17:01:51 OCA PAD AMENDMENT - PROJECT HEADER INFORMATION 10/11/95

Active

Project #: E-16-683 Cost share #: E-16-316 Rev #: 19
Center # : 10/24-6-R6934-0A0 Center shr #: 10/22-1-F6934-0A0 OCA file #:
Work type : RES
Contract#: N00014-90-J-1794 Mod #: A00003 Document : GRANT
Prime #: Contract entity: GTRC

Subprojects ? : Y CFDA: 12.AAA
Main project #: PE #: N/A

Project unit: AERO ENGR Unit code: 02.010.110
Project director(s):
PRASAD J V R AERO ENGR (404)894-3043

Sponsor/division names: NAVY / OFC OF NAVAL RESEARCH
Sponsor/division codes: 103 / 025

Award period: 900415 to 960605 (performance) 960605 (reports)

Sponsor amount	New this change	Total to date
Contract value	0.00	720,000.00
Funded	0.00	692,000.00
Cost sharing amount		177,443.00

Does subcontracting plan apply ?: N

Title: TOWARD A NEW GENERATION OF HIGH-PERFORMANCE GAS TURBINE ENGINE CONTROLS: A SY

PROJECT ADMINISTRATION DATA

OCA contact: E. Faith Gleason 894-4820

Sponsor technical contact	Sponsor issuing office
ERIC HENDRICKS	RUSSELLE DUNSON
(202)696-4715	(202)696-4508
OFFICE OF NAVAL RESEARCH	OFFICE OF NAVAL RESEARCH
CODE 1217	CODE 1512/RID
800 NORTH QUINCY STREET	800 NORTH QUINCY STREET
ARLINGTON, VA 22217-5000	ARLINGTON, VA 22217-5000

Security class (U,C,S,TS) : U ONR resident rep. is ACO (Y/N): Y
Defense priority rating : NA ONR supplemental sheet
Equipment title vests with: Sponsor GIT X

Administrative comments -
MODIFICATION NO. A00003 CHANGES P.I. FROM DR. NETT TO DR. PRASAD AND EXTENDS GRANT FOR ONE YEAR, TO 5 JUNE 1996.

21·

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

U

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 07/17/96

Project No. E-16-683 Center No. 10/24-6-R6934-0A0

Project Director PRASAD J V R School/Lab AERO ENGR

Sponsor NAVY/OFC OF NAVAL RESEARCH

Contract/Grant No. N00014-90-J-1794 Contract Entity GTRC

Prime Contract No.

Title TOWARD A NEW GENERATION OF HIGH-PERFORMANCE GAS TURBINE ENGINE CONTROLS:

Effective Completion Date 960605 (Performance) 960605 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	Y	
Final Report of Inventions and/or Subcontracts	Y	
Government Property Inventory & Related Certificate	N	
Classified Material Certificate	N	
Release and Assignment	N	
Other	N	

Comments

Subproject Under Main Project No.

Continues Project No.

Distribution Required:

Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Managment	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other	N
	N

NOTE: Final Patent Questionnaire sent to PDPI.

5/5/96 2

E-16-683

MEMORANDUM

To: OCA Reports Coordinator

From: Professor Carl N. Nett, Aerospace Engineering, 4-8200 CN

Subject: Deliverable Line Item No. 1 for Project No. E-16-683

Date: October 17, 1991

The subject deliverable, an annual performance (technical) report covering the period 900415-910414, has been distributed to the sponsor in the form of a videotape presentation of experimental facilities developed under support from the grant, at the sponsors request. For this reason only the letter of transmittal and this memo, and not the report itself, are being forwarded to OCA for archival purposes.

The master copy of the videotape will be retained in my lab for archival purposes.

If I do not hear from you promptly to the contary I will assume these arrangements are acceptable.

School of Aerospace Engineering

Georgia Institute of Technology
Atlanta, Georgia 30332-0150
404•894•3000
Fax: 404•894•2760

October 17, 1991

Dr. Eric Hendricks (Code 634)
Program Manager Assistant for Active Control
Naval Ocean Systems Center
San Diego, CA 92152–5000

(619) 553-1624

ehendrick@nosc.mil

Dear Dr. Hendricks,

Enclosed are two copies of the videotape I am submitting as the annual performance (technical) report covering the period 4/15/90 to 4/15/91 on ONR Grant No. N00014-90-J-1794. I have also sent a copy of this letter and the videotape directly to Dr. Hansen.

Copies of the videotape have also been sent to Professor Greitzer (MIT), Professor Ffowcs-Williams (Cambridge), GE Aircraft Engines (Lynn), GE Aircraft Engines (Evendale), GE Corporate Research and Development (Schenectady), Rolls-Royce (Atlanta), Pratt and Whitney (West Palm Beach), and United Technologies Research Center (Hartford).

The videotape describes in detail the experimental facilities developed thus far under the support of ONR. Though the videotape does not describe in detail the research that these facilities are being used to support, the papers I gave to you and Dr. Hansen at the last program review you hosted adequately describe the progress made in this research through 4/15/91.

Finally, I apologize for taking so long to get you the videotape. I grossly underestimated the effort required to generate it. Shooting the raw footage was easy; editing the raw footage was anything but easy. Nonetheless, I am very happy with the video, and I am confident that after viewing it both you and Dr. Hansen will feel it was well worth the wait.

Please contact me immediately if you have any questions concerning the videotape, or if you would like additional copies made from the master.

Sincerely,

Dr. Carl N. Nett
Associate Professor and Director
Laboratory for Identification and
Control of Complex Highly
Uncertain Systems (LICCHUS)

(404) 894-8200 Office
(404) 853-0173 Lab
(404) 924-1659 Home

(404) 853-0880 Fax

cn4@prism.gatech.edu

cc: Dr. R. J. Hansen

E-16-683 #2

FORM A2-2

UGMENTATION AWARDS FOR SCIENCE & ENGINEERING RESEARCH TRAINING (AASERT)

REPORTING FORM

e Department of Defense (DOD) requires certain information to evaluate the fectiveness of the AASERT program. By accepting this Grant Modification, ich bestows the AASERT funds, the Grantee agrees to provide the information quested below to the Government's technical point of contact by each annual niversary of the AASERT award date.

Grantee identification data: (R & T and Grant numbers found on Page 1 of Grant)

a. GEORGIA TECH RESEARCH CORPORATION
University Name

b. N00014-90-J-1794
Grant Number

c. 210f001---04
R & T Number

d. DR. J. PRASAD & DR. CARL NETT
P.I. Name

e. From: JUN 92 To: JUN 93
AASERT Reporting Period

E: Grant to which AASERT award is attached is referred to hereafter as "Parent Agreement."

Total funding of the Parent Agreement and the number of full-time uivalent graduate students (FTEGS) supported by the Parent Agreement during e 12-month period prior to the AASERT award date. (NOT SUPPORTED BY ASSERT)

a. Funding: $ 51,535.

b. Number FTEGS: 11

Total funding of the Parent Agreement and the number of FTEGS supported the Parent Agreement during the current 12-month reporting period. (NOT SUPPORTED BY ASSERT)

a. Funding: $ 22,865

b. Number FTEGS: 3

Total AASERT funding and the number of FTEGS and undergraduate students GS) supported by AASERT funds during the current 12-month reporting period.

a. Funding: $ 17,486.

b. Number FTEGS: 3 2

c. Number UGS: ______

RIFICATION STATEMENT: I hereby verify that all students supported by the SERT award are U.S. citizens.

____________________ 10/18/93
rincipal Investigator Date

E-16-683 #3

GRANT NUMBER: N00014-92-J-1794/P00003

FORM A2-2

AUGMENTATION AWARDS FOR SCIENCE & ENGINEERING RESEARCH TRAINING (AASERT) REPORTING FORM

The Department of Defense (DOD) requires certain information to evaluate the effectiveness of the AASERT program. By accepting this Grant Modification, which bestows the AASERT funds, the Grantee agrees to provide the information requested below to the Government's technical point of contact by each annual anniversary of the AASERT award date.

1. Grantee identification data: (R & T and Grant numbers found on Page 1 of Grant)

a. Georgia Tech Research Corp.
University Name

b. N00014-92-J-1794
Grant Number

c. 210F001---04
R & T Number

d. Dr. J.V.R. Prasad; Carl Nett
P.I. Name

e. From: 6/93 To: 6/94
AASERT Reporting Period

NOTE: Grant to which AASERT award is attached is referred to hereafter as "Parent Agreement."

2. Total funding of the Parent Agreement and the number of full-time equivalent graduate students (FTEGS) supported by the Parent Agreement during the 12-month period **prior to** the AASERT award date.

a. Funding: $ 51,535

b. Number FTEGS: 11

3. Total funding of the Parent Agreement and the number of FTEGS supported by the Parent Agreement during the current 12-month reporting period.

a. Funding: $ 0

b. Number FTEGS: 0

4. Total AASERT funding and the number of FTEGS and undergraduate students (UGS) supported by AASERT funds during the current 12-month reporting period.

a. Funding: $ 39,326

b. Number FTEGS: 5

c. Number UGS: 0

VERIFICATION STATEMENT: I hereby verify that all students supported by the AASERT award are U.S. citizens.

Principal Investigator

Date 7/17/95

FORM A2-2

AUGMENTATION AWARDS FOR SCIENCE & ENGINEERING RESEARCH TRAINING (AASERT) REPORTING FORM

The Department of Defense (DOD) requires certain information to evaluate the effectiveness of the AASERT program. By accepting this Grant Modification, which bestows the AASERT funds, the Grantee agrees to provide the information requested below to the Government's technical point of contact by each annual anniversary of the AASERT award date.

1. Grantee identification data: (R & T and Grant numbers found on Page 1 of Grant)

a. Georgia Tech Research Corp.
University Name

b. N00014-92-J-1794
Grant Number

c. Z10F001---04
R & T Number

d. Dr. J.V.R. Prasad
P.I. Name

e. From: 6/94 To: 6/95
AASERT Reporting Period

NOTE: Grant to which AASERT award is attached is referred to hereafter as "Parent Agreement."

2. Total funding of the Parent Agreement and the number of full-time equivalent graduate students (FTEGS) supported by the Parent Agreement during the 12-month period **prior to** the AASERT award date.

a. Funding: $ 51,535

b. Number FTEGS: 11

3. Total funding of the Parent Agreement and the number of FTEGS supported by the Parent Agreement during the current 12-month reporting period.

a. Funding: $ 0

b. Number FTEGS: 0

4. Total AASERT funding and the number of FTEGS and undergraduate students (UGS) supported by AASERT funds during the current 12-month reporting period.

a. Funding: $ 24,689.35

b. Number FTEGS: 1.5

c. Number UGS: 0

VERIFICATION STATEMENT: I hereby verify that all students supported by the AASERT award are U.S. citizens.

Principal Investigator

1/31/96
Date

E-16.683 #'s 5,6

FORM A2-2

AUGMENTATION AWARDS FOR SCIENCE & ENGINEERING RESEARCH TRAINING (AASERT) REPORTING FORM

ne Department of Defense (DOD) requires certain information to evaluate the ffectiveness of the AASERT program. By accepting this Grant Modification, hich bestows the AASERT funds, the Grantee agrees to provide the information equested below to the Government's technical point of contact by each annual nniversary of the AASERT award date.

. Grantee identification data: (R & T and Grant numbers found on Page 1 of Grant)

a. Georgia Tech Research Corp.
University Name

b. N00014-92-J-1794
Grant Number

c. 210F001---04
R & T Number

d. Dr. J.V.R. Prasad
P.I. Name

e. From: 6/95 To: 6/96
AASERT Reporting Period

TE: Grant to which AASERT award is attached is referred to hereafter as "Parent Agreement."

. Total funding of the Parent Agreement and the number of full-time quivalent graduate students (FTEGS) supported by the Parent Agreement during he 12-month period prior to the AASERT award date.

a. Funding: $ 51,535.

b. Number FTEGS: 11

. Total funding of the Parent Agreement and the number of FTEGS supported y the Parent Agreement during the current 12-month reporting period.

a. Funding: $ 0

b. Number FTEGS: 0

. Total AASERT funding and the number of FTEGS and undergraduate students JGS) supported by AASERT funds during the current 12-month reporting period.

a. Funding: $12,000

b. Number FTEGS: 1

c. Number UGS: 0

RIFICATION STATEMENT: I hereby verify that all students supported by the SERT award are U.S. citizens.

incipal Investigator

6/5/96
Date

E- 16- 683
#7

Toward a New Generation of High Performance Aircraft Gas Turbine Engine Controls: A Systems–Oriented Research Program

Report Submitted to:

Program Manager
Augmentation Award for Support of Students in
Engineering and Research Training
Office of Naval Research,
800 North Quincy, Arlington,
Virginia, 22217-5660.

by

Laboratory for Identification and
Control of Complex Highly Uncertain Systems
Georgia Institute of Technology,
School of Aerospace Engineering,
Atlanta, GA 30332-0150

Submission Date: August 1995

1 Executive Summary

In April 1990 a three year ONR funded research program spanning the duration FY91–FY93 was initiated at Georgia Tech. The stated goals of this research program encompass such diverse areas as mathematical modeling, nonlinear dynamics, bifurcation theory, and active control of the aerodynamic instabilities of surge and rotating stall in aircraft gas turbine engines.

In April 1992, a further extension for an additional two years was granted and approved to facilitate the completion of the original proposed research objectives.

This letter report outlines the details of the research accomplishments for the duration of the program amounting to the attainment of the stated goals in the original proposal.

The accomplishments in the first three years of the research program are numerous, and have made significant contributions to the current literature on theoretical, computational, and experimental approaches to the analysis and active control of compressor stall phenomena in aircraft gas turbine engines.

Some of the prominent accomplishments include:

- Development of a unique technique for detection, identification, and characterization of rotating stall and pre-stall waves in an axial compressor experimental rig.
- Development and dynamic analysis of a stall capable quasi two–dimensional model based on parallel compressor model concepts.
- Global nonlinear dynamic and bifurcation analysis of low–order two–dimensional rotating stall models that qualitatively predicts experimentally–observed stall inception scenarios.
- Design and experimental validation of linear and nonlinear active surge controllers on the LICCHUS axial compressor experimental rig.
- Experimental proof–of–concept validation of nonlinear control of rotating stall in an axial compressor rig based on 1D actuation and limited 2D sensing.
- Development, analysis and simulation validation of a Full–range integrated active surge control/rotating stall control scheme with a nonlinear observer on a two–dimensional rotating stall model.
- Design, development, instrumentation, and demonstration of a state-of-the-art turbojet engine experimental facility.
- Development of a quasi one–dimensional compressible viscous flow turbojet engine component models for generic engine modeling applications.
- Preliminary system identification, development and qualitative simulation validation of a one–dimensional model for the turbojet engine experimental rig.
- Experimental proof–of–concept of nonlinear feedback control of rotating stall with surge avoidance on a turbojet engine experimental rig based on 1D actuation and limited 2D sensing.
- Experimental Validation of the integrated surge control and rotating stall control scheme on the LICCHUS multistage axial compressor experimental facility.
- Development and parameter identification of a two–dimensional rotating stall model for the LICCHUS multistage axial compressor experimental facility.

- Parameter and system identification for the LICCHUS turbojet engine experimental facility for both quasi one–dimensional compressible flow dynamic models.
- Design and implementation of the surge control scheme on the LICCHUS turbojet engine experimental rig.

2 Research Program

2.1 Overview

The general goal of the research program has been to advance significantly beyond proof-of-concept control–based engine component technologies to establish the theoretical, computational, and experimental foundations for a new generation of high-performance aircraft gas turbine engine controls. In view of this goal, the principal technical objectives of the program can be summarized as follows:

- Determine the overall effectiveness and potential of active control for realizing full engine operating potential, while ensuring safe engine operation, in the face of significant engine system and environmental uncertainties.
- Develop overall engine control system design methodologies which realize full engine operating potential in the face of such uncertainties.

Consistent with the objectives, the primary issues delineated and investigated in the research program are:

- The robustness of active controls to uncertainties in the engine system and its operating environment.
- The robustness of active controls to both transient and quasi–steady engine operating point variations.
- The interaction/compatibility of active controls with other necessary engine controls.
- The extent to which active control schemes can be made fail–safe.

The overall approach taken in the research program has been based upon the latest work in the area of compressor stall phenomena. It has utilized expertise in nonlinear modeling, nonlinear dynamics, controls, diagnostics, and simulation of aircraft gas turbine engines. The tools of applied robust nonlinear multivariable control have been extensively used and applied to descriptive models of the phenomena under consideration. The physical data required for the execution of this approach has been gathered through a combination of theory, computation, and experimentation. This combination has been achieved through the development and utilization of the innovative simulation and experimental facilities listed below:

- Multistage Axial Compressor Experimental Facility.
- Extended First-Principles Theoretical Compression System Models Simulation Facility.
- Turbojet Engine Experimental Facility.
- Turbojet Engine Models Real-Time, Hardware-in-the-Loop, Simulation Facility.

These facilities are fully operational and are located at the LICCHUS at Georgia Tech.

2.2 Significance of Research Accomplishments

2.2.1 Accomplishments

The primary accomplishment in the research work during FY93 comprise of complete detailed nonlinear analysis, active control and simulation validation of the 1D unsteady, compressible viscous flow model against experimental data obtained from the LICCHUS axial compressor experimental rig. This represents a conclusive investigation of the varied nonlinear dynamic behavior exhibited by axial compression systems, and the development of suitable nonlinear control configurations for active surge stabilization and control in axial compression systems. This work has been accomplished based on a new approach for control-oriented high–frequency turbomachinery modeling. Models developed using this approach are inherently high–frequency, exhibiting expected high–frequency phenomena and transient behaviors, such as compressor surge and rotating stall. These models are also inherently control-oriented, i.e. well suited for control design, due to their relatively low complexity and accompanying uncertainty characterization, where complexity is measured in terms of flow dimensionality, variable dimensionality, and dynamic order. This approach is based on a novel dimensionless form of the governing equations for unsteady, compressible, viscous, quasi–1D flow, coupled with a novel discretization method and a new, valid, and very useful assumption concerning forcing term variable dependencies. Several important benefits accrue as a result of this approach. First, because the discretization method is matched to natural turbomachinery boundary conditions, the models developed using this approach are simply integrable. In other words, no nonlinear algebraic equations need be solved to compute solutions to the model; hence, the usual iterative process required to compute solutions of turbomachinery models is avoided. This leads to much computational simplification not only in the nonlinear models developed using this approach, but also in the computation of associated linear models, in that these models can be computed in closed form without iteration. Additionally, due to a matching of the forcing term assumption with the discretization method, forcing terms that cannot be determined by first principles modeling can be rigorously identified through steady state experiments. In other words, the unknown forcing terms can be rigorously replaced by steady state performance maps in the discretized equations. Interestingly, no assumptions pertaining to quasi-steady operation are required to validate this technique.

The approach has been applied to develop control–oriented high–frequency models for an externally–driven multi–stage axial compression system, and a single spool turbojet engine. For the compression system in particular, the model and experimental data have been compared with respect to both open-loop and closed-loop behaviors. The comparisons focus on forced transients and global nonlinear dynamics and bifurcations. In all cases, the comparison between model and experiment was found to be excellent. Of particular interest is the ability of the model, which does not include any hysteretic maps, to predict experimentally observed hysteresis with respect to the onset and cessation of surge. Also of interest is the fact that the controllers used for closed-loop comparisons were designed directly from the model with no subsequent tuning of controller parameters, that is, without recourse to an iterative design approach. The excellent comparison between model an experiment is largely attributed to the use of effective lengths instead of geometric lengths in the discretization approach mentioned above.

Significant progress has also been made in the analysis, simulation and validation of two-dimensional flow models. The Moore-Greitzer 2D theoretical model has been thoroughly assessed from the standpoint of nonlinear dynamic analysis and bifurcation theory. These studies have provided a better understanding of the rotating stall instability, particularly from the control point of view. A simple approach to control of rotating stall presented by Abed and Liaw in the *IFAC Symposia Series* (V. 7, 1993, pp 295–300) has been analyzed theoretically and implemented in experiment. This scheme, which is based on the Moore–Greitzer model, has been shown to increase the performance of compression systems by increasing the immunity of equilibrium points to persistent and impulsive disturbances. This control approach requires 2D sensing but only 1D actuation, resulting in relatively low bandwidth requirements. This advantage is attributed to the fact that in this approach one does not seek to extend the theoretical stable axisymmetric flow range of the compressor. Rather, one seeks to directly address persistent disturbances which would otherwise throttle the equilibrium into the unstable axisymmetric flow range of the compressor. In addition, one objective is to enlarge the domains of attraction of linearly stable axisymmetric equilibria, thereby addressing impulsive disturbances which would otherwise perturb the system state beyond the domain of attraction of the stable axisymmetric equilibrium.

The control concept has been tested on the experimental facility. The results are in complete qualitative agreement with the predictions of the theoretical control model. However, it has been found that a comparison with a system–fitted model is only of qualitative value, given that the base model seems to be inadequate in capturing some of the fluid dynamic phenomena. An assessment of this and other control approaches to rotating stall based on CFD is currently under way. The assessment focuses on the effectiveness of controllers based on low–order models in the detailed flow behavior.

In addition, new experimental techniques to detect rotating stall and pre-stall waves have been developed and applied to the data obtained from the facility. When applied, the technique allows the unique determination of the individual harmonics of stall and pre-stall waves from the data.

2.2.2 Relevance and Potential Applications

Previous work in the area of active control of compressor stall has been limited to isolated compressors operating in rather idealized environments. Consequently, little attention has been given to the issue of engine system/environmental uncertainty. It is clear that these and other issues, such as forced system transients, will have to be addressed before high-performance engine controls can be developed. In this sense, the turbojet engine model has the potential to become the first model put forth in the open literature that aptly addresses these points. Important aspects of the model include i) the inclusion of fuel addition and fuel combustion, ii) internal turbine driving of the compressor, and iii) the capability to properly simulate speed transients. This model differs markedly from the original Greitzer model which assumes i) that the plenum temperature is constant and equal to ambient temperature (no combustion allowed), and ii) the compressor speed is constant, in which case there are no speed transients and no distinction between internal and external driving of the compressor.

In addition, the global nonlinear dynamic behavior and bifurcation analyses of compression system models provide insights which could allow engineers to map the performance of components in situations where data cannot be obtained due to physical or system limitations. For example, a low-flow-rate steady-state point is difficult to measure since such point can be unstable by virtue of the system parameters and dynamics. One approach to this problem is to first characterize the qualitative nonlinear dynamic behavior of the model as a function of the qualitative features of the compressor steady-state performance map. Then, one can observe the qualitative nonlinear dynamic behavior exhibited by the physical system. Finally, one can combine the results of the previous steps to define a steady-state performance map so as to achieve qualitative agreement between the analysis and the experimental observations.

As a final remark, since the theoretical identification methods that have been developed in this program yield not only nominal design models, but also uncertainty specifications in exactly the form required by current popular approaches to robust control system design, those methods for system identification are inherently better-suited for control system design than existing methods. With further theoretical development, these identification methods may lead to significant advances in adaptive control by providing a direct link between the modeling.identification process and the robust control system design process. In any case, a direct link of this sort is crucial for realizing the full potential of the impressive robust control design methodologies that have been recently developed.

3 Future Research

In line with the overall goals and objectives of this research program, a number of research tasks have been planned, and currently being pursued, to complement the accomplishments of the original program. The stated objectives indicate that much of the theoretical, simulation, and control design aspects of the original research program has been completed. The development of a validated two–dimensional rotating stall model for the LICCHUS axial compressor rig, and the system identification and implementation of the integrated surge control and rotating stall control scheme on the LICCHUS axial compressor rig, constitute part of the ongoing research efforts. The CFD assessment of the rotating stall controllers is another aspect of current complementary research efforts. The completion of this work will close a new chapter on modeling, simulation and control of externally-driven compression sytems.

In addition, the success of the modeling and control concepts on the externally-driven compressor has turned our focus to the extension of the surge/rotating stall control methods to full engine systems. To that end, the development of validated two-dimensional flow model for the LICCHUS turbojet engine facility constitutes the remainder of the current research efforts.

4 Publications

Conference Presentations/Publications Listing Grant Support:

1. O. O. Badmus, S. Chowdhury, K. M. Eveker, C. N. Nett, and C. J. Rivera. 'A simplified approach for control of rotating stall, Part 2: Experimental results,' In *Proceedings of the 1993 AIAA Joint Propulsion Conference*, 1993. AIAA Paper No. 93-2234.

2. O. O. Badmus, S. Chowdhury, K. M. Eveker, and C. N. Nett. 'A simplified approach for control of compressor surge and rotating stall: Part 1, Theoretical development and Part 2, Experimental results,' 1993. Presented at the 1993 Joint Propulsion Conference.

3. O. O. Badmus. 'Nonlinear Dynamic Analysis and Control of Surge and Rotating Stall in Axial Compression System Models,' PhD thesis, Georgia Institute of Technology, 1994.

4. O. O. Badmus, S. Chowdhury, K. M. Eveker, C. N. Nett, and C. J. Rivera. 'A simplified approach for control of rotating stall, Part 1: Theoretical development,' In *Proceedings of the 1993 AIAA Joint Propulsion Conference*, 1993. AIAA Paper No. 93-2229.

5. O. O. Badmus, S. Chowdhury, K. M. Eveker, and C. N. Nett. 'Control–oriented high–frequency turbomachinery modeling: Single–stage compression system 1D model,' 1993. Presented at the 1993 ASME International Gas Turbine and Aeroengine Congress and Exposition.

6. O. O. Badmus and C. N. Nett. 'Dual stabilization and control of surge and rotating stall in axial compression systems' *draft*, January 1994.

7. C. N. Nett. 'Compressor stall phenomena: Modeling, dynamics, and active control - The 1992 Eckman Award Lecture.' *IEEE Control Systems Magazine*, 1993. Submitted for publication.

8. C. N. Nett. 'Toward a new generation of high performance aircraft gas turbine engine controls: A systems–oriented research program.' Research proposal for onr grant # N00014–90–J–1794, Georgia Institute of Technology, February 1990. Available from School of Aerospace Engineering Georgia Institute of Technology.

9. C. N. Nett. 'LICCHUS Experimental Facilities Summary: September 1991 (Months 1-18), October 1991.' Videotape presentation, available from School of Aerospace Engineering, Georgia Tech, 120 minutes.

10. K. M. Eveker and C. N. Nett. 'Control of compression system surge and rotating stall: A laboratory–based "hands-on" introduction.' In *Video Proceedings of the 1993 American Control Conference*, June 1993. Videotape presentation.

11. O. O. Badmus, K. M. Eveker, and C. N. Nett. 'Control–oriented high–frequency turbomachinery modeling, Part 1: Theoretical foundations.' In *Proceedings of the 1992 AIAA Joint Propulsion Conference*, July 1992. AIAA Paper No. 92-3314.

12. O. O. Badmus, K. M. Eveker, and C. N. Nett. 'Control–oriented high–frequency turbomachinery modeling: General 1D model development,' 1993. Presented at the 1993 ASME International Gas Turbine and Aeroengine Congress and Exposition.

13. O. O. Badmus, K. M. Eveker, and C. N. Nett. 'Control–oriented high–frequency turbomachinery modeling: Theoretical foundations,' 1993. Submitted for publication to the ASME Journal of Turbomachinery.

14. K. M. Eveker. 'Model Development for Active Control of Stall Phenomena in Aircraft Gas Turbine Engines.' PhD thesis, Georgia Institute of Technology, 1993.

15. S. Chowdhury. 'An Experimental Investigation of Active Stall Control in Compression Systems.' PhD thesis, Georgia Institute of Technology, 1994.

5 Participants

The research program is conducted by the collaborative efforts of a group of students and faculty from several academic backgrounds, consisting of aerospace engineers, chemical engineers, electrical engineers, and mathematicians.

The table below lists the student participants:

Name	Position	Status	Primary Support	Thesis and/or Project Topic
Badmus, Lanre	ChE PhD	Graduated	ONR	Modeling, Dynamics and Control of Industrial Process Compression Systems
Eveker, Kevin	AE MS/PhD Student	Graduated	AFOSR	Modeling, Dynamics and Control of Aircraft Gas Turbine Engines
Chowdhury, Subhradeep	AE MS/PhD Student	Graduated	ONR	Experimental Investigation of Active Gas Turbine Engine Controls
Liu, Ketao	AE MS/PhD Student	Active	ONR	System Identification and Model Validation of Turbojet Facility
Rivera, Carlos	AE MS/PhD Student	Active	ONR	System Identification and Validation of First-Principles Two-Dimensional Compression System Models

Toward a New Generation of High Performance Aircraft Gas Turbine Engine Controls: A Systems–Oriented Research Program

Report Submitted to:

Program Manager
Augmentation Award for Science
and Engineering Research Training
Office of Naval Research,
800 North Quincy, Arlington,
Virginia, 22217-5660.

by

Laboratory for Identification and
Control of Complex Highly Uncertain Systems
School of Aerospace Engineering,
Georgia Institute of Technology,
Atlanta, GA 30332-0150

Submission Date: June 1996

1 Executive Summary

In April 1990 a four year ONR funded research program was initiated at Georgia Tech. The stated goals of this research program encompass such diverse areas as mathematical modeling, nonlinear dynamics, bifurcation theory, and active control of the aerodynamic instabilities of surge and rotating stall in aircraft gas turbine engines.

In August 1995, a further extension for an additional year was granted and approved to facilitate the completion of the original proposed research objectives.

This letter report outlines the details of the research accomplishments for the duration of the program amounting to the attainment of the stated goals in the original proposal. In addition, the progress made during the year 1995 is outlined. The progress during this period is described in more detail in the attached PhD thesis proposal of one of the participants in the program.

The accomplishments in the first three years of the research program are numerous, and have made significant contributions to the current literature on theoretical, computational, and experimental approaches to the analysis and active control of compressor stall phenomena in aircraft gas turbine engines.

Some of the prominent accomplishments include:

- Development of a unique technique for detection, identification, and characterization of rotating stall and pre-stall waves in an axial compressor experimental rig.
- Development and dynamic analysis of a stall capable quasi two–dimensional model based on parallel compressor model concepts.
- Global nonlinear dynamic and bifurcation analysis of low–order two-dimensional rotating stall models that qualitatively predicts experimentally–observed stall inception scenarios.
- Design and experimental validation of linear and nonlinear active surge controllers on the LICCHUS axial compressor experimental rig.
- Experimental proof–of–concept validation of nonlinear control of rotating stall in an axial compressor rig based on 1D actuation and limited 2D sensing.
- Development, analysis and simulation validation of a Full–range integrated active surge control/rotating stall control scheme with a nonlinear observer on a two–dimensional rotating stall model.
- Design, development, instrumentation, and demonstration of a state-of-the-art turbojet engine experimental facility.
- Development of a quasi one–dimensional compressible viscous flow turbojet engine component models for generic engine modeling applications.
- Preliminary system identification, development and qualitative simulation validation of a one–dimensional model for the turbojet engine experimental rig.
- Experimental proof–of–concept of nonlinear feedback control of rotating stall with surge avoidance on a turbojet engine experimental rig based on 1D actuation and limited 2D sensing.
- Experimental Validation of the integrated surge control and rotating stall control scheme on the LICCHUS multistage axial compressor experimental facility.
- Development and parameter identification of a two–dimensional rotating stall model for the LICCHUS multistage axial compressor experimental facility.
- Parameter and system identification for the LICCHUS turbojet engine experimental facility for both quasi one–dimensional compressible flow dynamic models.
- Design and implementation of the surge control scheme on the LICCHUS turbojet engine experimental rig.

The work performed over the 1995 year period revolves around the simulation for active control development of rotating stall through Navier-Stokes-based analysis. The motivation of this part of the program is to account for inherent disturbances, such as rotor flapping interactions, in compression systems. The evolution in time of these disturbances has been shown experimentally to indicate the propensity of the compression system to exhibit unstable behavior. These disturbances are not accounted for in reduced-order models of compressors available in the open literature. By exploring the dynamics of stall inception through CFD, it is expected that new concepts involving the relevant quantities to be monitored will allow the development of improved reduced-order models suitable for control.

The relevant accomplishments of the program over the year 1995 are as follows:

- Modification of existent Navier-Stokes, multiblock flow solver for the problem of cascade and rotor flow analysis in two spatial dimensions.
- Validation of the flow solver against experimental and numerical benchmark data for stationary and oscillating cascades.
- Simulations and steady-state performance determination of a realistic rotor blade environment.
- Analysis of rotor-generated disturbances in the absence of stall.

2 Research Program

2.1 Overview

The general goal of the research program has been to advance significantly beyond proof-of-concept control-based engine component technologies to establish the theoretical, computational, and experimental foundations for a new generation of high-performance aircraft gas turbine engine controls. In view of this goal, the principal technical objectives of the program can be summarized as follows:

- Determine the overall effectiveness and potential of active control for realizing full engine operating potential, while ensuring safe engine operation, in the face of significant engine system and environmental uncertainties.
- Develop overall engine control system design methodologies which realize full engine operating potential in the face of such uncertainties.

Consistent with the objectives, the primary issues delineated and investigated in the research program are:

- The robustness of active controls to uncertainties in the engine system and its operating environment.
- The robustness of active controls to both transient and quasi–steady engine operating point variations.
- The interaction/compatibility of active controls with other necessary engine controls.
- The extent to which active control schemes can be made fail–safe.

The overall approach taken in the research program has been based upon the latest work in the area of compressor stall phenomena. It has utilized expertise in nonlinear modeling, nonlinear dynamics, controls, diagnostics, and simulation of aircraft gas turbine engines. The tools of applied robust nonlinear multivariable control have been extensively used and applied to descriptive models of the phenomena under consideration. The physical data required for the execution of this approach has been gathered through a combination of theory, computation, and experimentation. This combination has been achieved through the development and utilization of the innovative simulation and experimental facilities listed below:

- Multistage Axial Compressor Experimental Facility.
- Extended First-Principles Theoretical Compression System Models Simulation Facility.
- Turbojet Engine Experimental Facility.
- Turbojet Engine Models Real-Time, Hardware-in-the-Loop, Simulation Facility.

These facilities are fully operational and are located at the LICCHUS at Georgia Tech.

2.2 Significance of Research Accomplishments

2.2.1 Accomplishments in Previous Years

The primary accomplishment in the research work during the first four years of the program comprise of complete detailed nonlinear analysis, active control and simulation validation of the 1D unsteady, compressible viscous flow model against experimental data obtained from the LICCHUS axial compressor experimental rig. This represents a conclusive investigation of the varied nonlinear dynamic behavior exhibited by axial compression systems, and the development of suitable nonlinear control configurations for active surge stabilization and control in axial compression systems. This work has been accomplished based on a new approach for control-oriented high–frequency turbomachinery modeling. Models developed using this approach are inherently high–frequency, exhibiting expected high–frequency phenomena and transient behaviors, such as compressor surge and rotating stall. These models are also inherently control-oriented, i.e. well suited for control design, due to their relatively low complexity and accompanying uncertainty characterization, where complexity is measured in terms of flow dimensionality, variable dimensionality, and dynamic order. This approach is based on a novel dimensionless form of the governing equations for unsteady, compressible, viscous, quasi–1D flow, coupled with a novel discretization method and a new, valid, and very useful assumption concerning forcing term variable dependencies. Several important benefits accrue as a result of this approach. First, because the discretization method is matched to natural turbomachinery boundary conditions, the models developed using this approach are simply integrable. In other words, no nonlinear algebraic equations need be solved to compute solutions to the model; hence, the usual iterative process required to compute solutions of turbomachinery models is avoided. This leads to much computational simplification not only in the nonlinear models developed using this approach, but also in the computation of associated linear models, in that these models can be computed in closed form without iteration. Additionally, due to a matching of the forcing term assumption with the discretization method, forcing terms that cannot be determined by first principles modeling can be rigorously identified through steady state experiments. In other words, the unknown forcing terms can be rigorously replaced by steady state performance maps in the discretized equations. Interestingly, no assumptions pertaining to quasi-steady operation are required to validate this technique.

The approach has been applied to develop control–oriented high–frequency models for an externally–driven multi–stage axial compression system, and a single spool turbojet engine. For the compression system in particular, the model and experimental data have been compared with respect to both open-loop and closed-loop behaviors. The comparisons focus on forced transients and global nonlinear dynamics and bifurcations. In all cases, the comparison between model and experiment was found to be excellent. Of particular interest is the ability of the model, which does not include any hysteretic maps, to predict experimentally observed hysteresis with respect to the onset and cessation of surge. Also of interest is the fact that the controllers used for closed-loop comparisons were designed directly from the model with no subsequent tuning of controller parameters, that is, without recourse to an iterative design approach. The excellent comparison between model an experiment is largely attributed to the use of effective lengths instead of geometric lengths in the discretization approach mentioned above.

Significant progress has also been made in the analysis, simulation and validation of two-dimensional flow models. The Moore-Greitzer 2D theoretical model has been thoroughly assessed from the standpoint of nonlinear dynamic analysis and bifurcation theory. These studies have provided a better understanding of the rotating stall instability, particularly from the control point of view. A simple approach to control of rotating stall presented by Abed and Liaw in the *IFAC Symposia Series* (V. 7, 1993, pp 295–300) has been analyzed theoretically and implemented in experiment. This scheme, which is based on the Moore–Greitzer model, has been shown to increase the performance of compression systems by increasing the immunity of equilibrium points to persistent and impulsive disturbances. This control approach requires 2D sensing but only 1D actuation, resulting in relatively low bandwidth requirements. This advantage is attributed to the fact that in this approach one does not seek to extend the theoretical stable axisymmetric flow range of the compressor. Rather, one seeks to directly address persistent disturbances which would otherwise throttle the equilibrium into the unstable axisymmetric flow range of the compressor. In addition, one objective is to enlarge the domains of attraction of linearly stable axisymmetric equilibria, thereby addressing impulsive disturbances which would otherwise perturb the system state beyond the domain of attraction of the stable axisymmetric equilibrium.

The control concept has been tested on the experimental facility. The results are in complete qualitative agreement with the predictions of the theoretical control model. However, it has been found that a comparison with a system–fitted model is only of qualitative value, given that the base model seems to be inadequate in capturing some of the fluid dynamic phenomena.

In addition, new experimental techniques to detect rotating stall and pre-stall waves have been developed and applied to the data obtained from the facility. When applied, the technique allows the unique determination of the individual harmonics of stall and pre-stall waves from the data.

2.2.2 Current Year Accomplishments

Given the prediction capabilities of current stall-capable models, an assessment of the dynamics and control of rotating stall based on CFD is currently under way. The assessment focuses on the dynamic interactions during stall which are not captured by current reduced-order models, and the effectiveness of controllers based on low–order models in the detailed flow behavior. Research during 1995 has been devoted to this aspect of the problem. This work is expected to become one of the first few in its class, combining low level fluid dynamic analysis and simulation with control concepts.

One of the problems concerning the detection of rotating stall in axial flow compressors is the resolution of the so-called pre-stall or precursor waves. These disturbances are the small perturbations often seen to grow into fully developed rotating stall. There are a number of problems involving these waves. First, it has been demonstrated experimentally that, prior to stall, there is a multiplicity of precursors of uncorrelated harmonic content traveling around the compressor wheel. The tracking of the precursor which will eventually grow into stall becomes the problem to be solved for appropriate control action. Additionally, in high speed compressors, these waves tend to be very short lived. In other words, fully developed rotating stall occurs with little warning time. What is sought for through the numerical analysis is the relevant dynamic information that will need to be monitored by stall controllers. At the same time, the exploration of such dynamics will allow a better understanding of this complex flow phenomena.

The numerical approach launched during the last year is expected to yield a more complete picture of the dynamic events during rotating stall. This approach is crucial in allowing increased understanding, given the limitations of turbomachinery flow measurement. The overall objective is to establish a computational platform which can be used to identify system dynamics relevant to rotating stall, which involve propagation phenomena in the pitchwise or tangential direction, and their effects on throughflow properties. Analysis of such dynamics is expected to yield new concepts and rules for the formulation of better reduced-order models for control design.

Specific details on the progress in this leg of the research program are presented in the PhD thesis proposal submitted to the faculty by one of the program participants. The proposal is attached as an appendix to this letter report.

2.2.3 Relevance and Potential Applications

Previous work in the area of active control of compressor stall has been limited to isolated compressors operating in rather idealized environments. Consequently, little attention has been given to the issue of engine system/environmental uncertainty. It is clear that these and other issues, such as forced system transients, will have to be addressed before high-performance engine controls can be developed. In this sense, the turbojet engine model has the potential to become the first model put forth in the open literature that aptly addresses these points. Important aspects of the model include i) the inclusion of fuel addition and fuel combustion, ii) internal turbine driving of the compressor, and iii) the capability to properly simulate speed transients. This model differs markedly from the original Greitzer model which assumes i) that the plenum temperature is constant and equal to ambient temperature (no combustion allowed), and ii) the compressor speed is constant, in which case there are no speed transients and no distinction between internal and external driving of the compressor.

In addition, the global nonlinear dynamic behavior and bifurcation analyses of compression system models provide insights which could allow engineers to map the performance of components in situations where data cannot be obtained due to physical or system limitations. For example, a low-flow-rate steady-state point is difficult to measure since such point can be unstable by virtue of the system parameters and dynamics. One approach to this problem is to first characterize the qualitative nonlinear dynamic behavior of the model as a function of the qualitative features of the compressor steady-state performance map. Then, one can observe the qualitative nonlinear dynamic behavior exhibited by the physical system. Finally, one can combine the results of the previous steps to define a steady-state performance map so as to achieve qualitative agreement between the analysis and the experimental observations. An example of this approach will be presented in publication number 17 of the Publications section below.

Additionally, since the theoretical identification methods that have been developed in this program yield not only nominal design models, but also uncertainty specifications in exactly the form required by current popular approaches to robust control system design, those methods for system identification are inherently better-suited for control system design than existing methods. With further theoretical development, these identification methods may lead to significant advances in adaptive control by providing a direct link between the modeling, identification process and the robust control system design process. In any case, a direct link of this sort is crucial for realizing the full potential of the impressive robust control design methodologies that have been recently developed.

3 Future Research

In line with the overall goals and objectives of this research program, a number of research tasks have been planned, and currently being pursued, to complement the accomplishments of the original program. The stated objectives indicate that much of the theoretical, simulation, and control design aspects of the original research program has been completed. The CFD assessment of rotating stall dynamics is the principal aspect of the current complementary research efforts. The simulation of rotating stall will be demonstrated and the interactions of control and stall dynamics in the distributed-parameter simulations will be documented as the ongoing effort in the research program. The completion of this work will close a new chapter on modeling, simulation and control of externally-driven compression systems.

4 Publications

Conference Presentations/Publications Listing Grant Support:

1. O. O. Badmus, S. Chowdhury, K. M. Eveker, C. N. Nett, and C. J. Rivera. 'A simplified approach for control of rotating stall, Part 2: Experimental results,' In *Proceedings of the 1993 AIAA Joint Propulsion Conference*, 1993. AIAA Paper No. 93-2234.

2. O. O. Badmus, S. Chowdhury, K. M. Eveker, and C. N. Nett. 'A simplified approach for control of compressor surge and rotating stall: Part 1, Theoretical development and Part 2, Experimental results,' 1993. Presented at the 1993 Joint Propulsion Conference.

3. O. O. Badmus. 'Nonlinear Dynamic Analysis and Control of Surge and Rotating Stall in Axial Compression System Models,' PhD thesis, Georgia Institute of Technology, 1994.

4. O. O. Badmus, S. Chowdhury, K. M. Eveker, C. N. Nett, and C. J. Rivera. 'A simplified approach for control of rotating stall, Part 1: Theoretical development,' In *Proceedings of the 1993 AIAA Joint Propulsion Conference*, 1993. AIAA Paper No. 93-2229.

5. O. O. Badmus, S. Chowdhury, K. M. Eveker, and C. N. Nett. 'Control-oriented high-frequency turbomachinery modeling: Single-stage compression system 1D model,' 1993. Presented at the 1993 ASME International Gas Turbine and Aeroengine Congress and Exposition.

6. O. O. Badmus and C. N. Nett. 'Dual stabilization and control of surge and rotating stall in axial compression systems' *draft*, January 1994.

7. C. N. Nett. 'Compressor stall phenomena: Modeling, dynamics, and active control - The 1992 Eckman Award Lecture.' *IEEE Control Systems Magazine*, 1993. Submitted for publication.

8. C. N. Nett. 'Toward a new generation of high performance aircraft gas turbine engine controls: A systems–oriented research program.' Research proposal for onr grant # N00014–90–J–1794, Georgia Institute of Technology, February 1990. Available from School of Aerospace Engineering Georgia Institute of Technology.

9. C. N. Nett. 'LICCHUS Experimental Facilities Summary: September 1991 (Months 1-18), October 1991.' Videotape presentation, available from School of Aerospace Engineering, Georgia Tech, 120 minutes.

10. K. M. Eveker and C. N. Nett. 'Control of compression system surge and rotating stall: A laboratory–based "hands-on" introduction.' In *Video Proceedings of the 1993 American Control Conference*, June 1993. Videotape presentation.

11. O. O. Badmus, K. M. Eveker, and C. N. Nett. 'Control–oriented high–frequency turbomachinery modeling, Part 1: Theoretical foundations.' In *Proceedings of the 1992 AIAA Joint Propulsion Conference*, July 1992. AIAA Paper No. 92-3314.

12. O. O. Badmus, K. M. Eveker, and C. N. Nett. 'Control–oriented high–frequency turbomachinery modeling: General 1D model development,' 1993. Presented at the 1993 ASME International Gas Turbine and Aeroengine Congress and Exposition.

13. O. O. Badmus, K. M. Eveker, and C. N. Nett. 'Control–oriented high–frequency turbomachinery modeling: Theoretical foundations,' 1993. Submitted for publication to the ASME Journal of Turbomachinery.

14. K. M. Eveker. 'Model Development for Active Control of Stall Phenomena in Aircraft Gas Turbine Engines.' PhD thesis, Georgia Institute of Technology, 1993.

15. S. Chowdhury. 'An Experimental Investigation of Active Stall Control in Compression Systems.' PhD thesis, Georgia Institute of Technology, 1994.

16. C. J. Rivera. 'Numerical Simulation of Dynamic Stall in Axial Flow Compressor Blade Rows', PhD thesis, Georgia Institute of Technology, In preparation.

17. C. J. Rivera, L. N. Sankar, and J. V. R. Prasad. 'Numerical Simulation of Dynamic Stall in Axial Flow Compressor Blade Rows', Submitted for publication in the Proceedings of the 35^{th} Aerospace Sciences Meeting and Exhibit, Reno, NV., 1997

18. C. J. Rivera and J. V. R. Prasad. 'A Method for Unstable Steady-State Solutions of a Nonlinear Compressor Model', Submitted for publication in the Proceedings of the 2^{nd} MATHMOD Conference, Vienna, Austria, 1997.

5 Participants

The research program is conducted by the collaborative efforts of a group of students and faculty from several academic backgrounds, consisting of aerospace engineers, chemical engineers, electrical engineers, and mathematicians.

The table below lists the student participants:

Name	Position	Status	Primary Support	Thesis and/or Project Topic
Badmus, Lanre	ChE PhD	Graduated	ONR	Modeling, Dynamics and Control of Industrial Process Compression Systems
Eveker, Kevin	AE MS/PhD Student	Graduated	AFOSR	Modeling, Dynamics and Control of Aircraft Gas Turbine Engines
Chowdhury, Subhradeep	AE MS/PhD Student	Graduated	ONR	Experimental Investigation of Active Gas Turbine Engine Controls
Rivera, Carlos	AE MS/PhD Student	Active	ONR	System Identification and Validation of First-Principles Two-Dimensional Compression System Models Numerical Simulation of Dynamic Stall in Rotor Blade Rows.

Numerical Simulation of Dynamic Stall in Axial Flow Compressor Blade Rows

A Proposal for a Doctoral Dissertation
Presented to
The Academic Faculty
By

Carlos J. Rivera

School of Aerospace Engineering
Georgia Institute of Technology

May 2, 1996

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Summary

The recently demonstrated success of several control strategies for stall phenomena in axial flow compressors has prompted a renewed interest for better modeling of these complex flow systems. The basic motivation is to account for several interference effects and inherent disturbances in these systems whose evolution in time can indicate the propensity of the system to exhibit unstable behavior. The purpose of this work is to demonstrate that such increased knowledge can be gathered by the application of well established methods in Computational Fluid Dynamics, and the proper interplay with experimental techniques. Accordingly, the simulation of the phenomena of rotating stall in compressor blade rows will be demonstrated. In addition, the numerical calculations will be used in conjunction with one of the instability control concepts presented in the growing literature of stall control in an attempt to quantify the degree of success of such strategies.

CHAPTER 1

Introduction

A renewed interest in modeling and simulation of aircraft engine components has emerged during the past twenty years. One motivation for such interest is the prediction of unsteady stall phenomena in compressors and its relation to fluid dynamic stability. Analytical and experimental work in this area can be traced back to the middle of the century [1, 2, 3, 4, 5, 6, 7, 8]. More recently, the literature has grown considerably, expanding to embrace the field of modern control theory, in a logical search for mechanisms of instability suppression.

The pursue of simplified models stems from the motivation of reducing the dimensionality and associated complexity of complete aerodynamic simulations. This is achieved by exploiting the disparity in time and length scales found in turbomachinery, which allows the separate study of transient events which may be occurring simultaneously. Although different models capable of describing both fully-developed and transient stall phenomena are fairly numerous [7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22], only a handful of these yield accurate quantitative data, or do so only in comparison to certain experimental settings [20, 23]. Thus, while these efforts offer the advantage of computational economy, the simplifications made result in the residualization of physical system dynamics which might be of interest, and whose detailed study could reveal finer details of instability inception and development.

Nevertheless, models that reduce to systems of ordinary differential equations possess an additional advantage. The impressive advances in the numerical analysis, simulation, and control of complex, ODE-based nonlinear systems have allowed the development of control strategies for turbomachinery flow instabilities based on such models. This is an extremely important point when one considers the conventional approach for addressing instability in turbomachinery. Currently, compressors are designed to optimize point or steady-state performance. This involves limiting the operation of the compressor to a region where the plant operates safely. A stall line bounding the region of safe operation from unstable operation is specified early in the design [24, 25]. The use of such margin introduces a penalty on the pressure rise delivered by the compressor. Thus, the reduction or elimination of the stall margin through active control has become a major objective for addressing instability phenomena [26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 20, 37, 38]. Examples of detailed analysis and validation programs carried out in this area can be found in [18, 20, 22, 19, 39, 23, 40], which deal with the suppression of surge, rotating stall and simultaneous surge/rotating stall instabilities in compression systems.

In short, simplified models are able to explain instability phenomena by means of a relatively small set of primary descriptive parameters. The dynamic behavior embodied in these parameters has been tapped to the point of allowing the derivation of model-based control laws for instability suppression. In principle, the practical validation and implementation of these strategies will enable the design and manufacture of more robust, high-performance machinery. However, as pointed out earlier, most existing models lack quantitative description capabilities for most physical systems. When such models are employed, the physical system under consideration must be tailored to the model by means of system identification tools and a certain degree of empiricism, given the standing ideal-behavior assumptions present in these models. Such detailed numerical analysis, transient parameter adjustment, and map identification are required when the control law to be implemented in a physical system is derived from a mathematical model.

While no simplified model can adequately capture the behavior of a physical system, it is believed that the element of empiricism can be removed from a model formulation by studying detailed physical aspects of the modeled system. This statement can be elaborated through a motivating example. In a dynamic stall model validation program carried by the author[1], it is found that adequate transient matching is possible by introducing a lower bound on one of the states of a model or an unmodeled input term, whereas the model formulation itself does not account for such bound or the extra input. In other instances, one may find that assigning to a model constant a value different from that dictated by the model definitions (often given in terms of measurable geometric or physical parameters) also results in improved transient matching. However, some of these parameters are those which dictate the stability of the system, leaving an undesirable degree of free-play in the model.

Model development has been fueled primarily by limitations in computing and measuring power. While these difficulties have not been completely overcome, major advances have taken place in measurement and computation, especially in computation. A problem with experiments in complex flows is that it will be necessary to measure flow quantities in a finely distributed manner. Although many improvements are constantly being made in the area of turbomachinery flow measurement, they have yet to insure an increased understanding of the flow pattern [41]. The hostility of the flow environment in rotating machinery is responsible for this state of affairs. On the other hand, it has

[1] A condensed version of the cited validation work is presented in Chapter 2

been established in recent times that flow field information obtained from the numerical solution of the fundamental equations of fluid motion in their different levels of simplification has become a profitable means of obtaining and analyzing data for complex turbomachinery flows [42, 43, 44, 45, 46, 47, 48]. Given that the behaviors which can be analyzed by detailed flow simulations are quite often those which have a large impact on the response of these complex flow systems, one can identify an incentive for considering such calculations, as they provide information not completely conveyed by simplified models.

The purpose of the present work is twofold. The primary objective will be the successful simulation of rotating stall in an isolated rotor based on the solution of the Reynolds-averaged Navier-Stokes equations in two spatial dimensions. The simulation of time-varying stalled flow fields in turbomachinery is a virgin area of research. It is expected that this work will be one of the first few in its class. The two-dimensional flow simulations are in line with the most recent approaches to turbomachinery instability modeling and should not be considered an unavoidable limitation in the present effort. Second, as a demonstrative feature of the capabilities of the calculations, the code will be used in conjunction with the one of the model-based instability control ideas presented in the literature in an attempt to quantify the degree of success of such strategies. Particular attention will be paid to those aspects of the physical setup and system behaviors which are absent in simplified models.

Although the approach to be taken in this work may seem rather overwhelming and limited in its own, given the need for computing power to evolve many generations before full distributed-parameter engine simulations could be possible, it is expected that the validity of standing concepts will be verified. The major contribution of this work to the existent literature is expected to be the practical demonstration of the possibility of increasing our current knowledge of rotating stall instabilities by the application of well known CFD tools. It is also expected that exploring and analyzing simple simulation results will generate ideas for better guidance on post-stall modeling of rotating machinery.

1.1 Organization of the Proposal

The proposal is organized as follows. Chapter 2 presents a discussion on the compressor instability of rotating stall, which will be the primary unsteady phenomena to be discussed in this work. This discussion introduces a familiar model employed in simulating this phenomenon, along with recently proposed ideas for control. A comparison of model output versus experimental results in terms of model steady-state solutions and forced transients is also presented, in order to quantify the current state of the art in compressor instability modeling. A statement on the work to follow, based on the results presented, is also set forth. Chapter 3 demonstrates the capabilities of the flow solver by comparing numerical results against experimental or numerical benchmark data. In addition, an outline for the completion of the proposed work is presented.

CHAPTER 2

Phenomenological Modeling of Compression System Instabilities

Axial flow compressors are used in the majority of high thrust gas turbine engines employed in aerospace applications. As they control the pressure ratio, compressors have a dominant effect on the performance characteristics of the gas turbine engine. Enthalpy addition occurs in rotating blade rows by increasing the kinetic energy and static pressure of the flow. Stator rows remove some of the tangential velocity, thus decreasing the kinetic energy and consequently increasing the static pressure. The limiting pressure rise through an axial flow compressor is reached when the adverse streamwise pressure gradient on the suction side of the blades becomes so large that flow separation occurs. As the limiting operating condition is reached the entire power plant may surge or the compressor may enter a rotating–stall mode of operation. Surge is a predominantly one–dimensional flow phenomenon which involves limit–cycle type oscillations in flow and pressure rise. Rotating stall, on the other hand, is a non–axisymmetric distribution of axial velocity local to the compressor.

Surge can be explained via a simple mechanical argument. An externally–driven compression system, such as that shown in Figure B.1 serves to illustrate this point. A compressor operates in a duct and discharges into a plenum of dimensions larger than those of the compressor and its ducts. The flow through the compression system is controlled by a throttle at the exit of the plenum. In general terms, this compression system behaves as a nonlinear spring–mass–damper oscillator. This can be demonstrated from first principles, by applying one–dimensional fluid dynamic statements of mass and momentum conservation, and adding a modeling term for the compressor forcing [7]. The inertia is provided by the flow mass in the ducts, the stiffness comes from the compressibility of the fluid in the plenum, and damping is provided by the compressor and throttle. The damping of the throttle is always positive, since a pressure drop increase through a throttle is accompanied by a corresponding increase in the flow through the device. However, the damping provided by the compressor in combination with the compliance of the plenum could become negative, depending on the operating point, given that the compressor can operate in a region of positive pressure–flow map slope. In the critical damping unbalance case, the dissipation in the throttle may not be sufficient to eliminate the input of energy from the compressor and thus the ability of the plenum gas to utilize this energy to oscillate.

A qualitative explanation for the onset and propagation of stall in compressors has been given by Emmons [4]. As the blades approach limiting flow incidence and loading, a transient disturbance could cause a single blade to stall. The corresponding blade channel chokes momentarily, as the pressure rise remains almost constant. In this manner, the flow is diverted toward neighboring blades, causing an increase in the incidence of the blade next to the stalled blade. As this occurs, the stall condition is relieved on the stalled blade, the next blade stalls, and the effect propagates around the annulus.

The extent to which one type of instability triggers the other depends on the dynamic coupling between components and the geometry of the system. It has been shown experimentally [7, 36, 38] and could be argued by the analysis of existing models, that an externally driven compressor can be easily set up in a facility in such a way that one type of instability is more predominant than the other. In this spirit, the discussion in this work will be limited to the phenomenon of rotating stall; surge has been and will be mentioned hereafter to motivate the arguments set forth. There is, however, a common denominator to both types of instabilities. According to existent models, the appearance of either surge or rotating stall is a result of a bifurcation in the system dynamics. At critical values of system parameters, a formerly stable solution ceases to be stable, and, as a result, a new kind of stable solution appears. This issue is discussed below in its relation to rotating stall.

Several models for studying rotating stall have been proposed. For incompressible flow in externally driven compression systems, models given in [11, 14, 12] adequately describe the propagation phenomena of rotating stall. Out of these, in particular, the Moore–Greitzer model in [12] has received considerable attention from the research community since its publication. The linear model derived from it is presented in different forms in [23]. Inclusion of other effects, such as more elaborate blade row input–output relations concerning the flow deflection and compressor pressure rise, is presented in [21], in which a family of models based on the original Moore–Greitzer model is developed. An extension to compressible flow is made in [17]. Although comprehensive reviews of this model have been presented elsewhere [40, 39, 49, 50, 23], a brief presentation of the model is included in Appendix A, given that most recent rotating stall control strategies available in the open literature are based on this particular model.

2.1 Reduced–Order Stall Model Analysis and Validation

As part of the research efforts to implement a rotating stall controller for the axial compressor rig at the Georgia Tech LICCHUS, a steady–state and dynamic mode validation of the Moore–Greitzer model was undertaken. The validation approach is based upon the argument that, in order to obtain good transient predictions from a nonlinear model, steady–state maps must also be demonstrating good agreement with experiment. The analysis of the model, presented in [40], plays an important role in the steady–state validation. Here, we refer to steady–states as the limit sets of system trajectories from all possible initial conditions in the phase space. These limit sets include equilibrium points (time–invariant solutions), periodic and aperiodic orbits, and strange attractors. In this discussion, we focus the attention on equilibrium solutions and periodic orbits, as these are the relevant steady–state solutions in the the neighborhood of the high pressure operating regime of the model. A brief digression of the work in [40] is made here, in order to show its connection to the validation problem.

2.1.1 Model Analysis

The model equations for a single–harmonic flow disturbance and an axisymmetric compressor pressure map described by a polynomial in flow are given in Appendix A in Equation A.6 and given below for convenience:

$$
\begin{aligned}
\frac{da_1}{d\tau} &= \frac{2}{\mu+m_1}\left[\frac{\lambda}{2}b_1 + a_1\left(\sum_{n=1,n\ odd}^{N_p}\frac{\psi_c^{(n)}(\Phi)}{n!\,r_n}(a_1^2+b_1^2)^{\frac{n-1}{2}}\right)\right] \qquad (2.1)\\
\frac{db_1}{d\tau} &= \frac{2}{\mu+m_1}\left[-\frac{\lambda}{2}a_1 + b_1\left(\sum_{n=1,n\ odd}^{N_p}\frac{\psi_c^{(n)}(\Phi)}{n!\,r_n}(a_1^2+b_1^2)^{\frac{n-1}{2}}\right)\right]\\
\frac{d\Phi}{d\tau} &= \frac{1}{L_c}\left[\sum_{n=1,n\ even}^{N_p}\frac{\psi_c^{(n)}(\Phi)}{n!\,s_n}(a_1^2+b_1^2)^{n/2} - \frac{1-K_I}{4}(\tanh l_i)^2(a_1^2+b_1^2) - \Psi\right]\\
\frac{d\Psi}{d\tau} &= \frac{1}{4B^2L_c}\left(\Phi - K_T\sqrt{\Psi}\right)
\end{aligned}
$$

Here, Φ and Ψ represent the annulus–average flow and pressure respectively, and a_1 and b_1 correspond to the Fourier sine and cosine coefficients of the first spatial harmonic of the rotating flow disturbance. The parameter K_T is the principal input in this model and corresponds to the throttle area parameter, which sets the operating point of the system. As explained in Appendix A, the term K_I in the flow equation is attributed to a model for inlet guide vane loss. The remaining constants μ, λ, m_1, B, and L_c are geometric and physical parameters of the modeled system, as explained in the Appendix.

The ability to validate the steady–state solutions resides mainly upon the identification of the axisymmetric compressor map, which is the locus of points in the flow–pressure coordinates for which $a_1 = b_1 = 0$ (no rotating disturbance). As touched upon in [19], such a procedure could be carried out experimentally if the zero stall solution is stabilized in the throttle parameter range of interest. However, these dynamics pose complex control challenges, as they are unstable or very lightly damped in the vicinity of the high pressure rise region under consideration. Moreover, the axisymmetric solution is linearly uncontrollable and unobservable from one–dimensional (average) control and sensing. In other words, a multidimensional sensing and control scheme must be implemented to carry out this procedure.

A linear analysis of the model equations reveals that the axisymmetric steady–state solution, defined by

$$\Psi = \psi_c(\Phi) \ , \quad \Phi = K_T\sqrt{\Psi} \ , \quad a_1 = b_1 = 0 \ ,$$

is locally stable if the slope of the map, $\psi_c^{(1)}(\Phi) = \psi_c'(\Phi)$ is negative. At axisymmetric equilibrium points for which $\psi_c'(\Phi) > 0$, the local solution is unstable. The stability of points for which $\psi_c'(\Phi)$ is zero cannot be determined from the linearization, since the eigenvalues of the Jacobian matrix of the system in 2.1 corresponding to the Fourier coefficient equations have zero real part. The equilibrium point at which $\psi_c'(\Phi) = 0$ and a local maximum in the flow/pressure map occurs is called the stall inception point in [40]. In the neighborhood of this point, various steady–state solutions are possible. As the throttle parameter varies around the value

$$K_{T_c} = \Phi/\sqrt{\psi_c(\Phi)} \ , \quad \psi_c'(\Phi) = 0 \ , \quad \psi_c''(\Phi) < 0 \ ,$$

a non–zero initial condition simulation of the model out of the hyperplane $a_1 = b_1 = 0$ may settle into the axisymmetric solution, or the system may enter fully developed stall, in which case, a_1 and b_1 have oscillatory solutions. In critical cases, the model exhibits hysteresis with respect to the onset and cessation of rotating stall. When hysteresis is present, there will be a throttle parameter range over which both linearly stable axisymmetric and stalled solutions overlap. Whether the system settles in either axisymmetric or stalled operation in this throttle range depends on the initial condition of the model. Hence, either of the solutions possess a finite domain of attraction. The presence of hysteresis in the steady–state behavior is a characteristic of highly loaded compressors, notably those of the axial type [8]. Conditions for the occurrence of hysteresis in this model have been derived by Badmus [40] and are presented below.

The study of this particular multiple steady–state behavior shown by the model must be performed by means of bifurcation theory [51, 52]. Bifurcation theory addresses the possible changes in the structure of the orbits of differential equations as variable parameters in these equations are changed. The analysis of local bifurcations of nonlinear equations is accomplished by reducing the model vector field to a single scalar equation–the bifurcation equation–and determining singularity conditions by computing derivatives of this equation. While this standard mathematical approach is usually simple to carry out in low–dimensional systems, it becomes quite arduous for high–order, highly nonlinear systems. In such case, the use of numerical methods to compute steady states facilitates the analysis [53, 54]. The method employed for the results shown in this chapter is described by Doedel in [55]. The algorithm presented in this reference is capable of identifying various types of bifurcation points, as well as trace stable and unstable equilibrium and dynamic solutions.

Figure B.2 shows the numerical bifurcation diagrams obtained for the two–harmonic truncation of the model with a cubic axisymmetric map. These diagrams represent steady–state solutions of the system as a function of the throttle parameter, K_T, which, for a given axisymmetric map, is the parameter whose variations affect the steady state solutions. Shown are plots of the maximum values of the Fourier coefficients of the first and second harmonics (in other words, $A_1 = \sqrt{a_1^2 + b_1^2}$ and $A_2 = \sqrt{a_2^2 + b_2^2}$), as functions of the throttle position, annulus–averaged flow Φ and pressure Ψ as function of K_T and the compressor performance map of Ψ versus Φ by elimination of K_T. The locus of stable steady–state solutions is depicted by solid lines and the unstable solutions are shown in dash–dot lines. The numbers in the (K_T, A_1) plot represent the number of steady–state solutions in the regions bounded by the vertical lines. As can be seen, for large values of K_T, in this case, $K_T > 0.825$, there is a single stable axisymmetric equilibrium point for a given K_T value, in which case $a_n = b_n = 0$, $n = 1, 2$. Following the axisymmetric locus, as the system is throttled up by reducing the throttle area, a region is entered in which the stable axisymmetric equilibrium is no longer the unique equilibrium condition for this system. In this region, for which $K_T \in (0.5148, 0.825)$ and corresponding to the rightmost 3 in the (K_T, A_1) plot, there is a stable axisymmetric equilibrium point and a stable nonaxisymmetric periodic solution, in addition to an unstable nonaxisymmetric periodic solution. The stable nonaxisymmetric periodic solution corresponds to rotating stall. The region of throttle positions in which multiple stable steady–state solutions exists is the hysteresis region of the compressor. As mentioned above, when the throttle parameter takes on a value belonging to this region, the axisymmetric and nonaxisymmetric equilibrium solutions have finite domains of attraction and the system evolution in time is largely dependent on the initial conditions. Furthermore, if the system throttle is in this region and the system is in stalled steady–state, it takes larger values of K_T to get out of stall than those required to have the system enter into rotating stall from axisymmetric operation, if the throttling is relatively slow and perturbations from steady-state conditions are small (e.g., quasi-steady behavior). When K_T takes on the critical value $K_{T_c} = 0.5148$, the unstable nonaxisymmetric solution merges with the axisymmetric solution, becoming effectively a single equilibrium condition. At this point, the stalled solution still prevails, as can be seen in the amplitude diagrams. As K_T is further reduced below the critical value and enters region 2 in the A_1 diagram, the axisymmetric solution becomes linearly unstable, and the nonaxisymmetric steady state solution corresponding to rotating stall is the only stable steady state solution. The change in the stability of the axisymmetric solution occurs when $\psi_c'(\Phi) = 0$, that is, at the stall inception point, as can be seen in the plot of pressure versus flow. The corresponding axisymmetric steady state solution at which the slope of the map is zero represents a subcritical Hopf bifurcation point in this model, since the axisymmetric stall inception point is an unstable steady-state solution, as discussed below.

Since the linearization of the model at the critical point is singular, the stability of the stall inception point must be determined from a nonlinear analysis. In [40], explicit relations for the stability of the peak point are developed. There, the center manifold theorem and the reduction principle [51, 56] are employed to determine the stability. The condition derived by Badmus states that the stability of the stall inception point is determined by the sign definiteness of

$$q(\Phi_p) = \psi_c'''(\Phi_p) + \frac{\Phi_p}{\psi_c(\Phi_p)}[\psi_c''(\Phi_p)]^2 \ , \quad \{\Phi_p : \psi_c'(\Phi_p) = 0 \ , \quad \psi_c''(\Phi_p) < 0\} \ . \tag{2.2}$$

Fixing now the attention on the peak point, if $q(\Phi_p) < 0$, then the stall inception point is a locally stable steady-state solution and the Hopf bifurcation is a supercritical bifurcation. In this case, there will be no unstable branch in the interval $K_T \in (0.5148, 0.825)$ and rotating stall will be seen to bifurcate smoothly from the stall inception point. If, on the other hand, $q(\Phi_p) > 0$, the stall inception point is an unstable equilibrium solution, in which case the solution corresponding to nonaxisymmetric behavior in the immediate neighborhood of the stall inception point is also unstable. This is the case for the map chosen to generate these bifurcation diagrams, as evidenced by the presence of hysteresis, given the 3 equilibrium solutions in the interval $K_T \in (0.5148, 0.825)$.

The result shows that, for this particular model, the stability of the stall inception point can be determined simply from the derivatives of the compressor and throttle characteristics (the appearance of Φ_p is a result of the plenum/throttle steady-state relation) and is independent of the number of harmonics used to describe the rotating flow perturbation.

2.1.2 Steady–State and Dynamic Validation

The strategy for the determination of the axisymmetric polynomial map for the single–harmonic model, given a set of experimental data, can be formally posed as an exercise in nonlinear programming. The model steady–state relations, which are linear in the coefficients of the polynomial, define the function to be minimized. Specifically, one can expand out the steady–state relations for flow and disturbance amplitude, collect known, measurable terms such as flow, pressure and amplitude, and assemble a system of two equations for each data point. The equations for all experimental points to be used can be finally cast in the form

$$Ax = b \quad ,$$

where x denotes the unknown polynomial coefficients. The exact definition of A and b are left out for the sake of brevity. The important observation is that the system of equations for x is not necessarily square, given that the number of available experimental data points does not have to be related to the order of the matching polynomial. Since a solution to $Ax - b = 0$ is thus not feasible, an attempt is made to find an x such that the 2–norm of $Ax - b$, is minimized. The square of this norm, which is simply a standard form of a quadratic cost function, is given by

$$J(x) = x^T A^T Ax + (-2b^T A)x + b^T b \quad .$$

To match the stability requirements of the model, inequality constraints for the coefficients can be defined from the slope of the map in the unmeasurable unstable portion, which must be positive. The slope constraints are linear in the unknown coefficients. The combination of the quadratic objective function with linear constraints constitutes a quadratic programming problem, which can be solved routinely, for example, with the MATLAB *qp m*-file in the optimization toolbox. If, in addition, a peak point has been identified from the data or, otherwise, an estimate of this point can be made, the peak point stability criterion in 2.2 can used to force the coefficients to produce stability or instability. In that case, the problem becomes nonlinear in the coefficients, and a nonlinear programming technique must be used. The work presented here was carried out with DONLP, a code written by Spelluci [57], in the case where nonlinear constraints are used.

Figure B.3 shows the results obtained from a bifurcation analysis of the two–harmonic model after computing an optimal solution for the axisymmetric polynomial coefficients based on the first–harmonic equations. The data comes directly from an experimental axial flow compressor rig set up at LICCHUS and described in [38]. The optimal solution is based on selecting a reasonable value for the peak point flow, Φ_p and pressure $\psi_c(\Phi_p)$, which, due to the very low damping, cannot be measured. In addition, a lower bound on the value of the condition $q(\Phi_p) \geq 10$ is used and constraints on the first and second derivatives employed to impose linear stability requirements and control the shape of the polynomial. The resulting polynomial for the pressure is

$$\psi_c(\Phi) = 73.34\Phi^5 - 127.787\Phi^4 + 68.1558\Phi^3 - 13.2766\Phi^2 + 0.6546\Phi + 0.1802 \quad ,$$

with a peak point at $\Phi_p = 0.35, \psi_c(\Phi_p) = 0.173$. This plot shows a reasonable agreement of the steady-state solutions, at least in the region near the stall inception point. There are however certain issues that can be further addressed. The complexity of the model equations as presented above does not allow one to obtain exact steady-state matching at each operating condition, forcing the recourse to optimization. This is to be contrasted with the case of a parallel compressor model, such as that discussed in [40]. For that model, an exact steady-state match can be found and splines can be utilized to describe the maps in the model. The second point is that of the throttle steady-state relation, which is only approximately satisfied by the experimental data. This issue becomes very troublesome in the case shown in Figure B.4, which shows the simulation of a forced transient by performing a step input on the throttle parameter K_T, and a comparison with the experimental data. The model K_T input differs from the experimental value by an average of 0.04, which is also, roughly, the mean error in the throttle steady-state relation throughout the measurement domain. This is attributed to both the inefficacy of the ideal steady-state relation, in addition to calibration and measurement errors of the experimental K_T values, and the optimization method which was employed to obtain a map.

Successful validation of this model thus poses interesting questions regarding complexity. We have emphasized here the importance of bifurcations in the system dynamics. Thus, the nonlinear steady-state relations play a very important role in the pursue of a good model. The selection of a polynomial as an axisymmetric map in this section has been based on a desired level of simplicity in the model equations. It allows a relatively fast solution to the steady-state equations. Moreover, when a polynomial is assumed, the stability of the system can be expressed by a combination of the derivatives of this function, which provides an elegant analysis result, and remains valid even if other functional representation is used, as could be argued on the basis of a series expansion. The choice of other shapes based on different mathematical or numerical functions, not just for the axisymmetric map but also for the throttle forcing, is an area of this model which can be developed. The basic starting point will be to find zero solutions to the model equations with the flow and amplitude equations in integral form[12]:

$$
\begin{aligned}
\dot{\Phi} &= \frac{1}{L_c}\left[\frac{1}{2\pi}\int_0^{2\pi} \psi_c(\Phi + A\sin\theta^*)\,d\theta^* + \frac{K_I-1}{4}(\tanh l_i)^2 A^2 - \Psi\right] \\
\dot{A} &= \frac{2}{\mu + m_1}\left[\frac{1}{2\pi}\int_0^{2\pi} \psi_c(\Phi + A\sin\theta^*)\sin\theta^*\,d\theta^*\right] \\
\dot{\Psi} &= \frac{1}{4B^2 L_c}\left(\Phi - F_T^{-1}(\Psi)\right)
\end{aligned}
$$

The problem of finding a function $\psi_c(\cdot)$ such that the right-hand sides of these relations involving the integrals are minimized could probably be posed and analyzed through the calculus of variations. However, this approach has yet to be pursued.

2.2 Rotating Stall Control

Current methodologies for stability enhancement of rotating stall dynamics can be broadly classified as passive or active approaches. The basic premise of the passive approach is to prevent the plant to operate in a regime in which disturbances could cause stall by means of improvements in the aeromechanical design, in the form of variable geometries, or smart structures. An example of a commonly employed passive stall stabilization method is that of casing treatment, which consists of perforations over the tips of the rotors on the casing in axial flow compressors [8]. It has been found that the level of blockage resulting from predominantly endwall stall is reduced when such treatments are employed, and that the corresponding stall margin can be reduced. The philosophy of active control methods is, on the other hand, to enable stable operation at lower flows and higher pressure ratios than those allowable by the passive approaches by means of real-time sensing and actuation.

The possibility of operating at higher pressures through active control has been demonstrated in [23] and [37] both numerically and experimentally. This constitutes an exciting area of research, with a growing number of practitioners, as documented in the bibliography. However promising, it is believed that the full potential of active stall control will be realized through a revision of the current compressor design process. The implementation of a stall controller on a compressor design of current technology may result in an inefficient plant operation, given that, most likely, such compressor has been built to optimize thermal efficiency at design, while allowing the presently large transient and stall margins. As pointed out in [37], the design of a compressor with added active control would proceed by first specifying a smaller stall margin (afforded by active control) and then maximizing point efficiency based on the reduced stall margin.

The approaches to rotating stall stabilization can be further subdivided in terms of the physical means in which disturbances are introduced in the system to induce the desired performance. Currently, the methods employ either non-axisymmetric or axisymmetric effectors to achieve stabilization. In terms of present models and documented mathematical approaches, these physical means correspond to linear and nonlinear stability augmentation, respectively. Linear feedback stabilization involves the local stabilization of unstable axisymmetric points, thereby extending the theoretical stable axisymmetric flow range [26, 27, 36, 33, 23]. This approach involves adding extra terms to model distributed actuation around the circumference of the compressor, linearizing the resulting formulation and developing a linear feedback control system based on the model. However, the linear approach does not completely account for the magnitude of the impulsive disturbances which can be tolerated at the stabilized equilibrium condition. To do this, a calculation of the domains of attraction of the linearly stabilized equilibrium point must be performed, and this requires an account of the nonlinear terms in the model. In contrast, the nonlinear control approach does not attempt to alter the stability of unstable axisymmetric points. Rather, it increases the stability of stable, low-damping, operating regimes, where the pressure is at its highest. This is achieved by eliminating hysteresis with respect to the onset and cessation of rotating stall, thus allowing the stable axisymmetric steady-state points within the former hysteresis region to become global attractors.

A mathematical approach for the simplified nonlinear control of stall, presented in [39] and formally analyzed in [40] and implemented in [38, 37], consists in feeding back the square of the amplitude of the first harmonic of the stall to the throttle parameter according to the quadratic control law

$$K_T = K_{T_{nom}} + k(a_1^2 + b_1^2) = K_{T_{nom}} + kA_1^2 = K_{T_{nom}}(1 + rA_1^2) \ .$$

In [39, 40] it is shown that the bifurcation diagrams of the controlled system for the four-state model (first-harmonic truncation) is modified to that shown in Figure B.5, which allows a comparison with the uncontrolled system bifurcation diagrams. The achievement of this controller is not only the increased stability of the axisymmetric steady states in the former hysteretic region (in fact, these steady states become global attractors), but also the stabilization of the stall inception point in the case where $q(\Phi_p) > 0$, as given in Equation 2.2. Closed-loop simulations for non-equilibrium initial conditions on the amplitude and a nominal throttle parameter set at the critical value $K_{T_{nom}} = \Phi_p/\sqrt{\psi_c(\Phi_p)}$ are shown in [40, 37] to demonstrate this fact. Badmus [40] shows that if the controller gain ratio, r, is chosen to satisfy the inequality

$$r > -\frac{q(\Phi_p)}{8\psi_c'''(\Phi_p)\Phi_p} \; , \quad \{\Phi_p : \psi_c'(\Phi_p) = 0 \; , \; \; \psi_c''(\Phi_p) < 0 \; , \; \; q(\Phi_p) > 0\} \; , \tag{2.3}$$

then, the nonlinearly unstable stall inception point becomes asymptotically stable. For sufficiently high gain ratio, this has the effect of modifying the Hopf bifurcation occurring at the stall inception point from a subcritical to a supercritical bifurcation, with the consequence that the unstable periodic solutions emerging from the stall inception point become stable periodic solutions. This provides a mechanism for the elimination of hysteresis.

It should be emphasized here that the nonlinear controller does not eliminate the possibility of stalled operation. What it does, however, is to attenuate the magnitude of the stalled amplitude as compared to the uncontrolled magnitude. Hence, the stall that occurs in the controlled system is milder than that in the uncontrolled system, for the same nominal throttle parameter value. This can be seen in Figure B.6, which shows forced transients into stall with and without control, for a given nominal throttle step input. As could be noticed, the forced transient in the controlled case still causes the system to enter stall, but in this case, the flow and pressure are higher, and the amplitude is smaller, when compared to the uncontrolled counterpart. This demonstrates that, indeed, this controller is not a simple avoidance mechanism by means of which control is employed to back away from the stall condition. Furthermore, this outcome suggests that this controller can be used as a form of low-authority stall suppression mechanism, if, indeed, the stall dynamics are adequately captured by this model.

2.3 Concluding Remarks

The nonlinear dynamic behaviors demonstrated by simplified post-stall models appear to be quite generic, in the sense that such behaviors have been confirmed in a number of experimental facilities. However, as described in previous sections, the validation of these important nonlinear phenomena against experimental observations on the basis of a given model is an involved process. For models such as that of Moore and Greitzer, this problem can be, at best, treated by well known optimization techniques. These optimal results provide relatively good approximations to the actual observations which perhaps define a boundary between model predictions and model uncertainty. In general, one cannot expect a simplified mathematical model to generate a precise representation of a physical system, given unpredictable or unmodeled inputs and dynamics. In fact, during model-based control design, the use of system identification and robust control design tools can be formally applied to a partially validated model such as that in Section 2.1 to i) match frequency responses via parameter adjustment, ii) define model prediction bounds, and iii) design controllers which operate satisfactorily in the face of model uncertainty. Although part of the concern of this work is to elucidate input/ouput type of relations during stall events, the application of model-based uncertainty analysis and robust control are not considered here.

While this discussion might suggest that incorporating more detailed modeling of the compressor and other dynamic components will increase the accuracy of a model, the fluid dynamic phenomena leading to stall in turbomachinery is still not very well understood. Indeed, given the vast number of geometric and physical parameters encountered in turbomachinery, development of "generic" models that may suit very different plants is a quite difficult effort. In fact, models which have been proposed in the past have met incisive scrutiny and considerable amount of debate still exists over the current ideas. Even in the case of a more detailed model, the number of parameters and maps to be accounted for in the retrieval process from experimental data will certainly increase, and so will the number of measurements to be taken from an experimental facility in order to validate such model [21].

It appears that an improved understanding of the fluid dynamics of rotating stall is necessary in the further development of active stall control. It is necessary to account for the fact that nonlinearity has a strong effect on the behavior of the mathematical formulations which model these systems. The approach to be considered in this work is to study rotating stall based on the numerical solution of the governing fluid flow equations across a two-dimensional rotor blade row. As argued in the Introduction, full numerical simulation provides in many instances the only source of information available to the engineer in the analysis of turbomachinery flows. By tapping this source, we propose to investigate the inception of stall with rather modest objectives. In summary, these objectives are to validate those aspects of stall that are known from the analysis of simplified dynamic models. First, the successful simulation of rotating stall obtained from a Navier-Stokes flow solver will be pursued. It is expected that the introduction of flow nonuniformities combined with mild geometric differences from blade to blade and the application of periodic boundary conditions will be sufficient to result in a propagating stall scenario. Second, the concept of finite domains of attraction of certain operating conditions in a highly-loaded blade row will be investigated. Mapping various operating conditions from unstalled to stalled solutions through the variations of an artificial throttling device will allow the investigation of the sensitivity of those solutions to changes in the throttling parameter. This will provide an account of the importance of nonlinear phenomena during stall events. Finally, the rotating stall control law presented in the previous section will be implemented in the code to investigate the development of stall under a control effort. If the nonlinear phenomena associated with the finite domains of attraction of steady-state solutions is observed, it should be expected that such control law will have a definite impact on the evolution of the stall scenario.

Discerning other aspects of the blade–to–blade flow, such as the behavior of the pressure rise across the blade row during a transient and the evolution in time of rotor flapping modes will also help in the understanding and modeling for rotating stall. A section of this work will be devoted to this particular matter and its connection to the pressure modeling in the Moore–Greitzer approach.

CHAPTER 3

Preliminary Validation Results and Proposal Outline

In order to accomplish the objectives of this work, the solution of the problem of two–dimensional, unsteady, compressible, turbulent flow through stationary and moving cascades is required. The purpose of this chapter is to demonstrate the usefulness and validity of employing a numerical code to solve unsteady viscous flow fields in cascades. This is presented in the first section. The presentation is limited to descriptive, rather than elaborate details; a detailed presentation of the mathematical and numerical aspects will be included in the final documentation of the research. A second section describes the expected timetable for the completion of the work.

3.1 Application of Navier–Stokes Solver to Cascade Flows

Accounts for the presentation and discretization of the Navier-Stokes equations can be found in various references, for example [48]. In this work, the discretization is based on the Alternating Direction Implicit (ADI) scheme employing an Approximate Factorization (AF) on the time–linearized discrete equations [58, 59]. These schemes have been efficiently coded in a multiblock flow solver by a generation of students at Georgia Tech under the supervision of Prof. L. N. Sankar. The MFOIL program, as has been called, requires the user to supply a grid, boundary–condition routines and flow parameters. This code, originally intended for the solution of the flow past multicomponent airfoil systems, is very well suited for the problem under consideration. A grid generator program accompanying the original distribution of MFOIL is also utilized to create the cascade grids. The multiblock code solves for the flow in each of the zones or blocks of the grid at a given value of the discrete time. Rather than employing implicit boundary methods – an approach still under consideration – the boundary conditions are updated after the solution for each zone is obtained. These boundary conditions are employed at the next time level as the part of the starting solution.

The core flow solver itself or its input/output logic were not fundamentally altered from the original code distribution for use in the cascade applications. Rather, most of the work that has been performed over the last eleven months has been the comparison of the program output and selected experimental and numerical benchmark data. This has been accomplished by refining the grid generation and boundary-condition procedures through several parametric studies. The final form of the boundary conditions selected for the application will be presented in the final document. In this process of program tuning, many literature items have been perused. These include articles and reports associated with the numerical solution and boundary condition treatment of cascade flows, dynamic grid issues, experimental reports, and ideal-flow numerical methods based on potential flow, which are used for comparison with code output [60, 61, 45, 62, 63, 43, 44, 45, 46, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76].

3.1.1 Cascade Nomenclature and Numerical Results

A cascade can be defined as an infinite row of equidistant similar bodies [77]. Such bodies or blades are usually shaped as airfoils for the purpose of diverting a stream of fluid. The most common use of cascades is in the design and analysis of turbomachinery. This has been done by modeling the rotating blades as two-dimensional sections and studyi. ; the flow pattern in the simplified view. Compressors and turbines alike bear in their designs elements of correlations obtained from cascade experimental data. In fact, the so called meanline design of a turbomachine is restricted to the cascade plane [78, 41, 79, 80].

The basic nomenclature and parameters of a compression cascade are illustrated in Figure B.7. Regarding an individual blade row, such as that shown in the figure, the subscripts 1 and 2 correspond to inlet and exit, respectively. The stacking line or cascade axis is the direction perpendicular to the axial direction, x. The flow angle measured from the axial direction is β, and, when measured off the chord line of the blade, the angle is just the traditional angle of attack, α. In the figure, only α_1 and β_1, which correspond to the inlet section, are shown; the downstream angles are defined in a similar manner. The blades, of chord c, are spaced equally along the cascade axis by the distance s, called the blade spacing. The ratio $\sigma = c/s$ is the solidity of the cascade; it controls the efficiency of the flow turning in the cascade.

The airfoil chord makes an angle with respect to the axial direction. This angle is called the stagger angle, γ and plays an important role in moving cascades, since its value dictates how fast the blades can move in such a way that the α_1 that the blade sees (e.g relative angle of attack) allows adequate diffusion without separation. Note that, for the non-moving cascade, $\gamma = \beta_1 - \alpha_1 = \beta_2 - \alpha_2$.

The performance of a cascade is usually correlated in various forms; a typical representation is by the amount of flow deviation across the cascade. To this end, the differences between the flow angles β and the blade metal angles, which are measured between the axial line and the tangent to the blade camber line at the leading and trailing edges, are introduced. The incidence angle, ι, is defined as the difference between the inlet flow angle β_1 and the leading-edge metal angle, and the deviation angle, δ, is the corresponding angle difference in the exit plane or trailing edge. Just as for the isolated airfoil, if the metal angle lines are extrapolated as in the top blade of the figure, the angle included within these lines is the camber angle, φ, which, to first order, controls the flow turning of the airfoil.

A representative computational H-type grid consisting of various blocks or zones employed by the multiblock flow solver is shown in Figure B.8. This particular grid is the one employed in the rotor simulations described below. When geometry does not change from blade to blade, one can stack up the computed grid for one zone as many times as required by the number of blades. The solution to true two–dimensional stationary cascade flows without geometric differences from blade to blade can be obtained by just solving the flow in one zone, since the flow repeats itself every zone. This periodicity condition can be generalized for cases in which geometric differences are present, such as in the case of calculations involving mismatched cascades or dynamic pitch solutions. An example of a dynamic pitching solution is discussed below.

The cascade description in the previous paragraphs has been introduced to facilitate the interpretation of the results obtained with MFOIL for the solution of steady and unsteady cascade flows. A representative steady–state solution is shown in Figure B.9, which compares experimental and numerically computed loads on a NACA 65-410 profile cascade at zero angle of attack ($\alpha_1 = 0$), 30 degrees flow off the axial direction ($\beta_1 = 30°$), cascade solidity, σ, of 1.25, free–stream Mach number $M_\infty = 0.085$, and Reynolds number $Re = 245,000$. In these results, loading is reported in terms of total–to–static pressure difference,

$$S_{surface} = \frac{p_0 - p_{surface}}{\frac{1}{2}\rho_\infty V_\infty^2}$$

where p_0 is the stagnation pressure of the upstream flow. For incompressible flow, S is simply $1 - C_p$, where C_p is the standard static–to–static pressure difference coefficient. Otherwise, the relation

$$S + C_p = \frac{(1 + \frac{\gamma-1}{2}M_\infty^2)^{\gamma/(\gamma-1)} - 1}{\frac{\gamma}{2}M_\infty^2}$$

which approaches 1 in the limit as M_∞ goes to zero, holds in general for compressible flows. The experimental data comes from the NACA Report no. 1368 [76]. Other of the many cases listed in this report compared well with the output of the program. Minor discrepancies can be attributed to grid resolution near the surface and perhaps an unaccounted small bending of the blades under load in the experiment.

Figure B.10 shows a comparison of the amplitude, $|C_{p_s}|$ and phase relative to input of the lifting pressure coefficient $C_{p_s} = C_{p_{upper}} - C_{p_{lower}}$ of a flat plate cascade in which the blades undergo simple-harmonic pitching about their leading edges. The phase angle between the motion of neighboring blades, otherwise known as the interblade phase angle (IBPA), is zero in this case, as well as the mean angle of attack and blade stagger angle, $\gamma = \beta_1 - \alpha_1$. The free stream Mach number is 0.5 and the pitch angle amplitude, α_p is 0.01 degrees. The reduced frequency of oscillation based on semichord is $k = \omega c/(2V_\infty) = 0.5$, where ω, c and V_∞ are the circular oscillation frequency, blade chord length, and free-stream speed, respectively. The mean solidity of the cascade is $\sigma = 0.4$. The MFOIL results are obtained by suppressing the calculation of the viscous terms in the equations. The deforming grid technique employed in these calculations is a modified version of that presented by Huff in [70]. In the MFOIL implementation, the region up to three grid points away from the solid surfaces executes the full pitching motion, while the rest of the grid performs a weighted motion which becomes zero near the middle of the cascade channel and ten to fifteen points away from the leading and trailing edges. The results are compared with the output of LINSUB, a numerical program written by Whitehead for the calculation of linearized unsteady subsonic flows in flat plate cascades [75]. The nonlinearity of the Euler equations can be appreciated, as the output of MFOIL varies considerably more than that of LINSUB over the chord length. Overall, the code does predict the expected linear behavior quite adequately. This set of results gives confidence in the code for time accurate calculations involving grid motions.

Additional results have been obtained for the simulation of relative and absolute flows in a realistic rotor blade environment to demonstrate the ability of the code in computing a compressor characteristic. These calculations have been performed based on an approximation to the rotor blade shape of a small single stage axial fan manufactured by Eastern Air Devices in Dover, New Hampshire. These compressors have been extensively used on the Axial Compressor Rig at LICCHUS for the identification and control of stall phenomena. In fact, the experimental steady state and transient data presented in Chapter 2 comes from these compressors, operating in a rig very similar in appearance to that shown in Figure B.1. As the rotor of the device can be removed for inspection and service, a scan of the blade cross section reveals that it closely resembles a NACA 0012 thickness profile cambered with a NACA $a = 1.0$ meanline with zero-angle lift of $C_{l_o} = 1.5$ [81]. However, the thickness distribution employed in the numerical simulation is a fit to the scanned data, rather than the 0012 profile. The stagger angle at midspan can be estimated at $\gamma = 44.46°$ and the solidity is $\sigma = 1.465$.

The relative flow calculation method involves the conversion of inertial quantities, such as flow coefficient and wheel speed into corresponding angle of attack and inflow speed relative to the presummed moving blades. In these simulations, the wheel Mach number was held at $M_T = U/a_\infty = 0.1576$, which corresponds to a rotor speed $U = 54\ m/s$ at midspan or a frequency of 10500 RPM. For each value of tabulated inertial flow Mach numbers a table of corresponding relative Mach numbers and angles of attack was obtained for the selected rotor speed. The selection of Mach numbers involved some trial and error in order to demonstrate the interesting region of rising compressor performance near the peak of the characteristic. In its final form, the Mach number table involves samples of 0.025 in axial Mach number. The results are shown in Figure B.11. The basic quantities shown in the figure are flow coefficient, Φ and static-to-total pressure rise, Ψ, defined by

$$\Phi = \frac{u_\infty}{U}, \quad \Psi = \frac{p_2 - p_0}{\rho_\infty U^2}$$

The velocity u_∞ is the inertial flow velocity, which is assumed to be the only component of velocity far upstream. The outlet pressure p_2 is computed by averaging over the exit plane. The inlet total pressure is constant along the entire inflow plane. This is achieved by employing a set of boundary conditions suggested by Chima [45]. The curve with the "x" markers corresponds to the computed characteristic. Note that the trends of every basic compressor map are well predicted. As the axial flow decreases, the pressure rise increases up to the point of stall inception. Further decrease in axial flow beyond the peak causes the pressure to drop markedly, and consequently, to a loss in performance due to stall. Since this simulations involved only the rotor blade row, a first principles continuity argument shows that an isolated rotor will be able generate more flow than in the stage configuration with an accompanying stator downstream. To account for the smaller flow in the stage configuration, the flow coordinate is scaled by a factor $1/(\sigma_s \cos\gamma_s)$, where σ_s and γ_s are the solidity and stagger angle of the stator, respectively, which, for the device under consideration have approximate values $\sigma_s = 1.52$, $\gamma_s = 28.6°$. This x-scaled curve is shown in the figure with open circle markers. Near the peak of this curve, an overlay of the experimentally determined map for this compressor in rig operation is shown in dashed lines, with the maximum measured pressure rise point shown as a filled circle. The comparison shows that the code is indeed able to provide correct quantitative behaviors for the flow through a compressor rotor in steady state operation.

In order to explore further the validity of the results, an inertial calculation involving the displacement of the rotor blades in time along the negative cascade axis has been carried out for one of the cases computed in the relative frame simulations. The case compared consists of the rotor with a free-stream Mach number $M_\infty = 0.08274$, which, for the wheel speed $M_T = 0.1576$, corresponds to a flow coefficient $\Phi = 0.525$. A comparison of computed relative and moving frame surface pressure distributions near steady state is shown in Figure B.12. According to an inviscid calculation, convergence to time derivatives to the order 10^{-5} takes well over 32 units of normalized time, if the starting solution is chosen as the free stream condition everywhere in the domain. The results shown are obtained after integration over 33 units of time, equivalent to a rotor displacement of 7.7 units of pitch in the tangential direction, or 15000 time steps of calculation at the selected time sampling rate. As can be seen, pressures computed both in the moving grid calculation and the relative frame simulation are identical. Further postprocessing reveals that the pressure and density scalar fields are the same, in fact, to 9 decimal places in the normalized variables in the entire computational domain. When the velocities computed in the relative frame simulation are transformed into absolute velocities for proper comparison, the differences in computed velocities in both cases are of similar order of magnitude. In addition, the pressure rise Ψ computed with the results obtained from both calculations are identical, equal to about 0.1325 as shown in Figure B.11. Overall, the moving rotor calculation and the calculation in the frame relative to the rotor predict the same behavior in time.

3.2 Outline of Work to Follow

The numerical results presented in the previous sections demonstrate the capabilities of MFOIL in computing cascade flows. Not documented here is the ability of the code to compute separated and reversed boundary layer flows, as encountered in the low end flow of the computed compressor map in Figure B.11. These results will be presented in the final thesis document. The primary goal of demonstrating the feasibility analyzing rotating stall phenomena through MFOIL will be approached by parts. It has been argued that one of the possible causes of rotating stall in the absence of any upstream and downstream nonaxisymmetric perturbations is due to geometric mismatches from blade to blade. Hence, an apparently simple way of forcing the occurrence of stall is to incorporate geometric variations from blade to blade in the grid generation, for example, by mismatching one of the blades of a rotor to have a stagger angle larger than that of the other blades. Most often, however, inflow perturbations caused by aircraft attitude changes comprise a more serious cause of stall. To capture these, it is necessary to incorporate a logic to introduce defects at the boundaries, for example gusts or axial flow defects. Both of these methods will be worked out for a stationary cascade first, then generalized to the rotor motion case. Work is under progress in this area.

Once the the occurrence of rotating stall is established, the issue of comparing simulation outputs with simplified model outputs will be undertaken. To that end, is is necessary to establish a set of rules to perform interpolations from the moving grid solutions back to the inertial frame. This is necessary in order to obtain a diagnosis of the stall event in terms of averages and perturbations from average over the tangential direction, an approach commonly followed in experiments. In particular, rotor/flow interactions and the existence of finite domains of attraction of high-load operating conditions will be investigated. An attempt to implement the control approach presented in Chapter 2 in the numerical code will be made at a later time to assess the effectiveness of the control strategy and to point out the feasibility of improving modeling through CFD simulations. This part of the work is expected to take up the second part of the year and into the year 1997.

The tentative milestone table set up for the completion of this work looks as follows

Goal	**Completion Time, Dates**
Interpolation/Boundary Perturbation Coding	2 Months, May-June 96
Blade Row Dynamic Stall Analysis	4 Months, May-Aug 96
Blade Row Stall Control Analysis	6 Months, Aug 96 - Jan 97
Thesis Preparation and Defense	Dec 96 - Feb 97

The bulk of the remaining computational work will be performed on the computational facilities available at the Georgia Tech Campus. It is very likely that more computing power will be needed for a full five or seven blade rotor similar to those found in the LICCHUS lab. To that end, a proposal for request of computer time will be submitted to the Georgia Tech High-Performance Computing Group.

APPENDIX A

Moore–Greitzer Model

The Moore–Greitzer model is based on considerations of spatial and temporal flow scales. The flow asymmetry in rotating stall, which has a length scale of the order of the compressor diameter, will change over a time scale which is long compared to the response time of a blade passage (convection time). Moreover, the time scales in surge are even slower than those in rotating stall, as the length along a typical ducted compressor at least one to two diameters. This argument allows the development of a model in which accurate modeling of blade flow unsteadiness is arguably unnecessary, given that such processes are fast compared to the problem under consideration. These issues are discussed in more detail in [21].

The plants modeled by the mathematical formulation are externally–driven compression systems, such as that depicted in Figure B.1. A compressor operates in a duct and discharges into a plenum of dimensions larger than those of the compressor and its ducts. Forces in the plenum are only attributed to pressure, which is uniform in such volume. The flow through the compression system is controlled by a throttle at the exit of the plenum. It is assumed that the overall pressure rise in the system is not large compared to ambient pressure and that the air in the ducts is incompressible. The gas in the plenum, however, is considered compressible, since, as shown by Greitzer [7] it acts as a gas spring. The flow upstream of the compressor is assumed irrotational. In addition, the compressor and ducts are assumed to have a large hub–to–tip radius ratio, which allows the use of two–dimensional fluid flow equations, as gradients of properties in the radial direction, are negligible. Coriolis forces do not affect this model since they only enter in the radial momentum balance.

In the rest of this discussion the axial and circumferential coordinates are normalized by a representative mean radius, R, velocities are normalized by the wheel speed U and time is normalized by the time taken for a radian of wheel travel R/U. Normalized axial and circumferential dimensions are denoted by η and θ, respectively. Normalized time is given by τ. Pressures are normalized by ρU^2.

The most important issue pertinent to this modeling concept is the blade row performance model. The performance of a blade row is correlated to the quasi-steady performance, in addition to an inertial term

$$F_{row}(\phi_0) = F_{row,ss}(\phi_0) - \lambda_{row}\frac{d\phi_0}{d\tau} \ . \tag{A.1}$$

Here, $\phi_0 = u/U$ is the axial flow coefficient through the compressor, $F_{row} = (\Delta p)/(\rho U^2)$ is the pressure rise for each row, and the time derivative is taken in the frame of reference of the blades. Aside from this assumption, the model develops from distributed fluid-dynamic first principles in various levels of simplification. As argued in [11], a reasonable value of the blade row lag λ_{row} can be obtained by modeling the blade passages as parallel ducts at the mean stagger angle, in which case it becomes a measure of the convection time through the blade row.

The model is developed by writing out a momentum balance across the compression system. The pressure rise produced by the compressor and its ducts must be balanced by the throttle pressure drop, as it is assumed that the throttle discharges into a reservoir with the same total pressure as that in the inlet. First, one can proceed to assemble the compressor/duct pressure rise as follows. As the upstream flow is assumed irrotational, the unsteady Bernoulli equation is used to find the static-to-total pressure rise in the inlet duct. The velocity potential,

$$W(\tau,\eta,\theta) = \Phi(\tau)\eta + \varphi(\tau,\eta,\theta)$$

and its disturbance, $\varphi(\tau,\eta,\theta)$ satisfy Laplace's equation in the inlet, subject to a periodic boundary condition in θ, and a uniform axial flow condition at inlet ($\eta = -l_i$). The equation for the disturbance potential can be solved by simple separation of variables, with a boundary condition describing the axial velocity in terms of a Fourier series with an unknown time dependence of the form

$$\left.\frac{\partial\varphi}{\partial\eta}\right|_{\tau,\eta=0,\theta} = \sum_{n=1}^{\infty} a_n(\tau)\sin n\theta + b_n(\tau)\cos n\theta \ .$$

The compressor static pressure rise follows by adding Equation A.1 over all stages of the compressor, accounting for the inertial time derivatives in the rotor frame of reference, which amounts to adding a flow derivative in the tangential direction.

In the exit duct, the Poisson equation for the pressure is simplified to a Laplace equation, under the assumption that the quadratic terms in the velocity disturbance derivatives are small. Specifically, the Laplace equation for the quantity $\tilde{p}_p - \tilde{p}$ is developed, where $\tilde{p}_p$ is the plenum pressure. This equation is solved with a periodic boundary condition in angle, a vanishing boundary condition at discharge ($\eta = l_e$), and matched to the upstream solution by postulating that the instantaneous axial flow at the compressor exit is the same as that at the inlet. Employing the axial momentum equation and the matching assumption yields a solution which, evaluated at $\eta = l_c$, produces the static pressure rise in the exit duct.

Assembling these pressure contributions, one obtains an equation for the instantaneous total-to-static pressure rise across the compressor and its ducts

$$\Psi(\tau) = \psi_c(\phi_0) - \left[\frac{\partial\varphi}{\partial\tau} + \frac{1-K_I}{2}\left(\frac{\partial\varphi}{\partial\theta}\right)^2 + \mu\frac{\partial^2\varphi}{\partial\eta\partial\tau} + \lambda\frac{\partial^2\varphi}{\partial\eta\partial\theta}\right]_{\eta=0} + P_\varphi(\tau, l_c, \theta) - L_c\frac{d\Phi}{d\tau} \quad , \quad \text{(A.2)}$$

where

$$\psi_c(\phi_0) = \psi_c\left(\Phi + \left(\frac{\partial\varphi}{\partial\eta}\right)_{\tau,\eta=0,\theta}\right)$$

arises from the sum of the stage quasi–steady forcing functions and P_φ is part of the solution to the exit duct pressure equation. The parameters μ, λ, and L_c also accumulate in the sum of the three pressure contributions. The factor K_I arises from a model for inlet guide vane pressure loss. When $K_I = 1$, the vanes convert all the tangential kinetic energy into pressure rise. If a loss occurs in the vanes, then $K_I < 1$. The case of a compressor with no inlet vanes corresponds to $K_I = 0$. The important fact regarding this mathematical statement is that only the temporal dependencies are left as unknowns in the equation, as the spatial dependencies have been solved out.

The plenum, which eliminates spatial nonuniformities due to its size, receives mass at the compressor flow rate and discharges at the throttle flow rate. It is assumed that the throttle discharges into a state having the same total pressure as the system inlet. Since compressor and throttle flows are not necessarily equal in a dynamic case, the continuity equation is cast as an equation for the rate of change of pressure in the plenum in time, by assuming a local isentropic condition in the plenum. This gives

$$\frac{d\Psi}{d\tau} = \frac{1}{4B^2L_c}\left(\Phi(\tau) - \Phi_T(\tau)\right) \quad ,$$

where Φ_T is the (normalized) throttle mass flow and B is Greitzer's "B" parameter [7]

$$B = \frac{U}{2a_p}\sqrt{\frac{V_p}{A_cL_cR}} \quad .$$

In the throttle, the instantaneous pressure drop is given as a function of the flow inertia, as could be argued from the quasi–one dimensional momentum equation. Since the throttle discharges to a reservoir of stagnant air at total pressure $\tilde{p}_T$, equal to that at the inlet, then the throttle pressure drop must equal the pressure rise produced by the compressor and inlet/exit ducts. In addition, if the length of the throttle duct is assumed small, then the dynamic throttle equation can be residualized to

$$\Phi_T = F_T^{-1}(\Psi) = K_T\sqrt{\Psi} \ ,$$

where $K_T = \sqrt{2}A_T/A_c$ is the throttle area ratio parameter. The function F_T^{-1}, when inverted, gives the throttle pressure drop as a function of the throttle flow. The expression above in terms of the area ratio results from the ideal case obtained from the discretization of the momentum equation; thus it does not include non-ideal behaviors, such as orifice loss factors and the like. It is entirely possible to analyze the model with a general function F_T satisfying the physical constraints and, in fact, this is done in [40]. The ideal result is then inserted in the plenum pressure equation to yield

$$\frac{d\Psi}{d\tau} = \frac{1}{4B^2L_c}\left(\Phi(\tau) - K_T\sqrt{\Psi(\tau)}\right) \ . \tag{A.3}$$

The Moore–Greitzer model thus consists of Equations A.2 and A.3. As seen, the distributed influence is still present, as Equation A.2 is a partial differential equation involving derivatives of the boundary values of the velocity potential. The method adopted for the development of reduced order models is the Galerkin method [82]. For this problem, such procedure is relatively simple, as the solution to the potential flow problems has been formulated in terms of Fourier series. To carry out this procedures, the infinite sums are truncated down to N terms. Then, the time evolution of the mean flow Φ is found by computing the average of Equation A.2. The time dependence of the Fourier sine and cosine coefficients of the flow disturbance are found by computing the moments of Equation A.2 with the circular functions $\{\sin k\theta, \cos k\theta\}_{k=1,\dots N}$. The resulting system of $(2N+2)$ equations is

$$\begin{aligned}
\frac{d\Phi}{d\tau} &= \frac{1}{L_c}\left[\frac{1}{2\pi}\int_0^{2\pi}\psi_c(\phi_0)\,d\theta - \frac{1}{2\pi}\int_0^{2\pi}\frac{1-K_I}{2}\left(\frac{\partial\varphi}{\partial\theta}\right)^2_{\eta=0} d\theta - \Psi\right] && \text{(A.4)}\\
\frac{da_k}{d\tau} &= \frac{2k}{k\mu+m_k}\left[\frac{\lambda}{2}b_k + \frac{1}{2\pi}\int_0^{2\pi}\psi_c(\phi_0)\sin k\theta\,d\theta - \frac{1}{2\pi}\int_0^{2\pi}\frac{1-K_I}{2}\left(\frac{\partial\varphi}{\partial\theta}\right)^2_{\eta=0}\sin k\theta\,d\theta\right] \ , \ k=1,\dots,N\\
\frac{db_k}{d\tau} &= \frac{2k}{k\mu+m_k}\left[-\frac{\lambda}{2}a_k + \frac{1}{2\pi}\int_0^{2\pi}\psi_c(\phi_0)\cos k\theta\,d\theta - \frac{1}{2\pi}\int_0^{2\pi}\frac{1-K_I}{2}\left(\frac{\partial\varphi}{\partial\theta}\right)^2_{\eta=0}\cos k\theta\,d\theta\right] \ , \ k=1,\dots,N\\
\frac{d\Psi}{d\tau} &= \frac{1}{4B^2L_c}\left(\Phi - K_T\sqrt{\Psi}\right)
\end{aligned}$$

where

$$m_k = \tanh[k(l_e - l_c)] + \tanh(kl_i) \ .$$

Axisymmetric Polynomial Model and Single Harmonic Truncation The equations presented above attain a simple, yet insightful, structure when the axisymmetric map $\psi_c(\cdot)$ is assumed a polynomial function of its argument and only the first harmonic is retained in the series. In that case, one has

$$\begin{aligned}
\frac{d\Phi}{d\tau} &= \frac{1}{L_c}\left[\sum_{n=1,n\ even}^{N_p} \frac{\psi_c^{(n)}(\Phi)}{n!\ s_n}(a_1^2+b_1^2)^{n/2} - \frac{1-K_I}{4}(\tanh l_i)^2(a_1^2+b_1^2) - \Psi\right] \qquad (A.5)\\
\frac{da_1}{d\tau} &= \frac{2}{\mu+m_1}\left[\frac{\lambda}{2}b_1 + a_1\left(\sum_{n=1,n\ odd}^{N_p}\frac{\psi_c^{(n)}(\Phi)}{n!\ r_n}(a_1^2+b_1^2)^{\frac{n-1}{2}}\right)\right]\\
\frac{db_1}{d\tau} &= \frac{2}{\mu+m_1}\left[-\frac{\lambda}{2}a_1 + b_1\left(\sum_{n=1,n\ odd}^{N_p}\frac{\psi_c^{(n)}(\Phi)}{n!\ r_n}(a_1^2+b_1^2)^{\frac{n-1}{2}}\right)\right]\\
\frac{d\Psi}{d\tau} &= \frac{1}{4B^2L_c}\left(\Phi - K_T\sqrt{\Psi}\right)
\end{aligned}$$

where $\psi_c^{(n)}(\Phi)$ represents the n-th derivative of the axisymmetric map evaluated at Φ, and

$$r_n = \frac{\prod_{k=0}^{(n-1)/2}(n-2k+1)}{\prod_{k=0}^{(n-1)/2}(n-2k)}\ ,n\ \text{odd}\ ;\quad s_n = \frac{\prod_{k=0}^{(n-2)/2}(n-2k)}{\prod_{k=0}^{(n-2)/2}(n-2k-1)}\ ,n\ \text{even}\ .$$

The appearance of powers of the quadratic term $(a_1^2+b_1^2)$ in the equations suggests a transformation of coordinates in these states to polar coordinates

$$a_1 = A_1\cos r_1\ ,\quad b_1 = -A_1\sin r_1\ .$$

The resulting dynamic equation for r_1 is $dr_1/d\tau = \lambda/(\mu+m_1)$, which represents the frequency of oscillation of the rotating flow disturbance. Its solution can be obtained independently of the remaining equations in the model, since, according to the transformation, the instantaneous value of r_1 does not affect the remaining equations. In this case, the system is then reduced to three equations for the flow disturbance amplitude $A_1 = \sqrt{a_1^2+b_1^2}$, and mean flow and pressure. These are given by

$$\begin{aligned}
\frac{d\Phi}{d\tau} &= \frac{1}{L_c}\left[\sum_{n=1,n\ even}^{N_p}\frac{\psi_c^{(n)}(\Phi)}{n!\ s_n}A_1^n - \frac{1-K_I}{4}(\tanh l_i)^2A_1^2 - \Psi\right] \qquad (A.6)\\
\frac{dA_1}{d\tau} &= \frac{2}{\mu+m_1}\left[\left(\sum_{n=1,n\ odd}^{N_p}\frac{\psi_c^{(n)}(\Phi)}{n!\ r_n}A_1^n\right)\right]\\
\frac{d\Psi}{d\tau} &= \frac{1}{4B^2L_c}\left(\Phi - K_T\sqrt{\Psi}\right)
\end{aligned}$$

APPENDIX B

Figures

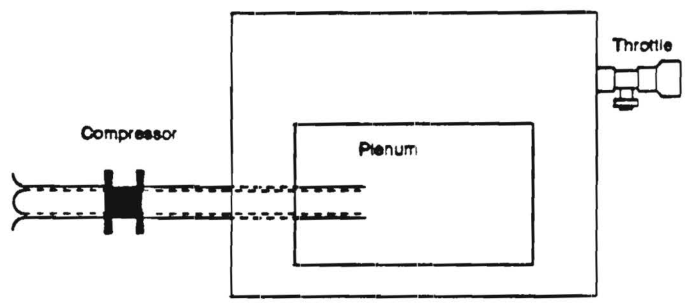

Figure B.1: Schematic of Generic Compression System

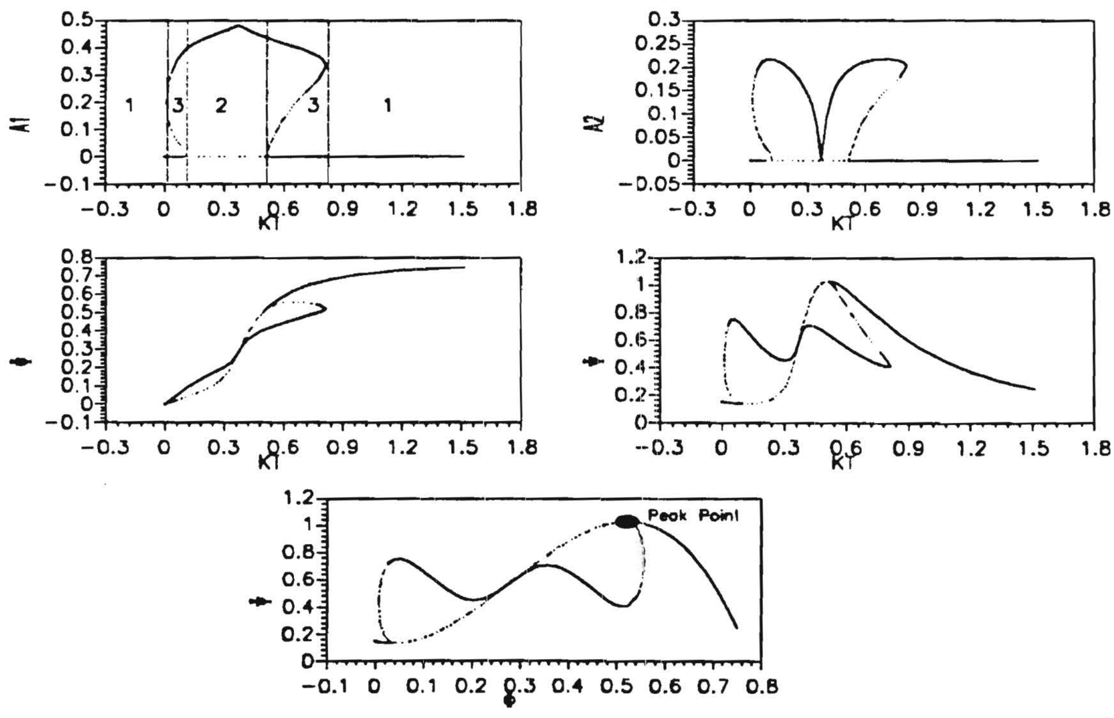

Figure B.2: Bifurcation Diagram for Two-Harmonic Truncation Model

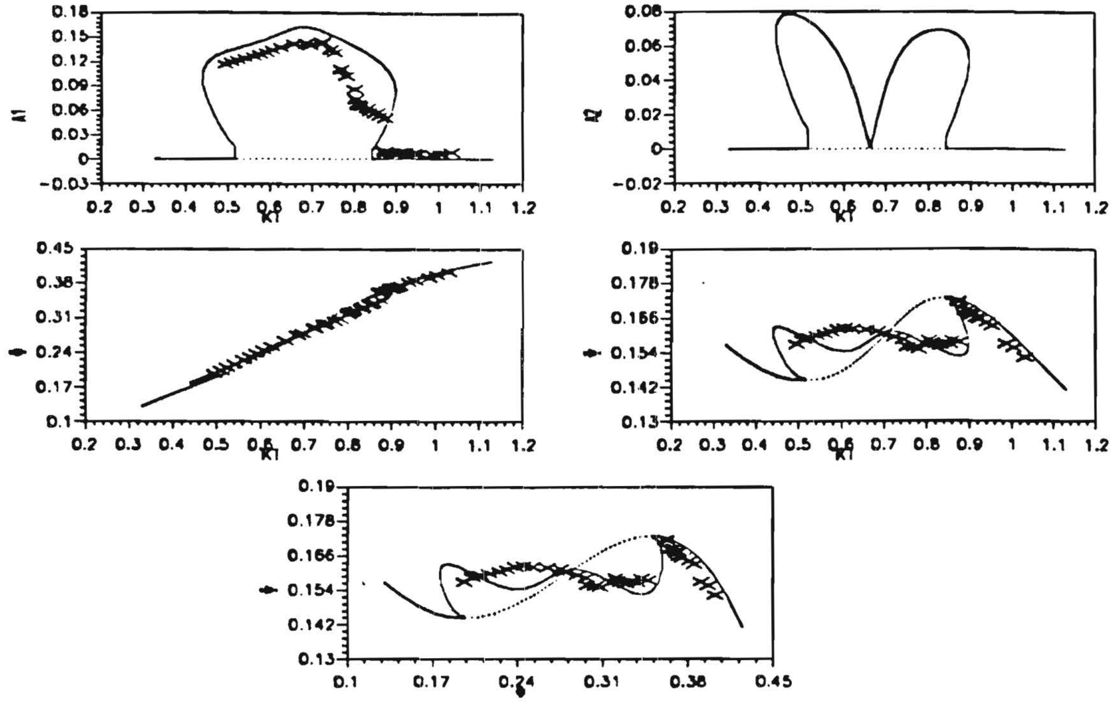

Figure B.3: Bifurcation Diagrams for Steady-State Optimal Axisymmetric Map and Comparison with Experiment (x).

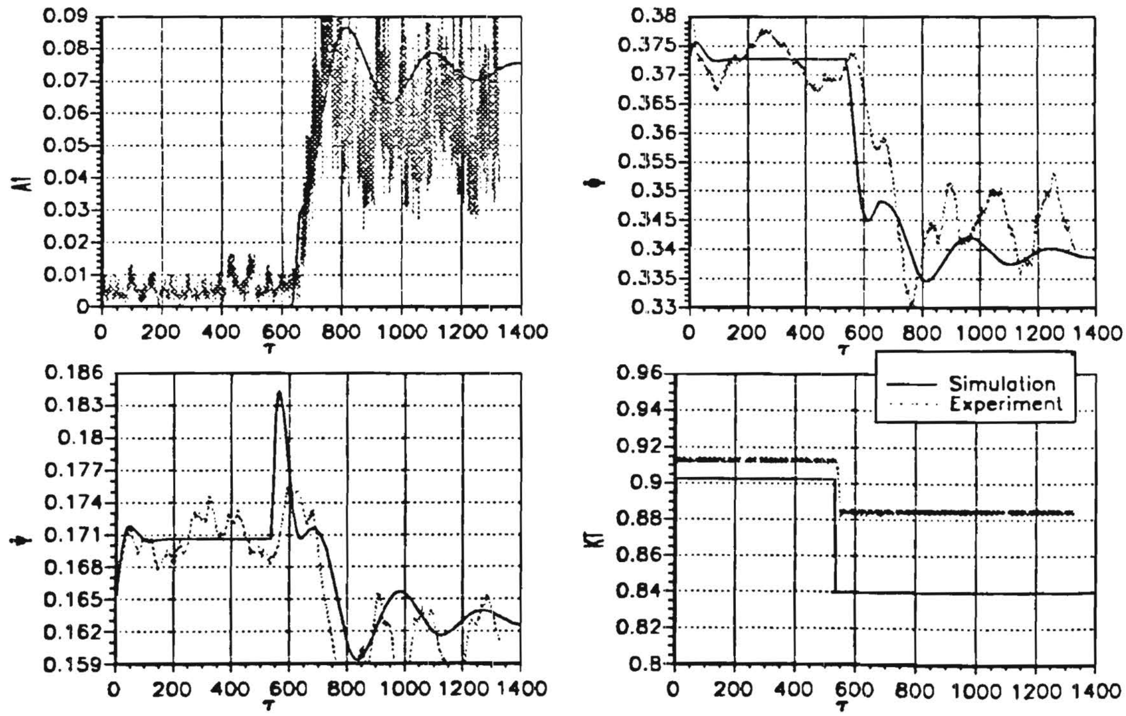

Figure B.4: Forced Transient into Stall for Steady-State Optimal Model

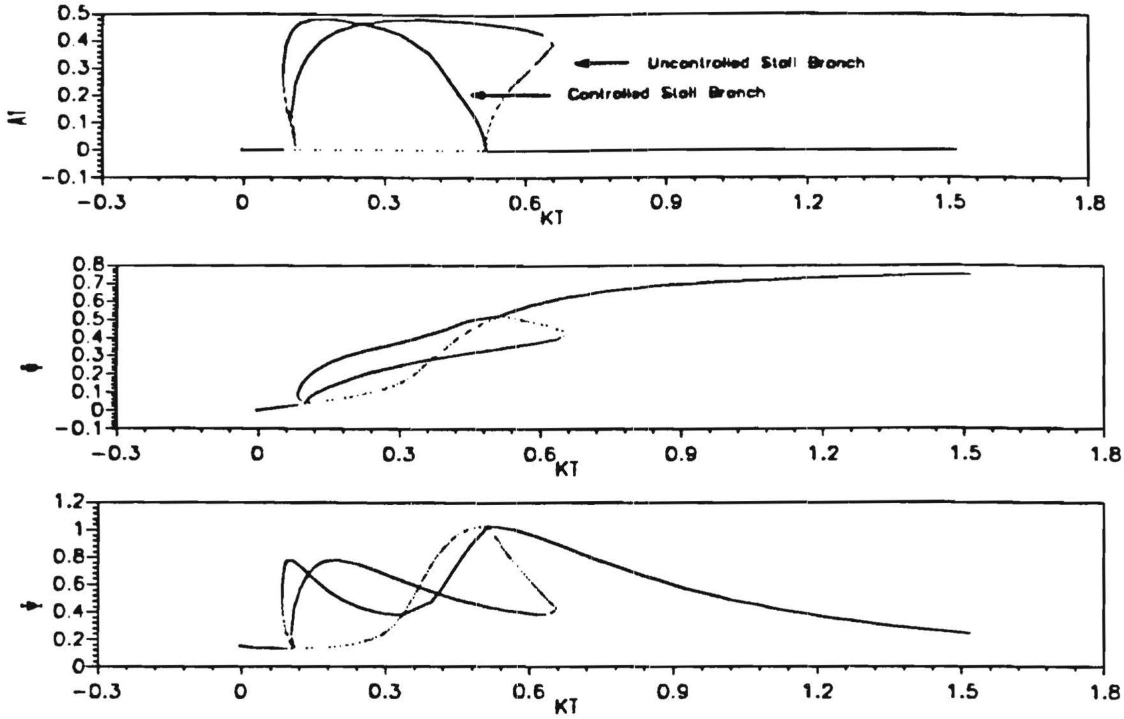

Figure B.5: Bifurcation Diagrams for Controlled System ($r = 5$) Overlaid on Uncontrolled System Diagrams.

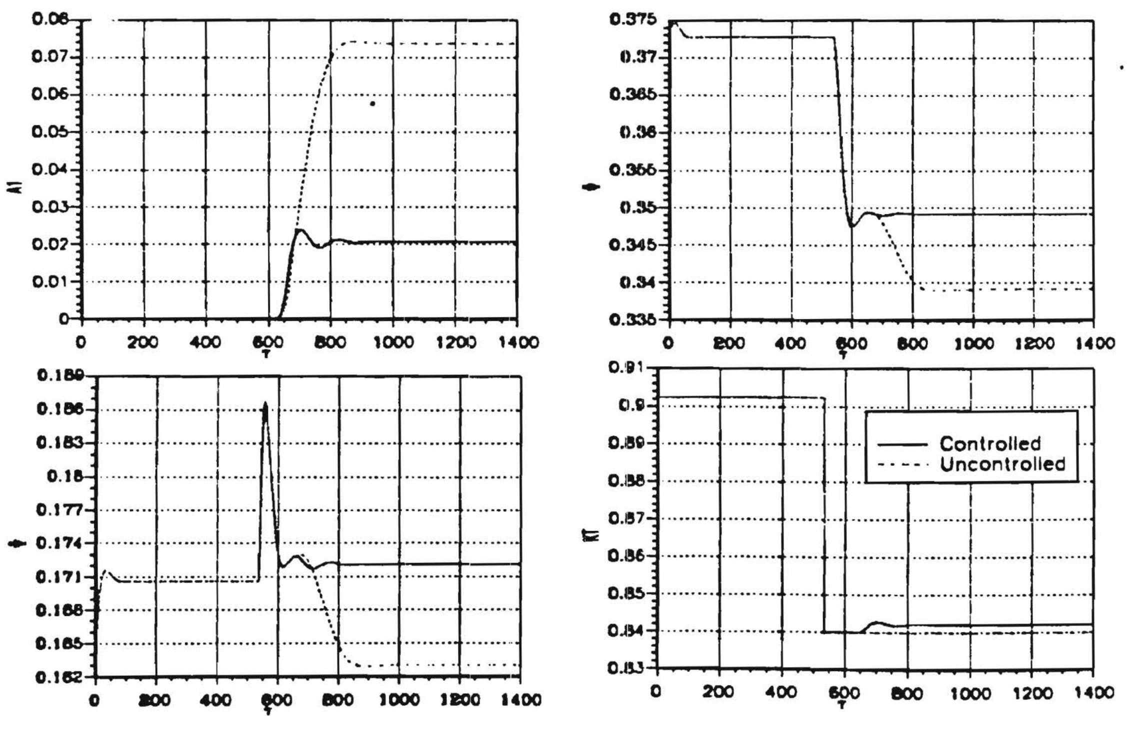

Figure B.6: Comparision of Uncontrolled and Controlled Forced Transient into Stall for Given Nominal Step Input in $K_{T_{nom}}$, $r = 5$ in Controlled Simulation.

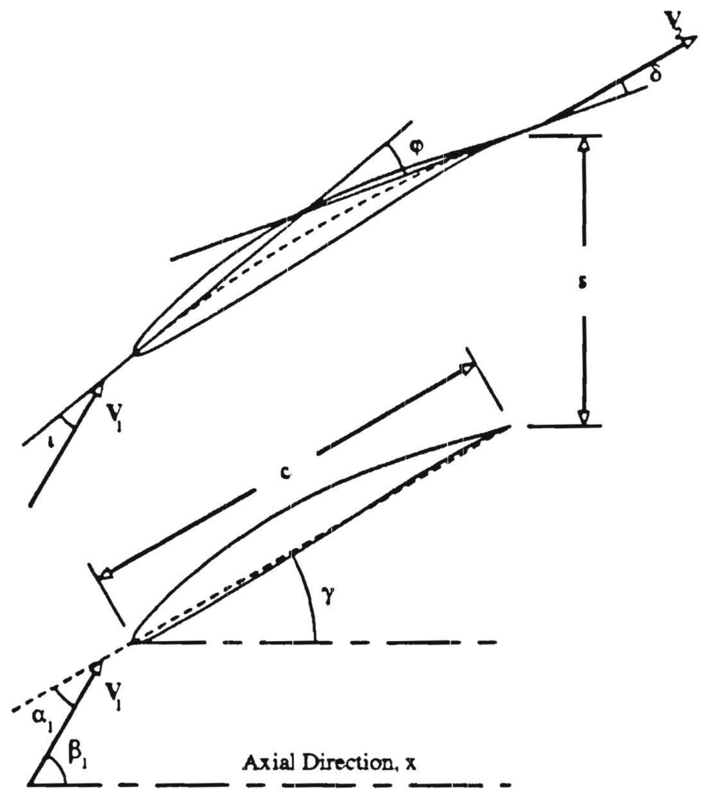

Figure B.7: Cascade Nomenclature and Parameters.

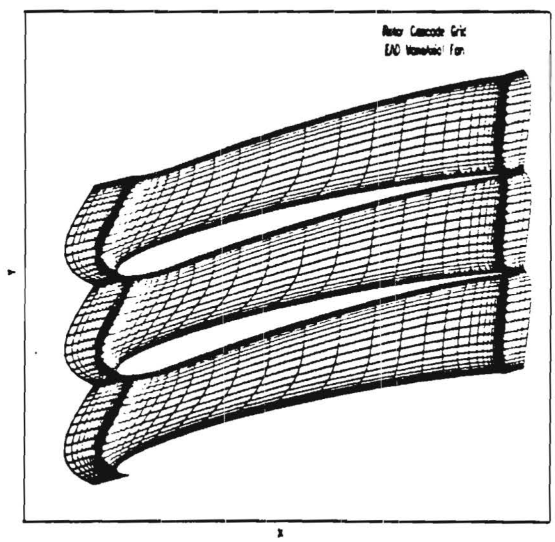

Figure B.8: Detail of Rotor Cascade Grid Near Blades, Showing Stacking Method

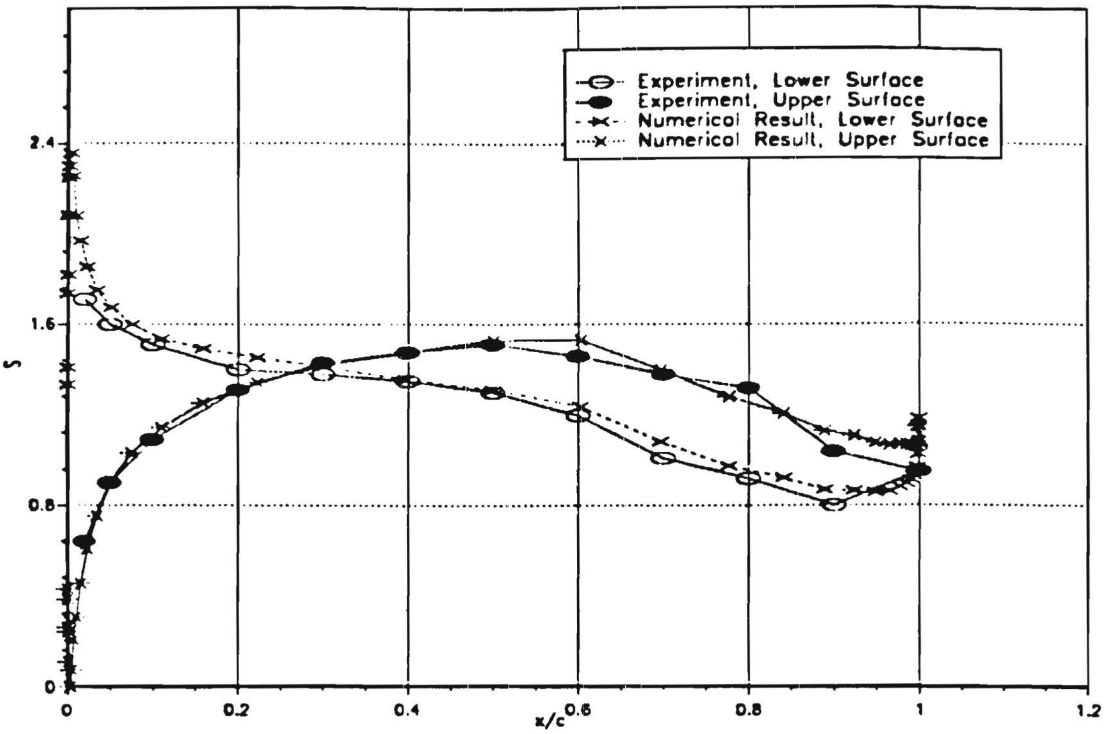

Figure B.9: Comparison of Experimental and Numerical Results for Cascade Loading on NACA 65-410 profile, $M_\infty = 0.085, Re = 245,000$, $\alpha_1 = 0$, $\beta_1 = 30^o$, $\sigma = 1.25$.

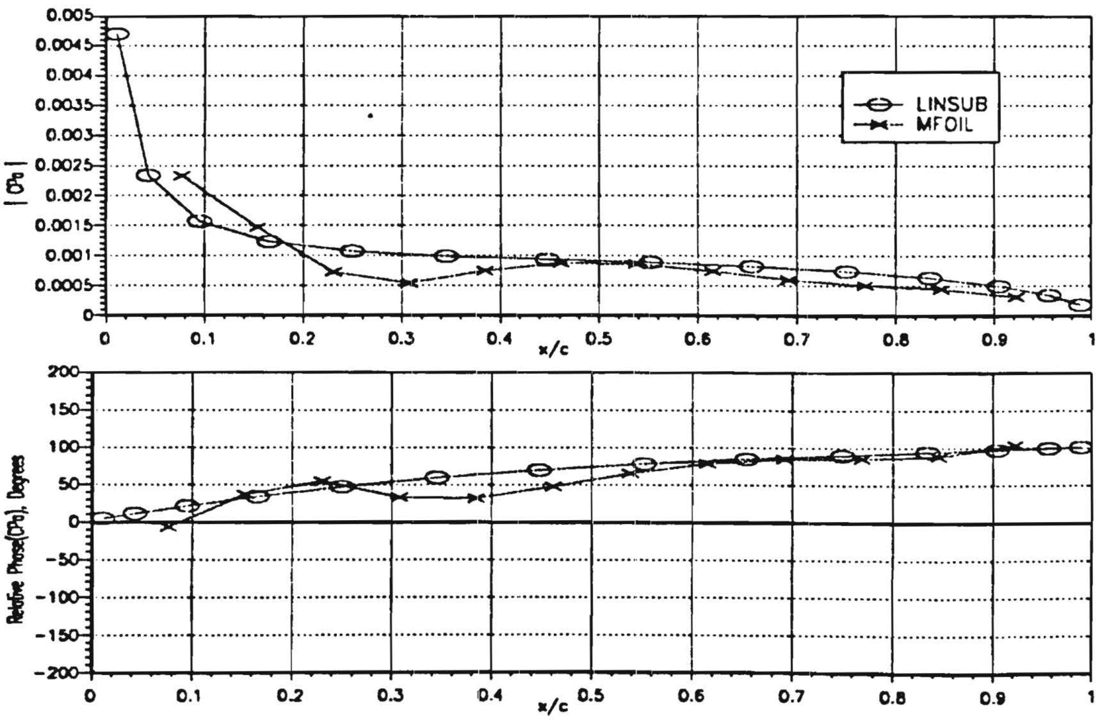

Figure B.10: Comparison of LINSUB and MFOIL Results for Lifting Pressure on Flat Plate Cascade Oscillating About Leading Edge, $k = 0.5$, $M_\infty = 0.5$, $IBPA = 0$, $\alpha_p = 0.01^o$, $\alpha_1 = \beta_1 = 0$, $\sigma = 0.4$.

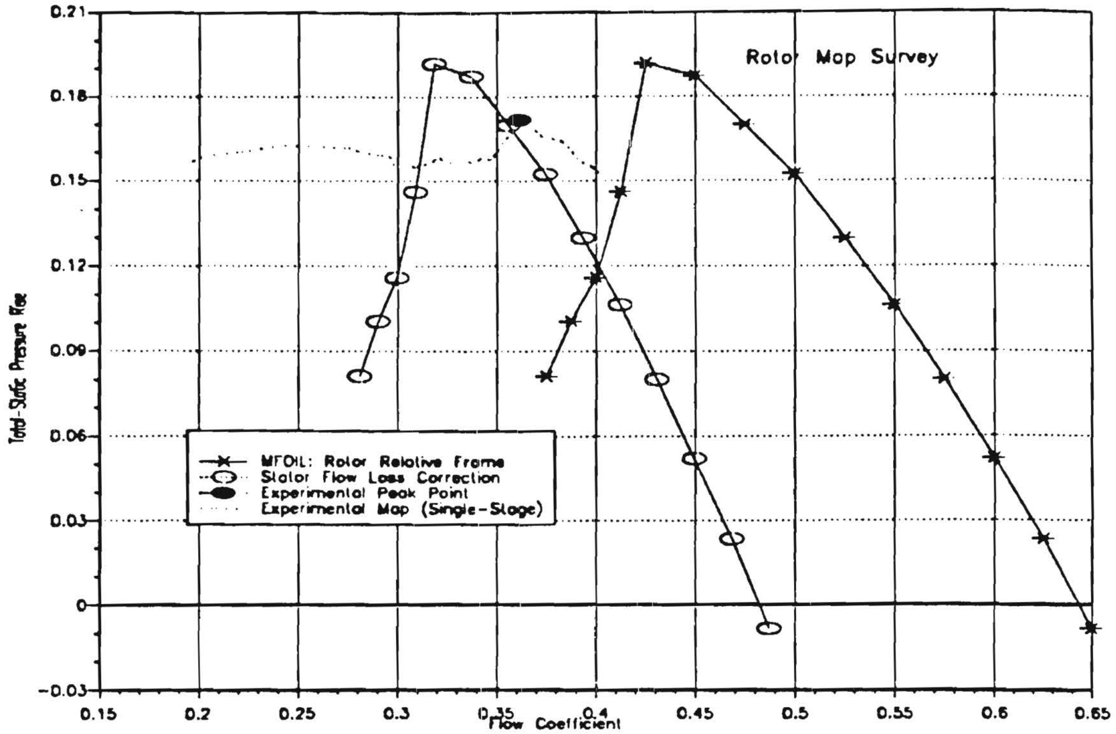

Figure B.11: Calculation of Compressor Rotor Performance Map

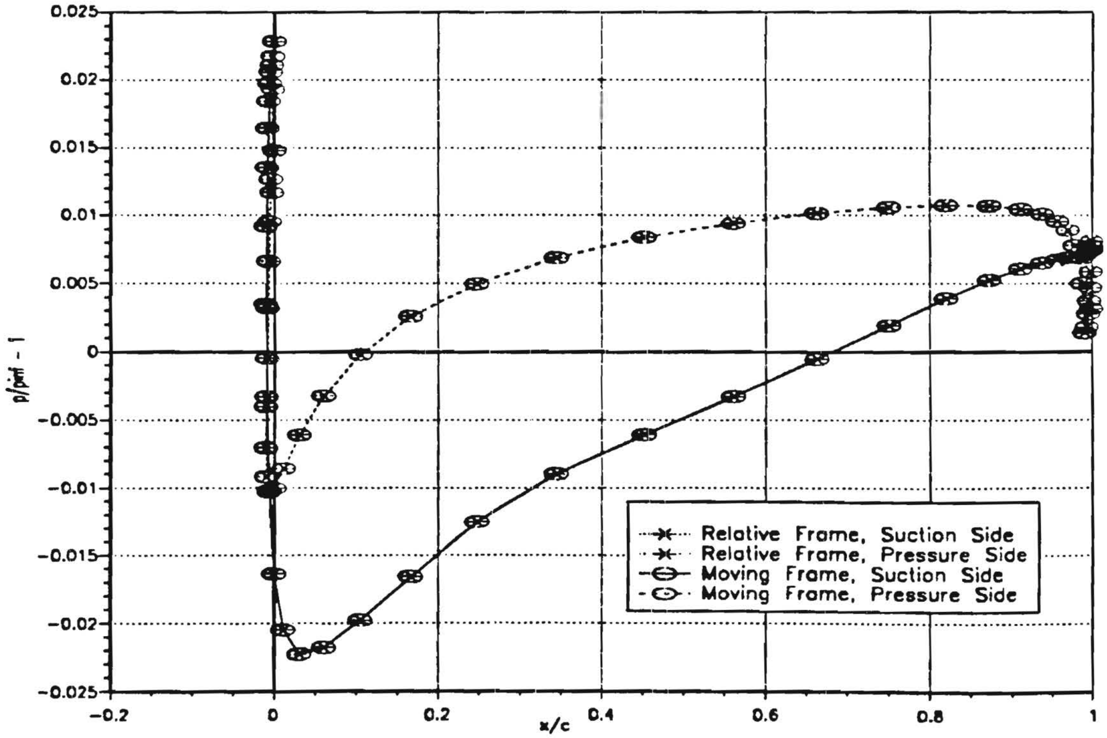

Figure B.12: Comparision of Pressure Distrubution $p/p_\infty - 1$ Over Surfaces in Relative Frame and Moving Grid Simulations Near Convergence

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