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A NEW METHOD FOR PLASTIC DESIGN
IN STRUCTURAL STEEL

A THESIS

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Master of Science in Civil Engineering

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Ralph Stanley McChesney, Jr.

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ABSTRACT

The ability of mild structural steel to deform plastically after the yield point stress has been reached, thereby permitting greater stress on previously lower-stressed members in a given structure, is the basis of the plastic methods of structural analysis. A given structural system may be proportionally loaded until sufficient points of yielding (plastic hinges) occur to transpose the system into a failure mechanism. Present practical methods of plastic analysis seem usually to assume several collapse mechanisms and test each for the lower load carrying capacity. The usual test is to draw moment diagrams to insure that the correct mechanism has been selected.

A new method is presented whereby it is possible to make a direct, systematic selection of the failure mechanism. A failure mechanism will be composed of one or more elementary mechanisms, or simple mechanisms operating under each separate load of the loading system. It is possible to derive equilibrium equations for each elementary mechanism. No method is currently available for combining directly the equilibrium equations of various elementary mechanisms. However, by utilizing the inequality resulting from a knowledge of the extreme values necessary for the formation of a plastic hinge, it is possible to set up a system of inequalities representing all possible elementary mechanisms in the structure. This system of inequalities can be reduced by a method suggested by a mathematician, L. L. Dines, for reducing a system of inequalities.

At each section where a plastic hinge may occur in an elementary mechanism, two inequalities may be derived; each section inequality represents the rotation of the hinge in a fixed direction. Section inequalities for sections where plastic hinges occur in a mechanism may be grouped together as a system of inequalities representing the mechanism. However, all inequalities in the system must represent consistent hinge rotation for a mechanism operation. Therefore, two systems of inequalities are possible; one system operating in a direction consistent with the elemental loading and the other system operating in a direction inconsistent with the elemental loading. The equilibrium equation for the mechanism may be substituted in one of the section inequalities. This introduces a load term into the system. The system of inequalities may be reduced until the formation of an I-minor matrix which is I-definite, ie. it has a column which contains either all negative or all positive terms. When this occurs, the I-minor representing the mechanism will contain only a plastic moment term and a load term. The value for the load term which just satisfies this inequality represents the minimum load, or the collapse load for the mechanism.

The same reduction may be performed for a combination mechanism. A numerical method has been established to systematically delete extraneous elementary mechanisms from a combination mechanism consisting of all possible elementary mechanisms. This numerical method has been successfully programmed for the ERA 1101 Digital Computer at the Rich Computer Center. This will facilitate additional research and study of the problems involved in Plastic Design.

CHAPTER I

INTRODUCTION TO PLASTIC DESIGN IN STRUCTURAL STEEL

If a system of loads acting on a structural system is increased in a gradual and proportional manner, some point within the system will reach its limiting elastic value of resisting moment. Further increase in load will result essentially in the plastification of such sections which have reached the limiting elastic value of resisting moment or the yield-point stress. Section plastification leads to the concept of a plastic hinge, which considers that such sections will act as if hinged except with a restraining moment known as the plastic moment. T he plastic hinge concept is reasonable because of the ability of mild structural steel to deform plastically after the yield stress has been reached. Increase of loads on the structural system will cause increased stress on those members which remain elastic until sufficient plastic hinges have formed to transform the structural system into a failure mechanism.

The concept of a structural system being converted into a mechanism is the basis of many proposed methods of analysis designed to take advantage of the capacity of structural steel to deform plastically, thereby allowing the structural system to draw upon reserve strength of components having lower stress under an elastic behavior assumption. Such methods would have the unique feature of basing design criteria on ultimate load, rather than yield stress. This should result in the more economic and efficient use of steel as a structural material.

Probably the best method of approach to the problem of using the mechanism concept would be one which considered both the elastic and plastic properties of the structure. Such a method should give the stress history of the structure so that factors such as excessive deflections, instability, fracture, fatique, and inelastic stress reversal, could be considered as well as the attainment of maximum plastic strength. At present no method of an elastic-plastic nature is available which can be practically and accurately applied to the analysis of structural systems of any complexity. It should also be emphasized that methods featuring steel in a plastic range are not always appropriate, particularly with loading conditions conducive to fatique-type failure and situations in which stress reversal is likely.

Another approach is to actually assume a mechanism and then test to see whether the mechanism assumed is the correct one. Such a method would concern itself almost exclusively with the attainment of maximum plastic strength because consideration of other factors generally necessitates a knowledge of the progress of collapse during the sequence of hinge formation. The method of dealing with an assumed collapse mechanism has the advantage of simplicity because the analysis of the collapse mechanism is essentially the analysis of a statically determinate system.

Present methods of quickly assuming the correct trial collapse mechanism are dependent greatly on the skill and judgement of the analyst. Some methods are rather involved due to geometric considerations necessary for the test of the mechanism and for the determination of the ultimate load. This paper proposes to show a method by which a direct, systematic selection of the correct collapse mechanism could be made. Features of this new method may also be readily adapted for purposes of simplification to present methods of analysis.

A collapse mechanism may be tested by the consideration of two fundamental principles of plastic design first established by H. J. Greenberg and W. Prager (1). A brief, concise statement of these principles as given by P. S. Symonds and B. G. Neal (2) will be given in the immediate following paragraphs.

The first principle is known as the statical principle. It may be stated briefly as follows:

> The actual collapse load is the largest load at which it is possible to find a system of bending moments satisfying all equilibrium conditions with that load and nowhere violating a plasticity condition.

$$W' \leq W_c$$

where

W_c = collapse load

W' = load satisfying all equilibrium conditions and fully plastic moment of no member is exceeded

The second principle is known as the kinematic principle. It may be stated briefly as follows:

> The actual collapse load never exceeds the load corresponding to any mechanism into which the frame is converted by a suitable disposition of plastic hinges.

$$W_c \leq W'$$

where

W_c = collapse load.

W' = load corresponding to any configuration of hinges which reduces the frame to a mechanism.

Most methods of analysis involving the selection of a mechanism are facilitated by making a number of simplifying assumptions. The remainder of this chapter will be concerned with some of the more important of these assumptions.

The yield stress may be considered constant (see fig. 1) with increased strain in the portion of the stress-strain curve normally utilized in plastic analysis. A number of investigators have demonstrated experimentally that the error introduced by such an assumption is minor for most grades of structural steel.[1]

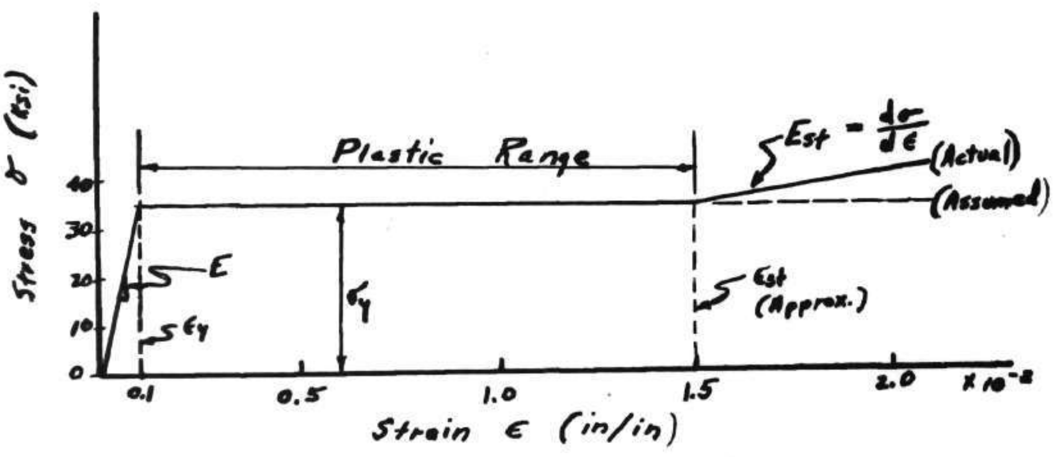


Fig. 1 IDEALIZED STRESS-STRAIN DIAGRAM

[1]Considerable research is currently being conducted to determine the validity of assumptions made in plastic analysis and the possible magnitude of error for each assumption. Much additional information can be obtained by a careful perusal of the references cited in the bibliography of this paper.

The idealized stress-strain relationship leads to an idealized Moment versus Curvature, or M-ϕ , curve (see fig. 2).

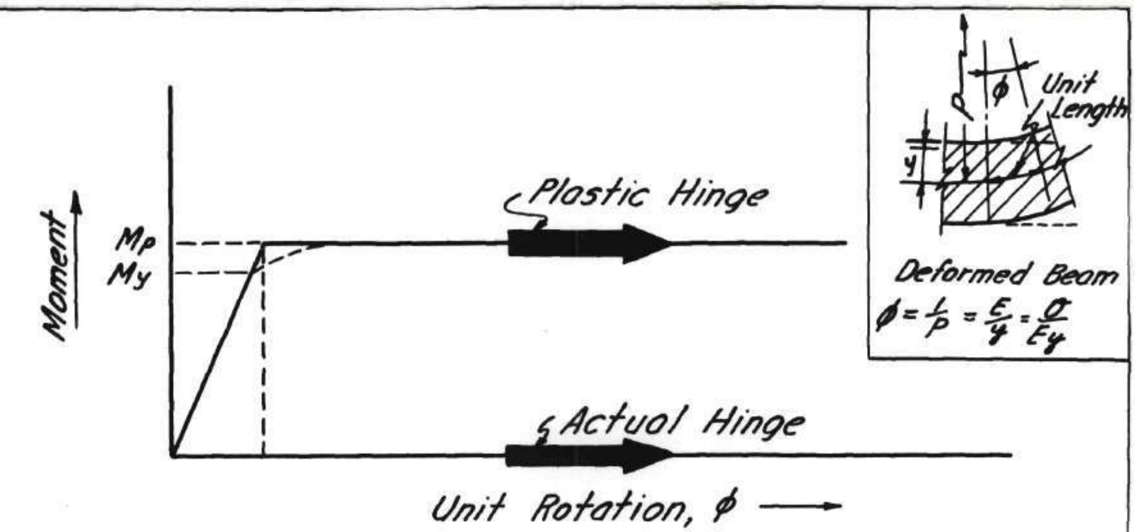


Fig. 2 IDEALIZED M - Ø CURVE

In the idealized M-ϕ curve, plastic hinges occur at distinct locations where all plastic rotation is assumed to occur such that the hinge length approaches zero. In actuality the extent of a hinge is dependent upon loading, shape of member cross section, and the geometry of the structural system. At a plastic hinge location a member acts as if it were hinged except with a constant restraining moment known as the plastic moment (3).

Shear forces may normally be neglected in plastic analysis. The maximum error introduced in the value of plastic moment due to the neglect of these forces is likely to be less than five per cent (2).

Axial or normal forces could contribute as much as fifteen per cent reduction in the value of the plastic moment. Such forces could also contribute buckling effects. Axial and buckling effects could perhaps be resolved by some scheme wherein the effective plastic modulus of a section may be reduced to compensate for this effect.

Most plastic methods of design utilize proportional loading. This means that all loads are defined by a single parameter such that they increase in fixed proportions.

Hinge locations are assumed to occur at points of loading, at changes of geometry of the structure, and at changes in the plastic section modulus of members. Hinge locations under uniform loads can be located with reasonable accuracy by methods involving successive approximations or by methods which involve representing the uniform load by a system of concentrated loads.

CHAPTER II

PLASTIC ANALYSIS BY THE METHOD OF EQUILIBRIUM INEQUALITY MATRIX

The plasticity condition at any section in a structural system can be represented mathematically by a pair of inequalities

$$- M_{pi} \leq M_i \leq M_{pi}$$

$$\text{or} \quad - M_i - M_{pi} \geq 0 \quad , \quad M_i - M_{pi} \geq 0$$

where M_{pi} , $- M_{pi}$ represent the extreme values necessary for the formation of a plastic hinge and M_i represents the actual moment at the section. The absolute values of M_{pi} and $- M_{pi}$ are equal in virtually all cases for steel members, and will be so assumed.

In a structural system having sufficient hinges formed so that a mechanism type collapse is imminent, changes in curvature at hinge locations may differ both in magnitude and in sense (sign). The sign convention to be used is such that increments of bending moment always have the same sign as increments of curvature at a given section, that is,

$$\frac{dM_i}{dk_i} \geq 0$$

where M_i represents the moment at the section and k_i represents the curvature. Consequently, when comparing the plasticity conditions at several sections, it is necessary to consider the relative effect of curvature at each section. This can be effected by introducing into the section plasticity inequality a non-dimensional constant k_r defined as

$$0 < k_r = \frac{k_i}{k_s}$$

where k_i represents the curvature at any section and k_s represents an arbitrarily assumed curvature standard for the structural system. It will be shown that the introduction of k_r to the section plasticity inequality, that is,

$$- k_r M_{pi} \leq k_r M_i \leq k_r M_{pi}$$

will have no deleterious effect in the analyses to follow.

P. S. Symonds and B. G. Neal (4) suggested a method of plastic design analysis which involved writing the pair of inequalities for each possible hinge location, combining these with equilibrium equations relating the bending moments, and then systematically reducing them so as to evaluate the largest value of the load for which all of the plasticity conditions could be satisfied. The method suggested had as its basis a method suggested by L. L. Dines (5) involving a matrix reduction of a system of linear inequalities. The method of Symonds and Neal, although probably systematic, is, by their own admission, too laborious for practical analysis of complex frames.

The new method presented in this paper utilizes the basic idea presented by Symonds and Neal but effects simplifications chiefly through the medium of stricter adherence to the methods and procedures of L. L. Dines.

Consider a fixed-end beam mechanism (fig. 3) having plastic hinges at sections A, B, and C.

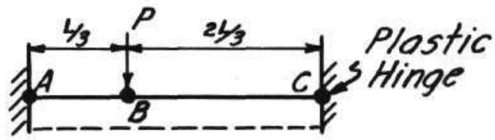


Fig. 3 BEAM MECHANISM

(The sign convention is such that positive moments put tensile stresses in the side of the member adjacent to the dashed line.)

@ Section A : $-M_A - M_p \geqslant 0$

@ Section B : $M_B - M_p \geqslant 0$

@ Section C : $-M_C - M_p \geqslant 0$

when each section is considered individually. If we are to compare the plasticity condition at each section, consideration must be made for the effects of curvature at the sections, readily determined from the geometry of the collapsing structure, as follows:

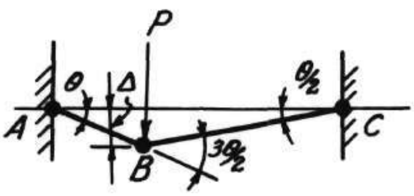


Fig. 4 GEOMETRY OF COLLAPSING MECHANISM

@ Section A : $k_A = \theta$

@ Section B : $k_B = 1.5\ \theta$

@ Section C : $k_C = 0.5\ \theta$

Let k_B = arbitrarily assumed curvature standard for the system

@ Section A : $k_r = \frac{k_A}{k_B} = \frac{2}{3}$

@ Section B : $k_r = \frac{k_B}{k_B} = 1$

@ Section C : $k_r = \frac{k_C}{k_B} = \frac{1}{3}$

Therefore, for comparison purposes, the plasticity condition is

@ Section A : $-\frac{2}{3} M_A - \frac{2}{3} M_p \geqslant 0$

@ Section B : $M_B - M_p \geqslant 0$

@ Section C : $-\frac{1}{3} M_C - \frac{1}{3} M_p \geqslant 0$

when all sections are considered collectively. The inequality, modified for curvature effects, representing the plasticity condition at a section will be called the "section inequality".

Consideration of the equilibrium of the fixed-end beam mechanism, collapse imminent, yields the equilibrium equation

$$M_B = \frac{2}{3} M_A + \frac{1}{3} M_C + \frac{2}{9} PL$$

where M_A , M_B , and M_C are the plastic moments at sections A, B, and C respectively, and $\frac{2}{9}$ PL is the simple moment at section B. The section inequality at section B may be modified, as follows,

$$M_B - M_p \geqslant 0$$

$$\frac{2}{3} M_A + \frac{1}{3} M_C - M_p + \frac{2}{9} PL \geqslant 0$$

to form a new inequality, which will be called the "equilibrium inequality". The remaining two section inequalities may now be written with the equilibrium inequality as a system of linear inequalities representing an elementary, or beam mechanism.

	column 1	column 2	column 3	column 4	
row 1	$\frac{2}{3} M_A$	$+ \frac{1}{3} M_C$	$- M_p$	$+ \frac{2}{9} PL$	$\geqslant 0$
row 2	$- \frac{2}{3} M_A$		$- \frac{2}{3} M_p$		$\geqslant 0$
row 3		$- \frac{1}{3} M_C$	$- \frac{1}{3} M_p$		$\geqslant 0$

The system of linear inequalities will now be solved for the minimum load, P, which will just satisfy the plasticity conditions at all hinge locations, that is, the plasticity conditions which will just satisfy the two fundamental principles of Plastic Design for the mechanism or combination of mechanisms under consideration.

The two fundamental principles of Plastic Design, stated briefly, are

<u>Statical Principle</u> : $W' \leq W_c$

<u>Kinematic Principle</u> : $W_c \leq W''$

where W_c is the collapse load; W' is the load satisfying all equilibrium conditions and the fully plastic moment of no member is exceeded; and W'' is the load corresponding to any configuration of hinges which reduces the frame to a mechanism.

The method of L. L. Dines will be used to solve for the minimum, ultimate load, P . Pertinent definitions, theorems, corollaries, and general information from Dines' paper (5) will be presented as needed in this chapter. This paper will attempt to show that the solution to the system of linear inequalities is a legitimate application of Dines' procedure.

Form the matrix of coefficients for the system of linear inequalities representing the mechanism illustrated in figure 3.

$$M = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{vmatrix} = \begin{vmatrix} \frac{2}{3} & \frac{1}{3} & -1 & \frac{2}{9} \\ -\frac{2}{3} & 0 & \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} & -\frac{1}{3} & 0 \end{vmatrix}$$

The following definition, as well as all subsequent information of this type in this chapter, will be quoted directly from Dines' paper (5) .

> A matrix will be said to be I-positive (or I-negative) with respect to a given one of its columns if all elements of that column are positive (or negative). In either case the matrix will said to be I-definite with respect to that column. A matrix will be said to be I-positive (or I-negative, or I-definite) if it possesses a column to which it is I-positive (or I-negative, or I-definite).

On the basis of this definition it is apparent that matrix M is I-negative, I-definite with respect to column 3 . Matrix M is not I-definite with respect to columns 1 , 2 , and 4 .

Consider any column r , such as column 1 of matrix M . The elements of this column can be divided into three classes, viz., those which are positive: $a_{ir} = a_{11} = \frac{2}{3}$

those which are negative: $a_{jr} = a_{21} = -\frac{2}{3}$

those which are zero: $a_{kr} = a_{31} = 0$

the number of elements in the respective classes being represented by P, N, and Z. In matrix M, column 1, $P = N = Z = 1$.

Form matrix $M_1^{(1)}$ (or $M_1^{(r)}$), derived from matrix M as follows:

To each pair of elements, a_{ir}, a_{jr} ; (a_{11}, a_{21}) ; the first positive and the second negative, corresponds one row of the derived matrix, the elements of which are second order determinants,

$$\begin{vmatrix} a_{ir} & a_{i1} \\ a_{jr} & a_{j1} \end{vmatrix}, \begin{vmatrix} a_{ir} & a_{i2} \\ a_{jr} & a_{j2} \end{vmatrix}, \dots, \begin{vmatrix} a_{ir} & a_{i\,r-1} \\ a_{jr} & a_{j\,r-1} \end{vmatrix}, \begin{vmatrix} a_{ir} & a_{i\,r+1} \\ a_{jr} & a_{j\,r+1} \end{vmatrix}, \begin{vmatrix} a_{ir} & a_{in} \\ a_{jr} & a_{jn} \end{vmatrix}.$$

To each zero element a_{kr} corresponds one row of the derived matrix,

$$a_{k1}, a_{k2}, \dots, a_{k\,r-1}, a_{k\,r+1}, a_{kn}.$$

The matrix $M_1^{(r)}$ will then consist of the rows so formed, their number being $P(N) + Z$.

$$M_1^{(1)} = \left\| \begin{matrix} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{14} \\ a_{22} & a_{24} \end{vmatrix} \\ |a_{32}| & |a_{33}| & |a_{34}| \end{matrix} \right\| = \left\| \begin{matrix} \begin{vmatrix} \frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & 0 \end{vmatrix} & \begin{vmatrix} \frac{2}{3} & -1 \\ -\frac{2}{3} & -\frac{2}{3} \end{vmatrix} & \begin{vmatrix} \frac{2}{3} & \frac{2}{9} \\ -\frac{2}{3} & 0 \end{vmatrix} \\ \left|-\frac{1}{3}\right| & \left|-\frac{1}{3}\right| & |0| \end{matrix} \right\|$$

Further simplification of matrix $M_1^{(1)}$ may be made through the medium of conventional determinant analysis.

$$M_1^{(1)} = \left\| \begin{matrix} |a_{11}a_{22} - a_{21}a_{12}| & |a_{11}a_{23} - a_{21}a_{13}| & |a_{11}a_{24} - a_{21}a_{14}| \\ |a_{32}| & |a_{33}| & |a_{34}| \end{matrix} \right\|$$

$$= \left\| \begin{matrix} \left|\left(\frac{2}{3}\right)(0)-\left(-\frac{2}{3}\right)\left(\frac{1}{3}\right)\right| & \left|\left(\frac{2}{3}\right)\left(-\frac{2}{3}\right)-\left(\frac{2}{3}\right)(-1)\right| & \left|\left(\frac{2}{3}\right)(0)-\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)\right| \\ \left|\frac{1}{3}\right| & \left|-\frac{1}{3}\right| & |0| \end{matrix} \right\|$$

If $a_{11} = -a_{21}$;

$$M_1^{(1)} = \left\| \begin{matrix} \left|a_{11}(a_{22}+a_{12})\right| & \left|a_{11}(a_{23}+a_{13})\right| & \left|a_{11}(a_{24}+a_{14})\right| \\ |a_{32}| & |a_{33}| & |a_{34}| \end{matrix} \right\|$$

$$= \left\| \begin{matrix} \left|\left(\frac{2}{3}\right)\left[0+\left(+\frac{1}{3}\right)\right]\right| & \left|\left(\frac{2}{3}\right)\left[-1+\left(-\frac{2}{3}\right)\right]\right| & \left|\left(\frac{2}{3}\right)\left[0+\left(+\frac{2}{9}\right)\right]\right| \\ \left|-\frac{1}{3}\right| & \left|-\frac{1}{3}\right| & |0| \end{matrix} \right\|$$

The I-rank of a matrix (basis of solution) is not altered if
(1) any two rows or any two columns are interchanged;
(2) all elements of any row or any column are multiplied by the same positive constant.

Multiplying combined rows 1 and 2 by $\frac{1}{a_{11}}$; $\frac{3}{2}$:

$$M_1^{(1)} = \left\| \begin{matrix} a_{22}+a_{12} & a_{23}+a_{13} & a_{24}+a_{14} \\ a_{32} & a_{33} & a_{34} \end{matrix} \right\| = \left\| \begin{matrix} \frac{1}{3} & -\frac{5}{3} & \frac{2}{9} \\ -\frac{1}{3} & -\frac{1}{3} & 0 \end{matrix} \right\|$$

Similarly, from $M_1^{(1)}$, form matrix $M_2^{(12)}$:

$$M_2^{(12)} = \left\| \begin{matrix} a_{23}+a_{13}+a_{33} & a_{24}+a_{14}+a_{34} \end{matrix} \right\| = \left\| \begin{matrix} -2 & +\frac{2}{9} \end{matrix} \right\|$$

The matrix $M_1^{(r)}$ (or $M_1^{(1)}$) will be called the I-complement of the r^{th} column of M.
The matrices $M_1^{(1)}$, $M_1^{(2)}$, ... , $M_1^{(n)}$ will be called the I-minors of $n\ \overline{1}$ columns of the matrix M .

A matrix will be said to be of I-rank k if it possesses at least one I-minor of k columns which is I-definite, but does not possess any I-minor of k 1 columns which is I-definite.

Derived matrices may be formed until there is no column which is not I-definite. Successive I-complements may be formed by considering each derived matrix operationally as the original matrix; this was done in deriving $M_2^{(12)}$ from $M_1^{(1)}$. Matrix $M_2^{(12)}$ has two columns which are I-definite and no columns which are not I-definite. Therefore matrix M is of I-rank 2.

Theorem I: A necessary and sufficient condition for the existence of a solution to the system (1) is that the I-rank of the matrix M be greater than zero.

Theorem II: If the I-rank of the matrix is k (>0), then the system (1) possesses a solution in which k-1 of the unknowns may be assigned values at pleasure.

Theorems I and II dictate that either of the columns in matrix $M_2^{(12)}$ may be assigned values at pleasure if the sum of both columns always exceeds zero. The two principles of Plastic Design dictate that values assigned must just satisfy the inequalities. Therefore the sum of the two columns may be equated to zero, and the minimum load P determined.

$$-2\,M_p + \frac{2}{9}PL = 0$$

$$P = 9\,\frac{M_p}{L}$$

The central feature of Dines' method for systems of linear inequalities is a concept analogous to the rank of a matrix, which he calls the inequality-rank or I-rank of the matrix. The application of plastic analysis to this method involves merely the formation of a sequence of I-minors of the matrix of coefficients, each I-minor being the I-complement of a column of its predecessor, the process to be continued until an I-definite matrix is obtained. The successive I-minors are the matrices of the successive systems of inequalities occurring in the elimination.

CHAPTER III

COMBINING INEQUALITY SYSTEMS OF ELEMENTARY MECHANISMS

The combination of inequality systems for elementary mechanisms is readily accomplished by considering all systems as one system and reducing by Dines' method. The resulting reduction will give the minimum load for all systems, provided no section where a plastic hinge may be possibly eliminated is used in forming an equilibrium inequality. Consider frame ABCDEF (see fig. 5) having possible mechanisms as beam mechanism BDE , beam mechanism BCE , and panel mechanism ABEF .

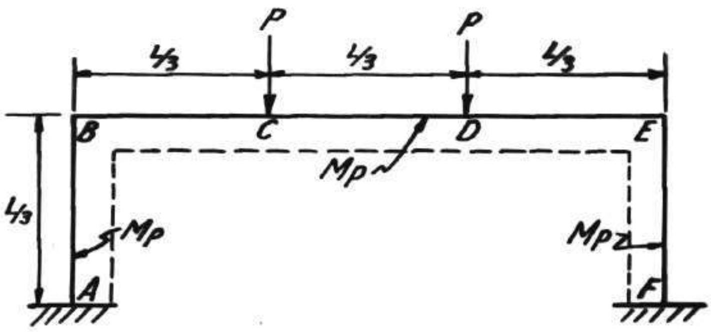


Fig. 5 FRAME ABCDEF

The inequality system representing a typical mechanism BCE may be derived by first developing the statical equilibrium equation,[2] then

[2]See the appendix for a brief explanation as to the derivation of statical equilibrium equations.

forming the equilibrium inequality at some hinge section in the mechanism, and finally writing in the remaining section inequalities. The equilibrium equation may be easily derived by consideration of the moment diagram (see fig. 6).

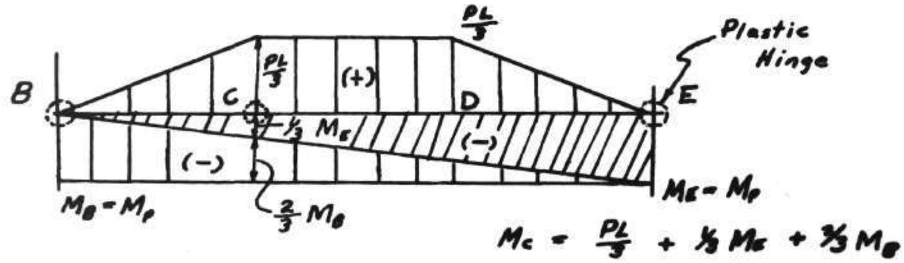


Fig. 6 MOMENT DIAGRAMS FOR MECHANISM BCE

Equilibrium Equation: $M_C = 0.67\ M_B + 0.33\ M_E + 0.33\ PL$

Section Inequality @C : $M_C - M_p \geqslant 0$

Equilibrium Inequality @C : $0.67\ M_B + 0.33\ M_E - 1.00\ M_p + 0.33\ PL \geqslant 0$

Section Inequality @B : $-0.67\ M_B \quad -0.67\ M_p \geqslant 0$

Section Inequality @E : $-0.33\ M_E - 0.33\ M_p \geqslant 0$

M_A , M_C , and M_D are selected as redundant moments because there is no possibility that a plastic hinge could be eliminated at sections A , C , or D . A similar operation is done to derive other mechanism inequalities. All systems may be written together as one system.

Column Number	1 M_B	2 M_E	3 M_A	4 M_p	5 PL	Row
Beam Mechanism	0.67	0.33	0	- 1.00	0.33	(1)
BCE	-0.67	0	0	- 0.67	0	(2)
	0	-0.33	0	- 0.33	0	(3)

Beam Mechanism	0.33	0.67	0	- 1.00	0.33	(4)
BDE	-0.33	0	0	- 0.33	0	(5)
	0	-0.67	0	- 0.67	0	(6)
Panel Mechanism	-1.00	1.00	-1.00	-1.00	0.67	(7)
ABEF	1.00	0	0	-1.00	0	(8)
	0	-1.00	0	-1.00	0	(9)
	0	0	1.00	-1.00	0	(10)

Row operation so that column 1 may be eliminated:

$$\begin{bmatrix} 1.00 & 0.50 & 0 & -1.50 & 0.50 \\ -1.00 & 0 & 0 & -1.00 & 0 \\ 0 & -1.00^{*} & 0 & -1.00 & 0 \\ 1.00 & 2.00 & 0 & -3.00 & 1.00 \\ -1.00 & 0 & 0 & -1.00 & 0 \\ 0 & -1.00 & 0 & -1.00 & 0 \\ -1.00 & 1.00 & -1.00 & -1.00 & 0.67 \\ 1.00 & 0 & 0 & -1.00 & 0 \\ 0 & -1.00 & 0 & -1.00 & 0 \\ 0 & 0 & 1.00 & -1.00 & 0 \end{bmatrix} \begin{array}{l} (1) \\ (2)\ \text{Omit because} \\ (3)\ \text{of duplication.} \\ (4) \\ (5,2) \\ (6)\ \text{duplication} \\ (7) \\ (8) \\ (9,3,6) \\ (10) \end{array}$$

Reduce column 1, considering all possible row combinations:

$$\begin{bmatrix} 0 & 0.50 & 0 & -2.50 & 0.50 \\ 0 & 1.50 & -1.00 & -2.50 & 1.17 \\ 0 & 2.00 & 0 & -4.00 & 1.00 \\ 0 & 3.00 & -1.00 & -4.00 & 1.67 \\ 0 & 0 & 0 & -2.00 & 0 \\ 0 & 1.00 & -1.00 & -2.00 & 0.67 \\ 0 & -1.00 & 0 & -1.00 & 0 \\ 0 & 0 & 1.00 & -1.00 & 0 \end{bmatrix} \begin{array}{l} (1) + (5,2) \\ (1) + (7) \\ (4) + (5,2) \\ (4) + (7) \\ (5,2) + (8) \\ (7) + (8) \\ (9,3,6) \\ (10) \end{array}$$

Row operation so that column 2 may be eliminated:

$$\begin{bmatrix} 0 & 1.00 & 0 & -5.00 & 1.00 \\ 0 & 1.00 & -0.67 & -1.67 & 0.78 \\ 0 & 1.00 & 0 & -2.00 & 0.50 \\ 0 & 1.00 & -0.33 & -1.33 & 0.56 \\ 0 & 0 & 0 & -2.00 & 0 \\ 0 & 1.00 & -1.00 & -2.00 & 0.67 \\ 0 & -1.00 & 0 & -1.00 & 0 \\ 0 & 0 & 1.00 & -1.00 & 0 \end{bmatrix} \begin{array}{l} (1) + (5,2) \\ (1) + (7) \\ (4) + (5,2) \\ (4) + (7) \\ (5,2) + (8) \\ (7) + (8) \\ (9,3,6) \\ (10) \end{array}$$

---

*Section inequalities may be converted to ± 1 immediately since they may be changed at pleasure to facilitate reduction.

Reduce column 2, considering all possible row combinations:

$$\begin{bmatrix} 0 & 0 & 0 & -6.00 & 1.00 \\ 0 & 0 & -0.67 & -2.67 & 0.78 \\ 0 & 0 & 0 & -3.00 & 0.50 \\ 0 & 0 & -0.33 & -2.33 & 0.56 \\ 0 & 0 & -1.00 & -3.00 & 0.67 \\ 0 & 0 & 1.00 & -1.00 & 0 \\ 0 & 0 & 0 & -2.00 & 0 \end{bmatrix} \begin{array}{l} (1)+(5,2)+(9,3,6) \\ (1)+(7)+(9,3,6) \\ (4)+(5,2)+(9,3,6) \\ (4)+(7)+(9,3,6) \\ (7)+(8)+(9,3,6) \\ (10) \\ (5,2)+(8) \end{array}$$

Row operation so that column 3 may be eliminated:

$$\begin{bmatrix} 0 & 0 & 0 & -6.00 & 1.00 \\ 0 & 0 & -1.00 & -4.00 & 1.17 \\ 0 & 0 & 0 & -3.00 & 0.50 \\ 0 & 0 & -1.00 & -7.00 & 1.67 \\ 0 & 0 & -1.00 & -3.00 & 0.67 \\ 0 & 0 & 1.00 & -1.00 & 0 \\ 0 & 0 & 0 & -2.00 & 0 \end{bmatrix} \begin{array}{l} (1)+(5,2)+(9,3,6) \\ (1)+(7)+(9,3,6) \\ (4)+(5,2)+(9,3,6) \\ (4)+(7)+(9,3,6) \\ (7)+(8)+(9,3,6) \\ (10) \\ (5,2)+(8) \end{array}$$

Reduce column 3, considering all possible row combinations:

$$\begin{bmatrix} 0 & 0 & 0 & -5.00 & 1.17 \\ 0 & 0 & 0 & -8.00 & 1.67 \\ 0 & 0 & 0 & -4.00 & 0.67 \\ 0 & 0 & 0 & -6.00 & 1.00 \\ 0 & 0 & 0 & -3.00 & 0.50 \\ 0 & 0 & 0 & -2.00 & 0 \end{bmatrix} \begin{array}{l} (1)+(7)+(9,3,6)+(10) \\ (4)+(7)+(9,3,6)+(10) \\ (7)+(8)+(10) \\ (1)+(5,2)+(9,3,6) \\ (4)+(5,2)+(9,3,6) \\ (5,2)+(8) \end{array}$$

The reduction has been completed and the remaining expressions should be equated to zero and solved for the minimum load.

Mechanism BCE - Mechanism ABEF = (1) + (7) + (9,3,6) + (10)

Mechanism BDE - Mechanism ABEF = (4) + (7) + (9,3,6) + (10)

Mechanism ABEF = (7) + (8) + (10)

Mechanism BCE = (1) + (5,2) + (9,3,6)

Mechanism BDE = (4) + (5,2) + (9,3,6)

Mechanism does not exist = (5,2) + (8)

BCE + ABEF	$-5.00\,M_p + 1.17\,PL = 0$	$P = 4.29\,\frac{M_p}{L}$
BDE + ABEF	$-8.00\,M_p + 1.67\,PL = 0$	$P = 4.80\,\frac{M_p}{L}$
ABEF	$-4.00\,M_p + 0.67\,PL = 0$	$P = 6.00\,\frac{M_p}{L}$
BCE	$-6.00\,M_p + 1.00\,PL = 0$	$P = 6.00\,\frac{M_p}{L}$
BDE	$-3.00\,M_p + 0.50\,PL = 0$	$P = 6.00\,\frac{M_p}{L}$
No Mechanism	$-2.00\,M_p + 0\,PL = 0$	$P = \infty$

Since BCE + ABEF gives the smallest minimum load, BCE + ABEF is the correct collapse mode (see fig. 7).

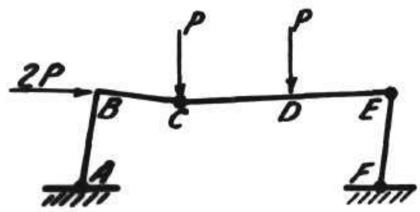


Fig. 7 COLLAPSE MECHANISM BCE - ABEF OR ACEF

Because each plastic moment term in the equilibrium inequality can be matched by its complement in the section inequality by utilizing the appropriate row operation, the reduction may be done by simultaneously adding the equilibrium inequality and the complemented section inequalities.

Consider the system of inequalities for the mechanism BCE .

$$\begin{array}{l} \text{row 1} \\ \text{row 2} \\ \text{row 3} \end{array} \begin{bmatrix} 0.67\,M_B + 0.33\,M_E & -1.00\,M_p & +0.33\,PL & \geq 0 \\ -0.67\,M_B & -0.67\,M_p & & \geq 0 \\ \quad\quad - 0.33\,M_E & -0.33\,M_p & & \geq 0 \end{bmatrix}$$

Add rows 1,2,&3 : $-2.00\,M_p + 0.33\,PL \geq 0$

Solve for minimum load: $P = 6.00\,\frac{M_p}{L}$

Let $(M_p)_B$ and $(M_p)_E$ represent the relative plastic stiffness at sections B and E respectively. The reduction may be done as follows using only the equilibrium inequality.

$$\text{Equilibrium Inequality:} \quad 0.67\, M_B + 0.33\, M_E - 1.00\, M_p + 0.33\, PL \geqslant 0$$

$$\text{Reduction:} \quad \left[(-|0.67\,(M_p)_B|) + (-|0.33\,(M_p)_E|) + (-1.00\, M_p)\right] + 0.33\, PL \geqslant 0$$

$$-2.00\, M_p + 0.33\, PL \geqslant 0$$

$$P = 6.00\, \frac{M_p}{L}$$

This method may also be applied to any combination of equilibrium inequalities. Consider the combination of mechanisms BCE and ABEF .

$$\begin{array}{lllllll}
\underline{BCE} & 0.67\, M_B & + 0.33\, M_E & & -1.00\, M_p & + 0.33\, PL & \geqslant 0 \\
\underline{ABEF} & -1.00\, M_B & + 1.00\, M_E & -1.00\, M_F & -1.00\, M_p & + 0.67\, PL & \geqslant 0
\end{array}$$

Utilize appropriate row operation on the equilibrium inequality representing BCE so that addition of inequalities will result in the elimination of the plastic hinge at section B in the resulting equilibrium inequality.

$$\begin{array}{lllllll}
\underline{BCE} & 1.00\, M_B & + 0.50\, M_E & & -1.50\, M_p & + 0.50\, PL & \geqslant 0 \\
\underline{ABEF} & -1.00\, M_B & + 1.00\, M_E & -1.00\, M_F & -1.00\, M_p & + 0.67\, PL & \geqslant 0 \\
\hline
\underline{BCE}\ \underline{ABEF} & 0 & + 1.50\, M_E & -1.00\, M_F & -2.50\, M_p & + 1.17\, PL & \geqslant 0
\end{array}$$

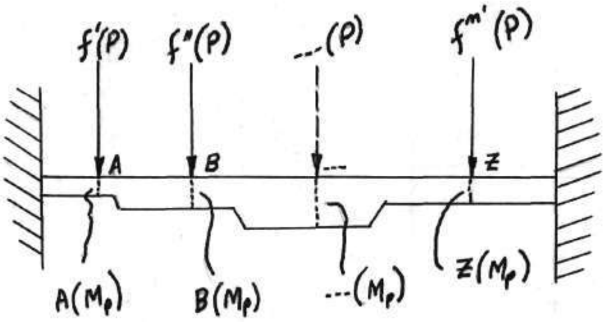
$$\text{Reduction:} \quad \left[(-|1.50\,(M_p)_E|) + (-|1.00\,(M_p)_F|) + (-2.50\,(M_p))\right] + 1.17\, PL \geqslant 0$$

$$-5.00\, M_p + 1.17\, PL \geqslant 0 \; ; \; P = 4.29\, \frac{M_p}{L}$$

A method has been demonstrated for the direct combination of elementary mechanisms and the computation of the minimum load for the new combined mechanism.

Difficulty may be encountered in setting up basic statical equilibrium equations and in selecting section to form equilibrium inequality. Care must be exercised so that section forming equilibrium inequality does not correspond to a section where a plastic hinge may possibly be eliminated by combination with other possible mechanisms.

The reduction process may be described by means of a reduction formula (see fig. 8).



for a mechanism, elementary or combined:

$$P = \frac{\left[\left(-\left|A(M_p)\right|\right) + \left(-\left|B(M_p)\right|\right) + \cdots\cdots + \left(-\left|Z(M_p)\right|\right)\right] - \left|x(M_p)\right|}{f(L)}$$

P = Minimum Load

$\left|A(M_p)\right|$ = Absolute Value of the Relative Plastic Stiffness @ any section, say A .

$\left|x(M_p)\right|$ = Total Effective Value of Plastic Stiffness for Section forming Equilibrium Inequality.

$f(L)$ = a function of Length

$f'(P)$ = a function of Loading

Fig. 8 REDUCTION FORMULA FOR MECHANISMS

CHAPTER IV

POSITIVE AND NEGATIVE ELEMENTARY MECHANISMS

Oftentimes it is difficult to predict the direction in which a combination mechanism will operate. Therefore it may be necessary to try combinations of elementary mechanisms considering some elementary mechanisms to operate in two directions. A mechanism operating in the direction of mechanism loading will be called a "positive mechanism", while a mechanism acting in a direction opposing mechanism loading will be called a "negative mechanism". For example, frame ABCDE (see fig. 9) may be converted into a positive mechanism $\underline{ABDE}$ (see fig. 10) or a negative mechanism $\overline{\underline{ABDE}}$. The negative mechanism (see fig. 11) will produce a negative ultimate load.

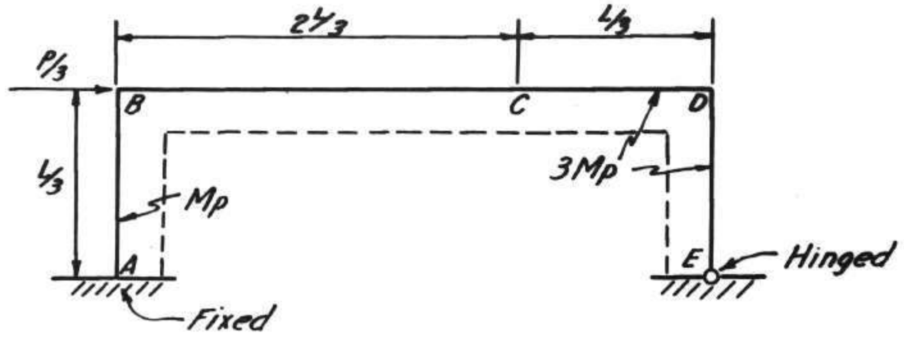


Fig. 9 FRAME ABCDE

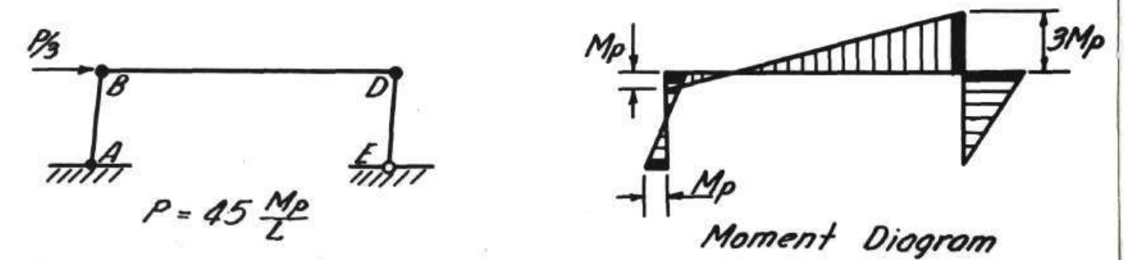


Fig. 10 POSITIVE MECHANISM ABDE & MOMENT DIAGRAM

Equilibrium Equation: $M_A = M_B - M_D - 0.11\ PL$

Section Inequality @ A: $-M_A - M_p \geqslant 0$

Equilibrium Inequality @ A: $-M_B + M_D - M_p + 0.11\ PL \geqslant 0$

Solved for Minimum P : $P = 45 \frac{M_p}{L}$

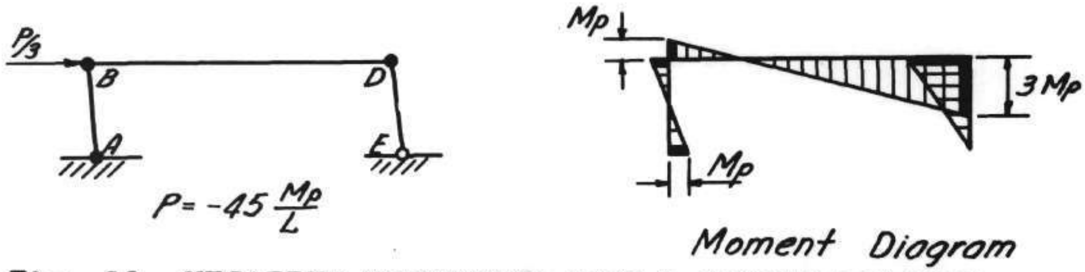


Fig. 11 NEGATIVE MECHANISM ABDE & MOMENT DIAGRAM

Equilibrium Equation: $-M_A = -M_B + M_D + 0.11\ PL$

Section Inequality @ A: $M_A - M_p \geqslant 0$

Equilibrium Equation @ A: $M_B - M_D - M_p - 0.11\ PL \geqslant 0$

Solved for Minimum P : $P = -45 \frac{M_p}{L}$

An example illustrating the occurrence of a negative mechanism follows by considering frame ABCDE (see fig. 12) with possible mechanisms ABDE , $\overline{\text{ABDE}}$, and BCD .

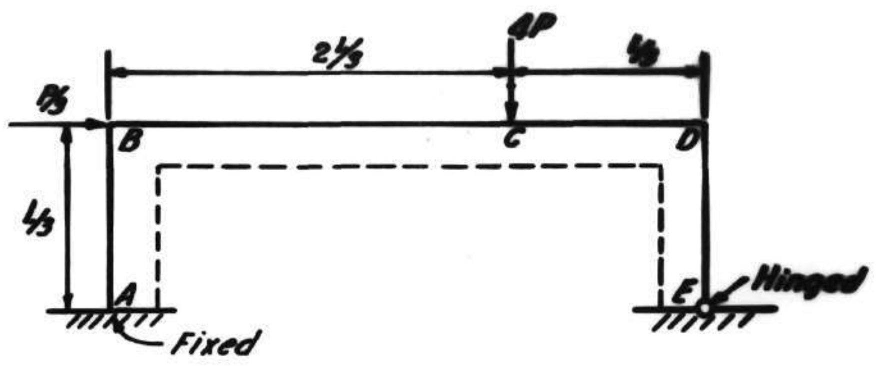


Fig. 12 FRAME ABCDE WITH LOADING AT B AND C

Mechanism BCD : $0.33M_B + 0.67M_D \; -3M_p + 1.00(PL) \geqslant 0$; $P = 5.33\frac{M_p}{L}$

Mechanism ABDE : $-1.00M_B + 1.00M_D \; - M_p + 0.11(PL) \geqslant 0$; $P = 45\frac{M_p}{L}$

Mechanism $\overline{\text{ABDE}}$: $1.00M_B \; -1.00M_D \; - M_p \; -0.11(PL) \geqslant 0$; $P = -45\frac{M_p}{L}$

Combine mechanisms BCD and ABDE (see fig. 13), eliminating the plastic hinge at section B .

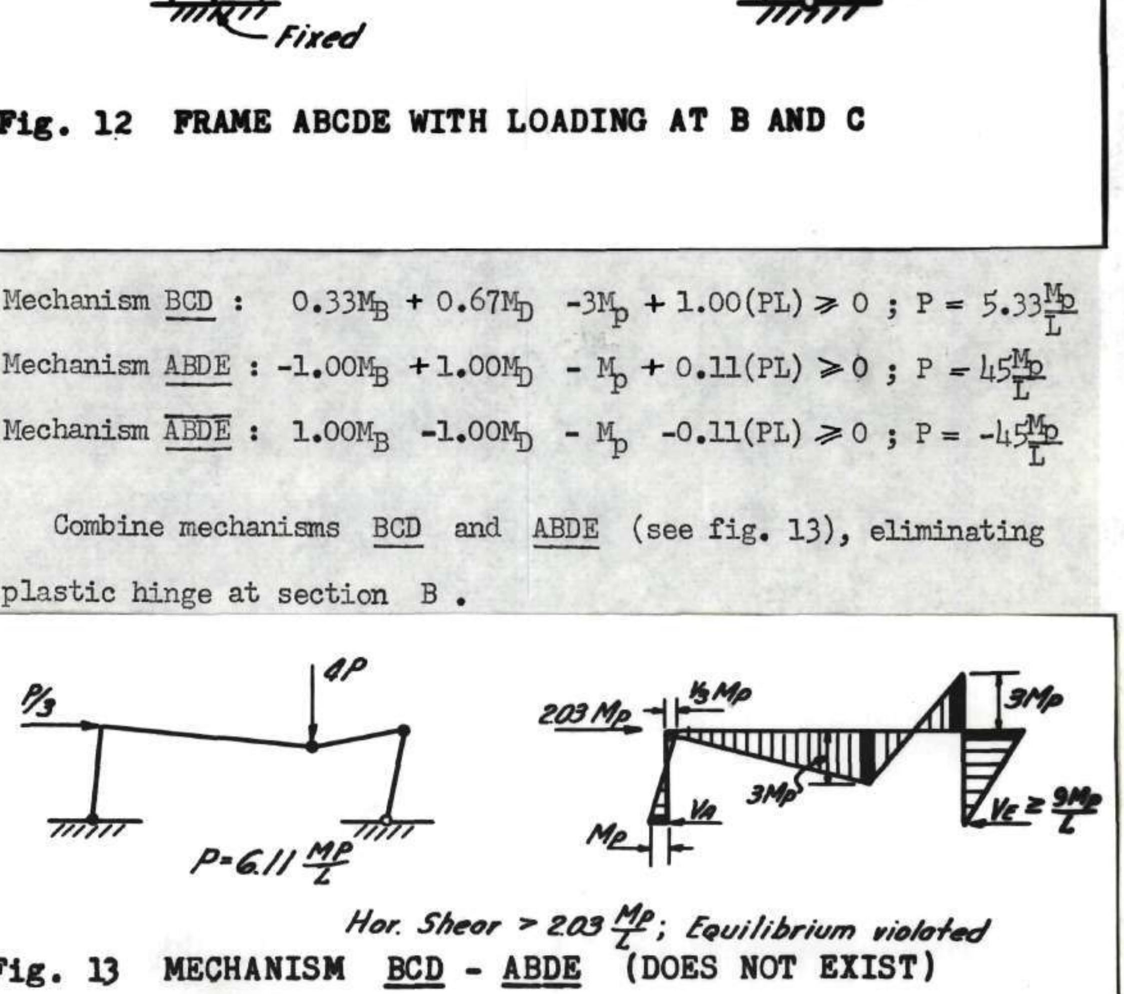


Fig. 13 MECHANISM BCD - ABDE (DOES NOT EXIST)

$$0.33\ M_B + 0.67\ M_D - 3.00\ M_p + 1.00\ PL \geqslant 0$$

$$-0.33\ M_B + 0.33\ M_D - 0.33\ M_p + 0.04\ PL \geqslant 0$$

$$(0)\ M_B + 1.00\ M_D - 3.33\ M_p + 1.04\ PL \geqslant 0 \quad ; \quad P = 6.11\frac{M_p}{L}$$

Combine mechanisms $\underline{BCD}$ and $\overline{ABDE}$ (see fig. 14), eliminating the plastic hinge at section D .

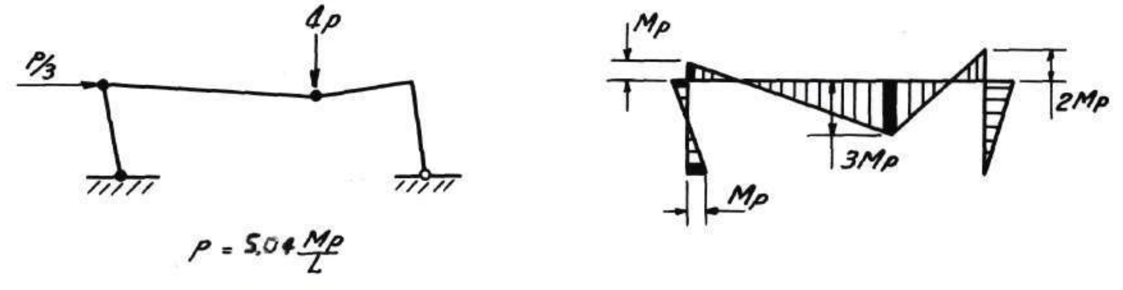


Fig. 14 MECHANISM $\underline{BCD}$ - $\underline{ABDE}$ & MOMENT DIAGRAM

$$0.33\ M_B + 0.67\ M_D - 3.00\ M_p + 1.00\ PL \geqslant 0$$

$$0.67\ M_B - 0.67\ M_D - 0.67\ M_p - 0.07\ PL \geqslant 0$$

$$1.00\ M_B - (0)\ M_D - 3.67\ M_p + 0.93\ PL \geqslant 0 \quad ; \; P = 5.04\ \frac{M_p}{L}$$

Therefore the correct collapse mode is due to the combination of mechanism $\underline{BCD}$ with negative mechanism $\overline{ABDE}$.

The determination of the correct collapse mode requires testing all possible mechanisms for all possible plastic hinge eliminations. A method will be given in a subsequent section for testing all possible hinge eliminations. The determination of the number of elementary mechanisms possible, considering positive and negative mechanisms, is given by the modified rule (6) which follows.

Rule: N = number of possible plastic hinges
X = redundancies

2 (N - X) = number of elementary mechanisms

Any possible combination of these mechanisms should be investigated to determine the smallest possible load or ultimate load.

CHAPTER V

FORMING EQUILIBRIUM INEQUALITIES

Equilibrium inequalities are formed from equilibrium equations using sections where there is no possibility that a plastic hinge may be eliminated when combining with other possible mechanisms. If no section meets this requirement it will be necessary to form sufficient equilibrium inequalities that all possible hinge eliminations may be considered. One section may be satisfactory for a positive mechanism and be completely unsatisfactory for a negative mechanism. Consider a frame ABCDE (see fig. 15) with mechanisms ACDE and ABC .

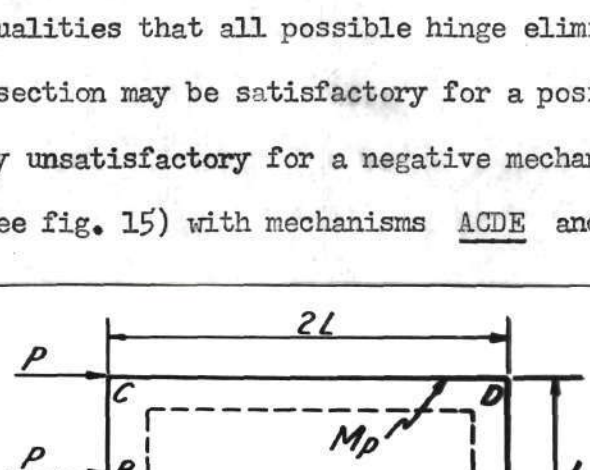


Fig. 15 FRAME ABCDE WITH LOADING AT B & C (SAME DIRECTION)

Mechanism ACDE :

Equilibrium Equation: $- M_A + M_C - M_D + M_E - 1\frac{1}{2}\,PL = 0$

Equilibrium Inequality @A : $- M_C + M_D - M_E - M_p + 1\frac{1}{2}\,PL \geqslant 0$

Equilibrium Inequality @E : $+ M_A - M_C + M_D - M_p + 1\frac{1}{2}\,PL \geqslant 0$

Using Reduction Formula, Chapter III ; $P = 2.67 \frac{M_p}{L}$

Mechanism ABC :

Equilibrium Equation: $-\frac{1}{2} M_A + M_B - \frac{1}{2} M_C - \frac{1}{2} PL = 0$

Equilibrium Inequality @B : $+\frac{1}{2} M_A + \frac{1}{2} M_C - M_p + \frac{1}{2} PL \geqslant 0$

Using Reduction Formula, Chapter III ; $P = 4.00 \frac{M_p}{L}$

Combine mechanism ABC with mechanism ACDE using the equilibrium inequality for ACDE in either form.

$$
\begin{array}{lrl}
\text{Mechanism } \underline{\text{ABC}} : & +M_A + M_C & -2M_p + PL \geqslant 0 \\
\text{Mechanism } \underline{\text{ACDE}} : & -M_C + M_D - M_E & - M_p + 1\tfrac{1}{2} PL \geqslant 0 \\
\hline
\underline{\text{ABC}}\ \ \underline{\text{ACDE}} : & +M_A + M_D - M_E & -3M_p + 2\tfrac{1}{2} PL \geqslant 0 \\
 & & P = 2.40 \frac{M_p}{L}
\end{array}
$$

$$
\begin{array}{lrl}
\text{Mechanism } \underline{\text{ABC}} : & +M_A + M_C & -2M_p + PL \geqslant 0 \\
\text{Mechanism } \underline{\text{ACDE}} : & +M_A - M_C + M_D & - M_p + 1\tfrac{1}{2} PL \geqslant 0 \\
\hline
\text{ABC}\ \ \text{ACDE} : & +2M_A + M_D & -3M_p + 2\tfrac{1}{2} PL \geqslant 0 \\
 & & P = 2.40 \frac{M_p}{L}
\end{array}
$$

Sections A , D , or E would be satisfactory for forming equilibrium inequalities because there is no possibility of eliminating a plastic hinge at these sections. A plastic hinge can be eliminated at section C and therefore section C is unsatisfactory in this particular application. Section A may be made unsatisfactory by reversing the load at section C (see fig. 16).

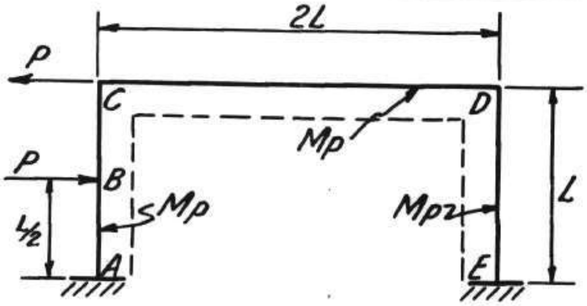


Fig. 16 FRAME ABCDE WITH LOADING AT B & C (OPPOSITE DIRECTION)

Mechanism ACDE :

Equilibrium Equation: $+M_A - M_C + M_D - M_E - \frac{1}{2}PL = 0$

Equilibrium Inequality @A : $+M_C - M_D + M_E - M_p + \frac{1}{2}PL \geqslant 0$

Equilibrium Inequality @E : $-M_A + M_C - M_D - M_p + \frac{1}{2}PL \geqslant 0$

$$P = 8.00 \frac{M_p}{L}$$

Combine mechanism ABC with mechanism ACDE using the correct equilibrium inequality formed at section E .

Mechanism ABC : $+M_A + M_C \quad - 2M_p + PL \geqslant 0$

Mechanism ACDE : $-M_A + M_C - M_D - M_p + \frac{1}{2}PL \geqslant 0$

ABC + ACDE : $+2M_C - M_D - 3M_p + 1\frac{1}{2}PL \geqslant 0$

(M_A eliminated) $\quad P = 4.00 \frac{M_p}{L}$

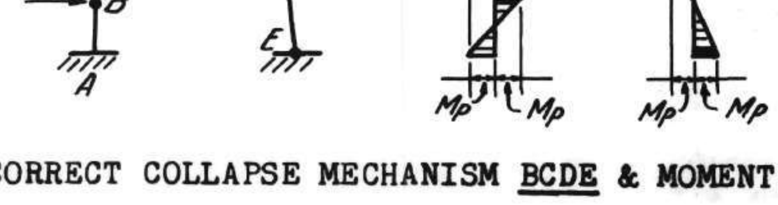


Fig. 17 CORRECT COLLAPSE MECHANISM BCDE & MOMENT DIAGRAM

Actually the combined mechanism failure (see fig. 17) occurs simultaneously with the local failure on column AC (see fig. 18) because both collapse modes have the same minimum value for P .

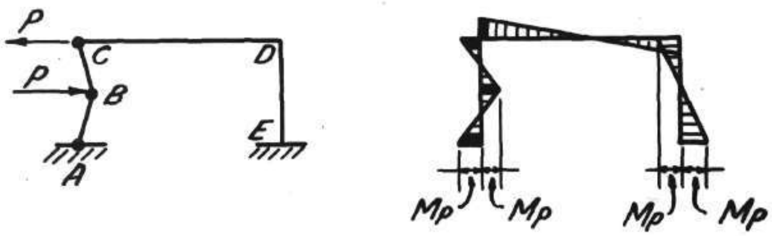


Fig. 18 CORRECT COLLAPSE MECHANISM ABC & MOMENT DIAGRAM

Combine mechanism ABC with mechanism ACDE using the incorrect equilibrium inequality formed at section A .

Mechanism ABC : $+ M_A + M_C \quad - 2 M_p + PL \geqslant 0$

Mechanism ACDE : $+ M_C - M_D + M_E - M_p + \frac{1}{2} PL \geqslant 0$

ABC + ACDE : $+ M_A + 2 M_C - M_D + M_E - 3 M_p + 1\frac{1}{2} PL \geqslant 0$

(M_A not eliminated) $\quad P = 5.33 \frac{M_p}{L}$

M_A can be eliminated in the combination of ABC with ACDE . Therefore, the equilibrium inequality at section A is not satisfactory. Unsatisfactory sections for forming equilibrium inequalities can usually be determined by inspection. When in doubt, it may be necessary to try several equilibrium inequalities formed at different sections.

When combining elementary mechanisms, derive as many equilibrium inequalities as possible from the same given section inequality so that this section may be effectively eliminated from the analysis, thus simplifying the computation. Before forming the equilibrium inequalities, no substitution of terms should be made from the equilibrium equation of one mechanism to that of another; an exception may be made in the case of a "joint mechanism" which has a load term of zero.

CHAPTER VI

COMBINING EQUILIBRIUM INEQUALITIES FOR A MINIMUM LOADING CONDITION

All possible equilibrium inequalities for a structural system may be considered as a collection of definite or separate objects which may be called the "aggregate of the system equilibrium inequalities". In turn, each equilibrium inequality will be the aggregate of the individual plastic moment terms, etc., operative in the respective mechanism. Therefore, the aggregate of the system equilibrium inequalities represented by K will be composed of partial aggregates K_1 , K_2 , K_3 , ... , K_n , each partial aggregate being equal to the respective equilibrium inequality aggregate, with n being equal to the total number of mechanisms available for combination in the structural system. Let k_1 , k_2 , k_3 , ... , k_n represent the aggregates formed by abstracting respectively K_1 , K_2 , K_3 , ... , K_n from aggregate K . The aggregate k_1 will will represent a finite cardinal number of aggregate K . Similarly, k_2, k_3 , ... , k_n will represent different finite cardinal numbers of aggregate K .

The aggregate systems previously defined are conformable to the following basic mathematical theorems.

(1) If K is any aggregate of different finite cardinal numbers, there is one, k_1 , amongst them which is smaller than the rest, and therefore the smallest of all.

(2) Each aggregate $K = \{k\}$ of different finite cardinal numbers can be brought into the form of a series

$$K = (k_1 , k_2 , k_3 , \ldots , k_n)$$

such that

$$k_1 < k_2 < k_3 \quad \ldots < k_n$$

All aggregates which have been considered may be assigned values according to a definite mathematical formulation. The value of aggregate K will be the minimum load determined for the equilibrium inequality resulting from adding all component equilibrium inequalities together. The value of aggregate k_1 , etc., will be the minimum load determined for the equilibrium inequality resulting from adding all component equilibrium inequalities together and subtracting the equilibrium inequality composing the abstracted K_i -type aggregate such as K_1 .

Upon determination of the aggregate such as k_1 , k_2 , k_3 , ... , or k_n having the smaller positive minimum load, this aggregate may now be considered as aggregate K and the process repeated with the exception that previously eliminated aggregates will not be eliminated again. When all k -type aggregates have been formed, their values will be compared with the value of the current aggregate K . If the value of the current aggregate K is less than the value of the smallest k -type aggregate formed, then the operation will cease and the current aggregate K represents the equilibrium inequality for the correct collapse mode. For purposes of comparison a negative minimum load may be considered as nonexistant.

Consider frame ABCDE (see fig. 19) having possible mechanisms $\overline{BCD}$, $\overline{ABDE}$, $\underline{ABDE}$, and $\underline{BCD}$.

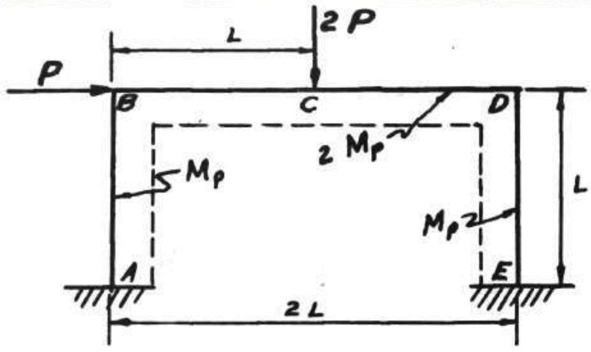


Fig. 19 FRAME ABCDE WITH LOADING AT B & C

Equilibrium Equations:

Mechanism $\overline{ABDE}$, $\underline{ABDE}$: $\pm M_A \mp M_B \pm M_D \mp M_E \pm PL = 0$

Mechanism $\overline{BCD}$, $\underline{BCD}$: $\pm \frac{1}{2}M_B \mp M_C \pm \frac{1}{2}M_D \pm PL = 0$

Equilibrium Inequalities:

Mechanism $\overline{ABDE}$, $\underline{ABDE}$: $\pm M_B \mp M_D \pm M_E - M_p \mp PL \geq 0$

Mechanism $\overline{BCD}$, $\underline{BCD}$: $\mp \frac{1}{2}M_B \mp \frac{1}{2}M_D - 2M_p \mp PL \geq 0$

Row operations on equilibrium inequalities to facilitate hinge elimination:

Mechanism $\overline{ABDE}$, $\underline{ABDE}$: $\pm M_B \mp M_D \pm M_E - M_p \mp PL \geq 0$

Mechanism $\overline{BCD}$, $\underline{BCD}$: $\mp M_B \mp M_D - 4M_p \mp 2PL \geq 0$

$$K = \left[\begin{array}{l} K_1 = \left[- M_B - M_D \qquad - 4M_p - 2PL \geq 0\right] \\ K_2 = \left[+ M_B - M_D + M_E - M_p - PL \geq 0\right] \\ K_3 = \left[- M_B + M_D - M_E - M_p + PL \geq 0\right] \\ K_4 = \left[+ M_B + M_D \qquad - 4M_p + 2PL \geq 0\right] \end{array}\right] \begin{array}{l} (\overline{BCD}) \\ (\overline{ABDE}) \\ (\underline{ABDE}) \\ (\underline{BCD}) \end{array}$$

Evaluate K : $0 + 0 + 0 - 10 M_p - 0 PL \geq 0$

$$P = \infty$$

$$k_1 = \begin{bmatrix} K_2 = & [+M_B & -M_D & +M_E & -M_p & -PL & \geqslant 0] \\ K_3 = & [-M_B & +M_D & -M_E & -M_p & +PL & \geqslant 0] \\ K_4 = & [+M_B & +M_D & & -4M_p & +2PL & \geqslant 0] \end{bmatrix}$$

$$\text{Evaluate } k_1 : +M_B + M_D \quad - 6M_p + 2PL \geqslant 0$$

$$P = 4\,\frac{M_p}{L}$$

$$k_2 = \begin{bmatrix} K_1 = & [-M_B & -M_D & & -4M_p & -2PL & \geqslant 0] \\ K_3 = & [-M_B & +M_D & -M_E & -M_p & +PL & \geqslant 0] \\ K_4 = & [+M_B & +M_D & & -4M_p & +2PL & \geqslant 0] \end{bmatrix}$$

$$\text{Evaluate } k_2 : -M_B + M_D - M_E - 9M_p + PL \geqslant 0$$

$$P = 12\,\frac{M_p}{L}$$

$$k_3 = \begin{bmatrix} K_1 = & [-M_B & -M_D & & -4M_p & -2PL & \geqslant 0] \\ K_2 = & [+M_B & -M_D & +M_E & -M_p & -PL & \geqslant 0] \\ K_4 = & [+M_B & +M_D & & -4M_p & +2PL & \geqslant 0] \end{bmatrix}$$

$$\text{Evaluate } k_3 : +M_B - M_D + M_E - 9M_p - PL \geqslant 0$$

$$P = -12\,\frac{M_p}{L}$$

$$k_4 = \begin{bmatrix} K_1 = & [-M_B & -M_D & & -4M_p & -2PL & \geqslant 0] \\ K_2 = & [+M_B & -M_D & +M_E & -M_p & -PL & \geqslant 0] \\ K_3 = & [-M_B & +M_D & -M_E & -M_p & +PL & \geqslant 0] \end{bmatrix}$$

$$\text{Evaluate } k_4 : -M_B - M_D \quad -6M_p - 2PL \geqslant 0$$

$$P = -4\,\frac{M_p}{L}$$

$k_1 < k_2 < K$; k_3 , k_4 nonexistant; let $k_1 = K'$.

$$k_{2'} = \begin{bmatrix} K_3 = & [-M_B + M_D - M_E - M_p + PL \geq 0] \\ K_4 = & [+M_B + M_D \quad - 4M_p + 2PL \geq 0] \end{bmatrix}$$

Evaluate k_2' : $+2M_D - M_E - 5M_p + 3PL \geq 0$

$$P = 2.67 \frac{M_p}{L}$$

$$k_{3'} = \begin{bmatrix} K_2 = & [+M_B - M_D + M_E - M_p - PL \geq 0] \\ K_4 = & [+M_B + M_D \quad - 4M_p - 2PL \geq 0] \end{bmatrix}$$

Evaluate k_3' : $2M_B + M_E - 5M_p + PL \geq 0$

$$P = 8 \frac{M_p}{L}$$

$$k_{4'} = \begin{bmatrix} K2 = & [+M_B - M_D + M_E - M_p - PL \geq 0] \\ K3 = & [-M_B + M_D - M_E - M_p + PL \geq 0] \end{bmatrix}$$

Evaluate k_4' : $-2M_p-(0)PL \geq 0$

$$P = \infty$$

$k_2' < k_3' < k_4' < K$; let $k_2' = K''$

$$k_3'' = \begin{bmatrix} K_4 = & [\pm M_B + M_D \quad - 4M_p + 2PL \geq 0] \end{bmatrix}$$

Evaluate k_3'' : $+M_B + M_D - 4M_p + 2PL \geq 0$

$$P = 3 \frac{M_p}{L}$$

$$k_4'' = \begin{bmatrix} K_3 = & [-M_B + M_D - M_E - M_p + PL \geq 0] \end{bmatrix}$$

Evaluate k_4'' : $-M_B + M_D - M_E - M_p + PL \geq 0$

$$P = 4 \frac{M_p}{L}$$

$K'' < k_3''$, $K'' < k_4''$; Therefore K'' represents the equilibrium inequalities for the elementary mechanisms composing the correct collapse mode (see fig. 20).

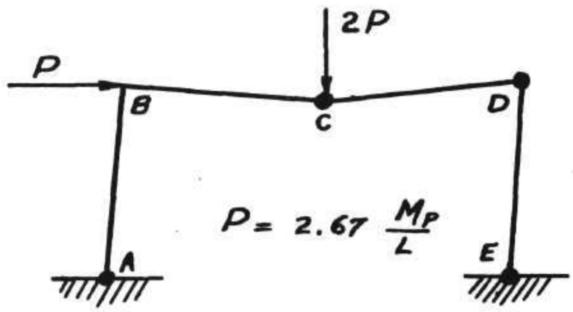


Fig. 20 CORRECT COLLAPSE MECHANISM ACDE

This method is absolute when all mechanisms, both positive and negative, are considered. However, because all mechanisms may not be recognized and consequently considered, it is extremely advisable to draw the moment diagram to ascertain that the correct collapse mode has been determined. This method has an extreme disadvantage inasmuch as row operations, which are necessary to facilitate hinge elimination, must be done prior to combining all mechanism equilibrium inequalities. The necessity for prior row operation may cause an inequality to appear several times in different forms in the aggregate of inequalities. Computation can be shortened by not considering mechanisms which obviously could not be a component mechanism of the final collapse mode. Again, it is advisable to draw the moment diagram to ascertain that the correct collapse mode has been determined.

Dispite some inherent disadvantages in the method outlined, it is superior in most instances to a system dependent upon trial of all combinations which may result in plastic hinge eliminations, the number of trials being roughly proportional to the factorial of the number of possible hinge eliminations.

A number of examples have been worked and are included in the appendix of this paper. These examples will demonstrate how to handle several problems of different types as well as to show simplifications through the use of tabular format and procedure.

CHAPTER VII

COMPUTER PROGRAM FOR DETERMINING MINIMUM LOADING CONDITIONS FOR A STRUCTURAL SYSTEM

One of the chief disadvantages of any method of plastic analysis is the necessity for repeating the entire analysis of a structural system for a variation in loading conditions or a variation in member size. The amount of work required can become prohibitive for more than a few such variations. Design by the plastic methods could be greatly facilitated by the development of design charts for a great many common types of structures. The computative work required for such design charts could be greatly reduced by the use of high-speed digital computing equipment.

For the most satisfactory results, design methods used with such equipment should involve a procedure which can be reduced to a systematic, arithmetic operation. The method of analysis described in Chapter VI of this paper satisfies the requirement for a systematic, arithmetic operation or procedure. A program for this method has been written for the Remington Rand 1101 Digital Computer in accordance with programming manual PX 77000-A.

The method for arranging data, the data required, and the location or address where this data will be stored in the computer memory will be described in this section. The program, together with operational data, is included in the appendix of this paper.

A typical equilibrium inequality may be divided into three types of parts - section plastic moment terms, the plastic stiffness term, and the load term. An aggregate composed of a number of equilibrium inequalities may be arranged in format so as to have columns corresponding to each plastic moment term at each section, a column corresponding to the plastic stiffness term, and a column corresponding to the load term. To adapt this aggregate to the computer program it is necessary to multiply the entire aggregate by -1 .

Consider frame 1-2-3-4-5 (see fig. 21) having possible mechanisms $\overline{1\text{-}2\text{-}4\text{-}5}$, $\underline{1\text{-}2\text{-}4\text{-}5}$, $\overline{2\text{-}3\text{-}4}$, and $\underline{2\text{-}3\text{-}4}$.

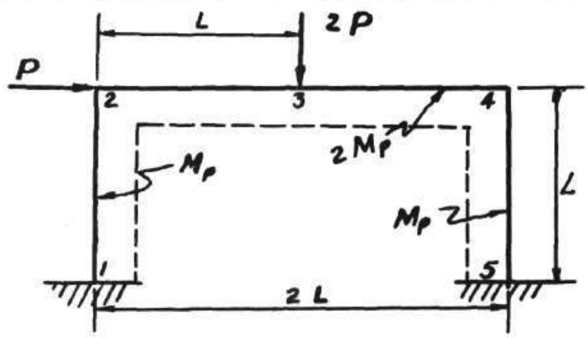


Fig. 21 FRAME 1-2-3-4-5

Equilibrium Inequalities:

Mechanism $\overline{1\text{-}2\text{-}4\text{-}5}$: $+M_2 - M_4 + M_5 - M_p - PL \geqslant 0$

Mechanism $\underline{1\text{-}2\text{-}4\text{-}5}$: $-M_2 + M_4 - M_5 - M_p + PL \geqslant 0$

Mechanism $\overline{2\text{-}3\text{-}4}$: $-M_2 - M_4 \quad -4M_p - 2PL \geqslant 0$

Mechanism $\underline{2\text{-}3\text{-}4}$: $+M_2 + M_4 \quad -4M_p + 2PL \geqslant 0$

Multiply the aggregate of equilibrium inequalities by -1 :

Mechanism $\overline{1\text{-}2\text{-}4\text{-}5}$: $-M_2 + M_4 - M_5 + M_p + PL \geqslant 0$

Mechanism $\underline{1\text{-}2\text{-}4\text{-}5}$: $+M_2 - M_4 + M_5 + M_p - PL \geqslant 0$

Mechanism $\underline{\overline{2\text{-}3\text{-}4}}$: $\quad M_2 + M_4 \qquad +4M_p + 2PL \geqslant 0$

Mechanism $\underline{2\text{-}3\text{-}4}$: $\quad -M_2 - M_4 \qquad +4M_p - 2PL \geqslant 0$

Table 1 represents the described column arrangement for the mechanisms in frame 1-2-3-4-5 .

Table 1. Arrangement of Terms for Computer Program

	Load Column	Plastic Stiffness Column	Section Plastic Moment Columns			Mechanism
	$-PL$	$-M_p$	$-M_2$	$-M_4$	$-M_5$	
Θ	1	1	-1	1	-1	$\underline{\overline{1\text{-}2\text{-}4\text{-}5}}$
Θ	-1	1	1	-1	1	$\underline{1\text{-}2\text{-}4\text{-}5}$
Θ	2	4	1	1	0	$\underline{\overline{2\text{-}3\text{-}4}}$
Θ	-2	4	-1	-1	0	$\underline{2\text{-}3\text{-}4}$
	0	10	0	0	0	Summation of all mechanisms.
		G	G	G	G	
			$(M_p)_2$	$(M_p)_4$	$(M_p)_5$	Section Plastic Stiffness
			1	1	1	

In table 1 the constant Θ represents the number of inequalities composing the aggregate. The constant G represents the total number of section plastic moment columns plus one.

The memory of the 1101 computer is a magnetic drum having 2^{14} or 16,384 locations (addresses) where numerical information may be stored in the form of a twenty-four binary digit number or word at each address used. The computer will interpret some of these words as instructions, but will use others as operands. The addresses allocated for the operands in the Plastic Design Routine are shown in the Storage Diagram (see fig. 22).

The values shown in table 1 may be assigned addresses in accordance with the Storage Diagram. Table 2 gives table 1 values with their respective address assignments.

Table 2. Address Assignment of Terms

	$-PL$	$-M_p$	$-M_2$	$-M_4$	$-M_5$	Mechanism	
θ 4 (20201)	1 (20202)	1 (20203)	-1 (20204)	1 (20205)	-1 (20206)	1-2-4-5	K_1
θ 4 (20301)	-1 (20302)	1 (20303)	1 (20304)	-1 (20305)	1 (20306)	1-2-4-5	K_2
G 4 (20401)	2 (20402)	4 (20403)	1 (20404)	1 (20405)	0 (20406)	2-3-4	K_3
G 4 (20501)	- 2 (20502)	4 (20503)	-1 (20504)	-1 (20505)	0 (20506)	2-3-4	K_4
P (20001)	0 (20002)	10 (20003)	0 (20004)	0 (20005)	0 (20006)		K

T. C.
high no. (20101)

$(M_p)_2$	$(M_p)_4$	$(M_p)_5$	Section Plastic Stiffness
1 (20104)	1 (20105)	1 (20106)	

It is necessary to multiply the values listed in table 1 and table 2 by appropriate scale factors in order to get the answer to a suitable number of decimal places. The scale factor may be expressed as

GROUP INDEX 1	2	3	4	5	6	-	77		ORBIT INDEX
P 20001*	$\lesssim(-PL)$ 20002	$\lesssim(-M_p)$ 20003	$\lesssim(-M_1)$ 20004	$\lesssim(-M_2)$ 20005	$\lesssim(-M_3)$ 20006	-	$-\lesssim(-M_{60})$ 20077	K	2000
T. C. 20101			$(M_p)_1$ 20104	$(M_p)_2$ 20105	$(M_p)_3$ 20106	-	$(M_p)_{60}$ 20177		
θ 20201	$-PL$ 20202	$-M_p$ 20203	$-M_1$ 20204	$-M_2$ 20205	$-M_3$ 20206	-	$-M_{60}$ 20277	K_1	2020
θ 20301	$-PL$ 20302	$-M_p$ 20303	$-M_1$ 20304	$-M_2$ 20305	$-M_3$ 20306	-	$-M_{60}$ 20377	K_2	2030
G 20401	$-PL$ 20402	$-M_p$ 20403	$-M_1$ 20404	$-M_2$ 20405	$-M_3$ 20406	-	$-M_{60}$ 20477	K_3	2040
G 20501	$-PL$ 20502	$-M_p$ 20503	$-M_1$ 20504	$-M_2$ 20505	$-M_3$ 20506	-	$-M_{60}$ 20577	K_4	2050
	$-PL$ 20602	$-M_p$ 20603	$-M_1$ 20604	$-M_2$ 20605	$-M_3$ 20606	-	$-M_{60}$ 20677	K_5	2060
content address	$-PL$ 20702	$-M_p$ 20703	$-M_1$ 20704	$-M_2$ 20705	$-M_3$ 20706	-	$-M_{60}$ 20777	K_6	2070
	$-PL$ 21002	$-M_p$ 21003	$-M_1$ 21004	$-M_2$ 21005	$-M_3$ 21006	-	$-M_{60}$ 21077	K_7	2100
	$-PL$ 21102- 37002	$-M_p$ 21103- 37003	$-M_1$ 21104- 37004	$-M_2$ 21105- 37005	$-M_3$ 21106-21177 - 37006-37077	-	$-M_{60}$	K_8- K_{118}	2110- 3700

P = Minimum Load
PL = Load Term
M_p = Plastic Stiffness Term
M_i = Section Plastic Moment Term
$(M_p)_i$ = Section Plastic Stiffness Term
θ = Total Number of Inequalities
G = Total Number of Plastic Moment Terms + 1
K_i = Equilibrium Inequality for Elementary Mechanism
K = Summation of Equilibrium Inequalities
T. C. = Test Constant

Fig. 22 STORAGE DIAGRAM

*Addresses are expressed in octal notation, ie. radix is eight.

power of ten. Suitable scale factors may be readily determined by an operation generalized somewhat as follows:

$$(P)(10^x) = \frac{\left|(M_p)(10^p)\right| + \left|(M_i)(10^r) \times (M_p)_i\ (10^s)\right|}{(PL)(10^n)}$$

10^p must equal 10^{rs} .

Therefore, 10^x is equal to 10^{p-n} .

$p, r, s, n, \& x =$ exponents of 10.
$P =$ Minimum Load.
$PL =$ Load Term.
$M_p =$ Plastic Stiffness Term.
$M_i =$ any Section Plastic Moment Term.
$(M_p)_i =$ Plastic Stiffness Term at section i .

Appropriate selection of scale factors so that x is equal to three has been found satisfactory in most cases and allows the program to converge properly.

The minimum load P equal to infinity for the initial problem setup may be satisfactorily handled by letting P equal the computer's maximum positive value which is octal 37777777 ; similarly, the test quantity C. T. will normally be assigned this maximum positive value.

The output from the computer will be printed in decimal form. The answer will consist of the minimum load P for the equilibrium inequality representing the mechanisms composing the collapse mode, an entry of one line consisting of ones and zero's to signify plastic hinge locations, and an entry of two lines consisting of two's and zero's to signify component equilibrium inequalities. In the lines signifying plastic hinge locations and component equilibrium inequalities, zero represents the absence of a hinge or an inequality. A one signifies the location of a plastic hinge and a two signifies the presence of an equilibrium inequality. The order and arrangement of these terms matches the original order and arrangement of terms.

Arranging scale factors so that the answer has a scale factor of $10^x \quad 10^3$, the answer to the example problem would appear as illustrated in figure 23.

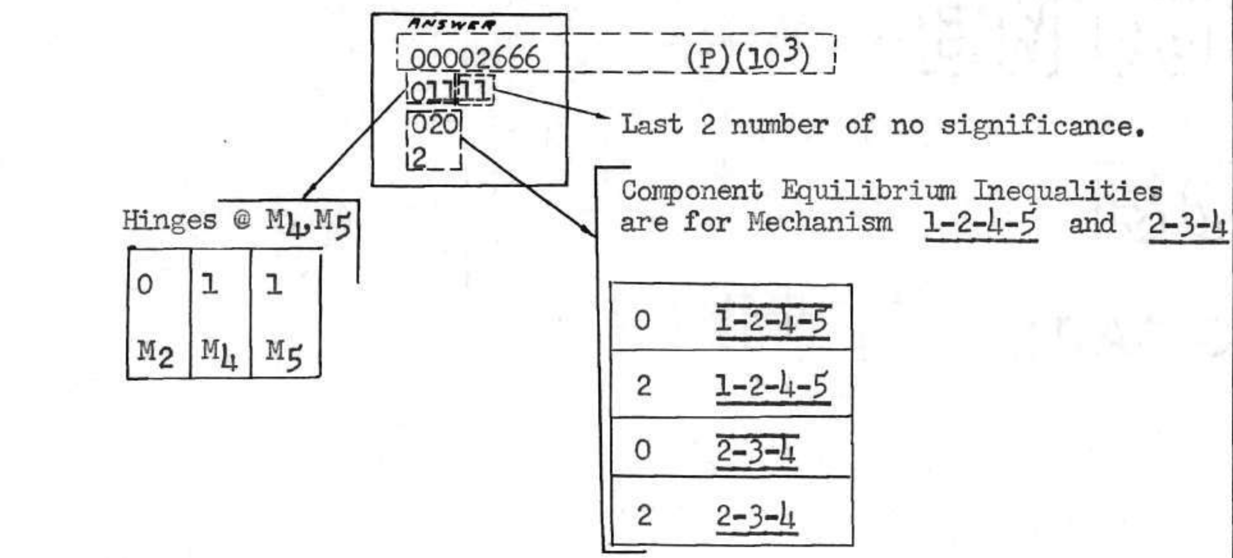


Fig. 23 ARRANGEMENT OF ANSWER.

APPENDIX "A"

COMPUTER PROGRAM

TITLE: Plastic Design Routine # 1 START: 37777

INTERLACE: C SKIPS: $b_{17}, 2^8$; $b_{16}, 2^7$; $b_{15}, 2^6$ STOP: 14373

ADDRESS	CONTENTS	ADDRESS	CONTENTS	ADDRESS	CONTENTS
37777	11100077				
00077	00000001	04077	35020301	10077	35037415
00177	35020103	04177	11000677	10177	11110277
00277	11100377	04277	35003177	10277	00000060
00377	00000100	04377	27003277	10377	35037416
00477	35037402	04477	11020401	10477	11110577
00577	11100677	04577	12104677	10577	00000033
00677	00020200	04677	00000002	10677	35037417
00777	35037400	04777	35037404	10777	56000000
01077	11101177	05077	35037405	11077	110(20000)
01177	00020202	05177	11105277	11177	350(37500)
01277	35037401	05277	00000037	11277	71011077
01377	11101477	05377	35037406	11377	27011077
01477	37777777	05477	11105577	11477	71011177
01577	35037301	05577	00000052	11577	27011177
01677	11101777	05677	35037407	11677	11037405
01777	00000000	05777	11106077	11777	47212077
02077	35020000	06077	00000074	12077	35037405
02177	35020102	06177	35037410	12177	45011077
02277	35037300	06277	11106377	12277	11037404
02377	35037403	06377	00000070	12377	35037405
02477	11020301	06477	35037411	12477	11112577
02577	47002777	06577	11106677	12577	00020000
02677	45003777	06677	00000064	12677	27011077
02777	35020301	06777	35037412	12777	11113077
03077	11103177	07077	11107177	13077	00037500
03177	(00020200)	07177	00000062	13177	27011177
03277	350(20200)	07277	35037413	13277	56000000
03377	12037402	07377	11107477	13377	11020301
03477	35003177	07477	00000066	13477	47013677
03577	27003277	07577	35037414	13577	45013074
03677	45002477	07677	11107777	13677	35020301
03777	11020201	07777	00000072	13777	12020103

ADDRESS	CONTENTS	ADDRESS	CONTENTS	ADDRESS	CONTENTS
14077	14020201	01074	27017477	06074	45013377
14177	47014377	01174	11020501		
14277	45015277	01274	47201374		
14377	11037402	01374	35020501		
14477	12037403	01474	45016777	06474	41000000
14577	35037403	01574	11020401	06574	14020001
14677	12037400	01674	35020501	06674	46007274
14777	27015377	01774	11102074	06774	47207074
15077	27015677	02074	00020002	07074	56000000
15177	27015777	02174	27016777	07174	45013377
15277	11020000	02274	11102374	07274	11037101
15377	140(20200)	02374	00037102	07374	14037301
15477	47015777	02474	27017177	07474	46010074
15577	11020102	02574	11102674	07574	47207674
15677	350(20200)	02674	00020102	07674	56000000
15777	110(20200)	02774	27017377	07774	45013377
16077	47016377	03074	11103174	10074	56000000
16177	56000000	03174	00037202	10174	11037400
16277	45013377	03274	27017477	10274	12037403
16377	11037401	03374	56000000	10374	35137100
16477	12037403	03474	37000000	10474	110(37100)
16577	56000000	03574	220(37202)	10574	350(37300)
16677	27017077	03674	43000000	10674	71010474
16777	110(20002)	03774	71003574	10774	27010474
17077	140(20202)	04074	27003574	11074	71010574
17177	350(37102)	04174	11020501	11174	27010574
17277	43000000	04274	47304374	11274	11037405
17377	610(20102)	04374	35020501	11374	47211474
17477	350(37202)	04474	41000000	11474	35037405
17577	71016777	04574	45003574	11574	45010474
17677	45000074	04674	36037201	11674	11037404
		04774	11020401	11774	35037405
00074	27016777	05074	35020501	12074	11112174
00174	71017077	05174	11105274	12174	00037100
00274	27017077	05274	00037202	12274	27010474
00374	71017177	05374	27003574	12374	11112474
00474	27017177	05474	56000000	12474	00037300
00574	71017377	05574	11037102	12574	27010574
00674	27017377	05674	46015073	12674	56000000
00774	71017477	05774	56000000	12774	45013377

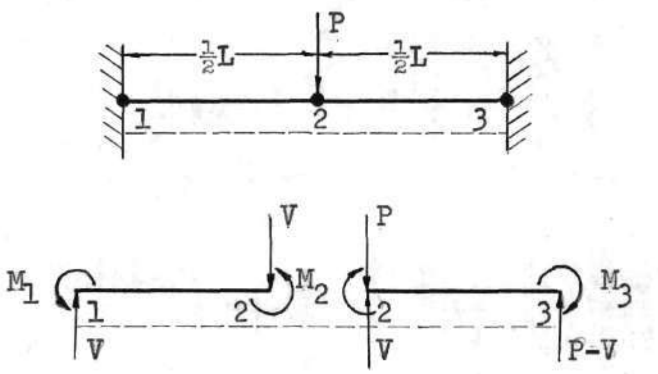
ADDRESS	CONTENTS	ADDRESS	CONTENTS	ADDRESS	CONTENTS
13074	11020201	00073	71017574	05073	11020501
13174	35020301	00173	27017574	05173	47205273
13274	11020102	00273	14100373	05273	35020501
13374	35037403	00373	35037430	05373	45003473
13474	11037400	00473	46200573	05473	11020401
13574	27015377	00573	41000000	05573	35020501
13674	27015677	00673	45017374	05673	53002773
13774	27015777	00773	11101073	05773	56000000
14074	11114174	01073	00037420	06073	11020301
14174	00020202	01173	27017574	06173	63004677
14274	27017077	01273	11101373	06273	36037430
14374	11037300	01373	00037406	06373	11020301
14474	14020000	01473	120(37427)	06473	47006673
14574	47015074	01573	27001673	06573	45010673
14674	56000000	01673	53037406	06673	35020301
14774	45017274	01773	11001473	06773	110(20200)
15074	110(37300)	02073	14102173	07073	47107173
15174	350(20000)	02173	12000001	07173	53137410
15274	71015074	02273	27001473	07273	53037406
15374	27015074	02373	14102473	07373	11037406
15474	71015174	02473	00037417	07473	47007773
15574	27015174	02573	47001273	07573	53002773
15674	11037405	02673	53102773	07673	11020201
15774	47216074	02773	00000045	07773	35037430
16074	35037405	03073	56000000	10073	11037402
16174	45015074	03173	11103273	10173	12037403
16274	11037404	03273	00037427	10273	35037400
16374	35037405	03373	27001473	10373	12037400
16474	11012577	03473	11020101	10473	27006773
16574	27015174	03573	240(20004)	10573	45006373
16674	11116774	03673	47004273	10673	11020201
16774	00037300	03773	110(20004)	10773	35020301
17074	27015074	04073	47204173	11073	11020102
17174	45013377	04173	53337407	11173	35037403
17274	11020001	04273	46004473	11273	11037400
17374	63117474	04373	53137406	11373	27006773
17474	00000012	04473	53037417	11473	56000000
17574	350(37420)	04573	71003573	11573	110(37500)
17674	45000073	04673	27003573	11673	350(20000)
		04773	27003773	11773	71011573

ADDRESS	CONTENTS
12073	27011573
12173	71011673
12273	27011673
12373	11037405
12473	47212573
12573	35037405
12673	45011573
12773	11037404
13073	35037405
13173	11012577
13273	27011673
13373	11013077
13473	27011573
13573	53002773
13673	53002773
13773	11114073
14073	00020004
14173	27003573
14273	27003773
14373	55000000
15073	15115173
15173	77777777
15273	35037200
15373	11037201
15473	63037200
15573	36037101
15673	45006474

APPENDIX "B"

FORMING STATICAL EQUILIBRIUM EQUATIONS

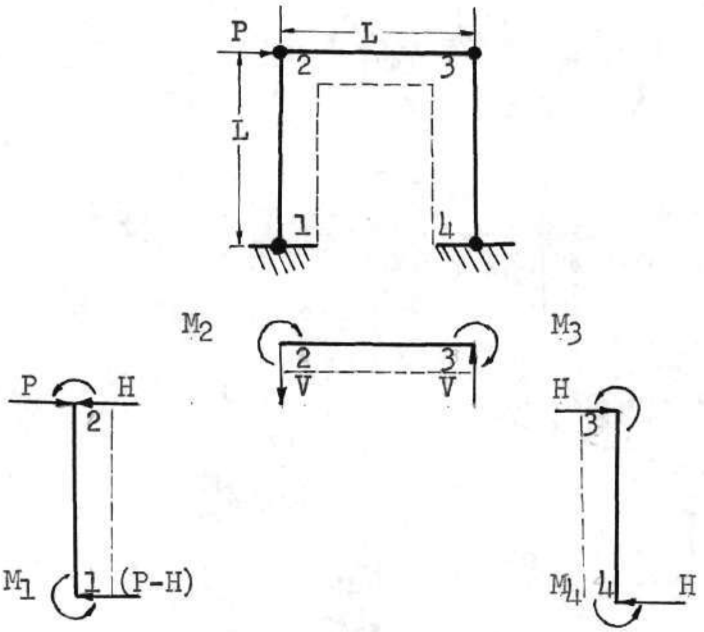
STATICAL EQUILIBRIUM EQUATION FOR A TYPICAL BEAM MECHANISM



$$-M_1 + M_2 = V(\tfrac{1}{2}L) \qquad + M_2 - M_3 = (P-V)(\tfrac{1}{2}L)$$

$$-M_1 + 2M_2 - M_3 = P(\tfrac{1}{2}L)$$

STATICAL EQUILIBRIUM EQUATION FOR A TYPICAL PANEL MECHANISM



$$-M_1 + M_2 = (P-H)L \qquad M_2 - M_3 = VL \qquad -M_3 + M_4 = HL$$

$$-M_1 + M_2 - M_3 + M_4 = PL$$

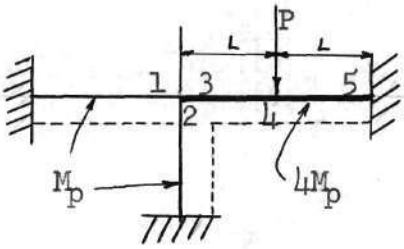
APPENDIX "C"

EXAMPLES

EXAMPLE I: TABULAR ARRANGEMENT FOR EXAMPLE PROBLEM OF CHAPTER VI

	M_B	M_D	M_E	M_p	PL	$\frac{M_p}{L}$	
1	-1	-1	0	-4	-2		$\overline{BCD}$
2	1	-1	1	-1	-1		$\overline{ABDE}$
3	-1	1	-1	-1	1		ABDE
4	1	1	0	-4	2		BCD
	0	0	0	-10	0	∞	Σ
Σ-1	1	1	0	-6	2	4.00	
Σ-2	-1	1	-1	-9	1	12.00	
Σ-3	1	-1	1	-9	-1	-12.00	
Σ-4	-1	-1	0	-6	-2	-4.00	
Σ-1	1	1	0	-6	2	4.00	Σ'
Σ'-2	0	2	-1	-5	3	2.67	
Σ'-3	2	0	1	-5	1	8.00	
Σ'-4	0	0	0	-2	0	∞	
Σ'-2	0	2	-1	-5	3	2.67	Σ''
Σ''-3	1	1	0	-4	2	3.00	
Σ''-4	-1	1	-1	-1	1	4.00	
3+4	0	2	-1	-5	3	2.67	

EXAMPLE II: MECHANISM FAILURE WITH JOINT ROTATION



INEQUALITIES:

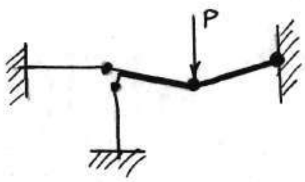
Joint Mechanism 1-2-3 : $M_1 - M_3 - M_p \geqslant 0$

$-M_1 + M_3 - M_p \geqslant 0$

Beam Mechanism 3-4-5 : $M_3 + M_5 - 8\,M_p + PL \geqslant 0$

M_p	$4M_p$	$4M_p$			Relative Plastic Stiffness	
M_1	M_3	M_5	M_p	PL	$\frac{M_p}{L}$	
1 -1 0	-1 1 1	0 0 1	-1 -1 -8	0 0 1	∞ ∞ 16	1-2-3 1-2-3 3-4-5
0	1	1	-10	1	18	
-1 1 0	2 0 0	1 1 0	-9 -9 -2	1 1 0	22 14 ∞	
1	0	1	-9	1	14	
0 1	1 -1	1 0	-8 -1	1 0	16	

FINAL COLLAPSE MECHANISM:



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