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PERFORMANCE OF LIGHTWEIGHT MANIPULATORS UNDER JOINT VARIABLE FEEDBACK CONTROL: ANALYTICAL STUDY OF LIMITATIONS

Dr. Sabri Cetinkunt
Assistant Professor
Dept. of Mechanical Engineering
University of Illinois at Chicago
Chicago, IL 60680

Dr. Wayne J. Book
Professor
The George W. Woodruff School of
Mechanical Engineering
Georgia Institute of Technology
Atlanta, GA 30332

Abstract

The performance limitations of joint variable feedback controlled manipulators due to manipulator flexibility are studied in fine and gross motions. A finite dimensional time-domain manipulator model is used in the study. Fine motion analysis results agree very well with the previously reported results based on infinite dimensional frequency domain models. The limitations of a class of adaptive controllers in high speed gross motion control are studied. Manipulation speeds are quantified as *low* , *medium*, or *high* with reference to the arm flexibility and dynamic nonlinearities.

I. Introduction

Demand for higher industrial productivity requires manipulators that manipulate with higher speed and precision for a wide range of operating conditions, i.e. payload variations, task requirement, and environment interaction changes. Lightweight manipulators emerge as one alternative tool in achieving higher productivity because of their high speed manipulation capabilities compared to the traditional industrial manipulators. The drawback, however, is the structural vibrations arising in high speed, lightweight manipulators. Therefore, the motion control problem of lightweight manipulators has two parts: 1. the joint variable control, 2. the structural vibration control. The significance of structural vibrations depends on the task. For instance, if the manipulation speeds and external interaction forces are small compared to the structural flexibility, there will not be any significant vibration. Hence, there will not be any need for explicit control of vibrations. In most cases, lightweight manipulators must operate with high speeds, for the need for high speed manipulation is the main reason for their use. This means that there will be many task conditions where flexibility will be significant, and there will also be cases where flexibility will not be significant enough to be concerned with.

It is not yet clearly understood when the flexibility becomes significant and when one must be concerned with flexible vibrations. Therefore, very conservative rule of thumb design rules are suggested to guarantee that the flexibility will not be significant even in the worst possible cases [1,2]. This results in the underutilization of the existing capabilities of a robot. Book supported his design rules by explicit analysis of flexibility in fine motion [3]. However, results cannot be generalized to fast gross motions where dynamic nonlinear effects become significant relative to other dynamic forces.

The first objective of this work is to determine when a manipulator must be considered flexible and when it can be considered rigid. *The second objective* is to study the best performance that can be achieved by control algorithms using joint position and velocity feedback, for a given manipulator with structural flexibility. The conditions at which flexibility becomes significant and, the conditions at which the best performance is achieved, are not totally independent of each other. Now, we will clarify the difference between them.

We will designate that the arm flexibility starts to become significant when the behavior of the flexible arm starts to deviate from the behavior of an equivalent rigid arm under the same conditions. The behavior comparison will be quantified using the root locus analysis in fine motion, and the time domain simulations in high speed gross motions. Here, an equivalent rigid arm means that it has the same geometric and inertial properties, but has no structural flexibility.

The *best* performance of a joint variable feedback controller is defined as the highest closed loop bandwidth possible with damping ratios more than 0.707 value. Clearly, there may be conditions where behavior of flexible arm is quite different than that of rigid arm, and yet all dominant modes are well damped such that flexibility does not pose a problem.

The significance of solving these problems is in two-fold: first, for a given manipulator, one can determine the range of closed loop bandwidth for which the arm flexibility can be safely ignored, and the range where the flexibility of the arm must be taken into account: Second, the *best* possible performance of joint variable feedback controllers can be determined and the designer may not attempt to achieve higher performances. Furthermore, this result can be used as a reference to evaluate the relative merits of more sophisticated control algorithms that may employ sensory information about the flexible behavior of the arm in addition to the joint variables.

The reader is refered to [7,8] for a recent literature review in the dynamics and control aspects of lightweight manipulators.

II. Mathematical Model

Given the constant geometric parameters, the kinematics of the manipulator (Fig.1) is described by the joint variables, $\theta = [\theta_1, \theta_2]$, and deformation coordinate variables $w_1(x_1,t)$, $w_2(x_2,t)$, which are functions of spatial variable x_i. The spatial variable dependence of the deformation coordinates leads to a mathematical dynamic model that is of partial integro-differential equation form. In order to simplify the model, the deformation coordinates are approximated by a finite series which consists of shape functions multiplied by time dependent generalized coordinates. This results in a finite order dynamic system. Since the spatial variable dependence is already specified through the shape functions, the mathematical model is of ordinary differential equation form.

The dynamic model of a rigid manipulator, in general, has the form

$$[M(\theta)]\ddot{\theta} + \underline{f}(\theta,\dot{\theta}) + \underline{g}(\theta) = \underline{u} \quad (2.1)$$

Let us order the generalized coordinates as $q = [\theta, \delta]$, where $\theta = [\theta_1, \theta_2]$, joint variables, and $\delta = [(\delta_{11}, ..., \delta_{1n_1}), (\delta_{21}, ..., \delta_{2n_2})]$, deformation variables. The dynamic model of the flexible manipulator has the form

$$\begin{bmatrix} m_r & m_{rf} \\ m_{rf}^T & m_f \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{\delta} \end{Bmatrix} + \begin{Bmatrix} f_r \\ f_f \end{Bmatrix} + \begin{Bmatrix} 0 \\ [K]\delta \end{Bmatrix} + \begin{Bmatrix} g_r \\ g_f \end{Bmatrix} = \begin{bmatrix} I \\ B_m \end{bmatrix} u \quad (2.2)$$

where; $m_r(\theta,\delta), m_{rf}(\theta,\delta), m_f(\theta,\delta)$ are partitioned elements of generalized inertia matrix which is always positive definite, and symmetric, $f_r(\theta,\dot{\theta},\delta,\dot{\delta})$, $f_f(\theta,\dot{\theta},\delta,\dot{\delta})$ are coriolis and centrifugal terms which are quadratic in the generalized coordinate velocities $(\dot{\theta},\dot{\delta})$; $g_r(\theta,\delta), g_f(\theta,\delta)$ are gravitational terms; and $[K]$ is the structural stiffness matrix associated with arm flexibility and mode shape functions, u represents the effective torque (or force) input vector at the joints. For the two link arm example considered here $\theta = [\theta_1, \theta_2]$, and $\delta = [(\delta_{11}, \delta_{12}), (\delta_{21}, \delta_{22})]$.

The equation (2.2) is a highly nonlinear and coupled ordinary differential equation set. This makes the controller synthesis and design problem difficult. Furthermore, experiments indicate that the mode shapes of the beams quickly converge to the mode shapes of clamped-base beam under joint variable feedback control for even low values of feedback gains of interest [4]. All mode shapes of a clamped-base beam have zero slope at the base, therefore the $B_m = 0$ for the dynamics of flexible manipulators under feedback control. That means the joint variable controller effects the flexible variables through the coupling from joint variables, but not directly.

III. Linear and Nonlinear Analysis of the Flexibility Effects

The question of when the arm flexibility becomes significant and what limitations it imposes on the performance of joint variable controllers are studied first using linear techniques. Linear analysis results are valid only for the fine motions where nonlinearities are negligible. In order to determine the effect of dynamic nonlinearities (coriolis and centrifugal forces), linear and nonlinear control algorithms are simulated on the nonlinear model (2.2) for motions where nonlinear effects are larger relative to other dynamic forces of the system, such as inertial and gravitational forces.

III.1 Linear Analysis

Nonlinear model (2.2) is linearized about a nominal configuration, $x_n = [\theta, \delta, \dot{\theta}, \dot{\delta}] = [\theta_{nominal}, 0, 0, 0]$ and nominal input u_n which compensates for the nominal gravitational loading. Since nonlinear coriolis and centrifugal terms are quadratic in $\dot{\theta}, \dot{\delta}$, they have no contribution to the model that is obtained by linearizing about a nominal configuration where nominal values of $\dot{\theta} = \dot{\delta} = 0$. Let $\theta = \theta_{nominal} + \Delta\theta$, $\delta = \delta_{nominal} + \Delta\theta$, and $u = u_{nominal} + \Delta u$, then the linear dynamic model about the nominal configuration $x_{nominal} = [\theta_{nominal}, 0, 0, 0]$ is given by (3.1),

$$\underbrace{\begin{bmatrix} m_r & m_{rf} \\ m_{rf}^T & m_f \end{bmatrix}}_{M_{eff}} \begin{Bmatrix} \Delta\ddot{\theta} \\ \Delta\ddot{\delta} \end{Bmatrix} + \underbrace{\begin{bmatrix} \partial g_r/\partial\theta & \partial g_r/\partial\delta \\ \partial g_f/\partial\theta & \partial g_f/\partial\delta + [K] \end{bmatrix}}_{K_{eff}} \begin{Bmatrix} \Delta\theta \\ \Delta\delta \end{Bmatrix} = \begin{Bmatrix} \Delta u \\ 0 \end{Bmatrix} \quad (3.1)$$

In compact form, let $\Delta x = [\Delta\theta, \Delta\delta, \Delta\dot{\theta}, \Delta\dot{\delta}]$, the linear dynamic model about the given nominal configuration can be expressed as,

$$\Delta\dot{x} = A\,\Delta x + B\,\Delta u \quad (3.2)$$

where;

$$A = \begin{bmatrix} 0 & I \\ -M_{eff}^{-1} K_{eff} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M_{eff}^{-1} \begin{pmatrix} I \\ 0 \end{pmatrix} \end{bmatrix} \quad (3.3)$$

The closed loop eigenstructure of the linear model under linear joint variable feedback controllers is studied as function of feedback gains. The linear joint variable feedback control has the general form

$$\Delta u = -[K_{ij}]\Delta\theta - [C_{ij}]\Delta\dot{\theta} \quad (3.4)$$

For independent joint control;

$$[K_{ij}] = diag\{k_{ii}\}$$
$$[C_{ij}] = diag\{c_{ii}\}$$

For decoupled joint control;

$$[K_{ij}] = m_r(\theta_{nominal}, 0)\ diag\{k_{ii}\}$$
$$[C_{ij}] = m_r(\theta_{nominal}, 0)\ diag\{c_{ii}\}$$

Independent joint control results are presented here in order to compare with the previously reported ones. Position and velocity feedback gains of joint 1, (k_{11}, c_{11}), are set to very high values in order to force the joint 1 behave like a clamped base. The locus of closed loop eigenvalues are studied as a function of joint 2 feedback gains, k_{22}, c_{22}, (Fig. 2.a, 2.b). The finite dimensional linear model should be able to predict at least the dominant behavior of the closed loop dynamics of the infinite dimensional actual system, despite the errors introduced due to truncated dynamics. Otherwise the truncated finite dimensional model would not be of any value.

By comparing the root locus behavior of a given flexible manipulator with that of an equivalent rigid manipulator, the conditions at which flexibility becomes significant and the range of conditions where the flexibility can be ignored can be determined. The study of dominant behavior of closed loop eigenvalues will determine the best possible performance in fine motion.

III.2. Nonlinear Analysis

The effect of nonlinear coriolis and centrifugal forces on the significance of flexibility and the best performance of joint variable feedback controllers are studied using high speed motion simulations. The fundamental challenge in the control of space and industrial robots is to provide high speed, high precision motions despite large payload variations and external disturbances. Extensive research in the past decade has shown that adaptive control methods are potentially more promising to meet that challenge than the non-adaptive control methods. Therefore, the nonlinearity effects will be studied with an adaptive controller in the closed loop.

The main objective is to study the effect of nonlinearities on the flexibility problem, not the adaptive controller. Here, the adaptive control algorithm is directly stated. The design details and analysis of the adaptive controller can be found in [8].

Let us call $x_\theta = [\,\theta, \dot{\theta}\,]$. The adaptive control algorithm is given by, (Fig. 3),

$$u = -K_{pn}\, x_\theta + K_{un}\, u_m + \Delta K_p(e,t)\, x_\theta + \Delta K_u(e,t)\, u_m \qquad (3.5)$$

where;

$$K_{pn} = m_r(\theta, \delta_{st})\,[[k_{ii}],[c_{ii}]] \qquad (3.6.a)$$

$$K_{un} = m_r(\theta, \delta_{st}) \qquad (3.6.b)$$

$$\Delta K_p = \int_0^t p_{pi}\, m_r(\theta_o, \delta_{st})\, \nu\, x_\theta^T\, d\tau \qquad (3.6.c)$$

$$\Delta K_u = \int_0^t p_{ui}\, m_r(\theta_o, \delta_{st})\, \nu\, u_m^T\, d\tau \qquad (3.6.d)$$

$[k_{ii}]$, and $[c_{ii}]$ are the reference model dynamic components chosen by the designer, δ_{st} is the static deflection values of flexible modes. Here, the reference model is chosen as a decoupled linear system of the form

$$\begin{bmatrix}\dot{\theta}_m \\ \ddot{\theta}_m\end{bmatrix} = \begin{bmatrix} 0 & I \\ [-kii] & [-c_{ii}]\end{bmatrix}\begin{bmatrix}\theta_m \\ \dot{\theta}_m\end{bmatrix} + \begin{bmatrix}0 \\ I\end{bmatrix} u_m \qquad (3.7)$$

The response of the reference model, $\theta_m(t)$, to the commanded input, $u_m(t)$, is the desired joint response. The reference model dynamics affects the control through equations (3.6.a, c, d). Using δ_{st} in the control algorithm does not require real-time feedback information about the flexible states. Therefore, the controller is still a joint variable feedback control algorithm. The use of δ_{st} as opposed to 0 (zero) for the flexible modes is more accurate and improves the decoupled control of the flexible manipulator without imposing any significant implementation difficulty. ν is the filtered tracking error e (Fig.3). p_{pi} and p_{ui} are arbitrary, scalar adaptive controller design parameters effecting the convergence rate of the adaptive control system and the transient response of the closed loop system. The design advantages, performance improvements, and stability aspects of this algorithm are discussed in detail in [8]. In order to see the effect of dynamic nonlinearities, the closed loop system is simulated for two classes of motions: first, slow motions where nonlinear forces are small, and secondly, fast motions where nonlinear forces are significantly larger or of same magnitude with the other dynamic forces.

IV. Results and Discussion

IV.1. Linear Analysis Results and Discussion

Let w_{cc1} be the lowest structural natural frequency of the arm when both joints are clamped (k_{11} and $k_{22} \rightarrow \infty$, and $c_{11} = c_{22} = 0$, Fig.2). Consider an equivalent rigid manipulator, with same inertial and geometric properties of flexible manipulator except it is rigid. The rigid system with first joint clamped will be a second order mass-spring system with feedback gains $k_{22} \neq 0$ and $c_{22} = 0$. Let w_{r1} be the undamped natural frequency of the rigid system for a given position feedback value.

It is the ratio of w_{r1}/w_{cc1} that determines the significance of flexibility and the dominant behavior of the closed loop system. In rigid arm case it is possible to achieve arbitrarily large closed loop bandwidth (undamped natural frequency) by increasing k_{22}, for $w_{r1} = \sqrt{(k_{22}/(J_{o2})_{eff}}$, where $(J_{o2})_{eff}$ is the effective moment of inertia of link 2 and payload about joint 2 axis of rotation. However when the same controller is applied to the flexible arm, the closed loop bandwidth, w_{f1}, will be definitely smaller than w_{cc1}, that is as $k_{22} \rightarrow \infty$, $w_{f1} \rightarrow w_{cc1}$ (Fig.4). If servo stiffness is low relative to arm flexibility, that is $w_{r1}/w_{cc1} << 1/2$, the locus of closed loop eigenvalues is indistinguishable from those of rigid arm as c_{22} increases. However, if velocity feedback c_{22} is further increased to too large values, the effective result is to stiffen the joint. One dominant eigenvalue meets with another and breaks away from the real axis converging to the w_{cc1} on the imaginary axis as c_{22} increases (Fig.4 and 5.a). In rigid arm case this phenomenon does not exist for any value of feedback gains. The root locus analysis is done as a function of c_{22} for many other values of w_{r1}/w_{cc1} (Fig. 5.a, b, c). It is seen that above a critical value of the w_{r1}/w_{cc1} ratio, the dominant eigenvalues no longer able to reach the real axis (Fig.4, curve b). Physically that means, if the joint position control is too stiff relative to the arm flexibility, it is not possible to provide well damped dominant modes no matter how large the velocity feedback is.

For a given manipulator and payload, w_{cc1} is determined by the geometric, inertial and structural flexibility properties. If a joint variable controller attempts closed loop bandwidth larger than $1/2\, w_{cc1}$, then the flexibility of the arm will be significant. Otherwise the flexibility of the arm can be reasonably ignored and controller can be designed as if the arm were rigid (Fig.4, 5.a-c).

The *best* performance of a joint variable feedback controller is defined here as the highest possible dominant eigenvalues with damping ratio of 0.707 or more. As shown in Fig. 5.b, approximately $2/3\, w_{cc1}$ closed loop bandwidth value can be achieved by appropriate choice of joint variable feedback gains, k_{22} and c_{22}. It is important to note that the dominant eigenvalue locations are very sensitive to the variation of joint velocity feedback gain around the best solution (Fig. 5.b, between each point velocity feedback gain is incremented a constant amount).

Based on our definition of best performance, a manipulator is best utilized if its speeds are high to the point where the flexibility becomes significant, yet does not pose a problem due to well damped dominant modes. The results, concerning the flexibility significance and dominant closed loop dynamics, agree very well with the results based on infinite dimensional models of [3].

IV.2. Nonlinear Analysis Results

Fig.7 shows the response of the manipulator with adaptive controller to the desired slow motion. Two different adaptive control results are shown for *slow* and *fast* adaptation, refering to small and large values of the adaptation parameters p_{pi} and p_{ui}. The appropriate values for these parameters are found by trial and error. This motion has two properties: 1. dynamic nonlinearities are not significantly large (Fig.6, curve (a)), 2. the bandwidth of the desired motion is about 1/4 of the lowest natural frequency of the arm. The bandwidth of the desired motion, w_{mi}, is defined as the bandwidth of the reference model which generates the desired motion in response to a step command input (u_m in Fig.3).

Since the adaptive controller essentially tries to make the closed loop dynamic behavior equivalent to that of the reference model, the function of w_{mi} in the nonlinear analysis content is similar to the function of the

w_{r1} in the linear analysis. Clearly figures 7 and 8 show that flexibility of the arm is not significant in terms of joint tracking and settling time of flexible vibrations at the end of motion, which is in agreement with the linear analysis results. When the same system is simulated for motion (b) where $w_{mi}/w_{cc1} = 1/2$ and nonlinearities are significant (Fig.6, curve (b)), the response deteriorates. Persistent, lightly damped oscillations occur in joint and flexible mode variables (Fig. 9 and 10). The response of the system predictably gets worse for motion (c). The difference here is the nonlinear forces. According to linear analysis results, the performance of the system should be very good and flexibility should not be a problem, for the closed loop bandwidth is not too high ($w_{mi}/w_{cc1} = 1/2$). However, the performance is unacceptably poor and this is due to the dynamic nonlinear forces in high speed gross motion. Therefore, nonlinear effects impose further restrictions on the performance of joint variable feedback controllers in gross motions.

The mechanism through which the nonlinear forces affects the joint controller performance can be described as follows. If the nonlinearities are significant, the adaptive controller automatically adjusts its feedback gains through integral adaptation (3.6.c-d) to compensate for the tracking errors caused by the nonlinear forces. Increasing the controller gains through the adaptation rule eventually leads to very stiff joints. Linear analysis has shown that very high joint stiffness relative to the flexibility of a given arm results in very lightly damped dominant modes (Fig.4 curve (c), and Fig.5.c). Thus, lightly damped dominant modes are generated by the adaptive controller, while it is trying to compensate for the joint tracking errors caused by the large nonlinear forces. It is important to note that this mechanism is valid for the class of model reference adaptive controllers that use integral adaptation only.

IV.3. Further Discussion of Linear Results

In section IV.1., the break-in behavior of the eigenvalues of one higher mode to the real axis (Fig.5.a-c) was not discussed in order to focus on the results relating to the main questions dealt with in this paper. An eigenvalue, approaching the dominant mode region from the negative real axis direction, is needed in order to reflect the dominant behavior of manipulator dynamics under joint variable feedback control. It would be impossible to predict the dominant behavior without such an eigenvalue. For example, the break-away behavior of figure 5.a (and Fig.4, curve a, point C) can only happen with the aid of an additional eigenvalue. Similarly, the eigenvalue on the real axis of figures 5.b, and 5.c would not have been observed without such an additional eigenvalue. Since model is finite dimensional, the additional eigenvalue needed must originate at one of the higher modes. However it is unrealistic to expect that one of the higher frequency modes would have overdamped behavior through only joint position and velocity feedback, while the other modes have relatively small damping ratios and stay close to the imaginary axis.

The same dominant behavior is predicted by infinite dimensional frequency domain models [3]. However, the question of where the additional eigenvalue originated from did not arise, for the model was infinite dimensional. The eigenvalue problem has infinite number of solutions. In numerical calculations only a finite range of the s-plane was searched for the roots of the characteristic equation. As the feedback controller parameters vary, the number of eigenvalues found in the searched range varied too. Since there were an infinite number of eigenvalues, the question as to where the additional eigenvalue was coming from did not arise. Physically, the dominant behavior was explained by an analogy shown in figure 11.

Therefore, we conclude that the finite dimensional linear time domain model predicts the dominant dynamic behavior under joint variable feedback control very well, but at the expense of loosing accuracy in predicting the behavior of some of higher modes.

V. Conclusions

In fine motions and gross motions where coriolis and centrifugal nonlinear forces can be neglected, a given manipulator can be considered rigid if the controller does not attempt to reach closed loop bandwidth more than 1/2 of w_{cc1}, the lowest natural frequency of the arm with joints clamped. If the coriolis and centrifugal forces have comparable magnitudes with gravitational and inertial forces, the above conclusion is further restricted to 1/4 of w_{cc1}. In fine motion, the *best* performance of joint variable feedback controllers can be achieved up to 2/3 of w_{cc1} with damping ratios greater than 0.707, with the appropriate choice of feedback gains. However, it is important to note that the sensitivity of the dominant eigenvalues to the variations of joint feedback gains is highest in the best performance region (Fig.5.c, locations 8,9,10). Therefore, it may be difficult to guarantee 2/3 w_{cc1} closed loop bandwidth due to model inaccuracies. The linear analysis results obtained based on a finite dimensional time domain model agree very well with the results based on infinite dimensional frequency domain models.

The performance of an adaptive control algorithm is limited to a range $1/2 - 1/4\ w_{cc1}$ in high speed gross motions due to nonlinear effects. If the speed of motion were slow such that dynamic nonlinear effects were negligible, the adaptive controller would achieve a closed loop bandwidth up to $2/3 w_{cc1}$ in gross motions as well as in fine motions. If the nonlinearities become significant relative to other dynamic forces, the adaptive controller with integral adaptation automatically increases its feedback gains to compensate for the tracking errors caused by the nonlinear forces. As a result, joint stiffness increases and lightly damped dominant modes are generated. Through that mechanism, the nonlinear forces impose further limitations on the performance of model reference *adaptive* joint variable feedback controllers that use *integral adaptation*.

Acknowledgments

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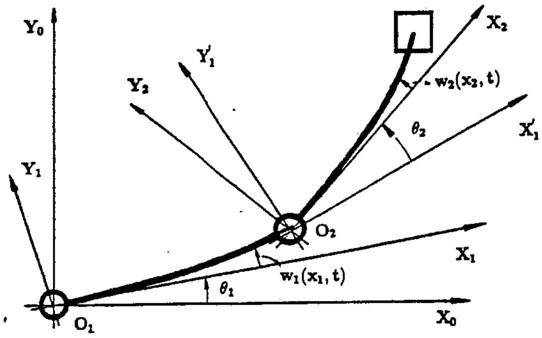


Fig.1 Two-link flexible manipulator example.

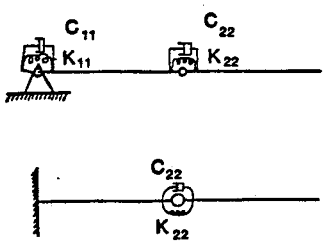


Fig.2 Locus of closed loop eigenvalues as function of joint feedback gains.

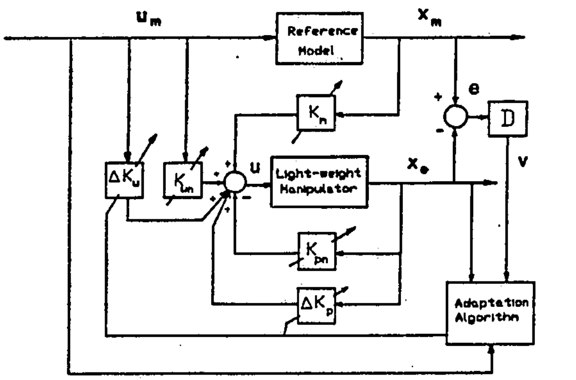


Fig.3 Generalized inertia matrix based AMFC

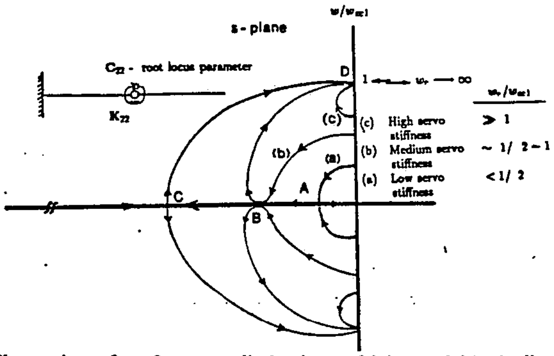
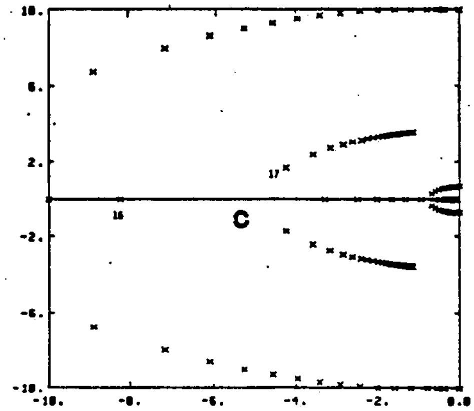
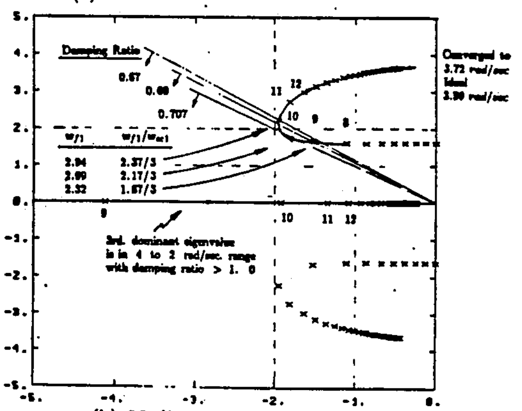


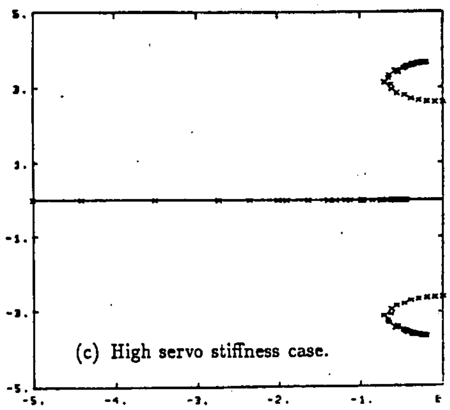
Fig.4 Illustration of performance limitations of joint variable feedback controller due to arm flexibility.



(a) Low servo stiffness case.



(b) Medium servo stiffness case.



(c) High servo stiffness case.

Fig.5 Locus of closed loop eigenvalues as function of joint 2 velocity feedback gain.

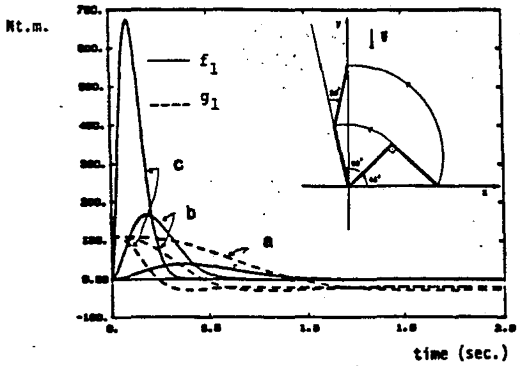


Fig.6 Relative importance of nonlinear (coriolis and centrifugal) forces and gravitational forces along different motions(only joint 1 responses presented, joint 2 responses are similiar).

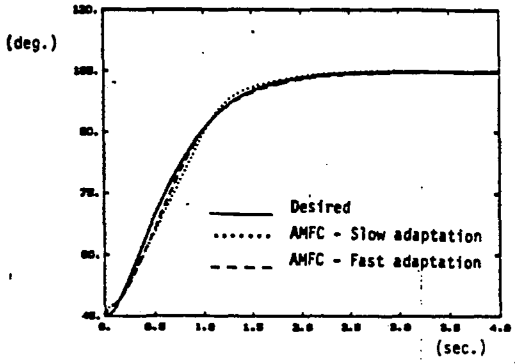


Fig.7 Joint angle 1 response under AMFC control for slow motions.

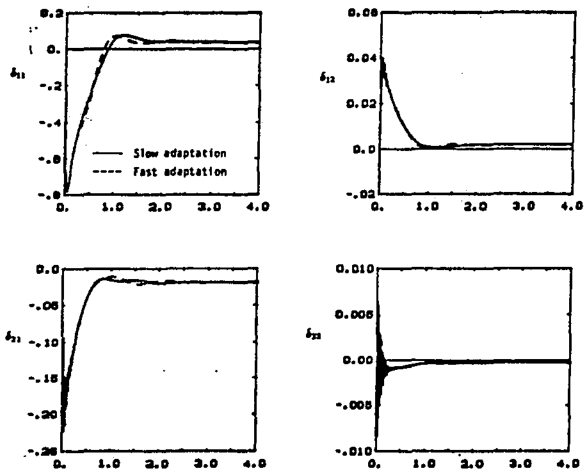


Fig.8 Flexible mode shape magnitude response along the motion shown in figure 7.

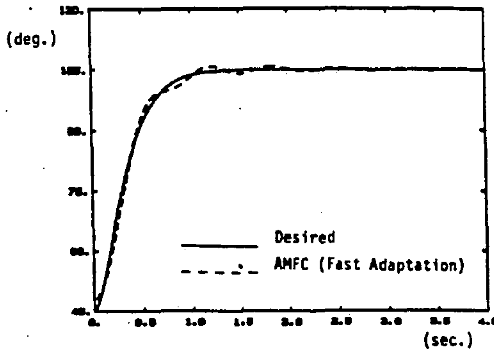


Fig.9 Joint angle 1 response under AMFC control for fast motions

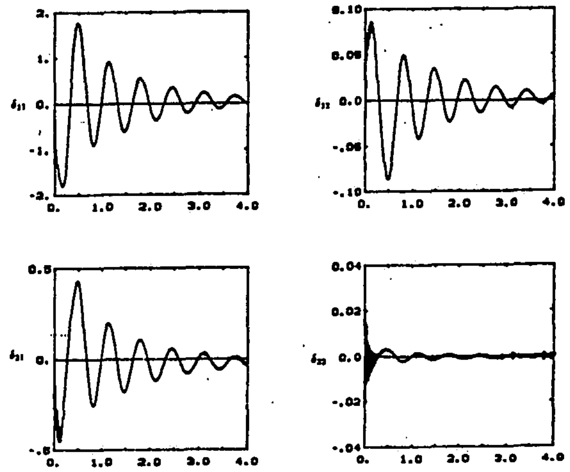


Fig.10 Flexible mode shape magnitude response along the motion shown in figure 9.

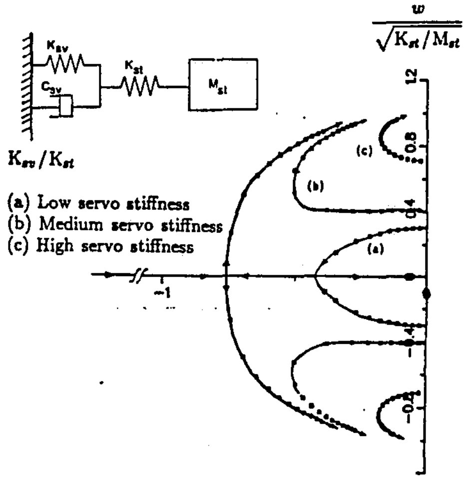


Fig.11 Analogy: Lumped parameter model exhibiting the similar root locus behavior as function of joint feedback gains.