In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institution shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the Dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

Emory Warren Farr

A THEORETICAL INVESTIGATION OF A CONICAL RADIAL AND THRUST BEARING

A THESIS

Presented to

the Faculty of the Graduate Division

Georgia Institute of Technology

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

By

Emory Warren Farr

November 1955

A THEORETICAL INVESTIGATION OF A CONICAL RADIAL AND THRUST BEARING

THRUST BEARING

Approved:

Date Approved by Chairman: Nov. 23, 1955

ACKNOWLEDGMENTS

I express my sincere thanks to the following for aiding me in the pursuance of this research. The Gulf Oil Corporation and Mr. J. H. Hooten; Mr. J. L. Morris and Dr. W. B. Harrison, Professors of Mechanical Engineering; Dr. M. B. Sledd, Professor of Mathematics; Dr. J. P. Vidosic, my thesis advisor; Mr. T. D. Sangster, Mechanic of the Mechanical Engineering Department. Finally I thank my wife, Laura Farr for her understanding and patience during the pursuance of the work.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

SUMMARY

Since a theory applicable to the design of conical radial and thrust bearings does not appear to exist, the investigation reported in this thesis attempts to develop such a theory. To accomplish the development, mathematical equations were derived to relate the involved variables, then sufficient experimentation was carried out to attempt verification of the derived equations.

The conical bearing was assumed to operate hydrodynamically; therefore Reynolds' differential pressure distribution equation was considered applicable. This equation was applied to the bearing and solved to yield an expression for pressure in the oil film. This pressure was then used to derive the bearing operating equations for radial load, thrust load, and friction.

The experimental verification of the derived equations was made using a setup that measured friction in the bearing while holding set quantities for the other variables. This setup also provided means to introduce oil to the bearing under external pressure. Data were taken for conditions of external pressure with varying speed, varying radial load, varying thrust load, and varying radial clearance between the bearing and the journal. To determine the significance of these data, they were combined into dimensionless factors; then the factors were plotted and represented by an average curve.

The comparison between the theoretical and experimental work was made using conventional McKee type graphs of dimensionless factors. From

the comparison, reasonable agreement was obtained considering the deviation between the curves attributed to end flow. End flow was neglected in deriving the theoretical equations thereby justifying the attribution.

An important result found from this investigation is that a conical bearing will not hydrodynamically support a radial load unless external oil pressure is supplied to support the thrust load. This fact is indicated from an analysis of the thrust load equation and was fully confirmed experimentally. From this result and from the comparison curves, there is considerable indication that the derived operating equations for radial load and friction are verified. However, before definite conclusions can be reached, more experimental data are needed. To complete the investigation, correction factors for end flow in the bearing should be determined.

CHAPTER I

INTRODUCTION

Since a theory applicable to the design of conical radial and thrust bearings does not appear to exist, the investigation reported in this thesis attempts to develop such a theory. First, mathematical equations were derived relating the bearing variables; then the proposed theory was verified experimentally.

Reynolds' differential equation (1)* for pressure distribution constituted the starting point for the investigation. To solve it, however, it was first necessary to describe mathematically the geometry of the oil film in the bearing. The substitution of this relation converted Reynolds' equation into one that specifically applied to the bearing under study. The specific differential equation then had to be solved by means of some mathematical technique.

Difficulty arose immediately because the complexity of the equation is such that exact methods of solution of differential equations cannot be applied. This difficulty necessitated approximations which do not permit an exact solution and which in turn affect the computed operating equations. Due to the complexity Reynolds' equation has only been applied to relatively simple bearings, the most common being the plane slider type and the cylindrical journal type. Even with these simple

---

*Numbers in parenthesis identify references given in Bibliography.

bearings, it is possible to impose operating conditions that are inconsistent with the computed equations by a considerable amount.

To verify the analytical work experimentally, a setup was used that permitted accurate measurement of the friction in a bearing while maintaining fixed values for the other variables involved. The variables were then combined and plotted in conventional dimensionless groups for comparison with the theory.

In the technical literature the words conical and pivot are used synonymously in describing bearings of the point-contact type found in such things as watches and instruments. These are not the same as the conical bearings with full surface contact investigated here. The two types appear to possess entirely different characteristics and are therefore mentioned here to avoid confusion later.

CHAPTER II

THEORETICAL DEVELOPMENT

General Hydrodynamic Theory

The so-called "perfectly" lubricated bearing is one that operates hydrodynamically. This means that during operation, the journal shifts itself to run eccentrically with the bearing and by so doing is able to form a pressure within the lubricant that will support an external load. During this kind of operation the journal and bearing are always separated by a fluid film of lubricant, the friction thus resulting from the viscous drag within the film.

A bearing that does not operate hydrodynamically must be in the so-called boundary range of lubrication. Here factors are such that internal pressure is not generated within the lubricant and consequently metal-to-metal contact occurs and increases friction and power loss. Certain dimensionless combinations can be used to distinguish the two operating ranges.

Derivation of the Operating Equations

Analysis of the Problem.--In this investigation, it was assumed that if the conical bearing operates hydrodynamically, the journal under a radial load must shift itself to a position eccentric with the bearing. In order to do this, however, the journal must also rise since at rest under a thrust load the journal and bearing are in full contact (see Figure 1).

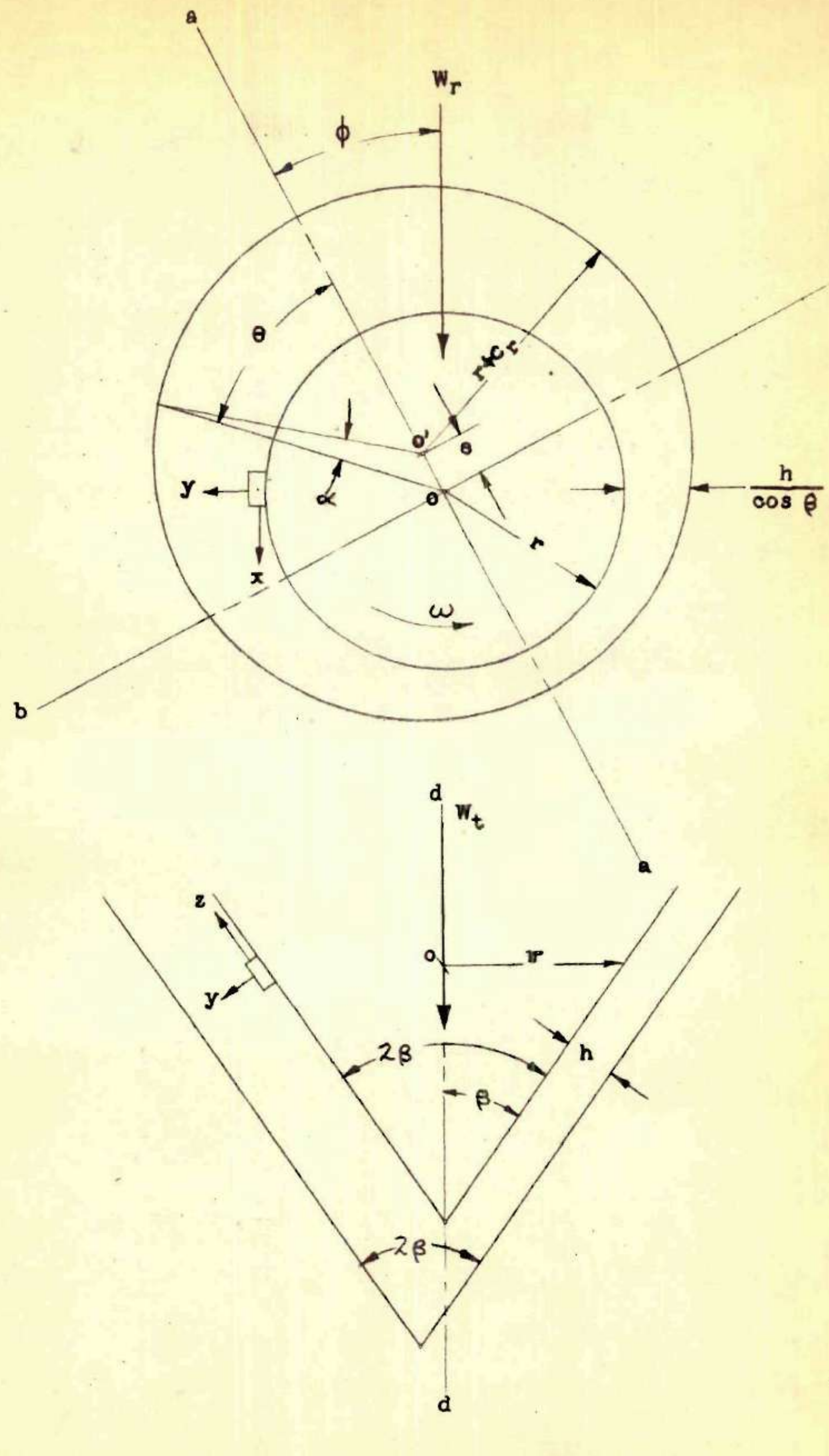

Figure 1. Diagram of Bearing

To analyze the bearing it was necessary to determine the internal pressure developed under this condition and use it to determine if or how much thrust and radial load could be supported. To determine the frictional drag, it was also necessary to know the shear stress within the lubricant. Whether or not the actual frictional drag is due to shear stress in the lubricant, of course, depends on the bearing operating hydrodynamically; that is, whether the radial and thrust loads can be supported by the pressure.

Derivation of Pressure Distribution Equation.--The pressure distribution relation was derived from Reynolds' equation as mentioned in Chapter I. The equation given in generalized form is (2)

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(h^3 \frac{\partial p}{\partial z}\right) = 6\mu r \omega \frac{\partial h}{\partial x},$$

where p = pressure in lb./sq. in.;

μ = viscosity in Reyns (lb.-sec./sq. in.);

h = oil thickness at any point in inches;

r = radius of the journal at any cross section in inches;

ω = angular velocity of the journal in radians per second;

x, z = coordinates as shown in Figure 1.

In the equation, μ and ω are considered constant; h is a function of x only (see Figure 1); and p is a function of x and z.

In order to obtain an expression for p, the equation was simplified by introducing the approximation that $\partial p / \partial z = 0$. This simplification in effect assumes that there is no pressure change in the oil in the z direction or that it is negligible in comparison to the pressure change in the x direction. In the case of the previously analyzed bearings (cylindrical journal and plane slider), this approximation was used and justified on the assumption of no end flow. The conical bearing, however, does not strictly permit justification even in the case of no end flow because of the pressure change due to the varying radius r. The approximation as used here permits the only presently known simple solution to Reynolds' equation.

The simplified equation is rewritten as

$$\frac{\partial}{\partial\theta}\left(h^3 \frac{\partial P}{\partial\theta}\right) = 6\mu r^2\omega \frac{\partial h}{\partial\theta},$$

where ∂x has been replaced by $r\,\partial\theta$. To solve this equation an expression for h in terms of θ was substituted into it. Then the equation was integrated and simplified to obtain the following (see Appendix A, Derivation 1, for the complete derivation):

$$P = \frac{6\mu\omega r^2}{c_r^2\cos^2\beta}\left[\frac{n(2+n\cos\theta)\sin\theta}{(2+n^2)(1+n\cos\theta)^2}\right] + P_o, \qquad (1)$$

where p_o = constant external supplied pressure in psi;

c_r = radial clearance between the journal and bearing in inches;

n = attitude of the journal = eccentricity/clearance;

β = one-half of the included angle describing the conical shape (degrees);

θ = angle around the bearing circumference as measured in Figure 1 in degrees.

Radial Loading.--To determine the radial capacity of the bearing, the forces on the journal acting along the mutually perpendicular axes aa and bb were summed and equated to zero (see Figure 1). To accomplish this purpose the pressure at any point on the journal was multiplied by an infinitesimal area and integrated over the entire conic surface to get the total normal force. Then the radial and aa components of the force were taken and set equal to the aa components of the load W_r (see Figure 1). Summing forces along aa gives the following equation:

$$W_r \cos\phi = \int_0^{2\pi}\int_0^{R} P \cos\theta \cos\beta \frac{r\,dr\,d\theta}{\sin\beta}$$

where R = maximum value of r;

ϕ = angle between the load and axis aa in degrees (see Figure 1).

From this relation it was found that $W_r (\cos\phi) = 0$, which meant that $\cos\phi$ must be zero and therefore $\phi = 90°$.

Similarly summing forces along axis bb gives:

$$W_r \sin\phi = \int_0^{2\pi}\int_0^{R} P \sin\theta \cos\beta \frac{r\,dr\,d\theta}{\sin\beta}$$

Hence (see Appendix A, Derivation 2, for the complete derivation),

$$W_r = \frac{\mu\omega R^4}{C_r^2\cos^2\beta}\left[\frac{3\pi n}{\tan\beta\,(2+n^2)(1-n^2)^{1/2}}\right]. \qquad (2)$$

Thrust Loading.--To determine the thrust capacity of the bearing, the components of forces on the journal acting along the axis dd (see Figure 1) were summed and equated to zero as in the radial load case. Summing the forces along this axis gives the equation

$$W_t = \int_0^{2\pi}\int_0^{R} P \sin\beta \, \frac{r\,dr\,d\theta}{\sin\beta},$$

from which (see Appendix A, Derivation 3, for the complete derivation)

$$W_t = P_o R^2 \pi. \tag{3}$$

Journal Friction.--To determine the frictional drag on the journal, the shear stress in the lubricant in the direction of motion was multiplied by the area of the journal. There were two components to this shear, τ_{xy} and τ_{xz}. However, in the pressure derivation τ_{xz} was assumed negligible in comparison to τ_{xy}. This assumption in effect neglects the velocity gradient of oil in the z direction, saying that it is very small in comparison to the velocity gradient of the oil in the y direction. In the conical bearing it is possible that such an assumption is not justified; however, since Reynolds' equation was the basis for this whole analysis, the assumption is at least consistent. The simplified shear stress is given as (3)

$$\tau_{xy} = \frac{\mu r \omega}{h} + \frac{h - 2y}{2}\frac{\partial P}{\partial x}.$$

At the journal surface, where the stress was wanted, $y = 0$ and $x = r\theta$.

This gave the shear on the journal as

$$\tau_J = \frac{\mu r \omega}{h} + \frac{h}{2r}\frac{\partial P}{\partial \theta}.$$

Upon multiplying this expression by an infinitesimal area and integrating over the entire conic surface, the friction on the journal was found to be (see Appendix A, Derivation 4, for the complete derivation)

$$F_J = \frac{\mu \omega R^3}{3 \sin\beta\, C_r \cos\beta}\left[\frac{4\pi(1+2n^2)}{(2+n^2)(1-n^2)^{1/2}}\right]. \qquad (4)$$

Preliminary Check and Analysis

Cylindrical bearing theory was used to obtain a preliminary check on the W_r and F_j equations by forming conical bearings of different sizes from a series of cylindrical bearings. For a given set of conditions, W_r and F_j were computed for each of the cylinders forming the bearing and their total compared to the W_r and F_j obtained from the derived equations. Cones made up by using 32 cylinders were formed with included angles varying from 20° to 160°. In all cases the derived equations checked the comparison to within 1.2 per cent.

The W_t equation could not be checked by using theory for a cylindrical bearing since loading of that kind cannot exist in a cylinder. This equation says in effect, however, that the thrust load must equal the external pressure times the projected area of the cone.

Assuming that the derived equations are correct, an analysis shows that the conical bearing will not support a thrust load except by external pressure. If external pressure is supplied to support the thrust load, however, a radial load can be imposed that will be supported hydrodynamically. Looking at it another way, it can be said that unless an external pressure is supplied, the bearing operates completely in the boundary lubrication range.

CHAPTER III

EXPERIMENTAL APPARATUS AND PROCEDURE

Instrumentation and Equipment

The apparatus used for the experimental verification of the theory was a bearing testing machine perfected and built in 1953 by R. D. Cheverton (4), a graduate student at the Georgia Institute of Technology. The machine was originally used in connection with a thesis investigation on a textile spindle. However, with several modifications it was adapted to the conical bearing investigation.

In general, the setup used enabled one to vary the load, speed or clearance on the journal for a particular oil and at the same time measure the friction force exerted on the bearing. The following paragraphs describe how these controls were accomplished.

The journal was a vertically mounted spindle with a conical point. A pulley was pressed on for driving. The bearing had a matching conical surface and was fastened to a cylindrical shell that extended approximately four inches up the spindle; this shell acted as a container for the oil used. The shell and bearing in turn were mounted on ball and roller bearings in a heavy aluminum casting. The assembly is shown in Figure 2; a closeup of the bearing and spindle components is shown in Figure 3.

In order to measure the frictional force generated by the journal during operation, the shell and bearing were prevented from turning on

FIGURE 2

Spindle Dynamometer-Assembly

1. Aluminum casting housing spindle components
2. Dial indicator for clearance measurement
3. Nine-speed transmission
4. Idler pulley

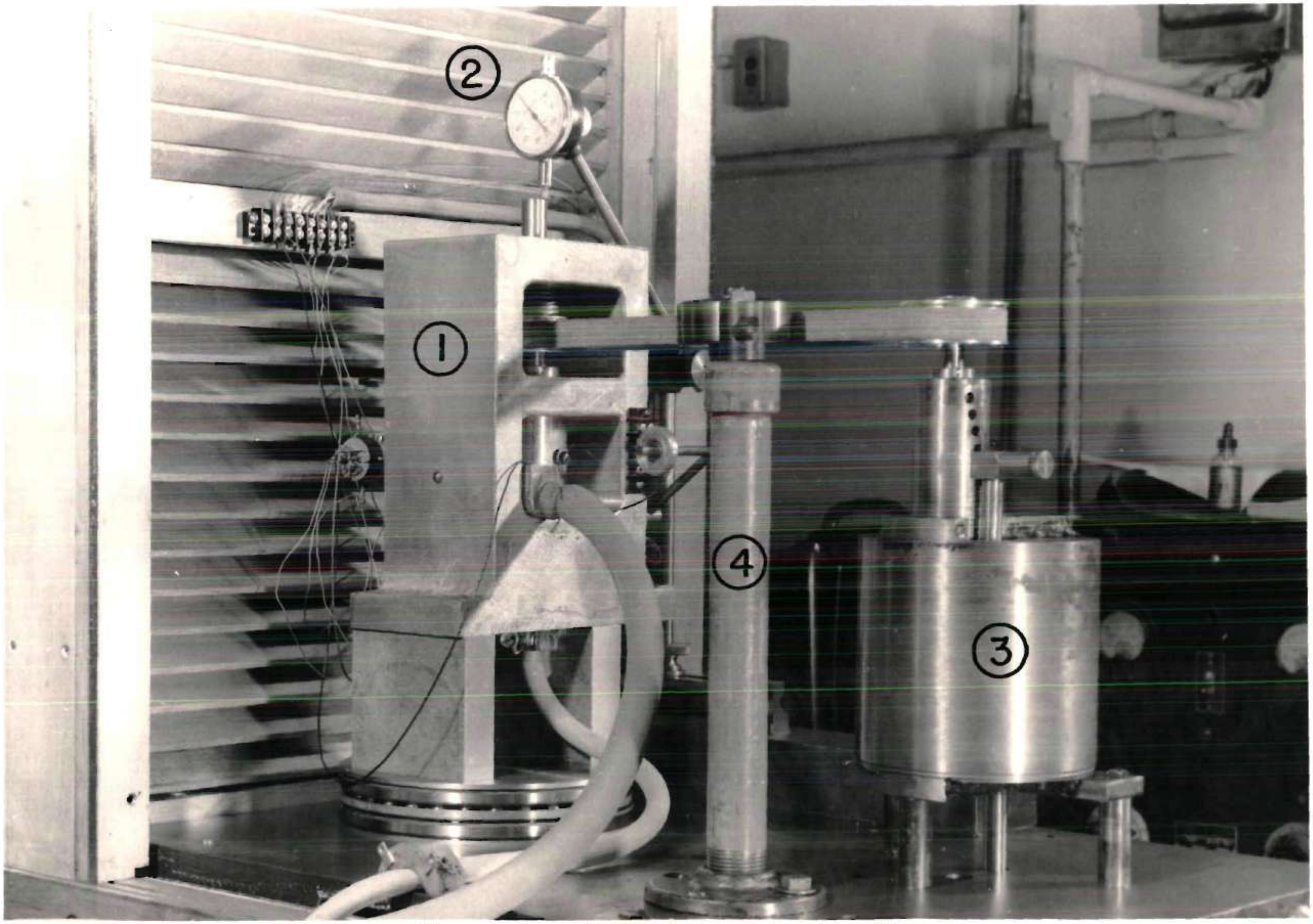

Figure 2. Spindle Dynamometer-Assembly

FIGURE 3

Spindle Dynamometer-Component Parts

1. Aluminum cantilever beam for friction measurement
2. Spindle with the conical journal at lower end
3. Cylindrical shell with mounting bearing
4. Conical bearing with mounting bearing
5. Trough to oil return line

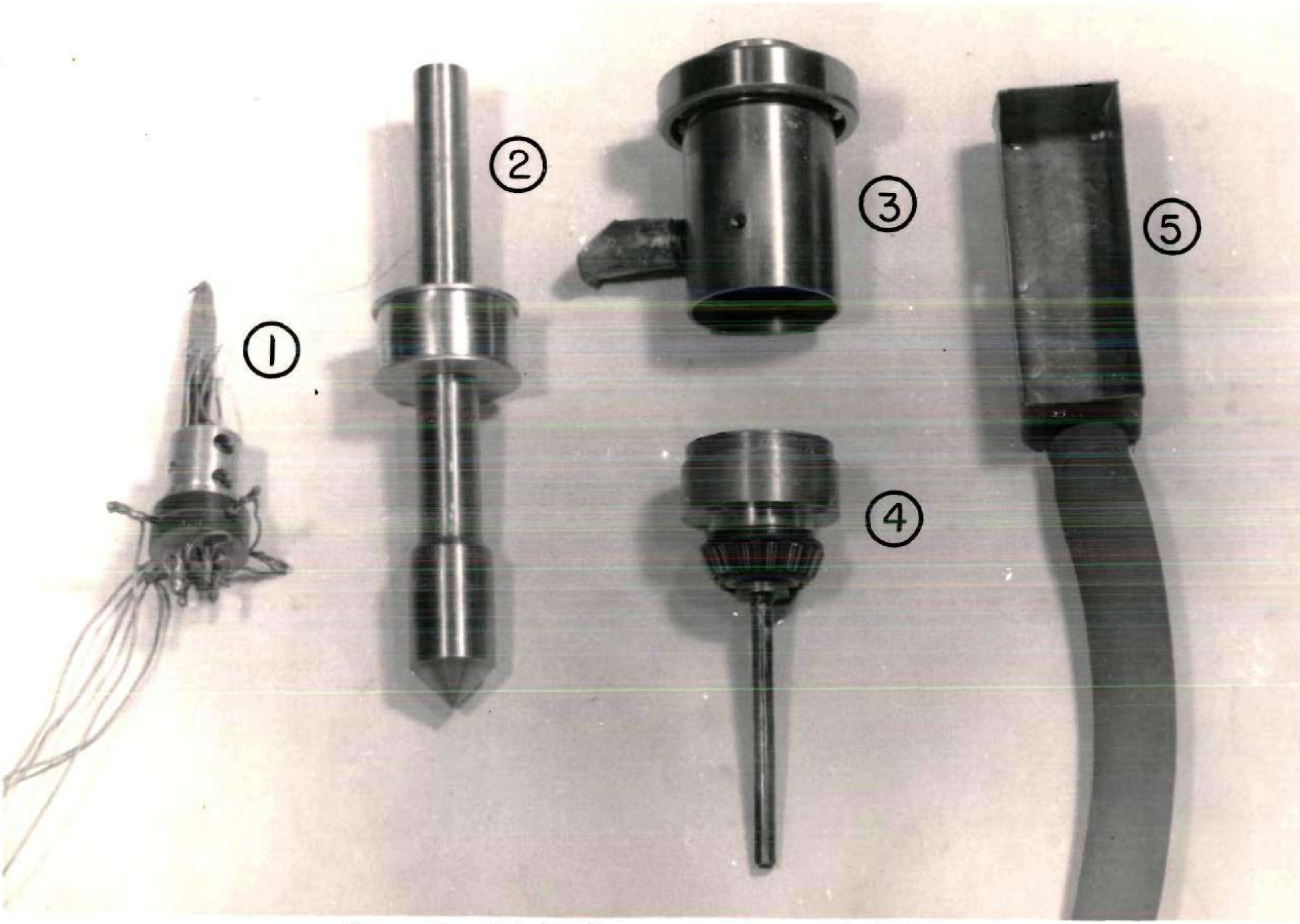

Figure 3. Spindle Dynamometer-Component Parts

the roller and ball bearings by a thread connected from the shell to the free end of a small aluminum cantilever beam (see Figure 3). This beam was mounted on the heavy aluminum casting (Figure 4) and had cemented to it two SR-4 electrical strain gages which were connected to a Foxboro strain recording instrument (see Figure 5). Though the instrument actually measured strain on the beam, it could be calibrated to read the frictional force on the bearing. For a constant power supply, the strain recorder was connected to a voltage regulator shown in Figure 6.

The heavy aluminum casting housing the conical bearing and spindle was itself mounted to a castiron table through a large ball type thrust bearing (see Figure 2). The thrust bearing was positioned eccentric to the spindle and thus allowed a load to be applied to the aluminum casting that would create a tension force in the spindle drive belt. This force or load was applied by a cord which, attached to the base, passed over a pulley and was hung with various weights as shown in Figure 7. A portion of this force was directly carried by the conical bearing as a radial load.

The bottom race of the thrust bearing could be moved parallel to the load line by means of a screw adjustment shown in Figure 4. This adjustment allowed the external force from the cord to be lined up with the tension force from the belt, thereby keeping the forces in the same vertical plane.

Since the equation derived in the previous chapter indicated that a thrust load could not be supported by internal pressure, a means of introducing oil under pressure was deemed necessary. The problem was

FIGURE 4

Spindle Dynamometer-Assembly

1. Thrust bearing adjustment handwheel
2. Nine-speed transmission
3. Aluminum casting housing spindle components
4. Cantilever beam mounting assembly

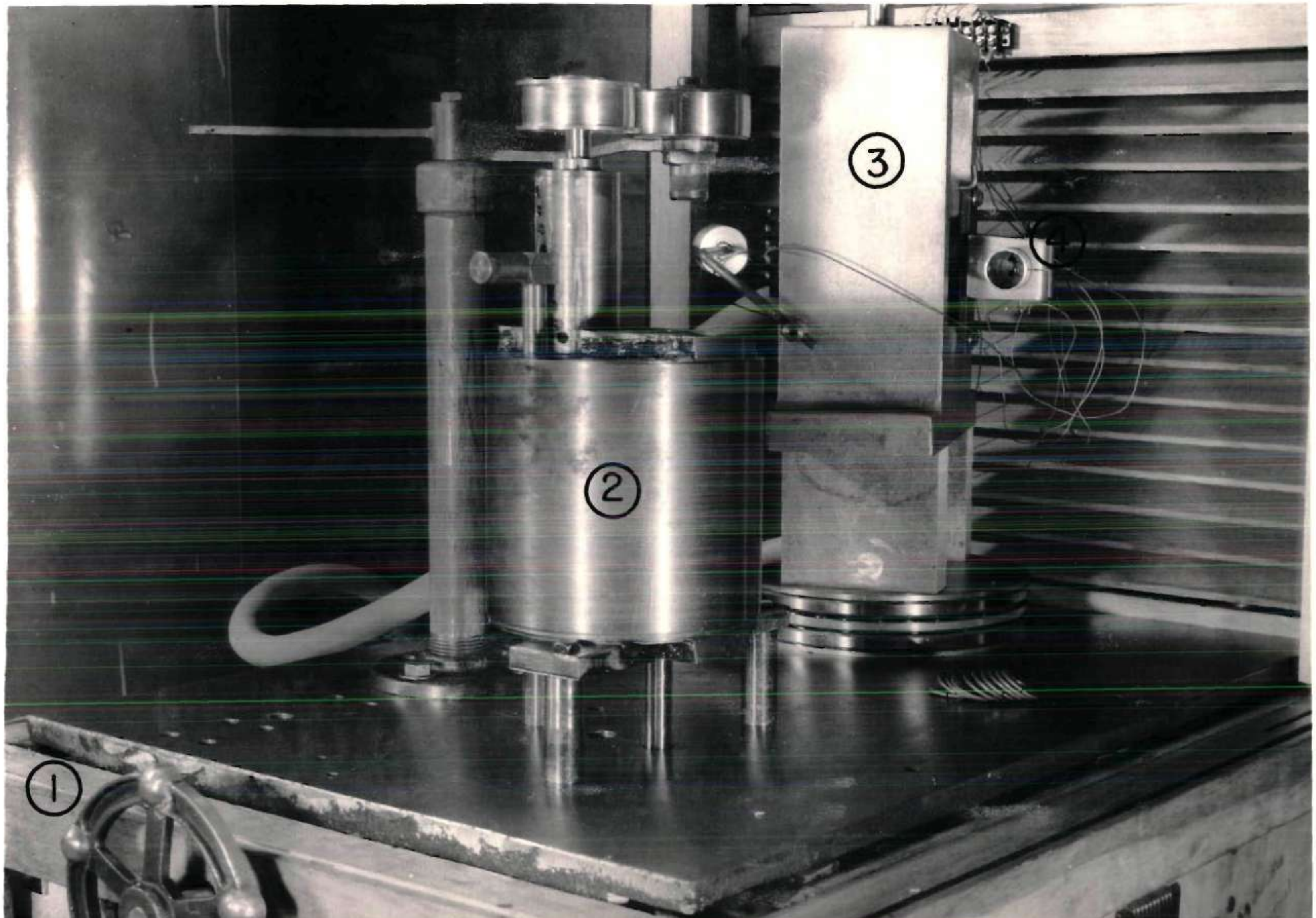

Figure 4. Spindle Dynamometer-Assembly

FIGURE 5

Spindle Dynamometer-Measuring Equipment

1. Chronometric tachometer
2. Foxboro temperature indicator
3. Foxboro strain recorder

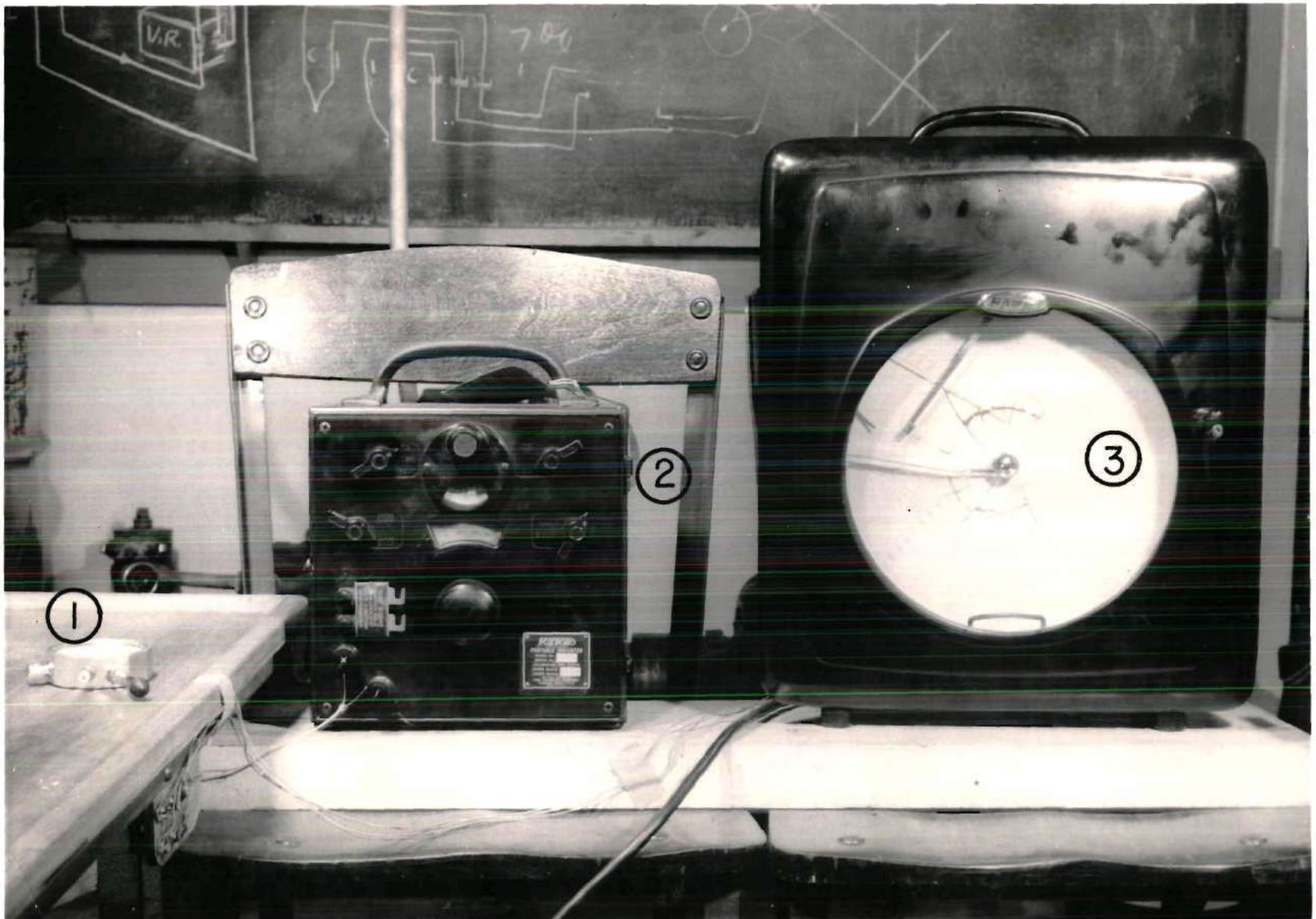

Figure 5. Spindle Dynamometer-Measuring Equipment

FIGURE 6

Spindle Dynamometer-Operating Equipment

1. Gear reduction unit
2. Driving motor
3. Voltage regulator

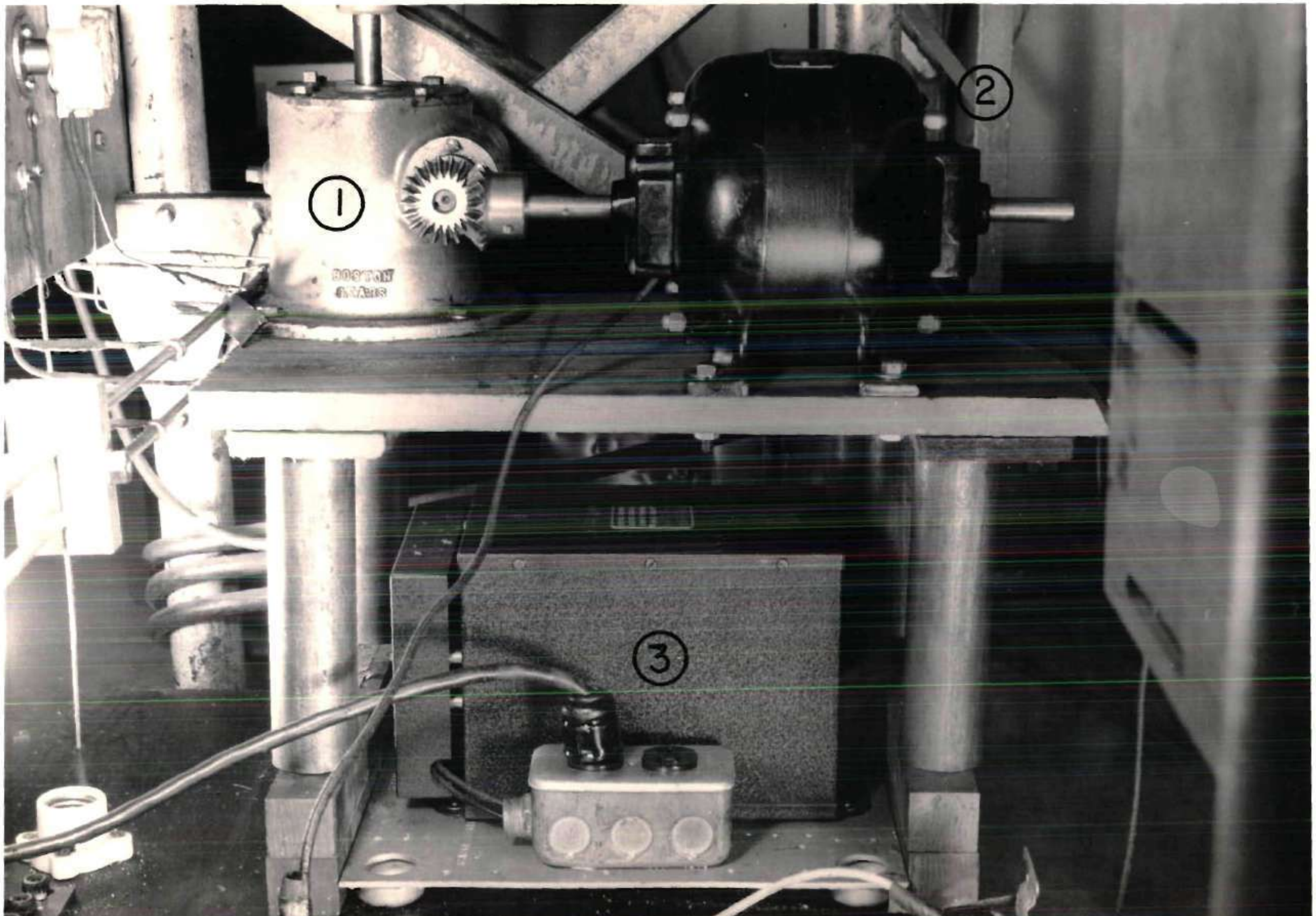

Figure 6. Spindle Dynamometer-Operating Equipment

FIGURE 7

Spindle Dynamometer-Pump Assembly

1. Pump motor
2. Pump
3. Pressure gage on supply line
4. By-pass valve
5. Oil reservoir
6. Cord for radial load application

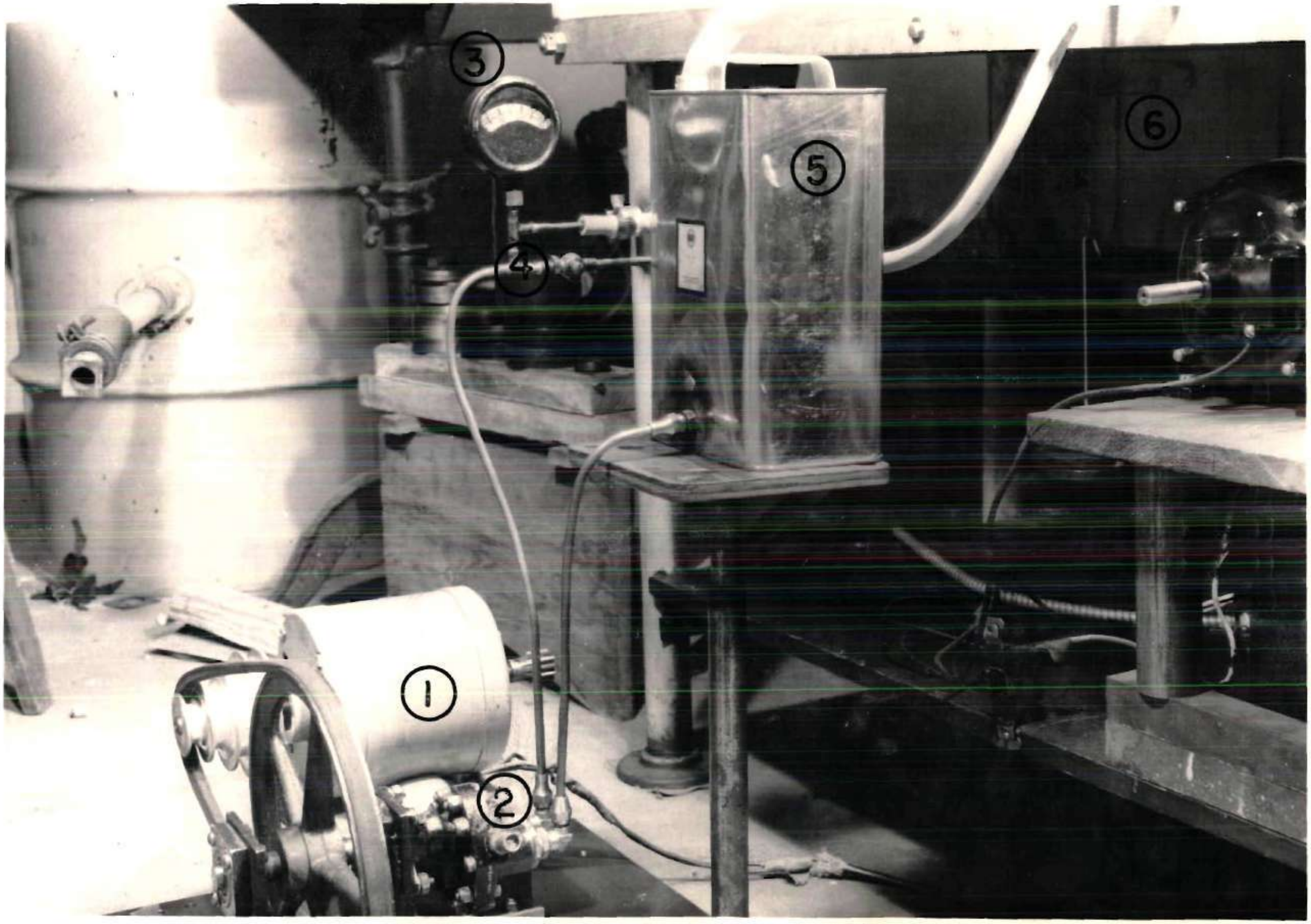

Figure 7. Spindle Dynamometer-Pump Assembly

solved by a small positive displacement gear pump which introduced oil to the bearing through a hole in the center of the cone. The oil was removed from the bearing by a spout in the cylindrical shell and dumped into a trough; from here the oil flowed back to the reservoir by gravity. The pressure of oil flowing to the bearing was controlled by means of a by-pass valve between the pump and the reservoir. Figures 3 and 7 show the pump assembly and detail components of the flow circuit.

The weight of the vertically mounted spindle provided the thrust load on the bearing for which the external pressure was needed. The thrust load was essentially fixed at this value but could be slightly varied by adding small coils of solder to the spindle.

To determine the viscosity of the oil used, thermocouples were installed in the bearing entrance at the top of the cone and also in the exit at the trough. A Foxboro temperature indicator (Figure 5) was used with the thermocouples.

To measure the radial clearance between the bearing and journal produced by a given flow of oil, a Starrett dial indicator set was used as shown in Figure 2. This indicator actually measured the vertical clearance between the bearing and journal, but radial clearance was easily computed from it.

Speed control on the journal was accomplished by a synchronous motor that was geared through a nine to one reduction unit to a nine-speed transmission (see Figures 2 and 6). This reduction unit was a modification of the original apparatus necessary to limit speeds to a range where data were needed. The speeds used ranged from 300 to 900 RPM.

Test Procedure

Calibration of Aluminum Beam.--Before tests were run, it was necessary to calibrate the aluminum cantilever beam with known weights so that a curve could be drawn relating the recorder readings to forces on the beam. For this purpose, the thread connecting the shell and bearing to the cantilever beam was extended around the shell and over a ball bearing pulley attached to the aluminum base as shown in Figures 2 and 4. Known weights were applied to the thread to cover the expected range of forces. In Figure 4, the weights used to form the calibration are shown lying on the castiron table. The data for the calibration are tabulated in Appendix B, Table 5; the conversion curve plotted from this data is in Figure 11, Appendix C.

It was very important during the calibration to have the driving motor running. The motor produced a vibration in the castiron table that was carried to the ball and roller mounting bearings, thereby helping to reduce the static friction in them. In an effort to reduce further this friction after the weights were applied, the shell was set in torsional vibration by hand and allowed to damp itself to an equilibrium position.

It was found later during the runs that small particles of dirt and dust accumulated in the bearings and tended to increase their frictional resistance. This difficulty was overcome by frequent cleaning with a solvent but necessitated frequent checks on the calibration.

Speed Determination.--The speed of the spindle for each run was determined directly by using a chronometric tachometer. This instrument timed the revolutions of the spindle for a period of five seconds and then gave a corrected reading in RPM. The need for frequent speed measurement arose from the fact that slippage occurred in the drive belt and varied as loading on the bearing was changed.

Data Taking.--To begin a run, loads, speed, and clearance was set on the bearing and then the driving motor was started. First, the dial indicator on top of the spindle that read the vertical clearance was removed so that the speed could be determined by the tachometer. With the indicator replaced, the chart motor on the strain recorder was activated, and a friction reading was recorded for approximately one minute. During this time, the thermocouple readings on the temperatures of the entering and leaving oil were recorded. A total of thirteen runs were made on the bearing, and the above data were taken for the following conditions:

1. Varying speed with constant clearance and load. Four different oils were used for this condition.

2. Varying radial load with constant clearance, speed and thrust load. Four different oils were used for this condition.

3. Varying clearance with constant speed and loads. Four different oils were used for this condition.

4. Varying thrust load with constant clearance, speed and radial load. One oil was used for this condition.

The data as recorded are tabulated in Tables 1, 2, 3, 4 of Appendix B.

CHAPTER IV

EVALUATION OF DATA

The evaluation of the data was made by a graphical representation involving as many of the bearing variables as possible. To accomplish this purpose, the variables were combined into two dimensionless factors which could be computed from the data, and then plotted as coordinates. A single continuous curve drawn through the plot was then used as the basis for all further experimental computations.

Determination of Dimensionless Factors

The dimensionless factors used were determined from equations 1 and 2. They were formed by dividing the respective equations through by W_r and F_j and then removing the following quantities:

$$M = \frac{\mu \omega R^4}{C_r^2 \cos^2\beta \, W_r} \qquad J = \frac{\mu \omega R^3}{C_r \cos\beta \, F_J}$$

All of the bearing variables are represented here except W_t. However, since the major part of the data was taken with W_t constant, this omission becomes unimportant. The one run taken with variable W_t will be evaluated separately in the next chapter.

Determination of Bearing Variables from Data

Before the dimensionless factors M and J could be computed, it was first necessary to determine the bearing variables from the observed and measured data. With the exception of R and ω which were found directly, they were determined as follows:

Viscosity Determination.--The viscosity (μ) of the oils used was found from the curves on Figure 12 (see Appendix C) by reading the viscosity value corresponding to each oil temperature. The oils used in the investigation were obtained through the courtesy of Gulf Oil Corporation, and the curves of Figure 12 were computed from information supplied by Gulf on the oils.

Since the oil was circulating at all times, its temperature remained approximately constant. Temperatures were taken as the oil entered and left the bearing, but no appreciable difference was noted in the readings.

Radial Clearance Determination.--The radial clearance (c_r) can be related to the vertical clearance (c_v) by a trigonometric function. The relation is:

$$c_r = c_v \times \tan \beta$$

In this case the angle β for the bearing investigated was 45°; therefore the radial clearance equaled the vertical clearance.

Radial Load Determination.--By considering the aluminum casting and spindle as simply supported beams, the force on the conical bearing (W_r) could be related to the external force applied by the cord (W_c) by elementary mechanics. To determine this relation, it was necessary to measure along the so-called "beams" and find the points where the different forces act. These points were readily found for all the loads except the one on the conical bearing; and there the point must be derived using equation 2. The point was assumed to act at the centroid of loading and was found to be at $r = .84R$ by computation (see Appendix A, Derivation 5). Then by a summation of moments, W_r was determined to be .483 times W_c.

Friction Determination.--The friction force on the journal (F_j) could be related to the measured force on the cantilever beam (F_b) by a summation of torque around the spindle. To determine this relation, however, as for the radial load, it was necessary to compute the point on the cone surface where F_j could be considered acting. This point was found by assuming that the torque exerted by the journal during running must equal the torque to restrain the bearing. It was then found to be at $r = .75R$ by means of equation 4 (see Appendix A, Derivation 6). Then by summing torques about the spindle, F_j was determined to be 2.45 times F_b.

To convert the arbitrary reading (Q) of the strain recorder to the actual F_b measured, the calibration curve on Figure 11 (see Appendix C) was used. This curve was obtained by the method described earlier in Chapter III.

Representation of Plotted Points by Average Curve

After the dimensionless factors were computed (see Appendix D, Sample Calculation 1) and plotted for all of the data, a representative curve was drawn through the plotted points. Before this curve could be drawn, however, it was necessary to narrow the dispersion of points by averaging the J values in the range of a particular M. If a point in the average had a residual from the average of more than twice the probable error at that point, then that point was discarded as accidental and the remaining points reaveraged (see Appendix D, Sample Calculation 2). Once this dispersion was narrowed, a representative curve was drawn by eye. The curve obtained by this method is shown in Figure 8. Also shown is the theoretical curve which was computed from equations 2 and 4 by assuming values for n.

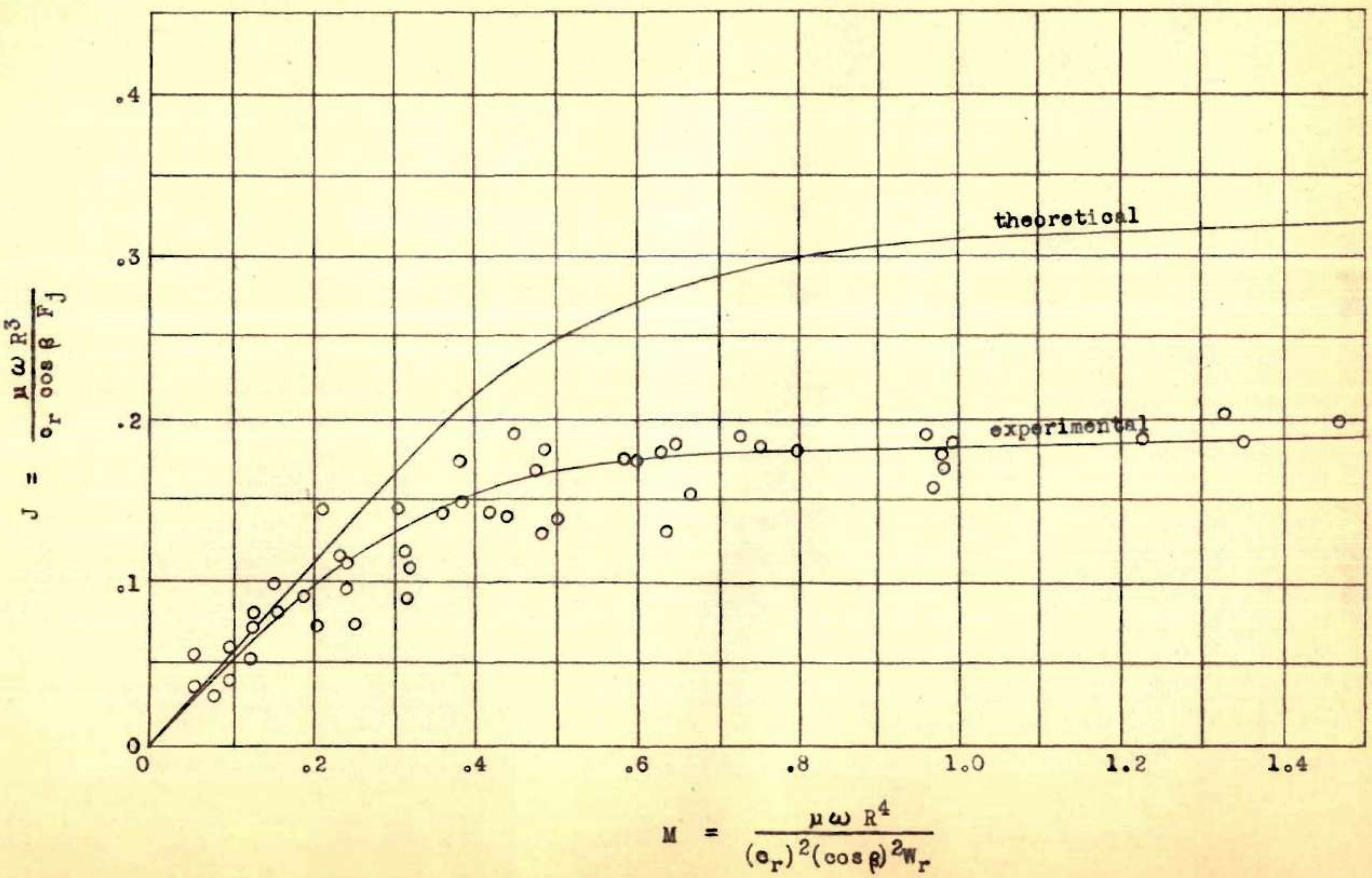

Figure 8. Data Evaluation Curve

CHAPTER V

DISCUSSION OF RESULTS

To compare the theoretical and experimental performance of the bearing, McKee type graphs were used. This type of graph employs certain dimensionless factors that when plotted enable one to identify regions of hydrodynamic and boundary lubrication in the bearing. The factors that produced this effect for the conical bearing are:

$$K = f\frac{R}{C_r}, \qquad M = \frac{\mu\omega R^4}{C_r^2 \cos^2\beta \, W_r}.$$

where f = coefficient of friction = $\dfrac{F_j}{W_r \cos\beta + W_t \sin\beta}$

These factors are somewhat similar to those used by Sommerfeld (5) to denote the performance of a cylindrical bearing, the factor M being analogous to the Sommerfeld number.

In order to compute the value f needed to determine K, it was necessary in the experimental case to use the curve on Figure 8. From this curve the "corrected" J corresponding to a particular M was read and then used with the original data to compute the corrected F_j. By knowing F_j and the loads enabled the factor f to be computed (see Appendix D, Sample Calculations 1 and 3).

<u>Curves with Varying Load and Positive Pressure</u>.--A curve of K vs. M plotted from all data taken with constant loads is shown in Figure 9 along with the theoretical curve. The curves are similar to those for

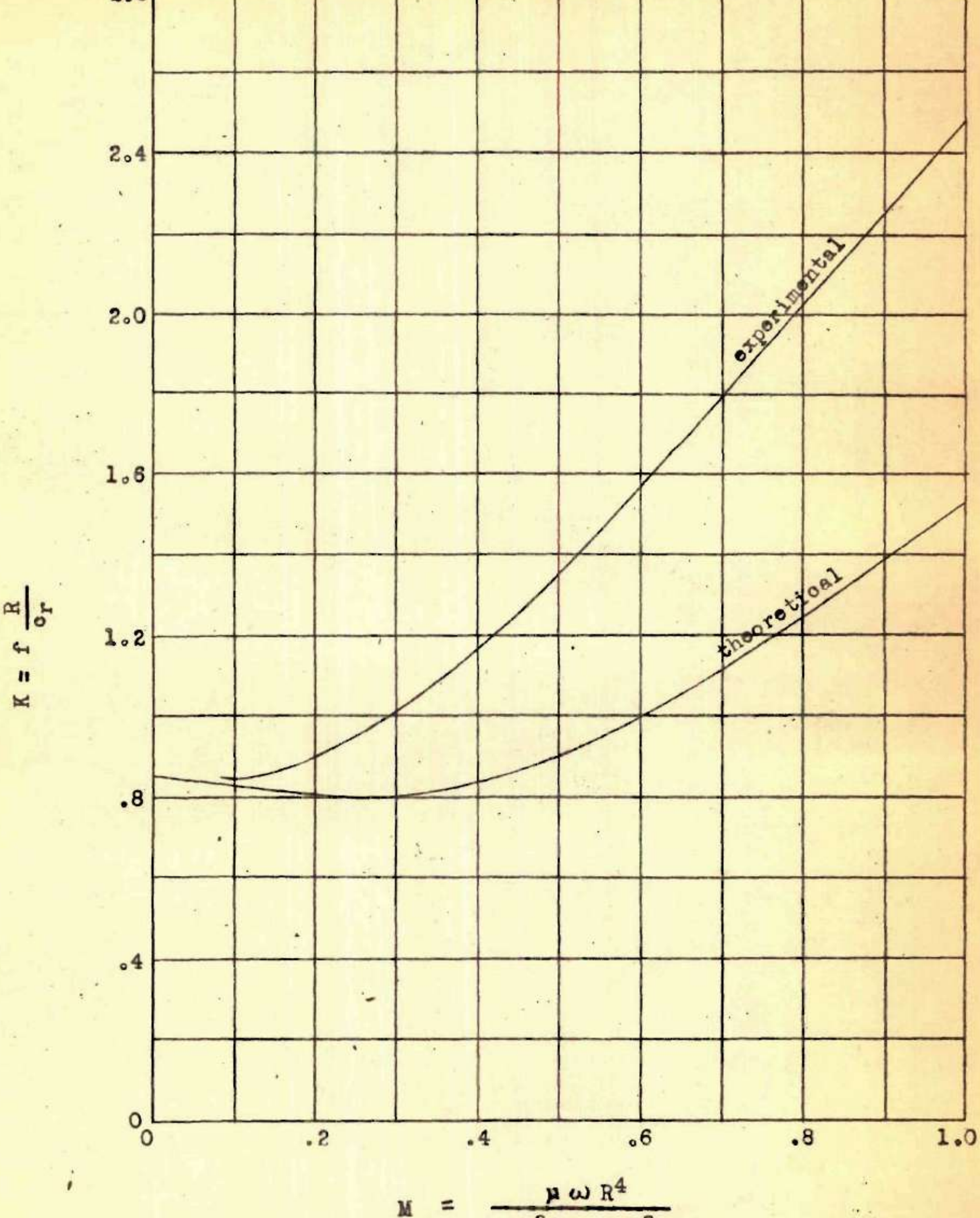

Figure 9. Friction Curves with Constant Load and Positive Pressure

a cylindrical bearing with the respective regions of boundary and hydrodynamic lubrication clearly defined to the left and right of the minimum or critical points. Comparing the curves shows that the deviation between the theoretical and experimental cases increases as the factor M increases.

The deviation between the two curves can be attributed to end flow which occurs in the actual bearing and results in pressure leakage and a consequent load reduction. In the theoretical bearing this leakage was neglected in deriving a pressure solution to Reynolds' equation; thus equations for pressure and load were obtained that gave much higher values than those in the actual bearing. The effect of correcting for this leakage, in the theory, would be to bring the curves closer together.

Neglecting end flow in effect also dropped a term in the friction equation that was concerned with the velocity gradient of the oil on the face of the cone. This force is directly proportional to viscosity and therefore will increase as M increases (assuming other variables constant). This velocity gradient is theoretically negligible as pointed out in Chapter II; however, a high rate of flow produced by the positive pressure may make this term significant.

Curves with Varying Load and Positive Pressure.--A plot of K vs. M for data taken with varying load produced the family of curves shown in Figure 10. These curves were plotted from data obtained with a constant thrust load and a varying radial load using four different oils.

The curve for the most viscous oil shows that typical McKee curve is obtained up to $M = 9$ with the critical operating point at $M = .25$

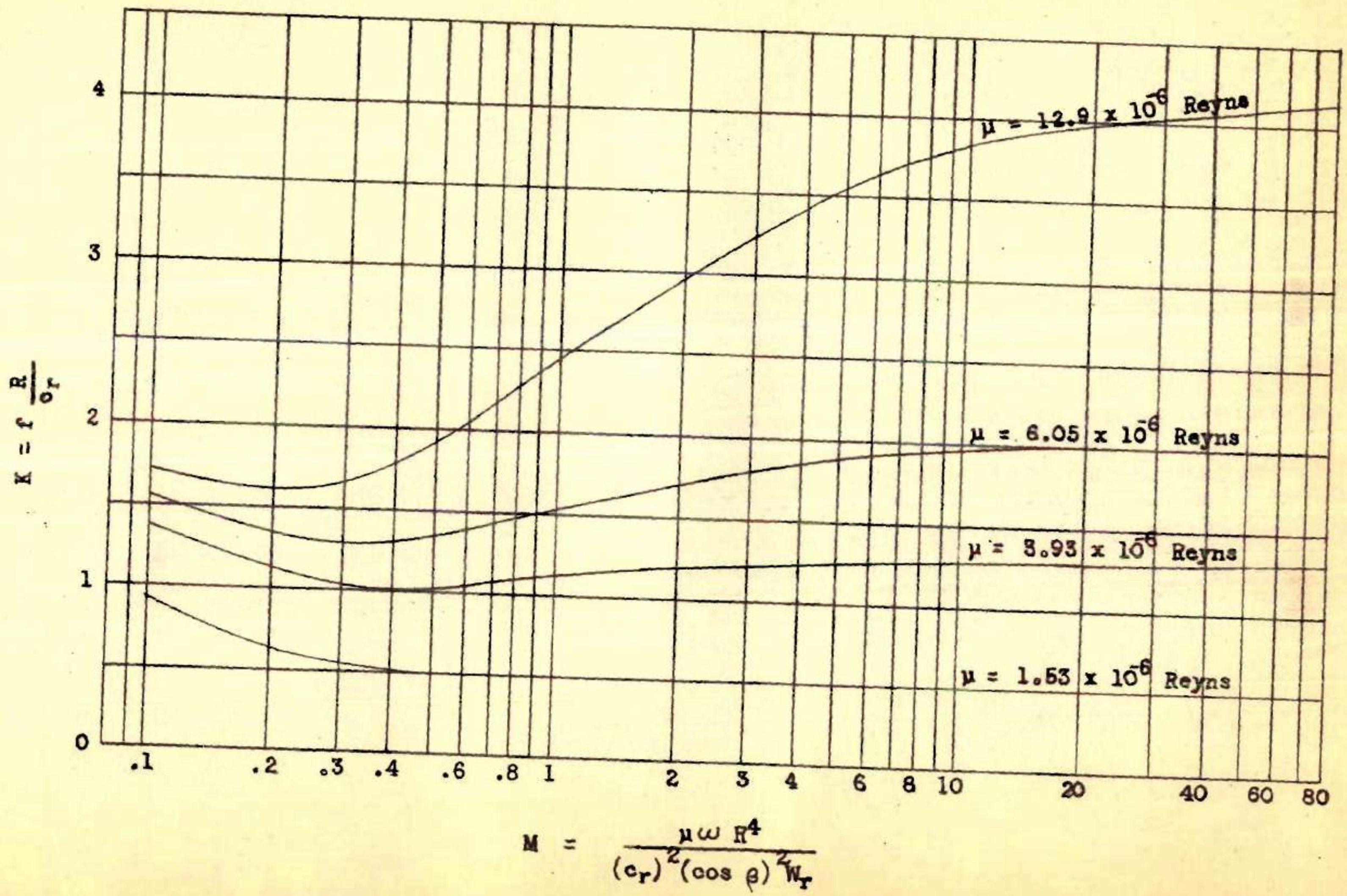

Figure 10. Friction Curves with Varying Load and Positive Pressure

dividing the regions of boundary and hydrodynamic operation. To the right of $M = 9$, however, the curve gradually begins to flatten out and approach a range where all the lubrication is due to the initial pressure of the oil. This region is analogous to that obtained from a hydrostatic bearing and results from an essentially constant friction force and a constant thrust load. It occurs when M is such that W_r is negligible in comparison to W_t. As the viscosity decreases, curves are obtained with the flat region closer and closer to the critical point until finally there is no hydrodynamic region at all. This condition is the limiting case.

Though not shown, the theoretical curves for varying load have the same deviation from the experimental as the constant load curves in Figure 9. The reasons for the deviation are also the same as those for the constant loads and were explained in the previous section.

Hydrodynamic Lubrication Related to Positive Pressure.--In Chapter II it was shown that the conical bearing could not operate hydrodynamically unless a measurable vertical clearance was produced between the journal and the bearing. To check this observation, the bearing was set with a given load, clearance, and speed, and then the driving motor was started. While running, the pressure was cut off and the supply hose clamped, thereby limiting the bearing to a static quantity of oil with comparatively no pressure. The result was that the clearance immediately dropped to zero and the friction increased sharply. The same results were obtained by starting the bearing from rest with zero clearance. This performance agreed exactly with the theoretical derivation.

Effect of Varying Thrust Load on Friction.--The equation derived for friction in Chapter II showed friction to be independent of thrust load. To check this observation, one run was taken in which the speed, clearance, viscosity, and radial load were maintained constant while the thrust load was varied. The result was that F_j increased as W_t increased. This is an apparent discrepancy but can be explained by the fact that increased W_t necessitated increased oil pressure to maintain constant clearance. The greater pressure increased end flow from the bearing and consequently increased friction as explained previously.

CHAPTER VI

CONCLUSIONS

Since all of the variables involved in the investigation were not extensively studied, complete generalization is not possible. However, insofar as the conditions of operating described herein are concerned, the following can be concluded.

1. A conical radial and thrust bearing of the type analyzed in this thesis will not hydrodynamically support a thrust load without positive oil pressure.

2. If positive oil pressure is supplied to the bearing to equalize the thrust load, a radial load can be applied that will operate hydrodynamically.

3. If condition 2 is met, then the equations derived for W_r and F_j can be used to obtain reliable results concerning the expected performance of the bearing.

CHAPTER VII

RECOMMENDATIONS

All of the variables involved in the derived operating equations were studied sufficiently except the cone angle β and the thrust load W_t. Before definite conclusions are made regarding verification of the equations, however, it is recommended that these variables be studied. To complete the investigation, it is also recommended that an attempt be made to determine correction factors for end flow in the bearing.

A more general conical bearing than the one studied here is one that has unequal cone angles on the journal and bearing. It is recommended that this case be studied both analytically and experimentally in order to establish more general conical bearing operating equations.

APPENDIX A

DERIVATIONS

1. Derivation of pressure distribution equation.--The pressure equation for the conical bearing is derived from Reynolds' equation in exactly the same way as the pressure equation for a cylindrical bearing is derived. The method of Sommerfeld (see reference 5) is used for this purpose and proceeds as follows.

A. The simplified Reynolds equation is

$$\frac{\partial}{\partial\theta}\left(h^3\frac{\partial P}{\partial\theta}\right)=6\mu\omega r^2\frac{\partial h}{\partial\theta}.$$

B. This equation must be integrated twice to obtain a solution for p. Perform the first integration and obtain:

$$h^3\frac{\partial P}{\partial\theta}=6\mu\omega r^2 h+D,$$

where D is some constant of integration.

Simplifying yields

$$\frac{\partial P}{\partial\theta}=\frac{6\mu\omega r^2}{h^2}+\frac{D}{h^3}.$$

C. An expression for h in terms of θ must be obtained and substituted into this equation. This expression is found to be (see Figure 1)

$$h=C_r\cos\beta\,(1+n\cos\theta).$$

Substitute.

$$\frac{\partial P}{\partial \theta} = \frac{6\mu\omega r^2}{C_r^2 \cos^2\beta}\left[\frac{1}{(1+n\cos\theta)^2} + \frac{E}{C_r\cos\beta(1+n\cos\theta)^3}\right]$$

$$\int_{P_0}^{P} dP = \frac{6\mu\omega r^2}{C_r^2\cos^2\beta}\int_0^{\theta}\frac{d\theta}{(1+n\cos\theta)^2} + \frac{E\,d\theta}{C_r\cos\beta(1+n\cos\theta)^3}$$

where E is some new constant.

D. In order to evaluate the integral in the right member of the equality, substitution is made that was devised and used by Sommerfeld in the case of a cylindrical bearing. This substitution is

$$\frac{1-n^2}{1-n\cos\gamma} = 1+n\cos\theta,$$

whence

$$d\theta = \frac{(1-n^2)^{1/2}\,d\gamma}{1-n\cos\gamma}$$

(θ and γ have the same limits, 0 to 2π).

Substitute.

$$\int_{P_0}^{P} dP = \frac{6\mu\omega r^2}{C_r^2\cos^2\beta}\int_0^{\gamma}\frac{(1-n\cos\gamma)\,d\gamma}{(1-n^2)^{3/2}}$$

$$+\frac{E(1-n\cos\gamma)^2\,d\gamma}{C_r\cos\beta\,(1-n^2)^{5/2}}$$

E. Now integrate and obtain p as a function of γ.

$$p - p_o = \frac{6\mu\omega r^2}{C_r^2\cos^2\beta}\left[\frac{\gamma - n\sin\gamma}{(1-n^2)^{3/2}} + E\left\{\frac{\gamma - 2n\sin\gamma + .5(n^2\gamma + n^2\sin 2\gamma)}{C_r\cos\beta(1-n^2)^{5/2}}\right\}\right]$$

where p_o = some initial pressure.

F. Evaluate E by knowing that p at 0 and 2π must be the same.

$$0 = \frac{2\pi}{(1-n^2)^{3/2}} + \frac{E(2+n^2)\pi}{C_r\cos\beta(1-n^2)^{5/2}}$$

$$E = -\frac{2C_r\cos\beta(1-n^2)}{2+n^2}$$

G. Substitute E and θ back into the equation and obtain the required expression for p.

$$p = \frac{6\mu\omega r^2}{C_r^2\cos^2\beta}\left[\frac{n(2+n\cos\theta)\sin\theta}{(2+n^2)(1+n\cos\theta)^2}\right] + p_o$$

2. Derivation of Radial Loading Equation.--The radial capacity of the bearing is determined by summing forces along the axes aa and bb of Figure 1. First, find the normal force against the cone that is exerted by the pressure; then take the radial and directional components of this force and set it equal to the radial and directional components of the load W_r.

A. First sum forces along axis aa in Figure 1.

$$W_r \cos\phi = \int_0^A P \cos\beta \cos\theta \, dA,$$

where p = pressure as derived in Derivation 1;

dA = element of surface area of the cone

(see Figure 1) = $r\,dr\,d\theta / \sin\beta$.

Substitute for dA.

$$W_r \cos\phi = \int_0^R \int_0^{2\pi} P \cot\beta \cos\theta \, r \, d\theta \, dr$$

B. Before integrating this expression, a simplification can be obtained by using the method of integration by parts.

Let u = p; du = dp;

dv = $\cos\theta \, d\theta$; v = $\sin\theta$.

$$W_r \cos\phi = \left[\int_0^R P \cot\beta \cancelto{0}{\sin\theta} \, r \, dr \right]_0^{2\pi} - \int_0^R \int_0^{2\pi} \frac{dP}{d\theta} \sin\theta \cot\beta \, r \, d\theta \, dr$$

where

$$\frac{dP}{d\theta} = \frac{6\mu\omega r^2}{C_r^2 \cos^2\beta} \left[\frac{1}{(1+n\cos\theta)^2} - \frac{2(1-n^2)}{(2+n^2)(1+n\cos\theta)^3} \right]$$

from Derivation 1.

Substitute for $dp/d\theta$.

$$W_r \cos\phi + \int_0^R \frac{6\mu\omega r^3 dr \cot\beta}{C_r^2 \cos^2\beta} \int_0^{2\pi} \frac{\sin\theta \, d\theta}{(1+n\cos\theta)^2} - \frac{2(1-n^2)\sin\theta \, d\theta}{(2+n^2)(1+n\cos\theta)^3} = 0$$

C. Integrate with respect to θ and obtain

$$W_r \cos\phi + \int_0^R \frac{6\mu\omega r^3 dr \cot\beta}{C_r^2 \cos^2\beta} \left[\frac{1}{n(1+n\cos\theta)} - \frac{1-n^2}{n(2+n^2)(1+n\cos\theta)^2} \right]_0^{2\pi} = 0$$

On evaluating, it is found that the bracketed term equals 0.

Therefore

$$W_r \cos\phi = 0 \text{ and } \phi = 90°.$$

D. Now sum forces along the axis bb in Figure 1.

$$W_r \sin\phi = \int_0^A P \sin\theta \cos\beta \, dA$$

Substitute for dA and ϕ.

$$W_r = \int_0^R \int_0^{2\pi} P \sin\theta \cot\beta \, r \, d\theta \, dr$$

E. Before integrating this expression, as before, a simplification is possible using the method of integration by parts.

Let $u = p$; $du = dp$;

$dv = \sin\theta \, d\theta$ $v = -\cos\theta$

$$W_r = \left[\int_0^R -P \cot\beta \cos\theta \, r\,dr\right]_0^{2\pi} + \int_0^R \int_0^{2\pi} \frac{dP}{d\theta} \cot\beta \cos\theta \, r\,d\theta\,dr$$

Substitute for $dp/d\theta$.

$$W_r = \int_0^R \frac{6\mu\omega r^3 dr \cot\beta}{C r^2 \cos^2\beta} \int_0^{2\pi} \frac{\cos\theta \, d\theta}{(1+n\cos\theta)^2} + \frac{2(1-n^2)\cos\theta \, d\theta}{(2+n^2)(1+n\cos\theta)^3}$$

F. Integrate with respect to θ and evaluate.

(a) Simplify and break down by partial fractions.

$$W_r = \int_0^R \frac{6\mu\omega r^3 dr \cot\beta}{C r^2 \cos^2\beta (n)} \int_0^{2\pi} \frac{d\theta}{1+n\cos\theta} - \frac{d\theta}{(1+n\cos\theta)^2} - \frac{2(1-n^2)}{2+n^2}\left[\frac{d\theta}{(1+n\cos\theta)^2} - \frac{d\theta}{(1+n\cos\theta)^3}\right]$$

(b) Integrate and evaluate by complex variables (theory of residues).

$$W_r = \int_0^R \frac{6\mu\omega r^3 dr \cot\beta}{C_r^2 \cos^2\beta\,(n)}\left[\frac{2\pi}{(1-n^2)^{1/2}} - \frac{2\pi}{(1-n^2)^{3/2}} - \frac{4\pi(1-n^2)}{(2+n^2)(1-n^2)^{1/2}} + \frac{2\pi}{(1-n^2)^{3/2}}\right]$$

$$W_r = \int_0^R \frac{12\mu\omega r^3 dr \cot\beta}{C_r^2 \cos^2\beta}\left[\frac{\pi n}{(2+n^2)(1-n^2)^{1/2}}\right]$$

G. Now integrate with respect to r, evaluate and get the required equation for W_r.

$$W_r = \frac{\mu\omega R^4}{C_r^2 \cos^2\beta \tan\beta}\left[\frac{3\pi n}{(2+n^2)(1-n^2)^{1/2}}\right]$$

3. Derivation of Thrust Loading.—The thrust capacity of the bearing is determined by summing forces along axis dd of Figure 1. First, find the normal force against the cone that is exerted by the pressure; then take its upward component and set it equal to the thrust load W_t.

A. Sum forces along axis dd in Figure 1.

$$W_t = \int_0^A P \sin\beta \; dA,$$

where p = pressure as determined in Derivation 1;

dA = element of surface area of the cone

(see Figure 1) = $r\,dr\,d\theta/\sin\beta$.

Substitute for dA and p.

$$W_t = \int_0^{2\pi}\int_0^R \left\{\frac{6\mu\omega r^3}{C_r^2\cos^2\beta}\left[\frac{n(2+n\cos\theta)\sin\theta}{(2+n^2)(1+n\cos\theta)^2}\right] + P_o r\right\} dr\, d\theta$$

B. Integrate with respect to r and evaluate.

$$W_t = \int_0^{2\pi} \left\{ \frac{3\mu\omega R^4}{2c_r^2\cos^2\beta}\left[\frac{n(2+n\cos\theta)\sin\theta}{(2+n^2)(1+n\cos\theta)^2}\right] + \frac{P_o R^2}{2} \right\} d\theta$$

C. Integrate with respect to .

(a) Simply by partial fractions.

$$W_t = \frac{3\mu\omega R^4 n}{2(2+n^2)(c_r^2\cos^2\beta)} \int_0^{2\pi} \frac{\sin\theta\, d\theta}{(1+n\cos\theta)^2} + \frac{\sin\theta\, d\theta}{1+n\cos\theta} + \int_0^{2\pi} \frac{P_o R^2}{2}\, d\theta$$

(b) Integrate.

$$W_t = \frac{3\mu\omega R^4 n}{2(2+n^2)(c_r^2\cos^2\beta)} \left[\frac{1}{n(1+n\cos\theta)} - \frac{\log(1+n\cos\theta)}{n}\right]_0^{2\pi} + \frac{P_o R^2 \theta}{2}\Bigg]_0^{2\pi}$$

D. Evaluate and obtain the required expression for W_t.

$$W_t = P_o R^2 \pi$$

4. Derivation of Friction Equation.--The frictional drag on the journal is found by multiplying the shear stress at the journal surface by the journal area.

A. Shear stress at the journal surface is

$$T_J = \frac{\mu r\omega}{h} + \frac{h}{2r}\frac{\partial P}{\partial \theta},$$

where $h = c_r \cos\beta\ (1 + n\cos\Theta)$ [see Derivation 1],

$$\frac{\partial P}{\partial \Theta} = \frac{6\mu\omega r^2}{c_r^2\cos^2\beta}\left[\frac{1}{(1+n\cos\Theta)^2} - \frac{2(1-n^2)}{(2+n^2)(1+n\cos\Theta)^3}\right]$$

Substitute and simplify.

$$\tau_J = \frac{\mu\omega r}{c_r\cos\beta}\left[\frac{4}{(1+n\cos\Theta)} - \frac{6(1-n^2)}{(2+n^2)(1+n\cos\Theta)^2}\right]$$

B. The friction force equals the shear stress times the journal area.

$$F_J = \int_0^A \tau_J \, dA,$$

where dA = element of surface area of a cone

= $r\,dr\,d\Theta/\sin\beta$ (see Figure 1).

Substitute for τ_J, dA.

$$F_J = \int_0^{2\pi}\int_0^R \frac{\mu\omega r^2}{(c_r\cos\beta)}\left[\frac{4}{(1+n\cos\Theta)} - \frac{6(1-n^2)}{(2+n^2)(1+n\cos\Theta)^2}\right]\frac{dr\,d\Theta}{\sin\beta}$$

C. Integrate with respect to r and evaluate.

$$F_J = \int_0^{2\pi} \frac{\mu\omega R^3}{3c_r\cos\beta}\left[\frac{4}{(1+n\cos\Theta)} - \frac{6(1-n^2)}{(2+n^2)(1+n\cos\Theta)^2}\right]\frac{d\Theta}{\sin\beta}$$

D. Integrate with respect to θ and evaluate using complex variables (theory of residues).

$$F_J = \frac{\mu\omega R^3}{3c_r \cos\beta \sin\beta}\left[\frac{8\pi}{(1-n^2)^{1/2}} - \frac{12(1-n^2)\pi}{(2+n^2)(1-n^2)^{3/2}}\right]$$

E. Simplify and obtain the required expression for F_j.

$$F_J = \frac{\mu\omega R^3}{3c_r \cos\beta \sin\beta}\left[\frac{4\pi(1+2n^2)}{(2+n^2)(1-n^2)^{1/2}}\right]$$

5. Derivation of the Point of Action of the Radial Load.--The point of action of W_r along the face of the cone is assumed to be at the centroid of loading. It is derived using the W_r equation as follows.

A. The total W_r was shown in Derivation 2 to be

$$W_r = \int_0^R \frac{\mu\omega r^3 dr \cot\beta}{c_r^2 \cos^2\beta}\left[\frac{12\pi n}{(2+n^2)(1-n^2)^{1/2}}\right]$$

B. The above integral can be expressed as the sum of two other integrals.

$$W_r = \frac{\mu\omega \cot\beta}{c_r^2 \cos^2\beta}\left[\frac{12\pi n}{(2+n^2)(1-n^2)^{1/2}}\right]\left\{\int_0^r r^3 dr + \int_r^R r^3 dr\right\}$$

C. To separate the total loading at its centroid, the following must hold:

$$C\int_0^r r^3 dr = C\int_r^R r^3 dr$$

where $C = \dfrac{\mu \omega \cot \beta}{C_r^2 \cos^2 \beta} \left[\dfrac{12 \pi n}{(2+n^2)(1-n^2)^{1/2}} \right].$

D. Integrate and evaluate the two expressions:

$$\frac{r^4}{4} = \frac{R^4}{4} - \frac{r^4}{4}$$

E. Solve for r and get the point of assumed action for the load.

$$r = .84\, R$$

6. Derivation of the Point of Action of the Friction Force.--To determine the point of action of F_j, it is assumed that the friction torque exerted by the journal must equal the friction torque to restrain the bearing. This point is computed by using the F_j equation as follows.

A. The total F_j could be shown in Derivation 4 to be:

$$F_J = \int_0^R \frac{\mu \omega r^2 dr}{C_r \cos\beta \sin\beta} \left[\frac{4\pi (1+2n^2)}{(2+n^2)(1-n^2)^{1/2}} \right].$$

B. The torque exerted on the bearing by this force is

$$T_J = \int_0^R \frac{\mu \omega r^3 dr}{C_r \cos\beta \sin\beta} \left[\frac{4\pi (1+2n^2)}{(2+n^2)(1-n^2)^{1/2}} \right].$$

C. The restraining torque must equal the total F_j times some radius x.

$$T_b = F_J(x) = \int_0^R \frac{\mu \omega r^2 dr}{C_r \cos\beta \sin\beta} \left[\frac{4\pi (1+2n^2)}{(2+n^2)(1-n^2)^{1/2}} \right] \times$$

D. Assuming the torque equal ($T_J = T_b$), the following equation must hold

$$D\int_0^R r^3\,dr = D\int_0^R r^2\,dr\,(x),$$

where D = $$\frac{\mu\,\omega}{G\cos\beta\sin\beta}\left[\frac{4\pi(1+2n^2)}{(2+n^2)(1-n^2)^{1/2}}\right].$$

E. Integrate and evaluate the two expressions.

$$\frac{R^4}{4} = \frac{R^3}{3}\,x$$

F. Solve for x and obtain the point of assumed action for the friction.

$$x = .75\,R$$

APPENDIX B

Table 1. Tabulated Data for Varying Spindle Speed

constant cord load (W_c) = 2 lb.

constant thrust load (W_t) = 1.17 lb.

constant vertical clearance (c_v) = .010 inches

Spindle Speed (N) RPM	Oil Temp. (T) F	Recorder Reading Q
	Gulfgem A	
294	83	58
437	83	57
591	83	57
733	83	59
891	83	60
	Gulfgem C	
293	84	58
443	84	59
592	84	60
731	84	60
872	84	61
	Gulfcrest A	
293	84	60
445	84	62
593	84	67
735	84	66
888	84	68
	Gulfcrest C	
294	85	67
440	85	69
573	85	76
721	85	80
861	85	85

Table 2. Tabulated Data for Varying Radial Load

Constant thrust load (W_t) = 1.17 lbs.

Constant vertical clearance (c_v) = .010 inches

Spindle Speed (N) RPM	Oil Temp. (T) °F	Cord Load (W_c) #	Recorder Reading (Q)
	Gulfgem A		
591	85	1.0	56
591	85	1.5	57
591	85	2.0	61
591	85	2.5	65
591	85	3.0	69
	Gulfgem C		
593	83	1.0	58
593	83	1.5	60
593	83	2.0	67
593	83	2.5	70
593	83	3.0	70
	Gulfcrest A		
593	84	1.0	62
593	84	1.5	62
593	84	2.0	62
593	84	2.5	65
593	84	3.0	71
	Gulfcrest C		
573	84	1.0	72
573	84	1.5	73
573	84	2.0	75
573	84	2.5	76
573	84	3.0	80

Table 3. Tabulated Data for Varying Clearance

Constant cord load (W_c) = 2 lb.

Constant thrust load (W_t) = 1.17 lb.

Spindle Speed (N) RPM	Oil Temp. (T) °F	Vert. Clear (c_v) Inches	Recorder Reading (Q)
	Gulfgem A		
591	85	.015	57
591	85	.010	58
591	85	.003	60
591	85	.002	63
	Gulfgem C		
593	83	.010	63
593	83	.007	66
593	83	.003	68
593	83	.0015	94
	Gulfcrest A		
593	84	.019	60
593	84	.010	62
593	84	.009	64
593	84	.007	70
593	84	.0025	78
	Gulfcrest C		
573	85	.015	84
573	85	.010	81
573	85	.007	85
573	85	.004	110

Table 4. Tabulated Data for Varying Thrust Load

Constant cord load (W_r) = 2 lb.

Constant vertical clearance (c_v) = .010 inches

Constant spindle speed (N) = 591 RPM

Thrust Load (W_t) lbs.	Oil Temp. (T) °F	Recorder Reading Q
	Gulfgem A	
1.17	82.5	57.5
1.25	82.5	59
1.38	82.5	60
1.44	82.5	61

Table 5. Calibration of Strain Recorder

Weight lb.	Summation of Wt. lb.	Recorder Reading (Q) Before Runs	After Runs
0	0	50	50
.01345	.01345	61	60
.01334	.02679	69	67
.01488	.04167	84	84
.01475	.05642	97	96
.01365	.07007	108	112

APPENDIX C

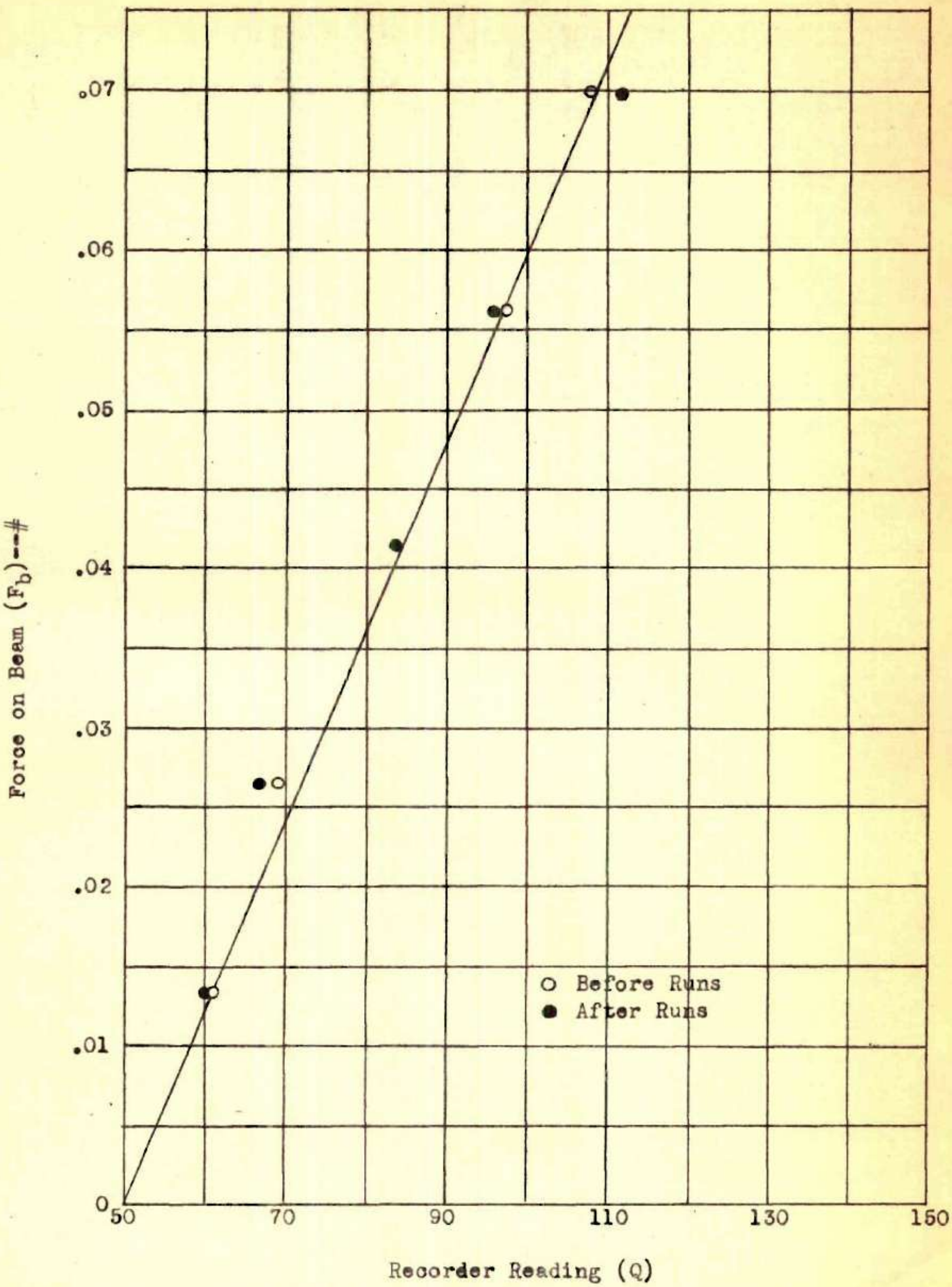

Figure 11. Calibration Curve for Strain Recorder

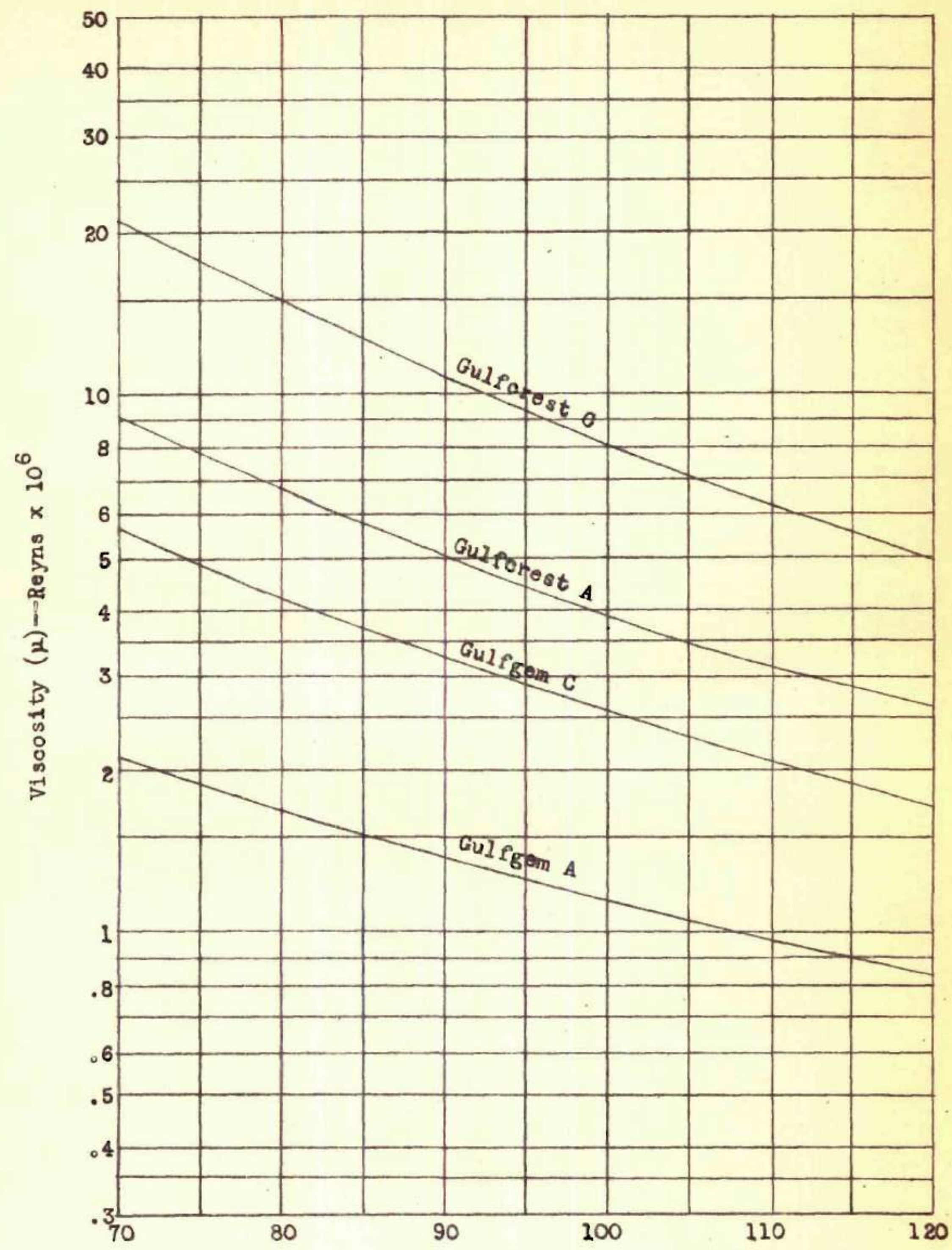

Figure 12. Viscosity-Temp. Conversion Curves

APPENDIX D

SAMPLE CALCULATIONS

1. Determination of Factors M, J from Data in Tables 1, 2, 3.

Given: the following data from Table 1

Cord load (W_c) = 2 lb.

Recorder reading (Q) = 58

Oil Temp. (T) = 84 °F

Vertical clearance (c_v) = .010 inches

Spindle speed (N) = 294 RPM

Compute M and J.

A. M and J are given as

$$M = \frac{\mu \omega R^4}{c_r^2 \cos^2\beta\, W_r} \qquad J = \frac{\mu \omega R^3}{c_r \cos\beta\, F_J}$$

where $\mu = 1.6(10^{-6})$ Reyns (Figure 12 using T);

$\omega = \frac{2\pi N}{60} = \frac{2\pi(294)}{60} = 30.8$ radians;

$R = .5$ inches (geometry of bearing);

$c_r = c_v = .010$ inches;

$F_j = 2.45(F_b) = 2.45(.00993) = .0243$ lb. (from Figure 11 using Q);

$W_r = .483\ (W_c) = .483(2) = .966$ lb.;

$\cos\beta = \cos 45° = .707$ (geometry of bearing).

B. Substitute the above and evaluate.

$$M = \frac{(1.6)\ (10^{-6})\ (30.8)\ (.5)^4}{(.966)\ (.707)^2\ (.010)^2} = .0637$$

$$J = \frac{(1.6)\ (10^{-6})\ (30.8)\ (.5)^3}{(.0243)\ (.707)\ (.010)} = .036$$

2. Determination of Average Experimental Curve for Figure 8.

Given: the following J's at M = .09 (approximately)

J = .0603, .0377, .0296

Compute J_{avg}, discarding points whose residual from the average is more than twice the probable error at M = .09.

A. Compute the average J from the given data.

$$\begin{array}{r|l} & .0603 \\ & .0377 \\ & \underline{.0296} \\ 3 & \underline{.1276} \\ & .0425 = J_{avg} \end{array}$$

B. Find residuals from the average for each point.

$$\begin{aligned} \text{residual } (\nu) &= .0603 - .0425 = + .0178 \\ & .0425 - .0377 = - .0048 \\ & .0425 - .0296 = - .0129 \end{aligned}$$

C. Compute the probable error at the point.

(a) From Scarborough (6), the probable error of a single measurement is given as

$$PE = .6745 \sqrt{\frac{\nu_1^2 + \nu_2^2 + --- \nu_n^2}{n-1}}$$

(b) Substitute (ν_s) and compute.

$$PE = .6745 \sqrt{\frac{(.0178)^2 + (.0048)^2 + (.0129)^2}{2}}$$

$$PE = .0106$$

D. Check points to see if residuals exceed twice the above quantity.

$$\begin{aligned} 2(.0106) = .0212 \quad &> .0178 \\ &> .0048 \\ &> .0129 \end{aligned}$$

All points are within the allowed range; therefore none are discarded.

3. <u>Determination of the Factor (K) from the Average Curve on Figure 8.</u>

Given: the following data from Table 1 except the recorder reading

Cord load (W_c) = 2 lb.

Thrust load (W_t) = 1.17 lb.

Oil Temp. (T) = 84 °F

Vertical clearance (c_v) = .010 inches

Spindle speed (N) = 294 RPM

Compute K where $K = f\ R/c_r$.

A. Compute M as in Sample calculation 1.

$M = .0637$

B. From the average curve on Figure 8, read the "corrected" J corresponding to the above M.

$J = .0337$

C. Use method of Sample Calculation 1 and compute from J the corrected F_j.

$F_j = .0259$ lb.

D. Compute K.

(a) K is given as

$$K = f\,\frac{R}{c_r} = \frac{F_j\ R/c_r}{W_r \cos\beta + W_t \sin\beta},$$

where W_r, R, c_r, $\cos\beta$ have the values given in Sample Calculation 1.

$\sin\beta = \sin 45° = .707$ (geometry of bearing)

(b) Substitute and compute.

$$K = \frac{(.0259)\ (.5)/(.010)}{(.966)\ (.707) + (1.17)\ (.707)} = .855$$

BIBLIOGRAPHY

1. Reynolds, O: "On the Theory of Lubrication and Its Application to Mr. Beaucamp Tower's Experiments, Including an Experimental Determination of the Viscosity of Olive Oil," Transactions Royal Society. (London) 177 (pt. 1), 157 (1886).

2. Shaw, M. C. and Mack, E. F: "Analysis and Lubrication of Bearings", McGraw-Hill Book Company, Inc., New York, 1949, p. 192.

3. Norton, A. E: "Lubrication," McGraw-Hill Book Company, Inc., New York, 1942, p. 114.

4. Cheverton, R. D: "A Basic Investigation of the Full Floating Textile Spindle Bearing," Unpublished Master's Thesis, Georgia Institute of Technology, 1953.

5. Sommerfeld, A: "Zur Hydrodynamischen Theorie der Schmiermittelreibung," Zeitschrift Fur Mathematik und Physik 50, 97 (1904).

6. Scarborough, J. B.: "Numerical Mathematical Analysis," The Johns Hopkins Press, Baltimore, 1930, p. 327.