

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF RESEARCH ADMINISTRATION

Date: 4 March 1970

RESEARCH PROJECT INITIATION

Project Title: Evolution Systems and Product Integration

Project No.: B-1901

Project Director: Dr. James V. Herod

Sponsor: Research Corporation, 405 Lexington Avenue, New York, N.Y. 10017

Agreement Period: From 1 March 1970 until 28 February 1971

Type Agreement: Grant Letter dated 16 February 1970

Amount: \$2,640

Grant Administrator

Mr. Sam C. Smith
Vice President - Grants
Research Corporation
405 Lexington Avenue
New York, N.Y. 10017

Reports Required

Informal progress and publication of
results in professional journals.

Assigned to: School of Mathematics

COPIES TO: -

- ☒ Project Director
- ☒ School Director
- ☒ Dean of the College
- ☒ Administrator of Research
- ☒ Associate Controller (2)
- ☒ Security-Reports-Property Office
- ☒ Patent Coordinator

- ☒ Library
- ☒ Rich Electronic Computer Center
- ☒ Photographic Laboratory
- ☐ EES Machine Shop
- ☐ EES Accounting Office

Other File B-1901

GEORGIA INSTITUTE OF TECHNOLOGY

OFFICE OF RESEARCH ADMINISTRATION

RESEARCH PROJECT TERMINATION

Date: March 17, 1971

Project Title: Evolution Systems and Product Integration

Project No: B-1901

Principal Investigator: J. V. Herod

Sponsor: Research Corporation

Effective Termination Date: February 28, 1971

Clearance of Accounting Charges: All charges have cleared

Grant/Contract Closeout Actions Remaining:

Assigned to: School of Mathematics

COPIES TO:

Principal Investigator
School Director
Dean of the College
Director of Research Administration
Deputy Controller (2)
Security-Reports-Property Office
Patent and Inventions Coordinator

Library, Technical Reports Section ✓
Rich Electronic Computer Center
Photographic Laboratory
Project File
Other B-1901

REPORT OF RESEARCH CORPORATION GRANT

(Please check one)

(Submit original and one legible copy)

☐ Interim Report

☒ Terminal Report

INSTITUTION AND ADDRESS Georgia Institute of Technology
Atlanta, Georgia 30332

PRINCIPAL INVESTIGATOR James V. Herod

PHONE 894-2715

ACADEMIC RANK AND DEPARTMENT Associate Professor, School of Mathematics

SHORT TITLE OF RESEARCH SUPPORTED BY GRANT

Evolution systems and product integration

STARTING DATE June, 1970

SUMMARY OR PRINCIPAL FINDINGS AND THEIR SIGNIFICANCE (State succinctly in language understandable to one not necessarily expert in this field. Include extent to which original goals have been realized and any changes to original plan made or contemplated.)

Suppose that S is a Banach space, A is a function such that, if t is a number, then $A(t, \cdot)$ has domain all of S and values in S , and M is a function such that, if $x \leq z$ then $M(x, z)$ is a function from S to S satisfying $M(x, y)M(y, z)P = M(x, z)P$ for all y between x and z and all P in S . With this grant, I studied the relation $M(x, y)P = P + \int_x^y A(t, M(t, y)P)dt$ between A and M . Earlier work in this area included the case that, for $a > b$ the function $M(a, b)$ is "generated" by a Lipschitz function $A(t, \cdot)$ from S to S and $\lim_{a \rightarrow b} |[M(a, b) - 1]P - [M(a, b) - 1]Q| = 0$. In case $A(t, \cdot)$ is (not necessarily Lipschitz) continuous and $\lim_{a \rightarrow b} |M(a, b)P - P| = 0$ and, especially in case M arises from a one-parameter semigroup of nonlinear function, many investigations had been made. However, none had made the complete pairing of solutions to evolution systems with their generators in the non-Lipschitz case. I was able to make such a pairing and the results appear in "A pairing of a class of evolution systems with a class of generators", Transactions of the American Mathematical Society, Vol. 157, 1971.

Also during the period of the grant, revisions were made in the paper "Coalescence of solutions for nonlinear Streltjes equations" which was to appear in Journal für die Reine and Angewandte Mathematik, Vol. 272, 1972.

REPORT OF RESEARCH CORPORATION GRANT

SUMMARY OR PRINCIPAL FINDINGS AND THEIR SIGNIFICANCE (continued)

Also, with S as above and R the real numbers, I found a necessary and sufficient condition that a function A from $R \times S$ to S should be the uniform limit of a sequence of Lipschitz continuous, dissipative functions. This answered two questions negatively: Was the class I had found in the outline above just the closure - in the sense of uniform limits - of earlier Lipschitz classes? and Can you get solutions for the system

$$M(x,y)P = P + \int_x^y A(t, M(t,P))dt$$

if A is the uniform limit of Lipschitz functions? These results are published in "The uniform limit of Lipschitz functions on a Banach space", The Journal of the Australian Mathematical Society, Volume 15, 1973.

REPORT OF RESEARCH CORPORATION GRANT

Page 2

STUDENT PARTICIPATION (Give names of students working on the project, their role in the research, their achievements and their career plans.)

There were no students supported by the project. However, I had lengthy discussions with Robert H. Martin during his last year at Georgia Tech. The importance of these discussions to me are indicated on page 255 of my paper in the Transactions indicated above. Professor Martin is now on the faculty at North Carolina State University in Raleigh.

PAPERS AND SCIENTIFIC TALKS (Give titles and references to papers or talks resulting from the work. Attach two copies of any reprints available, if not previously forwarded.)

Colloquium address, Colorado State University: "Evolution systems and their generators."

Research report, Annual Meeting of American Mathematical Society: "A pairing of a class of evolution systems with a class of generators."

Papers are referenced in the summary.

OTHER SUPPORT (List amounts and sources—including institutional—of other contributions received or expected for this work.)

None

EXPENDITURE OF RESEARCH CORPORATION GRANT FUNDS (The terminal report should be approved by an authorized officer of the institution.)

a. Equipment, supplies (Itemize major expenditures)

None

b. Stipends (Academic status, rates, periods of appointment)

\$2,640, from June 1970 until September 1970.

c. Other expenditures (Itemize and give purpose)

None

Signature of Principal Investigator

Signature of authorized official

(Terminal report only)

Date

January 4, 1974

Date

Milton W. Bennett, Acting Director, Research Administration

Jan 4, 1974

1/4/74