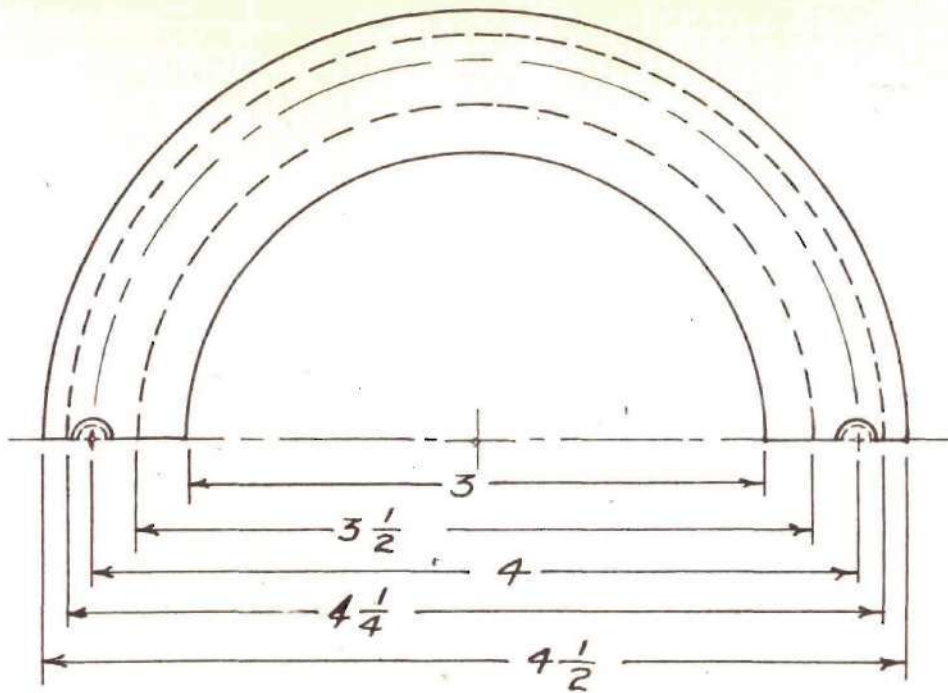
PART NO. 5

DRAWN BY C. SCOTT 8-49
TRACED BY C. SCOTT 8-49
CHECKED BY

FULL

D-205

⑨ TOP INNER CHAMBER; 1 REQD; BRASS; F.A.O.



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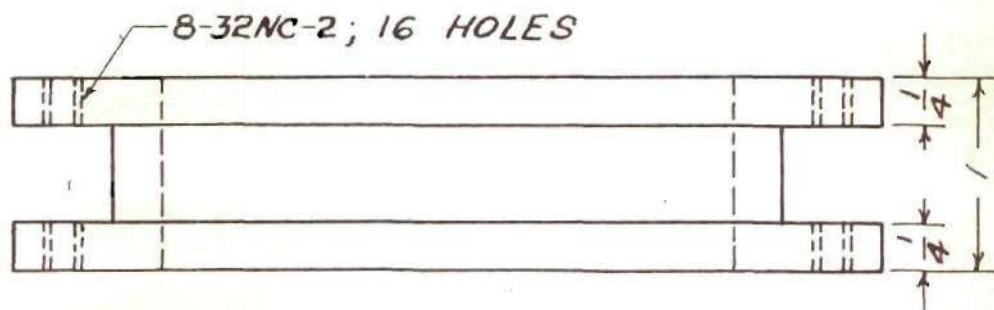
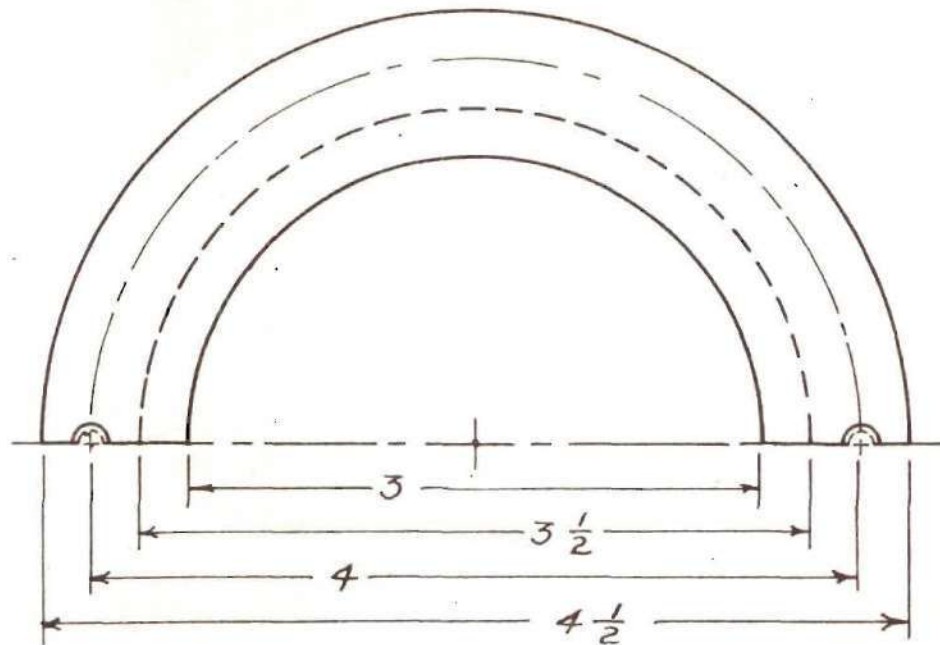
PART NO. 9

DRAWN BY C. SCOTT 8-49
TRACED BY C. SCOTT 8-49
CHECKED BY

FULL

D-209

⑪ CENTER INSIDE CHAMBER; 1 REQD; BRASS; F.A.O.



MACHINE INSIDE OF PARTS 9, 11, AND 13 AFTER ASSEMBLY

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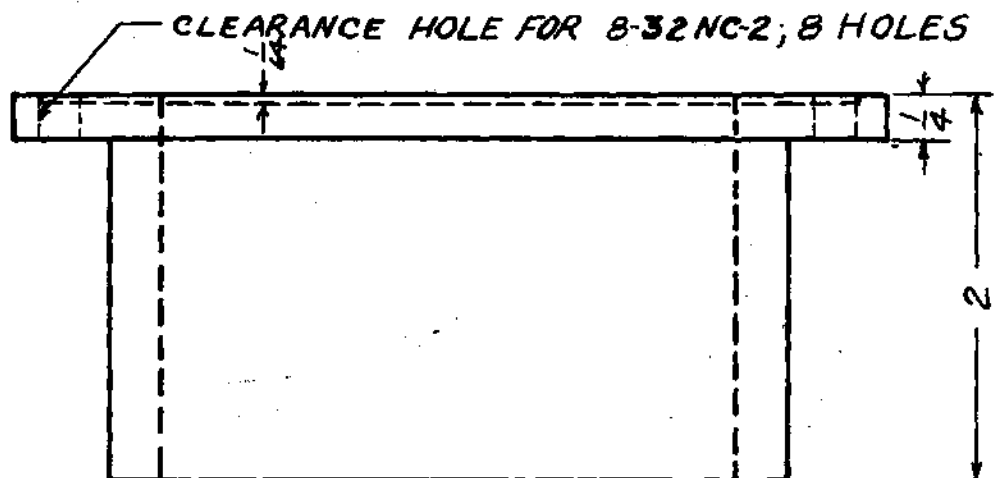
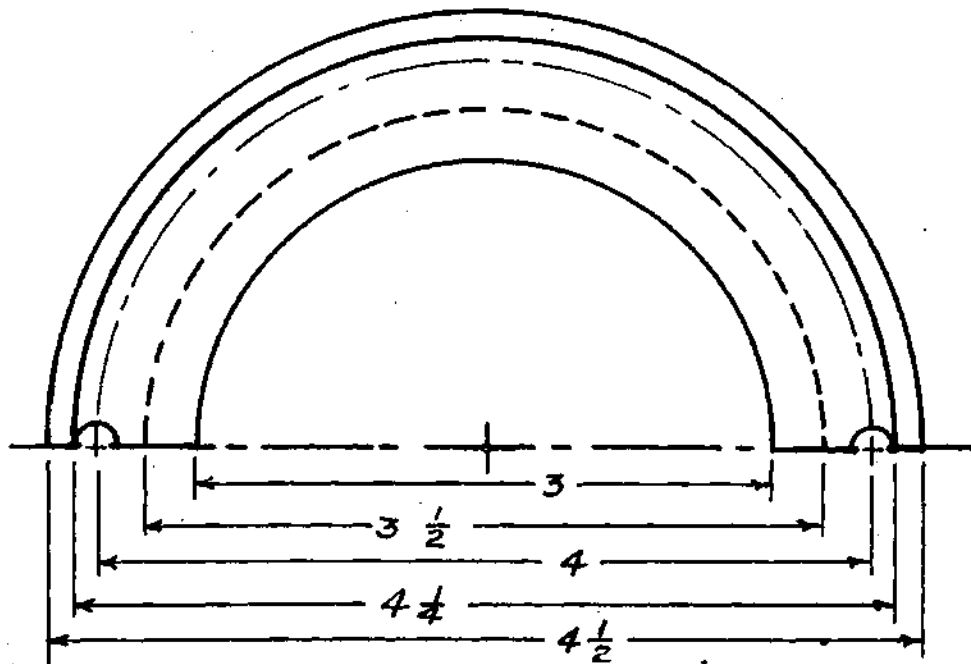
PART NO. 11

DRAWN BY C. SCOTT 8-49
TRACED BY C. SCOTT 8-49
CHECKED BY

FULL

D-211

⑬ LOWER INNER CHAMBER; 1 REQD.; BRASS; F.A.O



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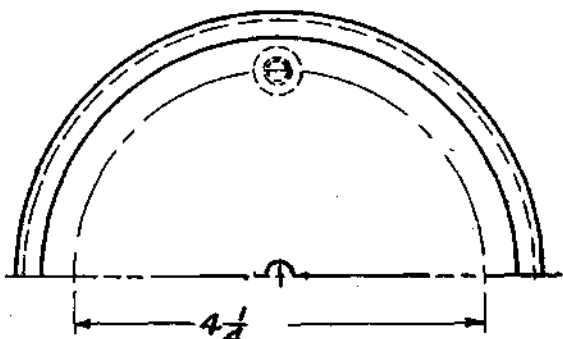
PART NO 13

DRAWN BY G. SCOTT 8-49
TRACED BY C. SCOTT 8-49
CHECKED BY

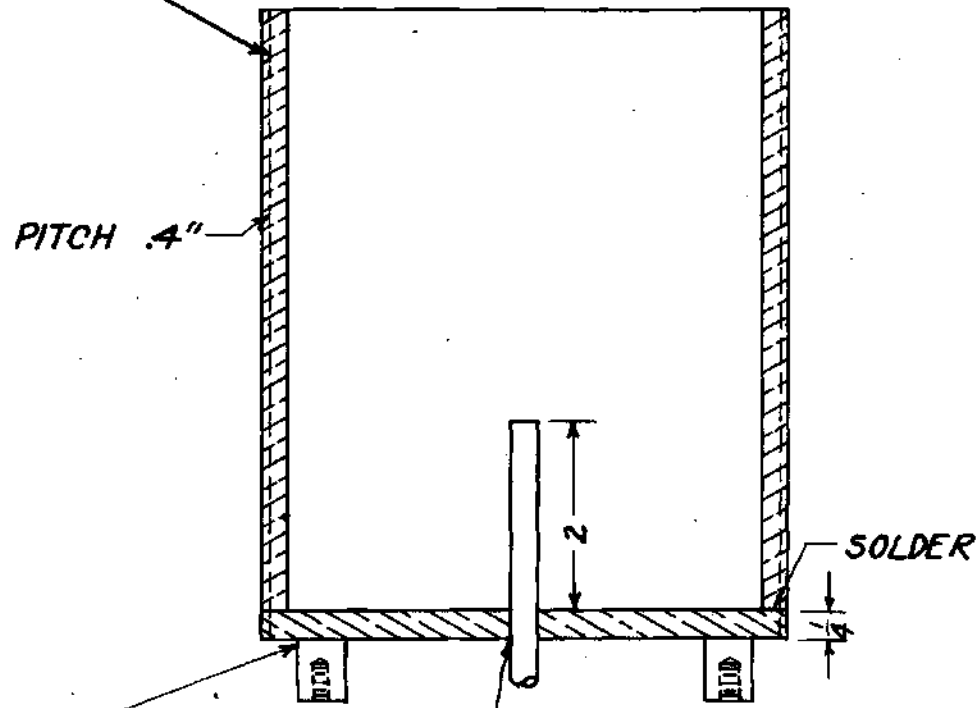
FULL

D-213

⑭ LINER; 1 REQD; BRASS, F.A.Q.



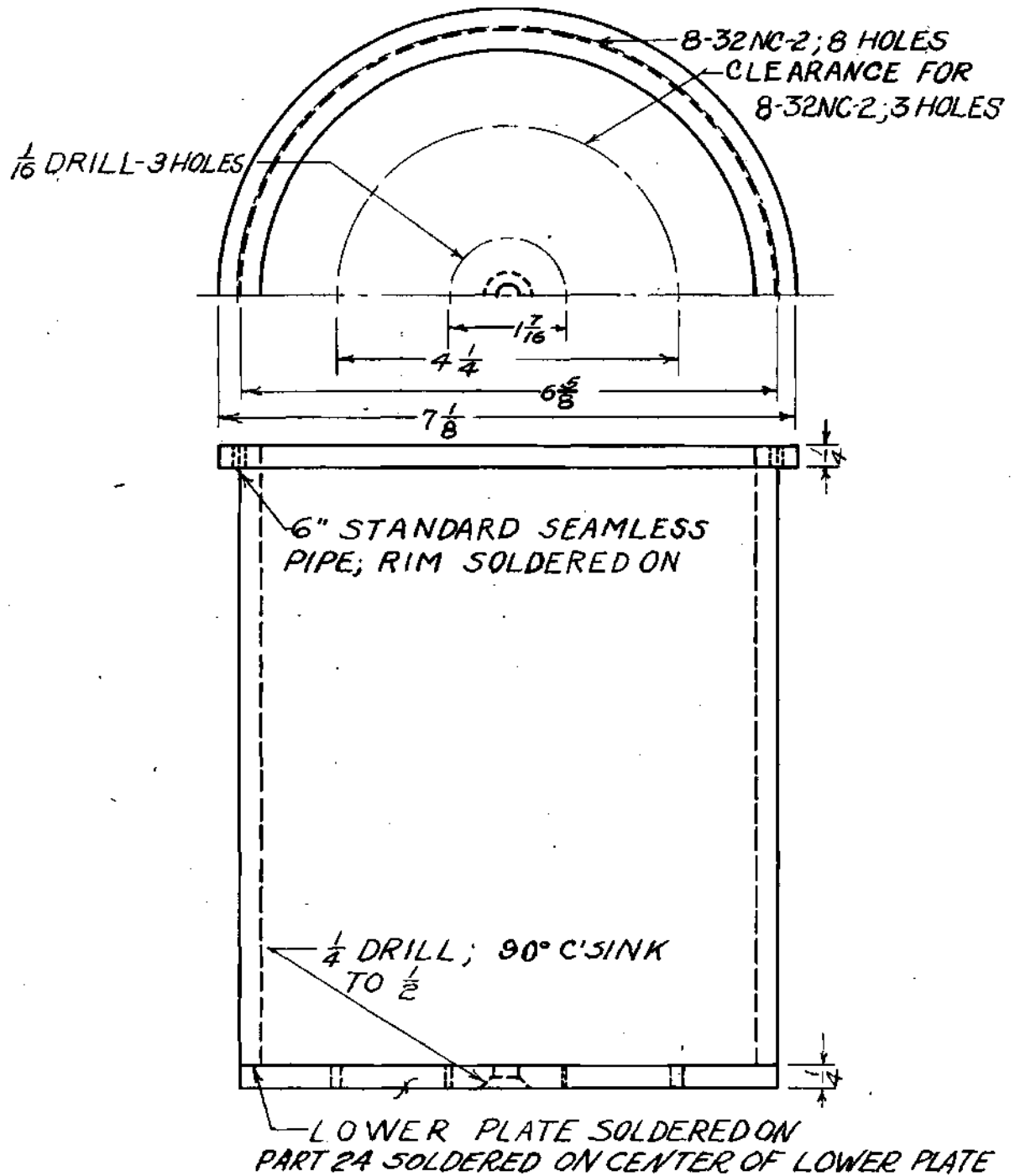
5" STANDARD SEAMLESS PIPE
 $\frac{1}{16}$ " DEEP SPIRAL GROOVE ENTIRE LENGTH



SOLDER 3- $\frac{1}{2}$ x $\frac{3}{8}$ BRASS SPACERS IN PLACE; 8-32 NC-2 TO FIT PART 15
 $\frac{1}{4}$ DRILL; SOLDER PART 19 IN PLACE; PART 19 IS $\frac{1}{4}$ x 4 $\frac{1}{2}$ COPPERTUBE

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PART NOS. 14 AND 19		
DRAWN BY C. SCOTT 8-49 TRACED BY C. SCOTT 8-49 CHECKED BY	6" = 1'	D-214

⑬ BUCKET; 1 REQD.; BRASS



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PART NO. 15

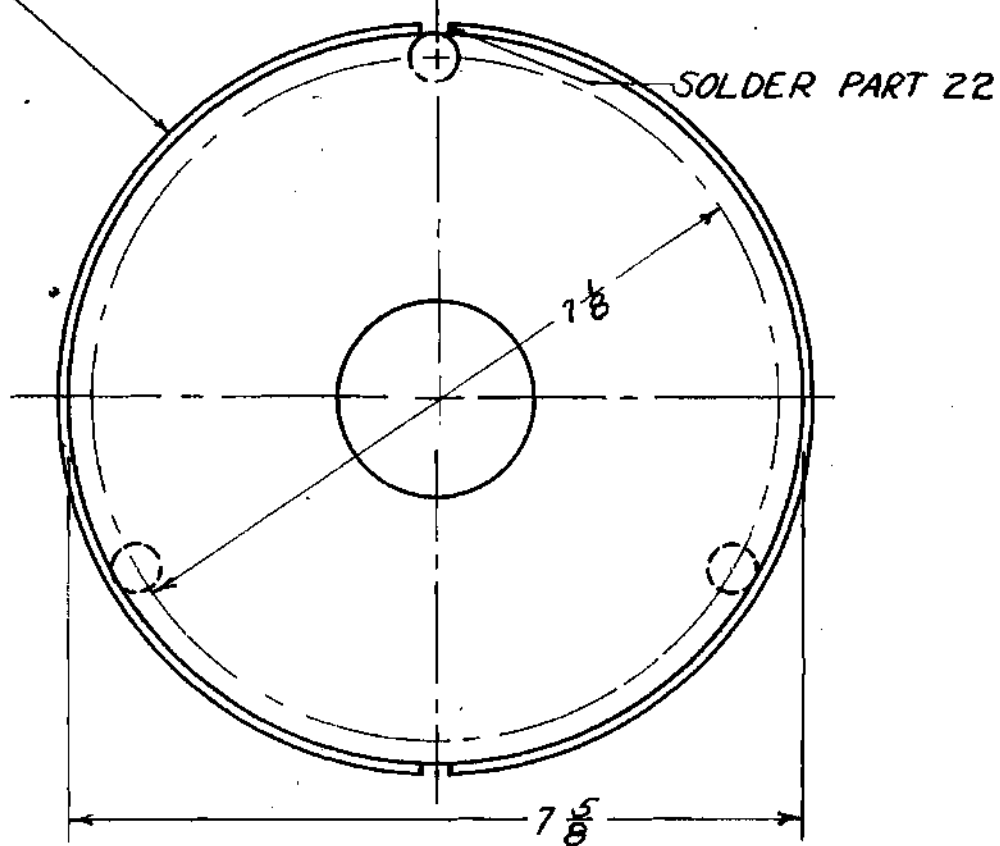
DRAWN BY C. SCOTT 8-49
 TRACED BY C. SCOTT 8-49
 CHECKED BY

6" = 1'

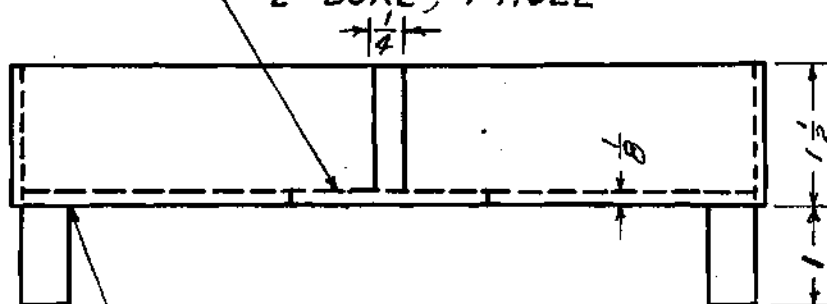
D-215

①⑥ BASE; BRASS; ONE REQD

$\frac{1}{16}$ COPPER STRIP SOLDERED ON AS SHOWN



2 BORE; 1 HOLE



SOLDER 3- $\frac{1}{2}$ x 1 BRASS LEGS

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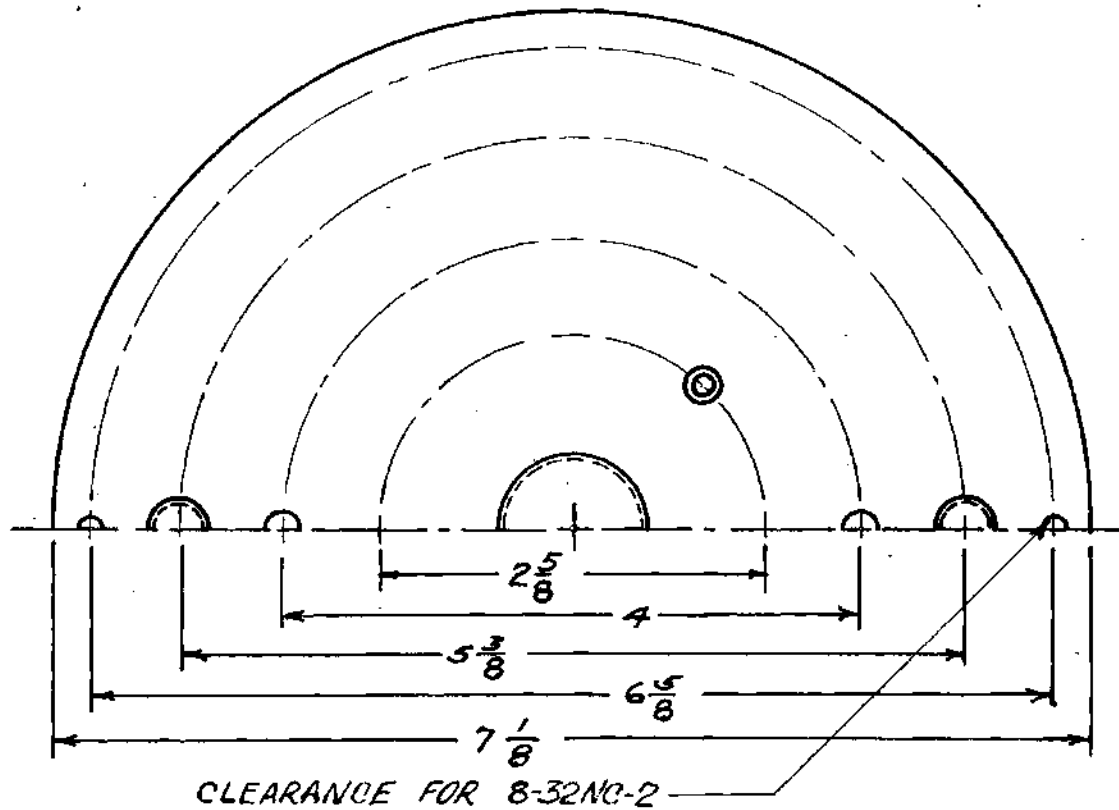
PART NO. 16

DRAWN BY C. SCOTT 8-49
TRACED BY C. SCOTT 8-49
CHECKED BY

5" = 1"

D-216

②3 COVER; 1 REQD.; BRASS; F.A.Q



14 MILLIMETER TAP FOR SPARK PLUG; 2 HOLES

1-14NF2

$\frac{1}{4}$ COUNTERBORE $\frac{1}{8}$ DEEP; $\frac{1}{16}$ DRILL CLEAR THROUGH;
SOLDER PART NO. 6 HERE WITH 50-50 SOLDER
PART NO. 6 IS $\frac{1}{4}$ X 1 HARD DRAWN COPPER TUBE

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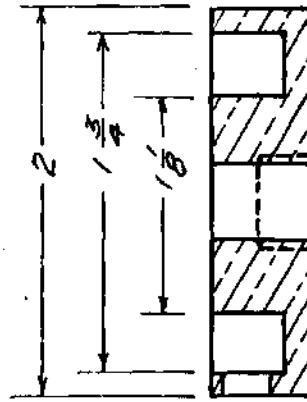
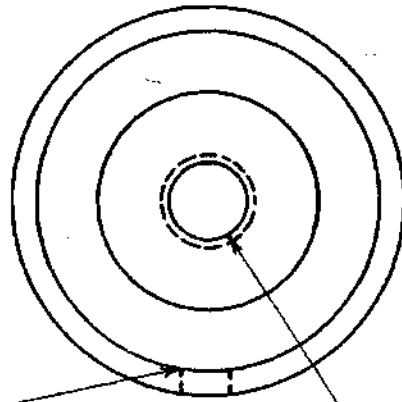
PARTS 6 AND 23

DRAWN BY C. SCOTT B-49
TRACED BY C. SCOTT B-49
CHECKED BY

9" = 1"

D-223

②④ DELIVERY CHAMBER; 1 REQD; BRASS



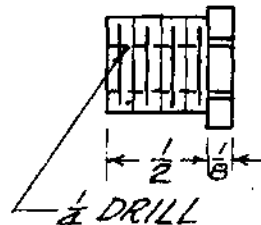
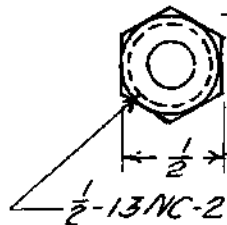
$\frac{1}{4}$ DRILL; 1 HOLE WITH $\frac{1}{4} \times 5$ COPPER TUBE SOLDERED IN. $\frac{7}{16}$ DRILL CLEAR THROUGH; $\frac{1}{2}$ -13NC-2; $\frac{5}{16}$ "

PART 20 IS A BRASS WASHER WITH $\frac{13}{32}$ O.D., $\frac{9}{32}$ I.D. AND $\frac{1}{8}$ THICK.

PART 22 IS $\frac{1}{4} \times \frac{1}{2} \times 14\frac{1}{2}$ BRASS STRIP WITH A $\frac{1}{4} \times 5$ BRASS ROD SOLDERED ON ONE END. NOTE THAT THIS PART MUST BE SOLDERED ON PART 16 SO THAT THE ROD IS $1\frac{1}{2}$ " OFF CENTER.

NOTE: ALL SOLDER USED IN FABRICATIONS IS TO BE 50-50 SILVER SOLDER.

①⑧ PACKING SCREW; 1 REQD; BRASS



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PARTS 24, 20, 22

DRAWN BY C. SCOTT B-49
TRACED BY C. SCOTT B-49
CHECKED BY

FULL

D-224

APPENDIX A

CALCULATION OF MAXIMUM HEAD DEFECT

The percentage total head defect has been defined as the ratio of $p_0 - p_2$ to $p_0 - p_1$ expressed as percent.

From equation (8) p_2 can be obtained in terms of p_0 . Any value of the ratio of p_0 to p_1 may be chosen. In order to be definite, select $p_0/p_1 = 1.531$, or $p_1 = .653p_0$, and a temperature of 100.9°F .

Since CO_2 is a linear molecule, the value of C_p' is $3.5R$ from earlier parts of this report. To find C_p , the formulas of Sweigert and Beardsley (Reference 24) are used.

$$\frac{c_p}{R} = \frac{1}{MJ} \left(16.2 - \frac{6.53 \times 10^6}{T} + \frac{1.41 \times 10^6}{T^2} \right)$$

where M is molecular weight and J is the mechanical equivalent of heat.

$$c_p \approx 4.55 R$$

Therefore

$$c_{\text{vib}} = 1.05 R$$

By substitution of the isentropic relationship

$$\left(\frac{T_0}{T_1} \right) = \left(\frac{p_0}{p_1} \right)^{R/c_p}$$

in equation (8) yields

$$\frac{p_0}{p_2} = \left[\frac{\frac{c_p}{c_{vib}} - 1}{\frac{c_p}{c_{vib}} - \left(\frac{p_1}{p_0}\right)^{R/c_p}} \right] \frac{c_p}{R} \left(\frac{p_0}{p_1}\right)^{\frac{c_{vib}}{c_p}}$$

From this equation with the values assumed and determined

$$\frac{p_0}{p_2} = 1.00608$$

or

$$p_2 = .994p_0$$

The total head defect is

$$\frac{p_0 - p_2}{p_0 - p_1} = \frac{p_0 - .994p_0}{p_0 - .653p_0} = 1.73 \%$$

In making the calculation of $\frac{p_0}{p_2}$, accuracy is best obtained by

first dividing through the complex fraction by its numerator. This division is made possible by adding and subtracting 1 in the denominator. The expression is then reduced to the form $\frac{1}{1-x}$ which may be expanded in a power series which converges rapidly. The second order term produces accuracy in the fourth decimal.