DESIGN OF MODEL REFERENCE ADAPTIVE CONTROL SYSTEMS USING LIAPUNOV THEORY

A THESIS

Presented to

The Faculty of the Graduate Division

by

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In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

in the School of Electrical Engineering

Georgia Institute of Technology

November, 1969

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Approved:

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Date approved by Chairman: December 1, 1969

ACKNOWLEDGMENTS

I wish to express my sincere appreciation to Dr. Carter B. Norman of Minden, Louisiana. His continual insistence over a period of several years influenced my decision to work toward this degree. I am deeply grateful to Dr. James R. Rowland of the Georgia Institute of Technology who served as my thesis advisor. His advice, his guidance, and his limitless optimism greatly lightened the burden of work which culminated in this thesis. I also wish to thank Dr. Roger P. Webb and Dr. Thomas M. White, both of Georgia Tech, who served as members of my reading committee.

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SUMMARY

This thesis presents several results on the design of model reference adaptive control systems. Such systems contain a plant, which is to be controlled, and a reference model, which is a computer representation of the desired dynamic response of the overall system. The objective of the design is to synthesize a control signal for the plant. The control signal forces the output of the plant to be identical to the output of the reference model. Furthermore, errors in the overall system obey a Liapunov stability theorem which guarantees that all errors approach zero asymptotically.

By considering the internal states of the plant and the model only as they affect the outputs, a control policy is derived which allows the plant and the model to be of different orders. Furthermore, the plant and model need not be linear, time-invariant, or continuous. They need not even be of the same form, e.g. both time-varying, except that both must be either continuous or discrete. The plant may have incompletely specified parameters which lie within known bounds. The obtained results apply both to single-input, single-output systems and to multiple-input, multiple-output systems. In the latter case, it is required that the plant and the model have the same number of outputs. In the case of multiple-input, multiple-output plant and models, it is possible to eliminate cross-coupling effects in the final system by choosing a non-interacting model. Examples are

presented throughout the thesis to illustrate the application of the various results.

CHAPTER I

INTRODUCTION TO THE BASIC ADAPTIVE CONTROL PROBLEM

In recent years as control system environments have been expanded by space age technology, it has become increasingly difficult to satisfy the design requirements which have been imposed on practical control systems. In many fixed-parameter problems a satisfactory compromise among the various design constraints can be reached. In those cases where the system mission is well known beforehand, it is usually advantageous to have programmed time variations of the controller parameters. However, sometimes no compromise is possible which will result in an acceptable fixed-parameter system. In many of these cases programmed adjustments cannot be made because of insufficient knowledge relating system performance to time. The existence of such problems has motivated adaptive control system development.

One of the more important types of adaptive control systems is the model reference system. An integral part of this system is the computer model of the desired dynamic response of the plant. The design goal is to develop a control strategy which causes the plant to track the model as closely as possible. The problem differs from the optimum tracking problem in three important ways:

1. In the adaptive control system there are usually parameters which vary in some unknown manner.

2. The input to the adaptive control system is not usually considered to be known a priori.

3. Instantaneous error, rather than a time integral of error, is often the quantity to be minimized in the adaptive problem.

The objective of this research was to develop a design technique for model reference adaptive control systems. The techniques developed depended upon synthesizing a feedback control signal for the plant. The controller was designed so that the system errors satisfied the conditions of a Liapunov function. Because of this requirement, any errors in the system were guaranteed to approach zero asymptotically. In addition, the technique allows the use of much broader classes of plants and models than any technique previously reported in the literature.

General Description of the Adaptive Control Literature

The term "adaptive" has been used to describe a wide variety of control systems [1,2,3,4,5]. In this thesis a control system is considered to be adaptive if the controller monitors the performance characteristics of the system and then uses this information to modify the control action to make the overall system perform acceptably. Most of the important adaptive control systems can be grouped into the following three categories:

1. High-gain schemes
2. Optimal-adaptive schemes
3. Model reference schemes.

The essence of the high-gain scheme [6,7] is that the gain in the feedback loop around the changing process is kept as high as possible. Keeping the gain high holds the input-output transference close to unity. Because stability problems arise, the signal in the control loop is monitored for oscillations. The loop gain is then continuously adjusted to keep the system on the verge of instability. A response close to that of a particular model is obtained by placing the model in front of the feedback loop. The main objection to the high-gain approach is that small oscillations are always present in the loop. However, the technique has been used in the autopilot of the F101-A and the X-15 aircraft with very satisfactory results [6].

Optimal-adaptive methods [8,9,10] are also used to solve some types of optimization problems. A control signal is synthesized on the assumption that the plant parameters and the states are known. However these quantities are unknown to some extent in systems requiring an adaptive controller. For this reason state estimation and plant identification are essential parts of the total optimal-adaptive method. For optimization it is also necessary that the input be known for the duration of the control operation. When the input is not known a priori, a predictor must be provided. Although much work has been done in each of the separate areas involved in this method, only a few practical optimal-adaptive techniques have been reported [11,12].

The third important category of adaptive control systems is the model reference scheme. Figure 1 shows a basic block diagram which encompasses most of the current model reference systems.

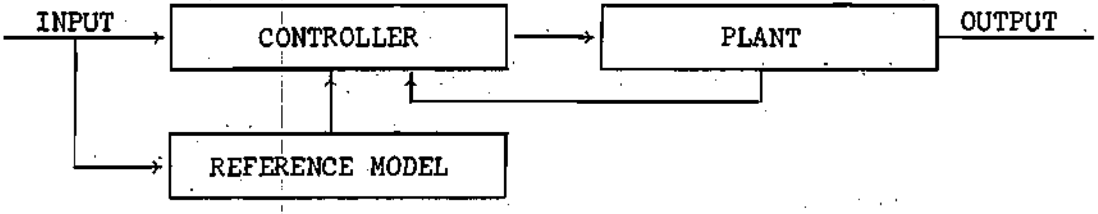

Figure 1. A Generalized Model Reference Adaptive Control System

The reference model is a computer representation of the desired dynamic response of the overall system. The controller compares the output of the plant with the output of the reference model. Using this knowledge, the controller then adapts itself to force the input-output transference of the controller and plant combination to closely match that of the reference model. It is convenient to divide the model reference adaptive techniques into three categories:

1. Parameter adjustment techniques
2. Parameter perturbation techniques
3. Control signal synthesis techniques.

In the parameter adjustment techniques [3,13,14] the input to the reference model is the same as the input to the plant. The model output is subtracted from the plant output to form an error signal. This error signal serves as the input to a mechanism which adjusts the parameters of the various compensation elements in the plant to reduce the error. In other words when changes occur in the plant, the control system adapts the values of the compensation elements to keep the overall control system response unchanged. Donalson [15] and

Margolis [3] adjust parameters using gradient techniques. Dressler [13], Parks [14], and Rang [16] use Liapunov stability theory to aid in the design of an adjustment policy. At the present state of development, the form selected for the reference model must be compatible with the basic form of the physical plant and the available compensation. Furthermore, for any values the parameters in the plant may assume, proper adjustment of the compensation elements must make the differential equation of the overall control system identical to that of the reference model. In the absence of an input, sizable errors between the actual values of the adjustable parameters and their desired values can result. If the parameter adjustment is slow after such errors build up, the control system output may deviate considerably from the desired output. However, simulation of simple systems [6] has shown that parameter adjustment can take place within 10 per cent of the basic time constant of the reference model.

To implement a parameter perturbation technique [17,18,19], an even function of error, such as e^2 or $|e|$, is generated. One parameter in the system is defined by

$$b = b_o + b_c + b_p \sin \omega t \tag{1.1}$$

where b_o is the basic system parameter, b_c is the correction applied by the adaptive portion of the system, and $b_p \sin \omega t$ is a sinusoidal perturbation signal used to create a continuous error. Since the parameter b is being perturbed in a sinusoidal manner, the error e will also exhibit an oscillatory component of the same frequency. The amplitude and sign of b_c are then chosen to reduce a short time average of the error function. McGrath [17] showed that it is possible to adjust a number of parameters simultaneously by applying perturbation signals at different frequencies to the parameters to be adjusted. If the plant and the model are of the same form, the perturbing signals may be applied to the model parameters [18]. A primary objection to the parameter perturbation technique is that, except when the plant and the model are of the same form, the perturbation signal must appear in the output of the system. A problem shared by both the parameter adjustment method and the parameter perturbation method is that if there are many parameters to be controlled simultaneously, the controller design becomes very complex and stability is difficult, if not impossible, to guarantee.

In the signal synthesis technique, the input signal is applied to both the reference model and the controller. The controller also has access to the outputs of the plant and the outputs of the reference model. On the basis of this knowledge, the controller produces a control signal which is applied to the plant. The purpose of this control signal is to cause the output of the plant to be the same as the output of the reference model. While the objective is easy to understand in these general terms, the design of the controller and the model are in general quite difficult.

Specific Background

There are in the literature five techniques which form the direct background for this thesis: one each by Rang [16], Hiza and

Li [21], and Monopoli [22], and two by Grayson [20]. All used Liapunov theory to design model reference adaptive control systems of the signal synthesis type.

Each investigator assumes a linear plant of the form

$$\dot{\underline{x}} = A\underline{x} + Bu \tag{1.2}$$

where $\underline{x}$ is an n-dimensional vector of plant states, A is an n by n constant matrix, B is an n by k constant matrix, and u is the k-dimensional plant input vector.

Rang's method and Grayson's second method allow the plant input u to be a vector, but results can be obtained only in special cases. The other three techniques consider a scalar u. Rang's method, which includes parameter identification as an integral part of the method, requires that A and B be fixed but unknown matrices. The other techniques assume time-varying matrices which have known bounds on the variations of each element.

In each technique a linear time-invariant single-input model of the following form was chosen:

$$\dot{\underline{y}} = F\underline{y} + \underline{G}r \tag{1.3}$$

where $\underline{y}$ is an m-dimensional vector of model states, F is an m by m constant matrix, $\underline{G}$ is a constant m-vector, and r is the scalar model input.

Hiza and Li require that the model be of order one less than the order of the plant. In all the other techniques the plant and the model are required to be of the same order.

The quadratic Liapunov function was defined as

$$V = \underline{e}^T P \underline{e} \tag{1.4}$$

where

$$\underline{e} = \underline{x} - \tilde{\underline{y}} \tag{1.5}$$

and P is a square matrix used to assign weights to each error product. The vector $\tilde{\underline{y}}$ is identical to $\underline{y}$ in all the techniques except that of Hiza and Li, who require that $\tilde{\underline{y}}$ be given by

$$\tilde{\underline{y}} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n-1} \\ \dot{y}_{n-1} \end{bmatrix} \tag{1.6}$$

By choosing the quadratic Liapunov function in (1.4), each of these authors was attempting to control n independent states by using one control variable u. They were, of course, unable to achieve this objective in general. Furthermore, the restrictions and simplifications necessary to arrive at a presentable result caused their systems to be of little practical value. The most confining requirements were that the plant and the model be of the same order, that the plant and the model be linear, and that the model be time-invariant. In addition, each author has other specific restrictions on the use of his method.

Rang [16] considered the plant input u to be a vector. However, his technique cannot generally be used unless either the individual components of u are independent or the product Bu is nonzero only in the nth position. An analog computer simulation of a second order system showed that the adaptive properties of the overall system were relatively slow.

Grayson's first method [20] requires that each of the time-varying elements of the A and B matrices be sign definite. Furthermore, the matrices A and F are required to be of the same form, as are the matrices B and G. In a second order example which Grayson presented, it was required that the input r and its derivative $\dot{r}$ both have known bounds. However, the vector input in Grayson's second technique cannot be handled unless the matrix B(t) is fixed, known, and nonsingular. Even the case of a scalar plant input can be handled in general only if there are no zeros in the numerator of the plant transfer function.

Only Hiza and Li [21] allow the plant and model to be of different orders. However, they permit only step inputs to the model, and require that the static gain of both the plant and the model be unity. Monopoli [22] made the restriction that his plant must have no numerator dynamics. Even so, his technique is still so complicated that the calculations for systems higher than second order are prohibitive.

While the signal synthesis technique offers a clever approach to the model reference adaptive control problem, the known results have been obtained at the expense of stringent requirements on the form of the plant, the model, and the model input. The basic difficulty with the above methods is that the authors consider each of the internal states of the plant and the model as a separate output. A separate error was defined between each plant state and the corresponding model state, thereby forcing the plant and the model to be of the same order. The results presented in this thesis eliminate many of the restrictions which the previous authors found necessary to impose, and therefore apply to much broader classes of systems.

Basic Design Philosophy

In this research the model reference adaptive control problem was considered from a more basic viewpoint than had been taken previously. The precise objective was that the output of the plant must follow the output of the model. The quantities which were of concern, therefore, were the outputs of the plant and the model and not their internal states. Once it was realized that the internal states are irrelevant except as they affect the outputs, it became possible to use Liapunov stability theory for designing broad classes of model reference adaptive control systems. A plant could follow any model which had the same number of outputs as the plant, regardless of the order of the plant or the model. It was no longer necessary to require that the model be time-invariant, or that the plant and the model be linear. Restrictions on the form of the system input were unnecessary, and no a priori knowledge of the input was needed. Thus it was shown that the signal synthesis technique could be used as a practical design procedure.

Organization of the Thesis

Following this introductory material, Chapter II begins with a derivation of the theory for the simplest case: a single-input, single-output system with a fixed linear plant and a fixed linear model, which are not necessarily of the same order. The technique is then extended to permit time varying plants and models, as well as special types of plant and model nonlinearities. In Chapter III, a more elaborate Liapunov function is used to expand the class of allowable plants. In addition, it is shown that proper use of this more complex Liapunov function will lead to improvements in systems of the type considered in the second chapter.

The extension of the theory to encompass multiple-input multiple-output systems is presented in Chapter IV. It is shown that the techniques of this chapter may be applied to yield the same effect as decoupling. The modifications necessary to handle sampled data systems are developed in Chapter V.

Finally, Chapter VI concludes the thesis with a qualitative discussion of the usefulness and limitations of the techniques which are set forth in the first five chapters. Recommendations for future research in this area are presented and discussed.

CHAPTER II

THE BASIC SYNTHESIS SOLUTION

The purpose of this research was to develop a practical design technique for model reference adaptive control systems. To achieve this objective, the following goals were established. The resulting system must handle a wide range of inputs, which need not be known a priori. Broad classes of plants and models must be permitted, including plants which have incompletely specified parameters. The plant must be able to follow models of different orders. In addition, the form of the model must be independent of the form of the plant. The resulting system must be stable, even when the plant itself is unstable.

A signal synthesis technique was developed to meet the above objectives. Liapunov theory was utilized in the design and the resulting system was guaranteed to be asymptotically stable.*

Mathematical Formulation of the General Problem

The general configuration of the system which was considered is shown in Figure 2. The plant in Figure 2 is nth order and in general is nonlinear and time-varying. Moreover, the inputs enter linearly and

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*In this dissertation, the term "asymptotically stable system" indicates only that the difference in the plant and model outputs approaches zero asymptotically.

the outputs are linear combinations of the states; i.e.

$$\dot{\underline{x}} = \underline{f}(\underline{x},t) + B(t)\underline{u} \tag{2.1}$$

$$\underline{z} = C^T(t)\underline{x}$$

In (2.1), $\underline{f}(\underline{x},t)$ is an n-dimensional vector whose components are nonlinear functions of the state vector $\underline{x}$ and time t. The matrix B(t) is an n by k time-varying matrix, where k is the number of plant inputs. The matrix C(t) is an n by ℓ time-varying matrix where ℓ is the number of plant outputs.

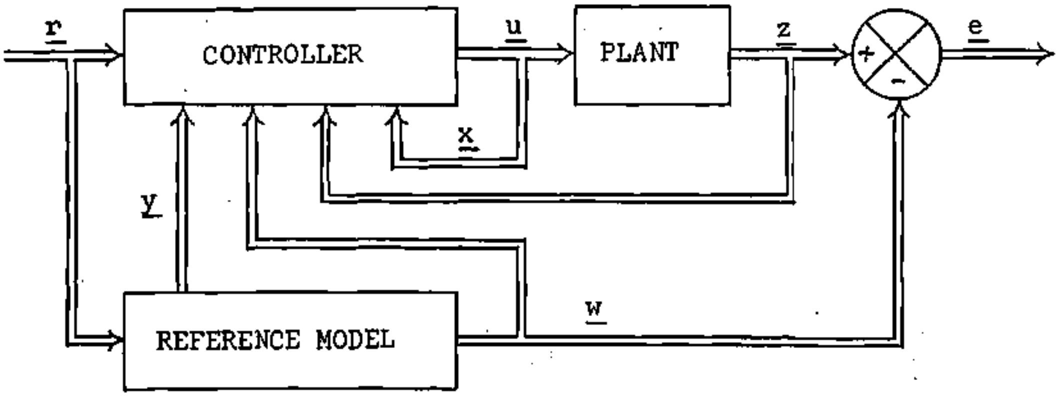

Figure 2. A Signal Synthesis Type of Model Reference Adaptive Control System

The following plant properties were assumed in this thesis:

1. All the states $\underline{x}$ as well as all the outputs $\underline{z}$ are accessible.

2. The number of plant outputs is less than or equal to the number of plant inputs, i.e. $\ell \le k$.

3. Upper and lower bounds are known for each incompletely specified parameter in (2.1).

4. An upper bound is known, when necessary, for the absolute value of each of the derivatives of incompletely specified parameters which are time-varying.

An mth order model was assumed, where m is not necessarily equal to n. The model can in general be time-varying and nonlinear. The outputs of the model are linear combinations of the states. It was required that the model have the same number of outputs as the plant.

In vector-matrix notation the most general allowable model differential equation may be written as

$$\dot{\underline{y}} = \underline{g}(\underline{y},\underline{r},t) \tag{2.2}$$

$$\underline{w} = H^T(t)\underline{y}$$

In (2.2), $\underline{g}(\underline{y},\underline{r},t)$ is an m-dimensional vector with each component a nonlinear function of the state vector $\underline{y}$, the input vector $\underline{r}$, and time. The matrix H(t) is an m by j time-varying matrix, where j is the number of model outputs. It was assumed that the model was completely known and that the model input and all the states of the model were accessible.

The synthesis problem was to use all available knowledge to determine a vector $\underline{u}$ which would cause $\underline{z}$ to follow $\underline{w}$, i.e., to find a $\underline{u}$,

$$\underline{u} = \underline{u}(\underline{x},\underline{f},B(t),C(t),\underline{z},\underline{y},\underline{g},H(t),\underline{w},\underline{r},t) \tag{2.3}$$

such that $\lim_{t\to\infty}(\underline{z}-\underline{w}) = \underline{0}$.

Liapunov's Stability Theorems

The theorems in this section [23] are fundamental to Liapunov theory. They provide a theoretical basis for the design procedures which are developed in this thesis. These theorems are stated in terms of the dynamical system

$$\dot{\underline{e}} = \underline{h}(\underline{e},t) \tag{2.4}$$

The equilibrium state for the system of (2.4) comes from

$$\underline{h}(\underline{e},t) = \underline{0} \tag{2.5}$$

which is satisfied by $\underline{e} = \underline{0}$.

Theorem 1

If there exists a real-valued continuous function $V(\underline{e})$ with the following properties:

1. $V(\underline{e})$ has continuous first partial derivatives
2. $V(\underline{e})$ is positive definite
3. $\lim_{|e_i|\to\infty} V(\underline{e}) = \infty$ for all e_i
4. There exists some region including the origin in the state space in which $\dot{V}(\underline{x}) \le 0$,

then the equilibrium state $\underline{e} = \underline{0}$ is stable in the sense of Liapunov.

Theorem 2

If there exists a real-valued continuous scalar function $V(\underline{e})$ which satisfies conditions 1, 2, and 3 of Theorem 1, and in some region including the equilibrium state the condition $\dot{V}(\underline{e}) < 0$ is satisfied, then the equilibrium state $\underline{e} = \underline{0}$ is asymptotically stable in the sense of Liapunov.

Theorem 3

If there exists a real-valued continuous scalar function $V(\underline{e})$ which satisfies Theorem 1 and, in addition, the curve $\dot{V}(\underline{e}) = 0$ is not a trajectory of the system, then the equilibrium state $\underline{e} = \underline{0}$ is asymptotically stable in the sense of Liapunov.

Throughout the remainder of this thesis, Theorem 2 is the one used when it is not specifically stated otherwise.

The Simplest Case

Before examining a solution for the most general case, it is instructive to examine certain special cases. This approach permits the technique, as well as the implications of the assumptions in the previous section, to be more readily understood.

Consider a plant that is nth order, linear, time-invariant, and has a single input and a single output. The plant differential equation may be written in the form

$$\dot{\underline{x}} = A\underline{x} + \underline{B}u \qquad (2.6)$$

$$z = \underline{C}^T\underline{x}$$

The single-input, single-output model is chosen to be linear, time-invariant, and mth order, with m not necessarily equal to n. It can be represented in vector-matrix form as

$$\dot{\underline{y}} = F\underline{y} + \underline{G}r \qquad (2.7)$$

$$w = \underline{H}^T\underline{y}$$

Define the error e to be the difference between the plant output and the model output:

$$e = z - w \qquad (2.8)$$

Choose for a Liapunov function the simplest quadratic form:

$$V = \frac{1}{2} e^2 \qquad (2.9)$$

The time derivative is

$$\dot{V} = \frac{dV}{dt} = e\dot{e} = (z-w)(\dot{z}-\dot{w}) \qquad (2.10)$$

The terms $\dot{z}$ and $\dot{w}$ are calculated as

$$\dot{z} = \underline{C}^T\dot{\underline{x}} = \underline{C}^TA\underline{x} + \underline{C}^T\underline{B}u \qquad (2.11)$$

$$\dot{w} = \underline{H}^T\dot{\underline{y}} = \underline{H}^TF\underline{y} + \underline{H}^T\underline{G}r$$

Thus, in terms of the states, $\dot{V}$ is expressed as

$$\dot{V} = (\underline{C}^T\underline{x}-\underline{H}^T\underline{y})(\underline{C}^TA\underline{x} + \underline{C}^T\underline{B}u - \underline{H}^TF\underline{y} - \underline{H}^T\underline{G}r) \tag{2.12}$$

The Liapunov function V is a positive definite function of the error e. Its time derivative $\dot{V}$ can be made negative definite by controlling the sign of $\dot{e}$. By Theorem 2 the system is guaranteed to be asymptotically stable in any region including the origin in which $\dot{e}$ can be so controlled. If this region is the whole space, then the system is globally asymptotically stable.

Since

$$\dot{V} = e\dot{e} \tag{2.10}$$

to maintain $\dot{V} < 0$ when $e > 0$, it is necessary to have $\dot{e} < 0$. In terms of the states this requirement becomes

$$\underline{C}^TA\underline{x} + \underline{C}^T\underline{B}u - \underline{H}^TF\underline{y} - \underline{H}^T\underline{G}r < 0 \tag{2.13}$$

When $e < 0$, the requirement is for $\dot{e}$ to be positive, or

$$\underline{C}^TA\underline{x} + \underline{C}^T\underline{B}u - \underline{H}^T\underline{F}y - \underline{H}^T\underline{G}r > 0 \tag{2.14}$$

If the system error is to remain at zero once that condition has been achieved, $\dot{e}$ must be held at zero, or

$$\underline{C}^T\underline{A}\underline{x} + \underline{C}^T\underline{B}u - \underline{H}^T\underline{F}\underline{y} - \underline{H}^T\underline{G}r = 0 \qquad (2.15)$$

Since u is the only quantity available to control (2.13)-(2.15) the matrix product $\underline{C}^T\underline{B}$ must be nonzero. For linear systems the quantity $\underline{C}^T\underline{B}$ equals the coefficient of s^{n-1} in the numerator of the plant transfer function. Thus the requirement that $\underline{C}^T\underline{B}$ be nonzero means that the nth order plant must have a numerator of order n-1. This restriction is removed in Chapter III, where plants with any numerator of order less than n are admissible.

To obtain the plant input in a readily usable form, (2.13)-(2.15) are combined to yield

$$u = \frac{1}{\underline{C}^T\underline{B}}\left(\underline{H}^T\underline{F}\underline{y} + \underline{H}^T\underline{G}r - \underline{C}^T\underline{A}\underline{x} - p \text{ sign } e\right) \qquad (2.16)$$

In (2.16), p is any positive constant or function, and sign e is defined to be

$$\text{sign } e = \begin{cases} +1 & \text{when } e > 0 \\ 0 & \text{when } e = 0 \\ -1 & \text{when } e < 0 \end{cases}$$

The control defined by (2.16) is a feedback control which in general involves the model input and every state of the plant and the model. This fact made necessary the assumptions that the plant and model states, and the model input be accessible.

The convergence factor p sign e is very important: it controls the stability properties of the model reference system. If p were zero, the system would be stable, but not asymptotically stable, in the sense of Liapunov. Any errors which were caused by the plant and model initial conditions would remain constant. By choosing p to be a positive constant, a positive function of time, or a positive definite function of the system error, the model reference system is forced to be asymptotically stable. In the examples which are presented in this chapter, p was chosen to be $k|e|$, where $k > 0$. Such a convergence factor caused the error to approach zero asymptotically as $e(0)\varepsilon^{-kt}$, where $e(0)$ was the system error at time $t = 0$.

The following example illustrates how (2.16) was used to yield a design for a simple model reference adaptive control system.

Example 1. Suppose the model which represents the desired response characteristics had the transfer function

$$G_M(s) = \frac{s + 0.5}{s^2 + 2s + 2} \tag{2.17}$$

The phase variable representation of this model is given by

$$\begin{aligned} \dot{y}_1 &= y_2 \qquad (2.18) \\ \dot{y}_2 &= -2y_1 - 2y_2 + r \\ w &= 0.5y_1 + y_2 \end{aligned}$$

In the vector-matrix notation of (2.7), the model matrices are

$$F = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} ; \quad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2.19}$$

$$H = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} .$$

The plant to be controlled has the transfer function

$$G_p(s) = \frac{100s(s+1)}{(s+2)[(s+0.1)^2 + 8^2]} = \frac{100s^2 + 100s}{s^3 + 2.2s^2 + 64.4s + 128} \tag{2.20}$$

Using the phase variable form of state variables, the state equations are

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -128x_1 - 64.4x_2 - 2.2x_3 + u \\ z &= 100x_2 + 100x_3 \end{aligned} \tag{2.21}$$

Therefore, the defining plant matrices are

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -128 & -64.4 & -2.2 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad C = \begin{bmatrix} 0 \\ 100 \\ 100 \end{bmatrix}. \qquad (2.22)$$

Referring to (2.16), the plant input is defined by

$$u = \frac{1}{\underline{C}^T\underline{B}} (\underline{H}^T F\underline{y} + \underline{H}^T\underline{G}r - \underline{C}^T A\underline{x} - p \text{ sign } e) \qquad (2.16)$$

Arbitrarily, the quantity p was chosen as

$$p = 3|e| = 3|\underline{C}^T\underline{x} - \underline{H}^T\underline{y}| \qquad (2.23)$$

so that the initial error would decay as $e(0)\epsilon^{-3t}$. When the matrix operations indicated in (2.16) are performed, the resulting feedback control is

$$u = 128x_1 + 61.4x_2 - 2.8x_3 - 0.005y_1 + 0.015y_2 + 0.01r \qquad (2.24)$$

Since the model input appears explicitly in the equation for the plant input, r must remain finite. However, no other a priori knowledge of r is required.

Figure 3 shows the plant and model outputs, the system error, and the control u which resulted from a simulation of the system. The model had the non-zero initial condition $y_1(0) = 1$, while the plant started from rest. The input to the model for $t \geq 0$ was $10(1-t)$.

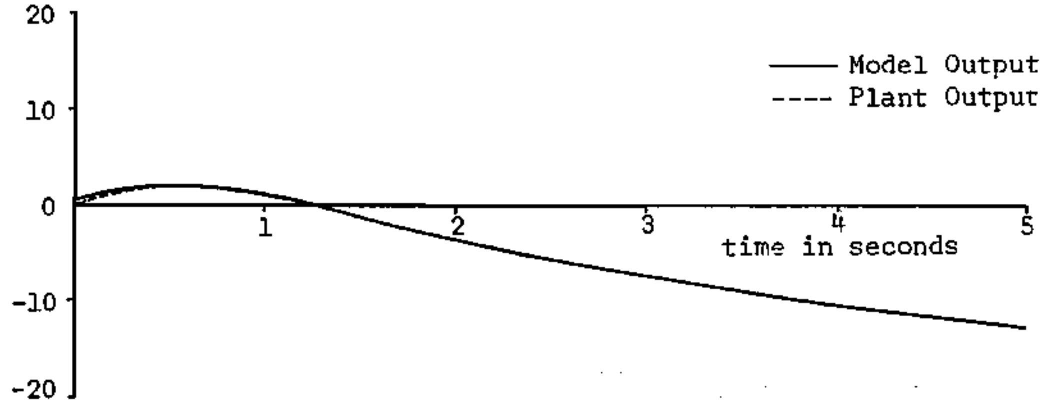

Figure 3(a). Plant and Model Outputs for Example 1

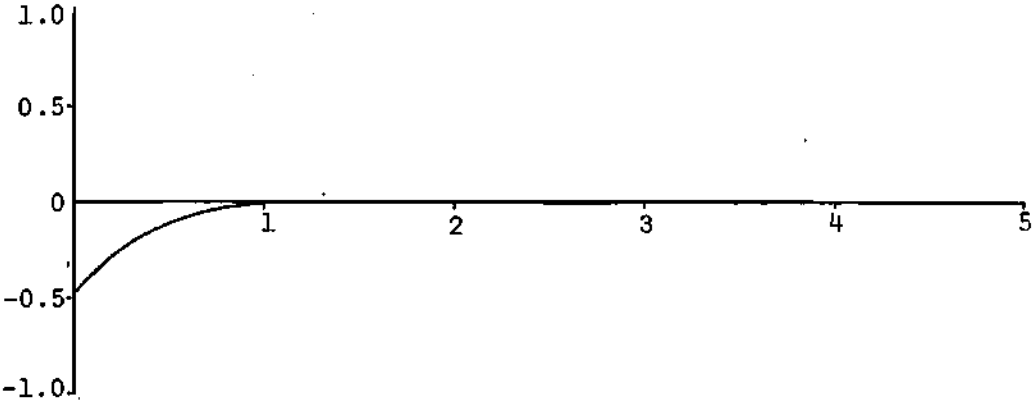

Figure 3(b). System Error for Example 1

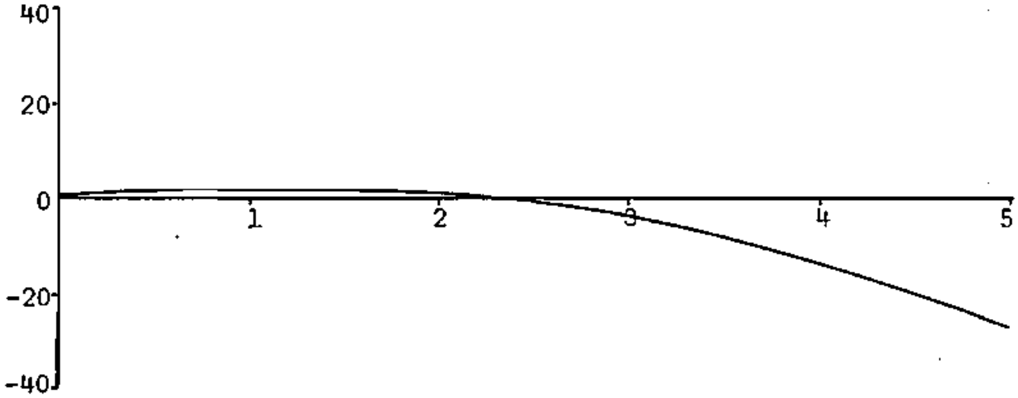

Figure 3(c). Synthesized Plant Input for Example 1

Because the input was continuous and smooth, the control was also continuous and smooth. The system error did indeed go to zero exponentially.

The convergence factor p was chosen to be $3|e|$ in this example. This choice was entirely arbitrary. If it had been desired that errors be reduced much more quickly, one could have chosen, for example, $50|e|$. If large errors had been important but small errors had not, one might have chosen $10e^2$ or $25e^4$ or some other even power law convergence factor.

The Nonlinear Time-Varying Plant

The set of permissible plants is actually much larger than the class of linear time-invariant plants. The plant may be time-varying and may possess one or more of a large number of permissible nonlinearities.

Consider first the linear time-varying plant:

$$\dot{\underline{x}} = A(t)\underline{x} + \underline{B}(t)u \qquad (2.25)$$

$$z = \underline{C}^T(t)\underline{x}$$

For this plant

$$\dot{z} = \underline{C}^T(t)\dot{\underline{x}} + \dot{\underline{C}}^T(t)\underline{x} = \underline{C}^T(t)A(t)\underline{x} + \underline{C}^T(t)\underline{B}(t)u + \dot{\underline{C}}^T(t)\underline{x} \qquad (2.26)$$

When (2.25) and (2.26) are substituted into (2.10), the result is

$$\dot{V} = [\underline{C}^T(t)\underline{x}-w][\underline{C}^T(t)A(t)\underline{x} + \underline{C}^T(t)\underline{B}(t)u + \underline{\dot{C}}^T(t)\underline{x} - \dot{w}] \le 0 \qquad (2.27)$$

In (2.27), w and $\dot{w}$ are the model output and its first derivative for some specific model. The control is

$$u = \frac{1}{\underline{C}^T(t)\underline{B}(t)} \{\dot{w} - \underline{C}^T(t)A(t)\underline{x} - \underline{\dot{C}}^T(t)\underline{x} - p\ \mathrm{sign}[\underline{C}^T(t)\underline{x}-w]\} \qquad (2,28)$$

As before, it must be required that $\underline{C}^T(t)\underline{B}(t)$ be non-zero for all $t \ge 0$.

Next consider the nonlinear plant

$$\underline{\dot{x}} = \underline{f}(\underline{x}) + \underline{B}u \qquad (2.29)$$

$$z = \underline{C}^T\underline{x}$$

For this plant

$$\dot{z} = \underline{C}^T\underline{\dot{x}} = \underline{C}^T\underline{f}(\underline{x}) + \underline{C}^T\underline{B}u \qquad (2.30)$$

The resulting control is then

$$u = \frac{1}{\underline{C}^T\underline{B}} \{\dot{w} - \underline{C}^T\underline{f}(\underline{x}) - p\ \mathrm{sign}[\underline{C}^T\underline{x}-w]\} \qquad (2.31)$$

The most general plant where u enters linearly is the nonlinear time-varying plant of the form

$$\dot{\underline{x}} = \underline{f}(\underline{x},t) + \underline{B}(t)u \tag{2.32}$$

$$z = \underline{C}^T(t)\underline{x}$$

The derivative of the plant output is computed as

$$\dot{z} = \underline{C}^T(t)\dot{\underline{x}} + \dot{\underline{C}}^T(t)\underline{x} = \underline{C}^T(t)\underline{f}(\underline{x},t) + \underline{C}^T(t)\underline{B}(t)u + \dot{\underline{C}}^T(t)\underline{x} \tag{2.33}$$

The control for this plant is then

$$u = \frac{1}{\underline{C}^T(t)\underline{B}(t)} \{\dot{w} - \underline{C}^T(t)\underline{f}(\underline{x},t) - \dot{\underline{C}}^T(t)\underline{x} - p \text{ sign}[\underline{C}^T(t)\underline{x}-w]\} \tag{2.34}$$

The Nonlinear Time-Varying Model

The single-input, single-output model may assume any of the forms just discussed for the plant. In addition, the model can be permitted to have nonlinearities which involve the model input r.

Consider the general time-varying model, whose equation is

$$\dot{\underline{y}} = F(t)\underline{y} + \underline{G}(t)r \tag{2.35}$$

$$w = \underline{H}^T(t)\underline{y}$$

For this model the output derivative is

$$\dot{w} = \underline{H}^T(t)F(t)\underline{y} + \underline{H}^T(t)\underline{G}(t)r + \dot{\underline{H}}^T(t)\underline{y} \tag{2.36}$$

In the model given by (2.35), there are no restrictions on the matrix product $\underline{H}^T(t)\underline{G}(t)$ except that it must remain finite for all $t \geq 0$. When w and $\dot{w}$ are substituted into any of the Equations (2.28), (2.31), or (2.34), a model reference adaptive controller is completely designed.

Two forms of nonlinear models are permissible:

$$\underline{\dot{y}} = \underline{g}(\underline{y}) + \underline{G}r \tag{2.37}$$

$$w = \underline{H}^T\underline{y}$$

and

$$\underline{\dot{y}} = \underline{g}(\underline{y},r) \tag{2.38}$$

$$w = \underline{H}^T\underline{y}$$

The model may also be nonlinear and time-varying, and there are three permissible types:

$$\dot{y} = \underline{g}(\underline{y}) + \underline{G}(t)r \tag{2.39}$$

$$w = \underline{H}^T(t)\underline{y}$$

$$\underline{\dot{y}} = \underline{g}(\underline{y},t) + \underline{G}(t)r \tag{2.40}$$

$$w = \underline{H}^T(t)\underline{y}$$

and

$$\dot{\underline{y}} = \underline{g}(\underline{y},r,t) \tag{2.41}$$

$$w = \underline{H}^T(t)\underline{y}$$

Any of the models represented by (2.35) and (2.37)-(2.41), as well as the linear time-invariant model of (2.7), may also have an input nonlinearity. The input nonlinearity may be fixed, e.g. $h(r)$, or may be time-varying, e.g. $h(r,t)$. The particular form of the nonlinearity is almost unrestricted. For example, it may have hysteresis, dead zone, or saturation. The only requirement is that the output of the nonlinearity must remain finite for every finite r for all $t \geq 0$. The input nonlinearity is handled by substituting $h(r)$ or $h(r,t)$ for r in the model equations and the corresponding $\dot{w}$ equations.

The flexibility of the approach used in this thesis becomes evident when it is noted that each of the aforementioned model forms may be used with any of the permissible plants. Furthermore, the plant and the model need not be of the same order.

The following three examples illustrate some of the kinds of permissible plants and models.

Example 2

Consider a second order, linear, and time-varying plant. The plant equations have the form given in (2.25) where

$$A(t) = \begin{bmatrix} 0 & 1 \\ -0.707-\sin 4t & -1-t+0.1t^2 \end{bmatrix} ; \quad B(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2.42}$$

$$C(t) = \begin{bmatrix} 1-0.5t \\ 1+0.5t \end{bmatrix}.$$

Let the model be first order, linear, and time-varying. The model equations are of the form given in (2.35) where y is a scalar, and

$$F(t) = -\cos^2 t \qquad (2.43)$$

$$G(t) = 1$$

$$H(t) = 2 + \sin t$$

The input to the model was chosen to be cos 3.1416t. Applying the design equations to this system, and requiring initial conditions to die out as ϵ^{-5t}, the control equation corresponding to (2.16) is

$$u = \frac{1}{\underline{C}^T(t)\underline{B}(t)} \{\underline{H}^T(t)F(t)\underline{y} + \underline{H}^T(t)\underline{G}(t)r + \dot{\underline{H}}^T(t)\underline{y} \qquad (2.44)$$

$$- \underline{C}^T(t)A(t)\underline{x} - \dot{\underline{C}}^T(t)\underline{x} - 5[\underline{C}^T(t)\underline{x} - \underline{H}^T(t)\underline{y}]\}$$

Substituting from (2.42) and (2.43), (2.44) becomes

$$u = (0.707\sin 4t)x_1 + (1+t-0.1t^2)x_2 \qquad (2.45)$$

$$+ \frac{1}{1 + 0.5t} \{0.5x_1 - (1.5-0.5t)x_2 - (2+\sin t)y \cos^2 t$$

$$+ y\cos t + (2+\sin t)\cos 3.1416t - 5[(1-0.5t)x_1 + (1+0.5t)x_2$$
$$- (2+\sin t)y]\}$$

A simulation was made with initial conditions

$$x_1(0) = 0 \tag{2.46}$$
$$x_2(0) = 0$$
$$y(0) = 2$$

The results of this simulation are given in Figure 4.

Example 3

Consider a second order, nonlinear, time-invariant plant of the form given in (2.29) where

$$\underline{f}(\underline{x}) = \begin{bmatrix} x_2 \\ -x_1-3x_1^2-2x_2 \end{bmatrix}; \quad \underline{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2.47}$$

$$C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Let the model be first order, nonlinear, time-varying, and of the form given in (2.41). For this particular model y is a scalar and

$$f(y,r,t) = -yrt \tag{2.48}$$
$$H = 1$$

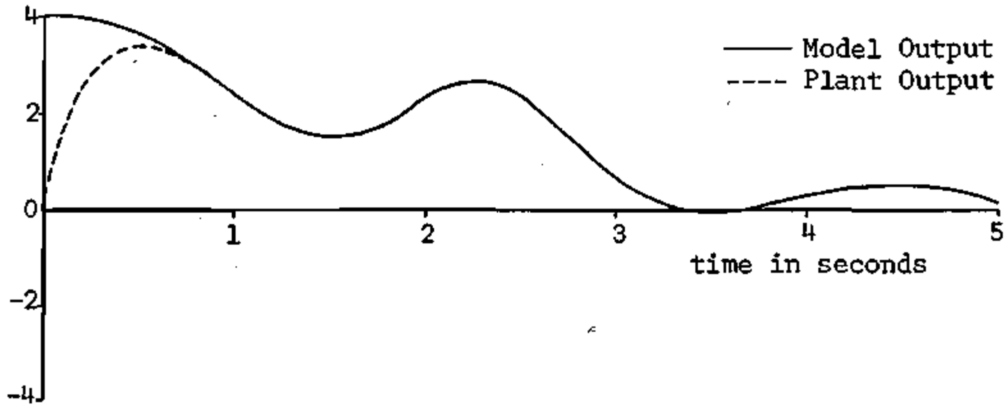

Figure 4(a). Plant and Model Output for Example 2

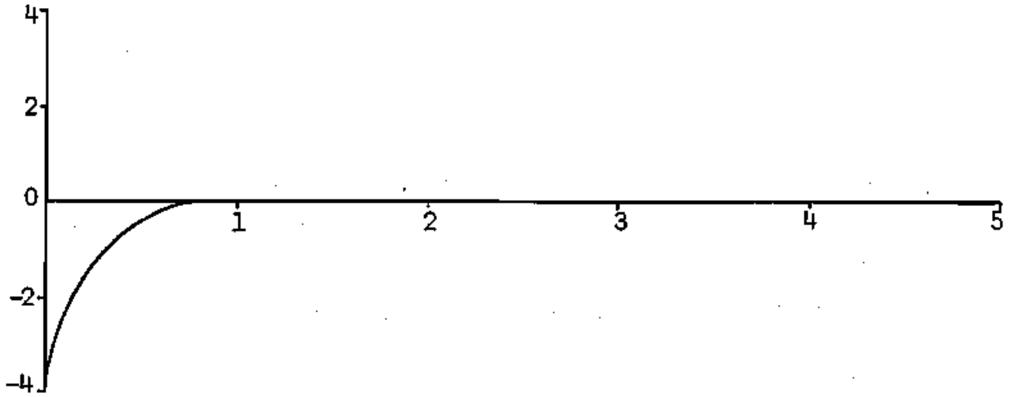

Figure 4(b). System Error for Example 2

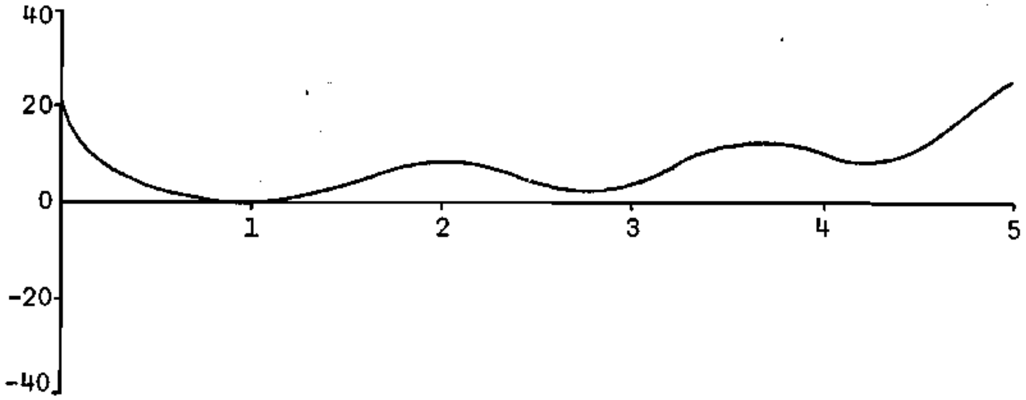

Figure 4(c). Synthesized Plant Input for Example 2

The equation for the plant input is

$$u = \frac{1}{\underline{C}^T\underline{B}} \{H^Ty + H^Tf(y,r,t) - \underline{C}^T\underline{f}(\underline{x}) - 5[\underline{C}^T\underline{x}-H^Ty]\} \qquad (2.49)$$

since it is desired that the initial error decay as ϵ^{-5t}. With sin 10t as the model input, the input to the plant is

$$u = x_1 + 3x_1^3 + 2x_2 - yt \sin 10t - 5(x_2-y) \qquad (2.50)$$

Figure 5 shows the results of the simulation of this system with initial conditions

$$\begin{aligned} x_1(0) &= 0 \qquad (2.51) \\ x_2(0) &= 0 \\ y(0) &= 1 \end{aligned}$$

Example 4

Consider a second order, nonlinear, and time varying plant of the form given in (2.32), where

$$\underline{f}(\underline{x},t) = \begin{bmatrix} x_2 \\ -x_1-x_1^3-0.04x_1^2x_2^3t \end{bmatrix}; \quad \underline{B}(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (2.52)$$

$$\underline{C}(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

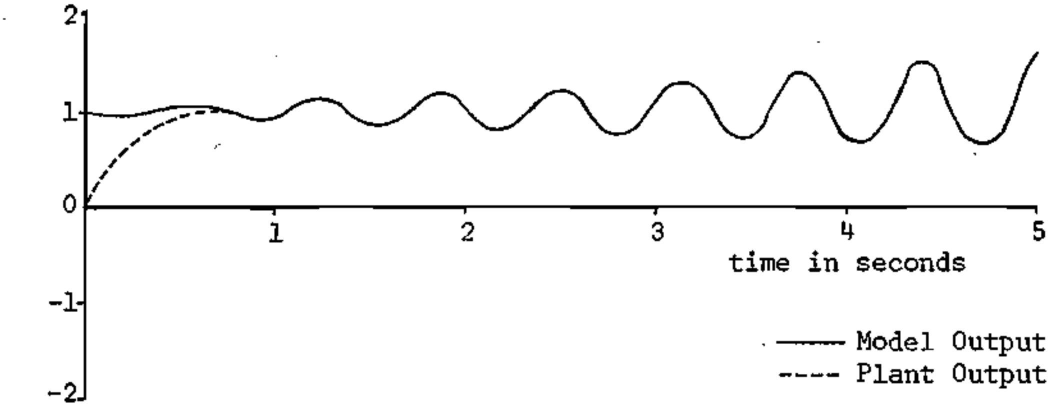

Figure 5(a). Plant and Model Outputs for Example 3

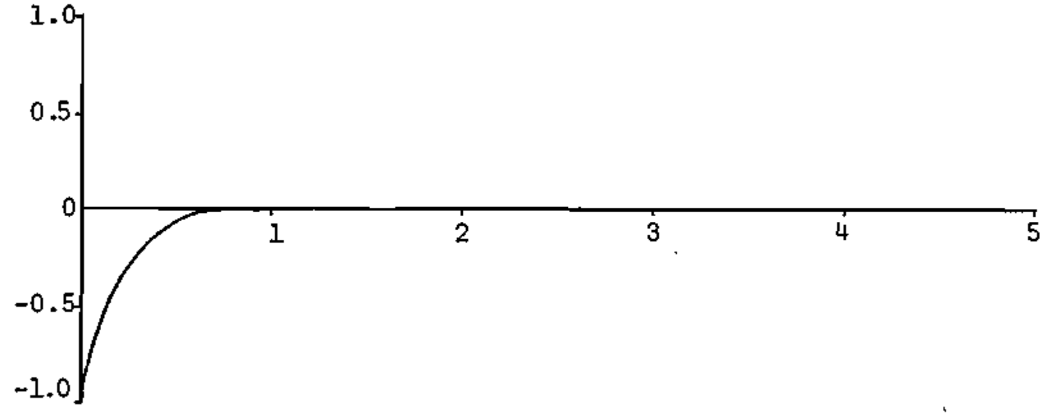

Figure 5(b). System Error for Example 3

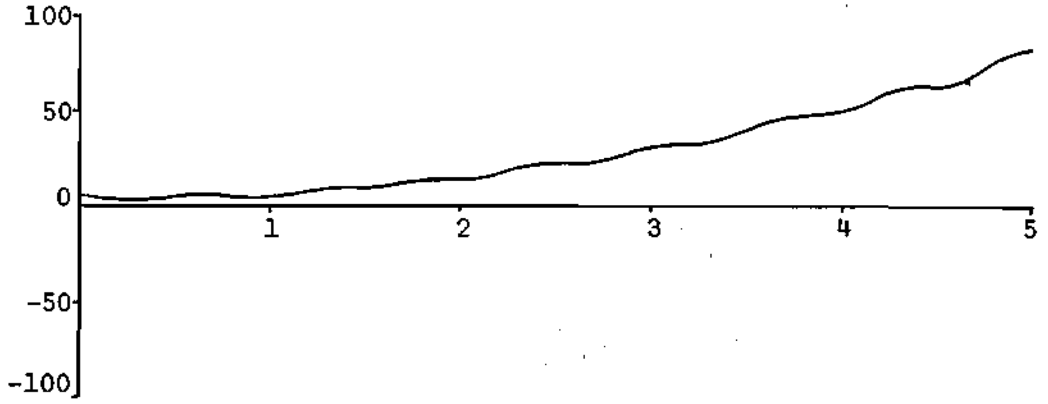

Figure 5(c). Synthesized Plant Input for Example 3

Let the model be fourth order, linear, and time-invariant, and have as its transfer function

$$G_p(s) = \frac{s^2 + s + 1}{(s+0.4)(s+0.6)(s+1)(s+2)} \tag{2.53}$$

In vector-matrix notation the model is of the form of (2.5) where

$$F = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.48 & -2.72 & -5.24 & -4 \end{bmatrix}; \quad G = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{2.54}$$

$$H = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

However, the model has a saturation type input nonlinearity. The equation for the plant input is

$$u = \frac{1}{\underline{C}^T(t)\underline{B}(t)} \{\underline{H}^T F\underline{y} + \underline{H}^T\underline{G}h(r) - \underline{C}^T(t)\underline{f}(\underline{x},t) - 4[\underline{C}^T(t)\underline{x}-\underline{H}^T\underline{y}]\} \tag{2.55}$$

because it is desired that the initial error decay as ϵ^{-4t}. Substituting the plant and model equations, (2.55) becomes

$$u = x_1 + x_1^3 + 0.04x_1^2x_2^3t - 4x_2 + 4y_1 + 5y_2 + 5y_3 + y_4 \tag{2.56}$$

The initial conditions were chosen as

$$x_1(0) = x_2(0) = 0 \tag{2.57}$$

$$y_1(0) = y_2(0) = y_3(0) = y_4(0) = 1$$

The input to the model nonlinearity was 2 sin 6t, and the output was SAT(r), where

$$\mathrm{SAT}(r) = \begin{cases} +1 \text{ for } r > 1 \\ r \text{ for } |r| \le 1 \\ -1 \text{ for } r < -1 \end{cases} \tag{2.58}$$

The results of the simulation are shown in Figure 6.

The previous three examples were presented to emphasize the flexibility of the signal synthesis technique. In Example 2 a linear, time-varying plant was made to follow a linear, time-varying model. In Example 3, a nonlinear, time-invariant plant followed a nonlinear, time-varying model. The plant was nonlinear and time-varying while the model was linear and time-invariant in Example 4. Each system was started with an initial error, and the error decreased exponentially toward zero.

Techniques for handling incompletely specified plants are discussed in the remainder of this chapter. Some of the plants and models that were used in Examples 1-4 are utilized in the examples of the next several sections to better illustrate the effects of having incompletely specified plant parameters.

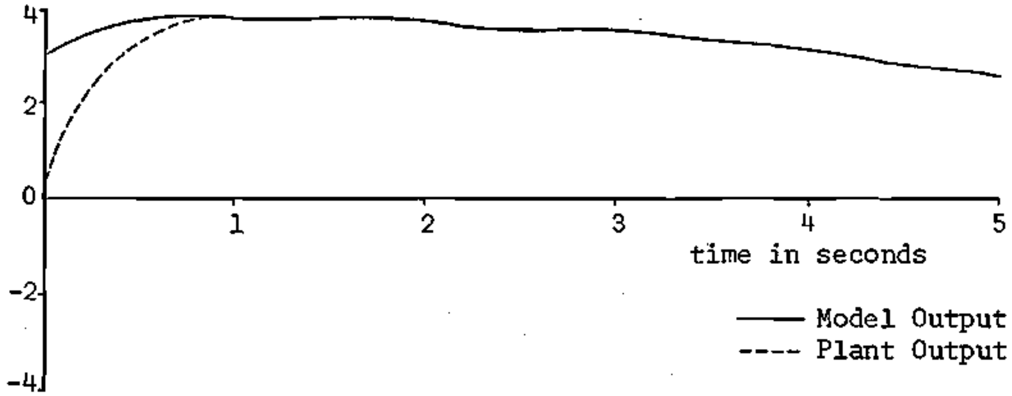

Figure 6(a). Plant and Model Outputs for Example 4

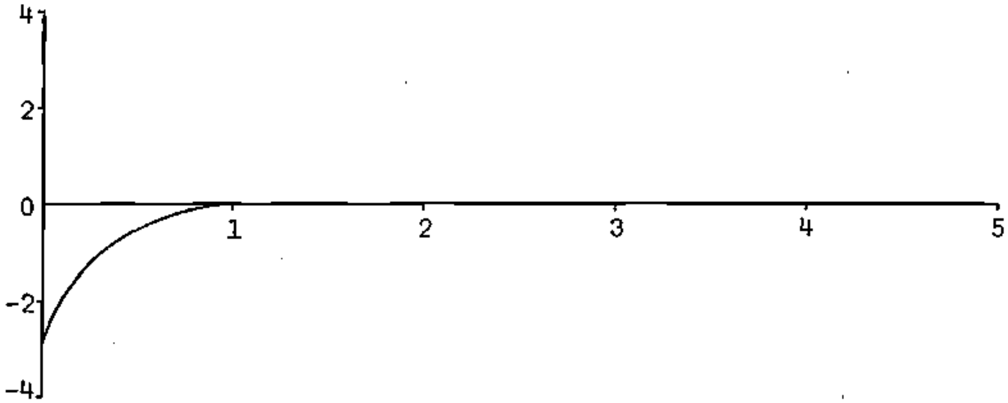

Figure 6(b). System Error for Example 4

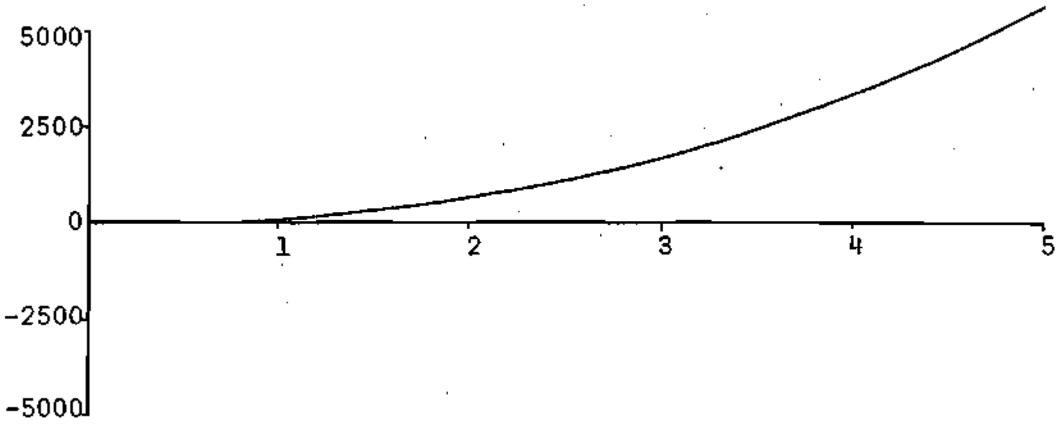

Figure 6(c). Synthesized Plant Input for Example 4

Plants with Unknown Parameters

In the presentation thus far it has been assumed that both the plant and the model are completely known. In this section the model is still considered to be completely known. However, the plant differential equation, or for the linear case the A, B, and C matrices, has parameters which lie in known ranges but either are unknown and fixed or are time-varying in some unknown manner. The resulting feedback law is nonlinear and discontinuous. The magnitude of the discontinuity is smaller for parameters which are known to lie in a small range, larger for parameters which lie in a large range. The discontinuity is smaller when the internal states of the plant are small, and is larger when these states are large.

Rather than beginning with a discussion of the general case with all its varied possibilities, the simplest single-input, single-output case is considered first. Let the plant be nth order, linear, and time-invariant. To keep the mathematics as simple as possible, let the model be mth order, linear, and time-invariant. Then referring to (2.12), the requirement for asymptotic stability is

$$\dot{V} = (\underline{C}^T\underline{x}-\underline{H}^T\underline{y})(\underline{C}^TA\underline{x} + \underline{C}^T\underline{B}u - \underline{H}^TF\underline{y} - \underline{H}^T\underline{G}r) \le 0 \qquad (2.12)$$

with equality only when V is zero. Fulfilling the requirement presented in (2.12) is somewhat more complicated than before because now the quantities $\underline{C}^TA$ and $\underline{C}^T\underline{B}$ are not completely specified. To make $\dot{V}$ negative definite when the plant has incompletely specified parameters, it is necessary to consider separately each term of the matrix products represented in (2.12). The resulting control u becomes a function not only of the states of the plant and the states and the input of the model, but also of the bounds on each unknown parameter in the plant. The procedure for handling unknown parameters in this linear, time-invariant case is explained within the context of the following example.

Example 5

Consider again the plant and the model used in Example 1. The plant matrices are of the form

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -d & -c & -b \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{2.59}$$

$$C = \begin{bmatrix} 0 \\ a \\ 100 \end{bmatrix}$$

It is assumed that knowledge about the plant is imperfect in that the exact values of a, b, c, and d are unknown. However, each of these coefficients is known to lie in some specific range. The known ranges are

$$50 \le a \le 100$$

$$1.6 \le b \le 3.6$$

$$24 \le c \le 102$$

$$36 \le d \le 300$$

A sufficient condition for the model reference system to be asymptotically stable is that (2.12) be satisfied. Consider first the case when $e = \underline{C}^T\underline{x}-\underline{H}^T\underline{y}>0$. For this case u must satisfy the equation

$$u < \frac{1}{\underline{C}^T\underline{B}}(\underline{H}^T\underline{F}\underline{y} + \underline{H}^T\underline{G}r - \underline{C}^T\underline{A}\underline{x}) \tag{2.60}$$

In terms of the plant and model parameters, (2.60) is

$$u < -0.02y_1 - 0.015y_2 + 0.01r + dx_1 + cx_2 + (100b-a)x_3 \tag{2.61}$$

However, a, b, c, and d are to some extent unknown and (2.61) must hold for any permissible values of a, b, c, and d.

It would be possible to satisfy (2.61) by choosing the following plant input:

$$u < -0.02y_1 - 0.015y_2 + 0.01r - |d|_{max}|x_1| \tag{2.62}$$

$$- |c|_{max}|x_2| - |100b-a|_{max}|x_3|$$

Equation (2.62) would serve as a good control when the error is large because it would cause the error to be reduced quickly. However, it would be a very poor control when the error is near zero because there would be a large jump discontinuity in the plant input each time the error changed sign. The reason that there would be a jump discontinuity is that the corresponding inequality to be met when the error is negative is

$$u > -0.02y_1 - 0.015y_2 + 0.01r + |d|_{max}|x_1| + |c|_{max}|x_2| \tag{2.63}$$

$$+ |100b-a|_{max}|x_3|$$

The best policy when the error is positive is to be certain that (2.61) is satisfied while keeping the jump discontinuity as small as possible. The procedure is explained below for the given bounds on the plant coefficients.

Consider the single term dx_1 in (2.61). The quantity which replaces dx_1 must be such that the inequality holds regardless of the value of d. To satisfy (2.61) use $-300|x_1|$ or $300x_1$ when x_1 is negative, and use $36x_1$ when x_1 is positive. The reasoning is similar for the other two terms:

for $x_2>0$, use $24x_2$;
for $x_2<0$, use $102x_2$;

for $x_3>0$, use $0.6x_3$;
for $x_3<0$, use $3.1x_3$.

These quantities can be combined without using logic statements. The resulting equation is the condition to be met by the plant input when the error is positive:

$$u < -0.02y_1 - 0.015y_2 + 0.01r + 168x_1 - 132|x_1| \tag{2.64}$$

$$+ 63x_2 - 39|x_2| + 1.85x_3 - 1.25|x_3|$$

If it is desired that the initial error decay at least as fast as ϵ^{-3t}, the inequality in (2.64) could be modified as

$$u^{+} = -0.02y_1 - 0.015y_2 + 0.01r + 168x_1 - 132|x_1| \qquad (2.65)$$

$$+ 63x_2 - 39|x_2| + 1.85x_3 - 1.25|x_3| - 3e$$

where u^{+} refers to the control when the error is positive. The case for negative error must be considered separately, since (2.65) holds only for positive errors.

When the error is negative it is necessary that u satisfy

$$u > \frac{1}{\underline{C}^T\underline{B}} (\underline{H}^T F\underline{y} + \underline{H}^T \underline{G}r - \underline{C}^T A\underline{x}) \qquad (2.66)$$

In terms of the plant and the model parameters, (2.66) was

$$u > -0.02y_1 - 0.015y_2 + 0.01r - dx_1 - cx_2 - (b-0.01a)x_3 \qquad (2.67)$$

Reasoning as before, the plant input when the error is negative is designated as u^{-} and obeys the equation

$$u^{-} = -0.02y_1 - 0.015y_2 + 0.01r + 168x_1 + 132|x_1| + 63x_2 \qquad (2.68)$$

$$+ 39|x_2| + 1.85x_3 + 1.25|x_3| - 3e$$

Equations (2.65) and (2.68) are combined by a simple procedure:

$$u = \frac{1}{2}\,(1+\text{sign } e)u^{+} + \frac{1}{2}\,(1-\text{sign } e)u^{-} \qquad (2.69)$$

where u is the complete plant input. When (2.65) and (2.68) are substituted into (2.69) and the terms are combined, the result is

$$u = -0.02y_1 - 0.015y_2 + 0.01r + 168x_1 + 63x_2 + 1.85x_3 \qquad (2.70)$$

$$-(\text{sign } e)(132|x_1| + 39|x_2| + 1.25|x_3|) - 3e$$

Comparison of (2.70) with (2.24), the corresponding control equation when the plant parameters were known, gives an indication of the increase in complexity that can be expected when there are incompletely specified plant parameters. This system was simulated with the same input and the same initial conditions as were used in Example 1. The results of the simulation are shown in Figure 7.

Careful study of Figure 7 reveals an important characteristic of the synthesized plant input u: the plant input behaved very nicely whenever the system error was away from zero. Indeed, the error in this example was quickly driven to zero, rather than approaching zero exponentially as in Example 1. However, because of the term $-(\text{sign } e)(132|x_1| + 39|x_2| + 1.25|x_3|)$ in (2.70), the input made large changes in magnitude each time the error e changed sign.

The magnitude of the discontinuity which occurred when the error changed sign was $2(132|x_1| + 39|x_2| + 1.25|x_3|)$, which is a function both of the states and of the ranges of each incompletely specified plant parameter. Since the controller has no means of controlling

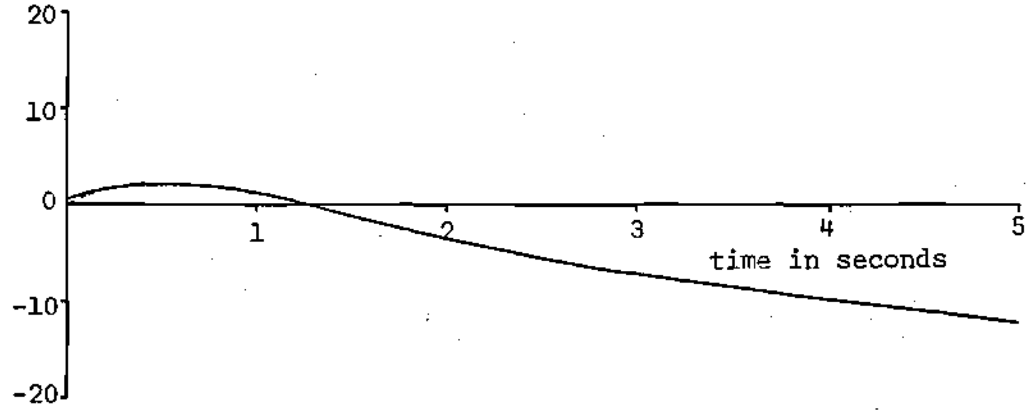

Figure 7(a). Plant and Model Outputs for Example 5

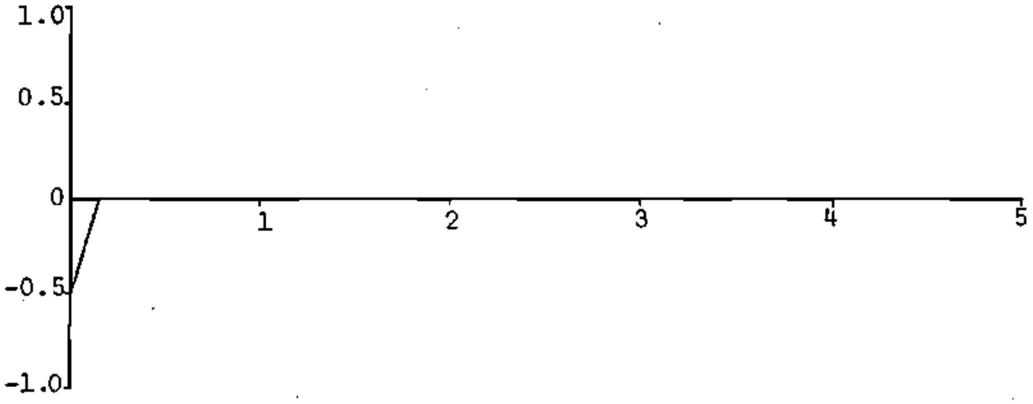

Figure 7(b). System Error for Example 5

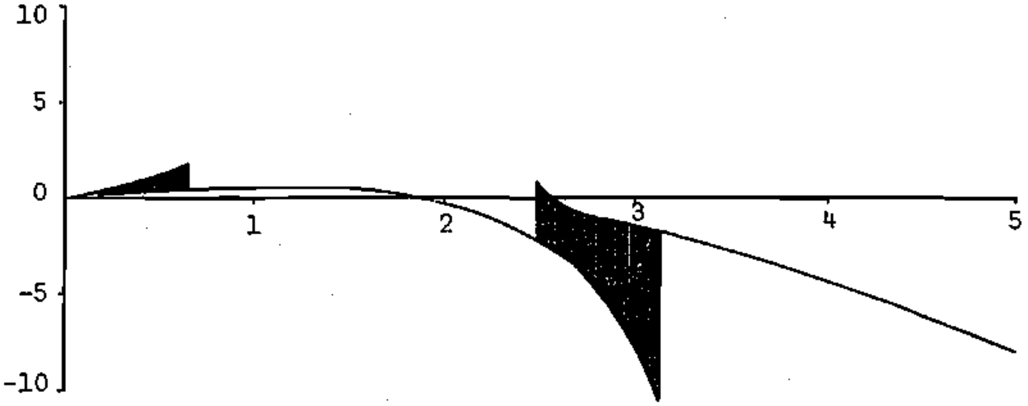

Figure 7(c). Synthesized Plant Input for Example 5

individual states in the plant, the only way to reduce the magnitude of the discontinuity would have been to have more accurate a priori knowledge about the plant parameters.

When the plant is nonlinear or time-varying, the method parallels that for the linear time-invariant plant of Example 5. The Liapunov function V is defined. The derivative of V is calculated, and each term of $\dot{V}$ is considered separately so that the control u will make $\dot{V}$ negative definite for all permissible values of each unknown parameter.

The following example illustrates the versatility of these techniques when they are applied to nonlinear and time-varying systems which have unknown plant parameters.

Example 6

The plant, model, and model input, and initial conditions are those used in Example 3. Recall that the equation for $\dot{x}_2$ was of the form

$$\dot{x}_2 = -x_1 - 3|x_1|^a - bx_2 + u \tag{2.71}$$

where in Example 3 the exponent a was 2 and the coefficient b was 2. For this example it is assumed that a is known to lie in the range $1.8 \le a \le 2.2$, but the exact value of a is not known. The coefficient b is known to be in the range $1 \le b \le 4$.

The requirements on u were

$$u < x_1 + 3|x_1|^a + bx_2 - ty \sin 10t \tag{2.72}$$

when $e > 0$, and

$$u > x_1 + 3|x_1|^a + bx_2 - ty \sin 10t \qquad (2.73)$$

when $e < 0$. The factor bx_2 is handled as in Example 5. The factor $|x_1|^a$ is handled as follows:

For $e>0$ and $|x_1|<1$, use $|x_1|^{2.2}$;
For $e>0$ and $|x_1|>1$, use $|x_1|^{1.8}$;
For $e<0$ and $|x_1|<1$, use $|x_1|^{1.8}$;
For $e<0$ and $|x_1|>1$, use $|x_1|^{2.2}$.

The resulting control law with convergence factor $5|e|$ is

$$\begin{aligned} u = x_1 &+ 1.5(|x_1|^{1.8} + |x_1|^{2.2}) + 1.5(\text{sign } e)\{|x_1|^{1.8}\text{sign}(|x_1|-1) \\ &-|x_1|^{2.2}\text{sign}(|x_1|-1)\} + 2.5\, x_2 - 1.5|x_2|\text{sign } e \\ &- ty \sin 10t - 5e \end{aligned} \qquad (2.74)$$

Comparison of (2.74) with (2.50), the corresponding control in Example 3, shows that the complexity of the control was greatly increased when there were incompletely specified plant nonlinearities.

The results of the simulation of this system are shown in Figure 8. Comparison with Figure 5 shows again that the error was driven to zero more quickly when there were unknown parameters.

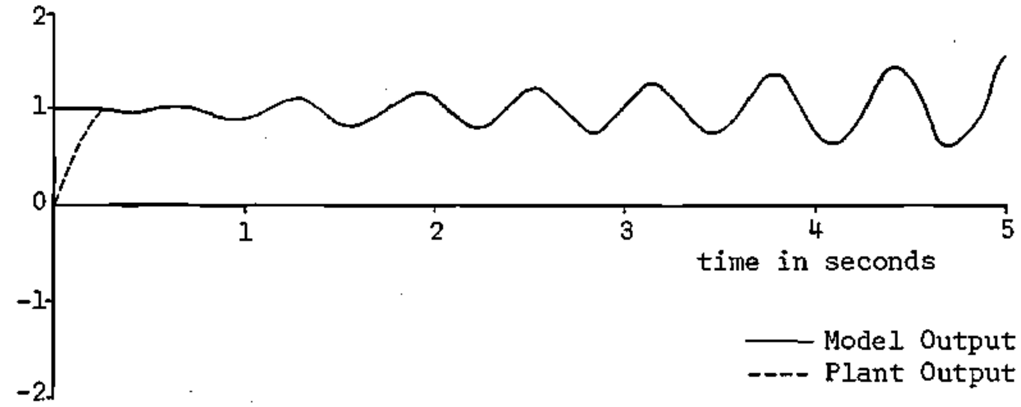

Figure 8(a). Plant and Model Outputs for Example 6

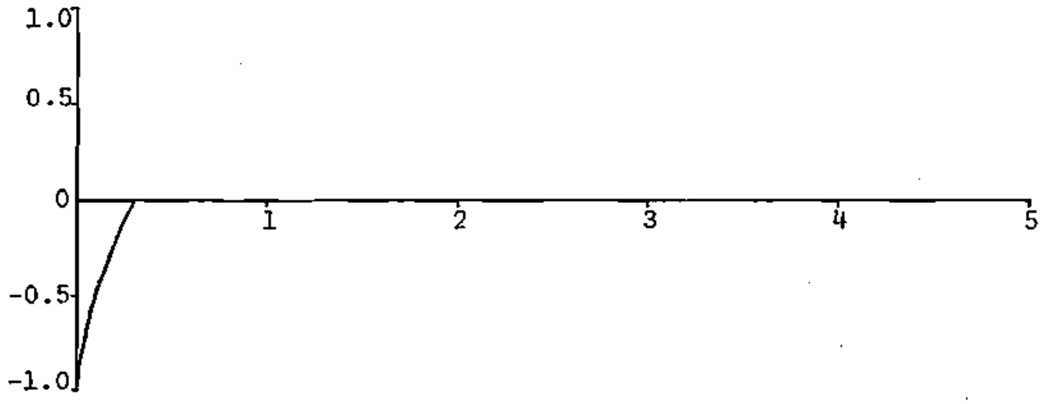

Figure 8(b). System Error for Example 6

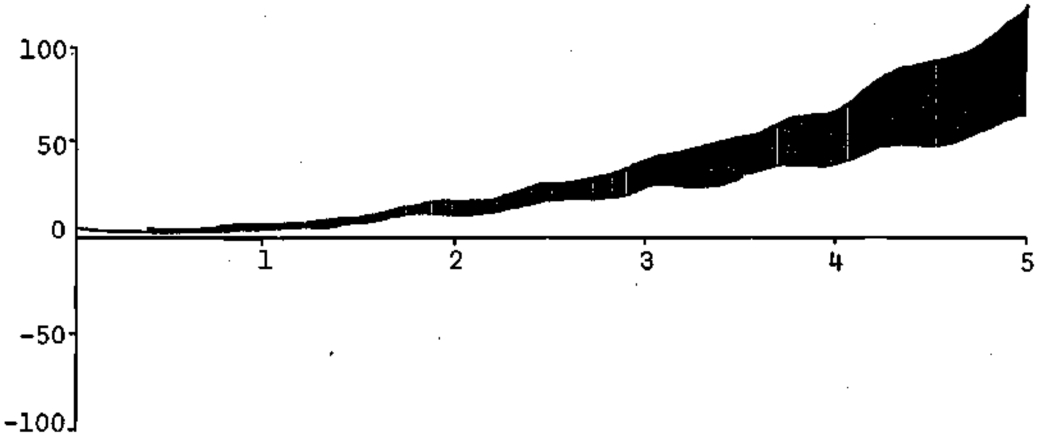

Figure 8(c). Synthesized Plant Input for Example 6

This was caused by the extra terms which insured that $\dot{V}$ was negative definite for all conditions of each unknown parameter.

Special Nonlinear Cases

There were two specific nonlinear cases which required special attention. When a separable nonlinearity occurred at the input of the plant, the plant could be controlled by making appropriate modifications to the techniques of the preceding sections. The ability to handle plants with separable input nonlinearities greatly increases the applicability of this thesis. The second type of nonlinearity which was considered was the system output nonlinearity where, instead of an output error, only a nonlinear function of the output error was accessible. In this case, an integral type Liapunov function was especially appropriate. Furthermore, discussion of the system output nonlinearity led naturally to the consideration of relaxed stability requirements to improve the overall system design.

The Plant Input Nonlinearity

Figure 9 shows an input-output plane for a separable nonlinearity with input u and output f(u). The technique which is presented in Example 7 is applicable to any input nonlinearity which can be contained in the shaded region of Figure 9, provided the k's and ℓ's are known. The k's and ℓ's are slopes and intercepts, respectively, of the boundary lines for the region. The form of the region permits a wide variety of nonlinearities, such as dead zone, hysteresis, and nonlinearities that do not necessarily satisfy the condition $f(0) = 0$.

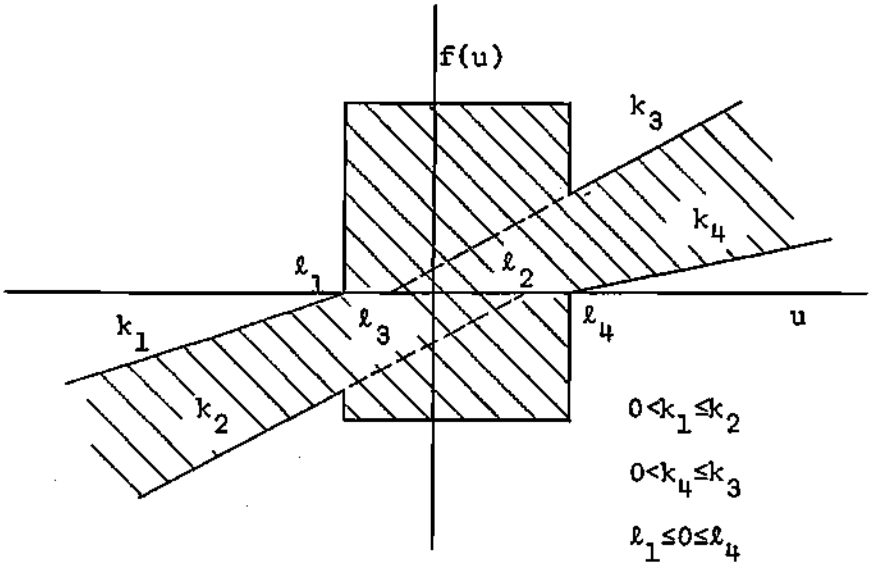

Figure 9. Bounds on the Plant Input Nonlinearity

Assuming an appropriate plant and model exist, the quadratic Liapunov function $V = \frac{1}{2} e^2$ is defined and the derivative $\dot{V}$ is calculated in terms of the plant and model states and inputs. Using the techniques which have been presented earlier, inequalities similar to (2.13) and (2.14) are obtained: one inequality applying when the error is positive, and the other when the error is negative. However, the inequalities are in terms of f(u), the output of the nonlinearity. Using these inequalities together with the available knowledge about the nonlinearity, an equation for the input u can be obtained. This equation for u must be such that the nonlinear function f(u) satisfies the inequality constraints. Such a control is possible if the nonlinearity can be obtained in the shaded region of Figure 9. The following example demonstrates the technique for a hysteresis-type input nonlinearity.

Example 7

Let the model be second order, linear, and time-varying of the form given in (2.35), where

$$F(t) = \begin{bmatrix} 0 & 1 \\ -1 & -0.8-10\sin t \end{bmatrix}; \quad G(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (2.75)$$

$$H(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The plant is second order and time-invariant, with equations

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -16x_1 - 3.2x_2 + f(u) \\ z &= 10x_1 + x_2 \end{aligned} \qquad (2.76)$$

The plant input nonlinearity, which is of the hysteresis type, is shown in Figure 10.

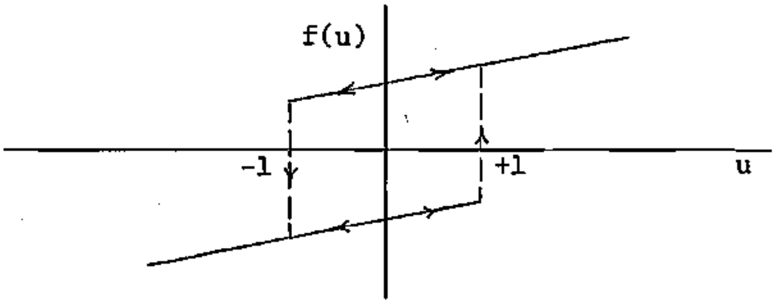

Figure 10. A Hysteresis Type Plant Input Nonlinearity

In terms of $f(u)$, the requirements for asymptotic stability are when $e > 0$,

$$f(u) < 16x_1 - 6.8x_2 - y_1 - (0.8 + \sin 10t)y_2 + r \qquad (2.77)$$

and when $e < 0$,

$$f(u) > 16x_1 - 6.8x_2 - y_1 - (0.8 + \sin 10t)y_2 + r \qquad (2.78)$$

With the choices of $2|e|$ for a convergence factor and $5 \sin 6t$ for a model input, (2.77) and (2.78) are satisfied with u defined as

When $f(u) \le -1.25$, then $u = 4[f(u) + 1]$
When $f(u) < -0.75$ and $f(u)\big|_{t^-} < 0$, then $u = 4[f(u) + 1]$
When $f(u) \ge 1.25$, then $u = 4[f(u) - 1]$
When $f(u) > 0.75$ and $f(u)\big|_{t^-} > 0$, then $u = 4[f(u) - 1]$
When $|f(u)| < 0.75$, then $u = -\text{sign } e$
When $0.75 \le |f(u)| < 1.25$ and $\text{sign } f(u)\big|_t = -\text{sign } f(u)\big|_{t^-}$,
then $u = \text{sign } f(u)$. (2.79)

The results of the simulation of this system are shown in Figure 11. The initial conditions in the simulation were

$$x_1(0) = x_2(0) = y_1(1) = 0 \qquad (2.80)$$

$$y_2(0) = -1$$

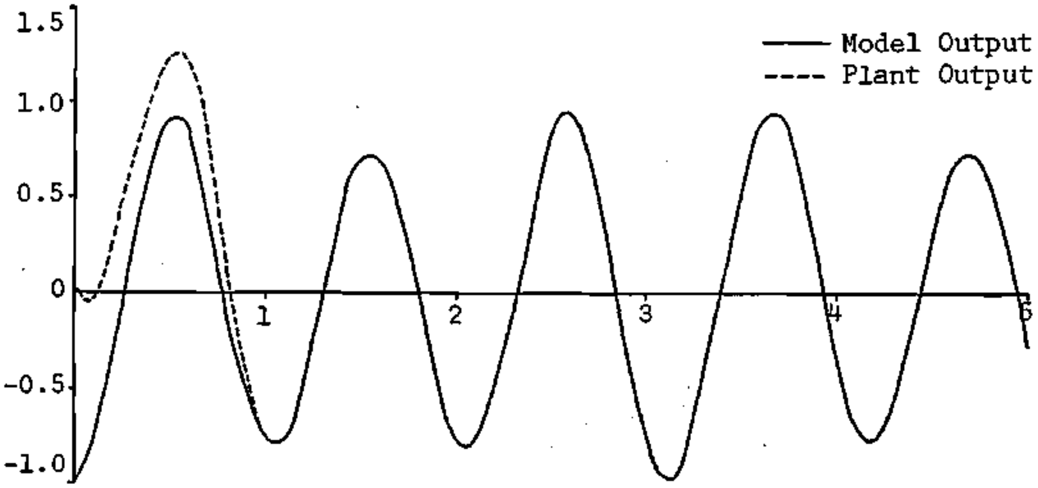

Figure 11(a). Plant and Model Outputs for Example 7

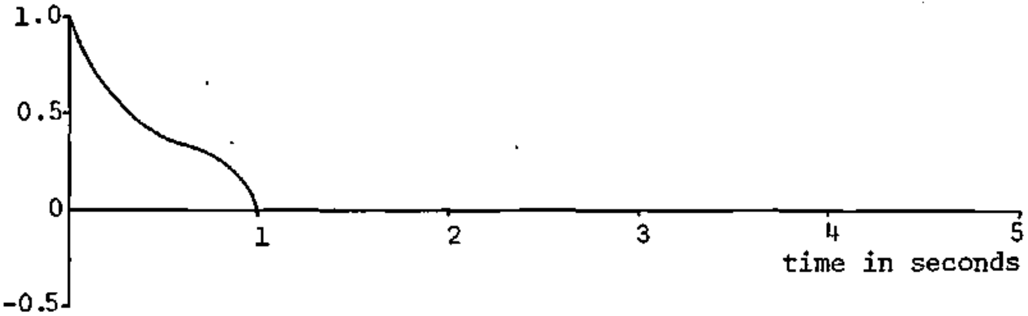

Figure 11(b). System Error for Example 7

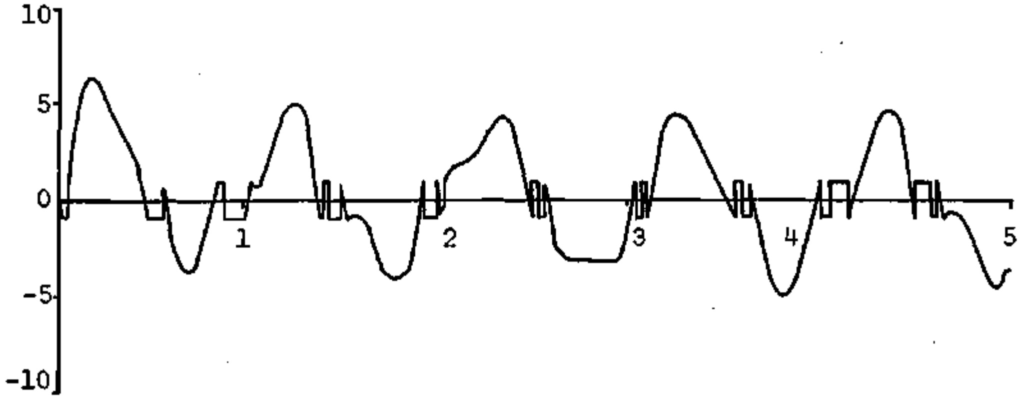

Figure 11(c). Synthesized Plant Input for Example 7

Examination of Figure 11(b) shows that the control of (2.79) did indeed cause this system to be asymptotically stable. In fact, the system error was forced to zero rather quickly. The peculiar shape of the control u versus time curve in Figure 11(c) was intentional. One model parameter and the model input were chosen to be sinusoid, thus providing a better test for the hysteresis plant input nonlinearity.

The System Output Nonlinearity

A system of special interest is shown in Figure 12. In this system neither the plant output nor the system error is known. Instead, a function of the error is available as the output of an incompletely specified time-varying nonlinearity satisfying for all $t \geq 0$ the conditions

1. $0 < k_{min} e^2 \leq ef(e,t)$ for $e \neq 0$
2. $f(0,t) = 0$
3. $\left|\frac{\partial f(e,t)}{\partial t}\right| \leq m|e|$.

The system can be controlled by choosing for a Liapunov function

$$V = \int_0^e f(e,t)de \qquad (2.81)$$

For this choice, the derivative is

$$\dot{V} = f(e,t)\dot{e} + \int_0^e \frac{\partial f(e,t)}{\partial t} de \leq 0 \qquad (2.82)$$

But

$$\int_0^e \frac{\partial f(e,t)}{\partial t} \le \frac{me^2}{2} \le \frac{m[f(e,t)]^2}{2k_{min}^2} \tag{2.83}$$

so asymptotic stability can be guaranteed by satisfying

$$\dot{V} \le f(e,t)\left[\dot{e} + \frac{mf(e,t)}{2k_{min}^2}\right] \le 0 \tag{2.84}$$

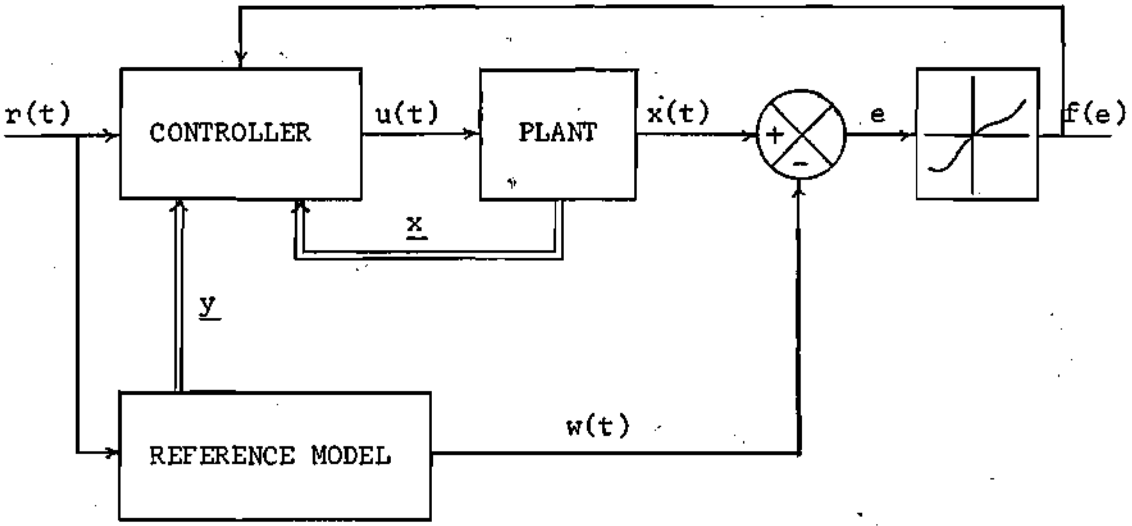

Figure 12. A Model Reference Adaptive Control System for Which the Error is Not Explicitly Available

The following example demonstrates how (2.84) was applied to a particular plant and model.

Example 8

The plant, the model, the model input, and the initial conditions are the same as those used in Example 2. Now, however, the error is available only as the output of an incompletely specified time-varying nonlinearity. The actual nonlinearity used in the simulation was

$$f(e,t) = (2-\epsilon^{-t})e \qquad (2.85)$$

Only m and k_{min} were assumed known. Equation (2.84) can be satisfied by using the term -pf(e,t) instead of the convergence term -pe of Example 2. Here p is a positive constant or function greater than 0.5. Figure 13 shows the results of a simulation of this system with p chosen as .5. Comparison with Figure 4 shows that in this instance the output nonlinearily affected the system performance very little.

Relaxed Stability Requirements Near the Origin

The techniques in this chapter have dealt with designing systems that are asymptotically stable to the origin. It was mentioned previously that when there were unknown parameters, the control which one could derive was a good one only until the error reached zero. Thereafter, as long as the error remained near zero, the control was a poor one because the term $f(\underline{x})$ sign e caused large jump discontinuities in u each time the error changed sign.

One way to improve the control near the origin is to relax the stability requirements when the error is small. Instead of requiring the error to remain at zero once it gets there, one may require instead that the error remain near zero, e.g., in the range $-0.05 \leq e \leq 0.05$.

The procedure is to first use the complete control equation to reduce the error to zero. The control equation is then modified by removing all terms which are multiplied by sign e. This modified

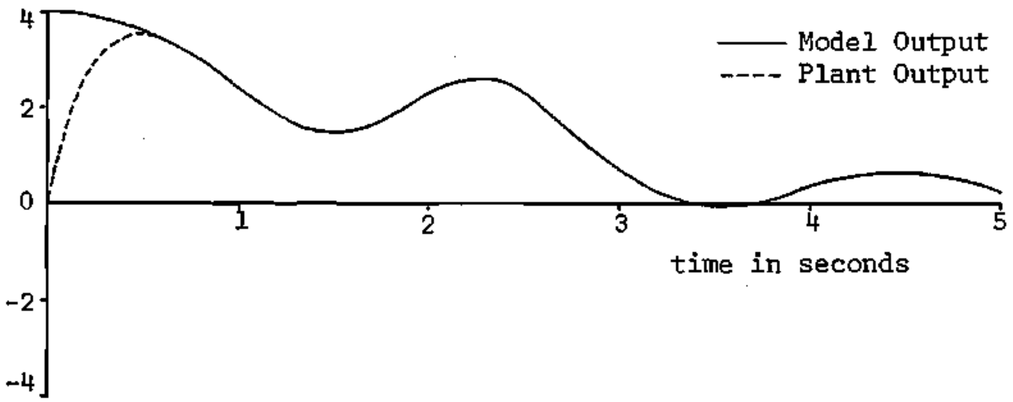

Figure 13(a). Plant and Model Outputs for Example 8

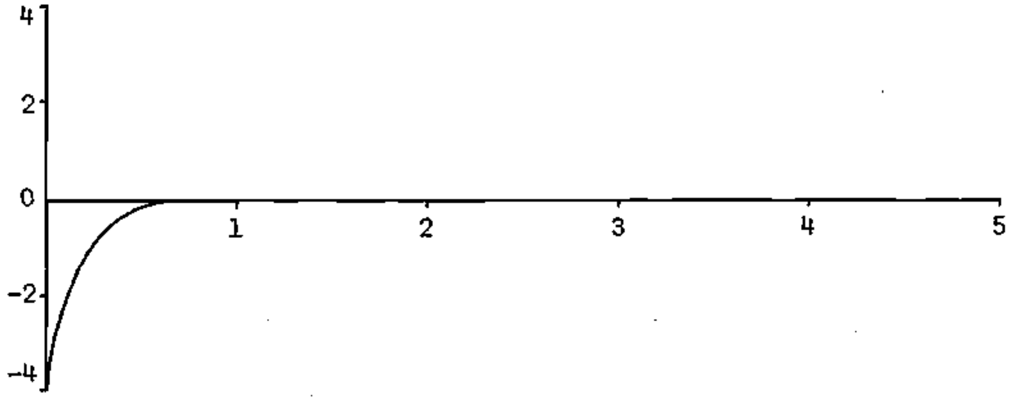

Figure 13(b). System Error for Example 8

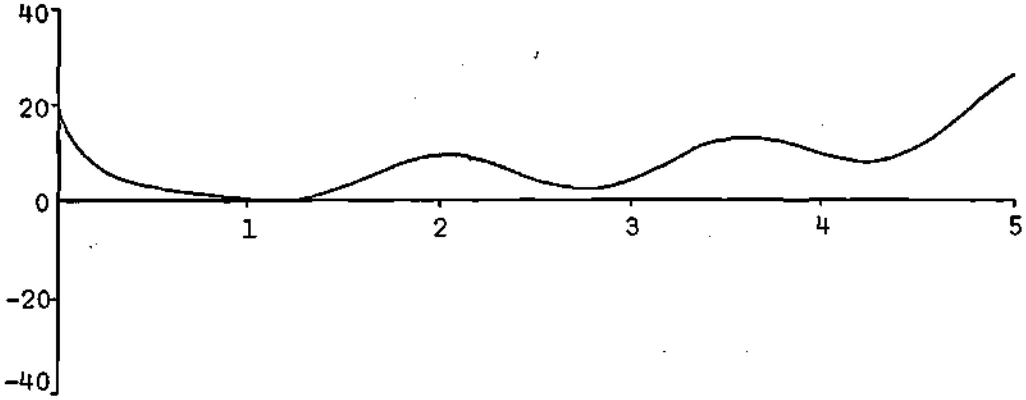

Figure 13(c). Synthesized Plant Input for Example 8

control is then used until the error becomes greater than 0.05 in absolute value. Once the error exceeds 0.05, the complete control equation is again used to drive the error to zero. This operation is easily implemented by adding the system output nonlinearity shown in Figure 14. The following example illustrates how this can be done, and what effects might be expected.

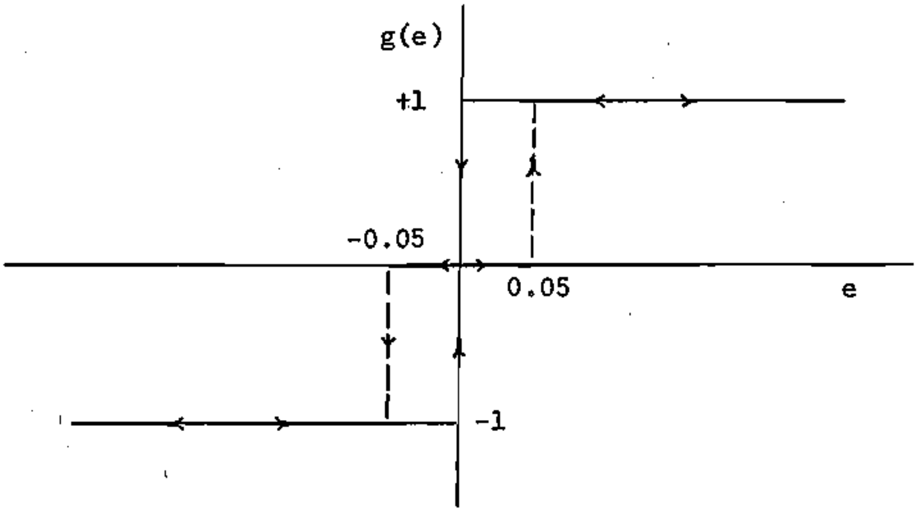

Figure 14. A System Output Nonlinearity for Relaxed Stability Requirements

Example 9

The system of Example 5 was altered by substituting g(e) for sign e in the equation for u. The results of a simulation of the system are shown in Figure 15. The improvement in u can be noted by comparing Figure 15 with Figure 7. Figure 15 was plotted on an expanded scale so that u could be better presented. Because the unknown parameters were within fairly large ranges, the modified

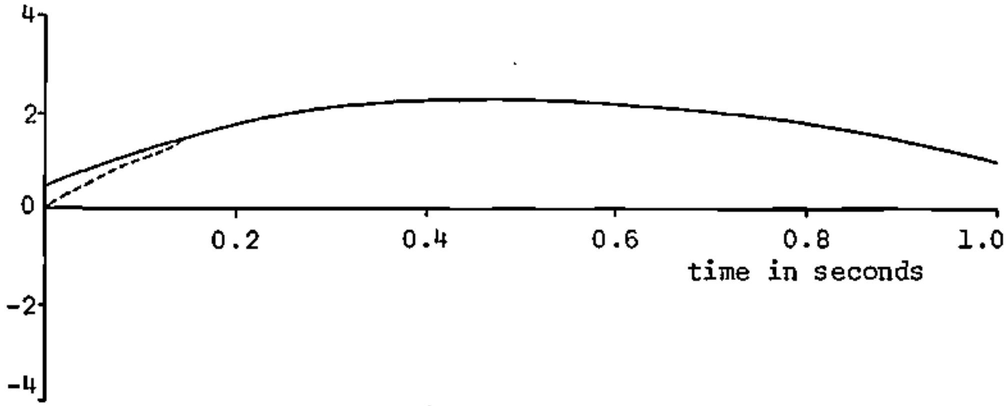

Figure 15(a). Plant and Model Outputs for Example 9

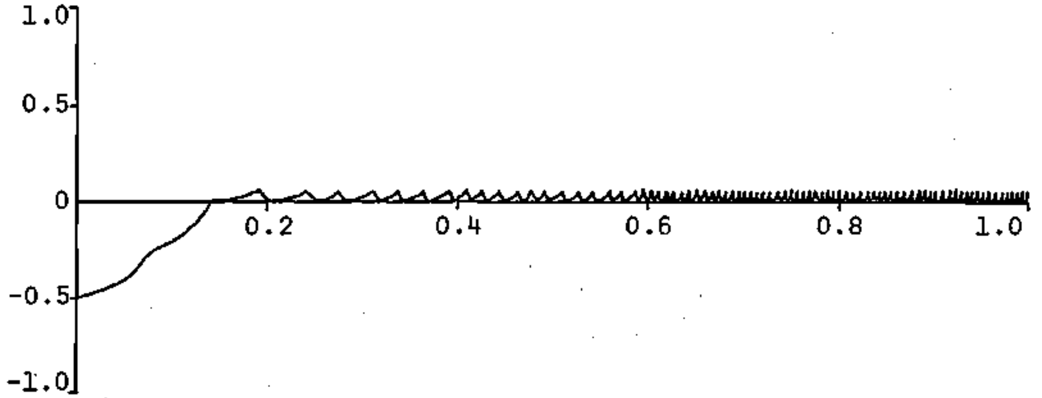

Figure 15(b). System Error for Example 9

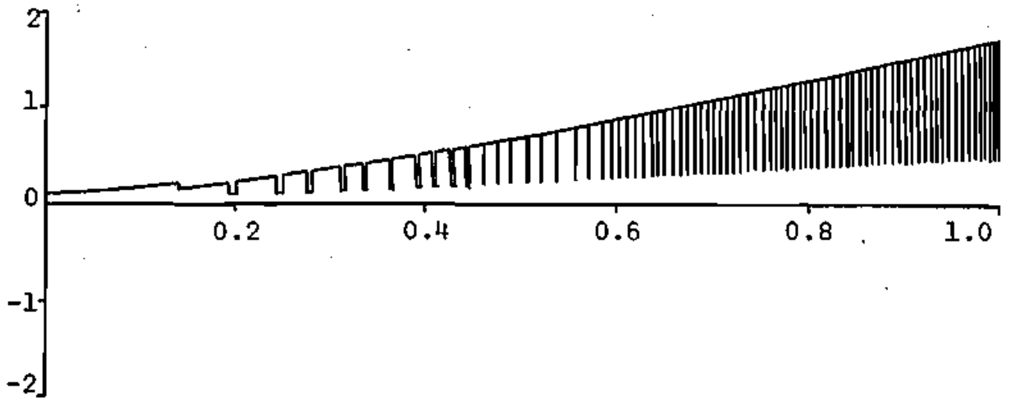

Figure 15(c). Synthesized Plant Input for Example 9

control policy quickly lost its advantage in this particular example. However, the modified control policy could be used to better advantage if the plant parameters were more precisely known.

The System Bandwidth

The original assumptions about the plant and model imply perfect measurements of the state variables. Considering systems whose measurements are corrupted by noise is beyond the scope of this thesis. However, some indication of how the system might behave in the presence of noise can be obtained by determining the closed loop poles of the plant as it responds to the model input. In the simpler cases, this information is equivalent to determining the system bandwidth.

For the linear time-invariant plant of (2.6) and the linear time-invariant model of (2.7), the control is

$$u = \frac{1}{\underline{C}^T\underline{B}} (\underline{H}^T F\underline{y} + \underline{H}^T G\underline{r} - \underline{C}^T A\underline{x} - ke) \qquad (2.86)$$

when the convergence factor is chosen as $k|e|$. Substituting (2.86) into (2.6) yields

$$\dot{\underline{x}} = A\underline{x} + \frac{\underline{B}}{\underline{C}^T\underline{B}} [\underline{H}^T F\underline{y} + \underline{H}^T G\underline{r} - \underline{C}^T A\underline{x} - k\underline{C}^T\underline{x} + k\underline{H}^T\underline{y}] \qquad (2.87)$$

Multiplying (2.87) by $\underline{C}^T$ and combining terms yields

$$\underline{C}^T\dot{\underline{x}} = \underline{H}^T F\underline{y} + \underline{H}^T G\underline{r} - k\underline{C}^T\underline{x} + k\underline{H}^T\underline{y} \qquad (2.88)$$

By substituting from (2.7) and combining terms, one obtains

$$\underline{C}^T\dot{\underline{x}} + k\underline{C}^T\underline{x} = \underline{H}^T\dot{\underline{y}} + k\underline{H}^T\underline{y} \tag{2.89}$$

Taking the Laplace transform of (2.89), one has

$$\begin{gathered}\underline{C}^T s\underline{X}(s) - \underline{C}^T\underline{x}(0) + k\underline{C}^T\underline{X}(s) = \\ \underline{H}^T s\underline{Y}(s) - \underline{H}^T\underline{y}(0) + \underline{H}^T\underline{Y}(s)\end{gathered} \tag{2.90}$$

or

$$\underline{C}^T\underline{X}(s) = \underline{H}^T\underline{Y}(s) + \frac{\underline{C}^T\underline{x}(0)}{s + k} - \frac{\underline{H}^T\underline{y}(0)}{s + k} \tag{2.91}$$

Substituting from (2.6) and (2.7), (2.91) becomes

$$Z(s) = W(s) + \frac{e(0)}{s + k} \tag{2.92}$$

Thus the closed loop poles of the plant are located at the poles of the model, with one additional pole defined by the convergence factor. Therefore, the system bandwidth remains finite and may be selected by choosing the model and the convergence factor. Analogous results, i.e., the dependence of bandwidth on the model and on the convergence factor, are valid when the plant and model are time-varying and nonlinear, but the corresponding analysis is considerably more complicated.

Summary

In this chapter the basic theory and techniques for the simplest single-input, single-output system were derived. The techniques were then extended to include both plants and models with nonlinearities and with time-varying parameters. Plants with incompletely specified parameters were then considered and control techniques for this case were presented. Two important special types of nonlinearities were handled, one type requiring the use of an integral type Liapunov function. Because the control can be abrupt when there are unknown plant parameters, a technique was presented which results in a smoother control.

The next chapter deals with using an expanded quadratic Liapunov function in the design of model reference adaptive control systems. One advantage is that a broad new class of plants can thereby be handled. Another advantage is that using additional terms in the Liapunov function can lead to the synthesis of a plant output u which has no discontinuities, even though the plant may have unknown parameters. Subsequent chapters deal with the extensions to multiple-input, multiple-output systems and to sampled-data systems.

CHAPTER III

RESULTS USING EXPANDED FORMS OF QUADRATIC LIAPUNOV FUNCTIONS

In Chapter II only the simplest quadratic Liapunov function, $V = \frac{1}{2} e^2$, was used. Proper application of the conditions for asymptotic stability of the system error resulted in useful synthesis techniques. These synthesis techniques were applied to a wide variety of plants and models to yield asymptotically stable model reference adaptive control systems.

The class of permissible plants has thus far been restricted to those plants whose matrix product $\underline{C}^T\underline{B}$ is nonzero. For linear nth order plants, this requirement means that the numerator of the transfer function must be of order n-1. A technique for handling a plant numerator of any order m less than n is presented in this chapter. The technique requires the use of a more elaborate quadratic Liapunov function.

It has been pointed out previously that the plant input has closely spaced jump discontinuities when there are unknown plant parameters and the system error is near zero. One way of circumventing this undesirable property was presented in Chapter II. A second method is presented in this chapter. The method is based on using a quadratic Liapunov function which involves successive derivatives of the system error.

A More General Quadratic Liapunov Function

The results presented in this chapter depend on choosing an expanded Liapunov function. The Liapunov function chosen is the more general quadratic form:

$$V = \frac{1}{2} e^2 + \frac{1}{2} P_1 \dot{e}^2 + \frac{1}{2} P_2 \ddot{e}^2 + \cdots \tag{3.1}$$

The number of terms that are used depends upon the order and form of the plant, the order and form of the model, and the desired behavior of the plant input.

The synthesis procedure is based on insuring the negative definiteness of the time derivative of the Liapunov function. For the system to be asymptotically stable, the plant input u must be such that

$$\dot{V} = e\dot{e} + P_1 \dot{e}\ddot{e} + P_2 \ddot{e}\dddot{e} + \cdots \leq 0 \tag{3.2}$$

with the equality holding only when V has been reduced to zero. Equation (3.2) is very unwieldy in general because of the number of matrix products which occur in the system error and its derivatives. A solution for the most general case is presented in the latter part of this chapter. However, to facilitate the presentation, some simpler special cases are first considered and discussed.

The Liapunov Function $V = \frac{1}{2}e^2 + \frac{1}{2}P\dot{e}^2$

Several of the effects of choosing an expanded quadratic Liapunov function can be observed when using the simple function

$$V = \frac{1}{2}e^2 + \frac{1}{2}P\dot{e}^2 \tag{3.3}$$

where P is a positive constant used as a weighting factor. The time derivative of the Liapunov function is

$$\dot{V} = e\dot{e} + P\dot{e}\ddot{e} \tag{3.4}$$

From Theorem 3, a sufficient condition for asymptotic stability is that (3.4) be negative semi-definite and that $\dot{V}$ not be zero along a system trajectory. Some care must be exercised when choosing a control u, or else it will be possible to drive $\dot{e}$ to zero and keep it there even though e is nonzero.

The control which forces V to zero asymptotically must satisfy the following four conditions:

$$\begin{aligned} &\text{When } \dot{e}>0\text{, make } \ddot{e}<-\frac{e}{P}\,; \\ &\text{When } \dot{e}<0\text{, make } \ddot{e}>-\frac{e}{P}\,; \\ &\text{When } \dot{e}=0 \text{ and } e>0\text{, make } \ddot{e}<0; \\ &\text{When } \dot{e}=0 \text{ and } e<0\text{, make } \ddot{e}>0. \end{aligned} \tag{3.5}$$

Thus $\dot{e}$ must be accessible for controlling this system. If the Liapunov function V is to remain at zero, the following condition must be satisfied:

$$\text{When } \dot{e} = 0 \text{ and } e = 0, \text{ make } \ddot{e} = 0. \qquad (3.6)$$

It is not possible to satisfy (3.6) when there are unknown parameters, just as it was not possible to make $\dot{e} = 0$ when $e = 0$ in Chapter II when there were unknown parameters.

There are a variety of ways which one might use to cause V to approach zero. Each depends upon the arbitrary function which is chosen to insure that the inequalities of (3.5) are satisfied. One effective way is to try to approach the point $e = 0$, $\dot{e} = 0$ along a straight line in the $\dot{e}$ versus e plane. For example, consider the line defined by

$$\dot{e} + e = 0 \qquad (3.7)$$

along which

$$\ddot{e} = -\dot{e} \qquad (3.8)$$

To make the system approach and then follow the line defined by (3.7), one proceeds in the following manner. Referring to Figure 16,

If the system trajectory is in Region I, make $\ddot{e} < 0$; (3.9)

If the system trajectory is in Region II, make $\ddot{e} > 0$;

If the system trajectory is on the boundary, make $\ddot{e} = -\dot{e}$.

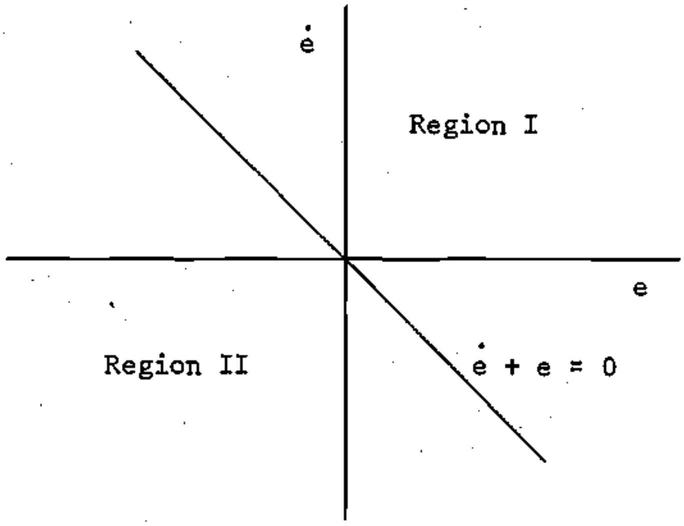

Figure 16. Regions in the $\dot{e}$ versus e Plane

In the exact case, (3.5)-(3.7) are satisfied with the control defined by

$$\ddot{e} = -\frac{e}{P} - \frac{2\dot{e}}{P} \qquad (3.10)$$

When there are unknown parameters, (3.5) and (3.7) can be satisfied only by considering the inequalities directly, since $\ddot{e}$ is not known when the plant is incompletely specified.

Consider the linear time-invariant plant given by

$$\underline{\dot{x}} = A\underline{x} + \underline{B}u \qquad (3.11)$$

$$z = \underline{C}^T\underline{x}$$

and the linear time-invariant model

$$\underline{\dot{y}} = F\underline{y} + \underline{G}r \tag{3.12}$$

$$w = \underline{H}^T\underline{y}$$

The system error e and its first two derivatives are calculated as

$$e = \underline{C}^T\underline{x} - \underline{H}^T\underline{y} \tag{3.13}$$

$$\dot{e} = \underline{C}^T A\underline{x} + \underline{C}^T\underline{B}u - \underline{H}^T F\underline{y} - \underline{H}^T\underline{G}r \tag{3.14}$$

$$\ddot{e} = \underline{C}^T A^2\underline{x} + \underline{C}^T A\underline{B}u + \underline{C}^T\underline{B}\dot{u} - \underline{H}^T F^2\underline{y} - \underline{H}^T F\underline{G}r - \underline{H}^T\underline{G}\dot{r} \tag{3.15}$$

There are four cases which occur. These cases are distinguished by whether the matrix products $\underline{C}^T\underline{B}$ and $\underline{C}^T A\underline{B}$ in (3.15) are nonzero. Case 1, when both matrix products are zero, occurs if an nth order plant has a numerator of order n-3 or less. An n-2 order numerator gives rise to Case 2, in which only $\underline{C}^T A\underline{B}$ is nonzero. One special type of n-1 order numerator results in Case 3, in which only $\underline{C}^T\underline{B}$ is nonzero. Case 4, when both matrix products are nonzero, is the general case of an nth order plant whose numerator is of order n-1. These four cases are discussed more thoroughly in the remainder of this section.

Case 1

When the numerator of an nth order plant is of order n-3 or less, the matrix products $\underline{C}^T\underline{B}$ and $\underline{C}^T A\underline{B}$ are both zero. Equation

(3.15) then reduces to

$$\ddot{e} = \underline{C}^T A^2 \underline{x} - \underline{H}^T F^2 \underline{y} - \underline{H}^T F \underline{G} r - \underline{H}^T \underline{G} \dot{r} \tag{3.16}$$

Since there are no terms involving u or $\dot{u}$ appearing in (3.16), the Liapunov function was not adequate for the plant chosen. The remedy is to add more terms to the Liapunov function. Terms must be added until u or its derivative appears explicitly when the highest derivative of e is calculated. This remedy is considered in more detail in a later section of this chapter.

Case 2

For nth order plants whose numerator is of order n-2, the matrix product $\underline{C}^T \underline{B}$ is zero, but the matrix product $\underline{C}^T A \underline{B}$ is nonzero. When this occurs, Equation (3.15) becomes

$$\ddot{e} = \underline{C}^T A^2 \underline{x} + \underline{C}^T A \underline{B} u - \underline{H}^T F^2 \underline{y} - \underline{H}^T F \underline{G} r - \underline{H}^T \underline{G} \dot{r} \tag{3.17}$$

The following example illustrates how (3.10) was applied to yield a control when the plant is known. In the example the slightly more complicated condition of a time-varying plant was considered.

Example 1. Let the plant be linear, time-varying, and of the form given in (2.25) with

$$A(t) = \begin{bmatrix} 0 & 1 \\ -2+\sin 8t & -2-\sin 4t \end{bmatrix}; \quad B(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{3.18}$$

$$C(t) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

The model is of the form given in (3.12), where

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}; \quad G = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{3.19}$$

$$H = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}.$$

The error and its first two derivatives are

$$e = z - w = 2x_1 - 6y_1 \tag{3.20}$$

$$\dot{e} = 2x_2 - 6y_2 \tag{3.21}$$

$$\ddot{e} = (-4+2 \sin 8t)x_1 - (4+2 \sin 4t)x_2 + 2u - 6y_3 \tag{3.22}$$

Choosing P = 1 and substituting (3.20)-(3.22) into (3.10), the defining equation for u becomes

$$(-4+2 \sin 8t)x_1 - (4+2 \sin 4t)x_2 + 2u - 6y_3 = \tag{3.23}$$

$$- 2x_1 + 6y_1 - 4x_2 + 12y_2$$

Thus the plant input is

$$u = (1 - \sin 8t)x_1 + x_2 \sin 4t + 3x_1 + 6y_2 + 3y_3 \qquad (3.24)$$

The results of a simulation of this system are shown in Figure 17. The input to the model was $1 + 0.1t^2$. The plant and model had zero initial conditions except for $y_1(0)$, which was $\frac{1}{6}$. The system behaved quite well, with the error approaching zero asymptotically.

Case 3

When the numerator of an nth order plant is of the form

$$a_1 s^{n-1} + a_3 s^{n-3} + a_4 s^{n-4} + \cdots \qquad a_1 \neq 0$$

the matrix product $\underline{C}^T\underline{B}$ is nonzero, but $\underline{C}^T A\underline{B}$ is zero. The equation for $\ddot{e}$ becomes

$$\ddot{e} = \underline{C}^T A^2 \underline{x} + \underline{C}^T \underline{B}\dot{u} - \underline{H}^T F^2 \underline{y} - \underline{H}^T F \underline{G} r - \underline{H}^T \underline{G} \dot{r} \qquad (3.25)$$

Since $\underline{C}^T\underline{B} \neq 0$, this case could have been handled by the techniques of Chapter II, by using the Liapunov function $V = \frac{1}{2} e^2$. The choice of a quadratic Liapunov function with two terms leads to a synthesis equation for $\dot{u}$, the first derivative of the plant input. The plant input u is then taken from the output of an integrator which is fed by the synthesized $\dot{u}$.

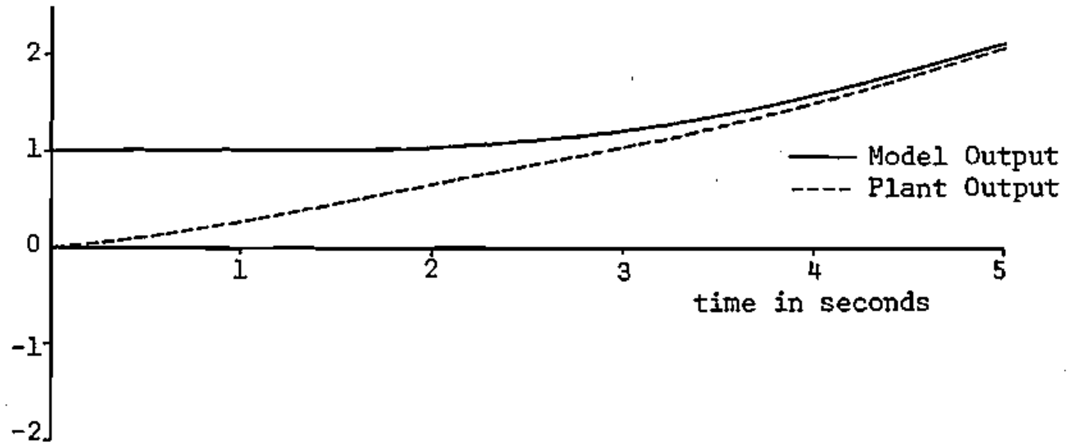

Figure 17(a). Plant and Model Output for Example 1

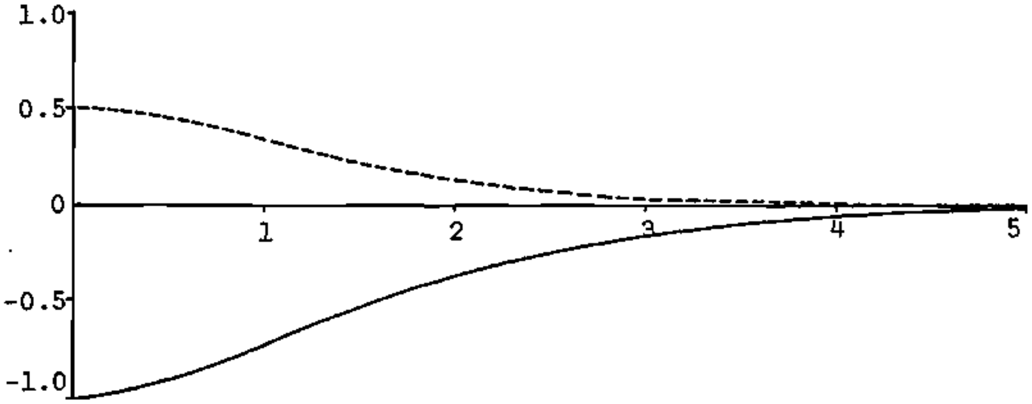

Figure 17(b). System Error and Liapunov Function for Example 1

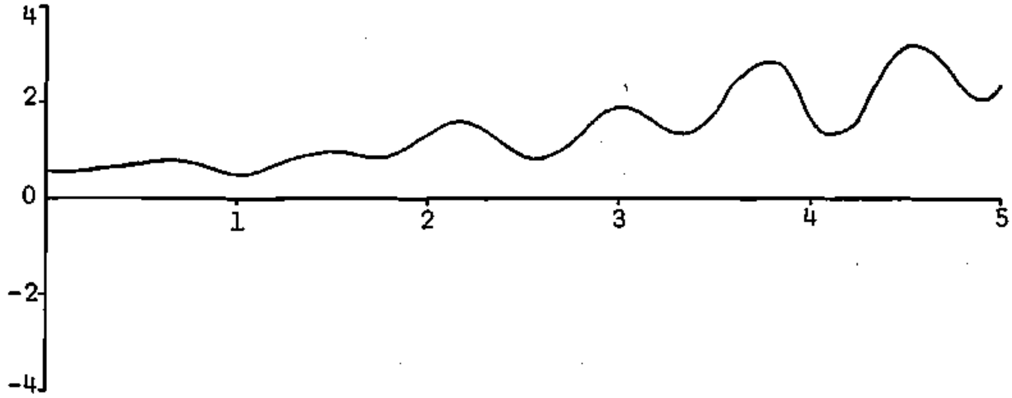

Figure 17(c). Synthesized Plant Input for Example 1

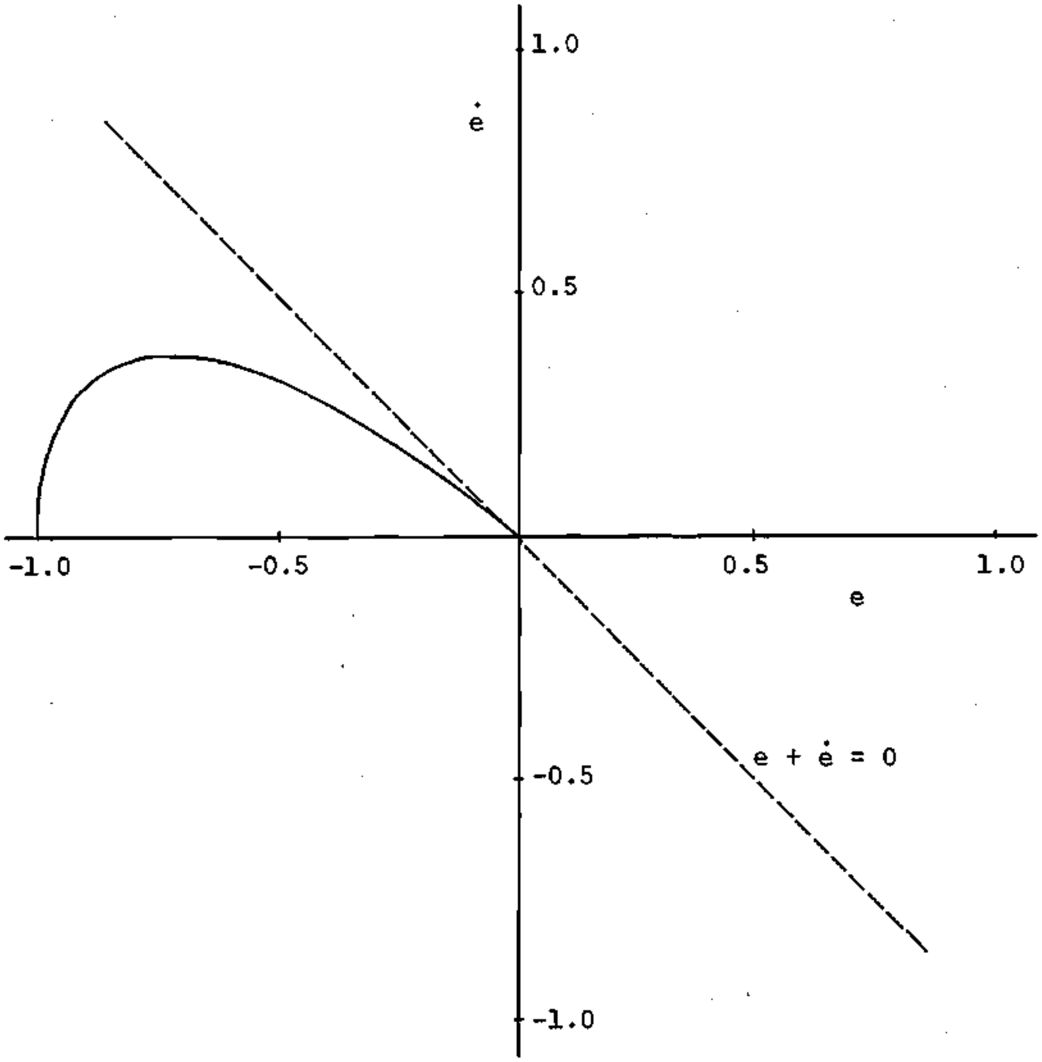

Figure 17(d). Phase Plane Trajectory of the System Error for Example 1

Case 4

For an nth order plant with a numerator of the form

$$a_1 s^{n-1} + a_2 s^{n-2} + \cdots \qquad a_1 \neq 0,\ a_2 \neq 0$$

the matrix products $\underline{C}^T\underline{B}$ and $\underline{C}^T A\underline{B}$ are both nonzero. In this case the equation for $\ddot{e}$ is the same as (3.15), i.e.

$$\ddot{e} = \underline{C}^T A^2 \underline{x} + \underline{C}^T A\underline{B}u + \underline{C}^T\underline{B}\dot{u} - \underline{H}^T F^2 \underline{y} - \underline{H}^T F\underline{G}r - \underline{H}^T\underline{G}\dot{r} \qquad (3.26)$$

Using (3.26) leads to an equation for $\dot{u}$. The implementation involves an integrator with feedback, since the equation also involves u. Example 2 illustrates how (3.26) is applied to such a problem.

Example 2. The plant and model used are the same as the plant and model used in Example 5 of Chapter II. Because the Liapunov function involves two terms and the plant numerator is of order n-1, the resulting u is the output of an integrator whose input is $\dot{u}$. For the plant

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -d & -c & -b \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad C = \begin{bmatrix} 0 \\ a \\ 100 \end{bmatrix}. \qquad (3.27)$$

with known bounds

$$50 \le a \le 100$$

$$1.6 \le b \le 3.6$$

$$24 \le c \le 102$$

$$36 \le d \le 300$$

For the model,

$$F = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}; \quad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad H = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}. \tag{3.28}$$

Because the matrix product $\underline{H}^T\underline{G}$ is non-zero, $\dot{r}$ is required to either be available or have known finite bounds. In this example, it is assumed to be available. Performing the indicated matrix operations and substituting $r = 10(1-t)$ and $\dot{r} = -10$ in (3.26) yields

$$\begin{aligned} \ddot{e} = {} & -d(b-0.01a)x_1 - [c(b-0.01a) - d]x_2 - [b(b-0.01a) - \\ & c]x_3 - (b-0.01a)u + 0.03y_1 + 0.01y_2 - 0.25 + 0.15t \end{aligned} \tag{3.29}$$

Choosing $P = 1$ and applying (3.5) and (3.7), $\dot{u}$ becomes

$$\begin{aligned} \dot{u} = {} & -475.8x_1 + 57x_3 + 1.85u + 0.03y_1 + 0.01y_2 - 0.25 + 0.15t \\ & -0.01e - 0.02\dot{e} - \{454.2|x_1| + 285.6|x_2| + 44|x_3| + 1.25|u|\} \end{aligned} \tag{3.30}$$

$$\{\operatorname{sign}\dot{e} + (1 - |\operatorname{sign}\dot{e}|)\operatorname{sign} e\}$$

The results of a simulation of this system are shown in Figure 18.

When the quadratic Liapunov function $V = \frac{1}{2}e^2 + \frac{1}{2}P\dot{e}^2$ is used, the term $\underline{H}^T\underline{G}\dot{\underline{r}}$ appears in the equation for $\ddot{e}$. By choosing an mth order model with a numerator of order m-2, the matrix product $\underline{H}^T\underline{G}$ is zero. If it is impractical to choose such a model, then $\dot{r}$ must be available.

An Alternate Liapunov Function

Figure 18(d) shows the results of a characteristic of the control when there are unknown parameters: the convergence terms which insured that $\dot{V}$ was negative semi-definite acted to force $\dot{e}$ to zero. This negated the effects of the convergence terms which were trying to force $\dot{e}$ to equal $-e$. The effect was that although the system error still approached zero asymptotically, it tried to do so along the line $\dot{e} = 0$. Thus the error decreased very slowly.

An effective means of circumventing this problem is to choose as a Liapunov function

$$V = \frac{1}{2}(e + \dot{e})^2 \tag{3.31}$$

which is positive except along the line $e + \dot{e} = 0$. The derivative of (3.31) is

$$\dot{V} = (e + \dot{e})(\dot{e} + \ddot{e}) \tag{3.32}$$

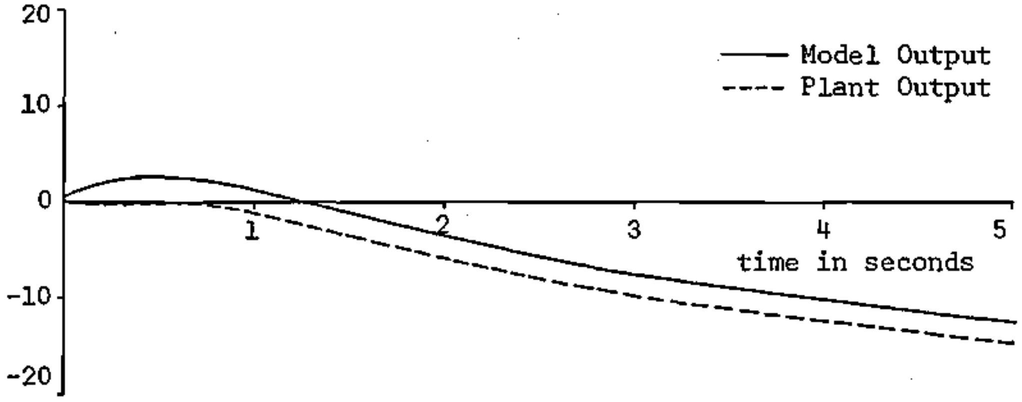

Figure 18(a). Plant and Model Output for Example 2

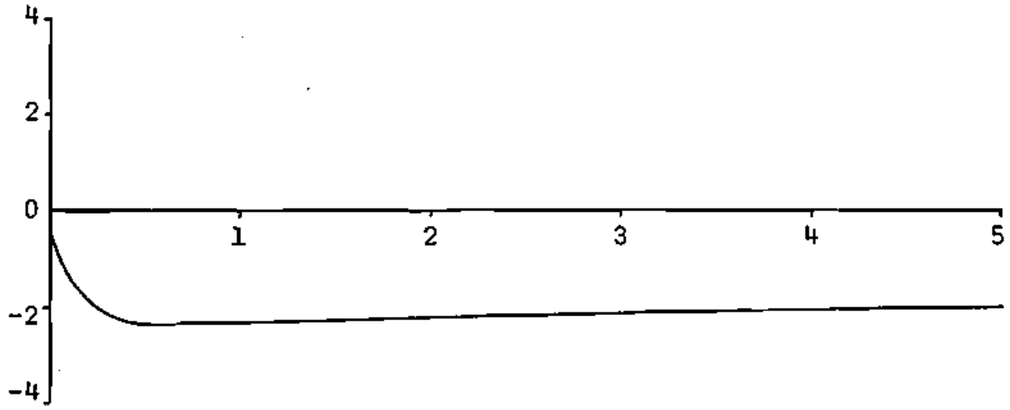

Figure 18(b). System Error for Example 2

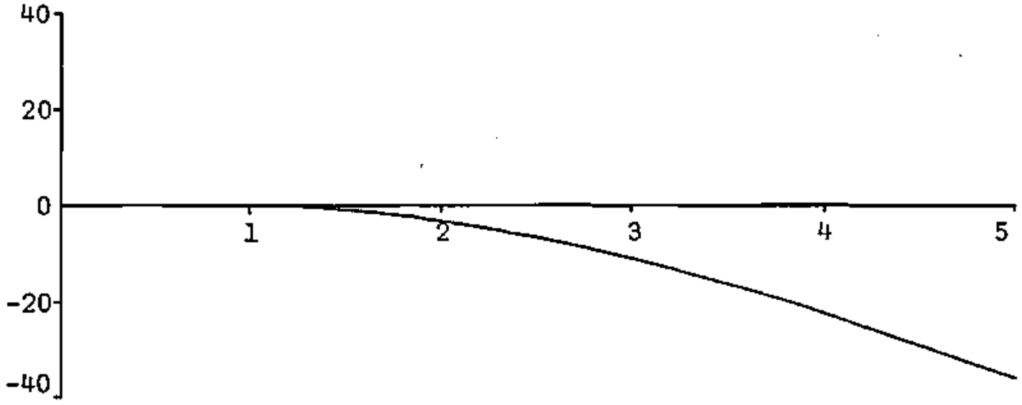

Figure 18(c). Synthesized Plant Input for Example 2

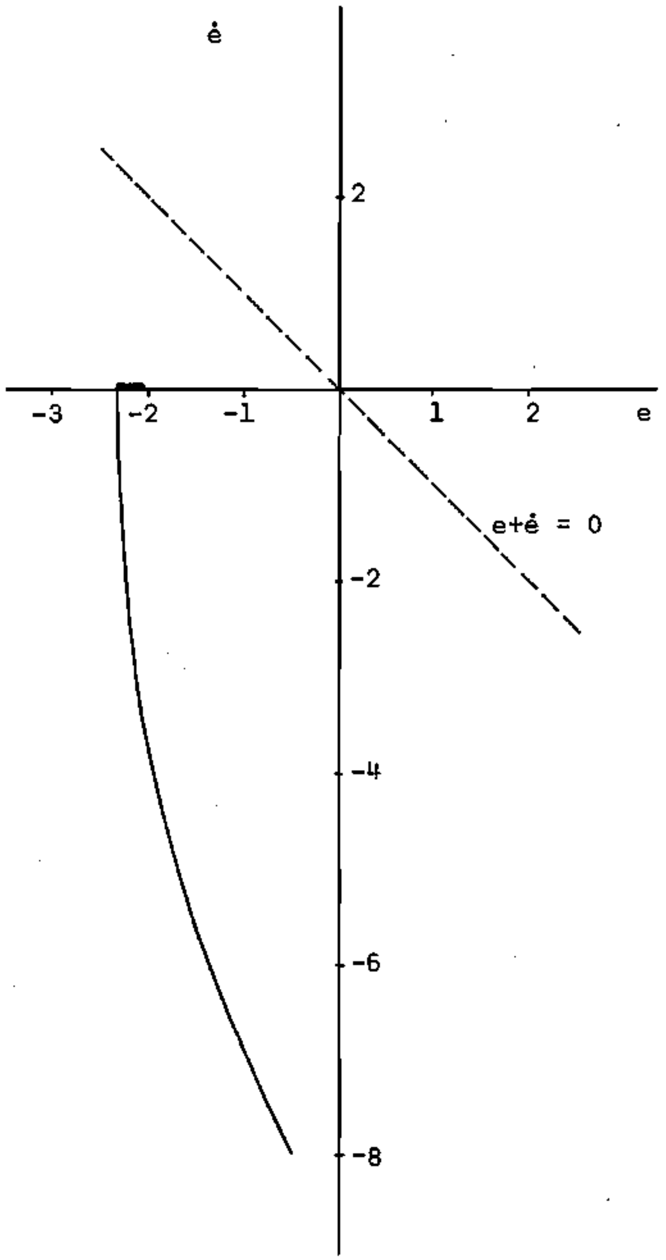

Figure 18(d). Phase Plane Trajectory of the System Error for Example 2

Equation (3.32) is made negative definite by controlling $\ddot{e}$ so that

$$\text{When } e+\dot{e}>0, \quad \ddot{e}<-\dot{e}; \tag{3.33}$$

$$\text{When } e+\dot{e}<0, \quad \ddot{e}>-\dot{e}.$$

However, if V is to remain at zero, the control must also make $\ddot{e}=-\dot{e}$ when $e+\dot{e}=0$. A control which satisfies these requirements will yield the desired system performance. It will bring the $\dot{e}$ versus e trajectory toward the line $e+\dot{e}=0$, and then toward the origin along the line. The following example illustrates this method on the familiar system of Example 2.

Example 3

The plant, model, model input, and initial conditions are the same as those of Example 2. Choosing as a Liapunov function $V = \frac{1}{2}(e+\dot{e})^2$, the control equation becomes

$$\dot{u} = -475.8x_1 + 57x_3 + 1.85u + 0.03y_1 + 0.01y_2 \tag{3.34}$$

$$-0.25 + 0.15t - 0.01\dot{e} + \{454.2|x_1|$$

$$+285.6|x_2| + 44|x_3| + 1.25|u|\}\text{sign}(e+\dot{e})$$

The results of the simulation are shown in Figure 19. Of particular significance is the $\dot{e}$ versus e trajectory in Figure 19(d). This figure shows that the control defined by (3.34) did indeed force the error and its first derivative to the origin.

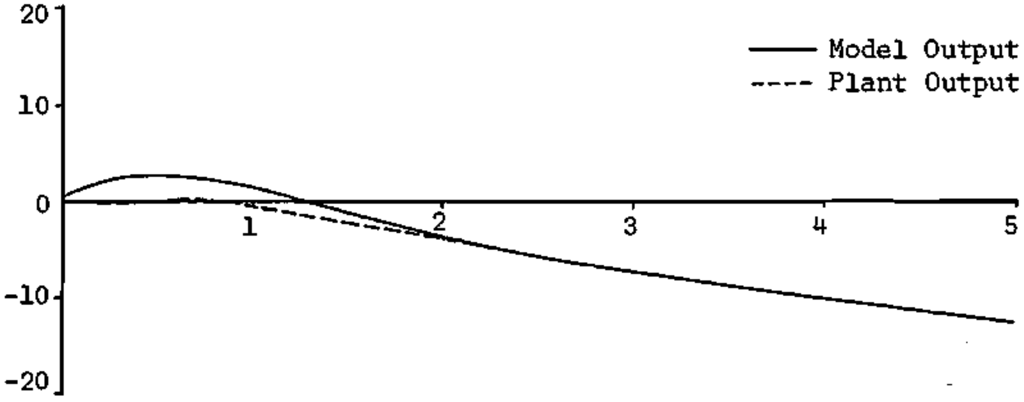

Figure 19(a). Plant and Model Output for Example 3

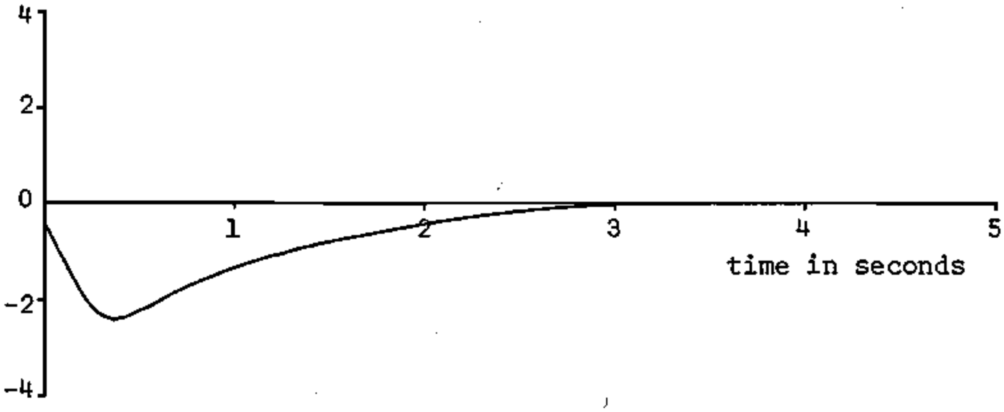

Figure 19(b). System Error for Example 3

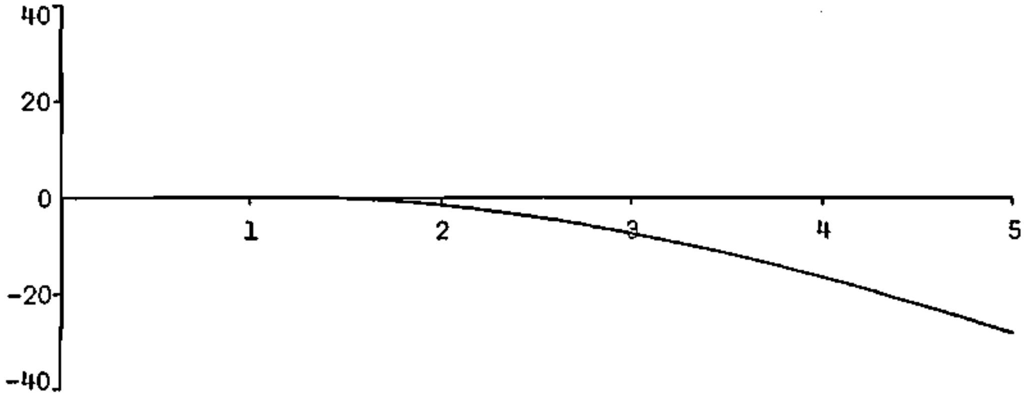

Figure 19(c). Synthesized Plant Input for Example 3

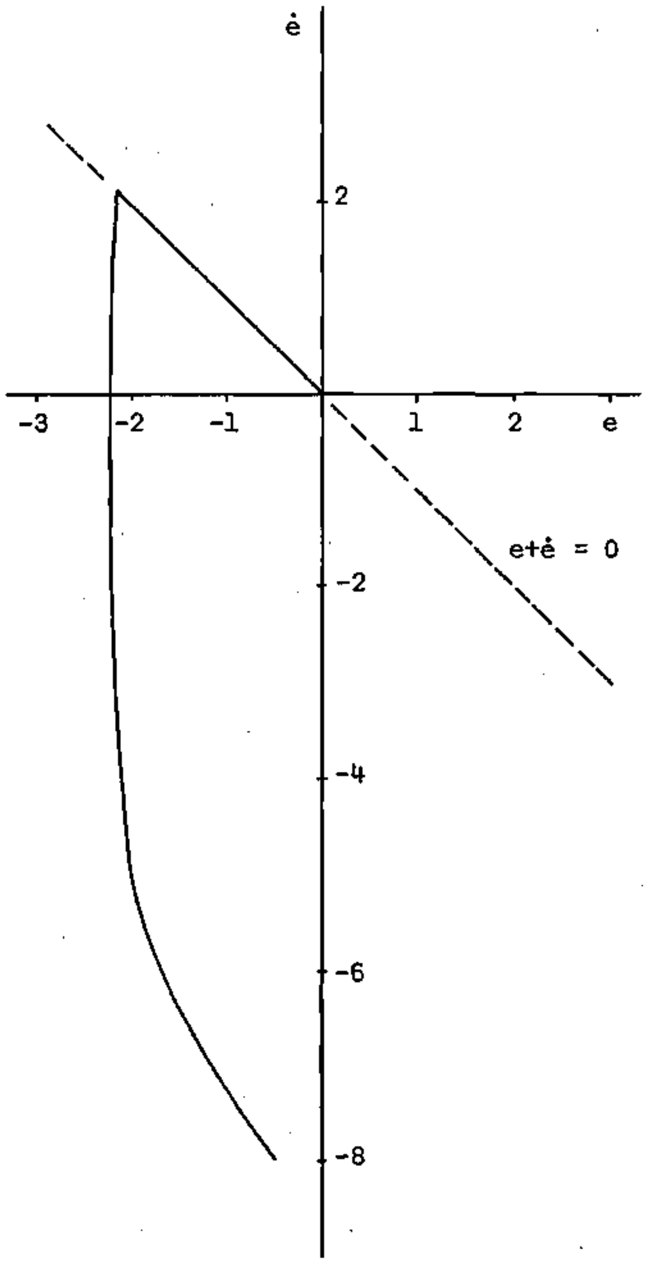

Figure 19(d). Phase Plane Trajectory of the System Error for Example 3

The Time-Varying Plant or Model

The derivations in the previous sections were for a linear time-invariant plant and a linear time-invariant model. The techniques which have been presented are also applicable to the case where either the plant or the model or both are time-varying. One difference which results from this extension is that there are many more terms in the equations for $\dot{e}$ and $\ddot{e}$. Another is that when the plant has unknown parameters, knowledge about the behavior of the first and second derivatives of some of these parameters are required. Consider the general time-varying plant

$$\underline{\dot{x}} = A(t)\underline{x} + \underline{B}(t)u \tag{3.35}$$

$$z = \underline{C}^T(t)\underline{x}$$

and the general time-varying model

$$\underline{\dot{y}} = F(t)\underline{y} + \underline{G}(t)r \tag{3.36}$$

$$w = \underline{H}^T(t)\underline{y}$$

The system error and its first two derivatives are

$$e = z - w = \underline{C}^T(t)\underline{x} - \underline{H}^T(t)\underline{y} \tag{3.37}$$

$$\begin{aligned} \dot{e} = {} & \underline{\dot{C}}^T(t)\underline{x} + \underline{C}^T(t)A(t)\underline{x} + \underline{C}^T(t)\underline{B}(t)u - \underline{\dot{H}}^T(t)\underline{y} \\ & - \underline{H}^T(t)F(t)\underline{y} - \underline{H}^T(t)\underline{G}(t)r \end{aligned} \tag{3.38}$$

$$\begin{aligned} \ddot{e} = {} & \underline{\ddot{C}}^T(t)\underline{x} + 2\underline{\dot{C}}^T(t)A(t)\underline{x} + 2\underline{\dot{C}}^T(t)\underline{B}(t)\dot{u} + \underline{C}^T(t)\dot{A}(t)\underline{x} \\ & + \underline{C}^T(t)A^2(t)\underline{x} + \underline{C}^T(t)A(t)\underline{B}(t)u + \underline{C}^T(t)\underline{\dot{B}}(t)u + \underline{C}^T(t)\underline{B}(t)\dot{u} \\ & - \ddot{H}^T(t)\underline{y} - 2\underline{\dot{H}}^T(t)F(t)\underline{y} - 2\underline{\dot{H}}^T(t)\underline{G}(t)r - \underline{H}^T(t)\dot{F}(t)\underline{y} \\ & - \underline{H}^T(t)F^2(t)\underline{y} - \underline{H}^T(t)F(t)\underline{G}(t)r - \underline{H}^T(t)\underline{\dot{G}}(t)r - \underline{H}^T(t)\underline{G}(t)\dot{r} \end{aligned} \tag{3.39}$$

When (3.37)-(3.39) are substituted into (3.10), the result is a lengthy equation for $\dot{u}$. If there are unknown parameters, the procedure becomes a test of the endurance of the designer.

The Nonlinear Plant and Model

The most general nonlinear plant is

$$\begin{aligned} \underline{\dot{x}} &= \underline{f}(\underline{x}) + \underline{B}u \\ z &= \underline{C}^T\underline{x} \end{aligned} \tag{3.40}$$

The most general nonlinear model is

$$\begin{aligned} \underline{\dot{y}} &= \underline{g}(\underline{y},r) \\ w &= \underline{H}^T\underline{y} \end{aligned} \tag{3.41}$$

The system error and its first two derivatives are

$$e = \underline{C}^T\underline{x} - \underline{H}^T\underline{y} \tag{3.42}$$

$$\dot{e} = \underline{C}^T\underline{f}(\underline{x}) + \underline{C}^T\underline{B}u - \underline{H}^T g(\underline{y},r) \tag{3.43}$$

$$\ddot{e} = \underline{C}^T\underline{f}\left(\underline{f}(\underline{x}) + \underline{B}u\right) + \underline{C}^T\underline{B}\dot{u} - \underline{H}^T \frac{d}{dt}\left(\underline{g}(\underline{y},r)\right) \tag{3.44}$$

Equations (3.41)-(3.43) are substituted into (3.10) to yield an equation involving u and $\dot{u}$. In general the situation is quite hopeless because of the nonlinear term $C^T f\left(f(x) + Bu\right)$ in (3.44). There are, however, special cases in which the procedure will be usable.

The nonlinear, time-varying plant and model are treated similarly to the nonlinear plant and model. There are, of course, additional terms occurring in the equations for $\dot{e}$ and $\ddot{e}$ because of the time dependence. Thus increased complexity is the price one must pay to control more general plants.

A quadratic Liapunov function with two terms has been considered in this section. The requirement that $\dot{V}$ be negative definite gave rise to four cases. These four cases were distinguished by whether the matrix products $\underline{C}^T\underline{B}$ and $\underline{C}^T A\underline{B}$ are zero or nonzero. Two improvements over the techniques of Chapter II should be noted:

1. Plants which could not be handled by the techniques of Chapter II can be handled when $\underline{C}^T A\underline{B}$ is nonzero.

2. The technique gave rise to a synthesis equation for $\dot{u}$ when $\underline{C}^T\underline{B}\neq 0$, thus causing the plant input to be continuous.

However, these improvements are obtained only with these additional system requirements:

1. If the matrix product $\underline{H}^T\underline{G}$ is nonzero, the first derivative of the model input is required in the synthesis equation.

2. The extra terms in the Liapunov function increase the complexity of the design, especially when there are nonlinearities or unknown parameters.

3. The technique requires that $\dot{e}$ be accessible.

Using a More General Quadratic Form

In the preceding sections the implications of using the quadratic Liapunov function $V = \frac{1}{2} e^2 + \frac{1}{2} P\dot{e}^2$ were discussed in detail. The procedure for using a quadratic form with more than two terms is straightforward. The Liapunov function V is defined and its first derivative $\dot{V}$ is calculated. The error e and its derivatives are calculated using plant and model matrices. These terms are substituted into the equation for $\dot{V}$, and the highest derivative of u is isolated. An equation is then written for this term such that $\dot{V}$ is negative definite. The highest derivative of u which appears is then integrated a sufficient number of times to get u, the synthesized plant input.

Conclusion

Using the techniques presented in this chapter, an nth order plant with any numerator of order less than n may be controlled by a model reference adaptive control scheme. This greatly extends the class of plants which can be handled. Adding more terms to the Liapunov function chosen can cause the plant input u to be continuous and have continuous derivatives. This feature can improve the performance of the system, but at the cost of increased complexity and slower convergence of the error to zero. It is important to give careful consideration to the model chosen. If the order of the model numerator and the model denominator differ by less than the number of terms in the Liapunov function, derivatives of the model input will be required in the synthesis procedure.

In the following chapter, the techniques of this chapter and the preceding one are applied to multiple-input, multiple-output systems.

CHAPTER IV

MULTIPLE-INPUT MULTIPLE-OUTPUT SYSTEMS

The material in Chapters II and III dealt with designing controllers for model reference adaptive control systems which have single-input, single-output plants and models. The techniques are applicable to very broad classes of such plants and models, but there are also many important plants which have more than one input and more than one output. In this chapter the techniques of the previous two chapters are extended to handle multiple-input, multiple-output plants and models.

Use of the Liapunov Function $V = \frac{1}{2}\underline{e}^T P \underline{e}$

The mathematics for treating multiple-input, multiple-output systems is similar in several ways to the mathematics for treating single-input, single-output control systems. The chief difference is the increase in the dimensionality of the various matrices. The inputs u and r and the outputs z and w become vectors, where formerly they were scalars. The vector quantities B, C, G, and H become matrices.

Consider the linear time-invariant plant

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{4.1}$$

$$\underline{z} = C^T\underline{x}$$

and the linear, time-invariant model

$$\dot{\underline{y}} = F\underline{y} + G\underline{r} \tag{4.2}$$

$$\underline{w} = H^T\underline{y}$$

Define the system error vector as

$$\underline{e} = \underline{z} - \underline{w} \tag{4.3}$$

For (4.3) to be meaningful, the vectors $\underline{z}$ and $\underline{w}$ must have the same dimensions. Thus the model which is chosen must have the same number of outputs as the plant. To control each plant output, it is generally necessary that there be at least as many plant inputs as there are plant outputs. It is assumed throughout the remainder of this thesis that the number of plant inputs and outputs are equal. There are no restrictions on the number of model inputs.

The synthesis procedure uses the Liapunov function

$$V = \frac{1}{2}\underline{e}^T P\underline{e} \tag{4.4}$$

where $\underline{e}$ is defined in (4.3) and P is a diagonal positive definite matrix. The derivative of (4.4) is

$$\dot{V} = \underline{e}^T P\dot{\underline{e}} \tag{4.5}$$

and the design procedure is to synthesize $\underline{u}$ so that $\dot{V}$ is made negative definite. Upon substituting the plant and model equations, the requirement on (4.5) becomes

$$\dot{V} = (C^T\underline{x} - H^T\underline{y})^T P(C^T A\underline{x} + C^T B\underline{u} - H^T F\underline{y} - H^T G\underline{r}) \leq 0 \qquad (4.6)$$

As in the single-input, single-output systems of Chapter II, the feasibility of the system design depends on the matrix product C^TB. The easiest system to handle is the one in which C^TB is diagonal with all nonzero elements on the diagonal. In that case, there is one and only one component of $\underline{u}$ associated with each component of $\underline{e}$. Each u_i is then synthesized to control its associated e_i via the methods of Chapter II, as if it were a single-input, single-output system. When the matrix product has nonzero off-diagonal elements, it is often not possible to use the techniques of this chapter to design a model reference adaptive control system for that particular plant.

It is possible to use a time-varying plant,

$$\dot{\underline{x}} = A(t)\underline{x} + B(t)\underline{u} \qquad (4.7)$$

$$\underline{z} = C^T(t)\underline{x}$$

a nonlinear plant,

$$\dot{\underline{x}} = \underline{f}(\underline{x}) + B\underline{u} \qquad (4.8)$$

$$\underline{z} = C^T\underline{x}$$

or a nonlinear, time-varying plant

$$\dot{\underline{x}} = \underline{f}(\underline{x},t) + B(t)\underline{u} \tag{4.9}$$

$$\underline{z} = C^T(t)\underline{x}$$

Similarly, the model may be time varying,

$$\dot{\underline{y}} = F(t)\underline{y} + G(t)\underline{r} \tag{4.10}$$

$$\underline{w} = H^T(t)\underline{y}$$

nonlinear,

$$\dot{\underline{y}} = \underline{g}(\underline{y},\underline{r}) \tag{4.11}$$

$$\underline{w} = H^T\underline{y}$$

or nonlinear and time-varying

$$\dot{\underline{y}} = \underline{g}(\underline{y},\underline{r},t) \tag{4.12}$$

$$\underline{w} = H^T(t)\underline{y}$$

In addition, the model may have separable input nonlinearities, in which case $\underline{r}$ is replaced by $\underline{h}(\underline{r})$, or by $\underline{h}(\underline{r},t)$ if the nonlinearity is time-varying. As in the single-input, single-output case, any model form may be used with any plant, subject to the restrictions on the numbers of inputs and outputs.

Treating Cross-Coupling Effects

One of the difficulties of controlling multiple-input, multiple-output plants is caused by cross-coupling: one input affects more than one output. When using classical design techniques, one would sometimes like to have each output affected by only one input. In such a case, the design of multiple-input, multiple-output systems is accomplished by treating the system as if it were composed of many independent single-input, single-output systems.

An interesting application of the techniques of this chapter is in the design of non-interacting multiple-input, multiple-output control systems. One simply selects a non-interacting model which has the desired dynamic characteristics. This model is then used, along with the given plant, in the model reference adaptive control system. Each plant output follows the corresponding model output as the model responds to the external inputs. The overall system, to the external observer, is a non-interacting multivariable system, with each subsystem having the desired dynamic characteristics. An important advantage of applying the techniques of this thesis to the problem of designing non-interacting control systems is that the design is possible, although difficult, even when the plant is nonlinear, time-varying, and has incompletely specified parameters.

The example which follows illustrates how the techniques are used to design a controller which effectively decoupled a multiple-input, multiple-output plant.

Example 1

The plant in (4.13) represents a particular turbo-prop engine with the following inputs and outputs [28]:

u_1 propeller blade angle
u_2 fuel rate
z_1 engine speed
z_2 turbine inlet temperature

The plant differential equations are

$$\dot{x}_1 = -x_1 + u_1 \tag{4.13}$$

$$\dot{x}_2 = -x_1 + u_1$$

$$\dot{x}_3 = -x_3 + u_2$$

$$\dot{x}_4 = -x_4 + u_2$$

$$z_1 = -2x_1 + 3x_3$$

$$z_2 = 4x_2 - 6x_4 + 8u_2$$

The non-interacting model is

$$\dot{y}_1 = -y_1 + r_1 \qquad (4.14)$$

$$\dot{y}_2 = -y_2 + r_2$$

$$w_1 = -2y_1$$

$$w_2 = -6y_2 + 8r_2$$

With P chosen as the 2 by 2 identity matrix, (4.5) becomes

$$\dot{V} = (z_1 - w_1)(\dot{z}_1 - \dot{w}_1) + (z_2 - w_2)(\dot{z}_2 - \dot{w}_2) \leq 0 \qquad (4.15)$$

which, upon substituting from (4.13) and (4.14) results in

$$\dot{V} = (-2x_1 + 3x_3 + 2y_1)(2x_1 - 3x_3 - 2u_1 + 3u_2 - 2y_1 + 2r_1) \qquad (4.16)$$

$$+ (4x_2 - 6x_4 + 8u_2 + 6y_2 - 8r_2)(-4x_2 + 6x_4 + 4u_1 - 6u_2$$

$$+ 8\dot{u}_2 - 6y_2 + 6r_2 - 8\dot{r}_2) \leq 0$$

From the first part of (4.16) a synthesis equation for u_1 is obtained, and from the second part of (4.16), a synthesis equation for $\dot{u}_2$ is obtained. These equations are

$$u_1 = x_1 - 1.5x_3 + 1.5u_2 - y_1 + r_1 + k_1e_1 \qquad (4.17)$$

and

$$\dot{u}_2 = 0.5x_2 - 0.75x_4 - 0.5u_1 + 0.75u_2 + 0.75r_2 + \dot{r}_2 - k_2e_2 \qquad (4.18)$$

The results of a simulation of this system are shown in Figure 20. The plant followed the non-interacting model with exponentially decreasing errors.

The General Quadratic Liapunov Function

It is quite possible that when (4.6) is calculated, the matrix product C^TB will be a diagonal matrix with one or more zeros on the main diagonal. When this occurs it is necessary to judiciously add more terms to the Liapunov function so that these channels may be controlled by means of the methods of Chapter III. The Liapunov function to be selected has the form

$$V = \underline{e}^T P_1 \underline{e} + \underline{\dot{e}}^T P_2 \underline{\dot{e}} + \underline{\ddot{e}}^T P_3 \underline{\ddot{e}} + \cdots \qquad (4.19)$$

where P_2 is a diagonal matrix having nonzero elements in those positions along the main diagonal where C^TB has zero elements. P_3 is diagonal and has nonzero elements where C^TB and C^TAB both have zeros. For instance, for

$$C^TB = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad (4.20)$$

choose

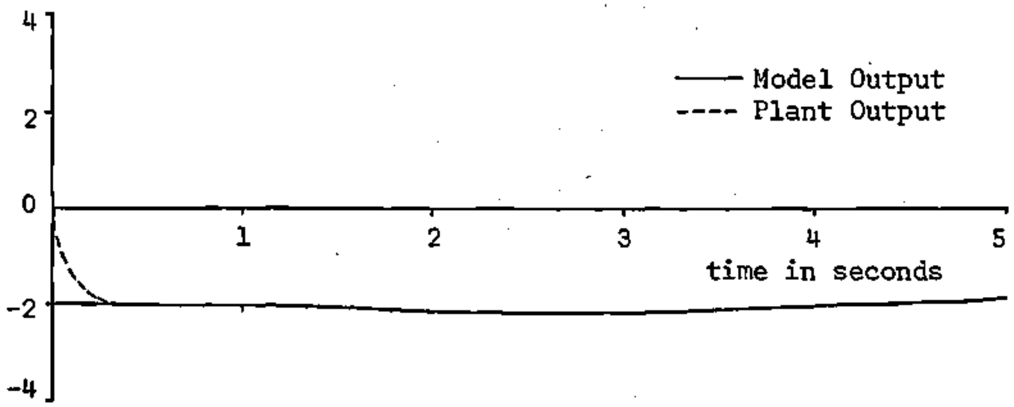

Figure 20(a). Plant and Model Output for Channel 1, Example 1

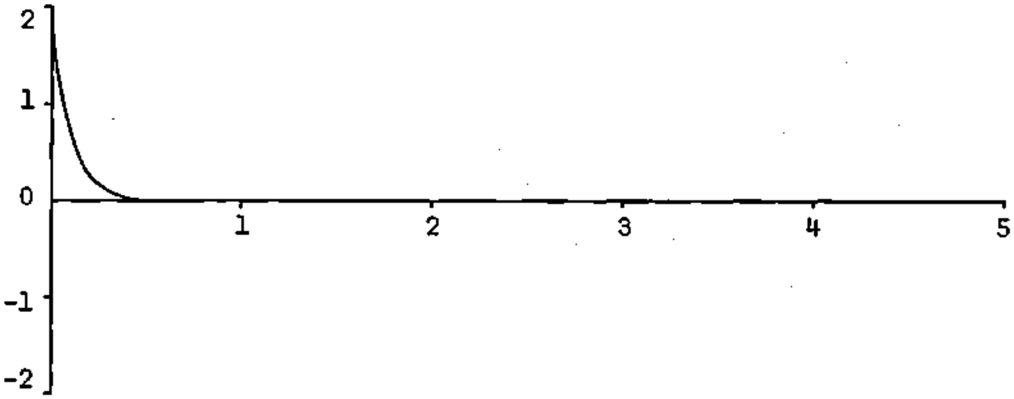

Figure 20(b). Channel 1 Error for Example 1

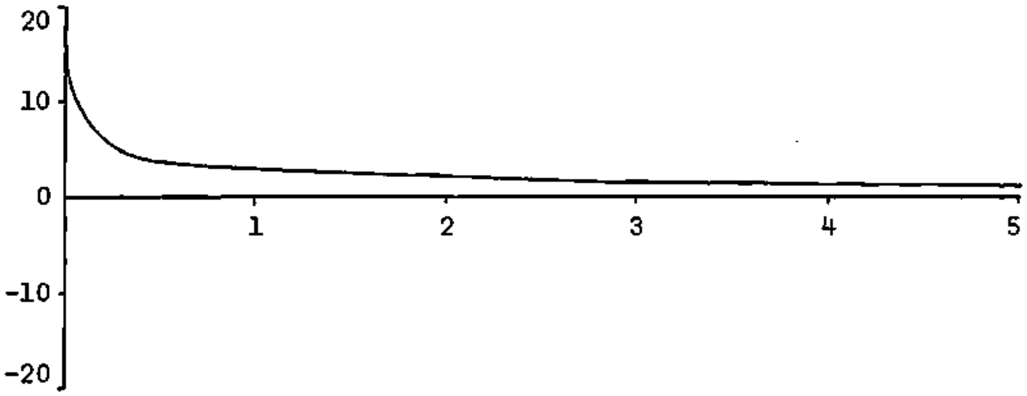

Figure 20(c). Channel 1 Plant Input for Example 1

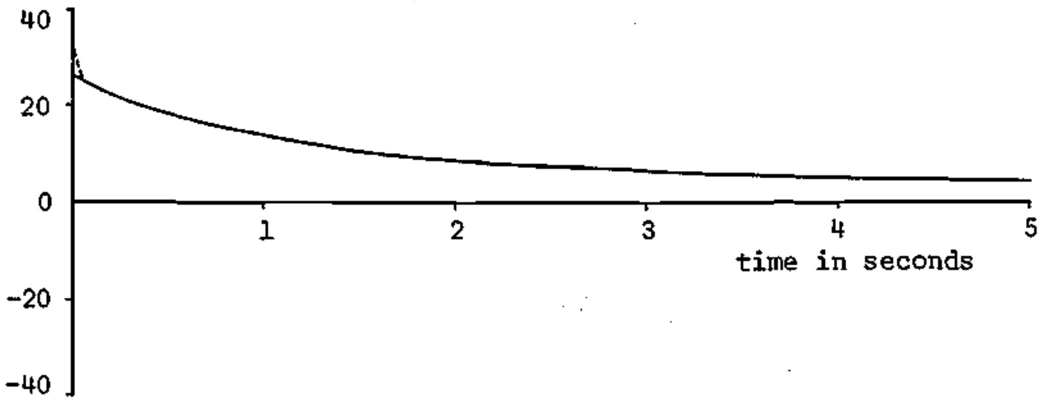

Figure 20(d). Plant and Model Output for Channel 2, Example 1

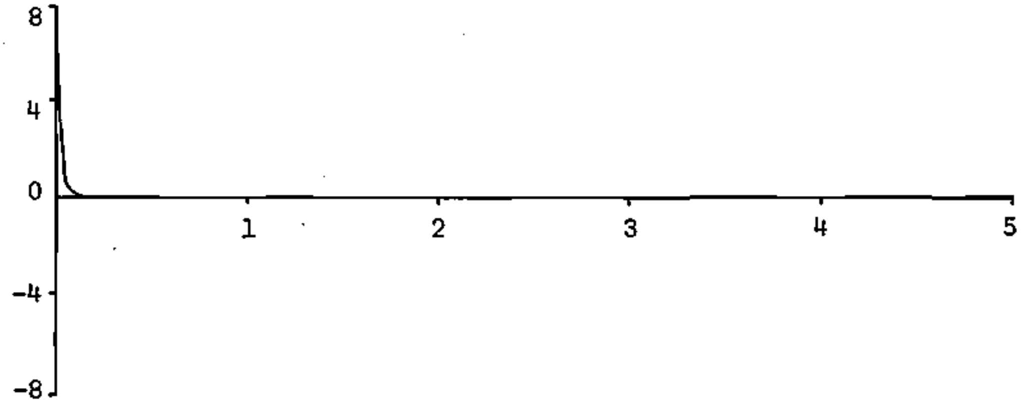

Figure 20(e). Channel 2 Error for Example 1

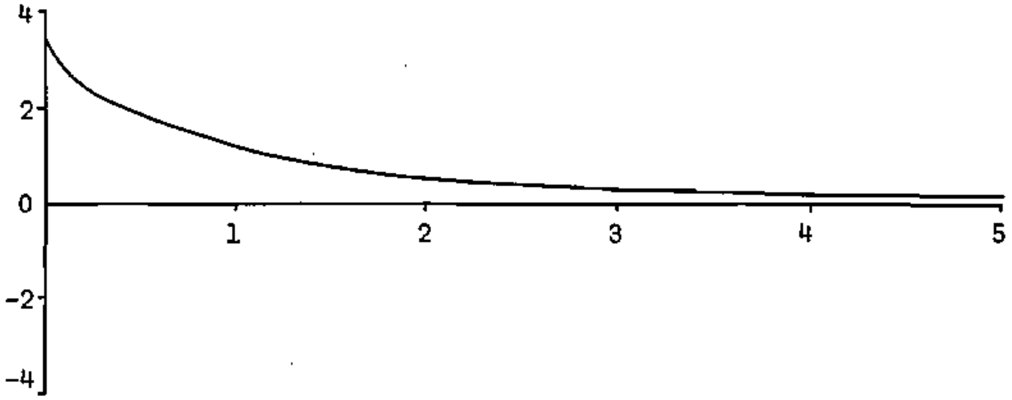

Figure 20(f). Channel 2 Plant Input for Example 1

$$P_2 = \begin{bmatrix} k_1 & 0 \\ 0 & 0 \end{bmatrix}; \quad k_1 > 0 \tag{4.21}$$

and *if*

$$C^T AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{4.22}$$

choose

$$P_3 = \begin{bmatrix} k_2 & 0 \\ 0 & 0 \end{bmatrix}; \quad k_2 > 0 \tag{4.23}$$

The above procedure permits each channel to be controlled using the fewest number of Liapunov terms. However, one may wish to add more terms, when there are unknown parameters, to synthesize a smoother control.

Example 2

The plant is linear, time-varying, and has two inputs and two outputs. The plant matrices are

$$A(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -2 & -a & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -b & -1 & 0 \\ 0 & 0 & 0 & 0 & -c \end{bmatrix}; \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4.24}$$

$$C = \begin{bmatrix} 0 & 2 \\ 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 3 & 0 \end{bmatrix}$$

The known bounds are

$$1.5 \le a \le 2.5$$

$$2.5 \le b \le 3.5$$

$$0.2 \le c \le 1$$

and the parameters used in the simulation were

$$a = 2 \qquad (4.25)$$

$$b = 3$$

$$c = 0.5 + 0.2 \cos 3t$$

The model has two inputs and two outputs, and is a non-interacting model. Both channels are second order, linear, time-invariant, and have input nonlinearities:

$$h_1(r_1) = \mathrm{SAT}(r_1) \qquad h_2(r_2) = \mathrm{SAT}(r_2) \qquad (4.26)$$

The model matrices are

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -5 & -6 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -3 & -2 \end{bmatrix}; \quad G = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{4.27}$$

$$H = \begin{bmatrix} 0 & 5 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}.$$

The matrix P_1 is chosen to be the identity matrix. Since

$$C^T B = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix} \tag{4.28}$$

the matrix P_2 is chosen to be

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{4.29}$$

Since the matrix $C^T AB$ is nonzero in the (1,1) position, no other Liapunov terms are needed.

There are two control equations, one for each channel:

$$u_1 = 2x_1 + 2x_2 - 12.5y_1 - 15y_2 + 2.5h_1(r_1) - \frac{1}{2}e_1 - \dot{e}_1 \qquad (4.30)$$

$$- 0.5|x_2|\text{sign } \dot{e} - 0.5(1 - |\text{sign } \dot{e}|)|x_2|\text{sign } e$$

$$u_2 = \frac{1}{3}\{x_4 - 3y_3 - y_4 + h_2(r_2) + 1.8x_5 - 1.2|x_5|\text{sign } e\} - 2e \quad (4.31)$$

A simulation was made using the inputs

$$r_1 = 3 \sin 3.1416t \qquad (4.32)$$

$$r_2 = 0.5 + 2 \cos 6.2832t$$

and nonzero initial conditions

$$y_1(0) = 0.2 \qquad (4.33)$$

$$y_2(0) = -0.4$$

$$y_3(0) = 1$$

The results of the simulation are shown in Figure 21.

Conclusion

The techniques of the previous chapters have been extended in this chapter to handle multiple-input, multiple-output plants with the signal synthesis type of model reference adaptive control system. The key to the procedure was the proper choice of a Liapunov function upon

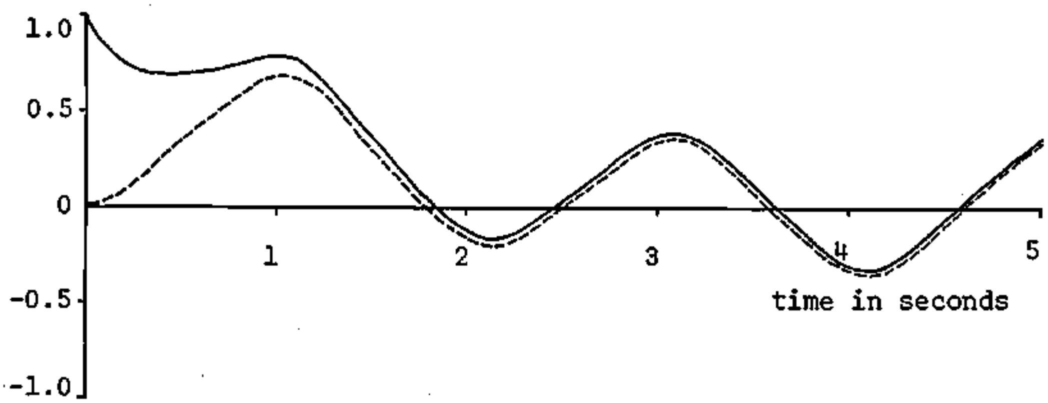

Figure 21(a). Plant and Model Outputs for Channel 1, Example 2

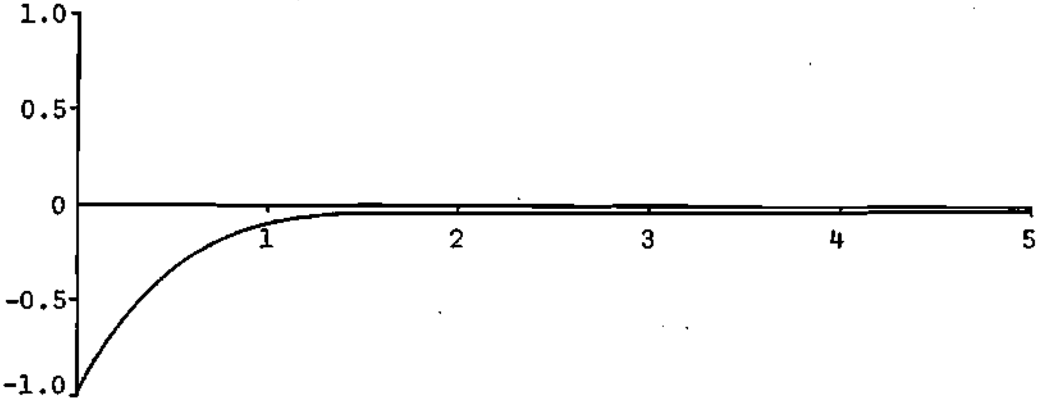

Figure 21(b). Channel 1 Error for Example 2

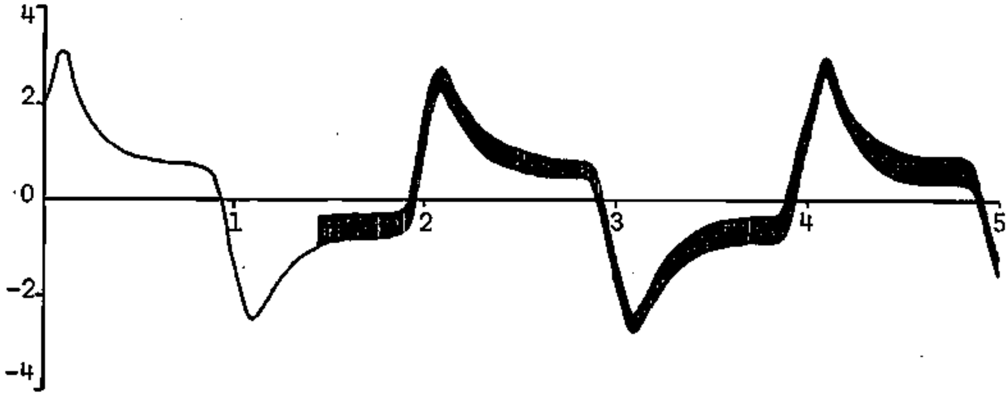

Figure 21(c). Channel 1 Plant Input for Example 2

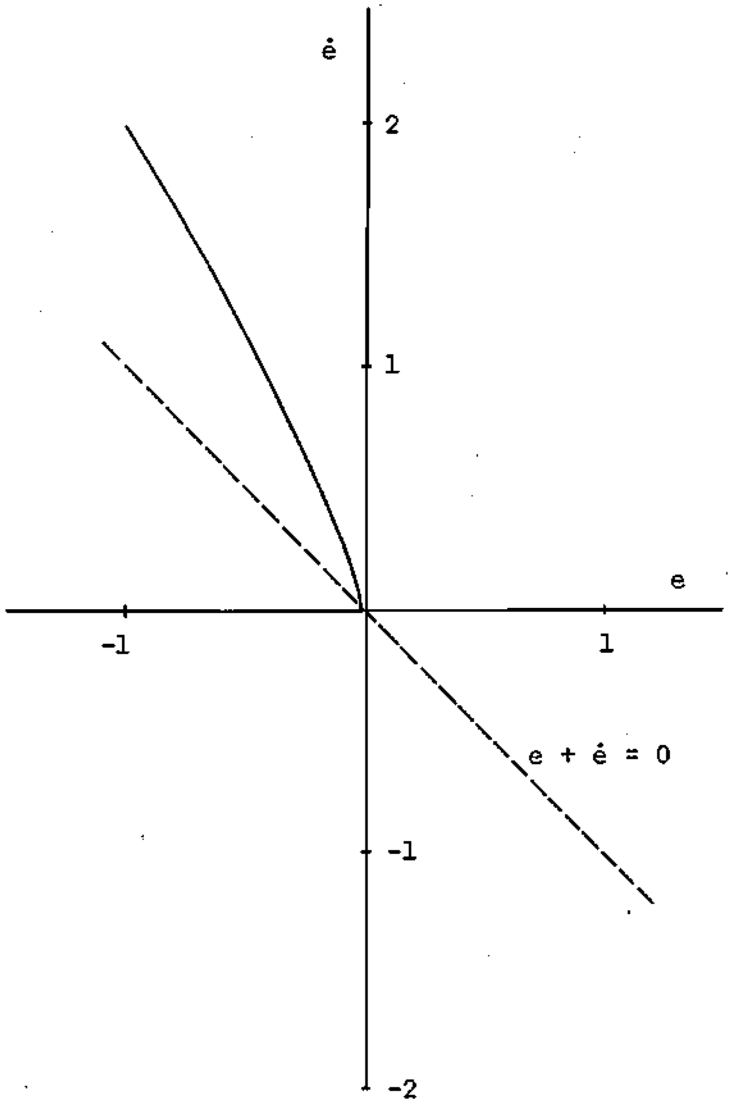

Figure 21(d). Phase Plane Trajectory for the Channel 1 Error for Example 2

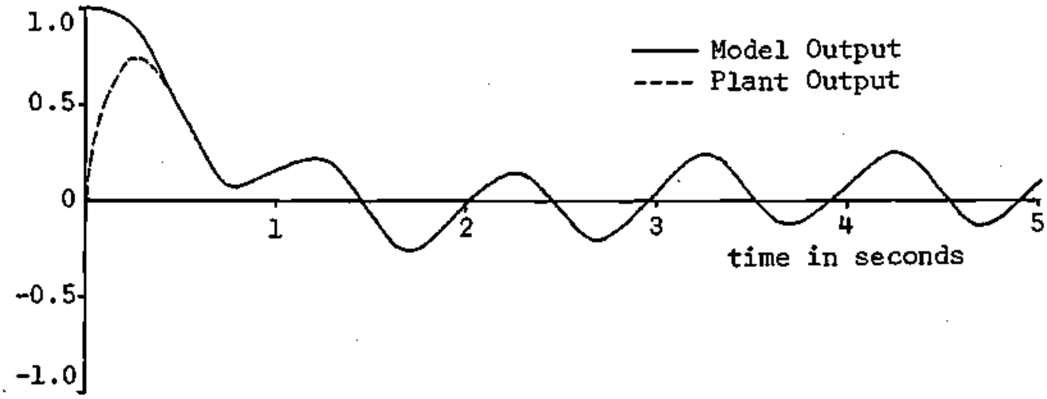

Figure 21(e). Plant and Model Output for Channel 2,Example 2

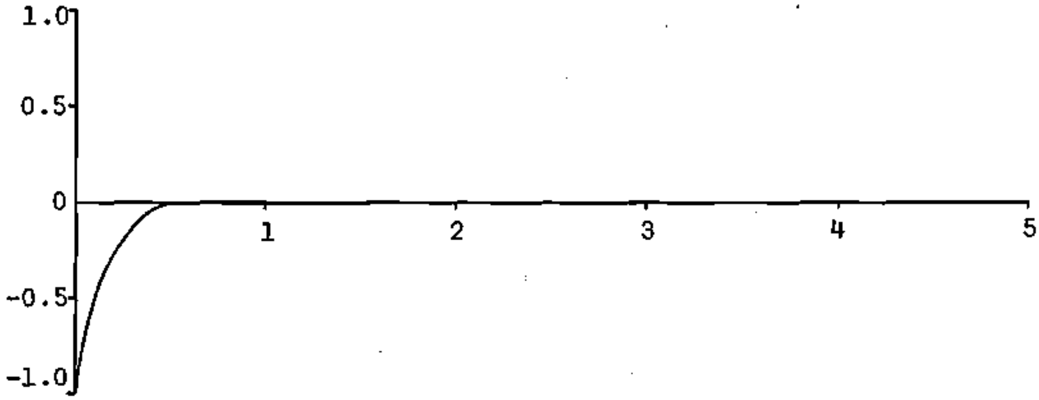

Figure 21(f). Channel 2 Error for Example 2

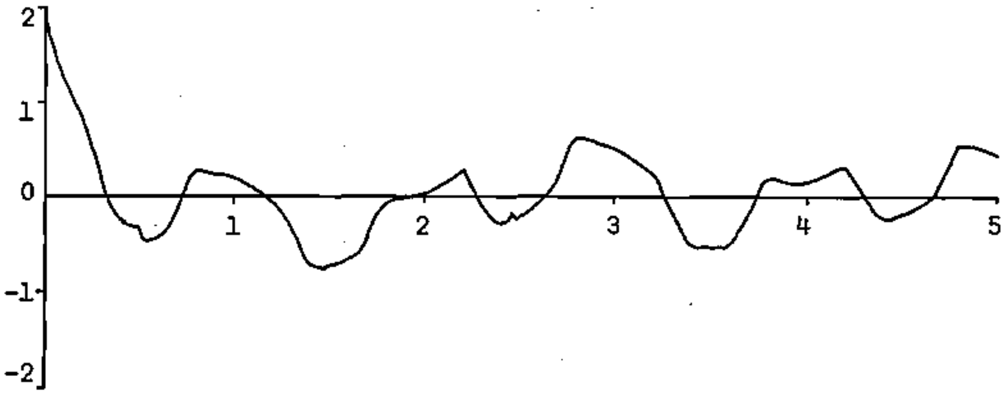

Figure 21(g). Channel 2 Plant Input for Example 2 which to build the design. An added feature obtained by using the techniques of this chapter was the design of noninteracting systems.

CHAPTER V

SAMPLED-DATA SYSTEMS

The systems considered so far in this thesis have been continuous systems, which means that all the state variables are continuously monitored for feedback. Another important category of control systems is one in which the signals are sampled and are thus available only at discrete instants of time. Whether such sampling is an inherent characteristic of the system or is intentionally added is not important to this thesis. The fact that sampled data systems are widely used is sufficient justification for developing techniques to handle them. In this chapter methods are presented for using discrete Liapunov theory to design sampled data model reference adaptive control systems of the signal synthesis type. Both single-input, single-output systems and multiple-input, multiple-output systems were investigated.

The Discrete Liapunov Function

The application of Liapunov theory to sampled data systems is considered in this chapter. As with continuous systems, the Liapunov function must be positive definite. However, the discrete Liapunov function is defined only at the sampling instants. To guarantee that the discrete system is asymptotically stable, a control must be synthesized which causes the Liapunov function at each sampling instant to be less than at the previous sampling instants.

The simplest quadratic form is

$$V_n = e_n^2 \tag{5.1}$$

for which

$$V_{n+1} = e_{n+1}^2 \tag{5.2}$$

The difference function is defined as

$$\Delta V_n = V_{n+1} - V_n = e_{n+1}^2 - e_n^2 \tag{5.3}$$

By making ΔV_n negative definite for all n, asymptotic stability at the sampling instants is guaranteed.

The Simplest Case

Consider the single-input, single-output plant

$$\underline{x}_{n+1} = A\underline{x}_n + \underline{B}u_n \tag{5.4}$$

$$z_n = \underline{C}^T\underline{x}_n$$

and the single-input, single-output model

$$\underline{y}_{n+1} = F\underline{y}_n + \underline{G}r_n \tag{5.5}$$

$$w_n = \underline{H}^T\underline{y}_n$$

Define the discrete system error as

$$e_n = z_n - w_n = \underline{C}^T\underline{x}_n - \underline{H}^T\underline{y}_n \tag{5.6}$$

For this plant and model, (5.1), (5.2), and (5.3) become

$$V_n = (\underline{C}^T\underline{x}_n - \underline{H}^T\underline{y}_n)^2 \tag{5.7}$$

$$V_{n+1} = (\underline{C}^T A\underline{x}_n + \underline{C}^T\underline{B}u_n - \underline{H}^T F\underline{y}_n - \underline{H}^T\underline{G}r_n)^2 \tag{5.8}$$

$$\Delta V_n = (\underline{C}^T A\underline{x}_n + \underline{C}^T\underline{B}u_n - \underline{H}^T F\underline{y}_n - \underline{H}^T\underline{G}r_n)^2 - (\underline{C}^T\underline{x}_n - \underline{H}^T\underline{y}_n)^2 \tag{5.9}$$

It is (5.9) which is to be made negative definite. As in the equivalent continuous case, this condition can be satisfied only if the matrix product $\underline{C}^T\underline{B}$ is nonzero. However in the sampled data system it is possible to reduce the system error to zero in one sampling instant when $\underline{C}^T\underline{B}$ is nonzero. This can be done by synthesizing u_n so that e_{n+1} is zero:

$$u_n = \frac{1}{\underline{C}^T\underline{B}}(\underline{H}^T F\underline{y}_n + \underline{H}^T\underline{G}r_n - \underline{C}A\underline{x}_n) \tag{5.10}$$

With this control the error is reduced to zero in one sampling instant and is held at zero thereafter. It might be mentioned that (5.10) is the same as the control equation one would obtain in the digital tracking problem.

As in the continuous case, it is permissible for the plant to be nonlinear, and time-varying:

$$\underline{x}_{n+1} = \underline{f}(\underline{x}_n, n) + \underline{B}(n)u_n \tag{5.11}$$

$$z_n = \underline{C}^T(n)\underline{x}_n$$

Similarly, the model may be nonlinear and time-varying:

$$\underline{y}_{n+1} = \underline{g}(\underline{y}_n, r_n, n) \tag{5.12}$$

$$w_n = \underline{H}^T(n)\underline{y}_n$$

It might be desirable in some cases to reduce the error gradually, rather than in one step. In those instances the discrete input u_n is synthesized so that e_{n+1} is not zero, but is less in absolute value than e_n. While such a control might offer more flexibility than the control in (5.10), it is more difficult to implement.

When there are incompletely specified plant parameters, the matrix products $\underline{C}^T\underline{B}$ and $\underline{C}^T A$ in (5.10) are unknown to some extent. It is therefore not possible to synthesize u_n so that e_{n+1} is exactly zero. What one should do in this instance is to choose some "best value" or "most likely value" for each unknown parameter, and synthesize u_n accordingly. If the chosen values are close to the actual values of the known parameters, the system error is held near zero. The following example illustrates this point.

Example 1

The plant to be controlled is second order, linear, and continuous, but is preceded by a sampler with sampling period 0.01 seconds, and a zero order hold. The continuous plant matrices are

$$A = \begin{bmatrix} -a & 0 \\ 1 & -b \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \tag{5.13}$$

$$C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

with known bounds

$$0.8 \le a \le 1.2$$

$$1.6 \le b \le 2.4$$

Using the state transition matrix, one may arrive at the sampled-data plant:

$$x_{1_{n+1}} = cx_{1_n} + 0.03u_n \tag{5.14}$$

$$x_{2_{n+1}} = x_{1_n} + dx_{2_n} + 0.01u_n$$

$$z_n = x_{1_n}$$

with known bounds

$$0.988 \leq c \leq 0.992$$

$$0.976 \leq d \leq 0.984$$

Choosing for a model the first order plant

$$y_{n+1} = 0.96y_n + 0.01r_n \tag{5.15}$$

$$w_n = y_n$$

and letting the input be

$$r_n = 1 + 0.01n \tag{5.16}$$

the control for one step convergence is

$$u_n = \frac{1}{0.03}\,(-0.990x_{1_n} + 0.96y_n + 0.01 + 0.0001n) \tag{5.17}$$

The results of the simulation are shown in Figure 22. Though the figure appears to be continuous it must be remembered that the pictured quantities are defined only at the sampling instants. In the simulation, the initial conditions were

$$x_1(0) = x_2(0) = 0 \tag{5.18}$$

$$y(0) = 1$$

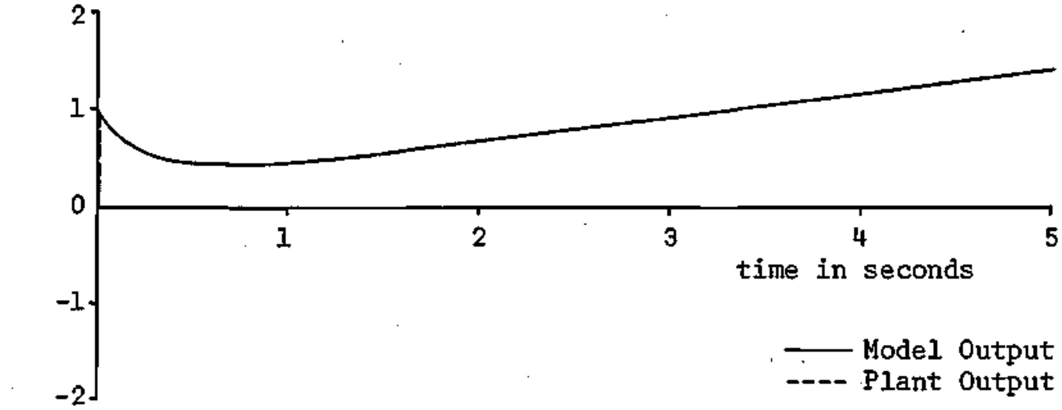

Figure 22(a). Plant and Model Output for Example 1

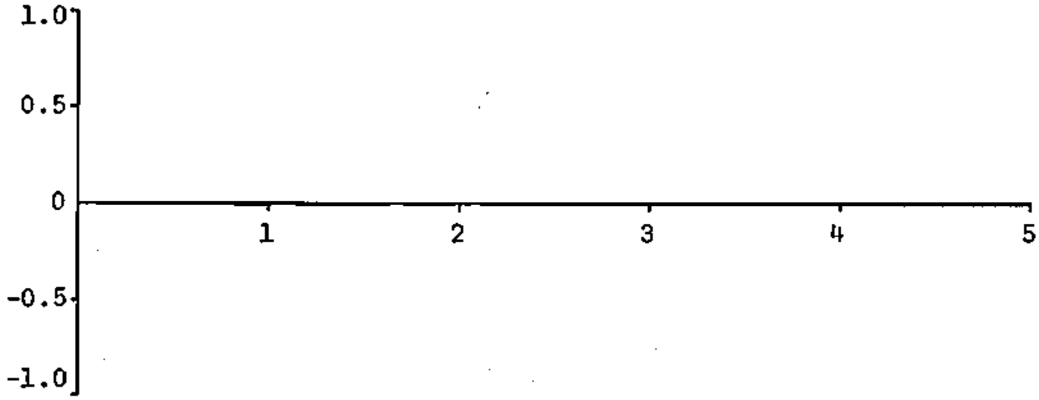

Figure 22(b). System Error for Example 1

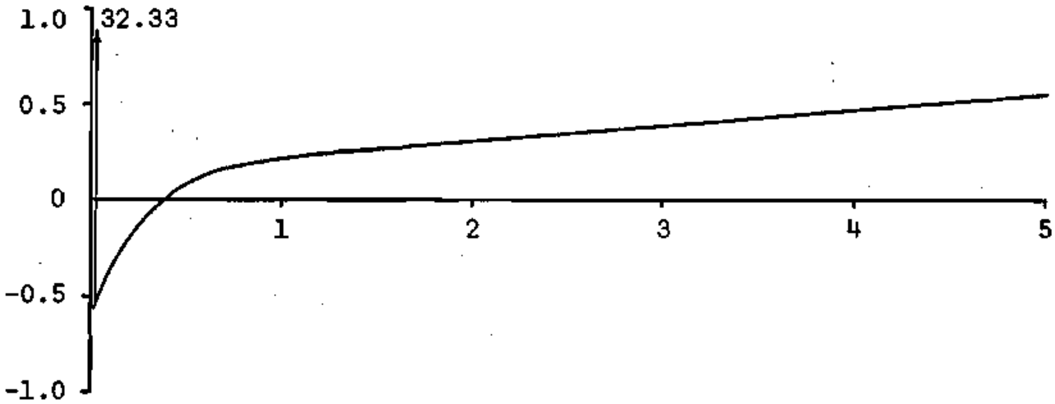

Figure 22(c). Synthesized Plant Input for Example 1

An Expanded Quadratic Form

When the matrix product $\underline{C}^T\underline{B}$ is zero, one step convergence is not possible. However, a control which causes multi-step convergence can be derived by using an expanded quadratic Liapunov function of the form

$$V_n = e_n^2 + P_1 e_{n+1}^2 + P_2 e_{n+2}^2 + \cdots \qquad (5.19)$$

The procedure is to use only the first term in (5.19) and then calculate V_{n+1}. When V_{n+1} contains u_n explicitly, then the Liapunov function is adequate and the design may proceed. If V_{n+1} has no term containing u_n, one begins again by trying the first two terms of (5.19). This procedure is repeated until an equation involving u_n appears when V_{n+1} is calculated. The discrete control u_n is then synthesized to make ΔV_n as negative as possible.

The preceding technique converges in the same number of steps as there are terms in the Liapunov function. However, unlike the procedure for continuous systems, there is nothing to be gained by using more terms than required in the Liapunov function. Furthermore, there is the disadvantage of increased complexity.

Multiple-Input Multiple-Output Systems

The extension to multiple-input, multiple-output sampled data systems is comparable to the corresponding extension for continuous systems. One begins by defining the quadratic Liapunov function

$$V_n = \underline{e}_n^T P_1 \underline{e}_n + \underline{e}_{n+1}^T P_2 \underline{e}_{n+1} + \underline{e}_{n+2}^T P_3 \underline{e}_{n+2} + \cdots \tag{5.20}$$

where the P_i are carefully chosen diagonal matrices and

$$\underline{e}_n = \underline{z}_n - \underline{w}_n \tag{5.21}$$

The elements of the P matrices are chosen as they were for continuous systems. P_1 is the identity matrix, and P_2 has positive entries in those positions where C^TB has zeros on the main diagonal. These entries may be chosen to be unity without loss of generality. The matrix P_3 has positive entries on the main diagonal only where C^TB and C^TAB both have zeros, and similarly for other added terms.

<u>Example 2</u>

The inputs to the plant and model of Example 2 in Chapter IV are preceded by a zero order sample-and-hold. Using a sampling interval of 0.01 seconds, the plant matrices become

$$A = \begin{bmatrix} 0.9999 & 0.0099 & 0 & 0 & 0 \\ -0.0198 & a & 0 & 0 & 0 \\ 0 & 0 & 0.9998 & 0.0100 & 0 \\ 0 & 0 & -b & 0.9899 & 0 \\ 0 & 0 & 0 & 0 & c \end{bmatrix}; \tag{5.22}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0.0016 & 0 \\ 0 & 0 \\ 0.0017 & 0 \\ 0 & 0.0017 \end{bmatrix}; \quad C = \begin{bmatrix} 0 & 2 \\ 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 3 & 0 \end{bmatrix}.$$

with bounds

$$0.9752 \le a \le 0.9850$$

$$0.0249 \le b \le 0.0348$$

$$0.9901 \le c \le 0.9980$$

The model matrices are

$$F = \begin{bmatrix} 0.9998 & 0.0097 & 0 & 0 \\ -0.0485 & 0.9416 & 0 & 0 \\ 0 & 0 & 0.9999 & 0.0099 \\ 0 & 0 & -0.0297 & 0.9801 \end{bmatrix}; \qquad (5.23)$$

$$G = \begin{bmatrix} 0 & 0 \\ 0.0016 & 0 \\ 0 & 0 \\ 0 & 0.0016 \end{bmatrix}; \quad H = \begin{bmatrix} 0 & 5 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}.$$

Since the matrix product C^TB was

$$C^T B = \begin{bmatrix} 0 & 0 \\ 0 & 0.0051 \end{bmatrix} \tag{5.24}$$

P_2 is chosen as

$$P_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{5.25}$$

No other P matrices are used since $C^T AB$ is nonzero in the (1,1) position. The resulting control gives two-step convergence for channel 1 and one-step convergence for channel 2. The actual controls are

$$u_{1_n} = \frac{1}{0.00003168}(-1.9992x_{1_n} - 0.0392x_{2_n} + 4.9974y_{1_n} + 0.0942y_{2_n} + 0.0001f(r_1)_n) \tag{5.26}$$

$$u_{2_n} = \frac{1}{0.0051}(0.9998x_{3_n} + 0.0100x_{4_n} - 2.9820x_{5_n} + 0.9702y_{3_n} + 0.9900y_{4_n} + 0.0016f(r_2)_n) \tag{5.27}$$

The simulation results are shown in Figure 23. Because there were incompletely specified parameters in (5.22), the errors were forced very close to, but were not exactly, zero.

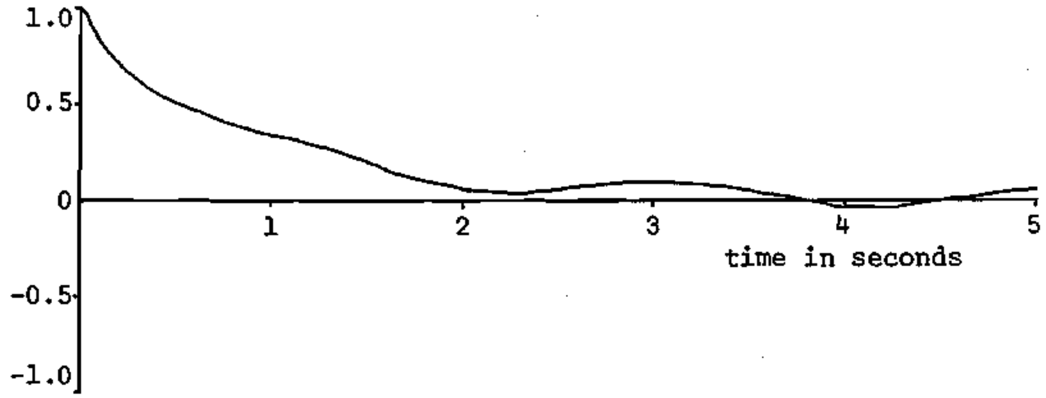

Figure 23(a). Plant and Model Output for Channel 1, Example 2

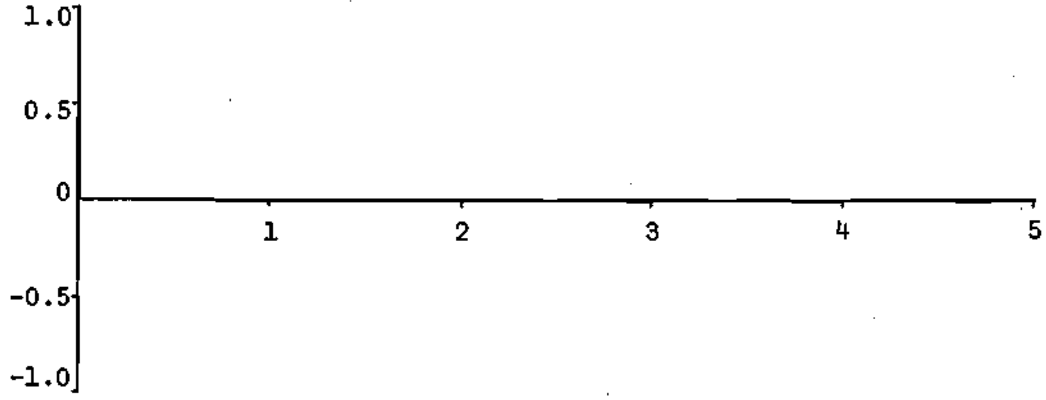

Figure 23(b). Channel 1 Error for Example 2

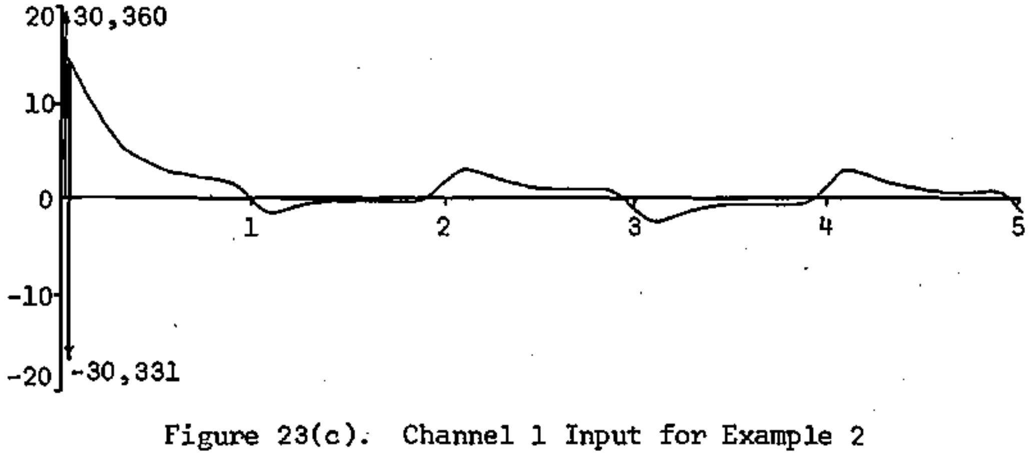

Figure 23(c). Channel 1 Input for Example 2

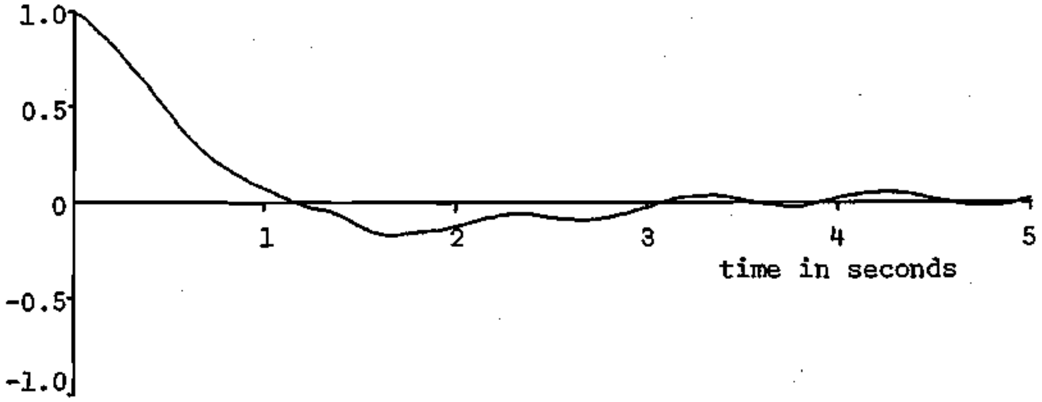

Figure 23(d). Plant and Model Output for Channel 2, Example 2

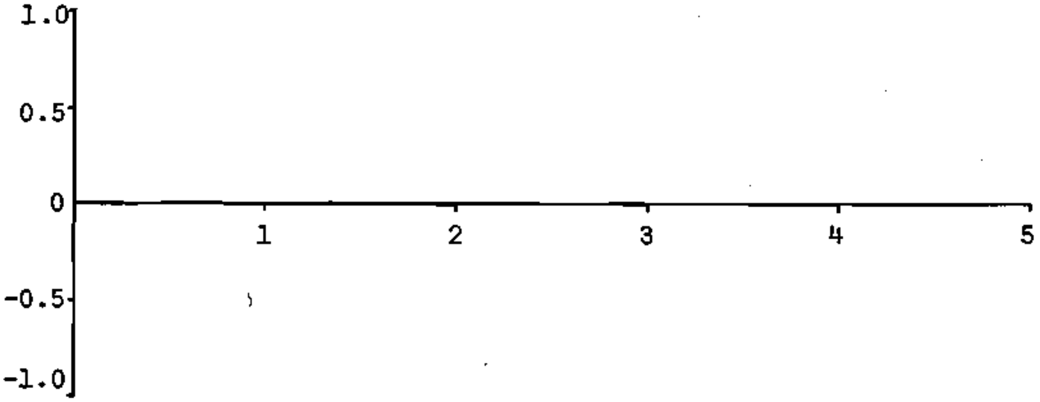

Figure 23(e). Channel 2 Error for Example 2

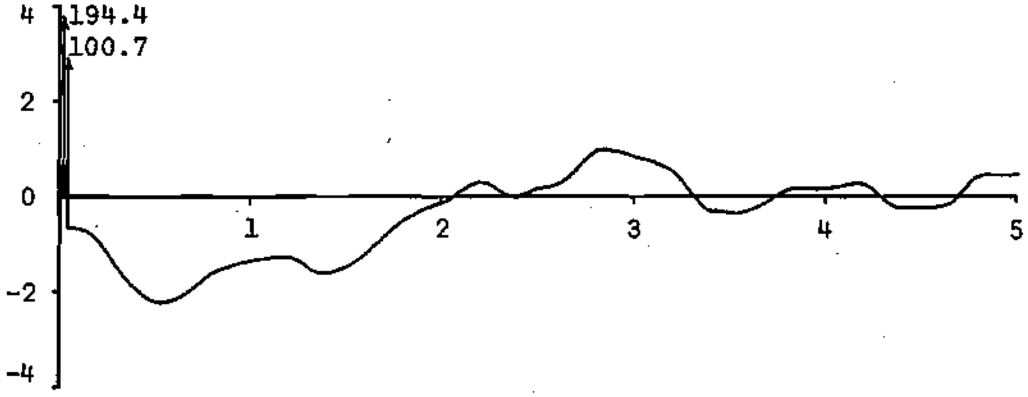

Figure 23(f). Channel 2 Input for Example 2

Summary

In this chapter a discrete Liapunov function was utilized in the design of sampled data model reference adaptive control systems. The procedure for controlling the error using this discrete function paralleled the procedure for controlling continuous systems. In the single-input, single-output case, one step convergence was possible if the matrix product $\underline{C}^T\underline{B}$ was nonzero and the plant was completely known. When there were unknown plant parameters, it was not possible to force the error to zero, and a small fluctuating error remained. For those plants whose matrix product $\underline{C}^T\underline{B}$ was zero, additional terms were added to the Liapunov function as was done with continuous systems. In those cases, convergence was possible in as many steps as there were terms in the Liapunov function. Multiple-input, multiple-output systems were handled by increasing the dimensions of the various quantities in the defining equations for single-input, single-output systems.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

This thesis has presented a number of techniques for designing of model reference adaptive control systems of the signal synthesis type. Perhaps the most significant characteristic of these techniques is that they are unencumbered by many of the restrictions which previous investigators found necessary to impose, particularly with regard to the form and the order of the plant and model.

The general approach is to synthesize a feedback type control signal for the plant. The control signal is in general nonlinear, and involves the model input, each state and each output of the plant and the model, and the bounds on each incompletely specified plant parameter. In addition, the control signal is such that the overall system satisfies the conditions of one of Liapunov's stability theorems. This latter requirement guarantees the global asymptotic stability of the resulting system design.

The signal synthesis techniques which were developed during the course of the research are applicable to plants and models which are of different orders and of different forms. The plant is permitted to be time-varying and nonlinear, provided the input enters either linearly or through a separable nonlinear block. The plant may have incompletely specified parameters which lie in known ranges. The model is permitted to be time-varying and nonlinear, and the model nonlinearities may involve the input. Any permissible model form may be used with any permissible plant, subject only to the requirement that the plant and the model have the same number of outputs. Several examples were presented to illustrate the broad range of permissible forms for the plant and the model.

Single-input, single-output continuous systems were considered first. It was by studying these simpler systems that the basic techniques were developed and presented. Examples were presented as each new development occurred, so that the implications of the development might be more fully understood. Of particular importance in developing a successful system design is the choice of the Liapunov function. A quadratic Liapunov function involving the system error and its higher derivatives was used throughout the thesis, with one exception. The exception was the case of the system output nonlinearity, for which an integral type function was chosen. The number of terms needed in the quadratic Liapunov function

$$V = \frac{1}{2} e^2 + \frac{1}{2} P_1 \dot{e}^2 + \frac{1}{2} P_2 \ddot{e}^2 + \cdots \qquad (6.1)$$

depends upon two considerations: (1) the sign definiteness of the plant matrix products $\underline{C}^T\underline{B}$, $\underline{C}^T A\underline{B}$, $\underline{C}^T A^2\underline{B}$, etc., and (2) the desired degree of continuity of the plant input. The sequence of matrix products is calculated until a nonzero product is obtained. The number of matrix products which were calculated equals the number of terms of (6.1) which are needed. If these are unknown plant parameters and it is desired that the plant input be continuous, one additional term must be used in the Liapunov function.

The techniques which were developed for single-input, single-output systems were then extended to multiple-input, multiple-output continuous systems. It was shown that an important advantage of using the procedures of this thesis to design controllers for multivariable systems is the possibility of designing a controller which will eliminate cross-coupling effects. This is accomplished by using a non-interacting model in the design.

Using a discrete Liapunov function makes it possible to handle sampled data plants with sampled data models. As with continuous systems, the design does not depend on the plant and the model being of the same order or of the same form. It is also permissible for the plant and model to be time-varying and nonlinear. However, for sampled data systems it is not possible to force the error completely to zero if the plant has incompletely specified parameters.

The results obtained have a broad scope of applicability. The method is relatively simple to use, and no significant restrictions are placed on the form of the plant, the model, or the model input. This accounts for the relative ease in obtaining results for systems which are too difficult to handle by other existing techniques.

Recommendations for Further Study

The approach taken in this research may be used to obtain additional results similar to those presented in this thesis.

An area which warrants further study is that of finding Liapunov functions which will yield improved time responses. This is especially important for those plants which require terms involving second or higher derivatives of the error when the quadratic form is used. One possibility for solving the problem when u appears in the equation for $\ddot{e}$ is to choose a Liapunov function which is zero along the line defined by

$$e + \dot{e} = \dot{e} + \ddot{e} \qquad (6\text{-}1)$$

The design may prove to be difficult because of the higher dimensions involved.

Another design procedure which could yield some important results is to combine a parameter identification scheme with this signal synthesis technique. For such a system it would be possible to synthesize a more nearly continuous input when the plant has incompletely specified parameters. This would offer an alternative to adding terms to the Liapunov function. Such a technique would be especially useful when the order of the plant numerator differs from the order of the plant denominator by two or more.

In this thesis all measurements were assumed to be noise free. By using a stochastic Liapunov function such as those presented by Kushner [34], a synthesis technique which guarantees stability may be possible when the measurements of the state variables are corrupted by noise.

This thesis has presented some new ideas on designing model reference adaptive control systems. The approach taken has resulted in design techniques to handle broad classes of plants and models, but some further work is indicated in the areas outlined above.

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VITA

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