A STUDY OF HEAT TRANSFER INVOLVING A

GAS WITH SUSPENDED PARTICLES

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A THESIS

Presented to

the Faculty of the Division of Graduate Studies

Georgia Institute of Techology

In Partial Fulfillment

of the Requirement for the Degree

Master of Science in Mechanical Engineering

by

Chia-San Chou

June 1949

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GAS WITH SUSPENDED PARTICLES

Approved:

Date Approved by Chairman June 2, 1949

ACKNOWLEDGMENTS

This problem was suggested by Professor M. J. Goglia. He assigned many valuable literature references and offered me continual, kind and helpful advice.

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A-------Cross-sectional area, square feet.

$A_p$------Projected area of particle, square feet.

a-------Acceleration, ft/sec.$^2$

c-------Drag coefficient.

$c_p$------Specific heat at constant pressure, B.T.U./lb-°F

$c_v$------Specific heat at constant volume, B.T.U./lb-°F

D-------Diameter, feet.

F-------Friction force of the pipe surface, pounds.

f-------Friction coefficient, defined as shearing stress on the wall of pipe is equal to $\frac{1}{2} f \rho V^2$.

g-------Gravitational acceleration, taken as the standard value, 32.2 ft./sec.$^2$

$g_o$------Dimensional constant, $\frac{lb_m\text{-}ft.}{lb_f\text{-}sec^2}$

H-------Enthalpy, B.T.U./lb.

$h_p$------Overall coefficient of heat transfer of the particles, B.T.U./hr.-ft.$^2$-°F.

$h_{cp}$-------Film coefficient of convection from particle to gas, B.T.U./hr.-ft.$^2$-°F.

$h_r$-------Radiation coefficient from particle to pipe wall, B.T.U./hr.-ft.$^2$-°F.

J-------Heat equivalent, 778 ft-lb./B.T.U.

$\bar{k}$-------Thermal conductivity, B.T.U./hr.-ft.-°F.

k-------Ratio of specific heat at constant pressure to specific heat at constant volume.

L-------Length of the pipe, ft.

M-------Mach number.

$m_p$------Mass of the particle, lb.-sec.$^2$/ft.

p-------Pressure, lb./ft.$^2$, or lb./in.$^2$

$Q$-------Rate of heat transfer, B.T.U./sec.

$R$-------Gas constant, ft.-lb./lb.°F.

$R_e$-------Reynolds number.

$r$-------Ratio of ashes to gases by weight.

$S$-------Distance traveled by particle, ft.

$s$-------Entropy, B.T.U./lb.-°R.

$T$-------Absolute temperature, degrees Fahrenheit absolute.

$t$-------Temperature, degrees Fahrenheit.

$V$-------Average velocity, ft./sec.

$V_r$-------Relative velocity of particle to gases, ft./sec.

$V_t$-------Terminal velocity of particle, ft./sec.

$v$-------Specific volume, ft.$^3$/lb.

$w$-------Mass rate of flow, lb./sec.

$X$-------Defined as $\frac{ca\mu_a}{2mD_p}$.

$Y$-------Defined as $\frac{D_p\rho_a a}{\mu_a}$.

$\theta$-------Time, seconds.

$\mu$-------Absolute viscosity, lb/sec.-ft.$^2$

$\rho$-------Mass density, lb-sec.$^2$/ft.$^4$

$\epsilon$-------Emissivity

$(\ )_1$-----Refers to section 1.

$(\ )_2$-----Refers to section 2.

$(\ )_i$-----Refers to the initial conditions of the primary flow.

$(\ )_o$-----Refers to the initial conditions of the secondary flow.

$(\ )'$-----Refers to the primary flow.

$(\ )''$-----Refers to the secondary flow.

$(\ )_p$-----Refers to particle.

$( )_a$------Refers to air, or gases.

$( )_w$-------Refers to pipe wall.

# A STUDY OF HEAT TRANSFER INVOLVING
# A GAS WITH SUSPENDED PARTICLES

## INTRODUCTION

### Origin of the problem

The Locomotive Development Committee[34] of Bituminous Coal Research, Inc. is developing a coal burning gas turbine power plant for locomotive use. A schematic diagram of this unit is shown as Fig. 1. The hot gases with burning ash particles at 1,300 degrees Fahrenheit from the fly ash separator pass through the ash discharge line; the ash is collected, for disposal later, in a storage tank. In order to be able to handle the ashes in the storage tank, the temperature of the ashes must be less than 500 degrees Fahrenheit, there. A simple device, (Fig.2,) essentially an ejector is used to aspirate atmospheric air into the mixing stream at the point where the hot gases and ashes enter the ash discharge line. Experimental evidence indicates that the use of this method of cooling is satisfactory.[32]

### Scope and purpose of the study

For the purpose of design, a knowledge of those criteria that affect the inter-relation between the aspirated air and the hot gas stream conveying the suspended particles must be known. Further, the mechanism of heat transfer from a stream of hot gases carrying hot ash particles when this stream is mixed with an aspirated stream of cold air must be analyzed in order that the design provide for the requisite heat transfer. There is growing interest and need to understand the mechanism of flow and heat transfer involving particles conveyed pneumatically. Information in this regard is very scarce at the present time.[8;26] The performance of the particle laden stream as it passes through the aspirator nozzle is not understood.

The scope of this thesis consists essentially of determining, from analysis of data available, the best combination of variables to effect the design of an unit giving the desired cooling of the hot ash particles.

When 72,000 pounds air and 642 pounds powdered coal per hour are supplied to the plant, 3,200 pounds hot gases are blown out from the fly ash separator to the air discharge line.[32] The weight ratio of the hot gases to the suspended ash particles is 20 to 1. The pressure in the fly ash separator is 40 p.s.i.a. To cool down the ash particles below 500 degrees Fahrenheit, a 15 feet long straight steel pipe has been used with an ejector which can aspirate 2.5 pounds air per pound hot gases.

There are four problems recognized in the problem being considered: (a) performance of a convergent nozzle passing a gas with suspended particles, (b) theory of the ejector, (c) motion of particles in a conveying stream,

(d) heat transfer from the particles. For simplicity of calculation, the gases are assumed to be air.

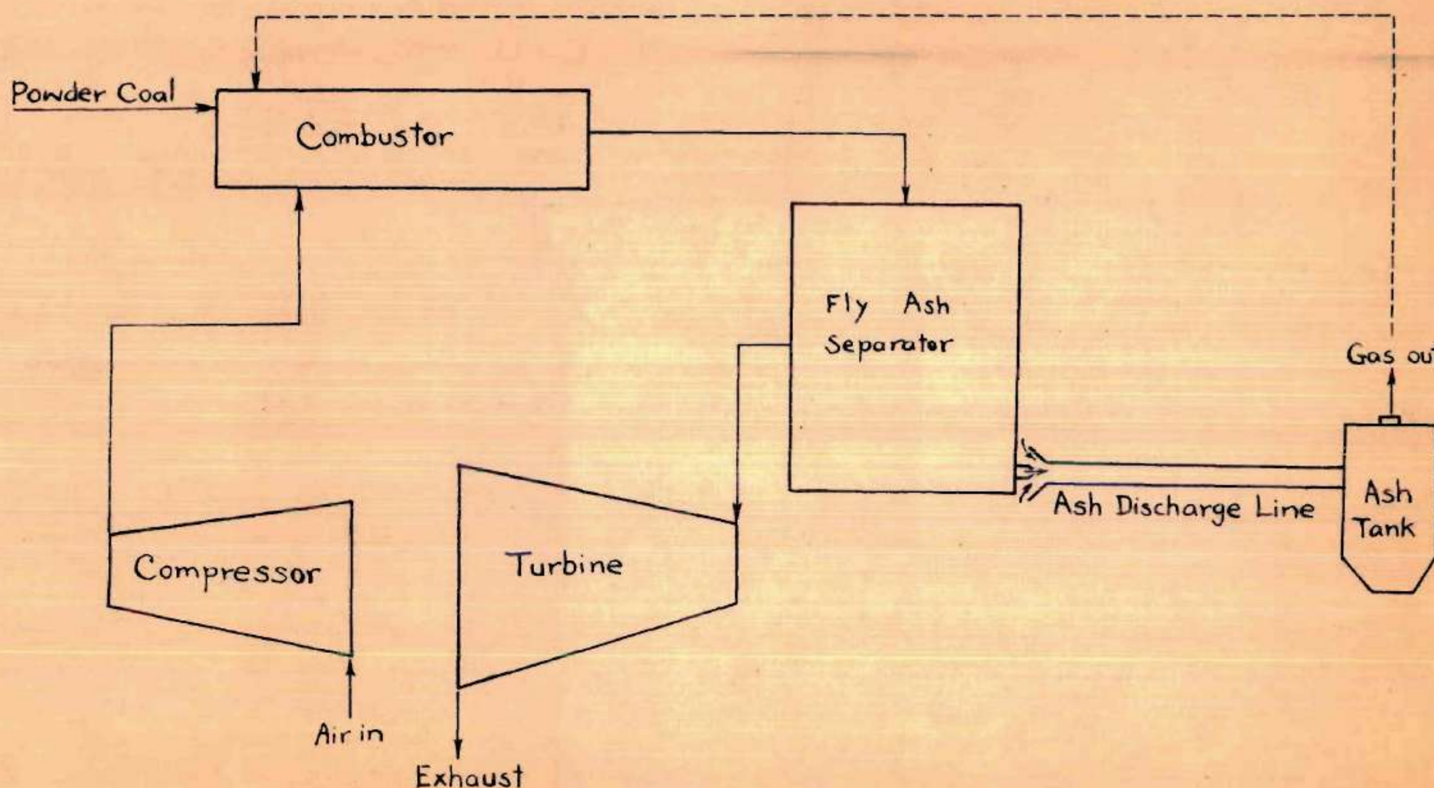



Fig. 1, Schematic Diagram of a Gas Turbine

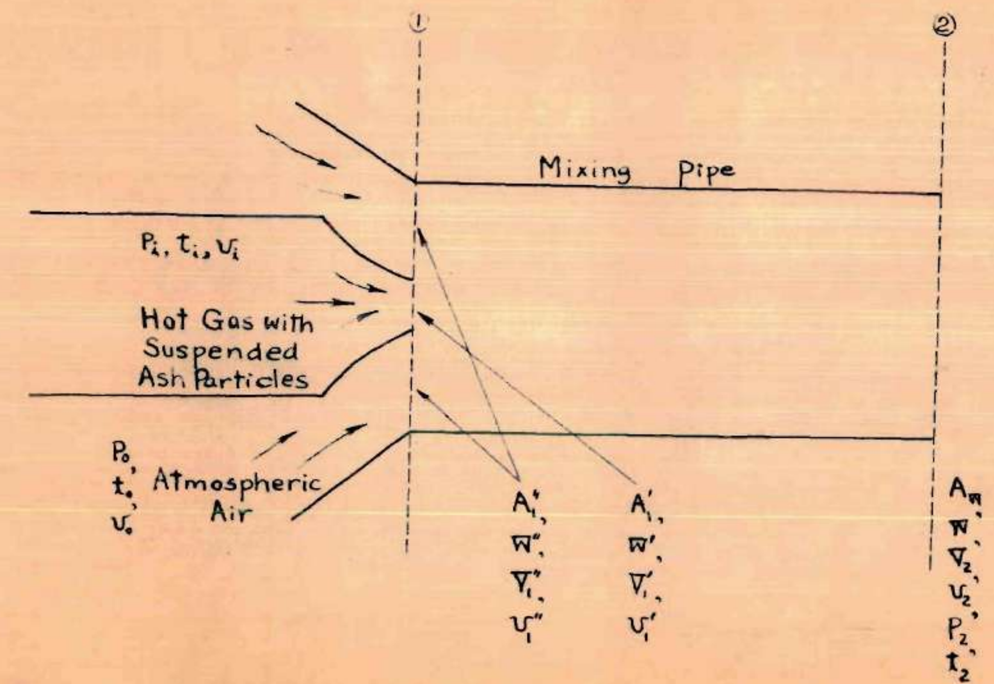



Fig. 2 Schematic Diagram of the Ejector

## HOW THE VELOCITY AND THE RATE OF FLOW OF THE HOT GASES ARE AFFECTED BY THE WEIGHT RATIO OF THE PARTICLES TO THE GASES

The performance of a nozzle, through which only a gas flows, is well known. Here, consider a gas conveying suspended particles and passing through a nozzle, ( an element of the ejector.) The gases and ashes pass through the nozzle to attain a high velocity; practically, it is an adiabatic process. If the process is assumed reversible,[14] then the velocity and the flow rate of the gases are affected by the weight ratio of ashes to the gases as, (Appendix 1,)

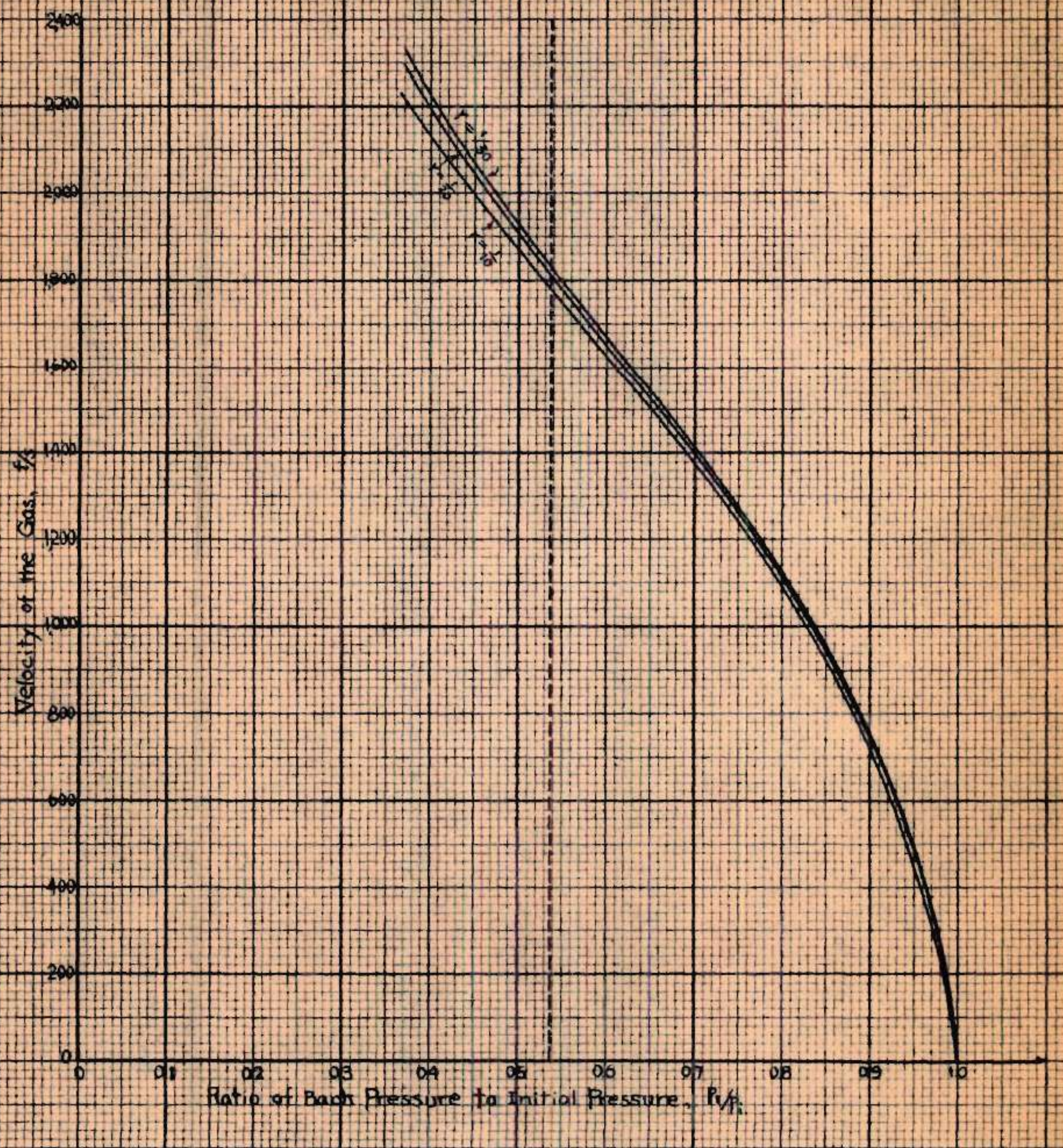
$$V_a = \sqrt{2g \frac{k}{k-1} P_i V_i \left[ 1 - \left(\frac{P_1}{P_i}\right)^{\frac{k-1}{k}\left(\frac{1}{r+1}\right)} \right]} \tag{1}$$

$$\frac{W_a}{A} = \sqrt{2g \frac{k}{k-1} \frac{P_i}{V_i} \left(\frac{P_1}{P_i}\right)^{\frac{2}{k\left[\frac{r+1}{kr+1}\right]}} \left[ 1 - \left(\frac{P_1}{P_i}\right)^{\frac{k-1}{k}\left(\frac{1}{r+1}\right)} \right]} \tag{2}$$

where r is the ratio of ashes to gases.

Assume $p_i$=40 p.s.i.a., $t_i = 1{,}300^\circ$ F, and R = 53.3, (the gas shall be assumed to be air.) Take three different r; the values of $V_a$ and $w_a/A$ are found as shown in Fig. 3 and 4. The higher the ratio of ashes to gases, the lower the gas velocity and its flow rate; however, the effect is small when the ash gas ratio is around 1:20. This is confirmed by test results of Yellott and Singh.[35] They tested a nozzle through which steam and coal particles flow; and found: (a) a reduction in steam flow as the solid feed is increased, (b) when the ratio of steam flow to solid flow is 1.0 or larger, the presence of the solid material has almost no effect upon the steam flow, (c) the effect of particle size is very slight; the most important factors are the initial steam pressure and the steam coal ratio.

Fig. 3 Velocity of the Gas at Different Weight Ratios, r, of the Particles to the Gases; for $p_1$=40 psia and $T_1$=1,300°F



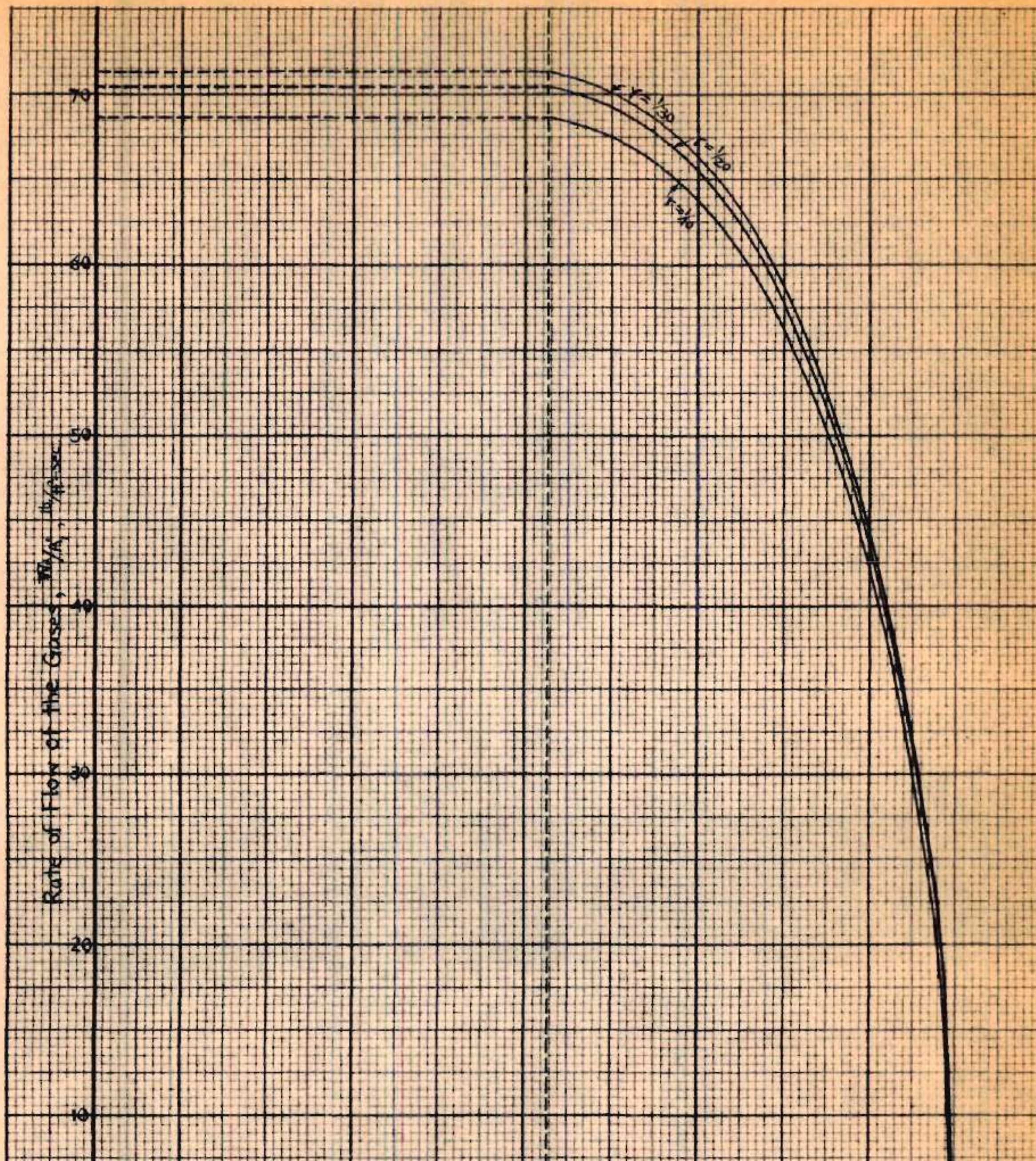



Fig. 4 Rate of Flow of the Gases at Different Weight Ratios, r, of the Particles to the Gases, for $P_c = 40$ psia and $t_c = 1300$°F.

## THEORY OF THE SIMPLE EJECTOR

### Ratio of secondary flow to primary flow

Analyses of ejectors that appear in the literature may be classified into two main groups: (a) one dimensional analysis, (b) two dimensional analysis. The latter was brought out by the hypothesis of mixing length, which was suggested by Prandtl, and has been studied by Tollmien, Kuethe,[24] Coogan and Goff.[13] But, since the two dimensional analysis involves many complicated mathematical problems, it is not well enough developed as yet, accordingly it can not be used for design purpose; the one dimensional analysis is used here.

For a simple air ejector, it is found:[23] (a) straight mixing pipe gives greater amount of aspirated air than the constant pressure mixing pipe, (b) the performance of a simple ejector can be calculated from the conditions imposed by conservation of matter, conservation of energy and the laws of motion, (c) experiments show the measured flow ratio is about 90% of calculated ratio.

Referring to Fig. 2, imagine a section 1, in front of the exit of the primary nozzle; the pressure there is at a uniform pressure $p_1$. Writing the equations for a reversible and adiabatic process from 0 to 1 and from i to 1,

$$V_1' = \sqrt{2gRT_i \frac{k}{k-1}\left[1-\left(\frac{p_1}{p_i}\right)^{\frac{k-1}{k}}\right]} \qquad (3)$$

$$a_1' = \frac{W' v_1'}{V_1'} = \frac{W' v_i}{V_1'}\left(\frac{p_i}{p_1}\right)^{\frac{1}{k}} \tag{4}$$

$$V_1'' = \sqrt{2gRT_0\frac{k}{k-1}\left[1-\left(\frac{p_1}{p_0}\right)^{\frac{k-1}{k}}\right]} \tag{5}$$

$$a_1'' = \frac{W'' v_1''}{V_1''} = \frac{W'' v_0}{V_1''}\left(\frac{p_0}{p_1}\right)^{\frac{1}{k}} \tag{6}$$

from 1 to 2,

$$W = W' + W'' \tag{7}$$

$$\frac{W'}{g_0}V_1' + \frac{W''}{g_0}V_1'' + p_1 A_W = \frac{W}{g_0}V_2 + p_2 A_W + F \tag{8}$$

$$W'H_i + W''H_0 = W\left(H_2 + \frac{V_2^2}{2g}\right) + Q \tag{9}$$

where F is the friction force of the pipe surface, Q is the heat transfer through the pipe and $A_W = A_1' + A_1''$. In this problem, both F and Q are relatively small in comparison with other items in the equations. By definition, $H = c_p \Delta t$; neglecting F and Q and rearranging equations 8 and 9, one gets

$$\frac{W'}{g_0}V_1' + \frac{W''}{g_0}V_1' + p_1 A_W = \frac{W}{g_0}V_2 + p_2 A_W \tag{10}$$

$$W'C_p t_i + W''C_p t_0 = W\left(C_p t_2 + \frac{V_2^2}{2g}\right) \tag{11}$$

If $p_0$ , $p_i$ , $p_2$ , $t_i$ and $t_0$ are fixed, we may use a trial method to find the ratio of secondary flow to primary flow, $w''/w'$ , for a corresponding area ratio $A_W/A_1'$ . Select a value $p_1$ ; $V_1''$ and $V_1'$ will be found from equations 3 and 5. For a fixed ratio of $A_W/A_1'$ , get $w''/w'$ from equations 4 and 6. If the values of $w''/w'$ and $A_W/A_1'$ do not satisfy equations 10 and 11 , a new value for $p_1$ shall be selected.

From equations 3, 4, 5, 6, 10, and 11, and assuming $p_1 = 40$ p.s.i.a., $p_0 = 14.7$ p.s.i.a., $t_1 = 1,300^\circ F$ and $t_0 = 70^\circ F$: $t_2$, $V_2$ and $w''/w'$ have been calculated for the corresponding values of $p_2$ and $A_m/A'_1$. (Fig. 5, 6, 7, 8, 9 and 10.) The optimum values of $A_m/A'_1$ for maximum flow ratio (Fig.8) are in agreement with Elrod's analysis.[10]

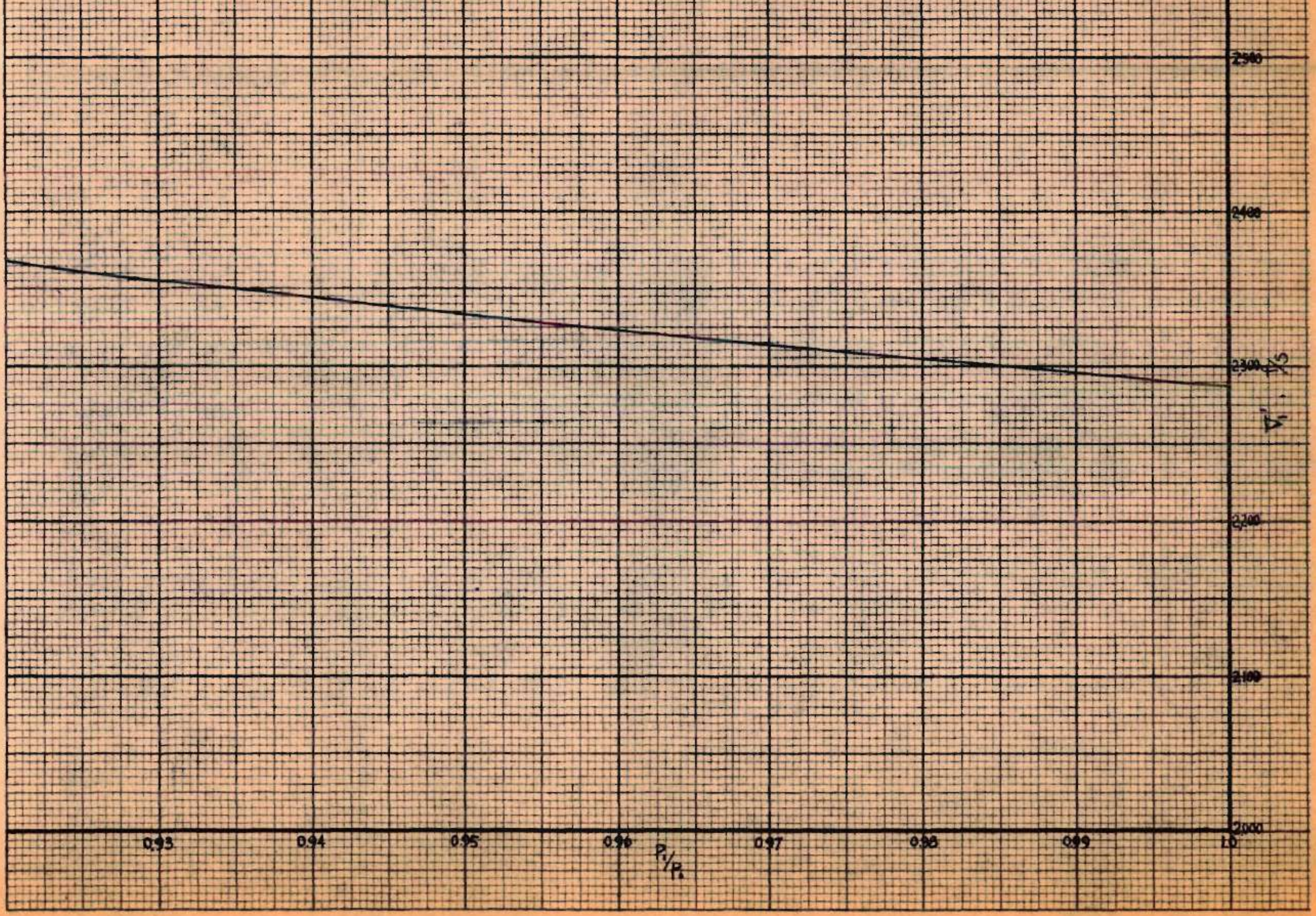
The best position of the exit of the primary nozzle should be one diameter to half diameter of the mixing pipe upstream from the throat of the secondary nozzle.[11] The difference in performance of a blunt nozzle (primary nozzle of the ejector) and that of a tapered nozzle is very slight.[23] The shape of the entry of the mixing pipe should be bell mouth to give the highest entrance coefficient.

A discussion of the mixing length in pipe

In two dimensional analysis, the boundaries of the mixing zone have been studied. Kuethe[24] proved that the core of a primary stream from a free jet is 4.76 diameters of the jet exit. There is no doubt that the mixing of two streams at high speed is caused by the momentum exchange.[4] By experiments, Keenan[23] and Engdahl[11] found that 7 to 7.5 diameters of the mixing pipe are the optimum length for maximum secondary flow; which means no momentum change occurs after 7.5 diameters of the mixing pipe. It may be true that mixing of two streams is accomplished before 7.5 diameters of the mixing pipe; and this is indicated in two dimensional analysis.

In a converging nozzle for subsonic flow, no shock wave occurs.[30] But, if the back pressure is lower than the critical pressure, the flow will have further sudden expansion after it leaves the exit of a converging nozzle. The sudden expansion causes an oscillating wave of pressure.[1;33] This phenomenon might be desirable for good mixing, as the pressure oscillation can create shear planes between the flow layers and make the flow pattern more complex. This of course, involves two dimensional flow considerations.

Fig. 5 $\bar{V}_1'$ versus $P_1/P_a$ of the Ejector; for $P_1 = 40$ psia, $P_a = 14.7$ psia, $t_1 = 1300°F$ and $t_0 = 70°F$.



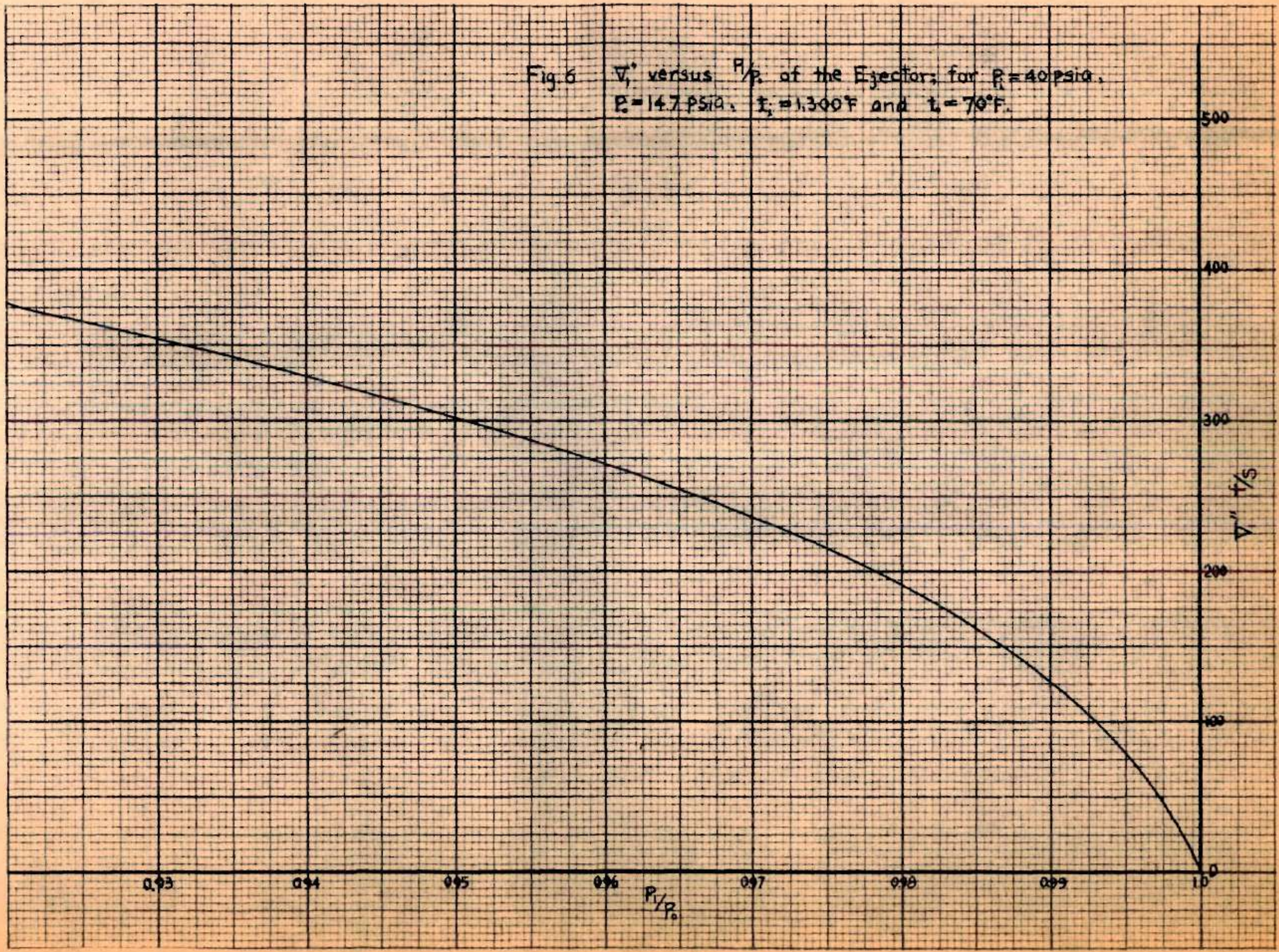



Fig. 6 $V_1'$ versus $P_1/P_0$ of the Ejector; for $P_i = 40$ psia, $P_0 = 14.7$ psia, $t_i = 1,300$°F and $t_0 = 70$°F.

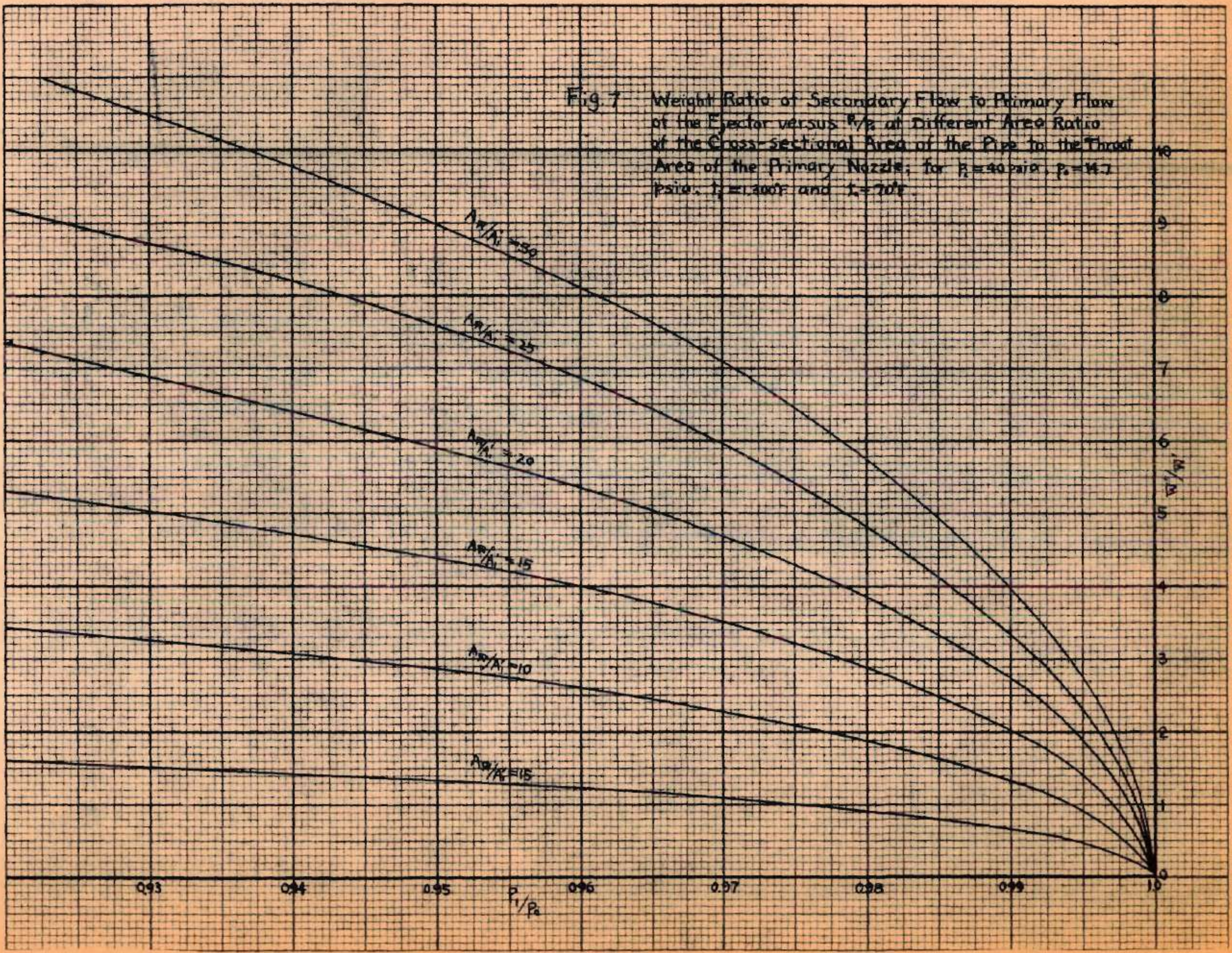



Fig. 7 Weight Ratio of Secondary Flow to Primary Flow of the Ejector versus $P_1/P_2$ at Different Area Ratio of the Cross-sectional Area of the Pipe to the Throat Area of the Primary Nozzle; for $P_1 = 40$ psia, $P_o = 14.7$ psia, $T_1 = 1{,}300°F$ and $T_2 = 70°F$.

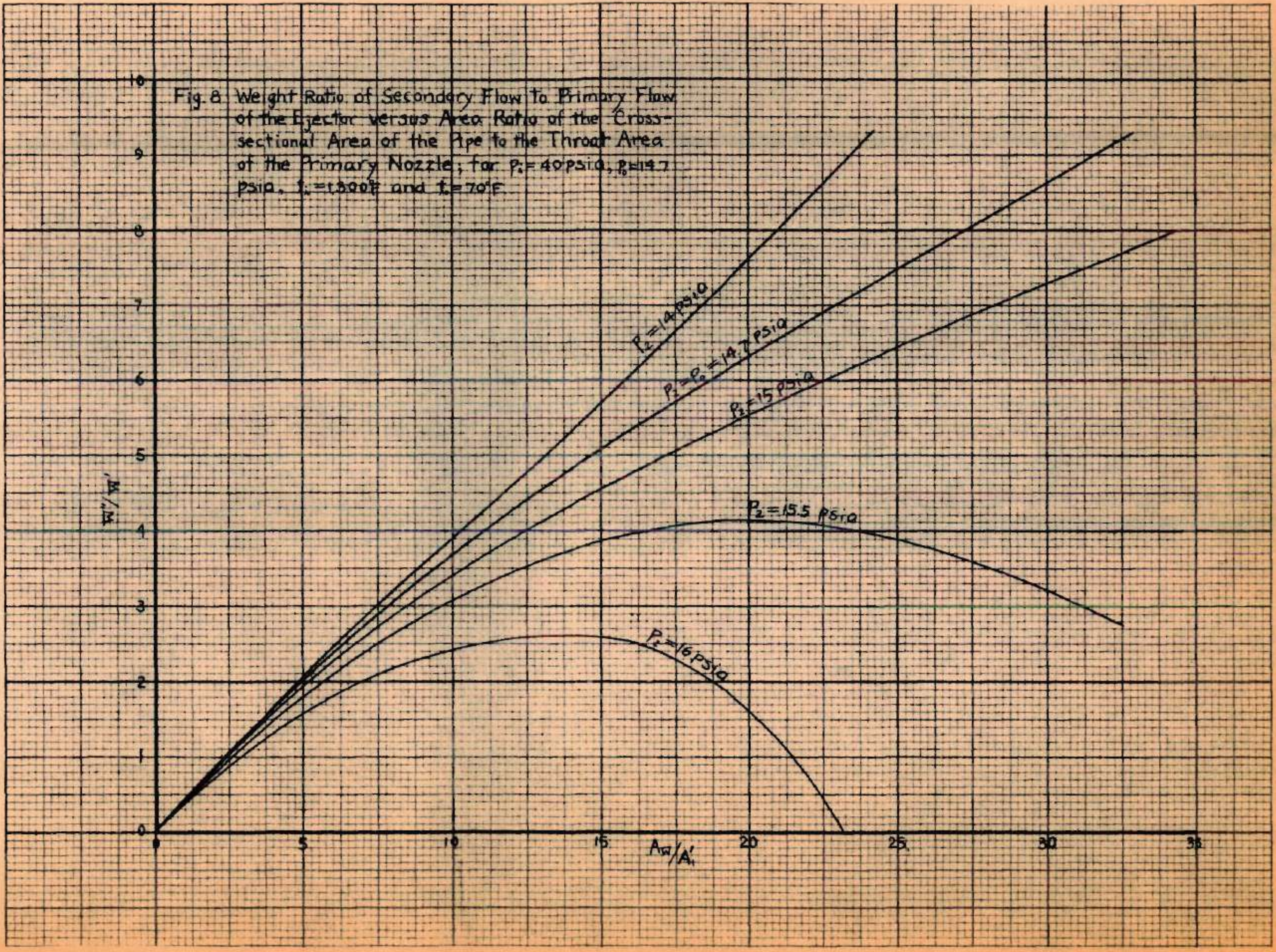



Fig. 8 Weight Ratio of Secondary Flow to Primary Flow of the Ejector versus Area Ratio of the Cross-sectional Area of the Pipe to the Throat Area of the Primary Nozzle; for $P_1 = 40$ psia, $P_0 = 14.7$ psia, $t_1 = 1300°F$ and $t_2 = 70°F$.

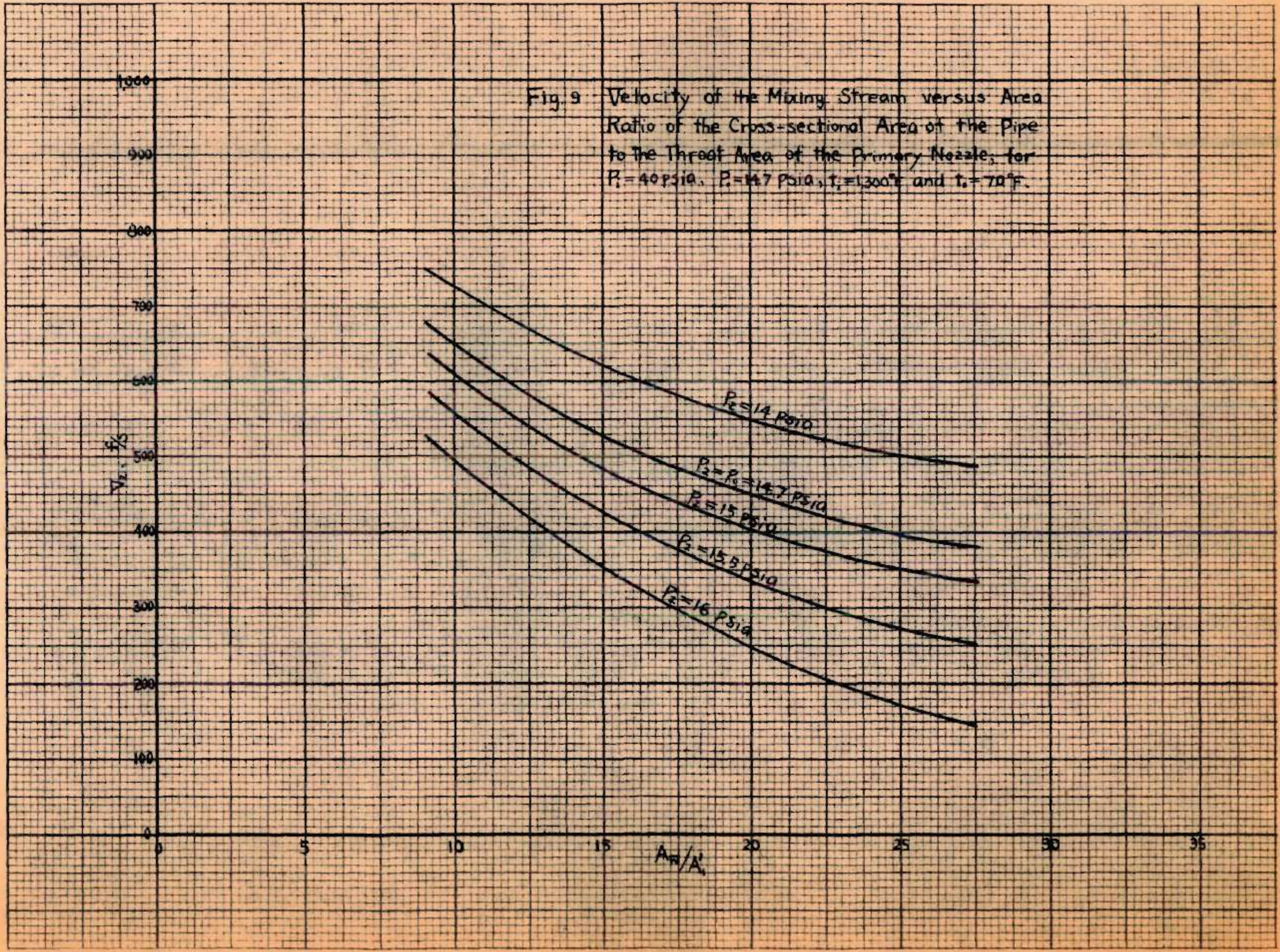



Fig. 9 Velocity of the Mixing Stream versus Area Ratio of the Cross-sectional Area of the Pipe to the Throat Area of the Primary Nozzle, for $P_1 = 40$ psia, $P_0 = 14.7$ psia, $t_1 = 1{,}300°F$ and $t_0 = 70°F$.

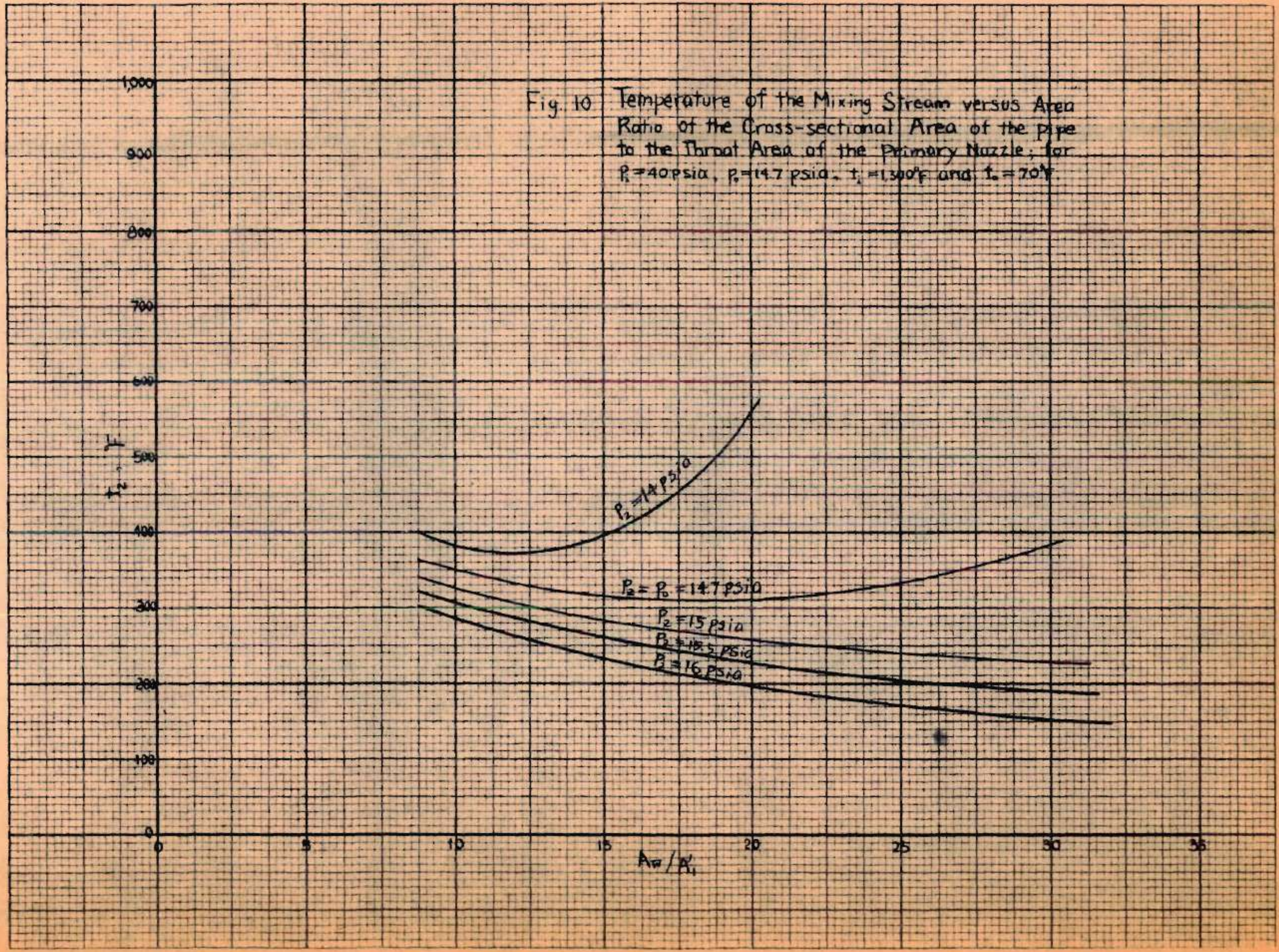



Fig. 10 Temperature of the Mixing Stream versus Area Ratio of the Cross-sectional Area of the pipe to the Throat Area of the Primary Nozzle; for $P_1 = 40$ psia, $P_0 = 14.7$ psia, $t_1 = 1{,}300°F$ and $t_0 = 70°F$.

## MOTION OF THE PARTICLES

It is well known that the film coefficient for convective heat transfer varies with velocity. Thus, the motion of the particles relative to the gas stream in which the particles are suspended must be understood before determination of the film coefficient applicable to the particles is possible. Various contributions have been made in the study of the motion of particles in a gas stream. Cramp[5] and Jennings[18] investigated for purpose of pneumatic transportation the motion of particles in a vertical and horizontal pipe. Stern,[29] and Drinker and Hatch[9] discussed how to separate dust from particle laden air. Ballisticians[6] solved the differential equations of motion of projectiles in air.

When air passes through a horizontal pipe at high velocity and pushes the suspended particles, the particles are accelerated. By the second law of motion,

$$m\frac{dV_p}{d\theta} = \frac{c}{2}\rho_a A_p V_r^2 \qquad (12)$$

Since $V_p = V_a - V_r$, obtain

$$m\left(\frac{dV_a}{d\theta} - \frac{dV_r}{d\theta}\right) = \frac{c}{2}\rho_a A_p V_r^2 \qquad (13)$$

The drag coefficient c, verified by a number of different authorities,[28;30] is a function of Reynolds number, (Fig.11.) By definition,

$$V_r = \frac{R_e \mu_a}{D_p \rho_a}$$

where $R_e$ is the Reynolds number describing the flow pattern about the particle. Consider a large stream of air at constant velocity $V_a$ and constant density $\rho_a$; and set limits as $V_r = V_{ro}$ and $R_e = R_{eo}$ when $\theta=0$. Rearranging and integrating equation 13, get

$$\frac{\mu_a A_p}{2D_p m_p}\theta = \int_{R_e}^{R_{eo}} \frac{dR_e}{C R_e^2} \tag{14}$$

Because c is a function of $R_e$ and $R_e$ is a function of $V_r$ , the relation between $\theta$ and $V_r$ may be found from equation 14. The distance traveled by a particle in a time interval $\theta$ is

$$S = \int_0^\theta (V_a - V_r)\, d\theta = V_a \theta - \int_0^\theta V_r \, d\theta \tag{15}$$

Thus, the velocity of the suspended particle relative to air in a horizontal pipe can be estimated when the particle is in acceleration if the air is assumed to have a constant velocity and constant density, and the initial velocity of the particle relative to the air is known.

In a converging nozzle, the air has high acceleration; thus the air velocity is not constant. Using pitot tubes, Nusselt[2] observed the behavior of a gas as it passes through a sharpedged orifice and through a nozzle, (Fig. 12.) In discussing the problem of coal metering, Blizard[2] assumed that the velocity of the gas increases linearly with the length of the nozzle. Nusselt's data, however, indicated the existence of a constant acceleration when the complete velocity distribution is examined as compared with the portion near the throat. Here, for the purpose of demonstrating the trend of variation of the velocity of particles relative to the air in a converging nozzle, the air is assumed at constant acceleration a, or

$V_a = a\theta$. From equation 13,

$$m\left(a - \frac{dV_r}{d\theta}\right) = \frac{C \rho_a A_p V_r^2}{2}$$

Substituting $R_e$ into the above equation,

$$\frac{D_p \rho_a a}{\mu_a} - \frac{dR_e}{d\theta} = \frac{C A_p R_e^2 \mu_a}{2 m D_p}$$

Set the limit: $V_r = 0$ and $R_e = 0$, when $\theta = 0$; i.e. upstream of the nozzle, by integrating,

$$\theta = \int_0^{R_e} \frac{dR_e}{-X R_e^2 + Y} = \frac{1}{2\sqrt{XY}} \log \frac{\sqrt{Y} + R_e\sqrt{X}}{\sqrt{Y} - R_e\sqrt{X}} \qquad (16)$$

where

$$X = \frac{C A_p \mu_a}{2 m D_p}, \qquad Y = \frac{D_p \rho_a a}{\mu_a}$$

Equation 16 gives the relation between $V_r$ and $\theta$, when the air is at constant acceleration and constant density passing through a nozzle. Assume the air is at constant acceleration and constant density; (take its average density and viscosity from the inlet and the exit of the nozzle,) for $p_i = 40$ p.s.i.a., $p_1 = 21.1$ p.s.i.a., (critical pressure,) $D_p = 20$ microns and $t_i =$ 1,300°F, the relation between the velocity of the particle relative to the air and the length of the nozzle is shown in Fig. 13.

Similarly, if the air passes through a gradually enlarging pipe (a diffuser) at constant deceleration (-a), and $V_p = V_a + V_r$; then,

$$\theta = \int_0^{R_e} \frac{dR_e}{X R_e^2 + Y} = \frac{1}{\sqrt{XY}} \tan^{-1}\left(R_e \sqrt{\frac{X}{Y}}\right) \qquad (17)$$

This equation shows that a high velocity of the particles relative to the air in a horizontal pipe may be obtained by using a diffuser immediately down from the section at which complete mixing has taken place.

Croft[7] gives the approximate equation,

$$V_r = 0.045\, V_a^{0.56}\, V_t$$

where $V_t$ is the terminal velocity of the particles. This equation shows that the velocity of the particles relative to the gases is of the order 0.06 to 0.08 fps, for a constant stream velocity $V_a$. The assumption is made that this equation applies at the section at wnich complete mixing has taken place.

Fig. 11 Drag Coefficient versus Reynolds Number for a Sphere

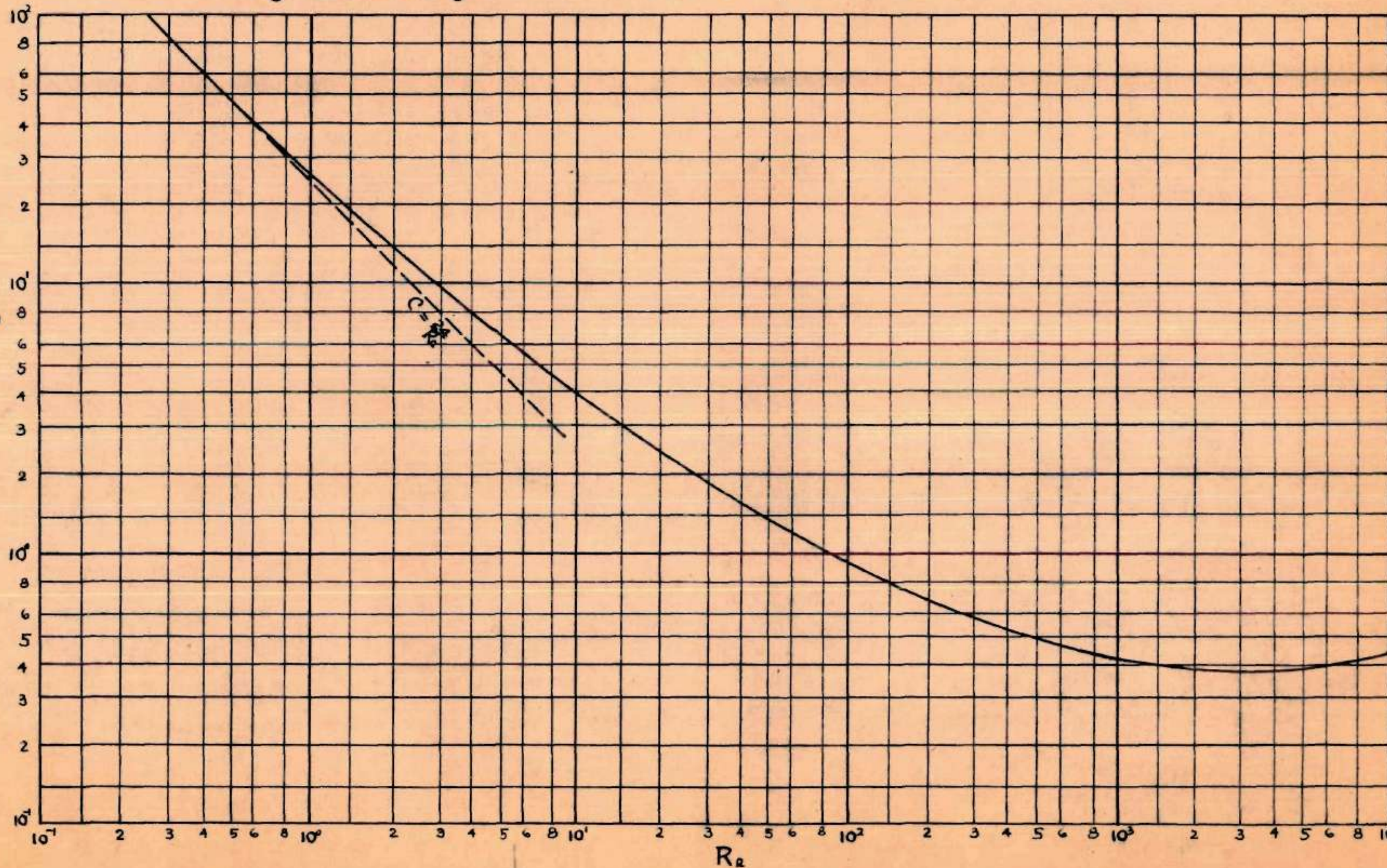


Fig. 12 Change in Velocity of Air Passing through in an Orifice and a Nozzle, by W. Nusselt[2]

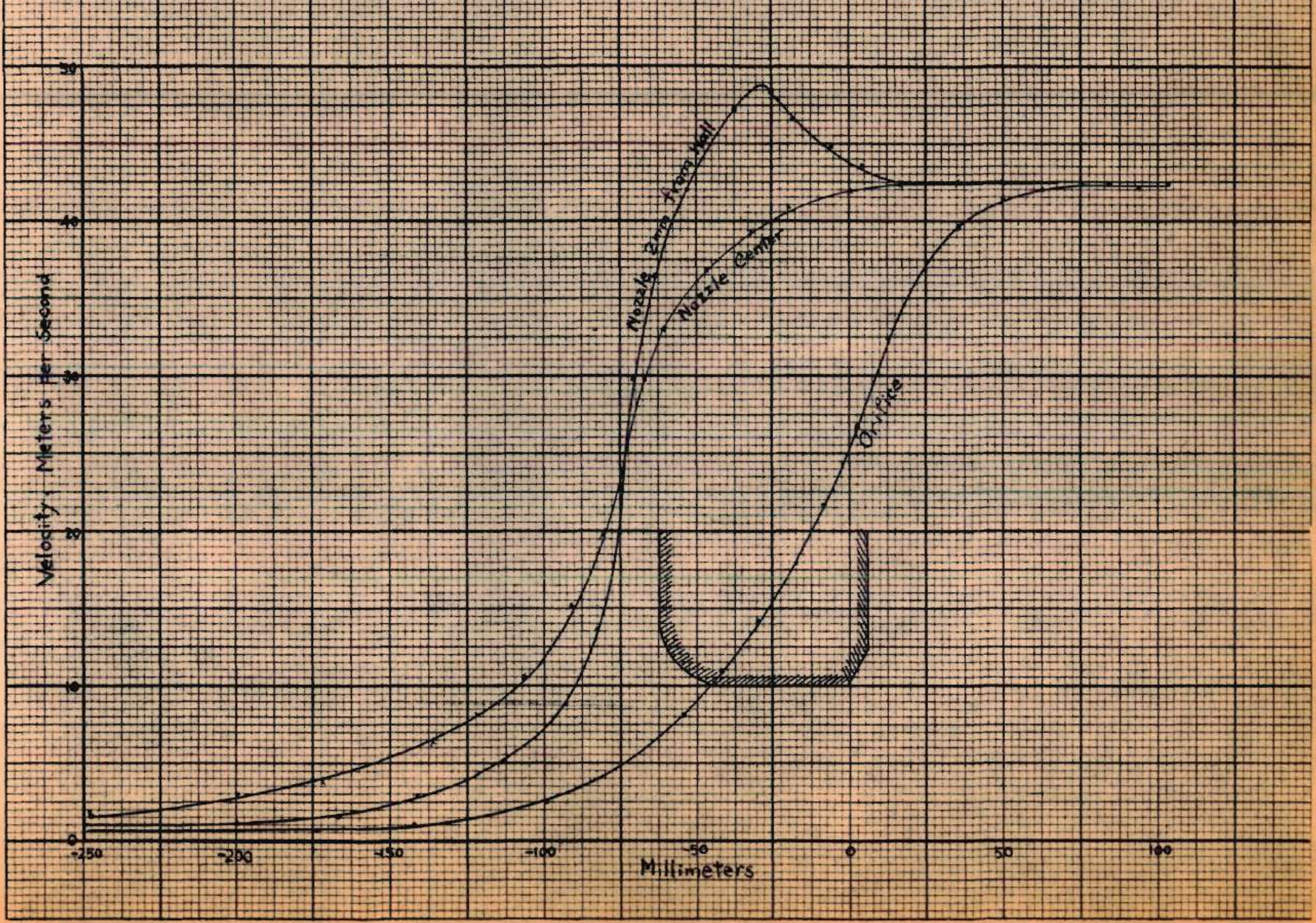
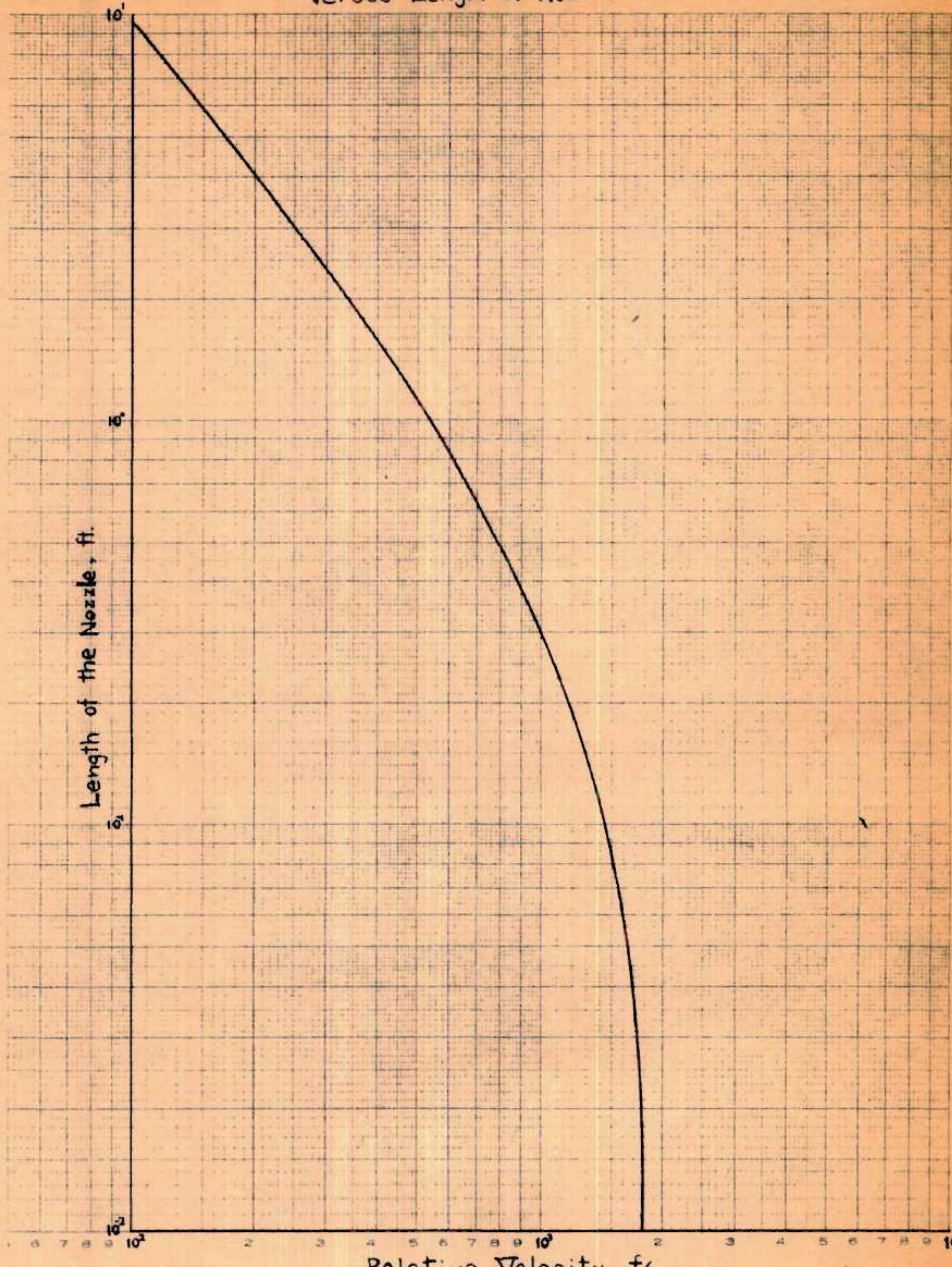


Fig. 13 Velocity of Particle Relative to Gases versus Length of Nozzle



HEAT TRANSFER COEFFICIENT OF THE PARTICLES

After the two streams of gases of different temperatures are properly mixed, the stream is in uniform temperature. The ash particles in the mixing streams take longer time to cool down. The heat transfer of a particle suspended in a stream of gases involves principally conduction in the particle, convection from the particle to the gases, radiation from the particle to the pipe wall, etc. If the minor effects are neglected, the heat transfer by conduction in the particle must be equal to the sum of the net radiation transfer and the convection, as the net radiation acts parallel with the convection. For a small particle with high conductivity, the heat transfer by conduction would not be important,[19] that can be verified by the following equation,[17]

$$t_c = t_s + (t_i - t_s)\, F\left(\frac{4\alpha\tau}{D_p^2}\right)$$

where $t_c$ is the temperature in the center of a spherical particle, $t_i$ is the initial temperature of the surface of the particle, $t_s$ is the suddenly imposed temperature of the surface of the particle, $\alpha$ is defined as $\bar{k}/\rho_a c_p$, (about 0.5 for the ash particle,) $\tau$ is the time of heat transfer in hours, and $D_p$ is the diameter of a spherical particle. From Fig. 14, if $4\alpha\tau/D_p^2$ is 0.5, then $F(4\alpha\tau/D_p^2)$ is about 0.01; and the center temperature of the particle is almost equal to the surface temperature. For a spherical ash particle of 20 microns in diameter, if $\mathcal{T}$ is $10.75 \times 10^{-10}$ hour, or $3.76 \times 10^{-6}$ second, then the surface temperature of the particle is almost same as the center temperature. Thus, the conduction in the particle is neglected, and the overall heat transfer coefficient of the particle becomes,

$$h_p = h_r + h_{cp}$$

Johnstone's experiments[19] show the film coefficient of convection from the particle to the gases as,

$$h_{cp} = 0.714 \sqrt{\frac{k V_r \rho g}{D_p}} \quad (18)$$

where $h_{cp}$ is independent of the temperature of the particle. For a particle of 20 microns diameter, suspended in air, $h_{cp}$ are plotted as Fig. 15. It shows that velocity of the particle relative to air is the controlling factor for $h_{cp}$. The radiation coefficient $h_r$ is given by[25]

$$h_r = \frac{0.173\epsilon}{T_p - T_w}\left[\left(\frac{T_p}{100}\right)^4 - \left(\frac{T_w}{100}\right)^4\right] \quad (19)$$

h at different temperatures are shown in Fig. 16, for an assumed $\epsilon$=0.9, (very conservative, see Fig.5 of Ref. 19) A comparison of the values of $h_{cp}$ and $h_r$ shows $h_{cp}$ to be more important in this problem. Unless the relative velocity of the particle to air is very low (this occurs after the particle travels a considerable distance in the horizontal pipe), the radiation effect is unimportant.

The time for heat transfer to the particle is the length of the pipe divided by the mean absolute velocity of the particle. In this aspect, we would prefer low absolute velocity of the particle, so that the length of the pipe can be reduced. The absolute velocity of particle is the difference of the absolute velocity of conveying gas and the velocity of the particle relative to gas. Thus, a low absolute velocity of the gases is desirable.

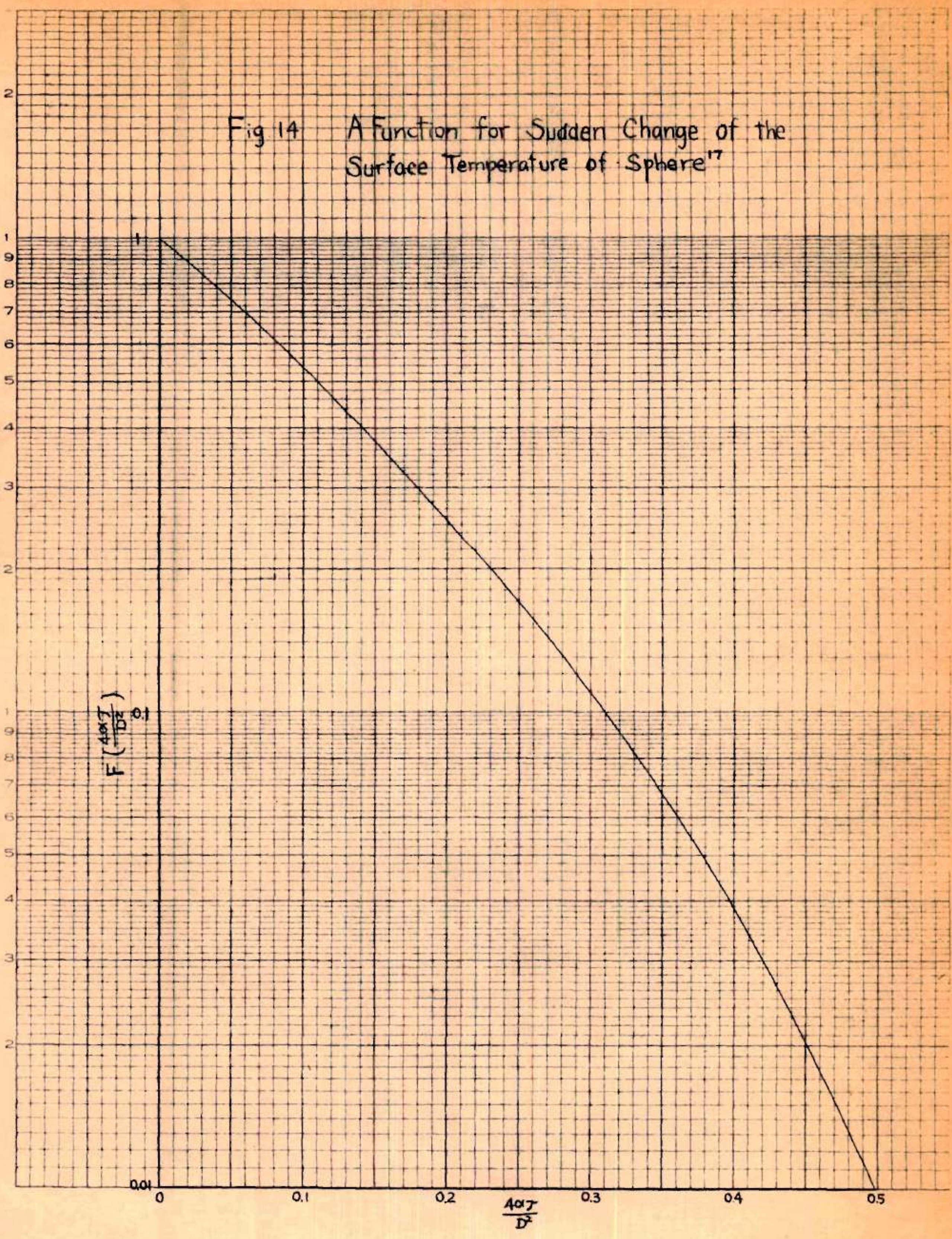


Fig 14 A Function for Sudden Change of the Surface Temperature of Sphere[17]
$F\left(\frac{4\alpha\tau}{D^2}\right)$
$\frac{4\alpha\tau}{D^2}$
1
0.1
0.01
0
0.1
0.2
0.3
0.4
0.5

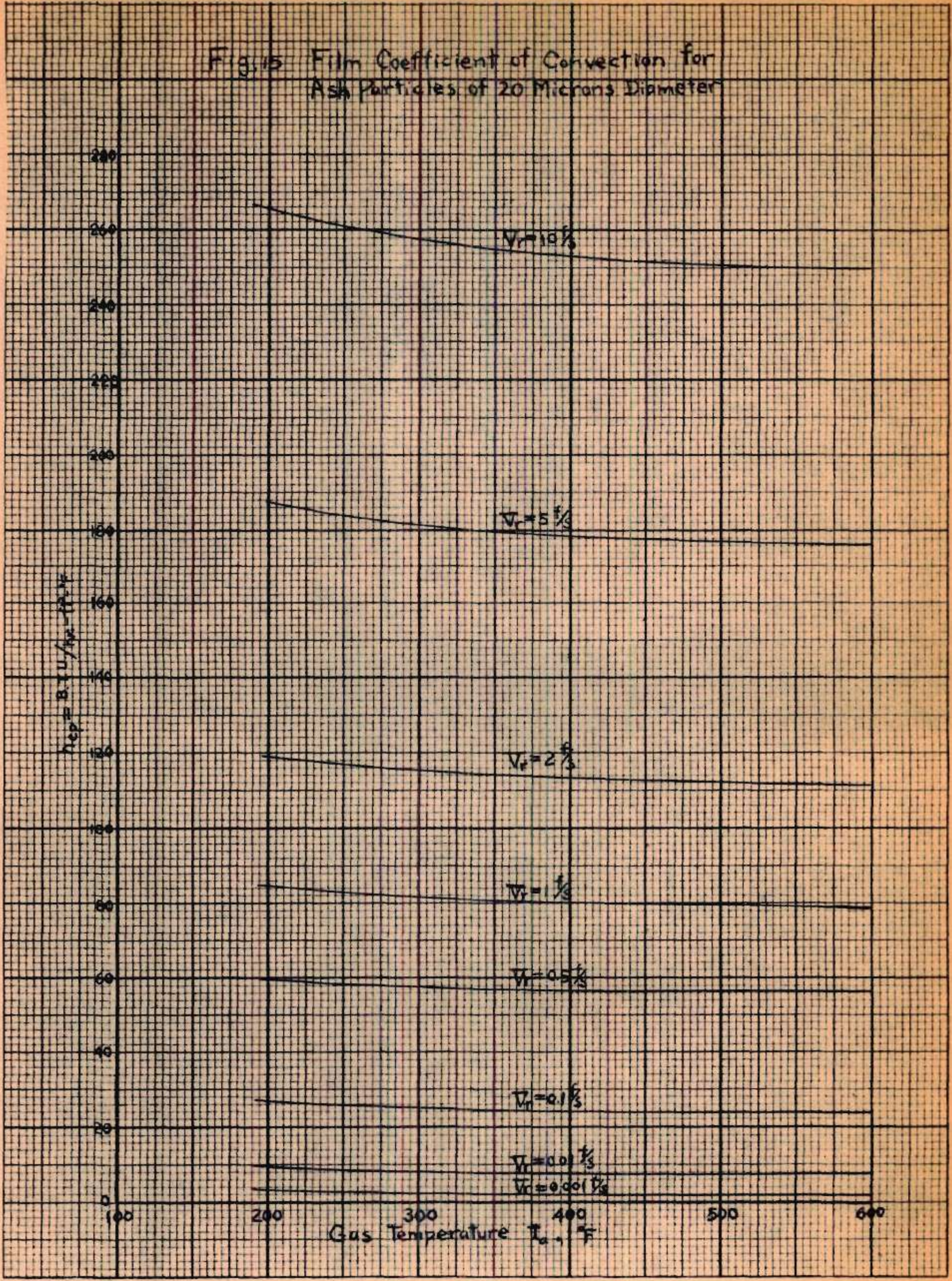


Fig. 15 Film Coefficient of Convection for
Ash Particles of 20 Microns Diameter
$V_r = 10$ f/s
$V_r = 5$ f/s
$V_r = 2$ f/s
$V_r = 1$ f/s
$V_r = 0.5$ f/s
$V_r = 0.1$ f/s
$V_r = 0.01$ f/s
$V_r = 0.001$ f/s
$h_{cp}$ = B.T.U./hr.-ft.²-°F
Gas Temperature $T_a$, °F
100
200
300
400
500
600
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280

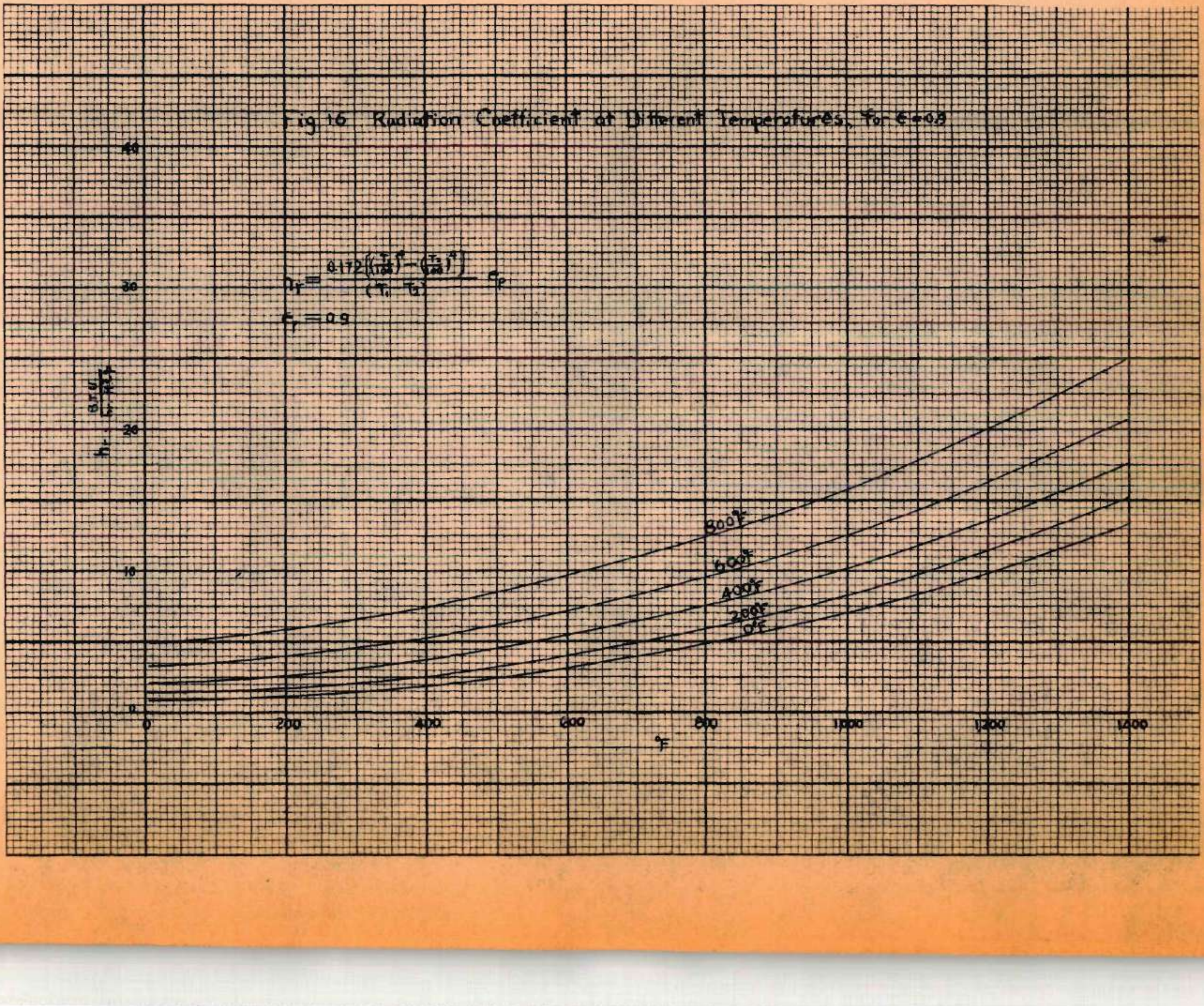



Fig 16 Radiation Coefficient at Different Temperatures, for ε=0.9

## CONCLUSION

An exact analysis of this problem is exceedingly difficult, because many complicated mechanisms are involved, whose complete description is unknown. The analysis of this problem has recognized at least four mechanisms: (1) performance of a convergent nozzle passing a gas with suspended particles; (2) theory of a simple ejector; (3) motion of particles in a conveying stream; (4) heat transfer from the particles.

As to the heat transfer the following analysis appears applicable. The film coefficient of convective heat transfer of a small particle is considerably high, (Fig.15.) The radiation effect is rather unimportant in this problem, (Fig. 16.) To improve the total heat transfer, a relative velocity of the particles to the gases as high as possible is desirable. There is no doubt that the particle laden gases have attained a uniform velocity after 7-7.5 diameters of mixing pipe; therefore, to obtain a higher velocity of the particles relative to the gases, it is necessary to have a variable cross-sectional area of the pipe after 7-7.5 diameters of the pipe. The heat transfer from the pipe wall to the ambient air effects the temperature of the mixing stream very little, less than 8 degrees Fahrenheit;[25] so, if necessary, the pipe can be insulated and regenerative heat exchange employed to advantage in the combustor-turbine cycle.

Considering problem 3, the velocity of the particles relative to the gases in a long nozzle is shown to be lower than that in a short nozzle, as the particles have enough time to accelerate; thus, a short nozzle is recommended for the primary nozzle. (A sharp-edged orifice may be used instead of a nozzle.)

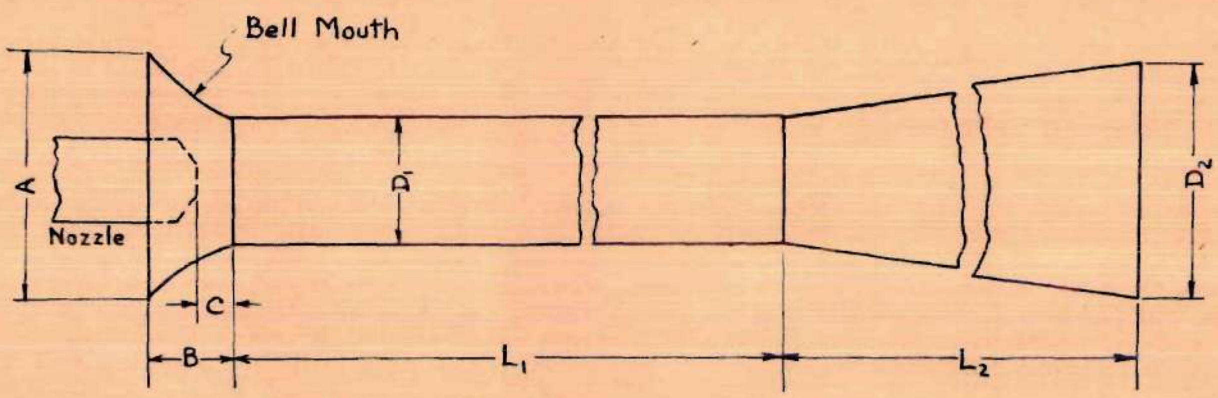
The performance of an ejector shows that for a given condition of pressure and temperature, the ratio of secondary flow to primary flow varies with the ratio of mixing tube area to nozzle throat area. There is an optimum area ratio for each $p_2$, such that the maximum flow ratio can be obtained. (Fig. 8.) The temperature and velocity of the mixing stream are determined also by the area ratio. (Fig. 9 and 10.) Practically, $p_2$ is the pressure in the ash tank, which is an important factor in determining the performance of the ejector. The pressure in the ash tank depends upon the vents on the ash tank.

The mass rate of flow and the velocity of gases leaving the nozzle are affected inappreciably by the presence of the ash particles, when the weight ratio of the solid to the gases is around 1:20. (Fig.3 and 4.)

For design purpose, some proportional dimensions are suggested as in Fig. 17. Fig. 8, 9 and 10 are very useful for design purpose; however, the exact length of the gradually enlarging part of the pipe should be determined by test.

This thesis indicates the need for further study of at least two problems: (1) the determination of temperature and velocity distribution in streams, subject to mixing as jets; (2) the study of the mechanism of motion of particles in a moving gas stream in a tube.

Fig. 17 Dimensions of the Ejector




$A$ = Any convenient length.

$B$ = Any convenient length.

$C = \frac{1}{2}D_1$ to $1 D_1$,

$L_1 = 7.5 D_1$.

$D_2 > D_1$.

$D_1$ ---- Determined by the weight ratio of the secondary flow to the primary flow

$L_2$ ---- Determined by test

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# APPENDIX

## Derivations[14]

Equations 1 and 2 show that the velocity and the rate of flow of the hot gases are influenced by the weight ratio of the particles to gas. The derivation follows:

by definition,

$$dS_p = (C_p)_p \frac{dT}{T}$$

$$dS_a = (C_p)_a \frac{dT}{T} - R\frac{dP}{P}$$

where $ds_p$ is the change of entropy of ash particles and $ds_a$ is that of gases. Since the gases and the ashes pass through the nozzle at high velocity, assume the process is isentropic, i.e.,

$$W_p\, dS_p + W_a\, dS_a = 0$$

or

$$\left[r(C_p)_a + (C_p)_p\right]\frac{dT}{T} - R\frac{dP}{P} = 0 \qquad (20)$$

When the ashes do not appear, equation 20 becomes

$$(C_p)_a\frac{dT}{T} - R\frac{dP}{P} = 0 \qquad (21)$$

For a perfect gas, the relation between the gas constant and the specific heat is $c_p - c_v = \frac{R}{J}$; rearranging equations 20 and 21,

$$\frac{1}{J}\left[\frac{r(C_p)_a + (C_p)_p}{(C_p)_a - (C_v)_a}\right]\frac{dT}{T} - \frac{dP}{P} = 0 \qquad (22)$$

$$\frac{1}{J}\left[\frac{(C_p)_a}{(C_p)_a - (C_v)_a}\right]\frac{dT}{T} - \frac{dP}{P} = 0 \qquad (23)$$

For adiabatic process of a perfect gas, the p-v relation can be expressed as $pv^k$= constant, where k is the ratio of the specific heat at constant pressure to that at constant volume. By observation of equations 22 and 23, the p-v relation of a perfect gas, in which the solid particles are suspended, may be expressed as $pv^{k'}$= constant, where

$$k' = k\frac{r\frac{(C_p)_p}{(C_p)_a}+1}{kr\frac{(C_p)_p}{(C_p)_a}+1} \tag{24}$$

The specific heat of ashes at constant pressure is very close to that of the gases. If $(c_p)_p = (c_p)_a$, then,

$$k' = k\left(\frac{r+1}{kr+1}\right) \tag{25}$$

Assume the initial velocity of the gases is zero. The velocity of the gases leaving the nozzle, $V_a$, can be expressed in the following equation as

$$\frac{V_a^2}{2g} = J(C_p)_a(T_i - T_1) = \frac{Rk}{k-1}(T_i - T_1) = \frac{k}{k-1}(P_i v_i - P_1 v_1) \tag{26}$$

Combining equations 25 and 26,

$$V_a = \sqrt{2g\frac{k}{k-1}P_i v_i\left[1-\left(\frac{P_1}{P_i}\right)^{\frac{k-1}{k}\left(\frac{1}{r+1}\right)}\right]} \tag{27}$$

Similarly,

$$\frac{W_a}{A} = \sqrt{2g\frac{k}{k-1}\frac{P_i}{v_i}\left(\frac{P_1}{P_i}\right)^{\frac{2}{k\left(\frac{r+1}{kr+1}\right)}}\left[1-\left(\frac{P_1}{P_i}\right)^{\frac{k-1}{k}\frac{1}{r+1}}\right]} \tag{28}$$

Note: The expansion of the air through the nozzle is, of course, irreversible because of the drag of the suspended particle; nevertheless the extent of the irreversibility is small.