Tone Reproduction for Realistic Computer Generated Images

by

Jack Tumblin and Holly Rushmeier

GIT-GVU-91-13
September 1991

Graphics, Visualization & Usability Center

Georgia Institute of Technology
Atlanta GA 30332-0280

TONE REPRODUCTION FOR REALISTIC COMPUTER GENERATED IMAGES

Jack Tumblin
Holly Rushmeier
Graphics, Visualization and Usability Center
Georgia Institute of Technology
July, 1991

ABSTRACT

Radiosity and other global illumination methods for image synthesis calculate the 'real world' radiance values of a scene instead of the display radiance values that will represent them. This causes 'display range' problems that are often solved by ad-hoc means ,giving little assurance that the evoked visual sensations (brightness, color, etc.) are truly equivalent. Workers in photography have studied such perception matching as 'tone reproduction', and devised correcting operators from both empirical and vision research data. Corrections are usually limited by the chemical/optical restrictions of film. These practical film methods were adopted by television systems and then by computer graphics, despite the ease of implementing better correction operators by computer.
In this paper we advocate the use of better tone reproduction for computer graphics. We give a general framework for tone reproduction, where mathematical models of the display device and human observers define an explicit conversion from real-world to display device radiance. These are used to review tone reproduction operators used for film and television. A brief summary of some applicable vision research literature leads to a simple example of an improved operator. We apply the Stevens & Stevens models of brightness .vs. luminance relations to our framework to create a new tone reproduction operator for black & white computer generated images. The new operator is shown to be a reasonable solution to the display range problem, and further extensions are suggested.

1. INTRODUCTION

Most computer graphics image synthesis algorithms don't know the difference between night and day -- differences that are obvious to any human eye. These differences are lost in the conversion of computed images to displayed images, and deserve more careful study. Though methods such as radiosity and stochastic ray tracing techniques can compute extremely accurate and wide-ranging scene radiances, these precise results must be converted to a very small range of displayed radiances for viewing on modern display devices. Despite good 'gamma correction', these conversions can be dubious, aphysical, and sometimes fail spectacularly for extreme lighting conditions because they ignore light-dependent changes in the way we see.

This paper calls attention to these conversions, not only because bad solutions can ruin the accuracy of displayed images, but because good solutions are implemented easily by computer from a mathematical description. A photograph is said to have good 'tone reproduction' if it faithfully reproduces the subjective brightnesses of the original scene, and tone reproduction is improved by careful control of the conversion between photographed and displayed radiance values. Only the simplest conversions are done, however, since more complicated changes are chemically or optically impractical. Free of these limits, computer graphics workers can not only apply the existing photographic tone reproduction operators, but may greatly extend them to include more complex and subtle effects of human vision, by simply exploiting the quantitative models and data published by vision researchers. Advanced methods of tone reproduction may eventually allow accurate reproduction of many familiar visual effects usually done by ad-hoc or artistic means, such as afterimages from a blinding flash, the eerie glow of a neon tube, or a gradual transition from daylight to night-vision.

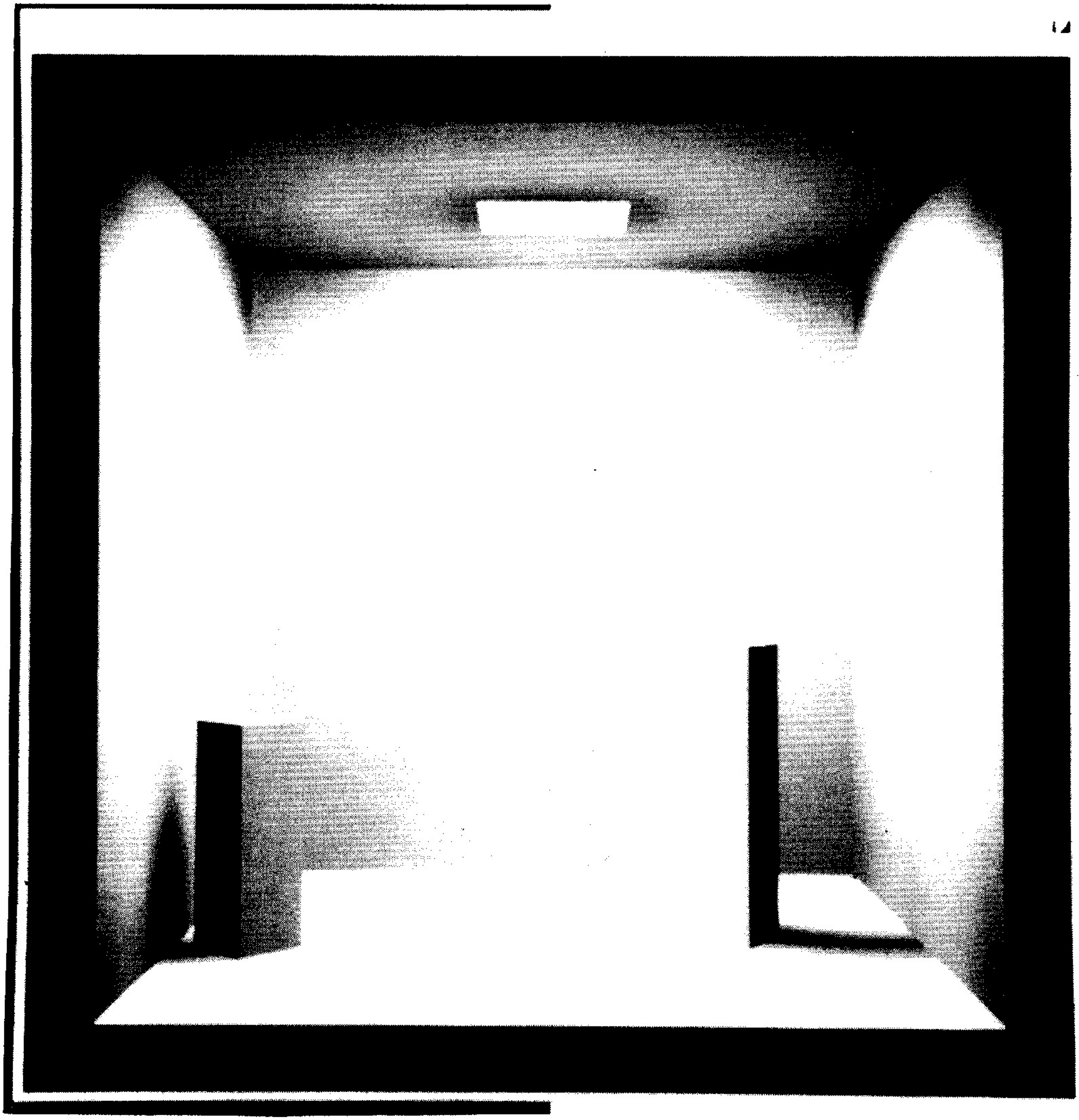

Figure 1: Display of radiosity solution using ad-hoc scale factor.
This display is valid for both the firefly- and the searchlight-illuminate case

2. Background

2.1 A Simple Example

A simple example of day-and-night confusion is shown in Fig. 1, and arises from the intuitive use of scale factors as a tone reproduction operator. Figure 1 depicts a room of uniformly diffuse surfaces lit by a single extended light source whose emitted power can be continuously varied between that of two squashed fireflies and that of an aircraft searchlight. Since the global illumination solution is linear in source radiance [ARVO90], if the image radiance values are known for one light source strength, results for any other light source strength are directly proportional; doubling the light source power doubles all image radiance values. Accordingly, the image of a room with firefly-powered lighting is identical to the image of a room lit by an aircraft searchlight, except for a scale factor of about 10^{11}.

How should such firefly- and searchlight-strength room images be displayed? One popular and widely used method normalizes all computed scene radiance values by the radiance of the strongest non-emitting surface in the image, and then these normalized values drive a gamma corrected display system. This method was used for figure 1, and gives the appearance of pleasant office lighting quite familiar to radiosity workers. When applied to the firefly- and searchlight-powered images, the 10^{11} scale factor that distinguishes them is removed by normalization, so Fig. 1 is the displayed result for BOTH of them!

Human observers would perceive these rooms quite differently. Given enough time for full dark-adaptation (up to an hour) [DAVS76], the firefly-powered room appears as little more than a very dim light source. Given time for the painful, squinting adjustments needed, a searchlight illuminator might allow a human observer to discern some harsh details in the darker shadow areas, but all else would be awash in glaring white. Neither observer would recognize Fig.1; despite careful computation of radiance, the displayed result is a wholly inaccurate representation of the two rooms.

Besides the clear failure of simple normalization to render extremes in illumination, our example also shows Fig. 1 to be of questionable accuracy even as a rendering of a typical indoor illumination. As discussed in [MEYE86], the rendering process has two steps -- the calculation of scene radiances, and the use of principles of perception to map these radiances to the display device. In order to be viewed as a science, each of these steps must be subject to verification by the scientific method. A tremendous amount of research has appeared recently dealing with rigorous, first principle methods for the first step (e.g. [HALL89]). The portion of the second step dealing with color transformations has also been been studied with a sound scientific basis (e.g. [DURR87]). However, the mapping of the range of real world radiances to the display device (tone reproduction) has been generally ignored. Currently, unless the scaling of an image is performed by a user who knows what the scene should look like, no justifiable model of the human vision system has been used to generate the image.

Figure 1 was computationally expensive, since the error tolerance of its radiosity solution was quite small and uniform across the entire image. Human observers in the firefly- and searchlight-illuminated rooms would see very little of the image detail present in Fig.1; all but the brightest features would be lost in blackness under firefly light, and all but the harshest shadows would be blasted away in the glare of a searchlight. In either case, images made with good tone reproduction would contain much less image detail than Fig. 1; so most of the accurately computed details presented there are wasted. A good tone reproduction operator can be used to predict the thresholds of error visibility in various regions of the displayed image, and the precision of the computed radiosity solution could be set accordingly. Thus a reliable tone reproduction operator can be used to answer one of the basic questions remaining in the area of global illumination -- how accurate does a solution have to be?

Figure 1 lacks the strongly light-dependent effects of human vision that are required for good tone reproduction. By including models of the complex, dramatic changes to human vision that occur over the firefly-to-searchlight range, the subjective accuracy of the displayed image might also be improved. There are many effects to consider, such as strong differences in sensitivity, acuity, contrast perception and color sensitivity; high contrast effects such as glare, dazzle, afterimages, color washout and diffraction; spatial effects such as Mach banding and hyperacuity; temporal effects such as adaptation, persistence of vision, patterns of gaze, and recognition time.

For genuinely accurate tone reproduction, all of these effects must eventually be considered, but useful results will be demonstrated here using just a few. In this introductory paper we wish to stress the importance of explicit tone reproduction operators and encourage their use, rather than to champion any one particular expression. The example we develop in section 5 is restricted to static black and white scenes; extensions for color and temporal dependence, while certainly possible and promising, will be considered in later papers.

2.2 Definitions

The eye is quite conceited; one can easily assume the scenes of the world around us are exactly as we see them, as an assemblage of radiance values that neatly match our sensations of them. This is an illusion: humans are very poor judges of absolute radiance ([DAVS76] cites experimentally measured errors exceeding 30%), and are easily fooled by simple tricks. Instead, the visual system is far more accurate as a detector of spatial and temporal CHANGES in radiance, and the sensations of radiance are apparently reconstructed from judgments of these changes. Since the sensation of light strength is often quite different from its measured strength, they are carefully defined by the Optical Society of America [JAME66];

> Let *'brightness'* define the magnitude of the subjective sensation produced by visible light;
>
> Let*'luminance'* define the physical measure of the magnitude of visible light.
>
> Luminance is found by averaging the radiance across the visible spectrum weighted by the normalized spectral sensitivity curve of the average eye. Thus 'brightness' is a measure of perception, while 'luminance' is a measure of radiance; the latter is physically measurable, the former is not.

While a 'quantitative' measure of a subjective value like brightness may seem specious, workers in psychophysics have deduced accurate, repeatable brightness measures from cleverly designed comparison experiments. We use the 'brils' units devised by Stevens [STEV60], where 1 bril equals the sensation of brightness induced in a fully dark-adapted eye by a brief exposure to a 5 degree white target of 1 microlambert luminance on a completely black background.

The purpose of 'tone reproduction' operators is to match brightness values of a real-world image by its reproduction on a display device. Two images that appear identical will by definition have the same values of brightness, regardless of their actual luminance values, which might be quite different. Image pairs that have measureably identical luminance values but whose brightnesses do not match are also possible, and form an interesting class of illusions (e.g. [CORN70]. Exploiting both makes tone reproduction possible; good tone reproduction recreates the sensations (brightnesses) of the computed scene radiances from luminance values entirely within the the tiny range of the display by exploiting visual illusions.

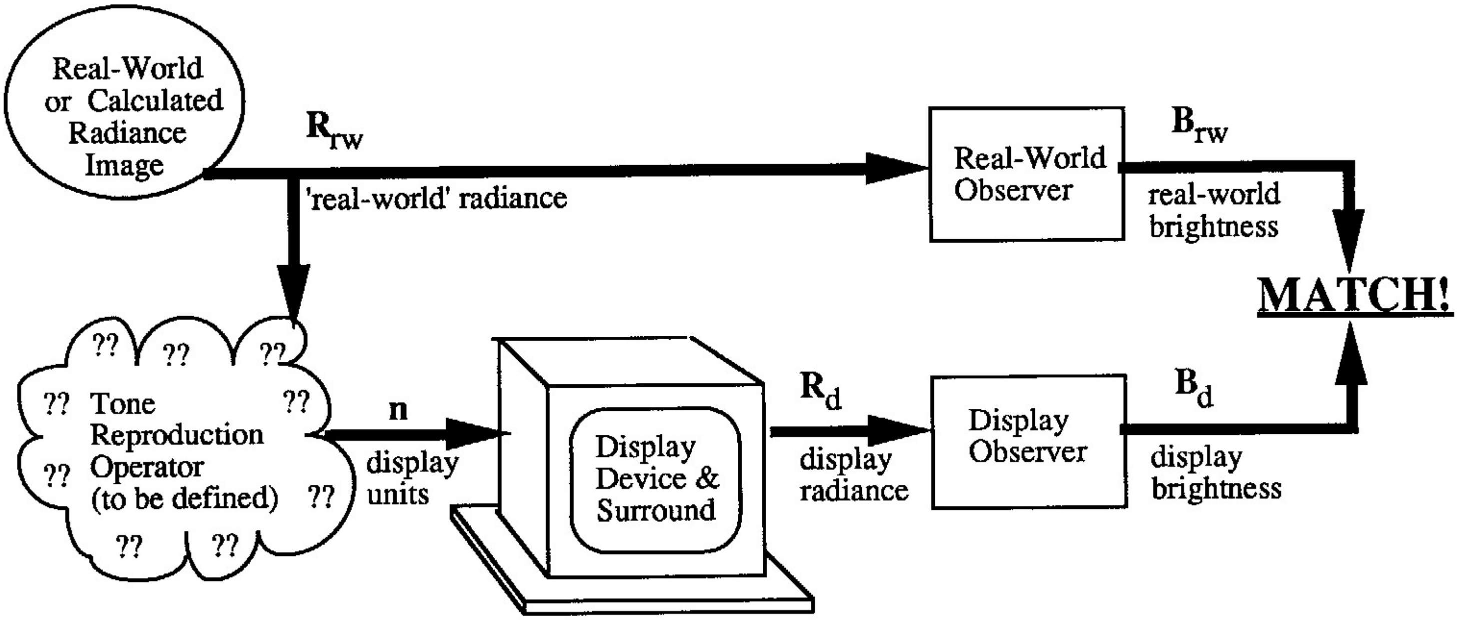

Figure 2a: The Tone Reproduction Problem

What tone reproduction function will cause displayed brightness sensations to equal or closely match the real-world brightness sensations?

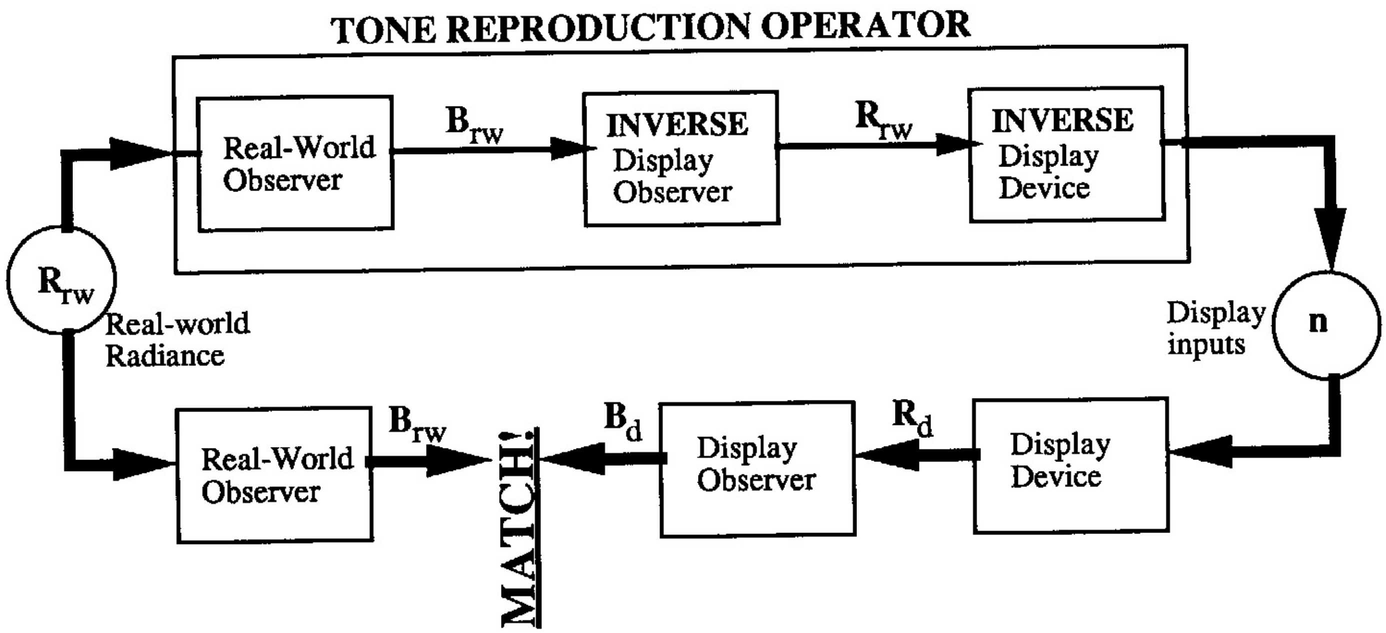

Figure 2b; A Simple Solution:
Make a simple tone reproducton operator by concatenating a real-world observer model, an inverse display observer model, and and inverse display device model. The resulting display brightness should be equal to the real-world brightness.

Tone reproduction is only necessary because the eye's input range dwarfs the output range of existing electronic displays. If computed scene radiances could be directly reproduced, then the light-dependent changes in the viewer's vision would be reproduced as well (though only approximately; the display's spatial bandwidths and the viewer's surroundings also have some effects[BART67]). Such a display device would be astounding and dangerous, for the output range of existing display devices is so narrow in comparison that direct reproduction is almost never possible. For example, shadows in a starlit forest are just barely visible at about 10^{-9} lamberts, while sun-glints on snowbanks are almost painful at about 10 lamberts, and nothing prevents synthetic images like Fig. 1 from exceeding this range. Meanwhile, CRTs in a dark room cover at best about 10^{-4} to 10^{-2} lamberts.

2.3 A General Framework

A general framework to define tone reproduction, as shown in Fig. 2a, and is built from mathematical models of the response of two observer models and a display system model. An 'observer model' is a mathematical model of the human visual system. This quantitative model, which includes all desired light-dependent visual effects, converts viewed radiance images to perceived brightness images.

Two such observer models are used. The 'real world observer' views the desired radiance image (R_{rw}) and corresponds to a human visitor to the rooms of Fig. 1, and the 'display observer' views the radiance values of the display device(R_d). The display device is also quantitatively modeled; the 'display model' converts display input values (RGB), to viewed radiance values (R_d); it includes effects of ambient room light, CRT performance, and compensations such as 'gamma correction'. The 'tone reproduction' operator we wish to find is a converter from 'real world' radiances (R_{rw}) to display input values (RGB), chosen so that the outputs of the two observer models, B_{rw} and B_d, are equal or well matched.

To solve for the tone reproduction operator, simply 'unfold' Figure 2a as shown in Figure 2b. The tone reproduction operator is thus defined as the concatenation of the real-world observer, the inverse of the display observer, and the inverse of the display model. If each of these operators are known and robust, then a tone reproduction operator follows easily.

3. Film & TV Tone Reproduction

All photographs perform some sort of tone reproduction; real-world radiances R_{rw} at the camera are translated to the display radiance R_d of the displayed print or projected transparency by chemical and optical means. When plotted on $\log_{10} - \log_{10}$ axes, film's response to light is linear near the center of its usable range, as shown in Figure 3a. Such plots are widely used to describe photographic emulsions, and are called 'D-Log-E' or 'H-D' plots, after Hurter and Driffield who devised them in 1890 [JAME66]. The plot axes are

Film Density $= D = -\log_{10}(T)$ where $T =$ transparency, $=0$ for perfect opacity
$=1$ for perfect transparency

Film Exposure $= E =$ light energy absorbed by the film.

By changing units on these axes, H-D plots can relate real-world to display radiances (R_{rw} to R_d), and thus describe the combined tone reproduction operator and display system operator in our framework of figure 2a. Note that film exposure E is directly proportional to real-world radiance R_{rw}, where proportionality constant a_0 is set by camera lenses, aperture, and exposure time.

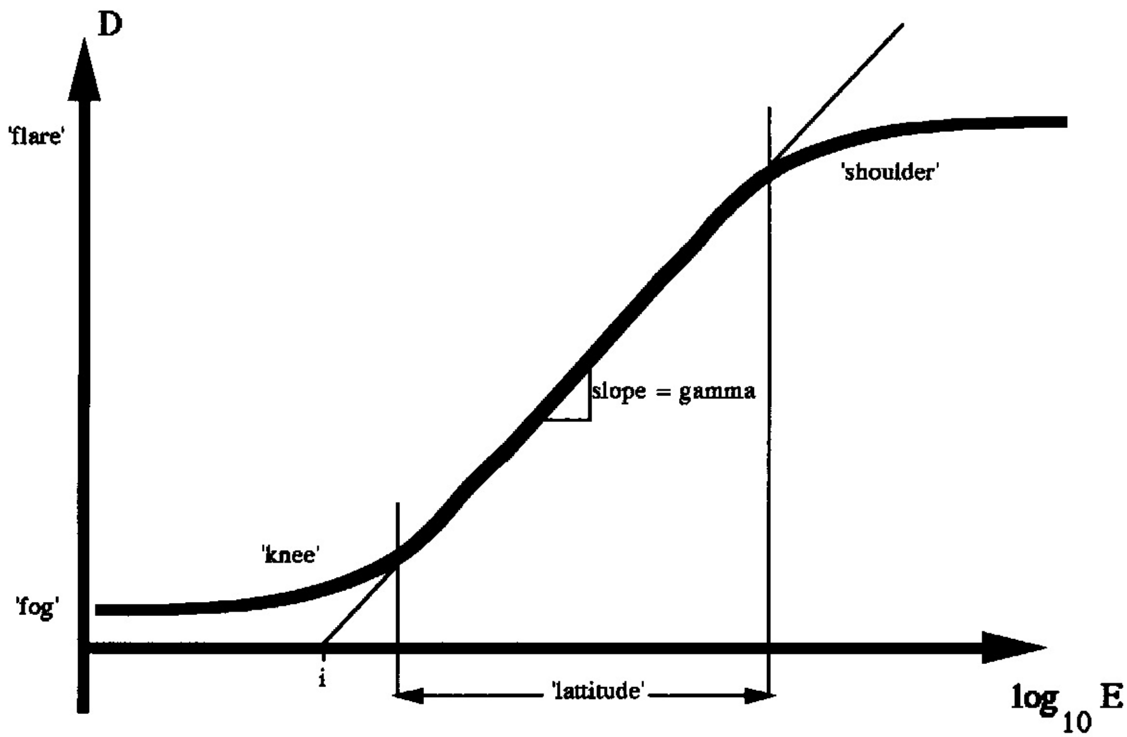

Figure 3a: Hurter & Driffield's "Characteristic Curve" for photographic film.

Where T = transparency: = 0 for perfectly opaque, =1 for perfect transparency
D = density = log(1/T)
t = exposure time, I = incident intensity
E = film exposure = t * I

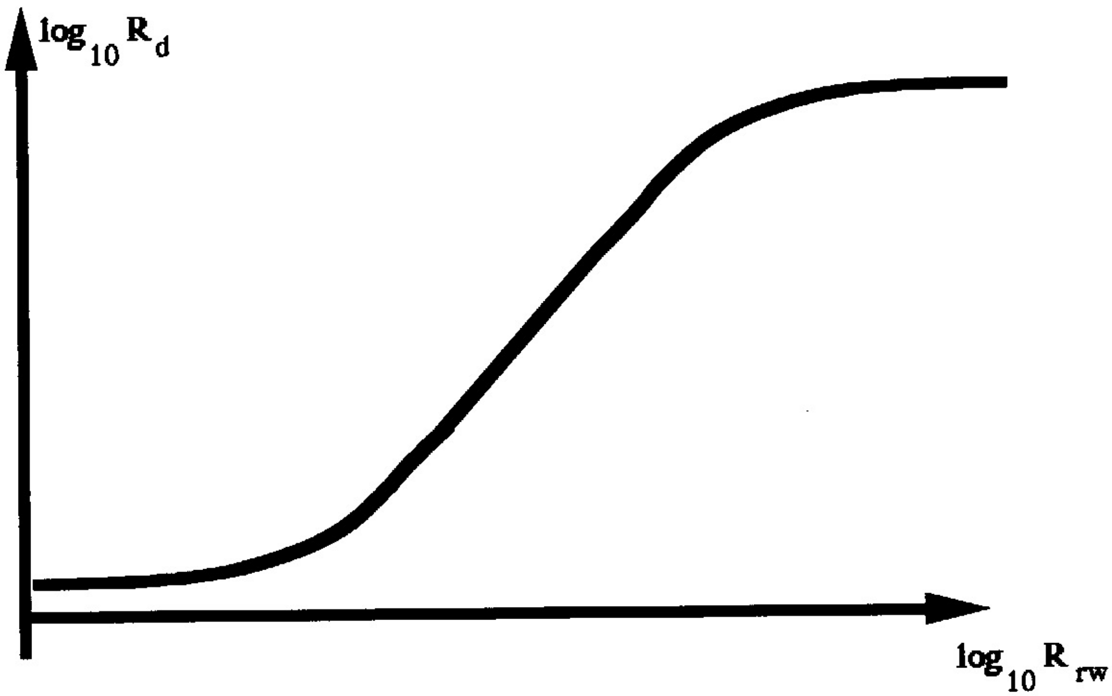

Figure 3b: Change units on both axes shows the H-D plot describes the relation between real-world radiance Rrw and the display radiance Rd of figure 2.

$$E = a_0 \cdot R_{rw} \qquad \text{so}$$
$$\log_{10}(R_{rw}) = \log_{10}(E) - \log_{10}(a_0) \qquad (1)$$

For the display system, suppose the film is viewed as a transparency placed on a light table -- a diffuse (lambertian) light source of L_d lamberts; then the resulting display image luminance is:

$$L_d = \text{light table luminance (lamberts)}$$
$$R_d = L_d \cdot T$$
$$D = -\log_{10}(T) = \log_{10}(L_d) - \log_{10}(R_d)$$

$$\log_{10}(R_d) = \log_{10}(L_d) - D \qquad (2)$$

Equations (1) and (2) can be applied to the H-D curve of figure 3a to find the graph in figure 3b, which directly relates real-world radiance R_{rw} to displayed radiance R_d.

The nearly linear center region of H-D plots usually spans about 2.0 to 3.0 log units in E for most film (100:1 to 1000:1) [JAME66]; outside this 'straight line' region sensitivity gradually vanishes into saturated black or white, and are used only for image highlights or deep shadow details. Hurter and Driffield parameterized this straight line region by its horizontal offset 'i', called the film sensitivity or film speed, and the 'middle-tone' or mid-region slope they named 'gamma' (γ); and the width of the region became known as the 'lattitude'. This concise description of film response has been widely used since these simple parameters can be robustly controlled by chemical and optical processes. Note that gamma is negative for film that produces negative images.

The 'replotting' in figure 3a & b can also be expressed mathematically. In the tone reproduction framework of figure 2b, the 'straight line' region of the H-D curve is

$$D = \gamma \cdot \log_{10}(i) - \gamma \cdot \log_{10}(E) \qquad (3)$$

where i = intercept of line with D=0 axis
(better measures of film speed are in current use [JAME66])

by substituting equations (1) and (2) into (3) we can express R_d in terms of R_{rw};

$$\log_{10}(L_d) - \log_{10}(R_d) = \gamma \cdot \log_{10}(i) - \gamma \cdot \log_{10}(R_{rw} \cdot a_0)$$

$$\log_{10}(R_d) = \gamma \cdot \log_{10}\left(\frac{R_{rw} \cdot a_0}{i}\right) + \log_{10}(L_d)$$

$$R_d = L_d \cdot \left(\frac{a_0}{i}\right)^{\gamma} \cdot R_{rw}{}^{\gamma}$$

combine constants into a_1;

$$R_d = a_1 \cdot L_d \cdot R_{rw}{}^{\gamma} \qquad (4)$$

Note that a_1 and γ are set by the photographers choice of film, lenses, exposure time, aperture, and darkroom processes, so artistic judgement can strongly affect the resulting image.

Extensive experiments in subjective image quality (such as [JONE48]) have shown the best choice for a_1 and γ depend strongly on both the illumination L_d and the surrounding luminances seen by the display observers. For viewing reflection prints in normal room light, most prefer γ between 1.1 and1.2, and the exposure constant a_1 must be made large enough to drive the image highlights above the straight-line region of the H-D curve into the saturated 'shoulder' region. Film reproduction with the simple scale factor relation as used in Figure 1, that is, with $\gamma = 1.0$, a_1 = nominal, are overwhelmingly rejected as too dark and lacking contrast when viewed as a reflection print. However, when viewed as a strongly back-lit transparency viewed in a dim room, such film is preferred over any other reflection print. Strong backlighting is needed, though; image highlights must be made much brighter than a diffuse white object would appear in the observing room. If the room lighting is then boosted so that the diffuse white object's luminance matches the transparency illuminator, the image will again look dark and flat[BART67]. Even though the image has not changed, the changed surrounds have made the eye's response very different. Such changes in eye response due to overall illumination changes are often called adaptation. A description of adaptation and other changes is needed to complete the framework of figure 2b, and is given in Section 4.1.

Now suppose we could safely approximate the eye's response by H-D plot-like curves; then the accuracy of tone reproduction can be assessed and controlled in (γ,i) terms. That is, suppose the observer models in figure 2b, like film, are reasonably modeled as line segments on log-log plots. Replace the exposure E with the viewed radiance value R, replace density D with $\log_{10}(B)$, where B is brightness expressed in brils. Then the observer models are simple equations of those lines:

$$\log_{10}(B) = \gamma_{eye} \cdot \log_{10}(i_{eye}) - \gamma_{eye} \cdot \log_{10}(R)$$

combine constants to get;

$$B = a_2 \cdot R^{\gamma_{eye}} \tag{5}$$

With this sort of observer model, a complete tone reproduction operator for film can be expressed entirely in terms of gamma and film speed, and hence can be robustly implemented. A very popular and successful graphical method to achieve this was introduced by [JONE20]. Jones drew eye-response curves on two quadrants adjacent to H-D plots, and transferred points to the 4th quadrant to find subjective tone reproduction curves. A good summary is found in [JAME66] or [HUNT75]. This film-like description of vision apparently led to the term 'visual gamma' as the inverse of the γ required for good tone reproduction, as in [DeMA72] and [BART67].

3.2 Extensions to Television

Response of cathode ray tubes (CRTs) to their control voltages are surprisingly similar to film. For normalized control voltage V where 0<= V =< 1, the display luminance R_d is approximately

$$R_d = a_3 \cdot V^{\gamma_{crt}} \tag{6}$$

where$2.8 \le \gamma_{crt} \le 3.0$ (standardized for television displays),
and typical peak luminace of $15 \le a_3 \le 40$ millilamberts [HUNT75]

Unfortunately the available contrast is generally much lower than film due to light reflections inside the glass envelope and veiling light on the CRT face from its surroundings. Television workers such as [DeMA72] have stated viewers preferred increasing γ as surrounding lights were dimmed; ranging from 1.0 in bright light, 1.2 for dim (4 foot-lambert) surrounds, to 1.5 for darkness. This is not terribly mysterious, in part because [SCHR86] suggests viewers crave contrast in most electronic displays; as the room light is reduced, the veiling glare on the CRT face falls. The peak luminance changes little, but reduced veiling glare allows darker blacks, approximating larger γ. Thus for dim surrounds the preferred system response is

$$a_3 \cdot R_{rw}^{\ 1.2} = R_d \tag{7}$$

Dim surroundings for viewing are typical, and to keep television receivers simple, 'gamma correction' is applied to the television signal (which becomes the 'control voltage' above) before broadcasting to reduce the effects of γ_{crt} values and thus achieve the response of equation (7). Thus the black & white broadcast signal is approximately

$$V = \left(a_4 \cdot R_{rw}\right)^{\left(\frac{1}{2.2}\right)} \tag{8}$$

so that when received signal V is applied as the CRT control voltage, the resulting response matches equation (7).

In short, television systems adopted the nomenclature and methods of film with little change, and introducted the notion of 'gamma correction' to both correct for CRT response and act as a simple tone reproduction operator at the same time. In both film and television, tone reproduction operators are primarily concerned with the light-dependent effects of the display observer alone; the real world observer effects, such as increased exposure to cause saturation of image highlights, are the responsibility of the photographer or videographer. Further, the purpose of these systems is to produce pleasing renditions rather than accurate ones.

4. Moving Beyond Film

4.1 The Applicability of Film Techniques to Computer Graphics

Simply reapplying film tone reproduction operators may be inadequate, since there are substantial differences in photographic image generation and synthetic image generation for computer graphics. These are basically the differences in the knowledge and goals of the persons generating the image, and the differences in the imaging media. For example:

A photographer can actually see the scene to be imaged. From prior experience, the photographer chooses film exposure for good reproduction of items of interest in the real-world scene. Often a computer image is synthesized because there is no way to actually see the object, so exposure choice imposes a scale factor that is undefined for computer graphics.

As an imaging medium, film has the advantage that its contrast range is far superior to the electronic displays in common use for computer graphics images. Film also suffers from a response to incident light is chemically controlled and tough to shape precisely. Tone reproduction functions for film must be simple to be achievable, so results are approximated by the best-fitting choice of gamma and exposure. In comparison, any tone operator of any complexity can be implemented for computer graphics. Furthermore, the sensitivity of film to light restricts most photography to the middle or upper range of human vision, where acuity and sensitivity to color and luminance are fairly constant, Photographic tone reproduction applied to starlit scenes is uncommon, but there is no such sensitivity restriction for computer graphics images. Precise spatial filtering is also impractical in film; therefore corrections for light-dependent acuity, diffraction in the eye and other effects are

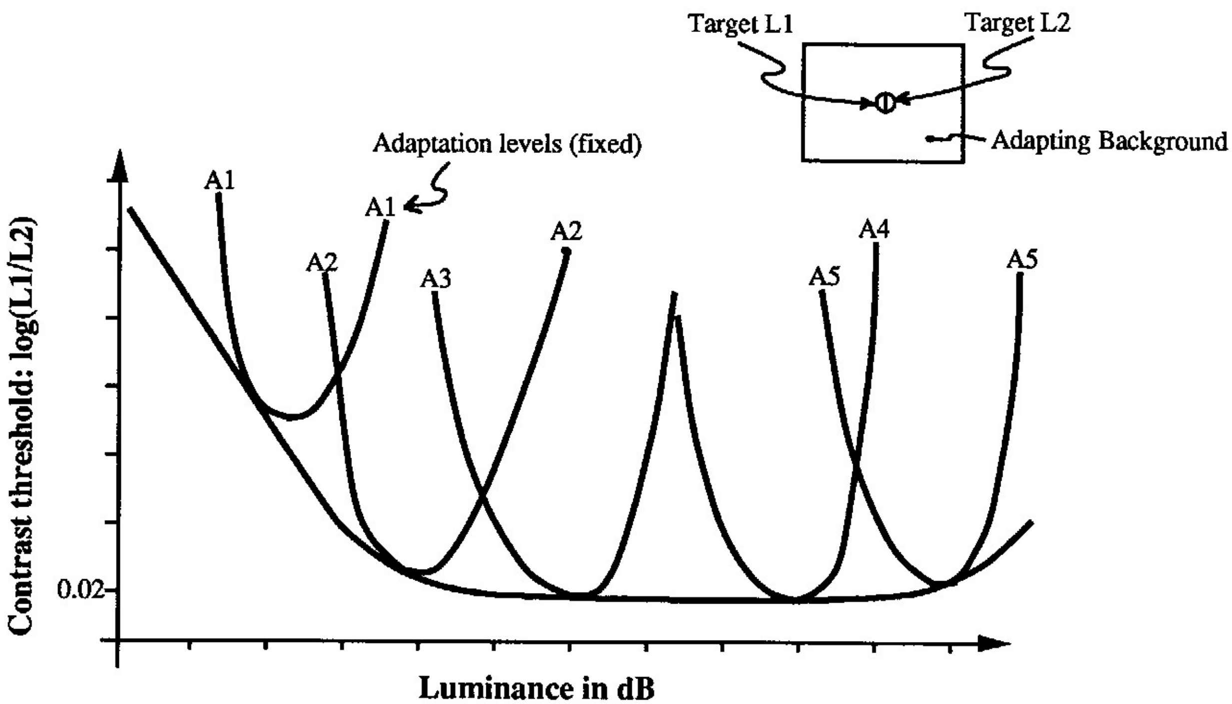

Figure 4: Split Target Contrast Thresholds
at various fixed adaptation levels A1 through A5

rarely attempted except by ad-hoc darkroom tricks. Some optical methods for spatial filtering are popular in offset printing (see [HUNT75]) but are also applied subjectively. In computer graphics such effects are more easily included in the image synthesis calculation; for example, eye diffraction calculations were used by [NAKA90].

4.2 Re-examining Applicable Vision Research

The eye's complete response to light is still not completely understood, despite over 100 years of research. It is difficult to accurately quantify because vision blends smoothly with higher brain functions, and because the eye's behavior is strongly dependent on the content of the viewed image. Brightness response is usually described by several processes, including at least adaptation, simultaneous contrast, brightness- and color - constancy, and image interpretation. Many of these are interdependent, self-adjusting, and difficult to measure separately: each tends to obscure the other, so that brightness rules inferred from simple tests often fail when applied to more complex images (see [BART67] and [CORN70]. Since brightness itself (and any other measure of perception) can only be measured indirectly, experienced workers such as [CAEL81] and [SCHR86] suggest that no quantitative model is above suspicion. A robust and accurate description of the entire gamut of visual response will probably come only when more of the biological mechanisms of perception are understood.

4.2.1 Basics: Adaptation

In simplest terms, the eye detects contrasts rather than absolute luminance, and absolutes appear to be inferred from detected contrasts [CORN70],[KING89]. The threshold of detectable contrast is near 2% for most viewing conditions ([WEBE1847], [BLAC46] & many others since], and gradually increases as surrounding luminance drops below about 10^{-3} lamberts; at these lower light levels the thresholds closely approach the theoretical quantum detection limits (nicely summarized by [SCHR86]). However, the recognition of small luminance differences, and the influence of the bordering luminances and their transition rates are still being actively debated (e.g. [KING89] & [YOUN87]); most of these effects are grouped as 'simultaneous contrast' and are discussed in the next section.

It is a popular mistake to treat these contrast thresholds as discrete differential units of sensation, or 'Just Noticeable Differences' and then integrate them to form a single logarithmic function for the luminance-brightness response, often attributed to Fechner's 1860 opus at the dawn of psychophysics, yet still persistent in the literature (for example, lucid texts by [RUBI69], and [GONZ87]). This "Weber-Fechner Law" was best put to rest by the definitive and entertaining article by Stevens [STEV61], who suggests the use of film-like sets of power-law rules instead. A single curve, regardless of shape, is clearly wrong; it ignores adaptation. For example, an automobile dome lamp's surface (about 1/2 lambert) is painfully bright when switched on suddenly at night, but as time passes our eyes adjust, and the apparent brilliance is reduced; clearly the brightness has changed with adaptation. Adaptation loosely names the slow adjustments (2 seconds to an hour or more) to sensitivity in response to surroundings. It is attributed to iris adjustment and some forms of photochemical equilibrium in the retina. Frequent movements of the eye assure that adaptation is approximately constant across the entire viewed scene.

The precise amount of adaptation is easy to define only for large fields of uniform luminance, so many experiments test visibility of small targets in the hope that adaptation is unchanged. Measurements of contrast thresholds between two small adjacent targets displayed on large uniform adapting fields have shown that contrast thresholds are not constant; thresholds for detecting differences between the two targets quickly increased as target luminance diverged from adaptation level, as shown in figure 4. As the differences from the adapting background grew large, the targets appeared completely black or as brilliant white, suggesting that if adaptation processes are fixed near

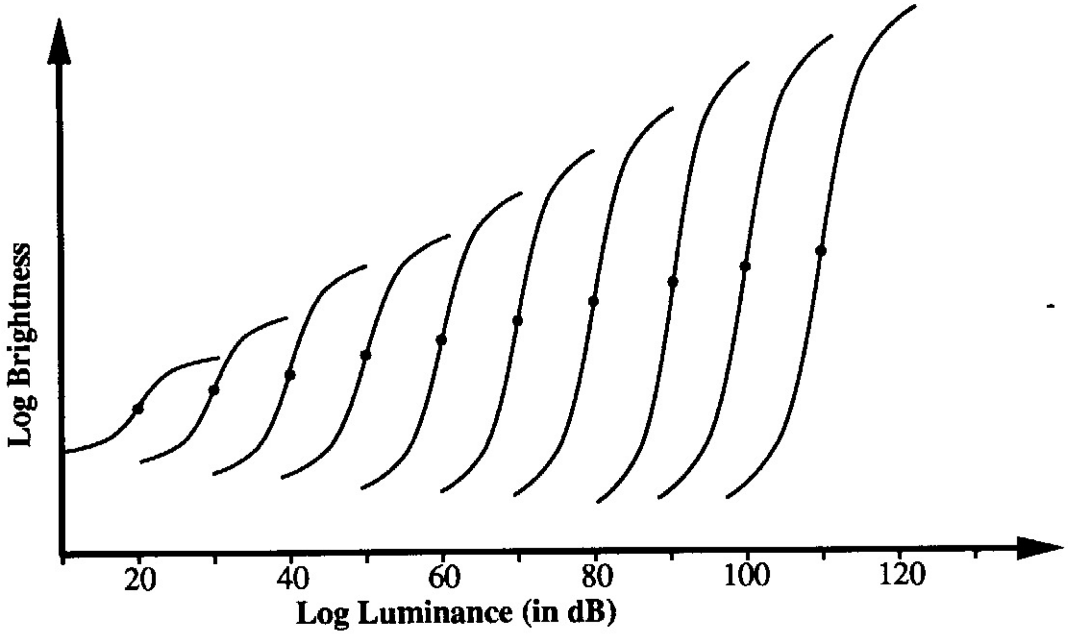

Figure 5: Postulated Brightness .vs. Luminance Response
(See text) with adaptation level fixed at dots

daylight conditions the usable luminance range of the eye spans only about 100:1 to 150:1, and this 'usable range' falls rapidly when adapted to lesser amounts of light. In this sense, the eye's performance is analogous to a camera using film with lattitude of about 2.2 $\log_{10}$ units, but with an automatic exposure constant a_2 (equation (5)) determined by adaptation amount.

If we repeat Fechner's dubious integration of threshold measurements with fixed adaptation, a set of sigmoid response functions is suggested (Figure 5), with one curve for each adaptation level. These curves must be regarded with suspicion, not only due to their origins and lack of corroborating experiments, but also because luminances that fall in the upper regions of such a curve are likely to increase the adaptation level and hence the choice of the curve itself. Regardless, such curves may describe the saturating effects of painfully bright lights, and workers such as [SCHR86] suspect these curves exist.

4.2.2 Simultaneous Contrast

Direct measurement of these fixed-adaptation response curves were attempted by diverse methods, though usually with targets that were darker than the adapting surrounds. Binocular matching methods used by [PITT39] were later adopted by [STEV63], while others used direct quantitative estimates made by the observers themselves: Newhall,Nickerson, & Judd used observer estimates of 'equal appearing intervals', and estimates of brightness ratios were used successfully by [STEV63] and [HANE49]. An excellent summary appears in [JAME66]. It was found that the shapes, sizes, and positions of the small targets used could strongly affect the resulting curves. These spatial dependencies in the measured luminance .vs. brightness curves are often called 'simultaneous contrast effects'.

Simultaneous contrast is a quick (0.1 to 2 seconds) localized adjustment in sensitivity, and an apparent sharing of local brightness signals due to the amount of local luminance. Simultaneous contrast effects increase the amplitude of brightness transitions at luminance edges, participate in curious effects such as edge acutance, vernier resolution or 'hyperacuity', and illusions such as the Craik/ Cornsweet illusion [KING89], Mach banding, and the Herman - Herring grid illusion of figure 6. Simultaneous contrast is usually attributed to neural processes in the retina.

Simultaneous contrast effects tend to exaggerate sharp brightness transistions and suppress slow-varying changes. Since these effects are frequency dependent, several workers in the 1960s attempted to model them by Fourier analysis. [LOWE60], [CAMP68], [NESS67], [OVER81], and others each attempted to model interactions between adjacent brightnesses by either direct frequency domain descriptions or by determining bandpass filter kernels for convolution; experiments included extensive measurements of the visibility thresholds of sine wave gratings and other methods. However, nonlinearity of both the brightness response and its local interactions meant only the measurements at detection threshold could be trusted. [CORN70] showed that such frequency response models formed a plausible fit to data when applied to log radiance instead of radiance itself, suggesting structures in which $\log(R_{rw})$ is applied to linear filters for modeling perceived brightness. Data for these filters are troublesome because most have no DC response; a separate 'adaptation' signal (or image) is also needed to reversibly describe the complete image. Some workers [FIOR68], [XIE89]) suspect that the adaptation level itself is not purely constant, but is localized, so that local adaptation is better modeled as a low-frequency image that affects spatial frequency response as well. Still others [MARR82] have proposed that perhaps only the edges, corners, and rate-of- change information is sensed directly and that perhaps intensity is only inferred without absolute sensing. Simultaneous contrast was also addressed by a neural network models to describe supra-threshold effects by [CAEL81], and [XIE89].

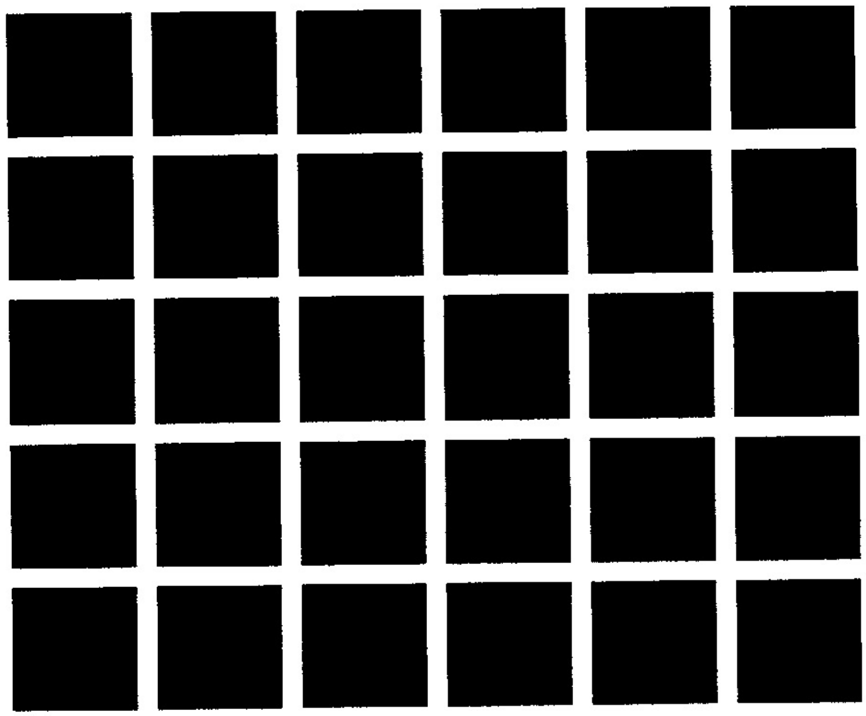

Figure 6: Herman Herring Grid Illusion

Note transient gray spots at corners of the black squares: this illsusion usually attributed to Simultaneous Contrast

Some of these models are troublesome in the tone reproduction framework of figure 2 because they appear to be irreversible, and an inverse observer model is needed in the tone reproduction operator shown. Perhaps the most intriguing model we found is the 'Intensity Dependent Spread' mechanism proposed by [CORN86]. They show this simple nonlinear mathematical model fits a wide range of visual data well, reproduces many disparate visual phenomena, and is very easy to compute. It does not appear to be easily reversible, but certainly merits further study; perhaps it can be reversed by preserving additional data with the brightness output.

4.2.3 Independent Measures of Adaptation and Simultaneous Contrast

Perhaps the most exhaustive experiments to measure the entire gamut of human brightness response were made by Stevens & Stevens between about 1953 and 1963, who showed that certain power law relations (equation (14) below) worked well for their very simple test images. Two reports on their work are of particular interest; an abstract of a paper apparently never published, giving equation (11) and Figure 8 to relate $\log_{10}$ (luminance) to $\log_{10}$(brightness) as a simple family of lines [STEV60], and later a paper describes further measurements in which the effects of simultaneous contrast have been stabilized and perhaps removed[STEV63], shown in figure 7. Though Jameson & Hurvich [JAME64] and later Bartelson and Breneman [BART67] showed the 1960 equation was inadequate for complex images, the separation of simultaneous contrast effects in the later paper suggests better models of simultaneous contrast might overcome these problems.

By 1963, Stevens & Stevens measured brightness versus luminance and adaptation by using 'haploscopic matching'[PITT37], that is, by the matching of brightnesses perceived when one eye is dark-adapted (as 'standard condtions' for measuring brightness) and the other is adapted to a test value. In the later experiments, once the left eye was fully dark-adapted and the right eye had adapted to its uniform test field, both were briefly presented with 5 degree test targets on completely black (light-tight) backgrounds. Brightness comparisons between the two eyes were made quickly, before either could change adaptation level significantly, and repeated experiments at different target brightnesses verified verified the brightness matches found.Since the target surrounds are always black during measurement, any interactions between adjacent brightnesses ('simultaneous contrast effects') are always between the target brightess and blackness, thus stabilizing and probably removing most of them. The results are given by equation (9), plotted on log-log axes in Figure 7.

$$B = K \cdot (R_{targ} - R_{thresh})^{m} \qquad (9)$$

where B = brightness in brils, where brils are a linear scale of brightness sensation described in [STEV63], where 1 bril is the sensation of brightness from a fully dark-adapted eye viewing a 5 degree target of 1 micro-Lambert for one second. Two brils appear exactly twice as bright as one bril, and ten brils appear five times as bright as two brils; thus brils are a linear scale of absolute brightness.

R_{targ} = radiance of target in millilamberts,

R_{thresh} = threshold of detectable radiance in millilamberts; this depends on the strength of the adapting field;

m , K = fixed parameters dependent on the strength of the adapting field.

For full dark-adaptation, R_{thresh} =0, m = 0.33, K = 10, and agrees with the data of Hanes (see [JAME66] pg475 eqn 22.12). Note that as adaptation level increases, R_{thresh} increases steadily, but with no other viewed luminances for comparison, the brightnesses above threshold are not strongly affected by adaptation amount; that is, K is slow-varying. [JAME66] notes that a millionfold increase in adapting luminance causes only a tenfold decrease in K.

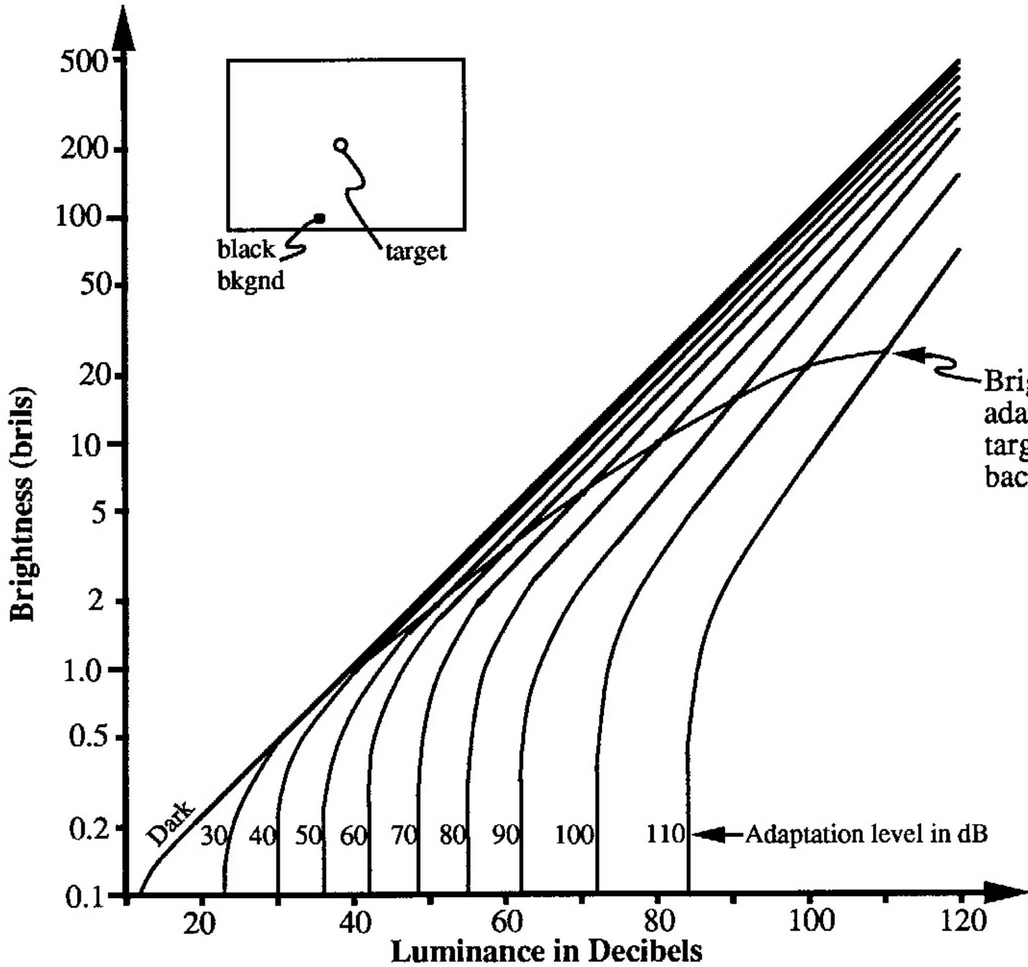

Figure 7: Stevens 1963 Experiment: brightness .vs. luminance at various adaptation levels, with effects of simultaneous contrast removed or stabilized.

The observer's eye was allowed to thoroughly adapt to a uniform background luminance. Then the background was switched to total darkness while a target was briefly presented, and its brightness was measured before significant adaptation could occur. Darkness surrounding the target stabilized and perhaps removed the simultaneous contrast effects.

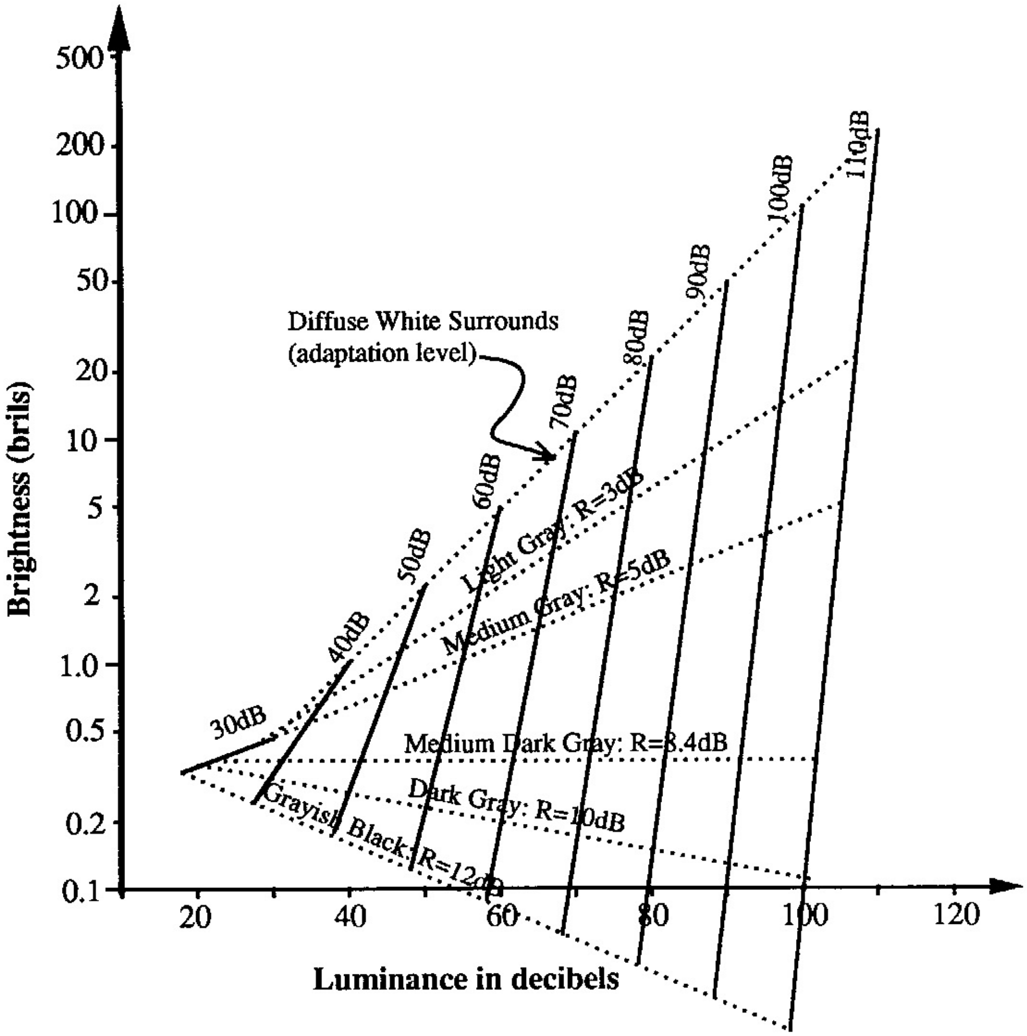

Figure 8: Stevens 1960 experiment: brightness .vs. luminance at various adaptation levels (as in Fig. 7) but now including effects of simultaneous contrast

The observer's eye was allowed to thoroughly adapt to a uniform background luminance. A target was briefly presented and its brightness measured, but unlike Figure 7, the adapting background was NOT removed. Simultaneous contrast between background and target strongly affected target appearance, resulting in sharply increased slopes, and "brightness constancy" for -8.4dB reflectivity.

The earlier experiments [STEV60] also measured 5 degree gray targets, but the targets were presented without switching off the adapting background; thus simultaneous contrast effects were included in the measurements; interactions occurred between perceived brightnesses the target and the white background used for adaptation. Stevens' equation for his data is given below and plotted in Figure 8;

$$\log_{10}(B) = 0.004\cdot[(S-27)\cdot(8.4-R)-108] \tag{11}$$

where

L_w = adapting white background luminance in lamberts and

L_{targ} = target luminance in lamberts,

$S = L_w$ in dB, where $0\text{dB} = 10^{-10}$ lamberts, so that

$$= 10\cdot\log_{10}\left(\frac{L_w}{10^{-10}\text{lamberts}}\right) \text{ or}$$

$$S = 100. + 10\cdot\log_{10}(L_w) \tag{12}$$

R = target luminance difference in dB,

$$= S - 10\cdot\log_{10}\left(\frac{L_{targ}}{10^{-10}\text{lamberts}}\right), \text{ so that}$$

$$R = 10\cdot\log_{10}\left(\frac{L_w}{L_{targ}}\right) \tag{13}$$

Alternately, if the adapting background S is formed by illumination of a 100% diffuse reflecting white surface, then the target under the same illuminant has reflectance given by $-R - \text{target reflectance in dB} = 10\cdot\log_{10}(\text{reflectance})$, where $0 <$ reflectance < 1. We can substitute expressions for S,R into equation(11) and express it in the same form as equation (5);

$$\log_{10}(B) = \alpha\cdot\log_{10}(L)+\beta \tag{14}$$

where

B = brightness in 'brils' (see equation(9))

L = viewed (target) radiance in lamberts,

L_w= luminance of white surround

$$\alpha = 0.4\cdot\log_{10}(L_w)+2.92 \tag{15}$$

$$\beta = -0.4\cdot(\log_{10}(L_w))^2 + (-2.584\cdot\log_{10}(L_w)) + (2.0208) \tag{16}$$

Simultaneous contrast effects have dramatically changed the eye's response to the target luminances. For the same adaptation levels of Figure 7, the eye's sensitivity to changes in target luminance has increased by several times, and the black thresholds R_{thresh} have grown much larger for the higher levels of adaptation. (Presume that adaptation level is fixed by the white background, and is not affected by changes in the target luminance). Also note that targets with -8.4dB reflectance (14%)

have constant brightness regardless of the adaptation amount, and thus this particular reflectance exhibits 'brightness constancy'. Reflectances below 14% act strangely; increased illumination (and hence increased adapting radiance) reduces their brightness; more light makes them look darker!

More importantly, notice that equation 14 and Figure 8 describe straight-line plots on log-log axes, so they are easily applied to tone reproduction for film. The graphical method of [JONE20] would show near-perfect tone reproduction could be achieved by adjusting (γ,i). This tends to verify the pleasing results of film.

Unfortunately, the Stevens & Stevens data is neither valid nor accurate when applied to more complex, interesting images, since their luminance .vs. brightness relations depend on localized image contents. By careful measurement of many test photographs, Bartelson & Breneman [BART67] found pleasing reproductions for complex scenes could be made with an additional exponential weighting term. They suggest (in different terms)

$$\log_{10}(B) = \alpha + \beta \cdot \log_{10}(R_{rw}) - \delta \cdot (R_{rw})^{\varepsilon} \qquad (17)$$

where α, β, δ and ε are parameters dependent on viewing conditions for 'best fit' to their data, and given graphically.

5. An Example Of A New Computer Graphics Tone Operator

Given the rich literature of vision research, it is tempting to begin building huge observer models immediately, but very complicated observer models would greatly increase the computational cost of generating an image. It is not clear that including all known effects would be worth the additional expense. As a starting point then, we present a simple tone reproduction operator to solve the display range problem described in section 1, using Steven's film-like luminance brightness relation of equation(14), and a few simple assumptions in the tone reproduction framework of figure 2b. We ignore Bartelson & Breneman's extensions to avoid root-finding problems.

5.1 Mathematical Description

Restating equations (14-16) as a set of simple line segment equations on log-log axes to act as an observer model yields:

$$\log_{10}(B) = \alpha \cdot \log_{10}(R) + \beta$$

where

B = brightness in 'brils' (see equation(9))

L = viewed (target) radiance in lamberts,

L_w = luminance of white surround for equivalent adaptation amount

$$\alpha = 0.4 \cdot \log_{10}(L_w) + 2.92$$

$$\beta = -0.4 \cdot (\log_{10}(L_w))^2 + (-2.584 \cdot \log_{10}(L_w)) + (2.0208)$$

As mentioned in section 4.2.1, adaptation level estimates for complex images are ill-defined, so the value of L_w is unclear. However [JAME66] notes that [MARI62] has found Stevens's data is well fitted by approximating adaptation with average luminance. Similarly, we approximate L_w for the real-world observer by using the mean (expected value) of the $\log_{10}$ of the real-world radiance, R_{rw}.

Let this correspond to the brightness constancy contour found by Stevens at 8.4dB = 0.84 $\log_{10}$ units below $\log_{10}(L_w)$. Thus the 'white adapting luminance' L_w for the real world observer is:

$$\log_{10}\left(L_{w(rw)}\right) = E\left\{\log_{10}(L_{rw})\right\} + 0.84 \tag{18}$$

Approximate L_w for a CRT display by using [HUNT75] pg 441 data claiming typical CRT peak luminance is ~50-120 cd/m^2. Since 1 cd/m^2 = π lamberts/10,000, then take a midrange value for L_w for the display observer:

$$\log_{10}(L_w) = -1.569$$

Collecting terms for the upper and lower paths of figure 2a gives:

Real-World: $$\log_{10}(B_{rw}) = \alpha_{rw} \cdot \log_{10}(L_{rw}) + \beta_{rw} \tag{19}$$

Display: $$\log_{10}(B_d) = \alpha_d \cdot \log_{10}(L_d) + \beta_d \tag{20}$$

As shown in figure 2b, we can find a tone reproduction operator by assuming these two equations are equal. Solving for L_d in terms of L_{rw},

$$L_d = \left[L_{rw}\right]^{\left(\frac{\alpha_{rw}}{\alpha_d}\right)} \cdot [10]^{\left(\frac{\beta_{rw}-\beta_d}{\alpha_d}\right)} \tag{21}$$

We can complete the operator by devising the inverse display model. A simple forward model is given by

$$\left(\frac{L_d}{L_{d\max}}\right) = n^{\gamma} + \left(\frac{BG}{L_{d\max}}\right) \tag{22}$$

Where:

L_d = screen radiance in lamberts,

$L_{d\max}$ = max possible screen luminance
  =L_w for display observer = 0.027 lambert typical.

γ = 2.8 to 3.0 for uncorrected CRTs; if display device includes built-in gamma correction, this value is typically 1.2.

n = frame buffer value ($0 < R=G=B= n < 1$) used as display device input

BG = screen background radiance
  = ambient_radiance * screen reflectance + secondary_CRT_internal_reflections

C_{max} = maximum contrast ratio possible between on-screen luminances. Typical values for direct view CRTs are about 35 (HUNT75).

Note that $\left(\frac{BG}{L_{d\,max}}\right)$ is approximately equal to $\left(\frac{1}{C_{max}}\right)$, because the brightest on-screen radiance is largely determined by $L_{d\,max}$, and the dimmest possible on-screen radiance is set by the screen background radiance. Dimmest/Brightest = maximum contrast = C_{max}. Now find the inverse display system model by simply solving for n:

$$n = \left[\frac{L_d}{L_{d\,max}} - \frac{1}{C_{max}}\right]^{\left(\frac{1}{\gamma}\right)} \tag{23}$$

Note that 'gamma correction' of the CRT's 2.8 to 3.0 value is not explicit; there is no (1/2.2) factor in the equations above. Instead, the preference for γ =1.2 is implicit in the inverse display observer model.

The foregoing results are combined to form the tone reproduction operator (24). This equation finds frame-buffer values 'n' from the calculated real-world radiance R_{rw}.

$$n = \left[\left(\frac{1}{L_{d\,max}}\right)\cdot[L_{rw}]^{\left(\frac{\alpha_{rw}}{\alpha_d}\right)}\cdot[10]^{\left(\frac{\beta_{rw}-\beta_d}{\alpha_d}\right)}\right]^{\left(\frac{1}{\gamma}\right)} \tag{24}$$

Despite its messy appearance, all symbols are constants except n and L_{rw} for any fixed adaptation levels. Once the adaptation levels for the two observer models are know, the tone reproduction operator quickly reduces to a simple power-law equation with constants aa and bb of the form $n = \beta \cdot L_{rw}{}^{\alpha}$.

5.2 Example Images

Figure 9 shows the results of this operator when applied to the radiosity solutions of Figure 1. We must stress that these images were entirely the result of our new tone reproduction operator developed in section 5.1; no 'adjustments' or 'magic scale factors' were used. For comparison, the lower left corner image 'stdfile.brt' was made by the popular ad-hoc method of normalizing all radiances to the strongest non-emitting surface in the room. The remaining images display the output of our tone reproduction operator when subjected to real-world radiances that range over the 'searchlight to firefly' range described in section 1. (The pure black and white background behind the framed images was used as a check of the photographic print). The brightest image is the displayed result for a radiosity solution with the overhead lamp luminance at 1000 Lamberts, and is awash in glaring whites. Succeeding images show display resluts as the lamp luminance is reduced by a factor of 100 with each successive image; thus the dimmest image is the result of a 10 microlambert light source. We apologize for any tone reproduction changes that may have occurred in the printing of this paper; please note the gray-scale strip across the center of figure 9.

The differences in these images show that this simple tone reproduction operator acts as an exposure control and as a contrast compressor. The 'stdfile.brt' image is much harsher than any of the tone-operator images, and is perhaps best matched by the Lamp0.1 image; a solution typical of ordinary

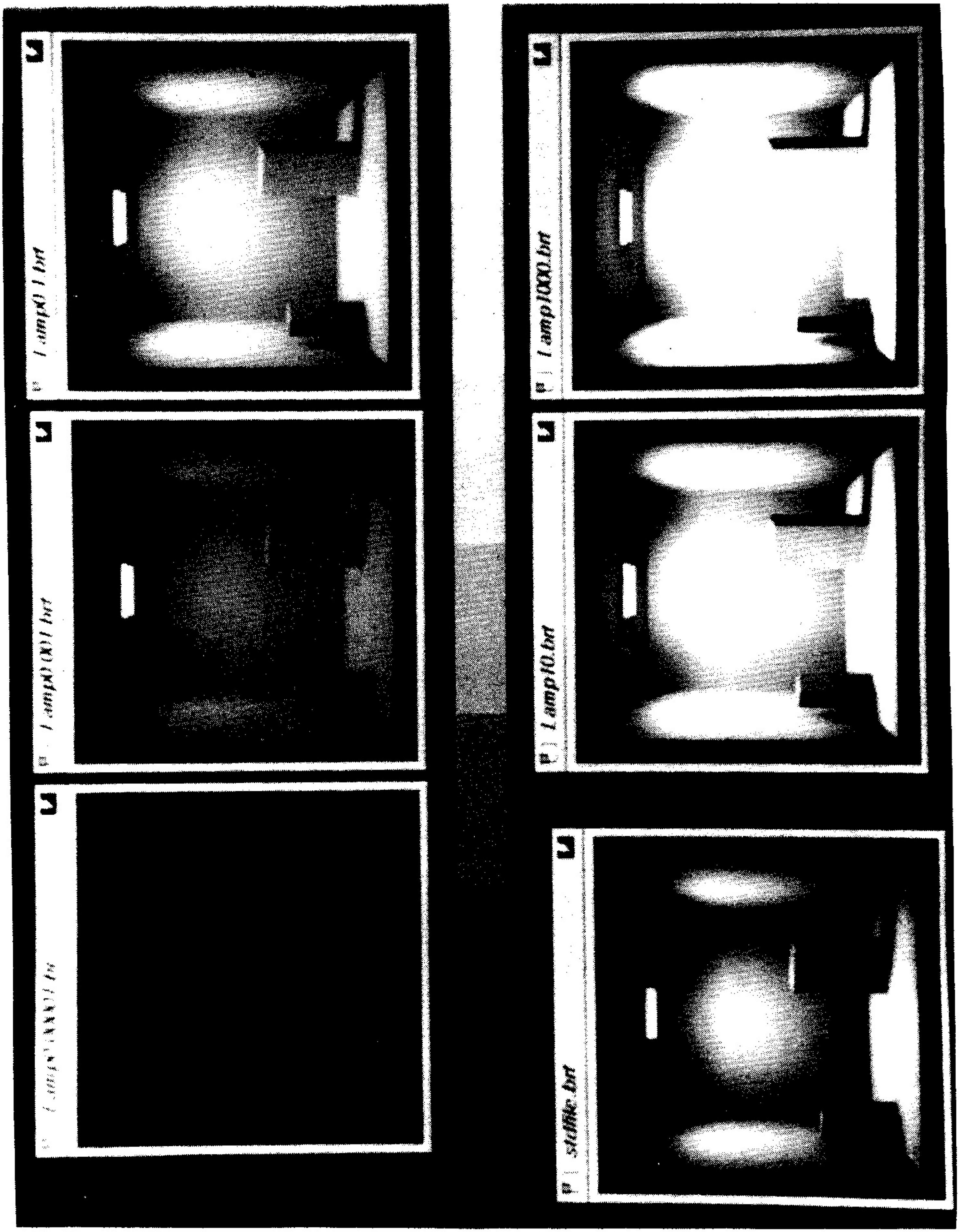

Figure 9: Example of Simple Tone Reproduction Operator results

office lighting. The tone operator is noticeably lacking in spatial effects: the very dim images should also be very blurry, as the eye's resolution fades with decreasing light, and the brilliant images should be more harsh, but glare and diffraction effects are missing. The operator is also flawed for very dim images; for $S < 27$dB (equation(11)) the slope of the R-B line is negative, and the last image in figure 9 is mostly dark grey instead of black. However, even this simple operator appears to be a consistent and plausible solution to the display range problem.

6. Conclusions and Future Work

We have shown by example that ad-hoc methods are not required for display of images whose computed radiances cannot be directly reproduced by the display device. Even the crude example operator developed here has proven useful for consistent image display, and was achieved with a fairly small amount of computation. More sophisticated observer models should increase the accuracy of the displayed image by including more light dependent effects of vision. Even without noticeable improvements in brightness reproduction, these methods remove some 'guesswork' from radiosity image display, and demonstrate that tone reproduction operators need not be constrained to the same gamma and exposure corrections used in film and television.

A truly practical tone reproduction operator should include more sophisticated models of simultaneous contrast, and must also include extensions to color and perhaps to time dependencies as well. Effects of color washout in bright light, the gradual loss of color vision as illumination fails, and models of 'color constancy' or 'color adaptation' seem especially important for realism.

Because any tone reproduction operator explicitly maps the results of a global illumination method to the display device, it also specifies the accuracy required in the illumination calculations themselves. Since computational costs rise rapidly as error tolerance falls, tone reproduction operators need to be explored for use in current efforts to develop efficient realistic image synthesis systems.

Acknowledgements

This work was funded, in part, by a grant from the National Science Foundation, ECS-9058389.

References

[ARVO90] James Arvo and David Kirk, "Particle Transport and Image Synthesis", *Computer Graphics*, Volume 24, number 4, ACM SIGGRAPH August (1990).

[BART67] C.J. Bartelson and E. J. Breneman, "Brightness Perception In Complex Fields" *Journal of the Optical Society of America,* volume 57 #7, page 953, July 1967.

[BART67] C. J. Bartelson and E. J. Breneman, "Brightness Reproduction in the Photographic Process", *Photgraphic Science And Engineering*, Volume 11, Number 4, July/August (1967)

[BLAC46] H. Richard Blackwell, "Contrast Thresholds of the Human Eye", *Journal of the Optical Society of America*, Volume 36, number 11, November (1946)

[CAMP68] F. W. Campbell and J. G. Robson "Application of Fourier Analysis to the Visibility of Gratings" *Journal Of Physiology*, Volume 197, pp. 551-566 (1968).

[CAEL81] Terry Caelli *Visual Perception, Theory, and Practice* Pergamon Press, New York (1981)

[CORN70] Tom N. Cornsweet *Visual Perception* Academic Press/ Harcourt Brace Jovanovich, New York, (1970)

[CORN86] Tom N. Cornsweet, "Prentice Award Lecture: A Simple Retinal Mechanism That Has Complex and Profound Effects on Perception", *American Journal of Optometry & Physiological Optics,* Volume 62, number 7, pp.427-438, (1986).
(also) Tom N. Cornsweet and John I Yellot, Jr. "Intensity-Dependent Spatial Summation" *Journal Of the Optical Society of America A*, Volume 2, number 10, pg 1769,October (1985)

[DAVS76] Hugh Davson, [editor] *The Eye, Volume 2A, "Visual Function in Man"* Part I 'On Human Vision', by H. Ripps and R. A. Weale. Academic Press/ Harcourt Brace Jovanovich, New York, (1976)

[DeMA72] L. E. DeMarsh "Optimal Telecine Transfer Characteristics", *Journal of the SMPTE* Society of Motion Picture and Television Engineers, Volume 81 pg 784, October (1972)

[DURR87] edited by H. John Durrett, *Color And The Computer*, Academic Press, Boston (1987)

[FIOR68] Adriana Fiorentini "Excitatory and Inhibitory Interactions in the Human Eye", *Visual Science: Proceedings of the 1968 Symposium*, p 269 (1968)

[GONZ87] Rafael C. Gonzalez and Paul Wintz, *Digital Image Processing*, Addison-Wesley Publishing, Reading Mass. (1987).

[HALL89] Roy Hall, Illumination and Color in Computer Generated Imagery, Springer-Verlag, New York (1989)

[HANE49] *Journal of Experimental Psychology,* volume 39, page 719, (1949)

[HUNT75] R. W. G. Hunt, *The Reproduction of Color*, Third edition, New York: John Wiley and Sons, (1975).

[JAME66] T. H. James, C. E. Kenneth Mees, *The Theory of The Photographic Process,* Third Edition, The Macmillan Company, New York (1966)

[JAME64] D. Jameson and L. M. Hurvich, *Journal of the Optical Society of America,* 1959 volume 49, pg 890.
also: D. Jameson and L. M. Hurvich, *Vision Research*, volume 4, page 135, (1964).

[JONE20] L. A. Jones *The Journal of the Franklin Institute* Volume 190, #39 (1920)

[JONE48] L. A. Jones and H. R. Condit, Journal of the Optical Society of America, Volume 38 pg 123, 1948
(also) volume 39, pg 94 (1949)

[KING89] Fred Kingdom and Bernard Moulden, "Border Effects on Brightness: A Review of findings, Models, and issues" *Spatial Vision*, Volume 3 number 4, pp.225-262, (1988).

[LOWE60] E. M. Lowry and J. J. DePalma "Sine-Wave Response of the Visual System. I. The Mach Phenomenon", *Journal of the Optical Society of America*, Volume 51, number 7 p. 740, July (1961).

[MARI62] Rosalind B. Marimont,"Model for Visual Response to Contrast", *Journal of the Optical Society of America*, volume 52, pg 800 (1962)

[MEYE86] Gary W. Meyer, Holly E. Rushmeier, Michael F. Cohen, Donald P. Greenberg and Kenneth E. Torrance. "An Experimental Evaluation of Computer Graphics Imagery" *ACM Transactions On Graphics*, Volume 5, number 1, pp30-50, January (1986).

[NAKA90] Eihachiro Nakamae, Kazufumi Kaneda, Takashi Okamoto, Tomoyuki Nishita "A Lighting Model Aiming at Drive Simulators" *Computer Graphics* , ACM SIGGRAPH, Volume 24, number 4, pg 395, August (1990)

[NESS67] F. L. Van Ness, J. J. Koenderink, H. Nas, and M. A. Bouman "Spatio-Temoral Modulation Transfer in the Human Eye" *Journal of the Optical Society of America*, Volume 57, number 9, p 1082, September (1967).

[OVER81] Ian Overington "Eye Modelling" *Assessment of Imaging Systems: Visible and Infrared(SIRA)*, T. L. Williams, editor, Proceedings of SPIE, Volume 274, pg182, April (1981).

[PITT39] F. H. G. Pitt, *Proceedings of the Physical Society* (London), Volume 51, pg 817,(1939).

[RUBI69] Melvin L. Rubin and Gordon L Walls, *Fundamentals Of Visual Science,*Charles C. Thomas Publishing, Springfield, Illinois (1969).

[SCHR1986] William F. Schreiber, *Fundamentals of Electronic Imaging Systems: Some Aspects of Image Processing*, Springer-Verlag, Berlin, (1986)

[STEV60] S. S. Stevens and J. C. Stevens "Brightness Function: Parametric Effects of Adaptation and Contrast", Program of the 1960 Annual Meeting, *Journal of the Optical Society of America*,Volume 53, #11, page 1139, November, (1960).

[STEV61] S. S. Stevens, "To Honor Fechner and Repeal His Law" *Science*, Volume 133, number 13, January (1961).

[STEV63] S. S. Stevens and J. C. Stevens "Brightness Function: Effects of Adaptation" *Journal of the Optical Society of America*, Volume 53, number 3, March (1963).

[WEBE1847] E. H. Weber *Wagner's Handwortenbuch de Physiolgie,* Volume III, p 481, Brunswick, (1846).

[XIE89] Zhenhua Xie and Thomas G. Stockham, Jr. "A Unification of Brightness Theories" *Human Vision, Visual Processing, and Digital Display,* Bernice E. Rogowitz, Editor, Proceedings of SPIE 1077 pg 124, (1989)

[YOUN87] Richard A. Young "The Gaussian Derivative Model for Spatial Vision: I. Retinal Mechanisms", *Spatial Vision*, Volume 2 Number 4, pp 273-293, (1987).

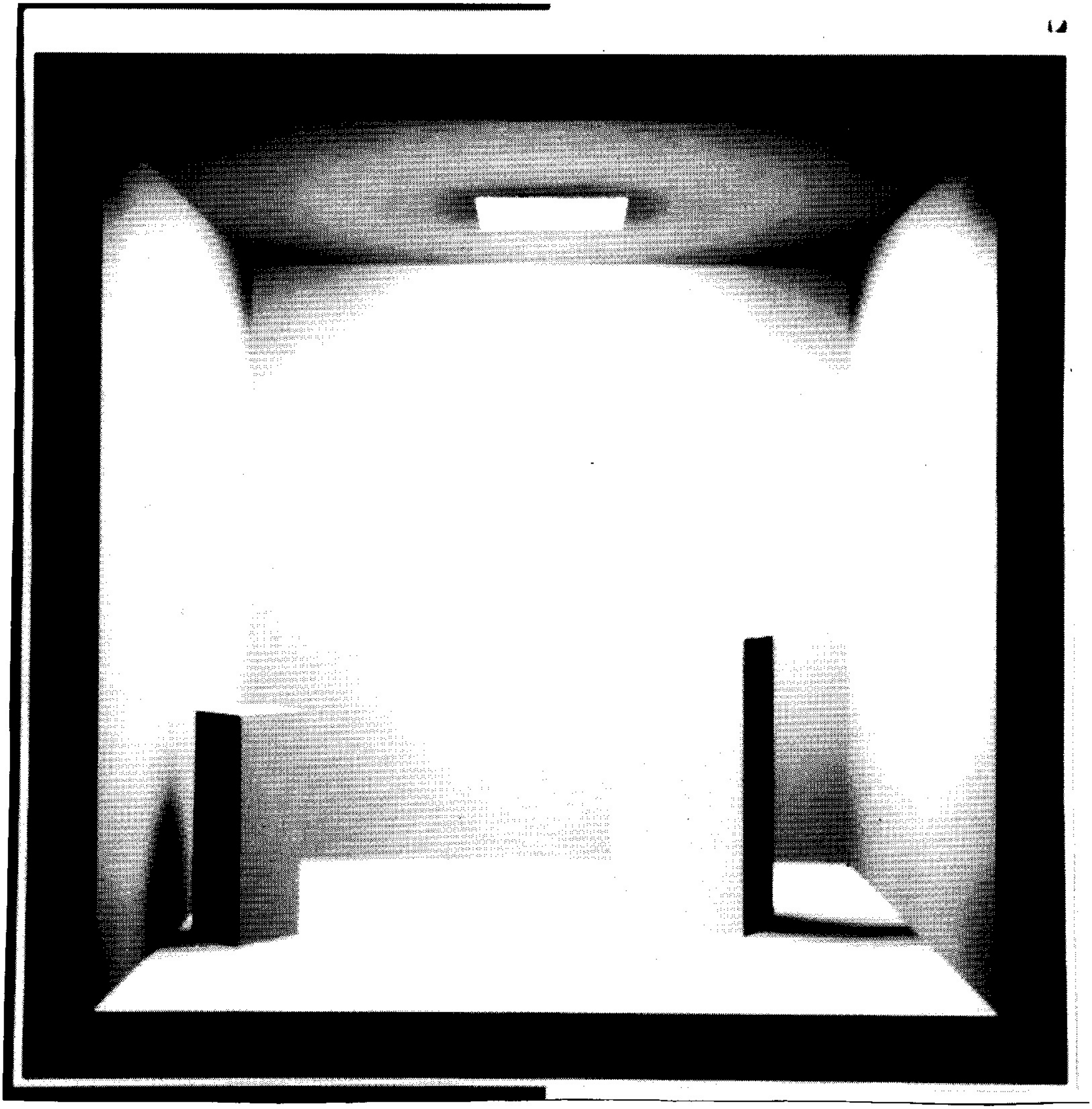

Figure 1: Display of radiosity solution using ad-hoc scale factor.
This display is valid for both the firefly- and the searchlight-illuminate case

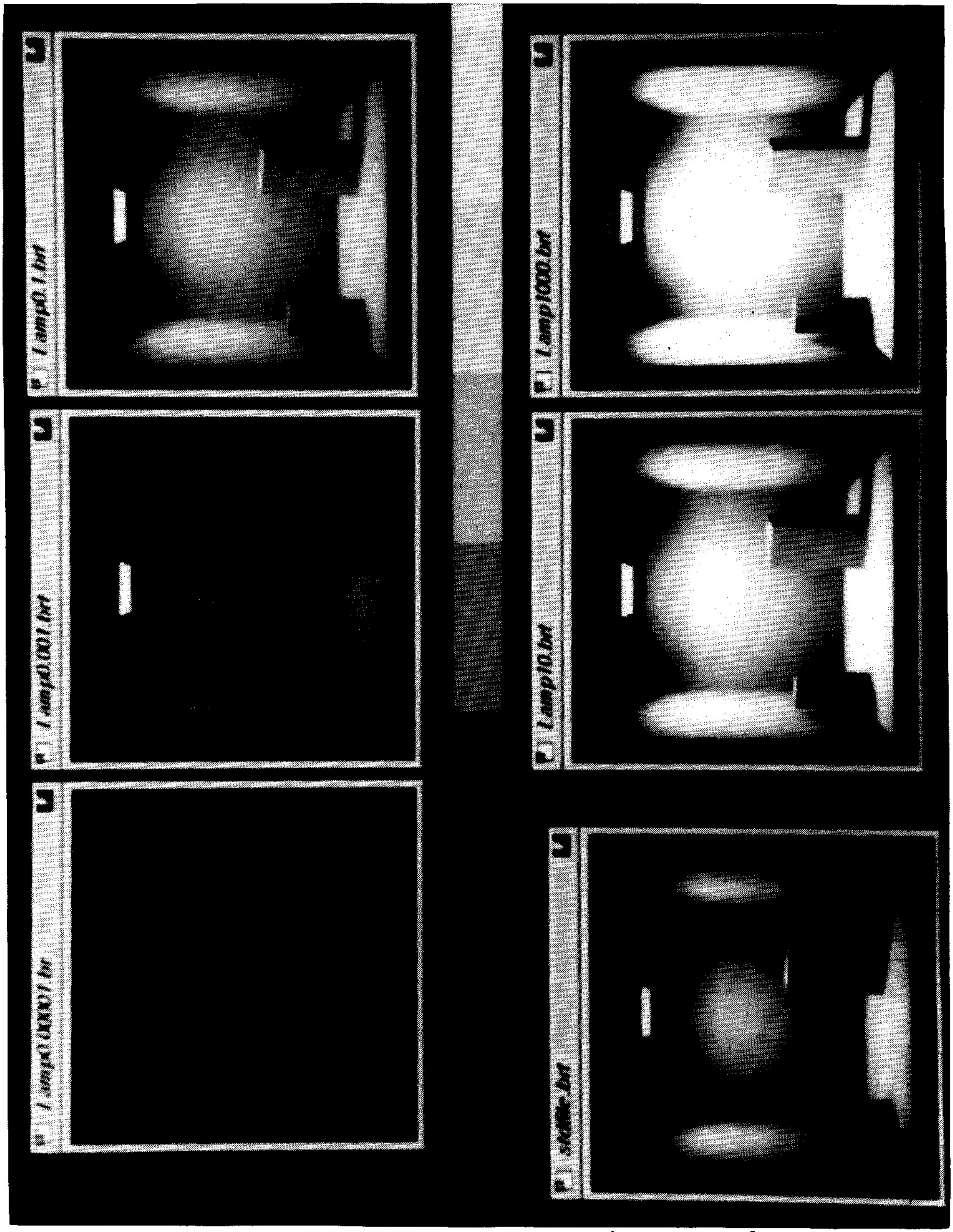

Figure 9: Example of Simple Tone Reproduction Operator results