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ACTIVE NETWORK SYNTHESIS USING THE GENERALIZED POSITIVE IMPEDANCE CONVERTER

A THESIS

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ACTIVE NETWORK SYNTHESIS USING THE

GENERALIZED POSITIVE IMPEDANCE CONVERTER

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SUMMARY

In this research an investigation is made into the feasibility of the use of the generalized positive impedance converter (GPIC) in the synthesis of the open-circuit voltage transfer function, the driving-point admittance function, the multi-port open-circuit voltage transfer matrix, and the short-circuit admittance matrix.

The active device, the GPIC, is defined in terms of its chain matrix. Several electronic circuits, two of which are thought to be novel, are presented. The circuits have as components differential-input operational amplifiers, resistors, and capacitors.

The approach used in developing the open-circuit voltage transfer function, T(s), synthesis procedure is similar to the one developed by Antoniou[5] in his T(s) synthesis procedure using the general immittance converter. The approach is to find a network whose short-circuit admittance parameters, $-y_{21}$ and y_{22}, can be equated respectively to the numerator and denominator polynomials of a real rational T(s) function. Such a network is found and a synthesis procedure is developed. No factorization of the voltage transfer function is needed, and the network elements are related simply to the transfer function coefficients. Each numerator coefficient of the voltage transfer function is proportional to a distinct network element and each denominator coefficient is proportional to the sum of two distinct network elements. The synthesis procedure will realize any arbitrary T(s) which is a real rational function in the complex variable s.

The approach used in developing the driving-point admittance function, Y(s), synthesis procedure is similar to the approach used by Hilberman and Joseph[7] for realizing Y(s) with a circuit that contained two unity-gain amplifiers and a network with an open-circuit voltage transfer function, T(s). The ideal unity-gain amplifiers used by Hilberman and Joseph are replaced by two special GPIC's. Use of these two GPIC's allows the synthesis of the desired Y(s) to be transferred to the synthesis of a particular T(s). This T(s) function can be synthesized according to the open-circuit voltage transfer function procedure. This procedure will realize any Y(s) which is a real rational function in the complex variable s.

The synthesized networks for the T(s) and Y(s) synthesis procedures use only GPIC's and resistors and are grounded. These realizations are producible by integrated-circuit fabrication techniques. The number of capacitors used in any synthesized network is equal to the order of T(s) or Y(s). Hence the synthesis procedures use the minimum number of capacitors. The capacitors are incorporated as internal elements in the GPIC devices.

Sensitivity is defined and discussed. A sensitivity investigation of the resulting realizations of the T(s) and Y(s) synthesis procedures is made.

For the network that realizes the unconditionally stable T(s), the coefficient sensitivity terms can always be made less than or equal to one in absolute value. In the case of the absolutely stable, second-order T(s) network, the selectivity and undamped natural frequency sensitivity terms can always be made less than or equal to one in absolute value. It is concluded that the T(s) synthesis realizations exhibit coefficient, selectivity, and undamped natural frequency sensitivities of the same order of magnitude as the realizations of Antoniou,[5] which are claimed to have low sensitivity. The T(s) realizations offer a definite improvement in selectivity sensitivity over a particular negative impedance converter circuit reported by Newcomb.[1] It is demonstrated that coefficient sensitivity terms of the resulting realizations of the Y(s) procedure can always be made less than or equal to one in absolute value, approximately.

The fact that each GPIC used in the T(s) and Y(s) synthesis procedures has two degrees of freedom, namely the specification of the active gain constants k_1 and k_2, can be used in the control of component values external to the GPIC devices. The unrestricted assignment of GPIC gain constants allows the network realizations of the T(s) and Y(s) synthesis procedures to be completed with component values external to the GPIC devices that fall into a prescribed range.

For completeness, a synthesis procedure is developed for realizing any multi-port open-circuit voltage transfer matrix whose elements are real rational functions in the complex variable s. The approach used to realize the matrix is to realize the matrix one row at a time. The network realization for each row is very similar to the network used to realize T(s). Again no factorization of the row elements is needed, and the network elements are simply related to the coefficients of each of the row elements of the transfer matrix. Each numerator coefficient of each row element is proportional to a distinct network element and each denominator coefficient of each row is proportional to the sum of (n+1) distinct network elements, where n is equal to the number of row elements in the multi-port open-circuit voltage transfer matrix. The network realization is grounded.

A synthesis procedure is developed for realizing any short-circuit admittance matrix whose elements are real rational functions in the complex variable s. The approach used in this development is similar to the work done by Hilberman and Joseph.[7] The ideal unity-gain amplifiers used by Hilberman and Joseph are replaced with special GPIC's. Use of these GPIC's allows the synthesis of the desired short-circuit admittance matrix to be transferred to the synthesis of a multi-port open-circuit voltage transfer matrix. Again, the network realization is grounded.

Numerical examples are included to illustrate each of the four synthesis procedures. The practicality of the T(s) and Y(s) synthesis procedures is demonstrated by experimental results. The results of the experimental realization through the use of the GPIC of the negative resistance and the inductance are presented.

CHAPTER I

INTRODUCTION

The rapid development of the transistor has generated considerable interest in active network theory. Low-cost active elements have made it possible for the network synthesist to replace the expensive, bulky inductor and to extend the range of realizable functions far beyond that possible with RLC networks.

The equally rapid development of thin-film and monolithic integrated circuits has encouraged the network synthesist to go one step forward--to eliminate the capacitor to the extent possible. Once a circuit configuration for an integrated or thin-film circuit is obtained, the capacitance of the circuit is determined by capacitor area. Consequently, it is desirable to minimize the total capacitance in the design of a circuit. Generally, this means a minimization in the number of capacitors; however, there may be special cases where this minimization in number may not yield the minimum total capacitance.[1]

It has been pointed out that processing and biasing for monolithic capacitors almost always make the choice of minimum number of capacitors over minimum capacitance the desirable one.[1] Also, it is often desirable to have all capacitors with a common plate (terminal).

Both the monolithic and thin-film integrated-circuit fabrication technologies require that resistor values and capacitor values be within a certain range. Thus it becomes desirable to be able to control the range of the passive element values in order to make the network producible with the new technologies.

In all active circuits, groundedness and low sensitivity with respect to both active and passive parameters are of much importance.

Kerwin, Huelsman, and Newcomb[2] addressed themselves to the integrated-circuit compatibility with an open-circuit voltage transfer function synthesis procedure based on the state-variable approach. The procedure renders a network that is grounded, contains the minimum number of capacitors, and uses only integrated-circuit operational amplifiers, resistors, and grounded capacitors. The synthesis technique has low sensitivity but appears to have no control over the range of passive element values.

Goldman and Ghausi[3] have developed a procedure for the synthesis of grounded active RC N-ports with a prescribed range of element values. The procedure uses resistors, operational amplifiers, voltage amplifiers, and more than the minimum number of capacitors.

The generalized positive impedance converter (GPIC) can be characterized by the chain matrix

$$\begin{bmatrix} \pm k_1 & 0 \\ 0 & \pm k_2 f(s) \end{bmatrix}$$

where the algebraic signs associated with k_1 and k_2 are the same and f(s) is a rational function of s. The GPIC has been used in synthesis procedures by Gorski-Popiel[4] and Antoniou[5] to realize the open-circuit voltage transfer function. Gorski-Popiel used the GPIC to realize the inductance in an L-C synthesis. Antoniou used it directly to realize the open-circuit voltage transfer function in terms of the short-circuit admittance parameters, $-y_{21}$ and y_{22}. Neither Antoniou nor Gorski-Popiel has attempted to minimize the number of capacitors in his synthesis procedure.

The purpose of this research is to investigate the feasibility and possible advantages of using the GPIC in the synthesis of the open-circuit voltage transfer function, the driving-point admittance function, the multi-port open-circuit voltage transfer matrix, and the short-circuit admittance matrix. In this thesis, open-circuit voltage transfer function and driving-point admittance function synthesis procedures will be developed, which realize arbitrary real rational functions in the complex variable s and have the following desirable characteristics:

1. The procedures require the minimum number of capacitors necessary for the synthesis.

2. The procedures are compatible with integrated-circuit fabrication technology.

3. The procedures have low sensitivity factors with respect to both active and passive parameters of certain functions of interest.

4. The procedures can exercise control over the range of element values external to the GPIC's.

5. The network realizations are grounded.

The open-circuit voltage transfer function synthesis procedure will be extended to a synthesis procedure that will realize an open-circuit voltage transfer matrix whose elements are real rational functions in the complex variable s. The driving-point admittance function synthesis procedure will be extended to a synthesis procedure that will realize a short-circuit admittance matrix whose elements are real rational functions in the complex variable s. The network realizations for these two synthesis procedures will be grounded.

The approach used in developing each of the synthesis procedures is similar. In the case of the open-circuit voltage transfer function synthesis, a network is found whose short-circuit admittance parameters, $-y_{21}$ and y_{22}, can be equated respectively to the numerator and denominator polynomials of a rational open-circuit voltage transfer function.

CHAPTER II

THE GENERALIZED POSITIVE IMPEDANCE CONVERTER

The generalized positive impedance converter (GPIC) can be characterized by the chain matrix

$$\begin{bmatrix} \pm k_1 & 0 \\ 0 & \pm k_2 f(s) \end{bmatrix} \tag{1}$$

where the algebraic signs associated with k_1 and k_2 are the same and $f(s)$ is a rational function of s. If port 2 of the device is terminated in Z_L, then the impedance looking into port 1 is

$$Z_{in} = \frac{k_1 Z_L}{k_2 f(s)} \tag{2}$$

If $f(s) = 1$, a special case of the GPIC known as the positive impedance converter (PIC) results.

Antoniou[5] has described a simple device (Figure 1) with the chain matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & \dfrac{Z_2 Z_4}{Z_1 Z_3} \end{bmatrix} \tag{3}$$

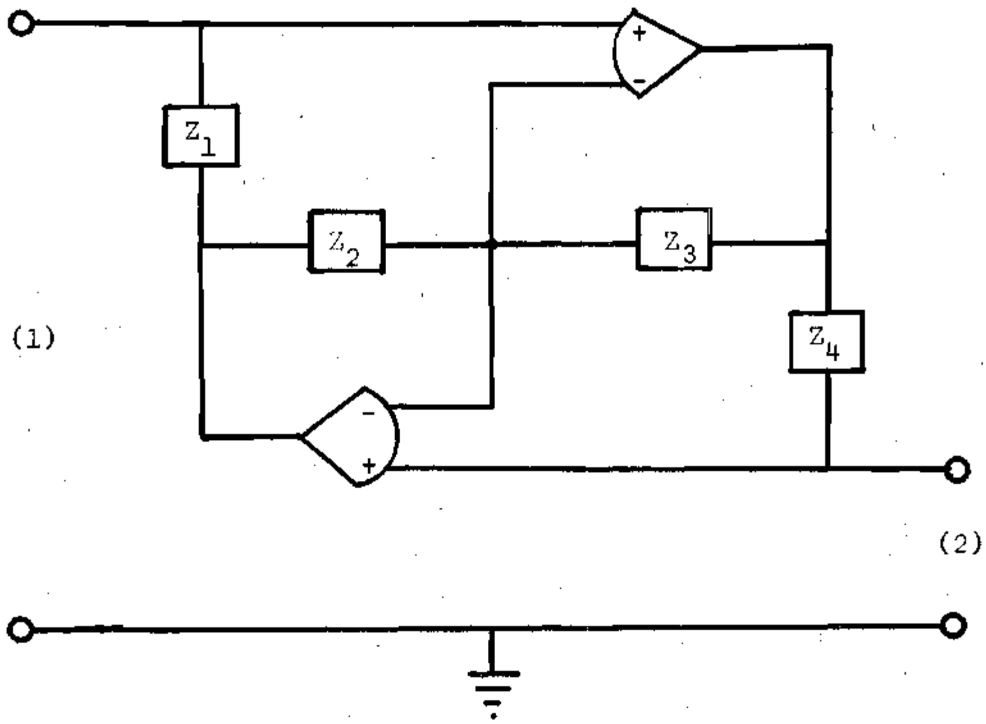


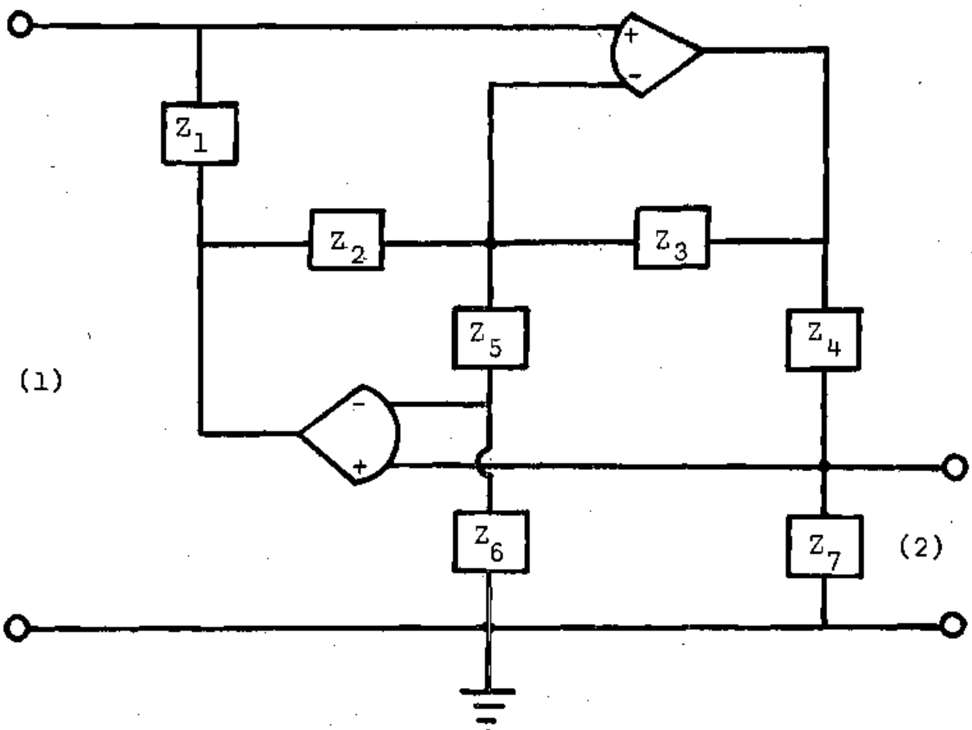
Figure 1. General Immittance Converter of Antoniou

He refers to this device as the general immittance converter (GIC). The GIC is also a special case of the GPIC. In order to obtain the results of (3), it is assumed that the operational amplifiers are ideal. Namely, the input impedance is infinite, the output impedance is zero, and the bandwidth is infinite. These same assumptions are made throughout the completion of this chapter in the analysis of any circuit which contains an operational amplifier.

The circuit of Figure 1 can be modified to yield a more versatile device with a chain matrix

$$\begin{bmatrix} \frac{Z_5+Z_6}{Z_6} & 0 \\ 0 & \frac{Z_2Z_4}{Z_1Z_3} \end{bmatrix} \tag{4}$$

The modified circuit is shown in Figure 2. The development or derivation of this novel circuit is given in detail in Appendix II.

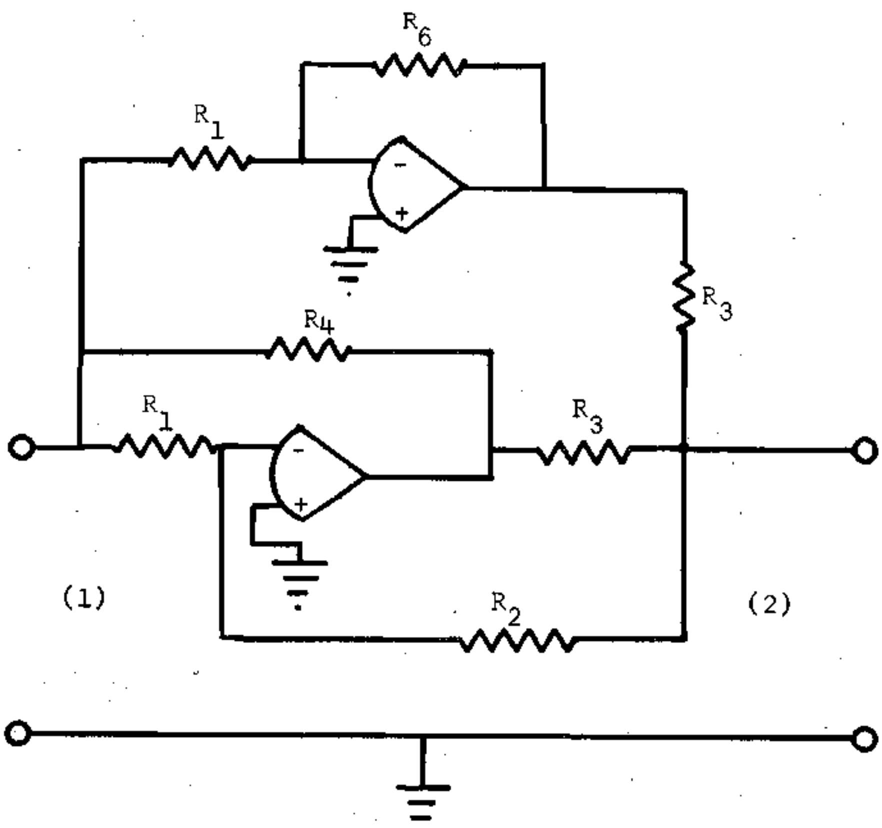


$$Z_7' = \frac{Z_4Z_6}{Z_3+Z_5}$$

Figure 2. Generalized Positive Impedance Converter with the Chain Matrix of (4)

In Figure 3 there is shown a novel positive impedance converter (PIC) circuit with the chain matrix

$$\begin{bmatrix} -\frac{R_1}{R_2} & 0 \\ 0 & -\frac{R_3}{R_4} \end{bmatrix} \tag{5}$$



$R_6 = R_1 + R_3 + 2(R_2+R_4)$

Figure 3. Positive Impedance Converter with the Chain Matrix of (5)

For a detailed analysis of this circuit, see Appendix III.

Cox, Su, and Woodward[6] have developed a circuit which they have named the universal impedance converter (UIC). It can serve as both a PIC and a negative impedance converter (NIC). Now the UIC can be modified to yield the most general GPIC with the chain matrix of (1). Figure 4 shows the UIC so modified. With S_1 open, S_2 closed, the circuit has the chain matrix

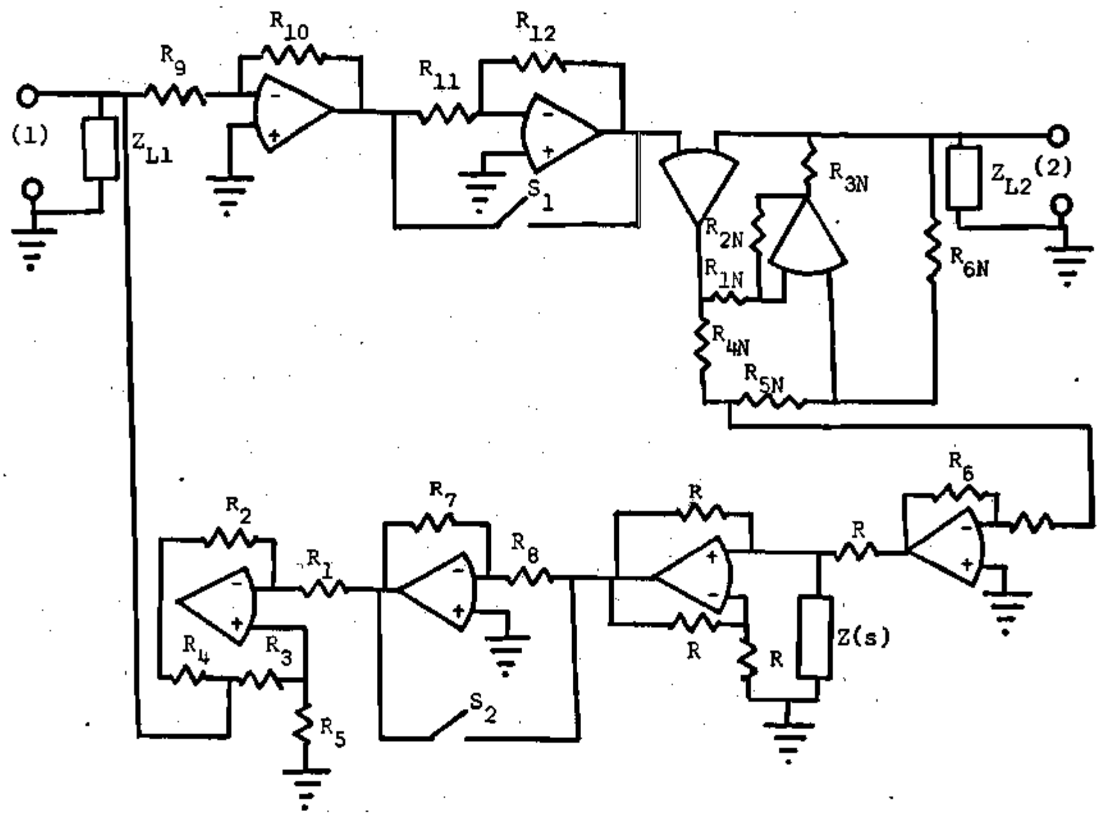
$$\begin{bmatrix} \frac{R_{10}R_{12}}{R_9R_{11}} & 0 \\ 0 & \frac{2R_2R_6}{aR_4R_5R} Z(s) \end{bmatrix} \tag{6}$$

With S_1 closed, S_2 open, it has the chain matrix

$$\begin{bmatrix} -\frac{R_{10}}{R_9} & 0 \\ 0 & -\frac{2R_2R_6R_7}{aR_4R_5R_8R} Z(s) \end{bmatrix} \tag{7}$$

Matrices (6) and (7) result provided R_9 is much larger than any loading impedance at port 1 of Figure 4.

If f(s) in (1) is made equal to $\frac{1}{s}$, then it will be possible to use the GPIC's developed in this chapter in open-circuit voltage transfer function and driving-point admittance function synthesis procedures



Z_{L1}, Z_{L2} - Loading at Ports 1 and 2, respectively.

$$\frac{R_{4N}}{R_{3N}} = \frac{R_{5N}}{R_{6N}} = \frac{R_{1N}}{R_{2N}}$$

$$\frac{R_2}{R_3+R_4} = \frac{R_1}{R_5} = a$$

$$R_9 >> Z_{L1}$$

Figure 4. Generalized Positive Impedance Converter Based on the UIC

that use the minimum number of capacitors necessary for the synthesis. With this choice of f(s), (1) becomes

$$\begin{bmatrix} \pm k_1 & 0 \\ 0 & \pm \frac{k_2}{s} \end{bmatrix} \tag{8}$$

It is obvious that the chain matrix of (8) can be realized by the modified UIC of Figure 4 if Z(s) is made the reactance of a capacitance. Note that this capacitor is grounded, an often desirable feature in hybrid integrated circuits.

Now somewhat less complex realizations of the chain matrix of (8) can be obtained from the previously discussed circuits of Figures 1, 2, and 3. If $Z_1 = R_1$, $Z_2 = \frac{1}{sC}$, $Z_3 = R_3$, and $Z_4 = R_4$ in Figure 1, the chain matrix of (3) becomes

$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{R_4}{sR_1R_3C} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{k_{21}}{s} \end{bmatrix} \tag{9}$$

If $Z_1 = R_1$, $Z_2 = \frac{1}{sC}$, $Z_3 = R_3$, $Z_4 = R_4$, $Z_5 = R_5$, $Z_6 = R_6$, and $Z_7 = \frac{R_4R_6}{R_3 + R_5}$ in Figure 2, the chain matrix of (4) becomes

$$\begin{bmatrix} \frac{R_5 + R_6}{R_6} & 0 \\ 0 & \frac{R_4}{sR_1R_3C} \end{bmatrix} = \begin{bmatrix} k_{12} & 0 \\ 0 & \frac{k_{22}}{s} \end{bmatrix} \tag{10}$$

Now the chain matrix of the circuit of Figure 3 is

$$\begin{bmatrix} -\frac{R_1}{R_2} & 0 \\ 0 & -\frac{R_3}{R_4} \end{bmatrix} = \begin{bmatrix} -k_{13} & 0 \\ 0 & -k_{23} \end{bmatrix} \tag{11}$$

The desired chain matrix of (8), when the positive algebraic signs and $k_1 \geq 1$ (the modification in Figure 2 necessary to set a $k_1 \leq 1$ is discussed in Appendix II) are required by the synthesis procedure, can be realized with the circuit of Figure 2. The chain matrix of (8) with associated negative signs can be realized by a cascade arrangement of the circuits of Figures 1 and 3. Such a cascaded circuit is shown in Figure 5. The chain matrix is given by

$$\begin{bmatrix} -\frac{R_4}{R_6} & 0 \\ 0 & -\frac{R_3R_7}{sR_1R_2R_5C} \end{bmatrix} = \begin{bmatrix} -k_{15} & 0 \\ 0 & -\frac{k_{25}}{s} \end{bmatrix} \tag{12}$$

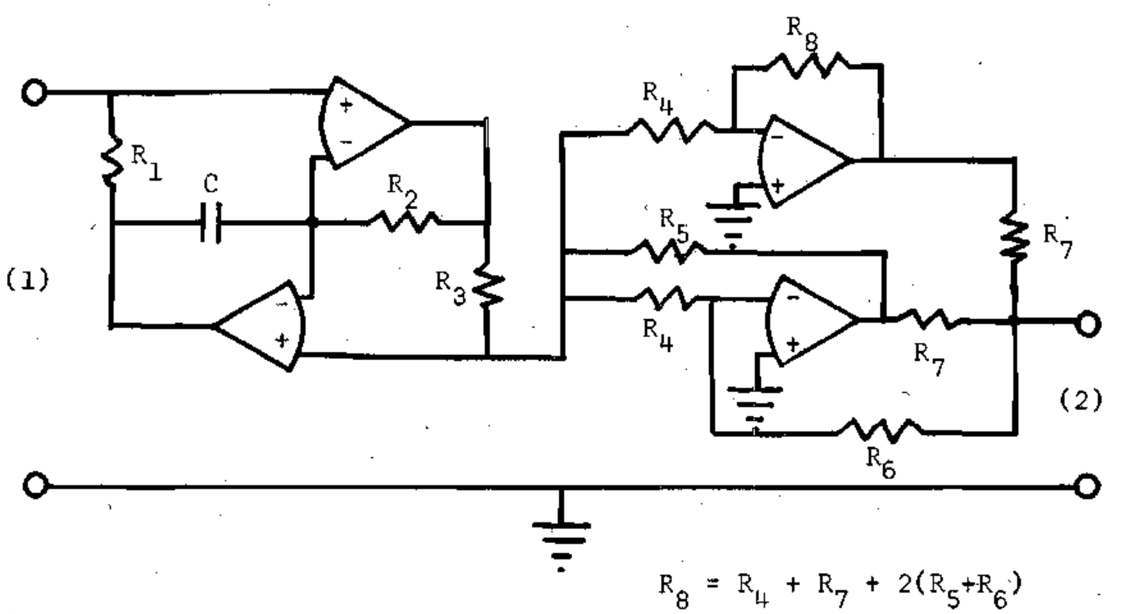


Figure 5. Generalized Positive Impedance Converter with Negative Chain Matrix Terms

The circuits of Figures 2 and 5 contain one capacitor each; however, the capacitor is not grounded.

One degenerate case of the GPIC is of interest in the driving-point synthesis procedure. It can be described by the chain matrix

$$\begin{bmatrix} k_1 & 0 \\ 0 & 0 \end{bmatrix} \tag{13}$$

If $k_1 = 1$, then the matrix of (13) can be realized with the circuit of Figure 1 with $Z_1 = R_1$, $Z_2 = R_2$, $Z_3 = R_3$, $Z_4 = 0$. Figure 1 so modified is shown in Figure 6.

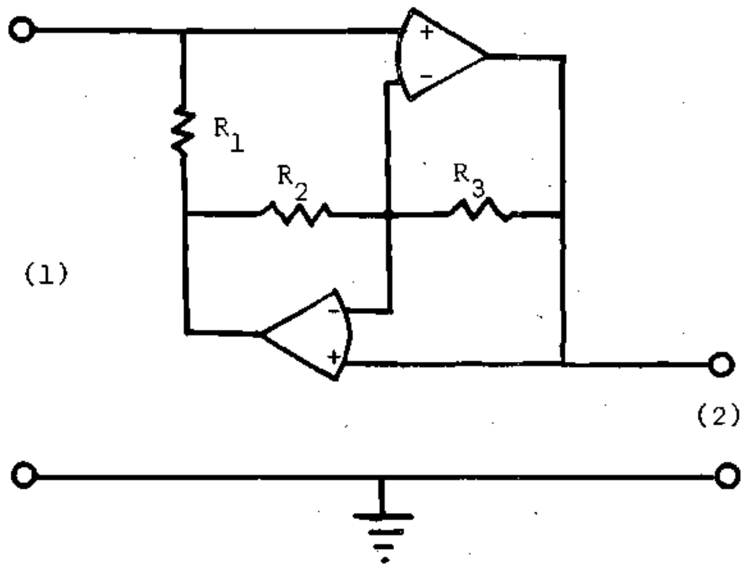


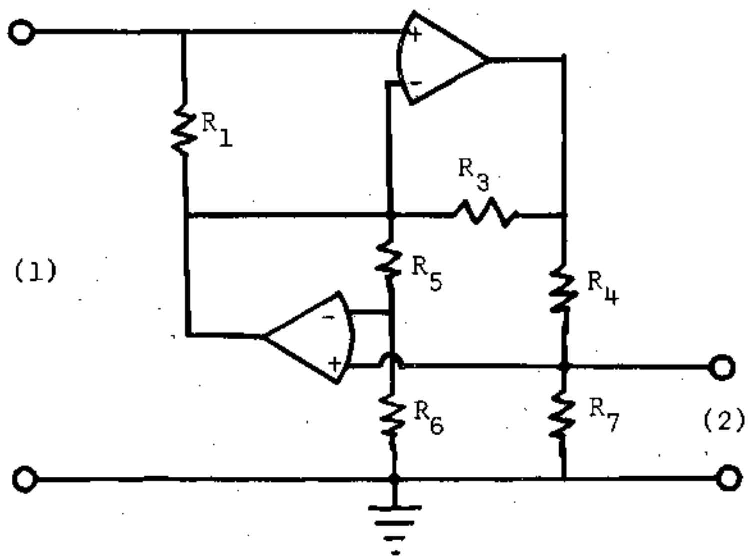
Figure 6. Degenerate GPIC with $k_1 = 1$

If $k_1 \neq 1$, the matrix of (13) can be realized with the circuit of Figure 2 with $Z_1 = R_1$, $Z_2 = 0$, $Z_3 = R_3$, $Z_4 = R_4$, $Z_5 = R_5$, $Z_6 = R_6$, and $Z_7 = \frac{R_4 R_6}{R_3 + R_5}$. Then

$$k_1 = \frac{R_5 + R_6}{R_6} \tag{14}$$

Figure 2 so modified is shown in Figure 7. Note that this circuit gives a k_1 greater than one. To get a k_1 less than one, interchange ports 1 and 2 in Figure 7. Then k_1 of (13) in terms of the elements of Figure 7 becomes

$$k_1 = \frac{R_6}{R_5 + R_6} \tag{15}$$



$$R_7 = \frac{R_4 R_6}{R_3 + R_5}$$

Figure 7. Degenerate GPIC with $k_1 \neq 1$

Note that each circuit that has been discussed for the possible realization of the GPIC devices contains only operational amplifiers, resistors, and capacitors. Thus each realization has the potential of being fabricated by the monolithic and/or thin-film integrated-circuit processes.

CHAPTER III

SYNTHESIS OF THE OPEN-CIRCUIT VOLTAGE TRANSFER FUNCTION

The approach used in developing the open-circuit voltage transfer function, T(s), synthesis procedure is similar to the one developed by Antoniou[5] in his T(s) synthesis procedure using the general immittance converter. The approach is to find a network whose short-circuit admittance parameters, $-y_{21}$ and y_{22}, can be equated respectively to the numerator and denominator polynomials of a real rational T(s) function. Such a network is found and a synthesis procedure is developed. No factorization of the voltage transfer function is needed, and the network elements are related simply to the transfer function coefficients. Each numerator coefficient of the voltage transfer function is proportional to a distinct network element and each denominator coefficient is proportional to the sum of two distinct network elements. The synthesis procedure will realize any arbitrary T(s) which is a real rational function in the complex variable s.

Consider first the open-circuit voltage transfer function

$$T(s) = \frac{E_2}{E_1} = \frac{\sum_{i=0}^{n} a_i s^i}{\sum_{j=0}^{m} b_j s^j} \qquad (16)$$

From two-port network theory,

$$T(s) = \frac{-y_{21}}{y_{22}} \tag{17}$$

where y_{21} and y_{22} are the network short-circuit admittance parameters. It is apparent that any T(s) can be realized for a particular two-port network if,

$$-y_{21} = \sum_{i=0}^{n} a_i s^i$$

and (18)

$$y_{22} = \sum_{j=0}^{m} b_j s^j$$

If the numerator and denominator of T(s) are divided by s^q where q is the order of the transfer function [q=max(m,n)], then T(s) is realized if,

$$-y_{21} = \sum_{i=0}^{n} a_i s^{i-q} \tag{19}$$

and

$$y_{22} = \sum_{j=0}^{m} b_j s^{j-q} \tag{20}$$

Now consider the circuit of Figure 8 which employs a GPIC described by the matrix equation

$$\begin{bmatrix} V_1' \\ \\ I_1' \end{bmatrix} = \begin{bmatrix} k_{1r} & 0 \\ \\ 0 & \frac{k_{2r}}{s} \end{bmatrix} \begin{bmatrix} V_2' \\ \\ -I_2' \end{bmatrix} \tag{21}$$

where k_{1r} and k_{2r} can be positive or negative constants but must have the same algebraic sign. The arrow in the GPIC block of Figure 8 and subsequent figures is directed from port 1 to port 2 of each individual GPIC. This will enable each GPIC to be described by its chain matrix.

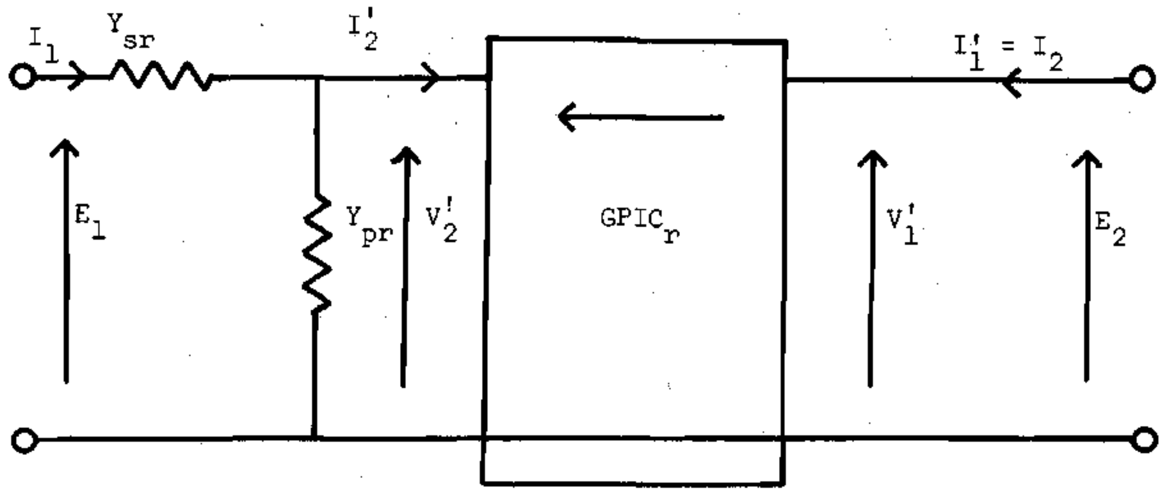


Figure 8. Basic Network for Generating T(s)

Analysis of the circuit of Figure 8 yields

$$y_{21} = -\frac{k_{2r}}{s} Y_{sr} \tag{22}$$

and

$$y_{22} = \frac{k_{2r}}{sk_{1r}} (Y_{sr}+Y_{pr}) \tag{23}$$

Consider next the (q+1) cascaded GPIC's in the grounded network of Figure 9 which is to be used to realize the open-circuit voltage transfer function of order q. The GPIC's of Figure 9 are characterized as follows by their chain matrices:

$$\mathrm{GPIC}_0 : \begin{bmatrix} k_{10} & 0 \\ 0 & k_{20} \end{bmatrix}$$

and (24)

$$\mathrm{GPIC}_r : \begin{bmatrix} k_{1r} & 0 \\ 0 & \frac{k_{2r}}{s} \end{bmatrix}$$

where $r=1,2,\ldots,q$. Analysis for the network of Figure 9 reveals that

$$y_{21} = - k_{20}Y_{s0} - \frac{k_{21}Y_{s1}}{s} - \frac{k_{21}k_{22}Y_{s2}}{s^2} - \ldots - \frac{k_{21}k_{22}\ldots k_{2q}Y_{sq}}{s^q} \tag{25}$$

and

$$y_{22} = \frac{k_{20}}{k_{10}} (Y_{s0}+Y_{p0}) + \frac{k_{21}}{sk_{11}} (Y_{s1}+Y_{p1}) \tag{26}$$

$$+ \frac{k_{21}k_{22}}{s^2k_{11}k_{12}} (Y_{s2}+Y_{p2}) + \ldots + \frac{k_{21}k_{22}\ldots k_{2q}}{s^qk_{11}k_{12}\ldots k_{1q}} (Y_{sq}+Y_{pq})$$

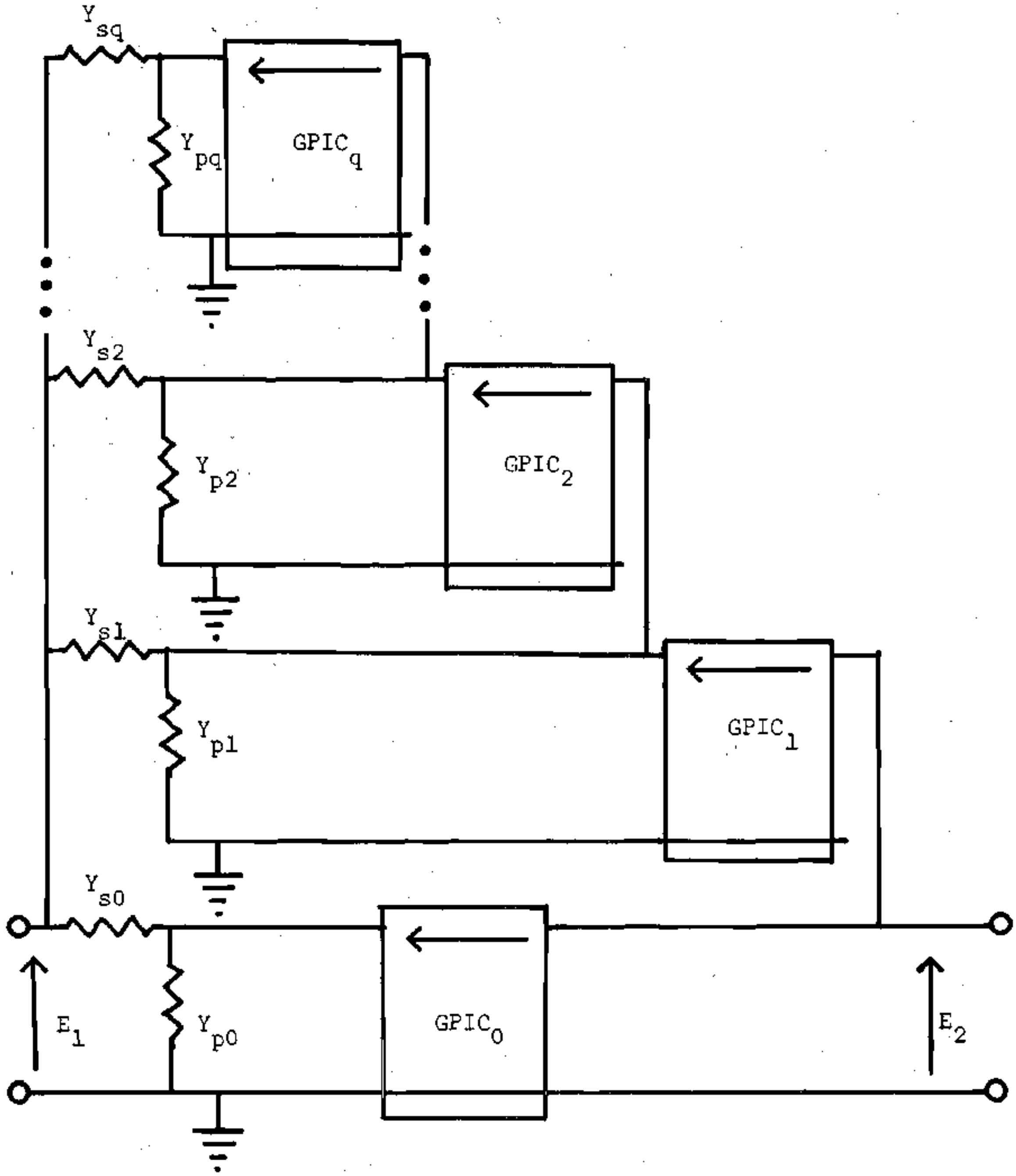


Figure 9. Network for Realizing qth Order T(s)

Hence,

$$T(s) = \frac{k_{20}Y_{s0} + \frac{k_{21}}{s}Y_{s1} + \frac{k_{21}k_{22}}{s^2}Y_{s2} + \ldots + \frac{k_{21}k_{22}\ldots k_{2q}}{s^q}Y_{sq}}{\frac{k_{20}}{k_{10}}(Y_{s0}+Y_{p0}) + \frac{k_{21}}{sk_{11}}(Y_{s1}+Y_{p1}) + \frac{k_{21}k_{22}}{s^2k_{11}k_{12}}(Y_{s2}+Y_{p2}) + \ldots + \frac{k_{21}k_{22}\ldots k_{2q}}{s^q k_{11}k_{12}\ldots k_{1q}}(Y_{sq}+Y_{pq})} \tag{27}$$

Now the admittance values of the network realization can be obtained in terms of the coefficients of the transfer function and gain constants k's of the GPIC's by equating term by term the equations of (25) and (19) and the equations of (26) and (20). Whence we have:

$$Y_{s0} = \frac{a_q}{k_{20}} \tag{28}$$

$$Y_{p0} = \frac{k_{10}b_q}{k_{20}} - \frac{a_q}{k_{20}}$$

$$Y_{s1} = \frac{a_{q-1}}{k_{21}}$$

$$Y_{p1} = \frac{k_{11}b_{q-1}}{k_{21}} - \frac{a_{q-1}}{k_{21}}$$

$$Y_{s2} = \frac{a_{q-2}}{k_{21}k_{22}}$$

$$Y_{p2} = \frac{k_{11}k_{12}b_{q-2}}{k_{21}k_{22}} - \frac{a_{q-2}}{k_{21}k_{22}}$$

$$\vdots$$

$$Y_{sr} = \frac{a_{q-r}}{k_{21}k_{22}\cdots k_{2r}}$$

$$Y_{pr} = \frac{k_{11}k_{12}\cdots k_{1r}b_{q-r}}{k_{21}k_{22}\cdots k_{2r}} - \frac{a_{q-r}}{k_{21}k_{22}\cdots k_{2r}}$$

where $r=1,2,\ldots,q$.

From (28), it is apparent that a negative coefficient in the numerator of $T(s)$, i.e., a_{q-r}, is accommodated by appropriately choosing the algebraic sign of k_{2r} and consequently k_{1r} in order to avoid a negative Y_{sr}. Another point is revealed in (28). If $b_{q-r} > 0$, and k_{1r} can take on any value, then Y_{pr} can always be made positive and real for $r=0,1,2,\ldots,q$. However, the range on k_{1r} may be limited such that Y_{pr} may need to be a negative admittance. Also, if $b_{q-r} \le 0$, then Y_{pr} may need to be negative. Negative Y_{pr}'s will require realizations by Kim's method (see Appendix I) which is grounded and employs a resistance and a GPIC with the chain matrix

$$\begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \tag{29}$$

Several observations can be made regarding the open-circuit voltage transfer function synthesis. By virtue of the synthesis technique, it is obviously grounded. The only GPIC circuits not shown in Figure 9 that may be required in the synthesis are those used in realizing negative admittances. These GPIC's do not require capacitors. Of the q+1 GPIC's shown in Figure 9, q require one capacitor each and can be realized by one of the configurations discussed in Chapter II. It was pointed out in Chapter II that each of the configurations are compatible with hybrid integrated-circuit fabrication technologies. The resistive network coupled to the GPIC's in Figure 9 is also easily handled by the integrated-circuit technology. Thus it is concluded that the overall network for the synthesis of the open-circuit voltage transfer function is compatible with hybrid integrated-circuit technology.

The minimum number of reactive elements for the transfer function has been discussed by Newcomb.[1] He has pointed out that the minimum number of reactive elements needed to realize a rational transfer function of order q is q. The T(s) GPIC synthesis procedure realizes a qth order transfer function with q capacitors--the minimum number required.

Since the minimum number of reactive elements is used in the synthesis, all poles and zeros in the synthesized network are accounted for. Hence the problem of unobservable or uncontrollable modes of possible oscillation does not exist. Therefore, if T(s) has stable poles, then the circuit does also.

Some thought reveals that $GPIC_0$ of Figure 9 is not always necessary in the synthesis procedure. $GPIC_0$'s functions are

(1) to eliminate the necessity for having to realize a negative admittance Y_{p0} for certain desired transfer functions,

(2) to control the range of admittance values of Y_{s0} and Y_{p0}.

These two functions of $GPIC_0$ may not be desired or needed in certain applications.

Elimination of $GPIC_0$ is equivalent to letting

$$k_{10} = k_{20} = 1 \tag{30}$$

in (28). Elimination of $GPIC_0$ results in a modification in Figure 9. This modification is shown in Figure 10. Hence, to synthesize the transfer function of (16) without $GPIC_0$ of Figure 9, the network of Figure 10 can be used. The element values of Figure 10 for the successful synthesis are given by the following:

$$Y_{s0} = a_q \tag{31}$$

$$Y_{p0} = b_q - a_q$$

$$Y_{sr} = \frac{a_{q-r}}{k_{21}k_{22}\cdots k_{2r}}$$

$$Y_{pr} = \frac{k_{11}k_{12}\cdots k_{1r}b_{q-r}}{k_{21}k_{22}\cdots k_{2r}} - \frac{a_{q-r}}{k_{21}k_{22}\cdots k_{2r}}$$

for $r=1,2,\ldots,q$.

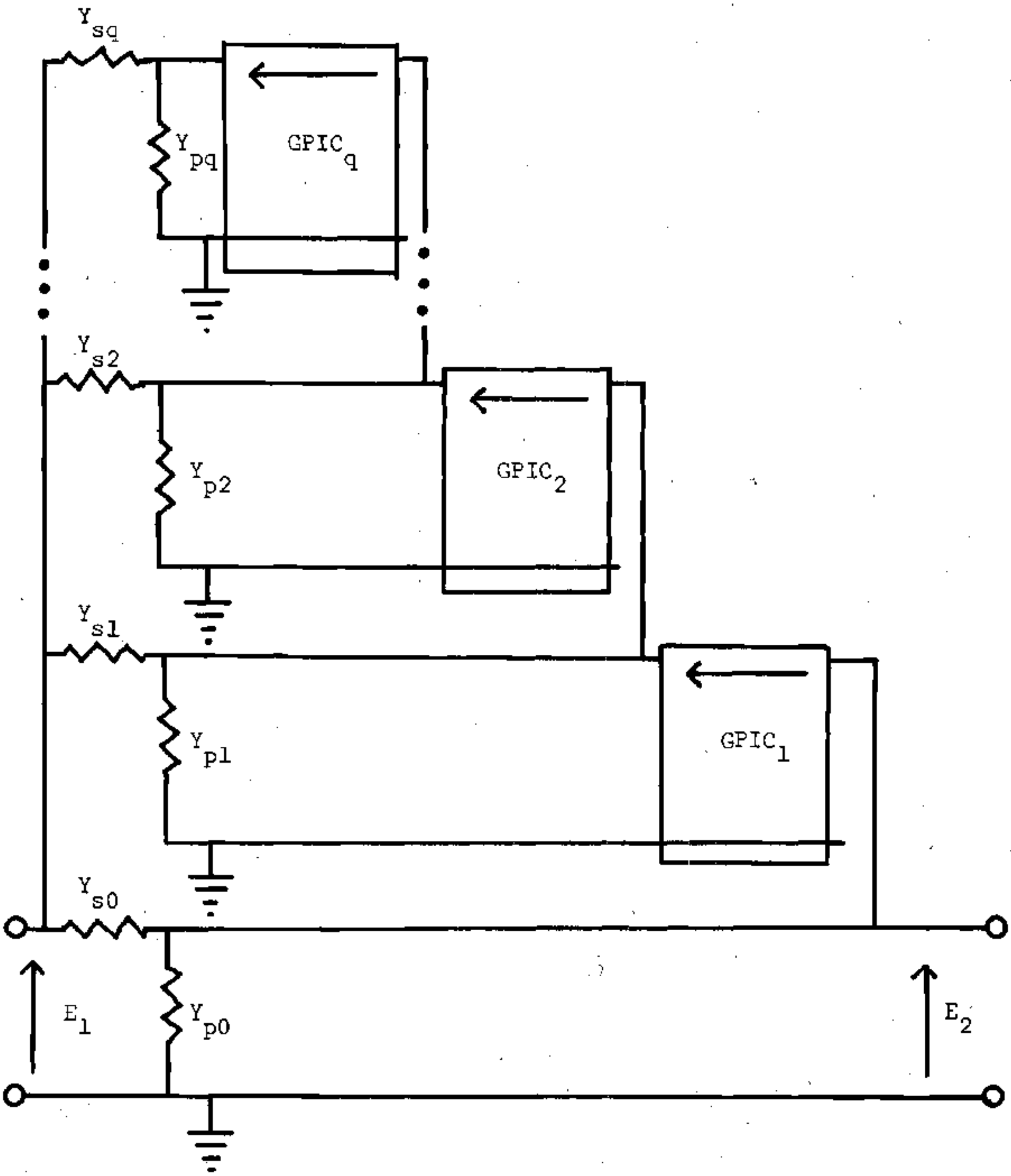


Figure 10. Modified Network for Realizing qth Order T(s)

An Example

To illustrate the open-circuit voltage transfer function synthesis,

$$T(s) = \frac{s^2 + 3s - 2}{s^2 + 2s + 3} \tag{32}$$

will be synthesized according to the theory of this chapter. Now the order, q, of the transfer function is two. From (28) we obtain the expressions for the necessary parameter values.

$$Y_{s0} = \frac{a_2}{k_{20}} \tag{33}$$

$$Y_{p0} = \frac{k_{10}b_2}{k_{20}} - \frac{a_2}{k_{20}}$$

$$Y_{s1} = \frac{a_1}{k_{21}}$$

$$Y_{p1} = \frac{k_{11}b_1}{k_{21}} - \frac{a_1}{k_{21}}$$

$$Y_{s2} = \frac{a_0}{k_{21}k_{22}}$$

$$Y_{p2} = \frac{k_{11}k_{12}b_0}{k_{21}k_{22}} - \frac{a_0}{k_{21}k_{22}}$$

Comparing the T(s) to be synthesized with (16), it is apparent that

$$a_0 = -2 \tag{34}$$

$$a_1 = 3$$

$$a_2 = 1$$

$$b_0 = 3$$

$$b_1 = 2$$

$$b_2 = 1$$

Let

$$k_{10} = k_{20} = k_{11} = k_{21} = 1$$
$$k_{12} = k_{22} = -1 \tag{35}$$

Then the synthesis can be completed with the following parameter values in units of mhos.

$$Y_{s0} = 1 \tag{36}$$

$$Y_{p0} = 0$$

$$Y_{s1} = 3$$

$$Y_{p1} = -1$$

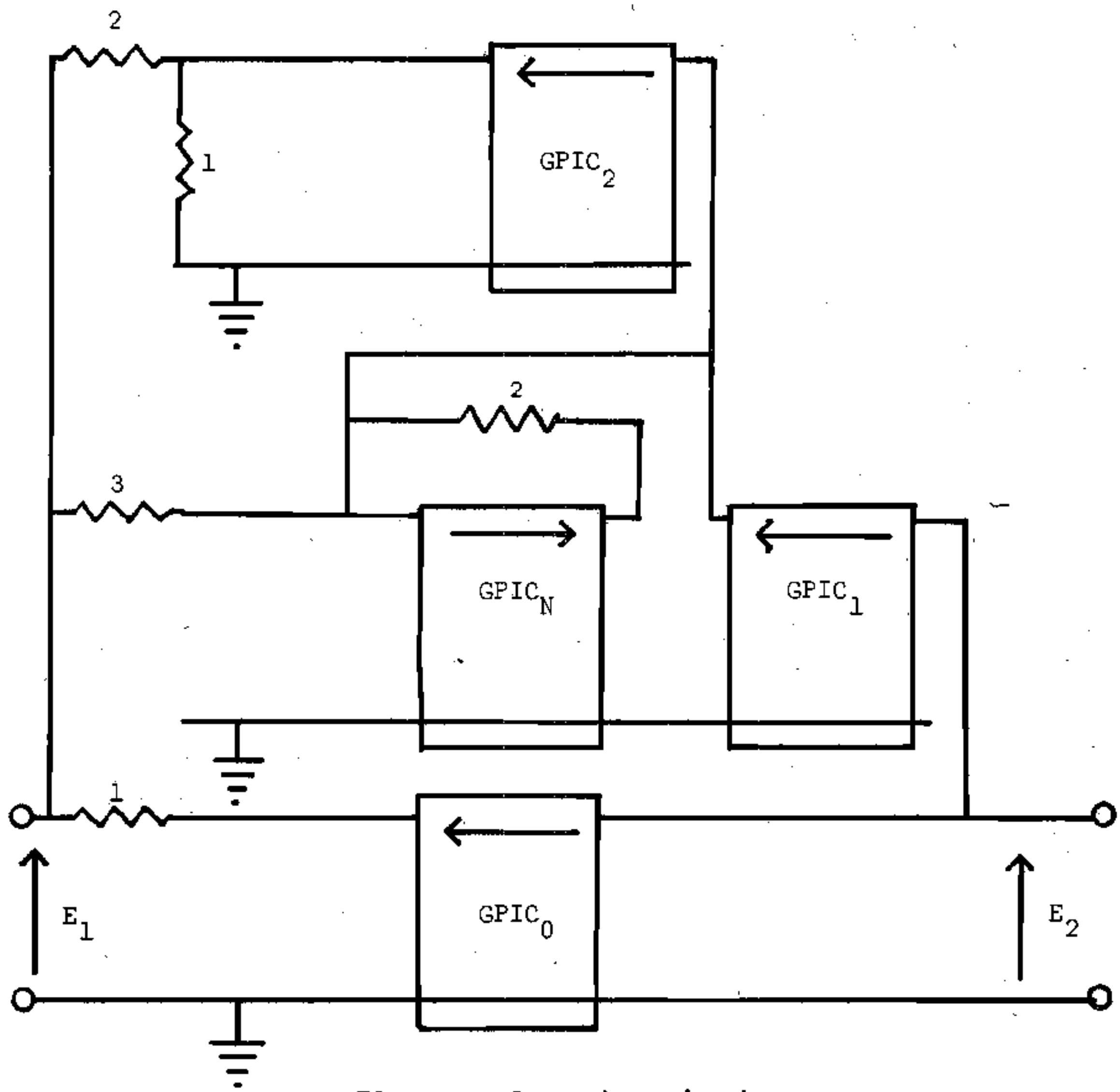
$$Y_{s2} = 2$$

$$Y_{p2} = 1$$

Now the negative admittance Y_{p1} can be handled by the negative resistance method discussed in Appendix I. If

$$k_{1N} = k_{2N} = 2 \tag{37}$$

for the negative resistance generating generalized positive impedance converter, $GPIC_N$, then the network shown in Figure 11 will realize the desired T(s).



Element values shown in mhos.

Figure 11. Network for Realizing the T(s) of (32)

CHAPTER IV

DRIVING-POINT FUNCTION SYNTHESIS

In this chapter a synthesis procedure for realizing any arbitrary driving-point admittance which is a real rational function in the complex variable s is discussed. The procedure shifts the synthesis of the driving-point function to the synthesis of the open-circuit voltage transfer function.

If the ideal unity-gain amplifiers in the network of Joseph and Hilberman[7] for realizing the driving-point admittance Y(s) from a network with an open-circuit voltage transfer function T(s) are replaced by GPIC's, the grounded circuit of Figure 12 is obtained.

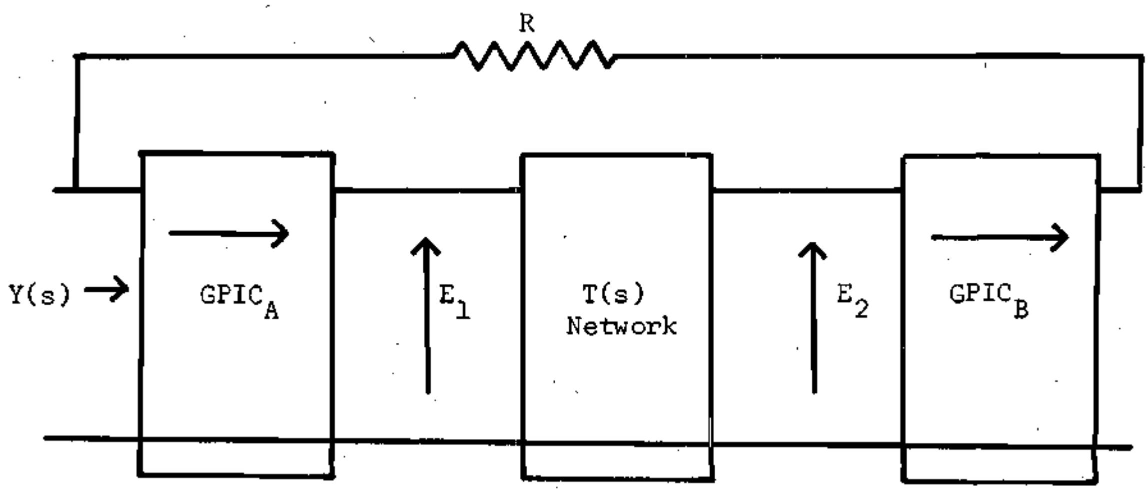


Figure 12. Network for Realizing any Rational Y(s)

If the chain matrices of $GPIC_A$ and $GPIC_B$ in Figure 12 are respectively

$$\begin{bmatrix} k_{1A} & 0 \\ 0 & 0 \end{bmatrix}$$

and (38)

$$\begin{bmatrix} k_{1B} & 0 \\ 0 & 0 \end{bmatrix}$$

then it can be shown that

$$Y(s) = \frac{1}{R}\,[1 - k_{1A}k_{1B}T(s)] \tag{39}$$

Now if the desired Y(s) is given by

$$Y(s) = \frac{P(s)}{Q(s)} \tag{40}$$

analysis will show that it is only necessary to synthesize the network whose open-circuit voltage transfer function is

$$T(s) = \frac{Q(s) - RP(s)}{k_{1A}k_{1B}Q(s)} \tag{41}$$

Suppose that the desired driving-point admittance Y(s) is given by

$$Y(s) = \frac{\sum_{i=0}^{n} c_i s^i}{\sum_{j=0}^{m} d_j s^i} \tag{42}$$

Then the open-circuit voltage transfer function to be synthesized is given by

$$T(s) = \frac{(d_0-Rc_0) + (d_1-Rc_1)s + \ldots + (d_q-Rc_q)s^q}{k_{1A}k_{1B}(d_0+d_1s+d_2s^2+\ldots+d_qs^q)} \tag{43}$$

where q is the order of Y(s).

Now T(s) can be synthesized by the procedure discussed in Chapter III and the network of Figure 9. The identification of parameter values for the T(s) network of Figure 9 in terms of the coefficients of (42) is as follows:

$$Y_{s0} = \frac{d_q - Rc_q}{k_{1A}k_{1B}k_{20}} \tag{44}$$

$$Y_{p0} = \frac{k_{10}}{k_{20}} d_q - \frac{d_q - Rc_q}{k_{1A}k_{1B}k_{20}}$$

$$Y_{s1} = \frac{d_{q-1} - Rc_{q-1}}{k_{1A}k_{1B}k_{21}}$$

$$Y_{p1} = \frac{k_{11}d_{q-1}}{k_{21}} - \frac{d_{q-1} - Rc_{q-1}}{k_{1A}k_{1B}k_{21}}$$

$$Y_{s2} = \frac{d_{q-2} - Rc_{q-2}}{k_{1A}k_{1B}k_{21}k_{22}}$$

$$Y_{p2} = \frac{k_{11}k_{12}d_{q-2}}{k_{21}k_{22}} - \frac{d_{q-2} - Rc_{q-2}}{k_{1A}k_{1B}k_{21}k_{22}}$$

$$\vdots$$

$$Y_{sr} = \frac{d_{q-r} - Rc_{q-r}}{k_{1A\ 1B}k_{21}k_{22}\cdots k_{2r}}$$

$$Y_{pr} = \frac{k_{11}k_{12}\cdots k_{1r}d_{q-r}}{k_{21}k_{22}\cdots k_{2r}} - \frac{d_{q-r} - Rc_{q-r}}{k_{1A}k_{1B}k_{21}k_{22}\cdots k_{2r}}$$

where $r=1,2,\ldots,q$.

$GPIC_A$ and $GPIC_B$ of Figure 12 can be realized with a circuit such as the one of Figure 7.

Note that the order of the open-circuit voltage transfer function, T(s), of (43) is equal to the order of the driving-point admittance function, Y(s), of (42). The two additional GPIC's needed in the synthesis of Y(s), $GPIC_A$ and $GPIC_B$ of Figure 12, require no capacitors. Hence to realize a Y(s) of order q, we must realize a T(s) of order q. It was shown in Chapter III that this T(s) can be realized with q capacitors. Newcomb[8] has pointed out that the minimum number of reactive elements needed to realize a real rational driving-point admittance of order q is q. Therefore, the driving-point admittance synthesis procedure of this chapter uses the minimum number of capacitors necessary for the synthesis.

The actual synthesis in the driving-point admittance procedure is that of synthesizing a network with a certain open-circuit voltage transfer function. This transfer function network is connected to another network with two more GPIC's and a single resistor to form the final network for the desired admittance function. Since the same types of active devices and network elements appear here as in the voltage transfer function synthesis, the same arguments for compatibility with fabrication by an integrated-circuit technology hold here as was used in Chapter III.

An Example

To illustrate the driving-point admittance function synthesis,

$$Y(s) = \frac{2s^2 + s + 2}{s^2 + 3s + 1} \tag{45}$$

will be synthesized according to the theory of this chapter. The order of Y(s), q, is equal to two. Comparing the Y(s) to be synthesized with (42), it is apparent that

$$\begin{aligned} c_0 &= 2 \\ c_1 &= 1 \\ c_2 &= 2 \\ d_0 &= 1 \\ d_1 &= 3 \end{aligned} \tag{46}$$

$$d_2 = 1$$

Arbitrarily letting

$$R = 1$$

$$k_{1A} = k_{1B} = 2 \tag{47}$$

the following admittance parameters are obtained from (44).

$$Y_{s0} = -\frac{1}{4k_{20}} \tag{48}$$

$$Y_{p0} = \frac{4k_{10} + 1}{4k_{20}}$$

$$Y_{s1} = \frac{1}{2k_{21}}$$

$$Y_{p1} = \frac{6k_{11} - 1}{2k_{21}}$$

$$Y_{s2} = -\frac{1}{4k_{21}k_{22}}$$

$$Y_{p2} = \frac{4k_{11}k_{12} + 1}{4k_{21}k_{22}}$$

In order to make all Y_{sr}'s positive and real, make k_{20} and k_{22} be negative. Since k_{20} and k_{22} are negative and are GPIC parameters, k_{10} and k_{12} must be negative also. All other k_{ij}'s must be positive. Arbitrarily make the absolute value of all k_{ij}'s be one. In other words, make

$$
\begin{aligned}
k_{10} &= k_{20} = k_{12} = k_{22} = -1 \\
k_{11} &= k_{21} = 1
\end{aligned}
\tag{49}
$$

Then from (48) and (49), the following admittance values are obtained:

$$
\begin{aligned}
Y_{s0} &= \frac{1}{4} \\
Y_{p0} &= \frac{3}{4} \\
Y_{s1} &= \frac{1}{2} \\
Y_{p1} &= \frac{5}{2} \\
Y_{s2} &= \frac{1}{4} \\
Y_{p2} &= \frac{3}{4}
\end{aligned}
\tag{50}
$$

The circuit of Figure 13 will realize the desired Y(s).

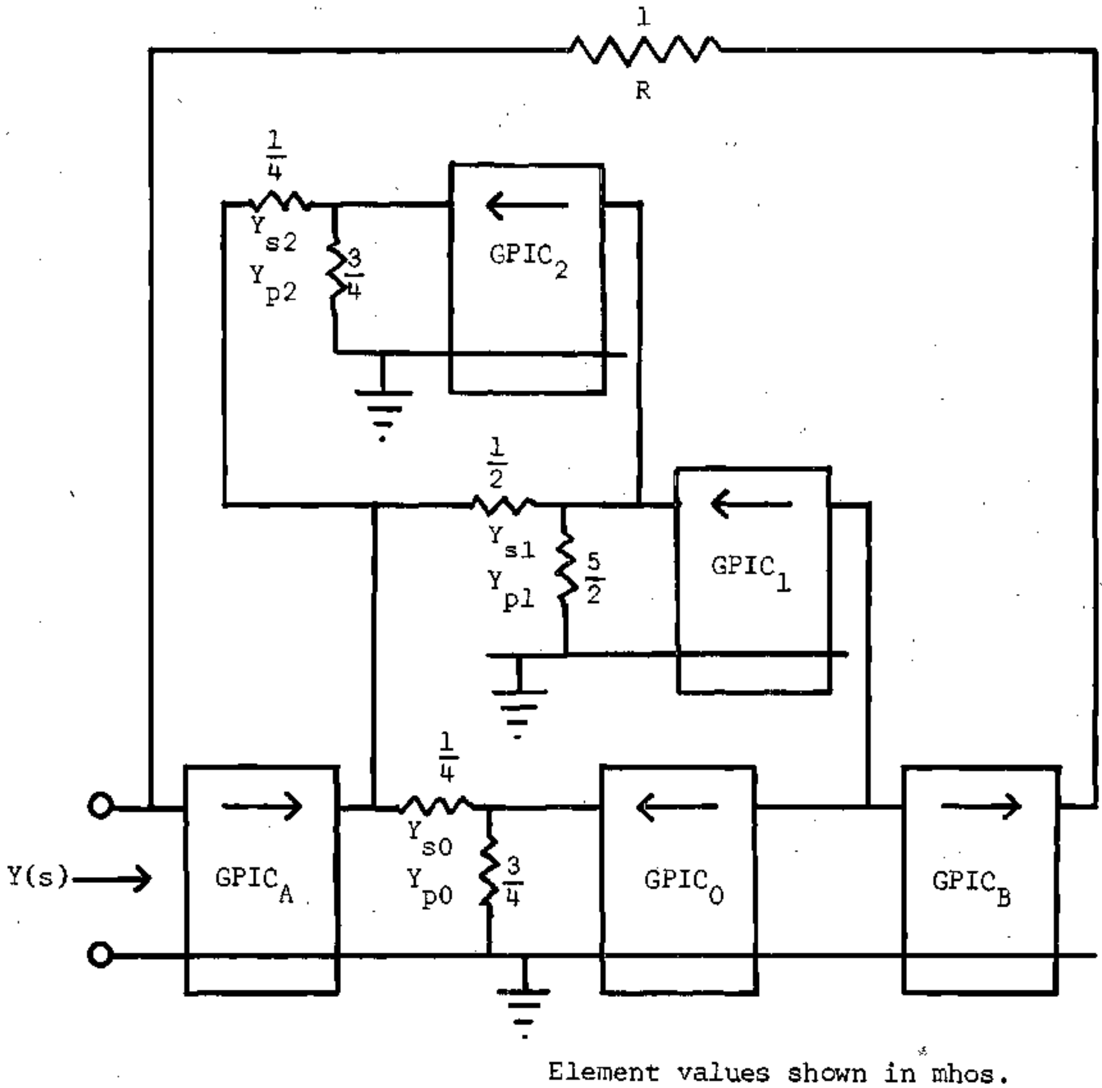


Figure 13. Network for Realizing Driving-Point Admittance Function of (45)

CHAPTER V

SENSITIVITY

In this chapter, sensitivity is defined and discussed. A sensitivity analysis of the resulting network realizations of the open-circuit voltage transfer function and driving-point admittance function synthesis procedures is made.

Definition

Sensitivity can be defined as the fractional change in performance resulting from a given fractional change in an independent variable of the system. Geffe[9] has presented an enlightening view on the topic of network sensitivity by comparing macroscopic sensitivity and differential sensitivity.

Suppose that we are interested in the change in the performance parameter T of a network with respect to some fractional change in the element value x. The macroscopic sensitivity of T to x can be defined by

$$\bar{S}_x^T = \frac{\frac{\Delta T}{T}}{\frac{\Delta x}{x}} \tag{51}$$

The macroscopic concept simply means that we intend to use realistic magnitudes of $\frac{\Delta x}{x}$. Realistic magnitudes may fall in the range of 1 to

10 per cent, i.e., lumped resistors. If $\bar{S}_x^T$ were obtainable, we could get to the final quantity of interest,

$$\frac{\Delta T}{T} = \bar{S}_x^T \cdot \frac{\Delta x}{x} \tag{52}$$

Unfortunately, $\bar{S}_x^T$ is mathematically intractable in most situations; therefore, we are generally forced to abandon the idea of realistic magnitudes of $\frac{\Delta x}{x}$. If we let Δx be a differential quantity, then we can define the differential sensitivity of T to x as

$$S_x^T = \frac{\frac{\partial T}{T}}{\frac{\partial x}{x}} = \frac{\partial T}{\partial x} \cdot \frac{x}{T} \tag{53}$$

As Δx approaches zero, (51) and (53) become identical. In the discussion of sensitivity which follows, sensitivity will have the meaning as defined by (53).

In the investigation of active circuits, it is of interest to check sensitivity factors with respect to passive elements as well as the active parameters. It is desirable to have all sensitivity factors low. High sensitivity with respect to passive elements is cause for alarm. High sensitivity with respect to active parameters, such as gain, may not be a severe defect in a network synthesis realization since gain can often be very well stabilized with feedback.

Coefficient Sensitivity of the Open-Circuit Voltage Transfer Function

Consider first the coefficient sensitivity of the network that realizes the qth order open-circuit voltage transfer function

$$T(s) = \frac{\sum_{i=0}^{q} a_i s^i}{\sum_{j=0}^{q} b_j s^j} \tag{54}$$

As discussed in Chapter III, the T(s) of (54) can be realized by the network of Figure 9. The element values needed are given in terms of the coefficients of the transfer function and the active gain constants of the GPIC's by (28). The equations of (28) can be solved to obtain the coefficients of the T(s) of (54) in terms of element values and GPIC gain constants. Doing so we obtain

$$\begin{aligned}
a_q &= k_{20} Y_{s0} \qquad (55)\\
a_{q-1} &= k_{21} Y_{s1}\\
a_{q-2} &= k_{21} k_{22} Y_{s2}\\
&\vdots\\
a_{q-r} &= k_{21} k_{22} \ldots k_{2r} Y_{sr}\\
b_q &= \frac{k_{20}(Y_{s0}+Y_{p0})}{k_{10}}
\end{aligned}$$

$$b_{q-1} = \frac{k_{21}(Y_{s1}+Y_{p1})}{k_{11}}$$

$$b_{q-2} = \frac{k_{21}k_{22}(Y_{s2}+Y_{p2})}{k_{11}k_{12}}$$

$$\vdots$$

$$b_{q-r} = \frac{k_{21}k_{22}\ldots k_{2r}(Y_{sr}+Y_{pr})}{k_{11}k_{12}\ldots k_{1r}}$$

where $r=1,2,\ldots,q$.

Using the definition of (53) for sensitivity, the following coefficient sensitivities are obtained:

$$S_{k_{2r}}^{a_{q-r}} = S_{Y_{sr}}^{a_{q-r}} = S_{k_{2r}}^{b_{q-r}} = - S_{k_{1r}}^{b_{q-r}} = 1 \tag{56}$$

$$S_{Y_{sr}}^{b_{q-r}} = \frac{Y_{sr}}{Y_{sr} + Y_{pr}}$$

$$S_{Y_{pr}}^{b_{q-r}} = \frac{Y_{pr}}{Y_{sr} + Y_{pr}}$$

for $r=0,1,2,\ldots,q$. All other coefficient sensitivities are equal to zero.

If all $b_{q-r} > 0$, and k_{1r} can take on any value, then Y_{pr} can always be made positive and real for $r=0,1,2,\ldots,q$. (This point was discussed in Chapter IV.) Hence, the absolute value of all the coefficient sensitivities of (56) will be less than or equal to one. These coefficient sensitivities are as low as the factors reported by Antoniou[5] in his RC-GIC realization for the open-circuit voltage transfer function of (54) restricted as follows:

$$\begin{aligned} &(1)\quad b_j > 0 \\ &(2)\quad b_j > |a_j| \end{aligned} \tag{57}$$

If we do not restrict the coefficients, b_{q-r}'s, and restrict the range of k_{1r}'s, then it may be necessary to generate a negative Y_{pr}. Examination of the equations of (56) indicates that care must be taken in this case to assure that $(Y_{pr}+Y_{sr})$ is as large in absolute value as possible in order to assure minimum coefficient sensitivity.

Selectivity and Undamped Natural Frequency Sensitivities of the Second-Order Open-Circuit Voltage Transfer Function

The general open-circuit voltage transfer function defined by (54) can be factored into first and second order poles and zeros. This factored transfer function can be realized by first and second order network sections cascaded through isolation buffer amplifiers. These first and second order sections can be realized by the GPIC synthesis procedure for T(s) discussed previously. This decomposition of the T(s) is often done and in some cases results in reduced sensitivity.[10]

Since transfer functions of the first order can often be handled by simple passive RC sections, the transfer function of the second order

$$T(s) = \frac{a_0 + a_1 s + a_2 s^2}{b_0 + b_1 s + b_2 s^2} \tag{58}$$

has received considerable special attention. When $b_0 > 0$, it is customary to write (58) in terms of the undamped natural frequency, ω_n, and the damping factor, σ, as follows.

$$T(s) = \frac{1}{b_2} \cdot \frac{a_0 + a_1 s + a_2 s^2}{s^2 + 2\sigma s + \omega_n^2} \tag{59}$$

where

$$\omega_n = \sqrt{\frac{b_0}{b_2}} \tag{60}$$

and

$$\sigma = \frac{b_1}{2b_2} \tag{61}$$

In terms of the selectivity, or Q, of the poles, (59) can be written as

$$T(s) = \frac{1}{b_2} \frac{a_0 + a_1 s + a_2 s^2}{s^2 + \left(\frac{\omega_n}{Q}\right) s + \omega_n^2} \tag{62}$$

where

$$Q = \frac{\omega_n}{2\sigma} = \frac{\sqrt{b_0 b_2}}{b_1} \tag{63}$$

If we consider the synthesis of the transfer function of (58) by the previously discussed open-circuit voltage transfer function synthesis technique, we can identify the coefficients of (58) with the network elements of Figure 9 through the equations of (55). Using the results of (55) for q equal to two and substituting into (60) and (61), the following is obtained.

$$\omega_n = \sqrt{\frac{Y_{s2} + Y_{p2}}{Y_{s0} + Y_{p0}}} \cdot \sqrt{\frac{k_{21}k_{22}k_{10}}{k_{11}k_{12}k_{20}}} \tag{64}$$

$$Q = \frac{\sqrt{(Y_{s0}+Y_{p0})(Y_{s2}+Y_{p2})}}{(Y_{s1}+Y_{p1})} \cdot \sqrt{\frac{k_{11}k_{22}k_{20}}{k_{12}k_{21}k_{10}}} \tag{65}$$

The selectivity and undamped natural frequency sensitivities of the resulting network realizations are often used to judge the merits of a particular synthesis procedure.

Again, using the differential sensitivity definition of (53), the following sensitivities are obtained.

$$S_{k_{10}}^{\omega_n} = S_{k_{21}}^{\omega_n} = S_{k_{22}}^{\omega_n} = \frac{1}{2} \tag{66}$$

$$S_{k_{11}}^{\omega_n} = S_{k_{12}}^{\omega_n} = S_{k_{20}}^{\omega_n} = -\frac{1}{2}$$

$$S_{Y_{s2}}^{\omega_n} = \frac{1}{2} \cdot \frac{Y_{s2}}{Y_{s2} + Y_{p2}}$$

$$S_{Y_{p2}}^{\omega_n} = \frac{1}{2} \cdot \frac{Y_{p2}}{Y_{s2} + Y_{p2}}$$

$$S_{Y_{s0}}^{\omega_n} = -\frac{1}{2} \cdot \frac{Y_{s0}}{Y_{s0} + Y_{p0}}$$

$$S_{Y_{p0}}^{\omega_n} = -\frac{1}{2} \cdot \frac{Y_{p0}}{Y_{s0} + Y_{p0}}$$

$$S_{k_{11}}^{Q} = S_{k_{22}}^{Q} = S_{k_{20}}^{Q} = \frac{1}{2}$$

$$S_{k_{12}}^{Q} = S_{k_{21}}^{Q} = S_{k_{10}}^{Q} = -\frac{1}{2}$$

$$S_{Y_{s0}}^{Q} = \frac{1}{2} \cdot \frac{Y_{s0}}{Y_{s0} + Y_{p0}}$$

$$S_{Y_{p0}}^{Q} = \frac{1}{2} \cdot \frac{Y_{p0}}{Y_{s0} + Y_{p0}}$$

$$S_{Y_{s2}}^{Q} = \frac{1}{2} \cdot \frac{Y_{s2}}{Y_{s2} + Y_{p2}}$$

$$S_{Y_{p2}}^{Q} = \frac{1}{2} \cdot \frac{Y_{p2}}{Y_{s2} + Y_{p2}}$$

$$S_{Y_{s1}}^{Q} = -\frac{Y_{s1}}{Y_{s1} + Y_{p1}}$$

$$S^{Q}_{Y_{p1}} = - \frac{Y_{p1}}{Y_{s1} + Y_{p1}}$$

If all Y_{pr}'s ≥ 0 as before (i.e., $b_j > 0$ for $j=0,1,2$ and k_{1r} unrestricted in value for integer values of r between 0 and 2), then the absolute value of all Q and ω_n sensitivities will be less than or equal to one. These sensitivity figures are almost the same as the low factors reported by Antoniou[5] in his RC-GIC synthesis realizations.

Newcomb shows that for high Q circuits, the INIC realization of Figure 7.2.2 with assigned values from Table 7.2.1 in his book[1] has a

$$|S^{Q}_{k}| = 2Q^2 \tag{67}$$

Further examination reveals that

$$|S^{Q}_{r}| \cong 2Q^2 \tag{68}$$

Now k is the current gain constant of the INIC and r is a circuit resistor value. Therefore, the circuit is very sensitive to both active and passive parameters.

It can be concluded that the resulting network realizations of the GPIC open-circuit voltage transfer function synthesis exhibit selectivity and undamped natural frequency sensitivity figures of the same order of magnitude as Antoniou's realizations, which are claimed to have low sensitivity. The GPIC realization offers a definite improvement in

Q function sensitivity over a particular INIC circuit reported by Newcomb.

Sensitivity of the Absolute Value of the Second-Order All-Pole Open-Circuit Voltage Transfer Function

The second-order all-pole transfer function

$$T(s) = \frac{a_0}{b_0 + b_1 s + b_2 s^2} \tag{69}$$

can be synthesized by the GPIC procedure described previously and realized with the network of Figure 9. Assume that the transfer function is absolutely stable (i.e., b_0, b_1, $b_2 > 0$) and that $a_0 > 0$. The equations of (55) enable us to relate the coefficients of (69) to the network admittances and GPIC gain constants needed for the synthesis with the network of Figure 9. Namely,

$$T(s) = \frac{k_{21}k_{22}Y_{s2}}{\dfrac{k_{21}k_{22}(Y_{s2}+Y_{p2})}{k_{11}k_{12}} + \dfrac{k_{21}(Y_{s1}+Y_{p1})s}{k_{11}} + \dfrac{k_{20}(Y_{s0}+Y_{p0})s^2}{k_{10}}} \tag{70}$$

Now

$$|T(j\omega)| = \frac{|k_{21}||k_{22}|Y_{s2}}{D(\omega)} \tag{71}$$

where

$$D(\omega) = \sqrt{A^2(\omega) + B^2(\omega)} \tag{72}$$

and

$$A(\omega) = \frac{k_{21}k_{22}(Y_{s2}+Y_{p2})}{k_{11}k_{12}} - \frac{\omega^2 k_{20}(Y_{s0}+Y_{p0})}{k_{10}} \tag{73}$$

and

$$B(\omega) = \frac{\omega k_{21}(Y_{s1}+Y_{p1})}{k_{11}} \tag{74}$$

Analysis will yield the following sensitivities.

$$S_{k_{11}}^{|T(j\omega)|} = \frac{1}{D^2(\omega)}\left[\frac{k_{21}k_{22}(Y_{s2}+Y_{p2})}{k_{11}k_{12}} \cdot A(\omega) + B^2(\omega)\right] \tag{75}$$

$$S_{k_{12}}^{|T(j\omega)|} = \frac{1}{D^2(\omega)}\left[\frac{k_{21}k_{22}(Y_{s2}+Y_{p2})}{k_{11}k_{12}} \cdot A(\omega)\right]$$

$$S_{k_{21}}^{|T(j\omega)|} = 1 - S_{k_{11}}^{|T(j\omega)|}$$

$$S_{k_{22}}^{|T(j\omega)|} = 1 - S_{k_{12}}^{|T(j\omega)|}$$

$$S_{k_{10}}^{|T(j\omega)|} = -\omega^2 \cdot \frac{k_{20}k_{11}k_{12}(Y_{s0}+Y_{p0})}{k_{10}k_{21}k_{22}(Y_{s2}+Y_{p2})} S_{k_{12}}^{|T(j\omega)|}$$

$$S_{k_{20}}^{|T(j\omega)|} = -S_{k_{10}}^{|T(j\omega)|}$$

$$S_{Y_{s0}}^{|T(j\omega)|} = \omega^2 \cdot \frac{k_{20}k_{11}k_{12}Y_{s0}}{k_{10}k_{21}k_{22}(Y_{s2}+Y_{p2})} \cdot S_{k_{12}}^{|T(j\omega)|}$$

$$S_{Y_{p0}}^{|T(j\omega)|} = \omega^2 \cdot \frac{k_{20}k_{11}k_{12}Y_{p0}}{k_{10}k_{21}k_{22}(Y_{s2}+Y_{p2})} S_{k_{12}}^{|T(j\omega)|}$$

$$S_{Y_{s1}}^{|T(j\omega)|} = -\frac{Y_{s1}(Y_{s1}+Y_{p1})(\omega k_{21})^2}{[k_{11}D(\omega)]^2}$$

$$S_{Y_{p1}}^{|T(j\omega)|} = -\frac{Y_{p1}(Y_{s1}+Y_{p1})(\omega k_{21})^2}{[k_{11}D(\omega)]^2}$$

$$S_{Y_{s2}}^{|T(j\omega)|} = 1 - \frac{Y_{s2}}{(Y_{s2}+Y_{p2})} S_{k_{12}}^{|T(j\omega)|}$$

$$S_{Y_{p2}}^{|T(j\omega)|} = -\frac{Y_{p2}}{(Y_{s2}+Y_{p2})} S_{k_{12}}^{|T(j\omega)|}$$

No general statement can be made regarding the relative magnitude of the previously calculated absolute magnitude of T(s) sensitivity terms. That the network used to realize T(s) can be made to have low sensitivity terms can best be demonstrated through the following example.

An Example

The following open-circuit voltage transfer function

$$T(s) = \frac{1}{s^2 + \frac{1}{Q}s + 1} \qquad (76)$$

will be synthesized according to the procedure of Chapter III and the sensitivity factors of (75) will be computed. For the example, the undamped natural frequency, ω_n, is 1. The selectivity of the function is Q. The T(s) synthesis procedure of Chapter III requires that

$$Y_{s0} = 0 \tag{77}$$

$$Y_{p0} = \frac{k_{10}}{k_{20}}$$

$$Y_{s1} = 0$$

$$Y_{p1} = \frac{k_{11}}{Qk_{21}}$$

$$Y_{s2} = \frac{1}{k_{21}k_{22}}$$

$$Y_{p2} = \frac{k_{11}k_{12} - 1}{k_{21}k_{22}}$$

Arbitrarily, make all k_{ij}'s = 1. Then in units of mhos

$$Y_{s0} = 0 \tag{78}$$

$$Y_{s1} = 0$$

$$Y_{s2} = 1$$

$$Y_{p0} = 1$$

$$Y_{p1} = \frac{1}{Q}$$

$$Y_{p2} = 0$$

The equations of (75) indicated that certain sensitivity factors are functions of the radian frequency ω. Since it would be impossible to check the sensitivity factors at every frequency, three frequencies are usually used: $\omega = 0$, $\omega = \omega_n = 1$, and $\omega = \infty$. Using the results of (75) and substituting the above network admittance values and GPIC gain constants, Table 1 was obtained.

Table 1. Sensitivities of Second-Order All-Pole Transfer Function of (76)

Element	Element Value	$\left\lvert S^{\lvert T(j\omega)\rvert}_{\text{Element}} \right\rvert$ $\omega=0$	$\omega=1$	$\omega=\infty$
k_{10}	1	0	0	1
k_{11}	1	1	1	0
k_{12}	1	1	0	0
k_{20}	1	0	0	1
k_{21}	1	0	0	1
k_{22}	1	0	1	1
Y_{p0}	1 mho	0	0	1
Y_{p1}	$\frac{1}{Q}$ mho	0	1	0
Y_{s2}	1 mho	0	1	1

Note that all sensitivity factors are less than or equal to one. Hence for realizing the T(s) of (76), the synthesis procedure yields a network whose transfer function magnitude is very insensitive to the passive network elements and GPIC gain constants. It is significant to note that the selectivity, Q, of the transfer function does not appear in any of the sensitivity terms in Table 1.

The transfer function of (76) can be realized with the INIC realization of Figure 7.2.2 in Newcomb's book[1] when the circuit is assigned the values given in Table 7.2.1 of the same book. Examination of $\left|S_{k,r}^{|T(j\omega)|}\right|$ of the transfer function of the INIC realization reveals that for high Q

$$\left|S_{k}^{|T(j1)|}\right| = 2Q^2$$

and (79)

$$\left|S_{r}^{|T(j1)|}\right| \cong 2Q^2$$

where k = current gain of the INIC and r = resistor value. The network is therefore very sensitive to both active and passive parameters.

From a sensitivity standpoint, the INIC network examined above is described by Newcomb as being typical of NIC type circuits. Very definitely, the GPIC network described previously that will realize the transfer function of (76) has superior sensitivity characteristics.

Coefficient Sensitivity of qth Order Driving-Point Function

Consider the coefficient sensitivity of the realization for the qth order driving-point admittance function

$$Y(s) = \frac{\sum_{i=0}^{q} c_i s^i}{\sum_{j=0}^{q} d_j s^j} \tag{80}$$

As discussed in Chapter IV, (80) can be realized by the circuit of Figure 12 which incorporates a network that must realize

$$T(s) = \frac{(d_0-Rc_0) + (d_1-Rc_1)s + \ldots + (d_q-Rc_q)s^q}{k_{1A}k_{1B}(d_0 + d_1 s + d_2 s^2 + \ldots + d_q s^q)} \tag{81}$$

Now this T(s) can be realized by the network of Figure 9. The equations of (44) can be solved to obtain the coefficients of (80) in terms of the network elements needed for the synthesis of Y(s). Solving these equations we obtain

$$c_q = \frac{k_{20}}{k_{10}R}\{(1-k_{10}k_{1A}k_{1B})Y_{s0} + Y_{p0}\} \tag{82}$$

$$d_q = \frac{k_{20}}{k_{10}}(Y_{s0}+Y_{p0})$$

$$c_{q-r} = \frac{(k_{21}k_{22}\ldots k_{2r})}{(k_{11}k_{12}\ldots k_{1r}R)}\{(1-k_{11}k_{12}\ldots k_{1r}k_{1A}k_{1B})Y_{sr} + Y_{pr}\}$$

$$d_{q-r} = \frac{k_{21}k_{22}\cdots k_{2r}}{k_{11}k_{12}\cdots k_{1r}} (Y_{sr}+Y_{pr})$$

for r=1,2,...,q. Applying the differential sensitivity formula of (53), the following sensitivity factors are obtained.

$$S_{k_{20}}^{c_q} = 1 \tag{83}$$

$$S_{R}^{c_q} = -1$$

$$S_{k_{10}}^{c_q} = -\frac{(Y_{s0}+Y_{p0})}{\{(1-k_{10}k_{1A}k_{1B})Y_{s0} + Y_{p0}\}}$$

$$S_{Y_{s0}}^{c_q} = \frac{Y_{s0}(1-k_{10}k_{1A}k_{1B})}{\{(1-k_{10}k_{1A}k_{1B})Y_{s0} + Y_{p0}\}}$$

$$S_{Y_{p0}}^{c_q} = \frac{Y_{p0}}{\{(1-k_{10}k_{1A}k_{1B})Y_{s0} + Y_{p0}\}}$$

$$S_{k_{1A},k_{1B}}^{c_q} = -\frac{k_{10}k_{1A}k_{1B}Y_{s0}}{\{(1-k_{10}k_{1A}k_{1B})Y_{s0} + Y_{p0}\}}$$

$$S_{k_{10}}^{d_q} = -S_{k_{20}}^{d_q} = -1$$

$$S_{Y_{s0}}^{d_q} = \frac{Y_{s0}}{Y_{s0} + Y_{p0}}$$

$$S_{Y_{p0}}^{d_q} = \frac{Y_{p0}}{Y_{s0} + Y_{p0}}$$

$$S_{k_{21},k_{22},\ldots,k_{2r}}^{c_{q-r}} = 1$$

$$S_{R}^{c_{q-r}} = -1$$

$$S_{k_{11},k_{12},\ldots,k_{1r}}^{c_{q-r}} = -\frac{(Y_{sr}+Y_{pr})}{\{(1-k_{11}k_{12}\ldots k_{1r}k_{1A}k_{1B})Y_{sr} + Y_{pr}\}}$$

$$S_{Y_{sr}}^{c_{q-r}} = \frac{Y_{sr}(1-k_{11}k_{12}\ldots k_{1r}k_{1A}k_{1B})}{\{(1-k_{11}k_{12}\ldots k_{1r}k_{1A}k_{1B})Y_{sr} + Y_{pr}\}}$$

$$S_{Y_{pr}}^{c_{q-r}} = \frac{Y_{pr}}{\{(1-k_{11}k_{12}\ldots k_{1r}k_{1A}k_{1B})Y_{sr} + Y_{pr}\}}$$

$$S_{k_{1A},k_{1B}}^{c_{q-r}} = -\frac{k_{11}k_{12}\ldots k_{1r}k_{1A}k_{1B}Y_{sr}}{\{(1-k_{11}k_{12}\ldots k_{1r}k_{1A}k_{1B})Y_{sr} + Y_{pr}\}}$$

$$S_{k_{11},k_{12},\ldots,k_{1r}}^{d_{q-r}} = -S_{k_{21},k_{22},\ldots,k_{2r}}^{d_{q-r}} = -1$$

$$S_{Y_{sr}}^{d_{q-r}} = \frac{Y_{sr}}{Y_{sr} + Y_{pr}}$$

$$S_{Y_{pr}}^{d_{q-r}} = \frac{Y_{pr}}{Y_{sr} + Y_{pr}}$$

for $r=1,2,\ldots,q$.

In order to better judge the relative sensitivity values given by (83), the results of (44) can be substituted into (83) by which is obtained

$$S_{k_{20}}^{c_q} = 1 \tag{84}$$

$$S_{R}^{c_q} = -1$$

$$S_{k_{10}}^{c_q} = -\frac{d_q}{Rc_q}$$

$$S_{Y_{s0}}^{c_q} = \left(1 - \frac{d_q}{Rc_q}\right)\left(1 - \frac{1}{k_{1A}k_{1B}k_{10}}\right)$$

$$S_{Y_{p0}}^{c_q} = \left(1 - \frac{1}{k_{1A}k_{1B}k_{10}}\right)\frac{d_q}{Rc_q} + \frac{1}{k_{1A}k_{1B}k_{10}}$$

$$S_{k_{1A},k_{1B}}^{c_q} = 1 - \frac{d_q}{Rc_q}$$

$$S_{k_{10}}^{d_q} = -S_{k_{20}}^{d_q} = -1$$

$$S_{Y_{s0}}^{d_q} = \frac{1}{k_{1A}k_{1B}k_{10}}\left(1 - \frac{Rc_q}{d_q}\right)$$

$$S_{Y_{p0}}^{d_q} = \left(1 - \frac{1}{k_{1A}k_{1B}k_{10}}\right) + \frac{Rc_q}{k_{1A}k_{1B}k_{10}d_q}$$

$$S_{k_{21},k_{22},\ldots,k_{2r}}^{c_{q-r}} = 1$$

$$S_{R}^{c_{q-r}} = -1$$

$$S_{k_{11},k_{12},\ldots,k_{1r}}^{c_{q-r}} = -\frac{d_{q-r}}{Rc_{q-r}}$$

$$S_{Y_{sr}}^{c_{q-r}} = \left(1 - \frac{d_{q-r}}{Rc_{q-r}}\right)\left(1 - \frac{1}{k_{1A}k_{1B}k_{11}k_{12}\cdots k_{1r}}\right)$$

$$S_{Y_{pr}}^{c_{q-r}} = \left(1 - \frac{1}{k_{1A}k_{1B}k_{11}k_{12}\cdots k_{1r}}\right)\frac{d_{q-r}}{Rc_{q-r}} + \frac{1}{k_{1A}k_{1B}k_{11}k_{12}\cdots k_{1r}}$$

$$S_{k_{1A},k_{1B}}^{c_{q-r}} = 1 - \frac{d_{q-r}}{Rc_{q-r}}$$

$$S_{k_{11},k_{12},\ldots,k_{1r}}^{d_{q-r}} = -S_{k_{21},k_{22},\ldots,k_{2r}}^{d_{q-r}} = -1$$

$$S_{Y_{sr}}^{d_{q-r}} = \frac{1}{k_{1A}k_{1B}k_{11}k_{12}\cdots k_{1r}}\left(1 - \frac{Rc_{q-r}}{d_{q-r}}\right)$$

$$S_{Y_{pr}}^{d_{q-r}} = \left(1 - \frac{1}{k_{1A}k_{1B}k_{11}k_{12}\cdots k_{1r}}\right) + \frac{Rc_{q-r}}{k_{1A}k_{1B}k_{11}k_{12}\cdots k_{1r}d_{q-r}}$$

for $r=1,2,\ldots,q$.

It is not immediately evident from (84) that the coefficient sensitivities for the networks realizing the driving-point functions can be made low. To demonstrate that they can be made low,

(1) choose R such that

$$\left|\frac{d_q}{Rc_q}\right| \le 0.1$$
$$\left|\frac{d_{q-r}}{Rc_{q-r}}\right| \le 0.1 \qquad (85)$$

for r=1,2,...,q.

(2) Choose k_{10}, k_{1r} in order that

$$|k_{1A}k_{1B}k_{10}| = 100 \left|\frac{Rc_q}{d_q}\right| \tag{86}$$

$$|k_{1A}k_{1B}k_{11}k_{12}\cdots k_{1r}| = 100 \left|\frac{Rc_{q-r}}{d_{q-r}}\right|$$

for r=1,2,...,q. Examination of the sensitivity terms of (84) which are not equal to one in absolute value with the restrictions of (85) and (86) reveals that

$$\left|S_{k_{10}}^{c_q}\right| \leq 0.1 \tag{87}$$

$$\left|S_{Y_{s0},k_{1A},k_{1B}}^{c_q}\right| \cong 1$$

$$\left|S_{Y_{p0}}^{c_q}\right| \leq 0.11$$

$$\left|S_{Y_{s0}}^{d_q}\right| \cong 0.01$$

$$\left|S_{Y_{p0}}^{d_q}\right| \cong 1$$

$$\left|S_{k_{11},k_{12},\ldots,k_{1r}}^{c_{q-r}}\right| \leq 0.1$$

$$\left|S_{Y_{sr},k_{1A},k_{1B}}^{c_{q-r}}\right| \cong 1$$

$$\left|S_{Y_{pr}}^{c_{q-r}}\right| \leq 0.11$$

$$\left|S_{Y_{sr}}^{d_{q-r}}\right| \cong 0.01$$

$$\left|S_{Y_{pr}}^{d_{q-r}}\right| \cong 1$$

Hence, applying the restrictions of (85) and (86) to (84), it can be concluded that all coefficient sensitivity terms are low and approximately less than or equal to one in absolute value. The following example demonstrates the above conclusion.

An Example

Let it be desired to synthesize the following Y(s) according to the procedure of Chapter IV. Also, suppose that it is desired to have all coefficient sensitivity terms as defined by (84) approximately less than or equal to one in absolute value. The desired driving-point function is

$$Y(s) = \frac{2s^2 + s + 2}{s^2 + 3s + 1} \tag{88}$$

Applying the synthesis procedure of Chapter IV and the restrictions of (85) and (86), it is found that the following parameter values will satisfy the synthesized network requirements (44) and the requirements for the low sensitivity [(85) and (86)].

$$k_{1A} = k_{1B} = 10 \tag{89}$$

$$k_{10} = -200$$

$$k_{20} = -10^3$$

$$k_{11} = -\frac{100}{3}$$

$$k_{21} = -10^3$$

$$k_{12} = 6$$

$$k_{22} = 1$$

$$R = 100 \text{ ohms}$$

$$Y_{s0} = 1.99\times10^{-3} \text{ mho}$$

$$Y_{p0} = 1.9801\times10^{-1} \text{ mho}$$

$$Y_{s1} = 0.97\times10^{-3} \text{ mho}$$

$$Y_{p1} = 9.903\times10^{-2} \text{ mho}$$

$$Y_{s2} = 1.99\times10^{-3} \text{ mho}$$

$$Y_{p2} = 1.9801\times10^{-1} \text{ mho}$$

The absolute value coefficient sensitivity table, Table 2, is obtained by computation of the sensitivity terms according to (83) or (84).

Conclusion

A sensitivity investigation of the resulting realizations of the GPIC open-circuit voltage transfer function and the driving-point admittance function synthesis procedures was made. The investigation revealed that the T(s) and Y(s) realizations are generally insensitive to changes in both passive element values and GPIC gain constants.

For the network that realizes the unconditionally stable T(s), the coefficient sensitivity terms can always be made less than or equal to one in absolute value. In the case of the absolutely stable, second-order T(s) network, the selectivity and undamped natural frequency sensitivity terms can always be made less than or equal to one in absolute value. It is concluded that the GPIC T(s) synthesis realizations exhibit coefficient, selectivity, and undamped natural frequency sensitivities of the same order of magnitude as Antoniou's synthesis realizations, which are claimed to have low sensitivity. The GPIC T(s) procedure realizations offer a definite improvement in selectivity sensitivity over a particular INIC circuit reported by Newcomb.

Table 2. Absolute Value of Coefficient Sensitivity Terms of the Y(s) of (88)

Coefficient Element	c_0	c_1	c_2	d_0	d_1	d_2
R	1	1	1			
k_{1A}	1^*	1^*	1^*			
k_{1B}	1^*	1^*	1^*			
k_{10}			$\frac{1}{200}$			1
k_{11}	$\frac{1}{200}$	$\frac{3}{100}$		1	1	
k_{12}	$\frac{1}{200}$			1		
k_{20}			1			1
k_{21}	1	1		1	1	
k_{22}	1			1		
Y_{s0}			1^*			$\frac{1}{100}^*$
Y_{p0}			$\frac{1}{200}^*$			1^*
Y_{s1}		1^*			$\frac{1}{100}^*$	
Y_{p1}		$\frac{3}{100}^*$			1^*	
Y_{s2}	1^*			$\frac{1}{100}^*$		
Y_{p2}	$\frac{1}{200}^*$			1^*		

*Approximate value.

It was demonstrated that coefficient sensitivity terms of the resulting realizations of the Y(s) procedure can always be made approximately less than or equal to one in absolute value.

CHAPTER VI

CONTROL OF ELEMENTS VALUES EXTERNAL TO GPIC'S

In both the open-circuit voltage transfer function synthesis and the driving-point admittance synthesis, the actual network to be synthesized was one with the qth order open-circuit voltage transfer function

$$T(s) = \frac{\sum_{i=0}^{q} e_i s^i}{\sum_{j=0}^{q} f_j s^j} \tag{90}$$

According to the network synthesis procedure discussed in Chapter III, it was required that the network elements have the following values:

$$Y_{s0} = \frac{e_q}{k_{20}} \tag{91}$$

$$Y_{p0} = \frac{k_{10} f_q}{k_{20}} - \frac{e_q}{k_{20}}$$

$$Y_{s1} = \frac{e_{q-1}}{k_{21}}$$

$$Y_{p1} = \frac{k_{11} f_{q-1}}{k_{21}} - \frac{e_{q-1}}{k_{21}}$$

$$\vdots$$

$$Y_{sn} = \frac{e_{q-n}}{k_{21} k_{22} \cdots k_{2n}}$$

$$Y_{pn} = \frac{k_{11}k_{12}\cdots k_{1n}f_{q-n}}{k_{21}k_{22}\cdots k_{2n}} - \frac{e_{q-n}}{k_{21}k_{22}\cdots k_{2n}}$$

for $n=1,2,\ldots,q$.

Let it be required that the non-zero admittance parameter values of (91) satisfy the following equations.

$$\alpha \le Y_{sn} \le \beta \tag{92}$$

and

$$\alpha \le Y_{pn} \le \beta \tag{93}$$

or

$$-\gamma \le Y_{pn} \le -\lambda \tag{94}$$

for $\alpha \le \beta < \gamma \ge \lambda$ and $n=0,1,2,\ldots,q$.

Recall that negative Y_{pn}'s can be realized by Kim's method as discussed in Appendix I. For example, from Appendix I, we see that if $k_1 = 3$, $k_2 = 4$, then $Y_{pn} = -2Y$. If $\alpha \le Y \le \beta$, then the negative Y_{pn} range would extend from -2β to -2α.

Examination of the Y_{sn} equations of (91) for $n=0,1,2,\ldots,q$ reveals that all Y_{sn}'s can be made to fall within a certain range by choosing the appropriate algebraic sign and magnitude of k_{2n}. The algebraic sign of k_{1n} must be made the same as that of k_{2n}. Combining the equations of (91) and (92) we obtain

$$\alpha \le \frac{e_q}{k_{20}} \le \beta \tag{95}$$

$$\alpha \leq \frac{e_{q-n}}{k_{21}k_{22}\ldots k_{2n}} \leq \beta$$

for n=1,2,...,q.

If

$$e_q = 0$$

then

$$Y_{s0} = 0$$

and

$$k_{20} = \text{arbitrary}.$$

If (96)

$$e_{q-n} = 0$$

then

$$Y_{sn} = 0$$

and

$$k_{2n} = \text{arbitrary}$$

for n=1,2,...,q.

If e_q, $e_{q-n} \neq 0$, then Y_{s0} and the Y_{sn}'s can be made to fall into the appropriate range by choosing k_{2n}[n=0,1,2,...,q] to satisfy the following restrictions:

$$\frac{1}{\beta} \leq \frac{k_{20}}{e_q} \leq \frac{1}{\alpha} \tag{97}$$

$$\frac{1}{\beta} \leq \frac{k_{21}k_{22}\ldots k_{2n}}{e_{q-n}} \leq \frac{1}{\alpha}$$

for n=1,2,...,q.

Once the k_{2n} values for n=0,1,2,...,q are selected according to the equations of (96) and (97), then the k_{1n} values must be selected to satisfy either (93) or (94). For all f_j's of (90) that are positive, the corresponding Y_{pn}'s can be made to satisfy (93). For those f_j's that are nonpositive, the corresponding nonzero Y_{pn}'s will be negative and can be made to satisfy (94).

First consider the case of the f_j's of (90) that are positive. Substituting the appropriate equations from (91) into (93) and performing some algebraic steps, the following equations are obtained.

$$\alpha + \frac{e_q}{k_{20}} \le \frac{k_{10} f_q}{k_{20}} \le \beta + \frac{e_q}{k_{20}}$$

$$\alpha + \frac{e_{q-n}}{k_{21}k_{22}\cdots k_{2n}} \le \frac{k_{11}k_{12}\cdots k_{1n} f_{q-n}}{k_{21}k_{22}\cdots k_{2n}} \le \beta + \frac{e_{q-n}}{k_{21}k_{22}\cdots k_{2n}} \tag{98}$$

for f_q, $f_{q-n} > 0$ and n=1,2,...,q. It is apparent from (98) that each k_{1n} [n=0,1,2,...,q] must be calculated in ascending order of n. Hence the results of (98) must be used in conjunction with the procedure for calculating the k_{1n}'s for nonpositive f_j's. This procedure will now be discussed.

Consider the case of the f_j's of (90) that are nonpositive. First consider the f_j's that are zero. Examination of (91) reveals that if

$$f_q = 0$$

then

$$Y_{p0} = -Y_{s0} \tag{99}$$

and

$$k_{10} = \text{arbitrary.}$$

If

$$f_{q-n} = 0$$

then

$$Y_{pn} = -Y_{sn} \tag{100}$$

and

$$k_{1n} = \text{arbitrary}$$

for n=1,2,...,q.

Next consider the negative f_j's. Substituting the appropriate equations from (91) into (94) and performing the appropriate algebra, the following equations are obtained.

$$-\gamma + \frac{e_q}{k_{20}} \leq \frac{k_{10} f_q}{k_{20}} \leq -\lambda + \frac{e_q}{k_{20}}$$

$$-\gamma + \frac{e_{q-n}}{k_{21}k_{22}\ldots k_{2n}} \leq \frac{k_{11}k_{12}\ldots k_{1n} f_{q-n}}{k_{21}k_{22}\ldots k_{2n}} \leq -\lambda + \frac{e_{q-n}}{k_{21}k_{22}\ldots k_{2n}} \tag{101}$$

for f_q, $f_{q-n} < 0$ and n=1,2,...,q.

The case of most interest to engineers is the case where all f_j's are positive, the absolutely stable system. For this case (96) and (97) can be used to calculate all k_{2n}'s and (98) can be used to calculate all k_{1n}'s. Using the values obtained from these three sets of equations, the non-zero admittance parameter values will be in the range indicated below.

$$\begin{aligned} \alpha \le Y_{sn} \le \beta \\ \alpha \le Y_{pn} \le \beta \end{aligned} \tag{102}$$

for $\alpha \le \beta$ and $n=0,1,2,\ldots,q$.

Example One

Let it be required to synthesize

$$T(s) = \frac{s^2 - 3s + 2}{s^2 + s + 100} \tag{103}$$

with the following restrictions on the non-zero admittance values.

$$\begin{aligned} 1 \le Y_{sn} \le 2 \\ 1 \le Y_{pn} \le 2 \end{aligned} \tag{104}$$

Since all coefficients are present in the numerator and denominator and those present in the denominator are positive, the equations of (97) and (98) apply. In the above example,

$$q = 2 \qquad (105)$$

$$e_0 = 2$$

$$e_1 = -3$$

$$e_2 = 1$$

$$f_0 = 100$$

$$f_1 = 1$$

$$f_2 = 1$$

$$\alpha = 1$$

$$\beta = 2$$

From (97), the results below are obtained.

$$\frac{1}{2} \le k_{20} \le 1$$

$$-3 \le k_{21} \le -\frac{3}{2} \qquad (106)$$

$$1 \le k_{21}k_{22} \le 2$$

The following values of k_{2n} will satisfy (106).

$$k_{20} = 1 \qquad (107)$$

$$k_{21} = -3$$

$$k_{22} = -\frac{2}{3}$$

To get the corresponding k_{1n} values, use (98) and obtain

$$2 \le k_{10} \le 3$$

$$-9 \le k_{11} \le -6 \tag{108}$$

$$\frac{1}{25} \le k_{11}k_{12} \le \frac{3}{50}$$

Choose the k_{1n} values as follows.

$$k_{10} = 2$$

$$k_{11} = -6 \tag{109}$$

$$k_{12} = -\frac{1}{150}$$

As a check to see if the above k_{1n}, k_{2n} values give Y_{sn} and Y_{pn} values within specification, we substitute the above calculated values into (91) and obtain in mhos

$$Y_{s0} = 1 \tag{110}$$

$$Y_{s1} = 1$$

$$Y_{s2} = 1$$

$$Y_{p0} = 1$$

$$Y_{p1} = 1$$

$$Y_{p2} = 1$$

All the admittance values fall within specifications. In fact, by appropriately choosing values for k_{1n} and k_{2n}, all admittance values have been made equal to one.

<u>Example Two</u>

Let it be required to synthesize

$$T(s) = \frac{s^2 - 3s}{s^2 + 100} \tag{111}$$

with the following restrictions on the non-zero admittance values:

$$1 \le Y_{sn} \le 2$$

and

$$-3 \le Y_{pn} \le -1 \tag{112}$$

or

$$1 \le Y_{pn} \le 2$$

Since the zero-order coefficient in the numerator and the first-order coefficient of the denominator are missing, (96), (97), (98), and (100) apply. For the above example,

$$
\begin{aligned}
q &= 2 \\
e_0 &= 0 \\
e_1 &= -3 \\
e_2 &= 1 \\
f_0 &= 100 \\
f_1 &= 0 \\
f_2 &= 1 \\
\alpha &= 1 \\
\beta &= 2 \\
\gamma &= 3 \\
\lambda &= 1
\end{aligned}
\tag{113}
$$

From (96), it is concluded that

$$\begin{aligned} Y_{s2} &= 0 \\ k_{22} &= \text{arbitrary} \end{aligned} \tag{114}$$

since $e_0 = 0$. From (97), it is found that

$$\begin{aligned} \frac{1}{2} \le k_{20} \le 1 \\ -3 \le k_{21} \le -\frac{3}{2} \end{aligned} \tag{115}$$

In order to satisfy (114) and (115), choose

$$\begin{aligned} k_{20} &= 1 \\ k_{21} &= -3 \\ k_{22} &= -\frac{1}{3} \end{aligned} \tag{116}$$

From (98), it is found that

$$2 \le k_{10} \le 3 \tag{117}$$

From (100), the following are obtained.

$$Y_{p1} = -Y_{s1} \tag{118}$$

and

$$k_{11} = \text{arbitrary} \tag{119}$$

since $f_1 = 0$. From (98), it is found that

$$\frac{1}{100} \leq k_{11}k_{12} \leq \frac{2}{100} \tag{120}$$

In order to satisfy (117), (119), and (120) choose

$$\begin{aligned} k_{10} &= 2 \\ k_{11} &= 1 \\ k_{12} &= \frac{1}{100} \end{aligned} \tag{121}$$

Using (91) as a check to make sure the admittance parameter values fall into specification, the results below are obtained.

$$\begin{aligned} Y_{s0} &= 1 \\ Y_{s1} &= 1 \\ Y_{s2} &= 0 \\ Y_{p0} &= 1 \end{aligned} \tag{122}$$

$$Y_{p1} = -1$$

$$Y_{p2} = 1$$

Once again the appropriate selection of values for k_{1n} and k_{2n} has resulted in all non-zero admittance values having an absolute value of one and thus within the given specifications.

Conclusion

It has been shown that for both the T(s) and Y(s) synthesis procedures the synthesis can be successfully completed for any prescribed range of admittance parameter values external to the GPIC devices. This remarkable property of the synthesis procedures depends on the unrestricted assignment of GPIC gain constant values.

CHAPTER VII

MULTI-PORT OPEN-CIRCUIT VOLTAGE TRANSFER MATRIX SYNTHESIS

A synthesis procedure is developed for realizing any multi-port open-circuit voltage transfer matrix whose elements are real rational functions in the complex variable s. The approach used to realize the matrix is to realize the matrix one row at a time. The network realization for each row is similar to the network used to realize the open-circuit voltage transfer function. No factorization of the row elements is needed, and the network elements are simply related to the coefficients of each of the row elements of the transfer matrix. Each numerator coefficient of each row element is proportional to a distinct network element and each denominator coefficient of each row is proportional to the sum of (n+1) distinct network elements, where n is equal to the number of row elements in the multi-port open-circuit voltage transfer matrix. The network realization is grounded.

Consider the synthesis of the matrix [T] which relates the input voltages to the open-circuit output voltages of a network by the following matrix equation

$$E_{\text{output}}\Big] = [T] \; E_{\text{input}}\Big] \tag{123}$$

or equivalently

$$\begin{bmatrix} E_{n+1} \\ \vdots \\ E_{n+h} \\ \vdots \\ E_{n+r} \end{bmatrix} = \begin{bmatrix} T_{(n+1)1} & T_{(n+1)2} & \cdots & T_{(n+1)n} \\ \vdots & \vdots & & \vdots \\ T_{(n+h)1} & T_{(n+h)2} & \cdots & T_{(n+h)n} \\ \vdots & \vdots & & \vdots \\ T_{(n+r)1} & T_{(n+r)2} & \cdots & T_{(n+r)n} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix} \tag{124}$$

where

$$\begin{aligned} n &= \text{number of input voltages of the network} \\ r &= \text{number of output voltages of the network} \end{aligned} \tag{125}$$

Each row of the matrix [T] of (124) is augmented if necessary in order that each element of the row has the same common denominator. The hth row of [T] can be realized by the network of Figure 14. In Figure 14 and in the discussion to follow, q is equal to the maximum order of the rational elements in the row. The $\pm E_k$ $[k=1,2,\ldots,n]$ needed in each row synthesis can be realized by the networks of Figure 15 or Figure 16. The active blocks shown in Figure 15 are ideal phase-inverting unity-gain amplifiers. The GPIC's shown in Figure 16 are each characterized by the chain matrix

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \tag{126}$$

The GPIC's of Figure 14 are characterized by the following chain matrices.

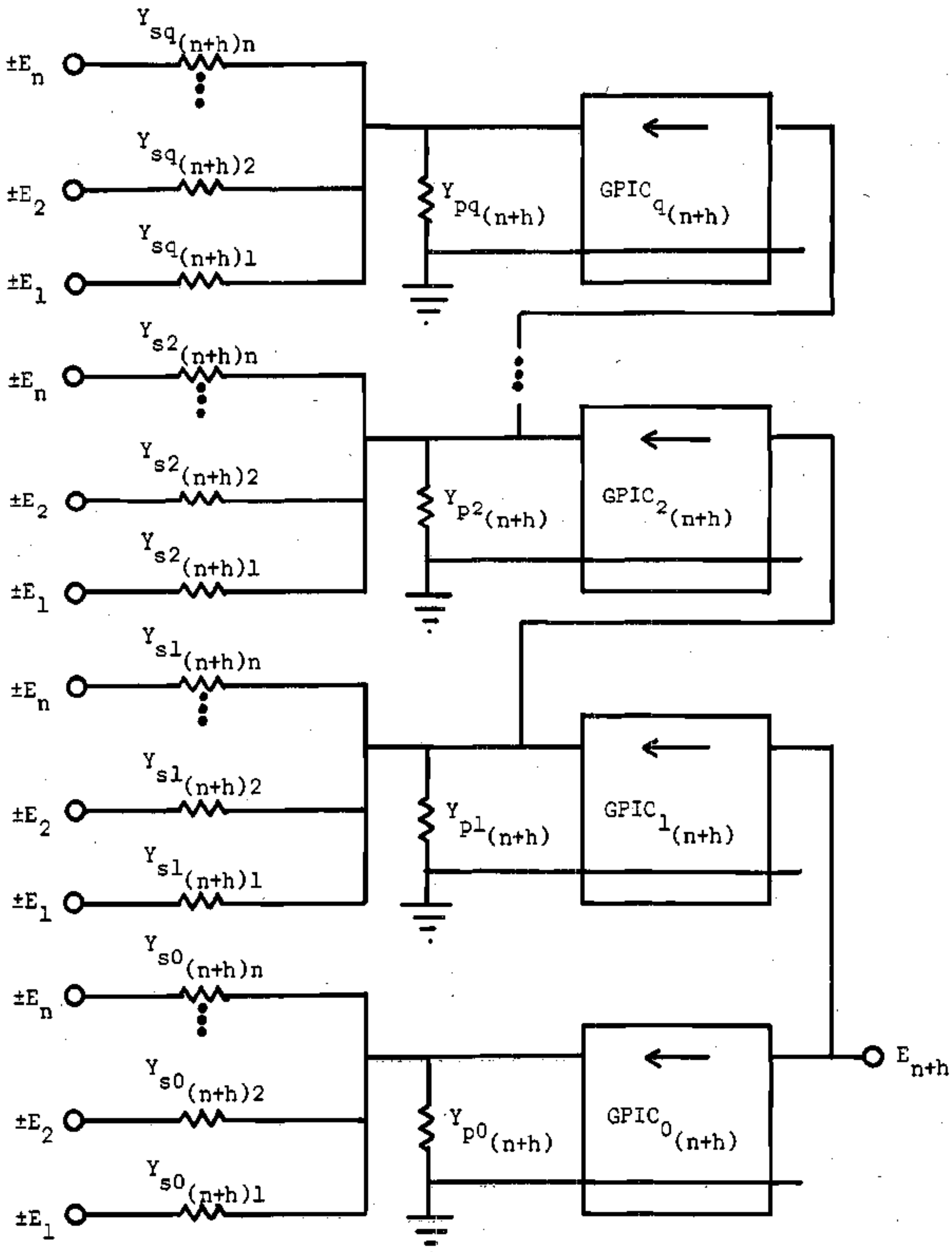


Figure 14. Network for Realizing hth Row of Matrix [T] of (125)

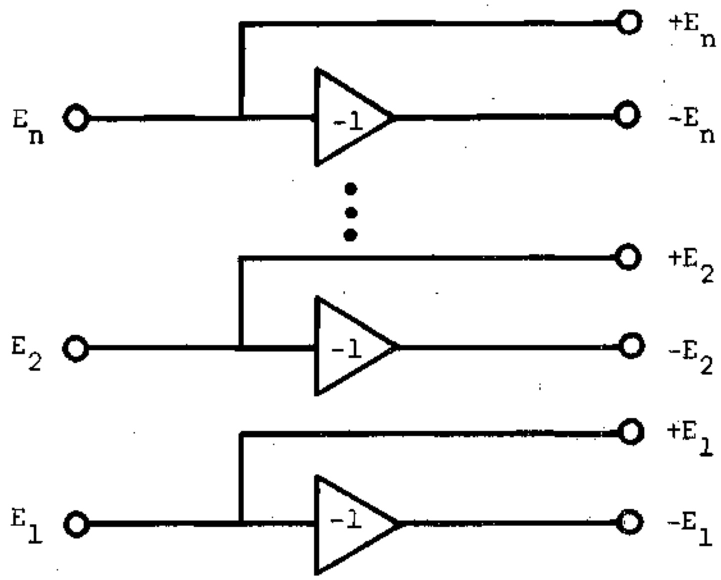


Figure 15. Unity-Gain Amplifier Network

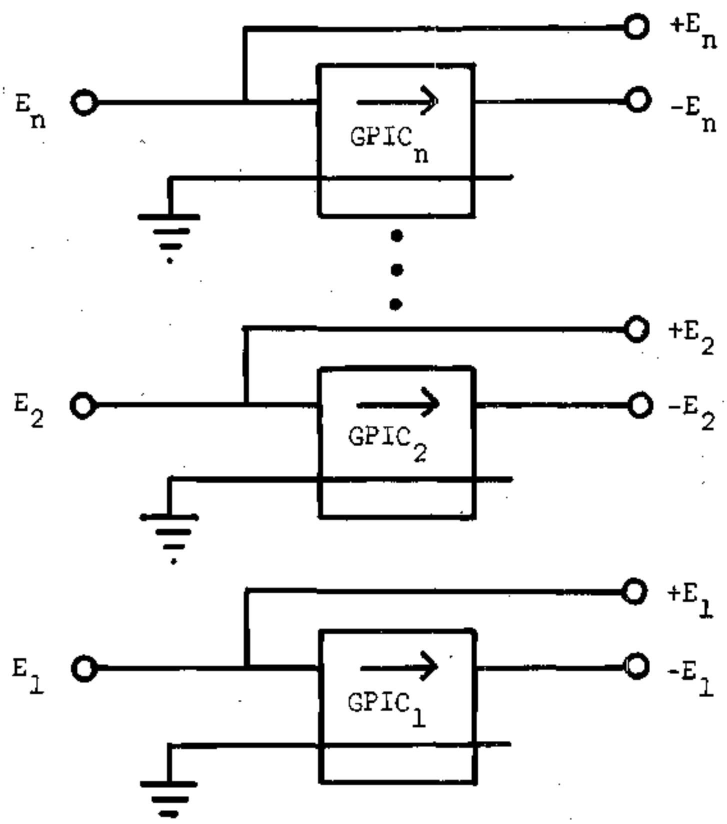


Figure 16. GPIC Voltage Network

$$\mathrm{GPIC}_{0_{(n+h)}} : \begin{bmatrix} k_{10_{(n+h)}} & 0 \\ 0 & k_{20_{(n+h)}} \end{bmatrix}$$

$$\mathrm{GPIC}_{m_{(n+h)}} : \begin{bmatrix} k_{1m_{(n+h)}} & 0 \\ 0 & \frac{k_{2m_{(n+h)}}}{s} \end{bmatrix} \tag{127}$$

for m=1,2,...,q.

Analysis of the network of Figure 14 reveals that for each i=1,2,...,n and h=1,2,...,r

$$T_{(n+h)i} = \frac{A(h,i)}{B(h)} \tag{128}$$

where

$$A(h,i) = k_{20_{(n+h)}} Y_{s0_{(n+h)i}} + \frac{k_{21_{(n+h)}} Y_{s1_{(n+h)i}}}{s} + \frac{k_{21_{(n+h)}} k_{22_{(n+h)}} Y_{s2_{(n+h)i}}}{s^2} + \dots + \frac{k_{21_{(n+h)}} k_{22_{(n+h)}} \dots k_{2q_{(n+h)}} Y_{sq_{(n+h)i}}}{s^q} \tag{129}$$

$$B(h) = \frac{k_{20_{(n+h)}}}{k_{10_{(n+h)}}}\left[\sum_{i=1}^{n} Y_{s0_{(n+h)i}} + Y_{p0_{(n+h)}}\right] + \tag{130}$$

$$\frac{k_{21_{(n+h)}}}{sk_{11_{(n+h)}}}\left[\sum_{i=1}^{n} Y_{s1_{(n+h)i}} + Y_{p1_{(n+h)}}\right] +$$

$$\frac{k_{21_{(n+h)}}k_{22_{(n+h)}}}{s^2k_{11_{(n+h)}}k_{12_{(n+h)}}}\left[\sum_{i=1}^{n} Y_{s2_{(n+h)i}} + Y_{p2_{(n+h)}}\right] + \dots +$$

$$\frac{k_{21_{(n+h)}}k_{22_{(n+h)}}\dots k_{2q_{(n+h)}}}{s^qk_{11_{(n+h)}}k_{12_{(n+h)}}\dots k_{1q_{(n+h)}}}\left[\sum_{i=1}^{n} Y_{sq_{(n+h)i}} + Y_{pq_{(n+h)}}\right]$$

Now each $T_{(n+h)i}$ of the matrix to be synthesized is the ratio of two polynomials in s as was the T(s) in Chapter III and is of the form

$$T_{(n+h)i} = \frac{\sum_{u=0}^{q} a_{u_i} s^{u-q}}{\sum_{v=0}^{q} b_v s^{v-q}} \tag{131}$$

Therefore the synthesis procedure becomes similar to that discussed in Chapter III, one of matching coefficients of a transfer function to network element values. Comparing (131) with Equations (128), (129), and (130), we see that we can realize (131) with the network of Figure 14 as follows. For h=1,2,...,r and i=1,2,...,n, make

$$Y_{s0_{(n+h)i}} = \frac{|a_{q_i}|}{k_{20_{(n+h)}}} \tag{132}$$

$$Y_{s1_{(n+h)i}} = \frac{|a_{q-1_i}|}{k_{21_{(n+h)}}}$$

$$Y_{s2_{(n+h)i}} = \frac{|a_{q-2_i}|}{k_{21_{(n+h)}} k_{22_{(n+h)}}}$$

$$\vdots$$

$$Y_{sq_{(n+h)i}} = \frac{|a_{0_i}|}{k_{21_{(n+h)}} k_{22_{(n+h)}} \cdots k_{2q_{(n+h)}}}$$

For any $a_{(q-k)_i}$ [k=0,1,...,q] encountered, which is algebraically negative, $Y_{sk_{(n+h)i}}$ must be connected to $-E_i$. All other $Y_{sk_{(n+h)i}}$ network elements are connected to $+E_i$. To complete the realization, make

$$Y_{p0_{(n+h)}} = \frac{k_{10_{(n+h)}} b_q}{k_{20_{(n+h)}}} - \sum_{i=1}^{n} Y_{s0_{(n+h)i}} \tag{133}$$

$$Y_{p1_{(n+h)}} = \frac{k_{11_{(n+h)}} b_{q-1}}{k_{21_{(n+h)}}} - \sum_{i=1}^{n} Y_{s1_{(n+h)i}}$$

$$Y_{p2_{(n+h)}} = \frac{k_{11_{(n+h)}} k_{12_{(n+h)}} b_{q-2}}{k_{21_{(n+h)}} k_{22_{(n+h)}}} - \sum_{i=1}^{n} Y_{s2_{(n+h)i}}$$

$$\vdots$$

$$Y_{pq_{(n+h)}} = \frac{k_{11_{(n+h)}} k_{12_{(n+h)}} \cdots k_{1q_{(n+h)}} b_0}{k_{21_{(n+h)}} k_{22_{(n+h)}} \cdots k_{2q_{(n+h)}}} - \sum_{i=1}^{n} Y_{sq_{(n+h)i}}$$

The synthesis procedure can be summarized as follows:

1. Identify the matrix elements of the multi-port open-circuit voltage transfer matrix [T] to be synthesized as in (124).

2. Identify each coefficient of the ith element of the hth row of [T] according to (131).

3. Evaluate all $Y_{sk_{(n+h)i}}$ [k=0,1,2,...,q] network elements of Figure 14 for the ith element of the hth row of [T] according to (132), noting that for any $a_{(q-k)_i}$ [k=0,1,2,...,q] encountered, which is algebraically negative, $Y_{sk_{(n+h)i}}$ must be connected to $-E_i$. All other $Y_{sk_{(n+h)i}}$ network elements are connected to $+E_i$.

4. Repeat 2 and 3 for each matrix element of the hth row of [T].

5. Evaluate all $Y_{pk_{(n+h)}}$ [k=0,1,2,...,q] network elements of Figure 14 for the hth row according to (133).

6. Repeat 2 through 5 for each row of [T].

An Example

Using the multi-port open-circuit voltage transfer matrix synthesis procedure, a network will be found that realizes the following:

$$\begin{bmatrix} E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} \frac{s}{s+1} & -\frac{s}{s+1} \\ -\frac{2}{s^2+1} & \frac{s^2-s+1}{s^2+1} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \tag{134}$$

For the synthesis procedure, n=2, r=2 and

$$\begin{bmatrix} T_{31} & T_{32} \\ T_{41} & T_{42} \end{bmatrix} = \begin{bmatrix} \frac{s}{s+1} & -\frac{s}{s+1} \\ -\frac{2}{s^2+1} & \frac{s^2-s+1}{s^2+1} \end{bmatrix} \tag{135}$$

For the row corresponding to h=1, it is obvious that q=1. For h=1 and i=1,

$$T_{31} = \frac{1}{1+\frac{1}{s}} = \frac{\sum_{u=0}^{1} a_{u_1} s^{u-1}}{\sum_{v=0}^{1} b_v s^{v-1}} \tag{136}$$

Hence,

$$a_{0_1} = 0 \tag{137}$$

$$a_{1_1} = 1$$

$$b_0 = 1$$

$$b_1 = 1$$

Using the results of (132) and (137), it is found that

$$Y_{s0_{(3)1}} = \frac{1}{k_{20_{(3)}}} \tag{138}$$

$$Y_{s1_{(3)1}} = 0$$

For h=1 and i=2,

$$T_{32} = \frac{-1}{1 + \frac{1}{s}} = \frac{\sum_{u=0}^{1} a_{u_2} s^{u-1}}{\sum_{v=0}^{1} b_v s^{v-1}} \tag{139}$$

Hence,

$$a_{0_2} = 0$$
$$a_{1_2} = -1 \tag{140}$$

Since a_{1_2} is algebraically negative, connect $Y_{s0_{(3)2}}$ to $-E_2$. Using the results of (132) and (140), it is found that

$$Y_{s0_{(3)2}} = \frac{1}{k_{20_{(3)}}} \tag{141}$$

$$Y_{s1_{(3)2}} = 0$$

To complete the row synthesis corresponding to h=1, use (133), (137), (138), and (141) in determining that

$$Y_{p0_{(3)}} = \frac{k_{10_{(3)}} - 2}{k_{20_{(3)}}} \tag{142}$$

$$Y_{p1_{(3)}} = \frac{k_{11_{(3)}}}{k_{21_{(3)}}}$$

For the row corresponding to h=2, q=2. For h=2 and i=1,

$$T_{41} = \frac{-\frac{2}{s^2}}{1 + \frac{1}{s^2}} = \frac{\sum_{u=0}^{2} a_{u_1} s^{u-2}}{\sum_{v=0}^{2} b_v s^{v-2}} \tag{143}$$

Hence,

$$a_{0_1} = -2 \tag{144}$$

$$a_{1_1} = 0$$

$$a_{2_1} = 0$$

$$b_0 = 1$$

$$b_1 = 0$$

$$b_2 = 1$$

Since a_{0_1} is algebraically negative, connect $Y_{s2_{(4)1}}$ to $-E_1$. By using (132) and (144) the following results are obtained.

$$Y_{s0_{(4)1}} = 0 \tag{145}$$

$$Y_{s1_{(4)1}} = 0$$

$$Y_{s2_{(4)1}} = \frac{2}{k_{21_{(4)}} k_{22_{(4)}}}$$

For h=2 and i=2,

$$T_{42} = \frac{1 - \frac{1}{s} + \frac{1}{s^2}}{1 + \frac{1}{s^2}} = \frac{\sum_{u=0}^{2} a_{u_2} s^{u-2}}{\sum_{v=0}^{2} b_v s^{v-2}} \tag{146}$$

Therefore,

$$a_{0_2} = 1 \tag{147}$$

$$a_{1_2} = -1$$

$$a_{2_2} = 1$$

Since a_{1_2} is negative, connect $Y_{s1_{(4)2}}$ to $-E_2$. Using the results of (132) and (147), it is found that

$$Y_{s0_{(4)2}} = \frac{1}{k_{20_{(4)}}} \tag{148}$$

$$Y_{s1_{(4)2}} = \frac{1}{k_{21_{(4)}}}$$

$$Y_{s2_{(4)2}} = \frac{1}{k_{21_{(4)}} k_{22_{(4)}}}$$

Using the results of (133), (144), (145), and (148), the following results are obtained.

$$Y_{p0_{(4)}} = \frac{k_{10_{(4)}} - 1}{k_{20_{(4)}}}$$

$$Y_{p1_{(4)}} = -\frac{1}{k_{21_{(4)}}} \tag{149}$$

$$Y_{p2_{(4)}} = \frac{k_{11_{(4)}} k_{12_{(4)}} - 3}{k_{21_{(4)}} k_{22_{(4)}}}$$

The network that realizes the desired multi-port open-circuit voltage transfer matrix is shown in Figure 17. The GPIC's of Figure 17 have the chain matrices given by (127).

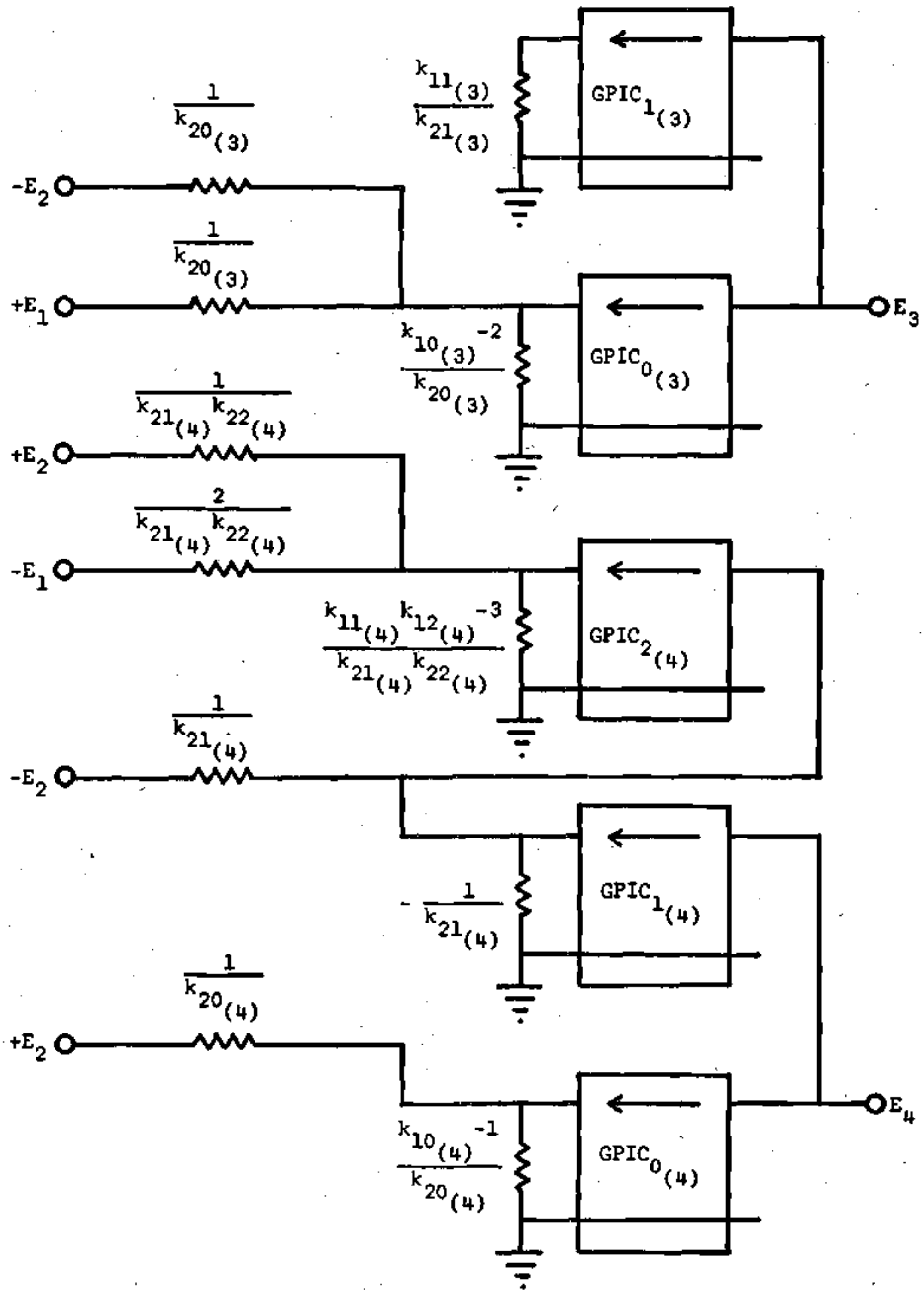


Figure 17. Multi-Port Transfer Matrix Network for Realizing the Example of (134)

CHAPTER VIII

SHORT-CIRCUIT ADMITTANCE MATRIX SYNTHESIS

A synthesis procedure is developed for realizing any short-circuit admittance matrix whose elements are real rational functions in the complex variable s. The approach used in this development is similar to the work done by Hilberman and Joseph.[7] The ideal unity-gain amplifiers used by Hilberman and Joseph are replaced with special GPIC's. Use of these GPIC's allows the synthesis of the desired short-circuit admittance matrix to be transferred to the synthesis of a multi-port open-circuit voltage transfer matrix. The network realization is grounded.

The N×N short-circuit admittance matrix [Y] relates the N port voltages and N port currents of a network such as the one in Figure 18 as follows:

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = [Y] \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix} \tag{150}$$

where

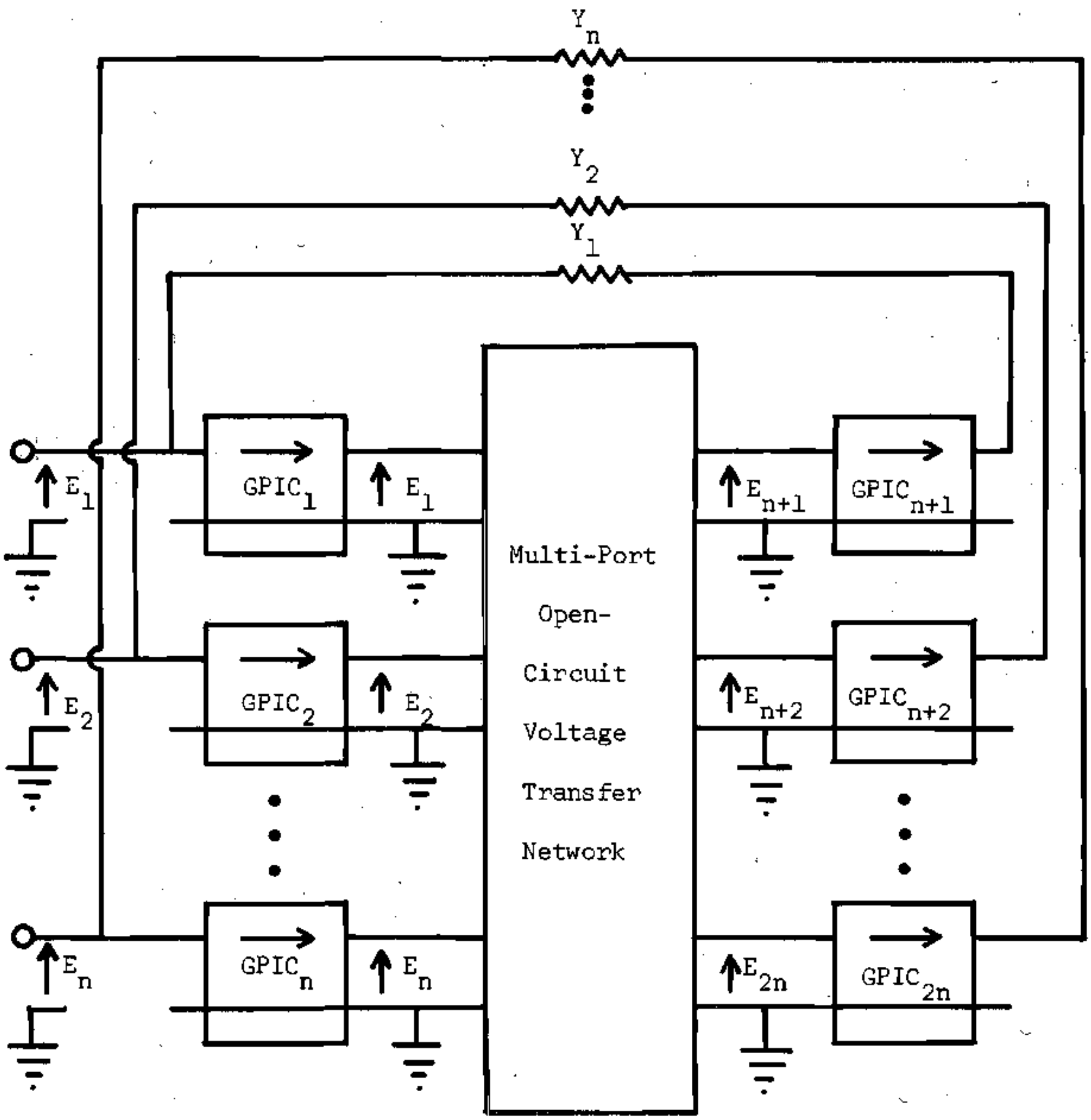


Figure 18. N×N Short-Circuit Admittance Matrix Network

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \qquad (151)$$

The network of Figure 18 is the N-port version of the network used to realize the driving-point admittance in Chapter IV. The network designated Multi-Port Open-Circuit Voltage Transfer Network performs the function as described in Chapter VII. The GPIC's shown in Figure 18 have the chain matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad (152)$$

Hence port 1 and port 2 voltages of each GPIC are identical and port 1 currents of each GPIC are zero. Therefore, it is meaningful to relate the voltages E_1 through E_n to E_{n+1} through E_{2n} by the multi-port open-circuit voltage transfer matrix, namely

$$\begin{bmatrix} E_{n+1} \\ E_{n+2} \\ \vdots \\ E_{2n} \end{bmatrix} = \begin{bmatrix} T_{(n+1)1} & T_{(n+1)2} & \cdots & T_{(n+1)n} \\ T_{(n+2)1} & T_{(n+2)2} & \cdots & T_{(n+2)n} \\ \vdots & \vdots & & \vdots \\ T_{(2n)1} & T_{(2n)2} & \cdots & T_{(2n)n} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix} \qquad (153)$$

Analysis of the network of Figure 18 reveals that

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix} \underline{|Y_1 Y_2 \ldots Y_n|} - \begin{bmatrix} E_{n+1} \\ E_{n+2} \\ \vdots \\ E_{2n} \end{bmatrix} \underline{|Y_1 Y_2 \ldots Y_n|} \tag{154}$$

If we substitute the results of (153) into (154) and carry out the matrix multiplication, Equation (155) is obtained.

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = [Y'] \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix} \tag{155}$$

where

$$[Y'] = \tag{156}$$

$$\begin{bmatrix} Y_1(1-T_{(n+1)1}) & -Y_1T_{(n+1)2} & -Y_1T_{(n+1)3} & \cdots & -Y_1T_{(n+1)n} \\ -Y_2T_{(n+2)1} & Y_2(1-T_{(n+2)2}) & -Y_2T_{(n+2)3} & \cdots & -Y_2T_{(n+2)n} \\ \vdots & \vdots & & & \vdots \\ -Y_nT_{(2n)1} & -Y_nT_{(2n)2} & \cdots & -Y_nT_{(2n)n-1} & Y_n(1-T_{(2n)n}) \end{bmatrix}$$

The matrix relating the currents and voltages in (156) must be equivalent to the admittance matrix relating the same voltages and currents of (151). From (156) and (151), we obtain

$$\begin{bmatrix} T_{(n+1)1} & T_{(n+1)2} & \cdots & T_{(n+1)n} \\ T_{(n+2)1} & T_{(n+2)2} & \cdots & T_{(n+2)n} \\ \vdots & \vdots & & \vdots \\ T_{(2n)1} & T_{(2n)2} & \cdots & T_{(2n)n} \end{bmatrix} = \begin{bmatrix} \frac{Y_1-Y_{11}}{Y_1} & -\frac{Y_{12}}{Y_1} & -\frac{Y_{13}}{Y_1} & \cdots & -\frac{Y_{1n}}{Y_1} \\ -\frac{Y_{21}}{Y_2} & \frac{Y_2-Y_{22}}{Y_2} & -\frac{Y_{23}}{Y_2} & \cdots & -\frac{Y_{2n}}{Y_2} \\ \vdots & \vdots & & & \vdots \\ -\frac{Y_{n1}}{Y_n} & -\frac{Y_{n2}}{Y_n} & \cdots & -\frac{Y_{n(n-1)}}{Y_n} & \frac{Y_n-Y_{nn}}{Y_n} \end{bmatrix} \tag{157}$$

Thus, the synthesis procedure for realizing the short-circuit admittance matrix [Y] is as follows:

1. Identify the elements of the matrix [Y] as in (151).

2. Use (157) to evaluate the elements of an open-circuit voltage transfer matrix [T].

3. Use the synthesis procedure of Chapter VII to find a network that will realize [T].

4. Use the network realization of [T] as in Figure 18 to obtain the [Y] network realization.

An Example

The following short-circuit admittance matrix will be realized by means of the synthesis procedure developed in this chapter.

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & \frac{s}{s+1} \\ \frac{2}{s^2+1} & \frac{s}{s^2+1} \end{bmatrix} \tag{158}$$

Assume that all Y_i's [i=1,2] in the short-circuit admittance matrix synthesis procedure discussed in this chapter are equal to one mho. Then from (157) in the synthesis procedure development, we get

$$\begin{bmatrix} T_{31} & T_{32} \\ T_{41} & T_{42} \end{bmatrix} = \begin{bmatrix} \frac{s}{s+1} & -\frac{s}{s+1} \\ -\frac{2}{s^2+1} & \frac{s^2-s+1}{s^2+1} \end{bmatrix} \tag{159}$$

The multi-port open-circuit voltage transfer network of Figure 18 needed in the synthesis must have the transfer matrix of (159). This particular transfer matrix was synthesized in Chapter VII and can be realized with the network of Figure 17.

The network of Figure 19 will realize the admittance matrix of (158). All $GPIC_i$'s [i=1,2,3,4] have the chain matrix of (152).

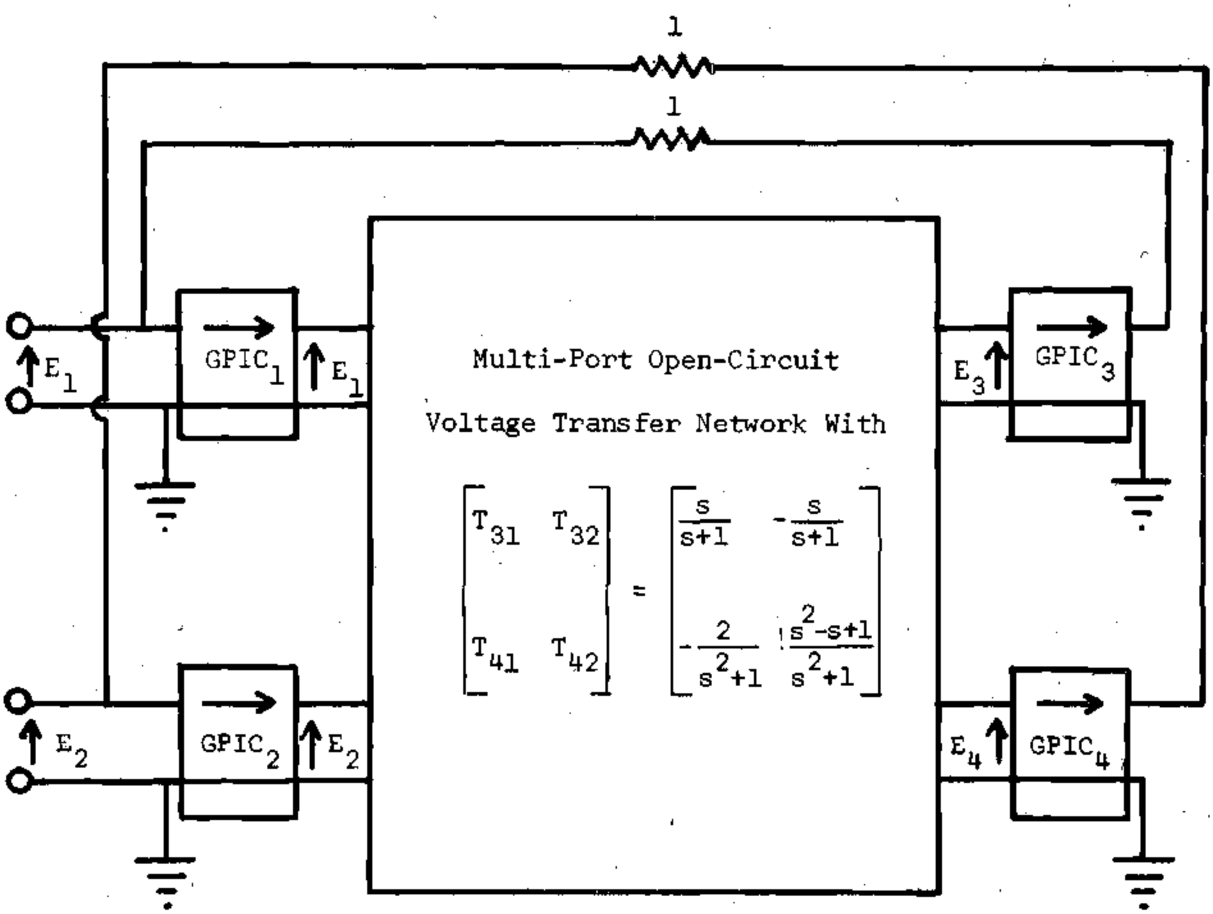


Figure 19. Network for Realizing the Admittance Matrix of (158)

CHAPTER IX

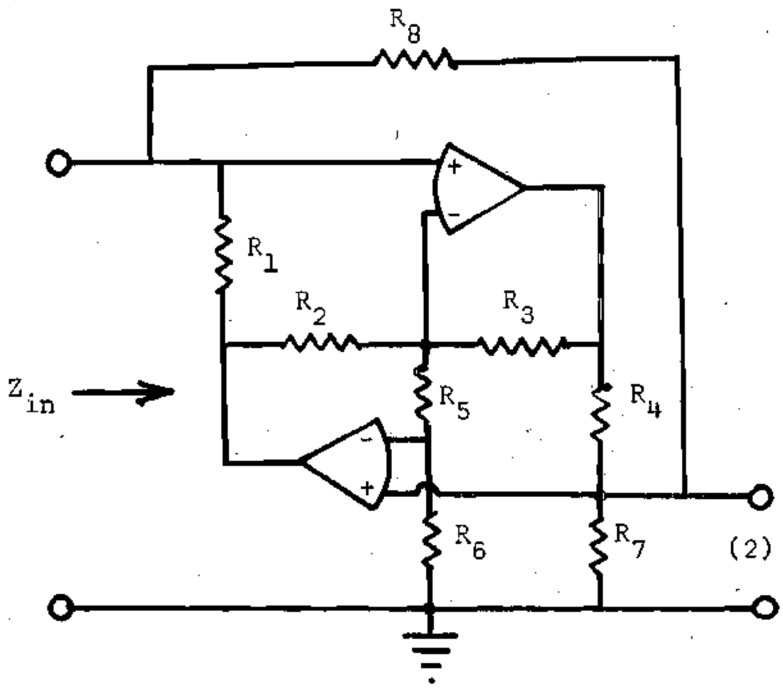
EXPERIMENTAL RESULTS

In order to demonstrate that the realization procedures which have been developed are not only correct, but also practical, examples were worked using the procedures, and the resulting networks were constructed and tested. The test data obtained were compared with the predicted behavior.

Example One

The complete generality of the synthesis procedures has depended upon the capability of being able to realize the negative resistance. The theoretical aspects of realizing the negative resistance by means of the GPIC are discussed in Appendix I. A circuit configuration with a GPIC that can be used in the negative resistance realization is shown in Figure 2. Using that particular circuit configuration and the discussion from Appendix I, it is obvious that the network of Figure 20 can be used in the realization of the negative resistance. The GPIC portion of the network in Figure 20 has the chain matrix

$$\begin{bmatrix} k_{1N} & 0 \\ 0 & k_{2N} \end{bmatrix} = \begin{bmatrix} \dfrac{R_5+R_6}{R_6} & 0 \\ 0 & \dfrac{R_2R_4}{R_1R_3} \end{bmatrix} \qquad (160)$$



$$R_7 = \frac{R_4 R_6}{R_3 + R_5}$$

Figure 20. Network for Realizing the Negative Resistance

From (A-7) of Appendix I, it is seen that the input impedance of the circuit under consideration is

$$Z_{in} = - \frac{k_{1N} R_8}{(k_{1N}-1)(k_{2N}-1)} \tag{161}$$

Three values of negative resistance were realized. In each of the three realizations, k_{1N} and k_{2N} of (161) were made equal to two. For a particular value of negative resistance, the following parameter values were assigned to the circuit of Figure 20:

a. For Z_{in} = -200 ohms, $R_1 = R_4 = R_8$ = 100 ohms, R_2 = 2 kilohms,

R_3 = 1 kilohm, $R_5 = R_6$ = 10 kilohms, and R_7 = 91 ohms were assigned.

b. For Z_{in} = -2 kilohms, $R_1 = R_3 = R_4 = R_8$ = 1 kilohm, R_2 = 2 kilohms, $R_5 = R_6$ = 10 kilohms, and R_7 = 909 ohms were assigned.

c. For Z_{in} = -4 kilohms, $R_1 = R_3 = R_4$ = 1 kilohm, $R_2 = R_8$ = 2 kilohms, $R_5 = R_6$ = 10 kilohms, and R_7 = 909 ohms were assigned.

The theoretical and measured values of magnitude and angle for the three realizations are shown in Figures 21 and 22. The measured curves are indicated by solid lines while the theoretical curves are indicated by broken lines.

Example Two

In order to demonstrate the practicality of the open-circuit voltage transfer function synthesis procedure of Chapter IV, the open-circuit voltage transfer function

$$T(s) = \frac{10^6}{10^{-2}s^2 + 2.5s + 10^6} \tag{162}$$

was realized according to the synthesis procedure. Note that this T(s) has a Q of 40. Q has the definition given in (63).

This transfer function can be synthesized with the network of Figure 10. The element values for the network are obtained from (16) and (31).

Since the order of the transfer function is two, two GPIC's are needed, namely $GPIC_1$ and $GPIC_2$ of Figure 10. Let $GPIC_1$ and $GPIC_2$ have the chain matrices

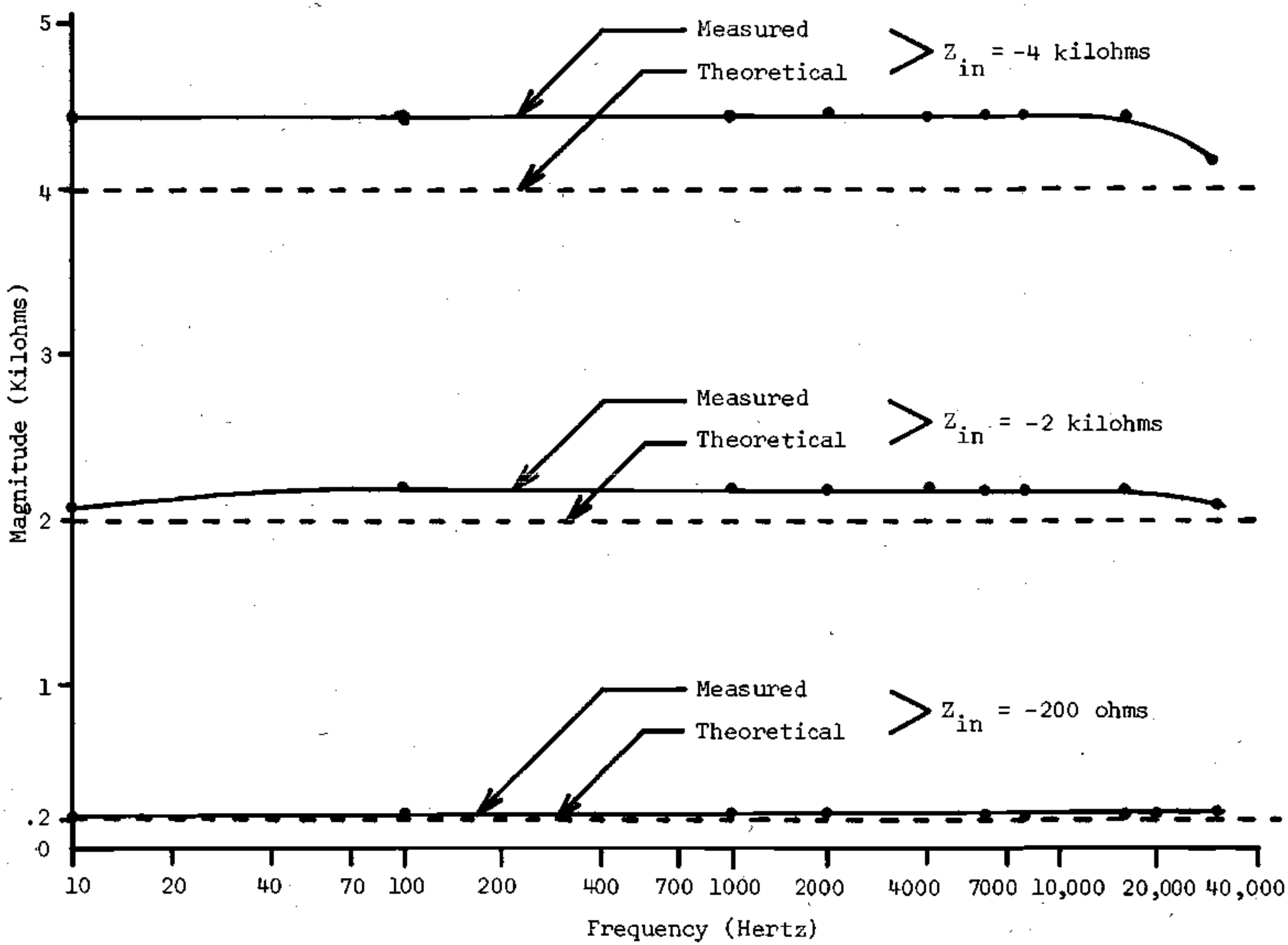


Figure 21. Magnitude, Example One

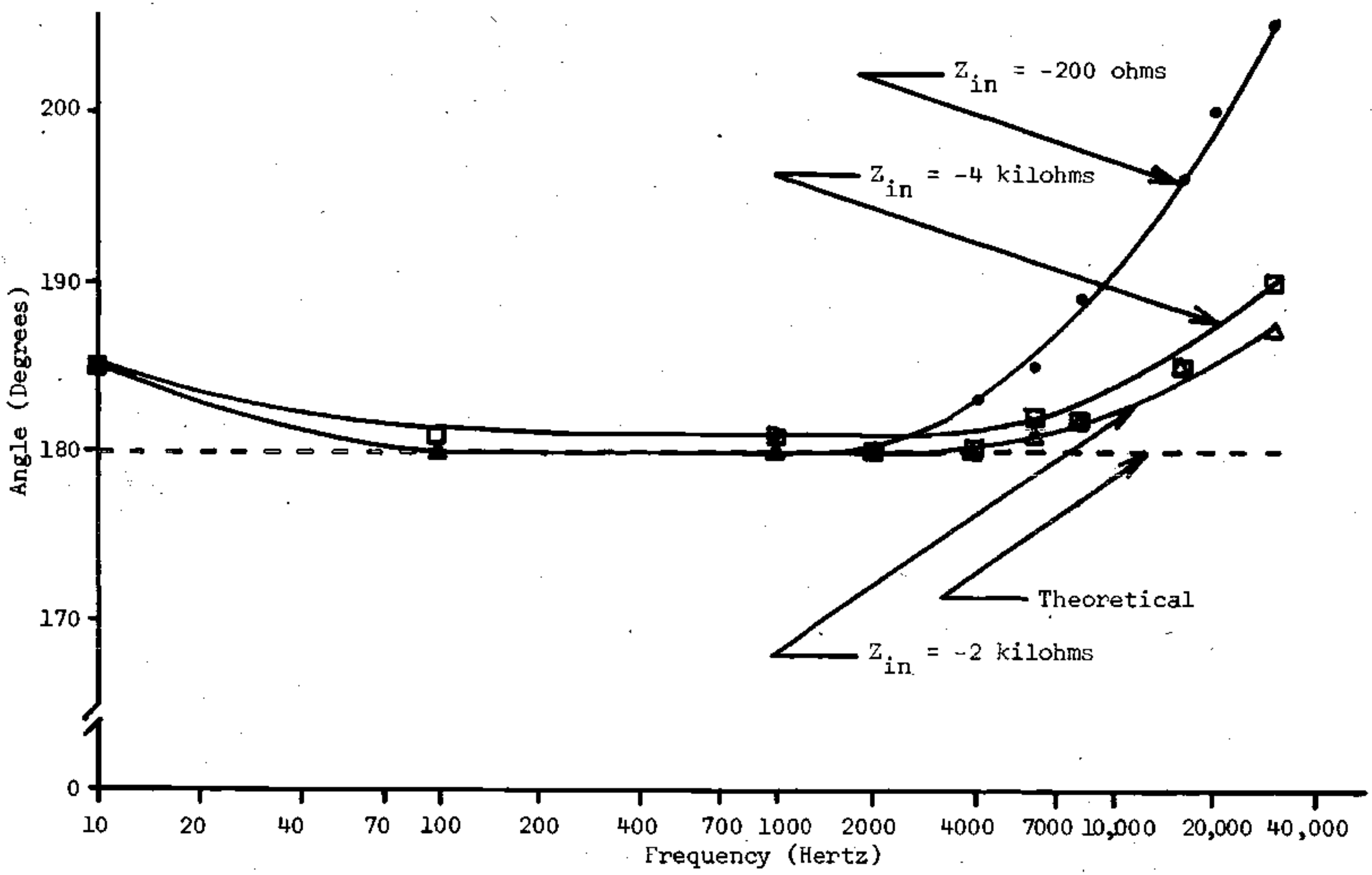


Figure 22. Angle, Example One

$$\begin{bmatrix} 2 & 0 \\ 0 & \frac{10^4}{s} \end{bmatrix} \tag{163}$$

and

$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{10^4}{s} \end{bmatrix} \tag{164}$$

respectively. Hence, in the synthesis procedure,

$$k_{11} = 2 \tag{165}$$

$$k_{21} = 10^4$$

$$k_{12} = 1$$

$$k_{22} = 10^4$$

From (16) and (162), the coefficient identification for the synthesis is made.

$$a_0 = 10^6 \tag{166}$$

$$a_1 = 0$$

$$a_2 = 0$$

$$b_0 = 10^6$$

$$b_1 = 2.5$$

$$b_2 = 10^{-2}$$

Substituting the results of (165) and (166) into (31), the element values external to the GPIC's are obtained and are given below in units of mhos.

$$Y_{s0} = 0 \tag{167}$$

$$Y_{s1} = 0$$

$$Y_{s2} = 10^{-2}$$

$$Y_{p0} = 10^{-2}$$

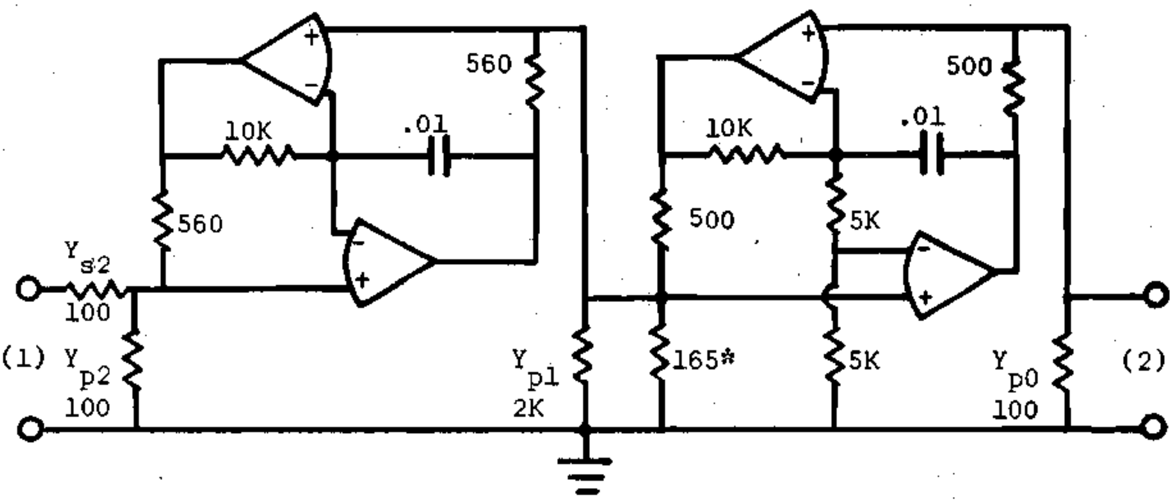
$$Y_{p1} = 5\times10^{-4}$$

$$Y_{p2} = 10^{-2}$$

$GPIC_1$ can be realized by the circuit of Figure 37, Appendix II, when the following element values are assigned: $R_1 = R_4 = 500$ ohms, $C = 0.01$ microfarad, $R_3 = 10$ kilohms, $R_5 = R_6 = 5$ kilohms, and $R_7 = 167$ ohms.

From (9) it is seen that $GPIC_2$ can be realized by the circuit of Figure 1 when $Z_1 = R_1$, $Z_2 = \frac{1}{sC}$, $Z_3 = R_3$, $Z_4 = R_4$ and the following values are assigned: R_1 = 560 ohms, C = 0.01 microfarad, R_3 = 10 kilohms, and R_4 = 560 ohms.

The circuit that was used to realize the transfer function of (162) is shown in Figure 23.



Element values in ohms or microfarads.

*Trim if necessary to account for tolerances of other resistors and non-idealness of operational amplifiers.

Figure 23. Realization of the Open-Circuit Voltage Transfer Function of (162)

Note that it was necessary to trim R_7 of $GPIC_1$, whose value was given as 167 ohms, to 165 ohms in the circuit of Figure 23. This was necessary in order to make $GPIC_1$ of Figure 23 have a k_2 value equal to 10^4 in its chain matrix. This trimming results from the fact that the circuit was constructed with resistors with 5 per cent tolerances and non-ideal operational amplifiers. The role that R_7 plays in the characteristics of $GPIC_1$ is discussed in Appendix II. The theoretical and measured values of magnitude and phase for the above example are shown in Figures 24 and 25, respectively.

Example Three

To demonstrate the practicality of the driving-point admittance synthesis procedure discussed in Chapter IV, the admittance

$$Y(s) = \frac{1}{s + 100} \tag{168}$$

was realized according to the synthesis procedure. Note that (168) is the admittance of a one henry inductor with 100 ohms resistance. Using the results of (38), (39), (40), and (41), the open-circuit voltage transfer function that must be synthesized is

$$T(s) = \frac{s + 100 - R}{k_{1A}k_{1B}(s+100)} \tag{169}$$

To make the synthesis of T(s) of (169) relatively simple, let R = 100 ohms and $k_{1A} = k_{1B} = 1$. Then (169) becomes

$$T(s) = \frac{s}{s + 100} \tag{170}$$

Equation (170) can be synthesized according to the method discussed in Chapter III or by recognizing that (170) is the transfer function of the form of the simple RC high-pass section shown in Figure 26.

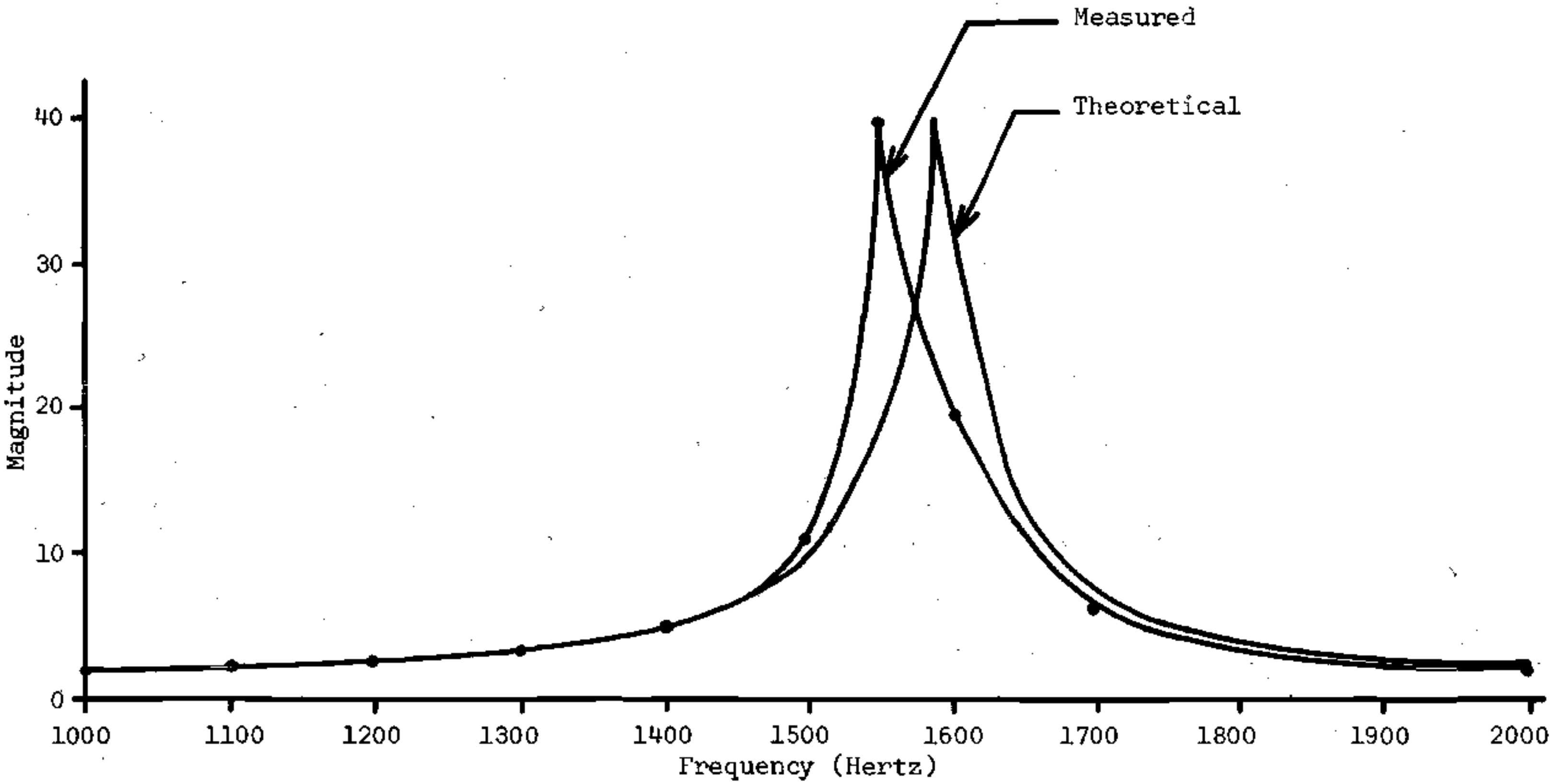


Figure 24. Magnitude, Example Two

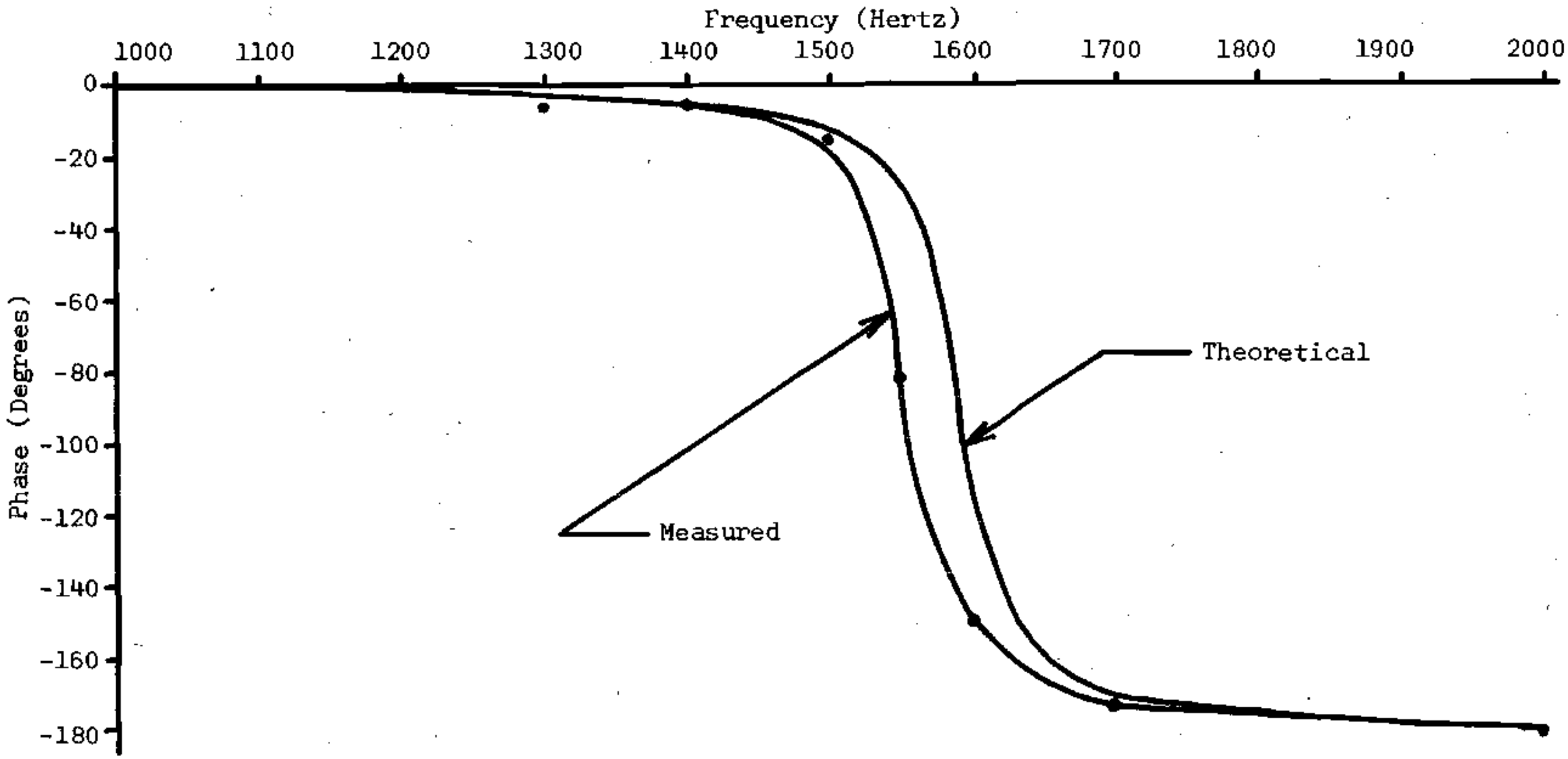


Figure 25. Phase, Example Two

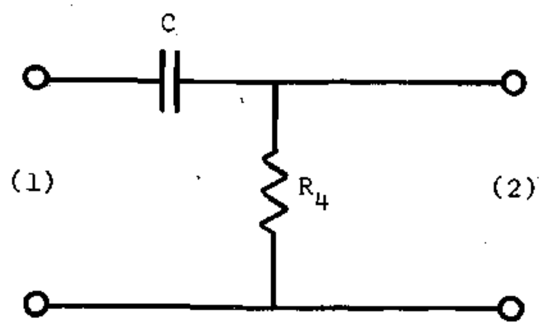


Figure 26. R-C High-Pass Network for Realizing the T(s) of (171)

The latter realization will be used. The open-circuit voltage transfer function of the network of Figure 26 is

$$T(s) = \frac{s}{s + \frac{1}{R_4 C}} \tag{171}$$

Equating (170) and (171), it is seen that $R_4C = 0.01$. To get the necessary transfer function, let $R_4 = 100$ kilohms and $C = 0.1$ microfarad.

Since k_{1A} and k_{1B} were made equal to one, $GPIC_A$ and $GPIC_B$ must have the chain matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{172}$$

A GPIC with the chain matrix of (172) is discussed in Chapter II and is shown in Figure 6. Arbitrarily, R_1, R_2, and R_3 were made equal to one kilohm in the circuit of Figure 6.

Combining $GPIC_A$, $GPIC_B$, R, and the T(s) network according to the theory of Chapter IV and the network of Figure 12, the desired driving-point function of (168) was realized with the network of Figure 27.

The theoretical and measured values of magnitude and angle for the above example are shown in Figures 28 and 29, respectively.

Example Four

It is not always necessary to resort to the driving-point admittance synthesis procedure of Chapter IV to realize driving-point functions with GPIC's. The synthesis of the inductance can be handled quite simply with a single GPIC and a resistor.

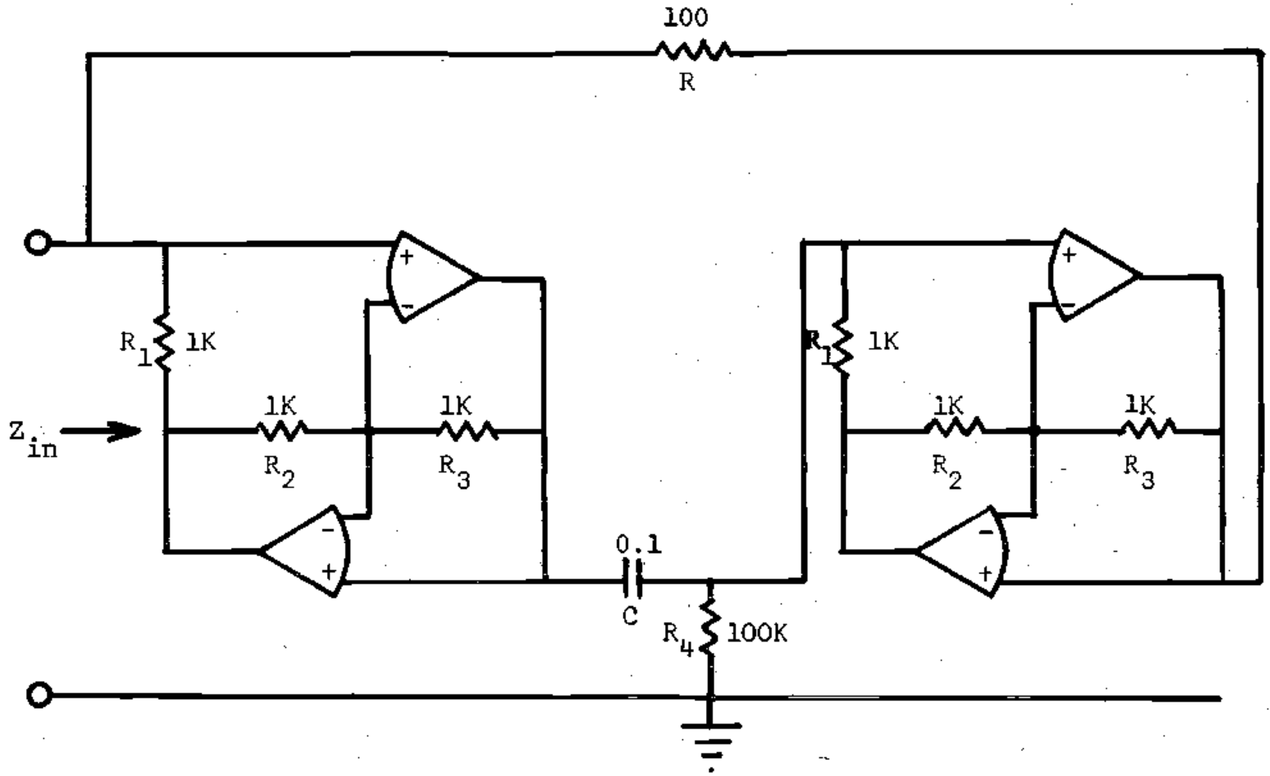
From (2), it is seen that when a GPIC described by the chain matrix

$$\begin{bmatrix} \pm k_1 & 0 \\ 0 & \pm \frac{k_2}{s} \end{bmatrix} \tag{173}$$

is terminated in R_L at port 2, then the input impedance looking into port 1 is given by

$$Z_{in} = \frac{sk_1R_L}{k_2} \tag{174}$$

Now k_1 and k_2 always have the same algebraic sign. Any one of the GPIC's discussed in Chapter II which has the complex D term in its chain matrix could be used to realize the inductance.



Element values shown in ohms or microfarads.

Figure 27. Network for Realizing the Driving-Point Admittance Function of (168)

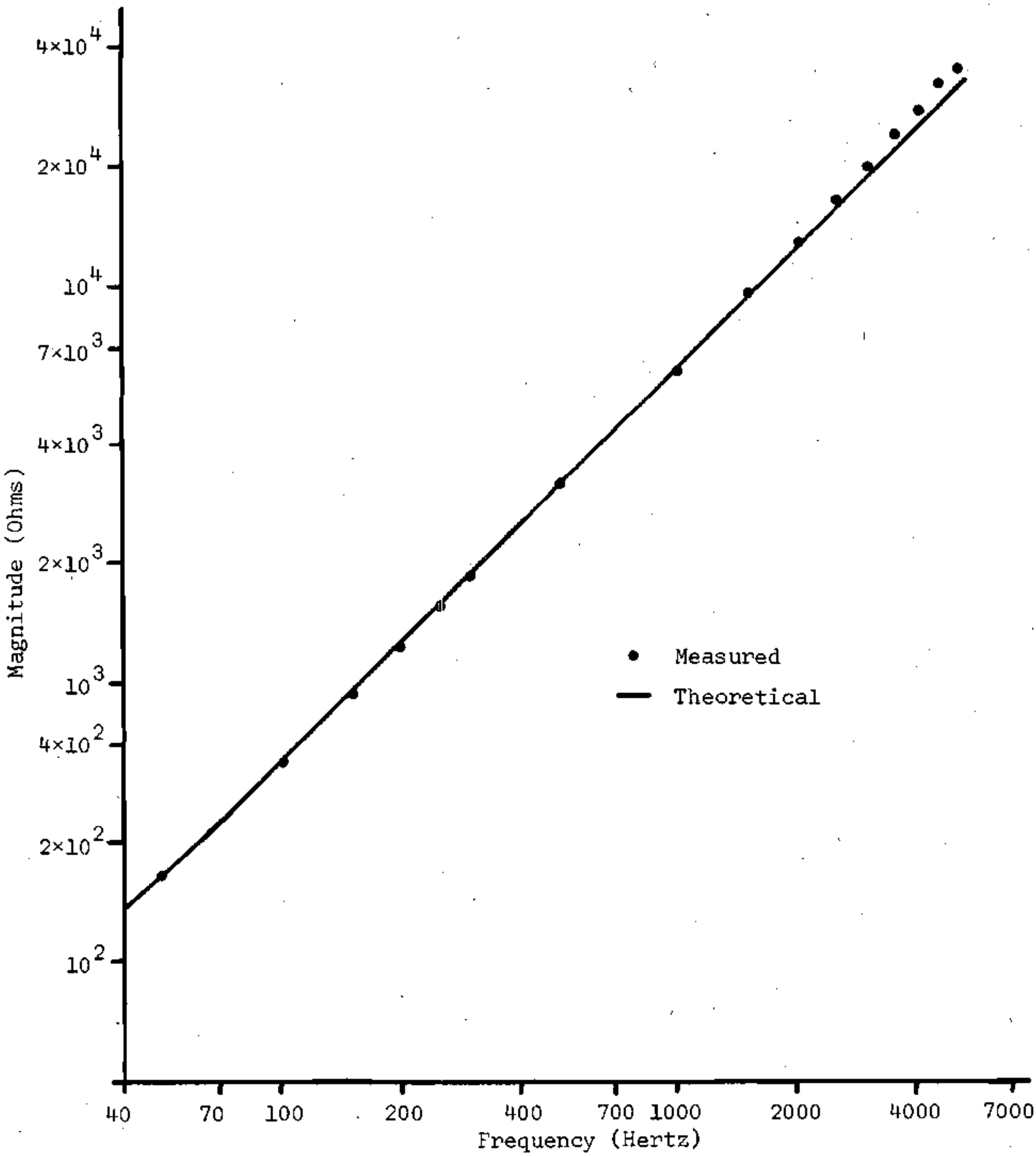


Figure 28. Magnitude, Example Three

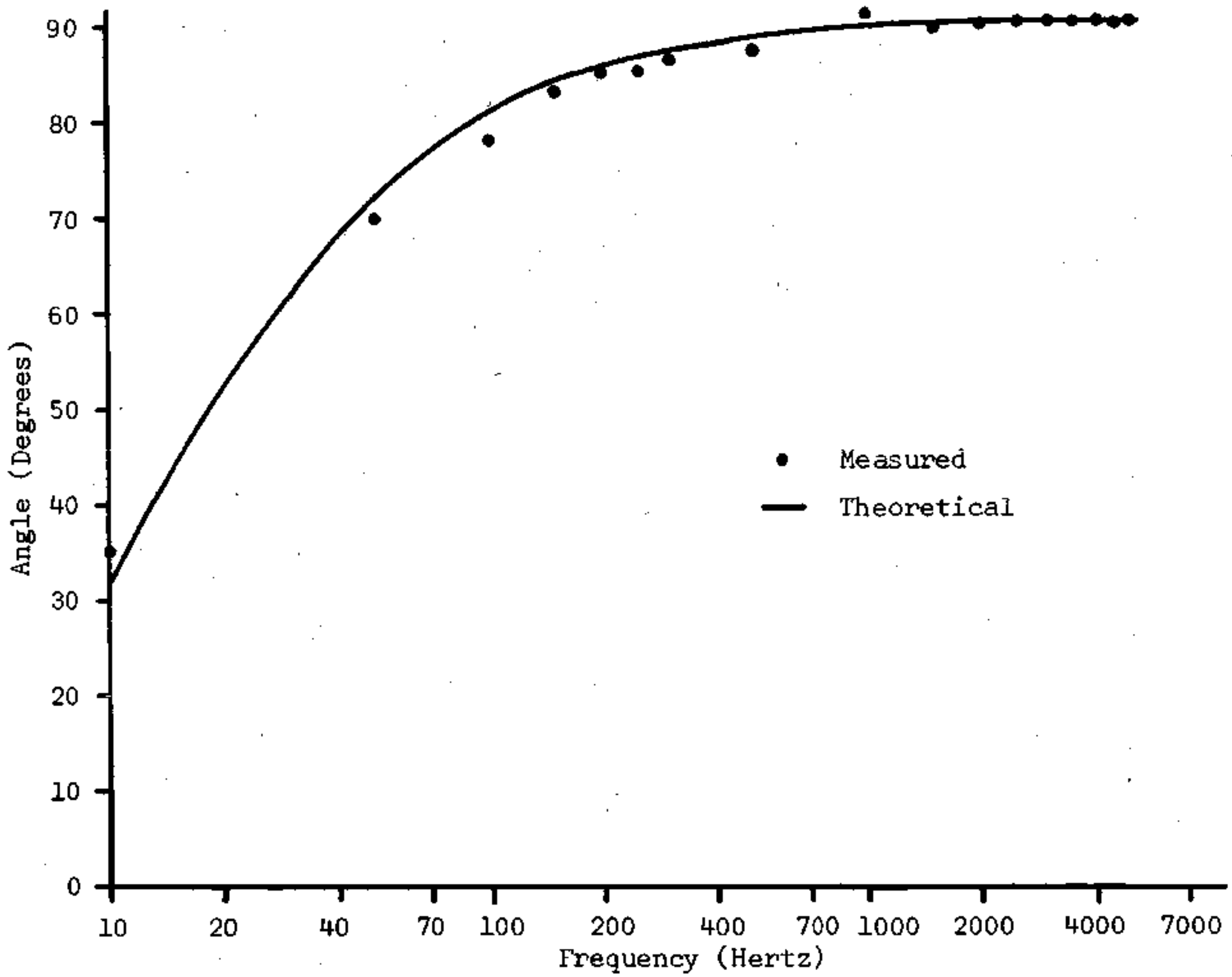


Figure 29. Angle, Example Three

To illustrate the feasibility and practicality of the circuit of Figure 5, the circuit of Figure 5 will be used. This circuit has the negative signs associated with its chain matrix terms.

Let it be required to simulate

$$Z_{in} = 0.1s \tag{175}$$

Let the chain matrix of the GPIC of Figure 5 be

$$\begin{bmatrix} -k_1 & 0 \\ 0 & -\frac{k_2}{s} \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -\frac{2\times10^4}{s} \end{bmatrix} \tag{176}$$

According to (12), this would place the following restrictions on the elements of the circuit of Figure 5:

$$\frac{R_4}{R_6} = k_1 = 2$$

and (177)

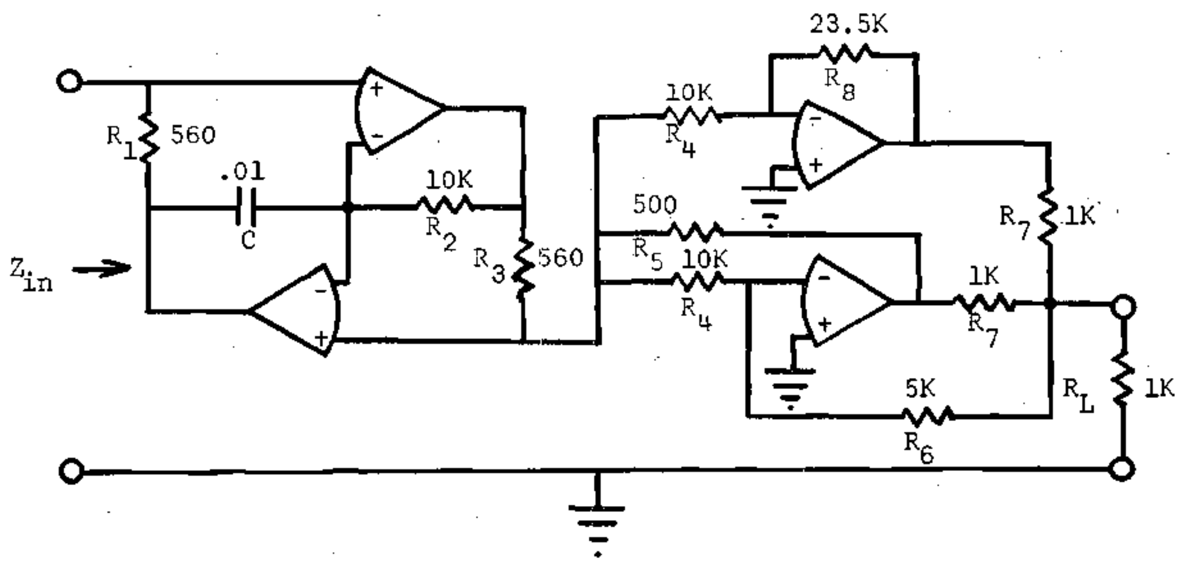
$$\frac{R_3R_7}{R_1R_2R_5C} = k_2 = 2\times10^4$$

Also, in the circuit of Figure 5,

$$R_8 = R_4 + R_7 + 2(R_5+R_6) \tag{178}$$

With the following element values in ohms and microfarads, (177) and (178) can be satisfied: R_1 = 560, R_2 = 10K, R_3 = 560, R_4 = 10K, R_5 = 500, R_6 = 5K, R_7 = 1K, R_8 = 22K, and C = 0.01. Substituting the values of k_1 and k_2 from (177) into (174) and equating this result to (175), it is seen that R_L must be made equal to one kilohm.

The network shown in Figure 30 was used to realize the desired driving-point function of (175).



Element values shown in ohms or microfarads.

Figure 30. Network for Realizing the 0.1 Henry Inductor of (175)

Note that R_8, which was given as 22 kilohms, was trimmed in the circuit of Figure 30 to 23.5 kilohms to make k_2 have a value of -2×10^4. That R_8 had to be trimmed to get the desired k_2 value can be explained by examining (178) and realizing that the resistors indicated by this equation and which were used in the construction of Figure 30 had 5 per cent tolerances. Also (178) was obtained by assuming that the operational amplifiers to be used in Figure 30 were ideal. (See Appendix III.)

Figures 31 and 32 show the theoretical and measured values of magnitude and angle for the above example.

Discussion of Techniques and Errors

As mentioned previously, the object of the experimental part of this research was to test the validity and practicality of the synthesis procedures, and not to develop sophisticated examples. Consequently, the construction and measurement techniques employed were somewhat crude. However, the results obtained indicate that they were sufficient for the purposes.

Unless otherwise noted on the circuit diagrams in this thesis, all the resistors used were carbon resistors with 5 per cent tolerances and all the capacitors were mylar capacitors with 10 per cent tolerances. The operational amplifiers used throughout the research were the Burr-Brown 3057/01 integrated-circuit operational amplifiers. The Burr-Brown 3057/01 operational amplifiers have a D.C. gain, input resistance, and output resistance of approximately 93 dB, 0.2 megohms, and 5 kilohms, respectively. Each of the operational amplifiers used in the synthesized circuits were phase compensated with a 470 ohm resistor in series with a 0.0025 microfarad capacitor. No attempt was made to select optimum phase compensation since the largest possible bandwidth of operation was not an objective. All the circuits were laid out on a specially designed test board similar to those used in analogue computers.

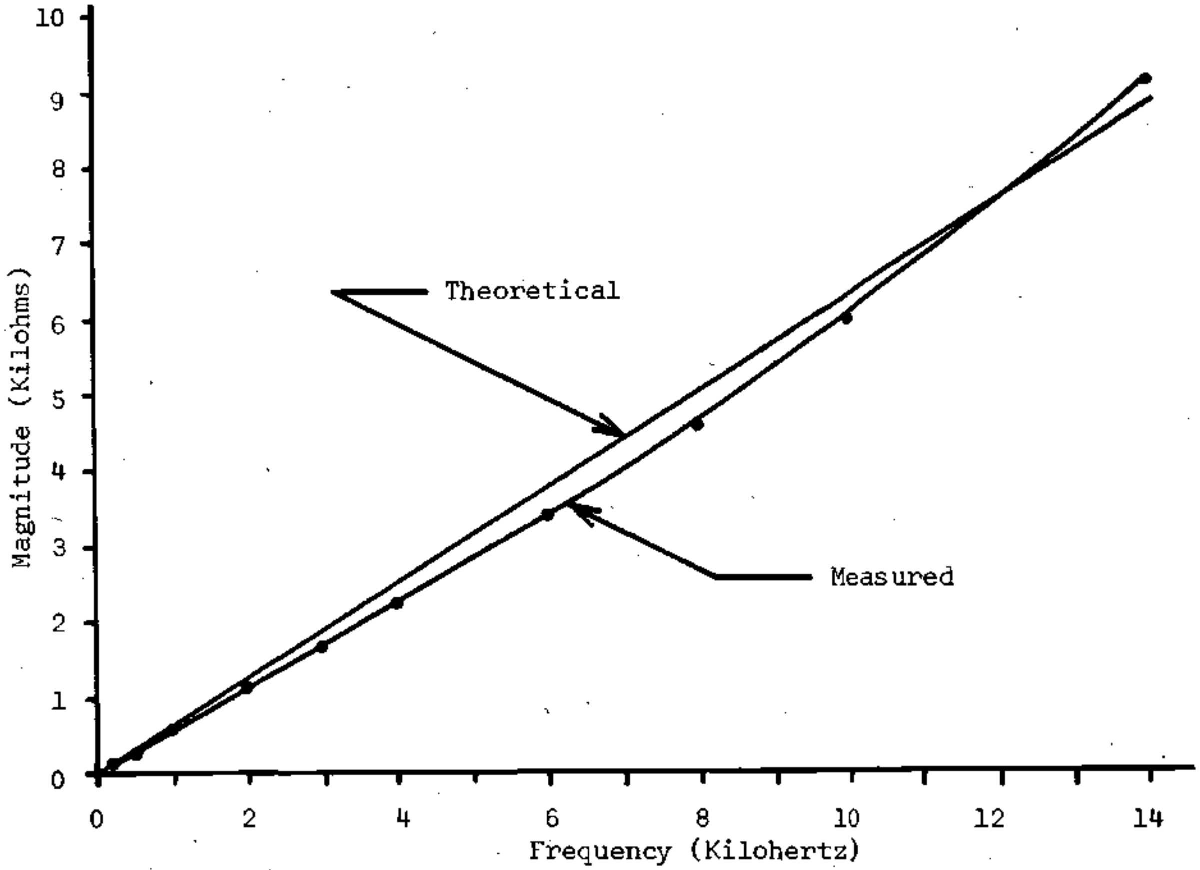


Figure 31. Magnitude, Example Four

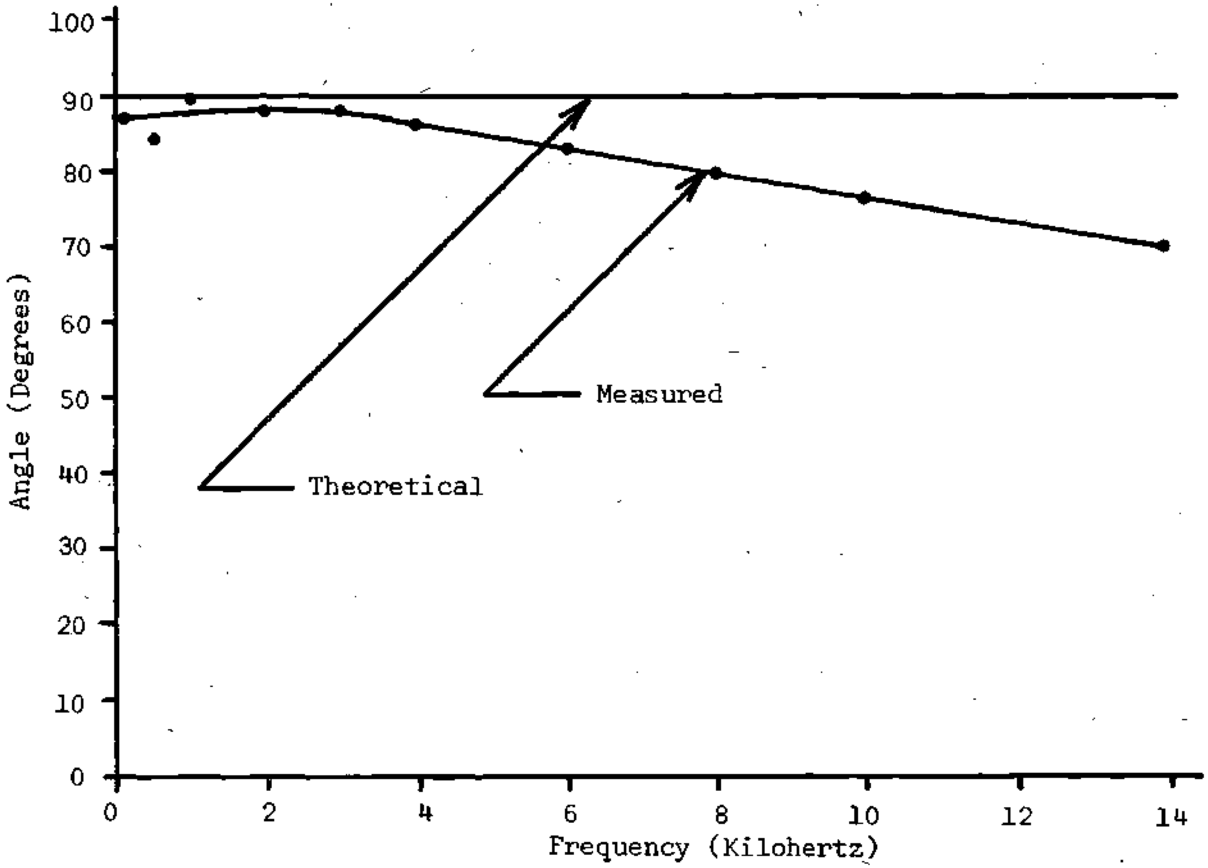


Figure 32. Angle, Example Four

The board had special connectors to accept the standard eight-pin 709 type operational amplifiers.

The magnitude and angle of the impedances of Examples One, Three, and Four were measured as a function of frequency in the following way. The network under test was connected to the appropriate test equipment as shown in Figure 33. A Hewlett-Packard 200 CD audio oscillator was connected to the network under test through a series resistance R. To obtain the magnitude of the impedance of the network under test, a Tektronix 545-B oscilloscope with a 1A2 dual trace plug-in was used in conjunction with a Hewlett-Packard 400 D vacuum tube voltmeter to measure the relative magnitudes of V_2 and V_1-V_2. The magnitude as a function of frequency of the input impedance of the network under test was then computed by

$$|Z_{in}| = \frac{|V_2|}{|V_1-V_2|} \cdot R \qquad (179)$$

To obtain the angle of the impedance of the network under test, V_1-V_2 from the vertical output of the Tektronix 545-B oscilloscope was fed into the horizontal input of the Hewlett-Packard Model 120-B oscilloscope while V_2 was fed into the vertical input. The phase difference between the two voltages, which was the angle of the impedance of the network under test, was then measured using a Hewlett-Packard Webb-Mask on the Hewlett-Packard oscilloscope.

The magnitude and phase of the open-circuit voltage transfer function of Example Two was measured as a function of frequency with

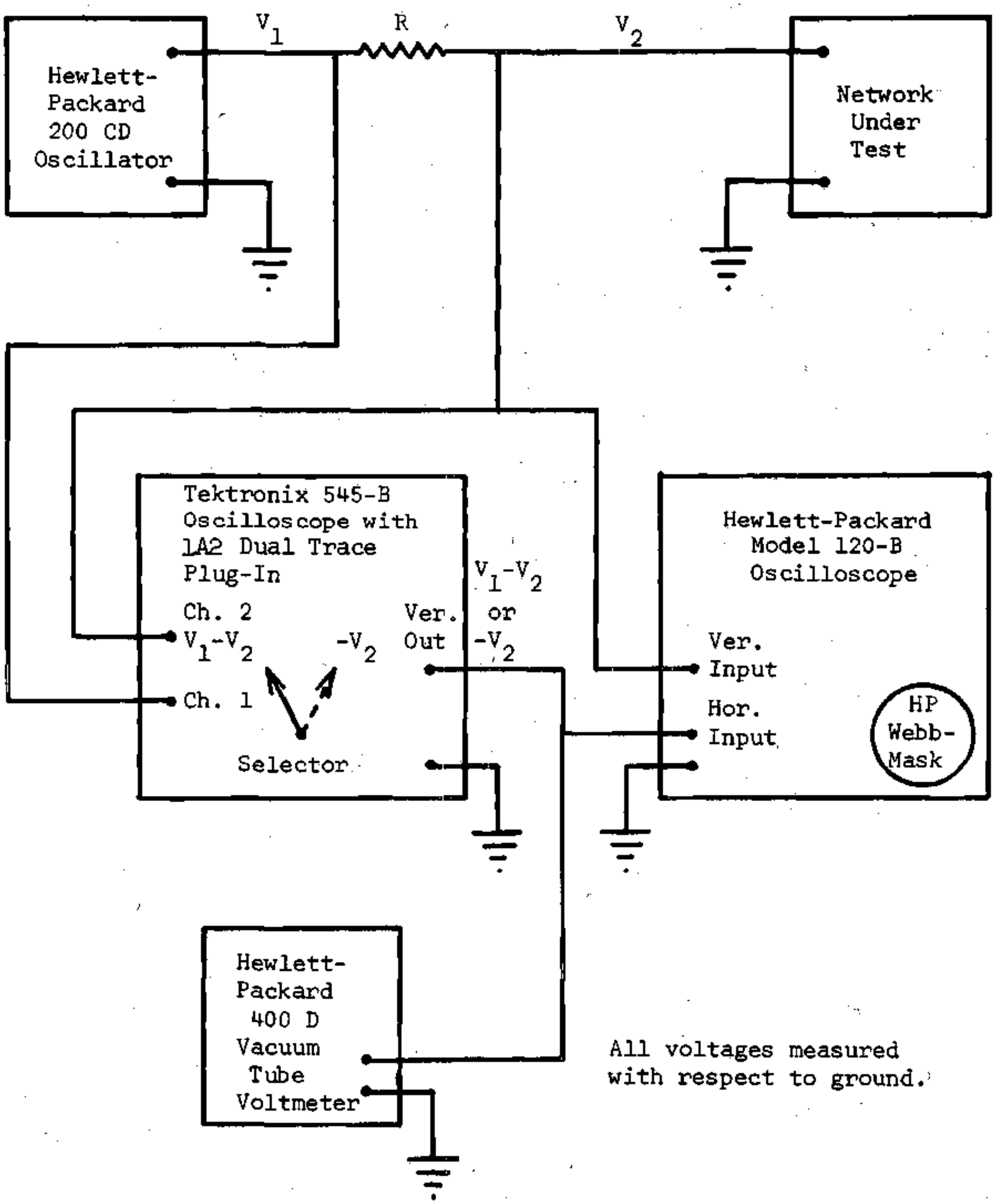


Figure 33. Test Set-Up for Measuring Input Impedance

the test set-up shown in Figure 34. The Burr-Brown 3057/01 operational amplifier in the circuit had the two-fold purpose of providing an almost ideal voltage source at port 1 of the network under test and of functioning as a voltage divider circuit to aid in the measurement of the low signal level voltage V_1. The magnitudes of the voltages 100 V_1 and V_2 were measured by reading the voltages from the Hewlett-Packard Model 120-B oscilloscope. The magnitude of the open-circuit voltage transfer function was then computed from

$$|T(s)| = \frac{|V_2|}{|100V_1|} \cdot 100 \tag{180}$$

The phase of T(s) was measured by applying $100V_1$ and V_2, respectively, to the horizontal and vertical deflection plates of the Hewlett-Packard 120-B oscilloscope and reading the phase difference from the Hewlett-Packard Webb-Mask placed on the screen of the oscilloscope.

The major difficulties noted in the experimental phase of this research were making accurate phase measurements in general and in measuring the magnitude and phase of low level signals. The later difficulty was particularly prevalent in the measurements of the transfer function in Example Two. Use of the Burr-Brown 3057/01 operational amplifier shown as used in Figure 34 greatly improved this troublesome situation. Measuring phase at very low frequencies required that phase correction be made for the errors introduced by the amplifiers of both oscilloscopes.

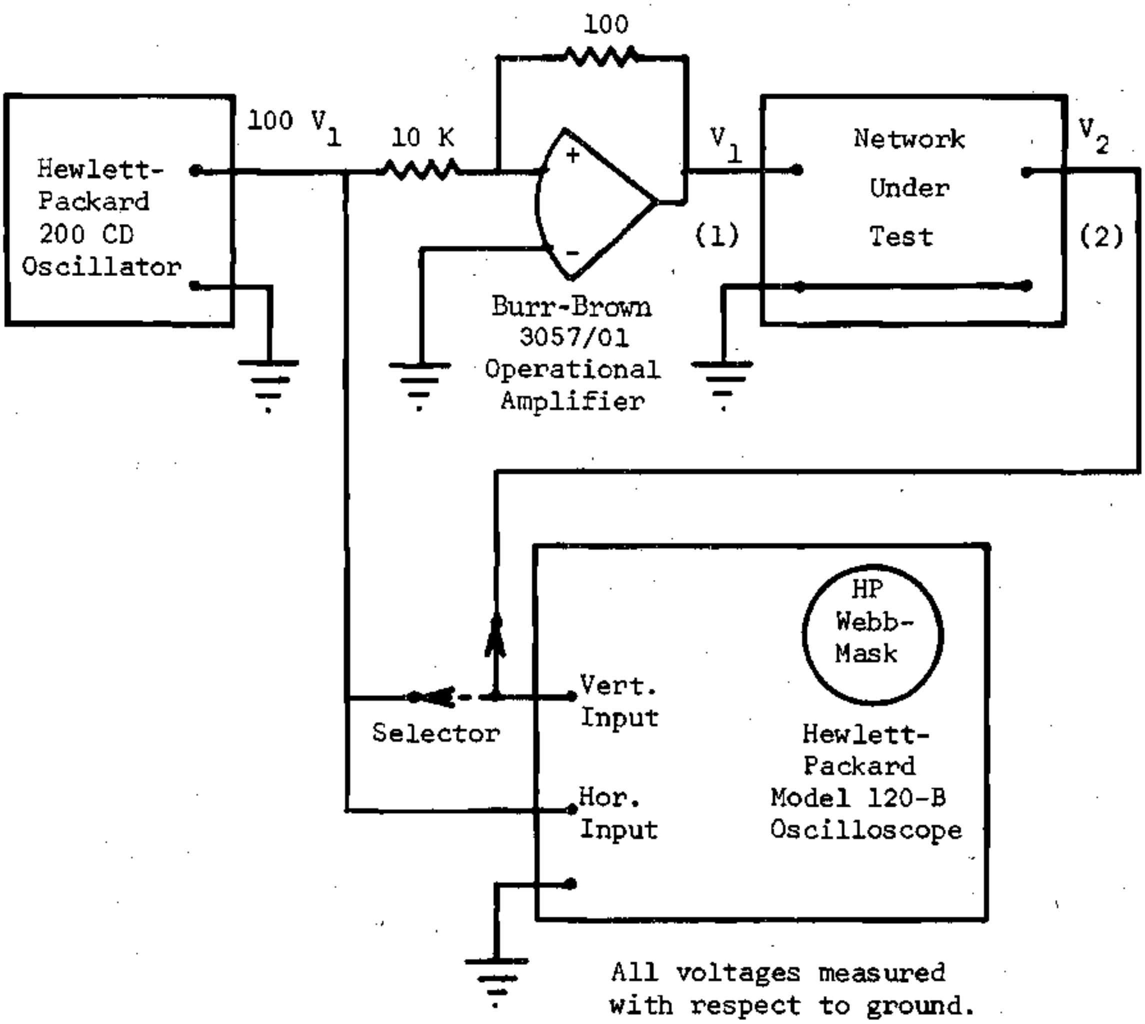


Figure 34. Test Set-Up for Measuring Open-Circuit Voltage Transfer Function

The data obtained from Example One which are shown in Figures 21 and 22 as solid curves, agreed fairly well with the theoretical predicted behavior which is shown as broken curves. From 10 to 20,000 hertz, the magnitude and angle of each of the negative resistances built were in error less than 12 per cent. From 20,000 to 30,000 hertz, the magnitude and angle of these same impedances were in no more error than 16 and 17 per cent, respectively.

The circuit constructed for Example Two also performed well, as can be seen by examining Figures 24 and 25. Examination of 24 reveals that the undamped natural frequency of the circuit of Example Two was 1550 hertz. Theoretically it should have been 1590 hertz. Hence, the measured undamped natural frequency was in error less than 3 per cent.

For Example Two, the measured Q, or selectivity, as defined by (63), was the magnitude of the transfer function at the resonant or undamped natural frequency. From Figure 24, it is observed that Q was 39.5. Theoretically, it should have been 40. Hence, the measured Q was in error less than 2 per cent.

An examination of the phase curves of Figure 25 reveals that the measured phase curve of Example Two behaves as predicted by the theoretical curve of the same figure. As expected from the magnitude curves of Figure 24, the measured phase curve is displaced slightly to the left of the theoretical phase curve. Note that the measured phase curve passes through -90 degrees at 1550 hertz, the measured undamped natural frequency, as it should.

The circuit constructed for Example Three was tested and found to be satisfactory. In Figures 28 and 29, the measured data are presented as dots and the predicted theoretical curves are presented as solid lines. Examination of Figure 28 reveals that there is very good agreement with the calculated curve and measured values of magnitude up to 2000 hertz. For example, at 100 hertz, the measured value is approximately 2 per cent in error as compared to the calculated. From 2000 to 5000 hertz, the measured values of magnitude deviate more from the calculated. The maximum error in this range of frequency is around 18 per cent.

Figure 29 indicates that the measured values of angle for Example Three agreed quite well with the calculated values. The largest percentage error in measured angle occurred at 10 hertz and was approximately 10 per cent.

The circuit constructed to simulate the inductance of Example Four performed very satisfactorily. The measured and calculated values of magnitude and angle of the impedance of Example Four are shown in Figures 31 and 32, respectively. From 200 to 14,000 hertz, the error in magnitude of the constructed circuit was less than 12 per cent. From 200 to 8000 hertz, the error in angle of the constructed circuit was less than 12 per cent. From 8,000 to 14,000 hertz, this angle error increased but still was less than 23 per cent.

Causes of the experimental errors are difficult to determine; however, most of them can be attributed to the following:

1. Deviation of elements from their design values.

2. Non-idealness of the operational amplifiers--primarily the decrease of the open-loop gain at high frequencies brought about by the phase compensation network.

3. Noise in the presence of low-level signals.

4. Errors in reading instruments, particularly in the case of reading the phase from the Hewlett-Packard Webb-Mask.

5. Electrical coupling due to the network layouts.

A thorough investigation of the sources of error was not made since the only purpose of the experimental work was to verify the realization procedures and demonstrate their practicality. Certainly the data taken and presented in this chapter demonstrate these two points well enough to avoid further investigation.

CHAPTER X

CONCLUSIONS AND RECOMMENDATIONS

The rapid development of integrated circuits has generated a tremendous interest in the application of active RC synthesis methods to solve circuit design problems. Unfortunately, many of the active RC synthesis procedures in existence today concentrate on minimizing the number of active elements at the expense of using an excessive number of capacitors. Capacitors use much area in integrated circuits and are often difficult to fabricate.

Many existing active RC synthesis procedures are very sensitive to both active and passive circuit parameter values. In integrated circuits, generally, active and passive parameter values have high tolerances. Hence, sensitivity figures with respect to these parameter values must be low in order to guarantee reasonable performance.

For fabrication purposes in integrated circuits, the acceptable spread in element values is usually given to the circuit designer. Many active RC synthesis procedures do not have enough degrees of freedom to allow the designer to stay within the allowable range for element values. Hence, these synthesis procedures are of little use.

Another group of the active synthesis procedures are impractical because of the fact that they are ungrounded.

This investigation has made use of an active device, the generalized positive impedance converter (GPIC), in network synthesis procedures which are practical. In Chapter II, the GPIC was defined and electronic circuits using operational amplifiers, resistors, and capacitors were developed which realized each GPIC discussed.

In Chapter III and Chapter IV, synthesis procedures for the open-circuit voltage transfer function, T(s), and the driving-point admittance function, Y(s), were developed, respectively. The synthesized networks are grounded, contain elements which are compatible with integrated circuits, and require the minimum number of capacitors necessary for the synthesis.

A sensitivity investigation of the T(s) procedure and the Y(s) procedure was made in Chapter V. For an unconditionally stable T(s), the coefficient sensitivity terms can always be made less than or equal to one in absolute value. In the case of the absolutely stable, second order T(s), the selectivity and undamped natural frequency sensitivity terms can always be made less than or equal to one in absolute value. It was demonstrated through an example that the absolute value of T(s) sensitivity terms can be made low in the case of the all-pole, second-order, absolutely stable transfer function. The coefficient sensitivity terms of the Y(s) function can always be made approximately less than or equal to one in absolute value. This investigation revealed that the T(s) and Y(s) synthesis procedures generally are insensitive to changes in both passive element values and GPIC gain constants.

In Chapter VI, it was shown that for both the T(s) and Y(s) synthesis procedures the synthesis could be successfully completed for any prescribed range of admittance values for the admittance parameters external to the GPIC devices. This remarkable property of the synthesis procedures was shown to depend on the unrestricted assignment of GPIC gain constant values.

Experimental verification of the T(s) and Y(s) synthesis procedures was presented in Chapter IX. The experimental results demonstrated the practicality and usefulness of the synthesis procedures.

For completeness, the T(s) and Y(s) synthesis procedures were extended to the multi-port open-circuit voltage transfer matrix synthesis procedure and the short-circuit admittance matrix synthesis procedure, respectively. The multi-port open-circuit voltage transfer matrix synthesis procedure is found in Chapter VII and the short-circuit admittance matrix synthesis procedure is found in Chapter VIII.

In the process of this investigation, several areas for further investigation have appeared. It is felt that more simplified circuits can be found for realizing the GPIC's than those presented in Chapter II. With a simpler and a more carefully designed GPIC circuit, the useful frequency range of the synthesis procedures probably can be extended.

Even though it appears that the synthesis procedures described in this research are generally insensitive to both passive and active parameter changes, the particular application should be examined carefully. The sensitivity discussed in Chapter V is the differential sensitivity. In a practical application, perhaps an incremental sensitivity analysis should be performed, if possible. There is no guarantee that low differential sensitivity always implies low incremental sensitivity.

The circuit design of the GPIC devices will determine the possible GPIC gain constants. There generally will be some range of gain constant values possible for each particular GPIC circuit design. Therefore, it is recommended that this range of gain constant values be incorporated into the work of Chapter VI in some future investigation.

The synthesis procedures described in this research can very easily be implemented on a digital computer since the procedures only require matching coefficients of transfer functions to circuit element values and GPIC gain constants. A digital computer implementation of the work in Chapter VI (CONTROL OF ELEMENT VALUES EXTERNAL TO GPIC'S) is recommended for anyone who may make much use of the synthesis procedures.

In summary, this work has resulted in another approach to the active RC synthesis problem. Synthesis procedures with practical electronic circuit realizations have been developed. The desirable characteristics of the T(s) and Y(s) synthesis procedures and the experimental results of the network realizations indicate that the procedures should find use in integrated-circuit applications.

APPENDICES

APPENDIX I

REALIZATION OF THE NEGATIVE RESISTANCE BY KIM'S METHOD

The following discussion is based on some work performed by C. D. Kim in the School of Electrical Engineering of the Georgia Institute of Technology. It will be presented by him in a later publication.[11]

Consider the circuit of Figure 35.

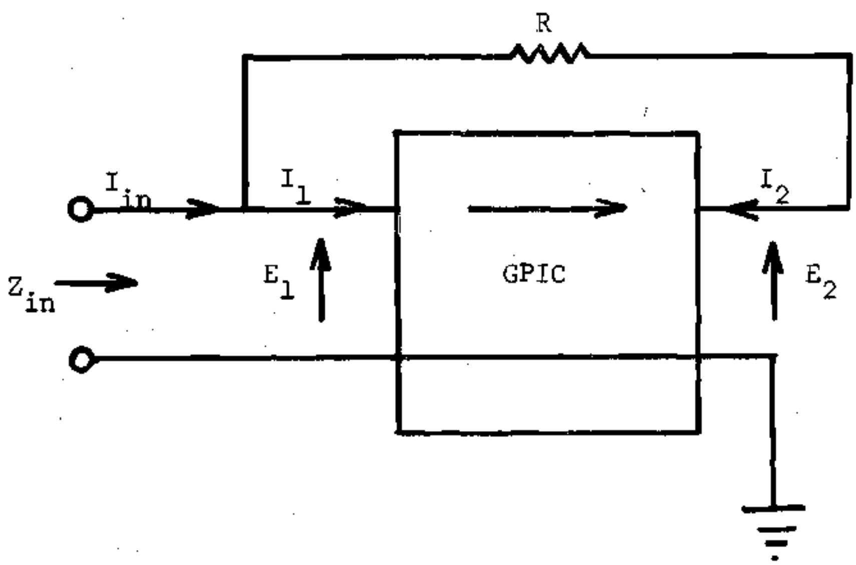


Figure 35. Negative Resistance Circuit

The chain matrix of the GPIC is assumed to be

$$\begin{bmatrix} k_{1N} & 0 \\ 0 & k_{2N} \end{bmatrix} \tag{A-1}$$

From the chain matrix description,

$$E_1 = k_{1N}E_2 \tag{A-2}$$

$$I_1 = -k_{2N}I_2 \tag{A-3}$$

Examination of the circuit of Figure 35 reveals that

$$E_1 = I_2R + E_2 \tag{A-4}$$

and

$$I_{in} = I_1 + I_2 \tag{A-5}$$

Now, the input impedance of the circuit is given by

$$Z_{in} = \frac{E_1}{I_{in}} \tag{A-6}$$

Equations (A-2), (A-3), (A-4), (A-5), and (A-6) can be solved to yield

$$Z_{in} = -\frac{K_{1N}R}{(k_{1N}-1)(k_{2N}-1)} \tag{A-7}$$

It is obvious from (A-7) that Z_{in} can be made a negative resistance. For example, make $k_{1N} = k_{2N} = 2$. Then

$$Z_{in} = -2R \tag{A-8}$$

APPENDIX II

GENERALIZED POSITIVE IMPEDANCE CONVERTER WITH POSITIVE CHAIN MATRIX TERMS

Consider the circuit shown in Figure 36.

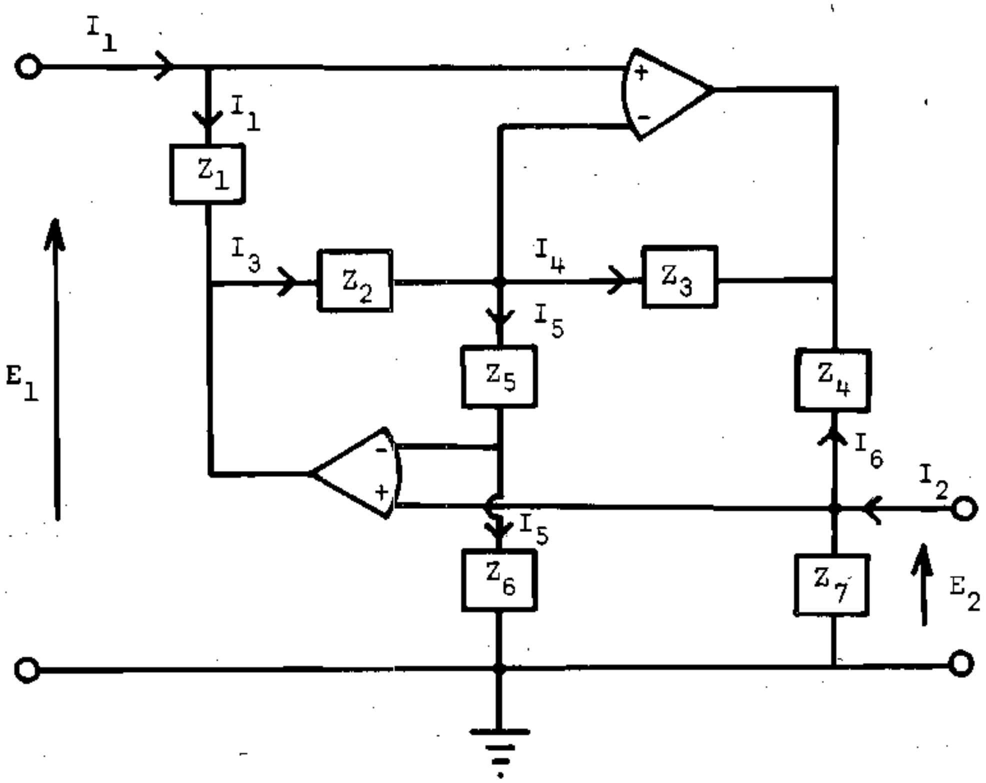


Figure 36. Generalized Positive Impedance Converter with Positive Chain Matrix Terms, for Analysis

Assume that the operational amplifiers are ideal, i.e., infinite input impedance, infinite gain, infinite bandwidth, and negligible output impedance. An analysis of the circuit yields that

$$I_1Z_1 + I_3Z_2 = 0 \tag{A-9}$$

which implies that

$$I_3 = -\frac{I_1Z_1}{Z_2} \tag{A-10}$$

Now,

$$I_5 = \frac{E_1}{Z_5 + Z_6} \tag{A-11}$$

and

$$E_2 = \frac{E_1Z_6}{Z_5 + Z_6} \tag{A-12}$$

Also,

$$I_4 = I_3 - I_5 \tag{A-13}$$

and

$$I_6 = I_2 - \frac{E_2}{Z_7} \tag{A-14}$$

It is seen that

$$I_4Z_3 - I_6Z_4 - I_5Z_5 = 0 \tag{A-15}$$

Substituting (A-13) and (A-14) into (A-15), the following expression results.

$$(I_3-I_5)Z_3 - \left(I_2 - \frac{E_2}{Z_7}\right)Z_4 - I_5Z_5 = 0 \tag{A-16}$$

Substituting (A-10), (A-11) and (A-12) into (A-16), the following result is obtained.

$$I_1 = -I_2 \frac{Z_2Z_4}{Z_1Z_3} + E_2 \frac{Z_2}{Z_1Z_3} \left\{ \frac{Z_4Z_6 - Z_7(Z_3+Z_5)}{Z_6Z_7} \right\} \tag{A-17}$$

From Equation (A-12), it is obvious that

$$E_1 = \frac{Z_5 + Z_6}{Z_6} E_2 \tag{A-18}$$

Now the desired results for the circuit of Figure 36 are

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} k_1 & 0 \\ 0 & \frac{k_2}{s} \end{bmatrix} \begin{bmatrix} E_2 \\ -I_2 \end{bmatrix} \tag{A-19}$$

If we make

$$\begin{aligned} Z_3 &= R_3 \\ Z_4 &= R_4 \\ Z_5 &= R_5 \\ Z_6 &= R_6 \\ Z_7 &= R_7 = \frac{R_4R_6}{R_3 + R_5} \end{aligned} \tag{A-20}$$

then Equations (A-18) and (A-17) become

$$E_1 = \frac{R_5 + R_6}{R_6} E_2 \tag{A-21}$$

and

$$I_1 = - \frac{Z_2 R_4}{Z_1 R_3} I_2 \tag{A-22}$$

respectively.

For a desired $k_1 \geq 1$, let

$$\begin{aligned} Z_1 &= R_1 \\ Z_2 &= \frac{1}{sC} \end{aligned} \tag{A-23}$$

then

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{R_5+R_6}{R_6} & 0 \\ 0 & \frac{R_4}{sR_1R_3C} \end{bmatrix} \begin{bmatrix} E_2 \\ -I_2 \end{bmatrix} \tag{A-24}$$

Therefore,

$$\begin{aligned} k_1 &= \frac{R_5 + R_6}{R_6} \\ k_2 &= \frac{R_4}{R_1 R_3 C} \end{aligned} \tag{A-25}$$

The desired circuit is shown in Figure 37.

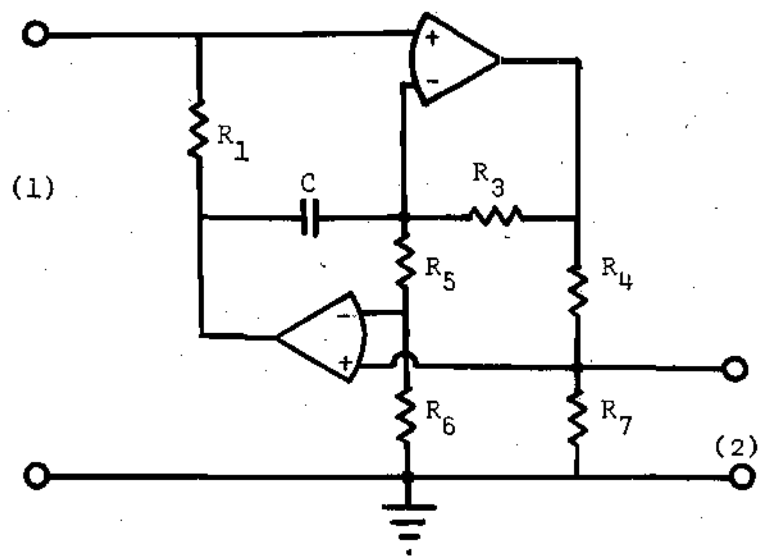


Figure 37. GPIC, $k_1 \geq 1$

For a desired $k_1 \leq 1$, interchange ports 1 and 2 of Figure 36, impose the restrictions of (A-20), and make

$$Z_1 = \frac{1}{sC}$$
$$Z_2 = R_2 \tag{A-26}$$

Then,

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{R_6}{R_5+R_6} & 0 \\ 0 & \frac{R_3}{sCR_2R_4} \end{bmatrix} \begin{bmatrix} E_2 \\ -I_2 \end{bmatrix} \tag{A-27}$$

Hence,

$$k_1 = \frac{R_6}{R_5 + R_6}$$

$$k_2 = \frac{R_3}{R_2 R_4 C} \tag{A-28}$$

The desired circuit is shown in Figure 38.

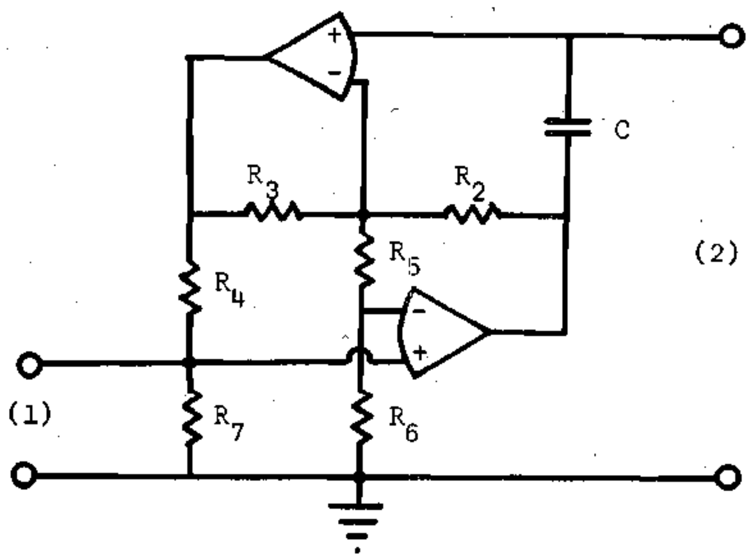


Figure 38. GPIC, $k_1 \leq 1$

APPENDIX III

POSITIVE IMPEDANCE CONVERTER
WITH NEGATIVE CHAIN MATRIX TERMS

Let it be desired to realize a two-port network described by

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} -k_1 & 0 \\ 0 & -k_2 \end{bmatrix} \begin{bmatrix} E_2 \\ -I_2 \end{bmatrix} \qquad \text{(A-29)}$$

A two-port described as above is a positive impedance converter.

It will be shown that the circuit of Figure 39 can be made to give the desired equation of (A-29). Assume that all operational amplifiers of Figure 39 are ideal. Analysis of the circuit reveals that

$$I_3 = \frac{E_1}{R_1} \qquad \text{(A-30)}$$

$$I_5 = \frac{E_1}{R_5} \qquad \text{(A-31)}$$

$$I_4 = I_1 - I_3 - I_5 \qquad \text{(A-32)}$$

Substituting (A-30) and (A-31) into (A-32), it is found that

$$I_4 = I_1 - \frac{E_1(R_1+R_5)}{R_1R_5} \qquad \text{(A-33)}$$

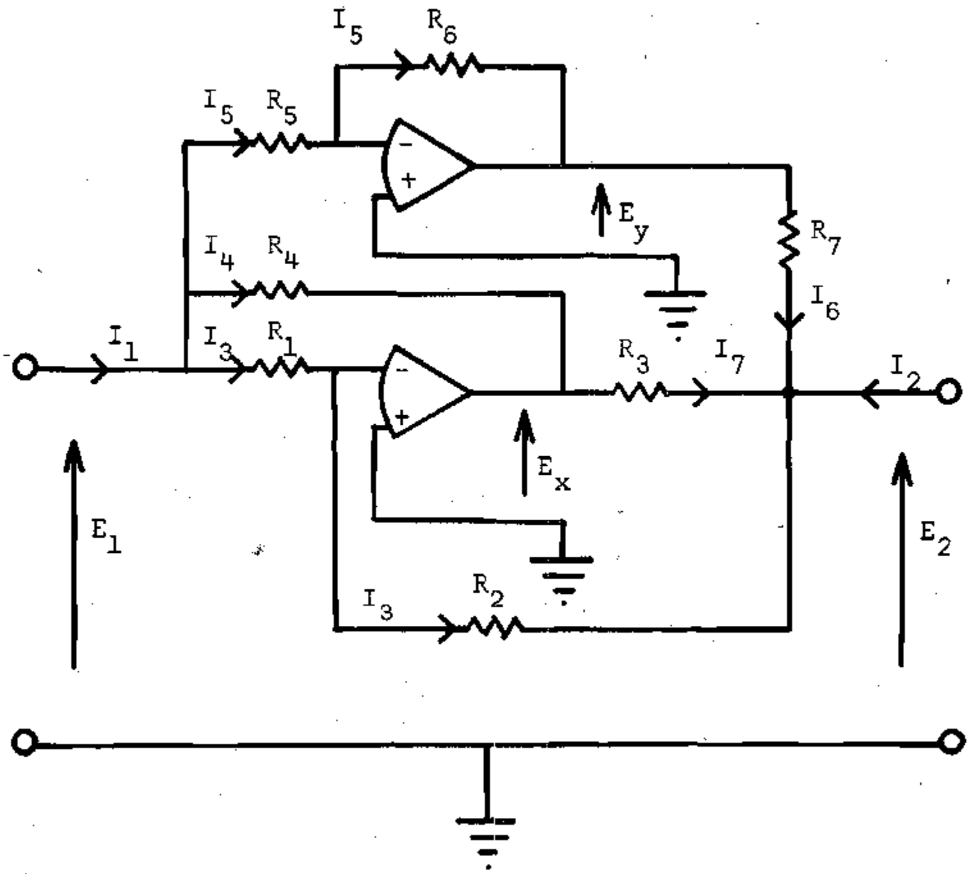


Figure 39. Positive Impedance Converter for Analysis

Now,

$$E_x = E_1 - I_4R_4 \tag{A-34}$$

Substituting (A-33) into (A-34), the expression to follow is obtained.

$$E_x = E_1\left[1 + \frac{(R_1+R_5)R_4}{R_1R_5}\right] - I_1R_4 \tag{A-35}$$

Analysis yields that

$$E_y = -\frac{E_1R_6}{R_5} \tag{A-36}$$

$$E_2 = -\frac{E_1R_2}{R_1} \tag{A-37}$$

$$I_6 = \frac{E_y - E_2}{R_7} \tag{A-38}$$

$$I_7 = \frac{E_x - E_2}{R_3} \tag{A-39}$$

$$-I_2 = I_3 + I_6 + I_7 \tag{A-40}$$

Substituting (A-36) and (A-37) into (A-38), it is found that

$$I_6 = \frac{E_1}{R_7}\left(\frac{R_2}{R_1} - \frac{R_6}{R_5}\right) \tag{A-41}$$

Substituting (A-35) and (A-37) into (A-39), the following result is obtained.

$$I_7 = \frac{E_1}{R_3}\left[1 + \frac{(R_1+R_5)R_4}{R_1R_5} + \frac{R_2}{R_1}\right] - \frac{I_1R_4}{R_3} \tag{A-42}$$

Substituting (A-30), (A-41) and (A-42) into (A-40), the following expression is obtained.

$$-I_2 = -\frac{I_1R_4}{R_3} + E_1\left\{\left[\frac{1}{R_1} + \frac{1}{R_3} + \frac{(R_1+R_5)R_4}{R_1R_3R_5} + \frac{R_2}{R_1R_3} + \frac{R_2}{R_1R_7}\right] - \frac{R_6}{R_5R_7}\right\} \tag{A-43}$$

To obtain the desired result of (A-29), it is required that

$$R_6 = \left(\frac{R_5R_7}{R_1} + \frac{R_5R_7}{R_3} + \frac{R_4R_7(R_1+R_5)}{R_1R_3} + \frac{R_2R_5R_7}{R_1R_3} + \frac{R_2R_5}{R_1}\right) \tag{A-44}$$

Then (A-43) becomes

$$-I_2 = -\frac{I_1R_4}{R_3} \tag{A-45}$$

or

$$I_1 = \frac{R_3I_2}{R_4} \tag{A-46}$$

From (A-37), it is obvious that

$$E_1 = -\frac{R_1E_2}{R_2} \tag{A-47}$$

A comparison of (A-29), (A-46), and (A-47) reveals that

$$k_1 = \frac{R_1}{R_2} \tag{A-48}$$

and

$$k_2 = \frac{R_3}{R_4}$$

Make

$$\begin{aligned} R_5 &= R_1 \\ R_7 &= R_3 \end{aligned} \tag{A-49}$$

Substituting (A-48) and (A-49) into (A-44) it is seen that

$$R_6 = R_1\left(1 + \frac{2}{k_1}\right) + R_3\left(1 + \frac{2}{k_2}\right) \qquad \text{(A-50)}$$

The circuit that will yield the results of (A-29) is shown in Figure 40.

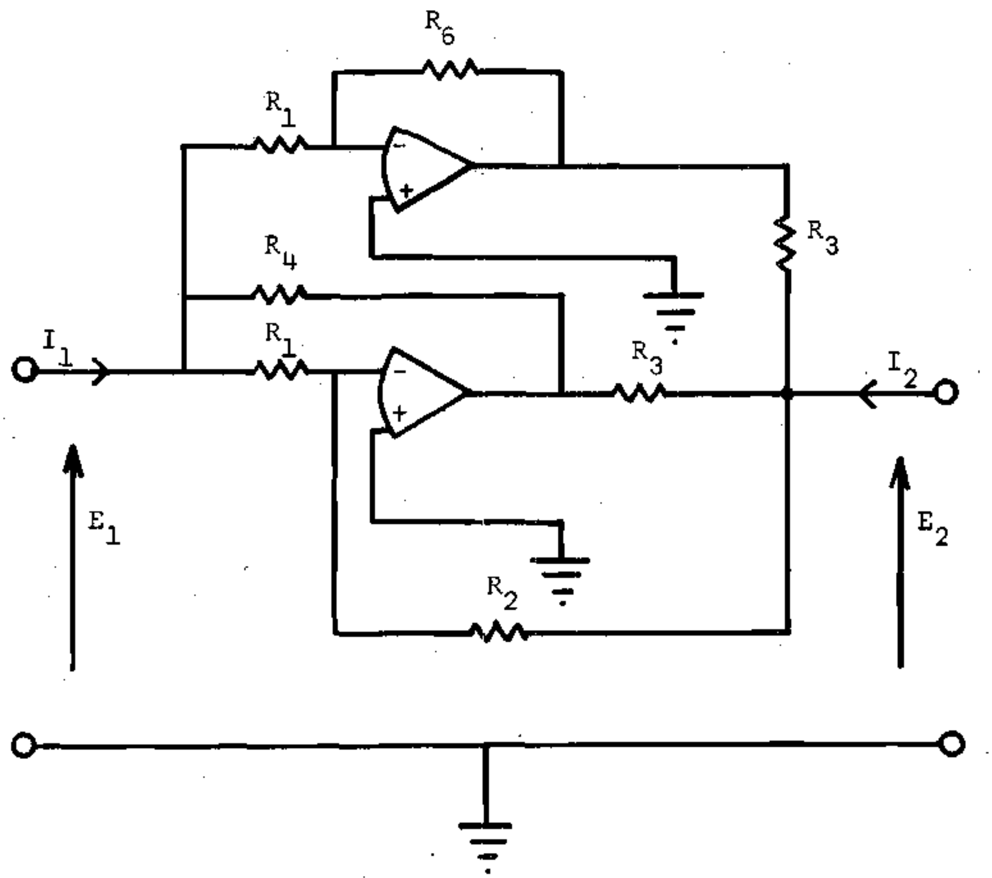


Figure 40. Positive Impedance Converter with Negative Chain Matrix Terms

Note that in this circuit

$$k_1 = \frac{R_1}{R_2}$$

$$k_2 = \frac{R_3}{R_4} \qquad \text{(A-51)}$$

only when (A-50) is satisfied.

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VITA

Douglas R. Cobb was born in Anderson, South Carolina, on May 6, 1942. He is the son of Edwin R. Cobb and Bobbie E. Cobb.

He attended public schools in Anderson, South Carolina, and was graduated from Anderson Boys' High School in 1960. Immediately following high school, he attended Clemson University. After receiving his B.S.E.E. degree from Clemson University in 1964, he attended the Massachusetts Institute of Technology under the One Year on Campus Program of Bell Telephone Laboratories, Incorporated. Upon graduation from M.I.T. with a S.M.E.E. degree in 1965, he returned to Bell Laboratories as a Member of Technical Staff and worked for the next three years as a circuit design engineer.

During his employment at Bell Laboratories, he completed the required B.T.L. technology courses and rotational assignments and was awarded the Communication Development Training Certificate in 1967. Also during this same period, he did post-master's work in electrical engineering at New York University on a part-time basis. In 1968 he began work on his doctorate degree in electrical engineering at the Georgia Institute of Technology.

He held the Civitan Club International Scholarship and the Higgins' Undergraduate Scholarship while at Clemson University. During his period of study at the Georgia Institute of Technology he held a National Defense Graduate Fellowship. He is a member of Phi Eta Sigma, Tau Beta Pi, Phi Kappa Phi, Sigma Xi, and the I.E.E.E.

In addition to his work at Bell Laboratories, he has held summer positions with the Western Electric Company, the Singer Manufacturing Company and the American Telephone and Telegraph Company. Since June of 1969, he has been a Graduate Research Assistant in the School of Electrical Engineering at the Georgia Institute of Technology.

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