

**Final Report for Period:** 09/2007 - 08/2008**Submitted on:** 12/01/2008**Principal Investigator:** Geronimo, Jeffrey S.**Award ID:** 0500641**Organization:** GA Tech Res Corp - GIT**Submitted By:**

Geronimo, Jeffrey - Principal Investigator

**Title:**

Collaborative Research: Multivariable Moments and Factorizations and Other Problems in Analysis

**Project Participants****Senior Personnel****Name:** Geronimo, Jeffrey**Worked for more than 160 Hours:** Yes**Contribution to Project:****Post-doc****Graduate Student****Undergraduate Student****Technician, Programmer****Other Participant****Research Experience for Undergraduates****Organizational Partners****Other Collaborators or Contacts**

Alexander Aptekarev, Moscow State University

Jorge Arvesu, University of Madrid Carlos III

Sergiy Borodachov, Towson University

Eric Carlen, Georgia Institute of Technology

Antonia Delgado, University of Almeria

Plamen Iliev, Georgia Institute of Technology

Ming-Jung Lai, University of Georgia

Michael Loss, Georgia Institute of Technology

Francisco Marcellan, University of Madrid Carlos III

Andrei Martinez-Finkelshtein, University of Almeria

Michael Loss, Georgia Institute of Technology

Walter Van Assche, University of Leuven

Hugo Woerdeman, Drexel University

Yuan Xu, University of Oregon

**Activities and Findings****Research and Education Activities:**

The goals and objectives over the past three years were to develop and extend the general theory of two variable polynomials orthogonal on the bicircle and the general theory of two variable polynomials orthogonal on the rectangle. The PI also sought to extend the bispectral problem associated with one dimensional Racah-Wilson polynomials to several variables. He also examined the role that orthogonal polynomials play in the Kac model in kinetic theory and conversely how this model could shed light on celebrated identities in orthogonal polynomials such as Gasper's and Gegenbauer's identity.

The first set of problems associated with bivariate orthogonal polynomials have application in factorization, engineering and solutions of nonlinear partial differential difference equations. The PI has presented algorithms that allow the computation of positive definite doubly Toeplitz and also doubly Hankel matrices. These algorithms also give a new parametrization of the two variable trigonometric and a new parametrization of the power moment problem. The PI has given lectures on these results at international conferences. He also has been co-director of the thesis of Antonia Delgado a Spanish graduate student and mentored Delgado for a semester in her post doctoral studies. He has also mentored Shannon Bishop a graduate student at Georgia Tech as well as REU student Carola Concess.

In the second problem mentioned above extended the bispectral problem associated the one dimensional Racah-Wilson polynomials to several variables. The Racah-Wilson polynomials are the most general hypergeometric polynomials in the Askey Scheme. The bispectral problem has had applications in limited angle tomography as well as in the theory of integrable systems.

In the third problem mentioned above the PI has used the theory of orthogonal polynomials to help calculate the spectral gap in the Kac model. He has also used the techniques developed in study of the Kac model to study orthogonal polynomials.

### **Findings:**

The PI's results have shed light on two variable polynomials orthogonal on the bi-circle and the role these polynomials play in the factorization of positive two variable trigonometric polynomials. He has also found recurrence formulas and difference equations satisfied by the multivariable Racah-Wilson polynomials. Finally using the techniques developed in the study of the Kac model he has found simple proofs of the celebrated identities of Gasper and Gegenbauer. The findings discussed above have been published or submitted for publication in the articles listed below.

### **Training and Development:**

### **Outreach Activities:**

### **Journal Publications**

J. S. Geronimo and M.J. Lai, "Factorization of Multivariate Positive Laurent Polynomials", JAT, p. 327-345, vol. 139, (2006). Published,

J. S. Geronimo and A. Martinez-Finkelshtein, "On extensions of a theorem of Baxter", JAT, p. 214-222, vol. 139, (2006). Published,

J. S. Geronimo and H. Woerdeman, "Two-variable polynomials: intersecting zeros and stability", IEEE CAS-1, p. 1130-1139, vol. 53, (2006). Published,

J. S. Geronimo and H. Woerdeman, "The Operator Valued Autoregressive Filter Problem and the Suboptimal Nehari Problem in Two variables", Int Eqs and Op Theory, p. 343-361, vol. 53, (2005). Published,

A. Delgado, J. Geronimo, P. Iliev, F. Marcellan, "Two Variable Orthogonal Polynomials and Structured Matrices", electronic version, p. 118-147, vol. 28, (2006). Published,

J. Geronimo and H. Woerdeman, "Two Variable Orthogonal Polynomials on the bi-circle and structured matrices", SIAM J Matrix Anal. (electronic version), p. 796-825, vol. 29, (2007). Published,

A. Aptekarev, J. Geronimo and W. Van Assche, "Varying weights for orthogonal polynomials with monotonically varying recurrence coefficients", JAT, p. , vol. , ( ). Accepted,

J. Geronimo, E. Carlen and M. Loss, "Determination of the spectral gap in the Kac model for physical momentum and energy conserving collisions", SIAM J. Math. Anal (electronic version), p. 327-364, vol. 40, (2008). Published,

J. Geronimo and P. Iliev, "Two variable deformations of the Chebyshev Measure", Contemporary Math, p. , vol. , ( ). Submitted,

J. Geronimo and P. Iliev, "Bispectrality of Multivariable Racah-Wilson Polynomials", Constructive Approximation, p. , vol. , (2008). Accepted,

E. Carlen, J Geronimo, and M. Loss, "The Markov sequence problem for Jacobi Polynomials", J AMS, p. , vol. , (2008). Submitted,

### **Books or Other One-time Publications**

### **Web/Internet Site**

#### **URL(s):**

[www.math.gatech.edu/~geronimo](http://www.math.gatech.edu/~geronimo)

#### **Description:**

In addition some of the papers are posted on other preprint servers, such as the ArXiv, as well as co-author Hugo Woerdeman's site [www.math.drexel.edu/~hugo/publ.html](http://www.math.drexel.edu/~hugo/publ.html). These postings reinforce the commitment to disseminate the research supported by this funding.

### **Other Specific Products**

### **Contributions**

#### **Contributions within Discipline:**

The theory of two variable orthogonal polynomials is still undeveloped. The PI and his collaborators have investigated the properties of orthogonal polynomials when the orthogonalization procedure uses the lexicographical ordering. This produces a moment matrix which is either doubly Toeplitz in the case of polynomials orthogonal on the bicircle or doubly Hankel in the case of real bivariate orthogonal polynomials. The PI and his collaborators have developed a new parametrization of the bivariate trigonometric and

power moment problem. He has given invited lectures at international conferences on these results as well as the results associated with the the bispectral problem and the Kac model discussed above.

#### **Contributions to Other Disciplines:**

This PI results have appeared in engineering journals and have shed light on how to compute bivariate stable polynomials. Such polynomials appear in the construction of filters used in signal processing.

#### **Contributions to Human Resource Development:**

As mentioned above the PI was co-director along with Francisco Marcellan of Antonia Delgado. She received her Ph.D from University of Madrid Carlos III this past May and she intends to do postdoctoral studies with the PI next year. The PI has mentored Shannon Bishop a graduate student in the School of Mathematics at Georgia Tech and REU student Carola Concess. The PI has partially sponsor the visits of Jorge Arvesu, Alexander Aptekarev, Sergiy Borodachov, and Antonia Delgado to Georgia Tech.

#### **Contributions to Resources for Research and Education:**

#### **Contributions Beyond Science and Engineering:**

#### **Categories for which nothing is reported:**

Organizational Partners

Activities and Findings: Any Training and Development

Activities and Findings: Any Outreach Activities

Any Book

Any Product

Contributions: To Any Resources for Research and Education

Contributions: To Any Beyond Science and Engineering