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THE DESIGN AND CONSTRUCTION OF
AN A-C INDUCTION RESOLVER FOR USE
WITH THE ELECTRONIC ANALOG COMPUTER

A THESIS

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THE DESIGN AND CONSTRUCTION
OF AN A-C INDUCTION RESOLVER FOR
USE WITH THE ELECTRONIC ANALOG COMPUTER

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SUMMARY

This study is concerned with the design, construction, and evaluation of an a-c induction resolver system for use in coordinate transformation of analog computer variables. The system was made compatible with the Berkeley EASE Model 1032 general-purpose analog computer equipment.

The purpose of this study was to seek to improve certain aspects of coordinate transformation of analog voltage variables using the a-c induction resolver. In particular, improvements were sought in the transformation of voltage variables in the x-y coordinate system to polar coordinates by extending the system bandwidth and operating range of the θ output variable over those of existing d-c zero resolver systems. These improvements were to be obtained with a minimum of compromise with the overall system accuracy.

The approach to the problem selected in this research entailed the design of suitable modulation and demodulation circuits to allow the a-c induction resolver to perform the necessary trigonometric computations on the d-c carrier analog voltage signals. A positional servomechanism was also required to drive the a-c resolver in the computation process.

The x and y coordinate analog voltage variables are amplitude modulated and applied to the two stator coil windings of the a-c resolver. The voltage induced in one resolver rotor winding is an amplitude-modulated voltage proportional to the variable $R = (x^2 + y^2)^{\frac{1}{2}}$. This voltage is demodulated and made available as a d-c carrier analog computer voltage. The voltage induced in the other rotor winding, wound in quadrature with the first rotor winding, is an error voltage resulting from an improper

position of the resolver rotor winding. This voltage is driven to zero when the resolver rotor angle assumes the position $\theta = \tan^{-1} (y/x)$. The error voltage is applied to the input of the positional servomechanism which drives the resolver rotor to the proper position, thus completing the conversion. The value of the shaft angle corresponding to the desired value of θ is obtained from a linear potentiometer supplied with the computer reference voltage and ganged to the resolver rotor shaft.

The results of the evaluation of the resolver system constructed for this study are presented in the text of this thesis and indicate that the coordinate conversion technique used here provides a practical method of obtaining the desired conversion improvements with certain adequate compromises. The system provides conversion accuracies of 0.5 per cent in obtaining the R output variable and 0.1 per cent in obtaining θ , for full-scale voltage inputs. The system possesses a bandwidth of approximately 3.5 cycles per second and an operating range of θ of 3600 degrees.

CHAPTER I

INTRODUCTION

This research was initiated through a need for improved methods of coordinate transformation at the Analog Computer Laboratory, Georgia Institute of Technology. The purpose of this study was to investigate the feasibility of employing the alternating-current induction resolver in the coordinate transformation of direct-current analog signals. A design for an a-c resolver system was proposed which offered an adequate compromise between several conflicting design criteria. A working model, made to be compatible with the general-purpose Berkeley EASE Model 1032 Analog Computer, was constructed and suitable tests performed to ascertain the efficiency of the design. The initial phase of this research concerned itself with a review of the state of the art in analog coordinate transformation and outlining the specifications for an a-c resolver system in the light of present technology.

The a-c induction resolver was used in some early general purpose electronic computers for performing coordinate transformation. However, owing to certain inherent problems the a-c resolver was definitely inferior and gradually gave way to the d-c resolver systems which are used almost universally today.¹

The present d-c resolver is a relatively simple and low-cost unit.²

¹C. L. Johnson, Analog Computer Techniques, New York, McGraw Hill, 1956, pp. 77-81.

²Ibid.

It is constructed using two tapered sine-wound potentiometers, two sliding wiper contacts accurately positioned at right angles to each other being used for each potentiometer card. These wipers are ganged on a single shaft so that appropriate sine and cosine functions of the input voltages and shaft angle are obtained from the potentiometers. These functions are instrumented to form the following relations:

$$x \cos \theta + y \sin \theta = R \quad (1)$$

$$x \sin \theta - y \cos \theta = 0 \quad (2)$$

A block diagram of the d-c system is shown in Figure 1. If the shaft angle of the potentiometer wiper contacts is not of the proper value to satisfy equations (1) and (2) (i.e. θ does not correspond to the angle associated with the cartesian coordinate variables x and y), then:

$$x \sin \theta - y \cos \theta = \varepsilon \quad (2-a)$$

where ε denotes an error voltage proportional to the inputs x and y and is a sinusoidal function of the shaft angle error. This error voltage is applied to a servomechanism which drives the potentiometer to a null. Now the relationships are correct and the computation is complete.³

This d-c system offers several advantages. It is relatively insensitive to drift and therefore requires infrequent calibration. The wire-wound potentiometers are generally rugged enough to insure a long life free

³Strictly speaking the R - θ variables will bear the correct relationship to the x - y input variables only if the proper feedback polarity is chosen. If the improper polarity is chosen θ will be in error by 180 degrees. This question of a stable null-point is discussed in detail by J. H. Meeks and R. S. Johnson in "Loop Gain Analysis for Implicit Function Generation," a paper delivered at the Southeastern Simulation Council meeting at the University of Alabama on October 2, 1959.

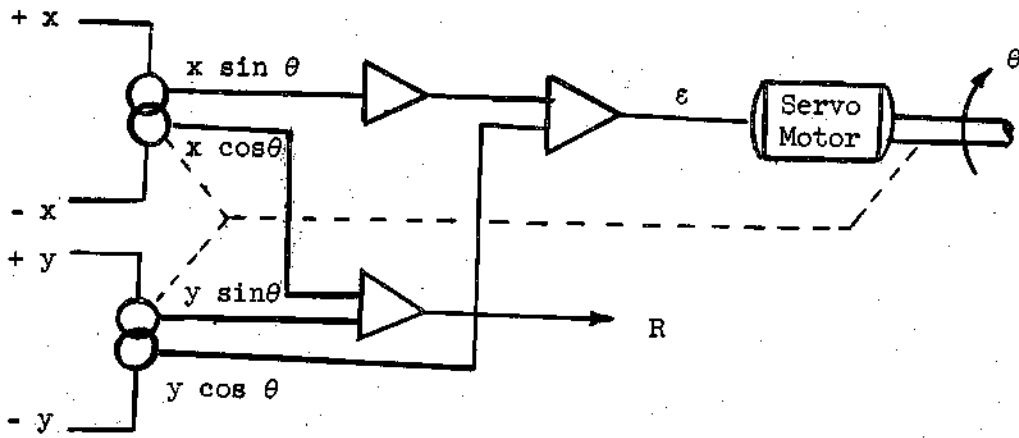


Figure 1. D-C Sine-wound Potentiometer Resolver.

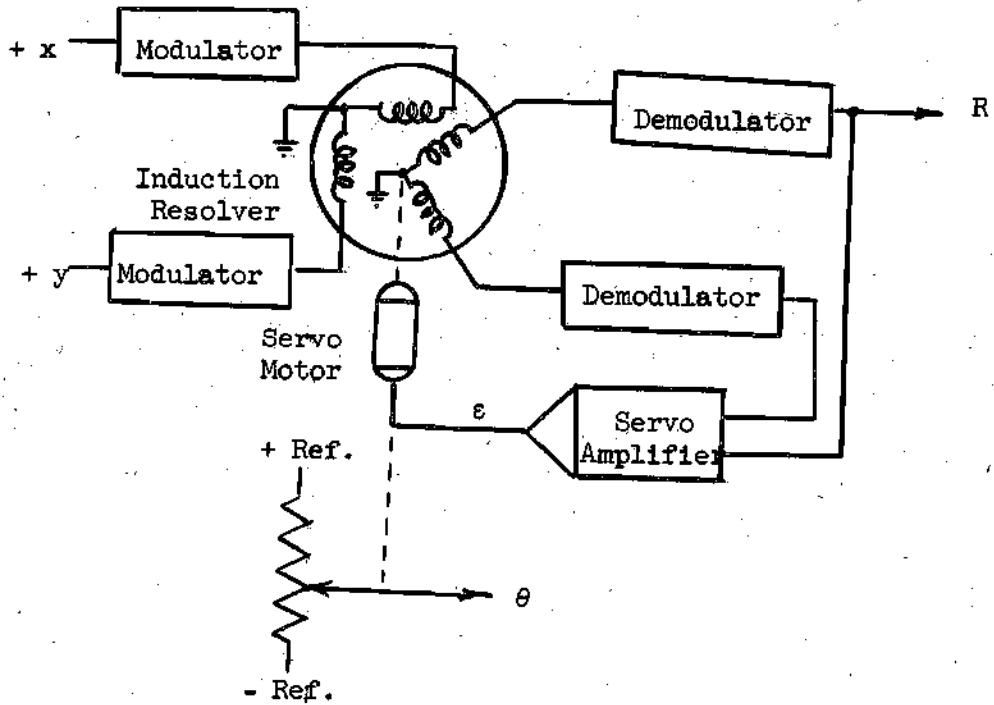


Figure 2. A-C Induction Resolver Block Diagram.

from mechanical deterioration resulting in incorrect computations. A typical accuracy figure for a one-turn unit is 0.25 per cent in obtaining R and 0.5 degree in obtaining θ . However, the d-c system possesses certain inherent disadvantages. In order to obtain the accuracies stated above, the sine and cosine-wound potentiometers must be rather large and cumbersome. For instance, the Electronic Associates Model 16-8C has a potentiometer with a diameter of approximately one foot. The excessive inertia of the potentiometer wiper arms and follow-up potentiometers results in a system bandwidth of approximately 1 to 2 cps, a figure far below the range of frequencies required by many real-time simulation problems. A second major performance limitation in the d-c system is the limited range of travel of the potentiometer wiper arms. In the Electronic Associates model cited previously this operating range is ± 540 degrees (i.e. 1 1/2 turns in either direction). Again this leaves much to be desired in many physical problems.

Several alternative approaches have been proposed to perform coordinate transformation. The all-electronic resolver utilizing sine-wave sampling techniques,^{4,5} and a diode function generator approach have been studied. Both systems lead to high speed computation but both possess major disadvantages. The sine-wave sampling techniques suffer from excessive drift rates, whereas the diode function generator is limited to a relatively small operating range of θ . The a-c induction resolver

⁴Ibid.

⁵Albert S. Jackson, Analog Computation, New York, McGraw-Hill, 1960, pp. 193-4.

approach has also been proposed. The resulting systems have been inferior to the d-c systems owing to drift associated with the modulator and demodulator circuits.⁶

However the a-c induction resolver approach to coordinate conversion possesses certain features which would be highly advantageous if a compromise were allowable which sacrificed some static accuracy for extended system bandwidth, operating range of θ , and reliability. First, the a-c resolver is inherently less complex than the d-c servo resolvers or all-electronic resolvers and therefore should result in a decidedly more reliable and less expensive device. Secondly, the small a-c resolver possesses much less inertia and hence provides a system bandwidth limitation far above that of the sine-wound potentiometer. Also, since the induction resolver, which performs the actual transformation, has no sliding contacts, the operation should be much more reliable.

Therefore, this study was undertaken to ascertain the extent of the design limitations in the a-c induction resolver approach to coordinate transformation and to outline a feasible system to be constructed and made compatible with existing general-purpose analog computer equipment. The scope of this research was to study the existing techniques of a-c resolution and establish a set of reasonable and realizable specifications. Then a design was proposed which offered an adequate compromise between these specifications and the existing equipment components. Finally, a working model for performing the rectangular-to-polar coordinate transformation was constructed and tested to determine the efficiency of the device.

⁶Johnson, op. cit., pp. 77-81.

CHAPTER II

DESIGN OF THE RESOLVER

General Requirements

The design of a high-accuracy extended-bandwidth resolver system for coordinate transformation originated with the need for such equipment in the Analog Computer Laboratory of the Georgia Institute of Technology Engineering Experiment Station. The initial period of the design concerned itself with outlining the desired performance specifications. Existing resolver equipments were of the d-c sine-wound potentiometer type described in the previous chapter. It was considered desirable that an a-c resolver system should possess static accuracies of the d-c system (i.e. 0.25 per cent accuracy in computing R and 0.5 degree error in shaft angle) as well as an extended frequency response and shaft-angle operating range. But a smooth, reliably operating servo was the most important design objective and, if necessary, was to be achieved with some sacrifice of dynamic performance. The tentative design specifications called for static computing accuracies of 0.25 per cent in R and 1 degree error in shaft angle, with overall system bandwidth of 10 cps and shaft-angle operating range of $\pm 1800^\circ$ (i.e. five turns of the follow-up potentiometer in either direction.)

Loop Configuration

To achieve the desired performance, a system configuration was chosen capable of providing high loop gain, low noise, low drift, and compatibility with the existing equipment components. A system block diagram is shown in Figure 2.

As mentioned above, the actual coordinate conversion is made in the resolver so that the equations (1) and (2) are satisfied.

$$x \cos \theta + y \sin \theta = R \quad (1)$$

$$x \sin \theta - y \cos \theta = \epsilon = 0 \quad (2)$$

The design procedure involved the selection of proper modulation circuits for amplitude modulating an a-c carrier with the d-c analog voltage, and phase-sensitive detection circuits for providing a d-c output voltage, R, and an error voltage, ϵ , which is applied to the servo loop for properly positioning the shaft angle.

A major factor limiting the system performance was the design of an accurate, high-speed servomechanism for the shaft angle position control. An equally important consideration was the automatic gain control (AGC) required for low signal-level operation. Each of these circuit elements will be described in detail in the following section.

System Description

Modulators and Demodulators.--Several forms of balanced modulators required to provide the phase sensitive, suppressed-carrier amplitude modulation are available. The form considered as the most advantageous was the four-diode bridge (sometimes called the ring modulator). It consists of four solid-state diodes connected as shown in Figure 3-a. The diodes are biased in the conducting state and alternately switched from the conducting to the non-conducting state by a vacuum-tube bistable multivibrator. The carrier frequency was chosen as 5 kc (for reasons presented in the following section). The 1N485 silicon diode was chosen to provide fast switching times and high leakage resistance.

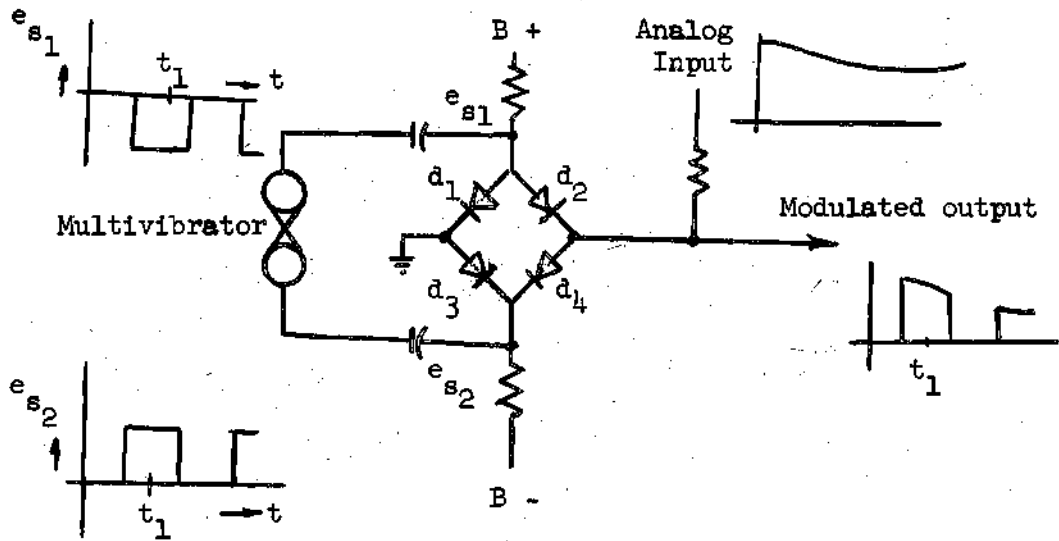


Figure 3-a. Four-diode Bridge Modulator Circuit.

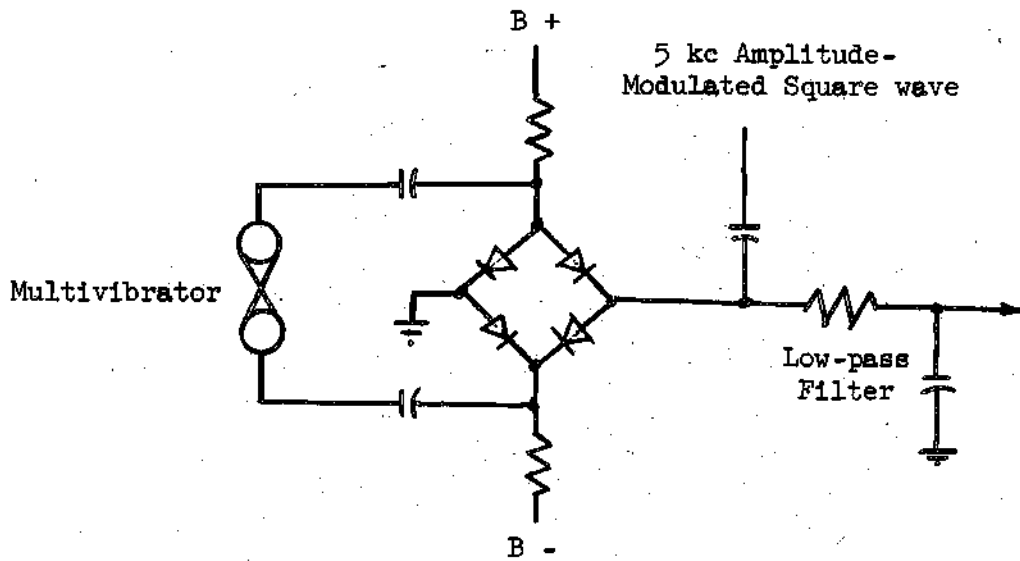


Figure 3-b. Four-diode Bridge Demodulator Circuit.

When the switching voltages are of the polarity shown at time t_1 (see Figure 3-a), the diodes are all conducting and the output voltage is clamped to ground potential. On the alternate half cycle the diodes are biased in their non-conducting state by the switching voltage and the output corresponds to the input analog signal. Since the analog signal frequencies are small compared to the carrier frequency (10 cps maximum compared to 5 kc) the output voltage waveform is essentially a square-wave amplitude modulated by the analog signal.

Phase sensitive demodulation is accomplished using a slight modification of the modulator circuit. As shown in Figure 3-b, the a-c square wave outputs of the resolver are alternately clamped to ground and switched to the amplitude of the demodulator input. The switching is synchronized to the modulation switching by employing the same multivibrator in both applications. Smoothing of the resulting square wave is provided by one stage of R-C filtering with a bandwidth set at approximately 50 cps which provides sufficient attenuation of the carrier frequency to produce negligible ripple in the output wave form.

Induction Resolver.--The a-c induction resolver is a precision rotatable electromagnetic device with an output whose amplitudes are proportional respectively to the sine or cosine of the shaft angle position. A symbolic diagram of a simple resolver is shown in Figure 4-a.

A more complicated resolver is shown symbolically in Figure 4-b and is the type used in this research. For the indicated inputs (with polarities as shown) the output voltages are given by the previous equations (1) and (2).

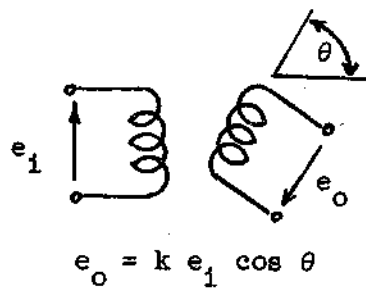


Figure 4-a. Single-phase Induction Resolver Diagram.

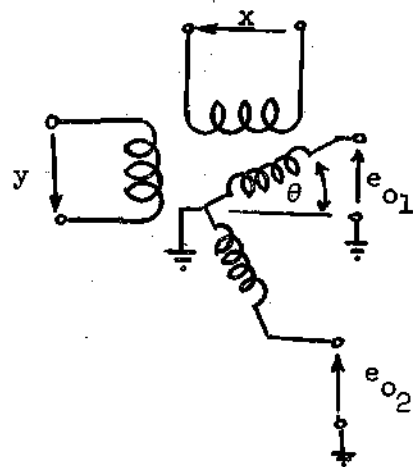


Figure 4-b. Two-phase Induction Resolver Diagram.

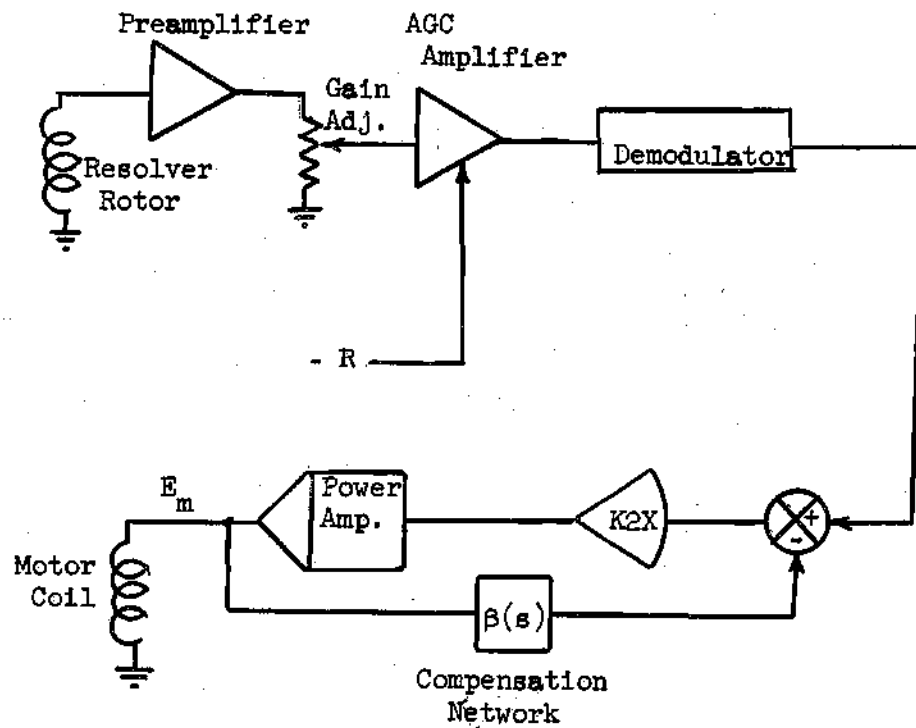


Figure 5. Servo Amplifier Block Diagram.

$$e_{o_1} = k_1(x \cos \theta + y \sin \theta) = k_1 R \quad (1-a)$$

$$e_{o_2} = k_2(x \sin \theta - y \cos \theta) = k_2 \epsilon \quad (2-a)$$

Notice that the complete expression for each output is obtained from a single rotor winding (as compared to the need for auxiliary operational amplifiers to produce the sum and difference voltages in a d-c system). These output voltages are used in a manner similar to the d-c systems. Equation (1-a) denotes the expression for the voltage corresponding to the polar vector amplitude, R , and equation (2-a) denotes the expression for the error voltage, ϵ , which is required to drive the servo loop to a null and hence position the shaft angle to the proper value of corresponding to the rectangular coordinate variables x and y .

A Bendix resolver Model No. AY-221-3-8 is used in the proposed system. This is a 400-cps type with a measured bandwidth of approximately 10 kc. Since the input signals applied to the resolver stator windings are amplitude-modulated square waves, a carrier frequency of 5 kc was chosen. This is required because the stator windings for this type resolver could not be tuned to prevent excessive transient oscillations of a 400-cps signal.

When one resolver stator is excited and the other is shorted, the rotor voltages vary sinusoidally with rotor angle. Any deviations from a true sinusoid constitute an angular error or an equivalent amplitude error. The accuracy of computing resolvers is customarily defined in terms of this equivalent amplitude error. The per cent error for the induction resolver used in this system is on the order of 0.05 per cent. Precise measurement

of this figure was impossible because of a lack of proper instruments to accurately position the rotor angle and measure the input and output voltages.⁷

Gears.--As mentioned previously, the induction resolver is capable of an infinite number of rotations and the limiting factor which determines the operating range of θ is the number of turns available in the read-out potentiometer. However, specifying a gear ratio between resolver and read-out potentiometer to offer a wide operating range of θ would necessarily sacrifice some resolution in the θ output voltage. Therefore a compromise between operating range and resolution of θ was made. The decision was made to sacrifice resolution for a 10 turn (± 1800 degrees) range of θ . This necessitated a 1:1 gear ratio between the resolver and 10-turn read-out potentiometer.

The servo motor shaft is equipped with a 36-tooth, 48-pitch spur gear. The resolver and read-out potentiometer are geared to the resolver with identical 57-tooth, 48-pitch, anti-backlash gears. The internal gear ratio of the d-c servo motor is 13:1 (after modification as mentioned below), yielding an overall ratio between motor and resolver of 20.6:1.

Servo Motor.--The basic requirements of the servo motor for fast, accurate positional control of the resolver rotor angle are high starting acceleration, low inertia, and minimum stall voltage. The motor used is a Delco 27-volt d-c type. The internal gearhead was altered to provide 770 rpm no-load speed. According to experimental data obtained in tests by the

⁷Resolver Manual, American Electronics, Inc., Instruments Division, p. 17.

author, the servo motor requires approximately 400 milliamperes current at 26 volts for maximum no-load acceleration at stall. This corresponds to a peak power requirement of approximately 1 watt. The no-load stall voltage is approximately 2 volts.

Servo Amplifier.--In order to provide the power requirements for the servo motor outlined in the previous section, a complementary emitter-follower transistor power amplifier was chosen. This d-c push-pull configuration is preceded by an a-c preamplifier, AGC amplifier, demodulator, and d-c operational amplifier. A block diagram of the servo amplifier is shown in Figure 5, and a detailed circuit diagram is included in the Appendix.

The a-c preamplifier provides a nominal overall voltage gain of five and is variable with a potentiometer adjustment. It consists of two stages of triode amplification connected in a common cathode arrangement with negative feedback, to assure a low drift rate, and a cathode-follower output.

It is noted in Figure 4-a that a voltage induced in the resolver rotor (the servo error voltage, for instance) is a function not only of the rotor shaft angle but also of the input voltages exciting the stator windings. Thus, when the amplitude of the voltage exciting the stator goes to zero the servo error voltage goes to zero and the positioning control is lost. Hence, in order to maintain the loop gain approximately constant over a 100:1 range of the input variable, an Automatic Gain Control (AGC) amplifier is needed. The desired expression for the overall servo loop is

$$\frac{100}{R} = K \quad (3)$$

where R is the magnitude of the polar variable corresponding to the resolver inputs x and y, and K is the gain of the AGC amplifier.⁸ Thus the AGC amplifier has a gain function inversely proportional to R. A single stage of amplification is employed to perform this AGC operation. A type 6BA6 remote cut-off pentode, which has an intrinsic gain versus control-grid voltage characteristic conforming closely to the reciprocal relationship of equation (3), was used as the amplifier.

The demodulator circuit was described previously under Modulators and Demodulators.

The d-c operational amplifier is required to provide sufficient buffering between the R-C filter section and the power amplifier. It must also provide a voltage gain and a peak current output of one milliamperere. A Philbrick model K2-X operational amplifier is used to satisfy these requirements.

The complementary emitter-follower configuration for the power amplifier was chosen to provide the desirable characteristics of a vacuum tube cathode-follower output (i.e. power gain and low output impedance) and also to provide the additional advantage of complementary transistor symmetry. Complementary symmetry refers to a method of constructing Class B amplifiers using two transistors, one a p-n-p type and the other an n-p-n, both having similar characteristics. With the transistors connected back-to-back and biased at cut-off in a push-pull circuit as shown in Figure 6, the grounded-emitter p-n-p (2N251) transistor requires a negative base current input and

⁸ Meeks and Johnson, op. cit.

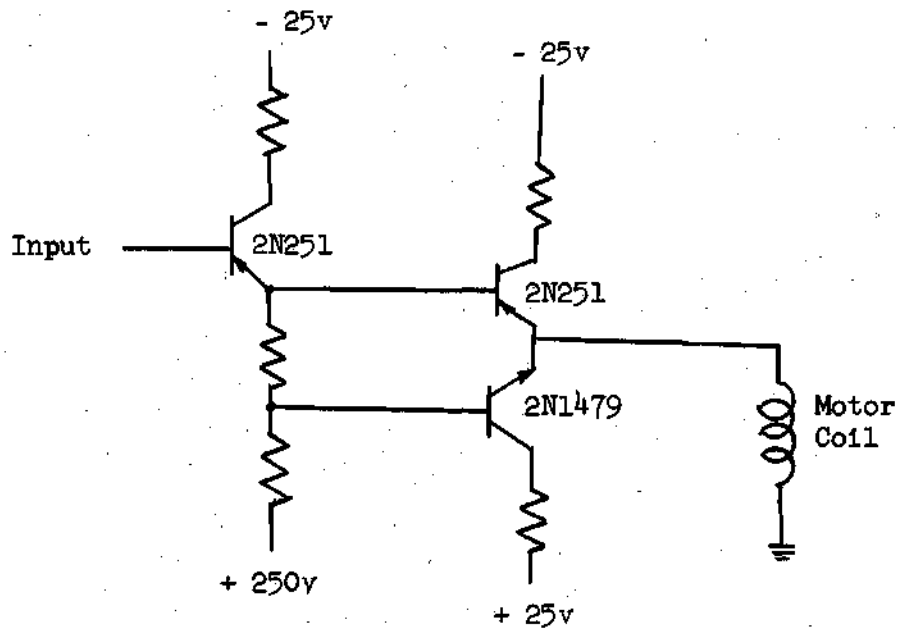


Figure 6. Complementary Emitter-follower Power Amplifier.

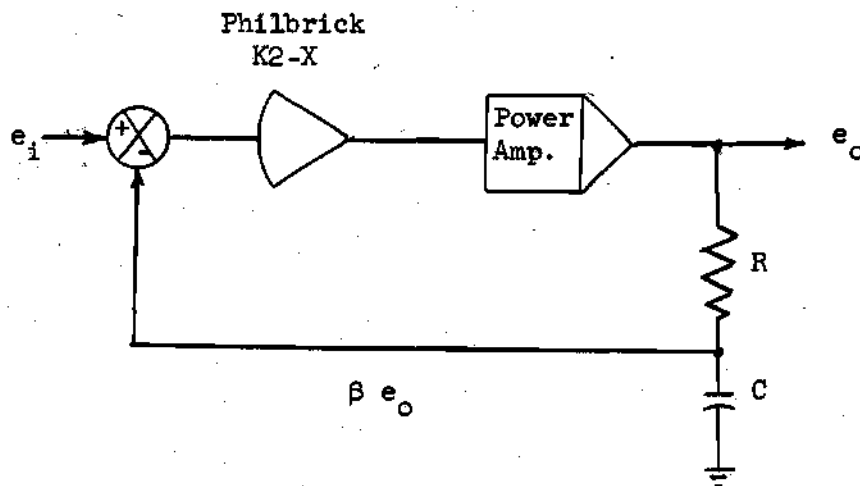


Figure 7. Servo Amplifier Compensation Network.

the n-p-n (2N1479) transistor requires a positive input. This allows the n-p-n unit to amplify the positive signals and the p-n-p unit to operate on the negative signals.⁹ These signals are added directly at the load. An additional stage of current amplification is inserted before the complementary emitter-follower to decrease the power requirements in the latter.

Adequate transient and steady state servo performance are obtained by inserting an integral compensation network in the feedback of the series d-c operational amplifier and power amplifier circuit. This configuration is shown in the block diagram of Figure 7. The loop transfer function of this configuration can be analyzed as follows.

Assume the operational amplifier has an open-loop gain A_1 and the power amplifier has an open-loop gain A_2 . Therefore, from Figure 7 the following relations can be written: (Assuming $A = A_1 A_2$ and $\beta(s)$ is the transfer function of the feedback network.

$$e_1 = e_i - \beta(s) e_o \quad (4)$$

$$e_o = A e_1 \quad (5)$$

$$\frac{e_o}{A} = e_i - \beta(s) e_o \quad (6)$$

⁹R. F. Shea, Transistor Circuit Engineering, New York, John Wiley and Sons, 1957, p. 128.

$$e_o \left(\frac{1}{A} + \beta(s) \right) = e_i \quad (7)$$

$$\frac{e_o}{e_i} = \frac{1}{\frac{1}{A} + \beta(s)} \quad (8)$$

If $\frac{1}{A} \ll |\beta|$

$$\frac{e_o}{e_i} = \frac{1}{\beta(s)} \quad (8-a)$$

Since the open loop gain of the Philbrick operational amplifier is approximately 30,000, equation (8-a) is a good approximation of the closed loop transfer function of the network. It can be shown that, for the particular feedback network shown in Figure 7, the servo amplifier transfer function is given by

$$\beta(s) = \frac{1}{RCs + 1} = \frac{1}{T_I s + 1} \quad (9)$$

If $R = 1$ Megohm

$C = 0.04$ mfd.

$$\frac{1}{\beta(s)} = \frac{e_o}{e_i}(s) = T_I s + 1 = 0.04 s + 1.$$

Assuming the servo motor to be an ideal d-c motor excited by a constant field, the motor transfer function has the form¹⁰

¹⁰ George M. Thaler and Robert G. Brown, Servomechanism Analysis, New York, McGraw-Hill, 1953, p. 120.

$$\frac{\theta_o(s)}{E_m} = \frac{1}{s(T_m s + 1)} \quad (10)$$

where θ_o is the motor output shaft angle E_m is the motor input voltage, T_m is the motor time constant.

Therefore the open-loop transfer function, resolver rotor to motor shaft angle, (see Figure 5) has the form

$$\frac{\theta}{e}(s) = \frac{G(T_f s + 1)}{s(T_m s + 1)} \quad (11)$$

where G is the open-loop gain of the system. Hence, the integral compensation network in the feedback of the servo amplifier serves to improve the high-frequency response of the positional servomechanism.

Read-out Potentiometer.--The steady-state position of the shaft angle of the induction resolver yields the proper value of θ corresponding to the rectangular variables x and y . This angle was obtained from a read-out potentiometer geared to the positioning servo motor. The potentiometer selected was a Helipot type UR, linear 10-turn, 20-kilohm potentiometer. The total linearity of the winding is 0.1 per cent. The potentiometer is excited with plus and minus 90-volt computer reference and a voltage whose magnitude corresponds to θ is obtained from the slider arm. With a gear ratio between resolver and potentiometer of 1:1, the 10 turn potentiometer is capable of ± 5 turns, corresponding to a ± 1800 degrees operating range of θ .

Input and Output Voltages

The analog voltages corresponding to the rectangular variables

x and y are attenuated by a factor of 10 before being applied to the induction resolver inputs. This attenuation is obtained in the coefficient potentiometers available in the analog computer. Similarly, the output voltage corresponding to the polar quantity is 0.1 R. This is amplified by a factor of 10 in a standard computer inverter-amplifier to produce the proper magnitude of R_{out} voltage. The voltage corresponding to the angle θ is loaded into a computer operational amplifier. The scale factor associated with the shaft angle output voltage is 20 degrees per volt.

CHAPTER III

SYSTEM EVALUATION

Objectives.--The a-c induction resolver described in the previous chapter was constructed and the working model subjected to a testing and evaluation program. The test results were obtained to specify the actual performance of the device and also to evaluate the general a-c induction-resolver conversion technique.

Of primary interest was the determination of the system accuracy in the conversion of fixed or slowly varying input voltages. Also of interest was the system dynamic response to high frequency input variations. Particular attention was given to the effects of drift on the conversion accuracy and repeatability. The evaluation program consisted of two major phases: (1) the determination of system static accuracy, and (2) determination of the system dynamic response. Both phases were carried out with the device serving as an integral component of the analog computer and performing under typical operating conditions.

Test Procedures.--To determine the static accuracy of the a-c induction resolver system, the device was programmed into the analog computer circuit as shown in Figure 8 to perform the desired conversion of fixed x and y voltages to R and θ voltages. The x and y input voltages were obtained from manually-adjusted ten-turn potentiometers supplied with plus and minus 100-volts from the computer reference. These input voltages were attenuated by a factor of ten in coefficient potentiometers to provide the necessary $0.1x$ and $0.1y$ input voltages to the resolver. The input voltages were

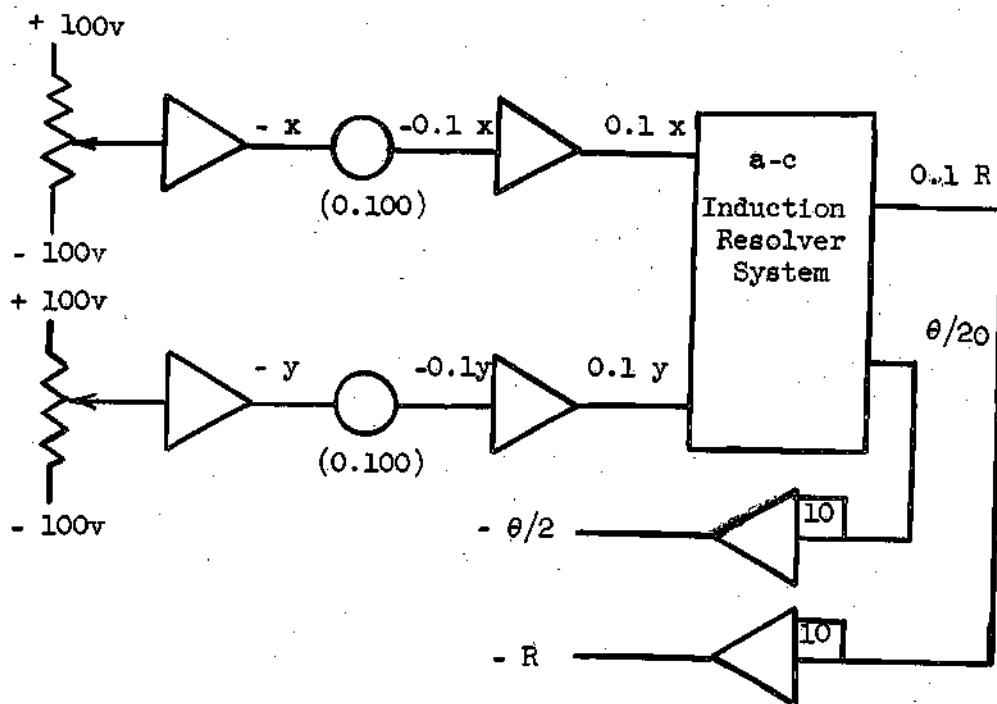


Figure 8. Analog Computer Program for Static Tests.

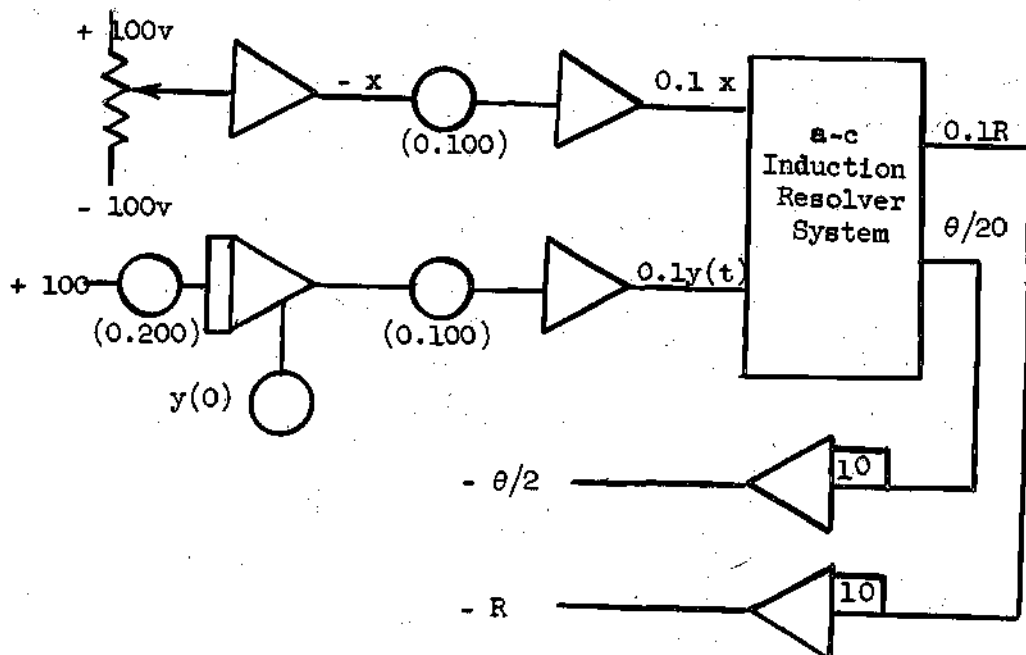


Figure 9. Analog Computer Program for Ramp Response Test.

passed through conventional inverter operational amplifiers to prevent any loading effects on the coefficient potentiometers. The values of x and y were measured on an Epsco digital voltmeter Model DV-103 (calibrated to the computer reference voltage) which has a maximum resolution of one millivolt.

The output voltages of the resolver were applied to operational amplifiers with gains of ten to yield the desired minus R and minus 10θ . The output voltages of these amplifiers were measured on the null-reading d-c voltmeter of the computer which has a resolution of better than 10 millivolts.

The static test procedure consisted of applying a known value of x and y input voltages to the resolver and monitoring the R and θ output variables. The x and y input voltages were varied to extend over the complete operating range of the computer voltages. The resolver output voltages were calibrated for input voltages of $x = y = 40.0$ volts. This corresponded to a value of $R = 56.6$ and $\theta = 2.25$ volts (45 degrees).

A typical set of data was obtained by setting $x = 40.0$ volts and varying y from zero to 80.0 volts in 20.0 volt steps. The complete static test data is given in the Appendix.

The dynamic characteristics of the a-c resolver were obtained by observing the system response to a step function input and a ramp function input. To obtain the step response, the x input voltage was fixed at $x = 40.0$ volts and the y variable was made initially zero. Then the y variable was stepped from zero to 60.0 volts by a snap-action switch closure, and the value of R was plotted as a function time on an Electro Instruments x-y recorder Model 400. The resulting plot of R versus time

is given in the Appendix. The x-y recorder has a dynamic accuracy of ± 0.2 per cent at ten inches per second and a slewing speed of 30 inches per second of the carriage and 20 inches per second of the pen. This provides plotting accuracies of ± 0.2 per cent for the range of R chosen.

The system ramp response was obtained by providing a fixed x input and making the y input a ramp function of time. The value of R was plotted as a function of y on the x-y recorder as y varied from zero to plus 80.0 volts. The analog computer program for the ramp response test is shown in Figure 9. The y input was obtained from the output of a computer integrator whose input was a constant chosen to make the slope of the y-voltage variation 20.0 volts per second. The curves of R versus y for the ramp response for $x = 20.0, 40.0, 60.0,$ and 80.0 volts are given in the Appendix.

The system drift rate was obtained by setting $x = y = 40.0$ volts (calibration inputs) and observing the change in R and θ .

Results.—The static and dynamic accuracy of the device in performing the proper computation of the R output was calculated as a per cent error of the plus and minus 100 volts computer operating voltage range. It was found that the static accuracy in obtaining R was approximately one per cent for small input voltages, and was typically 0.5 per cent accurate for full-scale input voltages. The per cent accuracy of the system dynamic response, as measured from the ramp response curves, was approximately one per cent over the entire operating range.

The absolute error in θ was found to vary from 0.1 degrees at high input voltages to 2.7 degrees at extremely low input voltages. Expressed as a per cent error of the full-scale operating range of plus and minus

1800 degrees, the maximum per cent error in θ was less than 0.1 per cent.

A measure of the system bandwidth was obtained from the rise time of the step response. By defining the rise time as the time required for the R output to go from 10 per cent to 90 per cent of the peak value as the y input was stepped from zero to 60.0 volts, and specifying the system bandwidth by the following approximate formula:¹¹

$$T_r B = 0.35 \quad (11)$$

where

T_r = rise time in seconds = 0.1 sec. (measured from the dynamic step response curve)

B = bandwidth in cps

it was found that the system bandwidth was approximately 3.5 cycles per second.

Sources of Error.--The primary source of conversion error was found in the diode gates which performed the modulation and demodulation in the circuit. For precise modulation of the input d-c voltage, all the diodes in the four-diode ring modulator are required to be clamped either in the non-conducting state or conducting state at precisely the same time. Any mismatch of diodes would result in a slight deviation from the optimum diode switching and a transient voltage spike would result. This transient, when applied to the stator windings of the induction resolver, would result in a differentiation of the spike transient. The overall

¹¹ John G. Truxall, Automatic Feedback Control System Synthesis, New York, McGraw-Hill, 1955, p. 80.

result would be a decided non-linearity of the modulated inputs at extremely low voltages. Every effort was made to experimentally match the diodes in the modulators by the judicious insertion of a small value of capacitance around diodes d_1 and d_3 shown in the modulator schematic of Figure 3-a.

A second major source of conversion error is the backlash in the internal motor gear train. This error was measured and found to contribute an absolute error of 0.1 volts in the R output and approximately two degrees of error in the θ output.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Conclusions.--The design and construction of an a-c induction resolver system was undertaken to ascertain the overall practicality of an a-c resolver performing in conjunction with a d-c general-purpose analog computer. The specific function of the device is to convert voltage variables in the rectangular coordinate system to the polar coordinate system. The allowable resolver input voltage range is plus and minus ten volt d-c computer voltages, to be obtained from the outputs of conventional operational amplifiers. The resolver outputs are applied to similar operational amplifiers as d-c computer voltages. The entire system was made compatible with existing general-purpose electronic analog computers with plus and minus 100-volt references.

The evaluation of the working model showed that every design specification was met for full-scale input voltages except the absolute error in obtaining the θ output and the system bandwidth. However, the per cent error of θ was quite acceptable. It was found that the actual static conversion accuracy for full-scale input variables was approximately 0.5 per cent in obtaining R and 0.1 per cent maximum in obtaining θ .

The system bandwidth, determined for approximately a 75 per cent amplitude variation of the output voltages, was approximately 3.5 cycles per second. The operating range of θ was plus and minus 1800 degrees.

Conversions of extremely small input voltages were somewhat less precise than the desired specifications, but the least accurate conversion

was found to be in error by approximately one per cent in obtaining R.

In general, it was found that the a-c induction resolver technique for converting analog time-varying voltages from rectangular coordinates to polar coordinates was acceptable in analog computation if certain operating compromises were allowable. First, some extension in system bandwidth, as compared with the existing d-c servo resolvers, was obtained at a slight decrease in static conversion accuracy at low-voltage inputs. Secondly, the increased operating range of θ was obtained at a decided sacrifice in absolute accuracy in the θ conversion. Finally, the simplicity of the system should result in a more reliable resolver than the existing devices.

Recommendations.---The experience gained in the course of this research has indicated that certain design modifications could provide some improvement in system performance. The incorporation of a 400 cycle induction resolver capable of accepting an amplitude modulated square wave input (manufactured, for example, by the Kearfott Division of General Precision, Inc.) would allow the use of a 400-cycle carrier frequency. Thus an a-c servo motor could have been used and the necessity for a demodulator in the servomechanism loop, along with the associated non-linearity and drift problems, would have been eliminated. The use of an inertia-damped servo motor in the system would have allowed an increased servo-loop gain and an improved static and dynamic conversion performance.

The present system could be altered to perform the inverse conversion from that outlined in this study, i.e. the conversion of voltages in polar coordinate system to rectangular coordinates. Such a system is outlined in block diagram form in Figure 10.

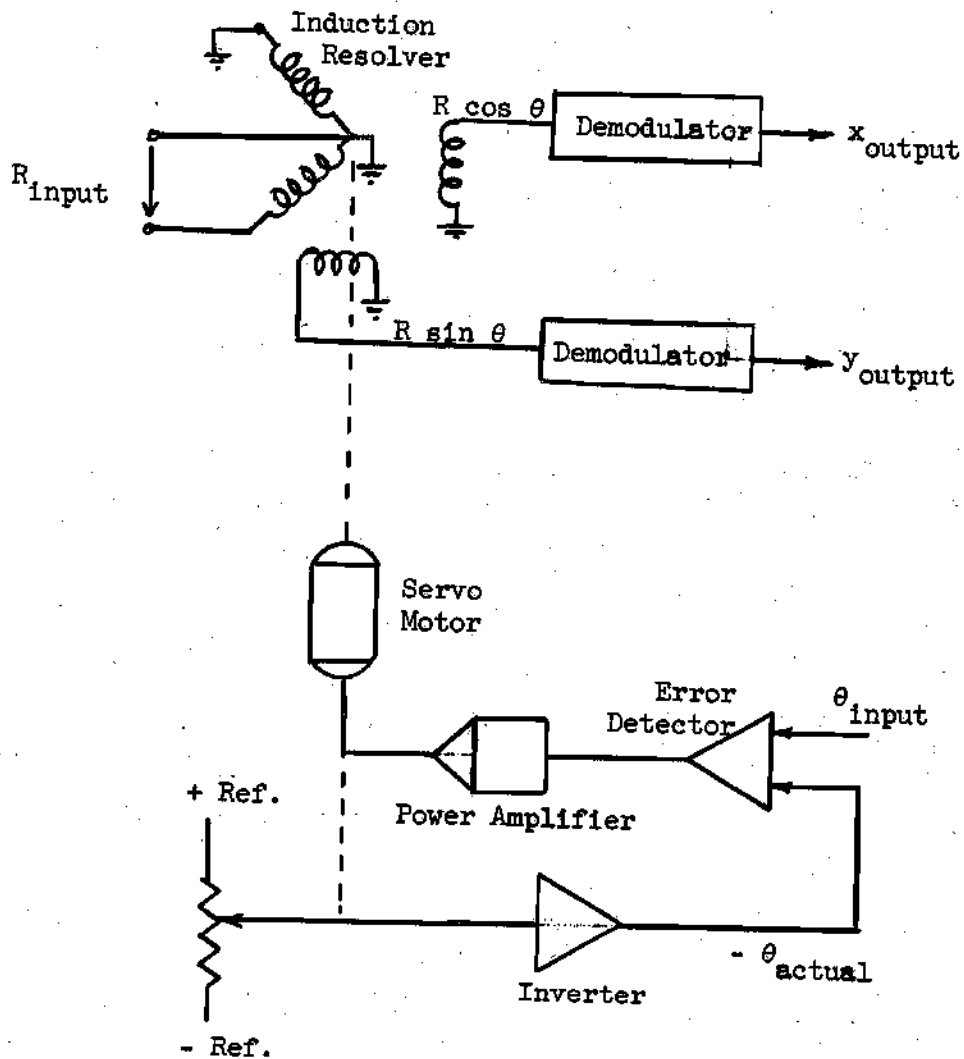


Figure 10. Block Diagram for Possible Polar-to-Rectangular Coordinate Conversion System Using the A-C Induction Resolver.

APPENDIX

Table 1. Induction Resolver Static Test Results, I.

Conversion	Input Voltages		R output Voltage	R theoretical Voltage	% Error*
	x	y			
1	0.0	20.0	17.3	20.0	1.35
2	0.0	40.0	38.5	40.0	0.75
3	0.0	60.0	59.2	60.0	0.40
4	0.0	80.0	79.0	80.0	0.50
5	0.0	99.8	97.8	99.8	1.00
6	20.0	0.0	18.8	20.0	0.60
7	40.0	0.0	40.0	40.0	0.00
8	60.0	0.0	59.9	60.0	0.05
9	80.0	0.0	78.8	80.0	0.10
10	99.8	0.0	96.9	99.8	1.45
11	20.0	20.0	26.4	28.3	0.95
12	20.0	40.0	43.7	44.8	0.55
13	20.0	60.0	62.8	63.2	0.20
14	20.0	80.0	81.8	82.6	0.40
15	40.0	20.0	44.3	44.8	0.25
16**	40.0	40.0	56.6	56.6	0.00
17	40.0	60.0	72.6	72.2	0.20
18	40.0	80.0	89.5	89.6	0.05
19	60.0	20.0	62.8	63.1	0.15
20	60.0	40.0	72.2	72.2	0.00
21	60.0	60.0	85.5	84.9	0.60
22	60.0	80.0	100.4	100.0	0.20
23	80.0	20.0	81.4	82.5	0.55
24	80.0	40.0	88.9	89.6	0.35
25	80.0	60.0	100.1	100.0	0.05

* % Error based on full-scale operating range of + 100 volts.

** Calibration point.

Table 2. Induction Resolver Static Test Results, II

Conversion	Input Voltages		θ output (Degrees)	θ theoretical (Degrees)	% Error* ($\times 10^{-1}$)
	X	Y			
1	20.0	0.0	2.70	0.0	0.75
2	20.0	20.0	45.6	45.0	0.16
3	20.0	40.0	61.0	63.5	0.69
4	20.0	60.0	69.2	71.6	0.67
5	20.0	80.0	73.8	76.0	0.61
6	40.0	20.0	26.6	26.5	0.03
7**	40.0	40.0	44.8	45.0	0.06
8	40.0	60.0	55.6	56.4	0.22
9	40.0	80.0	62.0	63.5	0.41
10	60.0	20.0	20.2	18.5	0.47
11	60.0	40.0	34.2	33.6	0.22
12	60.0	60.0	45.7	45.0	0.19
13	60.0	80.0	52.4	53.2	0.22
14	80.0	20.0	15.4	14.1	0.36
15	80.0	40.0	26.8	26.5	0.08
16	80.0	60.0	37.6	36.9	0.19

* % Error based on full-scale operating range of θ , $\pm 1800^\circ$.

** Calibration point.

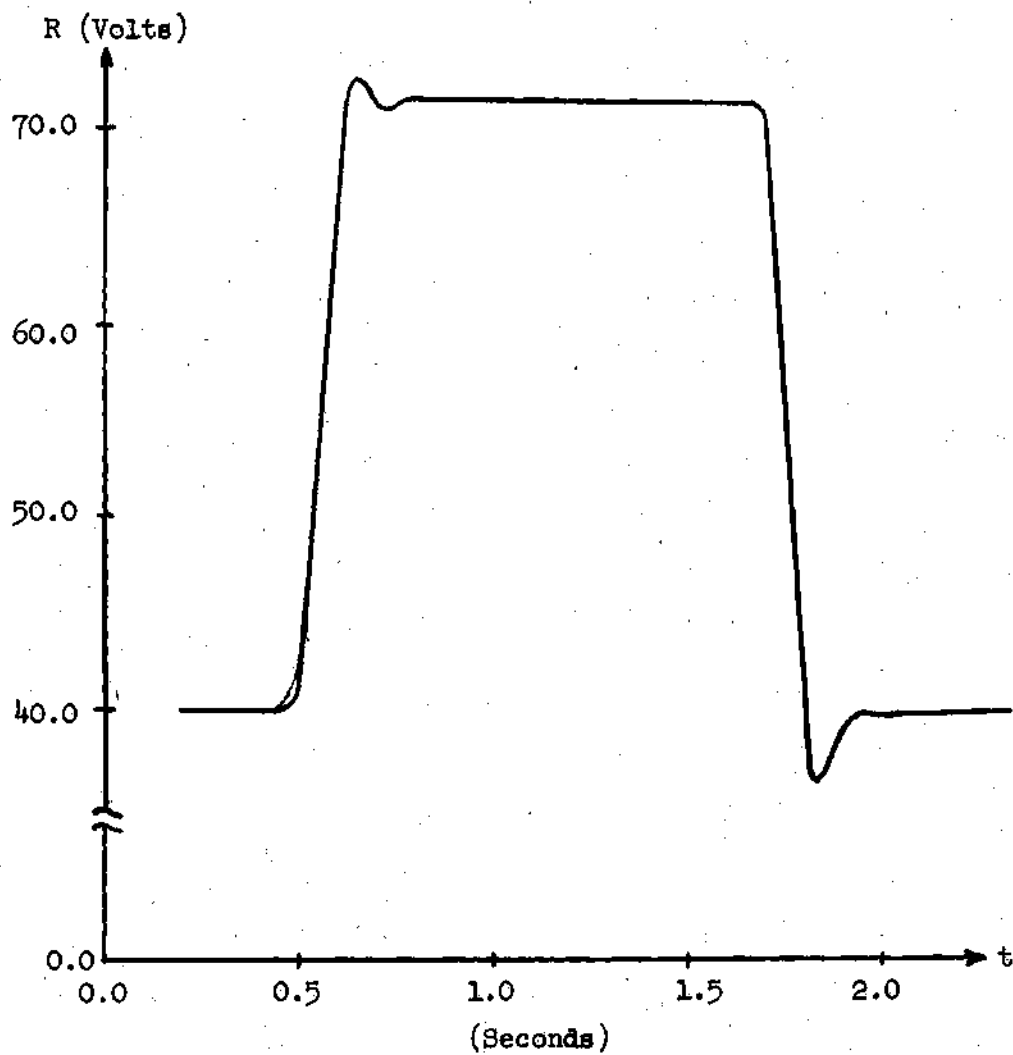


Figure 11. Induction Resolver System Step Response.

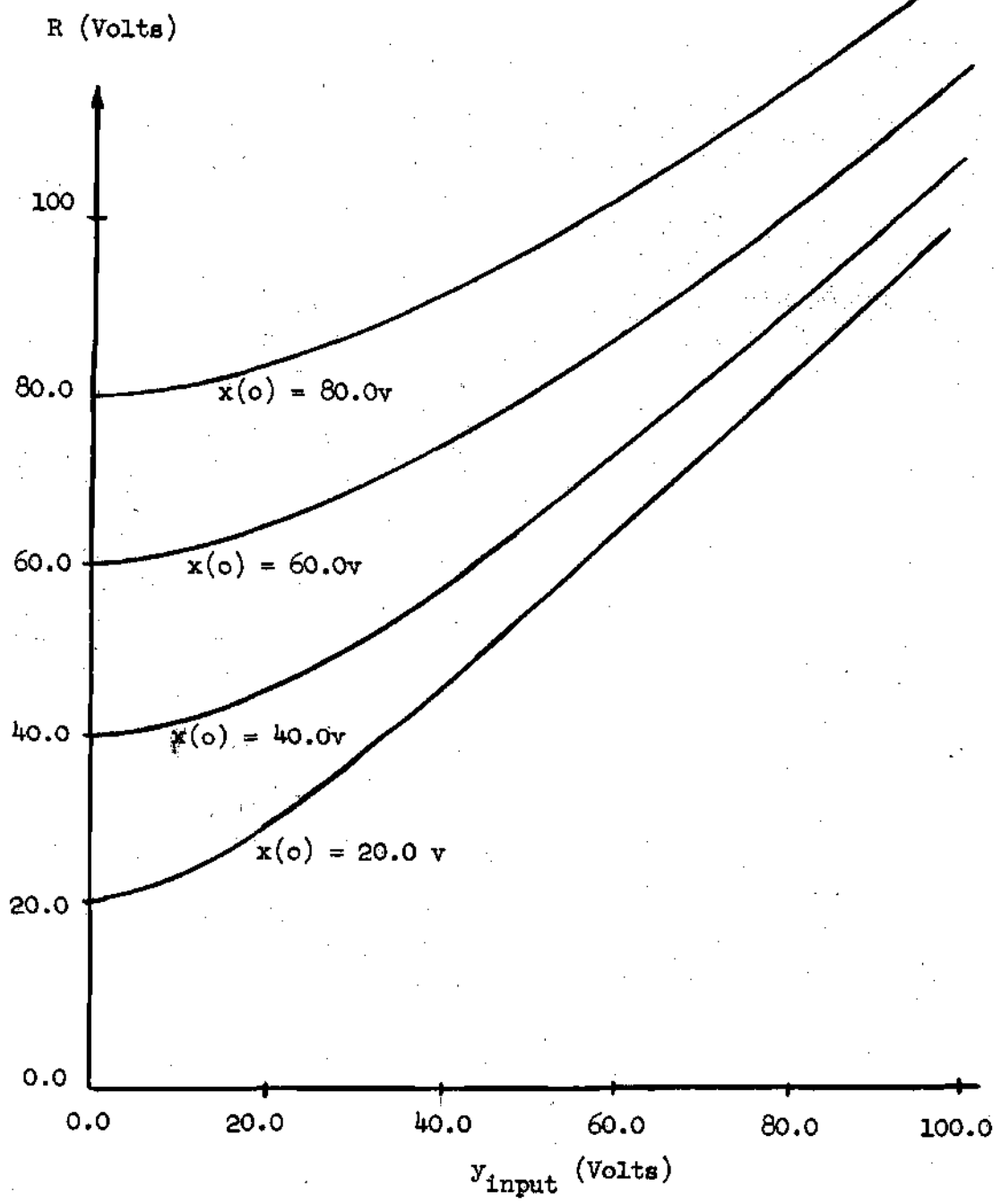


Figure 12. Induction Resolver System Ramp Response.

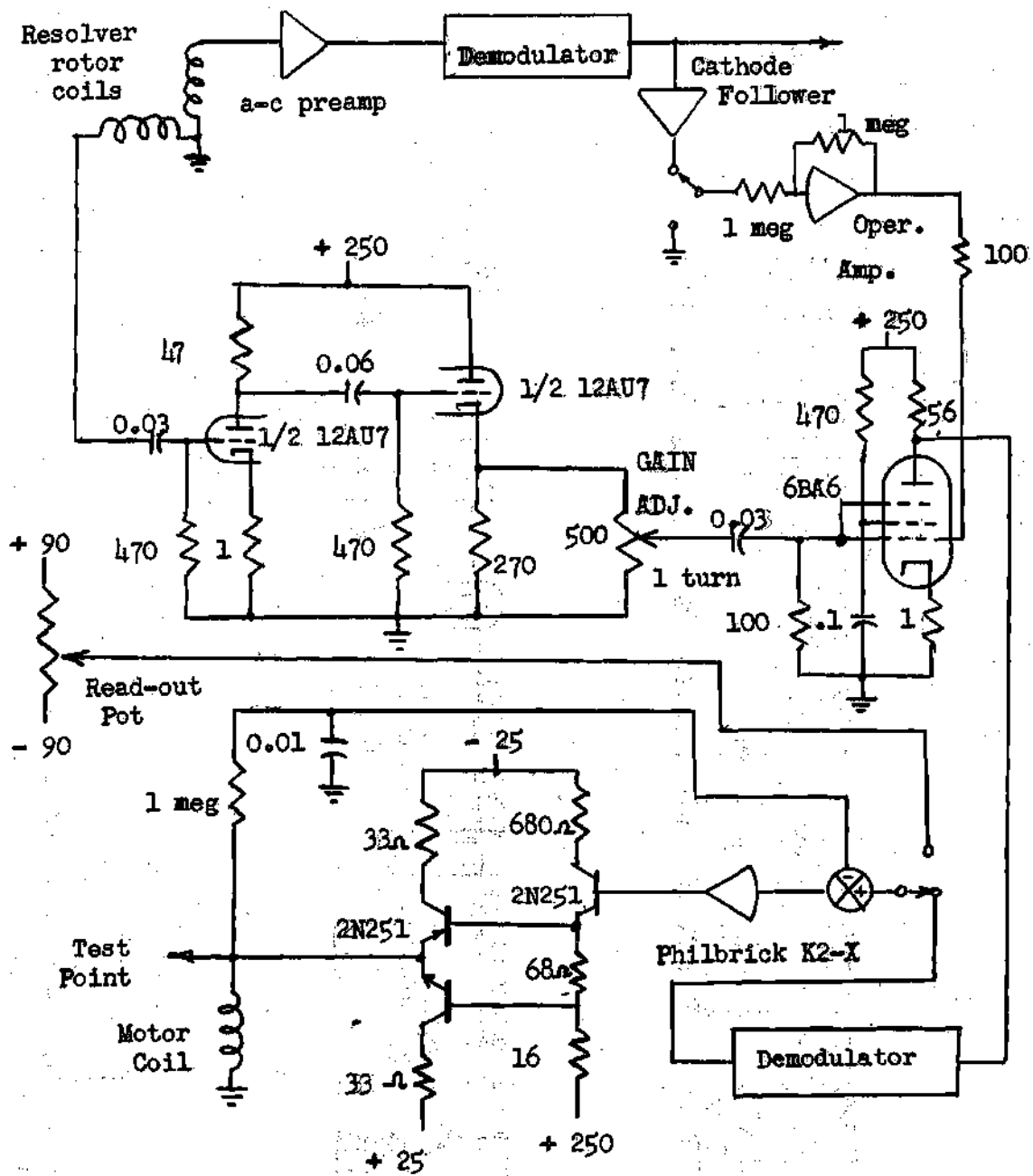


Figure 13. Detailed Servo Amplifier Circuit Diagram. (All resistance values given in kilohms and capacitances in microfarads unless otherwise noted.)

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