DYNAMIC TRANSFORMER PROTECTION

A NOVEL APPROACH USING STATE ESTIMATION

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Stephane Ntwoku Ntuomou

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Approved by:

Dr.A.P. "Sakis" Meliopoulos, Advisor
School of Electrical and Computer engineering
Georgia Institute of Technology

Dr. Santiago Grijalva
School of Electrical and Computer Engineering
Georgia Institute of Technology

Dr. Thomas G. Habetler
School of Electrical and Computer Engineering
Georgia Institute of Technology

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LIST OF SYMBOLS AND ABBREVIATIONS

$i_{1\phi}$	Transformer Current Vector
x	Transformer State Vector
v_1	Primary Voltage of Ungrounded Terminal
v_2	Primary Voltage of Grounded terminal
v_3	Secondary Voltage of Ungrounded terminal
v_4	Secondary Voltage of grounded Terminal
i_{L1}	Current through Primary leakage Inductance
i_{L2}	Current through Secondary leakage Inductance
e	Inner Core Voltage
λ	Non-linear Inductor Flux
i_m	Magnetizing Current
y_n	Additional Internal States from Quadratization
g_1	Primary winding Conductance
g_2	Secondary Winding Conductance
t	Transformer Turns Ratio
g_c	Magnetic Core Conductance
i_0	Nominal Magnetizing Current
G_1	Primary Stabilizing Conductance
G_2	Secondary Stabilizing Conductance
n	Exponent in Magnetizing Characteristic
Y_{eq}	Linear Component of Algebraic Companion Form
B_{eq}	Previous States Components of Algebraic Companion Form
F_{eq}	Non-Linear Component of Algebraic Companion Form

Q	Quadratic Coefficient
Z	Measurement Matrix
i_p	Primary current sample
i_s	Secondary Current Sample
v_p	Primary Voltage Sample
v_s	Secondary Voltage Sample
η	Statistical error on Measurement
$h(x)$	Objective Function of State Estimation
H	Jacobian Matrix of Objective Function
W	Weight Matrix
X	Computed State Estimate
σ	Standard deviation of Measurements
b_i	Measurement Component
μ	Total number of Measurement Equations
ρ	Total Number of Variables in State Vector
I_{pu}	Pickup Current
CT	Current Transformer
TD	Time Dial

SUMMARY

Transformers are present in almost every electric power system involving different voltage levels and not only do they occur in a wide variety of sizes but also they are used to transfer energy from one circuit to another at almost all the different kV levels of the power supply system. The largest transformers have prices ranging into the millions of dollars but their health affect the reliability of the power system as a whole.

In the advent of disturbances, transformers must be protected not only to ensure that they are not destroyed, but also to avoid the spread of the disturbance to adjacent electrical systems that may inherently lead to the entire system collapsing as has been the case in numerous of events.

Transformer protection schemes must primarily be able to detect internal faults in the transformer with a high degree of sensitivity, such that the transformer is subsequently de-energized to avoid damage to both the transformer itself and neighboring devices, while still being immune to faults external to the transformer such as through faults. Sensitive detection and de-energization enables the mitigation of the propagation of the fault to adjacent electrical systems and hence necessary repairs are reduced which is vital considering the lead time for repair and/or replacement of large transformers and their peripheral systems is costly.

In addition to providing appropriate protection, protection schemes should be highly insensitive to events such as inrush currents occurring during the energization of transformers which are not actual faults but have been found to cause false tripping of the traditional differential protection schemes on the transformer. This added complexity due to the different operation modes of the transformer leads to complex protection schemes to be implemented by protection engineers.

For this reason, we introduce a new protection scheme based on improvements in technology such as the availability of GPS synchronized measurements to provide settingless protection on the transformer. The method is based on the availability of measurements and their exact models on both sides of the transformer such that a state estimation procedure is implemented to determine the health of the transformer. Based on the estimate a statistical verification such as the chi square test is used and in real time measurements either fit the model and the transformer allowed to operate or not and the transformer is tripped.

The foundation upon which our protection scheme is built is the modeling of the single phase transformer system of equations. The transformer equations are composed of polynomial and differential equations and this system of equations involving the transformer's electrical quantities are modeled into a system of equations such that highest degree of each of the system's equations is quadratic—in a process named Quadratization. Then, the quadratized system of equations is integrated numerically into an algebraic companion form where the remaining dynamics/differential equations still present in the quadratized model is integrated numerically using a technique called Quadratic Integration to a set of algebraic equations involving previous and future states of the transformer.

The contributions of this research are:

1. Development of a quadratized model for describing single phase transformers.
2. Development of a quadratic integration procedure for converting differential equations to an algebraic system of companion equations-Algebraic Companion Form for single phase transformers
3. State Estimation procedure was applied to the Algebraic Companion Form of the single phase transformer and novel protection function for single phase transformers.
4. The scheme was tested on abnormal operating conditions of the transformer and the scheme performed well on determining whether the conditions warranted tripping or not.

CHAPTER 1: INTRODUCTION

Transformers are vital and expensive components of the power systems industry and due to the fact that they require long lead times for repair or replacement, their protection from damage is a priority.

During abnormal operating conditions, not only is the health of the transformer at risk but also a big portion of the system equipment might as well be damaged which is just a small consequence especially if you consider the fact that interruption of service will occur for customers. Since the lead time for repair and replacement of transformers is very long, limiting the damage of these precious pieces of equipment is vital to the proper operation of electrical systems.

The protection of the transformer requires that we protect it not only from electrical anomalies but also from mechanical abnormal conditions. In this thesis though, we will focus more on the protection against anomalous internal electrical phenomena and more specifically we will give motivation for the necessity for new transformer protection schemes using state estimation and modeling of the transformer. Internal transformer failures such as winding phase faults, inter-turn faults, core insulation faults and tank faults can all be dealt with differential protection schemes, making differential protection the most popular and effective transformer protection scheme(at least as far as internal faults are concerned) hence our interest in enhancing this technique.

1.1 Problem Statement

The reliability of electrical systems is greatly dependent on the reliability of its components. In today's vastly interconnected electric grid, a failure at one point can cause widespread devastating effects to other points on the grid, hence the importance of protection on devices such as transformers which are very common throughout power systems. Transformer failures occur on a frequent basis mainly due to strategies such as increased equipment utilization, deferred capital expenditures and reduced maintenance expenses on behalf of transformer owners. To make matters worse, world power consumption is increasing, and the load on transformers continues to grow.

It is also worth noting that the complex operation modes of the transformer also cause a fair share of disruption in the operation of power systems. This generally occurs as a result of false trips such as those caused by inrush currents associated with energization. These events are not actual faults but come as a result of the change in impedance of the magnetizing branch of the transformer. These currents trip the transformer and cause outages, and though the outages might be momentary they still cause economic stress on the consumer connected through that transformer.

1.2 Research Objectives

The problems associated with transformer protection that is described above will be addressed by the implementation of a protection scheme that has the following desirable features:

1. High degree of sensitivity to transformer internal faults.

2. Low sensitivity to inrush current that are not caused by internal faults or come as a result of normal operation of the transformer e.g. energization events.
3. Low sensitivity to external faults on the transformer.

CHAPTER 2: LITERATURE REVIEW

Initially we will provide a general overview of transformer protection as a whole to serve as foundation for further discussions.

The type of protection used on a transformer depends on the application and importance of the transformer. In general transformers are protected by fuses, over-current relays, differential relays and pressure relays. Which of the above devices will be used in any given situation will depend upon several factors discussed below.

Transformers that have ratings below 2500kVA are usually protected with fuses [1]. In transformers between 2500 and 5000kVA could be used but instantaneous and time-delay over-current relays are more desirable from the standpoint of sensitivity and coordination with protective relays on the high and low side of the transformer. Between 5000 – 10000kVA an induction disc over-current relay connected in a differential configuration is usually applied. Above 10MVA harmonic restraint differential protection is the most commonly used and recommended mode of action [2]. Pressure and temperature relays might also be applied with this transformer size as well.

Apart from the size of the transformer the location of the transformer within the overall network also plays a role in the type of protection scheme to be employed. If the transformer is an integral part of the bulk power system, it will probably require the more sophisticated relays in terms of design and redundancy. If it is say a distribution station step-down transformer, a single differential relay and an over-current backup relay might be enough.

The transformer voltage rating greatly affects the type of protection used, due to the fact that there is a direct relationship between the cost and the voltage rating, hence the rule of thumb: the larger the voltage rating of the transformer, the greater the sophistication of the protection scheme that will be used.

The issues described above depict a relatively straight forward way to choose the type of protection to be employed by transformers. This description is not complete without actually looking at how transformer abnormal behavior occurs and what types of protection will be used in this case so that we fully understand the issue of transformer protection. Transformer failures include:

- Winding faults due to short-circuits (turn to turn faults, phase-phase faults, phase-ground, open winding)).
- Core faults which include core insulation failure and shorted laminations.
- Terminal failures which include open leads, loose connections, short circuits.
- On load tap changer failures.
- Abnormal operating conditions such as overfluxing, overvoltage, overloading.
- External Faults.

These different failures can be faced using different strategies and the protection philosophy associated with each failure is summarized in the following table.

Table 2.1: Typical transformer failures and protection schemes

CONDITIONS	**PROTECTION PHILOSOPHY**
INTERNAL	
Winding Phase-Phase, Phase- Ground Faults	Differential(87T),Overcurrent(51,51N), Restricted Ground Fault Protection (87RGF)
Winding Inter-turn Faults	Differential(87T), Buchholz relay
Core Insulation Failure, Shorted laminations	Differential protection(87T), Buchholz relay, sudden pressure relay
Tank faults	Differential(87T), Buchholz relay, tank-ground protection
Overfluxing	Volts/Hz(24)
EXTERNAL	
Overloads	Thermal(49)
Overvoltage	Overvoltage(59)
Overfluxing	Volts/Hz
External system short circuits	Time Overcurrent(51,51G),Instantaneous Overcurrent(50, 50G)

2.1 Overview Of Transformer Differential Protection

As can be seen from Table 2.1 differential protection is the primary protection scheme that is used to ensure the protection of transformers against internal damage. Differential protection is based on the notion that, under normal operating conditions the ratio of the primary and secondary currents is constant and approximately inverse to the transformer turns' ratio. Hence, assuming for simplicity that we have a single phase transformer the quantity $I_0=I_1N_1-I_2N_2$ will remain nearly equal to zero (where I_1, I_2 represent the primary and secondary current and N_1, N_2 the primary and secondary turns' ratios) is approximately equal to zero unless an internal fault occurs. This fairly straightforward principle is very effective and with the improvements in numerical relays, protection functions can be implemented fairly easily. The factors affecting differential protection are [3]:

- Magnetizing inrush currents, over-excitation and Current Transformer (CT) saturation are conditions that can cause imbalance in the currents that are measured in the relay hence affecting the protection scheme.
- Differential Voltage levels affect differential protection since they lead to different current levels in the CTs.
- Transformer configuration affects differential protection due to phase shifts in wye and delta connected transformers.
- Tap changes in transformers also affect differential protection.

Despite its efficiency, differential protection is greatly challenged by the presence of inrush currents especially during the energization of the transformer causing the relay to trip despite the absence of an internal fault. As a matter of fact, any electrical phenomenon that has the property of lowering the magnetizing impedance of the transformer will create inrush currents and lead to non-proportional primary to secondary currents causing erroneous tripping of the numerical relay. Furthermore the use of digital relays has enhanced this problem even more due to the presence of CT used for instrumentation purposes. These CTs may saturate as well and cause false tripping just as the actual transformer they are monitoring.

2.2 Inrush Currents

Magnetizing inrush currents are generally responsible for the “fake-false” trips of transformer protection relays. The presence of inrush currents especially during the energization of the transformer causes the relay to trip despite the absence of an internal fault. As a matter of fact, any electrical phenomenon that has the property of lowering the magnetizing impedance of the transformer will create inrush currents several times the full load current [4] and lead to non-proportional primary to secondary currents causing erroneous tripping of the differential relay. Examples of such phenomena causing inrush currents [5] include and are not limited to:

- Occurrence of external faults
- Voltage recovery after clearing an external fault
- Change of the character of a fault (e.g. when a phase to ground fault evolves into a phase to phase to ground fault)
- Out of phase synchronizing of a connected generator

Furthermore, the use of digital relays inherently enhances this problem due to the presence of Current Transformers used for instrumentation purposes. These CTs may saturate under certain conditions and cause false tripping just as the actual transformer they are monitoring during energization. Inrush currents may be classified according to the following types [1]:

- Initial: Currents occurring during energization after a long period of de-energization which usually creates high value currents.
- Recover: They occur during return to normal of momentary dip in voltage. Such a phenomenon occurs when for example, a sold 3 phase fault occurs near a transformer

bank. When the fault is cleared such an inrush may occur, even though the current may not be as high as that in the case of initial inrush current.

- Sympathetic inrush current: Occurs in energized transformer when a nearby transformer is being energized e.g. paralleling a second transformer in a bank with one transformer already in operation which may saturate.

2.2.1 Factors Affecting Inrush Currents

Let us examine the factors affecting the inrush currents so as to provide some insight as to how this phenomenon can be handled [3].

- The size of the transformer
- The nature of the power system source
- Type of iron used in the fabrication of the transformer
- The previous history of the transformer— remnant flux especially
- L/R ratio of the transformer and system

2.3 Prevention Methods against False-Tripping

Due to the fact that false tripping of transformer relays is a common phenomenon, utilities and scientists have developed strategies to prevent this phenomenon from occurring.

Rectifier relays were initially used to avoid false tripping due to inrush currents. This method takes advantage of the fact that magnetizing inrush currents are in fact half frequency waves. This relays are hence designed using rectifiers such that they have two current sensing elements: one monitors the positive currents and the other monitoring the negative currents. Both elements must operate in order for a trip signal to be issued. On inrush though, only one element is activated and hence no trip occurs. Internal faults though cause sinusoidal current waves hence they are picked up by the relay [6]. A variation of the above method is a technique which measures the time (dwell time) the current waveform stays close to zero which indicates a full dc-offset [7]. This in turn is used to determine the inrush condition. The relays operating on this scheme typically expect dwell times of about $1/4$ cycle and will restrain tripping if this occurs.

Another unique method to prevent false tripping is the method whereby the flux- current relationship in the inductive element of the transformer is used [8]. In this method future states of the flux are calculated using previous known states and a restrain strategy is developed depending on the magnetizing characteristic profile.

Harmonic current restraint is by far the most used technique in dealing with magnetizing inrush currents. This technique has been extensively researched and has been implemented in a lot of different ways and all implementations are based on the fact that the magnetizing inrush current waveform is not a pure fundamental harmonic but instead consists of a high second harmonic content [9] usually of the order of 15-35% [10] and relatively low third harmonic [11]. Restrain schemes that take advantage of the presence of this harmonics to refrain tripping when a certain level of second harmonic is present. We will examine the two basic second harmonic restraint mechanisms which are: Simple 2nd harmonic restraint and shared 2nd harmonic restraint.

Despite these improvements, electrical engineers are faced with the difference in transformer design and magnetic core properties that lead to the presence of different 2nd harmonic content in different transformers. In addition, certain transformer inter-connections may attenuate 2nd harmonic inrush currents. This problem is made even worse by the fact that, recent advances in transformer design technology has led to significant drop in 2nd harmonic content to as much as 7% making the detection of inrush currents with this mechanism even more industrious. In addition, the presence of lower 2nd harmonic currents means lower current restraint levels which affects the performance of the differential protection.

Nowadays, with the advent of more powerful computers more and more transformer protection schemes take advantage of digital signal processing techniques. Historically, the first appearance of such techniques is the use of neural networks o identify internal faults [12, 13]. These techniques are based on training different architectures of artificial neural networks to identify and classify events such as inrush currents, internal faults, overexcitation with the second harmonic content as input. The disadvantage of this technique is mainly the large amount of time required to train the network for classifying the events. A variation of this method is that using fuzzy logic [14] where new rules involving harmonic content, flux-current relationships and other transformer parameters are used to create new rules in the process of “fuzzification” to identify internal faults. Still, this process though effective is made industrious by the need for creative criteria for defining events which must still be universal and be applicable to different types of transformers. Another such methods is using the Wavelet Transform [15]. In this technique, voltage and current signal are decomposed into different frequency bands in both the frequency and time domain so as to identify inrushes or other transformer events by classifying the events using artificial neural networks.

A different spin using signal processing is the application of Wavelet Packet Transforms [16]. This technique involves the use of digital filters that filter the current samples to detect second harmonic currents. These filers are cascaded and make sure that only signal with the characteristics of the inrush currents are detected.

Another innovative method is the time difference method [17]. It is implemented by detecting the time difference of the appearance of the abnormal state and differential current to distinguish between internal fault and non-internal fault conditions, that is, when the time difference is less than the threshold, an internal fault is detected and then the differential protection will operate, otherwise a non-internal fault is detected and the differential protection will not respond.

CHAPTER 3: PROPOSED METHOD

The shortcomings of the exiting methods can be overcome by the emergence of a new technique which takes advantage of the availability of modeling techniques, state estimation and the presence of redundant synchronized measurements to constantly monitor the "health" of the transformer. In traditional protection schemes measurements obtained from devices such as ammeters, voltmeters and digital relays are used directly as basis for protection logic, in our approach though we do not use the actual measurements (due to the fact that these measurements might not be reliable due to certain factors which have nothing to do with the transformer) but the state estimate to develop a protection scheme for our transformer. The difference between our approach and the traditional protection schemes is summarized in the below schematic.

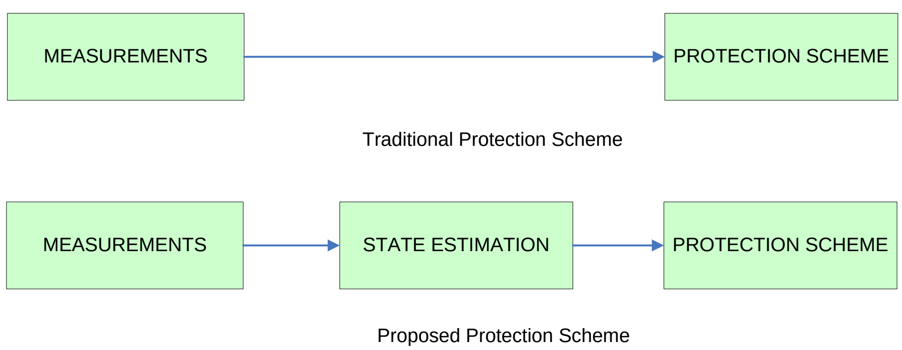

Figure 3.1: Comparison of traditional and the proposed protection scheme

The advantage of adding the State Estimation block is two-fold. Primarily, the measurements are used to set up a system of equations of degree at most quadratic such that each measurement can be described in terms of transformer state variables (this procedure will be described in detail in the following sections). Once this is done a State Estimation procedure is used such that the state of the transformer is determined and a statistical method applied to determine whether or not the measurements fit the model and hence the health of the transformer is determined depending on the goodness of fit of the statistical procedure. The second advantage of using the State Estimation block is that instead of using the measurements in traditional relaying functions we use the state estimate which is more accurate since the measurements might be flawed by the inaccuracy of the measuring devices. A legitimate question can be raised about the reliability/robustness of our additional State Estimation block, but this is entirely taken care of by the fact that our final transformer /measurement model is quadratic and due to the fact that we use the Newton iterative method convergence is guaranteed especially considering the fact that our initial point is the actual measurement.

In actual fact, though the idea behind this novel technique is rather simple, it entails a number of different steps. One of the backbones of our strategy is based on the fact that systems of differential equations can be changed from continuous time to discrete time systems using Quadratic Component Modeling and Quadratic Integration [18]. This technique can be illustrated using the following equations:

$$\begin{aligned} \frac{dx(t)}{dt} &= f\left(x(t), y(t), t\right) \\ 0 &= g\left(x(t), y(t), t\right) \end{aligned} \qquad (1)$$

Where x and y are the dynamic and algebraic states of the system respectively. Our goal is to approximate the above dynamic equations into algebraic equations at discrete time steps. Upon a process known as quadratization where the degree of all equations is reduced to no more than quadratic with the insertion of additional states, the model is modified into the following standard form.

$$\frac{dx(t)}{dt} = A_1 x(t) + A_2 y(t) + A_3 z(t) + C$$
$$0 = q\left(x(t), y(t), z(t)\right) \qquad (2)$$

Where

x(t): the dynamical states of the model

y(t): the algebraic states of the model

z(t): the additional algebraic states introduced for the quadratization of the model

It must be noted that there are no non-linearities in the dynamic part of the model. Any non-linearities are dealt with by introducing additional states in the algebraic states. It must also be notes that we have equations of degree no higher than two in the algebraic part of the equation.

Once this process is complete we can now assume that between consecutive time steps our functions vary quadratically, hence integrating on the interval [*t-h t*], the above dynamic model is described by the following algebraic equations.

$$x(t) - x(t-h) = A_1 \cdot \left(\frac{h}{6}x(t) + \frac{2h}{3}x(t_m) + \frac{h}{6}x(t-h)\right) + A_2 \cdot \left(\frac{h}{6}y(t) + \frac{2h}{3}y(t_m) + \frac{h}{6}y(t-h)\right)$$
$$+A_3 \cdot \left(\frac{h}{6}z(t) + \frac{2h}{3}z(t_m) + \frac{h}{6}z(t-h)\right) + h \cdot C$$
$$x(t_m) - x(t-h) = A_1 \cdot \left(-\frac{h}{24}x(t) + \frac{h}{3}x(t_m) + \frac{5h}{24}x(t-h)\right) + A_2 \cdot \left(-\frac{h}{24}y(t) + \frac{h}{3}y(t_m) + \frac{5h}{24}y(t-h)\right)$$
$$+A_3 \cdot \left(-\frac{h}{24}z(t) + \frac{h}{3}z(t_m) + \frac{5h}{24}z(t-h)\right) + \frac{h}{2} \cdot C$$
$$0 = q\left(x(t), y(t), z(t)\right)$$
$$0 = q\left(x(t_m), y(t_m), z(t_m)\right) \qquad (3)$$

With the above formulation the differential equations are modified to algebraic equations where the only unknowns are the present states of the device being modeled (transformer in our case). In the following sections we will show how this technique will be employed on the transformer and in the future this technique will be expanded to three phase transformers. All the other parts in these equations are either constants from the model being studied or are values from the previous states of the transformer which are known.

The goal of this thesis will be to develop a new relay that will take advantage of the presence of redundant synchronized measurements to monitor the health of any transformer. This will be done with using the dynamic model of the transformer and quadratic integration to be able to generate future operating states of the transformer from known past history such that we create a pr0ofile of the health. At this point a comparison will be made between the incoming measured sample and the mathematically generated state to determine whether or not the transformer should be tripped.

3.1 Proposed Methodology

The simple transformer network shown below will be used as an example on how this novel technique can be implemented. This example shows a Δ-Y transformer connected to a power source and a load. For simplicity we have only shown a few measurement devices but this is just for the sake of space. In actual fact measurement devices in an actual system are scattered all around even in the case of simplistic design as the one show in the example. The measurement devices include voltmeters, ammeters, watt meters and measurement from the digital relays placed to monitor the different devices.

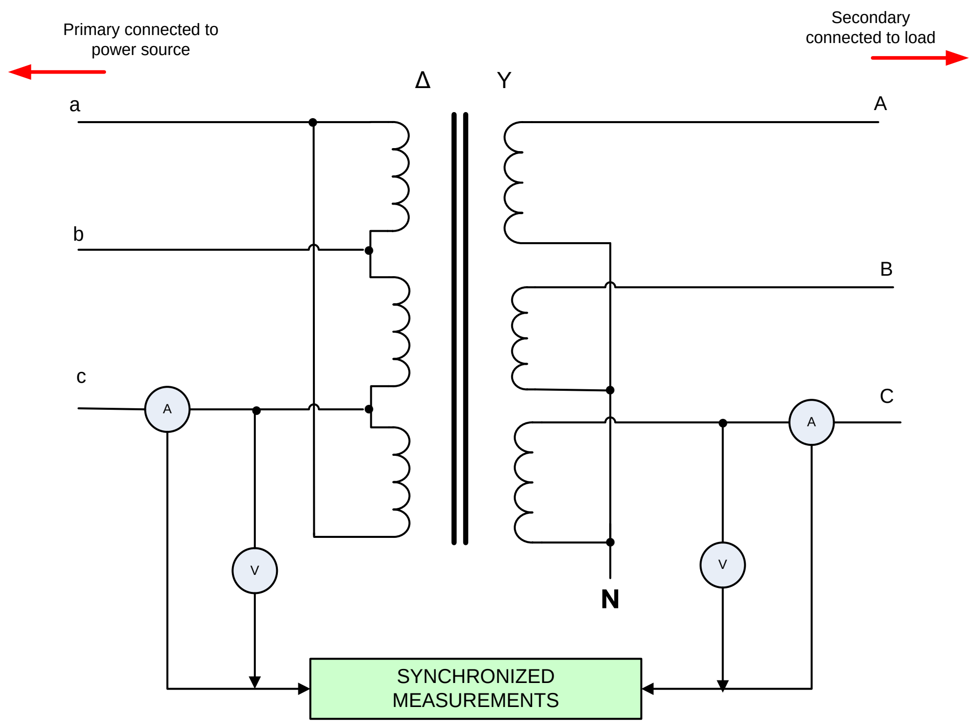

Figure 3.2: Three-phase Transformer connected to grid and load

On this diagram we have also shown a voltmeter and an ammeter that will be used to obtain voltage and current measurements on both the primary and secondary windings of the transformer. The voltmeters and ammeters provide the synchronized measurements to guarantee that when they will be used in the state estimation procedure the state of the transformer exactly represents the transformer's state at the specific time the measurements are made. It must be noted that voltmeters and ammeters are not the only measuring devices that can be used but are the only devices here that will be used for this example's sake.

Voltage and current samples will be taken at periodic time intervals such that the samples for the current time steps will be used to decide on whether or not to initiate the tripping mechanism before the next set of measurements are obtained. This example will be used just to understand qualitatively the process involved for our strategy. The first thing that has to be noted is that we will have a total of four equations that will describe the four measurements the transformer. In addition to these equations, we will have all the internal equations as well that are involved in the transformer model. The system that will result will result will be such that we can determine the state of the transformer.

In the demonstration of this principle on a single phase transformer we will see all the equations involved and kind of quantitatively examine what happens in the procedure described above.

3.2 Concept Demonstration on a Single Phase Transformer

To demonstrate our protection idea we will carry out the procedure on a single phase saturable core transformer. The equivalent circuit of the transformer is shown in the figure below. Our goal is to formulate the state estimation process for this relatively simple transformer and then later expand this concept to three phase transformers.

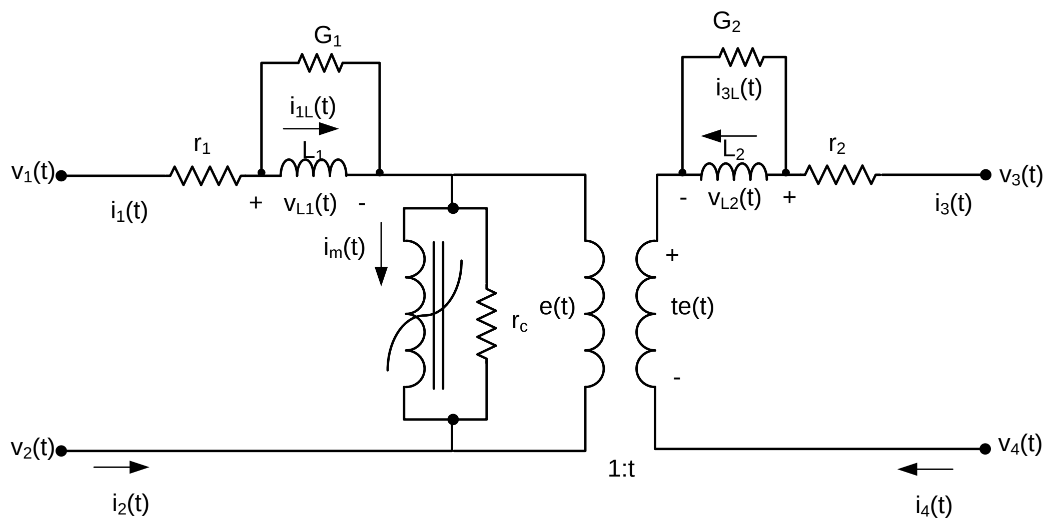

Figure 3.3: Single phase equivalent of saturable core transformer

The above equivalent circuit can be described in the following general form

$$i_{1\phi}(t) = A \cdot x(t) + B\frac{dx}{dt}$$
$$[0] = \begin{bmatrix} x^T(t) \cdot Q_1 \cdot x(t) \\ \vdots \\ x^T(t) \cdot Q_m \cdot x(t) \end{bmatrix} \qquad (4)$$

Where,

$$i_{1\phi}(t) = \begin{bmatrix} i_1(t) & i_2(t) & i_3(t) & i_4(t) & 0 & \cdots & 0 \end{bmatrix}^T$$

$$x(t) = \begin{bmatrix} v_1(t) & v_2(t) & v_3(t) & v_4(t) & i_{L1}(t) & i_{L2}(t) & e(t) & \lambda(t) & i_m(t) & y_1(t) & y_2(t) & \cdots & y_m(t) \end{bmatrix}^T$$

$$A = \begin{bmatrix} g_1 & -g_1 & 0 & 0 & 0 & 0 & -g_1 & 0 & 0 & & 0 \\ -g_1 & g_1 & 0 & 0 & 0 & 0 & g_1 & 0 & 0 & & 0 \\ 0 & 0 & g_2 & -g_2 & 0 & 0 & -t\cdot g_2 & 0 & 0 & & 0 \\ 0 & 0 & -g_2 & g_2 & 0 & 0 & t\cdot g_2 & 0 & 0 & & 0 \\ -g_1 & g_1 & 0 & 0 & 1 & 0 & g_1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & -g_2 & g_2 & 0 & 1 & t\cdot g_2 & 0 & 0 & & 0 \\ -g_1 & g_1 & -t\cdot g_2 & t\cdot g_2 & 0 & 0 & g_1+g_c+t^2 g_2 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & & -i_0 \end{bmatrix}$$

$$\dim(A) = 9 \times (9+m)$$

The dimension of A is 9 by (9+m) where $m = m_1 + m_2$, $m_1 = \mathrm{int}\left(\log_2(n)\right)$, $m_2 = \left(\#\ \text{of}\ 1's\ \text{in binary representation of} n\right) - 1$ and n is obtained from the equation $i_m(t) = i_0 \cdot \left|\frac{\lambda(t)}{\lambda_0}\right|^n sign\left(\lambda(t)\right)$ giving the relationship between the magnetizing current and the flux.

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & -g_1L_1 & 0 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & g_1L_1 & 0 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & -g_2L_2 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & g_2L_2 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & \left(G_1+g_1\right)L_1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & \left(G_2+g_2\right)L_2 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & g_1L_1 & g_2L_2 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 \end{bmatrix}$$

$$\dim(B) = 9 \times (9+m)$$

The dimension of B is 9 by (9+m) just as that of A_1. The second part represents the quadratic part of the model such that:

$$Q_k\left(1 \le k \le m,\ i,\ j\right) \neq 0$$

The variables used in modeling the transformer are:-

i_1 : is the terminal 1 current

i_2 : is the terminal 2 current

i_3 : is the terminal 3 current

i_4 : is the terminal 4 current

v_1 : is the terminal 1 voltage

v_2 : is the terminal 2 voltage

v_3 : is the terminal 3 voltage

v_4 : is the terminal 4 voltage

i_{L1} : is the current through inductance L_1

i_{L2} : is the current through inductance L_2

λ : is the nonlinear inductor flux

i_m : is the magnetizing current

e : is the emf across the inductor

t : is the transformation ratio

The size of the matrices A, B and F_{eq} is dependent on n the exponent in the magnetizing current equation and we can give general formulae for the above matrices for 2 different cases which are when n is odd and when n is even. The details of these differences are shown in Appendix A and here will just use the expressions derived from the appendix.

Performing Quadratic Integration and appending both parts of equation 4 we obtain an algebraic companion form for the single phase transformer of the form

$$\begin{bmatrix} i_{1\phi}(t) \\ i_{1\phi}(t_m) \end{bmatrix} = E \cdot F_1 \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - b_{eq}$$

$$[0] = \begin{bmatrix} x(t)^T & x(t_m)^T \end{bmatrix} \begin{bmatrix} Q_1 \\ \vdots \\ Q_{2m} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} \qquad (5)$$

Where,

$$i_{1\phi}(t) = \begin{bmatrix} i_1(t) & i_2(t) & i_3(t) & i_4(t) & 0 & \cdots & 0 \end{bmatrix}^T \quad i_{1\phi}(t_m) = \begin{bmatrix} i_1(t_m) & i_2(t_m) & i_3(t_m) & i_4(t_m) & 0 & \cdots & 0 \end{bmatrix}^T$$

$$x(t) = \begin{bmatrix} v_1(t) & v_2(t) & v_3(t) & v_4(t) & i_{L1}(t) & i_{L2}(t) & e(t) & \lambda(t) & i_m(t) & y_1(t) & y_2(t) & \cdots & y_m(t) \end{bmatrix}^T$$

$$x(t_m) = \begin{bmatrix} v_1(t_m) & v_2(t_m) & v_3(t_m) & v_4(t_m) & i_{L1}(t_m) & i_{L2}(t_m) & e(t_m) & \lambda(t_m) & i_m(t_m) & y_1(t_m) & y_2(t_m) & \cdots & y_m(t_m) \end{bmatrix}^T$$

$$b_{eq} = E \cdot F_2 \left[x(t-h) \right] + E \cdot F_3 \left[i_{1\phi}(t-h) \right]$$

$$E = \begin{bmatrix} \frac{4}{h}I & 0 & -\frac{8}{h}I & 0 \\ 0 & I & 0 & 0 \\ \frac{1}{2h}I & 0 & \frac{2}{h}I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \quad F_1 = \begin{bmatrix} \frac{h}{6}A + B & \frac{2h}{3}A \\ -\frac{h}{24}A & \frac{h}{3}A + B \end{bmatrix}, \quad F_2 = \begin{bmatrix} B - \frac{h}{6}A \\ B - \frac{5h}{24}A \end{bmatrix}, \quad F_3 = \begin{bmatrix} \frac{h}{6}I & 0 \\ 0 & 0 \\ \frac{5h}{24} & 0 \\ 0 & 0 \end{bmatrix}$$

Using the algebraic companion form in equation 5 we are able express the measurements on the transformer as a function of their states. The following categories of measurements can be created. Initially though for simplicity of symbolism we will assume that after appending both parts of equation we obtain:

$$\therefore \begin{bmatrix} i_{1\phi}(t) \\ i_{1\phi}(t_m) \\ 0 \end{bmatrix} = Y_{eq} \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - B_{eq} + F_{eq} \qquad (6)$$

Where

$$Y_{eq} = \begin{bmatrix} E \cdot F_1 \\ 0 \end{bmatrix}, \quad B_{eq} = \begin{bmatrix} b_{eq} \\ 0 \end{bmatrix}, \quad F_{eq} = \begin{bmatrix} x(t)^T & x(t_m)^T \end{bmatrix} \begin{bmatrix} 0 \\ Q_k^{(m)} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix}$$

Actual Measurements: The measurements we have about the transformer at every time instant are:

i_p: primary side current

i_s: secondary side current

v_p: primary side voltage

v_s: secondary side voltage

Then the measurement model can hence be described using the following matrix

$$Z=\left[i_p(t) \quad i_s(t) \quad v_p(t) \quad v_s(t) \quad i_p(t_m) \quad i_s(t_m) \quad v_p(t_m) \quad v_s(t_m)\right]^T \tag{7}$$

Then the relationship between the measurements and variables at time instant T can be described using the following relationships.

$$\begin{aligned} &i_p(t)=i_1(t)+\eta_1, \qquad i_s(t)=i_3(t)+\eta_3, \\ &v_p(t)=v_1(t)-v_2(t)+\eta_{2N+1}, \qquad v_s(t)=v_3(t)-v_4(t)+\eta_{2N+2} \end{aligned}$$

$$\begin{aligned} &i_p(t_m)=i(t_m)+\eta_{10}, \qquad i_s(t_m)=i(t_m)+\eta_{12} \\ &v_p(t_m)=v_1(t_m)-v_2(t_m)+\eta_{2N+5}, \qquad v_s(t_m)=v_3(t_m)-v_4(t_m)+\eta_{2N+6} \end{aligned}$$

The RHS of the current equations shown above are then developed into their respective functions of the states obtained from equation (6). (See Appendix on how these equations were obtained):

$$i_p(t)=\left[1^{st}\ row\, Y_{eq}\right]\begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix}-\left[1^{st}\ row\ B_{eq}\right]+\left[1^{st}\ row\ F_{eq}\right]+\eta_1(t)$$

$$i_S(t)=\left[3^{rd}\ row\, Y_{eq}\right]\begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix}-\left[3^{rd}\ row\ B_{eq}\right]+\left[3^{rd}\ row\ F_{eq}\right]+\eta_3(t)$$

$$i_p(t_m)=\left[10^{th}\ row\, Y_{eq}\right]\begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix}-\left[10^{th}\ row\ B_{eq}\right]+\left[10^{th}\ row\ F_{eq}\right]+\eta_{10}(t)$$

$$i_S(t_m)=\left[12^{th}\ row\, Y_{eq}\right]\begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix}-\left[12^{th}\ row\ B_{eq}\right]+\left[12^{th}\ row\ F_{eq}\right]+\eta_{12}(t)$$

$$v_p(t) = v_1(t) - v_2(t) + \eta_{2N+1}(t), \quad v_s(t) = v_3(t) - v_4(t) + \eta_{2N+2}(t)$$

$$v_p(t_m) = v_1(t_m) - v_2(t_m) + \eta_{2N+5}(t_m), \quad v_s(t_m) = v_3(tt_m) - v_4(t_m) + \eta_{2N+6}(t_m)$$

We can also assume that we have the measurements at intermediate time steps t_m, so we could as well write the current measurement equations for these time step similar to the ones above.

Virtual measurements: Since we have zeroes in the current matrix we can assume that these correspond to some virtual measurement that can be expressed as a function of the states as shown in the following equations.

$$0 = \left[5^{th}\ row\ Y_{eq}\right]\begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - \left[5^{th}\ row\ B_{eq}\right] + \left[5^{th}\ row\ F_{eq}\right] + \eta_5(t)$$

$$0 = \vdots$$

$$0 = \left[9^{th}\ row\ Y_{eq}\right]\begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - \left[9^{th}\ row\ B_{eq}\right] + \left[9^{th}\ row\ F_{eq}\right] + \eta_9(t)$$

$$0 = \left[14^{tht}\ row\ Y_{eq}\right]\begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - \left[14^{th}\ row\ B_{eq}\right] + \left[14^{th}\ row\ F_{eq}\right] + \eta_{14}(t)$$

$$0 = \vdots$$

$$0 = \left[18^{th}\ row\ Y_{eq}\right]\begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - \left[18^{th}\ row\ B_{eq}\right] + \left[18^{th}\ row\ F_{eq}\right] + \eta_{18}(t)$$

$$0 = \left[19^{tht}\ row\ Y_{eq}\right]\begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - \left[19^{th}\ row\ B_{eq}\right] + \left[19^{th}\ row\ F_{eq}\right] + \eta_{19}(t)$$

$$0 = \vdots$$

$$0 = \left[2N^{th}\ row\ Y_{eq}\right]\begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - \left[2N^{th}\ row\ B_{eq}\right] + \left[2N^{th}\ row\ F_{eq}\right] + \eta_{2N}(t)$$

Pseudo Measurements: These measurements represent quantities that are normally not measured such as voltages in the neutral. These equations for the transformer are as follows

$$v_G(t) = v_3(t) + \eta_{2N+3}(t) \quad v_g(t) = v_4(t) + \eta_{2N+4}(t)$$

$$v_G(t_m) = v_3(t_m) + \eta_{2N+7}(t_m) \quad v_g(t_m) = v_4(t_m) + \eta_{2N+8}(t_m)$$

Derived Measurements: These are measurements that are derived from other measurements such as the currents in the grounded branch in the transformer.

$$\begin{aligned}
-i_p(t) &= i_2(t) + \eta_2 \\
-i_s(t) &= i_4(t) + \eta_4 \\
-i_p(t_m) &= i_2(t_m) + \eta_{11} \\
-i_s(t_m) &= i_2(t_m) + \eta_{13}
\end{aligned}$$

The above system of equations set up an over-determined system with a total of 2N+8 equations with 2N unknowns, hence the state estimation procedure to determine the state of the transformer. To illustrate the above concept, let us assume for simplicity of notation that

$$Y_{eq} = \begin{bmatrix} A_{1,1} & \cdots & A_{1,2N} \\ \vdots & & \vdots \\ A_{2N,1} & \cdots & A_{2N,2N} \end{bmatrix}, \quad B_{eq} = \begin{bmatrix} B_1 \\ \vdots \\ B_{2N} \end{bmatrix}, \quad F_{eq} = \begin{bmatrix} F_{eq}^{(1)} \\ \vdots \\ F_{eq}^{(2N)} \end{bmatrix}$$

The Y_{eq} and B_{eq} shown above come from equation 5 basically model both the actual current and virtual measurements. In addition we can append the voltage measurements such that the state estimation can be performed. In the equations below we show the general structure of

$$\begin{bmatrix} i_p(t) \\ \vdots \\ -i_s(t) \\ 0 \\ i_p(t_m) \\ \vdots \\ -i_s(t_m) \\ 0 \\ \vdots \\ 0 \\ v_p(t) \\ v_s(t) \\ v_G(t) \\ v_g(t) \\ v_p(t_m) \\ v_s(t_m) \\ v_G(t_m) \\ v_g(t_m) \end{bmatrix} = \left[\begin{array}{cccccccccccccccccc} A_{1,1} & A_{1,2} & A_{1,3} & A_{1,4} & \cdots & & & & \cdots & A_{1,N+1} & A_{1,N+2} & A_{1,N+3} & A_{1,N+4} & \cdots & & & \cdots & A_{1,2N} \\ & & & & & & & & & & & & & & & & & \\ \vdots & & & \vdots & & & & & & \vdots & & & \vdots & & & & & \vdots \\ & & & & & & & & & & & & & & & & & \\ & & & & \cdots & & & & \cdots & & & & & & & & \cdots & \\ & & & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & & & & \\ \vdots & & & & & & & & & \vdots & & & & & & & & \vdots \\ & & & & & & & & & & & & & & & & & \\ A_{2N,1} & A_{2N,2} & A_{2N,3} & A_{2N,4} & \cdots & & & & \cdots & A_{2N,N+1} & A_{2N,N+2} & A_{2N,2N+3} & A_{2N,2N+4} & \cdots & & & \cdots & A_{2N,2N} \\ 1 & -1 & 0 & 0 & 0 & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & & & & 0 & 0 & & & & & & & & 0 \\ 0 & 1 & 0 & 0 & 0 & & \cdots & & 0 & 0 & & & & & & & & 0 \\ 0 & 0 & 0 & 1 & 0 & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & & 0 & & & & 0 \\ 0 & & & & & & & & 0 & 0 & 0 & 1 & -1 & 0 & & & & 0 \\ 0 & & & & & & & & 0 & 0 & 1 & 0 & 0 & 0 & & & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & & & & 0 \end{array}\right] \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \\ i_{1L}(t) \\ \vdots \\ \vdots \\ y_4(t) \\ v_1(t_m) \\ v_2(t_m) \\ v_3(t_m) \\ v_4(t_m) \\ \vdots \\ \vdots \\ y_1(t_m) \\ y_2(t_m) \\ y_3(t_m) \\ y_m(t_m) \end{bmatrix}$$

$$+ \begin{bmatrix} B_1 \\ \\ \\ \vdots \\ \\ B_{2N} \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} v_1(t) & \cdots & y_m(t_m) \end{bmatrix} \begin{bmatrix} F_{eq}^{(1)} \\ \\ \\ \\ \vdots \\ \\ F_{eq}^{(2N)} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} v_1(t) \\ \vdots \\ y_m(t_m) \end{bmatrix} + \begin{bmatrix} \eta_1(t) \\ \eta_2(t) \\ \eta_3(t) \\ \vdots \\ \eta_{2N+8}(t) \end{bmatrix} \tag{8}$$

$$Z = h(x) \tag{8*}$$

A state estimation procedure can now be set using the Newton iterative method such that a state estimate is obtained according to the following algorithm. It must be noted that h(x) is not linear but also contains a quadratic component hence the need for the iterative method.

$$X^{\upsilon+1} = X^{\upsilon} - \left(H^T W H\right)^{-1} H^T W \left(h(X^{\upsilon}) - Z\right) \tag{9}$$

In equation 8 above

X : is the state matrix

H : is the Jacobian of the RHS of equation 7

W : is a diagonal weight matrix with the standard devitions of the measur

Z : is the matrix of measurements or simply the LHS of equation 43

Analytically, the Jacobian H for the case transformer is of the from

$$
= \left[\begin{array}{cccccccccccccccccc}
A_{1,1} & A_{1,2} & A_{1,3} & A_{1,4} & \cdots & & & & \cdots & A_{1,N+1} & A_{1,N+2} & A_{1,N+3} & A_{1,N+4} & \cdots & & & \cdots & A_{1,2N} \\
\vdots & & & \vdots & & & & & & \vdots & & & \vdots & & & & & \vdots \\
& & & & \cdots & & & & \cdots & & & & & \cdots & & & \cdots & \\
\vdots & & & & & & & & & \vdots & & & & & & & & \vdots \\
A_{2N,1} & A_{2N,2} & A_{2N,3} & A_{2N,4} & \cdots & & & & \cdots & A_{2N,N+1} & A_{2N,N+2} & A_{2N,2N+3} & A_{2N,2N+4} & \cdots & & & \cdots & A_{2N,2N} \\
1 & -1 & 0 & 0 & 0 & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & & & & 0 & 0 & & & & & & & & 0 \\
0 & 1 & 0 & 0 & 0 & & \cdots & & 0 & 0 & & & & & & & & 0 \\
0 & 0 & 0 & 1 & 0 & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & & 0 & & & & 0 \\
0 & & & & & & & & 0 & 0 & 0 & 1 & -1 & 0 & & & & 0 \\
0 & & & & & & & & 0 & 0 & 1 & 0 & 0 & 0 & & & & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & & & & 0
\end{array}\right]
+ \frac{\delta}{\delta x_i}\left[v_1(t) \quad \ldots \quad y_m(t_m)\right]\Bigg|_{i=1\to 2N}
\left[\begin{array}{c} F_{eq}^{(1)} \\ \\ \vdots \\ \\ F_{eq}^{(2N)} \\ 0 \\ \vdots \\ 0 \end{array}\right]
\left[\begin{array}{c} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \\ \vdots \\ y_m(t_m) \end{array}\right]
$$

$$
+ \left[v_1(t) \quad v_2(t) \quad \cdots \quad y_m(t_m)\right]
\left[\begin{array}{c} F_{eq}^{(1)} \\ \\ \vdots \\ \\ F_{eq}^{(2N)} \\ 0 \\ \vdots \\ 0 \end{array}\right]
\frac{\delta}{\delta x_i}\left[\begin{array}{c} v_1(t) \\ \vdots \\ y_m(t_m) \end{array}\right]\Bigg|_{i=1\to 2N}
$$

The iterative process will hence give us the state of the transformer at each time step. Once the state estimation procedure has converged we now have to perform the chi-square test to determine whether or not the state estimate fits the model or not. The following actions will be performed for the last iteration

$$
I = \left(H\,WH\right)
$$

$$
\sigma = \sqrt{diag\ (I)}
$$

From the above matrix the standard deviations of the states are obtained by retrieving the diagonal elements of the above I_σ matrix which is also known as the information matrix. Now that we have the standard deviation we can now calculate the confidence level of our state estimate as shown below

$$\zeta = \sum_{i=1}^{2N} \left(\frac{h_i(x) - b_i}{\sigma_i} \right)^2$$

We then use the confidence level and the degrees of freedom $v=\mu-\rho=8$ where m is the number of equations (2N+8) and the number of unknowns is $2N$.

$$\Pr\left[\chi^2 \geq \zeta \right] = 1 - P(\zeta, \mu - \rho)$$

The whole process described above is summarized in the following block diagram.

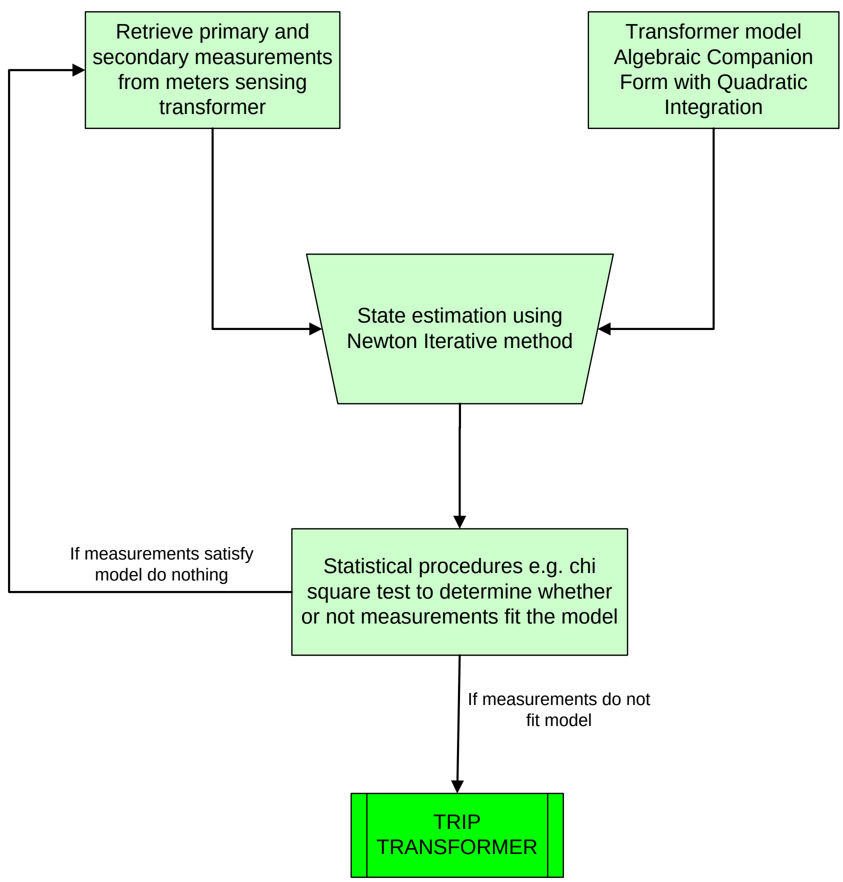

Figure 3.4: Transformer Protection proposed Algorithm

Our approach in this part has just been to describe the overall procedure on how to proceed to perform our state estimation process. It must be noted that the matrices involved are relatively sparse and hence a lot of our computations are really simplified.

3.3 Protection of Proposed Scheme

In the previous section we have shown how our novel technique works. In the present section we are going to examine the different types of protection that could be implemented with our approach.

Volts/Hz protection (24): It is very important to ensure that saturation is monitored on a power transformer. This comes as a result of the fact that, large electric currents are needed to maintain the flux in the magnetic core of the transformer. As a result of the large currents, overheating occurs, leading to improper functioning or even complete damage of the transformer. The way we can control the effects of saturation is based on Faraday's law.

$$v(t) = \frac{d\lambda}{dt}$$

In terms of phasors this equation could be rewritten as

$$\tilde{V} = j\omega\tilde{\Lambda}$$

This equation shows us that the magnitude of the flux linkage is proportional to the ratio of the voltage divided by the frequency.

$$\frac{V}{f} > 2\pi\Lambda_0$$

This shows that saturation occurs as soon as this ratio exceeds a certain threshold.

In our approach the voltage on both sides of the transformer can be computed very easily by the following equations.

$$v_1(t) - v_2(t) = V_p \qquad v_3(t) - v_4(t) = V_s$$

These values are components of the state vector obtained from our state estimation. The frequency can be calculated at every time instant by the following procedure where *x(i)* is the voltage sample. Then the following two sums can be generated.

$$Y_1(k) = \sum_{i=k}^{k+N-1} x(i)\cos\left(\omega_0 Ti\right)$$
$$Y_2(k) = \sum_{i=k}^{k+N-1} x(i)\sin\left(\omega_0 Ti\right)$$

Where,

x(i): Voltage sample

ω_0: Power base angular frequency ($2\pi f_0$)

T: Sampling period

N: Number of samples in one period ($1/f_0 T$)

At every time instant the phasor angle ϕ is computed as follows

$$\phi(k) = \tan^{-1}\left(\frac{Y_2(k)}{Y_1(k)}\right)$$

And the frequency can be calculated by

$$f = f_0 + \frac{\Delta\phi_k}{2\pi T}$$

Where

$$\Delta\phi_k = \phi(k) - \phi(k-1)$$

Once the frequency is computed then we can examine whether the ratios of the primary and secondary voltages are within our limits and based on this we issue a trip on the transformer or not.

$$k \cdot \lambda_{\min} < \left(\frac{V_p}{f}, \quad \frac{V_S}{f} \right) < k \cdot \lambda_{\max} \qquad \text{where} \quad k = \text{constant}$$

Undervoltage protection (27): The undervoltage protection unit is set such that a trip signal is issued when the voltage on the primary or secondary is below the normal minimum system load voltage.

Instantaneous Overcurrent protection (50): This is a quick intervention relay which trips the transformer whenever the current goes above a certain pickup value. Our state estimate at each instant is used to compute the currents through the primary and/or secondary side of the transformer which is then compared to this pickup value and based on whether the currents are above or below the pickup a trip signal is issued.

$$i_p(t) = \left[1^{st} \; row \, Y_{eq} \right] \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - \left[1^{st} \; row \; B_{eq} \right] + \left[1^{st} \; row \; F_{eq} \right]$$

$$i_S(t) = \left[3^{rd} \; row \, Y_{eq} \right] \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - \left[3^{rd} \; row \; B_{eq} \right] + \left[3^{rd} \; row \; F_{eq} \right]$$

$$i_p(t) > i_{\text{pickup}}, \qquad i_s(t) > i_{\text{pickup}}$$

It should be noted that the pickup is set at a value greater than the inrush currents and greater than the short circuit currents.

Phase Time Overcurrent protection (51): This function could be implemented using our protection by calculating the current estimate at each time instant and using it in estimating the time to trip using IEEE standard inverse time characteristics.

- Moderately Inverse: $t = \dfrac{0.0515 \cdot TD}{(\dfrac{I}{I_{pu}})^{0.02} - 1} + 0.114 \cdot TD$
- Very Inverse: $t = \dfrac{19.61 \cdot TD}{(\dfrac{I}{I_{pu}})^{2} - 1} + 0.491 \cdot TD$
- Extremely Inverse: $t = \dfrac{28.2 \cdot TD}{(\dfrac{I}{I_{pu}})^{2} - 1} + 0.1217 \cdot TD$

Where

t (i): trip time as a function of input current

I: Input Current

I_{pu}: Pickup Current

TD: Time Dial

Overvoltage protection (59): Overvoltage can be implemented fairly easily by comparing the voltage components of our state estimate with a predefined threshold. Once this voltage is surpassed then a trip signal is issued.

Differential protection (87): In this case our differential protection scheme is again based on the current estimates from our state estimate. His method is even more convenient than the traditional method due to the fact that we do not need to take into account any CT ratio. The currents are directly computed from our state estimate and can be used directly as follows.

$$I_0 = N_1 \cdot I_1 - N_2 \cdot I_2$$
$$I_R = I_1 + I_2$$

Where,

N_1: number of turns in primary of transformer

N_2: number of turns in secondary of transformer

I_1: current estimate in primary winding of transformer at particular instant

I_2: current in secondary winding of transformer at particular time instant

K: constant varying between 0.1-0.6

And a trip signal will be issued if

$$\frac{I_0}{I_R} > K$$

CHAPTER 4: NUMERICAL EXPERIMENTS

4.1 Experimental Procedure

In this section numerical experiments will be performed using the proposed protection function according to the procedure outlined below.

1) Different events describing the operation of the transformer will be simulated using the WINIGS program and recorded in COMTRADE format. The different events that will be captured include:

   a) An energization event

   b) Normal operation for a certain amount of time followed by a through fault which is successfully cleared.

   c) An internal fault on a transformer.

2) For each event, the operation of the relay will be simulated by reading the data in the COMTRADE file at a particular instant of time and performing our algorithm to determine whether or not the operation of the transformer is normal. Once this procedure is over for this time instant the algorithm is repeated for the next time instances.

3) Our results will be documented by comparing estimated and measures voltage and current waveforms. In addition chi-square test will be performed to measure the goodness of fit of our model at each instant of time upon which our protection scheme will either decide trip or not our transformer.

4.2 Measurement Analysis

The experiments' measurements are stored in a COMTRADE file. This file is composed of two parts namely: the configuration and the data file. The configuration file contains the multiplication coefficient and the offset used with the time stamped sample to obtain the actual measurement.

Initially the COMTRADE data file is checked whether it is in binary or ASCII format. In case it is stored in ASCII no conversion is needed, otherwise it is converted to its ASCII format using the XFM program. Once it is converted to ASCII format, the data file can be read as a matrix where each sample is converted to an actual measurement using the formula

$$Z_i = m_i z_i + c_i$$

Where Z_{i} represents the ith channel's measurement, z_i the measurement sample, m_i is the multiplication coefficient and c_i the offset. For each line of the matrix —which correspond to measurements with same time stamp— the actual measurement is computed using the above simple formula.

In order to test our protection scheme, our algorithm reads data for a particular time and generates a measurement matrix for that particular time instant. Once this has been done the parameters of the transformer such as the winding resistances, leakage inductances, core loss resistance, turns ratio, and the time step are used either to create the Y_{eq}, B_{eq} and F_{eq} matrices described in equation5. Using these parameters the Jacobian of the measurement equations is obtained and the state estimation process is carried out such that the sate estimate is obtained and the chi-square test carried out to determine the health of the transformer. If the chi square testis performed and the probability found is close enough to unity we do not trip the transformer. The procedure followed to carry out the state estimation is summarized in the following algorithm flow block diagram.

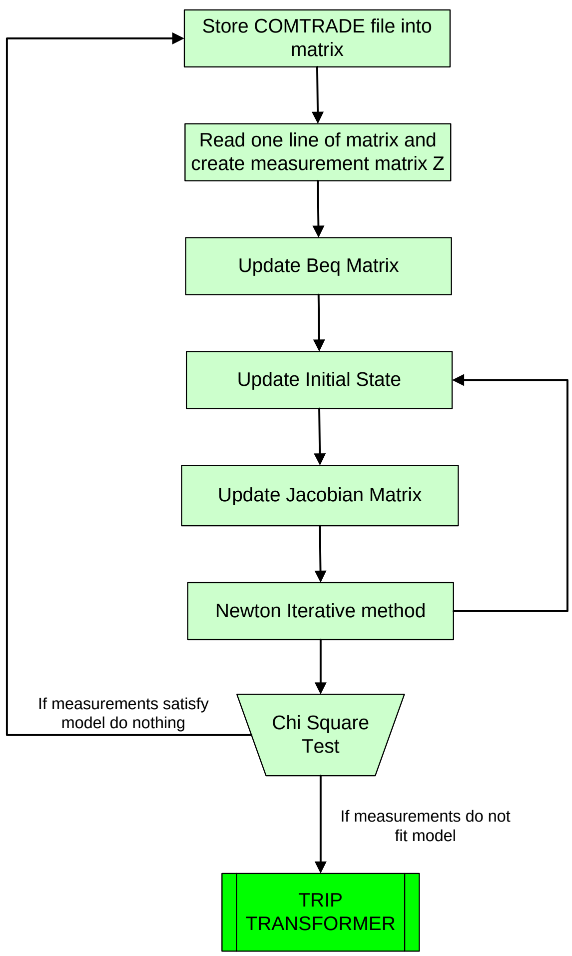

Figure 4.1: Algorithm Flow Diagram

The measurements used in these experiments can be put into the following categories.

1. **Actual Measurements**: These measurements include 2 currents (Phase A primary and secondary currents), 2 voltages (phase A to neutral primary and secondary voltages). For the above measurements the standard deviation was taken to be 0.01p.u. The model for the above measurements is as follows. Suppose that that

$I_A(T+1)$: is the primary side current measurement at time t

$I_a(T+1)$: is the secondary side current measurement at time t

$V_A(T+1)$: is the primary side voltage measurement at time t

$V_a(T+1)$: is the secondary side voltage measurement at time t

$I_A(T)$: is the primary side current measurement at time t_m

$I_a(T)$: is the secondary side current measurement at time t_m

$V_A(T)$: is the primary side voltage measurement at time t_m

$V_a(T)$: is the secondary side voltage measurement at time t_m

Then the measurement models for the respective above measurements is

$$I_A(T+1) = g_1 v_1(t) - g_1 v_2(t) - \frac{4g_1 L_1}{h} i_{L1}(t) - g_1 e(t) + \frac{8g_1 L_1}{h} i_{L1}(t_m) - g_1 v_1(t-h) + g_1 v_2(t-h)$$
$$- \frac{4g_1 L_1}{h} i_{L1}(t-h) + g_1 e(t-h) + i_1(t-h)$$

$$I_a(T+1) = g_2 v_3(t) - g_2 v_4(t) - \frac{4g_2 L_2}{h} i_{L2}(t) - t \cdot g_2 e(t) + \frac{8g_2 L_2}{h} i_{L2}(t_m) - g_2 v_3(t-h) + g_2 v_4(t-h)$$
$$- \frac{4g_2 L_2}{h} i_{L2}(t-h) + t \cdot g_2 e(t-h) + i_2(t-h)$$

$$I_A(T) = -\frac{g_1 L_1}{2h} i_{L1}(t) + g_1 v_1(t_m) - g_1 v_2(t_m) - \frac{2g_1 L_1}{h} i_{L1}(t_m) - g_1 e(t_m) + \frac{5g_1 L_1}{12} v_1(t-h)$$
$$- \frac{5g_1 L_1}{12} v_2(t-h) + \frac{5g_1 L_1}{2h} i_{L1}(t-h) - \frac{5g_1 L_1}{12} e(t-h) - \frac{1}{2} i_1(t-h)$$

$$I_a(T) = -\frac{g_2 L_2}{2h} i_{L2}(t) + g_2 v_3(t_m) - g_2 v_4(t_m) - \frac{2g_2 L_2}{h} i_{L2}(t_m) - t \cdot g_2 e(t_m) + \frac{5g_2 L_2}{12} v_3(t-h)$$
$$- \frac{5g_2 L_2}{12} v_2(t-h) + \frac{5g_2 L_2}{2h} i_{L2}(t-h) - \frac{5 \cdot t \cdot g_2 L_2}{12} e(t-h) - \frac{1}{2} i_3(t-h)$$

$$V_A(T+1) = v_1(t) - v_2(t) \qquad V_a(T+!) = v_3(t) - v_4(t)$$

$$V_A(T) = v_1(t_m) - v_2(t_m) \qquad V_a(T) = v_3(t_m) - v_4(t_m)$$

The above equations represent the model for the measurements, but we need to have the contributions of the above equations to the Jacobian in the iterative method.

$$\begin{bmatrix} g_1 & -g_1 & 0 & 0 & -\frac{4g_1L_1}{h} & 0 & -g_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{8g_1L_1}{h} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & g_2 & -g_2 & 0 & -\frac{4g_2L_2}{h} & -t\cdot g_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{8g_2L_2}{h} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & -\frac{g_1L_1}{2h} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g_1 & -g1 & 0 & 0 & -\frac{2g_1L_1}{h} & 0 & -g_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{g_2L_2}{2h} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g_2 & -g_2 & 0 & -\frac{2g_2L_2}{h} & -t\cdot g_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. **Virtual Measurements**: These measurements are the zero entries of the left-hand side of equation 8 which are actually the internal state equations of our transformer model. These entries are 20 in number, 10 at time instant t and 10 at the intermediate time step. For these measurements, the standard deviation was taken to be 0.001p.u.

$$0 = -\frac{g_1h}{6}v_1(t) + \frac{g_1h}{6}v_2(t) + \left(\frac{h}{6} + g_1L_1 + G_1L_1\right)i_{L1}(t) + \frac{g_1h}{6}e(t) - \frac{2g_1h}{3}v_1(t_m) + \frac{2g_1h}{3}v_2(t_m) + \frac{2h}{3}i_{L1}(t_m)$$
$$+\frac{2g_1h}{3}e(t_m) - \frac{g_1h}{6}v_1(t-h) + \frac{g_1h}{6}v_2(t-h) - \left(-\frac{h}{6} + g_1L_1 + G_1L_1\right)i_{L1}(t-h) + \frac{g_1h}{6}e(t-h)$$

$$0=-\frac{g_2h}{6}v_3(t)+\frac{g_2h}{6}v_4(t)+\left(\frac{h}{6}+g_2L_2+G_2L_2\right)i_{L2}(t)+\frac{t\cdot g_2h}{6}e(t)-\frac{2g_2h}{3}v_3(t_m)+\frac{2g_2h}{3}v_4(t_m)$$
$$+\frac{2h}{3}i_{L2}(t_m)+\frac{2t\cdot g_2h}{3}e(t_m)-\frac{g_2h}{6}v_3(t-h)+\frac{g_2h}{6}v_4(t-h)-\left(-\frac{h}{6}+g_2L_2+G_2L_2\right)i_{L2}(t-h)+\frac{t\cdot g_2h}{6}e(t-h)$$

$$0=-\frac{g_1h}{6}v_1(t)+\frac{g_1h}{6}v_2(t)-\frac{t\cdot g_2h}{6}v_3(t)+\frac{t\cdot g_2h}{6}v_4(t)+g_1L_1i_{L1}(t)+t\cdot g_2L_2i_{L2}(t)+\left(g_1+g_c+t^2g_2\right)\frac{h}{6}e(t)$$
$$+\frac{h}{6}i_m(t)-\frac{2g_1h}{3}v_1(t_m)+\frac{2g_1h}{3}v_2(t_m)-\frac{2t\cdot g_2h}{3}v_3(t_m)+\frac{2t\cdot g_2h}{3}v_4(t_m)+\left(g_1+g_c+t^2g_2\right)\frac{2h}{3}e(t_m)+\frac{2h}{3}i_m(t_m)$$
$$-\frac{g_1h}{6}v_1(t-h)+\frac{g_1h}{6}v_2(t-h)-\frac{t\cdot g_2h}{6}v_3(t)+\frac{t\cdot g_2h}{6}v_4(t)-g_1L_1i_{L1}(t-h)-t\cdot g_2L_2i_{L2}(t-h)$$
$$-\left(g_1+g_c+t^2g_2\right)\frac{h}{6}e(t-h)+\frac{h}{6}i_m(t-h)$$

$$0=-\frac{h}{6}e(t)+\lambda(t)-\frac{2h}{3}e(t_m)+\frac{h}{6}e(t-h)+\lambda(t-h)$$

$$0=\frac{h}{6}i_m(t)-\frac{i_0h}{6}y_5(t)+\frac{2h}{3}i_m(t_m)-\frac{2i_0h}{3}y_5(t_m)+\frac{h}{6}i_m(t-h)-\frac{i_0h}{6}y_5(t-h)$$

$$0=\frac{g_1h}{24}v_1(t)-\frac{g_1h}{24}v_2(t)-\frac{h}{24}i_{L1}(t)-\frac{g_1h}{24}e(t)-\frac{g_1h}{3}v_1(t_m)+\frac{g_1h}{3}v_2(t_m)+\left(g_1L_1+G_1L_1+\frac{h}{3}\right)i_{L1}(t_m)$$
$$+\frac{g_1h}{3}e(t_m)-\frac{g_1h}{6}v_1(t-h)+\frac{g_1h}{6}v_2(t-h)-\left(g_1L_1+G_1L_1-\frac{h}{6}\right)i_{L1}(t-h)+\frac{g_1h}{6}e(t-h)$$

$$0=\frac{g_2h}{24}v_3(t)+\frac{g_2h}{24}v_4(t)-\frac{h}{24}i_{L2}(t)-\frac{t\cdot g_2h}{24}e(t)-\frac{g_2h}{3}v_3(t_m)+\frac{g_2h}{3}v_4(t_m)+\left(\frac{h}{3}+g_2L_2+G_2L_2\right)i_{L2}(t_m)$$
$$+\frac{t\cdot g_2h}{3}e(t_m)-\frac{g_2h}{6}v_3(t-h)+\frac{g_2h}{6}v_4(t-h)-\left(g_2L_2+G_2L_2-\frac{h}{6}\right)i_{L2}(t-h)+\frac{t\cdot g_2h}{6}e(t-h)$$

$$
\begin{aligned}
0 &= \frac{g_1 h}{24} v_1(t) - \frac{g_1 h}{24} v_2(t) + \frac{t \cdot g_2 \cdot h}{24} v_3(t) - \frac{t \cdot g_2 \cdot h}{24} v_4(t) - \left(g_1 + g_c + t^2 g_2\right) \frac{h}{24} e(t) + \frac{h}{24} i_m(t) - \frac{g_1 h}{3} v_1(t_m) \\
&+ \frac{g_1 h}{3} v_2(t_m) - \frac{t \cdot g_2 \cdot h}{3} v_3(t_m) + \frac{t \cdot g_2 \cdot h}{3} v_4(t_m) + g_1 L_1 i_{L1}(t_m) + t \cdot g_2 L_2 i_{L2}(t_m) + \left(g_1 + g_c + t^2 g_2\right) \frac{h}{3} e(t_m) \\
&+ \frac{h}{3} i_m(t_m) - \frac{g_1 h}{6} v_1(t-h) + \frac{g_1 h}{6} v_2(t-h) - \frac{t \cdot g_2 \cdot h}{6} v_3(t) + \frac{t \cdot g_2 \cdot h}{6} v_4(t) - g_1 L_1 i_{L1}(t-h) - t \cdot g_2 L_2 i_{L2}(t-h) \\
&+ \left(g_1 + g_c + t^2 g_2\right) \frac{h}{6} e(t-h) + \frac{h}{6} i_m(t-h)
\end{aligned}
$$

$$0 = \frac{h}{24} e(t) - \frac{h}{3} e(t_m) + \lambda(t) - \frac{h}{6} e(t-h) - \lambda(t-h)$$

$$0 = -\frac{h}{24} i_m(t) + \frac{i_0 h}{24} y_5(t) + \frac{h}{3} i_m(t_m) - \frac{i_0 h}{3} y_5(t_m) + \frac{h}{6} i_m(t-h) - \frac{i_0 h}{6} y_5(t-h)$$

$$0 = y_1(t) + \lambda(t) \cdot \left[-\frac{1}{\lambda_0^2}\right] \cdot \lambda(t)$$

$$0 = y_2(t) + y_1(t) \cdot [-1] \cdot y_1(t)$$

$$0 = y_3(t) + y_2(t) \cdot [-1] \cdot y_2(t)$$

$$0 = y_4(t) + y_1(t) \cdot [-1] \cdot y_3(t)$$

$$0 = y_5(t) + \lambda(t) \cdot \left[-\frac{1}{\lambda_0}\right] \cdot y_4(t)$$

$$0 = y_1(t_m) + \lambda(t_m) \cdot \left[-\frac{1}{\lambda_0^2}\right] \cdot \lambda(t_m)$$

$$0 = y_2(t_m) + y_1(t_m) \cdot [-1] \cdot y_1(t_m)$$

$$0 = y_3(t_m) + y_2(t_m) \cdot [-1] \cdot y_2(t_m)$$

$$0 = y_4(t_m) + y_1(t_m) \cdot [-1] \cdot y_3(t_m)$$

$$0 = y_5(t_m) + \lambda(t_m) \cdot \left[-\frac{1}{\lambda_0} \right] \cdot y_4(t_m)$$

The contributions of each of the above equations to the Jacobian are as follows

$$\begin{bmatrix} -\frac{g_1 h}{6} & \frac{g_1 h}{6} & 0 & 0 & \frac{h}{6}+g_1 L_1+G_1 L_1 & 0 & \frac{g_1 h}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2g_1 h}{3} & \frac{2g_1 h}{3} & 0 & 0 & \frac{2h}{3} & 0 & \frac{2g_1 h}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -\frac{g_2 h}{6} & \frac{g_2 h}{6} & 0 & \frac{h}{6}+g_2 L_2+G_2 L_2 & \frac{t \cdot g_2 h}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2g_2 h}{3} & \frac{2g_2 h}{3} & 0 & \frac{2h}{3} & \frac{2t \cdot g_2 h}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{g_1 h}{6} & \frac{g_1 h}{6} & -\frac{t \cdot g_2 h}{6} & \frac{t \cdot g_2 h}{6} & g_1 L_1 & t \cdot g_2 L_2 & g_1+g_c+t^2 g_2 & 0 & \frac{h}{6} & 0 & 0 & 0 & 0 & 0 & -\frac{2g_1 h}{3} & \frac{2g_1 h}{3} & -\frac{2t \cdot g_2 h}{3} & \frac{2t \cdot g_2 h}{3} & 0 & 0 & g_1+g_c+t^2 g_2 & 0 & \frac{2h}{3} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -\frac{h}{6} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2h}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{6} & 0 & 0 & 0 & 0 & -\frac{i_0 h}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{3} & 0 & 0 & 0 & 0 & -\frac{2i_0 h}{3} \end{bmatrix}$$

$$\begin{bmatrix} \frac{g_1 h}{24} & -\frac{g_1 h}{24} & 0 & 0 & -\frac{h}{24} & 0 & -\frac{g_1 h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{g_1 h}{3} & \frac{g_1 h}{3} & 0 & 0 & g_1 L_1+G_1 L_1+\frac{h}{3} & 0 & \frac{g_1 h}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -\frac{g_2 h}{24} & \frac{g_2 h}{24} & 0 & -\frac{h}{24} & -\frac{t \cdot g_2 h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{g_2 h}{3} & \frac{g_2 h}{3} & 0 & \frac{h}{3}+g_2 L_2+G_2 L_2 & \frac{t \cdot g_2 h}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{g_1 h}{24} & -\frac{g_1 h}{24} & \frac{t \cdot g_2 \cdot h}{24} & -\frac{t \cdot g_2 \cdot h}{24} & 0 & 0 & -\left(g_1+g_c+t^2 g_2\right)\frac{h}{24} & 0 & \frac{h}{24} & 0 & 0 & 0 & 0 & 0 & -\frac{g_1 h}{3} & \frac{g_1 h}{3} & -\frac{t \cdot g_2 \cdot h}{3} & \frac{t \cdot g_2 \cdot h}{3} & g_1 L_1 & t \cdot g_2 L_2 i_{L2} & \left(g_1+g_c+t^2 g_2\right)\frac{h}{3} & 0 & \frac{h}{3} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{h}{3} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{h}{24} & 0 & 0 & 0 & \frac{i_0 h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{3} & 0 & 0 & 0 & 0 & -\frac{i_0 h}{3} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2\lambda(t)}{\lambda_0^2} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2y_1(t) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2y_2(t) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -y_3(t) & 0 & -y_1(t) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -y_5(t) & 0 & 0 & 0 & 0 & -\frac{\lambda(t)}{\lambda_0} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2\lambda(t)}{\lambda_0^2} & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2y_1(t) & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2y_2(t) & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -y_3(t) & 0 & -y_1(t) & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -y_5(t) & 0 & 0 & 0 & 0 & 0 & -\frac{\lambda(t)}{\lambda_0} & 1 \end{bmatrix}$$

3. **Pseudo Measurements**: These measurements represent quantities that are normally not measured such as voltages in the neutral. In this experiment the voltage of the neutral with respect to the ground on both sides of the transformer is taken to be zero (0). The standard deviation in this case was taken to be the same as that of voltage measurements i.e. 0.1p.u.

$$0 = v_2(t) \qquad 0 = v_4(t) \qquad 0 = v_2(t_m) \qquad 0 = v_4(t_m)$$

We can now determine the contributions of each of the above equations to the Jacobian matrix.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

4. **Derived Measurements;** These are measurements that are derived from other measurements such as the currents in the grounded branch in the transformer.

$$-I_A(T+1) = -g_1 v_1(t) + g_1 v_2(t) + \frac{4g_1 L_1}{h} i_{L1}(t) + g_1 e(t) - \frac{8g_1 L_1}{h} i_{L1}(t_m) + g_1 v_1(t-h)$$
$$-g_1 v_2(t-h) + \frac{4g_1 L_1}{h} i_{L1}(t-h) - g_1 e(t-h) - i_1(t-h)$$

$$-I_a(T+1) = -g_2 v_3(t) + g_2 v_4(t) + \frac{4g_2 L_2}{h} i_{L2}(t) + t\cdot g_2 e(t) - \frac{8g_2 L_2}{h} i_{L1}(t_m) + g_2 v_3(t-h)$$
$$-g_2 v_4(t-h) + \frac{4g_2 L_2}{h} i_{L2}(t-h) - t\cdot g_2 e(t-h) - i_2(t-h)$$

$$-I_A(T) = +\frac{g_1 L_1}{2h} i_{L1}(t) - g_1 v_1(t_m) + g_1 v_2(t_m) + \frac{2g_1 L_1}{h} i_{L1}(t_m) + g_1 e(t_m) - \frac{5g_1 L_1}{12} v_1(t-h)$$
$$+\frac{5g_1 L_1}{12} v_2(t-h) - \frac{5g_1 L_1}{2h} i_{L1}(t-h) + \frac{5g_1 L_1}{12} e(t-h) + \frac{1}{2} i_1(t-h)$$

$$-I_a(T) = \frac{g_2 L_2}{2h} i_{L2}(t) - g_2 v_3(t_m) + g_2 v_4(t_m) + \frac{2g_2 L_2}{h} i_{L2}(t_m) + t\cdot g_2 e(t_m) - \frac{5g_2 L_2}{12} v_3(t-h)$$
$$+\frac{5g_2 L_2}{12} v_2(t-h) - \frac{5g_2 L_2}{2h} i_{L2}(t-h) + \frac{5\cdot t\cdot g_2 L_2}{12} e(t-h) + \frac{1}{2} i_3(t-h)$$

The contributions of each of the above equations to the Jacobian are as follows

$$\begin{bmatrix}-g_1 & g_1 & 0 & 0 & \frac{4g_1L_1}{h} & 0 & g_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8g_1L_1}{h} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}0 & 0 & -g_2 & g_2 & 0 & \frac{4g_2L_2}{h} & t\cdot g_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8g_2L_2}{h} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}0 & 0 & 0 & 0 & \frac{g_1L_1}{2h} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g_1 & g1 & 0 & 0 & \frac{2g_1L_1}{h} & 0 & g_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}0 & 0 & 0 & 0 & 0 & \frac{g_2L_2}{2h} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g_2 & g_2 & 0 & \frac{2g_2L_2}{h} & t\cdot g_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

We can summarize the above with the following table

Table 4.1: Summary of Transformer Measurement Models by type

Measurement Type	**Measurement Model**	**Standard Deviation**	**Measurement**
Actual	$g_1v_1(t)-g_1v_2(t)-\frac{4g_1L_1}{h}i_{L1}(t)-g_1e(t)+\frac{8g_1L_1}{h}i_{L1}(t_m)-g_1v_1(t-h)+g_1v_2(t-h)$ $-\frac{4g_1L_1}{h}i_{L1}(t-h)+g_1e(t-h)+i_1(t-h)$	0.01pu	$I_A(T+1)$
Derived	$-g_1v_1(t)+g_1v_2(t)+\frac{4g_1L_1}{h}i_{L1}(t)+g_1e(t)-\frac{8g_1L_1}{h}i_{L1}(t_m)+g_1v_1(t-h)$ $-g_1v_2(t-h)+\frac{4g_1L_1}{h}i_{L1}(t-h)-g_1e(t-h)-i_1(t-h)$	0.01pu	$-I_A(T+1)$
Actual	$g_2v_3(t)-g_2v_4(t)-\frac{4g_2L_2}{h}i_{L2}(t)-t\cdot g_2e(t)+\frac{8g_2L_2}{h}i_{L2}(t_m)-g_2v_3(t-h)+g_2v_4(t-h)$ $-\frac{4g_2L_2}{h}i_{L2}(t-h)+t\cdot g_2e(t-h)+i_2(t-h)$	0.01pu	$I_a(T+1)$
Derived	$-g_2v_3(t)+g_2v_4(t)+\frac{4g_2L_2}{h}i_{L2}(t)+t\cdot g_2e(t)-\frac{8g_2L_2}{h}i_{L1}(t_m)+g_2v_3(t-h)$ $-g_2v_4(t-h)+\frac{4g_2L_2}{h}i_{L2}(t-h)-t\cdot g_2e(t-h)-i_2(t-h)$	0.01pu	$-I_a(T+1)$
Virtual	$-\frac{g_1h}{6}v_1(t)+\frac{g_1h}{6}v_2(t)+\left(\frac{h}{6}+g_1L_1+G_1L_1\right)i_{L1}(t)+\frac{g_1h}{6}e(t)-\frac{2g_1h}{3}v_1(t_m)+\frac{2g_1h}{3}v_2(t_m)+\frac{2h}{3}i_{L1}(t_m)$ $+\frac{2g_1h}{3}e(t_m)-\frac{g_1h}{6}v_1(t-h)+\frac{g_1h}{6}v_2(t-h)-\left(-\frac{h}{6}+g_1L_1+G_1L_1\right)i_{L1}(t-h)+\frac{g_1h}{6}e(t-h)$	0.001pu	0
Virtual	$-\frac{g_2h}{6}v_3(t)+\frac{g_2h}{6}v_4(t)+\left(\frac{h}{6}+g_2L_2+G_2L_2\right)i_{L2}(t)+\frac{t\cdot g_2h}{6}e(t)-\frac{2g_2h}{3}v_3(t_m)+\frac{2g_2h}{3}v_4(t_m)$ $+\frac{2h}{3}i_{L2}(t_m)+\frac{2t\cdot g_2h}{3}e(t_m)-\frac{g_2h}{6}v_3(t-h)+\frac{g_2h}{6}v_4(t-h)-\left(-\frac{h}{6}+g_2L_2+G_2L_2\right)i_{L2}(t-h)+\frac{t\cdot g_2h}{6}e(t-h)$	0.001pu	0
Virtual	$-\frac{g_1h}{6}v_1(t)+\frac{g_1h}{6}v_2(t)-\frac{t\cdot g_2h}{6}v_3(t)+\frac{t\cdot g_2h}{6}v_4(t)+g_1L_1i_{L1}(t)+t\cdot g_2L_2i_{L2}(t)+\left(g_1+g_c+t^2g_2\right)\frac{h}{6}e(t)$ $+\frac{h}{6}i_m(t)-\frac{2g_1h}{3}v_1(t_m)+\frac{2g_1h}{3}v_2(t_m)-\frac{2t\cdot g_2h}{3}v_3(t_m)+\frac{2t\cdot g_2h}{3}v_4(t_m)+\left(g_1+g_c+t^2g_2\right)\frac{2h}{3}e(t_m)+\frac{2h}{3}i_m(t_m)$ $-\frac{g_1h}{6}v_1(t-h)+\frac{g_1h}{6}v_2(t-h)-\frac{t\cdot g_2h}{6}v_3(t)+\frac{t\cdot g_2h}{6}v_4(t)-g_1L_1i_{L1}(t-h)-t\cdot g_2L_2i_{L2}(t-h)$ $-\left(g_1+g_c+t^2g_2\right)\frac{h}{6}e(t-h)+\frac{h}{6}i_m(t-h)$	0.001pu	0
Virtual	$-\frac{h}{6}e(t)+\lambda(t)-\frac{2h}{3}e(t_m)+\frac{h}{6}e(t-h)+\lambda(t-h)$	0.001pu	0
Virtual	$\frac{h}{6}i_m(t)-\frac{i_0h}{6}y_5(t)+\frac{2h}{3}i_m(t_m)-\frac{2i_0h}{3}y_5(t_m)+\frac{h}{6}i_m(t-h)-\frac{i_0h}{6}y_5(t-h)$	0.001pu	0
Actual	$-\frac{g_1L_1}{2h}i_{L1}(t)+g_1v_1(t_m)-g_1v_2(t_m)-\frac{2g_1L_1}{h}i_{L1}(t_m)-g_1e(t_m)+\frac{5g_1L_1}{12}v_1(t-h)$ $-\frac{5g_1L_1}{12}v_2(t-h)+\frac{5g_1L_1}{2h}i_{L1}(t-h)-\frac{5g_1L_1}{12}e(t-h)-\frac{1}{2}i_1(t-h)$	0.01pu	$I_A(T)$
Derived	$\frac{g_1L_1}{2h}i_{L1}(t)-g_1v_1(t_m)+g_1v_2(t_m)+\frac{2g_1L_1}{h}i_{L1}(t_m)+g_1e(t_m)-\frac{5g_1L_1}{12}v_1(t-h)$ $+\frac{5g_1L_1}{12}v_2(t-h)-\frac{5g_1L_1}{2h}i_{L1}(t-h)+\frac{5g_1L_1}{12}e(t-h)+\frac{1}{2}i_1(t-h)$	0.01pu	$-I_A(T)$
Actual	$-\frac{g_2L_2}{2h}i_{L2}(t)+g_2v_3(t_m)-g_2v_4(t_m)-\frac{2g_2L_2}{h}i_{L2}(t_m)-t\cdot g_2e(t_m)+\frac{5g_2L_2}{12}v_3(t-h)$ $-\frac{5g_2L_2}{12}v_2(t-h)+\frac{5g_2L_2}{2h}i_{L2}(t-h)-\frac{5\cdot t\cdot g_2L_2}{12}e(t-h)-\frac{1}{2}i_3(t-h)$	0.01pu	$I_a(T)$

Table 4.1 (continued): Summary of Transformer Measurement Models by type

Derived	$\frac{g_2 L_2}{2h} i_{L2}(t) - g_2 v_3(t_m) + g_2 v_4(t_m) + \frac{2 g_2 L_2}{h} i_{L2}(t_m) + t \cdot g_2 e(t_m) - \frac{5 g_2 L_2}{12} v_3(t-h)$ $+ \frac{5 g_2 L_2}{12} v_2(t-h) - \frac{5 g_2 L_2}{2h} i_{L2}(t-h) + \frac{5 \cdot t \cdot g_2 L_2}{12} e(t-h) + \frac{1}{2} i_3(t-h)$	0.01pu	$-I_a(T)$
Virtual	$\frac{g_1 h}{24} v_1(t) - \frac{g_1 h}{24} v_2(t) - \frac{h}{24} i_{L1}(t) - \frac{g_1 h}{24} e(t) - \frac{g_1 h}{3} v_1(t_m) + \frac{g_1 h}{3} v_2(t_m) + \left(g_1 L_1 + G_1 L_1 + \frac{h}{3} \right) i_{L1}(t_m)$ $+ \frac{g_1 h}{3} e(t_m) - \frac{g_1 h}{6} v_1(t-h) + \frac{g_1 h}{6} v_2(t-h) - \left(g_1 L_1 + G_1 L_1 - \frac{h}{6} \right) i_{L1}(t-h) + \frac{g_1 h}{6} e(t-h)$	0.001pu	0
Virtual	$\frac{g_2 h}{24} v_3(t) + \frac{g_2 h}{24} v_4(t) - \frac{h}{24} i_{L2}(t) - \frac{t \cdot g_2 h}{24} e(t) - \frac{g_2 h}{3} v_3(t_m) + \frac{g_2 h}{3} v_4(t_m) + \left(\frac{h}{3} + g_2 L_2 + G_2 L_2 \right) i_{L2}(t_m)$ $+ \frac{t \cdot g_2 h}{3} e(t_m) - \frac{g_2 h}{6} v_3(t-h) + \frac{g_2 h}{6} v_4(t-h) - \left(g_2 L_2 + G_2 L_2 - \frac{h}{6} \right) i_{L2}(t-h) + \frac{t \cdot g_2 h}{6} e(t-h)$	0.001pu	0
Virtual	$\frac{g_1 h}{24} v_1(t) - \frac{g_1 h}{24} v_2(t) + \frac{t \cdot g_2 \cdot h}{24} v_3(t) - \frac{t \cdot g_2 \cdot h}{24} v_4(t) - \left(g_1 + g_c + t^2 g_2 \right) \frac{h}{24} e(t) + \frac{h}{24} i_m(t) - \frac{g_1 h}{3} v_1(t_m)$ $+ \frac{g_1 h}{3} v_2(t_m) - \frac{t \cdot g_2 \cdot h}{3} v_3(t_m) + \frac{t \cdot g_2 \cdot h}{3} v_4(t_m) + g_1 L_1 i_{L1}(t_m) + t \cdot g_2 L_2 i_{L2}(t_m) + \left(g_1 + g_c + t^2 g_2 \right) \frac{h}{3} e(t_m)$ $+ \frac{h}{3} i_m(t_m) - \frac{g_1 h}{6} v_1(t-h) + \frac{g_1 h}{6} v_2(t-h) - \frac{t \cdot g_2 \cdot h}{6} v_3(t) + \frac{t \cdot g_2 \cdot h}{6} v_4(t) - g_1 L_1 i_{L1}(t-h) - t \cdot g_2 L_2 i_{L2}(t-h)$ $+ \left(g_1 + g_c + t^2 g_2 \right) \frac{h}{6} e(t-h) + \frac{h}{6} i_m(t-h)$	0.001pu	0
Virtual	$\frac{h}{24} e(t) - \frac{h}{3} e(t_m) + \lambda(t) - \frac{h}{6} e(t-h) - \lambda(t-h)$	0.01pu	I_A
Derived	$-\frac{h}{24} i_m(t) + \frac{i_0 h}{24} y_5(t) + \frac{h}{3} i_m(t_m) - \frac{i_0 h}{3} y_5(t_m) + \frac{h}{6} i_m(t-h) - \frac{i_0 h}{6} y_5(t-h)$	0.01pu	$-I_A$
Actual	$y_1(t) + \lambda(t) \cdot \left[-\frac{1}{\lambda_0^2} \right] \cdot \lambda(t)$	0.01pu	I_a
Derived	$y_2(t) + y_1(t) \cdot [-1] \cdot y_1(t)$	0.01pu	$-I_a$
Virtual	$y_3(t) + y_2(t) \cdot [-1] \cdot y_2(t)$	0.001pu	0
Virtual	$y_4(t) + y_1(t) \cdot [-1] \cdot y_3(t)$	0.001pu	0
Virtual	$y_5(t) + \lambda(t) \cdot \left[-\frac{1}{\lambda_0} \right] \cdot y_4(t)$	0.001pu	0
Virtual	$y_1(t_m) + \lambda(t_m) \cdot \left[-\frac{1}{\lambda_0^2} \right] \cdot \lambda(t_m)$	0.001pu	0
Virtual	$y_2(t_m) + y_1(t_m) \cdot [-1] \cdot y_1(t_m)$	0.001pu	0
Virtual	$y_3(t_m) + y_2(t_m) \cdot [-1] \cdot y_2(t_m)$	0.001pu	0
Virtual	$y_4(t_m) + y_1(t_m) \cdot [-1] \cdot y_3(t_m)$	0.001pu	0
Virtual	$y_5(t_m) + \lambda(t_m) \cdot \left[-\frac{1}{\lambda_0} \right] \cdot y_4(t_m)$	0.001pu	0
Actual	$v_1(t) - v_2(t)$	0.001pu	$V_A(T+1)$
Actual	$v_3(t) - v_4(t)$	0.001pu	$V_a(T+1)$
Pseudo	$v_2(t)$	0.1pu	0
Pseudo	$v_4(t)$	0.1pu	0

Table 4.1 (continued): Summary of Transformer Measurement Models by type

Actual	$v_1(t_m)-v_2(t_m)$	0.01pu	$V_A(T)$
Actual	$v_3(t_m)-v_4(t_m)$	0.01pu	$V_a(T)$
Pseudo	$v_2(t_m)$	0.1 pu	0
Pseudo	$v_4(t_m)$	0.01pu	0

4.3 Definition of Events

For the numerical experiments, a number of typical transformer events have been generated. These events will normally exercise the typical protection functions of a transformer i.e. 24, 27, 50, 51, 87.The objective of these numerical experiments is to determine whether the proposed algorithm will identify the condition and for any of the events that warrant tripping.

<u>Event 1-An energization event:</u> In this event we will examine the performance of our algorithm on one of the most common causes of fake trips in transformers, energization. We will simply run a simulation with a three phase Y-Y connected transformer for a few seconds and the COMTRADE file generated from this event will be tested for faults using our algorithm. Our test system is shown in Appendix B of this document.

<u>Event 2 –External Fault on a transformer:</u> This event represents a typical scenario that could lead to tripping of a transformer. In this scenario we have a normal operation of the transformer for a certain amount of time followed by an external fault which is cleared after some time and normal operation resumes. Three phase fault logic is added to our existing three phase Y-Y

connected transformer from the previous experiment and measurements rendered on both sides of our transformer.

Event 3-An internal fault: In this scenario, we will have normal operation of a transformer followed by a typical internal fault that will last for a 500 mss and then we will return to normal operation.

The following section will show the performance f our algorithm. It must be noted that a low probability or goodness of fit of our algorithm will be interpreted as a trip signal for our digital relay.

4.4 Results

The above events were simulated and we will show the performance of our algorithm by plotting the goodness of fit, from the chi square test as a function of time. In addition, we will also show voltage and current estimates from our state estimation and compare them to the actual measured values.

Enegization Event: The probability curve for the entire event shows according to our algorithm does not indicate any internal failure on our transformer as indicated by the goodness of fit which remains at a 100% all throughout the entire event.

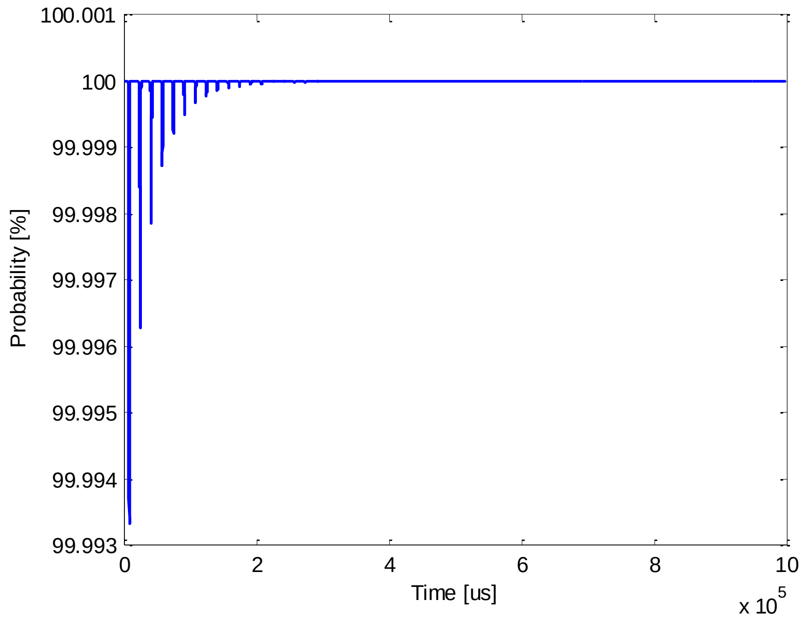

Figure 4.1: Goodness of fit of State Estimation for Energization Event

In Figure 4.1 we observe the goodness of fit of our state estimation during the first second of the energization. We clearly see that no trip signal will be issued during this event since our probability never falls below 99%. As a matter of fact, the probability always remains close to a 100% clearly indicating that the measurements match the normal operating transformer model to perfection. For the sake of clarity we have just shown only the very first second of the event but as we can observe, the chi square test continues to show a perfect fit between the measurements

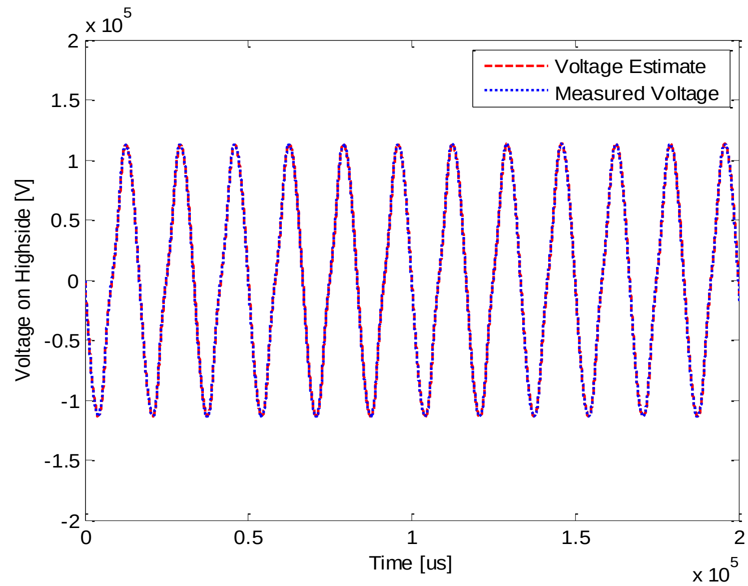

Figure 4.2: Comparison between Actual and Estimated Primary Voltages during Energization Event

In Figure 4.2 we see that both the estimated voltage and the measured voltage match perfectly during the energization event. These curves are interesting because it gives us an indication on the primary voltage indicating that no over-voltage or under-voltage is occurring but really is just one more component in the overall protection scheme because as we will see in future cases that the estimated and measured primary voltages might match but still we have an abnormal operating condition requiring a trip .

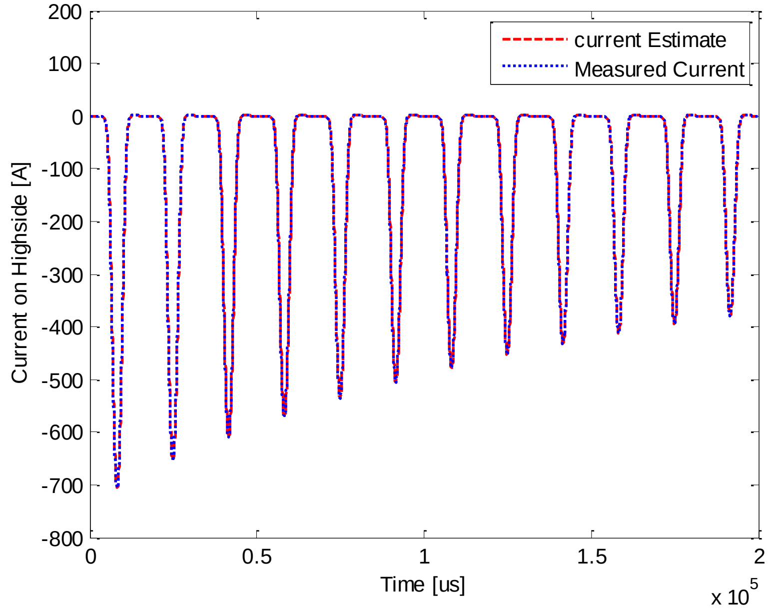

Figure 4.3: Comparison between Actual and Estimated Primary Current during Energization Event

Figure 4.3 may be the single most important representation of the success of our protection scheme during the energization event. Our methodology clearly matches the primary estimated and measured currents. The infamous inrush currents known for causing most, if not all of trips on traditional protection schemes is clearly identified and does not result in any trip by our protection scheme. The inrush currents are the main cause of the imbalance between the primary and the secondary currents which causes the trip in traditional differential protection schemes. Our protection scheme performed fine during excitation which is the Achilles heel of traditional transformer protection scheme due to the ability to capture and reproduce inrush currents.

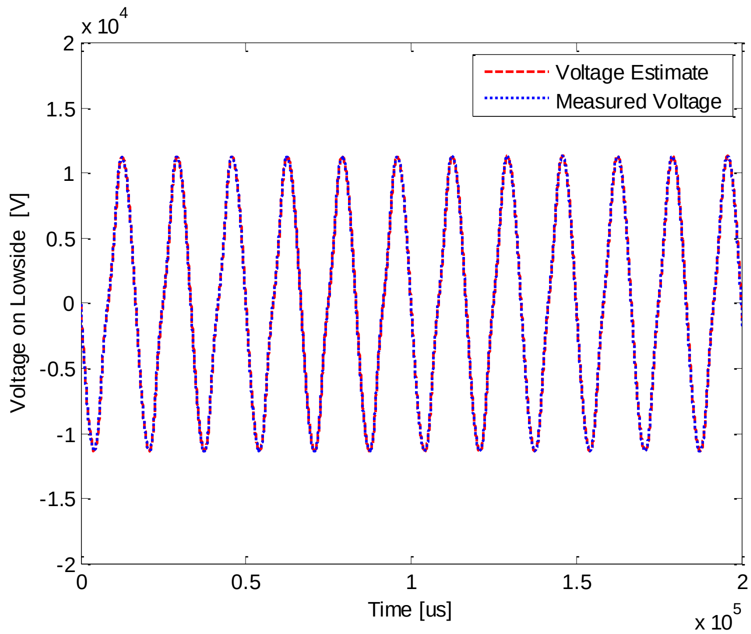

Figure 4.4: Comparison between Actual and Estimated Secondary Voltages during Energization Event

Figure 4.4 shows the estimated and measured secondary voltages matching perfectly, again which is no surprise considering the fact that the goodness of fit is near to a 100% during the duration of the energization event. The secondary voltage is perfectly sinusoidal as is the primary and in the exact ratio as the turns ratio.

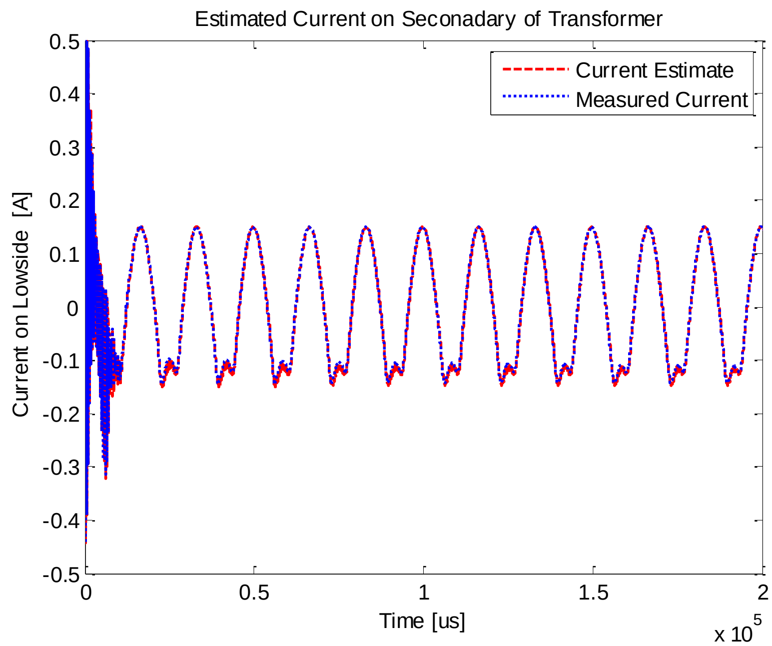

Figure 4.5: Comparison between Actual and Estimated Secondary Currents during Energization Even

The secondary estimated and measured currents shown in Figure 4.5 above are no exception to the curves shown in the previous figures. There is a perfect match between the estimated and measured currents. The primary and secondary currents do not match and is the reason for the tripping of traditional differential protection schemes. Our algorithm will not operate as the most advanced differential protection scheme. In addition the only parameters needed are the electrical parameters of our transformer and just our usual Kirchhoff equations. Another interesting part of our algorithm is that the estimates tend to follow even transient phenomena which is so important as far as not only initial inrushes but also sympathetic and recovery inrushes.

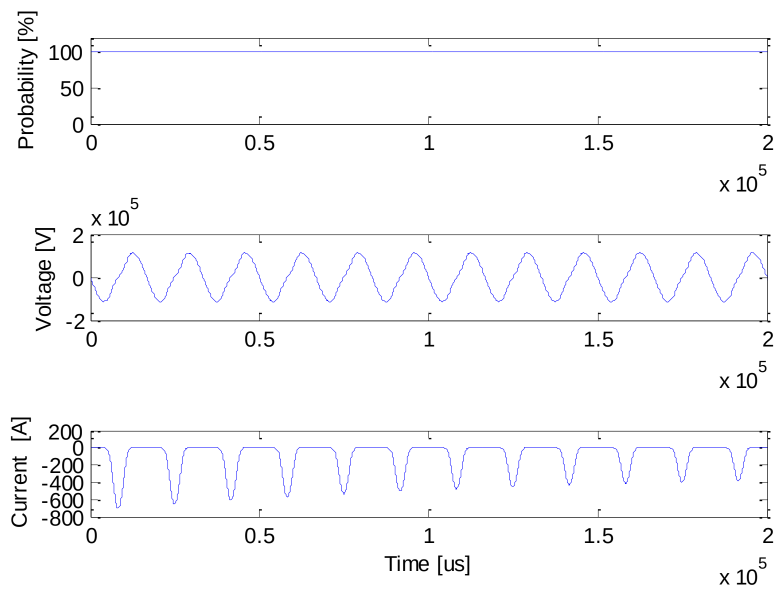

Figure 4.6: Goodness of Fit, Primary Voltage and Primary Current during Energization Event

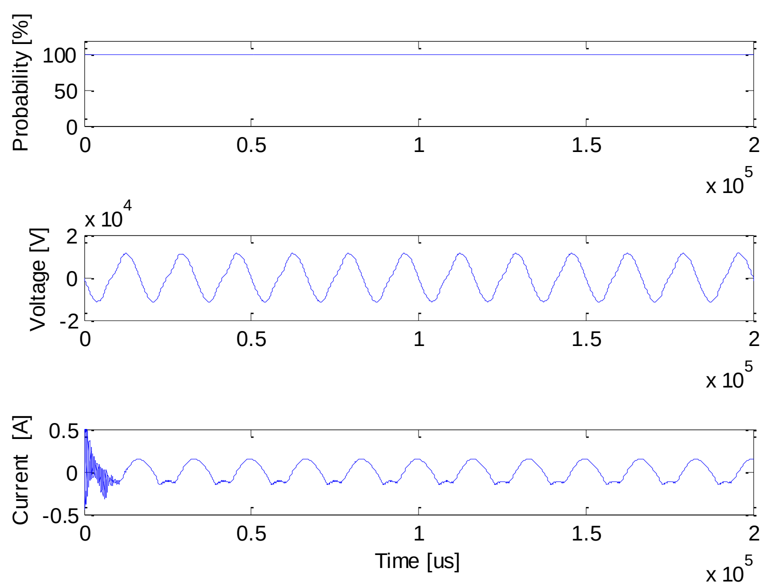

Figure 4.7: Goodness of Fit, Secondary Voltage and Secondary Current during Energization Event

The above curves just a simultaneous representation of our estimates and the goodness of fit. These curves might be redundant but they are very essential in visualizing at what times we our protection scheme issues a trip signal. In this case our relay does not trip hence the un-fluctuating 100% match through our event.

External fault event: In this event our test system was simulated with a fault logic that activated after 1500 ms of normal operation. The external fault then lasted for 500 ms and then the fault was cleared and the transformer returned to normal operation for 300ms.

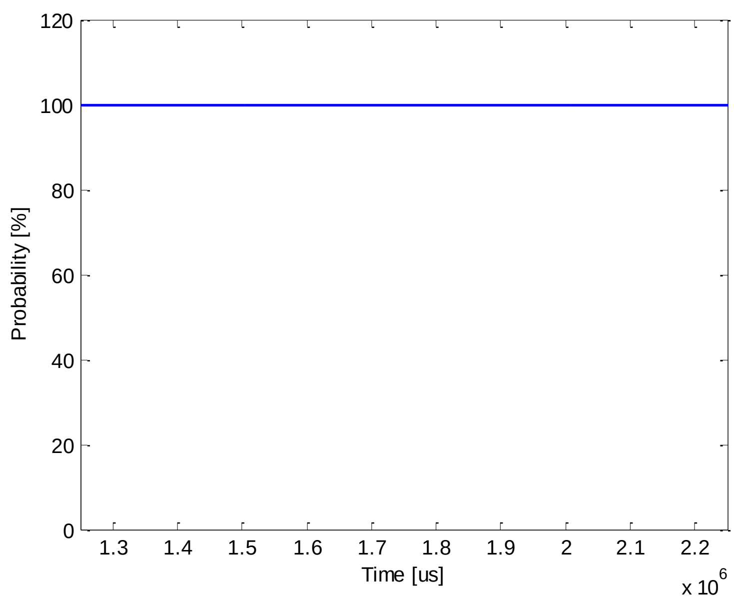

Figure 4.8: Goodness of fit of State Estimation for External Fault Event

Figure 4.8 shows the goodness of fit of the entire event and clearly shows that our protection scheme does not trip. We see a 100% goodness of fit for the whole event indicating that no internal fault occurred on the transformer.

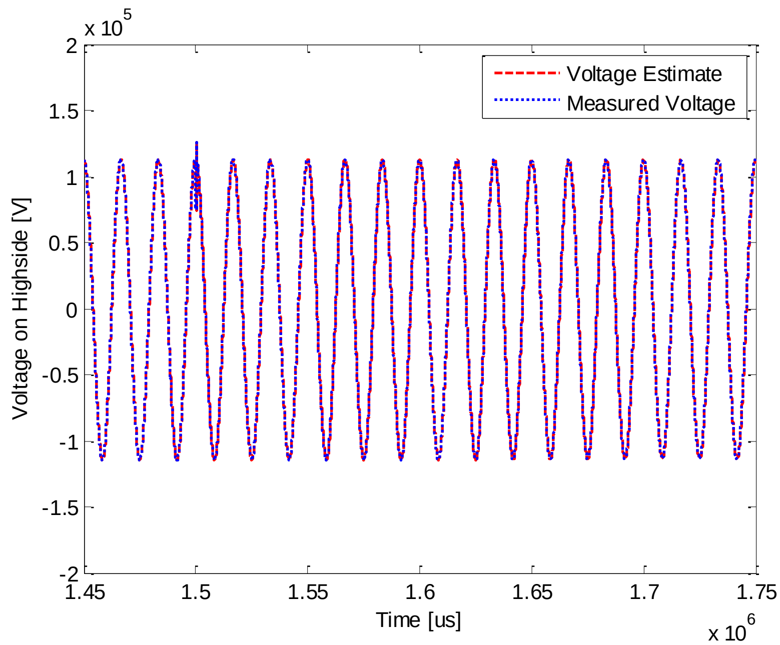

Figure 4.9: Comparison between Actual and Estimated Primary Voltages during External Fault Event

Shown in Figure 4.9 is a comparison between the estimated and measured primary voltage on the transformer, and as expected we have a perfect match between the two. The voltages match and even the time instant at which the fault takes place our protection scheme does not falter and still the probability remains unwavering at 100%. Even the transients, are picked up by the state estimation and no trip signal is rightfully issued.

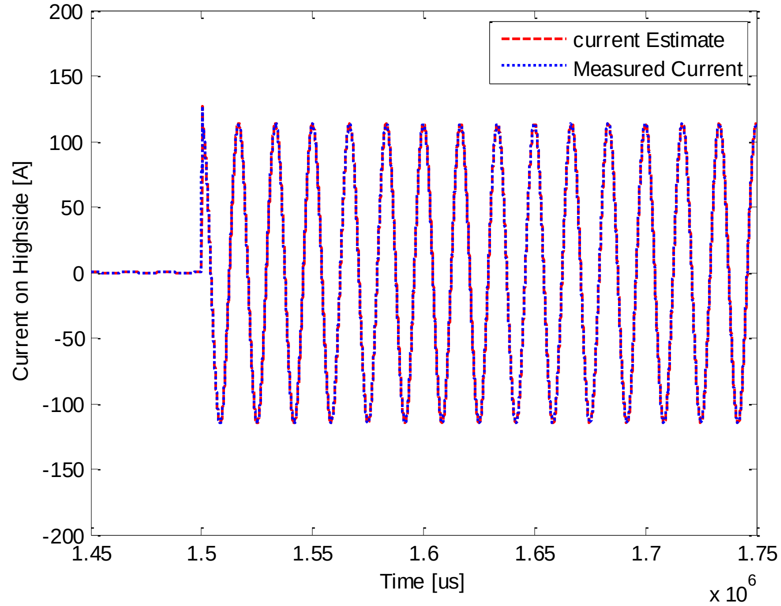

Figure 4.11: Comparison between Actual and Estimated Primary Currents during External Fault Event

External faults on a transformer may cause fake trips on transformer protection due to the fact that we have large instantaneous over-currents at the instant of the fault as can be seen in Figure 4.11. Regardless of this fact, the transformer might experience an instantaneous over-current which is not a result of an internal fault but rather caused by an external fault which our protection scheme detects with perfection as we observe that the estimated and measured currents match to perfection with no fault.

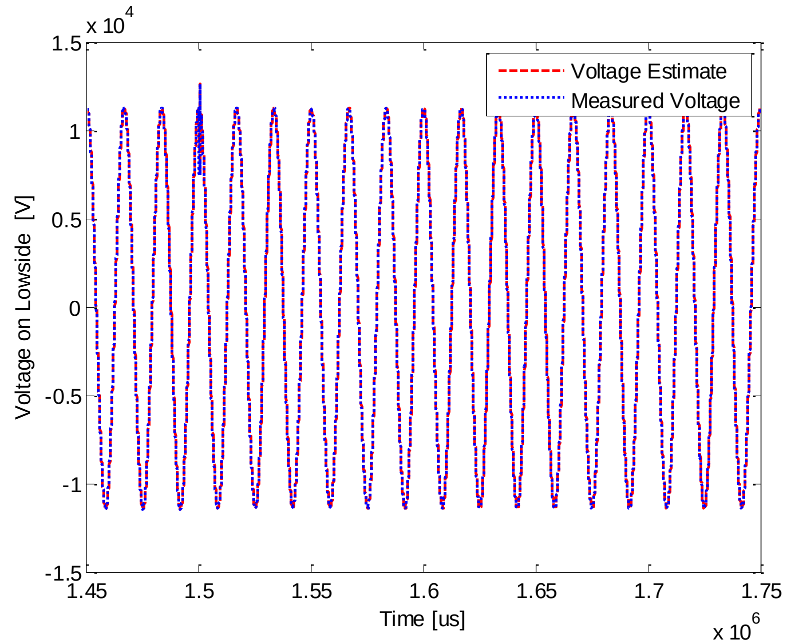

Figure 4.12: Comparison between Actual and Estimated Secondary Voltages during External Fault Event

In Figure 4.12 we compare the secondary measured and estimated voltages. As expected there is a perfect match between both curves which is expected considering the fact that our goodness of fit curve showing how closely our measurements match our model. An interesting thing in these curves is that even the minor transients at the instant of the fault are simulated with our state estimation procedure giving a lot of potential to our technique for other devices with even larger transients.

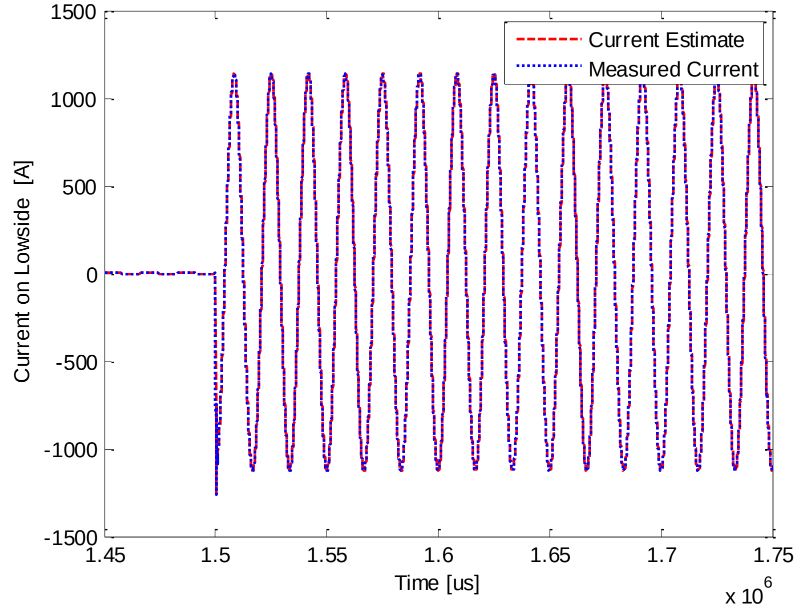

Figure 4.13: Comparison between Actual and Estimated Secondary Currents during External Fault Event

Figure 4.13 shows a comparison between the estimated and measured current on the secondary side of the transformer. Again this is consistent with our goodness of fit curve since the measurements match the model and at no point do we drop even below a 100%.

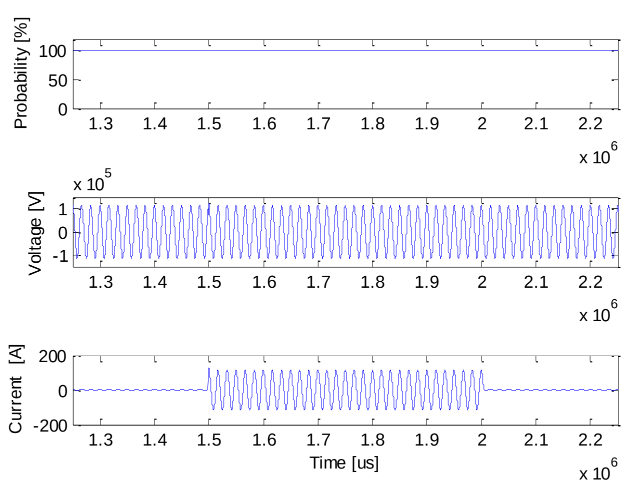

Figure 4.14: Goodness of Fit, Primary Voltage and Primary Current during External Fault Event

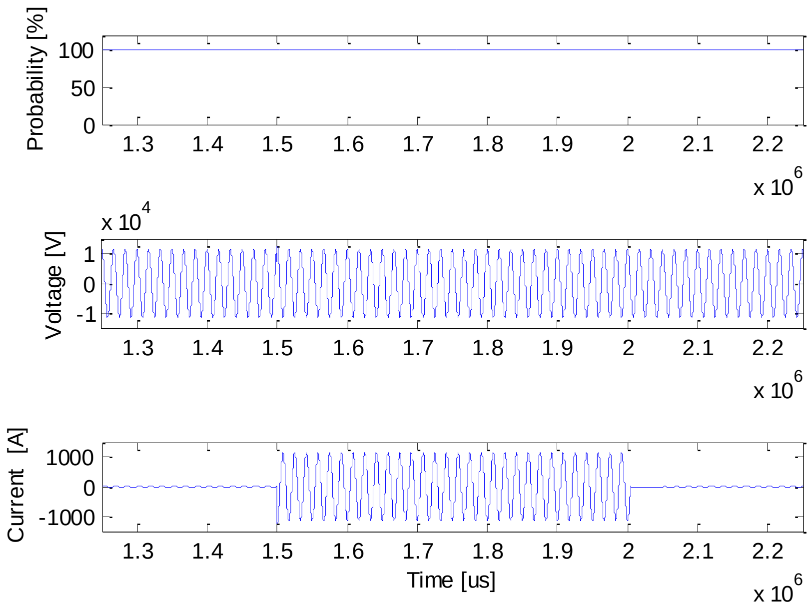

Figure 4.15: Goodness of Fit, Secondary Voltage and Secondary Current during External Fault Event

In our cumulative summary curves shown in Figures 4.14 and 4.15, we show our estimated primary and secondary voltage with the goodness of fit simultaneously and show that at no point does our protection scheme detect s an internal fault on the transformer during the course of our event.

Internal Fault Event: The internal fault event is very similar to the prior external fault scenario. The total event depicted is 3*s* in duration and starts with normal operation of the transformer and after 1500*ms* fault logic is activated internally in our simulation in Appendix B. After 500ms of operation, the logic is cleared and the transformer returns to normal operation.

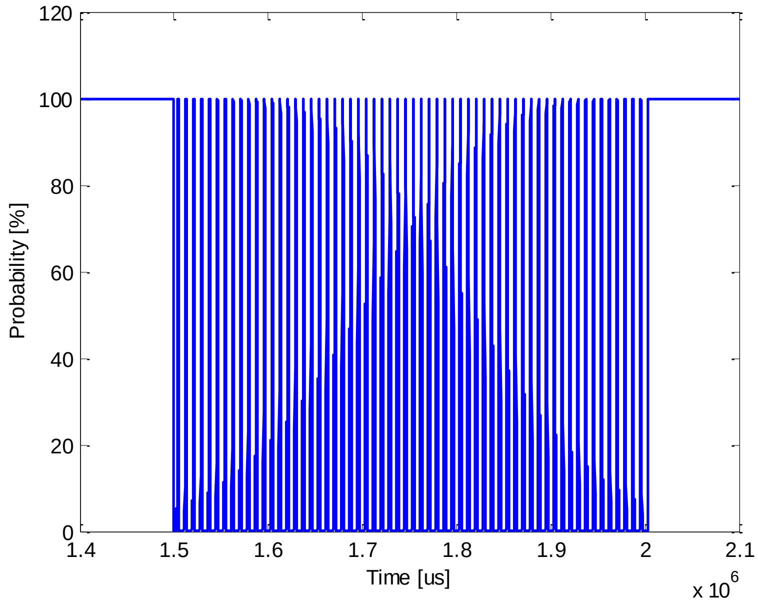

Figure 4.16: Goodness of fit of State Estimation for Internal Fault Event

In Figure 4.16 we observe the goodness of fit between our measurements and state estimates and we observe a series of drops not only at the instant the fault occurs but also during the entire fault until the fault is cleared after which the goodness of fit returns to a 100%. In this case drop in the goodness of fit clearly indicates the occurrence of an internal fault on the transformer.

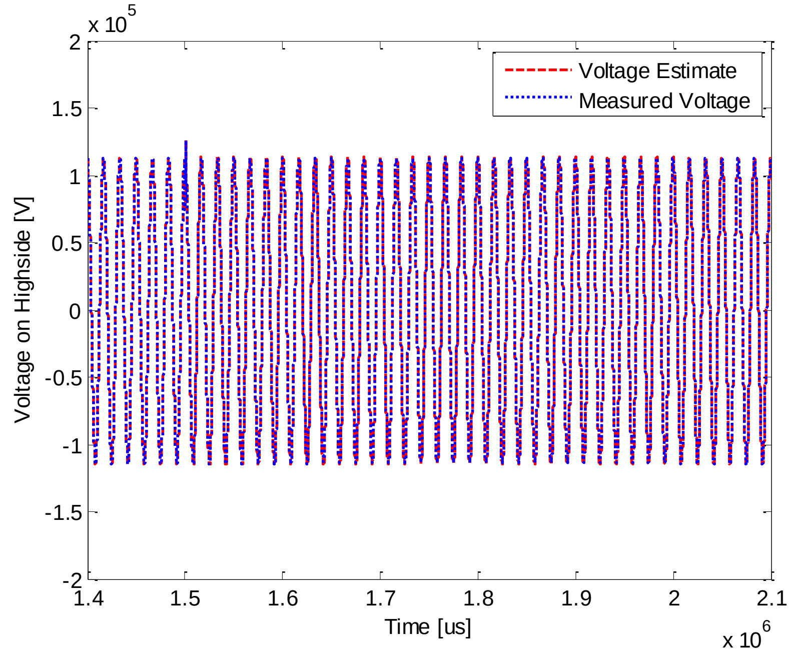

Figure 4.17: Comparison between Actual and Estimated Primary Voltages during Internal Fault Event

Figure 4.17 depicts the estimated and measured primary voltages, which match. It is clear that the voltage measured and estimated voltages match and this does not show us or explain the trip in our protection scheme.

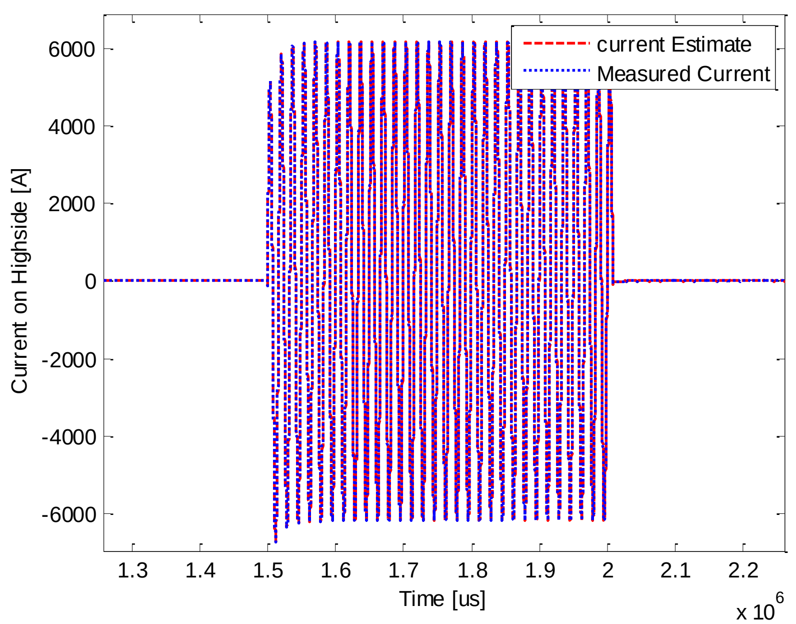

Figure 4.18: Comparison between Actual and Estimated Primary Currents during Internal Fault Event

Figure 4.18 shows a comparison between the estimated and measured primary current and still we observe a perfect match between the two during the entire duration of the event. As far as only this curve is concerned we cannot explain the fluctuation in the goodness of fit because both curves seem to perfectly fit one another.

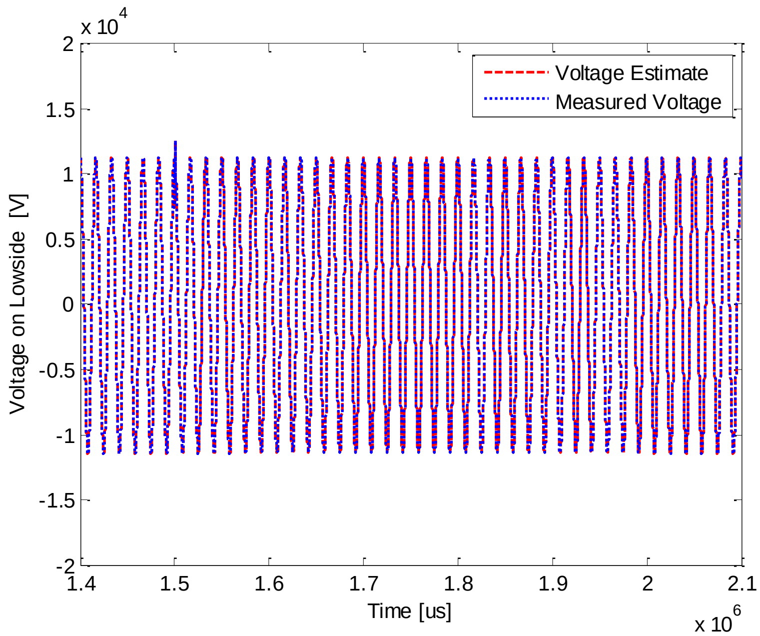

Figure 4.19: Comparison between Actual and Estimated Secondary Voltages during Internal Fault Event

Figure 4.19 shows the measured and estimated secondary voltages, these curves match to perfection. Just observing these two curves does not justify the reason for having a drop in the goodness of fit. But still this behavior is perfectly explainable by the fact that the measured and estimated voltages match but this might not be true for the other measurement models.

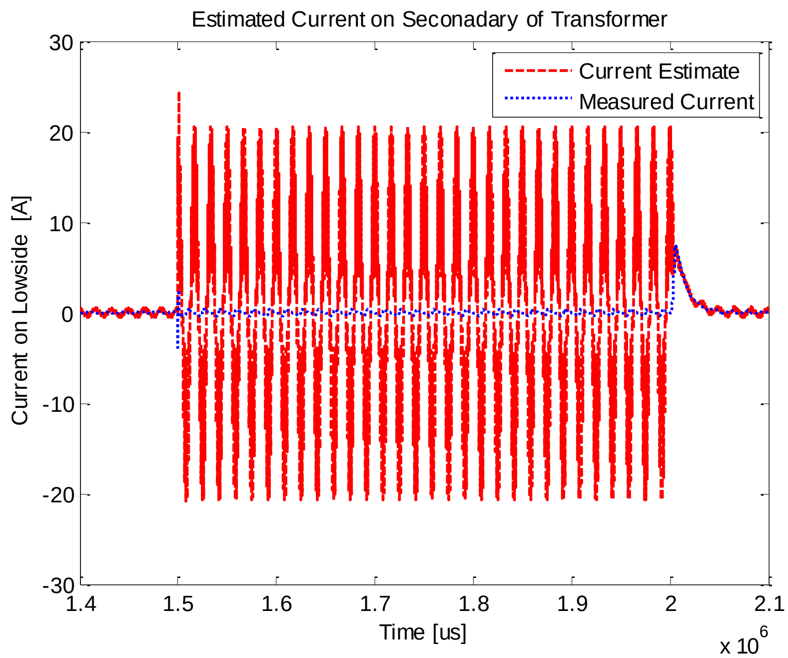

Figure 4.20: Comparison between Actual and Estimated Secondary Currents during Internal Fault Event

Figure 4.20 shows the estimated and measured secondary currents for our event. For the first time we observe a clear mismatch in the 2 curves from 1.5*s* to 2*s*. Our Current estimate is much higher than the measurement during this period of time which is definitely a contributing factor in the poor goodness of fit during this time period as we will later seen in the summary figures shown below.

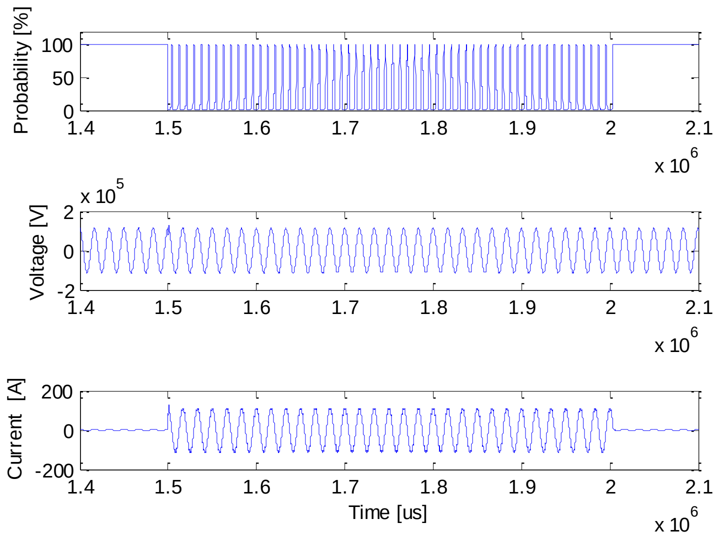

Figure 4.21: Goodness of Fit, Primary Voltage and Primary Current during Internal Fault Event

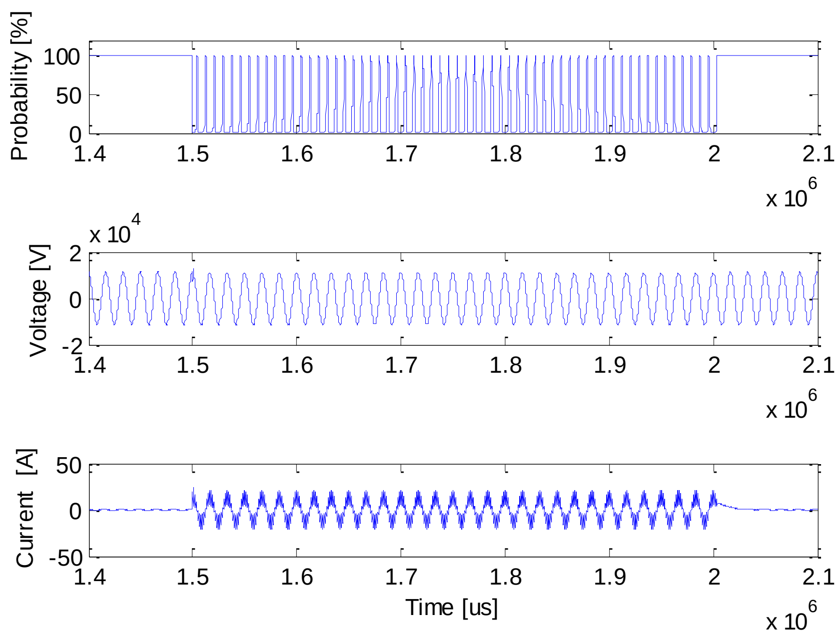

Figure 4.22: Goodness of Fit, Secondary Voltage and Secondary Current during Internal Fault Event

In summary as shown in Figures 4.21 and 4.22 where we show the primary voltage and currents with respect to the goodness of fit and the secondary voltage and current relative respectively our algorithm detects a fault between the 1500*ms* and 2000*ms*which is consistent with our experimental.

CONCLUSION AND FUTURE WORK

The research performed in this thesis focused on developing and implementing a transformer protection algorithm that is able to differentiate between normal operating and internal fault conditions. The algorithm was successfully implemented and was tested using different scenarios that usually occur during transformer operation.

The algorithm performed very well as far as the non-detection of any faults during events such as external faults on the transformer and energization. These events usually cause a lot of trouble for transformer protection schemes and our implementation is straightforward and does not need any training as would contemporary techniques.

During internal faults though, our algorithm detects the occurrence of faults but we have a lot of numerical oscillations causing the goodness of fit and the currents tend to oscillate which might be as a result of computational errors during programming which will be resolved during the commercial version of our software.

Despite our positive preliminary results, there is room for improving the accuracy of our some results in the future namely:

- Improving the software to remove any numerical oscillations
- Including the more equations such as mechanical equations to our model and take advantage of state components such as the flux, and magnetic core voltage and magnetizing currents for better and more complete protection.

- Expand the protection scheme not only to three phase transformer but also to other devices.

APPENDIX A: SINGLE PHASE TRANSFORMER MODEL

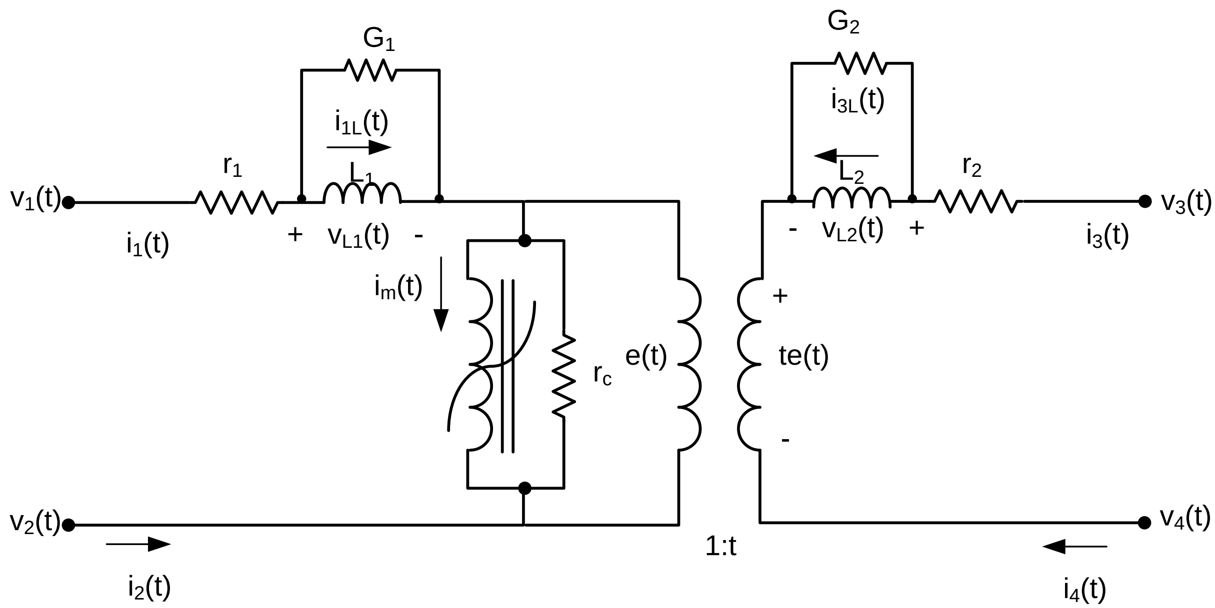

Figure A1: Single Phase Saturable Core Transformer Equivalent Circuit

The following equations describe the saturable core transformer model shown in the above circuit

$$i_1(t) = g_1\left(v_1(t) - v_2(t)\right) - g_1 e(t) - g_1 L_1 \frac{di_{L1}(t)}{dt} \tag{1}$$

$$i_2(t) = g_1\left(-v_1(t) + v_2(t)\right) + g_1 e(t) + g_1 L_1 \frac{di_{L1}(t)}{dt} \tag{2}$$

$$i_3(t) = g_2\left(v_3(t) - v_4(t)\right) - tg_2 e(t) - g_2 L_2 \frac{di_{L2}(t)}{dt} \tag{3}$$

$$i_4(t) = g_2\left(-v_3(t) + v_4(t)\right) + tg_2 e(t) + g_2 L_2 \frac{di_{L2}(t)}{dt} \tag{4}$$

$$0 = -g_1\left(v_1(t) - v_2(t)\right) + g_1 e(t) + \left(G_1 + g_1\right) L_1 \frac{di_{L1}(t)}{dt} + i_{L1}(t) \tag{5}$$

$$0 = -g_2\left(v_3(t) - v_4(t)\right) + g_2 t e(t) + \left(G_2 + g_2\right) L_2 \frac{di_{L2}(t)}{dt} + i_{L2}(t) \tag{6}$$

$$0 = -g_1\left(v_1(t) - v_2(t)\right) - t \cdot g_2\left(v_3(t) - v_4(t)\right) + g_1 L_1 \frac{di_{L1}(t)}{dt} + t \cdot g_2 L_2 \frac{di_{L2}(t)}{dt} + i_m(t) + \left(g_1 + g_c + t^2 g_2\right) e(t) \tag{7}$$

$$0 = e(t) - \frac{d\lambda(t)}{dt} \tag{8}$$

$$0 = i_m(t) - i_0 \cdot \left|\frac{\lambda(t)}{\lambda_0}\right|^n \cdot sign\left(\lambda(t)\right) \tag{9}$$

Where the variables used in these equations are,

i_1 : is the terminal 1 current

i_2 : is the terminal 2 current

i_3 : is the terminal 3 current

i_4 : is the terminal 4 current

v_1 : is the terminal 1 voltage

v_2 : is the terminal 2 voltage

v_3 : is the terminal 3 voltage

v_4 : is the terminal 4 voltage

i_{L1} : is the current through inductance L_1

i_{L2} : is the current through inductance L_2

λ : is the nonlinear inductor flux

i_m : is the magnetizing current

e : is the emf across the inductor

t : is the transformation ratio

Model Quadratization: Equation 9 is the general equation representing the magnetizing current of the single phase transformer with the exponent n taking integer values between 7 and 11. For these different cases of *n*, it should be recognized that the degree of equation (9) is greater than 2 hence it must be substituted with extra equations with rank not greater than 2. This process is referred to as quadratization and can be done by substituting equation by the following equations.

$$0 = i_m(t) - y_m(t) \tag{10}$$

$$0 = y_1(t) - \left(\frac{\lambda(t)}{\lambda_0}\right)^2 \tag{11}$$

$$0 = y_2(t) - y_1^2(t) \tag{12}$$

$$\vdots$$

$$0 = y_{m_1}(t) - y^2_{\frac{2}{m_1-1}}(t) \tag{12+m$_1$}$$

$$0 = y_{m_1+1}(t) - y_{i1}(t) \cdot y_{j1}(t) \tag{12+m$_1$+1}$$

$$0 = y_{m_1+2}(t) - y_{m_1+1}(t) \cdot y_{j2}(t) \tag{12+m$_1$+2}$$

$$\vdots$$

$$\begin{cases} 0 = y_m(t) - \left|y_{m-1}(t) \cdot y_{j3}(t)\right| \cdot sign\left(\lambda(t)\right), \quad \text{if n even} \\ 0 = y_m(t) - \left|y_{m-1}(t) \cdot \lambda(t)\right| \cdot sign\left(\lambda(t)\right), \quad \text{if n odd} \end{cases} \tag{12+m$_1$+m$_2$}$$

Based on the above formulation, the number of additional internal states and equations $m+1$ is computed as follows:

$$m = m_1 + m_2$$

Where,

$$m_1 = \mathrm{int}(\log_2(n))$$

$$m_2 = (\text{\# of ones in the binary representation of } n) - 1$$

The sets of indices *i* and *j* in the last set of equations (13+m1+1 to 13+m) are provided by positions of ones in the binary representation of *n*. The values of these indices are equal to the values of the power of 2 corresponding to that position, meaning that the right most positions is indexed 0 and the left most indexed $\mathrm{int}(\log_2(n))$.

After this substitution the model will be:

$$i_1(t) = g_1\left(v_1(t) - v_2(t)\right) - g_1 e(t) - g_1 L_1 \frac{di_{L1}(t)}{dt} \tag{1*}$$

$$i_2(t) = g_1\left(-v_1(t) + v_2(t)\right) + g_1 e(t) + g_1 L_1 \frac{di_{L1}(t)}{dt} \tag{2*}$$

$$i_3(t) = g_2\left(v_3(t) - v_4(t)\right) - t g_2 e(t) - g_2 L_2 \frac{di_{L2}(t)}{dt} \tag{3*}$$

$$i_4(t) = g_2\left(-v_3(t) + v_4(t)\right) + t g_2 e(t) + g_2 L_2 \frac{di_{L2}(t)}{dt} \tag{4*}$$

$$0 = -g_1\left(v_1(t) - v_2(t)\right) + g_1 e(t) + \left(G_1 + g_1\right) L_1 \frac{di_{L1}(t)}{dt}(t) + i_{L1}(t) \tag{5*}$$

$$0 = -g_2\left(v_3(t) - v_4(t)\right) + g_2 t e(t) + \left(G_2 + g_2\right) L_2 \frac{di_{L2}(t)}{dt} + i_{L2}(t) \tag{6*}$$

$$0 = -g_1\left(v_1(t) - v_2(t)\right) - t \cdot g_2\left(v_3(t) - v_4(t)\right) + g_1 L_1 \frac{di_{L1}(t)}{dt} + t \cdot g_2 L_2 \frac{di_{L2}(t)}{dt} + i_m(t) + \left(g_1 + g_c + t^2 g_2\right) e(t) \tag{7*}$$

$$0 = i_m(t) - i_0 y_m(t) \tag{8*}$$

$$0 = e(t) - \frac{d\lambda(t)}{dt} \tag{9*}$$

$$0 = y_1(t) - \frac{1}{\lambda_0^2}\lambda^2(t) \tag{10*}$$

$$0 = y_1(t) - \left(\frac{\lambda(t)}{\lambda_0}\right)^2 \qquad (11^*)$$

$$0 = y_2(t) - y_1^2(t) \qquad (12^*)$$

$$\vdots$$

$$0 = y_{m_1}(t) - y_{m_1-1}^2(t) \qquad ((9+m_1)^*)$$

$$0 = y_{m_1+1}(t) - y_{i1}(t) \cdot y_{j1}(t) \qquad ((9+m_1+1)^*)$$

$$0 = y_{m_1+2}(t) - y_{m_1+1}(t) \cdot y_{j2}(t) \qquad ((9+m_1+2)^*)$$

$$\vdots$$

$$\begin{cases} 0 = y_m(t) - \left|y_{m-1}(t) \cdot y_{j3}(t)\right| \cdot sign\left(\lambda(t)\right), & \text{if } \ n \text{ even} \\ 0 = y_m(t) - \left|y_{m-1}(t) \cdot \lambda(t)\right| \cdot sign\left(\lambda(t)\right), & \text{if } \ n \text{ odd} \end{cases} \qquad ((9+m)^*)$$

We can now rewrite equations 1* - (9+m)* in the following matrix (compact) form

$$\begin{aligned} i_{1\phi}(t) &= A \cdot x(t) + B\frac{dx(t)}{dt} \\ [0] &= \begin{bmatrix} x^T(t) \cdot Q_1 \cdot x(t) \\ \vdots \\ x^T(t) \cdot Q_m \cdot x(t) \end{bmatrix} \end{aligned} \qquad (13)$$

Where,

$$\begin{aligned} i_{1\phi}(t) &= \begin{bmatrix} i_1(t) & i_2(t) & i_3(t) & i_4(t) & 0 & \cdots & 0 \end{bmatrix}^T \\ \dim\left(i_{1\phi}(t)\right) &= 9 \times 1 \end{aligned}$$

$$\begin{aligned} x(t) &= \begin{bmatrix} v_1(t) & v_2(t) & v_3(t) & v_4(t) & i_{1L}(t) & i_{3L}(t) & e(t) & \lambda(t) & i_m(t) & y_1(t) & y_2(t) & \cdots & y_m(t) \end{bmatrix}^T \\ \dim\left(x(t)\right) &= (9+m) \times 1 = N \times 1 \end{aligned}$$

$$A=\begin{bmatrix} g_1 & -g_1 & 0 & 0 & 0 & 0 & -g_1 & 0 & 0 & & 0 \\ -g_1 & g_1 & 0 & 0 & 0 & 0 & g_1 & 0 & 0 & & 0 \\ 0 & 0 & g_2 & -g_2 & 0 & 0 & -t\cdot g_2 & 0 & 0 & & 0 \\ 0 & 0 & -g_2 & g_2 & 0 & 0 & t\cdot g_2 & 0 & 0 & & 0 \\ -g_1 & g_1 & 0 & 0 & 1 & 0 & g_1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & -g_2 & g_2 & 0 & 1 & t\cdot g_2 & 0 & 0 & & 0 \\ -g_1 & g_1 & -t\cdot g_2 & t\cdot g_2 & 0 & 0 & g_1+g_c+t^2g_2 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & & -i_0 \end{bmatrix}$$

$$\dim(A)=9\times(9+m)$$

$$B=\begin{bmatrix} 0 & 0 & 0 & 0 & -g_1L_1 & 0 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & g_1L_1 & 0 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & -g_2L_2 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & g_2L_2 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & (G_1+g_1)L_1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & (G_2+g_2)L_2 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & g_1L_1 & g_2L_2 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 \end{bmatrix}$$

$$\dim(B)=9\times(9+m)$$

	n even	n odd
$Q_k\left(1\le k\le m,\ i,\ j\right)\ne 0$	$Q_1\left(1,8,8\right)=-\frac{1}{\lambda_0^2}$	$Q_1\left(1,8,8\right)=-\frac{1}{\lambda_0^2}$
$N=9+m$	$Q_k\left(1<k\le m-1\ ,i\ ,j\right)=-1$	$Q_k\left(1<k\le m-1\ ,i\ ,j\right)=-1$
	$Q_k\left(m,i,N-1\right)=-1$	$Q_k\left(m,8,N-1\right)=-\frac{1}{\lambda_0}$

Numerical Integration of the Single Phase Transformer Model

Now that all the equations are in the quadratized form, we can now perform quadratic integration on the equations such that we obtain an algebraic companion form of the single phase transformer composed of two appended parts.

The first is shown below and is obtained by Quadratic Integration of equations 1*-9* which is shown below

$$\begin{bmatrix} i_{1\phi}(t) \\ i_{1\phi}(t_m) \end{bmatrix} = E \cdot F_1 \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - b_{eq}$$

$$b_{eq} = E \cdot F_2 \left[x(t-h) \right] + E \cdot F_3 \left[i_{1\phi}(t-h) \right] \qquad (14)$$

Where

$$E = \begin{bmatrix} \frac{4}{h} I & 0 & -\frac{8}{h} I & 0 \\ 0 & I & 0 & 0 \\ \frac{1}{2h} I & 0 & \frac{2}{h} I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \;,\; F_1 = \begin{bmatrix} \frac{h}{6} A + B & \frac{2h}{3} A \\ -\frac{h}{24} A & \frac{h}{3} A + B \end{bmatrix} \;,\; F_2 = \begin{bmatrix} B - \frac{h}{6} A \\ B - \frac{5h}{24} A \end{bmatrix} \;,\; F_3 = \begin{bmatrix} \frac{h}{6} I & 0 \\ 0 & 0 \\ \frac{5h}{24} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\dim(E) = 18 \times 18 \;,\; \dim(F_1) = 18 \times 2(9+m) \;,\; \dim(F_2) = 18 \times (9+m) \;,\; \dim(F_3) = 18 \times (9+m)$$

The second part consists of the compact form is the quadratic part (10*-(9+m)*) of the above system which can be written as

$$[0] = [D \quad 0] \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} + [I \quad 0] \begin{bmatrix} f(t) \\ f(t_m) \end{bmatrix} \;,\; [0] = [0 \quad D] \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} + [0 \quad I] \begin{bmatrix} f(t) \\ f(t_m) \end{bmatrix}$$

$$\dim(D) = m \times (9+m) \;,\; \dim(I) = m \times m \qquad (15)$$

Where,

$$i_{1\phi}(t) = \begin{bmatrix} i_1(t) & i_2(t) & i_3(t) & i_4(t) & 0 & \cdots & 0 \end{bmatrix}^T$$

$$i_{1\phi}(t_m) = \begin{bmatrix} i_1(t_m) & i_2(t_m) & i_3(t_m) & i_4(t_m) & 0 & \cdots & 0 \end{bmatrix}^T$$

$$x(t) = \begin{bmatrix} v_1(t) & v_2(t) & v_3(t) & v_4(t) & i_{L1}(t) & i_{L2}(t) & e(t) & \lambda(t)) & i_m(t) & y_1(t) & y_2(t) & \cdots & y_m(t) \end{bmatrix}^T$$

$$x(t_m) = \begin{bmatrix} v_1(t_m) & v_2(t_m) & v_3(t_m) & v_4(t_m) & i_{L1}(t_m) & i_{L2}(t_m) & e(t_m) & \lambda(t_m) & i_m(t_m) & y_1(t_m) & y_2(t_m) & \cdots & y_m(t_m) \end{bmatrix}^T$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ & & & & & \vdots & & & & & & & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \cdots & 1 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$\dim(D) = m \times (9 + m) \;\;, \;\; \dim(I) = m \times m$$

$$f(t) = \begin{bmatrix} x^T(t) \cdot Q_1 \cdot x(t) \\ \vdots \\ x^T(t) \cdot Q_m \cdot x(t) \end{bmatrix} \qquad f(t_m) = \begin{bmatrix} x^T(t_m) \cdot Q_{m+1} \cdot x(t_m) \\ \vdots \\ x^T(t_m) \cdot Q_{2m} \cdot x(t_m) \end{bmatrix}$$

Proof: Equations 14 and 15 above are derived from the compact form in equation 13 by recognizing the fact that the matrices A, B and the quadratic matrix can be partitioned into two different row categories which when integrated using Quadratic Integration result in equations 14 and 15 which is the Algebraic Companion Form for the transformer model. The following paragraph describes the mathematical procedure leading to equations 14 and 15.

The matrices involved in equation 15 can be partitioned into 2 categories depending on the type of variables involved namely: Differential equations and non-linear or quadratic equations.

1) Differential Equations: Rows 9-11 of equation 18 (Equations 9*-11*) can be expressed in the following differential form.

$$i_{1\varphi}(t) = A \cdot x(t) + B\frac{dx(t)}{dt}$$

When Quadratic Integration is performed on the above equations we obtain

$$\begin{bmatrix} \frac{h}{6}I & 0 & \frac{2h}{3}I & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{h}{24}I & 0 & \frac{h}{3}I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} i_{1\phi}(t) \\ i_{1\phi}(t_m) \end{bmatrix} = \begin{bmatrix} \frac{h}{6}A+B & \frac{2h}{3}A \\ -\frac{h}{24}A & \frac{h}{3}A+B \end{bmatrix}\begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - \begin{bmatrix} B-\frac{h}{6}A \\ B-\frac{5h}{24}A \end{bmatrix}\left[x(t-h)\right] - \begin{bmatrix} \frac{h}{6}I & 0 \\ 0 & 0 \\ \frac{5h}{24} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i(t-h) \\ 0 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} i_{1\phi}(t) \\ i_{1\phi}(t_m) \end{bmatrix} = E \cdot F_1 \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - b_{eq}$$

$$b_{eq} = E \cdot F_2\left[x(t-h)\right] + E \cdot F_3\left[i_{1\phi}(t-h)\right]$$

Where

$$E = \begin{bmatrix} \frac{4}{h}I & 0 & -\frac{8}{h}I & 0 \\ 0 & I & 0 & 0 \\ \frac{1}{2h}I & 0 & \frac{2}{h}I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \quad F_1 = \begin{bmatrix} \frac{h}{6}A+B & \frac{2h}{3}A \\ -\frac{h}{24}A & \frac{h}{3}A+B \end{bmatrix}, \quad F_2 = \begin{bmatrix} B-\frac{h}{6}A \\ B-\frac{5h}{24}A \end{bmatrix}, \quad F_3 = \begin{bmatrix} \frac{h}{6}I & 0 \\ 0 & 0 \\ \frac{5h}{24} & 0 \\ 0 & 0 \end{bmatrix}$$

2) Non-linear (Quadratic) equations: Equations 10*-(9+m)* are the non-linear and after performing Quadratic Integration can be written in the form:

$$[0]=[D \quad 0]\begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix}+[I \quad 0]\begin{bmatrix} f(t) \\ f(t_m) \end{bmatrix}$$

$$[0]=[0 \quad D]\begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix}+[0 \quad I]\begin{bmatrix} f(t) \\ f(t_m) \end{bmatrix}$$

Where,

$$D=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ & & & & & \vdots & & & & & & & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \cdots & 1 \end{bmatrix} \qquad I=\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$f(t)=\begin{bmatrix} x^T(t)\cdot Q_1\cdot x(t) \\ \vdots \\ x^T(t)\cdot Q_m\cdot x(t) \end{bmatrix} \qquad f(t_m)=\begin{bmatrix} x^T(t_m)\cdot Q_{m+1}\cdot x(t_m) \\ \vdots \\ x^T(t_m)\cdot Q_{2m}\cdot x(t_m) \end{bmatrix}$$

$$Q_1(1,8,8)=-\frac{1}{\lambda_0^2} \qquad Q_1(m+1,8+N,8+N)=-\frac{1}{\lambda_0^2}$$

$$\vdots \qquad \qquad \vdots$$

$$Q_k(1<k\le m-1,\ i,\ \ j)=-1 \qquad Q_k(m+1<k\le 2m-1,\ i,\ \ j)=-1$$

$$\vdots \qquad \qquad \vdots$$

$$\begin{cases} Q_m(m,i,N-1)=-1, & n \ \ even \\ Q_m(m,8,N-1)=-\dfrac{1}{\lambda_0}, & n \ \ odd \end{cases} \qquad \begin{cases} Q_{2m}(2m,i,2N-1)=-1, & n \ \ even \\ Q_{2m}(2m,8+N,2N-1)=-\dfrac{1}{\lambda_0}, & n \ \ odd \end{cases}$$

Appending the 2 categories above gives us the algebraic companion form described in the form below.

$$\begin{bmatrix} i_{1\phi}(t) \\ i_{1\phi}(t_m) \end{bmatrix} = E \cdot F_1 \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - b_{eq}$$

$$[0] = \begin{bmatrix} x(t)^T & x(t_m)^T \end{bmatrix} \begin{bmatrix} Q_1 \\ \vdots \\ Q_{2m} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix}$$

Where,

$$x(t) = \begin{bmatrix} v_1(t) & v_2(t) & v_3(t) & v_4(t) & i_{L1}(t) & i_{L2}(t) & e(t) & \lambda(t) & i_m(t) & y_1(t) & y_2(t) & \cdots & y_m(t) \end{bmatrix}^T$$

$$x(t) = \begin{bmatrix} v_1(t_m) & v_2(t_m) & v_3(t_m) & v_4(t_m) & i_{L1}(t_m) & i_{L2}(t_m) & e(t_m) & \lambda(t_m) & i_m(t_m) & y_1(t_m) & y_2(t_m) & \cdots & y_m(t_m) \end{bmatrix}^T$$

$$i_{1\phi}(t) = \begin{bmatrix} i_1(t) & i_2(t) & i_3(t) & i_4(t) & 0 & \cdots & 0 \end{bmatrix}^T$$

$$i_{1\phi}(t_m) = \begin{bmatrix} i_1(t_m) & i_2(t_m) & i_3(t_m) & i_4(t_m) & 0 & \cdots & 0 \end{bmatrix}^T$$

<u>Example:</u> We can now illustrate the above procedures on an example where the value of n the exponent of the magnetizing current is 11. The equations that would quadratize the model are

$$i_1(t) = g_1\left(v_1(t) - v_2(t)\right) - g_1 e(t) - g_1 L_1 \frac{di_{L1}(t)}{dt}$$

$$i_2(t) = g_1\left(-v_1(t) + v_2(t)\right) + g_1 e(t) + g_1 L_1 \frac{di_{L1}(t)}{dt}$$

$$i_3(t) = g_2\left(v_3(t) - v_4(t)\right) - t g_2 e(t) - g_2 L_2 \frac{di_{L2}(t)}{dt}$$

$$i_4(t) = g_2\left(-v_3(t) + v_4(t)\right) + t g_2 e(t) + g_2 L_2 \frac{di_{L2}(t)}{dt}$$

$$0 = -g_1\left(v_1(t) - v_2(t)\right) + g_1 e(t) + \left(G_1 + g_1\right) L_1 \frac{di_{L1}(t)}{dt}(t) + i_{L1}(t)$$

$$0 = -g_2\left(v_3(t) - v_4(t)\right) + g_2 t e(t) + \left(G_2 + g_2\right) L_2 \frac{di_{L2}(t)}{dt} + i_{L2}(t)$$

$$0 = -g_1\left(v_1(t) - v_2(t)\right) - t \cdot g_2\left(v_3(t) - v_4(t)\right) + g_1 L_1 \frac{di_{L1}(t)}{dt} + t \cdot g_2 L_2 \frac{di_{L2}(t)}{dt} + i_m(t) + \left(g_1 + g_c + t^2 g_2\right) e(t)$$

$$0 = i_m(t) - i_0 y_m(t)$$

$$0 = e(t) - \frac{d\lambda(t)}{dt}$$

$$0 = y_1(t) - \frac{1}{\lambda_0^2}\lambda^2(t)$$

$$0 = y_2(t) - y_1^2(t)$$

$$0 = y_3(t) - y_2^2(t)$$

$$0 = y_4(t) - y_1(t) y_3(t)$$

$$0 = y_5(t) - \frac{1}{\lambda_0}\lambda(t) y_4(t)$$

These equations can be written in the compact form:

$$i_{1\phi}(t) = A \cdot x(t) + B\frac{dx}{dt}$$
$$[0] = \begin{bmatrix} x^T(t) \cdot Q_1 \cdot x(t) \\ \vdots \\ x^T(t) \cdot Q_5 \cdot x(t) \end{bmatrix}$$

Where,

$$i_{1\phi}(t) = \begin{bmatrix} i_1(t) & i_2(t) & i_3(t) & i_4(t) & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$x(t) = \begin{bmatrix} v_1(t) & v_2(t) & v_3(t) & v_4(t) & i_{L1}(t) & i_{L2}(t) & e(t) & \lambda(t) & i_m(t) & y_1(t) & y_2(t) & y_3(t) & y_4(t) & y_5(t) \end{bmatrix}$$

$$
A=\begin{bmatrix}
g_1 & -g_1 & 0 & 0 & 0 & 0 & -g_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-g_1 & g_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & g_2 & -g_2 & 0 & 0 & -t\cdot g_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -g_2 & g_2 & 0 & 0 & t\cdot g_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-g_1 & g_1 & 0 & 0 & 1 & 0 & g_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -g_2 & g_2 & 0 & 1 & t\cdot g_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-g_1 & g_1 & -t\cdot g_2 & t\cdot g_2 & 0 & 0 & g_1+g_c+t^2 g_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -i_0
\end{bmatrix}
$$

$$
B=\begin{bmatrix}
0 & 0 & 0 & 0 & -g_1 L_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & g_1 L_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -g_2 L_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & g_2 L_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \left(G_1+g_1\right)L_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \left(G_2+g_2\right)L_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & g_1 L_1 & g_2 L_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

The quadratic part of the equations can be written as

$$Q_1\left(1,8,8\right)=-\frac{1}{\lambda_0^2}$$

$$Q_2\left(2,10,10\right)=-1 \quad , \quad Q_3\left(3,11,11\right)=-1$$

$$Q_4\left(4,10,12\right)=-1 \quad , \quad Q_5\left(5,8,13\right)=-\frac{1}{\lambda_0}$$

Upon Quadratic Integration of the differential part of the above equations we obtain

$$\begin{bmatrix} i_{1\phi}(t) \\ i_{1\phi}(t_m) \end{bmatrix} = E\cdot F_1 \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} - b_{eq}$$

$$b_{eq} = E\cdot F_2\left[x(t-h)\right] + E\cdot F_3\left[i_{1\phi}(t-h)\right]$$

Where,

$$E=\begin{bmatrix}\frac{4}{h}I & 0 & -\frac{8}{h}I & 0\\ 0 & I & 0 & 0\\ \frac{1}{2h}I & 0 & \frac{2}{h}I & 0\\ 0 & 0 & 0 & I\end{bmatrix}\ ,\ F_1=\begin{bmatrix}\frac{h}{6}A+B & \frac{2h}{3}A\\ -\frac{h}{24}A & \frac{h}{3}A+B\end{bmatrix}\ ,\ F_2=\begin{bmatrix}B-\frac{h}{6}A\\ B-\frac{5h}{24}A\end{bmatrix}\ ,\ F_3=\begin{bmatrix}\frac{h}{6}I & 0\\ 0 & 0\\ \frac{5h}{24} & 0\\ 0 & 0\end{bmatrix}$$

$$\dim(E)=18\times18\ ,\ \dim(F_1)=18\times28\ ,\ \dim(F_2)=18\times14\ ,\ \dim(F_3)=18\times14$$

The quadratic or non-linear part of the when quadratic integration is performed gives us

$$[0]=[D\quad 0]\begin{bmatrix}x(t)\\ x(t_m)\end{bmatrix}+[I\quad 0]\begin{bmatrix}f(t)\\ f(t_m)\end{bmatrix}\ ,\ [0]=[0\quad D]\begin{bmatrix}x(t)\\ x(t_m)\end{bmatrix}+[0\quad I]\begin{bmatrix}f(t)\\ f(t_m)\end{bmatrix}$$

$$\dim(D)=5\times14\ ,\ \dim(I)=5\times5$$

Where,

$$D=\begin{bmatrix}0&0&0&0&0&0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&1\end{bmatrix}\qquad I=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}$$

$$f(t)=\begin{bmatrix}x^T(t)\cdot Q_1\cdot x(t)\\ \vdots\\ x^T(t)\cdot Q_5\cdot x(t)\end{bmatrix}\qquad f(t_m)=\begin{bmatrix}x^T(t_m)\cdot Q_6\cdot x(t_m)\\ \vdots\\ x^T(t_m)\cdot Q_{10}\cdot x(t_m)\end{bmatrix}$$

APPENDIX B: SIMULATED SYSTEM

The system that is used as our algorithm is simulated using the program WINIGS is shown below. The system is composed of a 138 KV three phase equivalent source, a 138kV overhead transmission line which is connected to our 138kV:13.8kV Y-Y connected transformer, a 13.8 kV overhead transmission line and finally a load. The other devices in the simulation are the meters used to measure the primary/secondary voltages and currents and the relative ground.

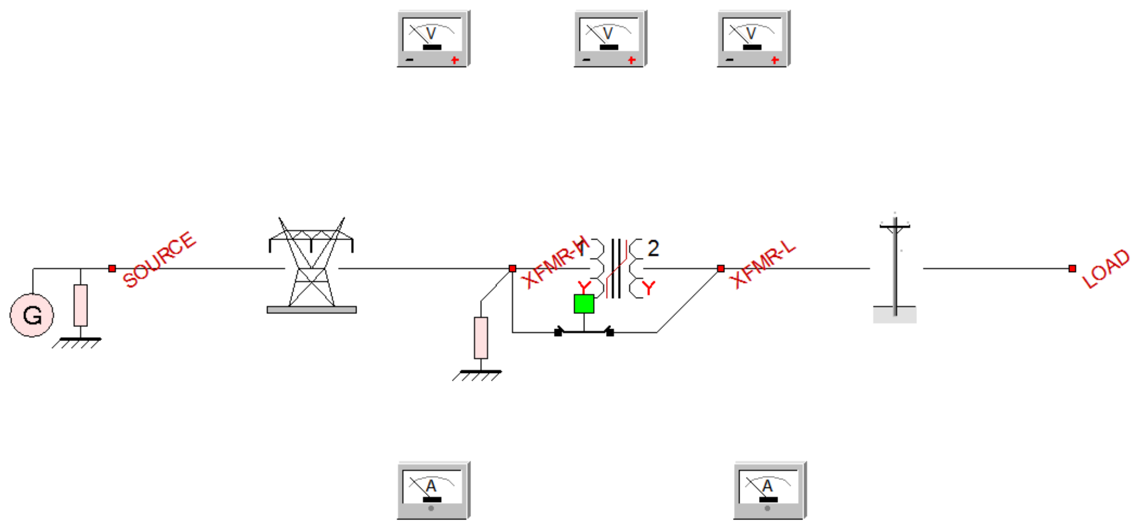

Figure B1: Simulated three phase transformer system for energization event

The three phase transformer has the following parameters.

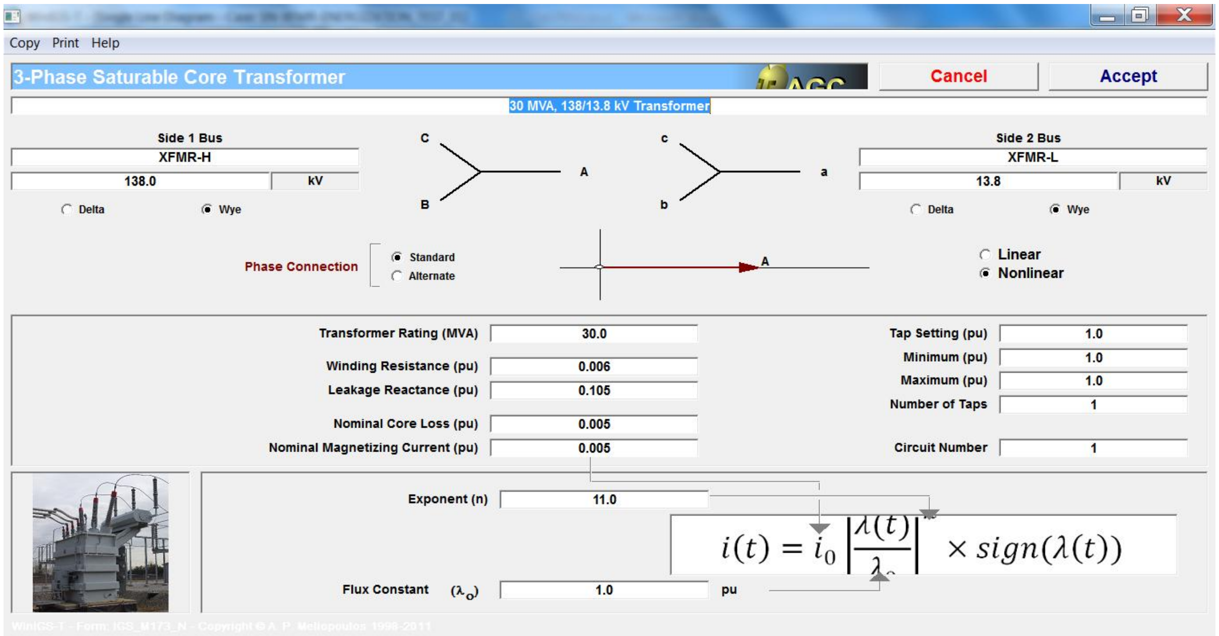

Figure B2: Parameters of Transformer used in Simulation

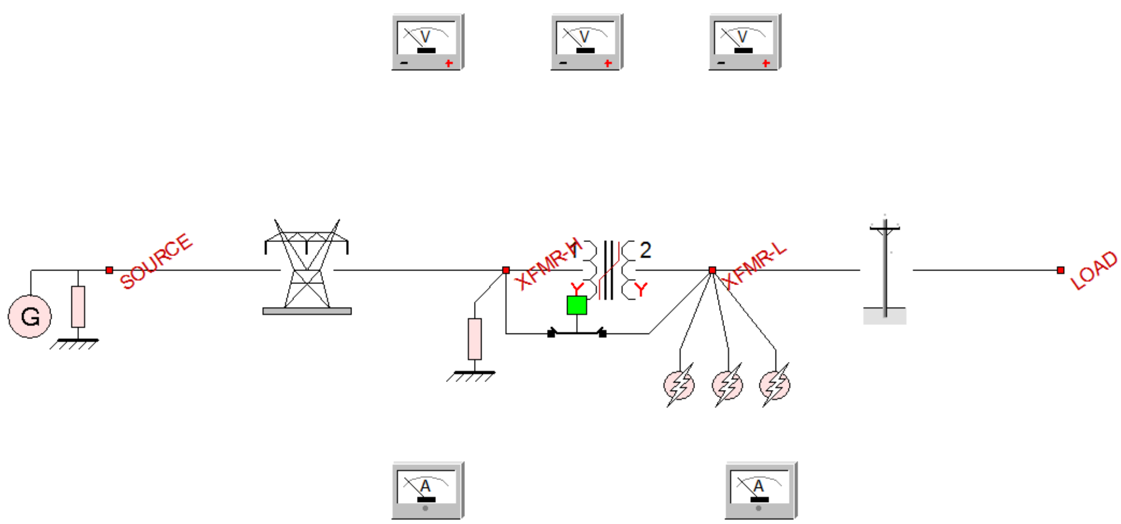

Figure B3: Simulated three phase transformer system for external fault event

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