Preview of Award 0901304 - Final Project Report

Cover

Federal Agency and Organization Element to Which Report is Submitted:	4900
Federal Grant or Other Identifying Number Assigned by Agency:	0901304
Project Title:	Collaborative Research: Variational Problems and Dynamics
PD/PI Name:	Michael Loss, Principal Investigator
Submitting Official (if other than PD\PI):	N/A
Submission Date:	N/A
Recipient Organization:	Georgia Tech Research Corporation
Project/Grant Period:	06/01/2009 - 05/31/2013
Reporting Period:	06/01/2012 - 05/31/2013
Signature of Submitting Official (signature shall be submitted in accordance with agency specific instructions)	N/A

Accomplishments

*** What are the major goals of the project?**

The work performed under this grant can be divided into three areas, kinetic theory, random Schrodinger operators and sharp functional inequalities. Most of the work on kinetic theory and some concerning functional inequalities was done in collaboration with Eric Carlen. The scientific objective of the grant was three fold. One goal was to finish a program initiated by Mark Kac concerning approach to equilibrium for a model of colliding hard spheres. The second goal was to push ahead on the problem of proving localization for the random displacement model a problem that has been open for 20 years. The third part consisted in establishing some classical Sobolev type inequalities for fractional derivatives.

*** What was accomplished under these goals (you must provide information for at least one of the 4 categories below)?**

Major Activities:

Specific Objectives:

Significant Results:

1. Kinetic theory. The Kac master equation is an excellent model for understanding certain aspects of non-equilibrium statistical mechanics, in particular the notion of approach to equilibrium. Two problems in this area were addressed in this grant.
2. Random displacement model. Another main outcome and in some sense the highlight among the results of this proposal is the PI's work on the random displacement model. As described in the proposal, this model describes an electron that interacts with an array of potentials that are randomly displaced from a regular lattice configuration. This way to model structural disorder goes back to the late 80-ies. The ultimate goal is to prove that at energies near the bottom of the deterministic spectrum, the electron states are localized, i.e., the transition from a localized state to a location at a large distance away is small uniformly in time. Such a result has been proved for the Anderson model in the 70-ies and around 2000 for the poisson model, but for the Random Displacement model this has been open except in one dimension. The problem is hard mainly because one of the main estimates, the Wegner estimate depends crucially on the fact the eigenvalues move if the parameters of the potential change. This is relatively straightforward for the Anderson

of the potential change. This is relatively straightforward for the Anderson model with sign definite potentials, but to prove this for the Random Displacement model is a major obstacle. It is therefore gratifying that jointly with Klopp, Nakamura and Stolz, the PI managed to prove localization in the strong form in any dimension. This is the culmination of a five year long investigation.

3. Sharp functional inequalities. Technical details given in attached file.

Key outcomes or Other achievements:

*** What opportunities for training and professional development has the project provided?**

The PI supported two graduate students on this grant. Craig Sloane and Amit Einav. Sloane received his PhD in Summer 2011 and Einav received his PhD in December 2011. Sloane obtained a position at Atlanta Metropolitan College and Einav took a post doctoral position at the University of Cambridge. With this grant the PI also supported Ranjini Vaydayanathan another graduate student working in kinetic theory.

*** How have the results been disseminated to communities of interest?**

Under this grant 22 papers were written of which 18 have appeared and 4 have been submitted. All the papers can be found on the LANL archive. The PI gave about 25 invited talks about the findings in Chile, France, Germany, Italy, Japan, the United Kingdom and the US. The PI also wrote a number of review papers on kinetic theory and the random displacement model.

Supporting Files

Filename	Description	Uploaded By	Uploaded On
DMS901304.pdf	Technical data describing research supported by this award and listed in the Significant Results section.	Michael Loss	06/10/2013

Products

Journals

Eric Carlen, Maria C. Carvalho, Michael Loss (1/1/14). Spectral gap for the Kac master equation with hard collisions. *Journal of Functional Analysis*. TBD TBD.

Status = SUBMITTED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Federico Bonetto, Michael Loss (1/1/14). Entropic Chaoticity for the Steady State of a Current Carrying System. *Journal of Mathematical Physics*. TBD TBD.

Status = SUBMITTED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Jean Dolbeault, Maria J. Esteban, Michael Loss (5/14/12). Symmetry of extremals of functional inequalities via spectral estimates for linear operators. *J. Math. Phys.*. 53 on-line.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Maria J. Esteban, Jean Dolbeault, Michael Loss (1/1/13). Sharp interpolation inequalities on the sphere: new methods and consequences. *Chinese Annals of Mathematics*. B34 99-112.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Jean Dolbeault, Maria Esteban, Michael Loss (1/1/14). Nonlinear flows and rigidity results on compact manifolds. *Journal of Functional Analysis*. TBD TBD.

Status = SUBMITTED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Jean Dolbeault, Maria Esteban, Ari Laptev, Michael Loss (1/1/14). Spectral properties of Schr\"odinger operators on compact manifolds; rigidity, flows, interpolation and spectral estimates. *Comte Rendue*. TBD TBD.

Status = SUBMITTED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Michael Loss (4/15/11). On Villani's conjecture concerning entropy production for the Kac master equation. *Kinet. Relat. Models*. 4 479-497.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Frederic Klopp, Michael Loss, Shu Nakamura, Gunter Stolz (3/15/12). Localization for the random displacement model. *Duke Math. J.*. 161 587-621.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Rafael Benguria, Marie-Cristina Depassier, Michael Loss (12/17/12). Rigorous results for the speed of Kolmogorov--Petrovskii--Piscounov fronts with a cutoff. *J. Math. Phys.*. 53 on-line.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Rafael Benguria, Gonzalo Bley, Michael Loss (3/15/12). A new estimate on the indirect term. *The International Journal of Quantum Chemistry*. 112 (6), 1579-1584.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

A. Einav, Michael Loss (4/5/12). `Sharp trace inequalities for fractional Laplacians. *Proc. Amer. Math. Soc.*. 140 (12), 4209-4216.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Rupert Frank, Michael Loss (1/31/12). Hardy-Sobolev-Maz'ya inequalities for arbitrary domains. *Journal de Mathematiques pures et appliquees*. 97 39-54.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Craig A. Sloane, Michael Loss (3/21/11). A fractional Hardy-Sobolev-Maz'ya inequality on the upper half-space. *Proc. Amer. Math. Soc.*. 139 (11), 4003-4016.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes

Eric A. Carlen, Jeffrey S. Geronimo, Michael Loss (3/1/11). On the Markov sequence problem for Jacobi polynomials. *Advances in Mathematics*. 226 (4), 3426-3466.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Michael Loss, Craig Sloane (9/15/10). Hardy inequalities for fractional integrals on general domains. *J. Funct. Anal.*. 259 1369-1379.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

E. A. Carlen, M. C. Carvalho, J. Le Roux, M. Loss, C. Villani (1/15/10). Entropy and chaos in the Kac model. *Kinetic and Related Models*. 3 85-122.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Eric Carlen, Jose A. Carrillo, Michael Loss (11/19/10). Hardy-Littlewood-Sobolev Inequalities via Fast Diffusion Flows. *Proceedings of the National Academy of Sciences*. 46 19696.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Jean Dolbeault, Maria J. Esteban, Michael Loss, Gabriella Tarantello (11/16/09). On the symmetry of extremals for the Caffarelli-Kohn-Nirenberg inequalities. *Advanced Nonlinear Studies*. 9 713-726.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Michael Loss, Tadahiro Miyao, Herbert Spohn (7/15/09). Kramers degeneracy theorem in nonrelativistic QED. *Letters in Mathematical Physics*. 89 21-31.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

Books

Book Chapters

F. Klopp, M. Loss, S. Nakamura, G. Stolz (2012). Understanding the Random Displacement Model: From Ground State Properties to Localization. *Operator Theory: Advances and Applications* 224. R. D. Benguria, E. Friedman, M. Mantoiu. Basel, Birkhauser. 183-220.

Status = PUBLISHED; Acknowledgement of Federal Support = Yes

Michael Loss, Gunter Stolz (2011). Localization for the random displacement model. *Mathematical results in Quantum Physics* Pavel Exner. World Scientific. 169-175.

Status = PUBLISHED; Acknowledgement of Federal Support = Yes

E. Carlen, M. C. Carvalho, M. Loss (2011). Kinetic theory and the Kac Master equation. *Entropy and the Quantum II* 552. American Mathematical Society. Providence, RI USA. 1-20.

Status = PUBLISHED; Acknowledgement of Federal Support = Yes

Thesis/Dissertations

Conference Papers and Presentations

Other Publications

Technologies or Techniques

Nothing to report.

Patents

Nothing to report.

Inventions

Nothing to report.

Licenses

Nothing to report.

Websites

Title: Dissemination Website

URL: http://people.math.gatech.edu/~loss/Papers.html

Description:

The papers that are listed in this report are posted at the URL and all the papers acknowledge NSF support. In addition, many of the papers are posted on other preprint servers, such as the ArXiv, further enhancing the dissemination of research supported by this grant.

Other Products

Nothing to report.

Participants

Research Experience for Undergraduates (REU) funding

What individuals have worked on the project?

Name	Most Senior Project Role	Nearest Person Month Worked
Michael Loss	PD/PI	2

What other organizations have been involved as partners?

Nothing to report.

Have other collaborators or contacts been involved? N

Impacts

What is the impact on the development of the principal discipline(s) of the project?

Nothing to report.

What is the impact on other disciplines?

Nothing to report.

What is the impact on the development of human resources?

Nothing to report.

What is the impact on physical resources that form infrastructure?

Nothing to report.

What is the impact on institutional resources that form infrastructure?

Nothing to report.

What is the impact on information resources that form infrastructure?

Nothing to report.

What is the impact on technology transfer?

Nothing to report.

What is the impact on society beyond science and technology?

Nothing to report.

Changes

Changes in approach and reason for change

Nothing to report.

Actual or Anticipated problems or delays and actions or plans to resolve them

Nothing to report.

Changes that have a significant impact on expenditures

Nothing to report.

Significant changes in use or care of human subjects
Nothing to report.

Significant changes in use or care of vertebrate animals
Nothing to report.

Significant changes in use or care of biohazards
Nothing to report.

Final Report NSF Grant DMS 901304
Collaborative Research: Variational Problems and Dynamics
Michael Loss PI

1. Significant results

1.1. **Kinetic theory.** The Kac master equation is an excellent model for understanding certain aspects of non-equilibrium statistical mechanics, in particular the notion of approach to equilibrium. Two problems in this area were addressed in this grant.

1.1.1. *Bound on the gap.* In [1] a bound, uniform in the number of particles, was given for the gap of a model of hard-sphere collisions. More precisely, on $L^2(\mathbb{S}^{N-1}(\sqrt{N}))$ consider the quadratic form

$$\mathcal{E}(f,f) = \frac{1}{\binom{N}{2}} \sum_{i<j} \int_{\mathbb{S}^{N-1}(\sqrt{N})} (v_i^2 + v_j^2)^\gamma (f - f^{i,j})^2 d\sigma \ . \tag{1}$$

Here $0 \le \gamma \le 1$, σ is the uniform probability measure on the sphere $S^{N-1}(\sqrt{N})$, and

$$f^{i,j}(\vec{v}) := \frac{1}{2\pi} \int_{-\pi}^{\pi} f(R_{i,j,\theta}\vec{v}) d\theta \ , \tag{2}$$

where $R_{i,j,\theta}$ is the rotation in the i,j plane by an angle θ. Define the gap Δ_N by

$$\Delta_N = \inf\{\mathcal{E}(f,f) : f \perp 1 \ , \|f\|_2 = 1\} \ . \tag{3}$$

It is proved in [1] that

$$\Delta_N \ge c > 0 \tag{4}$$

where c is a constant independent of N, the number of particles. Previous results on the gap were only available for Maxwellian Molecules for which the scattering rate is independent on the energy of the particles. Solving the problem for the model with hard sphere collisions has been much more difficult because the scattering of particles at low energy is suppressed and the high energy particles have, so to speak, make up for it. A large part of the work has gone into computing reasonable bounds for the gap, and we were able to give a abound that is about one order of magnitude smaller than the gap for Maxwellian molecules. The questions concerning the gap were asked by Kac in 1956 and our result can be considered as the capstone for this program.

1.1.2. *Entropy production.* It is well known that the gap does not give full insight for approach to equilibrium. It gives useful results for states that are close to equilibrium, in a regime where the dynamics for the marginals of the states can be described by the linearized Boltzmann equation. In general, approach to equilibrium has to be understood in the sense of entropy. Define the entropy production by

$$\Gamma_N = \inf \frac{\langle \log(\psi_N) \, , \mathcal{L}\psi_N \rangle}{H_N(\psi_N)}$$

where the infimum is taken over all probability densities ψ_N on $\mathbb{S}^{N-1}(\sqrt{N})$ which are symmetric in all their components and $\mathcal{L}$ is the generator of the Kac semigroup. Villani proved that $\Gamma_N \ge \frac{2}{N}$ and conjectured that this was the right behavior. Amit Einav, a student of the PI, proved in [7] that there is an upper bound of the same type. More precisely he showed the following theorem:

Theorem 1.1. *For any $\beta > 0$ there exists a constant $C_\beta > 0$ depending only on β such that*

$$\Gamma_N \leq \frac{C_\beta \log N}{N^{1-\beta}} \tag{5}$$

The proof of this theorem is via a trial state and it is a tour the force in asymptotic expansions. The significance of this result is that there exist states whose entropy production is very small.

1.2. **Random displacement model.** Another main outcome and in some sense the highlight among the results of this proposal is the PI's work on the random displacement model. As described in the proposal, this model describes an electron that interacts with an array of potentials that are randomly displaced from a regular lattice configuration. This way to model structural disorder goes back to the late 80-ies. The ultimate goal is to prove that at energies near the bottom of the deterministic spectrum, the electron states are localized, i.e., the transition from a localized state to a location at a large distance away is small uniformly in time. Such a result has been proved for the Anderson model in the 70-ies and around 2000 for the poisson model, but for the Random Displacement model this has been open except in one dimension. The problem is hard mainly because one of the main estimates, the Wegner estimate depends crucially on the fact the eigenvalues move if the parameters of the potential change. This is relatively straightforward for the Anderson model with sign definite potentials, but to prove this for the Random Displacement model is a major obstacle. It is therefore gratifying that jointly with Klopp, Nakamura and Stolz, the PI managed to prove localization in the strong form in any dimension [8]. This is the culmination of a five year long investigation.

The solution of this problem hinges to a large extent on a quantitative estimate of the ground state energy as a function of the position of the potential. From this one deduces then a Lifshitz tail estimate and a Wegner estimate, the key ingredients for a multiscale analysis. The PI jointly with Klopp, Nakamura and Stolz wrote two review articles, a short one [13] explaining the problem in simple terms, and a longer one [12] in an attempt to present all the ingredients in a pedagogical way.

1.3. **Sharp functional inequalities.**

1.3.1. *Symmetry in the Caffarelli-Kohn-Nirenberg inequalities.* Symmetry breaking or the absence thereof is an important issue in many applications. It is very often easy to decide whether certain optimizers are stable but it is much harder to decide whether this local stability translates into global stability.

It is very often the case that symmetric variational problems have symmetric minimizers. There are a number of standard techniques for proving this, such as rearrangement inequalities, the moving plane method and convexity. A number of problems, however, are not amenable to these techniques. The sharp constant in the Caffarelli-Kohn-Nirenberg inequalities has been a long standing problem in the Calculus of Variations with contributions for a number of researchers.

$$\left(\int_{\mathbb{R}^N} \frac{|w|^p}{|x|^{b\,p}}\,dx\right)^{2/p} \leq C_{a,b}^N \int_{\mathbb{R}^N} \frac{|\nabla w|^2}{|x|^{2\,a}}\,dx$$

for w in a suitable function space and for

$$p = p(a,b) := \frac{2\,N}{N-2+2\,(b-a)}\ .$$

After previous efforts [21] the PI jointly, with Maria Esteban and Jean Dolbeault, established the radial symmetry of optimizers in a large range for for the parameters a and b in [3]. No results were previously available for this situation because standard techniques like rearrangement inequalities cannot be applied. The authors prove a rigidity result for the Euler-Lagrange equations for this functional, thereby establishing the symmetry of the optimizers.

1.3.2. *Proof of sharp inequalities by means of flows.* In joint work with Carlen, Carillo the PI developped a novel proof for certain cases of the sharp Hardy-Littlewood-Sobolev inequality orginally due to Lieb. One can use a fast diffusion flow to show on the relation of fast diffusion flows and the Hardy-Littlewood-Sobolev inequality [20]. In joint work with Dolbeault and Esteban, Loss [5] has been able to a porous medium equations on d-dimensional manifolds that can be used to prove rigidity results for the equation

$$-\Delta u + \frac{\lambda}{p-2} u = u^{p-1}$$

where

$$2 < p \le \frac{2d}{d-2} \ .$$

Some of these cases have been treated by number of researchers, like Bidaut-Véron and Véron, Licois and Véron, Bakry and Ledoux using fairly involved methods. The idea is to use a porous medium flow to drive any function to the constant function while certain nonlinear functionals decrease. In this fashion one obtains uniqueness results of certain non-linear partial differential equations in addition also sharp inequalities on manifolds. A similar idea was used previously by Demange but our proofs are considerably more transparent when compared with the previous ones. In [6] these results were used to get spectral estimates for Schrödinger operators on compact manifolds.

2. Additional outcomes

2.1. **Sharp inequalities for fractional derivatives.**

2.1.1. *Sharp Hardy inequalities for fractional derivatives.* In [18] the PI jointly with Sloane graduate student proved a conjecture of Bogdan and Dyda. They proved that for any smooth function of compact support in C

$$\frac{1}{2} \int_{C \times C} \frac{|f(x) - f(y)|^2}{|x-y|^{n+\alpha}} dx dy \ge \kappa_{n,\alpha} \int_C \frac{|f(x)|^2}{d_C(x)^\alpha} dx \ . \tag{6}$$

Here $1 < \alpha < 2$,

$$\kappa_{n,\alpha} = \pi^{\frac{n-1}{2}} \frac{\Gamma(\frac{1+\alpha}{2})}{\Gamma(\frac{n+\alpha}{2})} \frac{1}{\alpha} \left[\frac{2^{1-\alpha}}{\sqrt{\pi}} \Gamma(\frac{2-\alpha}{2}) \Gamma(\frac{1+\alpha}{2}) - 1 \right] \ , \tag{7}$$

C is an arbitrary convex domain and $d_C(x)$ is the distance from the point $x \in C$ to the boundary of C. In fact stronger inequalities hold, for details see [18].

2.1.2. *Sharp Hardy-Sobolev-Maz'ya inequalities for fractional derivatives.* Craig Sloane, under the PI's supervision,derived Hardy-Sobolev-Mazya type inequalities for fractional integrals. In [16] he proved the following theorem for functions supported in the upper half space $\mathbb{H}^n$

$$\frac{1}{2}\int_{\mathbb{H}^n\times\mathbb{H}^n}\frac{|f(x)-f(y)|^2}{|x-y|^{n+\alpha}}dxdy-\kappa_{n,\alpha}\int_{\mathbb{H}^n}\frac{|f(x)|^2}{x_n^\alpha}dx\geq C_{n,\alpha}\|f\|_q\ , \tag{8}$$

where $q=\frac{2n}{n-\alpha}$ and $\kappa_{n,\alpha}$ is the sharp Hardy constant.

2.1.3. *Sharp trace inequalities.* Jointly with his student Amit Einav, the PI managed proved sharp trace inequalities for fractional operators. These include higher derivatives and traces on hyperplanes of smaller dimensions. In particular they proved the following theorem.

Theorem 2.1. *Let* $1\leq j<n$ *and* $\frac{j}{2}<\alpha<\frac{n}{2}$. *For any* $f\in S(\mathbb{R}^n)$ *we define* $\tau_j f(x')=f(x',0)$ *where* $x'\in\mathbb{R}^{n-j}$. *Then*

$$\|\tau_j f\|^2_{L^{\frac{2(n-j)}{n-2\alpha}}}\leq C_{j,\alpha,n}\left(f,(-\Delta)^\alpha f\right) \tag{9}$$

where

$$C_{j,a,n}=\frac{1}{2^{2\alpha}\pi^\alpha}\frac{\Gamma\left(\frac{n-2\alpha}{2}\right)\Gamma\left(\frac{2\alpha-j}{2}\right)}{\Gamma(\alpha)\Gamma\left(\frac{n+2\alpha-2j}{2}\right)}\left\{\frac{\Gamma(n-j)}{\Gamma\left(\frac{n-j}{2}\right)}\right\}^{\frac{2\alpha-j}{n-j}}$$

The work appeared in [11].

2.1.4. *Hardy-Sobolev-Maz'ya inequalities on general domains.* Related to this circle of ideas is a result of Rupert Frank and the PI [15]. This concerns an extension of the Hardy-Sobolev-Mazy'a inequality to general domains. let Ω be any domain. One among many results is the following theorem.

Theorem 2.2. *Let* $N\geq 3$. *There is a constant* $K_N>0$ *such that for any domain* $\Omega\subsetneq\mathbb{R}^N$ *and any* $u\in C_0^\infty(\Omega)$

$$\int_\Omega\left(|\nabla u|^2-\frac{|u|^2}{4D_\Omega^2}\right)dx\geq K_N\left(\int_\Omega|u|^{\frac{2N}{N-2}}\,dx\right)^{\frac{N-2}{N}}\ . \tag{10}$$

Here

$$D_\Omega(x):=\left(N|\mathbb{S}^{N-1}|^{-1}\int_{\mathbb{S}^{N-1}}d_e(x)^{-2}\,de\right)^{-\frac{1}{2}},$$

where $d_e(x):=\inf\{|t|:\ x+te\in\Omega^c\}$ for $e\in\mathbb{S}^{N-1}$ The constant K_N does not depend on the domain.

2.2. **A new bound ont the indirect Coulomb term.** In the atomic Hamiltonian, the difficult many-body term is the Coulomb repulsion which involves two particle correlations. It is a standard approach in density functional theory to try to replace this term by the direct term with an estimate on the indirect term. In a famous paper, Lieb and Oxford were able to bound the indirect by $1.68\int\rho(x)^{4/3}dx$ where $\rho(x)$ is the one particle density. The constant was later improve to 1.63 by Chan and Handy using a computer approach. It has been speculated in the chemistry literature that the estimate is better when the density $\rho(x)$ is nearly constant. In joint work with Rafael Benguria, the PI has succeeded in doing that [10]. More precisely the indirect terms is bounded by $1.45\int\rho(x)^{4/3}dx+C(\sqrt{\rho},\sqrt{-\Delta}\sqrt{\rho})$.

2.3. **Reaction diffusion equations.** Jointly with Depassier and Benguria the PI has proved estimates on the speed of travelling waves of reaction diffusion equations [9]. Rigorous and rather precise results in terms of upper and lower bounds on the speed for reaction equations with cutoffs are obtained, thereby putting error bars on some of the asymptotic calculations of the previous authors. The key tool is a variational formulation of this problem originally invented by Benguria and Depassier. The PI realized that this variational problem can be used to establish the existence of traveling waves under very general assumptions on the non-linearity. In particular these results hold for non-linearities with cutoff.

References

[1] " Spectral gap for the Kac master equation with hard collisions" (with Eric Carlen and Maria C. Carvalho), submitted to Journal of Functional Analysis. http://lanl.arxiv.org/abs/1304.5124

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[5] Nonlinear flows and rigidity results on compact manifolds (with Jean Dolbeault and Maria Esteban) submitted to Journal of functional analysis.

[6] Spectral properties of Schrödinger operators on compact manifolds; rigidity, flows, interpolation and spectral estimates (with Jean Dolbeault, Maria Esteban and Ari Laptev) submitted to Comte Rendue.

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[9] "Rigorous results for the speed of Kolmogorov–Petrovskii–Piscounov fronts with a cutoff" (with Rafael Benguria and Marie-Cristina Depassier). J. Math. Phys. 53, 123705 (2012). **Online Publication Date: 17 December 2012**

[10] "A new estimate on the indirect term", (with Rafael Benguria and Gonzalo Bley), The International Journal of Quantum Chemistry "The International Journal of Quantum Chemistry", Volume 112, Issue 6, **Publication date: 15 March 2012**, Pages: 1579-1584, http://lanl.arxiv.org/abs/1103.2162

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[14] "Kinetic theory and the Kac Master equation" (with E. Carlen and M.C. Carvalho), "Entropy and the Quantum II", Contemporary Mathematics **552**, 2011, pp 1-20. **Publication date: December 4, 2011** http://arxiv.org/abs/1107.1856

[15] "Hardy-Sobolev-Maz'ya inequalities for arbitrary domains" (with Rupert Frank) "Journal de Mathématiques pures et appliquées", **97**, 39-54, 2012. **Publication date: January 31, 2012** http://arxiv.org/abs/1102.4394

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