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INTERINDUSTRY ANALYSIS

WITH LINEAR PROGRAMMING

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To my patient and loving wife

MARIA

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

CHAPTER I

INTRODUCTION

Interindustry analysis is one of the most fruitful methods of linear economics and its contribution to both theoretical and empirical economics is transcendental, exercising a stimulating influence on economists and statisticians in many countries.

Originally, the interindustry analysis was developed by Professor Wassily Leontief with the name of "input-output analysis." Leontief published the first clear statement of the method in 1936. In its original version, input-output analysis dealt with an entirely closed economy system where all goods were intermediate goods, consumables being regarded as the intermediate goods needed in the production of personal services. After World War II a different view of Leontief's model has been developed, considering final demand as being exogenously determined and using input-output analysis to find levels of activity in the various sectors of the economy consistent with the specified pattern of final demand. In particular, it studies the interrelations among producers as buyers of each outputs, as users of scarce resources, and as sellers to final consumers.

Obviously, one of the most worthy contributions of the first Leontief study was to stimulate empirical work on interindustry relations in different countries such as Norway, Denmark, the Netherlands and Italy. Furthermore, as new countries in Asia, Africa, and Latin America embarked on development programs, they mostly used input-output techniques.

On the other hand, linear programming was developed by George B. Dantzig in 1947 as a technique for planning the diversified activities of the U. S. Air Force. As it is discussed in Chapter III, there are a number of important similarities between these two branches of linear economics; in each case there is an end-means connection. Input-output analysis may be thought of as a special case of linear programming in which there is no scope for choice once the desired pattern of final outputs has been determined. Linear programming offers a way of getting around the restrictive assumption of constant input coefficients in each sector, while retaining a formulation which permits statistical measurement.

The importance of the study of interindustry analysis with linear programming is due to the fact that it is a powerful tool for development programming in underdeveloped countries. As Chenery has pointed out in a paper presented at the third international input-output conference in Geneva in 1961, there is a compromise between the two methods and he suggests joint use of them so as to check and to modify each other's results.

Economists in many countries are concerned with the practical application of the input-output model to national economic planning and they agree with the combination of programming and input techniques. Probably the most complete mathematical discussion about the combination of linear programming and input-output analysis was done by Dorfman, Samuelson and Solow in their <u>Linear Programming and Economic Analysis.</u> Chenery and Clark in <u>Interindustry</u>

Economics present interesting findings in resource allocation and projection of the economic structure in different countries such as Italy, Japan, Argentina, and Colombia, where input-output analysis and linear programming has made unique contribution. Finally, to summarize the immense literature concerned with this topic, A. Ghosh in Experiments with Input-Output Models, analyzes empirically the structure of production in post-war Britain based on the input-output table for 1948 and suggests tentatively how production models and allocation models could be combined for programming purposes and how this approach may lead to formulating a linear programming model whereby the coefficients in an input-output matrix would be modified under the pressure of changes in final demand.

Besides all of the great advantages of interindustry analysis, Dorfman in his paper "The Nature and Significance of Input-Output" in The Review of Economics and Statistics, No. 2, Vol. 36, May 1964, pp. 121-133, pointed out that it appears now that input-output is not likely to supplant traditional methods for studying industrial or price problems, or even to replace the Walrasian conceptual framework for thinking in terms of general equilibrium. Interindustry analysis in its present imperfect form is a promising approach to the analysis of our complicated industrial structure and the most feasible technique yet developed for over-all industrial planning.

CHAPTER II

THE INPUT-OUTPUT MODEL

According to the purpose of this study, the analysis of the input-output model is referred to as the Leontief open, static model because it is the core of all forms of interindustry analysis. This model, in general, determines the relations between autonomous demands and levels of production. In other words, this model is based on the premise that it is possible to divide all productive activities in an economy into sectors whose interrelations can be meaningfully expressed in a set of simple input functions. It shows sales to all intermediate customers as well as sales to final demand. So long as there is at least one sector remaining in the autonomous category the input-output system will be an open model; if all of the sector outputs are regarded as dependent variables, the system will be a closed model. The input-output model is basically a theory of production.

The Accounting Framework

The purpose of accounting is to select the relevant aspects of economic events, record them, and subsequently to organize and summarize these records for further analysis. Thus, the consolidation of accounts of businesses to form any industry account is accomplished by adding up similar asset, liability, and net worth items or similar debit and credit items and entering the sums in an industry account with a similar format. The source and use accounts for all firms in an industry may be consolidated to form an industry source and use account. The source and use accounts for several industries may be arranged in a special table called an input-output table, sometimes called an interindustry account.

The Accounting Conventions

There are some accounting conventions required for the construction of the input-output model. These have been stated succinctly by Chenery and Clark as follows:

> (1) Transactions are usually recorded at the producer's price rather than at the purchaser's cost, which means that trade and transport margins are ascribed to the using sectors.
>
> (2) In principle, the flows should correspond to the use of inputs for current production rather than to the time when they are purchased. The difference between purchase and use are reflected in stock changes, which are part of final use.
>
> (3) Purchases on capital account are normally charged entirely to final use, and depreciation allowances are therefore included with primary inputs (1).

The input-output model could be regarded as a simple transaction model and consequently, it could be best viewed within the framework of the national accounts (2).

The model involves a matrix of transactions representing the flows of commodities between the industries engaged in current production, a matrix of injections showing the demands placed on the producing sectors by the non-producing sectors, and a matrix of dependent responses based on the assumption of functional dependence between the inputs and the total transactions of the interindustry block.

The General Assumptions of the Input-Output Model

The Leontief input-output model is concerned with the procedure of explaining the magnitudes of the interindustry flows in terms of the levels of production in each sector. But several assumptions are required for such a procedure in order to be theoretically meaningful (3). The hypothetical economy described for this model states that its main purpose is to produce a number of different commodities, each of which is manufactured by a different industry. Each industry uses only one process of production, and each process is one of simple addition in which a set of inputs is combined in fixed proportions to produce certain output.

Basic Assumptions of the Open Model

In order to obtain reasonable and practical results with the model under consideration, it is necessary to make a number of assumptions which will cover certain specified areas. These assumptions may be summarized as follows:

(a) Each commodity is supplied by a single industry or sector of production.

(b) The inputs purchased by each sector are a function only of the level of output of that sector.

(c) The total effect of carrying on several types of production is the sum of the separate effects (4).

The Formal Logic of Input-Output Tables

The analysis of a basic table of an input-output system -- as a formal economic model -- is related with the assumptions about economic behavior and definitions of the variables in the model. The level of aggregation of such a table is determined by data availability rather than by any set of rules. As a system of transactions the input-output table is best viewed within the framework of the national accounts. Thus the reduction of the combination between the assumptions and the accounting definitions to algebraic form, will set up the structure of the input-output model.

Differences Between the Input-Output Tables and Input-Output Analysis

In any interindustry analysis there is an analytical system to which is related an input-output table. The table is the statistical description -- double-entry bookkeeping operation -- of the inputs and outputs of the different branches of an economic system during a particular period of time. The input-output system is the theoretical scheme, the set of simultaneous linear equations in which the unknowns are the levels of output of the various branches, and in which the parameters are to be empirically estimated from the information contained in the input-output table.

Flow of Goods and Services in the Input-Output Model

Three types of organization -- households, productive organizations (firms and farms), and government -- and the rest of the world interconnected by the flow of goods and services give the best picture of an input-output table. Figure 1 demonstrates the interconnected flows which take place. It should be noted that the direction of cash flows and the flows of goods and services, if any, are opposite to each other.

The Transactions Table

The transactions table, which is called transactions matrix, contains all

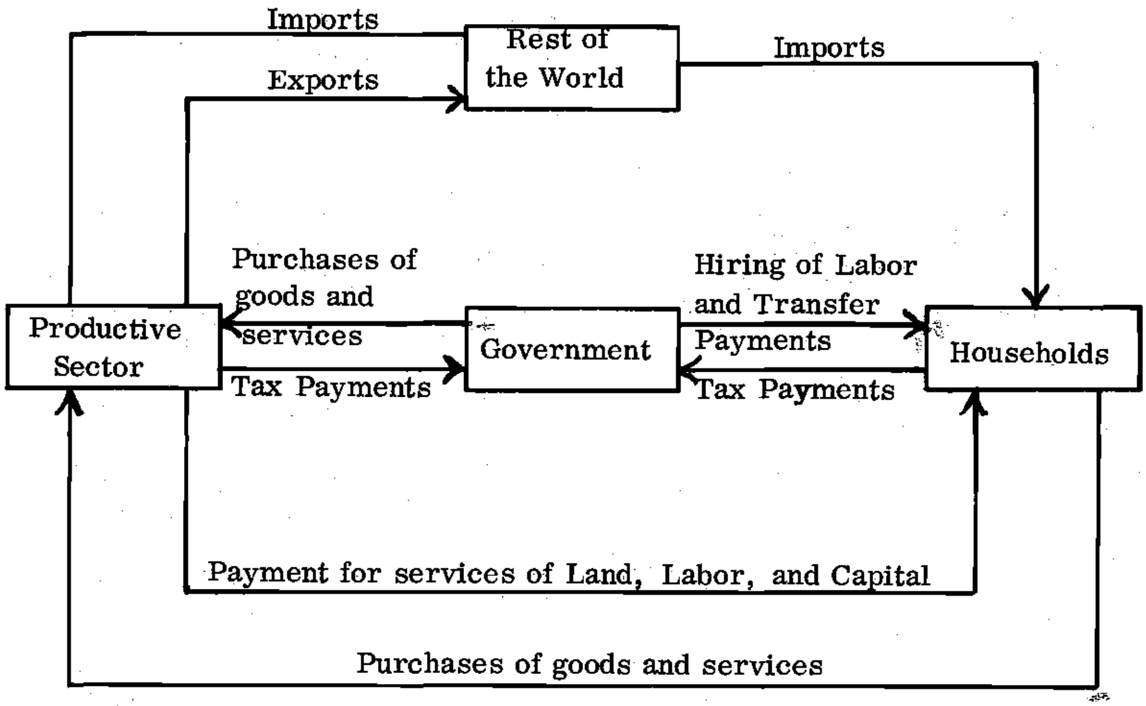

Figure 1. Flows of Cash.

the goods and services produced in an economy and is the basis of all input-output analysis. This table can be described symbolically and schematically as in Table 1 which shows the formal properties of the accounting system. The economic system is considered as a number of sectors each of which is represented in the table by a row and column. The output of a sector is distributed along each row, while the corresponding column records the current inputs to that sector. The entry in the cell at the intersection of the i^{th} row and j^{th} column represents the quantity of the output of sector i absorbed as input by sector j. Thus the symbol x_{ij} represents the transaction between two sectors of production, while the total output of sector i is written X_i.

The sectors of the input-output table are divided into two groups: the production or intermediate sectors on one hand and the final sectors on the other. This leads to four types of transactions, which are shown in the four quadrants of Table 1.

Quadrant I. All of the columns in this quadrant are referred to collectively as the final demand sector. When Quadrant I is considered alone it is often referred to as the "bill of goods" to distinguish it from final demand which consist of both Quadrants I and IV.

Quadrant II. This sector of a transactions table is typically known as the processing sector. It shows the interindustry transactions, or the sales of intermediate goods and services. The general term, X_{ij}, shows the sales by the i^{th} sector at the left to the j^{th} sector at the top, or conversely it shows purchases by the j^{th} sector from the i^{th} sector. The total intermediate use of any commodity

Table 1. Interindustry Transactions Table

Sector Purchasing → / Sector Producing ↓		Intermediate Use				Final Use					Total Use = Total Supply	Supply	
		Sector 1	j	n	Total Intermediate Use	Investment	Consumption	Government	Exports	Total Final Use		Imports	Production
Intermediate Inputs	1	X_{11}	X_{1j}	X_{1n}	W_1	I_1	C_1	G_1	E_1	Y_1	Z_1	M_1	X_1
	⋮	Quadrant II				Quadrant I							
	i	X_{i1}	X_{ij}	X_{in}	W_i	I_i	C_i	G_i	E_i	Y_i	Z_i	M_i	X_i
	⋮												
	n	X_{ni}	X_{nj}	X_{nn}	W_n	I_n	C_n	G_n	E_n	Y_n	Z_n	M_n	X_n
	Total Produced Inputs	U_1	U_j	U_n									
Primary Inputs	Value Added	Quadrant III				Quadrant IV							
		V_1	V_j	V_n		V_I	V_C	V_G	V_E		V		V
Total Production		X_1	X_j	X_n		I	C	G	E	Y	Z	M	X

is identified as W_i, and the total purchases from other sectors by a given industry as U_j.

Quadrant III. The rows in this sector are referred to collectively as the payment sector. Corresponding to the final demand columns there are rows showing the primary inputs. These inputs are "primary" because they are not part of the output of current production. Primary inputs into current production represent the value added in each sector of production as well as imports. The term value added (V_j) will be employed for the total use of primary inputs.

Quadrant IV. This quadrant is sometimes omitted from published tables. It records the primary inputs into final demand sectors, including such typical entries as the income of government employees and imports consumed directly by households.

The Input-Output System

The input-output system is derived from a set of accounting identities and from a special assumption concerning the relations between the sectors of production in an economy. Thus, after having described the table of transactions, it is appropriate to have a look at the underlying theoretical scheme.

The Accounting Symbols. The basic elements of the formal structure of the input-output accounts are defined as follows:

X_i = Total production of commodity i.

M_i = Import of commodity i.

X_{ij} = Amounts of commodity i used in sector j.

Y_i = Final demand for commodity i.

W_i = Total intermediate use of commodity i.

U_j = Total use by sector j of inputs purchased from other industries.

V_j = Total use of primary inputs in sector j.

It is important to emphasize that these variables represent flows which in theory are physical units, and not in money value. Not only product flows, but also the primary inputs of factor services are considered to be measured in physical units.

In order to set up the balance of the table, two equations are obtained. The first corresponding to the rows, which states that for each commodity, total supply is equal to total demand (intermediate demand plus total demand):

$$\underbrace{Z_i = M_i + X_i}_{\text{Supply}} = \underbrace{\sum_{j=1}^{n} X_{ij} + Y_i = W_i + Y_i}_{\text{Demand}} \qquad (1)$$

The second equation is related to the columns and it states that the total production in each sector is equal to the value of the inputs purchased from other sectors plus value added in the sector:

$$X_j = \sum_{i=1}^{n} X_{ij} + V_j = U_j + V_j \qquad (2)$$

Relationship Between Input-output Accounts and National Income Aggregate. From equation (1) the final demand (Y_i) is defined as the difference between the total supply of a commodity available and the amount used up in production and hence it includes changes in stocks. From equation (2) the value of primary inputs (V_j) is defined as the difference between the value of production in a sector and payments for inputs purchased from other productive sectors.

Thus these definitions are closely related with the concepts of final output and value added used in national income analysis.

Adding up each row in equation (1) and considering imports as a deduction from final demands, it follows that:

$$\sum_{j=1}^{n} X_i = \sum_{i=1}^{n} \sum_{j=1}^{n} X_{ij} + \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} M_i \tag{3}$$

and adding across all columns:

$$\sum_{j=1}^{n} X_j = \sum_{i=1}^{n} \sum_{j=1}^{n} X_{ij} + \sum_{j=1}^{n} V_j \tag{4}$$

and since $$\sum_{i=1}^{n} X_i = \sum_{j=1}^{n} X_j \tag{5}$$

Combining equations (3), (4) and (5) the expression for the basic national account is obtained:

$$\sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} M_i = \sum_{j=1}^{n} V_j \tag{6}$$

<u>The Input-Output Analytical Model</u>. The transactions table described above is a complete and detailed accounting system for an economy. For analytical purpose,

however, it is necessary to go beyond this table and compute a table of technical coefficients, which are computed for processing sectors only, and are defined as the direct purchases by each sector from every other sector per dollar of output.

Given an economy divided into n sectors, then the total disposition of the physical output of each sector can be described by the following set of n equations:

$$\begin{aligned} X_1 &= X_{11} + X_{12} + \ldots\ldots\ldots + X_{1n} + Y_1 \\ &\vdots \\ X_i &= X_{i1} + X_{i2} + \ldots\ldots \quad + X_{in} + Y_i \\ &\vdots \\ X_n &= X_{n1} + X_{n2} + \ldots\ldots + X_{nn} + Y_n \end{aligned} \tag{7}$$

The i^{th} equation in this system states that the total output of sector i is equal to the sum of the quantities consumed by each production sector, including sector i itself, plus the quantity consumed by all components of final demand. This row balance is necessary whatever units are chosen for each sector. However, it should be noted that it is impossible to sum the elements of each column since each represents different physical units.

As noted above, one of the assumptions of the input-output model is that the input from production sector i to production sector j is directly proportional to the output of sector j. This assumption can be expressed in the following equation:

$$X_{ij} = a_{ij} X_j \tag{8}$$

where a_{ij} are the technical coefficients or marginal input coefficients.

The set of n^2 such equations is known as the set of structural equations, while the n equations (7) are known as balance equations.

Combining equations (1) and (8), the original Leontief model is obtained. Thus the balance equation for each commodity or sector is:

$$X_i - \sum_{j=1}^{n} a_{ij} X_j = Y_i - M_i \tag{9}$$

In this system of n equations, there are n unknown production levels (X_j), n^2 parameters (a_{ij}) describing the input functions, and two sets of n autonomous variables (Y_i and M_i) whose values are specified for a given problem.

When trade is important, it is often desirable to make imports dependent variables. As a first approximation, it can be assumed that the level of imports (M_i) is a function of the total supply of that commodity (Z_i) and hence is related to the level of domestic production (X_i). Assuming a linear function over a certain range, we obtain:

$$M_i = \bar{M}_i + m_i X_i \tag{10}$$

where m_i is the import coefficient.

Combining equations (9) and (10), we obtain:

$$(1 + m_i) X_i - \sum_{j=1}^{n} a_{ij} X_j = \bar{Y}_i \tag{11}$$

where

$$\bar{Y}_i = Y_i + \sum_{j=1}^{n} \bar{X}_{ij} - \bar{M}_i$$

The variable $\bar{\bar{Y}}_i$ is the total autonomous demand which is equal to final demand (Y_i) when the other two terms are zero. The term $\bar{X}_{ij}$ is a constant which includes any fixed-cost elements which do not vary with the level of output. Thus equation (11) is the basic equation of the interindustry system in general case.

The general solution for the n unknown X's in terms of the given Y's can be written:

$$X_i = r_{i1}\bar{\bar{Y}}_1 + r_{i2}\,\bar{Y}_2 + \ldots\ldots + r_{in}\,\bar{Y}_n \;(i=1\ldots n) \tag{12}$$

where each coefficient r_{ij} is derived from a_{ij} and m_i and represents the amount of sector i required directly and indirectly to satisfy one unit of final demand of sector j.

<u>The Input-Output Model in Matrix Form.</u> From equations (7) and (8) and solving for Y_i, we obtain:

$$\begin{array}{l} X_1 - (a_{11}X_{11} + a_{12}X_{12} + \ldots + a_{1n}X_{1n}) = Y_1 \\ \cdot \\ \cdot \\ X_i - (a_{i1}X_{i1} + a_{i2}X_{i2} + \ldots + a_{in}X_{in}) = Y_i \\ \cdot \\ \cdot \\ X_n - (a_{n1}X_{n1} + a_{n2}X_{n2} + \ldots + a_{nn}X_{nn}) = Y_n \end{array} \tag{13}$$

In matrix form this expression becomes:

$$X - AX = Y$$

since $IX = X \quad (I = 1, \text{ unit matrix})$

then $IX - AX = (I - A)\,X = Y$ (14)

which is equivalent to:

$$\begin{bmatrix} (1-a_{11}) & -a_{12}\cdots\cdots & -a_{1n} \\ \vdots & \vdots & \vdots \\ -a_{i1} & (1-a_{i2})\cdots\cdots & -a_{in} \\ \vdots & \vdots & \vdots \\ -a_{n1} & -a_{n2}\cdots\cdots & (1-a_{nn}) \end{bmatrix} \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} Y_1 \\ \vdots \\ Y_i \\ \vdots \\ Y_n \end{bmatrix}$$

The matrix (I – A) is called the Leontief matrix.

When imports are added to the system as in equation (11), the corresponding matrix form is:

$$(I + M - A)\,X = Y \qquad (15)$$

The general solution for equation (14) gives:

$$X = (I - A)^{-1}Y$$

or

$$\begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12}\cdots\cdots & r_{1n} \\ \vdots & \vdots & \vdots \\ r_{i1} & r_{i2}\cdots\cdots & r_{in} \\ \vdots & \vdots & \vdots \\ r_{n1} & r_{n2}\cdots\cdots & r_{nn} \end{bmatrix} \begin{bmatrix} Y_1 \\ \vdots \\ Y_i \\ \vdots \\ Y_n \end{bmatrix} \qquad (16)$$

where the elements r_{ij} correspond to the <u>inverse matrix</u>

$$(I - A)^{-1} = R$$

In the general system which includes induced imports, the matrix form is:

$$R = (I + M - A)^{-1} \tag{17}$$

The matrix representations of the interindustry system showed here are only elementary considerations.

<u>Economic Meaning of the Inverse Matrix.</u> The inverse matrix plays a central part in the analysis of interindustry models. This inverse matrix is also known as the <u>Leontief's matrix multiplier</u> and like Keynes's scalar multiplier, is convergent and can be expanded as an infinite series:

$$(I - A)^{-1} = I + A + A^2 + A^3 + \ldots. \tag{18}$$

Thus solving for X in (14) it gives:

$$X = (I - A)^{-1}Y = (I + A + A^2 + A^3 + \ldots) Y \tag{19}$$

The coefficients of the matrix multiplier tell us how much output any industry is to produce in order that a specific amount of final demand may be satisfied. Thus the final output of any industry, say coal, will require for its production certain inputs and these inputs in their turn will require coal for their production. In this way a specific amount of final demand for coal will generate a series of indirect outputs, represented by the appropriate elements in the matrix sum

$$(A + A^2 + A^3 + \ldots\ldots)$$

Any vector of final demand Y will in turn, generate a total demand

$$(I + A + A^2 + A^3 + \ldots\ldots) Y$$

each stage in the series representing the inputs required by the earlier stage until it comes to the final demand itself.

The calculation of output requirements for a given vector of final demand can be obtained either by taking a sufficient number of terms in the sum $\sum_{n=0}^{\infty} A^n$ or by inverting the Leontief's matrix (I - A).

CHAPTER III

THE LINEAR PROGRAMMING APPROACH

Interindustry Analysis and Linear Programming

In the preceding chapter it was assumed that the use of the open, static Leontief's model does not exclude the possibility that the choice of technology, source of supply and pattern of demand are independent of the outcome of the analysis. The use of the Leontief system assumes that these choices are not dependent on the level of output in each sector and can, therefore, be fixed in advance.

In this chapter it will be assumed that the system under consideration takes account of the fact that there are many different ways of producing goods and satisfying wants and the choices in one part of the economy may be dependent on choices on other parts. In order to handle this general assumption, it is necessary to use the mathematical technique of linear programming.

With regard to the relationship between input-output theory and linear programming, Dorfman, Samuelson, and Solow pointed out that:

> The theory of input-output can also be regarded as a peculiarly simple form of linear programming: in the simplest Leontief system, in which no substitutions of inputs are technologically feasible, the optimizing solution is the one and only efficient solution possible; but in more general models, in which substitutions are possible, the system can be made determinate only by solving an appropriately formulated linear-programming problem (or by requiring the solution to satisfy some restrictive outside conditions) (5).

For the foregoing considerations so far pointed out, it is important to see that the linear programming approach includes alternative sources of supply as separate activities, and the level at which each is utilized becomes a variable in the model. Thus the system appears to have more variables than equations and many possible solutions. Upon these circumstances a criterion for preferring one solution to another should be established. This criterion may be cost minimization, welfare maximization, or any other function of the activity levels.

The Activity Matrix Form

In the linear programming analysis any possible transformation of fixed proportions of commodity output is called an activity and the extent to which an activity is utilized is called the activity level.

An activity, as in the input-output model, could be represented mathematically by a column of coefficients with one coefficient for each input and each output. Thus activity j is represented as the column vector:

$$A_j = \begin{bmatrix} a_{1j} \\ a_{2j} \\ \cdot \\ \cdot \\ a_{mj} \end{bmatrix}$$

The positive coefficient represents an output and a negative one an input.

If all the possible activities are gathered together, the next matrix is obtained:

$$A = (A_1 A_2 \ldots\ldots A_n) = \begin{array}{c} \left[\begin{array}{cccc} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_{m1} & a_{m2} & \cdots\cdots & a_{mn} \end{array}\right] \text{ m commodities} \\ \text{n activities} \end{array}$$

The above matrix is called the <u>technology matrix</u>.

The amount of commodity i used in sector j, as represented in the input-output system (X_{ij}), will represent the total of each input used or output produced by the activity with outputs positive and inputs negative. It gives the same equation as in (8):

$$X_{ij} = a_{ij} X_j$$

where X_j represents the activity levels and the set of such activity levels is called a <u>program</u>.

Restrictions or Constraints

It is not possible to carry out any arbitrary program. There are facts which have to be taken into account. Constraints are the mathematical formulations of facts which limit the arbitrary choice of programs. They include not only the final demands of input-output but also the quantities of resources available. Thus each type of commodity has a restriction which will be positive for final outputs, negative for primary inputs, and zero for intermediate commodities.

The set of restrictions may be written as the column:

$$B = \begin{bmatrix} B_1 \\ B_2 \\ . \\ . \\ B_m \end{bmatrix}$$

The Linear Programming Model

The general problem of production planning can be expressed as that of selecting from among the feasible alternatives one which is economically optimal. In the area of interindustry analysis, a typical programming problem is the choice of production techniques.

The standard form for the programming problem consists of three parts:

1) The function (i.e., profits or costs) whose value is to be maximized or minimized, which is called the objective function, being a function of the activity levels. Mathematically, the objective function is written as:

$$C = \sum_{j=1}^{n} c_j X_j \tag{20}$$

The objective function will permit to choose one solution as better than another. In interindustry analysis the objective is generally to maximize total output or to minimize cost.

2) The ordinary structural constraints which set up the limitations of the inputs in accordance with the requirements of the outputs. They can be written as:

$$\sum_{j=1}^{n} a_{ij} X_j \geq B_i \quad (i = 1 \ldots m) \tag{21}$$

3) The nonnegative conditions on the variables (or the activity levels) can be written as:

$$X_j \geq 0 \ (j = 1.\ldots..n) \qquad (22)$$

Any set of activity levels which satisfies only Eqs. (21) is called a solution. If, in addition, it satisfies the nonnegative conditions of Eqs. (22) it is a feasible solution. The feasible solution which maximizes the objective function, Eq. (20), is called the optimal solution.

In order to establish the programming equivalent of the Leontief system it is necessary to recall that interindustry programming distinguishes between the two types of restrictions: the final demands (Y_i) and the available supply of primary factors ($\bar{F}_h$). Thus considering the demand analysis, the inequalities of (21) can be written in the form of equations:

$$\sum_{j=1}^{n} a_{ij}X_j - D_i = Y_i \ (i = 1.\ldots m) \qquad (23)$$

$$\sum_{j=1}^{n} f_{hj}X_j - D_h = \bar{F}_h \ (h = m+1.\ldots.m+l) \qquad (24)$$

where $Y_i \geq 0$ and $\bar{F}_h < 0$.

Equations (23) are the programming equivalent of the Leontief system.

Differences and Similarities Between Linear Programming and Input-Output Models

Taking into account the assumptions of the input-output model stated in Chapter II under Basic Assumptions of the Open Model, and the requirements of the linear programming model, the following comparisons can be made:

(1) Input-Output: Each commodity is supplied by a single sector of production.

Linear Programming: Any commodity can be produced by any number of activities and each activity may have several outputs.

(2) Input-Output: Linearity is assumed in the inputs used by an activity as function of the level of that activity.

Linear Programming: Proportionality is assumed considering that a linear homogeneous function is required.

(3) In both models the additivity property is considered.

(4) In both models the nonnegative condition of activity levels is considered.

Thus, we can see that the main difference is in the first assumption. Furthermore, the linear programming model is more general because it can be applied to choices which are not considered by the input-output theory.

Implications in the Use of Linear Programming

From the preceding section it was pointed out that linear programming differs from Leontief's input-output model in its consideration of alternative sources of supply for given commodities and in its use of a comprehensive optimizing procedure to determine the best combination of supply activities.

It is a general agreement among many authorities in economic science -- Chenery, Dorfman, Stone, Sengupta, and others -- that the Leontief's input-output model works better as an instrument of planning than as a tool for analyzing the operation of a free economy. Thus, Chenery, for the purposes of development programming, shows how the input-output and the programming techniques may be used jointly, particularly to the long-term development of non-industrial countries. Chenery establishes a compromise between the original input-output formulation and the linear programming (6).

Table 2. Suggested Compromises Between Input-Output Analysis and Linear Programming

Nature of Model	Price Calculations	Choice Criterion	Feasibility of Quantity Solution
1. Input-Output	None	Partial analysis for for each sector.	Commodities only.
2. Mixed	Exogeneous determined accounting prices for foreign exchange, labour, capital.	Profitability of projects using accounting prices for direct inputs.	Commodities only.
3. Mixed	Accounting prices calculated from the model for factors only.	Same	Commodities and primary factors.
4. Linear Programming	Shadow prices for factors and commodities calculated from the model.	Profitability of activities (Simplex Criterion).	Commodities and factors.

The first step toward linear programming is the assumption of a number of activities for producing or importing the commodities included in a given industrial sector. The second step is the application of a uniform criterion to the choice of activities in each sector. The next step toward linear programming is the further adjusting of the choice of activities to be consistent with the factor limitations. The last step is the computation of the shadow prices for commodities as well as for primary inputs. In this way the simplex criterion of linear programming can be applied to all inputs instead of considering only the costs of the factors used directly.

The choice between a predetermined Leontief-type model of change and a programming formulation would seem to depend primarily on whether the purpose is prediction or planning. It is possible that a model which allocates resources in the most efficient manner will approximate the actual behavior of some sectors of the economy. On the other hand, where the model is to be used as a guide to government policy, it is desirable to include those choices over which the government has some control whenever government action depends to some extent on the nature of the solution.

Programming Solutions

In order to facilitate the discussion of the linear programming approach to interindustry analysis so far explained, it will be useful to have a look at some numerical examples whose data will be presented in Table 3 and Table 4. The choices to be considered will be on the demand side and on the supply side with a graphical interpretation and the explanation of the simplex method.

Graphical Analysis

For the sake of simplicity we will consider a Leontief model with only two

Table 3. Choice Among Final Uses

		Disposal Activities							
		Production Activities		Unused Resource Activities			Final Use Activities		
Equations	Commodities	A_1	A_2	A_3	A_4	A_5	A_6	A_7	Restrictions
(1)	Commodity 1	1.0	-0.5				-1		0
(2)	Commodity 2	-0.25	1.0					-1	0
(3)	Labor (L)	-7.5	-5.0	-1					-2000
(4)	Capital (K)	-1.25	-2.5		-1				- 600
(5)	Natural Resources (N)	-1.0				-1			- 180
(6)	Objective Function (C)	0	0	0	0	0	1.25	1.0	Maximum
Activity Level		X_1	X_2	X_3	X_4	X_5	X_6	X_7	

activities, i.e., two produced commodities and one primary input:

$$
\begin{aligned}
1.0X_1 - 0.5X_2 &= Y_1 \\
-0.25X_1 + 1.0X_2 &= Y_2 \\
-7.5X_1 - 5.0X_2 &= -L
\end{aligned}
$$

$$
\text{or} \qquad A_1X_1 + A_2X_2 = \begin{bmatrix} Y_1 \\ Y_2 \\ -L \end{bmatrix}
$$

where the numerical values correspond to A_1 and A_2 from Table 3 and to A_2 and A_5 from Table 4.

If the activity levels are used as axes and assuming $Y_1 = 10$ and $Y_2 = 50$, then the solution is given for $X_1 = 40$ and $X_2 = 60$ corresponding to the point s in Figure 2. The equation which determines the use of labor gives a family of straight lines for constant labor supplies. For L = 600, a line shows other combinations of the two activity levels which would use the same amount of labor and includes the solution for the given values of Y_1 and Y_2. Similarly, any other primary input can be shown in the same way as labor. For example, for capital use (from Table 3):

$$-1.25X_1 - 2.5X_2 = -K$$

The capital required by the given solution is K = 200 and the line shows other combinations of activity levels.

In order to show the alternative final output it is necessary to take Y_1 and Y_2 as variables and eliminate the activity levels from the system because they

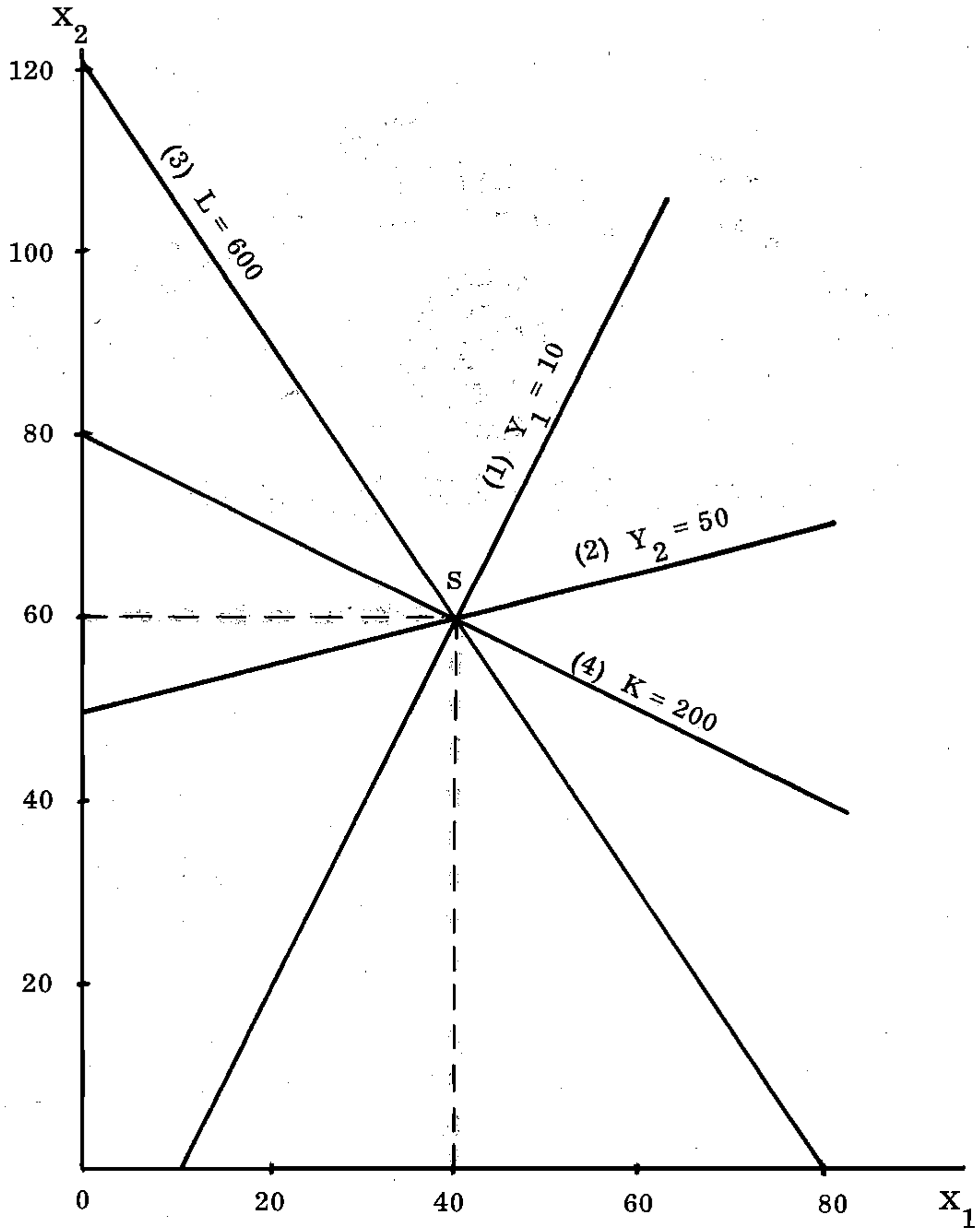

Figure 2. Restrictions (Activity Axes).

only illustrate the meaning of the restraints in linear programming.

After an algebraic manipulation we obtain:

$$L = 10.0Y_1 + 10.0Y_2$$

$$K = 2.143Y_1 + 3.571Y_2$$

Figure 3, with Y_1 and Y_2 as final output axes, shows the graphs of labor and capital use equations. The original solution for $Y_1 = 10$, and $Y_2 = 50$, is again given at point s. The other solution for Y_1 and Y_2 yield for K = 200 and I = 600. For these values of labor and capital, the maximum amounts of outputs that can be produced are determined by whichever factor is exhausted first, offering in this way the production possibility curve and all the feasible solutions lie in the area 0asb0 (please see either Figure 2 or 3). The line asb represents the transformation function for the economy and an optimum combination for Y_1 and Y_2 can be found from this diagram knowing the valuation assigned to each commodity.

The isoquant or production function is another useful graphical representation for the production possibilities. This kind of diagram shows alternative ways of producing the same output. Thus, for instance, from Table 4 we have:

$$A_4 = \begin{bmatrix} -0.2 \\ 1.0 \\ -15.0 \end{bmatrix}, \quad A_5 = \begin{bmatrix} -0.5 \\ 1.0 \\ -5.0 \end{bmatrix}, \quad A_6 = \begin{bmatrix} -0.8 \\ 1.0 \\ -4.0 \end{bmatrix}$$

The isoquant corresponding to commodity 2 is based on the inputs Y_1 and L as axes and the values of $X_{ij} = a_{ij}X_j$. The isoquants for three levels of output

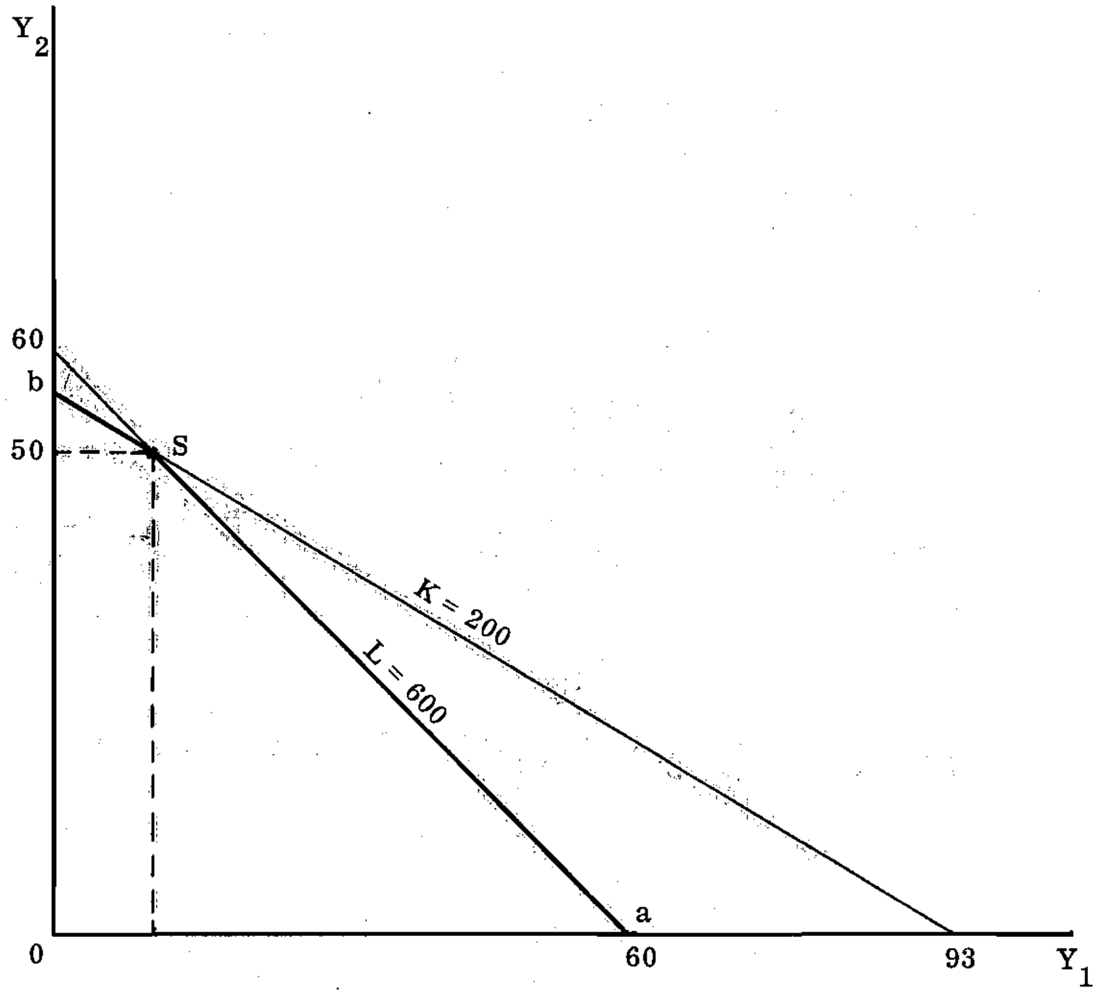

Figure 3. Restrictions (Output Axes).

Table 4. Choice of Technology

	Production Activities							Disposal Activities			
	Industry 1			Industry 2				Unused Resources		Final Use	
Commodities	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Restrictions
(1) Commodity 1 (Y_1)	1.0	1.0	1.0	-0.2	-0.5	-0.8	-0.75			-0.667	0
(2) Commodity 2 (Y_2)		-0.25	-0.50	1.0	1.0	1.0	1.0			-0.333	0
(3) Labor (L)	-12.5	-7.5	-6.0	-15.0	-5.0	-4.0	-4.5	-1			L
(4) Capital (K)	-1.10	-1.25	-0.30	-1.0	-2.50	-0.60	-0.80		-1		R
(5) Criterion (C)	0	0	0	0	0	0	0	0	0	1.0	Maximum

of commodity 2 are shown in Figure 4.

In a general sense, the choice of the axes for graphical analysis is determined by the problem and the nature of the model. Thus, activity axes are used to represent any number of restrictions in two variables or in general, for models in which the number of equations (including the slack variables) is two less than the number of variables. With respect to the commodity axes, they are useful when there is a greater number of activities and the problem can be expressed in two or three restrictions. Any other type of model which does not conform to these specifications cannot be represented in two dimensions, although sometimes a part of the optimization problem can be analyzed graphically as, for example, in Figure 4.

The Programming Model with Alternative Demands and Resource Limitations

The formulation or testing of economic programs by means of interindustry analysis is concerned with choices related to the level and composition of the final output that can be produced. Thus, the input-output system can be transformed into a programming model with alternative demands and resource limitations. In such cases, the following additional requirements should be considered: 1) A set of equations for resource use -- Eq. 24 -- with disposal activities to measure the amount of each resource not used; 2) Disposal activities to measure the final use of each commodity; 3) An objective function stating the value placed on the final use of each commodity.

The model of this type is shown on Table 3. There are six equations where each column represents an activity, the first two being Leontief production

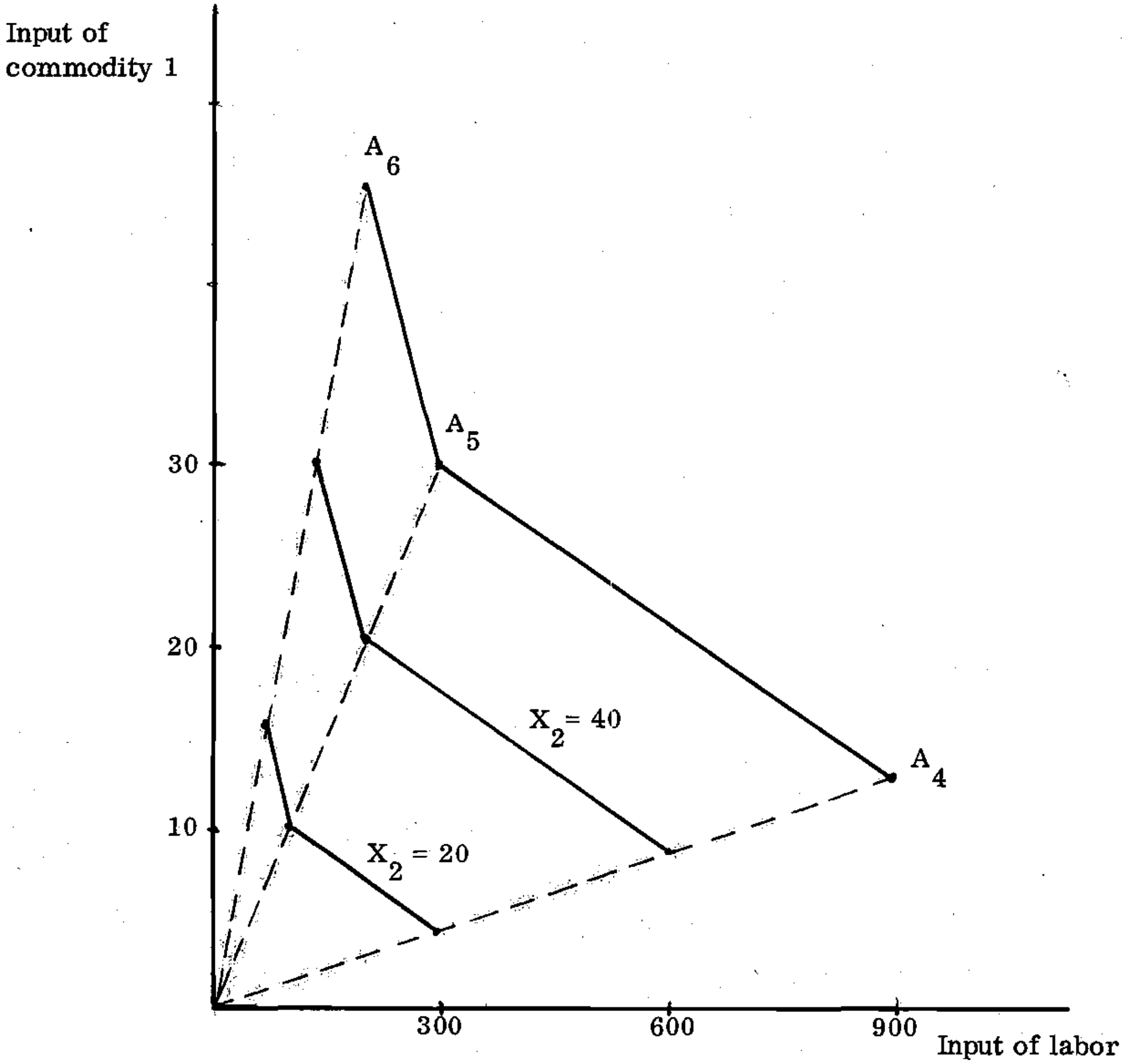

Figure 4. Isoquants (Input Axes).

activities and the others disposal activities for the resources and final products. The total output for final use is represented by the activity levels X_6 and X_7 and the two equations for the input-output system are now:

$$(1)\quad 1.0X_1 - 0.5X_2 - X_6 = 0$$

$$(2)\quad -0.25X_1 + 1.0X_2 - X_7 = 0$$

The resource equations have restrictions representing the total amount of each resource available:

$$(3)\quad -7.5X_1 - 5.0X_2 - X_3 = -200 \quad (L)$$

$$(4)\quad -1.25X_1 - 2.5X_2 - -X_4 = -600 \quad (K)$$

$$(5)\quad -X_1 \qquad -X_5 = -180 \quad (N)$$

The objective function is given by:

$$(6)\quad C = 1.25X_6 + 1.0X_7$$

because the activities A_1 through A_5 do not produce final output.

Furthermore, the objective function can be expressed in terms of X_1 and X_2 substituting from equation (1) and (2) in equation (6):

$$(7)\quad C = X_1 + 0.375X_2$$

where the new coefficients are related to the value added in producing each of the two commodities.

If it is desired, a feasible program can be found and later on alternative combinations of final output can be considered. Using the same values for A_1 and A_2 as in the graphical analysis before, and substituting values, the objective function is determined, and again, a graphical solution can be obtained.

Up to this point the Leontief assumptions have not been abandoned and only additional activities have been added to describe possible combinations of demand and to insure a feasible solution.

Choice Among Possible Supply Activities

Under the considerations of the choices among possible supply activities the original Leontief model does not provide all the necessary assumptions for a complete analysis.

The choices on the supply side can be summarized as follows:

1) In expanding productive capacity, a choice can be made among the alternative production techniques.
2) A choice can be made between the imports and domestic production. (This choice is often considered in development planning).
3) When considering supplies from existing plants or region, which choice of proportions should be made.
4) The choice between current production and depletion of inventory.

Each type of choice affects a particular set of parameters in the input-output model: (1) the input coefficients; (2) the imports coefficients; (3) regional supply coefficients; and (4) capital coefficients.

All these types of choice together with the choices on the demand side convert the input-output model into a general activity analysis. The data presented in Table 4 is related only to the choice among production activities and it is the model that will be considered for our illustration. As before, the objective is the maximization of national income and it is assumed that the proportion required for the two commodities is equal to two units of commodity 1 to one unit of commodity 2.

The analysis of this model is carried out in three stages. First, only one primary factor is considered taking into account the Leontief's assumption of a single output per activity. Thus, the activities can be grouped into industries and the maximizing problem can be broken down into a choice of technique in each industry. Second, the efficient set of activities for an industry and later for the whole economy, is considered. Finally, the assumption of a single primary factor is abandoned to investigate the production function for the whole economy in terms of the two primary inputs.

Derived Activities and Technological Efficiency Within an Industry. From Table 4, ignoring the capital inputs and considering only the first three coefficients in each activity, it is possible to use the activities to generate a whole set of isoquants because of the assumed properties of divisibility and additivity. For instance, if it is desired to produce 50 units of commodity 2 by activity 4 and 50 by activity 5, each coefficient of each activity is multiplied by 50 and the results are added to form a new derived activity:

$$50A_4 + 50A_5 = 50\begin{bmatrix} -0.2 \\ 1.0 \\ -15.0 \end{bmatrix} + 50\begin{bmatrix} -0.5 \\ 1.0 \\ -5.0 \end{bmatrix} = \begin{bmatrix} -10 \\ 50 \\ -750 \end{bmatrix} + \begin{bmatrix} -25 \\ 50 \\ -250 \end{bmatrix} = \begin{bmatrix} -35 \\ 100 \\ -1000 \end{bmatrix}$$

In order to construct a production function for each activity it is necessary to use the concept of derived activities and the definition of technological efficiency. It can be said that an activity or combination of activities is technologically inefficient and should be excluded if there exists some other combination of activities which will produce a given output with less use of one input and no greater use of others. The technological aspect is related to the omission of relative prices from the exclusion. If a graph of the isoquants is desired, we plot all the points producing a given output and draw straight lines connecting those that lie closest to the origin. The points lying above this line can be shown to be inefficient.

Technological Efficiency for the Economy. The main difference between the efficiency of a single industry and the whole economy is that for the whole economy it is necessary to consider several outputs at the same time and it is more convenient to take combinations of activities with a constant input of the single primary factor rather than constant output combinations. It can be done if in Table 4 the production activities are transformed setting the labor input at a convenient level, let us say 1000 units, and increasing the other coefficients proportionately. The new values are shown in Table 5.

The activities for both industries can be plotted in the same diagram with commodity axes since the labor input is now held constant. Figure 5 shows in the fourth quadrant all the activities in industry 1 with positive coefficients for

Table 5. Production Activities in Labor Input Units

	Industry 1			Industry 2			
Inputs	A_1	A_2	A_3	A_4	A_5	A_6	A_7
(1) Commodity 1	80	133	167	-13	-100	-200	-167
(2) Commodity 2	0	-33	-83	67	200	250	222
(3) Labor (L)	-1000	-1000	-1000	-1000	-1000	-1000	-1000

commodity 1 and negative ones for commodity 2; in the second quadrant are all activities in industry 2. Because of the joint production, no activities can occur inside the first quadrant.

The line comprising efficient combinations in each industry can be found as explained before. The line Odef in Figure 5 shows the effect of diminishing returns in industry 2 beyond point d as the input of commodity 1 is increased, as does Oabc in industry 1.

Obviously, any combination of one of the three activities in industry 1 with one of the industry 2 will yield some positive output of both commodities lying in the first quadrant. Any movement away from the origin constitutes an improvement since it is an increase in the availabilities of one or both final products with a given amount of labor input. Thus, line be, representing combinations of A_2 and A_5, is superior to any other combination. Also, the segment rs contains all the efficient combinations of Y_1 and Y_2 that can be produced with 1000 units of labor and the available technology. The equation for this production possibility

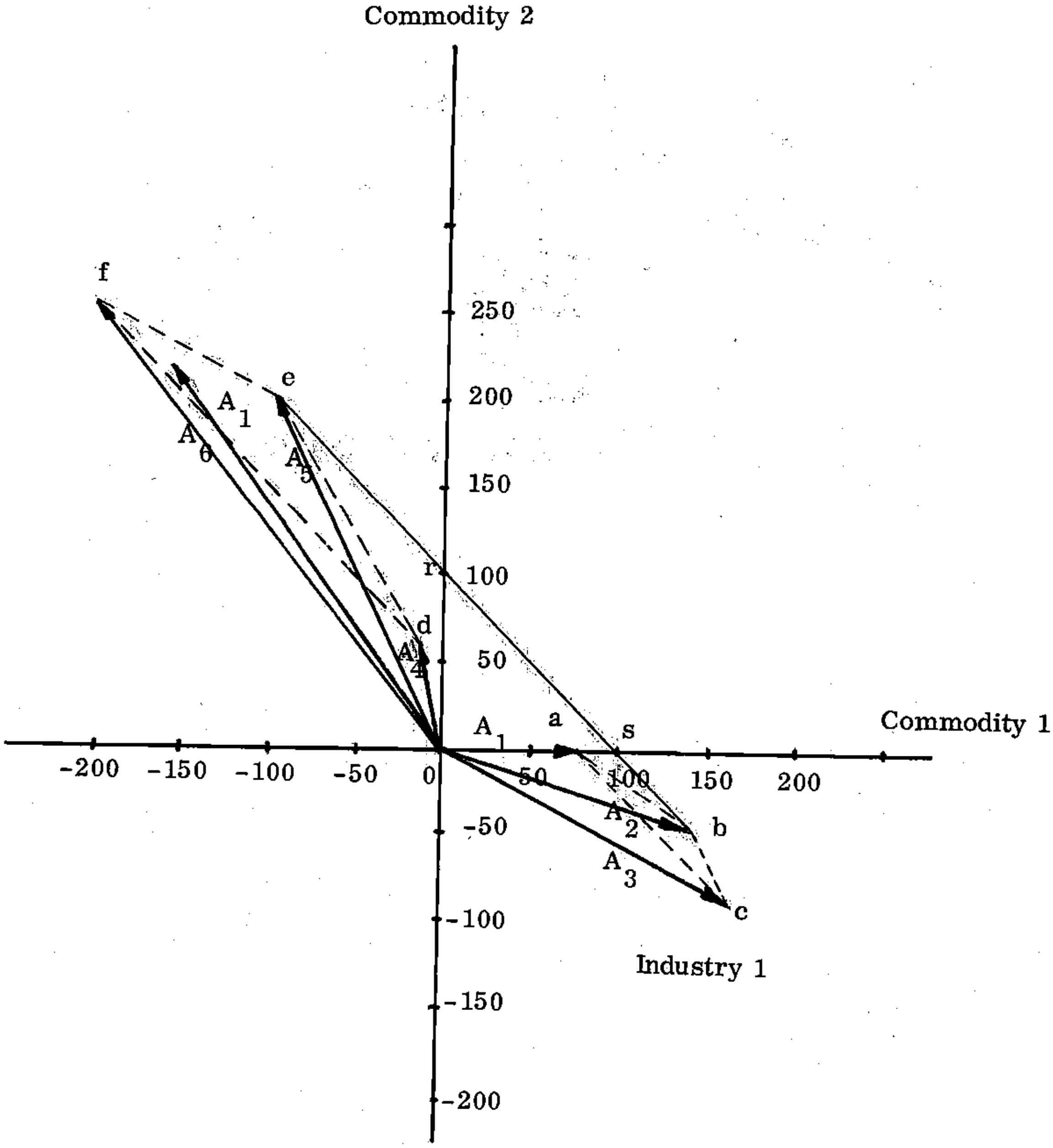

Figure 5. Efficient Activities (Labor Units).

curve can be derived from Table 5:

$$133X_2 - 100X_5 = Y_1$$

$$-33X_2 + 200X_5 = Y_2$$

$$-1000X_2 - 1000X_5 = -1000$$

Hence, $$10Y_1 + 10Y_2 = 100$$

This transformation function has the property that the particular combination of final demands has no effect on the choice of activities. So, the input coefficients are fixed in this case despite the fact that other technological choices exist. This result is known as the substitution theorem which is a remarkable implication of the Leontief system that even if there were available several different processes for each industry, only one of them should ever be observed. It does mean that with a given technology there is one preferred set of inputs ratios which will continue to be preferred no matter what the desired bill of final consumption happen to be. It does not mean that changes in relative prices will not induce changes in proportions. These prices do not depend on the levels of each activity but only on the input coefficients. Thus, the prices are determined as follows from Table 4:

$$P_1 - 0.25P_2 - 7.5P_L = 0$$

$$-0.5P_1 + P_2 - 5.0\,P_L = 0$$

For $P_L = 1.0$

Then, $P_1 = 10.0$ and $P_2 = 10.0$

Using the price of commodity 2, for instance, it is easy to determine the cost of production for each industry from Table 4:

	Industry 1			Industry 2			
Activity	A_1	A_2	A_3	A_4	A_5	A_6	A_7
Unit Cost	12.5	10.0	11.0	17.0	10.0	12.0	12.0

This calculation is valid for all combination of final demand since it does not involve the levels of output. Activities A_3 and A_4 provide the most efficient combination and commodity 2 is more expensive than commodity 1.

Solution by the Simplex Method. The simplex method is the most practical procedure suitable to the solution of a wide range of linear programming problems. With regard to interindustry problems which involve production functions with two primary factors where there may be more possible activities than restrictions, Chenery and Clark present a revised version of the Dantzig's two basic theorems of linear programming (7).

The Dantzig's revised simplex method applied to interindustry models using prices can be stated in the following steps:

1. Find feasible basis (those satisfying the restrictions of Equations 21 and Equations 22) and compute the corresponding activity levels.

2. Determine the shadow prices of the basis. The units in which shadow prices are measured are those of the criterion function. If the problem is as one of cost minimization, the gross value of an activity is defined as the excess of

the value of its output over the cost of its purchased inputs, all valued at the shadow prices of a given basis:

$$Z_j = \sum_{i=1}^{n} a_{ij} P_i + \sum_{h=1}^{n} f_{hj} P_h \tag{25}$$

where Z_j = Gross value of activity j.

P_i = Price of each commodity Y_i;

P_h = Price of each primary factor F_h.

The shadow price of a factor is a measure of its <u>opportunity cost</u> or its <u>marginal product.</u>

In an interindustry programming model the shadow prices for all commodities and factors can be defined by the expression:

$$c_j + \sum a_{ij} P_i + \sum f_{hj} P_h = 0 \quad \begin{matrix} (i=1 \ldots m) \\ (h=m+1 \ldots m+l) \\ (j=1 \ldots n) \end{matrix} \tag{26}$$

where c_j is the direct effect of activity j on the criterion function C;

$(m+l) = n$ is the number of unknown prices;

a_{ij} are the coefficients of the outputs of each activity j and they are positive;

f_{hj} are the coefficients of the direct use of primary factor h by sector j and they are negative.

3. Use the shadow prices to evaluate the profitability of the activities not

included in the basis, and select the most profitable one to introduce into the next basis.

The <u>profitability</u> of any activity is the difference between gross value and direct cost (minimizing problem):

$$\underset{\substack{\text{Unit}\\\text{profit}}}{\pi} = \underset{\substack{\text{Gross}\\\text{value}}}{Z_j} - \underset{\substack{\text{Direct}\\\text{cost}}}{c_j} \tag{27}$$

In a maximizing problem the criterion c_j reverses its sign.

4. Determine the activity to be replaced in forming the new basis.

The four steps above can be grouped in two parts: first, steps 1 and 4 called the quantity solution; and second, steps 2 and 3 called the price solution. Both involve solution to the set of (m + 1) simultaneous equations. The price solution and its economic significance will be considered in this case.

<u>Computation Applying the Simplex Method</u>. The data used is that from Table 4. The problem is one of minimizing the use of capital. The level of final output is 150 and the corresponding activity level, X_{10}, is equal to this value. The restrictions are 100 units of commodity 1, (B_1) and 50 of commodity 2, (B_2). The labor restriction, (B_3), is 2000 units with negative sign. The capital restriction is dropped and the capital coefficients become the coefficients of the objective function. Activities A_7 (inefficient) and A_9 (disposal activity for capital) are dropped.

The shadow prices and the profitability are computed from Table 5.

In any interindustry model it is always possible to find a basis that is feasible in the commodity equations by selecting one activity per industry plus the disposal activities for the primary factors.

The activity levels $A_2A_5A_8$ are the starting feasible basis (step 1) shown in Table 6, section III, row a (first basis).

To obtain the shadow prices (step 2) it is necessary to solve the three simultaneous equations of the form of (26). They are:

$$P_1 - 0.25P_2 - 7.5P_3 = 1.25 \qquad (\text{for } A_2)$$

$$-0.5P_1 + P_2 - 5.0P_3 = 2.5 \qquad (\text{for } A_5)$$

$$- \qquad P_3 = 0.0 \qquad (\text{for } A_8)$$

Solution:

$$P_1 = 2.14;\ P_2 = 3.57 \text{ and } P_3 = 0$$

These values are recorded at the right of the section I, row a in Table 6.

The profitability of the remaining activities A_1, A_3, A_4 and A_6 (step 3) is calculated using equations (25) and (27). For example, the profit for A_3 is:

$$\pi_3 = 1\ (2.14) - 0.5\ (3.57) - 6\ (0) - 0.3 = 0.06$$

Using the data from Table 6 this calculation is carried out in the following way: 1) multiply the prices at the right by the corresponding input coefficients for the activity, 2) add down the column, and 3) subtract c_j from this total. Finally, the most profitable activity is selected (A_4) for introducing into the next basis which implies that one of the old activities is dropped out to retain a basis

Table 6. Summary of Price Solution

Inputs	Basis No.	Activities A_1	A_2	A_3	A_4	A_5	A_6	A_8	Restrictions B_i	Prices P_i	Total Capital P_iB_i
I. Price Analysis											
a_{1j}		1.0	1.0	1.0	-0.2	-0.5	-0.8		100		
	a	2.14	2.14	2.14	-0.43	-1.07	-1.71			2.14	214
	b	5.20	5.20	5.20	-1.04	-2.60	-4.16			5.20	520
(1) Commodity 1	c	3.24	3.24	3.24	-0.65	-1.62	-2.59			3.24	324
$(a_{1j}P_1)$	d	2.03	2.03	2.03	-0.41	-1.02	-1.62			2.03	203
	e	2.60	2.60	2.60	-0.52	-1.30	-2.08			2.60	260
a_{2j}			-0.25	-0.5	1.0	1.0	1.0		50		
	a		-0.89	-1.79	3.57	3.57	3.57			3.57	179
	b		-1.66	-3.32	6.63	6.63	6.63			6.63	332
(2) Commodity 2	c		-0.94	-1.88	3.76	3.76	3.76			3.76	188
$(a_{2j}P_2)$	d		-0.63	-1.26	2.52	2.52	2.52			2.52	126
	e		-0.79	-1.58	3.16	3.16	3.16			3.16	158
f_{3j}		-12.5	-7.5	-6.0	-15.0	-5.0	-4.0	-1.0	-2000		
	a	0	0	0	0	0	0	0		0	0
	b	-3.83	-2.30	-1.84	-4.59	-1.53	-1.22	-0.31		0.306	-612
(3) Labor	c	-1.76	-1.06	-0.85	-2.12	-0.71	-0.56	-0.14		0.141	-218
$(f_{3j}P_3)$	d	-0.93	-0.56	-0.45	-1.12	-0.37	-0.30	-0.07		0.074	-149
	e	-1.50	-0.90	-0.72	-1.80	-0.60	-0.48	-0.12		0.120	-240
(4) Capital (c_j)		1.10	1.25	0.3	1.0	2.5	0.6	0			

Table 6. Summary of Price Solution (Continued)

Inputs	Basis No.	Activities A_1	A_2	A_3	A_4	A_5	A_6	A_8	Restrictions B_i	Prices P_i	Total Capital P_iB_i
II. Profit (π_j)	a	1.04	0	0.06	2.14	0	1.26	0			
	b	0.28	0	-0.25	0	0	0.65	-0.31			
	c	0.38	0	0.22	0	-1.07	0	-0.14			
	d	0	-0.41	0.02	0	-1.37	0	-0.07			
	e	0	-0.34	0	-0.16	-1.24	0	-0.12			
III. Activity Level (X_j)	a	0	142.9	0	0	85.7	0	500.0			
	b	0	118.3	0	71.4	8.2	0	0			
	c	0	124.3	0	67.6	0	13.5	0			
	d	131.4	0	0	14.3	0	35.7	0			
	e	80.0	0	100.0	0	0	100.0	0			
IV. Total Capital (c_jX_j)	a		178.6			214.3					393
	b		148.0		71.4	20.4					240
	c		155.4		67.6		8.1				213
	d	144.6			14.3		21.4				180
	e	88.0		30.0			60.0				178

solution. The activity that is replaced must be either another activity in the same industry. Thus the second basis (b) becomes $A_2A_4A_5$.

The remaining rounds are computed in the same way and the quantity solution can be found by substitution, iteration, or using the inverse. The procedure continues in the same way until all of the excluded activities become unprofitable. The simplex method guarantees that this result represents the optimum solution.

So far the analysis has been carried out with the purpose to facilitate hand calculation of illustrative programming models. Any realistic interindustry model will probably require electronic computer facilities.

CHAPTER IV

CONCLUSIONS

From the analysis carried out between interindustry models and linear programming models, the essential point is the fact that there is no inherent contradiction between the two approaches, and that they can be combined whenever it seems both desirable and practicable. A given set of parameters in an input-output model presents one of many possible solutions to a more general linear programming model and the choice of activities can be either predetermined or based on a criterion function or these alternatives can be used for different sectors in the same model.

Input-output analysis has two important objectives. The first is to develop a special kind of accounting system in which the transactions of the industrial sectors are given a full and detailed treatment, and the second is to develop a simple econometric model for the economy as a whole. Thus the accounting framework supplies the empirical basis on which the model is built by assuming relationship of proportionality between the inputs and the outputs of individual industries.

Input-output analysis is above all an analytical tool. The static, open Leontief model is operational as it stands for a wide variety of purposes. It has won international acceptance as an analytical tool which is an important guide to policy-makers. The main problem facing input-output analysts is the collection and processing of data for the construction of the transactions table, but the development of high-speed electronic computers makes possible the practical applications of input-output models to empirical problems.

On the other hand, the principal objections to the technological postulate of the model can be summarized as follows: (a) There is a problem of time because the usual input-output model abstracts from the time sequence of production and exchange and it applies only to a stationary equilibrium; (b) There is an aggregation problem because it is permitted that any industry includes firms whose technical methods or products are not identical; (c) There is a substitution problem because the conditions of production are such that once the level of output of each sector is given the quantity of each of its inputs is uniquely determined, but the production theory postulates that the amount of each input used in producing a given output will respond to changes in relative input prices; and (d) There is an investment problem because all purchases made by a productive sector may be classified into purchases for purposes of current production and purchases for purposes of investment, which means an increase and replacement of capital plant and equipment.

Besides these objections, the most valuable characteristic of the input-output model is the vast amount of organized empirical knowledge concerning the interrelations of industrial sectors, and the information related to where the products of various industries go and where their raw materials come from.

If it is true that the theoretical interindustry analysis and linear programming are very closely related, the model loses much of its sharpness when it descends from the level of abstract theory to that of application. Input-output has several advantages for descriptive analysis but linear programming requires a lot more information than will be available for many sectors in the near future and its assumption of optimization as a way of describing the complex effects of actual market forces is as yet untested. But, as a matter of fact, linear programming is clearly the better formulation in choosing among alternative decision policies because it provides a systematic method for reaching an optimum solution.

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