BENDING PARABOLAS: Formwork for Compression-only Structures

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Abstract: The "elastic" curves formed by a uniform buckled strut are not optimal shapes as guidework and formwork for compression-only structures. In this paper, we adapt the family of elastic curves to vaults and arches, by changing the stiffness of the strut to force it to buckle as a parabola. The approximation of elastica to parabola in a bent strip makes it useful to form-find, support, and guide the construction of vaults. Consequently, an average variation of the stiffness will form a strip that always generates parabolic arches as it moves, opens, or closes. Hence, the strip becomes a tool that always finds and describes multiple vaulted geometries that otherwise require complicated, one-use, and bulky formwork systems. The system was tested with thin-tile vaulting through building three thin-tile vaults using the bending system for simple in-situ construction.

Finding simple in-situ solutions for compression-only structures advocates local grassroots construction that seeks alternatives, not only to the way we build now, but also to the way we think about design. The production of the built environment is not always in the hands of architects and engineers; a dialogue between high-knowledge analysis and low-tech everyday construction is much needed. In this particular context, the paper proposes optimizing on-site technology through design analysis that focuses on the dialogue between material behavior and craftsmanship.

Keywords: Active bending, thin-tile vault, construction, geometry

INTRODUCTION

In both traditional and contemporary architecture, making compression-only shells requires a set of temporary structures that either support or guide the construction. Shells and the formwork with which they are built have critical reciprocity. The selection of materials and techniques has immediate implications on the type and intensity of the formwork; some structures require complete shuttering, while, for others, specific techniques can be used to mitigate shuttering. This can be observed in the way that different construction cultures respond to the abundance or scarcity of materials by adopting different formwork techniques for vaults. The abundance of wood and labor in the western areas of the Roman Empire resulted in concrete vaults with full shuttering (Lancaster 2015). The lack of timber in Mesopotamia and Egypt produced an architecture of mud blocks that relinquishes formwork altogether; it cheats gravity by changing the paths of the successive bedding so that a brick is always leaning on a previous brick (Ramírez Ponce and Ramírez Melendez 2004; Wendland 2007). In the current application of this construction, builders often resort to a hanging rope or chain fixed on both ends of a vault to help them visualize the perfect arch even in a reversed fashion. In thin-tile vault construction, a masonry construction technique of Mediterranean origin, heavy formwork is avoided with the use of fast-setting plaster of Paris and light tiles to build a first layer that serves as the formwork for more layers (Truño i Rusiñol et al. 2004; Ochsendorf 2010).

Today, we aim for construction that is conscious of material and energy consumption, as well as waste production; hence, reducing or recycling the materials used for formwork becomes critical. Many approaches to "rethinking formwork" have been suggested since the beginnings of mechanized construction. Geometry-oriented solutions offered ruled surfaces that reach complex structures by repeating a linear element. Examples of this approach can be found in the architecture of Antoni Gaudí, who used a hyper-parabola for thin-tile vaulting guidework, and Felix Candela, who used similar systems for the shuttering of concrete shells. Use-oriented solutions re-employ formwork materials for furniture or other building components, or for a stay-in-place part of the shell (Ramage et al. 2010; López López, Van Mele, and Block 2018). There are many recent technology-oriented solutions, including: the use of three-dimensional printing for making shells over curved surfaces composed of piles of gravel or soil; robotic arms that temporarily carry vault bricks; and net-cable systems with fabric that are made for concrete shells (Zivkovic and Battaglia 2018; Wu 2018; Van Mele and Block 2011).

In this paper, we return to the raw status quo of the formwork problem, that is: How can the perfect form of structural arches be captured? In other words, since the perfect form for compression-only structures is shaped, in Robert Hook's words, "as hangs the flexible line", the main objective of this paper is to flip Hook's flexible line to serve as formwork. For this purpose, we investigate the use of the elastic curve of a buckled strut as a method for a flexible formwork for compression-only shells.

1. BACKGROUND: BENDING FOR FORMWORK

If we could flip a hanging chain and keep it solid, this would be a perfect guide or formwork system for masonry or cast vaulting. However, we usually engineer such temporary structures using wood, steel, or other materials. One such system is based on elasticity: strips or surfaces are bent for vaults. Bending to create guidework in construction is not a new technique. In fact, the construction of thin-tile vaults incorporates bent elements: traditionally, reed and, recently, steel reinforcing bars (rebar). Diagonally oriented rebar is wedged between the vault's corners to mark the curvatures for sail or cross vaults.

Bending follows an elastic curve, whereas the line of thrust in compression-only structures follows a catenary, or a parabolic curve, if loaded with equally distributed loads. At moments of minimum bending, the two curves are very similar. Thus, a slightly bent steel bar aids the thin-tile vault builder, as it is close to the parabolic section. However, when bending with more acute angles, the two curves begin to diverge drastically, and the bent steel bar is no longer valid as a reference for vaulting. Figure 1 shows the different behaviors between bending steel bar and hanging a chain, where the deviation between the two is shown in relation to different angles of bending.

From mapping the behavior of the two curves, the divergence becomes noticeable when the height of the elastic curve is more than half of its span (figure 1). Traditional builders recognize this problem and have solved it by anchoring two sides of the steel rebar with strings, bringing the elastica back to a parabola. For more recent advanced fabrication techniques, the controlled active bending of wood is being explored through kerf bending or using bi-layer structures in which joints have two parts, one of which can move while the other restrains movement (Capone and Lanzara 2019; Baseta and Bollinger 2018). In both approaches, the freedom provided by the joints or cuts is what drives the movement; however, they are usually predesigned for a specific "target curve" and require intensive advanced fabrication work.

2. METHOD: BENDING TO A PARABOLA

Our approach to control bending is easy to accomplish with regular tools. We propose that merely changing the stiffness of the material along the strut leads to an approximation of the bending curvature to a parabola. This can be achieved by adding and subtracting material at a given location along the strut, which modifies the bending stiffness, and thus the curvature at that location. The stiffness profile along the strut can be chosen to alter the shape of the curve, and with the appropriate profile, it is physically possible to form a parabola. Two main equations were used to analyze the stiffness change along the section and determine the variations: the buckling equation (elastic curve) and the parabola equation. The two equations were set to give a stiffness variation for a curve at a specific height and span (figure 2). For the resultant strip, the overall shape is the graph of the stiffness (figure 2).

A parabola is defined by Equation (1), where x and y are cartesian coordinates and a and b are constants; a is the maximum height, y, and if the span between zero heights is l, then b is given by $b=4a/l^2$. Differentiating this equation twice gives an expression for the required radius of curvature, R, at a given x coordinate, in Equation (2).

1. $$y=a-bx^2$$

2. $$R(x)=\frac{(1+(2bx)^2)^{2/3}}{2b}$$

If a force, P, is applied horizontally along the dotted line in figure 1, the moment at a given point in the strut is P*y*. For an elastic strut to buckle into this shape, its stiffness must satisfy the bending equation, Equation (3).

3. $$M=Py=\frac{EI(x)}{R(x)}$$

Substituting (1) and (2) into (3) gives an equation for the bending stiffness of the strip per unit of applied force P, as shown in Equation (4).

4. $$EI/P=(a-bx^2)\left(\frac{(1+4b^2x^2)^{2/3}}{2b}\right)$$

To find the required stiffness at a particular point along the length of the strut, we must convert from the cartesian coordinate, x, to a distance along the strut, s. The incremental distance, ds, is defined in terms of the incremental coordinates dx and dy by Equation (5).

5. $$ds=\sqrt{dx^2+dy^2}$$

Since y=a-bx2, Equation (5) can be used to give an expression for s, Equation (6), yielding the relationship between x and s given by Equation (7).

6. $$s=\int_0^x\sqrt{1+4b^2x^2}\,dx$$

7. $$s=\frac{2bx\sqrt{4b^2x^2+1}+\sinh^{-1}(2bx)}{4b}$$

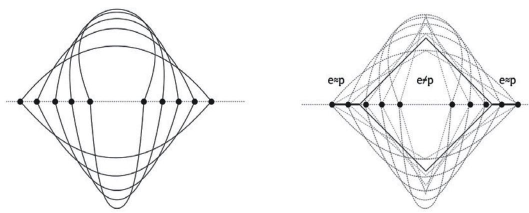

Figure 1: Elastica and parabola comparison (left). Divergence between elastica and parabola with more bending (right). (Author 2019)

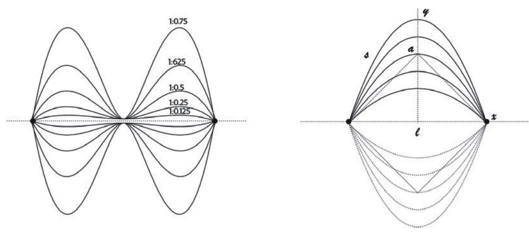

Figure 2: Changing stiffness diagrams in relation to rise-to-span ratio of curves (left). Results from approxmation (right). (Author 2019)

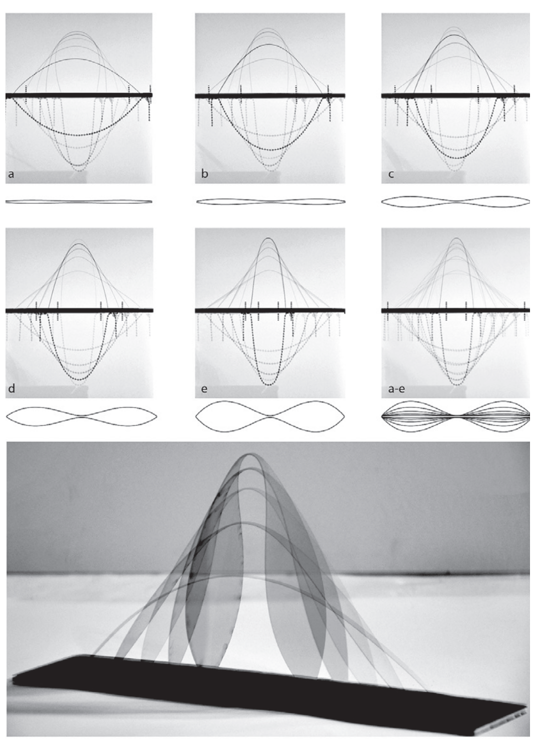

Figure 3: Testing: geometrical study of bending with different fusiformed strips (Author 2019)

Equation (4) thus gives the required stiffness, EI, at any horizontal coordinate, and this horizontal coordinate, x, can be transformed into a distance along the strut using Equation (7). These two equations give a relationship between the coordinate along the strut and its stiffness that allow the strut to be cut to the required shape. This can be done either by altering the width of the strip, which is proportional to EI, or the thickness of the strip, which is proportional to $\sqrt[3]{EI}$.

The shape of the strip is now two fusiform geometries. The maximum thickness is at the middle of each half before decreasing towards the center (figure 2). The change in thickness is moderate for nearly flat bending. In other words, at ratios of span-to-height of (1:0.25) or (1:0.5), the fusiform shapes are almost unrecognizable. A difference emerges with significant bending at a ratio of (1:1) or (1:2). There is no particular shape that can always give an exact parabola at any moment of bending, which is a limitation of such methods. However, an acceptable range of deviations can be accepted, primarily when the approximated strips are used as guidework or formwork for vaults, which usually have thicker sections.

To look for an average shape whose strip can cover a broad spectrum of parabolas that are constructionally acceptable as formwork, pieces of polypropylene with different span-to-height ratios were laser cut into parabolas with ratios of (1:0.25), (1:0.5), (1:0.75), (1:1), and (1:1.25). The physical texting showed that the range of shapes with curves with ratios from (1:0.75) to (1:1) can give a parabola, even for much steeper ratios (figure 3). For higher ratios, when the fusiform is very pronounced, shallow bending will create straight lines on the side, and the curvature will be restricted to the middle area, where the material is very thin.

The tool in hand, now called the *bending parabola*, can be used to devise an elastic guidework for a parabolic arch as it closes and opens. Sweeping, revolving, and shifting create sections of the bending parabola and generate multiple geometries of vaults. This strip then offers the possibility for an autonomous in-situ form-finding of structures, using simple materials such as rebar, wood, and bamboo.

The bending parabola can be used as a generative tool for many shapes. If we consider using only a linear element of the bending parabola (a strip) and move it on a rail to make a curve extrusion, we encounter several iterations of vaults with parabolic sections. The geometrical study of possible tracks in figure 4 shows various possibilities of vaulting that can be generated with the same bending parabola. Structural and design studies of such typologies become inherent in the tool, and they can be made without thinking about or designing one-time-use molds.

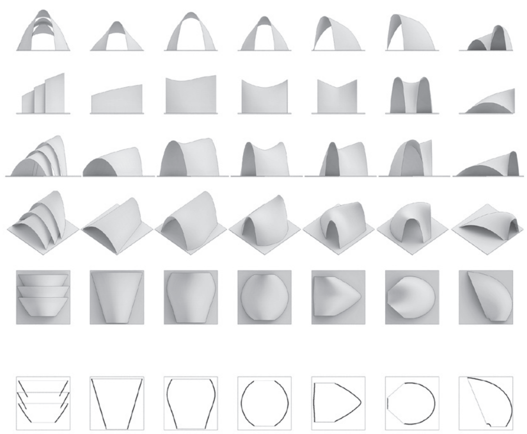

Figure 4: Possible vaulted geometries using the bending parabola (Author 2019)

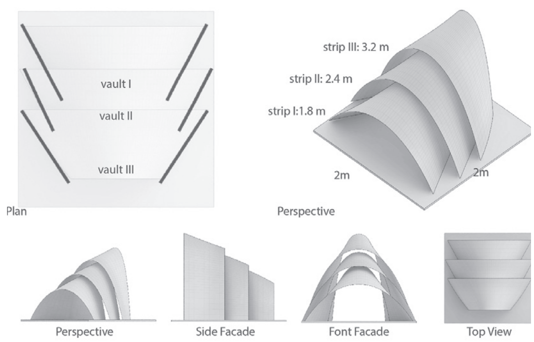

Figure 5: Vault composition from three bending parabolas (Author 2019)

3. RESULTS: AN APPLICATION

3.1. DESIGN: THIN-TILE VAULT

An application of the bending parabola was tested for use with the thin-tile vault technique. The traditional thin-tile vault construction can be found in Spain. Using terracotta tiles and plaster, builders construct various traditional vault typologies, such as ceilings of small vaults on steel or wooden beams, barrel vaults, and vaulted stairs. The thin-tile vault technique requires skill, but the few steps of construction, namely adding the plaster to the tile and then placing it on the vault, can be quickly learned. However, it is difficult to regulate the overall curvature of the vault or arch by controlling the placement of each tile. For this reason, a guidework to help visualize a structure is recommended for complex designs or for novice builders. Here, the bending parabola becomes useful.

The geometrical configuration of a vault was selected from the options in the previous geometrical study in figure 4. A structure containing three vaults with three different parabolic sections arranged from small to large was selected because it offers the possibility to test the tool's limitations for construction. The structure comprises three vaults, each of which is built with a strip of a specific length: 2.8, 3.4, and 4 meters. Consequently, each vault has a maximum height of 1.2, 1.5, and 1.8 meters, respectively. Due to the self-generated shape of the vault, no sections or drawings with precise dimensions were prepared as the change of height of each vault is generated by the bent strip (figure 5).

3.2. MATERIAL

An inquiry into a suitable material to make the strips was required. In an effort to avoid construction using advanced fabrication machinery, we adopted a lamination method to change the section of the bending parabola. The first option we examined was wood, which bends with an acceptable range of small thickness and long length. However, wood knots present an obstacle that can obstruct a uniform bending curve, and formed weak points are fragile against strong bending forces. Bamboo has nodes but no knots, which results in an enormous curve when bent, making it the most suitable material.

A small verification test was performed with bamboo blades, which were first bent without any additions and bent again with additional layers that were manually added to the middle quarters of the length and attached with tape. The test proves that the approximation by laminas helps bring an elastic curve to parabolas. Manual lamination also provided greater control of the ratio of thicknesses to achieve the target arch, since laminas can be added or removed as needed (figure 6).

3.3. CONSTRUCTION

The construction was done with the help of an expert in thin-tile vaults and stair making from Valencia. During an introductory meeting, we discussed the method and process of the construction, which involved a simple plan involving rails along which the parabolic strips would sweep. No plans or sections were introduced to the builder, and only the bamboo strips were provided.

To build a thin-tile vault with a sequence of arches, the first arch should be supported by formwork or a wall (Truño i Rusiñol et al. 2004), while later arches can be formwork-free because each tile has at least two edges with plaster, which is sufficient to support the tile. In the case of this parabolic vault, the bamboo worked as both formwork and guidework. Switching between the two roles was possible by adding or removing bamboo strips underneath the vault. Three bamboo strips, for instance, were sufficient to create a robust formwork for the smaller vault, whereas only one strip was needed for the guidework. The flexibility of changing between the two functions was advantageous in terms of accelerating the work and saving the formwork material (figure 7).

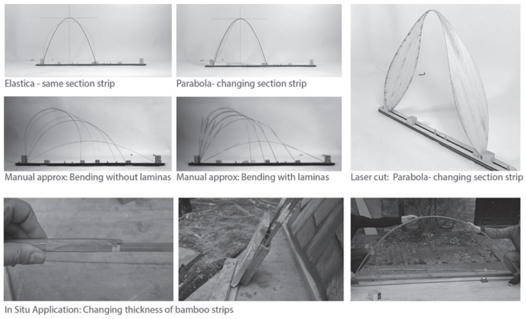

Figure 6: Approximation to parabola using laser cutting for changing the strip's thickness (top). Manual thickness variation using bamboo laminas (bottom). (Author 2019)

Figure 7: Thin-tile vault construction details (Author 2019)

3.4. A LEARNING TOOL

During construction of the largest vault, the mason found that the surface of the vault was reachable through an incremental three-dimensional rotation of tiles along the arch. This made construction possible without bent bamboo beneath the arches. Instead, we placed one bamboo strip at the end of the tile to serve as a visual guide for construction. In this instance, the tool become obsolete, but only after it served as a teaching element even for a skilled mason. It helped him to acquire the skills and knowledge required for building the specific thin-tile vault typology of a diverging parabolic vault.

The three parabolic vaults prove that the inherent properties of materials, bamboo and tiles, together provide several solutions to find a construction language using simple in-situ tools. In addition to the bending parabola, we used a conventional Catalan trowel, plaster, and tiles. The waste material from the formwork and guidework were only the tape used to fasten the bamboo strips together, as the strips themselves were returned to the material workshop almost as usable as when we first obtained them.

To avoid movement at the base of the bamboo during construction, we anchored the poles at each end in the holes of bricks, which were fixed to the ground with plaster of Paris (figure 6). After setting the base of the three vaults, we began with the smallest to learn how the bamboo would act during construction. The bamboo generated a parabolic arch and performed well for construction. When we reached the largest vault, the strip became very long, and the manual lamination of the strips resulted in deviations and asymmetries in the parabolic shape that necessitated delaminating the strips and correcting the positions of the laminas to perfect the curve (figure 7).

CONCLUSION

Bending a homogenous element creates an elastic curve. Elasticas cannot be used as, or for, compression-only structures, but the change of stiffness, or the amount of the material, of the bent element can provide a rough approximation for compression-only structural shells. Therefore, by changing the shape of strips, we explored different bending behaviors, one of which is a specific a strip with the shape of two fusiforms that creates a parabolic curve when bent. With a strip whose stiffness is high towards the first quarter and low towards the end and middle, one can form-find the compression-only structures in-situ by merely bending the strip: we call this the "the bending parabola". Adding stiffness to specific areas in the strip can be done by cutting, lathing, or laminating. Using the laminating method, three thin-tile vaults were built to examine the effectiveness of the bending parabola. The construction techniques and tools were the same for each vault: the strip, which served as a form-finder so there was no need for drawings, a formwork, which supported the thin-tile vault during construction, and a learning tool, which showed the artisan how the tiles should be positioned to achieve a specific form.

Rethinking materials for new systems of guidework and formwork can result in new methods of construction, especially when linked to geometry. When structures are understood as the materialization of their inherent material properties, bending for compression-only structures not only produces buildable vaults but also makes possible geometries that result from, and are described by, simple in-situ applications. Finding simple in-situ solutions for compression-only structures can become a driver of local grassroots construction that seeks alternatives not only to the way we build now but also to the way we think about design. The production of the built environment is not always in the hands of architects and engineers. In 2030, one in four people will be living in informal contexts, so an examination of and dialogue between high-knowledge analysis and low-tech everyday construction are much needed. In this particular context, the role of architecture is to mediate

Figure 8: Thin-tile vault result (Author 2019)

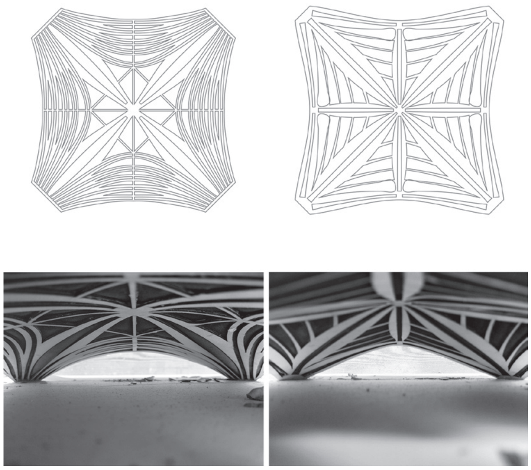

Figure 9: Bending plates for structural shells, patterns are studied to approximate bending curves to catenary sections (Author 2019)

Figure 10: Thin-tile vault result (Author 2019)

between materials, craftsmanship, and physics. An obvious example of this is the construction of Gaudí's ruled-surface vaults, where he combined ruled-surface logic with traditional string guidework. Gaudí's alteration of fabrication was not significant, in that the tools were still everyday strings, but he transformed them into a somewhat unprecedented mechanism for producing complex geometrical compositions.

Originally, the research focused on splines or the sequence of sections to generate geometries of compression-only shells. The next step is to work with sets of elements that work together as one system to describe a complex surface of a compression-only structures (figure 9). The move into bending plates offers a divergence from thin-tile vaulting (where the bent structure is a guidework) towards considering using bending-active systems as flexible formwork for lightweight compression-only concrete slabs or as lightweight flat-packed structures.

"Bend and build" should be explored more as it is underused given the ubiquity of linear elements in nature—bamboo and reed canes—and in the engineering and construction industries. This paper focuses on bent linear elements and their movements to produce thin-tile shells with parabolic sections (figure 9). Entirely new possibilities can now be opened in the study of surfaces, which is the topic of our ongoing research.

ACKNOWLEDGEMENTS

The authors would like to thank the master vault maker Salvador Gomis Aviño. This research is part of Al Asali's MPhil and Ph.D. on thin-tile vaulting, both funded by the Cambridge Trust-Saïd Foundation's scholarship.

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