

# **RESIDENTIAL DEMAND RESPONSE USING A HOUSE AS A BATTERY**

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by

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# **RESIDENTIAL DEMAND RESPONSE**

## **USING A HOUSE AS A BATTERY**

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## **LIST OF SYMBOLS AND ABBREVIATIONS**

CO Colorado

gal gallon

F degrees Fahrenheit

FL Florida

HVAC heating, ventilation, and air conditioning

kW kilowatt

kWh kilowatt-hour

PV Photovoltaic

TX Texas

W watt

## **SUMMARY**

The growth in the number of residential intelligent electrical appliances and home energy management systems creates the potential to provide residential demand response services to the electricity grid. Simultaneously, direct control of individual devices by utilities can cause coordination and privacy concerns. A method to overcome this challenge is to combine all devices in a house into a single unit for the purposes of demand response. It allows to provide demand response service without giving up information on or control over specific appliances.

The contribution of this thesis is the development of a proof of concept for a home energy management system that can combine several devices into a single unit of demand response. This study suggests an adjusted 2d bin packing problem with partial trimming and a recursive join algorithm to optimize bidding of an individual house. It tests the algorithm with the use of controllable devices in an experimental house. It further uses simulations to establish whether the use of a house as a battery causes a reduction in available demand response capacity and whether demand response can provide financial incentives to individual users. It also solves an additional problem that emerged along the way: the problem of predicting the charge of HVAC systems.

The thesis serves as an intermediate step between existing theoretical research and possible future steps, such as commercial prototyping of systems that provide residential demand response services at the point of common coupling.

# CHAPTER 1. INTRODUCTION

## 1.1 Motivation

In the last several years, there has been significant growth in the number of intelligent electrical appliances and home energy management systems within residential sector. Intelligent appliances and home energy management systems can allow utilities to manage system coincident peak and thus achieve cost reduction (Siano, 2014). Further, there exists a potential for residential devices to provide ancillary services (Ma et al., 2013) that can benefit the electricity grid.

The control that utilities can exercise over individual in-home appliances can cause coordination and privacy concerns. Access to individual appliance usage history can reveal unwanted detail about user intra-day routines, thus compromising user privacy. Direct control of individual appliances may also imperfectly align with user preferences. For instance, a utility or an aggregator may be interested in interrupting HVAC system operations while users would instead be more willing to turn off a water heater.

There exists a theoretical solution to combine all devices in a house into a single unit for the purposes of demand response (Hao et al., 2015). As a result of combining devices, a utility only receives information regarding total available capacity at the point of common coupling without specific knowledge of the supporting assets. Thus, demand response service is offered to the utility without giving up any sensitive information or control over specific appliances. No proof of concept for such a system has been attempted so far, which justifies research in the implementation area.

## 1.2 Problem statement

This study attempts to understand whether demand response at the point of common coupling can be implemented in a house. It further attempts to understand whether a home energy management system can interface with appliances and handle the necessary calculations in a reasonable amount of time. The research is done in collaboration with Oak Ridge National Laboratory. The Laboratory already had in place the hardware and communication systems for all devices in the experimental house. The objective was to propose a system that would access information from those devices, pack their loads into a single unit of demand response, and send the necessary connecting and disconnecting commands. The system was required to provide a result within a certain acceptable threshold of time and computational power.

In order to address these requirements, the study first adjusts an existing 2d bin packing algorithm to serve the purposes of demand response. Then it develops a Python solution which processes numerical data from individual devices. As it proceeds through the development of software it finds a side problem, the lack of HVAC data. It suggests a fixed effects model to implement load forecasts in HVAC. Once all data is available, the proposed algorithm is finalized in the form of a Python macro that communicates with individual devices and packs their capacity into a single bid. The solution is then tested through a controlled experiment in a dedicated facility.

Once practical implementation of a house as a battery is proven feasible, the study moves on to the second step. It investigates how combining devices into a single unit of demand response affects the available demand response capacity in the system. If a certain

percentage of capacity is lost under house as a battery conditions, there is social cost to maintaining privacy. Further, if a significant amount of capacity is lost, bidding a whole house may be prohibitively expensive from the system standpoint.

Finally, the study makes a basic attempt to estimate the efficiency of the system not just for a utility, but for an individual user. It builds a one-year hourly simulation to understand if the system has the potential to provide electricity bill savings and, if yes, which devices or hours of the day are the driver of these savings.

In sum, the present research addresses the problem of combining several devices into a single unit of demand response that a utility can access at the point of common coupling. It develops and tests an algorithm to predict the state of charge of selected residential devices. It develops and tests an algorithm to optimize the combined residential load of a single house. It estimates the social cost of combining individual devices into a single unit of demand response to help understand if the approach is overall economically efficient. Finally, it tests the ability of the system to pay off from the viewpoint of an individual user that shifts its load to provide demand response services.

## **CHAPTER 2. LITERATURE REVIEW**

### **2.1 Residential demand response from individual devices**

There is a very broad body of literature that studies residential demand response. The majority of papers dealing with residential demand response relate to the use of individual devices. Specifically, research highlights four main devices as having a high potential of contributing to demand response.

#### *2.1.1 Heating, ventilation, and air conditioning*

The first device is an HVAC system. Its participation in demand response was investigated as early as in 2006 (Katipamula, Lu, 2006). The research of HVAC contribution to demand response has highlighted benefits and shortcomings of the device. Specifically, HVAC is favored for large volume of demand response and interruptible cycle (Siano, Sarno, 2016). HVAC has the highest rated power among all devices found in a regular residential dwelling. Further, it is possible to disconnect an HVAC system without cycle interruption, unlike a clothes washer or a dryer.

The HVAC system also has a number of limitations. The most prominent limitation relates to user comfort: HVAC is only able to respond within user temperature settings (a “deadband”) and may require pre-cooling (Adhikari et al., 2016, Erdinc et al., 2017, Yoon et al., 2014). Further, HVAC is heavily dependent on the difference between outdoor and indoor temperatures; its ability to provide the right volume of demand response is determined by the accuracy of forecast of weather (Due et al., 2018) and occupancy (Sandels et al., 2015). Finally, the use of HVAC triggers a technical limit of compressor

ramp rate, which limits its ability to provide instant demand response (Berardino, Nwankpa, 2013). Currently, a disconnected HVAC system is believed to require 15 minutes of rest before it can be switched back on, and vice versa.

Overall, researchers agree that HVAC is a high-volume device that can contribute strongly to demand response, but it requires careful planning and scheduling and additional attention to user comfort. These characteristics of HVAC will become important later in this study: high rated power makes HVAC a desirable device, but the duration of its response will be contingent on user comfort settings. This makes prediction of remaining charge difficult in the event of weather forecast errors and occupancy forecast uncertainty.

### *2.1.2 Water heater*

The second device is a water heater. The research of water heaters in demand response is equally old. Initially it was considered one of the “big two” residential appliances that are able to provide demand response services (Tiptipakom, Lee, 2007). A water heater has one particular advantage. It has no ramp constraints and can disconnect and reconnect immediately and for multiple short intervals. This characteristic allows a water heater to supply demand response for additional purposes such as regulation (Kondoh et al., 2011, Motalleb et al., 2016). Efficient insulation of a water tank also allows it to disconnect for long periods: it takes about 2 hours for temperature to change by 1° C (Ericson, 2009).

The shortcomings of a water heater as a device for demand response relate to user comfort (Lu, Katipamula, 2005). The water heater was found to be able to respond only

within user comfort settings. Further, forecasting the performance of a water heater requires predicting when a user is going to continuously draw a large volume of water.

As the literature shows, a water heater is well suited for the purposes of residential demand response: it can provide reasonably large capacity, can be interrupted more often than HVAC, and provides continuous demand response services for a longer period. On the negative side, it still depends on user comfort settings. Predicting charge would require predicting water usage patterns.

### *2.1.3 Electric vehicles and home batteries*

Residential level storage appeared in demand response research a little later (Mallette and Venkataramanan, 2010). Electric vehicles and batteries accompanying rooftop PV systems immediately caught attention of researchers as easily dispatchable devices with an acceptable length of discharge, therefore able to provide both energy and ancillary services (Sortomme and El-Sharkawi, 2012, Zhao et al., 2013). Unlike the HVAC and water heaters discussed above, these devices represent batteries (not battery equivalents), which made it easier to plan their use and eliminated the need for approximation. Besides, the mobility of electric vehicle batteries has caused researchers to study their effect on the grid in a variety of locations (Daina et al., 2017).

The only significant shortcoming of residential batteries, performance degradation, is reflected in the additional component to their optimization algorithms: optimizing storage to prolong battery lifetime (Castelo-Becerra et al., 2017). Thus, batteries represent the most convenient device in residential demand response, capable of providing predictable demand response for a sufficiently long period.

#### *2.1.4 Simultaneous demand response from several devices and privacy issues*

The papers reviewed above mostly study the mentioned devices separately. Some studies see them as interacting within a home energy management system (Fotouhi Ghazvini et al., 2017, Hubert, Grijalva, 2012, Wu et al., 2014, Zhou et al., 2014). In those studies, a home energy management system is seen as a tool to optimize individual schedules of devices. The system analyses the schedule of each device on its own. It then uses constrained optimization to determine the optimal schedule for each device. Finally, it superimposes optimized schedules of individual devices to generate the optimal schedule for the house. When a utility calls upon the home energy management system to provide demand response, the home energy management system makes a number of decisions about which devices to disconnect. By exploiting specific advantages of individual devices and accommodating their limits, a home energy management system is able to provide the highest possible payoff for each device.

While this is certainly the mainstream approach to residential demand response now, it has some shortcomings. The most pronounced shortcoming is the possible loss of household privacy (Kou et al., 2019). If a utility is allowed to see the use of individual devices, it has a better opportunity to learn about usage patterns and underlying household routines. Using a home energy management system to provide a single house-level device equivalent is one way to solve this problem. An overview of pertinent research is provided in the next section.

## **2.2 Residential demand response at the point of common coupling**

The stream of research dealing with building-level load equivalents is not very large. The first attempt to combine residential devices into a house-level battery was made in (Hao et al. 2013) and was extended in (Hao et al. 2015). The latter paper became the key reference for all subsequent research in the field. It provides the mathematical formulation for a battery equivalent of thermostatically controlled devices and highlights that the eventual optimization of these battery equivalents has to be done based on Minkowski sum. A few more papers came out in 2015-2018. All of them build on or extend the 2015 paper. Specifically, (Zhao, Zhang, 2016) provides a detailed analysis of necessary and sufficient conditions of the application of polytopes for the purposes of combining devices. (Madjidian et al., 2016) and (Madjidian et al., 2018) explore the boundary values for the solution space and discuss how these boundary conditions affect the ability of deferrable loads to provide grid services.

Despite the existence of research related to theoretical foundations of combining devices, no attempts were found to build a proof of concept for an actual system. The existing experimental research of demand response is focused on investigating demand response from individual devices, but not from a combination of devices. One possible explanation is that the combinatorial nature of the problem results in a very high computational complexity, which makes studying combinations of devices less attractive. Importantly, lab environments capable of testing a home as a combination of devices are not widespread. This study aims to provide this missing contribution by constructing a system and testing it in a controllable house.

## **CHAPTER 3. MATHEMATICAL PROBLEM OF A BATTERY EQUIVALENT**

### **3.1 Introduction**

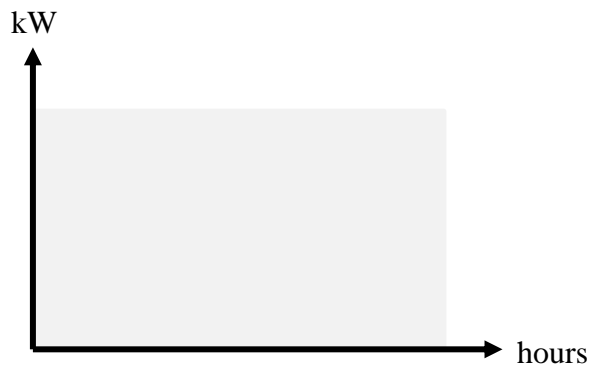
It follows from the previous section that the theoretical basis for creating house battery equivalents is not broad but is long existing and solid. However, a proof of concept for the theoretical model is missing. This thesis aims to construct an empirical proof of concept of the existing theoretical propositions. If such an empirical implementation can be provided, this will be an important intermediate step between theory and later stages such as prototyping or mass rollout. If the implementation stage shows unsurmountable obstacles, this would indicate that the approach is not feasible. In this event utilities would need to proceed with alternative approaches to demand response mentioned in the literature review.

Chapter 3 is designed for two main purposes. The first is to refresh the fundamentals of geometric sum. The second is to formulate a mathematical model that would later be converted into the prototype software. To serve these two purposes, we go through a number of steps, as described in the following subsections. Section 3.2 serves to review the use of geometric sum for the purposes of demand response. Section 3.3 provides a mathematical formulation of the problem. It further offers necessary adjustments to better fit the geometric sum approach to the empirical problem at hand. Finally, a special case of devices (thermostatically controlled devices) presents additional modelling challenges. Section 3.4 discusses the necessary extensions that would allow to use this type of devices

along with the rest of devices. Once the mathematical formulation for all devices is complete, Chapter 4 proceeds with computational and experimental implementation of the model.

### 3.2 Individual devices and the use of geometric sum

The demand response service that a utility expects from a residential user has two main properties: the magnitude of response (the amount of kW), and the duration of response in hours. These properties can be conveniently represented as two sides of a rectangle, as illustrated in Figure 1. Representing devices as rectangles (convex representation) is most convenient for the purposes of getting a dense aggregation, as will be seen in section 3.1. Further, it best represents the on/off nature of a thermostatically controlled device, which will be important later on in the study.

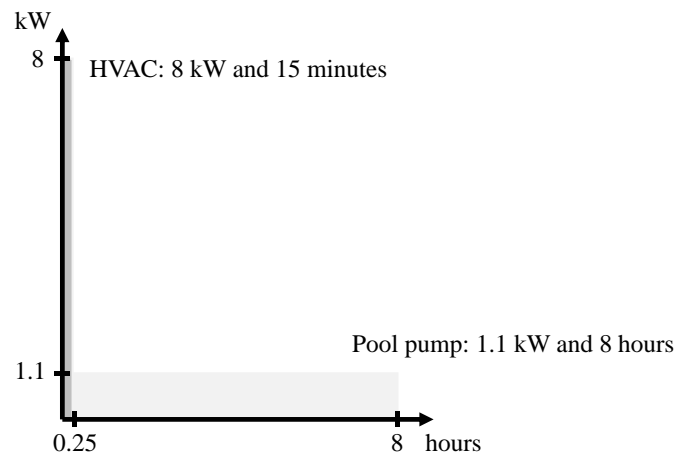


**Figure 1 – Representation of demand response as a rectangle.**

The total area of the rectangle then represents the forgone electric service, kWh, if a load is disconnected, or the provided electric service, kWh, if a battery is discharged. The selection of horizontal and vertical axes is arbitrary and does not affect the quality of

representation. For the purposes of this thesis, the horizontal axis will represent hours and the vertical axis will represent active power.

Each individual device, such as a coffee maker or a pool pump, has its own size of a rectangle, where length can range from minutes to hours and height can range between several W and several kW. A device which can be disconnected for a long time and has a low rated power would be represented by a wide and thin rectangle. An example of such a device is a pool pump, which has no comfort or ramp restrictions and can remain disconnected for multiple hours. Similarly, a device which has high rated power and can remain disconnected for a very short time would be represented by a narrow tall rectangle. An HVAC system on a very hot day is an example of such device. It can disconnect for only a few minutes, but then the indoor temperature would rise and force it to start again. The examples are illustrated in Figure 2.



**Figure 2 – Comparison of HVAC and pool pump dimensions of demand response.**

If several devices need to be combined into a single unit of demand response, this triggers a question of how to fit several device rectangles into a single rectangle. The

properties of such a rectangle, magnitude in kW and duration in hours, would be communicated to the utility as offered demand response service. In formal model language, a device rectangle is a special case of a convex polytope and the combined area of several device rectangles is called geometric sum of convex polytopes. The geometric sum is often called Minkowski sum after Polish mathematician Hermann Minkowski who used geometric representations to solve problems in combinatorics and theory of probabilities. The mathematical problem that addresses this question and limitations of the mathematical model are discussed in the next subsection.

### 3.3 Packing problem formulation

#### 3.3.1 Adaptation of traditional geometric sum to household demand response

Let us mathematically describe demand response from a house. In line with the current literature consensus on combining devices, we use geometric sum to maximize the inner area of the polytope that represents demand response offer:

$$\text{maximize } SOC_j = \sum_i P_i \text{ charge}_i \quad (1)$$

s.t.

$$P_i = P_{rated\ i}$$

$$\text{charge}_i \leq \text{charge}_{max\ i}$$

Where

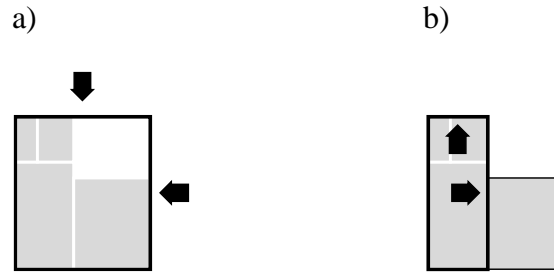
$SOC_j$ : the optimal amount bid by house  $j$ , equivalent to its state of charge as house battery, kWh

$P_{rated}$ : rated power of a device  $i$ , kW

$\text{charge}$ : the remaining charge of device  $i$ , hours to discharge

$\text{charge}_{max}$ : the maximum possible charge of device  $i$ , hours.

The polytope, here and later in the study a rectangle, is made of several smaller rectangles, each representing a separate device, as illustrated in Figure 3. The devices can be stacked next to each other along the horizontal axis (*charge*), hours, or on top of each other, along the vertical axis ( $P_{rated}$ ), rated power.



**Figure 3 – Existing (a) and modified (b) bin packing problem.**

The implementation of geometric sum for the purposes of residential devices results in a 2d bin packing problem, a subclass of combinatorial optimization problems. The traditional bin packing approach (Figure 3a), however, needs to be adjusted for the purposes of combining devices for three reasons.

- The traditional problem minimizes the bin area (outer sum) as shown in Figure 3 (a). Its objective is to find the smallest possible bin so that all shapes are inscribed into it. The objective of the home energy management system would be to maximize the inner sum because this is where a user makes profit. The logic is shown in Figure 3 (b). The objective is to make the bin as large as possible while having no blank spaces within it.
- The traditional problem is very general and allows to rotate and cut rectangles, as well as to fit them in multiple bins. The physics of electrical energy does not allow to rotate device rectangles or cut through rated power. As a result, the sum is very rigid on

vertical axis, rated power. However, it is possible to cut through the horizontal axis, hours of charge, by supplying less or more charge within  $charge_{max}$ .

- The traditional problem allows for empty inclusions in a bin. While it is possible to smoothen and derate some of the “home batteries”, an accurate problem should not allow to subscribe non-existent capacity. If a battery is allowed to have an empty corner like that in the illustration (Figure 3a), claiming full rated power for the full duration of charge would create oversubscription of capacity and, possibly, voltage or frequency drops when the oversubscribed capacity is not able to respond.

With the respective adjustments, geometric sum can serve as a valid theoretical foundation for the optimization problem at hand. Let us discuss the dimensions of the resulting polytope and underlying assumptions in more detail.

### 3.3.2 *Vertical axis: rated power*

Rated power, kW, is a very straightforward value. It is set by the manufacturer and typically cannot be changed. The only exception is a variable speed air conditioner, which is uncommon in residential settings and is therefore excluded from this analysis. The duration of charge, hours, is more complicated. It differs between two large groups of devices, thermostatically controlled and schedule-based.

### 3.3.3 *Horizontal axis: duration of charge for schedule-based devices*

Schedule-based devices include car chargers, pool pumps and similar devices. They are required to be on for a certain number of hours a day, but the schedule is usually not specified. A pool pump is believed to be “charged” almost always as it is required to run

for 8 hours during the day, but those 8 hours can be anywhere from late night to day peak. It is therefore assumed that a pool pump is always fully charged. An electric vehicle needs to charge for 4 or 5 hours a day, but those hours are also not determined. We assume that users prefer to charge their electric vehicles during late evening or the night. Home batteries have a similar property as car batteries: their charging hours are not specified. Besides, home batteries are usually kept full and do not have to charge unless they have been discharged earlier. For the purposes of this study, we assume that home batteries and electric vehicle batteries are at least partially charged and are available for discharge if a utility is requesting electric on ancillary service from a given house.

As a result, schedule-based devices are usually devices with the potential to provide demand response for long periods. The exact expected duration of charge is modelled using stochastic charge disturbances, such as the use of a car or solar irradiation.

#### *3.3.4 Horizontal axis: duration of charge for thermostatically controlled devices*

Thermostatically controlled devices include water heaters and HVAC systems. They were considered of higher importance for the purposes of this study for two main reasons. First, they are more common. Second, those are usually devices with high rated power and highest load shedding ability. The “charge” of such devices depends on the current air or water temperature and user-defined setpoint. For instance, suppose a user has set the comfortable water temperature between 100 and 120 F. Once the water heater reaches 120 F, it can disconnect and stay idle until temperature reaches 100 F. A water heater would be considered “fully charged” when water temperature is 120 F and “fully

discharged” when temperature is 100 F. After that point it will have to reconnect and start heating (“charging”) again to bring the temperature back to comfort limits.

Modelling the charge of water heaters is usually done in accordance with simplified versions of HVAC models, so they are not discussed here in detail. HVAC modelling, in turn, needs to be reviewed more closely.

### 3.4 Estimating duration of charge for thermostatically controlled devices

HVAC charge depends on air temperature and user setpoint. If a user prefers indoor temperature to be between 68 F and 72 F, the HVAC could be “charged” if air temperature is at 68 F, and “fully discharged” if air temperature is 72 F and the air needs to be cooled. But there are more factors that influence charge of HVAC systems. Conventional models, already discussed in the literature review, account for physical factors such as thermal capacitance, thermal resistance, and external disturbances. A general model adopted from (Hao et al., 2018) is presented in equation (2).

$$\frac{dtemp}{dt} = \frac{temp_0 - temp}{RC} - \frac{is\_on * P * c_{performance}}{C} - w \quad (2)$$

Where

*temp*: indoor temperature, F

*t*: time, h

*temp<sub>0</sub>*: temperature setpoint, F

*R*: thermal resistance of a house envelope, F/W

*C*: thermal capacitance of a house, W/F

*is\_on*: the status of HVAC system, 1 if on and 0 if off, which depends on whether

*temp*: higher or lower than *temp<sub>0</sub>*

*P*: rated power of a device, kW

*c<sub>performance</sub>*: coefficient of efficiency, %

*w*: disturbance, F.

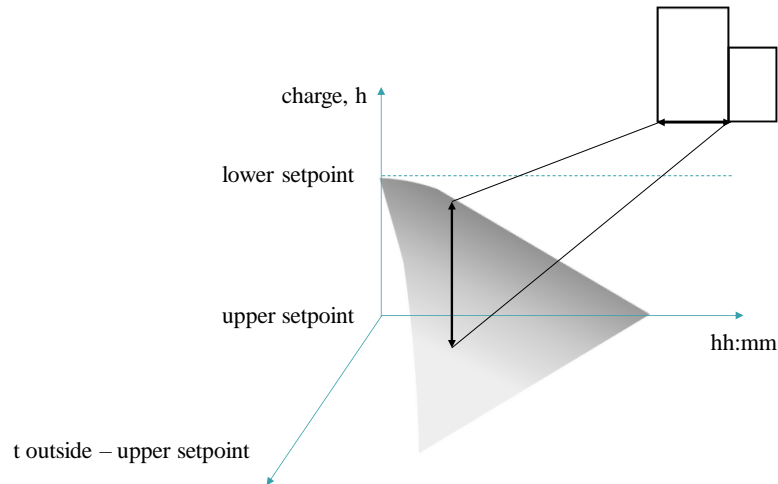
The model, as it can be seen from the equation, is designed to represent the effect of external factors on temperature increase, F/hour. However, for the purposes of this paper

we are interested in the effect of external factors on charge, hours. The required charge can be found using the fact that  $t(temp)$  is the inverse of  $temp(t)$ , while  $temp(t)$  is the integral of  $\frac{dtemp}{dt}$ , where  $temp$  is defined on the interval set by the user,  $[temp_l, temp_h]$ . The resulting charge as a function of temperature can be represented in a simplified form in equation (3).

$$t_{remaining}(temp) = \left( \int \left( \frac{temp_h - temp}{RC} - \frac{is\_on * P * c_{performance}}{C} - w \right) dt \right)^{-1} \Big|_{temp}^{temp_h} \quad (3)$$

Where  
 $temp \in [temp_l, temp_h]$   
 $t_{remaining}$ : remaining charge, h.

Let us briefly stop on the last component of the equation,  $w$ . Disturbance from weather is of very little importance if thermal resistance of the house is sufficiently high. However, it turns out to be almost as important as user preferences when it comes to empirical results. First, it contributes a lot to the actual duration of charge, with a strong negative relationship between outside temperature and charge. Second, the relationship between charge and user setpoint becomes nonlinear. The logic of the model is illustrated in Figure 4.



**Figure 4 – Relationship between charge and key control variables.**

There is a natural decline in remaining charge due to slow warming of the house through walls. However, the closer indoor temperature to outdoor temperature, the slower the discharge. This nonlinearity can be illustrated through the following observation. If the gap between outside temperature and room temperature is high (for instance, 98 F outside and 70 F as a setpoint), HVAC would have to run almost constantly. By contrast, if the difference is small enough, up to 5 F, the charge can hold for as long as 8 hours.

Thus, the remaining charge is assumed to be a function of current temperature, user setpoints, building and HVAC properties, and disturbances caused by outside temperature and human activities. For the purposes of this study, we establish this relationship empirically through a Least Squares regression. The details of the regression are beyond the scope of theoretical modelling and are provided in the next chapter.

## **CHAPTER 4. EXPERIMENTAL IMPLEMENTATION OF THE PACKING PROBLEM**

### **4.1 Summary of theoretical findings and introduction to the experiment**

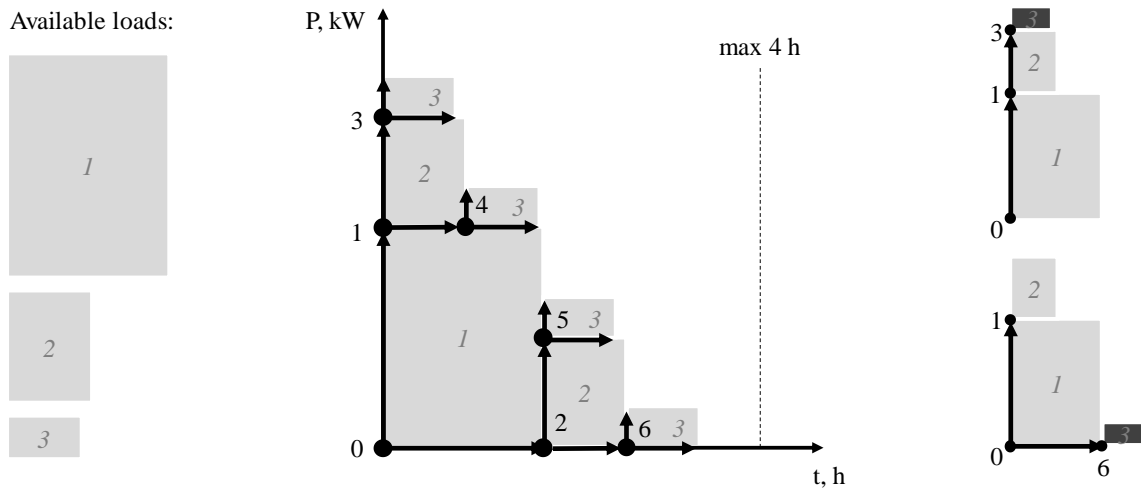
Chapter 3 was dedicated to the review and adaptation of geometric sum for the purposes of combining individual appliances into a single house battery. It established that each device can be viewed as a rectangle in a two-dimensional plane with rated power, kW, on the vertical axis and remaining charge, hours, on the horizontal axis. Further, it established the physical restrictions and the approach to dealing with thermostatically controlled devices.

The resulting bin packing problem (Figure 3b) uses rated power and charge to optimize the inner Minkowski sum. Each rectangle in the bin is allowed to change along axis  $t$ , hours, and is kept at nominal value along axis  $P$ , kW, thus being a representation of a two-dimensional bin packing algorithm with partial trimming. The target size of the bin is the maximum area which allows uninterrupted supply of demand response to the grid. Each bin represents a single bid that is submitted by a household to the respective utility at the point of common coupling. This single bid prevents the utility from learning anything about the underlying devices.

Chapter 4 proceeds with the experimental validation of the approach. Section 4.2 provides a computational algorithm. Section 4.3 describes the location of the experiment, as well as procedures and expected results. Section 4.4 discusses the findings. Section 4.5 concludes.

## 4.2 Algorithm and computational commands

Despite mentioning Minkowski sum as an approach to find the optimal battery size, known literature does not provide a specific implementation algorithm for it. Existing literature (Lodi et al., 2002) is focused on optimizing outer sum described in Figure 3 (a), but it not directly applicable to the maximization of inner sum as described in Figure 3 (b). This algorithm had to be developed. The full approach to the problem is illustrated in Figure 5.



**Figure 5 – Search tree for the maximum geometric sum.**

The algorithm picks a device and places it onto a hypothetical coordinate plane. The first device, device *1*, starts at a corner point, indicated as point 0 in Figure 5. Each following device can be placed either on the top or to the right of the preceding devices, resulting in a binary search tree. For instance, in Figure 5 device 2 can be placed either on top of device *1*, in node 1, or to the right of device *1*, in node 2.

This tree does not require the simultaneous existence of all branches. Once device 2 is placed in a desirable space, e.g. node 1, the search tree for device 3 is limited. It can still be placed in nodes 3 and 4. The nodes 2 and 5 are eliminated because device 2 is placed in node 1. But the path to node 6 still exists because device 3 can be placed to the right of device 1.

The complete list of paths was obtained using a discretized recursive join algorithm. Once a device is placed in a specific node, the algorithm checks whether the resulting inner sum satisfies the three requirements listed in section 3.3.1. Then it moves on to place a new shape. Once all shapes are placed in their nodes, the algorithm takes measurements, erases the recently developed shape, and starts constructing a new shape using different nodes.

#### Algorithm

##### Input:

- remaining charge in each device, hours
  - rated power of each device, kW
1. Initialize
    - a list of devices as rectangles with a specific width and height
    - an empty archive to later append with shapes and measurements
  2. For every hour in a day:
    3. Sort shapes by rated power
    4. While unused shapes exist:
      5. Place next shape in position i
      6. For the available positions:
        7. Place next shape in position i
        8. If needed: adjust width
        9. Calculate area according to equation (1), store area, width, height, copy of the polytope in the archive
      10. Clean the memory
    11. Reiterate
  12. Find the polytope with largest area, copy its width and height

##### Output:

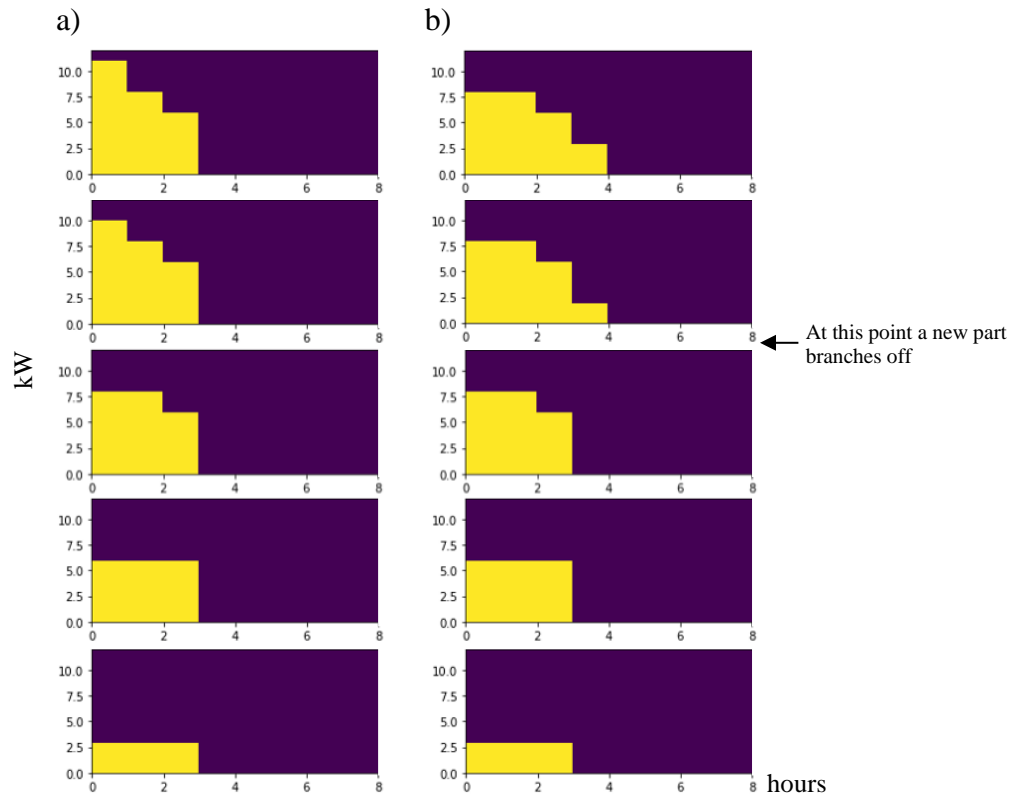
- charge of the combined load, hours
- nominal power of the combined load, kW

Python code is provided in Appendix A. It is helpful to provide an illustration of the actual work of the algorithm using two different branches. For the purposes of this illustration, branches 1 and 4 were selected arbitrarily, because they are sufficiently far apart from the perspective of constructing a final shape. The sample input is as follows:

```
shapes = [[3.0, 3.0],
          [3.0, 3.0],
          [2.0, 2.0],
          [2.0, 1.0],
          [1.0, 1.0]]
```

Where the first column of the input matrix represents height of the rectangle and the second column of the input matrix represents the width of the rectangle.

The resulting branches are provided in Figure 6. It can be noticed that branch 1 (Figure 6a) places all rectangles strictly on top of each other, while branch 4 (Figure 6b) places rectangle 4 to the left of rectangle 1 and rectangle 5 on top of rectangle 4.



**Figure 6 – Python results for a search tree for sample data, branches 1 (a) and 4 (b).**

It is important to highlight that the proposed algorithm does not guarantee optimal computation time or scalability. It relies on the performance of the existing packing algorithm adopted from (Lodi et al., 2002). For instance, this algorithm uses partial enumeration. When selecting the order in which to pack devices, it does not go through full enumeration and uses an insertion sort algorithm to pre-process data. Devices arrive at the beginning of the packing algorithm sorted by rated power, thereby eliminating enumeration in terms of which devices goes first, second, etc. The sorting stage was adopted in accordance with the mentioned literature to limit the computation demands of the algorithm. It reduces the number of scenarios the software needs to process. Insertion sort algorithm has the performance of  $n^2$ . For  $n > 3$  this approach is preferred to  $n!$ . Additional tests were performed to compare the sorted packing and the complete enumeration packing. The sorted packing algorithm was found to find the same optimal solution as enumeration packing in 100% of simulations.

The devices still go through enumeration in terms of position. This adds  $n!$  to the size of the problem, which makes the algorithm lose its speed quickly. The reason it is possible to use the sorted two-dimensional bin packing algorithm for the purposes of home energy management system is that the number of devices in a typical home is usually well within 10. But the nature of the algorithm makes it unfit for use in broader contexts such as distribution feeder level.

## 4.3 Background and description of the experiment

### 4.3.1 *Description of the house and devices*

The ability of the system to work in real-life settings was tested using HVAC thermostat, a pool pump, and a PV system battery in a controlled house. The readiness of the pool pump and the battery was ensured by scheduling these devices in advance. As was already discussed above, schedule-based devices are easy to control and tend to have full charge throughout the day. The most challenging device was the house HVAC system. HVAC systems tend to be less charged during peak hours because they have to constantly cool the house during the presence of people. As was mentioned above, the charge is affected by outside weather, thermostat setpoint, and the insulation of the house. Let us discuss the experiment in more detail.

The experiment was performed in a two-bedroom house owned by the Department of Energy and operated by Oak Ridge National Laboratory. The house is located at 1362 Yarnell Station Blvd, Knoxville.



**Figure 7 – House of experiment (Source: CRADA/NFE-17-06741).**

It has a ground floor with a front porch, an entrance hall, two extra rooms, a sitting room, a kitchen with walk-in closet, a bathroom and a two-car garage. The second floor has two bedrooms, two bathrooms, and a sunroom.

- The Hayward pool pump is located in a small bucket inside the house. Because water temperature and quality does not affect the schedule of a pool pump, this location is believed not to affect the quality of data. It is supposed to work 8 hours a day but is flexible as to when exactly these 8 hours are placed in the schedule. For most experiments in the house, the pool pump was a flexible device that was dispatched whenever necessary.
- The house has two PV batteries by LG. The total energy capacity is 6 kWh. The batteries are located in one of the rooms on the ground floor, in the ambient temperature, which ensures that their charge is close to nameplate capacity. The state of charge can be monitored and is usually close to nominal.
- The HVAC system of the house has an externally controllable Ecobee thermostat. The two floors of the house have two separate thermostats, the ground floor being the usual place of experiments. The second floor thermostat is usually programmed to one or two degrees above the ground floor because warm air accumulates on the second floor. The house has an attic. The southern part of the roof has a rooftop PV system. These two properties allow to prevent some further overheating of the indoor space. Due to its experimental nature, the house has more devices than a normal residential dwelling. There are several water heaters in the garage, two PV system batteries, and numerous smart household devices in the kitchen, such as an externally controllable fridge. Extra devices may create a small increase in indoor temperature compared to a regular house.

The house is furnished, which makes it similar to a residential dwelling from the perspective of heat accumulation and the air volume inside the house. The usual practice is to run experiments when the house is not occupied, which ensures that there is no additional flow of air from opening windows or doors, or no additional heat from people that are in the house.

#### *4.3.2 Background and description of the experiment*

The objective of the experiment was to test the ability of the suggested packing algorithm to perform a full range of bidding activities:

- collect data,
- transform collected data into model devices,
- combine model devices into a single polytope, and
- extract the dimensions of the polytope to produce a demand response bid.

At the point of collecting data, the software was expected to connect to devices inside the house and receive the data necessary to establish the current level of charge. The software that established connection with the devices was already in place by the beginning of this work. The HVAC system was the only device for which the connection and data gathering code was rewritten. Rewriting the code allowed to access additional fields in the data, as was required for the purposes of predicting charge. For the most part, connections were established through manufacturer websites and Application Programming Interface.

Once the individual data files were received, the objective of the next step was to establish the current level of charge. This stage had different inputs, depending on the type

of device, but had a uniform output: remaining charge in minutes. This task was very easy to accomplish for the batteries, which had the current state of charge readily available. The remaining charge could be approximated by dividing the received value by nameplate capacity and subtracting 10%. The input data for a pool pump in a real swimming pool would be the number of minutes already worked. The remainder would be the unused part of required 8 hours and would represent the desired output. In the event of the experimental pool pump, it always had full charge because it was never used for full 8 hours a day. Estimating the level of charge for the HVAC system was more difficult because the input data represented various temperature measurements, while the outputs were still minutes of charge. This device is discussed in more detail in the next subsection.

After obtaining rated power and estimates of charge for each device, these values were sent to the software module provided in Appendix 1. The algorithm behind this software was discussed in Chapter 3. The inputs to the algorithm, as was already discussed in Chapter 3, were rated power and duration of charge for individual devices. The output was a combined shape in a discretized coordinate plane.

Finally, the shape was measured for width and height. These measurements represent the final parameters of the bid. The height of the shape is the stacked height of multiple devices, if appropriate, and the length is the combined number of minutes for which this service can be provided.

#### *4.3.3 Peculiarities of collecting and processing HVAC data*

While obtaining height of an HVAC battery equivalent was simple, obtaining its width, or remaining charge in hours, was more difficult. The main difficulty posed by the

HVAC system is how to estimate the remaining charge in the house based on air temperature data. The Ecobee thermostat only displayed a variety of temperature measurements, but not the charge. Charge had to be predicted using temperature setpoints and weather forecast according to equation (3). However, in order to predict the charge, it was important to first produce reliable estimates on the impact of main factors affecting it. The initial main specification representing a statistical model for equation (3) is provided in equation (4).

$$t_{remaining} = b_0 + b_1(temp_{t=t} - temp_h) + b_2R + b_3C + b_4P + b_5c_{performance} + b_6temp_{outdoor} \quad (4)$$

Where

$t_{remaining}$ : remaining charge, h

$temp_{t=t}$ : current indoor temperature, F

$temp_h$ : maximum setpoint, F

$R$ : thermal resistance of a house envelope, F/W

$C$ : thermal capacitance of a house, W/F

$P$ : rated power of a device, kW

$c_{performance}$ : coefficient of efficiency, %

$temp_{outdoor}$ : outdoor temperature, F.

It is visible that a large part of this equation cannot be estimated using time series data because of the presence of fixed variables. Specifically,  $R$ ,  $C$ ,  $P$ ,  $c_{performance}$  are constant properties of the building and HVAC system. Their use thus will not affect regression results in a meaningful way. These variables were excluded from the analysis.

Furthermore,  $temp_{outdoor}$  by itself does not provide enough information to identify the effect of outdoor temperature on remaining charge. The reason behind this insufficiency is the nonlinearity of discharge already discussed above. The simpler way to look at the problem is to see two examples.

Example 1: outside temperature is 95 F, upper setpoint is 80 F, lower setpoint is 76 F. The house temperature is 19 F lower.

Example 2: outside temperature is 85 F, upper setpoint is 70 F, lower setpoint is 66 F. The house temperature is 19 F lower.

While the outside temperature would be considered very high in Example 1 and average in Example 2, the actual effect of outside temperatures is the same. Therefore, these two examples are identical for the purposes of disconnecting HVAC system. If in Example 1 the upper setpoint were 70 F, the house temperature would be lower by 29 F, which would significantly affect the duration of discharge. Therefore, external temperature was not included in the model as a separate variable, but rather, the difference between average external temperature and the upper setpoint was used as a control variable.

The corrections resulted in a much smaller model to estimate, represented by equation (5).

$$t_{remaining} = b_0 + b_1(temp_{t=t} - temp_h) + b_2(temp_{outdoor} - temp_h) \quad (5)$$

Where

$t_{remaining}$ : remaining charge, h

$temp_h$ : maximum setpoint, F

$temp_t$ : current temperature on the ground floor, F

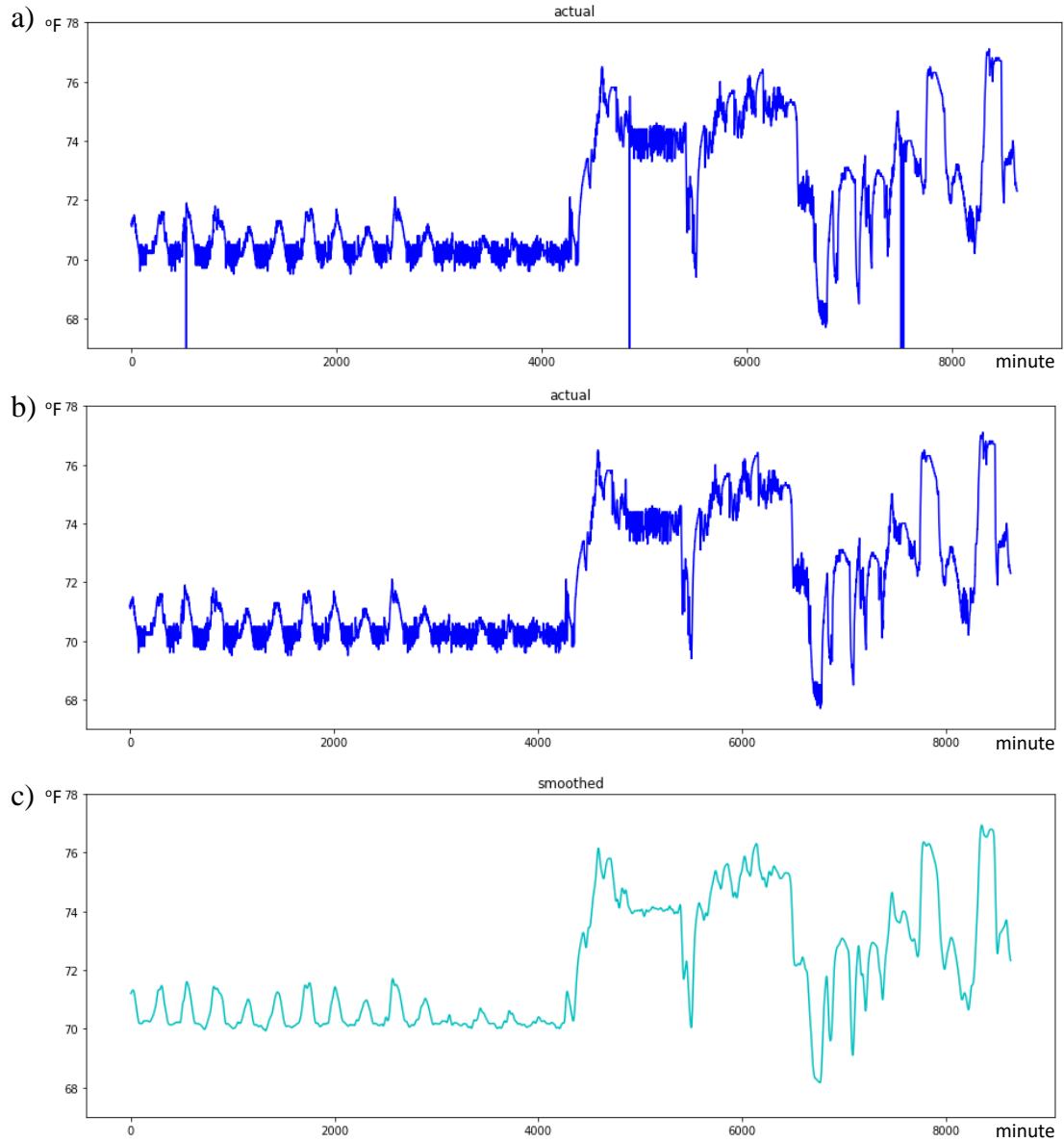
$temp_{outdoor}$ : outdoor temperature, F.

The expected estimation outcomes were as follows:

- The difference between current indoor temperature and upper setpoint was expected to positively affect charge. The explanation is intuitive. If the house has to recool after

- 2 F, it will have to switch on the HVAC system sooner. By contrast, if it recools after 4 F, the idle period would be much longer.
- The difference between outdoor temperature and upper setpoint was expected to have a negative effect on charge. However, the magnitude of the effect and extent of its nonlinearity would require additional testing through alternative, e.g. polynomial, specifications.

Obtaining specific estimates required a dataset with time series record of temperatures and response of the HVAC system. Data on current temperature, minimum setpoint and maximum setpoint were collected by Ecobee thermostat based on actual measurements and settings. Outdoor temperature was similarly obtained from Ecobee thermostat but was received by the thermostat from a Knoxville meteo station. Hence outdoor temperature was not precisely the temperature outside the house but can be assumed to be relatively close to it. Historical data were collected throughout July and the first half of August in a series of control actions. Because the house was occasionally used by other researchers, it was difficult to establish a one-month line of uninterrupted data gathering. But as data were collected on random days and at random points during the day, possible errors related to collecting data were considered non-systematic. The indoor temperature values for 1 minute intervals are shown in Figure 8.



**Figure 8 – Indoor temperatures on the ground floor of the experimental house in January-June 2019: raw (a), filtered for missing values (b), moving average (c).**

The obtained estimates were collected into an external training dataset, which allowed establishing the following coefficients: intercept 133.17,  $b_1$  96.26,  $b_2$  -21.82, and  $R^2$  of 0.8.

The suggested regression was found fit for further use from two perspectives. First, the sign of both  $b_1$  and  $b_2$  indicates that the effects work in the expected direction. The difference between the setpoint and the current temperature indoors is contributing positively to the remaining duration of charge. The difference between temperature indoors and temperature outdoors contributes negatively to the remaining duration of charge. Second, the resulting  $R^2$  suggests that the two mentioned effects explain 80% of all variance in observations. This is in line with the theoretical specifications discussed in Equations 4 and 5, as the two temperature effects are expected to be the largest when all other effects are fixed. Because regressor coefficients have the expected sign and that the current regression specification explains 80% of variance in observations, we believe that the goodness of fit is sufficient to apply the suggested regression in the experiment.

#### **4.4 Results and discussion**

The experiment took place between August 13, 10 a.m. and 1 p.m. The scheme of the experiment is presented below.

##### **Experiment**

1. Ensure that the house is not occupied and the pool pump and the PV system battery are ready
2. Set the HVAC system on a cool mode such that the deadband is 4 F
3. At 10 a.m. extract the cooling data from HVAC and predict the remaining discharge of HVAC system
4. Run the packing algorithm based on data extracted from the house

The difference between temperature indoors and outdoor temperature at the moment of disconnecting HVAC was 15 F. 20 F were taken for estimate due to expected increase in outdoor temperature (92 F later in the day). The total number of hours of charge was therefore estimated at 1.36 or 81 minutes. The HVAC system was expected to switch

back on at 11.20 a.m. The code for controlling the HVAC system is provided in Appendix B. The execution of algorithm took less than one minute on a 1.8 GHz Intel i5-8250U CPU.

The expected value based on the packing algorithm was 15.6 kWh of demand response. The experiment allowed the home to provide a total of 25.4 kWh of demand response, mainly because the HVAC system was switched off until 12.10 p.m. The difference can be attributed to two main reasons.

The first reason was, 20 F was taken as a conservative estimate of temperature difference. In fact, the difference was 15 F initially and only reached 18 F by 12.10 p.m. The weather turned out to be less hot than forecasted. Although it is not correct to substitute a more recent estimate of temperature difference into the previous estimate, the substitution of 20 F for 18 F explains some of the difference. The estimate of 20 F between outdoor and indoor temperatures resulted in prediction of 81 minutes. The estimate of 18 F brings the predicted duration of discharge to 125 minutes. This number appears much closer to the actual duration of 130 minutes. This finding is in line with research literature mentioned in Chapter 2: correct prediction of demand response of HVAC is very sensitive to weather forecast. The second possible reason was a relatively low accuracy of the forecast.  $R^2$  of 0.8 still leaves a large unexplained variance, which allows further investigation of prediction methods.

## **4.5 Conclusions**

The actions described in Chapter 4 can be summarized as follows. We developed an algorithm that allows us to combine individual devices into a single battery equivalent and maximize its capacity. This algorithm was further used to generate an estimate of

demand response service that a specific house could offer to a utility. The predictions were then experimentally tested by setting up a controlled experiment in the house.

The experiment proved that devices can indeed be combined into a battery equivalent. This can be achieved with a small piece of code, limited use of computer resources and in reasonable time. Hence, it is possible to conclude that there is practical potential in the theoretical suggestions of house battery equivalents. The experiment also revealed a number of potential issues which need further investigation. First, the algorithm leaves out some capacity, causing the combined battery to be smaller than the sum of devices. Second, the results depend heavily on the quality of weather forecast. Finally, the cost-benefit trade-off of the system is unclear. Chapter 5 investigates two of these questions, those which are in control of engineers as opposed to the weather service. It attempts to establish what part of capacity is lost due to combining devices, and whether a rollout of the system would be commercially viable under the existing residential tariffs.

## **CHAPTER 5. COST-BENEFIT ANALYSIS OF HOUSE BATTERY EQUIVALENT TECHNOLOGY**

### **5.1 Systemic cost considerations: loss of capacity**

#### *5.1.1 Introduction: background, inputs, expected outputs*

The initial findings from the experiment suggest that the algorithm leaves out some capacity, reducing the total volume of demand response service a utility can get from residential users. This finding is intuitive since the inner geometric sum is always less than or equal to the arithmetic sum of areas of individual polytopes. To verify how much capacity is left out on average, we run an additional simulation.

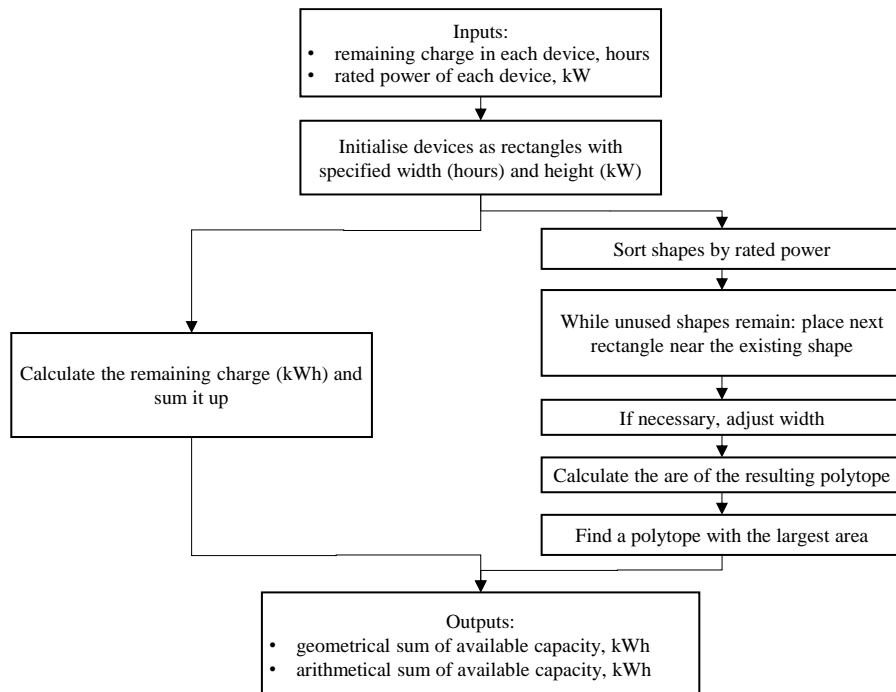
The simulation compares the geometric sum of four devices (HVAC, water heater, battery or car charger, pool pump) with the arithmetic sum of their capacity. Respectively, it provides two outputs: the battery equivalent capacity of individual devices and the battery equivalent capacity when those devices are combined into a single house as a battery. To do so, the simulation requires a number of inputs. The first input is rated power of a device, kW, as per manufacturer label. The second input is standard duration of charge. Where possible, this input was taken from manufacturer specifications (8 hours for LG Chem, 16 hours for Tesla Powerwall). Where it was not possible to establish a firm number, an estimate was taken based on industry practice (6 hours for pool pump is the minimal possible duration of circulation per day, 8 hours are required for the low-power car charger selected for the purposes of this study). Finally, for thermostatically controlled devices the

discharge estimates were based on discussions with the project team (1 hour for HVAC, 12 hours for water heater).

The simulation is performed for three scenarios, indicating varying possible sizes of a neighborhood. The methodology and algorithm behind the calculations are discussed in the next subsection, followed by results and discussion.

### 5.1.2 Methodology: summary of the algorithm and rationale for starting conditions

Since the simulation exactly follows the principles outlined in Chapter 4 and uses the same code, we only summarize the key methodological points here and put the main emphasis on selection of starting conditions and simulated devices. As was discussed in more detail in the previous sections, the simulation follows the scheme outlined in Figure 9.



**Figure 9 – Simulation scheme for establishing the difference between geometric sum and arithmetic sum.**

The packing algorithm sorts individual devices in the order of decreasing rated power. Then it selects the largest device and places it in the corner of a hypothetical coordinate plane. The second largest device is placed to the left of it or on top of it, then the third largest device is placed to the left or on top, all the way to  $n$ th device. The resulting search tree is evaluated for the inner geometric sum of devices, and devices can be trimmed in terms of duration of discharge. The highest possible geometric sum is selected as the battery equivalent. Independently of the packing algorithm, the initialized devices are analyzed separately to include the arithmetic sum of their areas into the estimate of the sum of available capacity.

The simulation used the following common devices as an input (Table 1). These devices are common to US households and are advertised by home improvement stores such as Lowes and Home Depot, so their installation in the near future is likely to continue.

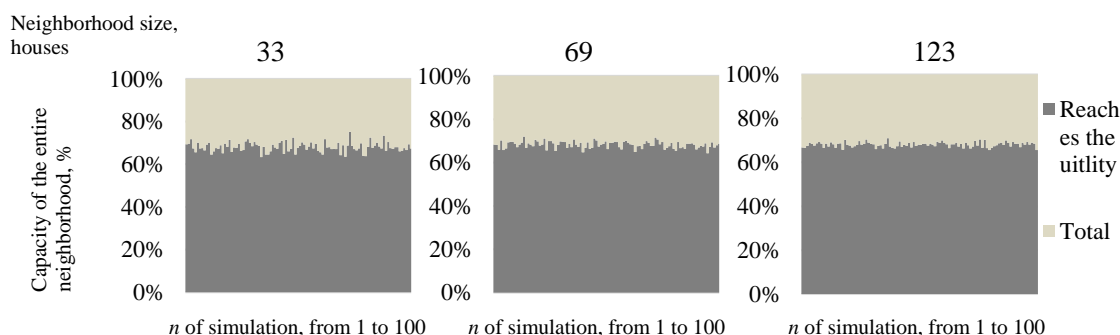
**Table 1 – Simulated devices for establishing the difference between geometric sum and arithmetic sum.**

Device	Rated power, kWh
HVAC Royalton 2.5 ton	8.2
HVAC Winchester 2 ton	7.0
Water heater Rheem 80 gal	5.0
Water heater Rheem 50 gal	4.5
Pool pump FlowXtreme II	1.1
Pool pump Hayward Super Pump	0.7
Tesla Powerwall	5.0
LG Chem	3.0
Charger	3.8

Each house was assumed to have an HVAC system, a water heater, a pool pump, and either an electric vehicle with a low power charger or a rooftop PV system with a battery. The devices were initialized with random uniformly distributed charge between 10% and 100% of maximum charge. Random rate of discharge was applied to each device separately. This approach was used to reflect the physical nature of a battery, which may not be fully charged in a given moment and is not allowed to discharge completely. The effective average charge of a single house was found between 40% and 60%. The results were found for simulations of 100 iterations with 33, 69 and 123 houses.

### 5.1.3 Results for neighborhoods of varying sizes and discussion

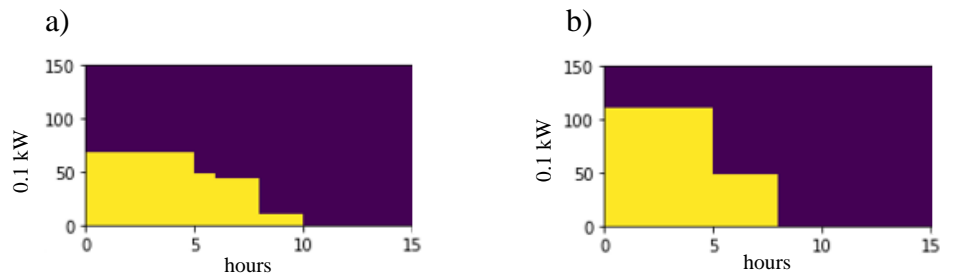
The share of capacity that reaches a utility for each setting is provided in Figure 10. It can be seen that if houses are allowed to submit combined bids, only about 70% of capacity would reach a utility. The results are robust across neighborhood sizes, although variance decreases in larger neighborhoods.



**Figure 10 – A comparison of bidding of device fleets and house fleets, where device fleets represent 100% of available capacity.**

The fluctuation in the share of capacity can be better understood when illustrated by examples of some of the shapes used in the simulation. Figure 11 shows a polytope

constructed from the devices mentioned above. Figure 11 (a) shows an example of devices that provide a small share of demand response. The first device is a 7 kW HVAC, and it has remaining charge of 50 minutes. But the next shape, a 5 kW PV battery, is almost discharged. Putting it on top of the HVAC would make a smaller inner sum, so it is placed to the right. It is followed by a 4.5 kW water heater, which also has the charge for only 20 minutes. The 1.1 kW pool pump concludes. This combination of device rated power and extent of charge causes the system to bid only the HVAC, or about 60% of available capacity. By contrast, Figure 11 (b) shows a different combination. First, the 7 kW HVAC system with a charge of 50 minutes is placed on the plane. But the 4.5 kW water heater now has a much higher charge, which allows to put it on top of HVAC, and the 1.1 kW pool pump is placed on top of it. The 5 kW battery has a small charge. Putting it on top of other devices would reduce the optimal area, so it remains to the right. The system now bids the sum of HVAC, water heater and a pool pump, bringing the total sum for the given house to almost 80%.



**Figure 11 – An example of conditions that affect the share of capacity a single house submits to a utility.**

This analysis shows that devices with higher charge make larger final polytopes, primarily because of their flexibility for trimming. The rated power of devices also plays a role in the size of a final polytope. A 5 kW water heater placed next to a 5 kW battery make a

union of a 5 kW device with larger remaining charge. Placing a 3 kW battery next to a 5 kW water heater does not allow for a similarly smooth polytope and reduces the optimal size.

Because the inner Minkowski sum is equal to or less than the total area of all rectangles that make it, some of the capacity is almost inevitably left out beyond the borders of the house as a battery. This capacity can be set aside for subsequent bid, but part of it would be lost due to self-discharge, or never bid due to optimization. The fact that bidding at a house level cuts about a third of available capacity (the actual share is 31.79-31.96%) shows that primary causes of bidding homes as batteries, privacy and simplified communications, come at their own price. More research is needed to estimate if this approach is economically justified.

## **5.2 Cost-benefit analysis for an individual household**

### *5.2.1 Introduction: background, inputs, expected outputs*

Despite the loss of capacity indicated in the previous section, an individual user may find it imperative to use a house battery equivalent for privacy or controllability reasons. If such a user still wishes to proceed with demand response services, it will face the next question: what are the costs and benefits associated with the use of the system.

To answer this question, we use a user-level simulation that estimates the yearly electricity bill under a baseline scenario and compares it to the yearly electricity bill when a demand response system is in place. The difference between the two numbers would represent the savings associated with demand response of the system based on current rates

and without additional payments such as incentives for demand response events. This value is tested for scenarios with varying levels of price penalty for consumption during half peak and peak hours.

The inputs required for the simulation include equipment characteristics, baseline hourly load profile for one calendar year, rate schedules, and constraints. Savings from the system in question would greatly depend on electricity prices and rate structures. Therefore, the simulation uses different rate structures to identify how savings change. There are in total three scenarios determined by three different residential electricity rates: a flat rate, a time-of-use rate, and a time-of-use rate with critical peak pricing option. The description of rate structures and load data is provided in the next subsection. It is followed by a description of constraints, methodology, and calculations. The subsection of results and discussion concludes this analysis. Conclusions are presented in section 5.3.

#### *5.2.2 Data: tariff structures, load schedules and peak events, constraints*

This section attempts to establish the benefits of a battery equivalent on a user level and to identify whether the system can be financially rewarding for an individual user. We use three scenarios to do so. The first is baseline scenario which uses a flat rate. The second scenario uses a time-of-use rate with moderate half peak and peak penalty. Further, an additional scenario of time-of-use rate with critical peak pricing is used to model high peak penalty and extremely high penalty. The rate structures for the suggested scenarios represent real tariffs selected in three states in 2019, followed by the rationale behind the selection of these specific examples.

Flat rate was adopted from United Power (CO). The rate is 10.15 c/kWh. It should be noted that true flat rates, i.e. those that do not change throughout the year or with consumption volumes are difficult to find. Most utilities use adaptations of volume-based rates in which they charge a smaller rate for all consumption under certain threshold and a higher rate for all consumption in excess. While the true flat rate was used for the purpose of the analysis, the same logic would apply to two-bracket flat rates, as long as the household stays within its consumption bracket for the most part of the year. Time-of-use rate was adopted from Austin Energy. Since Texas is the first state that deregulated retail electric service, most utilities in the state experiment with some forms of time-based or volume-based rates. Specifically, Austin Energy offers a very sophisticated rate structure that is summarized in Table 2.

**Table 2 – Rate structure for Austin Energy.**

Season	Weekday/weekend	Peak	Rate, c/kWh
Summer	Weekday	Off peak	2.893
		Half peak	2.915
		Peak	5.867
	Weekend	All day	2.893
Winter	Weekday	Off peak	2.732
		Half peak	2.952
		Peak	3.196
	Weekend	All day	2.732

Such a complex structure should allow to establish the benefits of demand response in general and helps understand where exactly the savings are coming from. Critical peak

pricing rate was adopted from Gulf Power (FL). The rate structure is as follows: off peak 6.5 c/kWh, half peak 9.6 c/kWh, peak 18.3 c/kWh, critical peak 78.5 c/kWh.

Time-of-use rates were applied to load data using the following logic. For every timestamp of the data, it was determined whether the month was summer or winter, and which rate applied to this hour of the day. Then a corresponding value of the rate schedule was selected. Critical peak rates which were available in Florida schedule could not be assigned by studying the time of day. Critical peak was believed to take place on hot summer days during the hours that would otherwise qualify as regular peak. It was assumed that a total of 5 critical peak events take place during the summer season, and those were scattered randomly at 6 p.m. throughout June, July, and August.

Using rate structures from three different states would raise the question of different cost of energy. It is evident that peak rate in Florida is much higher than in Texas, and the flat rate of Colorado is as high as half peak in Florida. This raises the question of endogeneity of consumption. If a household observes certain rates, it may adjust its behavior to accommodate rate peculiarities. This issue is handled through the use of behavior-neutral data. Baseline load profile is coming from OpenEI Datasets. It is a Department of Energy dataset that contains simulated data for a variety of locations in the US as of 2013 and reflects characteristics of a typical residential building for a given location. Its main advantage for the purpose of this study is exogeneity of data. The data is simulated based on external factors such as size of household, building properties or weather, and excludes behavioral patterns. Therefore, it is not rate dependent. Exogeneity of this dataset with respect to rates allows to impose any external rate on it and net out rate response effects. While the dataset is not very recent, it is built to the Department of Energy

methodology and offers sufficient granularity of data for the purposes of this study. The dataset contains total hourly load for a house and hourly load for largest load categories, such as HVAC, water heater, lighting, other appliances.

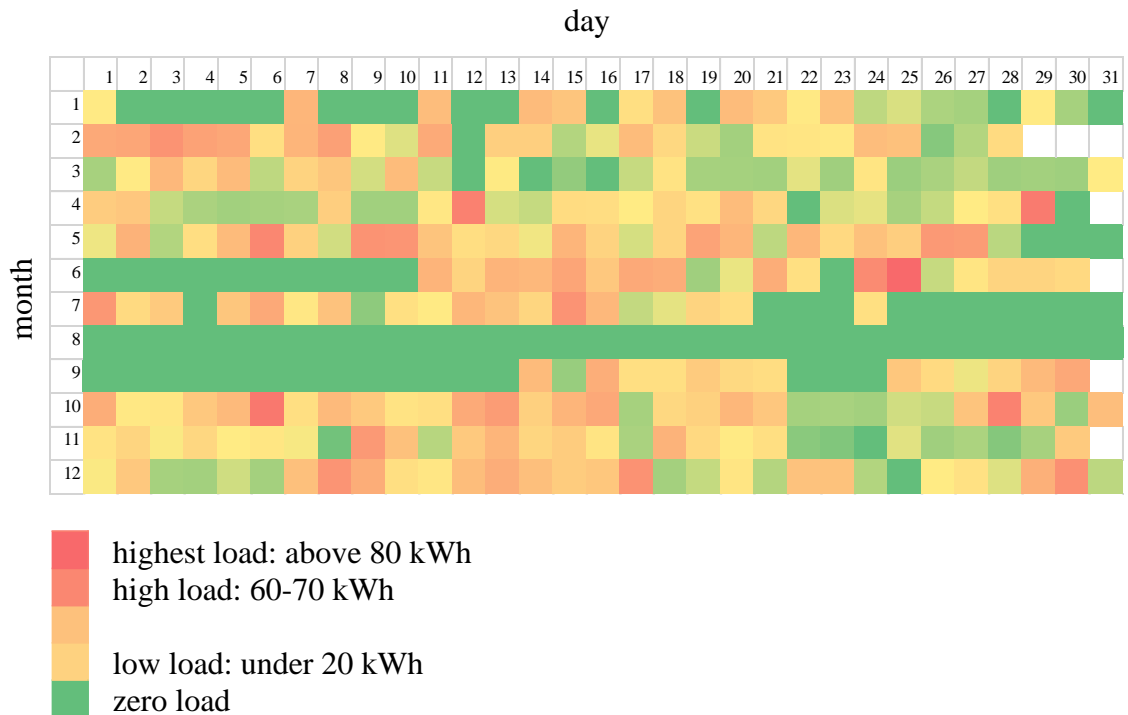
Additional devices, specifically a car charger, were not included in the original dataset. The rated power and hourly load of car chargers were adopted from a separate dataset. Pecan Street data contains detailed historical data on individual devices, including Level 2 car chargers, for 2018. The data on car chargers comes from Austin, TX. Since this is actual data from a real household, it must have been affected by the household's rate response and is endogenous from the perspective of behavior. This makes data less universal compared to the OpenEI files, but it seems to be the best option available. The original dataset contains entries for 15 minute intervals for 25 houses. However, data (car1) was present for only 8 houses (nn. 661, 1642, 2335, 4373, 4767, 6139, 7719, 8156). The remaining houses were excluded. Further, houses 2335 and 7719 had a very small consumption of electricity at car chargers, possibly indicating that chargers were idle, house 4767 had a consistent hourly consumption that was double the average, possibly indicating the presence of a fast charger or another car. Those houses were also excluded from the data. The remaining data were checked for outliers and appear in order, except a few missing hours. House 1642 was selected as the source of data because it had a sample average amount of kWh of charging during the year.

The final category of data that serves as input for the optimization problem is technical and user-related constraints. There is no precise information on the technical characteristics of the equipment used for the purposes of building the dataset. Those had to be estimated based on load profiles of individual devices. Nominal capacity was

established based on peak load of a given device. For HVAC, the estimated nominal capacity was 9 kW, for the water heater the estimated capacity was 2 kW, for the car charger the estimated nominal capacity was 12 kW.

User related constraints were difficult to establish. OpenEI did not contain a comprehensive description of modelling methodology, and Pecan Street had historical data, from which the constraints could only be guessed. Many of the previous studies listed in the Literature Review model comfort constraints by assuming user routines and assuming or analyzing weather patterns. While this approach is widespread, we chose not to impose a layer of additional assumptions about comfort constraints. Instead, we take the existing dataset and attempt to infer constraints from the data.

The only schedule-based device in the dataset is the car charger.

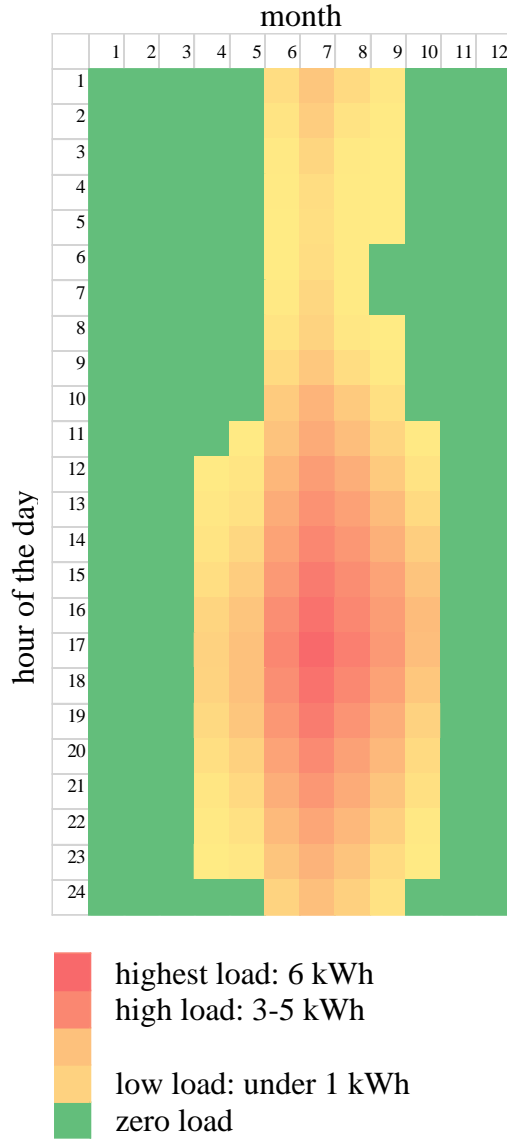


**Figure 12 – Patterns of energy consumption for the car charger by season and day.**

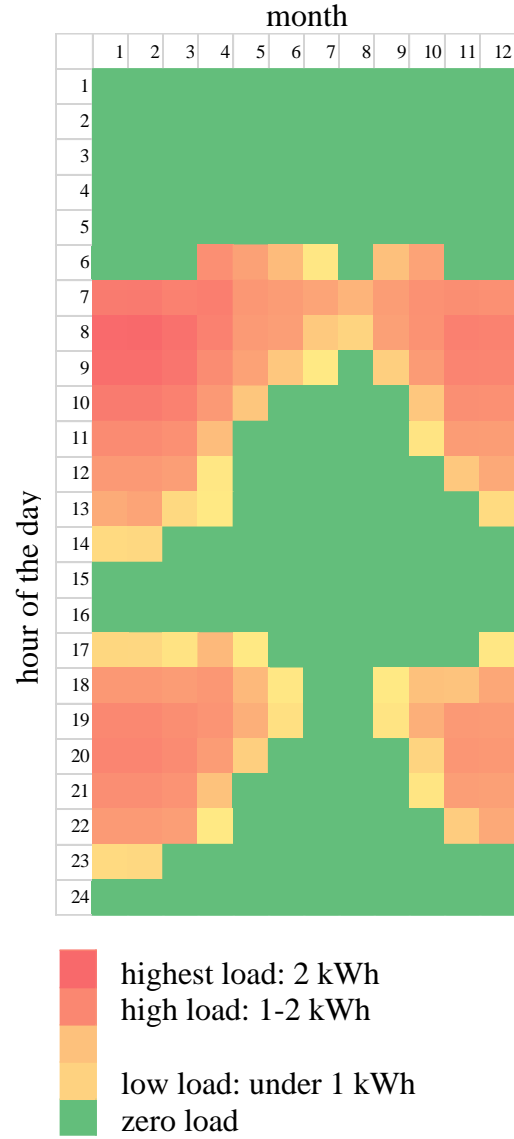
Since the car charger can work at any hour if it works for a total of 5 hours a day, the exact time of the day is less important. No specific pattern was observed in the data except that nobody used the car for almost two months in summer. This allowed to impose only day constraints.

Let us now observe thermostatically controlled devices. For them comfort constraints are more pressing because their charge is not enough to carry through to the night. Figure 13 shows the density of energy usage of HVAC (a) and water heater (b) for each hour of the day throughout the year. The usage allows to split the daily routine into several stages. The first stage is night, from hour 1 to hour 5. During these hours, devices are typically not used. The second stage starts at hour 6 and lasts through hour 10. This stage is related to morning activities. The HVAC is not active yet, but the water heater is used actively and reheats water after use. Hours 11 through 17 indicate day routine. The HVAC is active during summer. The water heater is less active during winter and inactive during summer. Hours 18 through 22 represent evening peak. The HVAC is most active, as is the water heater, indicating the intensive use of water and indoor spaces. Finally, hours 23 and 24 show a decline in the energy use of the HVAC and the water heater. These stages were used as comfort boundaries for each device. The optimization solver was allowed to shift load between hours as long as the sum of hours worked in each stage was kept equal to the sum in the same stage of baseline scenario. This discussion of constraints concludes the description of inputs to the optimization and allows us to move on the optimization problem itself.

a)



b)

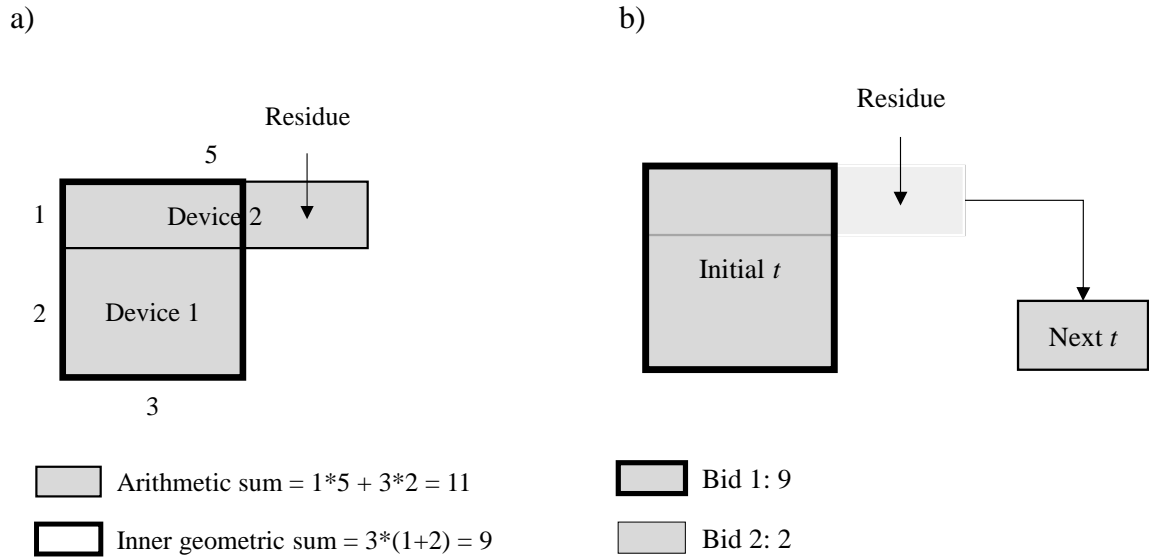


**Figure 13 – Patterns of energy consumption for heating, ventilation, and air conditioning (a) and water heater (b) by season and hour of the day.**

### 5.2.3 Methodology: constrained optimization of utility bills

The simulation of yearly savings from the system requires solving a constrained optimization problem. The objective function of the problem would represent the total cost of electricity for a given household, while the constraints would relate to physical and

behavioral nature of demand response. Before proceeding to the description of the objective function and constraints, it is important to emphasize the limits of conventional optimization algorithms for the purposes of this study. The previous chapters focused on maximization of geometric sum of demand response volume from various devices. Standard solvers that come in packages for programming languages and applications are designed to maximize arithmetic sum of demand response volume. The two numbers are not the same, as illustrated in Figure 14 (a).



**Figure 14 – The difference between arithmetic and geometric sum (a) and force bidding of residues of geometric sum to reach the volume of arithmetic sum (b).**

To handle this discrepancy between geometric and arithmetic sum, we impose an additional requirement for the purposes of this study by slicing demand response and forcing the system to bid residues after the main bid has been processed, as indicated in Figure 14 (b). This solution is not for granted in real life bidding. It potentially overstates the flexibility

of household bidding and in this study is treated as the upper bound of savings from the system.

The optimization is done for each day separately. Total cost of electricity for a given day is comprised of three separate values, each describing the cost of operating a specific device for 24 hours. The comfort and operational constraints are device specific and were partially discussed in the previous subsection. For all devices, the exact hours of operation are not specified. A car can charge for 1 hour several times a day or for several hours straight as long as it provides the same volume of charge as under normal operating conditions. The water heater and HVAC system are supposed to provide a specified amount hot water and cool air for user comfort but can shift their load within a specified stage of the day. For the purposes of this simulation, the water heater and the HVAC system are supposed to consume as many kWh of energy per stage of the day as they used to in the base scenario. The purpose of optimization for all devices is to keep the total load constant but shape it in a way that reduces the cost of energy for the entire day. For the car charger, this can be achieved by charging during off peak prices. The HVAC and water heater can shift load by precooling or preheating air or water.

The resulting problem formulation is as follows:

$$cost = \sum_i^3 \sum_t^{24} price_t * P_{it} \quad (6)$$

Subject to:

$$\sum_t^{24} P_{car\ t} = \sum_t^{24} P_{car\ base}$$

$$\sum_t^{T_j} P_{hvac\ t} = \sum_t^{T_{jbase}} P_{hvac\ t}$$

$$\sum_t^{T_j} P_{heater\ t} = \sum_t^{T_{jbase}} P_{heater\ t}$$

$$0 \leq P_{car\ t} \leq \max(P_{car})$$

$$0 \leq P_{hvac\ t} \leq \max(P_{hvac})$$

$$0 \leq P_{heater\ t} \leq \max(P_{heater})$$

Where

*cost*: total cost for the three devices in a given day

*price<sub>t</sub>*: residential electricity rate during hour *t*, c/kWh

*P<sub>it</sub>*: load of device *i* during hour *t*, kW

The total load of a car charger during the day equals its total load during the baseline day

*T<sub>j</sub>*, *j* = {1, 2, 3, 4, 5}: the stage of the day as deduced from data in subsection 5.2.2

The HVAC load during stage *T<sub>j</sub>* equals its load during corresponding baseline *T<sub>jbase</sub>*

The heater load during stage *T<sub>j</sub>* equals its load during corresponding baseline *T<sub>jbase</sub>*

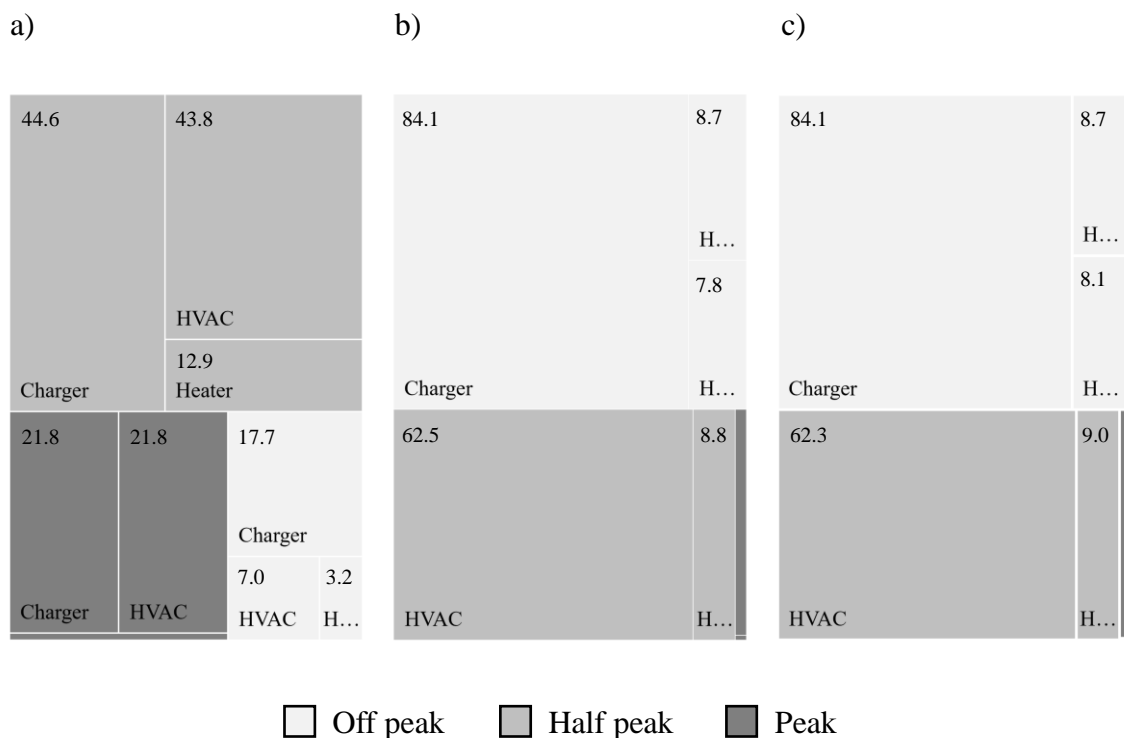
*max(P<sub>car</sub>)*, *max(P<sub>hvac</sub>)*, *max(P<sub>heater</sub>)*: maximum values of load for car charger, HVAC, and water heater in the dataset, used as a proxy for capacity constraints for respective devices.

It indicates the linear problem with step constraints for each of the intervals *T<sub>j</sub>*. There are a total of 365 days in the optimization. The optimization procedure was performed using VBA (Basic application) module in Excel. The code for the optimization procedure is provided in Appendix C.

#### 5.2.4 Results for different rate structures and discussion

With the methodology outlined in the previous subsection implemented, it is possible to compare the results of demand response. Figure 15 shows the breakdown of total load per device and category of peak. For the purposes of this breakdown, off peak hours were defined as per Austin Energy rate schedule: off peak hours 1-7 and 23-24, half

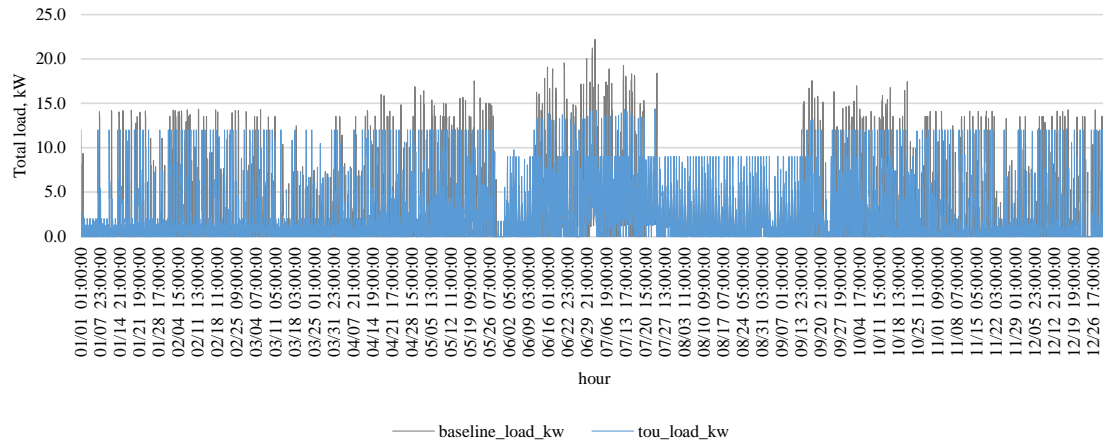
peak hours 8-15 and 19-22, peak hours 16-18. It is possible to see that charger load was pushed from peak or half peak hours to off peak. Both time-of-use and critical peak pricing scenarios still have some load taking place during half peak and peak hours. This load is due to comfort constraints on HVAC and water heater. Still, HVAC and water heater load have been seen to shift from peak to half peak, and only a tiny fraction of peak load remains due to HVAC constraints.



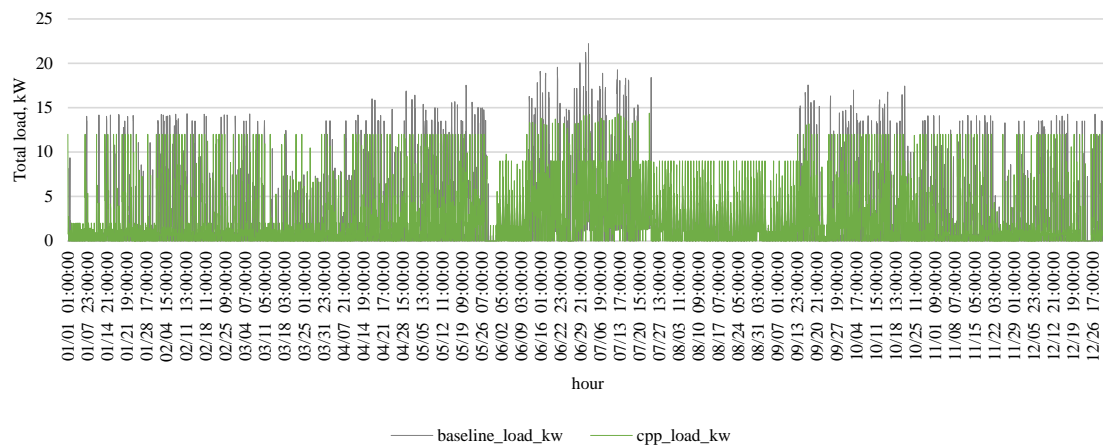
**Figure 15 – Share of load under flat rate (a), time-of-use rate (b) and critical peak pricing rate (c).**

A detailed hourly load profile is provided in Figure 16 (a – time of use, b – critical peak pricing). The graph shows that deviations mainly take place during peak and off peak hours and illustrates the peak shaving effect of the system.

a)



b)



**Figure 16 – Baseline and optimized load for time-of-use (a), critical peak pricing rate (b).**

A separate short paragraph is required to discuss the effect of critical peak pricing events in Figure 15 (b). The load during the five critical peak hours was 36.0 kWh in the baseline scenario and 3.3 kWh in the time-of-use scenario. In the critical peak pricing scenario this load declined to 0 kWh. This finding is intuitive and shows that extremely

high penalties force the system to allocate load to any possible hour outside of the critical peak.

Once the load estimates are available, it is possible to estimate savings from demand response. To do so, we adjust peak rates from the three states to the base of Colorado. In the baseline scenario, the total load is approximately 17423 kWh, resulting in the total electricity bill of about USD 1768, or about USD 147 per month. Much of this bill is due to the car charger, which alone consumes about 48% of electricity in the house. It is known that the ratio of peak to half peak to off peak for Texas is 2.14:1.17:1.08:1.07:1 in summer and winter. If the rates reflected the relative penalties of Texas and the baseline rate of Colorado, the demand response system would generate an electricity bill of USD 862, or about USD 72 per month. In the Florida example of high penalty, with the ratio of peak to half peak to off peak rates of 2.82:1.48:1 and 12.08:1 for critical peak, the electricity bill would equal USD 765 per year, or USD 63 per month. These findings are in line with the volume changes discussed above. The main driver of savings, car charger, allows for shifting the entire load from peak or half peak to off peak hours. But once charger-related savings are realized, the savings from thermostatically controlled devices are of incremental character. User comfort constraints prevent these devices from disconnecting, mainly in half peak hours.

The discussion of volumes and energy costs allows us to conclude that in the upper bound estimate, shifting load through demand response saves between 51% and 57% of total electricity cost for the three mentioned devices. Since these savings are mainly realized through shifting one device from evening peak to night off peak, it can be argued that in the presence of geometrical sum these numbers would not change much. Making

exact estimates of system payoff and break-even point would require the exact use of rates and a more thorough analysis of behavior patterns. But the preliminary findings provided in this section show that there exists a monetary incentive for users to adopt demand response systems. These findings are in line with the literature discussed above, which indicated the presence of savings from individual devices within home energy management systems.

## CHAPTER 6. CONCLUSIONS

This study was intended to answer three questions about the concept of house as a battery. The first question was, can the theoretical concept be implemented in practice. The study suggested a 2d bin packing problem with partial trimming and a recursive join algorithm to maximize the bid of an individual house. It proved that the algorithm is sufficiently compact and time-efficient to be used with basic hardware under the conditions of limited computational capacity. Further, the algorithm was tested by interacting it with controllable devices in an experimental house through the use of API.

The second question was, does the use of geometric sum as opposed to arithmetic sum reduce the total capacity from which a utility would obtain demand response service. The simulation on several typical devices showed that about 30% of total demand response capacity is lost due to packing. While it may be recovered through separate bidding of residues, the home as a battery approach certainly presents potential challenges in the form of capacity loss.

The third question was, will the demand response system be financially rewarding for an individual user. The one-year simulation based on endogenous load profile and a variety of potential rate structures shows that in the upper bound estimate, the approach allows for substantial savings, mainly due to shifting car charger load from peak and half peak to off peak hours. Other devices, including HVAC systems and water heaters, have been found to be less flexible due to user comfort constraints.

The future work in the stream of house as a battery could include more extensive practical research of the home energy management systems and development of a commercial prototype. It would be important to establish how to implement the proposed system in a secure cyber-physical environment. For instance, it would be important to understand how the system could transmit information between a utility and a home energy management system through a secure communication channel. It may be helpful to investigate the possibility to move from the current internet-based connection to a more reliable radio connection. It would be important to conduct a more thorough study of costs associated with implementing and supporting the system. This would include assessing hardware needs if the code is to run on a dedicated Raspberry Pi and use its own current sensors and breakers. The time and cost associated with retrofitting a house and preserving visual appeal would be of interest as well. Other areas of research include communication challenges of a demand response system, such as utility interface, or market implications of demand response such as user incentives.

## APPENDIX A. PACKING ALGORITHM

```
#####

# Run packing algorithm

#####

from copy import deepcopy
import matplotlib.pyplot as plt
import numpy as np

import time
def process_shapes(shapes):
    # Create lists for dimensions and b
    areas = []
    stored_shapes = []
    stored_height = []
    stored_width = []
    stored_areas = []
    storage = [stored_areas,
               stored_height,
               stored_width]

    # Create b that changes its shape depending on loads. It is a np.array of 0 that
    # provides the background grid
    # for shapes which are defined as 1
    a = []
    for i in range(len(shapes)):
        a.append(max(shapes[i]))
    dim = int(sum(a) + 1)
    b = np.zeros([dim, dim])

    # Store separate dimensions of each load, they might be optimal if used separately
    for i in range(len(shapes)):
        stored_height.append(shapes[i][1])
        stored_width.append(shapes[i][0])
        stored_areas.append(shapes[i][0] * shapes[i][1])
        b[0:int(shapes[i][1]), 0:int(shapes[i][0])] = np.full((int(shapes[i][1]),
int(shapes[i][0])), 1)
        b = np.copy(b)
        stored_shapes.append(b)
        b = np.zeros([dim, dim])

    find_next_shape(shapes, b, areas, storage) # this is the code that starts everything

    max_area = max(stored_areas)
    stored_areas = np.array(stored_areas)
    location_of_max_area = int(min(min(np.where(max_area <= stored_areas))))
    respective_height = stored_height[location_of_max_area]
    respective_width = stored_width[location_of_max_area]
    return max_area, respective_height, respective_width

# Remove the parts that stick out by axis h (1) to be able to add something at the right
def trim(b):
    # Find the convexity
    # The vector of nonzero value gives how wide is the device area
    a1 = np.zeros([len(b)])
    for i in range(len(b)):
        a1[i] = np.count_nonzero(b[i])
    # Additional vector that is used to locate where the next device sticks out
    a2 = np.zeros(len(b))
    for i in range(1, len(b)):
        a2[i] = a1[i - 1]
    a2[0] = a1[0]
    # The location that sticks out can be found by finding row and column
    # Suppose
    # this is the number that needs to be found
    # a1 = [5. 5. 5. 5. 4. 4. 4. 5. 5. 0. 0. 0. 0. 0.]
    # a2 = [5. 5. 5. 5. 5. 4. 4. 4. 5. 5. 0. 0. 0. 0.]
    try:
        row = int(np.array([np.where(a2 - a1 < 0)]))
        column = int(a1[row - 1])
        b[row][column]
```

```

        # This location indicates where to put a zero array
        a2 = np.zeros([int((len(b) - row)), int(len(b[0]) - column)])
        b[row:len(b), column:len(b[0])] = a2
    except TypeError:
        pass
    return b

# Find corners
def find_corners(b):
    # Find corners
    # Changes columns
    B1 = np.transpose(np.copy(b))
    c1 = np.zeros([len(B1)])
    for i in range(len(B1)):
        c1[i] = np.count_nonzero(B1[i])
    # Changes in rows
    B2 = np.copy(b)
    c2 = np.zeros([len(B2)])
    for i in range(len(B2)):
        c2[i] = np.count_nonzero(B2[i])
    c1 = np.unique(c1) # these are coordinates along P axis
    c2 = np.unique(c2)
    c2 = np.flip(c2) # these are coordinates along h axis
    return c1.tolist(), c2.tolist()

# Place a rectangle, trim so that it is within battery
def place_shape(c1, c2, current_shape, b):
    b[int(c1):int(c1) + int(current_shape[1]), int(c2):int(c2) + int(current_shape[0])] =
    np.full(
        (int(current_shape[1]),
         int(current_shape[0])),
        1)
    trim(b)
    return b

# Find area of the shape
def find_area_extended(new_b, storage):
    # Find all corners
    B1 = np.transpose(np.copy(new_b))
    c1 = np.zeros([len(B1)])
    for i in range(len(B1)):
        c1[i] = np.count_nonzero(B1[i])
    # changes in rows
    B2 = np.copy(new_b)
    c2 = np.zeros([len(B2)])
    for i in range(len(B2)):
        c2[i] = np.count_nonzero(B2[i])
    c1 = np.unique(c1)
    c2 = np.unique(c2)
    c2 = np.flip(c2)
    # Compare sizes
    area = np.zeros(len(c1))
    height = np.zeros(len(c1))
    width = np.zeros(len(c1))
    for i in range(len(area) - 1):
        area[i] = (c2[i] - c2[i + 1]) * c1[i + 1]
        height[i] = c1[i + 1]
        width[i] = c2[i] - c2[i + 1]
    max_area = max(area)
    # Find the respective height and width
    location_max = np.where(area >= max(area))
    width = int(min(width[location_max]))
    height = int(min(height[location_max]))
    storage[0].append(max_area) # area
    storage[1].append(height) # height
    storage[2].append(width) # width
    return width, height, max_area

# Call the code
def find_next_shape(shapes, b, areas, storage):
    if not shapes:
        pass
    else:
        c1, c2 = find_corners(b)
        for ix in range(len(c1)):
            new_shapes = deepcopy(shapes)
            new_b = np.copy(b)

```

```

        current_shape = new_shapes[0]
        new_shapes.pop(0)
        new_b = place_shape(c1[ix], c2[ix], current_shape, new_b)
        fflag = check_outlier(new_b)
        if fflag == 1:
            continue

        find_area_extended(new_b, storage)
        areas = find_next_shape(new_shapes, new_b, areas, storage)

    return areas

def check_outlier(new_b):
    b = np.array(new_b)
    array = np.sum(b, axis = 0)
    b1 = array
    nonconvex = any(np.diff(array) > 0)
    if nonconvex == True:
        return 1
    else:
        return 0

```

## APPENDIX B. CODE FOR CONTROLLING THE HVAC SYSTEM

```
#####  
# Check the current temperature and switch on or off depending on command  
#####  
  
import requests  
import json  
import time  
import pandas as pd  
  
# Create a file to store the access data  
credentials = json.load(open('C:/Users/Gaudi/Documents/thermostat_access.txt', 'r'))  
  
def renew_keys():  
    # Enter the system  
    data = {  
        "grant_type": "refresh_token",  
        "code": credentials['refresh_token'],  
        "client_id": credentials['api_key']  
    }  
    r = requests.post('https://api.ecobee.com/token', data = data)  
    r = r.json()  
    # Generate new code  
    credentials['access_token'] = r['access_token']  
    credentials['refresh_token'] = r['refresh_token']  
    # Write the code into txt  
    with open('thermostat_access.txt', 'w') as storage:  
        json.dump(credentials, storage, ensure_ascii = False, indent = 4)  
  
# Find current temperature  
def find_t():  
    header = {"Authorization": "Bearer {}".format(credentials['access_token'])}  
    req = {"selection":  
        {  
            "selectionType": "registered",  
            "selectionMatch": "",  
            "includeRuntime": True  
        }  
    }  
    param = {"format": "json", "body": json.dumps(req)}  
    r = requests.get('https://api.ecobee.com/1/thermostat', headers = header, params =  
param)  
    r = r.json()  
    current_t = r['thermostatList'][2]['runtime']['actualTemperature']/10.0  
    print(current_t)  
    return current_t  
  
# Change setpoint:  
def set_off():  
    header = {"Authorization": "Bearer {}".format(credentials['access_token']),  
        "Content-Type": "application/json",  
        "Accept-Charset": "UTF-8"}  
    commands = json.load(open("off.txt", 'r'))  
    r = requests.post('https://api.ecobee.com/1/thermostat', headers = header, data =  
json.dumps(commands))  
    #print(r.text)  
def set_cool():  
    header = {"Authorization": "Bearer {}".format(credentials['access_token']),  
        "Content-Type": "application/json",  
        "Accept-Charset": "UTF-8"}  
    commands = json.load(open("on.txt", 'r'))  
    r = requests.post('https://api.ecobee.com/1/thermostat', headers = header, data =  
json.dumps(commands))  
    #print(r.text)  
  
# Check the temperature and change setpoint  
status = []  
while True:  
    try:  
        current_t = find_t()  
        if current_t >= 72:  
            set_cool()
```

```
        status.append(1)
        print('switched on to cool {}'.format(time.time()))
    elif current_t < 68:
        set_off()
        status.append(0)
        print('switched off {}'.format(time.time()))
    else:
        pass
except Exception as e:
    print(e)
    renew_keys()
time.sleep(180)
```

## APPENDIX C. CODE FOR INTRADAY OPTIMISATION

```

Sub Macro1()
'
' Macro1 Macro
Application.DisplayAlerts = False
Application.EnableEvents = False
'
i = 3
While i < 9000

    SolverOk SetCell:="$V$" & i - 1, MaxMinVal:=2, ValueOf:=0, ByChange:="$G$" & i - 1 &
    ":$I$" & i + 23, _
    Engine:=1, EngineDesc:="GRG Nonlinear"

    SolverAdd CellRef:="$Z$" & i, Relation:=1, FormulaText:="$W$" & i          ' hvac
    between 0 and 9 kw
    SolverAdd CellRef:="$Y$" & i, Relation:=3, FormulaText:="$V$" & i
    SolverAdd CellRef:="$Z$" & i + 1, Relation:=1, FormulaText:="$W$" & i + 1 ' heater
    between 0 and 2 kw
    SolverAdd CellRef:="$Y$" & i + 1, Relation:=3, FormulaText:="$V$" & i + 1
    SolverAdd CellRef:="$Z$" & i + 2, Relation:=1, FormulaText:="$W$" & i + 2 ' charger
    between 0 and 12 kw
    SolverAdd CellRef:="$Y$" & i + 2, Relation:=3, FormulaText:="$V$" & i + 2

    SolverAdd CellRef:="$W$" & i + 5, Relation:=2, FormulaText:="$V$" & i + 5 ' hvac 1_5
    SolverAdd CellRef:="$W$" & i + 6, Relation:=2, FormulaText:="$V$" & i + 6 ' hvac 6_10
    SolverAdd CellRef:="$W$" & i + 7, Relation:=2, FormulaText:="$V$" & i + 7 ' hvac
    11_17
    SolverAdd CellRef:="$W$" & i + 8, Relation:=2, FormulaText:="$V$" & i + 8 ' hvac
    18_22
    SolverAdd CellRef:="$W$" & i + 9, Relation:=2, FormulaText:="$V$" & i + 9 ' hvac 23-
    24
    SolverAdd CellRef:="$W$" & i + 12, Relation:=2, FormulaText:="$V$" & i + 12 ' heater
    1_5
    SolverAdd CellRef:="$W$" & i + 13, Relation:=2, FormulaText:="$V$" & i + 13 ' heater
    6_10
    SolverAdd CellRef:="$W$" & i + 14, Relation:=2, FormulaText:="$V$" & i + 14 ' heater
    11_17
    SolverAdd CellRef:="$W$" & i + 15, Relation:=2, FormulaText:="$V$" & i + 15 ' heater
    18_22
    SolverAdd CellRef:="$W$" & i + 16, Relation:=2, FormulaText:="$V$" & i + 16 ' heater
    23-24
    SolverAdd CellRef:="$W$" & i + 18, Relation:=2, FormulaText:="$V$" & i + 18 '
    charger

    SolverSolve UserFinish:=True
    Range("AB" & i - 1).Value = 1

    Application.Wait (Now + TimeValue("0:00:02"))
    SolverReset
    i = i + 24
Wend

End Sub

```

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