AN INVESTIGATION OF THE RELATIONSHIP BETWEEN
THE COMPACTED DENSITY OF A COHESIVE
SOIL, LAYER THICKNESS AND COMPACTION FOOT WIDTH

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ABSTRACT

Recent research on the efficiency of different methods of soil compaction carried out at the Georgia Institute of Technology indicated that for a given soil at a constant water content and subject to a given contact pressure the density attained depended on the size of compaction device and the thickness of the layer being compacted.

The object of the investigation undertaken was to investigate the influence of the factors of compaction device size and compacted layer thickness on the density of a cohesive soil. The investigation was carried out on an analytical and experimental basis using circular compaction feet. The soil used was an orange-brown, sandy, silty clay of moderate compressibility.

In the analytical investigation the soil was assumed to be a perfectly elastic, homogeneous and isotropic material. Initially a purely analytical approach was attempted but this was later replaced by a method involving the use of a Newmark influence chart for vertical pressures beneath a surface loading. The mean pressure over a layer immediately below the compaction foot was calculated based on cylindrical and conical assumptions of pressure distribution throughout the layer. Curves showing the relationship between this pressure and the ratio of compaction foot radius to compacted layer thickness were developed.

In the experimental investigation the soil was compacted statically by circular compaction feet varying in diameter from one to four inches.

Contact pressures of 150, 200 and 250 pounds per square inch were used in the tests. Cylindrical sampling was used throughout. The dry density versus the ratio of foot radius to layer thickness relationships for each foot were found at 150 pounds per square inch and for the two intermediate feet at the two higher pressures.

The theoretical and experimental mean pressures over the compacted layer were compared.

These investigations led to the following conclusions. For a constant applied pressure the density increases with decrease of compacted layer thickness relative to foot radius but at a decreasing rate. The rate at which density increases and the densities attained depends on the foot size. As the compacted layer thickness decreases relative to foot radius the density attained approaches a constant value. This maximum density is attained with a relatively thicker compacted layer with the larger compaction feet. For compacted layer thicknesses roughly less than the foot radius the greatest densities are attained by the smallest feet. For compacted layer thicknesses roughly greater than the foot radius the greatest densities are attained by the largest feet. In addition, the layer thickness for each foot at which the density approaches the maximum increases as the contact pressure increases. Increase in pressure does not necessarily involve corresponding increases in density.

INTRODUCTION

Compaction as referred to soil means the act of densifying the soil. The soil grains themselves are largely incompressible but pressing of the soil particles closer together expels air and water from the soil mass and decreases the void spaces in the soil.

The excavation of soil masses in their natural state and the subsequent redeposition of these soil masses without compaction increases the average porosity, permeability and compressibility of the soil and greatly reduces the resistance to internal scour by water veins. Therefore since ancient times it has been customary to compact fills to be used as dams or levees. However no special attention was given to the compacting of highway embankments as the road surfaces were flexible enough to remain unharmed by the settlement of the fill.

This practice did not have any serious disadvantages until the beginning of the twentieth century when the demand for hard-surfaced roads was increased by the advent of the automobile. It was soon realized that roads on these uncompacted fills were liable to break up or become very uneven. The necessity of avoiding these conditions led to attention being focused on methods of economically and efficiently compacting the soil. At the same time the development of earth dam construction provided additional incentive for the development of construction techniques in the compaction of rolled fills.

Thus the object in compacting a soil is to improve its physical properties. In particular to increase its strength and bearing capacity, to reduce its compressibility and decrease its ability to hold water or to allow the passage of water through the soil.

The degree of compaction is expressed quantitatively in terms of dry density. This is defined as the weight in pounds of soil particles per cubic foot of moist soil.

It was not until a relatively recent date that means were developed for measuring and controlling the degree of compaction. It was Proctor (1) in 1933 who first published data to show that the effect of compaction on the dry density of a soil is dependent on the moisture content of the soil and the amount of compactive effort applied. The compactive effort may be defined as the work done on the soil per unit volume. He also showed that for a given compactive effort for each soil there is an "optimum moisture content" at which maximum dry density is obtained (Fig. 1).

A. A. Kelso, an Australian engineer, obtained similar results about the same time. His results are described in a paper submitted in January 1931 for publication but not published until 1934. (2)

In addition further investigation by the Corps of Engineers (3) and the Road Research Laboratory in Britain (4) among others revealed that by increasing the compactive effort an increase is obtained in the maximum density and a decrease in optimum moisture content.

Knowledge of the effect of compaction on the behavior of soil; the degree of compaction needed for various soils in various parts of a structure, the relative permanence of compaction; the degree of compaction which it is practicable to obtain; and methods for controlling compaction has increased rapidly since Proctor's early work. However much work is still required on the basic factors which control the effectiveness of compaction.

Recent research at the Georgia Institute of Technology on the efficiency of different

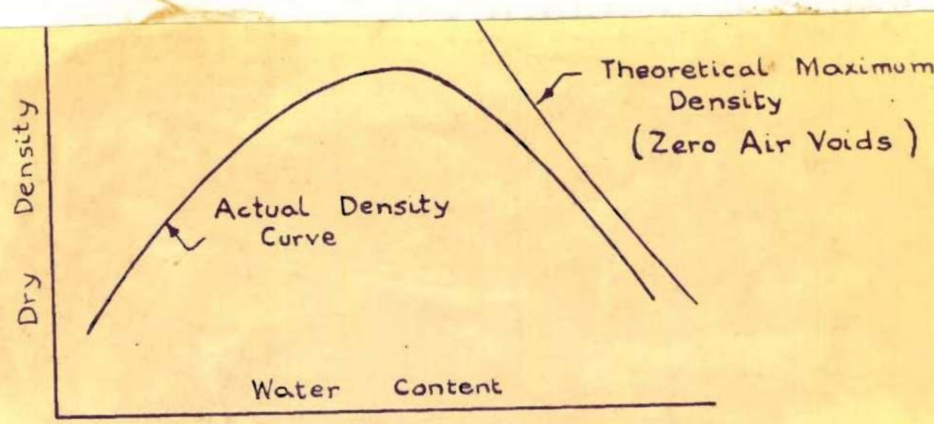

FIG. I
MOISTURE —DENSITY CURVE

methods of soil compaction (5) indicated that the density was influenced by the size of the compaction device and the thickness of the layer being compacted.

The purpose of this investigation was to find the effect of these factors on the compacted dry density of the soil.

When a pressure is applied to the surface of a soil mass this pressure is transmitted through the soil structure through the points of contact of the grains. This pressure between the grains causes many of them to move into more stable positions or squeezes them into the voids in the soil mass. The net effect is a reduction in the void space of the soil, that is, a densifying or compacting of the soil. If the soil were perfectly elastic,release of this applied pressure would result in an expansion of the soil back to its original volume. In actual fact there is no appreciable rebound of the soil after release of pressure.

However in the analytical investigation the soil was assumed to be a perfectly elastic material and using this assumption the stresses throughout the soil due to an applied pressure at the soil surface were calculated. This assumption was felt to be a justifiable one as soil does exhibit elastic properties to a limited degree. For example it is probably quite as elastic as concrete which is usually assumed to be elastic in structural analysis.

In the experimental investigation a pressure was applied statically by a compaction foot to the surface of an uncompacted layer of soil and the resulting density measured by sampling.

EQUIPMENT

The major items of equipment used in carrying out the experimental investigation are listed below. These included:

A. Hydraulic testing machine.

The compacting of the soil was carried out in a hydraulic testing machine (Fig. 2) having a total capacity of 4,000 pounds. The testing machine was fitted with a calibrated proving ring with an Ames gage, a reversing gear, and a speed-controlling mechanism.

B. Compaction mold.

The mold used (Fig. 2) was a large steel cylinder of ten and one-eighth inches internal diameter, five-sixteenths inch thick and twelve inches in height fitted with a detachable base plate.

C. Compaction feet.

Four steel feet were used having diameters of one, one and fifteen-sixteenths, two and fifteen-sixteenths, and three and thirteen-sixteenths inches. These feet were designated numbers one, two, three and four respectively and all had screwed attachments whereby they could be attached to the underside of the proving ring (Fig. 2).

D. Volume device.

The volume of the samples coated in paraffin wax was found by means of a small overflow tank which consisted simply of a open tin container with a short length of brass tube protruding from its side which acted as the overflow outlet. The volume of water displaced by the sample was collected in a graduated glass cylinder.

E. Soil.

The soil used in the tests was obtained from a pit behind the Civil Engineering building at the Georgia Institute of Technology. It was an orange-brown, sandy, silty clay of low plasticity and moderate compressibility.

Its specific physical properties are listed below.

Specific Gravity	2.70
Liquid Limit	38.7
Plastic Limit	23.4
Plasticity Index	15.3
Grain Size Distribution	(See Appendix C)
Standard Proctor Maximum Density	106.2 pounds per cubic foot
Optimum Moisture Content	19.6

Revised Bureau of Public Roads Classification A-6

Corps of Engineers Classifieation CL (an inorganic clay of low plasticity).

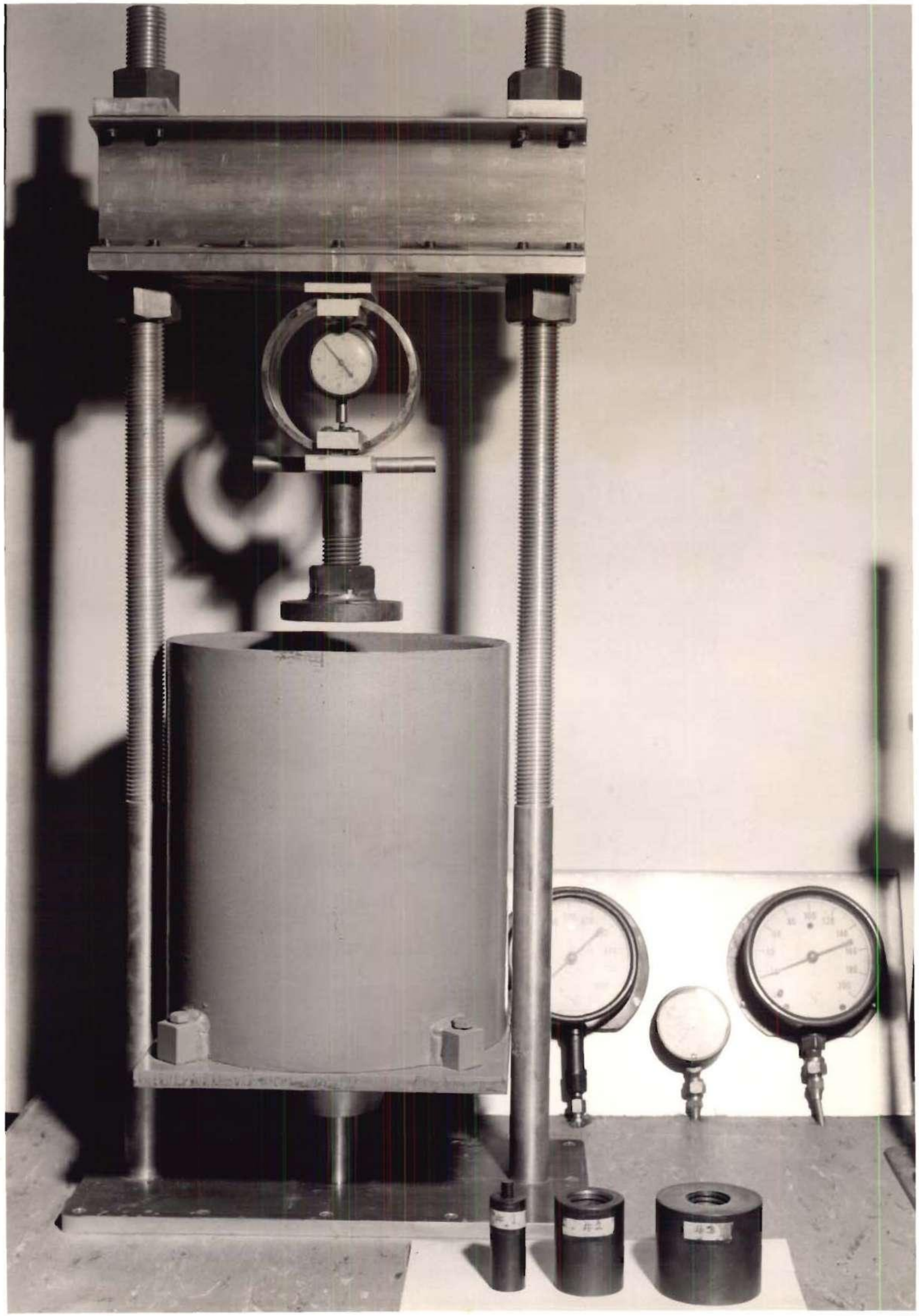

FIG.2 HYDRAULIC TESTING MACHINE WITH MOLD AND FOOT NO.4 IN POSITION. FOOT NOS. 1,2,3, ARE SHOWN IN THE FOREGROUND.

PROCEDURE

ANALYTICAL INVESTIGATION

The objective of this work was to investigate the relationship between the mean stress throughout a layer immediately below the compaction foot and the surface contact pressure.

The factors of importance can be established by dimensional analysis.

For if $\bar{p}$ = mean stress throughout layer of thickness $\underline{z}$,

p = surface contact pressure applied by compaction foot of radius $\underline{a}$.

Then for a given soil at a constant water content, $\bar{p}$ will depend on $\underline{p}$, $\underline{a}$, and $\underline{z}$.

Dimensionally,

$$\bar{p} = p^{\alpha} . a^{\beta} . z^{\gamma}$$

That is,

$$ML^{-2} = M^{\alpha}.L^{-2\alpha}.L^{\beta}L^{\gamma}$$

$$\alpha = 1 \quad , \quad \beta = -\gamma$$

Therefore $\bar{p} = p\,(a/z)^{\beta}$

That is $\bar{p}$ is a function of p and a/z or the mean stress depends on the surface pressure and the ratio of foot radius to layer thickness. The soil was assumed, for the purpose of analysis, to be a perfectly elastic, isotropic and homogeneous solid. The surface of the soil upon which the circular, uniformly loaded area acts was assumed to be the boundary of a semi-infinite solid.

Stress Distribution in Soil Mass. The problem was to find the stress distribution throughout a perfectly elastic, homogeneous, isotropic and semi-infinite solid due to a circular, uniformly loaded area acting on the plane which forms the boundary of the semi-infinite solid.

Solutions for the strains involved have been given by A. E. H. Love (6) and H. Lamb (7) but the conversion of these to give an expression for vertical stress was thought to be outside the scope of this thesis.

However an expression for vertical stress has been developed by K. Terazawa (8). It was

$$\widehat{zz} = -\frac{\Pi z}{\pi a}\int_0^\infty e^{-kz} J_0(kr) \cdot J_1(ka) \cdot k \, dk - \frac{\Pi}{\pi a}\int_0^\infty e^{-kz} J_0(kr) \cdot J_1(ka) \, dk \qquad (1)$$

where $\widehat{zz}$ = vertical stress at depth $\underline{z}$ below surface and at radius $\underline{r}$ from centerline of foot.

a = foot radius.

Π = total applied load

By using the substitution $R = \sqrt{(a^2 - 2ar\cos\theta + r^2)}$

these integrals are considerably simplified.

$$\int_0^\infty e^{-kz} \cdot J_0(kr) \cdot J_1(ka) \cdot dk = \frac{1}{\pi}\int_0^\pi \frac{(a - r\cos\theta) \cdot d\theta}{R^2} - \frac{z}{\pi}\int_0^\pi \frac{(a - r\cos\theta) \cdot d\theta}{R^2\sqrt{(R^2 + z^2)}}$$

and $$\int_0^\infty e^{-kz} \cdot J_0(kr) \cdot J_1(ka) \cdot K \cdot dk = \frac{1}{\pi}\int_0^\pi \frac{(a - r\cos\theta) \cdot d\theta}{(R^2 + z^2)^{3/2}}$$

therefore,

$$\widehat{zz} = -\frac{\Pi z}{\pi^2 a}\int_0^\pi \frac{(a - r\cos\theta) \cdot d\theta}{(R^2 + z^2)^{3/2}} - \frac{\Pi}{\pi^2 a}\int_0^\pi \frac{(a - r\cos\theta)\, d\theta}{R^2\sqrt{(R^2 + z^2)}}$$

$$- \frac{z\Pi}{\pi^2 a}\int_0^\pi \frac{(a - r\cos\theta) \cdot d\theta}{R^2\sqrt{(R^2 + z^2)}} \qquad (2)$$

For the case when r equals zero, that is for the vertical stress along the centerline of the circular area,
Therefore (2) becomes

$$\widehat{zz} = -\frac{\Pi z}{\pi^2}\int_0^\pi \frac{d\theta}{(a^2 + z^2)^{3/2}} - \frac{\Pi}{\pi^2 a}\int_0^\pi \frac{d\theta}{a} + \frac{\Pi z}{\pi^2}\int_0^\pi \frac{d\theta}{a^2\sqrt{(a^2 + z^2)}}$$

If p = surface contact pressure, then $\Pi = \pi a^2 p$

If $\widehat{zz}$ is replaced by the more customary symbol p_z then,

$$p_z = -\frac{p z a^2}{(a^2 + z^2)^{3/2}} - p + \frac{pz}{\sqrt{(a^2 + z^2)}}$$

$$= -p + \frac{p z^3}{(a^2 + z^2)^{3/2}}$$

Therefore $$p_z/p = -1 + \frac{1}{(1 + (a/z)^2)^{3/2}}$$ (tensile stress is taken as being positive.)

or, $$p_z/p = 1 - \frac{1}{(1 + (a/z)^2)^{3/2}} \qquad (3)$$

(where compressive stress is taken as being positive.)

This is the familiar expression for the vertical pressure p_z beneath the centre of a circular area of radius a uniformly loaded with intensity p on the surface of an elastic, homogeneous and isotropic material bounded by a plane. It is normally derived by the integration over a circular area of Boussinesq's point load formula.

Though the expression for vertical stress given by equation (2) is considerably simpler than (1), it is still exceedingly complicated as the three integrals involved are elliptic in form. Initially it was decided to give r and z specific values in terms of a and graph the functions under the integral signs. The area under each of these functions was to be found. The sum of the product of these areas and the factors outside the integral signs would then have given the value of vertical stress for the designated r and z value. This method was rejected because of the excessive amount of labor involved.

The method finally adopted was based on the use of Newmark's influence chart for the computation of vertical pressures beneath a surface loading (9). Newmark constructed this chart by use of equation (3). He gave various values to pz/p and found the corresponding values of r/z. Then after assigning a scale value z, the various radii corresponding to the different values of pz/p were calculated and drawn. The resulting rings were further subdivided radially. A typical Newmark chart is shown in Figure 3.

Using this chart, curves of stress as a percentage of surface contact pressure versus depth in radii were drawn for various distances from the center of the loaded area.

To draw one such curve, for example the curve corresponding to variation of stress with depth at a distance of three times the radius of the foot from the center of the loaded area the procedure would be as follows (Fig.3). Firstly assign a value of a/2 say, to the scale length and using this scale draw a circle of radius a on the influence chart at a distance of 3a from the center of the chart. The product of the number of areas within the foot area and the influence value for the chart would give the stress at a depth of a/2 below the surface and at a distance of 3a from the center of the circular foot. By assigning values of a/2, a, 3a/2, 2a----etc., to the scale length and repeating the above procedure the required curve can be drawn.

Similar curves can be drawn for various distances from the center of the foot. Because of the rapid decrease in stress with depth a semi-log plot was used. (Appendix A. Fig. 11).

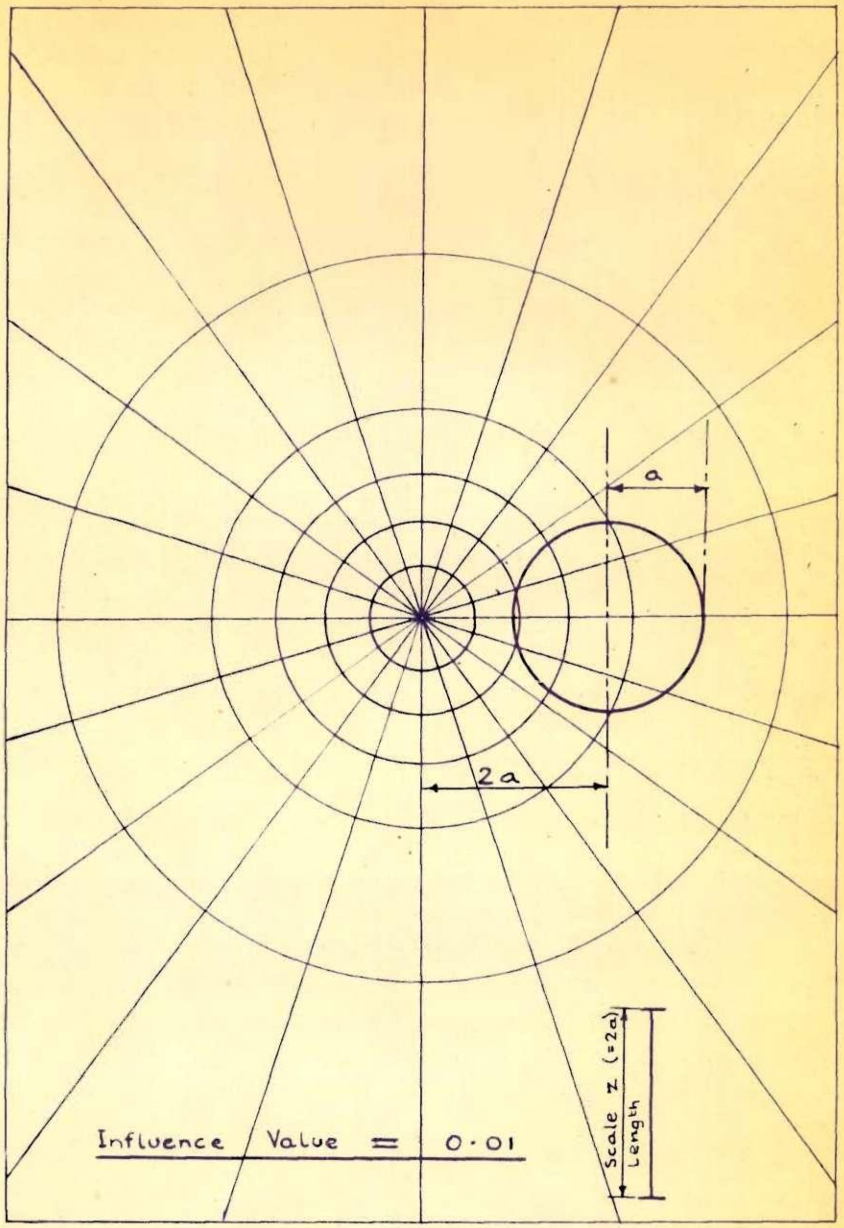

FIG. 3
VERTICAL STRESS DISTRIBUTION BY
NEWMARK INFLUENCE CHART.

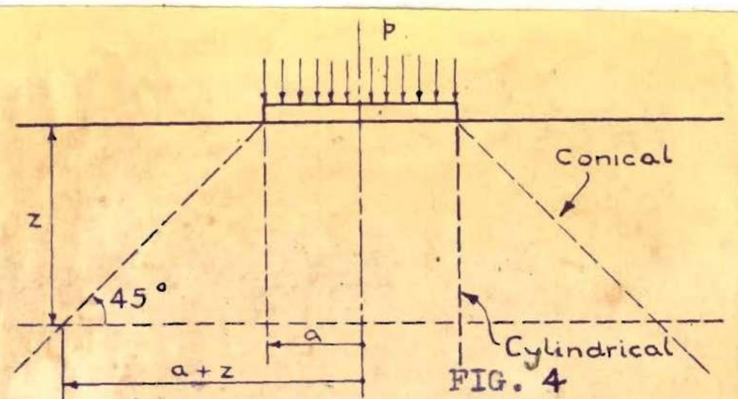

FIG. 4

MEAN OR AVERAGE PRESSURE AT DEPTH Z .

Mean or Average Stress at Depth z. Using the curves of stress versus depth for various values of the distance from the center of the loaded area it was possible to draw curves showing the variation in pressure with distance from the foot center for various depths. This was accomplished by plotting, for each depth being considered, the values of stress corresponding to the various radial distances against these radial distances (Appendix A, Figures 12, 13).

It was next necessary to find the mean stress at each of these depths. This was done on the basis of:

(i) all of the stress being within a truncated cone with a side slope of forty-five degrees. The assumption is that all of the load is supported by the stresses within the truncated cone. This assumption compares very closely with the distribution of actual significant stresses, as can be observed both from the stress curves (Figures 12, 13) and by observation of the compacted samples (Figure 9).

(ii) the stress within a cylinder of soil immediately below the circular area and having the same radius as this area, alone being used to calculate the mean stress. This approach was used to obtain a theoretical equivalent to the densities obtained by cylindrical sampling.

The formulae used were derived as follows:

From Fig. 4

If $\underline{p}$ = stress at a distance $\underline{r}$ from the center line of the foot and depth $\underline{z}$.

Then the load carried by a ring of radius $\underline{r}$ and thickness $\underline{\delta r}$

$$= p.\,2\pi r.\,\delta r$$

Therefore the total load $= \int_0^\infty p.\,2\pi r.\,dr$

= applied load.

If $\bar{p}$ = mean stress at a depth z and this stress acts out to radius $\bar{r}$,

$$\bar{p}\bar{r}^2 = 2\int_0^\infty p.r.\,dr$$

Using the conical assumption, $\bar{r} = z + a$

Therefore,

$$\bar{p} = \frac{2\sum_0^\infty p.r.\,\Delta r}{(a+z)^2} \tag{4}$$

Based on the cylindrical assumption, $\bar{r} = a$

$$\bar{p} = \frac{2\sum_0^a p\,r.\,\Delta r}{a^2} \tag{5}$$

Using these relationships the mean pressure at various depths was calculated based on a unit contact pressure. Curves of mean pressure versus the depth z expressed in radii were drawn (Appendix A Fig. 14) based upon both conical and cylindrical assumptions.

Mean or Average Pressure over Layer. To find the mean pressure over a layer of thickness z immediately below the loaded foot, the thickness z was divided into the area bounded by the curve of mean pressure vs. depth, the axes, and the line corresponding to the depth of the layer. These areas were measured by planimeter. The mean pressures for various layer thicknesses were calculated for both assumptions and finally curves of mean pressure based on a unit contact pressure versus the ratio of foot radius to layer thickness were drawn. This was done for both conical and cylindrical assumptions.

EXPERIMENTAL INVESTIGATION

The soil to be used in the tests was obtained from a pit behind the School of Civil Engineering. It was brought indoors and allowed to air dry by placing in large flat pans. The soil was passed through a U. S. Standard Number 4 Sieve. (square 0.185 inches openings). The larger lumps were broken up by hand. The soil retained was broken up and resieved as far as possible. After resieving the soil retained on the screen was discarded while all passing was mixed thoroughly to insure uniformity.

Standard classification tests were carried out on the soil including a grain size test, a specific gravity tests and liquid and plastic limit determinations (10). Tests were run to determine the water content of the soil. (10)

A standard Proctor test was carried out to determine the maximum density and optimum moisture content of the soil (10).

The amount of water to bring the soil up to a water content about four percent below the optimum moisture content was calculated. This water was added to the soil and mixed thoroughly to achieve a uniform distribution of moisture throughout the soil. The soil was placed in sealed containers and the water content checked by standard tests.

Soil was placed in the mold in thin layers and compacted with a standard Proctor hammer until a compacted layer having a total thickness of approximately five inches was obtained. Care was taken to achieve a flat level surface of compacted soil. A layer of uncompacted soil was next placed in the mold and levelled by hand. The mold was then positioned in the hydraulic testing machine and by the use of a wooden piston which fitted loosely inside the mold a preconsolidation pressure of ten pounds per square inch was applied to the soil to ensure uniform conditions throughout the uncompacted layer.

The wooden piston was removed and the compaction foot to be used fitted on the proving ring. The foot was then applied to the soil until the requisite pressure was attained. The pressures used throughout the tests were 150, 200 and 250 pounds per square inch which are typical of the range of widely used compaction equipment. The foot was withdrawn from the soil and the mold removed from the testing machine. On removal the loose soil around the compacted portion of the upper layer was scooped out until this portion and the permanently compacted layer below were all that remained. The remaining part of the upper layer in general resembled a truncated cone with bulging sides as would be expected from theoretical considerations. Initially attempts were made to sample this truncated cone but this proved impracticable due to the loose nature of the soil on its outside. Thereafter cylindrical sampling alone was used and this proved simple and efficient. The sample was separated from the lower layer by exerting a light lateral pull applied by the fingertips to the sample sides.

After removal the sample was trimmed and its thickness measured. This thickness was that of the newly compacted layer. The sample was weighed accurately, coated in paraffin wax and weighed again. The volume of the coated sample was found by the displacement of water. For each sample three or more volume determinations were made and the mean value tabulated. This volume less the volume of the paraffin wax was taken as the volume of the sample.

Two water content determinations were made for each sample where the sample size permitted this.

The dry density of each sample was calculated from the relationship,

$$\gamma_D = \frac{\gamma}{1+w} \quad ,$$

where γ_D = dry density in pounds per cubic foot ;

γ = unit weight of soil in pounds per cubic foot ;

w = water content.

Curves of dry density versus the ratio of compaction foot radius to layer thickness were drawn for the four compaction feet at each of the three pressures. (Figs. 5, 6, 7)

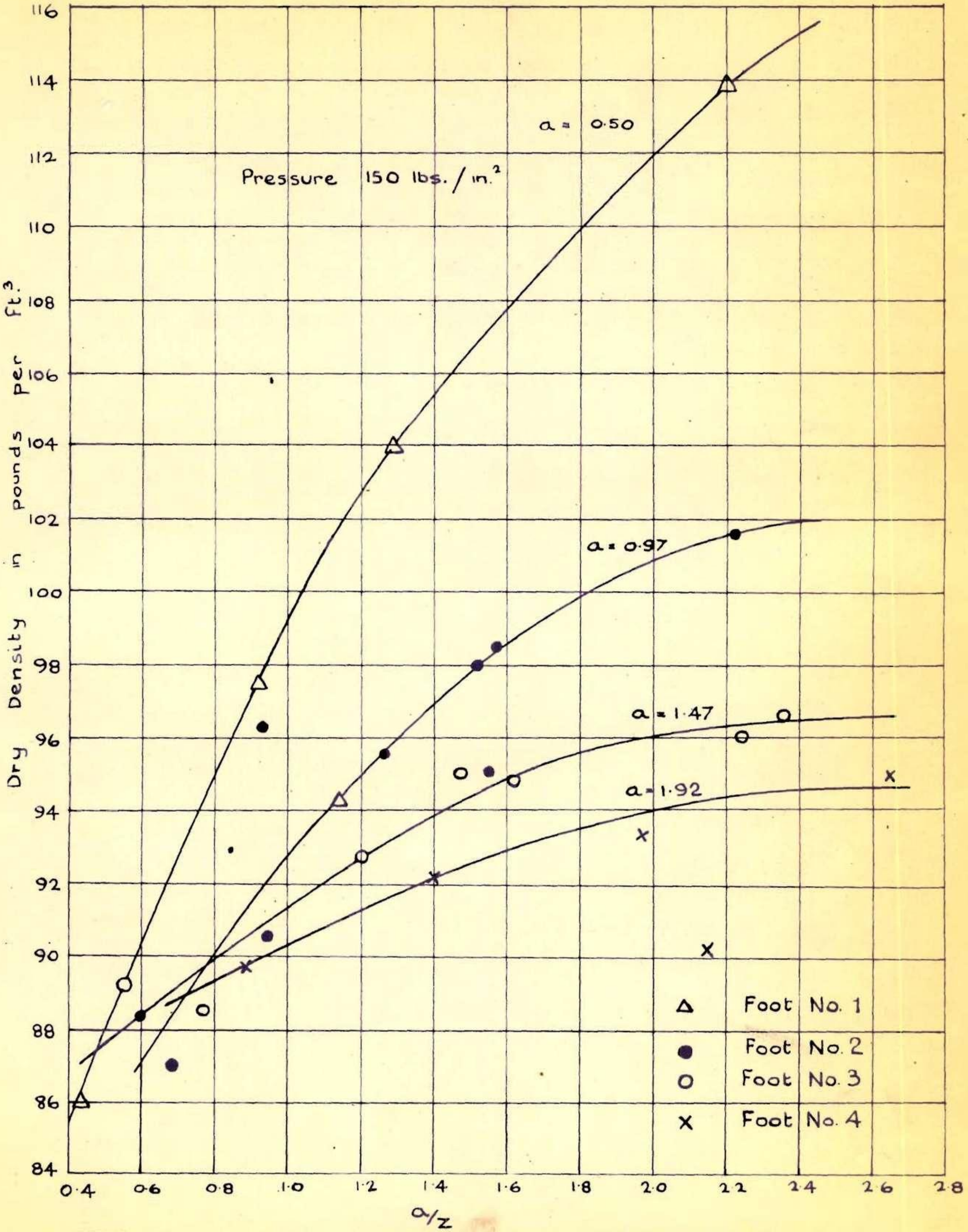

FIG. 5 EXPERIMENTAL DENSITY, LAYER THICKNESS, AND FOOT RADIUS RELATIONSHIP FOR 150 POUNDS PER SQUARE INCH CONTACT PRESSURE.

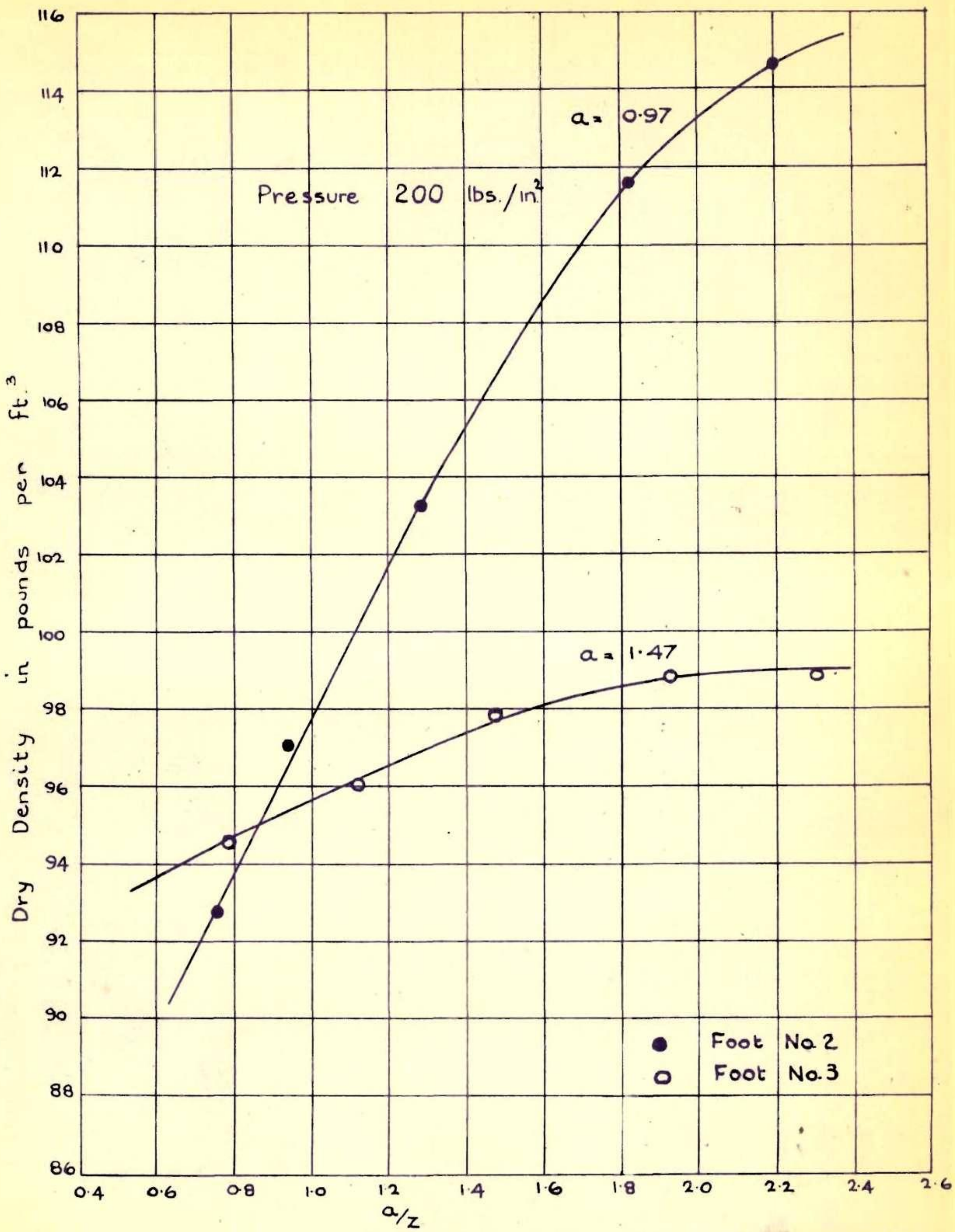

FIG. 6 EXPERIMENTAL DENSITY, LAYER THICKNESS, AND FOOT RADIUS RELATIONSHIP FOR 200 POUNDS PER SQUARE INCH CONTACT PRESSURE.

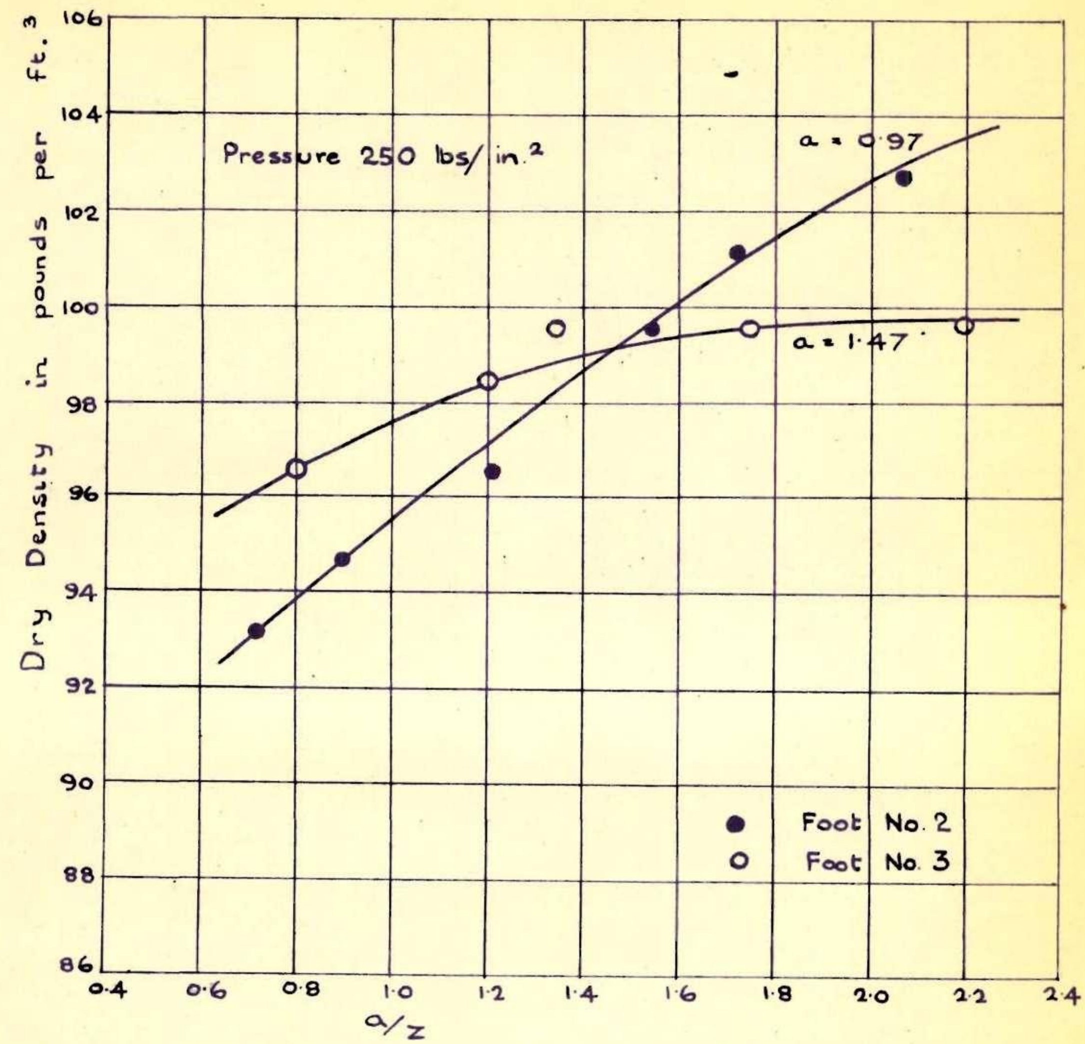

FIG. 7 EXPERIMENTAL DENSITY, LAYER THICKNESS, AND FOOT RADIUS RELATIONSHIP FOR 250 POUNDS PER SQUARE INCH CONTACT PRESSURE.

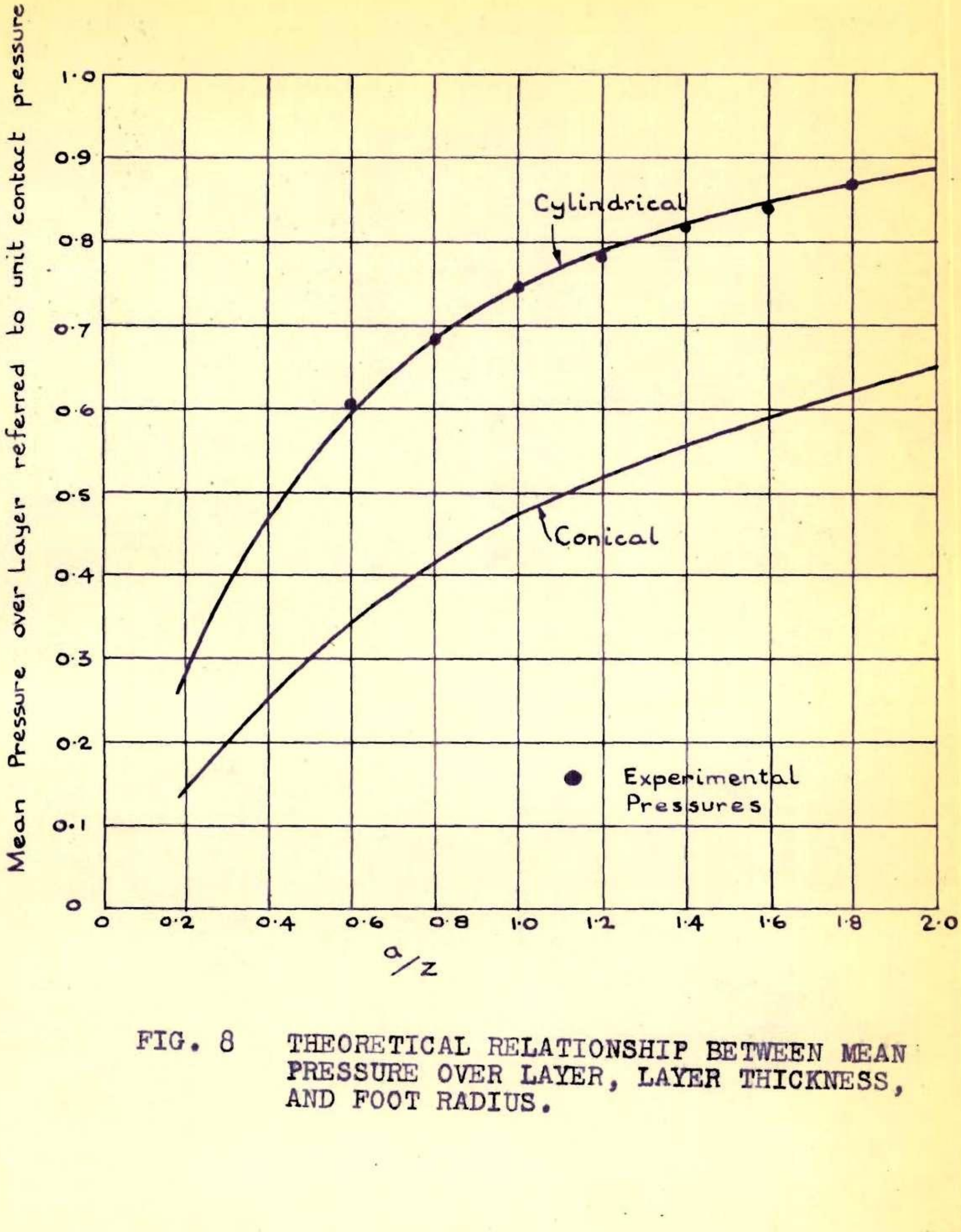

FIG. 8 THEORETICAL RELATIONSHIP BETWEEN MEAN PRESSURE OVER LAYER, LAYER THICKNESS, AND FOOT RADIUS.

DISCUSSION OF RESULTS

Experimental Results.

Graphical relationships for all four feet were found only for a pressure of 150 pounds per square inch. Certain trends then became apparent and it was felt that it would be sufficient to verify these trends at two higher pressures by the use of the two intermediate feet. In addition it was found extremely difficult to sample and accurately determine the volume of the one inch diameter samples. With the largest foot it was found impossible to obtain layer thicknesses as thick as would have been desirable. The range of pressures used (150-250 pounds per square inch) is roughly equivalent to light compaction equipment or roughly comparable with the pressures encountered in the standard Proctor test.

Inspection of the density - a/z curves for a pressure of 150 pounds per square inch reveals several points of interest which are noted and discussed in the following paragraphs.

The density--a/z relationship varies with the foot used in compacting the soil but all four curves are similar in character. Their common characteristic is that with the decrease of layer thickness relative to foot radius the density increases but at a decreasing rate. With further decrease in layer thickness the density attained approaches a constant value and does not increase with decrease in layer thickness. This maximum density would appear to be attained when the layer thickness approaches one-third of the foot radius but it occurs at lower a/z ratios for the larger feet.

From dimensional analysis, different curves would be expected from each foot since the density can be shown in fact to be a function of the reciprocal of the foot diameter. Thus for a given a/z ratio the smaller the foot used the greater the corresponding density would be expected to be. For values of a/z greater than about one, that is when the layer thickness becomes less than the foot radius this can be seen to be true. However for a/z less than one, that is when the layer thickness is greater than the foot radius this theoretical deduction is not borne out by the experimental results. In fact for layer thicknesses of two and one-half times the foot radius and greater the reverse seems to be the case viz. the larger the foot used the greater the density obtained. This can be attributed to a bearing capacity failure of the layer being compacted, in turn attributable to a layer thickness sufficient to enable full shear zones to develop (Fig. 10). This shear zone becomes fully developed when the layer thickness is about twice the foot radius. This bears out previous practical observations of the same failure action.

In addition the smaller the foot the greater the increase in density per unit increase in the a/z ratio.

Using these curves it is possible to show that for a constant a/z ratio equal to or greater than one, that is with layer thicknesses equal to or less than the foot radius, the relationship between density and the reciprocal of the foot radius are linearlly related. (Appendix C). This relationship is only true up to the point where bearing capacity failure begins to take place.

Inspection of the curves at 200 and 250 pounds per square inch yields some more interesting information.

It can be seen the a/z ratio at which the density attained is beginning to approach the maximum decreases as the pressure increases. For example for the 3" diameter foot, this a/z ratio at 150 pounds per square inch is about three whereas in the case of 250 pounds per square inch pressure this ratio is reduced to about two, that is when the layer thickness is about one-half of the foot radius. It is difficult to find a possible reason for this phenomenon.

These curves also indicate that as the pressure increases the value of a/z at which bearing capacity failure begins to occur increases. That is a shear zone may be developed with a thinner layer at higher pressures.

It is also interesting to note that increase in pressure does not necessarily involve corresponding increases in density. For example in the case of the two inch diameter foot the densities obtained with a pressure of 250 pounds per square inch were in fact less than those obtained with a pressure of 200 pounds per square inch. This phenomenon did not appear in connection with the three inch diameter foot suggesting that it depends on foot size as well as pressure. This is in line with the theory of bearing capacity failure. This phenomenon would seem to indicate a squeezing of the soil from beneath the foot due to a shearing failure of the soil.

Theoretical Results.

As the procedure in obtaining all of the theoretical curves is fully detailed little need be said about the theoretical results other than in their relation to the experimental results.

CORRELATION OF THEORETICAL AND EXPERIMENTAL RESULTS

It has already been shown in the analytical procedure that at a constant applied pressure the mean pressure over the layer is a function of the ratio a/z. In a similar fashion the dry density of the compacted layer can be shown to be a function of the ratio a/z and the reciprocal of the foot radius for a given surface contact pressure. From the curves of density versus a/z at 150 pounds per square inch contact pressure it was found that for a constant value of a/z the density was a linear function of the reciprocal of the foot radius. This was only true for values of a/z greater than one. Below one such a linear relationship could not be said to exist. This would seem to be due to the fact that bearing capacity failure tends to take place when the layer thickness is relatively greater than the foot radius. The tendency then is for the greatest densities to be attained by use of the larger feet.

The linear relationship between density and 1/a for constant values of a/z greater than one could be expressed by the equation,

$$\gamma_D = 86 + K/a \qquad \text{(See Appendix C)}$$

where k is a constant. This constant had a different value for each value of a/z at the contact pressure of 150 pounds per square inch.

K varies between about six when a/z equals one to about eighteen when a/z is greater than three.

Thus by drawing the curve of k or $(\gamma_D - 86)a$ versus a/z a unique curve was obtained from all four curves of density versus a/z.

$(\gamma_D - 86)a$ can be seen to have the dimensions of pressure. In fact it is a linear function of the mean theoretical pressure over the compacted layer for the given soil at a contact pressure of 150 pounds per square inch (Appendix C). Hence it may be reasonably assumed that $(\gamma_D - 86)a$ is a linear function of the mean experimental pressures over the compacted layer.

These experimental and theoretical pressures were compared at corresponding a/z ratios assuming that as a/z approaches two the theoretical and experimental values for the mean pressure become equal. The experimental values were plotted on the same graph as the curves showing the theoretical relationship between contact pressure, foot radius and larger thickness for a comparison of trends.

The calculations involved in correlating the experimental and theoretical results are included in Appendix C.

The values of the experimental pressures plotted on the same graph as the theoretical pressures agreed very closely with the cylindrical theoretical curve. However, it must be remembered that the assumption of equal theoretical and experimental pressures at a/z equal to about two may not be strictly correct. It is fairly evident that the assumption in treating the soil as a homogeneous isotropic, fully elastic solid is a good one in this case. The experimental results would be expected to be close to the cylindrical theoretical curve as all of the samples used in the tests were cylindrical samples.

It is obvious that if conical sampling had been possible the resulting densities in all cases would have been smaller and probably nearer the actual field densities.

FIG. 9 DENSIFICATION OF SOIL WITHIN PRESSURE BULB.

FIG. 10 BEARING CAPACITY FAILURE OF LAYER DURING COMPACTION.

CONCLUSIONS

The results may be summarized as follows:

For a constant applied pressure,

(a) The density - a/z relationship varies with foot size but all of the curves showing this relationship are similar in character. Their common characteristic is that with decrease of layer thickness relative to foot radius the density increases but at a decreasing rate.

(b) With further decrease in layer thickness the density attained approaches a constant value. This maximum density is attained at lower values of the a/z ratio with larger compaction feet. At 150 pounds per square inch surface contact pressure this maximum density is attained when the layer thickness approaches roughly one-third of the foot radius.

(c) For layer thicknesses less than the foot radius at a given a/z ratio the smaller the foot used the greater the corresponding density. For layer thicknesses greater than the foot radius a bearing capacity failure of the soil beneath the foot tends to take place and when this happens, the larger the foot used the greater the density attained.

(d) With layer thicknesses at which there is no bearing capacity failure, for any constant a/z ratio there is a linear relationship between density and the reciprocal of foot radius.

In addition,

(e) The ratio a/z at which the density attained approaches the maximum decreases as the pressure increases. That is for each foot the maximum layer thickness at which the maximum density is attained increases with increasing surface contact pressure.

(f) With increasing pressure the value of a/z at which bearing capacity failure occurs, increases. That is a complete shear zone can be developed with a thinner layer at higher pressures.

(g) Increase in pressure does not necessarily involve corresponding increases in density.

(h) The assumption that the soil is a perfectly elastic, homogeneous and isotropic material appears to be a valid one in this case judging by the similarity of trends between the analytical and experimental mean pressure curves (fig. 8).

RECOMMENDATIONS

There were several suggestions for further study arising from the investigation carried out.

(a) For a given compaction foot, the foot radius--layer thickness ratio at which the maximum density is attained with varying contact pressures is worthy of further study. From such a relationship it might be possible to find for a given foot size and contact pressure, the maximum layer thickness at which maximum density would be attained throughout the compacted layer.

(b) For a given contact pressure the variation with foot size of the foot radius--layer thickness ratio at which the maximum density is attained would also seem to be well worthy of further study.

(c) Bearing capacity failure of the soil being compacted leads to poor compaction. For this reason the variation of the a/z ratio at which bearing capacity begins to take place, with varying foot size and/or varying contact pressure is of interest and would appear to fully justify further investigation.

(d) The investigation revealed that even when there was no exterior evidence that a bearing capacity failure was taking place nevertheless increasing pressure did not necessarily involve increasing density. This would suggest possibly a local shearing of the soil in the immediate region of compaction foot. The pressure at which this begins to occur and its variation with foot size would seem to justify even more than the previous recommendations, a complete investigation.

APPENDIX A

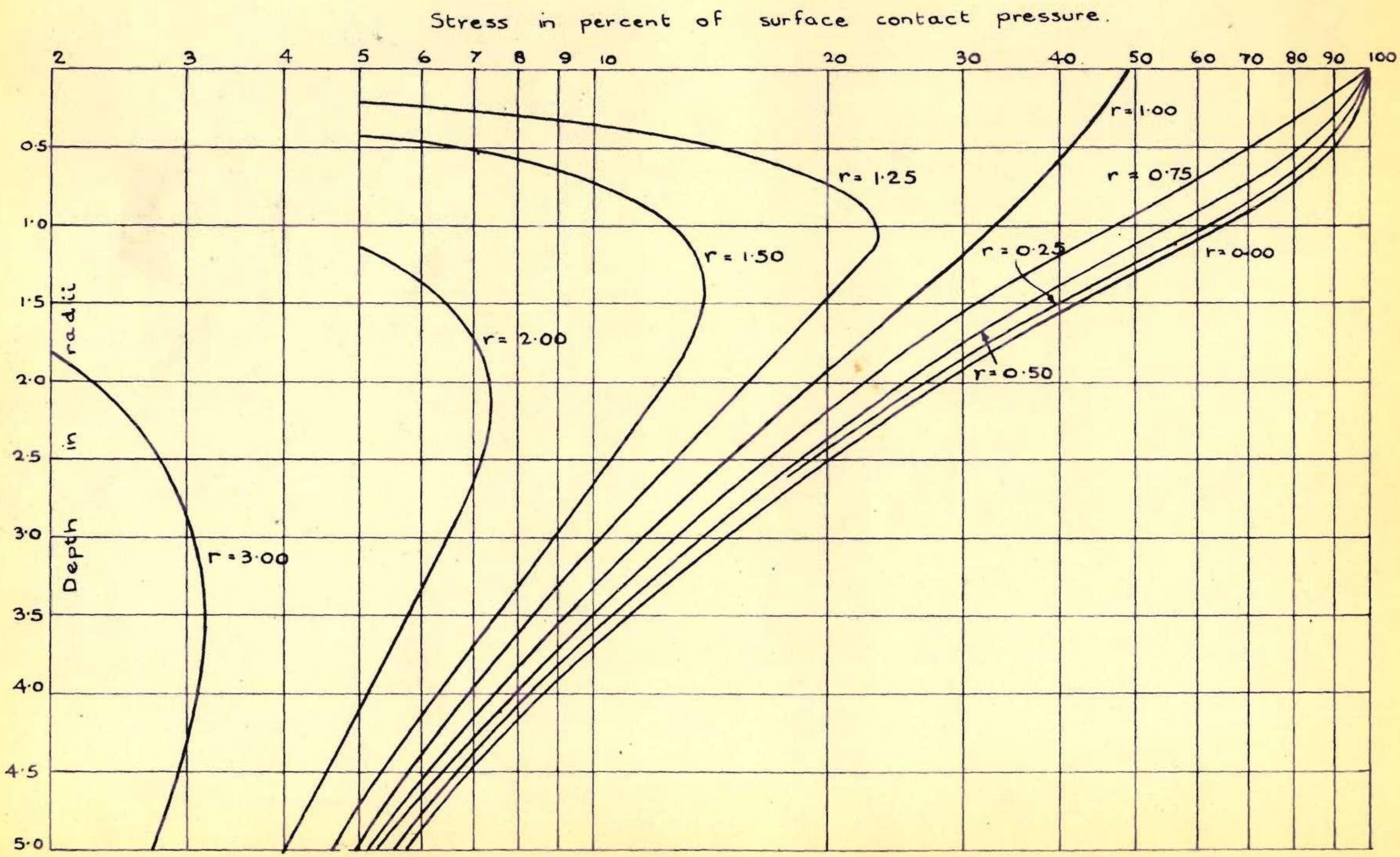

FIG. 11 VERTICAL STRESS DISTRIBUTION.

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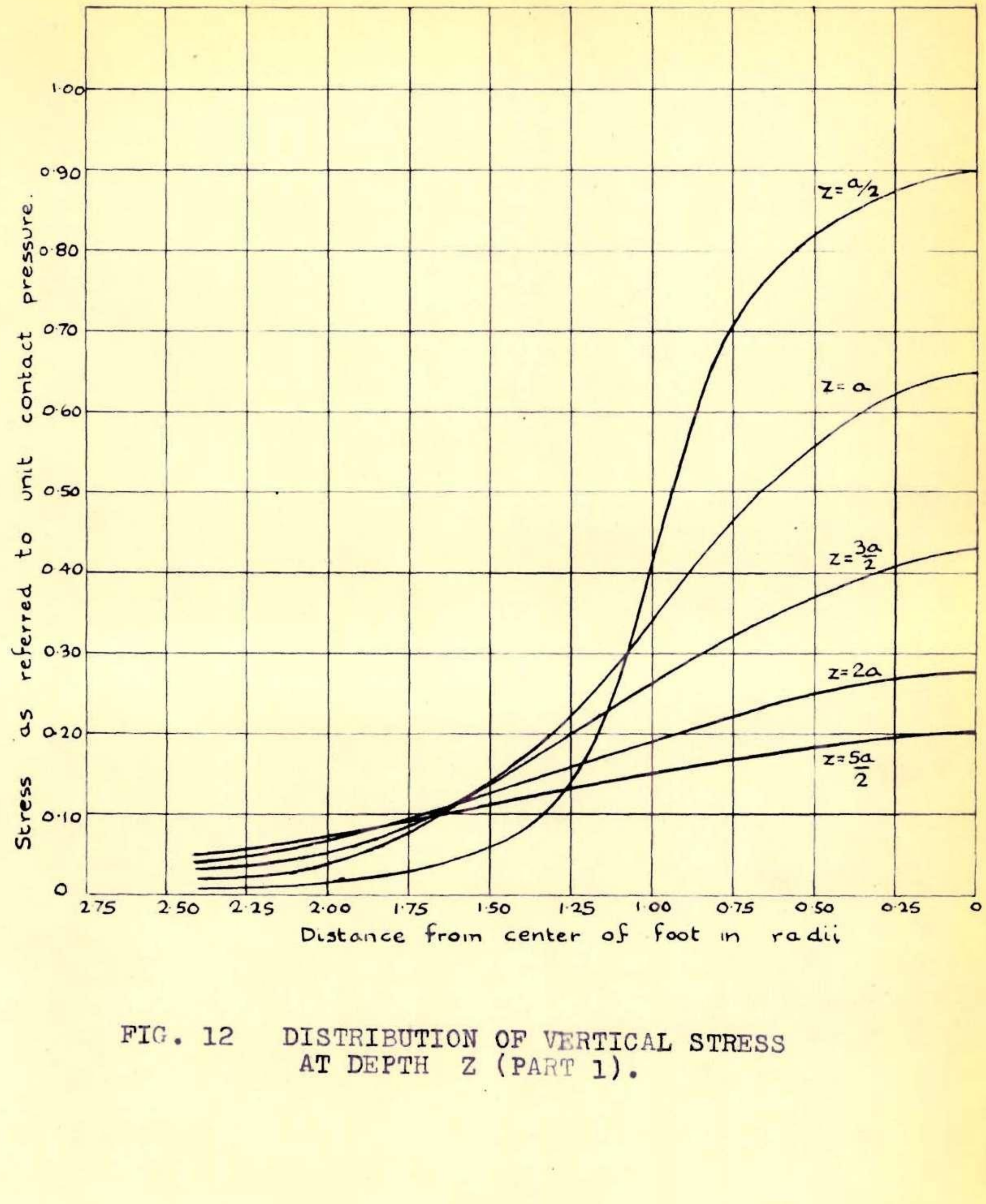

FIG. 12 DISTRIBUTION OF VERTICAL STRESS AT DEPTH Z (PART 1).

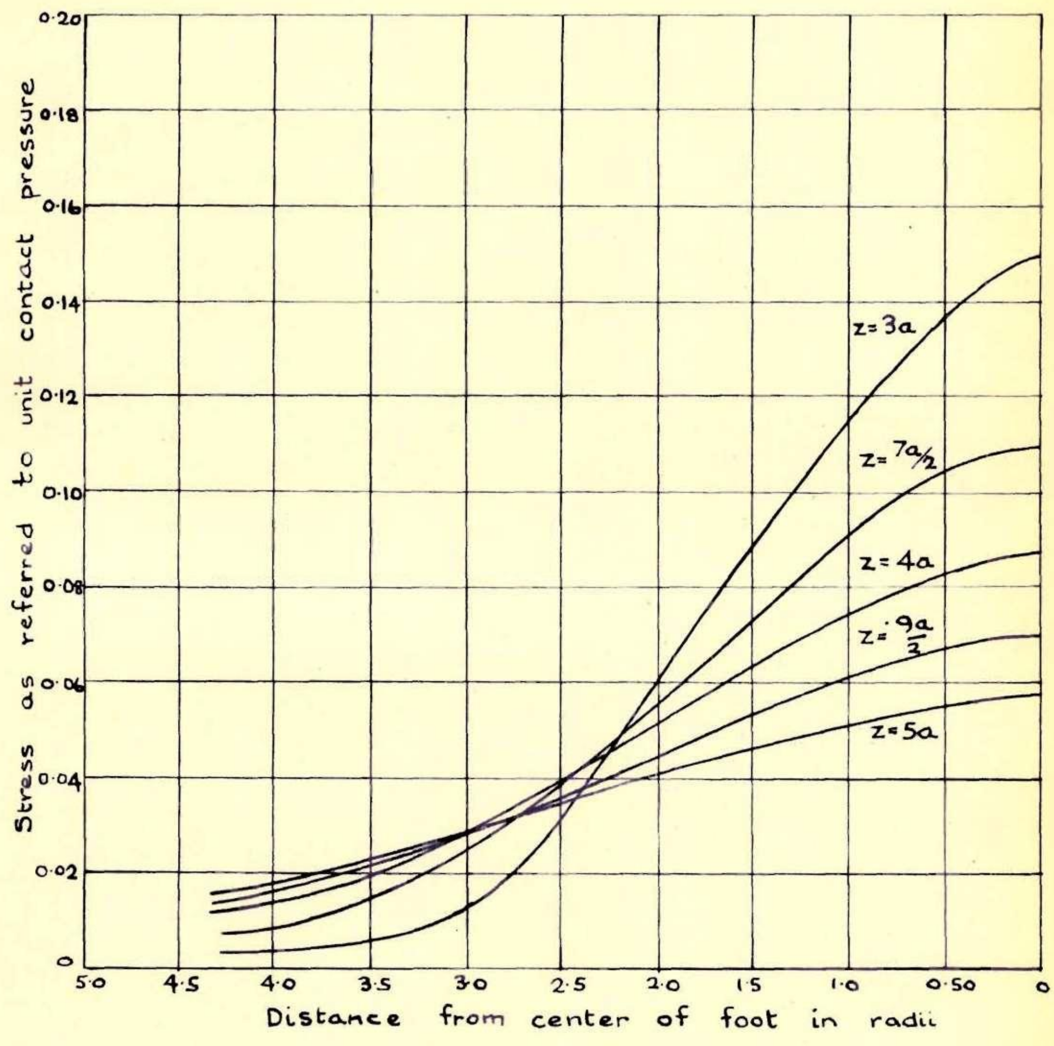

FIG. 13 DISTRIBUTION OF VERTICAL STRESS AT DEPTH Z (PART 2).

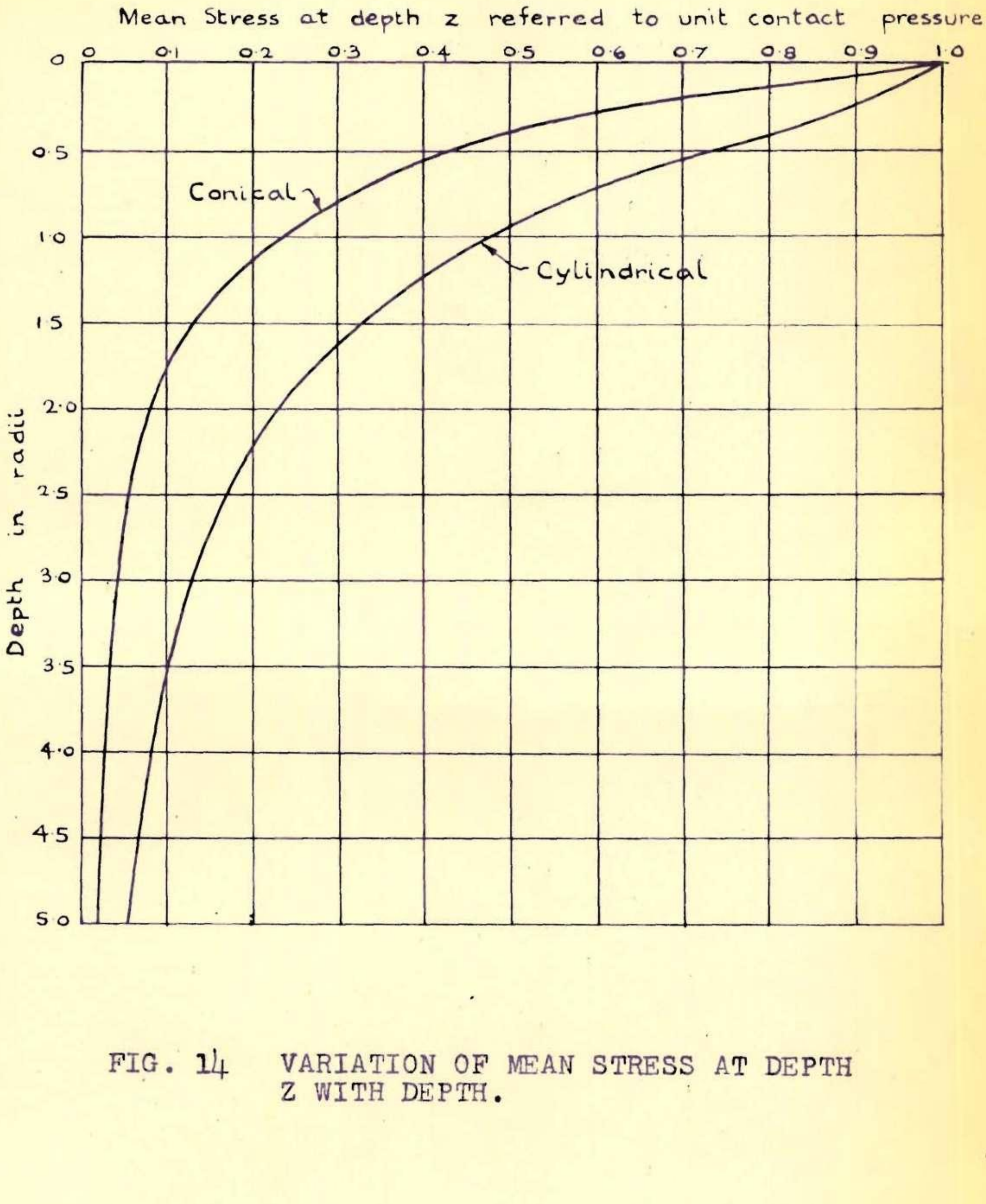

FIG. 14 VARIATION OF MEAN STRESS AT DEPTH Z WITH DEPTH.

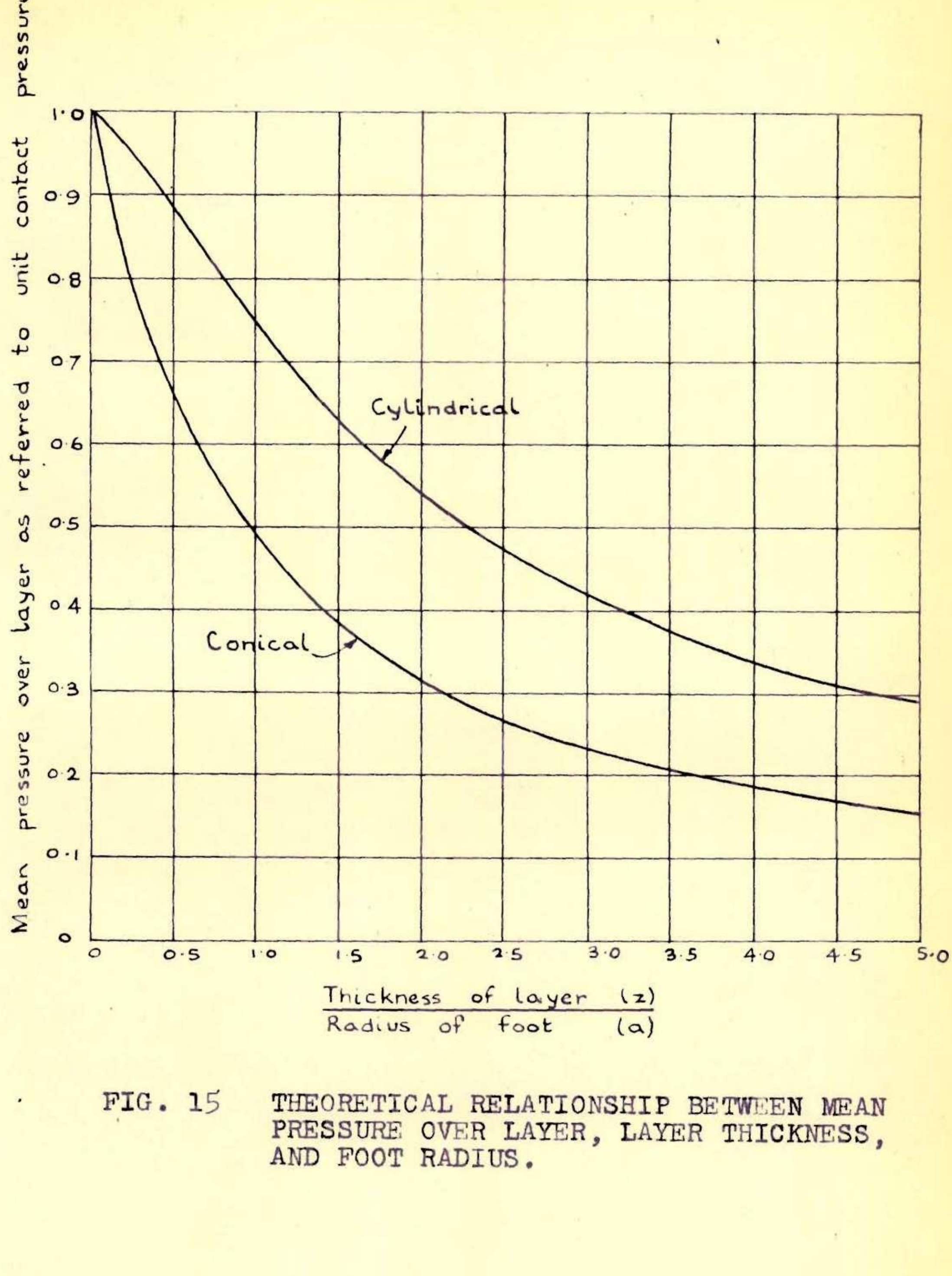

FIG. 15 THEORETICAL RELATIONSHIP BETWEEN MEAN PRESSURE OVER LAYER, LAYER THICKNESS, AND FOOT RADIUS.

APPENDIX B

PRESSURE IN LBS. / IN.^{2}	WATER CONTENT IN PER CENT	THICKNESS OF LAYER IN INCHES	FOOT RADIUS IN INCHES	$\frac{\text{FOOT RADIUS}}{\text{LAYER THICKNESS}}$	SAMPLE WT. IN GRAMS	SAMPLE VOL. IN CUBIC CMS.	DRY DENSITY IN LBS./FT.^{3}	$(\gamma_D - 86)$ IN LBS./FT.^{3}	$(\gamma_D - 86)a$ IN LBS. INS./FT^3
150	15.6	2.65	1.47	0.55	479.1	290.0	89.2		
150	14.9	1.44	0.97	0.68	162.2	101.0	87.0	1.0	0.97
150	14.7	2.16	1.92	0.88	626.0	379.0	89.7	3.7	7.1
150	14.1	0.44	0.50	1.14	7.0	4.2	94.2		
150	14.1	1.05	0.97	0.93	82.6	47.0	96.3		
150	14.9	0.64	0.97	1.52	24.1	13.8	98.0	12.0	11.6
150	14.6	0.91	1.47	1.62	117.9	67.5	94.8	8.8	13.9
150	13.4	0.89	1.92	2.15	176.9	108.0	90.2		
150	14.5	0.39	0.50	1.29	4.9	2.6	104.0	18.0	9.0
150	15.3	1.63	0.97	0.60	120.6	74.0	88.4		
150	13.7	1.94	1.47	0.77	367.8	227.7	88.5		
150	14.0	0.97	1.92	1.97	196.2	115.2	93.5	7.5	14.4
150	13.6	0.72	1.92	2.65	117.2	67.7	95.0	9.0	17.3
150	14.0	1.16	0.50	0.43	28.8	18.2	86.0	0.0	0.0
150	13.0	0.63	0.97	1.55	34.9	20.2	95.0		
150	14.3	0.44	0.97	2.22	19.4	10.5	101.5	15.5	15.0
150	13.0	0.77	0.97	1.27	48.3	27.8	95.5	9.5	9.2
150	14.0	1.00	0.97	0.97	74.2	45.0	90.5		
150	14.0	0.62	0.97	1.57	24.5	13.6	98.5		
150	13.4	0.66	1.47	2.24	64.1	36.7	96.0		
150	13.4	1.00	1.47	1.47	119.1	69.0	95.0		
150	14.1	1.37	1.92	1.40	223.4	133.0	92.2	6.2	11.9
150	13.6	1.21	1.47	1.20	130.0	77.0	92.7	6.7	9.8
150	13.2	0.63	1.47	1.55	59.0	33.6	96.6	10.5	15.5
150	14.5	0.54	0.50	0.92	9.8	5.6	97.5	11.5	5.8
150	14.6	0.23	0.50	2.20	4.9	1.8	113.0		

TABLE 1.

PRESSURE IN LBS. / IN.^{2}	WATER CONTENT IN PER CENT	THICKNESS OF LAYER IN INCHES	FOOT RADIUS IN INCHES	FOOT RADIUS / LAYER THICKNESS	SAMPLE WT. IN GRAMS	SAMPLE VOL. IN CUBIC CMS.	DRY DENSITY IN LBS. / FT.^{3}
200	14·4	0·77	1·47	1·92	98·0	54·0	98·8
200	15·2	0·66	1·47	2·24	83·8	48·2	94·1
200	14·5	1·00	1·47	1·47	135·6	75·5	97·7
200	14·4	1·33	1·47	1·11	220·4	126·7	96·0
200	13·9	1·88	1·47	0·78	390·6	228·6	94·5
200	13·9	1·28	0·97	0·76	118·3	69·7	92·7
200	14·3	1·03	0·97	0·94	81·9	46·2	97·0
200	14·4	0·75	0·97	1·29	48·9	25·9	103·2
200	14·4	0·53	0·97	1·82	34·4	17·2	111·6
200	15·1	0·64	1·47	2·30	67·6	36·3	98·8
200	14·5	0·41	0·97	2·40	27·5	13·1	114·6
200	14·0	0·36	0·50	1·39	5·4	3·6	81·7
200	13·0	0·49	0·50	1·03	10·2	5·4	103·5
200	13·0	0·56	0·50	0·89	14·2	8·6	90·0
200	14·0	0·95	0·50	0·53	24·5	15·6	86·1
200	14·0	0·27	0·50	1·88	2·7	1·6	89·5
200	14·0	0·37	0·50	1·36	5·2	2·9	98·6
250	13·9	0·56	0·97	1·73	34·5	18·4	101·1
250	13·5	0·80	0·97	1·22	59·8	34·0	96·5
250	14·5	0·95	0·97	1·02	72·0	39·4	99·5
250	13·6	1·25	0·97	0·78	112·1	64·0	95·8
250	14·9	0·63	0·97	1·55	37·0	20·2	99·5
250	14·3	0·47	0·97	2·07	25·4	13·5	102·7
250	13·6	1·08	0·97	0·90	100·2	58·0	94·6
250	13·6	1·34	0·97	0·72	132·6	78·2	93·1
250	13·1	1·84	1·47	0·80	378·5	216·0	96·5
250	13·2	1·24	1·47	1·19	190·0	106·0	98·4
250	13·5	1·09	1·47	1·34	142·3	78·4	99·4
250	13·0	0·67	1·47	2·19	66·4	36·8	99·5
250	13·5	0·84	1·47	1·74	84·9	46·6	99·5

TABLE 2.

APPENDIX C

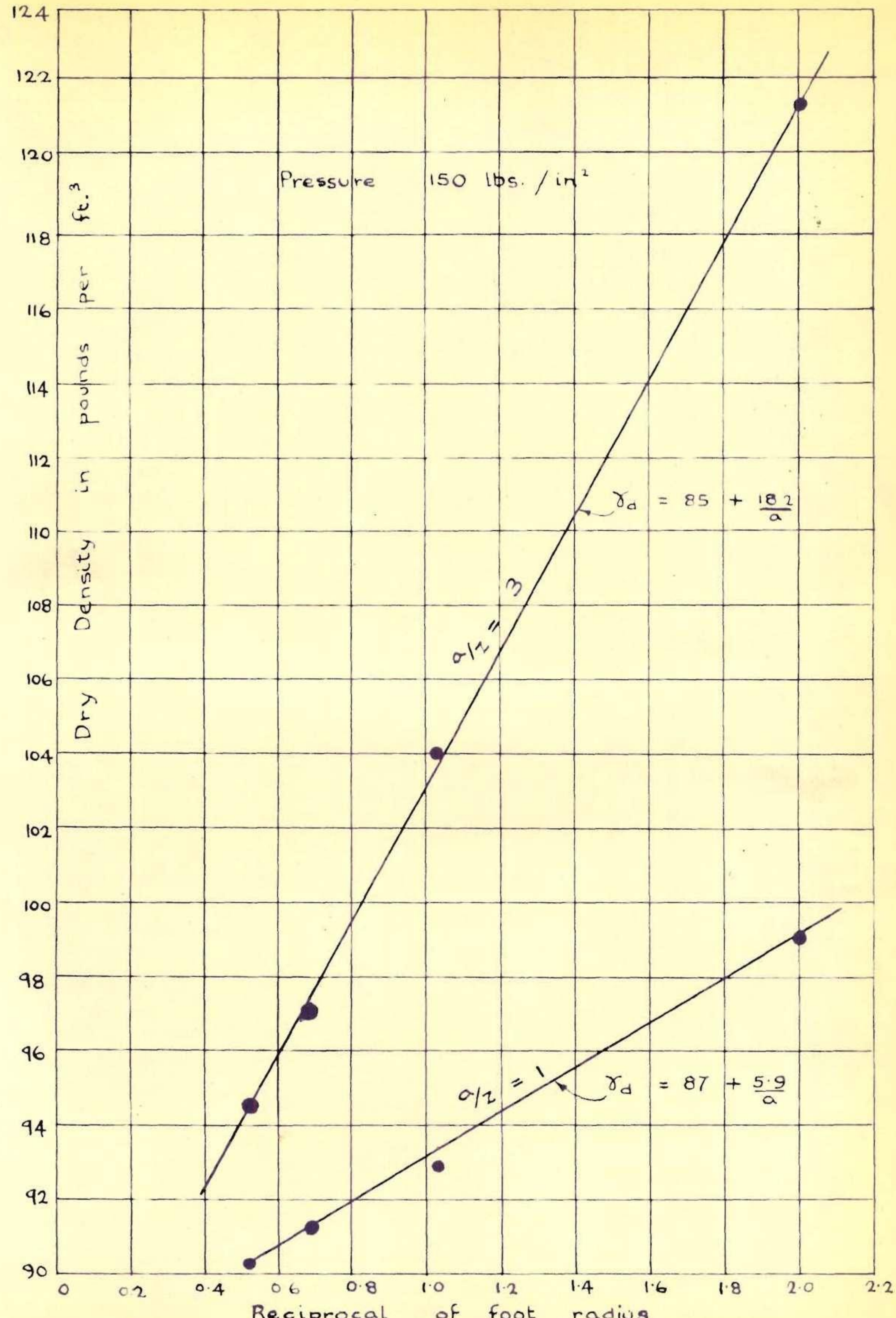

FIG. 16 EXPERIMENTAL RELATIONSHIP BETWEEN DENSITY AND THE RECIPROCAL OF FOOT RADIUS.

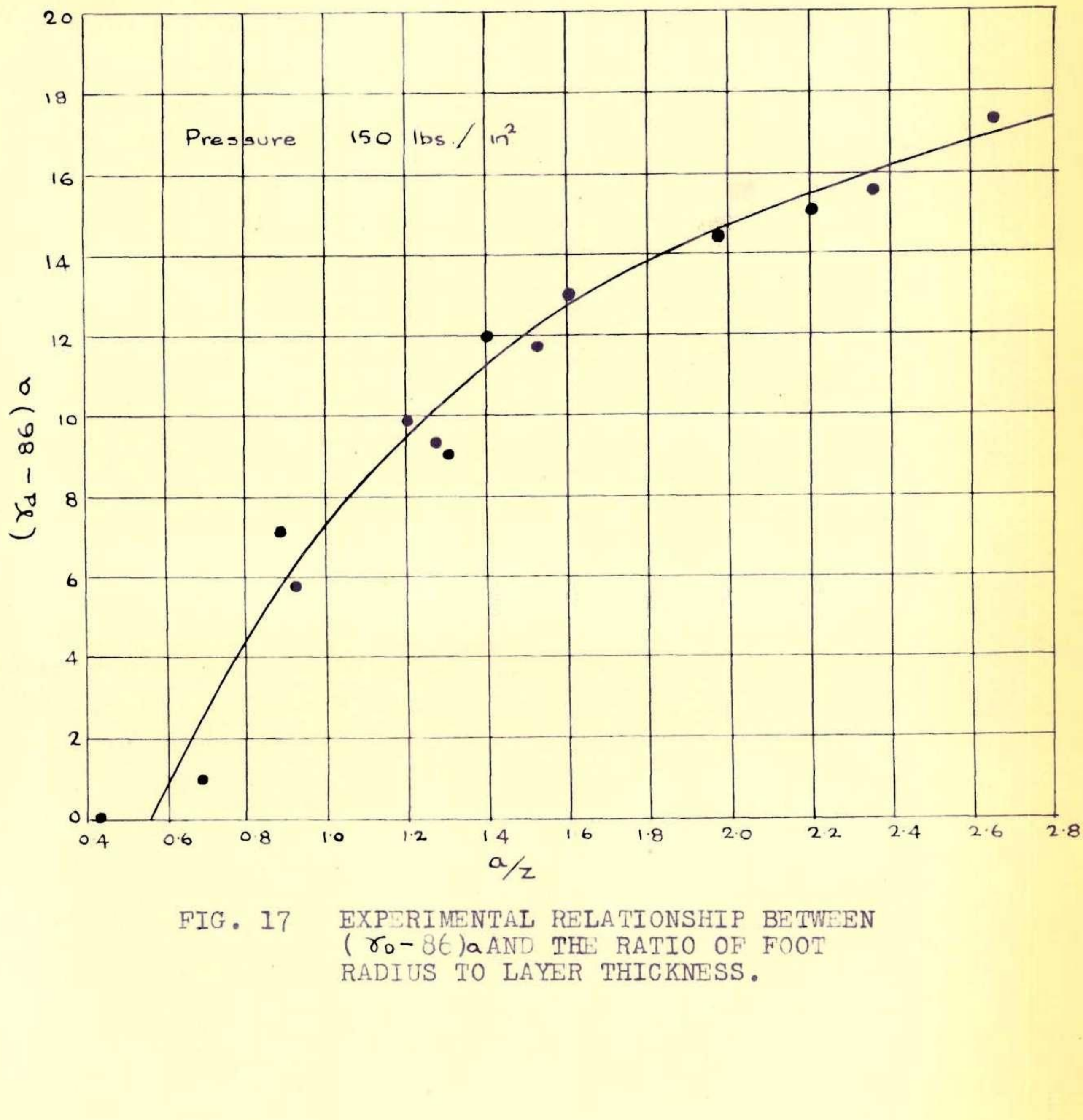

FIG. 17 EXPERIMENTAL RELATIONSHIP BETWEEN (γ_o - 86)a AND THE RATIO OF FOOT RADIUS TO LAYER THICKNESS.

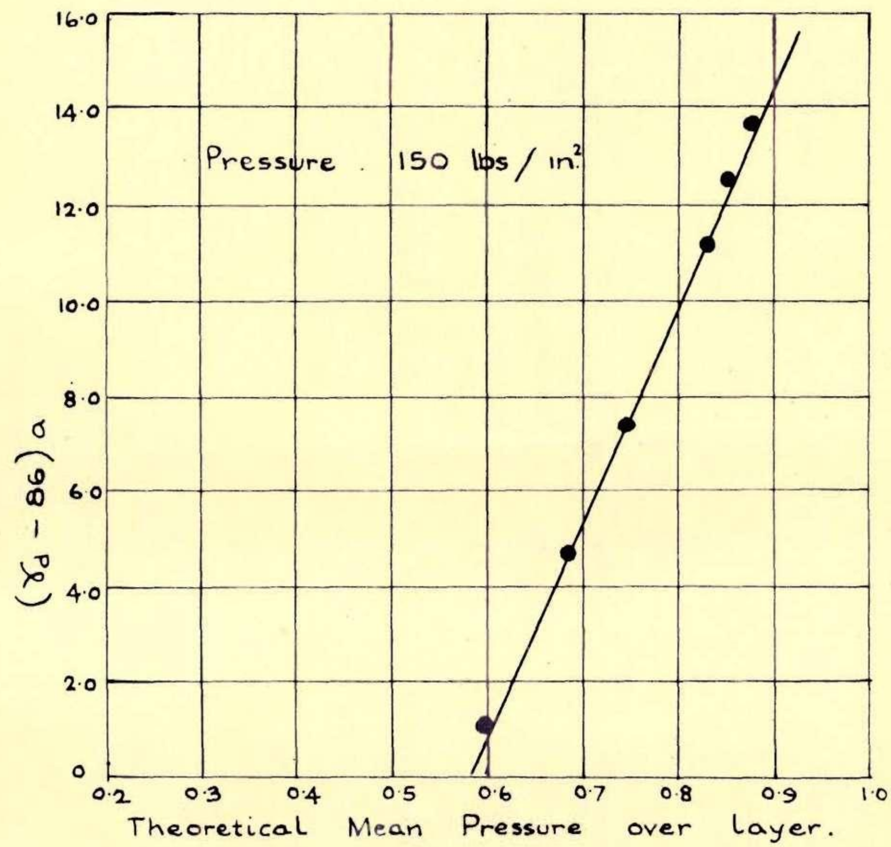

FIG. 18 RELATIONSHIP BETWEEN $(\gamma_D - 86)a$ AND THE THEORETICAL MEAN PRESSURE OVER LAYER.

$\frac{\text{FOOT RADIUS}}{\text{LAYER THICKNESS}}$	ANALYTICAL MEAN PRESSURE OVER LAYER	INCREMENTAL INCREASE IN ANALYT. PRESSURE	$(\gamma_d-86)a$	INCREMENTAL INCREASE IN $(\gamma_d-86)a$	$\frac{\text{INCR. ANAL PRESS.}}{\text{INCR. } (\gamma_d-86)a}$	INCREMENTAL INCREASE IN EXPT. PRESSURE	EXPERIMENTAL MEAN PRESSURE OVER LAYER
0·6	0·590		1·0				0·606
		0·090		3·6	4·00	0·075	
0·8	0·680		4·6				0·681
		0·060		2·8	4·70	0·059	
1·0	0·740		7·4				0·740
		0·050		2·0	4·00	0·042	
1·2	0·790		9·4				0·782
		0·035		1·7	4·85	0·036	
1·4	0·825		11·1				0·818
		0·025		1·4	5·60	0·029	
1·6	0·850		12·5				0·847
		0·020		1·1	5·50	0·023	
1·8	0·870		13·6				0·870

Σ 28·65

Mean 4·78

TABLE 3.

THIS TABLE WAS MADE UP ON THE ASSUMPTION THAT AS THE RATIO OF FOOT RADIUS TO LAYER THICKNESS APPROACHES TWO, THE THEORETICAL AND EXPERIMENTAL MEAN PRESSURES OVER THE COMPACTED LAYER BECOME EQUAL.

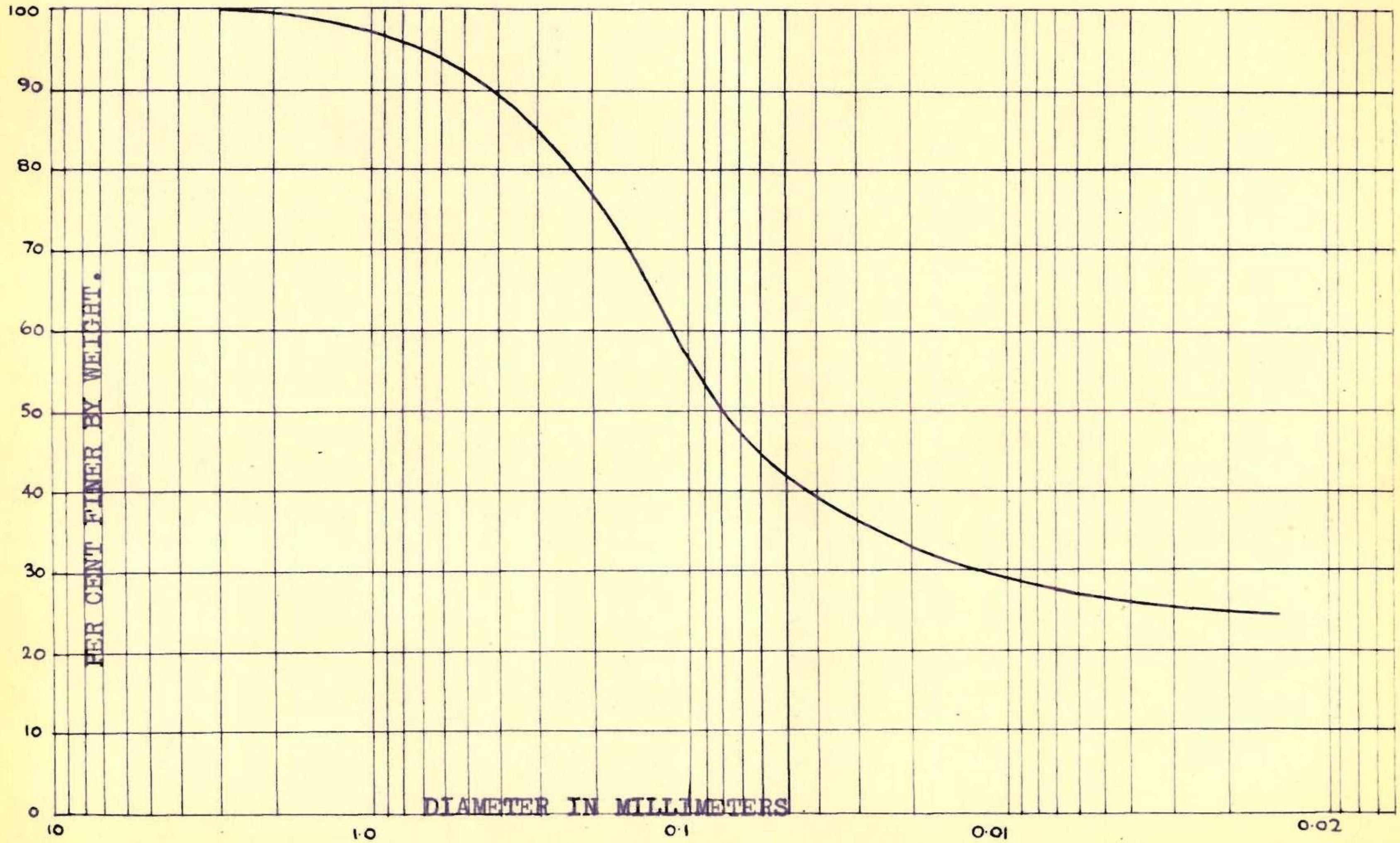

FIG. 19
GRAIN SIZE DISTRIBUTION OF TEST SOIL.

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