

THE DETERMINATION OF SHEAR LOAD DISTRIBUTION IN BOLT GROUPS
BY MEASUREMENT OF SURFACE STRAINS IN THE HEADS
WITH ELECTRIC RESISTANCE STRAIN GAGES

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SUMMARY

A brief review is given of the history of studies of load distribution among rivets and bolts in groups. A proposal for experimental determination of distribution is considered, based upon variation of surface strain in the heads of bolts. Electric resistance strain gages are used as a means of measurement of strains. Effects of various factors on magnitudes of head strains are considered in Chapter II, leading to calibration procedure for bolts to be used in group tests.

Studies of two bolts in line are presented in Chapter III. Results of single and double shear tests are given in tables. Two bolts loaded unsymmetrically are considered in Chapter IV. Determination of indicated loads and their directions is attempted by use of strain gage rosettes. Comparison is made with theoretical loads and directions. Conclusions on the usefulness of the method and recommendations for further investigation are given in Chapter V. Figures describing equipment and results are found in the Appendix.

CHAPTER I

INTRODUCTION

Statement of the Problem.-- When two or more bolts or rivets are used to connect parts of a loaded structure it is customary to make certain assumptions as to the distribution of shear loads among the individual bolts or rivets. Such assumptions are considered to be fairly accurate for high loads near the ultimate strength of the joint, after local yielding of the parts has occurred. They are less accurate for low loads, since dimensional differences can cause unequal loadings by allowing some parts to be brought into use before others.

The assumption for joints where the line of action of the load passes through the centroid of the bolt or rivet pattern is that each unit carries a direct proportionate part of the load. Where the line of action of the total load does not pass through the centroid of the pattern it is customary to assume that the loading on each bolt or rivet is a combination of a "direct" load and an "eccentric" load. The direct load is assumed to be parallel to the line of action of the applied load, with a magnitude proportionate to the strength of the bolt or rivet. The eccentric load is dependent on the area of the individual bolt or rivet and on the eccentricity of the applied load, or its perpendicular distance from the centroid of the pattern.¹

¹David J. Peery, Aircraft Structures, First Edition, New York: McGraw-Hill Book Co., Inc., 1950. pp. 306-309.

The aim of this paper is the investigation of an experimental method of determining the distribution of loads into individual bolts in a group. A successful experimental method will assist in verification of the assumptions customarily used.

The investigations reported herein are limited to two-bolt patterns, loaded symmetrically and unsymmetrically. A method which proves suitable for these should be satisfactory for investigation of joints employing more than two bolts. It is desirable to prove suitability of the method in the simpler types of bolt patterns.

Historical Background.-- The history of work on distribution of loads in rivets appears to begin in about 1867, when the belief was expressed by Schmedler in Germany that uniform distribution did not exist. In 1885 J. T. Milton stated definitely that in lap joints outer rivets carried more load than inner rivets. Discussions of the problem continued until 1904, but there was lack of systematic experiment. In 1908 a very general mathematical analysis was made by Arnolevic in Austria. Batho in 1916 derived theoretical formulas and checked results with strain gages. His results agree well with those of Arnolevic, although obtained differently.

In 1922 A. Hertwig in Germany proposed using the deflection of rivet heads as a means of determining the load distribution, and he and H. Peterman attempted to do so in 1929, using optical means of measuring angular deflections. The attempt was unsuccessful, and well-fitting pins were substituted for the rivets. In 1931 Wornle in Germany drilled holes

longitudinally through the rivets and inserted tightly fitted pins, the bending of which was measured by mirrors. He concluded that friction played an important part in the load distribution.²

F. Vogt in 1944 presented a theoretical method of calculating the load distribution in bolted or riveted joints where a line of bolts or rivets is used. He concluded that further experimental data were needed to check his theory.³

The successful use of electric resistance strain gages for measuring small strains suggested to this author that they might be used on rivet or bolt heads for determination of load distribution through head deformation. It was thought that this relatively new tool might succeed where the optical methods of Hertwig and Peterman failed. Preliminary work along this line was done by the author in 1950, using aluminum alloy rivets in plates of the same metal. Small strain gages were cemented to the flat heads of the rivets and the unit strains recorded for different shear loads. The tests indicated that proportionality of head strain to shear load existed, but agreement between specimens was poor. The conclusion was reached that the behavior of the head of one rivet loaded alone could not be used to predict the loads in other similar rivets in a pattern. The process of upsetting the rivet introduced unknown strains in its head an unknown amounts of friction between plates. It was decided to continue the tests on bolts rather than on rivets. Suffi-

²A. E. R. DeJonge, "Discussion of Paper by A. Hrennikoff", A. S. C. E. Transactions, Vol. 99, 1934. p. 474.

³F. Vogt, "Load Distribution in Bolted or Riveted Joints in Light Alloy Structures", U. S. N. A. C. A. Technical Memorandum No. 1135, 1947. p. 38.

cient results were obtained to suggest that a calibration curve could be prepared by plotting unit strains on the surface of the head against corresponding shear loads on the bolt. When that bolt was used as one of a group its head strain indications could be referred to the calibration curve for determination of shear load on the bolt.⁴ The work reported in this paper is a continuation of the work begun in 1950.

Principle Involved.-- When a bolt is loaded in single shear as in Figure 1 it is assumed that the line of action of the load at the ends of the specimen lies at the shearing surface. However by Saint-Venant's Principle the stress distribution in the strap at point A will be uniform over the thickness of the plate. Thus at this point the load P may be considered to be acting along the center of the strap. Assuming that this condition holds at the bolt and that bearing stress is uniformly distributed, the bolt itself is subjected to a moment Pt as in Figure 2(a).

For equilibrium to exist on a free body of half the bolt as in Figure 2(b), it is seen that an axial load T, reacted by a load on the rim of the bolt head, forms a couple to balance the shear loads P. At higher values of P local deformation of the plate due to bearing stress will allow the bolt to tilt, causing a change in bearing stress distribution and in the assumed point of application of P on the bolt. There will be an increase in T, accompanied by tensile elongation of the bolt. This results in separation of the plates and an increase in the magni-

⁴John W. Hoover, Investigations of Relationship between Shear Loads and Surface Strains in Heads of Rivets and Bolts, unpublished Advanced Airplane Design Problems report, Georgia Institute of Technology, 1950.

tude of the couple. The head rim reaction to the tension load in the bolt causes bending stress in the head. The surface strain can be measured with electric resistance strain gages bonded to the center of the head. The strain parallel to the line of action of the shearing load is compressive, and that normal to the load line is tensile. Experiments verify existence of these strains.

In the consideration of bolts loaded in double shear, bending strain in the head can be attributed mostly to bending of the bolt itself under three loads. Angular deflection of the head causes one side of the rim to bear upon the plate, resulting in bending strain in the head. In comparison to the plates, the bolts used in the tests herein reported had relatively high bending rigidity. It was suspected that head strains under double shear loads would be smaller than those in the same bolt loaded in single shear. Experiment verified this, showing that double shear head strains were of the order of one-half to two-thirds the magnitude of single shear strains.

Calculation of the compressive stress in the head would be very difficult if not impossible, due to the many factors that affect it. Sufficient work was done to convince the author that the most convenient application of the principle consists of the calibration of a given bolt in a given hole, the preparation of a load-strain curve, and the use of measured strains evaluated from the curve to find unknown loads.

The information presented will be of two general types: the behavior of a single bolt and effect of various factors on its head strain, and the division of shear load among two bolts loaded simultaneously.

Equipment and Materials.-- Bolts used in the tests were made in the machine shop of 24S-T aluminum alloy, according to Figure 3. These were used rather than standard bolts in order that closer tolerances might be held in the investigation of fit and other factors.

Strain gages used were SR-4 type A-7, having an effective gage length of one-fourth inch, a gage factor of 1.95 and resistance of 120 ohms.

The majority of the testing involved measurement of strains with the SR-4 Wheatstone Bridge Control Box. The arrangement and equipment used is illustrated in Figure 7. Loads were applied with a Riehle Universal Testing Machine having a capacity of 40,000 pounds. Other equally satisfactory equipment used in some tests included the SR-4 Type K Strain Indicator and SR-4 multichannel automatic strain recorder, and a 10,000 pound screw-operated testing machine.

Plate specimens for loading the bolts in single and double shear are shown in Figures 4, 5, 6 and 17.

Procedure and Accuracy.-- Following usual practice in the use of electric resistance strain gages, the circuit was balanced and readings taken representative of the zero-load condition. New readings were taken as each increment of load was applied, and the increments in readings converted to unit strains in micro-inches per inch. Values of unit strain were plotted against corresponding shear loads to provide calibration curves for each bolt.

It was found advisable to allow temperature to stabilize in the specimen after installation in the testing machine before taking initial readings. Even though a compensating gage in the circuit was kept in

contact with the machine, currents of air and body heat transferred in handling of the specimens caused fluctuations in the readings.

The first loading cycle generally produced different readings from those recorded for subsequent cycles. Calibration runs were therefore repeated until good agreement between successive readings was obtained. Most data presented represents such readings.

Agreement of results may depend upon accuracy of measurements. For most tests the testing machine was operated in its 0-4000 pound range, allowing reading of the load scale to within approximately five pounds. The accuracy of the strain measurement equipment should be within plus or minus ten micro-inches per inch. If the slope of the load-strain curve lies between 3 and 5 pounds per microinch per inch, a variation of 20 microinches per inch would represent from 60 to 100 pounds of load. It is reasonable to assume that the accuracy of prediction of load from a load-strain curve is within that range, providing accuracy of two to four per cent at a 3000 pound shear load.

CHAPTER II

CALIBRATION TESTS ON SINGLE BOLTS

Single Shear Tests. Effects of Variables.-- Preliminary tests mentioned in Chapter I indicated that head strain varied with single shear loads. It was desirable to determine whether the strain continued to increase up to shear failure. Bolt number 1 was loaded in 250 pound increments with intention of shearing it. Head unit strains were recorded at each interval of loading. The bolt did not fail in normal shear. The rim of the head, whose height had been made low to allow bending, failed by shearing off around the bolt shank. As it moved upward the rim lifted the strain gage from the surface of the bolt, so that the recorded unit strains reach a maximum and decrease with further load. Figure 8 shows two bolts which failed in this manner. Figure 9 presents a plot of shear load against unit strain as indicated by the strain gage.

For determination of the nature of the head strain up to shear failure a smaller bolt with higher head was made. It was loaded in plates designed to make the bolt critical in shear. Variation of head unit strain with shear load on this bolt is also presented on Figure 9. Load was carried up to 2750 pounds, released, then carried to failure of the bolt at 4650 pounds. The difference between the first and second loadings mentioned in Chapter I is evident here.

The change in slope which occurs at a load of approximately 3500 pounds may be associated with the yield stress of the material, since the ratio of that load to the ultimate load is approximately the ratio

of yield stress to ultimate stress of aluminum alloy. Although the bolt sheared cleanly, tensile stress is clearly evidenced by necking down at the sheared surface.

There are a considerable number of variables affecting the magnitude of unit strain on the top surface of the bolt head. Efforts were made to evaluate some of them, with little success. Among them are: fit of the bolt in the holes, alignment of the strain gage on the bolt head, fit of the bolt head against the plate, thickness of the plates compared to bolt diameter, and tightness of the nut.

Investigation of the effect of fit of the bolt in the holes was made by loading each of five bolts in different plates of $1/4$ inch thickness. Holes in the plates were reamed to provide clearances up to 0.0040 inch. Loads were carried only to 3000 pounds to avoid yielding of the material in the parts. The results, given in Figure 10, indicate insufficient consistency to permit a clear conclusion on the effect of fit. Combinations of factors may be responsible for the inconsistency.

The effect of alignment of the strain gage on the bolt head may be observed by recording the head strains, first with the strain gage parallel to the line of action of the load, then with the gage rotated 180 degrees. If the strain gage is slightly off center toward the edge of the head against which the plate is bearing (that is, the edge of the bolt head toward the loaded end of the plate), higher unit strains will be recorded than if the gage is off center toward the opposite edge. This may be observed on Figure 11, where plots of load against unit strain are made for various angles of strain gage orientation. The curves of zero and 180 degree alignment and those for 30 and 150 degrees

differ due to alignment of the strain gage on the bolt.

If the bolt head does not fit uniformly down upon the plate, the manner in which head strain changes with shear load is affected. By calibrating a bolt first in a plate in which the holes are perfectly normal to the plate surface, then in plates in which the holes are slightly inclined to the normal, it is seen that the magnitude of head strain is sharply affected by the manner in which the edge of the bolt head bears on the plate. If the hole is inclined toward the free end of the plate the application of shear load may actually reduce the unit strain in the head by tilting the bolt and lifting the bearing edge of the head off the plate. If the nut is tightened, high strains are produced without application of any shear load. The effect of the shear load then is to reduce the magnitude of the strain. Figure 13 shows the effect of nut tightness on bolts in both normal and slightly inclined holes. In the plates with normal holes tightening of the nut to various degrees of torque produced sharp changes in the shape of the curve in the low load region. However, for higher loads the curves are substantially the same.

Calibrating the same bolt in plates of different thickness but with similar fits shows little variation that can be attributed directly to plate thickness. Figure 12 shows that the shape of the curves are slightly different. In these tests on a bolt $1/2$ inch in diameter, it is seen that the curve for $3/16$ inch plates maintains a rate of change throughout its length, while the curves for thicker plates have fairly constant slopes above a load of about 500 pounds.

The conclusion drawn from these tests is that there is no defi-

nite association of certain effects with particular variables. There is need for more detailed study of the various factors and their relationships to the head unit strain. Further work of this nature is beyond the scope of this investigation.

In view of this conclusion, it was clear that bolts to be loaded as members of a two-bolt group should first be calibrated by loading in the holes in which they would be tested in the group. Accordingly, each of two bolts was placed in its assigned hole and loaded alone, a small bolt equipped with washers being used in the unloaded hole to hold the plates together at that point. This was done to simulate bending conditions which would exist in the two-bolt test. It has been observed in tests of single shear that the plates bend so that the line of action passes through the centroid of the group rather than through the centers of the bolts. This bending of the plates effects the head strain, as can be seen by comparing magnitudes of single shear bolt head strains with those of bolts in double shear. Data on calibration of two bolts for use in single shear tests are given in the (a) parts of Figures 14-1 through 14-5.

Double Shear Tests.-- The calibration of bolts in double shear was also done in the holes in which bolts were to be placed for distribution tests. The procedure used in calibrating the single shear plates was repeated with plates similar to Figure 5. Nuts were run up snugly against the plates, but not sufficiently tight to change the zero-load strain reading. Calibration data for bolts 2 and 8 are presented in Figures 15-1(a) and 15-2(a).

CHAPTER III

LOAD DISTRIBUTION AMONG TWO BOLTS IN A LINE

Single Shear Tests.-- The distribution of load among two bolts in line is usually assumed to be an equal division, with fifty per cent of the load carried by each bolt. Tests were conducted on two-bolt groups, utilizing previously described calibration curves for determining loads on each bolt.

Each bolt was calibrated by being loaded alone, in its own holes, while values of unit strain in the head were recorded for single shear loads up to 3000 pounds. After calibration, both bolts were installed and the specimen loaded in 1000 pound increments, unit strains in each head being recorded for each increment of loading. Table 1 gives the unit strains in each bolt head for a number of tests. These values were referred to calibration curves for the proper bolt in the hole in which it was loaded, and the indicated loads corresponding to the unit strains were indicated on a distribution plot.

In test number 1, a specimen similar to figure 4(b) was used, with both bolt heads on the same side of the plates. Results of the test are plotted in Figure 14-1(b). It is noted that the indicated loads carried by the bolts are not equal, nor do all the pairs of loads add up to the total loads applied. Percentage distribution is obtained by dividing the indicated load in the bolt by the sum of the indicated loads. Distribution in per cent is given in Table 2.

Test number 2 differed from number 1 in that bolt 2 was reversed in its hole. The new position placed the bolt heads on opposite sides of the plates, point symmetry now indicating that more nearly similar bending conditions would exist on the heads. Results are given in Figure 14-2. In both tests the distribution appears to become more nearly uniform as loads increase, although the maximum loading of 4000 pounds in these tests was not sufficient to bring about plastic yielding of the material, with consequent equalization of loads.

Test number 3 was conducted with plates similar to Figure 4(b) except that instead of offset blocks on the specimen itself, the two plates of Figure 4(a) were bolted to the ends of the two-bolt straps. This assures the application of load along a known line of action. The results of the test, as shown in Figure 14-3, are fairly good. The indicated loads are within 250 pounds of half the total applied load, and the variations of loads are practically linear.

Test number 4 uses the same plates as number 3, but both bolt heads were on the same side of the plates. Results, in Figure 14-4, are not as good as those of number 3.

Test number 5 was a repetition of number 2, except that the bolt positions were reversed in the plates. The per cent distribution was nearer the theoretical equal division, and indicated a tendency to approach it at higher loads. At 5000 pounds a percentage of 52.5 in bolt 2, as compared to 54.5 per cent for 4000 pounds, shows this trend.

Double Shear Tests.-- Tests numbers 6 and 7 consisted of loading the two bolts simultaneously in the holes in which they were calibrated. The tests differed in that the bolts were reversed in the holes. That

is, in test number 7 the head of bolt 2 was on the opposite side of the plates from its position in test number 6. In each test the heads of the two bolts were on opposite sides of the plates from each other.

Values of unit strains measured on the two heads are recorded in Table 2, and the distribution of load indicated in Figures 15-1 and 15-2. In both tests the sum of the indicated loads is less than the applied load, for higher loadings. The percentage distribution, calculated as previously described, is better in test 7 than in test 6. At a total load of 6000 pounds in test 7, bolt 2 carries only 2.5 per cent more than one half the load, and the decreasing percentage with increasing load indicates that a prediction of equal distribution at loads of 8000 pounds or more might be reasonable.

Since bending of the free ends of the plates on the outside of the double shear specimen would logically result in higher strains in the bolt head near that end of the specimen, a small C-clamp was used to restrict this bending. This was done to guard against undue errors in using extrapolated calibrated curves, since bending of the plates would alter the curves.

Test 8 was conducted by calibrating each bolt in its respective hole up to 5000 pounds, with the C-clamp in place on the ends of the outside plates. The clamp was not tightened to the point where its friction would be appreciable. In addition, when the bolt farthest from the free ends was being calibrated, a small bolt with washers was used in the other hole to maintain approximately the same condition of bending as might be expected in the two-bolt test. Figure 16 gives calibration curves for this test.

Both bolts were installed and the specimen was loaded to 14,300 pounds, where it failed. Failure did not occur in the bolt group itself. The single bolt attaching an auxiliary loading plate to the $1/8$ inch outside plates failed in bearing in the thin plates. Strains, corresponding loads from the extrapolated calibration curves and per cent load distribution are presented in Table 4.

It will be seen that bolt 2 carried more than fifty per cent of the load up to a total loading of 13,000 pounds. This is natural, since all the load is carried in the two outer plates up to bolt 2, at which point part of the load is transferred into the center plate through bolt 2. The remaining load in the outer plates is delivered to the center plate by bolt 8, so that it now carries the entire load. Inspection of the plates after the test showed that the holes in the outer plates, directly under the head of bolt 2, were elongated to a diameter difference five times as great as were the holes in the same plates at bolt 8. The same larger elongation occurred in the center plate hole at bolt 8, since more load exists at the hole nearest the loaded end of either plate. Since the bolt head strain is due to the effect of the plate in contact with the head it seems logical that bolt 2 would indicate a higher load than bolt 8.

Determination of load distribution as per cent of the sum of indicated loads gives fair to good agreement with the assumption of equal distribution at high loads, after yielding of the plates.

CHAPTER IV

DISTRIBUTION OF UNSYMMETRICALLY APPLIED LOADS

Theoretical Distribution.-- When a load is applied to a group of bolts in such fashion that its line of action does not pass through the centroid of the group, it is usually convenient to superimpose the effect of a force of equal magnitude and direction, passing through the centroid, upon the effect of a moment about the centroid which is equal to the product of the actual force and its perpendicular distance from the centroid. It is assumed that all plates are rigid and that the bolts are critical in shear.

For the concentric load effect all bolt shearing stresses are assumed to be equal, so that the force on any bolt for the concentric loading would be:

$$P_c = \frac{P A}{\sum A}$$

where A is the cross-sectional area of the bolt and $\sum A$ is the total cross-sectional area of all bolts. If the bolts are of equal area the equation reduces to

$$P_c = \frac{P}{n}$$

for a joint carrying a load P and consisting of n bolts.

The application of a moment to the joint produces shearing stresses which are assumed to be proportional to the distances r of

the bolt centers from the centroid of the bolt areas. The force applied to the bolt by the moment is

$$P_e = \frac{P_e r A}{\sum r^2 A}$$

where e is the perpendicular distance of the line of action of the load from the centroid of the bolt pattern, and r is the distance of each bolt center from the centroid. The eccentric load is assumed to act normal to r .

The total shear load on a bolt is then equal to the vector sum of the concentric and eccentric loads.⁵

The single shear plates in Figure 6 provide for calibration of the bolts by loading directly above and below each bolt, and for the unsymmetrical loading the line of action of the load is three inches from the centroid of the two-bolt group. In this case the eccentric loads parallel the concentric loads, and add algebraically to them. In the general case, as in the double shear plates in Figure 17, the two loads have different directions.

In Figure 17 the theoretical loads for the two eccentrically loaded specimens herein reported are calculated.

Calibration of Bolts.-- Since the loads on eccentrically loaded groups of bolts act at various angles, it was desirable to investigate the possibility of utilizing bolt head strains for determination of directions as well as magnitudes of unknown loads, and since Figure 11 shows

⁵Peery, op. cit., pp. 308, 309.

that head unit strains vary with orientation of the strain gage to the line of action of the load, it appeared that locating the axis of principal compressive strain would determine the line of action of a shear load on the bolt.

Since ninety degree orientation of the gage to the line of action of the load gave indications of tensile strain while a gage placed parallel to the line of action indicated compression of the head of a bolt in single shear, the use of a strain gage rosette offered a possible means of locating load direction and measuring magnitude. If the surface stresses in the head could be classified as two-dimensional, permitting use of available reduction formulas, and if proportionality between principal strains and shear loads could be shown, the unknown shear load on a bolt could be determined both as to magnitude and direction.

Reduction formulas for use with rectangular strain gage rosettes are⁶

$$e = \frac{1}{2} \left(1 - \frac{1}{b}\right) (R_1 + R_3) \pm \sqrt{\frac{(R_1 - R_2)^2 + (R_2 - R_3)^2}{2}} \left(1 + \frac{1}{b}\right)$$

$$\tan 2\alpha = \frac{R_1 + R_3 - 2R_2}{R_1 - R_3}$$

where e represents the principal strains, tension or compression as determined by the sign, R is the strain gage reading for a particular gage in the rosette, b is an auxiliary gage factor supplied by the

⁶R. Baumberger and F. Hines, "Practical Reduction Formulas for Use on Bonded Wire Strain Gages in Two-Dimensional Stress Fields", Experimental Stress Analysis, Vol. II, No. I, 1944. p. 113.

manufacturer, and α is the angular location of the major principal strain, measured counterclockwise from gage number 1 as shown in Figure 3(d). For the rosettes used in these tests the factor b was unknown, as the rosettes were made up of individual type A-7 strain gages applied one above the other so that their centers were over the center of the bolt head. Ignoring the auxiliary gage factor undoubtedly introduced a small error in the use of the formula.

Bolts 6, 9, 10 and 11 were equipped with rosettes as shown in Figure 3(d). In the first test bolt 6 was loaded in single shear with its head rotated to several positions. Calculations of principal strains and their angles were made for loads of one, two and three thousand pounds. Results of the test are given in Table 5.

For similar results in double shear, bolt 11 was tested in six head positions sixty degrees apart. Table 6 gives values of calculated principal strains and directions for various angles of loading.

From these results it appears that the rosette can give fairly accurate indication of direction of shear load application on a bolt. The errors in indicated direction varied to a maximum of about 16 degrees for higher loads in single shear, and to a maximum of about 6 degrees for higher loads in double shear. In most instances the variation was of the order of five degrees or less.

Poor agreement was obtained between curves of load versus principal strains for various angles of loading. Curves of principal tensile strain show more nearly straight-line form than do compression strain curves, hence could be more easily extrapolated. Figure 18 shows this difference, both for single shear and double shear bolts.

For the single shear bolt a 37 per cent decrease in compressive head strain is noted between the 3000 pound load applied at zero degrees or parallel to the index gage (gage number 1) of the rosette, and the same load applied at sixty degrees to the index gage. Reference to Table 5 shows that the calculated angle of compressive strain is very close to the actual angle of applied load for the 45 and 60 degree loadings, while the calculated angles for zero and 240 degrees loadings are not so accurate. This may be due to physical conditions of the experiment.

In the plot of strains for double shear in Figure 18(b) it is seen that the tensile strain curves show less variation with angle than do compressive strain curves. There is still a variation of plus or minus about 16 per cent caused by rotating the bolt in the hole. This is probably due to inaccurate location of the strain gages on the bolt head and to neglect of the auxiliary gage factor previously mentioned. There is also the possibility of creep in the cement causing an effect on the gage factor of the outer gages in the stack. These latter possibilities might be partially eliminated by use of rosettes made up to order by the manufacturer, to usual standards. Absence of suitable standard rosettes in lists supplied by the manufacturer prompted the use of stacked gages.

Distribution of Load among Bolts.-- In preparation for the first two-bolt eccentric loading test, bolts 9 and 10 were calibrated in their respective holes in the single shear specimen shown in Figure 6. Bolt 10 was used in hole 1, and bolt 9 was used in hole 2. Calibration

curves appear as Figure 19.

Both bolts were installed after calibration and the eccentric loads applied. Table 7 presents the results. Wide differences between the indicated loads and loads expected from analysis show that unknown bending effects are influencing the head strains in single shear tests, and more investigation is needed to develop the usefulness of the method for this type of test.

The indicated direction of the bolt load is close to the expected direction for bolt 10, but does not agree closely for bolt 9. The index gage for each rosette was parallel to the applied load.

Double shear loading on two bolts was done in a specimen illustrated in Figure 17(b). Results of this test are summarized in Table 8. Rosettes were aligned so that the index gages were normal to the centerline of the plate under the bolt head.

Extrapolated values of load from Figure 18(b) are within eight per cent of calculated loads for bolt 11, but the indicated direction is in poor agreement with the theoretical direction. The directions for bolt 10 agree closely for the higher loads, but the indicated loads are far too low.

It is recalled that the eccentric loading theory assumes that the plates are rigid and that the bolts are critical in shear. Neither of these assumed conditions actually applies to these tests. Deformation of the plates and small dimensional differences could easily account for indicated load directions other than those predicted by theory.

CHAPTER V

CONCLUSIONS

As a result of the tests conducted in this study, the author would draw the following conclusions:

- (1) The use of bolt head strain, measured with electric resistance strain gages, for determination of unknown loads in a single bolt is possible. The bolt must have been previously calibrated in the same hole under conditions as near as possible to those of the desired test. Accuracy within about five per cent is obtainable.
- (2) Percentage distribution among two bolts in line with the applied load can be determined under the same conditions of calibration. Each bolt must be calibrated by individual loading.
- (3) Determination of angular direction of applied shear load may be made to within a few degrees with strain gage rosettes at the center of the bolt heads.
- (4) The method appears to produce better results in double shear tests than in single shear tests.
- (5) The use of the method for studying eccentrically loaded joints offers possibilities and deserves further investigation. In groups of several bolts the determination of shear load direction alone would be of interest.
- (6) The accuracy of load indication is dependent upon the slope of the

load-strain curve. Better accuracy is possible with curves of lower slope values. A great deal depends on care in applying strain gages. Aside from this the method is fairly simple.

It is recommended that further investigation be made of effects of factors such as hole fit and plate thickness. Behavior of the head strain at loads above the proportional limit of the bolt in shear should be studied, as it would be useful in the extrapolation of calibration curves and in prediction of failing loads.

The method should prove useful in the study of load distribution among three, four, and five bolts in line, for verification of theory such as that presented by Vogt. This study is particularly recommended as a continuation of the work of this report.

Where rosettes are used, it is recommended that they be procured on special order from the manufacturers of strain gages, and that they be applied with extreme care. If rosettes are made up as described in this report, pilot tests should be made on simple structures where principal strains are known.

In evaluation of various factors that affect strain, it is necessary to maintain close control of fits and dimensions. There is a possibility that time of load application may influence strain indications. Investigation of the use of different materials in bolts and plates should be made, along with bolt head height and head diameter effect. The method also offers a possible means of studying friction effects in the joint.

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APPENDIX

Table 1. Head Unit Strains in Single Shear Two-bolt Tests.

Test No.	1		2		3		4		5	
Bolt No.	2	8	2	8	2	8	2	8	2	8
Load (Lbs.)										
1000	0.48	0	0	0.47	0.16	2.06	0.04	1.57	1.42	1.37
2000	2.20	0.65	0.04	1.03	1.41	3.28	1.21	3.92	3.00	2.79
3000	3.85	1.46	0.77	1.69	2.79	4.45	2.66	6.15	4.47	4.44
4000	5.31	2.20	1.74	2.46	4.16	5.43	4.03	8.49	5.95	6.10
5000									7.13	7.84

(Unit Strains are in Inches per Inch times 10^{-4})

Table 2. Head Unit Strains in Double Shear Two-bolt Tests.

Test No.	6		7	
	2	8	2	8
Load				
(Lbs.)				
1000	0.72	0.78	1.49	0.47
2000	1.81	1.14	2.32	1.07
3000	2.79	1.53	2.94	1.54
4000	3.58	1.80	3.42	2.06
5000			3.94	2.53
6000			4.51	3.00

(Unit Strains are In Inches per Inch $\times 10^{-4}$)

Table 3. Percentage Distribution of Shear Loads in Two-bolt Groups.

Load	1000		2000		3000		4000		5000		6000		Shear Load Type
Bolt	2	8	2	8	2	8	2	8	2	8	2	8	
Test No.													
1	100.0	0	56.5	43.5	55.5	44.5	55.5	44.5					Single
2	0	100.0	35.0	65.0	32.2	57.8	43.0	57.0					Single
3	48.2	51.8	46.5	53.5	46.0	54.0	46.5	53.5					Single
4	41.0	59.0	39.5	60.5	41.0	59.0	40.0	60.0					Single
5	57.2	42.8	58.5	41.5	56.0	44.0	54.5	45.5	52.5	47.5			Single
6	55.5	44.5	62.5	37.5	60.8	39.2	62.5	37.5					Double
7	70.5	29.5	63.2	36.8	59.0	41.0	55.2	44.8	53.2	46.8	52.5	47.5	Double

Table 4. Load Distribution in Two-bolt Double Shear Test.

Total Applied Load	Bolt 2			Bolt 8		
	Strain	Indicated Load	Per Cent	Strain	Indicated Load	Per Cent
500	1.77	550	100.0	0	0	0
1000	2.61	950	100.0	0	0	0
1500	3.15	1225	76.8	0.17	370	23.2
2000	3.54	1440	70.3	0.48	580	29.7
3000	4.30	1880	63.9	1.11	1060	36.1
4000	4.90	2260	60.1	1.66	1500	39.9
5000	5.37	2570	55.3	2.20	2000	44.7
6000	5.82	2900	54.3	2.63	2440	45.7
7000	6.17	3150	52.3	3.04	2870	47.7
8000	6.59	3450	51.3	3.40	3270	48.7
9000	6.94	3730	50.5	3.72	3650	49.5
10000	7.55	4240	50.8	4.06	4100	49.2
10500	7.89	4520	50.4	4.34	4450	49.6
11000	8.30	4870	50.9	4.54	4700	49.1
11500	8.70	5200	52.0	4.62	4810	48.0
12000	9.29	5700	51.5	5.07	5360	48.5
12500	9.95	6250	51.2	5.51	5950	48.8
13000	10.86	7000	51.5	6.05	6600	48.5
13500	11.59	7630	47.8	7.42	8370	52.2
14000	12.36	8250	49.2	7.56	8550	50.8

Table 5. Analysis of Rosette Strain Data, Single Shear.

Load	1000			2000			3000		
Angle of Load Line from Gage No. 1	θ	e_m	e_M	θ	e_m	e_M	θ	e_m	e_M
0°	23.1°	-2.75	+1.31	18.8°	-5.42	+2.50	15.9°	-7.61	+3.61
	12.1°	-3.06	+0.80	9.1°	-5.23	+2.73	10.0°	-7.92	+3.94
45°	50.0°	-1.66	+1.42	43.5°	-3.26	+2.12	43.8°	-4.84	+3.42
	47.1°	-1.66	+1.24	43.1°	-3.75	+2.75	42.3°	-5.82	+4.22
60°	61.5°	-1.50	+1.60	59.4°	-3.14	+3.06	57.6°	-4.88	+4.50
	60.4°	-1.66	+1.56	56.6°	-3.52	+3.12	57.9°	-4.63	+4.53
240°	255.5°	-2.95	+1.67	253.2°	-4.61	+3.17	252.6°	-7.61	+4.37

$\theta = \alpha \pm 90^\circ$ for location of Principal Compressive Strain from Gage No. 1.

e_m = Principal Compressive Strain, inches per inch $\times 10^{-4}$.

e_M = Principle Tensile Strain, inches per inch $\times 10^{-4}$.

Table 6. Analysis of Rosette Strain Data, Double Shear.

Load	1000			2000			3000		
Angle of Load Line from Gage No. 1	θ	e_m	e_M	θ	e_m	e_M	θ	e_m	e_M
0°	11.4°	-0.64	+0.22	- 1.0°	-1.12	+1.00	- 2.7°	-1.61	+1.95
	- 0.7°	-0.95	+0.22	- 5.3°	-1.58	+1.64	- 6.5°	-2.10	+2.62
60°	61.8°	-0.81	+0.83	56.6°	-1.44	+1.68	57.7°	-1.85	+2.51
	54.3°	-0.68	+0.72	56.3°	-1.19	+1.63	58.0°	-1.66	+2.54
120°	119.9°	-0.76	+0.68	120.8°	-1.23	+1.43	122.6°	-1.52	+2.22
	125.2°	-0.74	+0.68	125.0°	-1.08	+1.46	126.0°	-1.33	+2.21
180°	176.7°	-1.07	+0.77	177.5°	-1.94	+1.74	178.1°	-2.55	+2.67
	172.0°	-1.17	+0.97	173.6°	-1.94	+1.94	174.1°	-2.06	+3.00
240°	232.7°	-0.76	+0.58	234.1°	-1.37	+1.41	236.5°	-1.76	+2.16
	246.6°	-0.82	+0.66	245.7°	-1.41	+1.53	244.7°	-1.79	+2.37
300°	306.2°	-0.97	+0.93	301.0°	-1.67	+2.03	300.4°	-2.14	+2.98
	298.3°	-0.78	+0.90	297.4°	-1.46	+2.00	297.7°	-1.99	+2.09

$\theta = \alpha \pm 90^\circ$ for location of Principal Compressive Strain from Gage No. 1.

e_m = Principal Compressive Strain, inches per inch $\times 10^{-4}$.

e_M = Principal Tensile Strain, inches per inch $\times 10^{-4}$.

Table 7. Two-bolt Unsymmetrical Single Shear Test.

Applied Load	Bolt 9						Bolt 10					
	Strain	<u>Indicated</u>		<u>Calculated</u>			Strain	<u>Indicated</u>		<u>Calculated</u>		
		Load	Angle	Load	Angle			Load	Angle	Load	Angle	
1000	-5.98	2900	-26.5°	1000	0°		-7.14	2700	0.4°	2000	0°	
2000	-9.59	4550	-17.9°	2000	0°		-14.48	6500	-3.0°	4000	0°	

Table 8. Two-bolt Unsymmetrical Double Shear Test.

Applied Load	Bolt 10						Bolt 11					
	Strain	<u>Indicated</u>		<u>Calculated</u>			Strain	<u>Indicated</u>		<u>Calculated</u>		
		Load	Angle	Load	Angle			Load	Angle	Load	Angle	
1000	+0.46	350	-69°	794	-27°		+1.14	1450	5°	1459	14°	
2000	+0.35	300	-24°	1588	-27°		+2.76	3150	-1°	2918	14°	
3000	+1.50	1200	-23°	2382	-27°		+4.84	4300	0°	4377	14°	

(All Unit Strains in Tables 7 and 8 are in Inches per x 10^{-4} .)

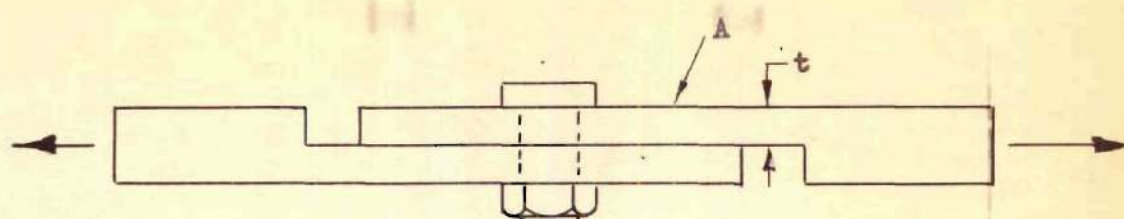


Figure 1. Bolt in Single Shear

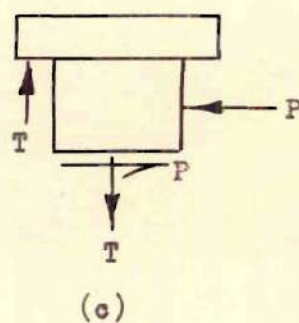
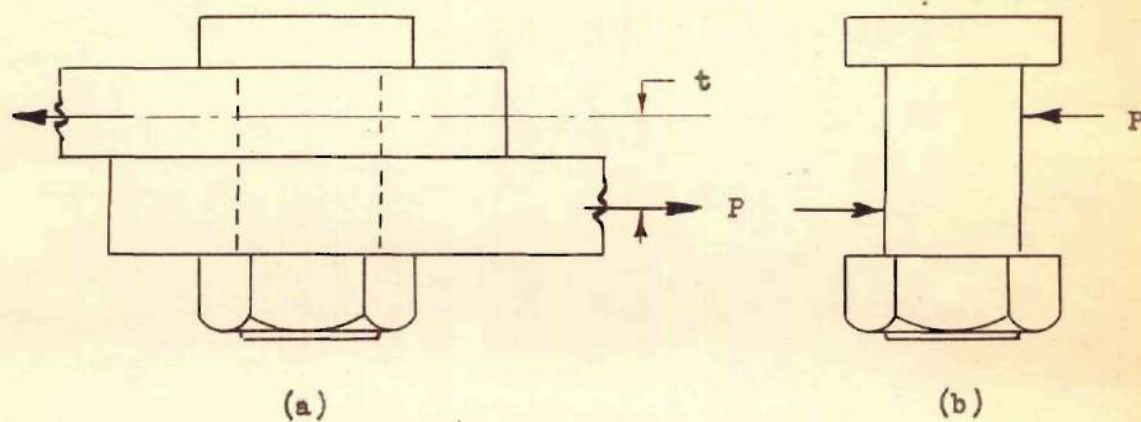
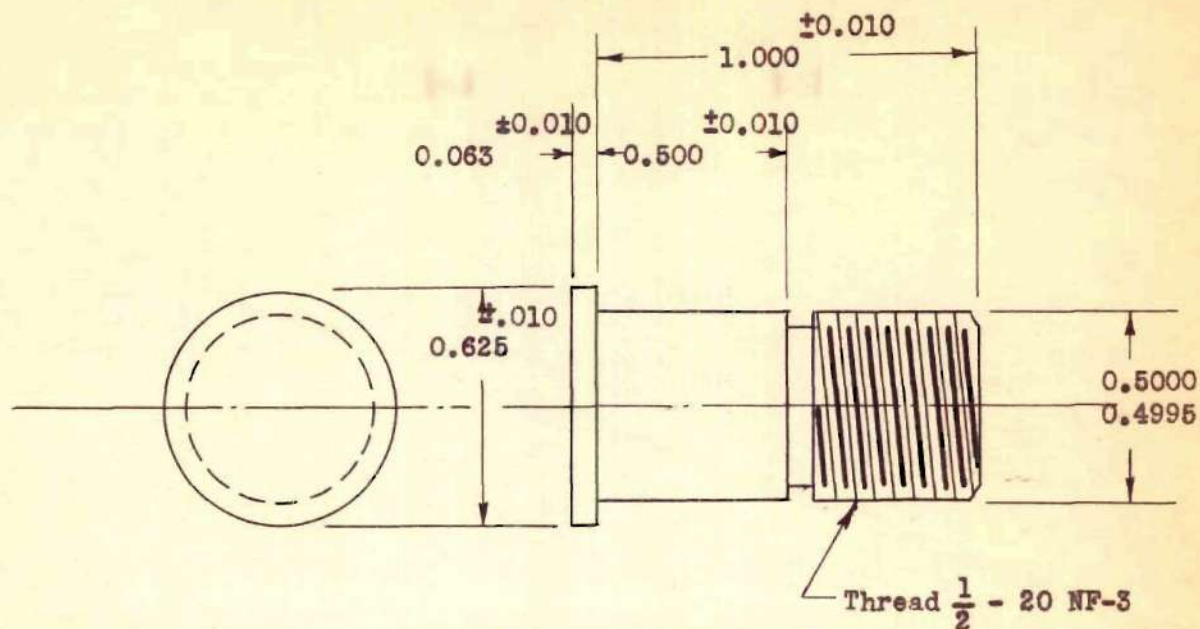
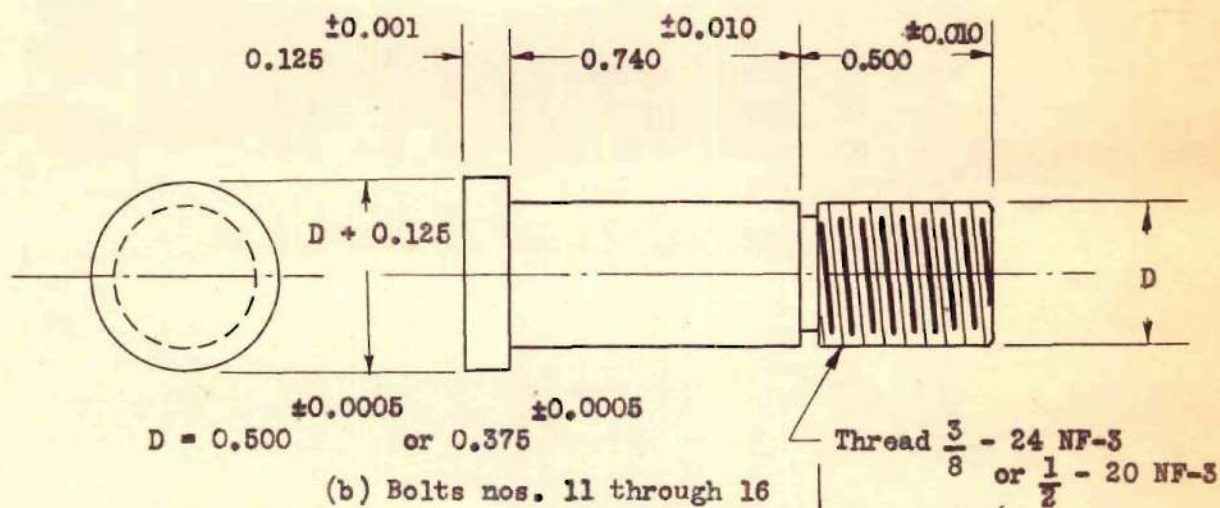


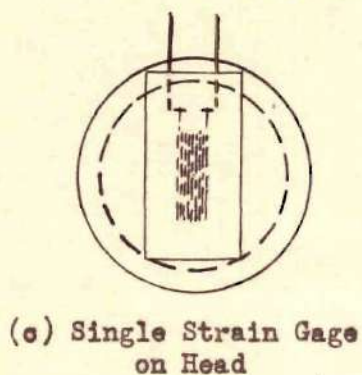
Figure 2. Loads on Bolt



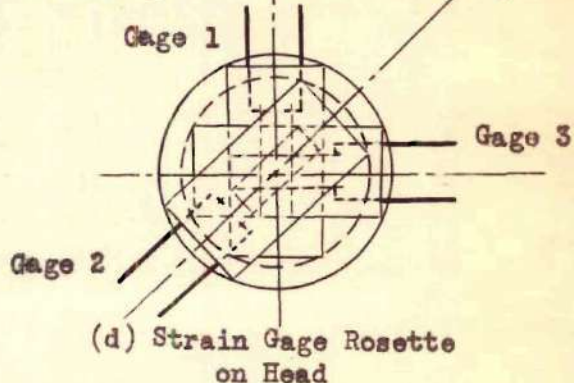
(a) Bolts nos. 1 through 10



(b) Bolts nos. 11 through 16

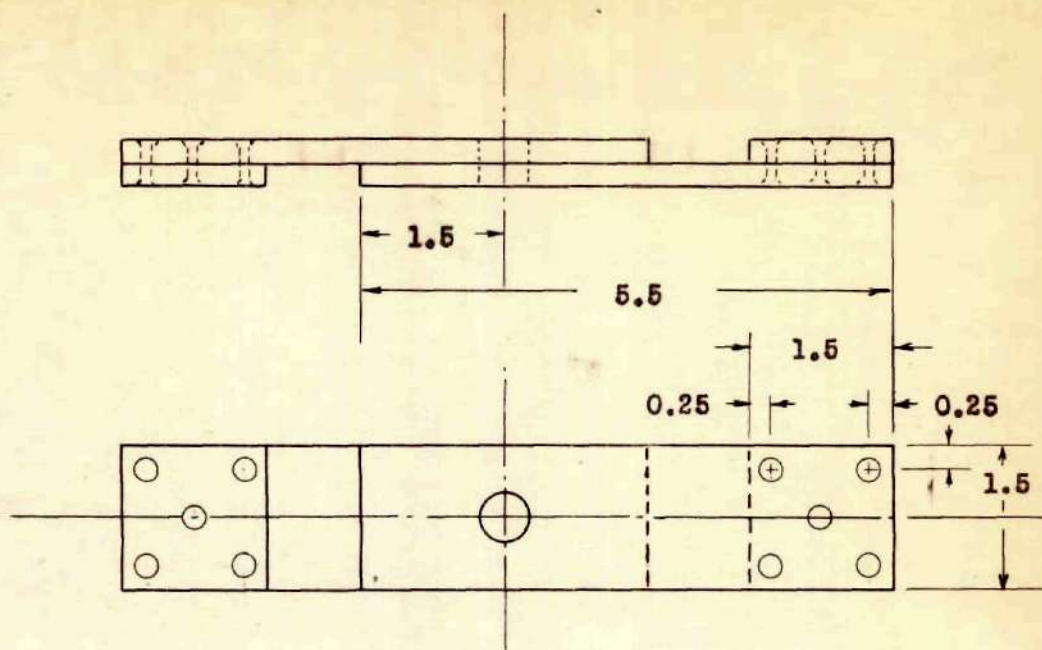


(c) Single Strain Gage on Head

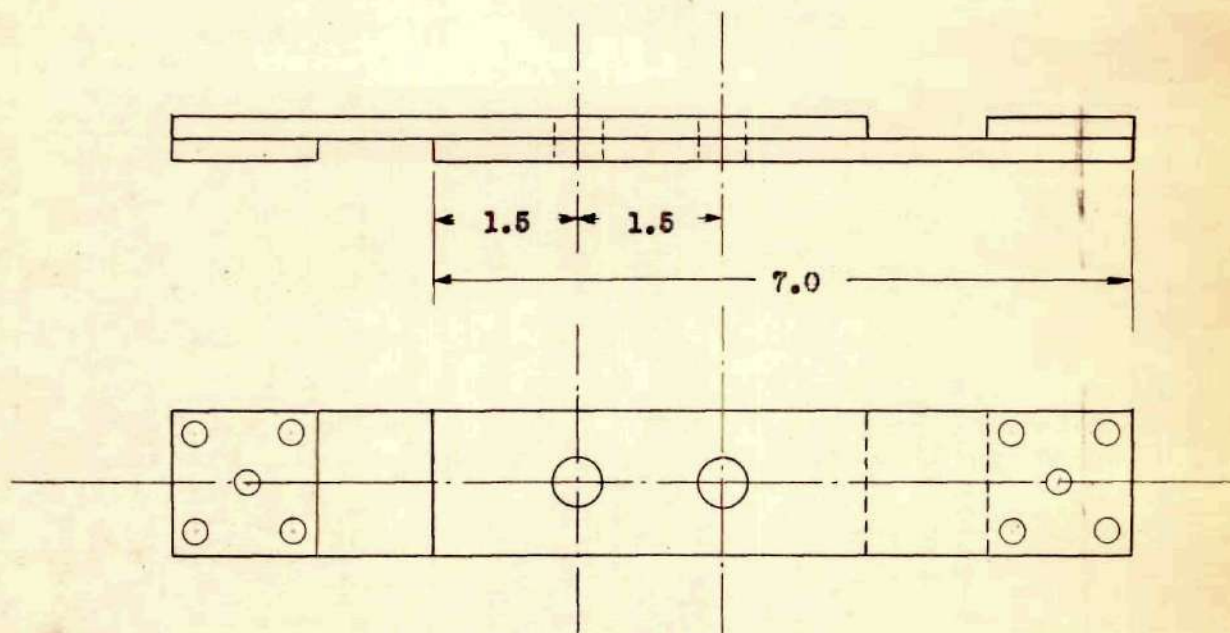


(d) Strain Gage Rosette on Head

Figure 3. Detail Drawings of Bolts



(a)



(b)

Figure 4. Single Shear Test Plates

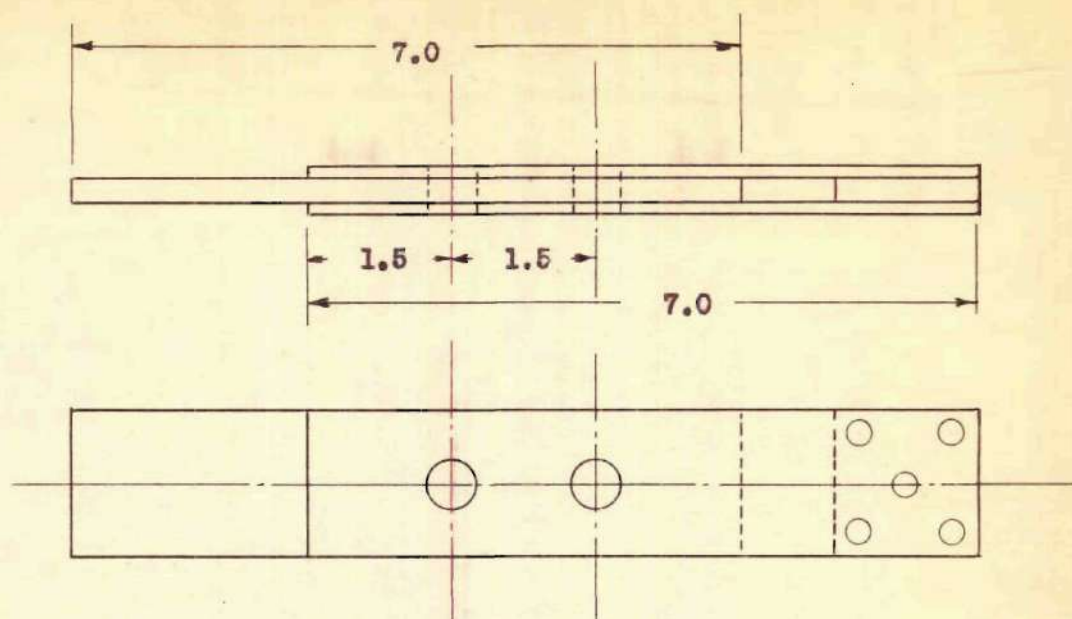


Figure 5. Double Shear Test Plates

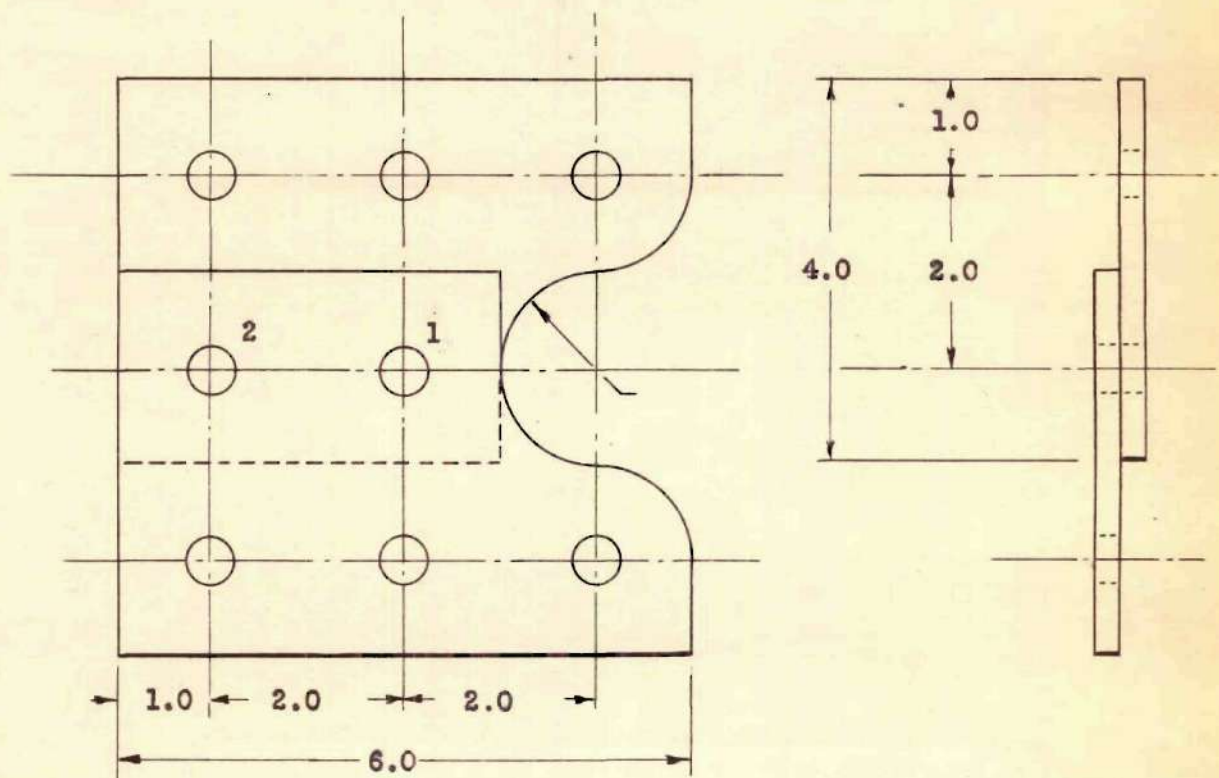


Figure 6. Eccentric Loading Plates

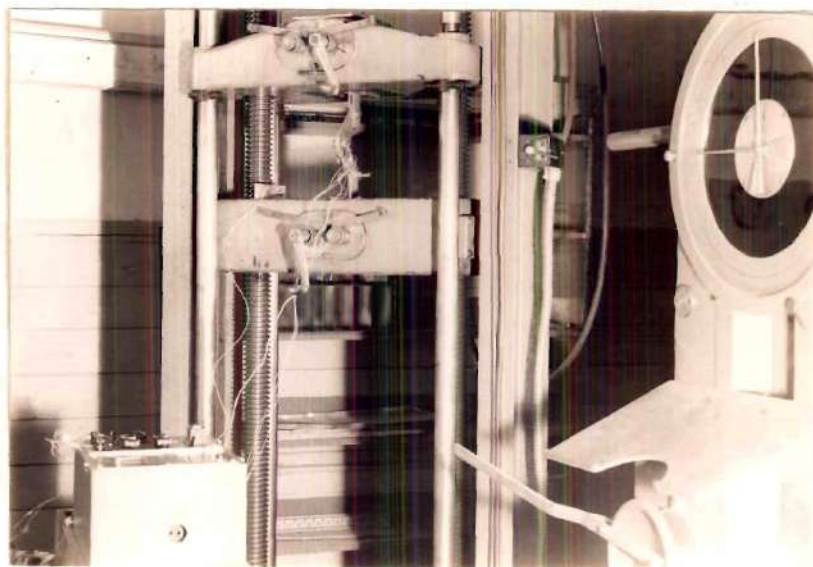


Figure 7. Equipment and Arrangement for Test.

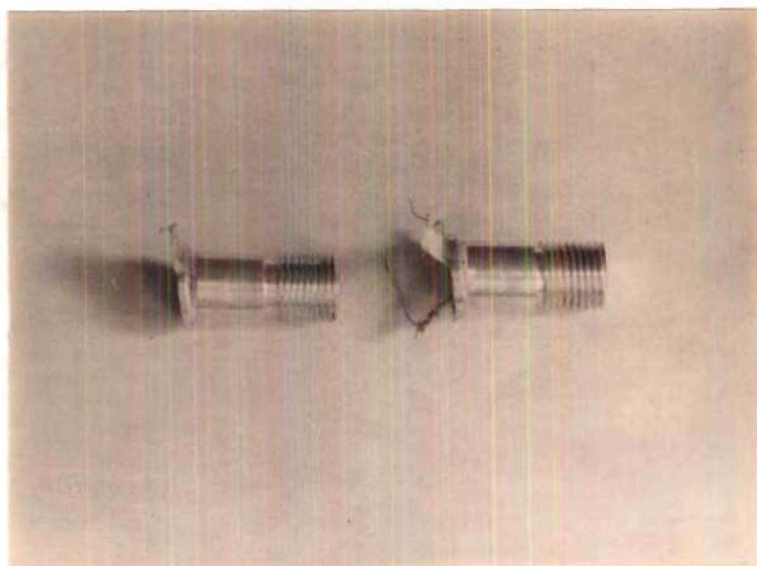


Figure 8. Bolt Failure by Shearing of Head Rims.

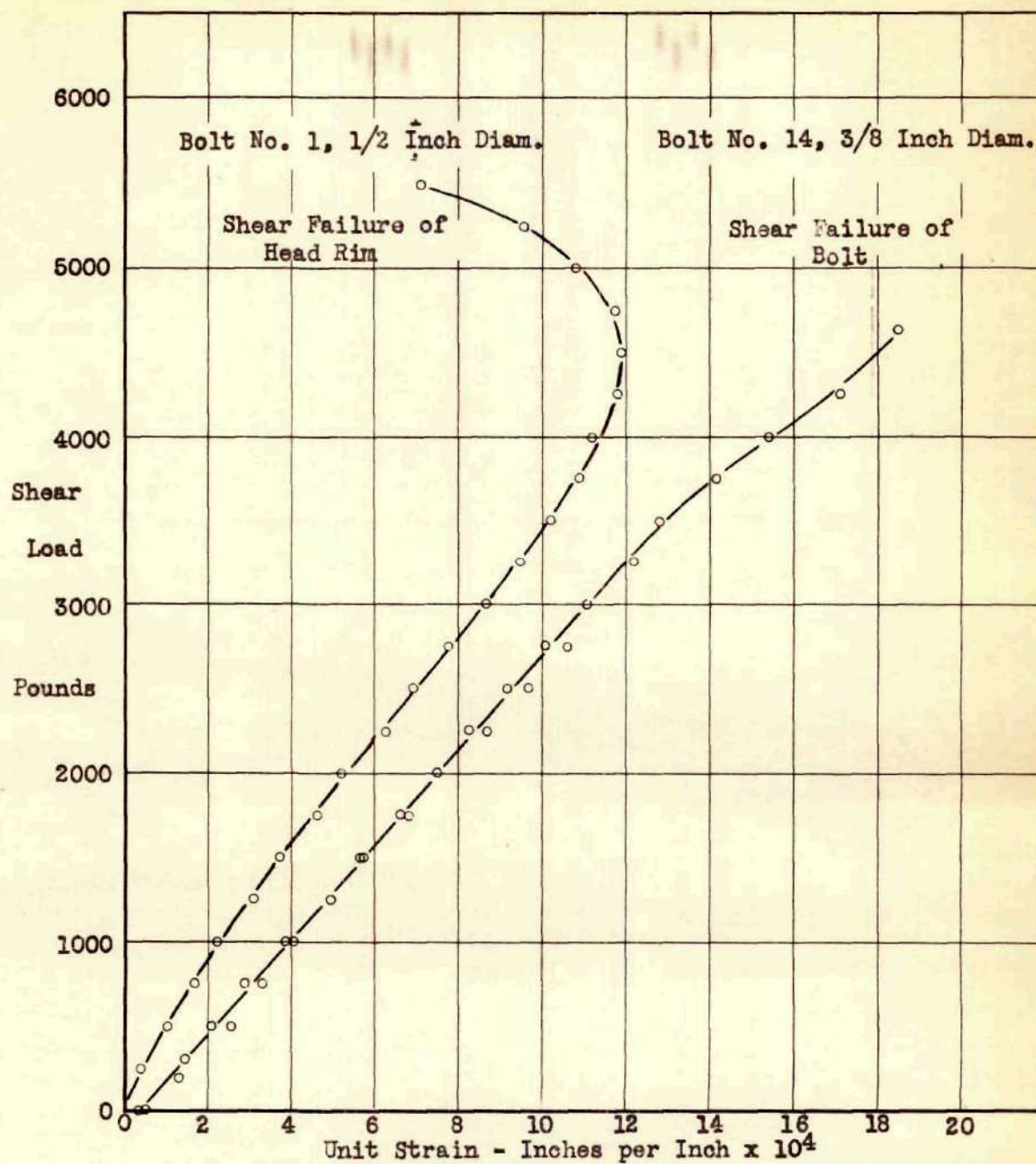
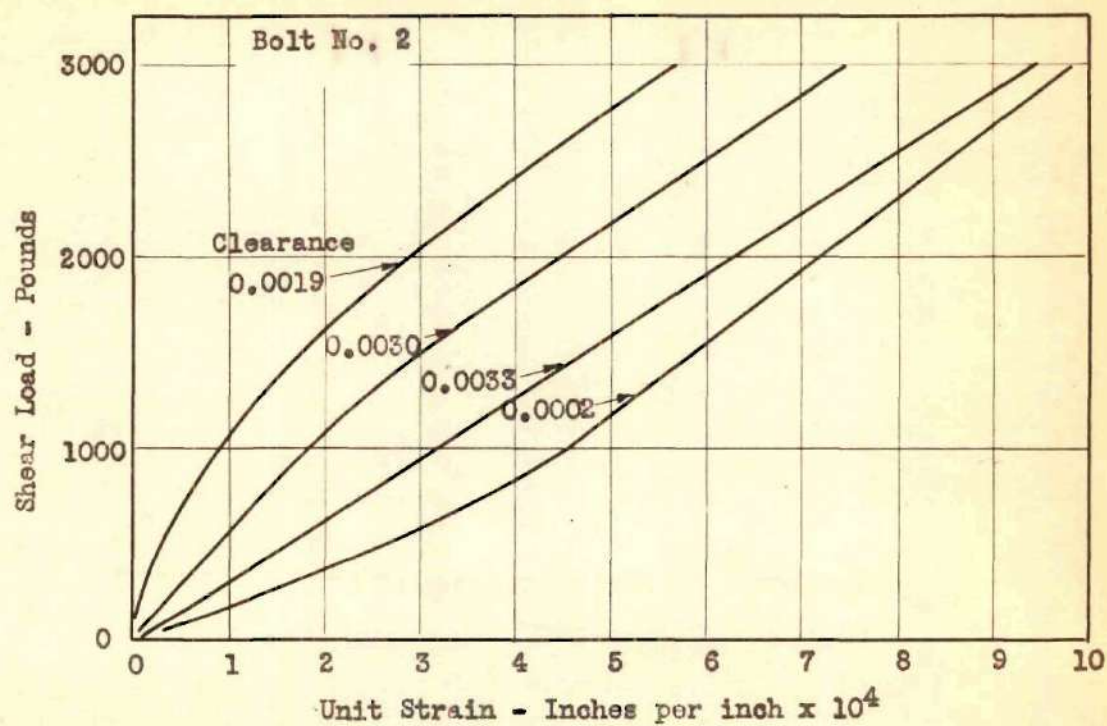
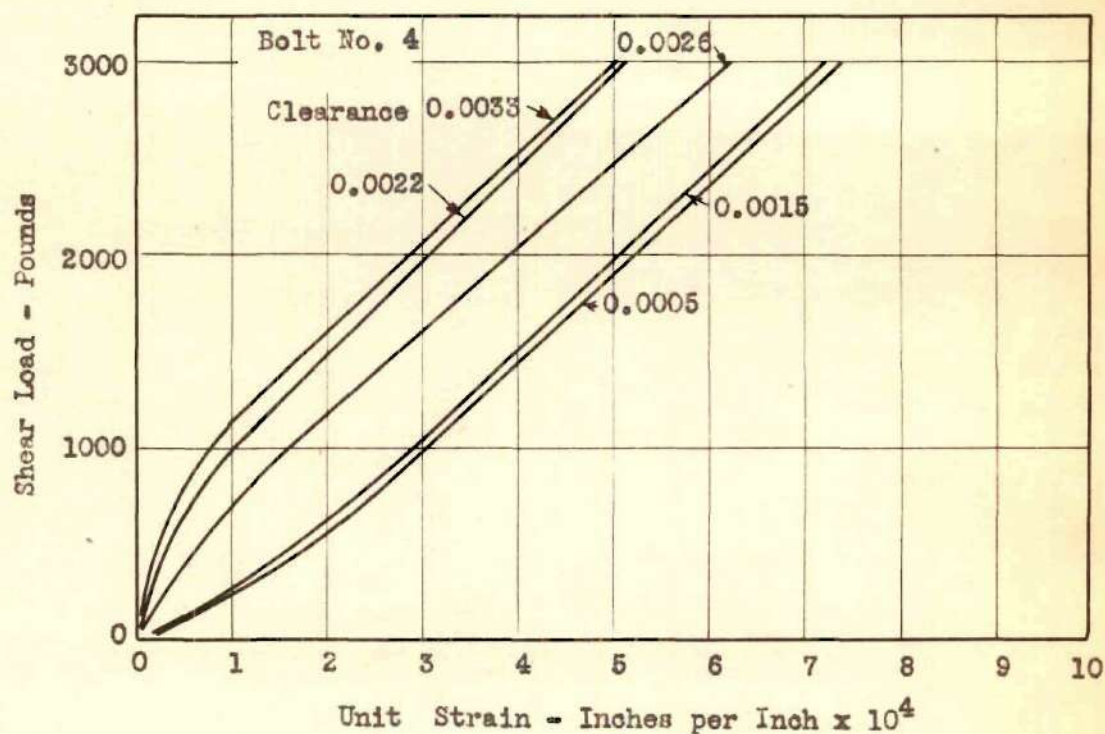


Figure 9. Variations of Head Unit Strain with Shear Load



(a)



(b)

Figure 10. Effect of Hole Fit

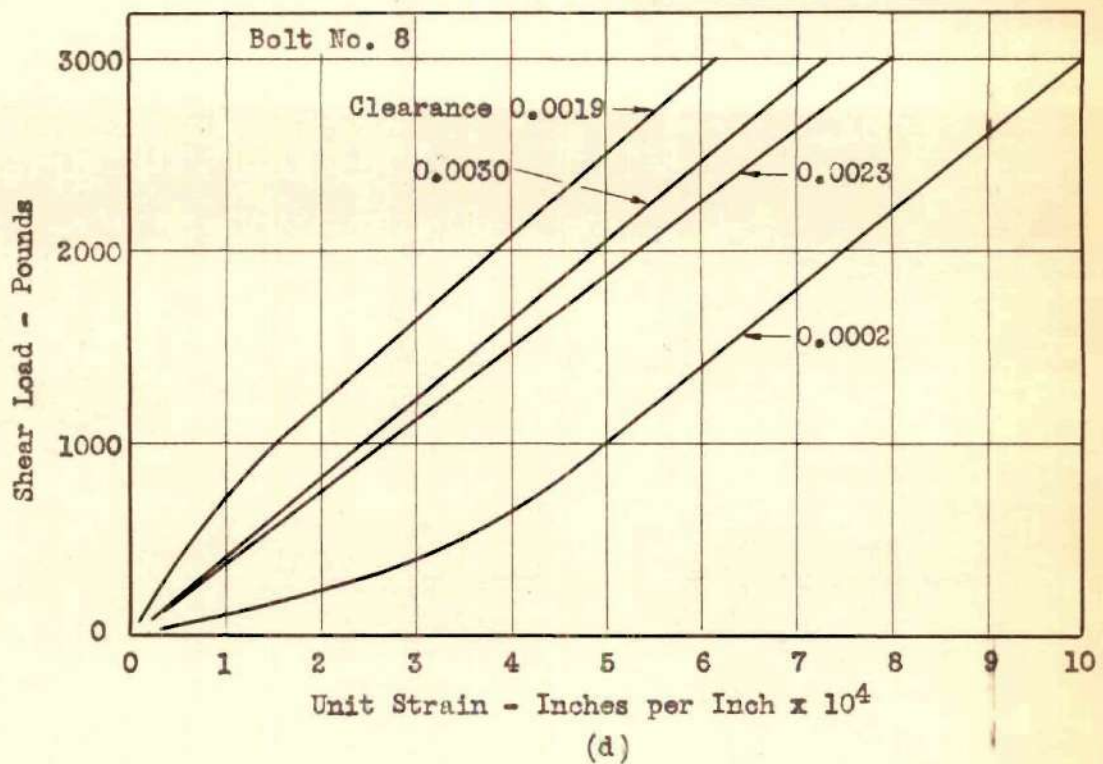
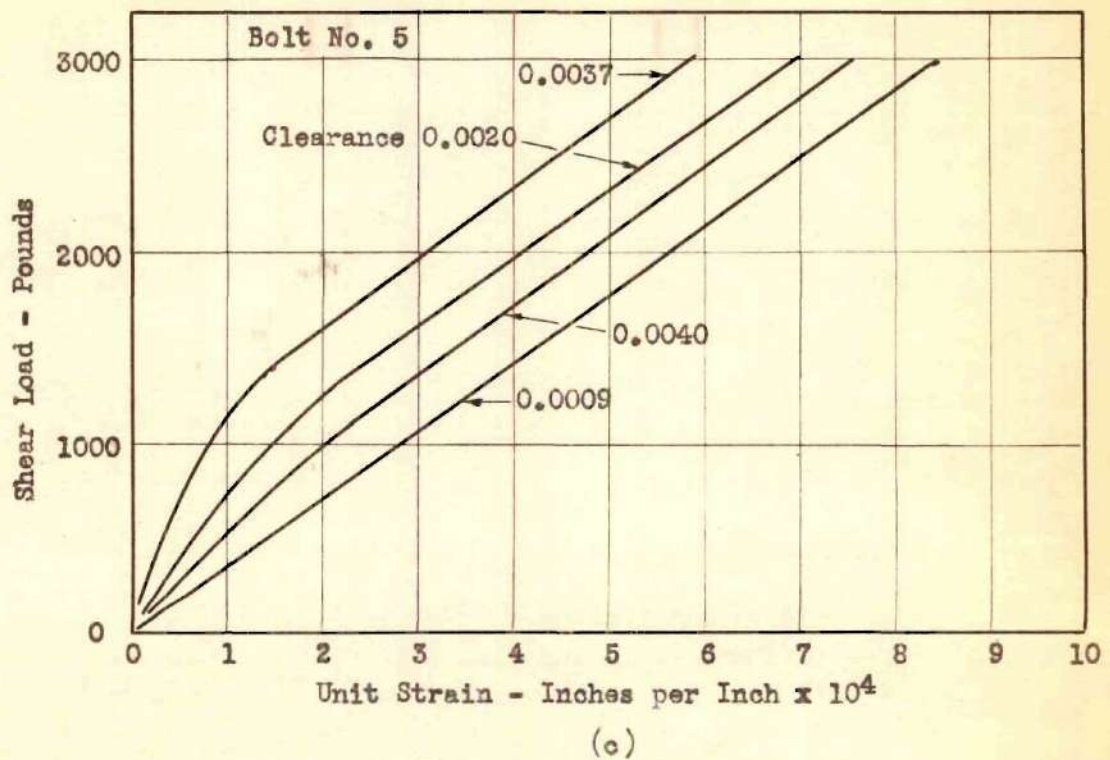


Figure 10. Effect of Hole Fit

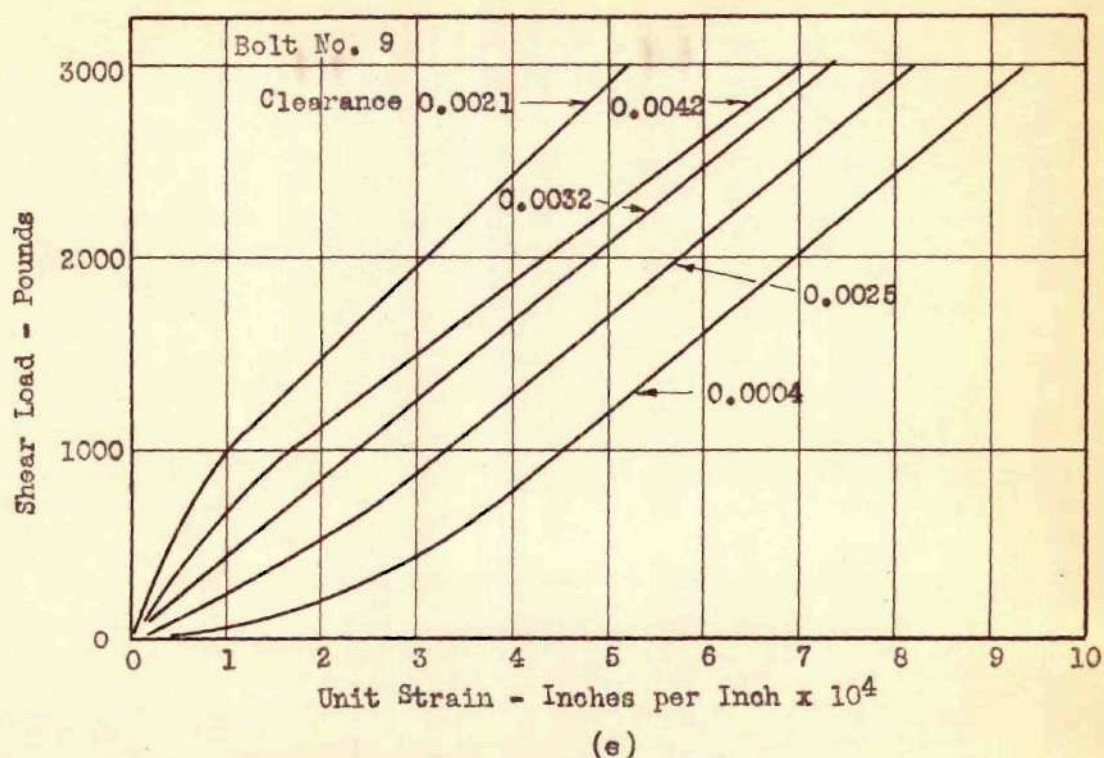


Figure 10. Effect of Hole Fit

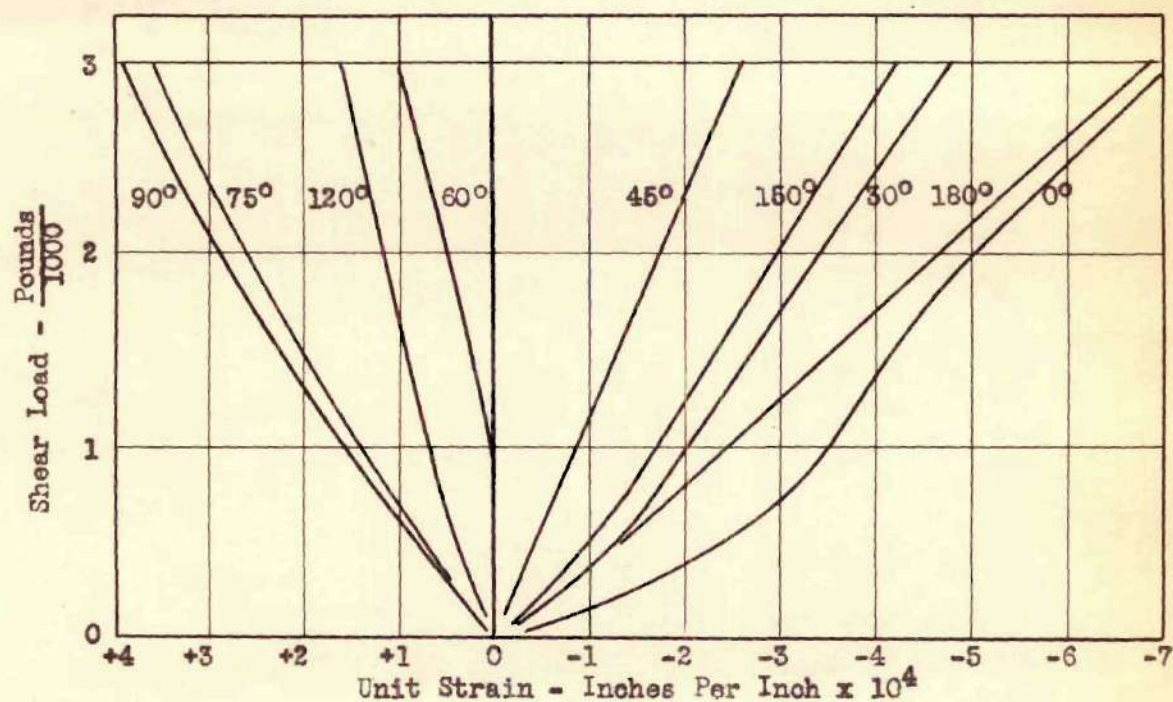


Figure 11. Effect of Angle of Strain Gage to Line of Load

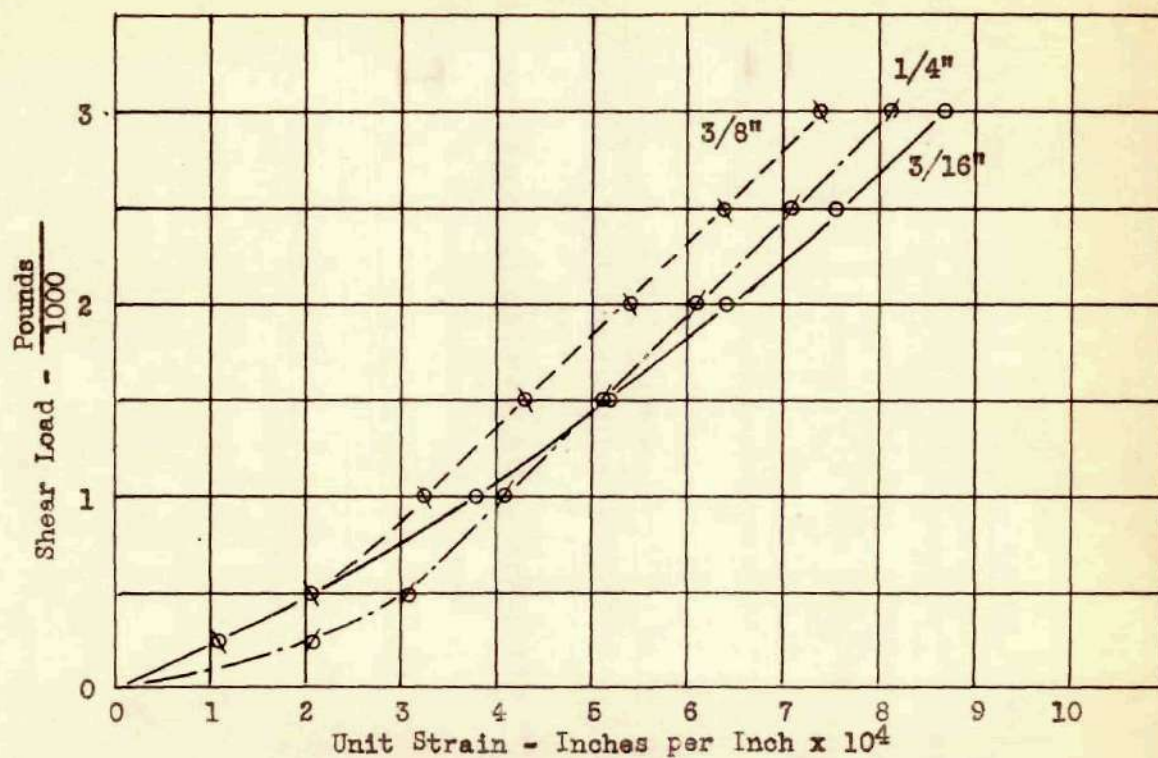


Figure 12. Effect of Plate Thickness

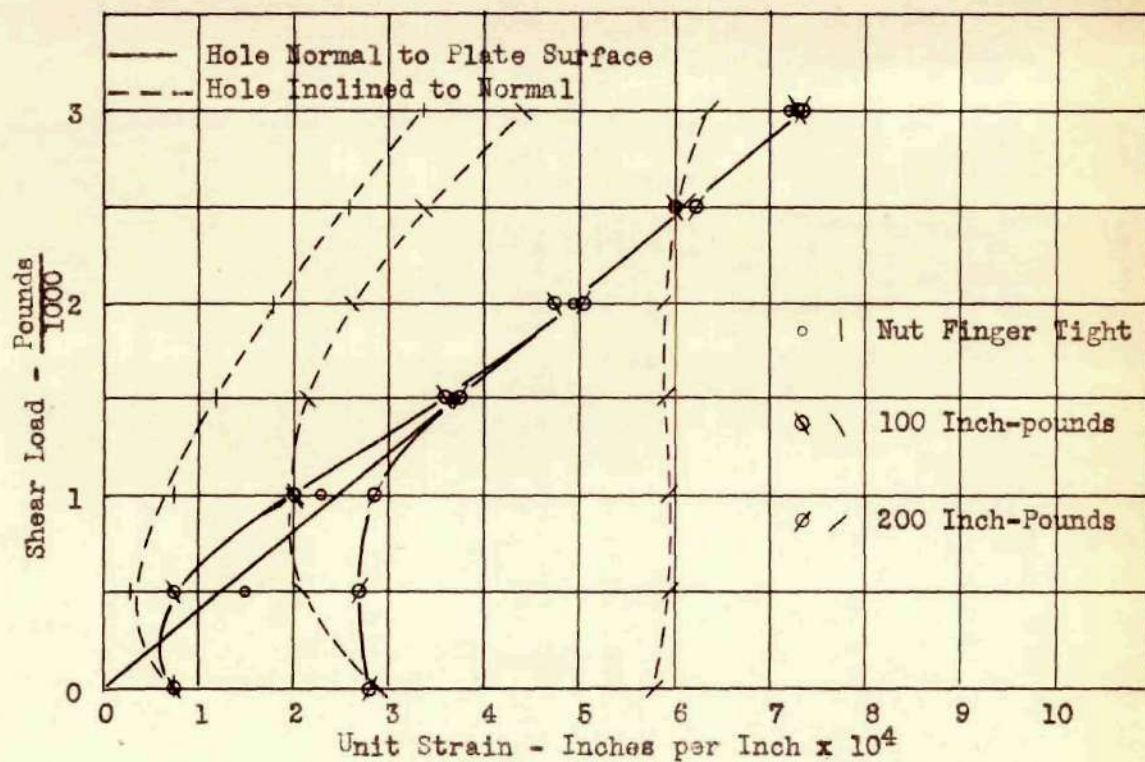


Figure 13. Effect of Hole Inclination and Nut Tightness

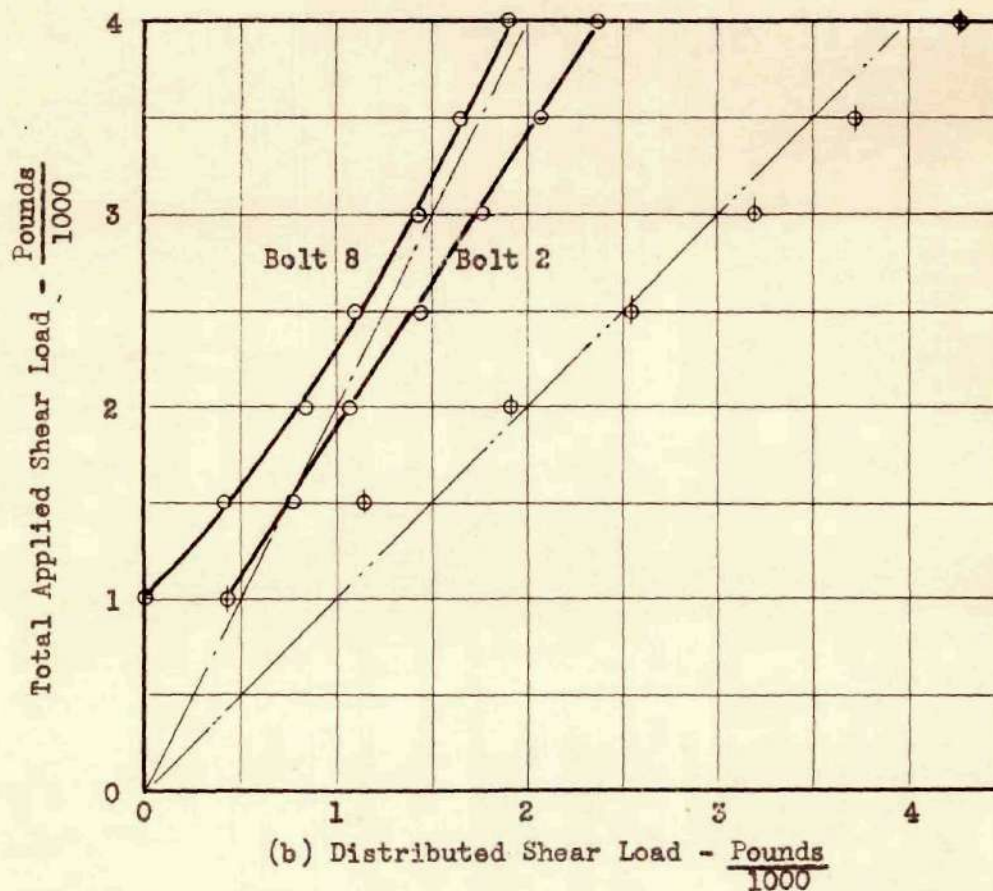
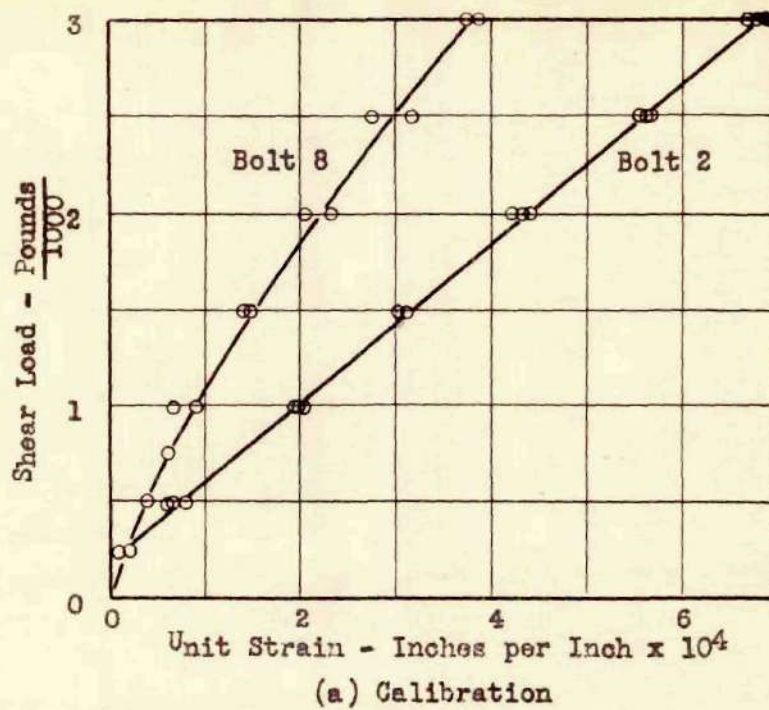


Figure 14-1. Two Bolts in Single Shear. Calibration and Distribution.

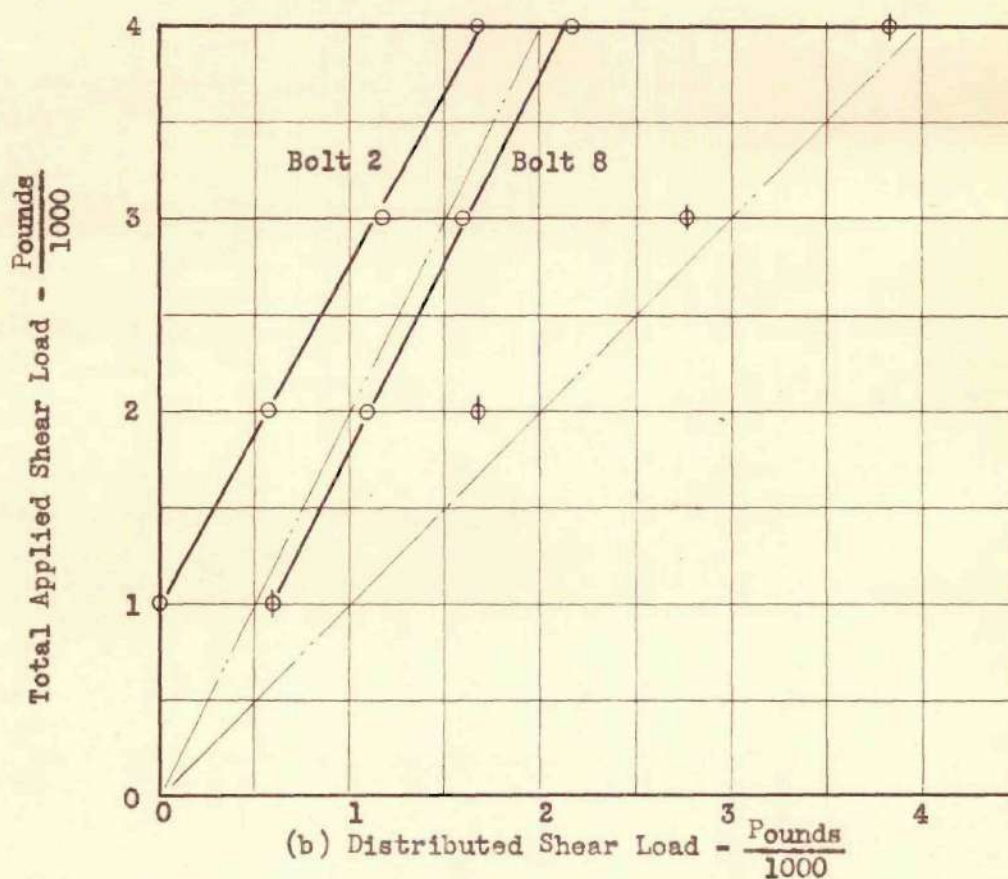
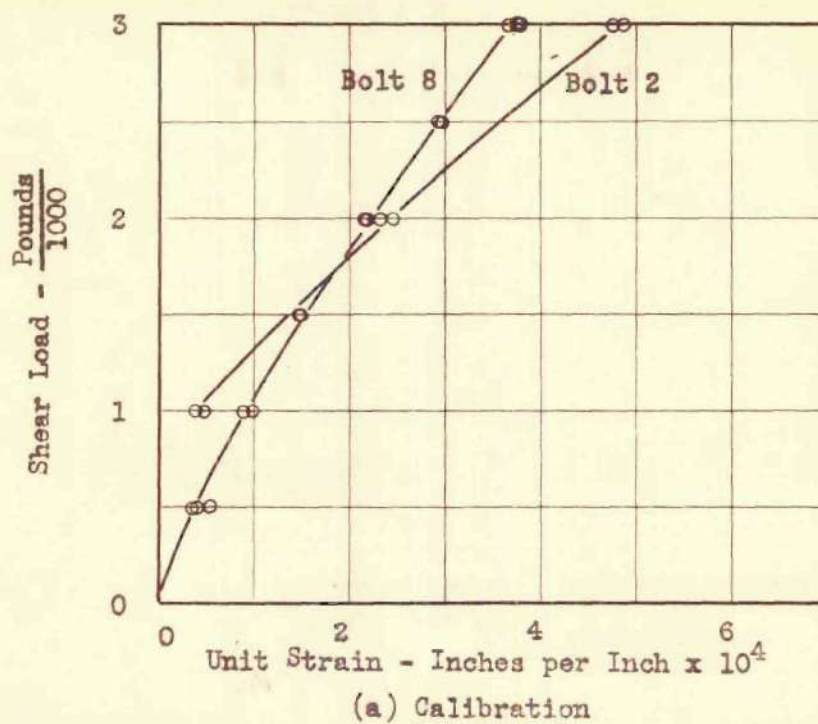


Figure 14-2. Two Bolts in Single Shear. Calibration and Distribution.

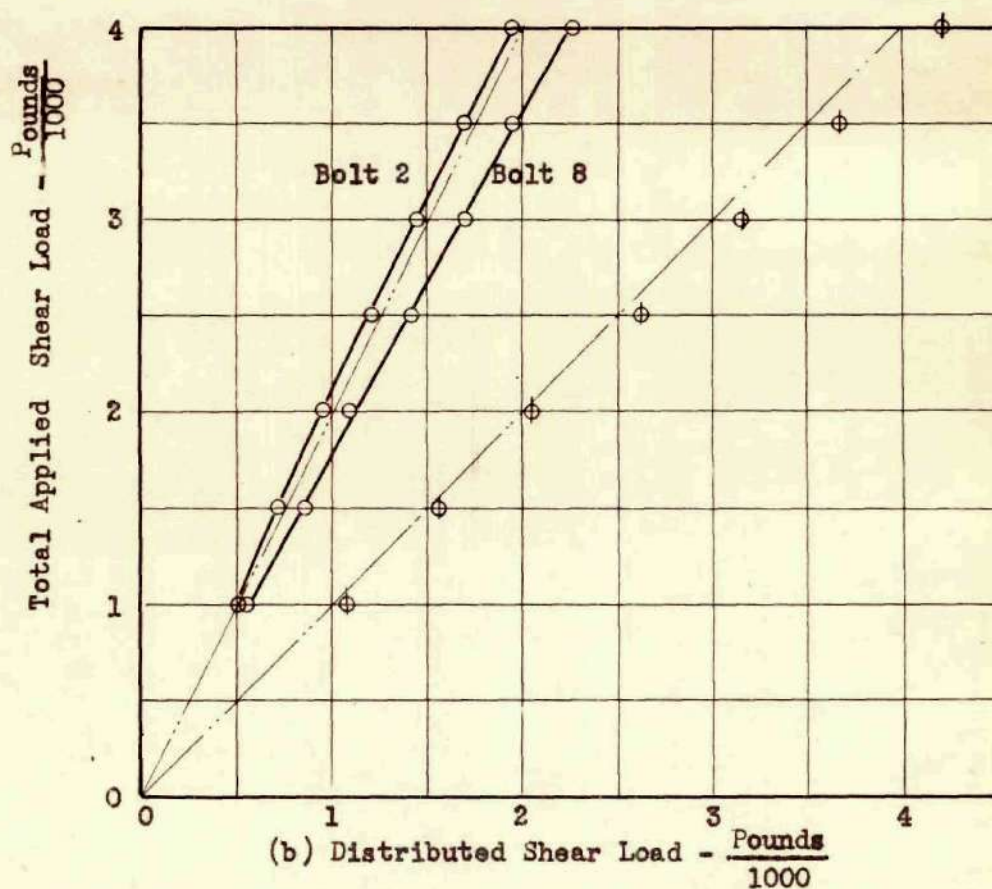
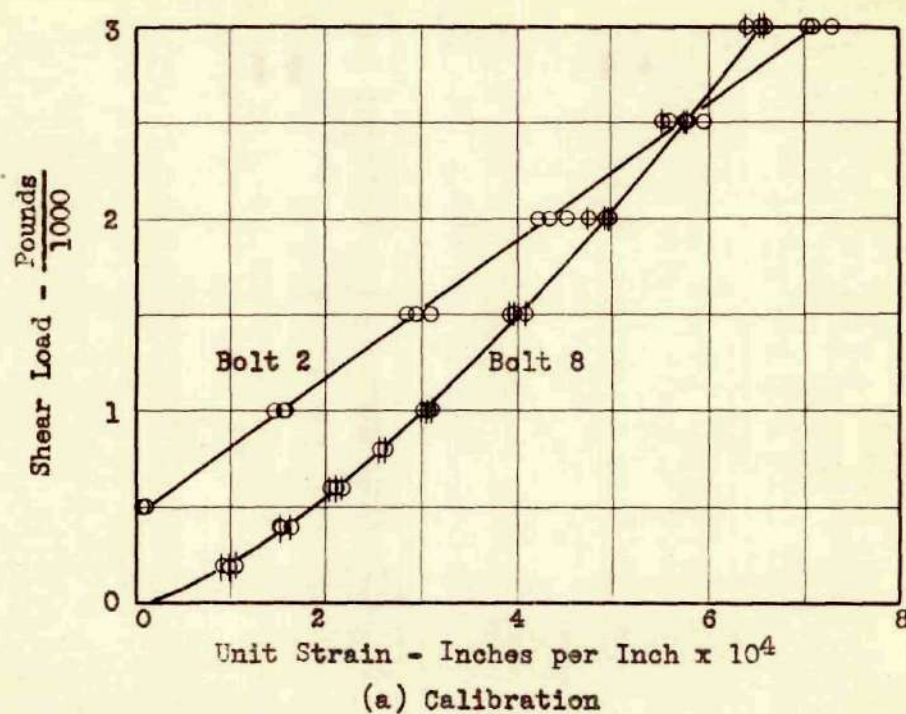


Figure 14-3. Two Bolts in Single Shear. Calibration and Distribution.

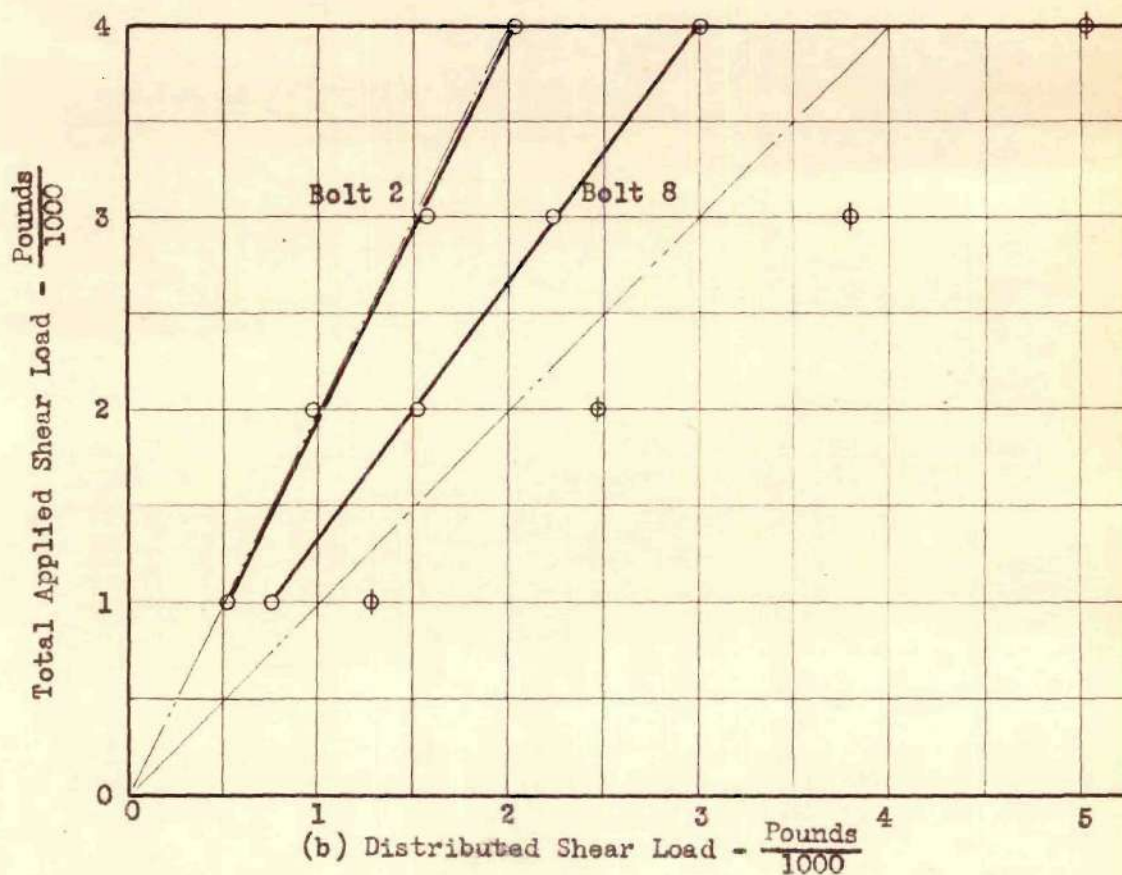
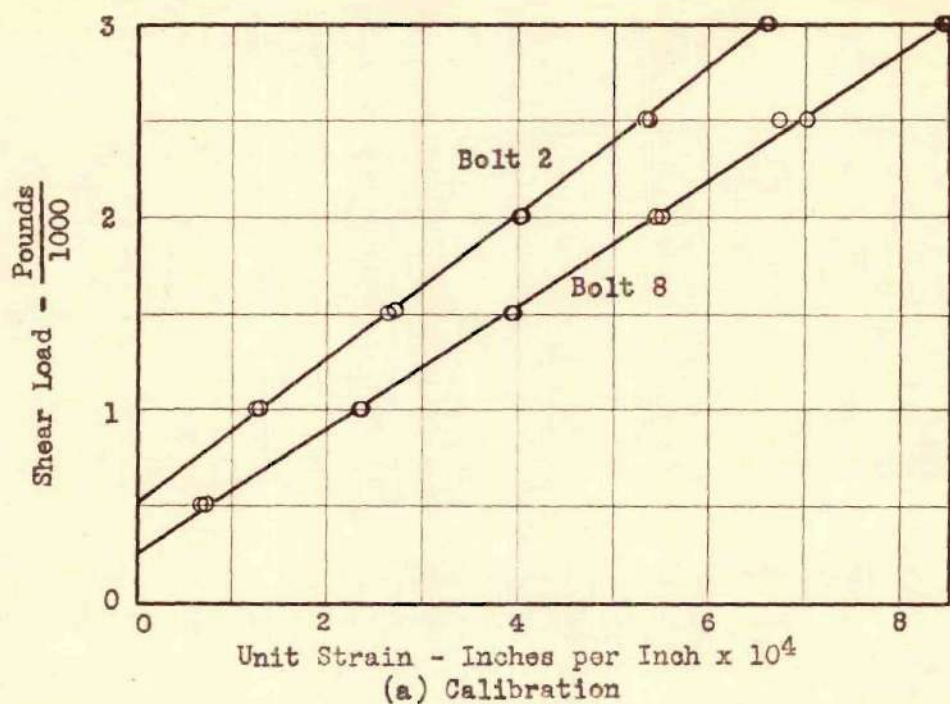


Figure 14-4. Two Bolts in Single Shear. Calibration and Distribution.

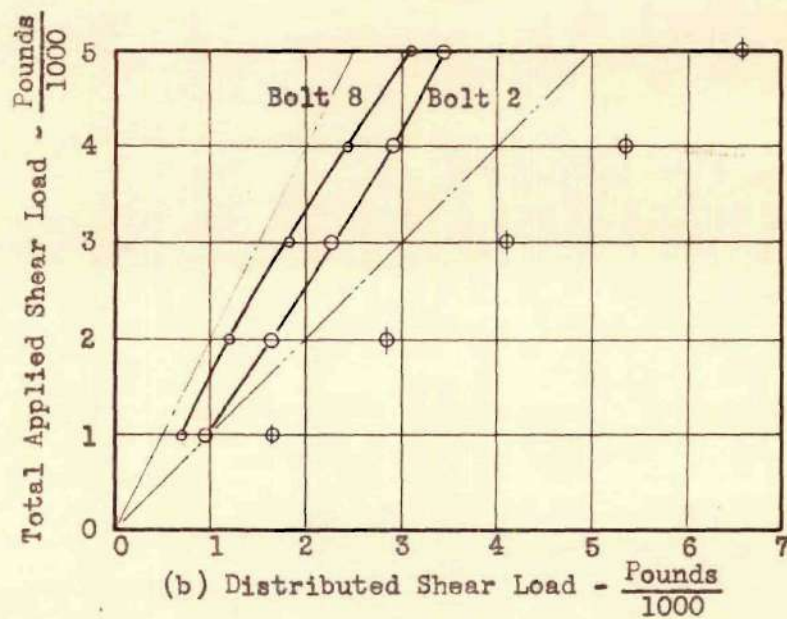
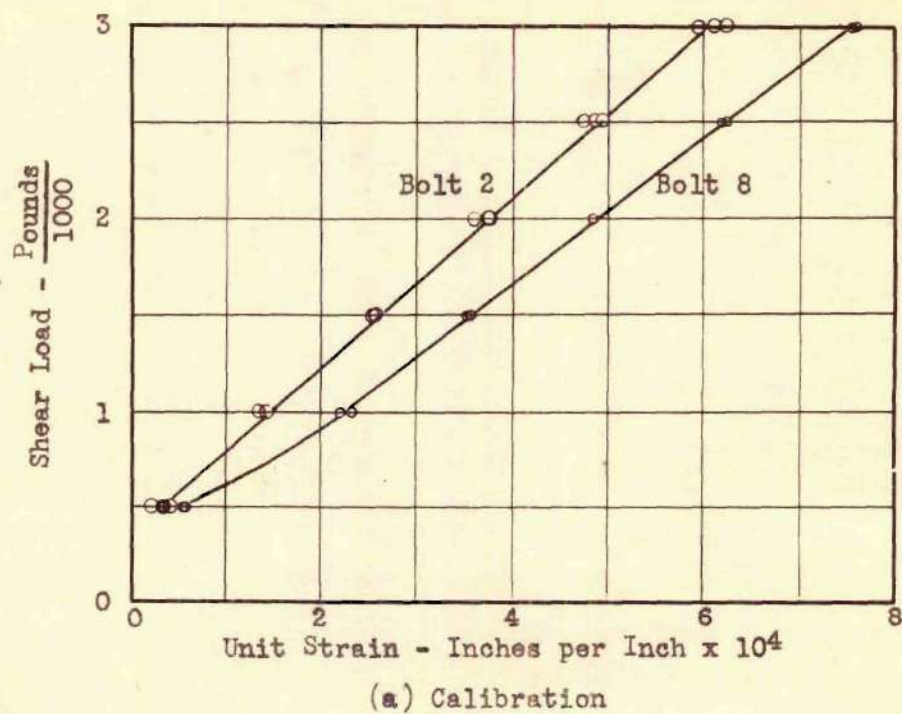


Figure 14-5. Two Bolts in Single Shear. Calibration and Distribution.

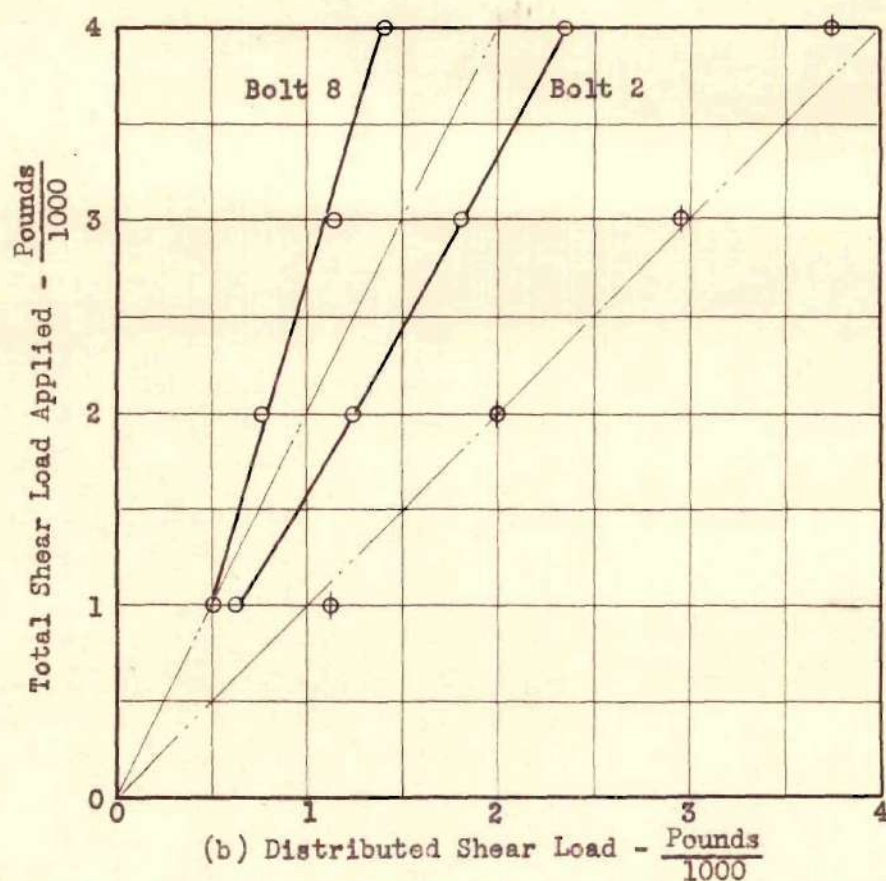
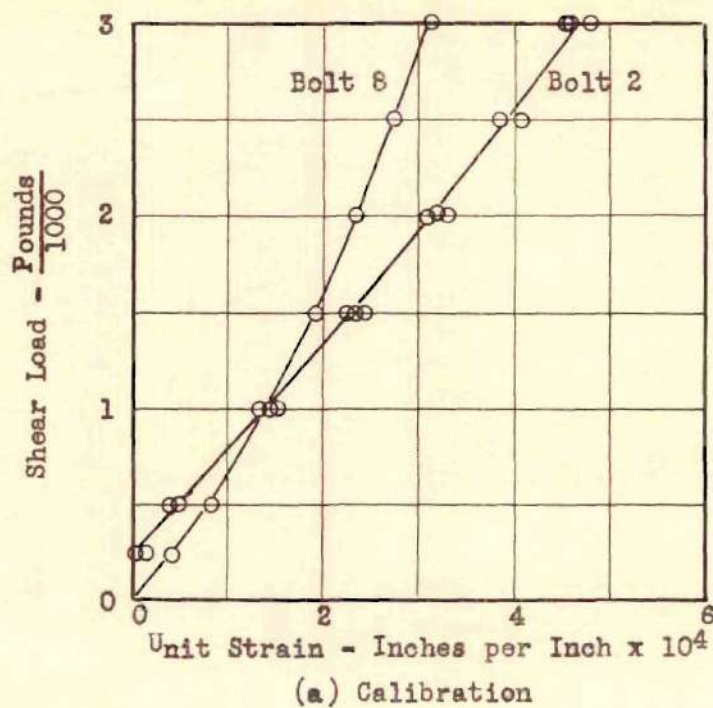


Figure 15-1. Two Bolts in Double Shear. Calibration and Distribution.

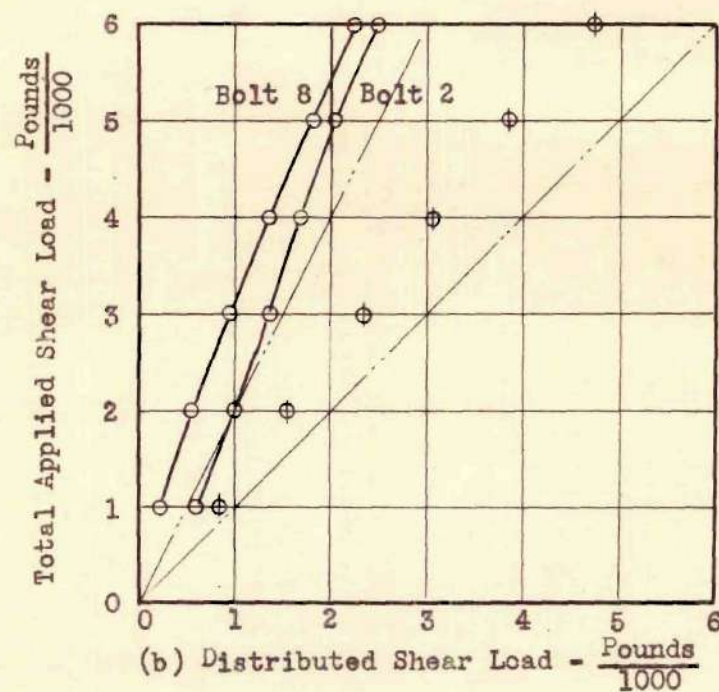
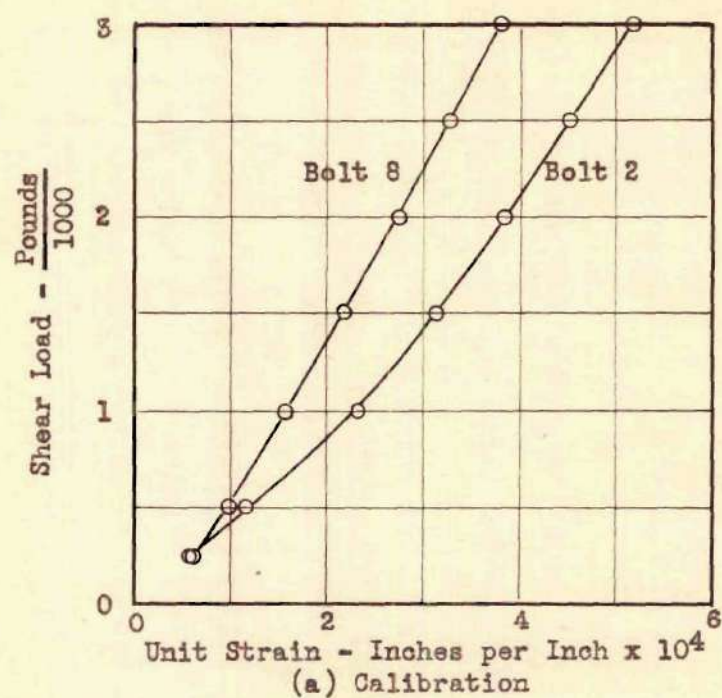


Figure 15-2. Two Bolts in Double Shear. Calibration and Distribution.

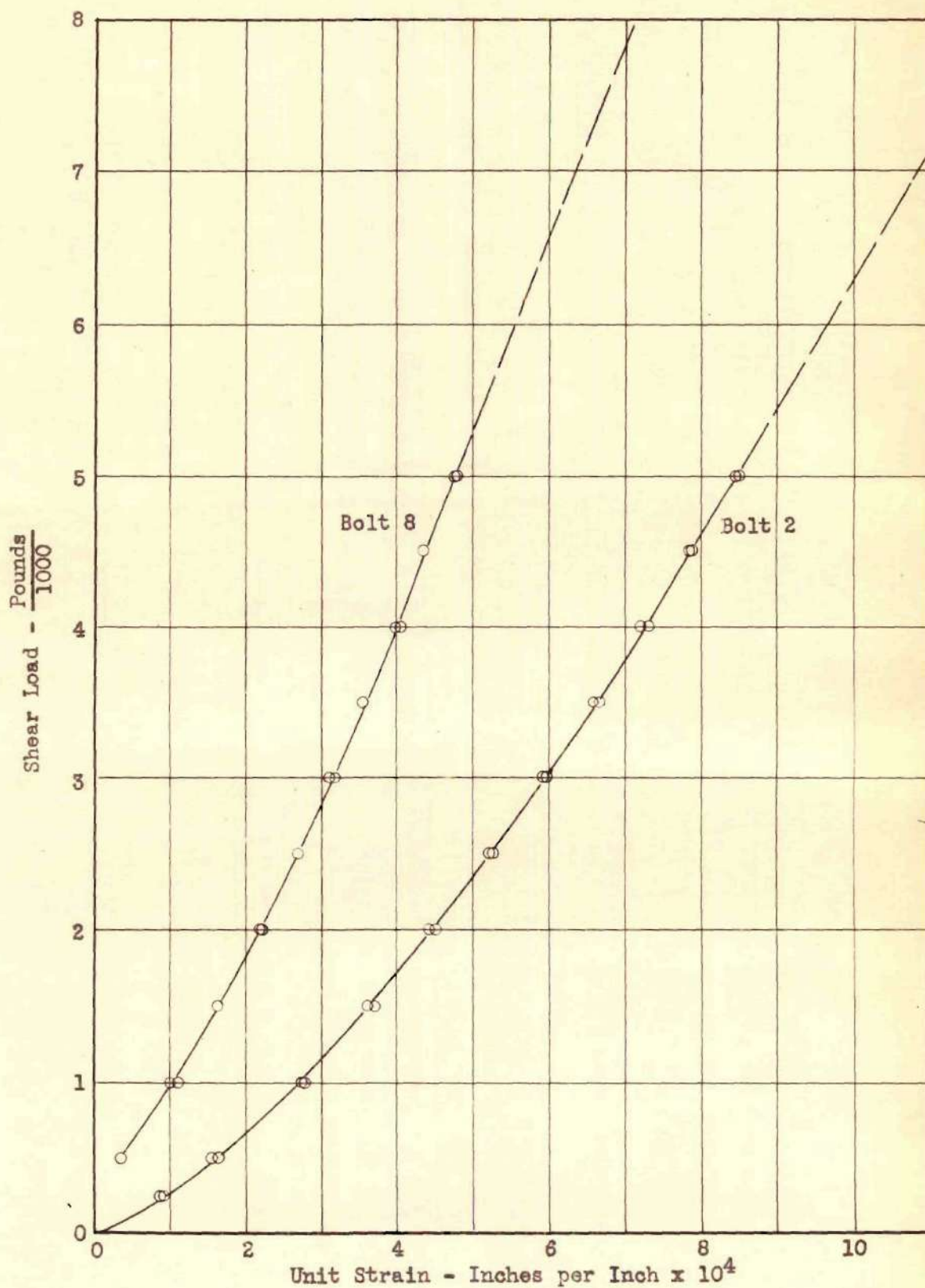
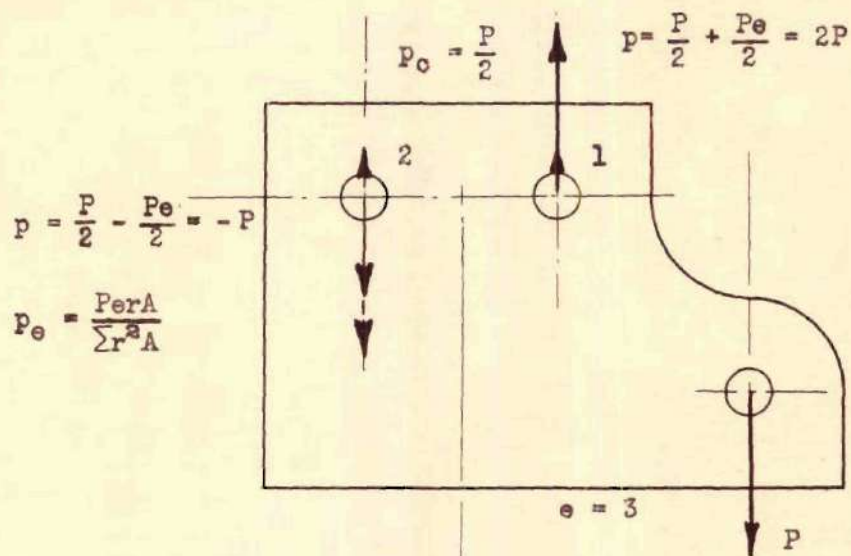
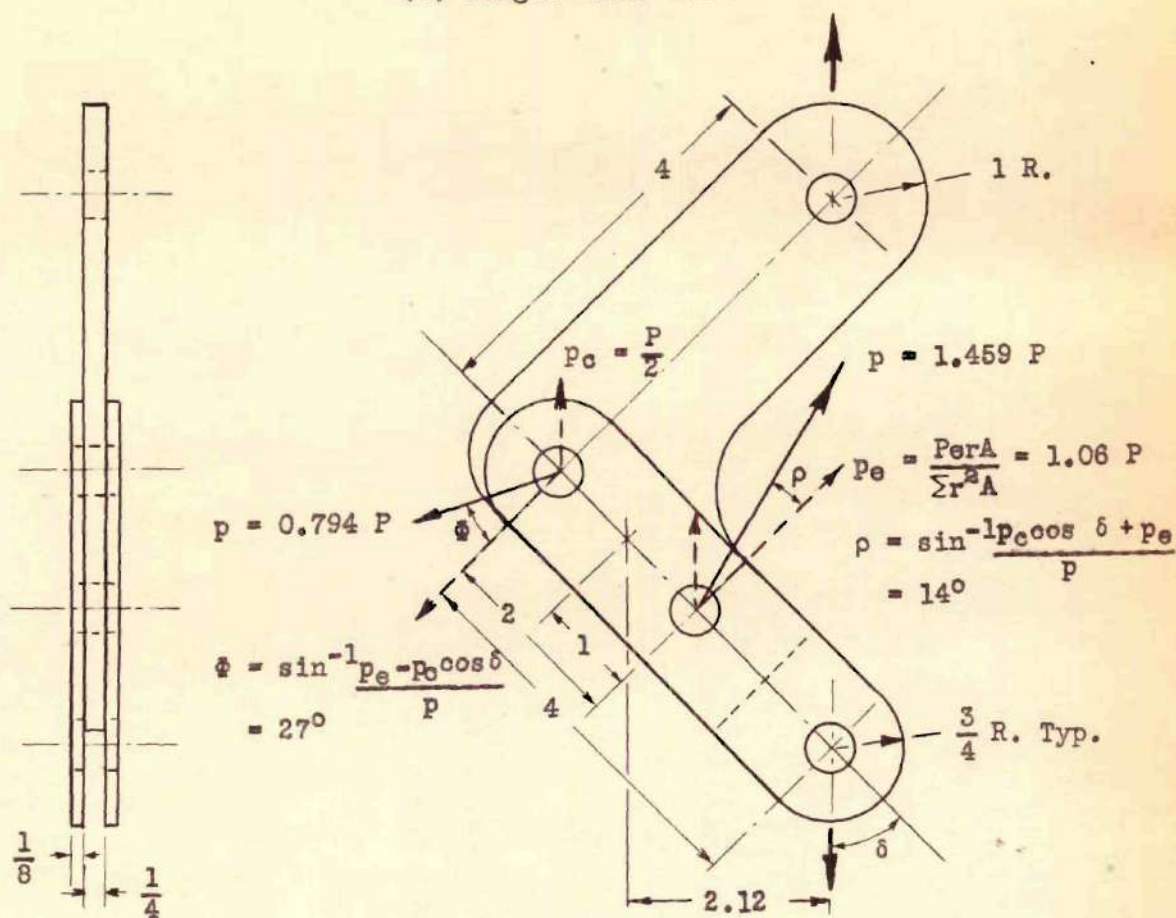


Figure 16. Extrapolated Calibration Curves for Bolts in Double Shear.

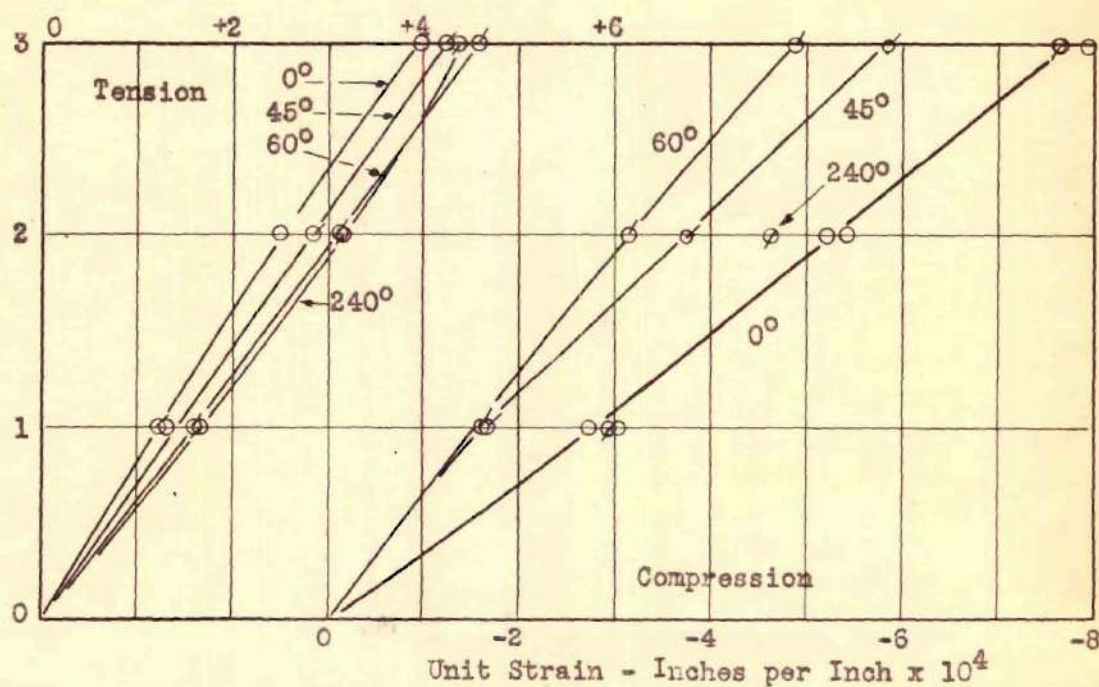


(a) Single Shear Test

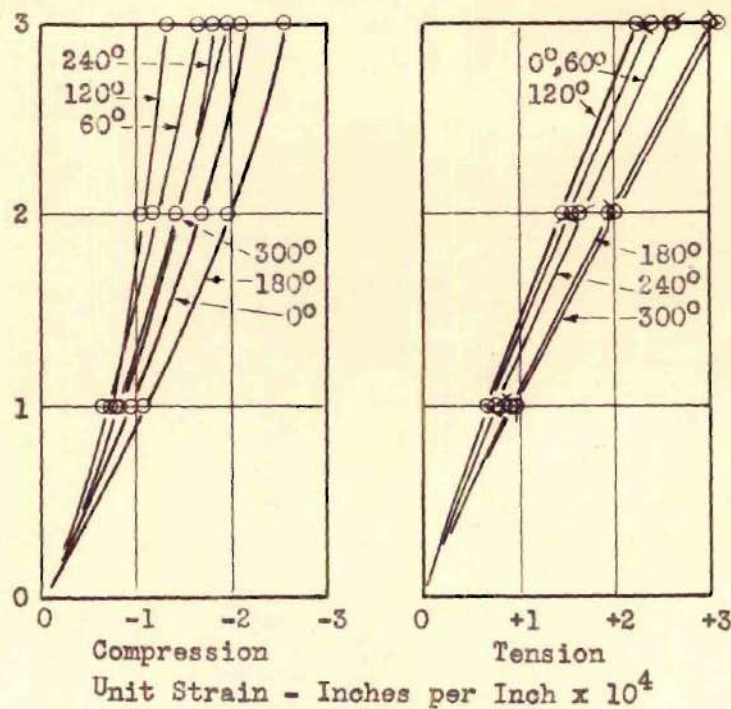


(b) Double Shear Test

Figure 17. Theoretical Loads on Eccentrically Loaded Bolts.



(a) Bolt 6, Single Shear.



(b) Bolt 11, Double Shear

Figure 18. Effect of Load Angle on Principal Head Strains.

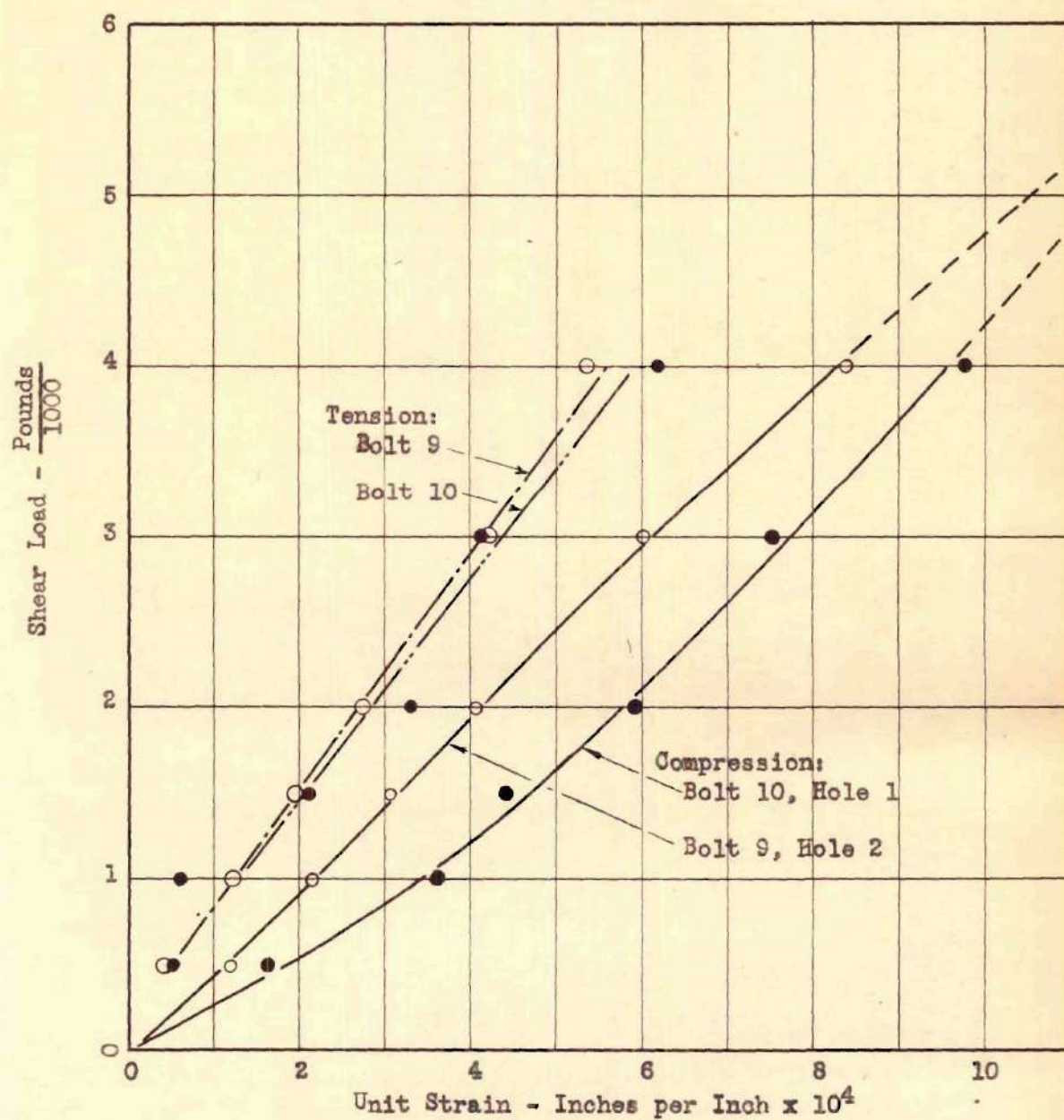


Figure 19. Calibration of Bolts for Single Shear Eccentric Load Tests