The Energy-Limited Capacity of Wireless Networks

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Abstract— **The performance of large-scale wireless ad hoc networks is often limited by the broadcasting nature of the wireless medium and the inherent node energy constraints. While the impact of the former on network capacity has been studied extensively in the literature, the impact of energy constraints has not received as much attention. In this paper, we study the capacity limitations resulting from the energy supplies in wireless nodes. We define the energy-limited capacity of a wireless network as the maximum amount of data the network can deliver before the nodes run out of energy. This energy-limited capacity is an important parameter in networks where operating lifetime is critical, such as ad hoc networks deployed in hazardous environments and sensor networks. We study two types of static networks, networks without any infrastructure support and networks where base stations with unlimited energy are deployed to support data forwarding. We consider two kinds of traffic models motivated by ad hoc networks and sensor networks. We derive upper and lower bounds on the energy-limited capacity of these networks. While throughput has been shown to not scale with node density in static networks by previous studies, our results show that, depending on the energy consumption characteristics of wireless communication, the energy-limited capacity *can* scale well under both traffic models. In addition, we show that the deployment of base stations can improve the energy-limited capacity of the network, especially for networks with sensor traffic.**

I. Introduction

Wireless ad hoc networks enable devices to communicate with each other without pre-installed infrastructure. These networks have a wide range of applications such as environmental sensing, battlefield support and disaster relief [19], [33]. However, the performance of wireless ad hoc networks is often limited by the broadcasting nature of the wireless medium, which results in interference when nearby nodes attempt to simultaneously transmit. In addition, since these networks often consist of devices that are powered by batteries, power management is of great importance.

Recently there has been significant effort in studying the fundamental limits in the *throughput capacity* of wireless ad hoc networks, defined as the total data rate (bits/second) that can be transferred between nodes, accounting for interference caused by the broadcast medium. In a seminal paper [15], Gupta and Kumar study a model of ad hoc networks with stationary nodes and show that when the number of nodes n per unit area increases, the per node throughput decreases as $\Theta(\frac{1}{\sqrt{n \log n}})$. This result implies that large scale ad hoc networks would not be scalable for non-local traffic patterns. Grossglauser and Tse [13] study mobile ad hoc networks and show that with loose delay constraints, node mobility can dramatically improve network capacity. They prove that the per node throughput can be kept constant as the number of nodes per unit area increases, assuming mobile nodes that carry packets. Recently, others have investigated the throughput capacity of hybrid networks with infrastructure support [27], [22]. There is also other work in the capacity of ad hoc networks under different models [12], [2], [32], [30], [11].

While the impact of broadcast interference on network capacity has been studied extensively in the literature, the effect of energy constraints has not received as much attention. In this paper, we study the capacity limitations resulting from the energy supplies in wireless nodes. In particular, we define the *energy-limited capacity* as the maximum amount of data (bits) the network can deliver before the nodes run out of energy[1]. This type of capacity constraint is an important parameter in networks where operating lifetime is critical, such as ad hoc networks deployed in hazardous environments and sensor networks that are significantly battery-constrained.

We study two types of static (i.e., stationary) networks, pure ad hoc networks without any infrastructure support (which we call *homogeneous networks*), and *hybrid networks* where base stations with unlimited energy are deployed to support data forwarding. In both types of networks, we consider two traffic models, an *ad hoc model* intended to reflect uniform traffic patterns as might arise in ad hoc networks, and a *sensor model* motivated by sensor networks that collect data and then deliver to a single destination.

We derive upper and lower bounds on the energy-limited capacity for these two networks and two traffic models. In general, our lower bounds are within a factor of $O(\log^k n)$ from the corresponding upper bounds for some constant k. Based on the bounds, we can reach conclusions about the scaling behavior of wireless networks under energy constraints. In particular:

- We find that the energy-limited capacity of wireless networks *can* scale well under both traffic models, for certain properties of the energy consumption characteristics. This result sharply contrasts with the behavior of throughput capacity, which does not scale with increasing node density.
- The capacity of networks supporting sensor traffic is less than the capacity of networks supporting ad hoc traffic, by a factor of $O(\sqrt{n} \log^c n)$ for some constant c.
- The deployment of base stations can improve the network capacity, especially with sensor traffic, where the addition

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[1]Nodes may certainly be involved in application-specific activities that consume energy. However, for generality, we restrict our attention to the consumption of energy that directly results from communication.

of even a single base station has an immediate positive effect.

- Reducing the constant energy that is required independent of transmission distance is key to desirable scaling.

The rest of this paper is structured as follows. Section II describes the network and traffic models we use in this paper and defines the energy-limited capacity. We study the energy-limited capacity of homogeneous networks with sensor traffic in Section III. Hybrid sensor networks where base stations are deployed to support data forwarding are considered in Section IV. We study the energy-limited capacity of networks with ad hoc traffic in Section V and VI, which focus on homogeneous and hybrid networks respectively. In Section VII, we compare our results with previous results on throughput capacity and discuss some related issues. Related work is reviewed in Section VIII and the paper is concluded in Section IX.

II. Network Models

We first describe the network models and traffic models we use in this paper. Then we define the energy-limited capacity.

A. Network Models

We consider two types of static networks, *homogeneous networks* and *hybrid networks*. In homogeneous networks, nodes rely on their own resources for communication and no infrastructure support exists. In contrast, hybrid networks consist of base stations (BSs) as well as normal nodes as in homogeneous networks. Nodes are assumed to be identical and have limited energy supplies; base stations are equipped with unlimited energy and deployed to support data forwarding for nodes. A hybrid network consists of two layers, the ad hoc layer formed by nodes, and the infrastructure or BS layer. In a homogeneous network, only the ad hoc layer exists.

In our analysis, the deployment area of the network is scaled to a unit disk. Nodes in the ad hoc layer are distributed on the unit area uniformly and independently. The number of nodes in the network is n and the initial energy of each node is a constant A.

In the infrastructure layer, m BSs are sparsely distributed in the same area as nodes. We assume that the number of base stations grows slower than the number of nodes[2], i.e., $\lim_{n\to\infty} \frac{m}{n} = 0$. In addition, we assume that BSs are deployed regularly in the unit disk such that the BSs divide the area into a tessellation of hexagon cells (see Fig. 1). The BSs are at the center of each cell. Without considering the boundary effects, the area of each cell is $\frac{1}{m}$.

In the ad hoc layer, data is forwarded from the sender to the destination in a multi-hop fashion. With the support of BSs in a hybrid network, data can be transmitted in two modes: the *ad hoc mode* and the *infrastructure mode*. In the ad hoc mode, a node transmits its data to the destination in the ad hoc layer without using any base station. In the infrastructure mode, data is forwarded via the infrastructure layer. However, nodes might still use multi-hop routing in the ad hoc layer in order to reach a BS. Thus in hybrid networks, there is an issue of how to route data via these two transmission modes.

[2]In Section IV, we require $m \leq n/(200 \log n)$ as $n \to \infty$ in the proof of the lower bound.

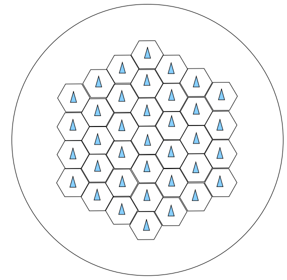

Fig. 1. A hybrid network.

B. Energy Model

To conserve energy, nodes may use different radio ranges in data transmission. In this paper, we consider only energy used in communication and do not account for the idle energy consumption. With efficient idle energy management schemes like [7], [43], [37], the impact of idle energy would be minimized.

Data transmission consumes energy in both the sender and the receiver. We define $e_t(r)$ as the minimum energy required in the sender to transmit a unit of data (called *bit*) directly over a distance r. Similarly, $e_r(r)$ is the minimum energy required in the receiver to receive a unit of data from a sender that is a distance r away. We define the *energy cost* function $e(r)$ as

$$e(r) = e_t(r) + e_r(r). \tag{1}$$

In this paper we assume that the energy consumption of the sender is $e(r)$ when the transmission range used is r and no energy is consumed in the receiver. This actually reallocates the energy consumption in a routing path by "moving" the energy consumption of each hop to its sender. Since the total energy of the routing path remains the same, this simplification would not affect our results on the energy-limited capacity[3].

We consider two specific models which are often used in the literature. The first one is a linear model in which

$$e(r) = c_a(c_b + r^\alpha) \tag{2}$$

where c_a and c_b are constants and $\alpha \geq 2$ is the path loss component. This is the same as the form of $e(r) = c' + c'' r^\alpha$ used in [16]. We adopt this form to emphasize the impact of c_b on the scaling behavior of the transport capacity, as will be clear in the following sections. The second model is

$$e(r) = c_a r^\alpha. \tag{3}$$

This actually is a special case of (2) by setting c_b to zero. We consider this separately because these two models have different scaling properties and are treated differently in the analysis.

[3]It can be shown that, if the receiving energy $e_r(r)$ is no more than a constant times of the transmission energy $e_t(r)$, which is normally true in practice, this simplification does not change our results by more than a constant factor.

C. Traffic Models

We consider two types of traffic models which are motivated by ad hoc networks and sensor networks. In the *ad hoc traffic model*, nodes can be a sender or destination and communication occurs in a one-to-one fashion. Specifically, each node chooses a random location in the area and selects the closest node to the location as its destination. In the *sensor traffic model*, all nodes are senders of data transmission but communicate with a common entity called the *sink*. We assume that the sink is at the center of the unit disk and has infinite energy. In both traffic models, we assume nodes have an infinite amount of data to send.

These traffic models try to capture the traffic patterns normally present in ad hoc networks and sensor networks. For example, in a sensor network, sensor nodes transfer collected data to a gateway node which processes and relays data to users. The sensor traffic model captures the effect of traffic concentration in sensor networks. As we will see later, traffic patterns have significant impacts on the energy-limited capacity of the network. Note that in sensor networks, data may be aggregated within the network to reduce traffic load. However, we do not consider the effect of data aggregation in this paper and leave it as a topic of future work.

D. Definition of Energy-Limited Capacity

Now we define the energy-limited capacity. We adopt the asymptotic notations of O, o, Ω, ω and Θ as defined in [8].

Since we are considering the maximum amount of data the network can deliver under node energy limits, we assume that an ideal transmission schedule is used, i.e., no collision occurs and nodes can avoid idle energy consumption. For example, by using contention-free MAC protocols and efficient idle energy management schemes, the impact of interference and idle energy would be minimized. We first define the *feasible transfer volume* of networks.

Definition 2.1 (Feasible transfer volume): For a network consisting of n nodes, a transfer volume of $\lambda(n)$ bits is *feasible* if each node can send $\lambda(n)$ bits to its corresponding destination under energy limitation of nodes.

The definition of the feasible transfer volume requires that every node is able to transmit $\lambda(n)$ bits of data. Therefore, it measures the transport capacity of the network until the first node runs out of energy. We now define the energy-limited capacity which is similar to the definition of throughput capacity in [15].

Definition 2.2 (Energy-Limited Capacity): The *Energy-Limited Capacity* (or *capacity* for short) of networks is of order $\Theta(f(n))$ bits if there are deterministic constants $c > 0$ and $c' < +\infty$ such that

$$\lim_{n\to\infty} \text{Prob}\{\lambda(n) = cf(n) \text{ is feasible}\} = 1, \text{ and}$$
$$\lim_{n\to\infty} \inf \text{Prob}\{\lambda(n) = c'f(n) \text{ is feasible}\} < 1.$$

The definition of energy-limited capacity does not consider throughput or delay. As we will see in later sections, minimum energy consumption in general requires using shorter transmission ranges, which tends to improve throughput because of the reduced interference [15]. So achieving optimal transfer volume does not pose limitations on the achievable throughput. However, it might result in larger delay because of the increased number of hops. We will study this tradeoff in future work.

For simplicity of presentation, we will use the term capacity in the rest of this paper to refer to energy-limited capacity unless stated otherwise.

III. Energy-Limited Capacity of Homogeneous Networks with Sensor Traffic

In this section, we study the energy-limited capacity of homogeneous networks with sensor traffic. In these networks, all nodes transmit data to a single node called the sink which is at the center of the unit area. We now obtain upper and lower bounds on the capacity.

A. Upper Bound on Energy-Limited Capacity

We first derive an upper bound on the energy-limited capacity which is stated in the following lemma.

Lemma 3.1: In a homogeneous network with n nodes and sensor traffic, the feasible transfer volume $\lambda(n)$ is no more than

$$c_0 \int_{\frac{1}{\phi_n\sqrt{n}}}^{\frac{1}{\sqrt{\pi}}} \frac{2\pi A x}{e(x)}\, dx \tag{4}$$

with high probability where ϕ_n is a sequence that satisfies $\lim_{n\to\infty} \phi_n \to \infty$ and c_0 is a constant.

Proof: We prove the upper bound of the transfer volume by bounding the total amount of data that nodes can transmit directly to the sink. We first consider how close nodes are to the sink when the network consists of n nodes. Let $P(d)$ be the probability that there is no node within a distance d to the sink. Since nodes are distributed randomly in the unit disk, we have $P(d) = (1-\pi d^2)^n$. Let ϕ_n be a sequence that satisfies $\lim_{n\to\infty} \phi_n \to \infty$. Then we have

$$P(\frac{1}{\phi_n\sqrt{n}}) = \left(1 - \frac{\pi}{\phi_n^2 n}\right)^n \to e^{-\frac{\pi}{\phi_n^2}}$$

which approaches 1 as $n \to \infty$. Thus with high probability, no node lies within a distance $\frac{1}{\phi_n\sqrt{n}}$ to the sink.

Consider any node X_i. Let d_i be the distance between X_i and the sink. Due to the uniform distribution of X_i, the probability density function of d_i is $P\{d_i = x\} = 2\pi x$. Let T_i be the amount of data that X_i can send directly to the sink under the energy limit. The mean of T_i is

$$E[T_i] = \int_{\frac{1}{\phi_n\sqrt{n}}}^{\frac{1}{\sqrt{\pi}}} P\{d_i = x\}\frac{A}{e(x)}\, dx = \int_{\frac{1}{\phi_n\sqrt{n}}}^{\frac{1}{\sqrt{\pi}}} \frac{2\pi A x}{e(x)}\, dx.$$

Here we take into account the fact that X_i is not within a distance of $\frac{1}{\phi_n\sqrt{n}}$ to the sink with high probability. By the Strong Law of Large Numbers, for a network of n nodes, we have

$$\frac{\sum_{i=1}^n T_i}{n} \to E[T_i]$$

with probability 1 as $n \to \infty$. In other words, the total amount of data that all nodes can send directly to the sink is no more than $c_0 n E[T_i]$ with high probability. Thus the feasible transfer volume per node $\lambda(n)$ is no more than $c_0 E[T_i]$. This completes the proof. ■

In this lemma, we assume that $\frac{x}{e(x)}$ is integrable over $(0, \frac{1}{\sqrt{\pi}})$. This does not pose much limitation since most energy cost functions in practice fall into this category. We now obtain upper bounds on the feasible transfer volume for the two specific cases.

Lemma 3.2: In a homogeneous network with sensor traffic, if the energy cost function is $e(r) = c_a r^\alpha$, the feasible transfer volume $\lambda(n)$ satisfies

$$\lambda(n) \leq \begin{cases} \frac{c_1 A}{c_a} \log(\phi_n^2 n) & \text{if } \alpha = 2; \\ \frac{c_2 A}{c_a(\alpha-2)} \phi_n^{\alpha-2} n^{\frac{\alpha}{2}-1} & \text{if } \alpha > 2 \end{cases} \tag{5}$$

with high probability.

Lemma 3.3: In a homogeneous network with sensor traffic, if the energy cost function is $e(r) = c_a(c_b + r^\alpha)$, the feasible transfer volume $\lambda(n)$ satisfies

$$\lambda(n) \leq \begin{cases} \frac{c_3 A}{2c_a} \left(\ln(c_b + \frac{1}{\pi}) - \ln(c_b + \frac{1}{\phi_n^2 n}) \right) & \text{if } \alpha = 2; \\ \frac{c_3 A}{c_a} \int_{\frac{1}{\phi_n \sqrt{n}}}^{\frac{1}{\sqrt{\pi}}} \frac{x\, dx}{c_b + x^\alpha} & \text{if } \alpha > 2 \end{cases} \tag{6}$$

with high probability.

In the case of $e(r) = c_a(c_b + r^\alpha)$, it is easy to prove $\lambda(n) = O(1)$. This is because for any transmission, the minimum energy required is $c_a c_b$. Given energy supply A, each node can transmit no more than $\frac{A}{c_a c_b}$ units of data. Therefore, the capacity of these networks is $O(1)$. However, this does not rule out the possibility that the feasible transfer volume can increase with the node density. To understand how $\lambda(n)$ evolves with n, we explicitly condition the feasible transfer volume $\lambda(n)$ on n in Lemma 3.3, which holds as long as n is large.

B. Lower Bound on Energy-Limited Capacity

In this section, we provide a lower bound on the energy-limited capacity. Our proof constructs a spatial tessellation and a transmission strategy that achieves the lower bound. In our proof, data transmissions use the same transmission range.

We first describe the spatial tessellation used in the proof. Let r_n be the data transmission range and define $w_n = r_n/8$. We set r_n to be at least $c_4\sqrt{\frac{\log n}{\pi n}}$ where c_4 is a constant. c_4 is set to be 160 which will be clear in the proof of Lemma 3.4. So the network is connected with high probability according to [14]. We partition the unit disk into regions using circles centered at the sink with radii iw_n for $1 \leq i \leq \frac{1}{w_n\sqrt{\pi}}$. Let B_i be the region between circles with radii iw_n and $(i+1)w_n$. For each i, we further divide region B_i evenly into $6i$ smaller cells (see Fig. 2). So the angle subtended by a cell at the sink is $\frac{\pi}{3i}$ and the area of a cell is $\Theta(w_n^2)$. Intuitively, nodes are distributed evenly in the unit disk due to the uniform distribution of nodes, i.e., every cell contains $\Theta(nw_n^2)$ nodes. By using similar techniques as in [15], we prove this claim in the following lemma, the proof of which can be found in the Appendix.

Lemma 3.4: Every cell contains $\Theta(nw_n^2)$ nodes with high probability.

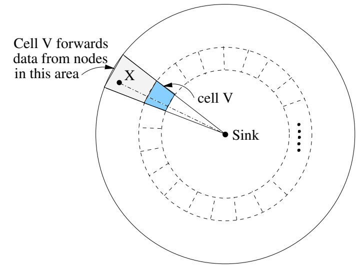

Fig. 2. Spatial tessellation in homogeneous networks with sensor traffic.

Given the spatial tessellation, data is forwarded geographically from the sender towards the sink. More precisely, consider any node X. Let L be the line segment connecting X and the sink. Data from X is forwarded along the sequence of cells intersecting L until reaching the sink. Since the transmission range r_n equals to $8w_n$ and each cell can be contained in a disk of radius $2w_n$, a node in any cell can transmit directly to nodes in adjacent cells. In addition, each cell contains $\Theta(nw_n^2)$ nodes. Therefore, for any node, a routing path along the sequence of cells described above exists with high probability.

Now we study the transfer volume achieved using this routing strategy. For a transfer volume of $\lambda(n)$, each cell must be able to forward the offered traffic load with high probability. Consider a cell V in region B_i. If data from a node, say X, is forwarded by cell V, the line segment connecting X and the sink must intersect V. So X must lie within the sector subtended by V at the sink and outside cell V, i.e., in a region B_j, $j > i$ (see Fig. 2). Noting that the angle of the sector is $\frac{\pi}{3i}$, the total number of nodes whose traffic is handled by V, including nodes in V itself, is no more than $\frac{c_5(1-\pi i^2 w_n^2)n}{i}$ with high probability. So for a transfer volume of $\lambda(n)$, the amount of traffic that cell V needs to transmit is no more than $\frac{c_5(1-\pi i^2 w_n^2)n\lambda(n)}{i}$.

Since nodes in a cell can transmit to any node in adjacent cells, nodes within a cell are exchangeable in forwarding traffic. Given that there are $\Theta(nw_n^2)$ nodes in V and the transmission range is r_n, the total amount of traffic that cell V can transmit is $\Theta(\frac{nw_n^2 A}{e(r_n)})$. Thus, for a transfer volume of $\lambda(n)$ to be feasible, we have

$$\frac{c_5(1-\pi i^2 w_n^2)n\lambda(n)}{i} \leq \frac{c_6 n w_n^2 A}{e(r_n)} \tag{7}$$

for all $1 \leq i \leq \frac{1}{w_n\sqrt{\pi}}$. In addition, nodes in the region that is within the circle of radius r_n need to transmit all traffic. So we have

$$n\lambda(n) \leq \frac{c_6 n w_n^2 A}{e(r_n)}. \tag{8}$$

Note that if $\lambda(n)$ satisfies (8), the minimum of $\lambda(n)/c_5$ and $\lambda(n)$ would satisfy both (8) and (7) for all i. That means either $\lambda(n)/c_5$ or $\lambda(n)$ is feasible. So combining with $w_n = r_n/8$, we prove that

$$\lambda(n) = \frac{c_7 r_n^2 A}{e(r_n)}$$

is feasible for some constant c_7. To maximize $\lambda(n)$, we should choose the transmission range r_n such that $\frac{r_n^2 A}{e(r_n)}$ is maximized. Taking into account the fact that $r_n \geq c_4\sqrt{\frac{\log n}{\pi n}}$, we have the following result.

Lemma 3.5: In a homogeneous network with sensor traffic, a transfer volume of

$$\lambda(n) = \max\{\frac{c_7 x^2 A}{e(x)} : c_4\sqrt{\frac{\log n}{\pi n}} \leq x \leq \frac{1}{\sqrt{\pi}}\}$$

is feasible with high probability.

Using the above lemma, we obtain lower bounds of $\lambda(n)$ for the specific cases of energy cost functions.

Lemma 3.6: In a homogeneous network with sensor traffic, if the energy cost function is $e(r) = c_a r^\alpha$, a transfer volume of

$$\lambda(n) = \begin{cases} \Omega(1) & \text{if } \alpha = 2; \\ \Omega((\frac{n}{\log n})^{\frac{\alpha}{2}-1}) & \text{if } \alpha > 2 \end{cases}$$

is feasible with high probability.

Proof: Let $f(x) = \frac{x^2 A}{e(x)}$. When $\alpha = 2$, $f(x)$ is a constant independent of x. So by Lemma 3.5, we have $\lambda(n) = \Omega(1)$. When $\alpha > 2$, $f(x)$ is a decreasing function of x. So we have $\lambda(n) = \Omega((\frac{n}{\log n})^{\frac{\alpha}{2}-1})$. ■

Lemma 3.7: In a homogeneous network with sensor traffic, if the energy cost function is $e(r) = c_a(c_b + r^\alpha)$, the feasible transfer volume is $\Omega(1)$ when $\alpha = 2$. If $\alpha > 2$, a transfer volume of

$$\lambda(n) = \begin{cases} \frac{c_8 A}{c_a} f((\frac{2c_b}{\alpha-2})^{\frac{1}{\alpha}}) & \text{if } c_4\sqrt{\frac{\log n}{\pi n}} \leq (\frac{2c_b}{\alpha-2})^{\frac{1}{\alpha}}; \\ \frac{c_8 A}{c_a} f(c_4\sqrt{\frac{\log n}{\pi n}}) & \text{if } c_4\sqrt{\frac{\log n}{\pi n}} > (\frac{2c_b}{\alpha-2})^{\frac{1}{\alpha}} \end{cases}$$

is feasible with high probability where $f(x) = \frac{x^2}{c_b + x^\alpha}$.

Proof: When $\alpha = 2$, $f(x)$ is an increasing function of x. So by Lemma 3.5, we have $\lambda(n) = \Omega(1)$.

We now consider the case of $\alpha > 2$. In this case, $f(x)$ is maximized when $x^* = (\frac{2c_b}{\alpha-2})^{\frac{1}{\alpha}}$. That is, the transfer volume is maximized when using the optimal radio range x^*. However, for a given n, the minimum transmission range used is $x' = c_4\sqrt{\frac{\log n}{\pi n}}$. Thus when x' is larger than the optimal range x^*, nodes can not use range x^*. In this case, the feasible transfer volume is $\frac{c_8 A}{c_a} f(x')$. When n is large enough such that $x' \leq x^*$, the maximum transfer volume is achieved which is $\frac{c_8 A}{c_a} f(x^*)$ by using the optimal range x^*. This completes the proof. ■

C. Scaling Properties

In this section, we discuss the scaling properties of the energy-limited capacity in homogeneous networks with sensor traffic. We focus on the class of networks where the energy cost function is $e(r) = c_a r^\alpha$ or $e(r) = c_a(c_b + r^\alpha)$. As we can see from previous results, the achievable lower bounds are within a factor of $O(\log^k n)$ from the corresponding upper bounds for some constant k. Thus we present only the results for the upper bounds here.

Based on our results, we classify these networks into two categories, depending on the scaling properties of the capacity. In the first category, the capacity of the network increases infinitely with the density of the network. These networks are said to have infinite scaling regions and include networks with $e(r) = c_a r^\alpha$. For these networks, minimum energy consumption is achieved by transmitting data using the shortest range possible, even at the cost of forwarding data in more hops. Thus the capacity benefits from the increase of node density.

In the second category, the capacity of the network can only increase finitely. These networks are said to have finite scaling regions and include networks with $e(r) = c_a(c_b + r^\alpha)$. The capacity of these networks is $O(1)$. However, this does not rule out the possibility that the feasible transfer volume can increase with node density. Intuitively, for these networks, there is an optimal transmission range which achieves minimum energy consumption. On the other hand, to ensure network connectivity, the transmission range must be large enough. So the transfer volume increases with the node density n when n is small, and saturates when n is large such that nodes can use the optimal transmission range while maintaining network connectivity.

As we can see in the next section, the scaling behavior of networks with $e(r) = c_a(c_b + r^\alpha)$ is similar to that of $e(r) = c_a r^\alpha$, but in a finite region. Thus we can apply the results for the case of $e(r) = c_a r^\alpha$, which is self-explained, to help understand the behavior of networks with $e(r) = c_a(c_b + r^\alpha)$.

D. Numerical Results

We now study in more detail the scaling properties of networks with $e(r) = c_a(c_b + r^\alpha)$ via numerical results. Specifically, we would like to answer the following questions.

1) What is the upper bound of energy-limited capacity? How does it relate to the parameters c_a, c_b and α?
2) How does the transfer volume evolve with the node density? In other words, what is the scaling region in which the capacity improves as the node density increases?

Our analytic results shows that the feasible transfer volume is inversely proportional to c_a. To quantify the effects of c_b and α, we provide numerical results of $\lambda(n)$ using Lemma 3.3. We choose $\phi_n = \ln n$ and set all constants in Lemma 3.3 to 1. The real upper bounds should be scaled by a constant factor. However, this does not affect the scaling behavior.

Fig. 3 shows the optimal transfer volume, i.e., $\lambda(n)$ when n is infinity, with different c_b and α. We can see that the feasible transfer volume increases as c_b decreases. This result suggests that reducing the constant energy consumption can lead to significant capacity improvement. Fig. 3 also shows that $\lambda(n)$ increases with α when c_b is small.

Fig. 4 shows how the transfer volume $\lambda(n)$ grows with the node density when α is 3. As the node density increases, $\lambda(n)$ also increases until saturating at the maximum value. We can see that with a smaller c_b, $\lambda(n)$ increases over a larger region of node density. Fig. 4 also shows that the scaling behavior of $e(r) = c_a(c_b + r^\alpha)$ is similar to that of $e(r) = c_a r^\alpha$ (the line with label "$c_b = 0$" in the figure) before $\lambda(n)$ reaches the maximum capacity. We observe similar results with different values of α.

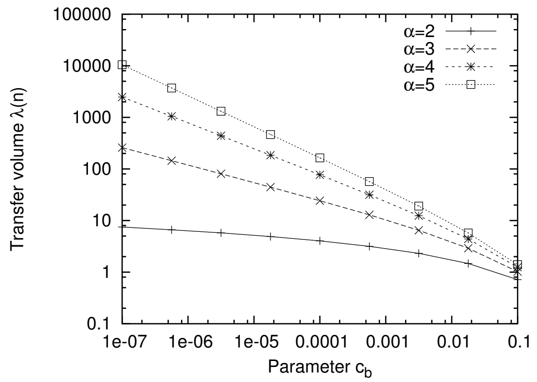

Fig. 3. Effects of parameters c_b and α on the transfer volume in homogeneous networks with sensor traffic.

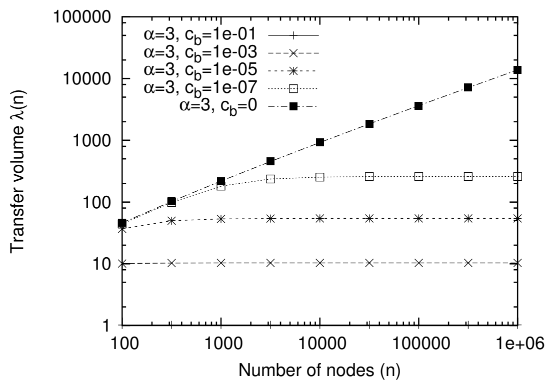

Fig. 4. Transfer volume vs. number of nodes in homogeneous networks with sensor traffic.

In summary, while the upper bound is $O(1)$ when $e(r) = c_a(c_b + r^\alpha)$, the energy-limited capacity can scale with the node density, although within a finite region. In addition, reducing the constant energy both improves the network capacity and increases the scaling region.

IV. Energy-Limited Capacity of Hybrid Networks with Sensor Traffic

In this section, we consider the energy-limited capacity in hybrid networks where base stations (BSs) with unlimited energy are deployed to support data forwarding. We study the capacity gain obtained with the deployment of base stations. Since the base stations have unlimited energy, the capacity limitation lies in the nodes due to their energy constraints. Intuitively, by transmitting data via the BSs, the traffic load on the nodes is reduced, resulting in better energy-limited capacity. On the other hand, the use of the BSs may cause hot spots and overload nodes near the BSs, leading to supoptimal transfer volume. Thus it is important to balance these two factors and make efficient use of the BSs.

In hybrid networks, data can be forwarded in the ad hoc mode where the BSs are not involved in data forwarding, or in the infrastructure mode. Node energy is shared by these two transmission modes. In this paper, we consider an energy allocation strategy where each node uses p fraction of its energy for the ad hoc mode and the remaining $(1-p)$ fraction for the infrastructure mode, $0 \leq p \leq 1$. In the infrastructure mode, nodes should transmit data to the closest BS, i.e., the BS in the same cell as the node, in order to conserve energy. In addition, data should enter and exit the BS layer once. Thus, routing in the infrastructure mode consists of three phases, from the sender to the closest BS, forwarding within the BS layer, and from the BS in the destination's cell to the destination. Node energy is only consumed in the first phase, i.e., forwarding data from the sender to the BS, possibly in multiple hops.

Let $\lambda_a(n)$ be the transfer volume of a network when all energy is used for the ad hoc mode. Similarly, define $\lambda_i(n,m)$ as the transfer volume if all energy is used for the infrastructure mode. Because the transfer volume is proportional to node energy, given any energy allocation with parameter p, the total transfer volume is

$$\Lambda(n,m) = p\lambda_a(n) + (1-p)\lambda_i(n,m). \quad (9)$$

Note that energy consumption in the ad hoc or infrastructure mode is isolated. So we can obtain an upper (lower) bound for $\Lambda(n,m)$ by summing up the upper (lower) bounds of $\lambda_a(n)$ and $\lambda_i(n,m)$.

A. Area Scaling

We first determine the transfer volume in both transmission modes, namely $\lambda_a(n)$ and $\lambda_i(n,m)$. In the ad hoc mode, nodes forward data without using any base station. We can obtain $\lambda_a(n)$ by applying the results of homogeneous networks in previous sections. In the infrastructure mode, all nodes in a cell transmit data to the BS within the cell. To compute $\lambda_i(n,m)$, we extend the analysis developed for homogeneous networks with sensor traffic in Section III. The difference is that $\lambda_i(n,m)$ considers only nodes within a cell, instead of the unit area in previous analysis.

For the upper bound of $\lambda_i(n,m)$, it is sufficient to consider a single cell. This is because, by definition, a transfer volume of $\lambda_i(n,m)$ requires every node to be able to transmit that amount of data to its destination. Let V be any cell. Because the BSs are placed regularly, the area of V is $\frac{1}{m}$. Since nodes are randomly distributed, the probability that a node is in V is $\frac{1}{m}$. By the Strong Law of Large Numbers, the number of nodes in V is $\frac{n}{m}$ with probability 1 as $n \to \infty$. To determine $\lambda_i(n,m)$, we adapt the proof in Lemma 3.1 to account for the fact that nodes are within a cell[4]. Specifically, the claim that no node lies within a distance of $\frac{1}{\phi_n\sqrt{n}}$ to the BS with high probability remains true. The probability density function of a node's distance to the BS becomes $P\{d_i = x\} = 2\pi m x$ by conditioning on the fact that the node is within the cell. So the upper bound $\lambda_i^u(n)$ of the feasible transfer volume is

$$\lambda_i^u(n,m) = c_9 m \int_{\frac{1}{\phi_n\sqrt{n}}}^{\frac{1}{\sqrt{m\pi}}} \frac{2\pi A x}{e(x)}\,dx. \quad (10)$$

[4]We approximate a hexagon cell as a disk of the same area to apply the analysis in Section III. We argue that this would not impact our results. Specifically, if the energy cost function $e(r)$ satisfies $e(c'r) \leq c''e(r)$ for some constants c', c'' and $0 \leq r \leq 1/\sqrt{\pi}$, which is the case in practice, it can be shown that the results obtained by approximation differ only by a constant factor.

For the lower bound of $\lambda_i(n,m)$, we need to prove that nodes in every cell can transmit enough data to the sink. To bound the number of nodes in each cell, we assume that $m \leq \frac{n}{200 \log n}$. So the area of each cell is at least $\frac{200 \log n}{n}$. Using the same techniques as in Lemma 3.4, the number of nodes in every cell can be shown to be $\Theta(\frac{n}{m})$. So to ensure connectivity among node within a cell, the transmission range r_n must be at least of order $\sqrt{\frac{\log(n/m)}{\pi n}}$. Following the same arguments in the proof of Lemma 3.5, we can prove a lower bound $\lambda_i^l(n,m)$ of the feasible transfer volume below.

$$\lambda_i^l(n,m) = \max\{\frac{c_{10} m x^2 A}{e(x)} : c_{11}\sqrt{\frac{\log(n/m)}{\pi n}} \leq x \leq \frac{1}{\sqrt{m\pi}}\}. \tag{11}$$

B. Upper Bound on Energy-Limited Capacity

We now study the capacity of hybrid networks with sensor traffic. For this class of networks, transmissions in the ad hoc mode and the infrastructure mode are the same except at different scales, i.e., in the unit disk or in a cell. In the following we present the upper bounds on the energy-limited capacity.

Given the energy allocation parameter p, we can obtain the total transfer volume by combining $\lambda_a(n)$ and $\lambda_i(n,m)$. We now derive upper bounds for the specific cases of energy cost functions.

Case 1. $e(r) = c_a r^\alpha$.

Noting that $\lim_{n\to\infty} \frac{m}{n} = 0$, we derive $\lambda_i^u(n,m)$ from (10).

$$\lambda_i^u(n,m) = \begin{cases} O(m \log n) & \text{if } \alpha = 2; \\ O(mn^{\frac{\alpha}{2}-1}\phi_n^{\alpha-2}) & \text{if } \alpha > 2. \end{cases} \tag{12}$$

When $\alpha > 2$, the infrastructure mode transfer volume $\lambda_i^u(n,m)$ is $O(mn^{\frac{\alpha}{2}-1}\phi_n^{\alpha-2})$ by (12). And the ad hoc mode transfer volume $\lambda_a(n)$ is $O(n^{\frac{\alpha}{2}-1}\phi_n^{\alpha-2})$. $\lambda_i^u(n,m)$ dominates the total transfer volume. So $\Lambda(n,m)$ is maximized if $p \to 0$, i.e., traffic should be forwarded via the BSs. In this case, the transfer volume scales with the number of BSs. Similarly, when $\alpha = 2$, the total capacity is $O(m \log n)$ as compared to the $O(\log n)$ capacity in networks without BSs. So the capacity increases as m increases.

In both cases, the deployment of BSs in networks with sensor traffic has immediate improvement on the network capacity. The capacity gain scales linearly with the number of BSs. This suggests that BS deployment in sensor networks is a promising approach to improve capacity.

Case 2. $e(r) = c_a(c_b + r^\alpha)$.

We first determine $\lambda_i^u(n,m)$ by (10).

$$\lambda_i^u(n,m) = \begin{cases} \frac{c_{12} m A}{2c_a} \ln \frac{c_b + \frac{1}{m\pi}}{c_b + \frac{1}{\phi_n^2 n}} & \text{if } \alpha = 2; \\ \frac{c_{12} m A}{c_a} \int_{\frac{1}{\phi_n \sqrt{n}}}^{\frac{1}{\sqrt{m\pi}}} \frac{x\,dx}{c_b + x^\alpha} & \text{if } \alpha > 2. \end{cases} \tag{13}$$

It is easy to see that $\lambda_i^u(n,m)$ is $O(1)$ regardless of α and m. This is because for any transmission, the minimum energy required is $c_a c_b$. Given energy supply A, each node can transmit no more than $\frac{A}{c_a c_b}$ units of data. Therefore, this class of networks has finite scaling regions.

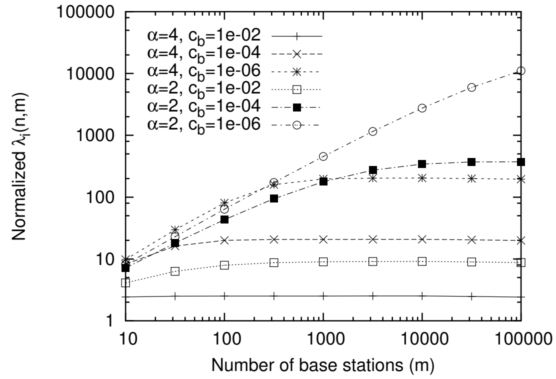

Fig. 5. Normalized $\lambda_i(n,m)$ vs. number of base stations in hybrid networks with sensor traffic.

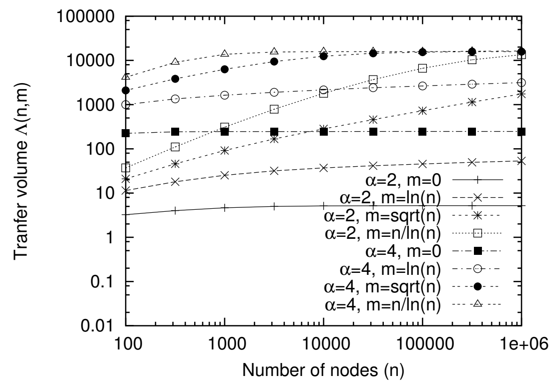

Fig. 6. Transfer volume $\Lambda(n,m)$ vs. number of nodes in hybrid networks with sensor traffic.

We now study the scaling properties of networks with $e(r) = c_a(c_b + r^\alpha)$ by numerical results. In the calculation, we set all constants to 1. Fig. 5 shows how the infrastructure mode capacity grows with the number of BSs when n is ∞. We normalize $\lambda_i^u(n,m)$ to the ad hoc mode transfer volume $\lambda_a(n)$. We can see that the normalized $\lambda_i^u(n,m)$ is always greater than 1, suggesting that $\lambda_i^u(n,m)$ always dominates the total transfer volume. $\lambda_i^u(n,m)$ increases with the number of BSs until it saturates. In addition, the maximum normalized $\lambda_i^u(n,m)$ increases with smaller c_b and smaller α. So the capacity gain is more significant when c_b and α is small.

Fig. 6 shows how the total transfer volume grows with the number of nodes when c_b is 10^{-5}. We compare $\Lambda(n,m)$ when the number of BSs m is 0, $\ln n$, $\sqrt{n}$ and $\frac{n}{\ln n}$. Specifically, we calculate $\lambda_a(n)$ using (6) and $\lambda_i(n,m)$ using (13), and set $\Lambda(n,m)$ as the larger of these two. We can see that these networks have finite scaling regions, i.e., the transfer volume saturates when n is large enough. Fig. 6 also confirms that the total transfer volume increases with the number of BSs, even the number of BSs is relatively small, e.g., $\ln n$.

C. Lower Bound on Energy-Limited Capacity

We now obtain lower bounds using similar techniques as in the previous section. We focus on the specific cases of energy cost functions. In general, the achievable lower bounds are within a factor of $O(\log^k n)$ from the corresponding upper bounds for some constant k and the scaling behavior is similar.

Case 1. $e(r) = c_a r^\alpha$.

We first determine $\lambda_i^l(n,m)$ by (11).

$$\lambda_i^l(n,m) = \begin{cases} \Omega(m) & \text{if } \alpha = 2; \\ \Omega(m(\frac{n}{\log n})^{\frac{\alpha}{2}-1}) & \text{if } \alpha > 2. \end{cases} \tag{14}$$

Comparing $\lambda_i^l(n,m)$ with the ad hoc mode transfer volume $\lambda_a(n)$ obtained in Lemma 3.6, we can see that $\lambda_i^l(n,m)$ will dominate the total transfer volume $\Lambda(n,m)$. So traffic should be transmitted using the infrastructure mode. And $\Lambda(n,m)$ increases linearly with the number of BSs. This is similar to the results on the upper bound in the previous section.

Case 2. $e(r) = c_a(c_b + r^\alpha)$.

We obtain the following results about $\lambda_i^l(n,m)$ using similar techniques as in Lemma 3.7. When $\alpha = 2$, we have

$$\lambda_i^l(n,m) = \frac{c_{13}}{c_b + \frac{1}{m\pi}}. \tag{15}$$

When $\alpha > 2$, we have

$$\lambda_i^l(n,m) = \begin{cases} c_{14} m f((\frac{2c_b}{\alpha-2})^{\frac{1}{\alpha}}) & \text{if } r' \le (\frac{2c_b}{\alpha-2})^{\frac{1}{\alpha}}; \\ c_{15} m f(r') & \text{if } r' > (\frac{2c_b}{\alpha-2})^{\frac{1}{\alpha}} \end{cases} \tag{16}$$

with high probability where $f(x) = \frac{x^2}{c_b + x^\alpha}$ and $r' = c_{11}\sqrt{\frac{\log(n/m)}{\pi n}}$.

With $\lambda_i^l(n,m)$, we can obtain a lower bound of the total transfer volume $\Lambda(n,m)$ by combining $\lambda_i^l(n,m)$ and the lower bound of $\lambda_a(n)$ obtained in Lemma 3.7.

V. Energy-Limited Capacity of Homogeneous Networks with Ad Hoc Traffic

In this section, we present the energy-limited capacity of homogeneous networks with ad hoc traffic. In these networks, nodes communicate with randomly chosen nodes, instead of a common sink. We will obtain upper and lower bounds of the energy-limited capacity.

A. Upper Bound on Energy-Limited Capacity

To establish an upper bound on the energy-limited capacity, we first determine the minimum energy that is required to transfer data over a given distance. Given limited energy supplies in nodes, we then obtain an upper bound of the feasible transfer volume. In our proof, we extend the techniques presented in [30]. The work in [30] studies the throughput capacity of networks where bandwidth is infinite but the maximum transmit power of each node is limited. Under these assumptions, the authors show that the interference is negligible, which renders the transmit power of nodes as the main constraint. This is similar to our analysis of energy-limited capacity where the transfer volume is limited by node energy supplies.

For any routing path, the total energy used to deliver a unit of data is the sum of energy consumed in each hop. Thus the number of hops and the transmission range of each hop determine the total energy of the routing path. For a network with n nodes, there is an upper bound on the number of hops that a routing path can have. The work in [30] has proved the following result.

Lemma 5.1: (Result from [30]) For a network with n nodes, the number of nodes on a routing path of length d is upper bounded by $N(n,d) = c_{16}\log n + c_{17} d\sqrt{n \log n}$ for some constants c_{16} and c_{17} with high probability.

We define $\gamma(n,d)$ as the minimum energy for routing a unit of data over a distance d within $N(n,d)$ hops where the relaying nodes can be arranged arbitrarily to minimize the total energy consumption. Now we prove the following result.

Lemma 5.2: For a network with n nodes, the minimum energy to transmit a unit of data over a distance d is no less than $\gamma(n,d)$ defined above with high probability.

Proof: Let R be a routing path between two nodes that are of distance d apart and $\{r_i\}_{i=1}^{k^*}$ be the transmission ranges used in R. By the triangle inequality, we have $\sum_{i=1}^{k^*} r_i \ge d$. Consider a routing path $\{r_i\}_{i=1}^{k}$ where k is the minimum number that satisfies $\sum_{i=1}^{k} r_i \ge d$. If we replace the last hop r_k with another range r' such that $\sum_{i=1}^{k-1} r_i + r' = d$, we have a routing path that is of length d and has k hops. Obviously we have $r' \le r_k$. By Lemma 5.1, k is no more than $N(n,d)$ with high probability. Because $\gamma(n,d)$ is the minimum energy for routing a unit of data over a distance d within $N(n,d)$ hops, we have $\sum_{i=1}^{k-1} e(r_i) + e(r') \ge \gamma(n,d)$. So the total energy consumption of R is no less than $\gamma(n,d)$. This completes the proof. ∎

For general energy cost functions $e(r)$, we can obtain $\gamma(n,d)$ by solving an optimization problem as follows. Because a routing path can consist of up to $N(n,d)$ hops, we use binary variables I_i to indicate whether transmission range r_i is used in the routing path, $1 \le i \le N(n,d)$. We can formulate the problem as follow.

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{N(n,d)} I_i e(r_i) \\ \text{subject to} \quad & \sum_{i=1}^{N(n,d)} I_i r_i = d, \\ & r_i \ge 0, I_i = \{0,1\} \text{ for } 1 \le i \le N(n,d). \end{aligned}$$

Then $\gamma(n,d)$ is the optimal value of the objective function, i.e., the minimum value of $\sum_{i=1}^{N(n,d)} I_i e(r_i)$.

Now we derive an upper bound of the energy-limited capacity. In a network with n nodes, data transmission for each node consumes energy in the network. The sum of energy consumption in all n routing paths must be no more than the total energy supply in the network which is nA. Let d_i be the distance between node X_i and its destination. To achieve a transfer volume of $\lambda(n)$, we need to have

$$\lambda(n)\sum_{i=1}^{n} \gamma(n,d_i) \le nA.$$

Since d_i is i.i.d., by the Strong Law of Large Number, we have $\sum_{i=1}^{n} \gamma(n,d_i)/n \to E[\gamma(n,d_i)]$ with probability 1 as $n \to \infty$. So with high probability, we get

$$\lambda(n) \le \frac{c_{18} A}{E[\gamma(n,d_i)]}. \tag{17}$$

For general energy cost functions, the analysis of $\gamma(n, d_i)$ and consequently $\lambda(n)$ would be difficult. In the following, we focus on the two specific cases.

Lemma 5.3: In a homogeneous network with ad hoc traffic, if the energy cost function is $e(r) = c_a r^\alpha$, the feasible transfer volume $\lambda(n)$ satisfies

$$\lambda(n) = O(\frac{A}{c_a}(n \log n)^{\frac{\alpha-1}{2}}) \quad (18)$$

with high probability.

Proof: Let d be the distance between a node and its destination. We first determine $\gamma(n, d)$. Since $\alpha \geq 2$, the energy cost function $e(r)$ is convex. So for any route of k hops, the energy consumption is at least $\frac{c_a d^\alpha}{k^{\alpha-1}}$ which is a decreasing function of k. Therefore, we can reduce energy consumption by transmitting data in more hops. Noting that $k \leq N(n, d)$, we have

$$\gamma(n, d) \geq \frac{c_a d^\alpha}{N(n, d)^{\alpha-1}}.$$

Recall that $N(n, d) = c_{16} \log n + c_{17} d\sqrt{n \log n}$. When $d > \epsilon$ for some constant ϵ, the $\sqrt{n \log n}$ term will dominate $N(n, d)$ for large n, i.e., $N(n, d) \simeq c_{17} d\sqrt{n \log n}$. We have $\gamma(n, d) \geq \frac{c_a d}{c_{17}^{\alpha-1}(n \log n)^{\frac{\alpha-1}{2}}}$ when $d > \epsilon$. Thus

$$\begin{aligned} E[\gamma(n, d)] &\geq P\{d > \epsilon\} E[\gamma(n, d) | d > \epsilon] \\ &\geq (1 - \pi\epsilon^2) E[\gamma(n, d) | d > \epsilon] \\ &\geq \frac{c_{19} c_a E[d | d > \epsilon]}{(n \log n)^{\frac{\alpha-1}{2}}}. \end{aligned}$$

Since $E[d | d > \epsilon]$ is a constant, by (17) we have

$$\lambda(n) = O(\frac{A}{c_a}(n \log n)^{\frac{\alpha-1}{2}}).$$

This completes the proof. ∎

Lemma 5.4: In a homogeneous network with ad hoc traffic, if the energy cost function is $e(r) = c_a(c_b + r^\alpha)$, the feasible transfer volume $\lambda(n)$ satisfies

$$\lambda(n) \leq \begin{cases} \frac{c_{20} A}{c_a f(k^*)} & \text{if } \sqrt{n \log n} \geq \frac{k^*}{c_{17}}; \\ \frac{c_{20} A}{c_a f(c_{17}\sqrt{n \log n})} & \text{if } \sqrt{n \log n} < \frac{k^*}{c_{17}} \end{cases} \quad (19)$$

with high probability where $k^* = (\frac{\alpha-1}{c_b})^{\frac{1}{\alpha}}$ and $f(x) = c_b x + \frac{1}{x^{\alpha-1}}$.

Proof: Consider any node and let d be its distance to the destination. We first determine $\gamma(n, d)$. Suppose that the routing path consists of k hops. Since $e(r)$ is convex, the energy consumption of the routing path is at least $c_a(kc_b + \frac{d^\alpha}{k^{\alpha-1}})$. So the minimum energy is achieved when the number of hops in the route is $d(\frac{\alpha-1}{c_b})^{\frac{1}{\alpha}}$, i.e., $k^* d$.

Recall that $N(n, d) \simeq c_{17} d\sqrt{n \log n}$ for large n when $d > \epsilon$ for some constant ϵ. If $N(n, d) \geq k^* d$, or $\sqrt{n \log n} \geq \frac{k^*}{c_{17}}$, data can be forwarded in $k^* d$ hops which consumes a minimum amount of energy. In this case, we have

$$\gamma(n, d) \geq c_a(k^* c_b + \frac{1}{{k^*}^{\alpha-1}})d = c_a f(k^*) d.$$

Similar to the proof of Lemma 5.3, we can prove

$$\lambda(n) \leq \frac{c_{20} A}{c_a f(k^*)}.$$

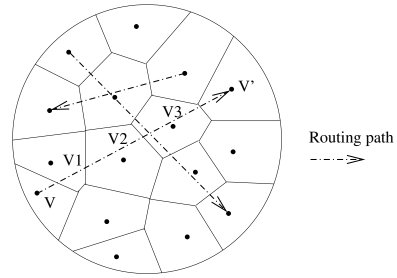

Fig. 7. Voronoi tessellation and routing paths.

If $N(n, d) < k^* d$, or $\sqrt{n \log n} < \frac{k^*}{c_{17}}$, data should be routed in as many hops as possible to conserve energy, i.e., in $N(n, d)$ hops. We can prove the result similarly. ∎

B. Lower Bound on Energy-Limited Capacity

In this section we provide a constructive proof of a lower bound on the energy-limited capacity, which is similar to the proof in [15].

In our proof, data is transmitted using the same radio range. To ensure that the network is connected with high probability, the radio range is at least of order $\sqrt{\frac{\log n}{n}}$ according to [14]. In our proof, the radio range r_n is set to be at least $c_{21}\sqrt{\frac{\log n}{\pi n}}$. We set c_{21} to 80 which will be clear in the proof of Lemma 5.5.

We divide the unit area into smaller regions. Specifically, we use a Voronoi tessellation $\mathcal{V}_n$ with the following properties which is shown to exist in [15].

1) Every Voronoi cell contains a disk of radius ρ_n where $\rho_n = \frac{r_n}{8}$.
2) Every Voronoi cell is contained in a disk of radius $2\rho_n$.

An example of a Voronoi tessellation is depicted in Fig. 7. We obtain the following result about the number of nodes in each cell. The proof is skipped here because it is very similar to that of Lemma 3.4.

Lemma 5.5: Every cell in $\mathcal{V}_n$ has $\Theta(nr_n^2)$ nodes with high probability.

We consider a geographic routing strategy in which data is forwarded from a node to its destination along Voronoi cells intersected by the line segment connecting the source and its destination (see Fig. 7). Specifically, consider a node X_i and its randomly chosen location Y_i. Let V_i and V_i' be the cells that contain X_i and Y_i respectively. Data originating at X_i will be forwarded from V_i to V_i' along the cells intersecting the line segment connecting X_i and Y_i. After reaching cell V_i', data will be sent to the destination.

To achieve a transfer volume of $\lambda(n)$, every cell should be able to transmit the amount of traffic offered by the routing algorithm. Thus it is important to balance traffic load among cells. The following result shows that this is achieved by using the spatial tessellation and routing algorithm described above. The proof of this lemma can be found in the Appendix.

Lemma 5.6: For a transfer volume of $\lambda(n)$, the amount of traffic every cell is responsible for forwarding is $O(nr_n\lambda(n))$ with high probability.

Now we are ready to derive a lower bound on the energy-limited capacity. By Lemma 5.5, the total energy in each cell is $\Theta(nr_n^2 A)$. For a transfer volume of $\lambda(n)$, since the radio range is r_n, the amount of energy required in each cell is $O(nr_n\lambda(n)e(r_n))$ by Lemma 5.6. Thus we have

$$nr_n\lambda(n)e(r_n) \geq c_{22}nr_n^2 A$$

for some constant c_{22}. So we have

$$\lambda(n) \geq \frac{c_{22}r_n A}{e(r_n)}.$$

Noting that the transmission range r_n is at least $c_{21}\sqrt{\frac{\log n}{\pi n}}$, we have the following result.

Lemma 5.7: For homogeneous networks with n nodes and ad hoc traffic, a transfer volume of

$$\lambda(n) \geq \max\{\frac{c_{22}xA}{e(x)} : c_{21}\sqrt{\frac{\log n}{\pi n}} \leq x \leq \frac{1}{\sqrt{\pi}}\} \qquad (20)$$

is feasible with high probability.

The above lemma can be applied to any energy cost function. We now derive the lower bounds for the two specific cases of energy cost functions.

Lemma 5.8: In a homogeneous network with ad hoc traffic, if the energy cost function is $e(r) = c_a r^\alpha$, a transfer volume of

$$\lambda(n) = \Omega(\frac{A}{c_a}(\frac{n}{\log n})^{\frac{\alpha-1}{2}}) \qquad (21)$$

is feasible with high probability.

Lemma 5.9: In a homogeneous network with ad hoc traffic, if the energy cost function is $e(r) = c_a(c_b + r^\alpha)$, a transfer volume of

$$\lambda(n) \geq \begin{cases} \frac{c_{23}A}{c_a g(\sqrt[\alpha]{\frac{c_b}{\alpha-1}})} & \text{if } c_{21}\sqrt{\frac{\log n}{\pi n}} \leq \sqrt[\alpha]{\frac{c_b}{\alpha-1}}; \\ \frac{c_{23}A}{c_a g(c_{21}\sqrt{\frac{\log n}{\pi n}})} & \text{if } c_{21}\sqrt{\frac{\log n}{\pi n}} > \sqrt[\alpha]{\frac{c_b}{\alpha-1}} \end{cases} \qquad (22)$$

is feasible with high probability where $g(x) = \frac{c_b + x^\alpha}{x}$.

Proof: Note that $g(r) = (c_b + r^\alpha)/r$. So $g(r)$ is minimized when the transmission range is $r^* = \sqrt[\alpha]{\frac{c_b}{\alpha-1}}$. That is, the feasible transfer volume is maximized by using radio range r^*. However, for a given n, the minimum transmission range used is $r' = c_{21}\sqrt{\frac{\log n}{\pi n}}$. Thus when the minimum range r' is larger than the optimal range r^*, nodes can not use range r^*. In this case, the feasible transfer volume is $\frac{c_{23}A}{c_a g(r')}$. When n is large enough such that $r' \leq r^*$, the maximum transfer volume can be achieved which is $\frac{c_{23}A}{c_a g(r^*)}$. ■

C. Scaling Properties

We now discuss the scaling properties of the capacity in networks with the specific energy cost functions. As we can see in previous sections, the achievable lower bounds are within a factor of $O(\log^k n)$ from the corresponding upper bounds for some constant k. Thus we present only the results for the upper bounds here.

We can see that the energy-limited capacity can scale well in homogeneous networks with ad hoc traffic. For example, when $e(r) = c_a r^\alpha$, the transfer volume scales as $O((n\log n)^{\frac{\alpha-1}{2}})$. When $e(r) = c_a(c_b + r^\alpha)$, the transfer volume scales with the number of nodes, but within a finite region, which parallels the results for networks with sensor traffic. Comparing networks with ad hoc and sensor traffic, we can see that the capacity of networks supporting sensor traffic is less than the capacity of networks supporting ad hoc traffic, by a factor of $O(\sqrt{n}\log^c n)$. This capacity penalty is due to the effect of traffic aggregation near the sink in networks with sensor traffic.

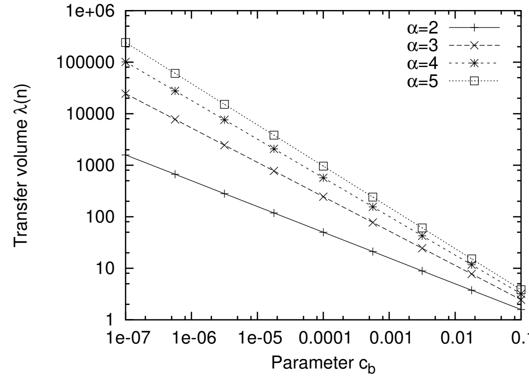

Fig. 8. Effect of parameter c_b on transfer volume in homogeneous networks with ad hoc traffic.

Now we provide some numerical results for the case where $e(r) = c_a(c_b + r^\alpha)$. We set all constants to 1. Fig. 8 depicts how the transfer volume $\lambda(n)$ changes with c_b. We can see that $\lambda(n)$ increases as c_b decreases. So reducing the constant energy consumption can increase the capacity of the network. In addition, $\lambda(n)$ increases with α. This behavior is similar to that of networks with sensor traffic.

Fig. 9 shows how the transfer volume $\lambda(n)$ grows with the node density when $\alpha = 3$. We can see that $\lambda(n)$ can increases with node density, as in networks with sensor traffic. In addition, with a smaller c_b, the transfer volume increases over a larger region of node density. Fig. 9 also shows that the scaling behavior for the case of $e(r) = c_a(c_b + r^\alpha)$ is similar to that of $e(r) = c_a r^\alpha$, which scales as $O(n\log n)$, before $\lambda(n)$ reaches the maximum transfer volume.

To quantify the scaling region, we define the *saturating density* as the minimum node density that achieves the maximum transfer volume. By Lemma 5.4, the saturating density is the minimum n that satisfies $c_{17}\sqrt{n\log n} \geq (\frac{\alpha-1}{c_b})^{\frac{1}{\alpha}}$. Fig. 10 shows the saturating density under different c_b. As expected, the saturating density increases with the decrease of c_b. It also shows that the saturating density is larger with a smaller α. Thus for a given c_b, while the maximum transfer volume is less with a smaller α, it requires higher node density to reach that maximum transfer volume.

In summary, while the upper bound is $O(1)$ when $e(r) = c_a(c_b + r^\alpha)$, the energy-limited capacity can scale with the node density, although within a finite region. In addition, reducing the constant energy both improves the network capacity and increases the scaling region.

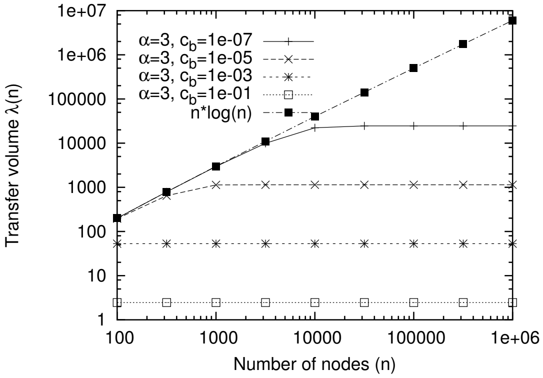

Fig. 9. Transfer volume vs. number of nodes in homogeneous networks with ad hoc traffic.

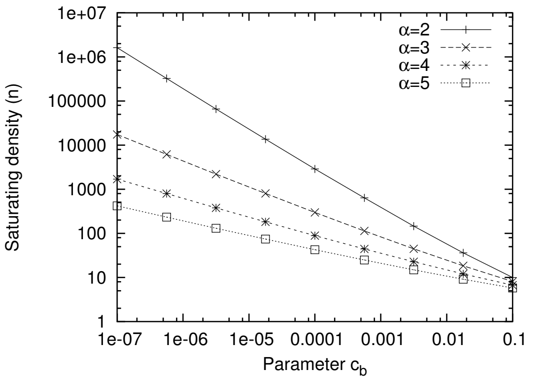

Fig. 10. Saturating density vs c_b in homogeneous networks with ad hoc traffic.

VI. Energy-Limited Capacity of Hybrid Networks with Ad Hoc Traffic

We now study hybrid networks with ad hoc traffic. In these networks, base stations with unlimited energy are deployed to help data forwarding and data can be routed via the ad hoc mode or the infrastructure mode. As in Section IV, we obtain the total transfer volume by combining the transfer volume contributed by the ad hoc and infrastructure modes. In the following, we focus on the specific cases of energy cost functions.

A. Upper Bound on Energy-Limited Capacity

We first obtain upper bounds of the energy-limited capacity.

Case 1. $e(r) = c_a r^\alpha$.

When $\alpha > 2$, by (18) and (12), the total transfer volume is

$$\Lambda(n, m) = pO((n \log n)^{\frac{\alpha-1}{2}}) + (1-p)O(mn^{\frac{\alpha}{2}-1}\phi_n^{\alpha-2}).$$

If $m = o(\sqrt{n}(\log n)^{\frac{\alpha-1}{2}})$, the ad hoc mode transfer volume $\lambda_a(n)$ dominates $\Lambda(n, m)$. So $\Lambda(n, m)$ is maximized if $p \to 1$. In other words, traffic should be forwarded using the ad hoc mode. Therefore, there is no significant performance gain when m is small, i.e., $m = o(\sqrt{n}(\log n)^{\frac{\alpha-1}{2}})$. If $m = \omega(\sqrt{n}(\log n)^{\frac{\alpha-1}{2}})$, the infrastructure mode transfer volume will be the dominating term in $\Lambda(n, m)$. So $\Lambda(n, m)$ is maximized if $p \to 0$, i.e., traffic should be routed via the BS layer. In this case, the transfer volume increases linearly with m.

Similarly, when $\alpha = 2$, we have

$$\Lambda(n, m) = pO(\sqrt{n \log n}) + (1-p)O(m \log n).$$

So the total transfer volume is $O(\sqrt{n \log n})$ if $m = o(\sqrt{\frac{n}{\log n}})$ and $O(m \log n)$ if $m = \omega(\sqrt{\frac{n}{\log n}})$.

In summary, when the energy cost function is $e(r) = c_a r^\alpha$, the capacity gain due to the deployment of BSs is bimodal. Specifically, when the number of BSs grows asymptotically slower than some threshold which is of the order $\sqrt{n}\log^k n$ for some constant k, the total capacity is dominated by the ad hoc mode capacity. Thus the improvement due to the deployment of BSs is insignificant. When the number of BSs grows faster than that threshold, the total capacity is dominated by the infrastructure mode capacity which increases linearly with the number of BSs. This suggests that to achieve non-negligible capacity gain, the number of BSs deployed should be large enough.

Case 2. $e(r) = c_a(c_b + r^\alpha)$.

Similarly, we can obtain the total transfer volume by combining the transfer volume contributed by the ad hoc and infrastructure modes using (19) and (13).

B. Lower Bound on Energy-Limited Capacity

We now consider the lower bound of the energy-limited capacity. In general, the lower bounds are within a factor of $O(\log^k n)$ from the corresponding upper bounds and the scaling behavior is similar.

Case 1. $e(r) = c_a r^\alpha$.

When $\alpha = 2$, by (14) and (21), we can get

$$\Lambda(n, m) = p\Omega(\sqrt{\frac{n}{\log n}}) + (1-p)\Omega(m).$$

If $m = o(\sqrt{\frac{n}{\log n}})$, the ad hoc mode transfer volume dominates $\Lambda(n, m)$. So there is no significant performance gain in this case. If $m = \omega(\sqrt{\frac{n}{\log n}})$, however, the total transfer volume will be dominated by the infrastructure mode transfer volume, which scales linearly as the number of BSs.

When $\alpha > 2$, we have

$$\Lambda(n, m) = p\Omega((\frac{n}{\log n})^{\frac{\alpha-1}{2}}) + (1-p)\Omega(m(\frac{n}{\log n})^{\frac{\alpha}{2}-1}).$$

So the total transfer volume $\Lambda(n, m)$ will be $\Omega((\frac{n}{\log n})^{\frac{\alpha-1}{2}})$ if $m = o(\sqrt{\frac{n}{\log n}})$. And $\Lambda(n, m)$ will be $\Omega(m(\frac{n}{\log n})^{\frac{\alpha}{2}-1})$ if $m = \omega(\sqrt{\frac{n}{\log n}})$.

Case 2. $e(r) = c_a(c_b + r^\alpha)$.

For this case, we can compute the total transfer volume by (16) and (22).

VII. Discussion

In this section, we compare our results on energy-limited capacity with previous results on throughput capacity and discuss the issues of idle energy consumption and network lifetime.

A. Comparison with Throughput Capacity

In [15], Gupta and Kumar study the throughput capacity of static random networks. They show that the per node throughput decreases as $\Theta(\frac{1}{\sqrt{n \log n}})$ as the number of nodes n increases. This implies that large scale wireless networks may have performance problems for non-local traffic patterns. Using the same network model, we analyze the energy-limited capacity, and our results indicate that the energy-limited capacity has fundamentally different scaling properties from those of throughput capacity. We show that the energy-limited capacity can scale with the node density, either infinitely or finitely. Therefore, from the energy consumption perspective, this demonstrates positively the feasibility of the deployment of large scale wireless networks, especially those networks where energy is of paramount importance and bandwidth demand is of less concern. By combining the results on both types of capacities, we can obtain a lower bound on network lifetime under ideal conditions, i.e., data transmission is optimal such that energy consumption is minimum. For example, when $e(r) = c_a r^\alpha$, the energy-limited capacity is $\Omega((\frac{n}{\log n})^{\frac{\alpha-1}{2}})$. Note that the throughput capacity is $O(\frac{1}{\sqrt{n \log n}})$. Therefore, a lower bound of the lifetime of the network is $\Omega(n^{\frac{\alpha}{2}}(\log n)^{1-\frac{\alpha}{2}})$.

For networks with sensor traffic, the per node throughput is on the order of $O(\frac{1}{n})$ due to the traffic aggregation at the sink. This implies that without data fusion or in-network processing, sensor networks are not scalable in data transmission performance. Our results show that the energy-limited capacity can scale with the node density, similar to the case with ad hoc traffic, but at a slower pace.

To overcome the throughput limitations in homogeneous networks, hybrid networks have been proposed and studied in previous work. Liu et al. [27] show that in order to achieve non-negligible throughput capacity gain, the number of BSs deployed should be large enough. Specifically, the authors show that the number of base stations must grow faster than $\sqrt{n}$, whereupon the overall capacity increases linearly with m. This result parallels our result that when $e(r) = c_a r^\alpha$, the number of base stations must grow faster that $\sqrt{n} \log^c n$ to achieve effective improvement in the energy-limited capacity where c is a constant.

In this paper, we also study the capacity of hybrid networks with sensor traffic. In this case, the capacity always benefits from the deployment of base stations. This suggests sensor network lifetime can be effectively improved by installing base stations, even in small numbers.

B. Energy Issues

Network lifetime. As pointed out in [5], the network lifetime depends on the services the network is designed to provide, thus is application-specific. For example, the goal of an ad hoc network is mainly to provide connectivity among nodes; in a sensor network, the requirement for coverage should also be considered. On the other hand, the capability to communicate is a basic requirement in every network. Thus, for general networks, we can consider the energy-limited capacity defined in this paper as a measure of network lifetime. More specifically, by the requirement of uniform transfer volume of all nodes, the energy-limited capacity measures the transport capacity until the first node runs out of energy. In many applications, the loss of a single node would not render the network useless. Thus this measure of network lifetime would be too conservative for these applications.

Idle energy consumption. In this paper, we consider only the transmission energy consumption. In other words, we assume the existence of an optimal scheduling algorithm which has perfect knowledge about the network and does not waste any energy by shutting down nodes whenever they are idle. In practice where such perfect information or perfect control of devices is not available, the capacity will be smaller. To minimize the idle energy consumption, efficient schemes have been developed in [7], [43] which turn off redundant nodes in the network while maintaining network connectivity. In addition, by aggressively putting nodes into sleep mode, we can further reduce idle energy consumption by trading off latency for energy saving [37]. Thus by using these mechanisms, the impact of idle energy consumption on the energy-limited capacity can be minimized.

VIII. Related Work

In this section, we review the related work which is broadly classified into two categories, throughput capacity and energy issues.

A. Throughput Capacity of Wireless Ad Hoc Networks

In a seminal paper, Gupta and Kumar [15] study a model of ad hoc networks with fixed nodes and show that when the number of nodes per unit area n increases, the per node throughput decreases as $O(\frac{1}{\sqrt{n}})$. Grossglauser and Tse [13] show that with loose delay constraints, node mobility can dramatically improve network capacity. They prove that the per node throughput can be kept constant as the number of nodes per unit area increases, although at the price of increased delay. In the work of [2], [32], the authors study the issue of delay and capacity in mobile networks. In recent work [12], the authors study the throughput-delay tradeoff in both stationary and mobile networks.

To address the limitation on throughput capacity in static networks, the work in [22] and [27] studies hybrid networks where base stations connected with a high-bandwidth wired network are deployed to support data forwarding in the wireless network. The work in [27] considers two routing strategies and shows that when the number of base stations grows slower than $\sqrt{n}$, the benefit of adding base stations is minimum, which suggests that the investment in base stations should be high enough to achieve effective improvement. In the work of [22], the authors show that the throughput capacity scales as $\Theta(\frac{1}{\log n})$ when the number of base stations grows linearly with the number of nodes in the network.

In [25], Li et al. conduct an experimental study of throughput capacity and confirm that throughput capacity of networks with non-local traffic pattern does not scale with the network size. The work in [24] evaluates the performance improvement due to the use of multi-channel in ad hoc networks via simulations and presents the scaling properties of per-node throughput. Toumpis and Goldsmith [39] study the capacity region for given network settings. Similarly, the work in [21] and [18] investigates the maximum achievable rates in ad hoc networks.

In recent work [30], Negi and Rajeswaran study the throughput capacity in networks where each node is constrained by a maximum transmit power but can utilize unlimited bandwidth and show that the throughput per node scales as $O((n \log n)^{\frac{\alpha-1}{2}})$, which demonstrates the effects of physical layer properties on network capacity. In addition, the capacity improvement by using directional antennas is studied in [44] and [31].

In [11], Duarte-Melo and Liu study the throughput capacity of networks with sensor traffic and present the conditions under which the trivial upper bound $\frac{1}{n}$ can not be achieved. They also show that the use of clustering can improve the throughput and discuss the tradeoff between capacity and energy consumption.

B. Energy Issues

In wireless environments, nodes are often equipped with limited energy. Thus energy management is important and receives much attention in the research community.

Singh et al. [38] propose power-aware routing and describe a number of routing metrics used in the routing algorithm. Chang and Tassiulas [6] consider the problem of maximizing the network lifetime when the traffic demand is known and propose algorithms to select the routes and the corresponding transmission ranges. They point out that traffic load should be balanced among the nodes in order to maximize the lifetime. In [26], Li et al. study online power-aware routing where data arrival is not known and develop an approximation routing algorithm. The work in [20] presents an improved algorithm for online power-aware routing which achieves a better approximation factor and requires less overhead.

To achieve optimal performance, it is important to reduce idle energy consumption which is shown to dominate the overall energy consumption if nodes are idle most of the time. Kravets and Krishnan [23] propose to shut down the communication device when idle to reduce power consumption. The work in [7] and [43] considers sensor networks and exploits node redundancy to minimize idle energy while maintaining network connectivity. In [5], Blough and Santi investigate the upper bound of network lifetime extension due to the cell-based energy conservation schemes. The work in [37] proposes a scheme to further reduce idle energy consumption by aggressively putting nodes into sleep mode, thus trading off latency for energy saving.

Recently there has been significant research on the energy issues of sensor networks [17], [1], [34]. In [16], Heinzelman et al. study energy-efficient communication protocol in sensor networks and propose a clustering-based protocol that utilizes randomized rotation of cluster-heads to evenly distribute the energy load among sensors. In [3], Bhardwaj and Chandrakasan derive upper bounds on the lifetime of sensor networks that transmit data from a point, a line or an area source. In [4], they further formulate the network lifetime problem as a network flow problem via optimal role assignment. In [9], Duarte-Melo and Liu analyze the energy consumption in networks with sensor traffic. They consider and compare flat and clustering network structures. In [10], they examine the energy consumption and lifetime of hybrid sensor networks based on a clustering mechanism and study the optimal number of clusters used. While the above work considers energy consumption and network lifetime in sensor network, our work is significantly different. We focus on the scaling properties of the energy-limited capacity, which is important in large scale networks, and formally derive upper and lower bounds on the capacity. In addition we analyze the capacity in both homogeneous and hybrid networks, with either ad hoc or sensor traffic.

In [29], Marco et al. study the transport capacity of sensor networks subject to a constraint on the quality of the reconstructed data. They show that as the node density approaches infinity, no data compression scheme is sufficient to transport enough data to achieve a given quality.

There is also some research on *topology control* in wireless multi-hop networks [36], [28], [35], [42]. This body of work focuses on adjusting the transmission power of nodes in a multihop wireless network in order to create a topology with desired properties, e.g., maintaining network connectivity.

In [15], Gupta and Kumar consider the minimum power in random networks that maintains network connectivity and show that nodes are connected with high probability when the radio range $r(n)$ satisfies $r(n) = \sqrt{\frac{\log n + \kappa_n}{\pi n}}$, where n is the number of nodes and $\kappa_n \to +\infty$.

IX. Conclusion

In this paper, we studied the energy-limited capacity in wireless networks. We derived upper and lower bounds for homogeneous and hybrid networks, under both ad hoc and sensor traffic models. We showed that the energy-limited capacity can scale well in all these environments. This implies that large scale wireless networks can be scalable in terms of energy consumption, despite the decrease of throughput capacity. Table I summarizes the results for networks with energy cost function $e(r) = c_a r^\alpha$. The scaling behavior of networks with energy cost function $e(r) = c_a(c_b + r^\alpha)$ is similar, but within a finite region. We found that reducing c_b can both improve the capacity and increase the scaling region. Thus reducing the constant energy consumption is key to desirable scaling. In addition, the capacity of networks supporting sensor traffic is less than the capacity of networks supporting ad hoc traffic, by a factor of $O(\sqrt{n} \log^c n)$ for some constant c. This is because of the effects of traffic concentration at the sink.

We also analyzed the capacity gain due to the deployment of base stations in hybrid networks. For ad hoc traffic, we showed that the capacity gain is bimodal. Specifically, when $e(r) = c_a r^\alpha$, to achieve non-negligible capacity gain, the number of base stations needs to grow faster than $\sqrt{n} \log^k n$. So a minimum investment in base stations is required to improve capacity in this case. This parallels the result in throughput

α	Capacity Bound	Homogeneous network with ad hoc traffic	Hybrid network with ad hoc traffic	Homogeneous network with sensor traffic	Hybrid network with sensor traffic
$\alpha = 2$	Upper	$O(\sqrt{n \log n})$	$O(\sqrt{n \log n})$ if $m = o(\sqrt{\frac{n}{\log n}})$ $O(m \log n)$ if $m = \omega(\sqrt{\frac{n}{\log n}})$	$O(\log n)$	$O(m \log n)$
	Lower	$\Omega(\sqrt{\frac{n}{\log n}})$	$\Omega(\sqrt{\frac{n}{\log n}})$ if $m = o(\sqrt{\frac{n}{\log n}})$ $\Omega(m)$ if $m = \omega(\sqrt{\frac{n}{\log n}})$	$\Omega(1)$	$\Omega(m)$
$\alpha > 2$	Upper	$O((n \log n)^{\frac{\alpha-1}{2}})$	$O((n \log n)^{\frac{\alpha-1}{2}})$ if $m = o(\sqrt{n}(\log n)^{\frac{\alpha-1}{2}})$ $O(mn^{\frac{\alpha}{2}-1}\phi_n^{\alpha-2})$ if $m = \omega(\sqrt{n}(\log n)^{\frac{\alpha-1}{2}})$	$O(\phi_n^{\alpha-2} n^{\frac{\alpha}{2}-1})$	$O(mn^{\frac{\alpha}{2}-1}\phi_n^{\alpha-2})$
	Lower	$\Omega((\frac{n}{\log n})^{\frac{\alpha-1}{2}})$	$\Omega((\frac{n}{\log n})^{\frac{\alpha-1}{2}})$ if $m = o(\sqrt{\frac{n}{\log n}})$ $\Omega(m(\frac{n}{\log n})^{\frac{\alpha}{2}-1})$ if $m = \omega(\sqrt{\frac{n}{\log n}})$	$\Omega((\frac{n}{\log n})^{\frac{\alpha}{2}-1})$	$\Omega(m(\frac{n}{\log n})^{\frac{\alpha}{2}-1})$

TABLE I

ENERGY-LIMITED CAPACITY OF NETWORKS WITH $e(r) = c_a r^{\alpha}$. ϕ_n IS A SEQUENCE THAT SATISFIES $\lim_{n\to\infty} \phi_n \to \infty$.

capacity [27]. For sensor traffic, however, the deployment of base stations has immediate improvement on the network capacity. This suggests that hybrid deployment would be a promising approach to extend the lifetime of sensor networks.

In this paper, we obtained lower bounds that are a factor of $O(\log^k n)$ from the corresponding upper bounds. This indicates effective scaling of the capacity is achievable. In the future, we would like to extend this study to consider other kinds of energy consumption in the network. For example, it would be interesting to investigate the impact of idle energy consumption and energy conservation schemes on the network capacity. In addition, we would like to design decentralized routing algorithms to achieve the energy-limited capacity.

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APPENDIX

We first describe Vapnik-Chervonenkis theorem [41], [40] which will be used in the proof of Lemma 3.4 and 5.6.

Let $\mathcal{F}$ be a set of subsets. A finite set of points A is said to *shattered* by $\mathcal{F}$ if for every subset B of A there is a set $F \in \mathcal{F}$ such that $A \cap F = B$. The *VC-dimension* of $\mathcal{F}$, denoted as $VC - d(\mathcal{F})$, is defined as the supremum of the sizes of all finite sets that can be shattered by $\mathcal{F}$. For sets of finite VC-dimension, one has uniform convergence in the weak law of large numbers as stated in the following theorem.

Theorem 1.1 (Vapnik-Chervonenkis Theorem): If $\mathcal{F}$ is a set of finite VC-dimension $VC - d(\mathcal{F})$, and $\{X_i\}$ is a sequence of i.i.d. random variables with common probability distribution P, then for every ϵ, $\delta > 0$,

$$\text{Prob}\left(\sup_{F\in\mathcal{F}}\left|\frac{1}{N}\sum_{i=1}^{N} I(X_i \in F) - P(F)\right| \leq \epsilon\right) > 1 - \delta$$

whenever

$$N > \max\left\{\frac{VC - d(\mathcal{F})}{\epsilon}\log\frac{16e}{\epsilon}, \frac{4}{\epsilon}\log\frac{2}{\delta}\right\}.$$

Proof of Lemma 3.4 - Let $\mathcal{F}$ be the set of disks with radius $w_n/2$, or of area $\pi w_n^2/4$. As shown in [15], the VC-dimension of the set of disks on the plane is 3. Thus, by the Vapnik-Chervonenkis Theorem, we have

$$\text{Prob}\left(\sup_{B\in\mathcal{F}}\left|\frac{\text{number of nodes in B}}{n} - \frac{\pi w_n^2}{4}\right| \leq \epsilon(n)\right) > 1 - \delta(n)$$

whenever $n > \max\left\{\frac{3}{\epsilon(n)}\log\frac{16e}{\epsilon(n)}, \frac{4}{\epsilon(n)}\log\frac{2}{\delta(n)}\right\}$. This condition is satisfied when $\epsilon(n) = \delta(n) = \frac{50\log n}{n}$.

Note that $r_n \geq 160\sqrt{\frac{\log n}{\pi n}}$. We have $w_n = \frac{r_n}{8} \geq 20\sqrt{\frac{\log n}{\pi n}}$. Let H be the number of nodes in any disk in $\mathcal{F}$. With probability at least $1 - \delta(n)$, we have

$$\frac{\pi w_n^2}{4} - \epsilon(n) \leq \frac{H}{n} \leq \frac{\pi w_n^2}{4} - \epsilon(n),$$

or $H = \Theta(n w_n^2)$.

Similarly, the number of nodes in any disk with radius $2w_n$ is also $\Theta(n w_n^2)$ with high probability.

In the spatial tessellation described in Section III-B, it can be shown that each cell contains a disk with radius $w_n/2$ and is contained within a disk with radius $2w_n$. So every cell contains $\Theta(n w_n^2)$ nodes with high probability. This completes the proof. ■

Proof of Lemma 5.6 - Recall that data is forwarded from a node to its destination along Voronoi cells intersected by the line segment connecting the source and its destination. To bound the amount of traffic handled by a cell, we first provide an upper bound on the number of line segments intersecting a cell.

Let L_i be the line segment connecting node X_i and its randomly chosen location Y_i. Denote $\{L_i\}_{i=1}^n$ as the set of all n line segments. We now prove that for any cell $V \in \mathcal{V}_n$,

$$E[\text{Number of lines in } \{L_i\}_{i=1}^n \text{ intersecting } V] \leq c_{24} n r_n$$

for some constant c_{24}.

Consider any node X_i. According to the properties of $\mathcal{V}_n$, V is contained in a disk of radius $2\rho_n$. So the angle θ subtended at X_i by this disk is no more than $\frac{c_{25}\rho_n}{x}$ where x is the distance of X_i from the disk. For the line L_i to intersect V, Y_i must lie in this sector. The area of the section is no more than $\frac{c_{26}\rho_n}{x}$. By the uniform distribution of Y_i, the probability that L_i intersects V is no more than $\frac{c_{26}\rho_n}{x}$. Since the probability density of x is upper bounded by $c_{27}\pi(x + 2\rho_n)$, we have

$$\begin{aligned} & \text{Prob}\{\text{Line } L_i \text{ intersects } V\} \\ \leq\; & \int_{2\rho_n}^{\frac{1}{\sqrt{\pi}}} \frac{c_{26}\rho_n}{x} \cdot c_{27}\pi(x + 2\rho_n)\, dx \\ \leq\; & c_{28}\rho_n. \end{aligned}$$

Since $\rho_n = r_n/8$, with a total of n lines, the average number of lines in $\{L_i\}_{i=1}^n$ that intersect a cell in $\mathcal{V}_n$ is no more than $c_{24} n r_n$.

In [15], the authors show that the random sequence of line segments $\{L_i\}_{i=1}^n$ is i.i.d. and prove that the Vapnik-Chervonenkis Theorem holds when counting the number of lines intersecting with a cell. So there is a $\delta(n) \to 0$ such that

$$\text{Prob}\left(\sup_{V\in\mathcal{V}_n}(\text{Number of lines } L_i \text{ intersecting } V) \leq c_{29} n r_n\right) > 1 - \delta(n).$$

Because the amount of traffic handled by a cell is proportional to the number of lines intersecting the cell, for a transfer volume of $\lambda(n)$, the amount of traffic every cell is responsible for forwarding is $O(n r_n \lambda(n))$ with high probability. This completes the proof. ■