The Institute of Paper Chemistry

Appleton, Wisconsin

Doctor's Dissertation

Correlation of Certain Morphological and Hydrodynamic Aspects of Loblolly Pine Bleached Kraft Pulp

Anthony P. Binotto

January, 1977

CORRELATION OF CERTAIN MORPHOLOGICAL AND HYDRODYNAMIC ASPECTS OF LOBLOLLY PINE BLEACHED KRAFT PULP

A thesis submitted by

Anthony P. Binotto

B.S. 1970, The Pennsylvania State University

M.S. 1972, The Pennsylvania State University

in partial fulfillment of the requirements
of The Institute of Paper Chemistry
for the degree of Doctor of Philosophy
from Lawrence University,
Appleton, Wisconsin

January, 1977

I am extremely appreciative for the secretarial and technical skills of my wife, Barbara, for whom this undertaking has resulted in many years of struggling with little more than basic necessities. Without her help, encouragement, and patience, I would not have made it through these trying times.

TABLE OF CONTENTS

	Page
ABSTRACT	1
INTRODUCTION	3
Objective	4
Review of Pertinent Literature Concerning Morphological Aspects of Pulp Fibers	4
Wall Fraction Variation	5
Fiber Length Variation	7
Review of Pertinent Literature Concerning Hydrodynamic Evaluation of Pulps	9
Wet Mat Compressibility	10
Specific Filtration Resistance	18
Specific Surface and Specific Volume	20
EXPERIMENTAL PROCEDURES	24
Isolation of Fiber Populations	24
Raw Materials	24
Preparation of Pulp	24
Earlywood and Latewood Separation	26
Fiber Length Separation	29
Fiber Analysis	29
Hydrodynamic Measurements	30
Wet Mat Compressibility	30
Constant Rate Filtration	32
Permeation	32
RESULTS AND DISCUSSION	34
Measured and Calculated Morphological Properties	34
Fiber Separation and Dimensions	34
Number of Fibers per Gram	44
Calculated Moments of Inertia	45

Page

Calculated Surface and Volume to Mass Ratios 48

Fiber Morphology and Wet Mat Compressibility 50

Compressibility Constants M and N 50

Compressibility 55

Wet Mat Density 56

Correlation of M and Fiber Dimensions 59

Mathematical Procedure for Determination of Derivatives from Curves of Experimental Data 66

Least Squares Solution of $y = b_0 + b_1x^m + b_2x^n$ 66

Computer Program 70

Fiber Morphology and Specific Filtration Resistance 71

Average Specific Filtration Resistance 71

Local Specific Filtration Resistance 74

Feasibility of Determining S_w and v as Functions of Pressure 81

Refinement of Calculation Procedures 82

Analysis of Assumption in Determining S_w and v as Functions of Pressure 83

Fiber Morphology and Average Specific Surface and Volume 84

Average Specific Surface 87

Average Specific Volume 88

CONCLUSIONS 91

IMPLICATIONS OF RESULTS 93

SUGGESTIONS FOR FUTURE WORK 95

LIST OF SYMBOLS 96

ACKNOWLEDGMENTS 99

LITERATURE CITED 100

APPENDIX I. ORIGINAL MAT DENSITY DATA 103

APPENDIX II. THREE-DIMENSIONAL PLOTTING PROGRAM 108

Page

APPENDIX III. COMPUTER PROGRAM USED TO CALCULATE AN EQUATION OF THE FORM $\underline{y} = \underline{b}_0 + \underline{b}_1\underline{x}^{\underline{m}} + \underline{b}_2\underline{x}^{\underline{n}}$ 109

APPENDIX IV. TWO-DIMENSIONAL PLOTTING PROGRAM 111

APPENDIX V. COMPUTER PROGRAM FOR THE ANALYSIS OF CONSTANT RATE FILTRATION DATA 112

APPENDIX VI. OUTPUT FROM PROGRAM CRFILT USING AVERAGED PRESSURE VS. TIME DATA FROM CONSTANT RATE FILTRATION, AND AVERAGED COMPRESSIBILITY DATA 119

APPENDIX VII. HISTORICAL PROGRAM AND OUTPUT FOR CALCULATION OF $<\underline{S}_{\underline{W}}>$ AND $<\underline{v}>$ AS FUNCTIONS OF PRESSURE FROM FILTRATION RESISTANCE AND COMPRESSIBILITY DATA 131

APPENDIX VIII. HISTORICAL PROGRAM AND OUTPUT FOR CALCULATION OF $\underline{S}_{\underline{W}}$ AND $\underline{v}$ AS FUNCTIONS OF PRESSURE FROM MULIPLE PRESSURE TAP PERMEATION 150

APPENDIX IX. ANALYSIS OF INHERENT ASSUMPTION IN DETERMINING $\underline{S}_{\underline{W}}$ AND $\underline{v}$ AS FUNCTIONS OF PRESSURE SIMULTANEOUSLY 171

ABSTRACT

Unbeaten, bleached loblolly pine kraft pulp is separated into fiber populations characterized by differences in wall-fraction and mean fiber length, $\underline{L}_{\underline{f}}$. Briefly, the procedure first involves separation of the bleached pulp into predominantly earlywood and latewood fiber fractions (low and high wall-fractions, respectively) using a Jacquelin apparatus. These fractions are next subdivided into fiber populations according to mean fiber length using a Bauer-McNett classifier. Variation in mean fiber length within the set of earlywood and latewood fiber populations correlates with wall thickness and fiber diameter in a manner similar to that within a tree. Static compressibility data show the relationship between mat density, $\underline{c}$, and pressure, $\underline{P}_{\underline{f}}$, follows $\underline{c} = \underline{M}\, \underline{P}_{\underline{f}}^{\underline{N}}$ for $\underline{P}_{\underline{f}}$ of about 10 to 150 cm H_2O. In this expression the compressibility constant $\underline{N}$ is found to equal 0.373 for earlywood and latewood fiber populations and the compressibility constant $\underline{M}$ correlates with fiber length for all fiber fractions. Compressibility, $d\underline{c}/d\underline{P}_{\underline{f}}$, is greatest for shortest earlywood fibers and least for longest latewood fibers. $\underline{M}$ is linearly related to $(1/\underline{I}_{\underline{F}})^{1/3}$, where $\underline{I}_{\underline{F}}$ represents the moment of inertia for a flattened fiber model, in agreement with the simple compressibility model originally developed by Wilder. The linearity of the relationship supports bending as the dominant mechanism in compressibility of wood pulp, and suggests that the wood pulp fiber is essentially flattened prior to bending. Since $\underline{I}_{\underline{F}}$ is defined as $2/3\ \underline{WT}^3\ \underline{d}_{\underline{f}}$, where $\underline{WT}$ is wall thickness and $\underline{d}_{\underline{f}}$ fiber diameter, it appears that the wood pulp fiber dimensions influencing compressibility are primarily wall thickness and to a lesser degree fiber diameter. Trends in $\underline{M}$ with changes in mean fiber length of earlywood and latewood fibers tend to follow previously reported changes in dynamic modulus of earlywood and latewood in successive growth rings, and also in elastic moduli and fibril angle. Average specific filtration resistance, $\langle\underline{R}\rangle$, obtained from constant rate filtration data at a given pressure drop, $\Delta\underline{P}_{\underline{f}}$, in the range 10 to 90 cm H_2O also correlates

with wall fraction and fiber length. Percentage change in $\langle \underline{R} \rangle$ is about comparable to those trends observed for $\underline{c}$; however, change in $\langle \underline{R} \rangle$ with $\underline{L_f}$ is much greater. Smallest earlywood fibers have highest $\langle \underline{R} \rangle$ values and these increase most with increase in $\Delta \underline{P_f}$ compared with other fiber fractions. At constant mat density, $\underline{c}$ = 0.100 g/cc, $\langle \underline{R} \rangle / \Delta \underline{P_f}$ has a sixfold change arising from fiber morphological variation. Local specific filtration resistance, $\underline{R}$, is calculated from pressure vs. time data obtained from constant rate filtration using a newly developed statistical procedure for determining derivatives. Changes in $\underline{R}$ with fiber wall fraction and length are similar to those found for $\langle \underline{R} \rangle$, but $\underline{R}$ values were much higher and increased significantly more with pressure. The square of average hydrodynamic specific surface, $\langle \underline{S_W} \rangle^2$, is proportional to $\langle \underline{R} \rangle$, and this relationship is comparatively insensitive to changes in $\underline{c}$ and average specific volume, $\langle \underline{v} \rangle$. Calculated geometric specific surface is closest to $\langle \underline{S_W} \rangle$ for latewood with greatest fiber length, probably because these fibers most closely approximate circular fibers. The swollen volume calculated from filtration and compressibility data is considerably less than that of a cylindrical model, indicating fiber collapse under fluid drag forces. The ratio of the two corresponding volumes is almost 3 for earlywood and about 1.6 for latewood. Data for $\langle \underline{v} \rangle$ also indicate immobilized water varies from 1.04 to 1.97 cc/g with morphological changes. Apparently most of this water is within the fibers and not elsewhere.

INTRODUCTION

Interest in the genetic and silvicultural improvement of wood fiber properties has been increasing for many years, especially with respect to the correlation of fiber improvements with the dry sheet properties of pulp. Changes in sheet properties have been related to changes in morphological factors such as cell wall thickness, length, and width (1-3). A comprehensive study on the relationship between fiber morphology and kraft paper properties for loblolly pine was also used as part of the basis for Tappi committee activities concerned with the aim of assigning economic values to specific methods of alterning wood and fiber properties (4). From such studies and activities it is apparent that there is significantly more known about the relationships between fiber morphology and products than is known about morphology and processes.

One process related area in which the role of wood fiber morphology is unclear concerns the hydrodynamic (water related) properties of pulp; specifically wet mat compressibility, filtration resistance, average specific surface, and average specific volume. Previous work has related these hydrodynamic properties to the structure of model fibers; i.e., the effects of glass or synthetic fiber dimensions and shape on compression and resistance to flow of fluids through fiber mats have been reported (5-8). On the basis of such studies it is to be expected that correlations would exist between wet mat compressibility and resistance to fluid flow, as reviewed by Han (9), and certain aspects of wood fiber morphology. One aspect is that thick-walled latewood fibers with relatively high wall fraction and thin-walled earlywood fibers with relatively low wall fraction would vary significantly in the compressibility of their wet mats. The latewood fibers, which have thicker cell walls and greater axial elastic moduli (10-14), might be expected to compress less readily into flattened cross sections and bend to a lesser degree, thereby providing lower wet mat density and less resistance to fluid flow. Another correlative aspect would be variation in fiber length (9). Short fibers which also have smaller diameters might be expected to pack tighter than long fibers and give mats of higher density. In addition to the dimensional aspects of wood fibers it is also necessary to take into account degree of delignification since this also relates to mat compressibility and flow of fluids through fiber mats (15,16).

In the past these hydrodynamic properties were of interest primarily in understanding paper machine processes such as drainage. Recently, however, technological developments in the area of displacement washing and bleaching have made use of wet mat compressibility and filtration resistance data in the design and operation of equipment (17). Correlation of variations in gross fiber morphology, such as wall fraction and fiber length, to these hydrodynamic properties should aid in equipment design and development of efficient operating conditions.

OBJECTIVE

The aim of this study is to obtain correlations between the aforementioned hydrodynamic pulp properties and wood fiber morphological variation, including wall fraction and fiber length, using unbeaten bleached loblolly pine kraft pulp.

REVIEW OF PERTINENT LITERATURE CONCERNING MORPHOLOGICAL ASPECTS OF PULP FIBERS

Studies involving model systems of synthetic fibers (5-9) have demonstrated the relative importance of fiber characteristics such as fiber cross-sectional shape and length in influencing the hydrodynamic properties of pulp slurries. Unfortunately, the pulp and paper industry does not work with "ideal" fiber systems. Wood, the principle raw material of the industry, is morphologically complex. The length of fibers varies within a given tree, and wood produced in temperate climates contains cells of greatly varying wall thickness. The largest difference, however, between wood fibers and the solid synthetic fibers previously used in model studies is that wood fibers contain a lumen.

WALL FRACTION VARIATION

Collapse of the lumens of wet wood pulp fibers under pressure increases fiber conformability by altering the cross-sectional shape of the fiber. At relatively low pressures fibers can collapse to varying degrees, and the degree of collapse appears dependent upon the fiber wall fraction (percentage of the fiber radius that consists of fiber wall).

For a softwood pulp major differences in wall fraction naturally occur between thin-walled earlywood fibers which have lower wall fraction and thick-walled latewood fibers. Earlywood fibers collapse at significantly lower compressive stress than latewood fibers, and a comparison of the apparent transverse elastic moduli (compressive modulus of the fiber wall) of wet earlywood and latewood spruce kraft pulp fibers also revealed earlywood modulus to be significantly lower (14).

In order to study the effects of these differences between earlywood and latewood wall fraction, the two fiber populations must be separated.

Usually the separation of large amounts of earlywood and latewood fiber for laboratory investigation is a difficult process. Conventional separation is achieved by splitting growth rings with a knife. Although the degree of separation using this technique is excellent, the job is tedius and the time required often prohibitive. Thus, large scale investigations are often impractical.

A mechanical method of fiber classification, which proposedly utilizes the modulus differences between earlywood and latewood pulp fibers, was discovered by Jacquelin (18) and has been successfully used at the Institute (19,20). Briefly, Jacquelin's procedure involves the slow rotating agitation of a pulp slurry in an inclined cylindrical container as shown in the schematic of Fig. 1. The rotating agitation causes the thick-walled latewood fibers to felt into flocs while the more flexible thin-walled earlywood fibers remain in the field fraction. Each fraction can then be isolated, redispersed, and reagitated to increase the degree of separation.

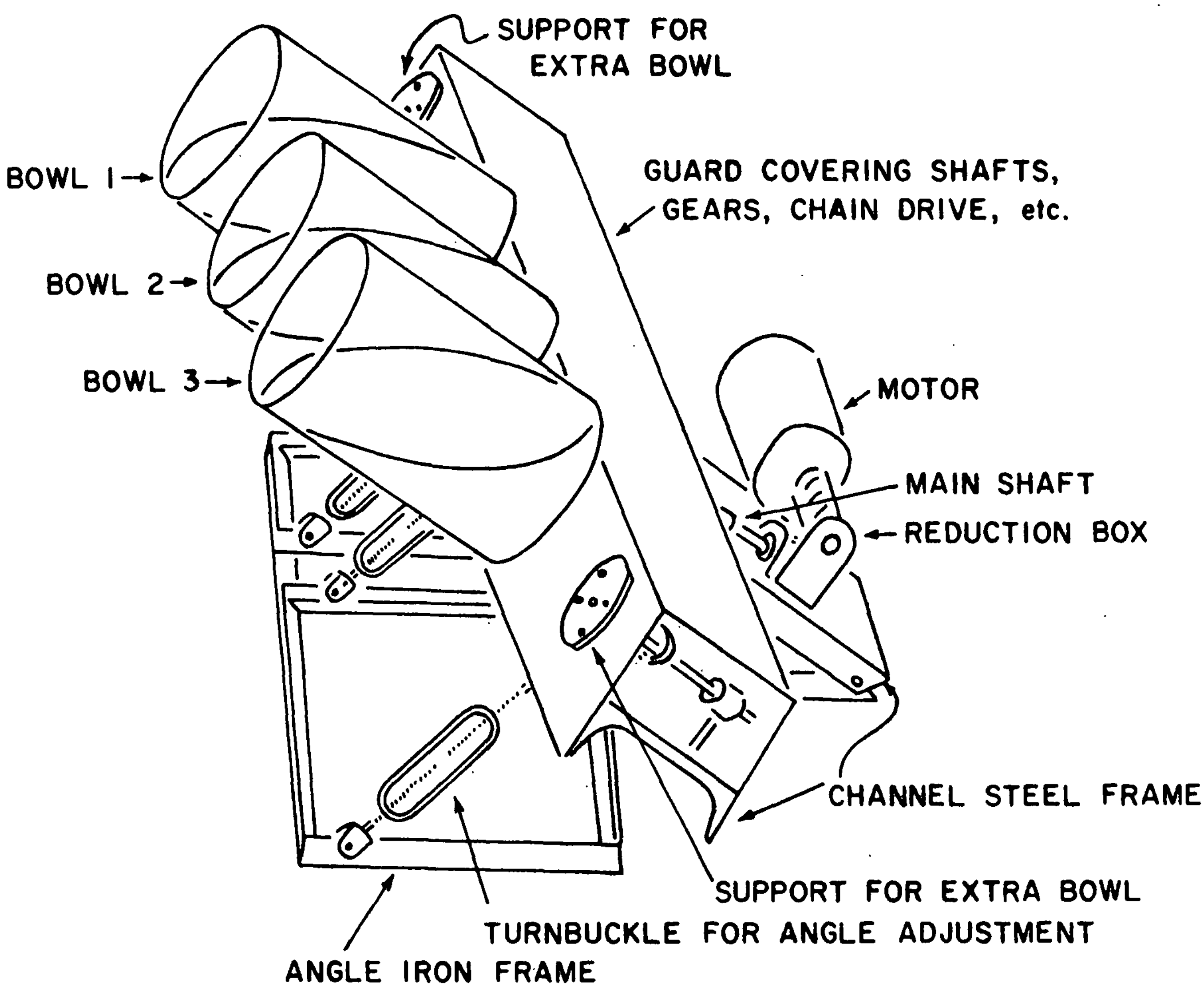

Figure 1. Schematic of Jacquelin Apparatus

FIBER LENGTH VARIATION

Fiber length distributions for unbleached loblolly pine earlywood and latewood fibers have been compiled and show for a range of specific gravities that earlywood and latewood mean fiber length increases with increasing distance from the pith. Thus, juvenile wood is composed of shorter fibers than mature wood. This trend is generally true for all conifer species, and is exemplified by the data in Fig. 2.

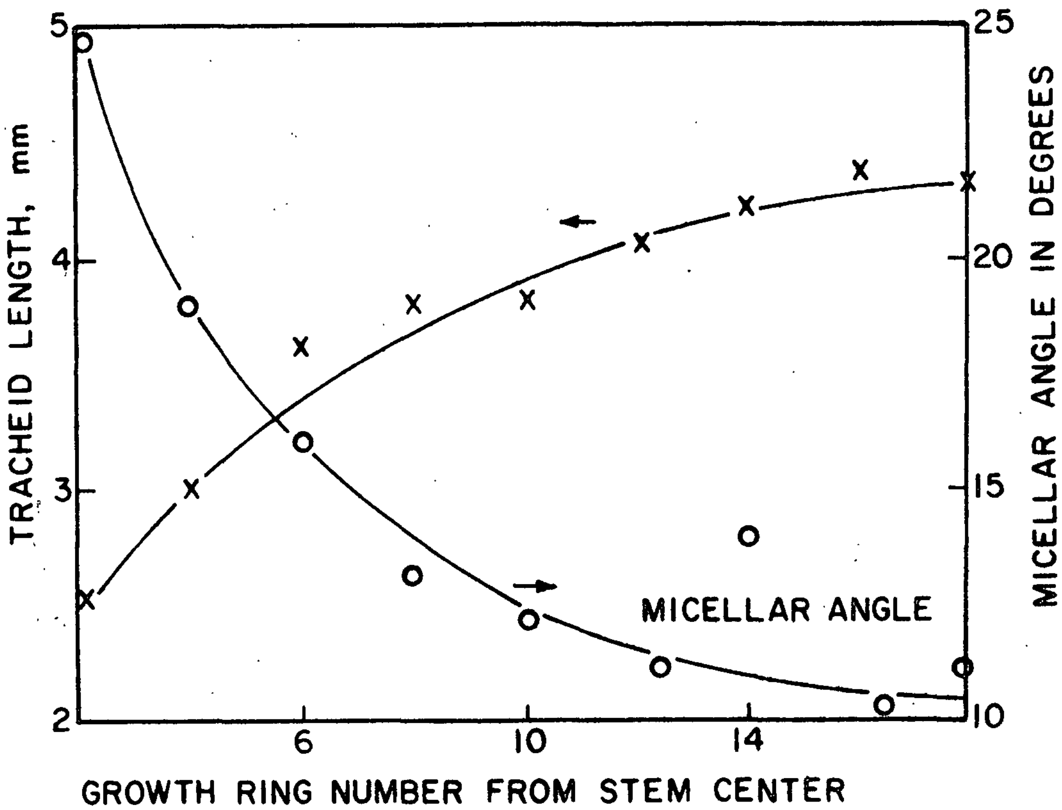

Figure 2. Variation in Tracheid Length and S_2 Fibril Angle in Successive Growth Rings of Pinus radiata (21)

Latewood fibers for a given growth ring and specific gravity are longer than earlywood, but within a given tree the range of earlywood and latewood fiber lengths overlaps (22). Therefore, a single tree produces earlywood and latewood fibers of the same length; however, the respective fibers may not occur within the same growth ring.

Unlike synthetic fibers, the lengths of unbroken wood pulp fibers are related to other fiber dimensions (22,23). As the average length of a wood pulp fiber increases:

1. average fiber diameter increases,
2. average wall-thickness increases, and
3. S_2 fibril angle decreases.

The decrease in S_2 fibril angle, θ, with increasing fiber length, L_f, shown in Fig. 2 can be described by Equation (1),

$$L_f = a + b \cot \theta \tag{1}$$

where a and b are constants. This relationship is important since the S_2 layer comprises the majority of the cell wall, and decreasing the fibril angle increases the apparent axial modulus of elasticity of the wall material (13,23,24).

The strong correlation of fiber length to these properties is a result of the growing process of the tree. Fibers near the pith are influenced by a rapidly growing apical meristem. They are generally short, narrow, thin-walled, and have high S_2 fibril angles. These fibers are called juvenile wood. As distance from the pith increases, the influence of the apical meristem decreases and an increase in the girth of the vascular cambium occurs (permitting the diameter of the tree to increase with age). In order for the girth of the cambium to increase, the cells composing the cambium (called fusiform initials which through repeated division give rise to a radially directed row of fibers) increase in number and alter their shape by increasing tangential diameter and length (23). Fibers in turn become longer, wider, thicker walled, contain higher wall fractions, and have lower S_2 fibril angles and subsequently higher modulus values with increasing age of the tree (23-25). The change from juvenile wood to mature wood is a gradual one and varies from tree to tree; however, wood is generally considered mature when the ratio of earlywood to latewood is approximately equal. This occurs after about 10-years growth, i.e., distances greater than 10 growth rings from the pith (22,23).

The isolation of fiber populations of corresponding length can be achieved through a second mechanical process — Bauer-McNett classification. This fiber length separation is based on the statistical probability that fibers of a certain length will be retained by a given size screen during agitation and controlled water flow.

REVIEW OF PERTINENT LITERATURE CONCERNING HYDRODYNAMIC EVALUATION OF PULPS

The flow of water through pulp fiber mats is of great importance since it is involved in both pulp washing and sheet formation. Technically, washing and sheet formation may be described by a process of filtration and/or permeation. Filtration generally refers to the retention of fibers on a screen (mat formation), whereas permeation describes the flow of water through a previously formed mat.

These flow processes may be quantified using mathematical expressions based on the well known empirical relationship, the Darcy equation (described below). In the pulp and paper industry, evaluations of this type are generally referred to as hydrodynamic evaluations.

A detailed review of the development of the field of hydrodynamics with respect to the pulp and paper industry has been presented by Han (26,27). For the sake of clarity, however, the development of the equations used to calculate wet mat compressibility, filtration resistance, specific surface, and specific volume is presented below.

WET MAT COMPRESSIBILITY

The wet mat compressibility of a wood pulp mat is generally defined by the correlation of wet mat density to static load. Compressibility data is obtained for first compression using the equipment shown in Fig. 3 in conjunction with the procedure developed by Ingmanson and Andrews (29) as presented in the Experimental section. Briefly, a fiber slurry is poured into the cylinder, agitated, and allowed to settle. The porous piston is placed on top of the mat and loaded with brass weights at equally spaced time intervals. Mat thickness (and subsequently mat density) is measured as a function of pressure with the dial micrometer.

The empirical correlation between wet mat density and static load used by Campbell (30) for kraft and groundwood pulps has repeatedly been shown to apply to other pulps for the pressure range of 10 to 100 g/cm^2 (26,27,31,32) and appears applicable up to pressures of 10^4 g/cm^2 (9). The empirical correlation which was modified by Ingmanson (31) is of the form:

$$c = c_o + M P_f^N \qquad (2)$$

where c_o is the mat density at zero stress which is usually about 0.02 to 0.04 g/cm^3, c is defined as the wet mat density at a pressure P_f, and M and N are empirical constants. The equation is the result of the linearity of a log-log plot of c vs. P_f.

In an attempt to define the physical significance of the compressibility constants M and N in Equation (2), Wilder (33) formulated a simplified mathematical model to describe the compressibility of a synthetic fiber mat. Wilder's model was refined by Han (9), resulting in a more realistic though still oversimplified description of compressibility. Development of this refined model with discussion

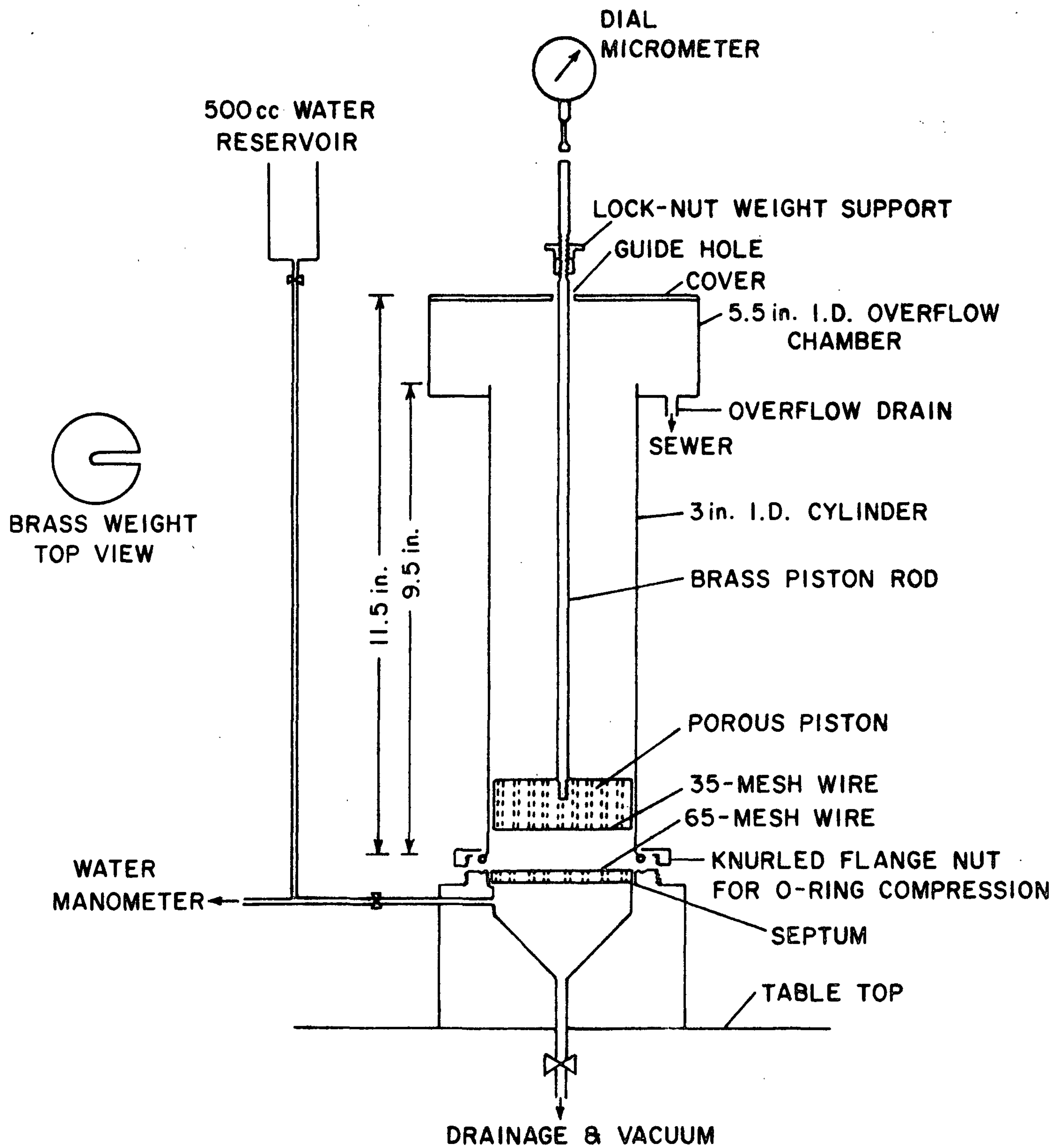

Figure 3. Compressibility Apparatus (28)

of the applicability of the necessary assumptions to wood pulp fiber mats is presented below.

The model is based on the statistical arrangement of fibers in a bed such that the fiber to fiber contact points are alternately arranged above and below a given fiber, as in Fig. 4. The structure of the mat is assumed to consist of horizontal layers (all fibers oriented in the x-y plane), and each layer supports the applied load equally. End effects are neglected.

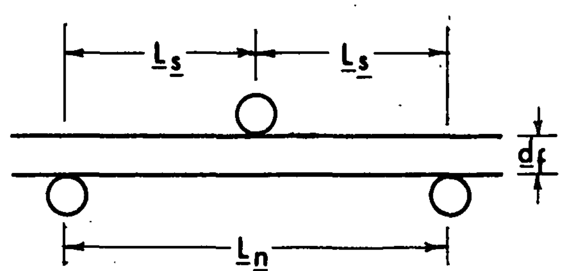

Figure 4. Alternate Arrangement of Fiber to Fiber Contacts

At any state of compaction there is a constant distance between fiber to fiber contacts called the segment length, L_s. Applying the Onagi-Sassaguri equation (62) to the unstressed structure, the initial segment length, $L_{s,o}$, is related to the initial solid fraction by the following equation,

$$\frac{c_o}{\rho_f} = \frac{\pi^3}{16} \frac{d_f}{L_{s,o}} \tag{3}$$

where ρ_f is the fiber density. The segment length is assumed to be statistically the same everywhere in the mat and is constant for a specific level of applied stress.

The initial mat density in a mat of unit area consisting of n similar layers would be:

$$c_o = \frac{W'}{L_o} = \frac{(\pi d_f^2/4)\ N_f\ L_f\ \rho_f}{n\ d_f} \tag{4}$$

where W' is the mass of the fibers, L_o is the initial mat thickness and N_f is the number of fibers per unit area.

Upon compacting a structural element, no deformation is assumed to occur at the contact points (an assumption necessary to facilitate solution of the resulting mathematical equations). Therefore, increase in mat density with increase in pressure is due to an increase in the number of contacts brought about by fiber bending. Increasing the number of contacts decreases L_s; this may be related to mat density using the simplest solution of the Onagi-Sassaguri theory:

$$\frac{c}{c_o} = \frac{L_{s,o}}{L_s} \tag{5}$$

The number of contacts per layer, n_c, adjacent to two other layers is:

$$n_c = \frac{N_f\ L_f}{2L_s\ n} \tag{6}$$

If the elastic deformation of the fibers is small, the deformation may be assumed to be governed by the equations of beam deflection. This enables the deflection, δ, of the fibers in the z-direction to be described by:

$$\delta = \frac{L_n^3\ P_n}{K_n\ E\ I} \tag{7}$$

where L_n is the free span between two supports (Fig. 4), P_n is the magnitude of the total load, E is the elastic modulus of the fibers, I is the second moment of the fiber cross-sectional area or moment of inertia. The product EI represents the flexural rigidity of the fibers, and K_n is a parameter dependent upon load distributions. The subscript n refers to the number of spans.

An infinitesimal load uniformly applied to the top of the mat is uniformly transmitted through the mat via the interfiber contact points. The incremental force sustained by each contact in the layer is dP_f/n_c. This force causes the fibers to bend, when the simple beam theory described by Equation (7) is applied, the reduction in thickness is:

$$-\frac{dL}{n} = \frac{L_n^3}{K_n\, E\, I}\;\frac{dP_f}{n_c} \tag{8}$$

where:

$$L = W'/c \tag{9}$$

From Fig. 4, L_n is twice L_s, and from Equation (4), $n = W/(c_o d_f)$. This information plus Equation (9) substituted into Equation (8) yields:

$$-\frac{d(W'/c)}{W'/(c_o d_f)} = \frac{(2L_s)^3}{K_n\, E\, I}\;\frac{dP_f}{n_c} \tag{10}$$

which reduces to:

$$\frac{dc}{c^2} = \frac{(2L_s)^3}{K_n\, E\, I\, c_o\, d_f}\;\frac{dP_f}{n_c} \tag{11}$$

From Equations (5) and (3):

$$L_s = L_{s,o}\,\frac{c_o}{c} = \frac{\pi^3\, \rho_f\, d_f}{16\, c} \tag{12}$$

Substituting Equations (4) and (12) into Equation (6) yields:

$$n_c = \frac{32\, c_o\, c}{\pi^4\, \rho_f^2\, d_f^2} \tag{13}$$

The final differential equation is obtained by combining Equations (11), (12), and (13):

$$c^2\,dc = \frac{\pi^{13}\,\rho_f^5\,d_f^4}{4^7\,K_n\,E\,I\,c_o^2}\,dP_f \tag{14}$$

For a wood pulp fiber system $\underline{K_n}$, $\underline{d_f}$, and $\underline{I}$ are functions of $\underline{P_f}$. Resolution of Equation (14), therefore, leads to Equation (15) which has little practical value since the integral cannot be evaluated.

$$c^3 - c_o^3 = \frac{3\pi^{13}\,\rho_f^5}{4^7\,E\,c_o^2}\int_o^{P_f}\frac{d_f^4}{K_n\,I}\,dP_f \tag{15}$$

However, for synthetic fiber systems in which the fibers do not appreciably deform under pressure, $\underline{d_f}$ and $\underline{I}$ are essentially constant with respect to changes in $\underline{P_f}$. Furthermore, $\underline{K_n}$ has been shown to be a strong function of $\underline{P_f}$ (6) such that,

$$K_n = \alpha(P_f/E)^\beta \tag{16}$$

where α and β are constants. With these contentions, Equation (15) reduces to:

$$c^3 - c_o^3 = \frac{3\pi^{13}\,\rho_f^5\,d_f^4}{4^7\,\alpha(1-\beta)c_o^2\,I\,E^{1-\beta}}\,P_f^{1-\beta} \tag{17}$$

Lacking understanding of the mechanism in Equation (16), the possibility of $\pm\beta$ may be assumed; and when $\underline{c_o} \ll \underline{c}$, Equation (17) becomes:

$$c = \left[\frac{3\pi^{13}\,\rho_f^5\,d_f^4}{4^7\,\alpha(1\pm\beta)c_o^2\,I\,E^{1\pm\beta}}\right]^{1/3} P_f^{(1\pm\beta)/3} = M\,P_f^N \tag{18}$$

Although Equation (18) is only an approximate description of the complex system of compressibility, it correlates well with experimental data, thereby giving at least a qualitative indication how the compressibility constant $\underline{M}$ is complexly related to the physical properties of the fibers comprising the mat. Through development of this model, $\underline{N}$ appears to be significantly less dependent on these properties.

From Equation (18) it is apparent that the most important factors influencing wet mat density are fiber dimensions and pressure. Wilder (33) in a study of compression, creep, and creep recovery showed that time was also an important factor, with initial changes in mat density primarily due to the resistance of the mat to the flow of water as pressure is applied very rapidly. Equation (18), therefore, applies primarily to relatively long periods of loading (several minutes) where the mat has, for practical purposes, reached an equilibrium.

The effects of fiber dimensions on compressibility were also studied by Jones (5) and Elias (6).

Jones (5) in a thesis on compression recovery response studied the compressibility effects of fiber length, diameter, and modulus of elasticity (M.O.E.). Using glass, Nylon, and Dacron fibers he was able to show that changes in wet mat compression response are independent of synthetic fiber diameter, and that mat compressibility at constant pressure increases with decreasing fiber length and M.O.E. Southern pine summerwood pulp fibers were shown to have a similar length *vs.* compressibility relationship; however, fiber M.O.E. was not measured.

Although the data compiled for the synthetic fibers was extensive, data compiled for the wood pulp was minimal. Specifically, Jones' study incompletely examined the effect of wood pulp fiber dimensions on compressibility.

Elias (6) further examined the factors relating to the mechanism of compressibility of fibrous mats by developing equipment and techniques which allowed individual fibers in the interior of thick glass fiber mats to be visually observed while the mat was subject to compression. By observing the arrangement and configuration of fibers within the mats, he was able to show how the internal structure of the mat was influenced by fiber dimensions and how the fibers respond to compression. As previously shown by Jones (5), fiber length was a critical fiber dimension in the glass system used by Elias. Beds of longer fibers compressed more readily, i.e., showed a greater change in solids fraction for a given increase in applied stress, then beds containing fibers of shorter length. By observing the distance between fiber contacts (segment length) this phenomenon could be explained. Elias found that segment length was proportional to fiber length; the mean number of fibers touching a given fiber per millimeter of fiber length decreased as the fiber length increased. This increase in segment length allows fibers to bend more readily, thereby making the mat more easily compressed.

In wood pulp fiber systems the process parameters of cooking and beating have also been shown to be important factors influencing wet mat compressibility. Gren (15) has shown that the compressibility constant, *N*, of fiber beds decreased slightly with increasing kappa number. If the assumption is made that wood pulp fiber stiffness decreases with lignin content, it may be noted that the compressibility of a pulp mat (like synthetic fibers) also increases with decreasing stiffness.

Han (9) has shown that wet mat density at a given applied stress increases with increase in time of Valley beating. The effect of beating on fibers is difficult to analyze, but Valley beating generally decreases mean fiber length, and on this basis would be expected to influence compressibility. This contention is supported by observation that ball milling does not appear to affect compressibility constants *M* and *N*.

For the above discussion, it may be hypothesized that the respective wet mat density of earlywood and latewood at a given applied stress would decrease with increasing fiber length; and that the slope of a plot of wet mat density *vs.* pressure should increase more rapidly with pressure for thin-walled earlywood fibers than for thick-walled latewood fibers.

SPECIFIC FILTRATION RESISTANCE

Specific filtration resistance is a reliable index of drainage, and, therefore, an important property of the pulp slurry (31); and like wet mat compressibility, constant rate filtration data from which specific filtration resistance is calculated, is relatively easy to obtain.

The equipment used is schematically shown in Fig. 5. Briefly, a dilute suspension of fibers, from an agitated holding tank, flows into the filtration tube, and the fibers are retained on a septum (wire screen). The water is pumped out of the tube through a rotameter at a constant rate, thus the name constant rate filtration. As the mat gets thicker, the pressure drop increases and is recorded with time on an electronic recorder.

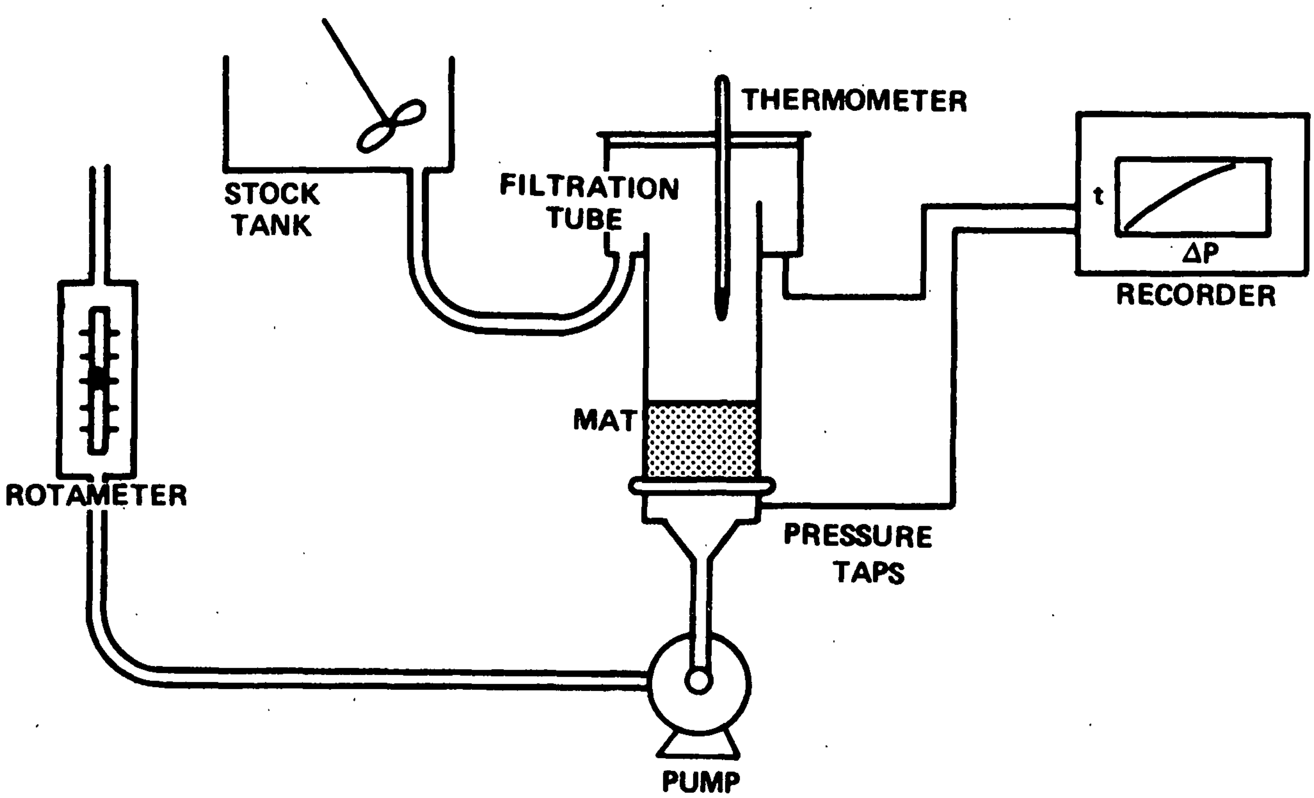

Figure 5. Filtration Apparatus

The equation from which specific filtration resistance, $\underline{R}$, is calculated is based on the differential form of the Darcy equation (31,32,34,35),

$$-\frac{d\Delta P_f}{dz} = \frac{1}{K}\frac{q\mu}{A} \qquad (19)$$

where $\underline{q}$ is the volumetric flow rate of a noncompressible fluid of viscosity, μ, through a fiber bed of cross-sectional area, $\underline{A}$, and thickness, $\underline{dz}$, which develops a frictional pressure drop, $d\Delta P_f$. The negative sign indicates flow in the downward (negative $\underline{z}$) direction.

The permeability coefficient, $\underline{K}$, is related to the specific filtration resistance, $\underline{R}$, by the following expression:

$$K = \frac{1}{R(dW/dz)} = \frac{1}{Rc} \qquad (20)$$

where $\underline{W}$ is the mass of fibers per unit area of mat; therefore dW/dz represents a local mat density, $\underline{c}$.

$\underline{W}$ can also be expressed in terms of the filtration time, $\underline{t}$, and stock consistency, $\underline{C}$.

$$W = \frac{q}{A}\int_0^t C\,dt \qquad (21)$$

Systematic substitution of Equations (20) and (21) into Equation (19) results in an equation describing the local specific filtration resistance,

$$R = \frac{A^2}{q^2\,\mu C}\,\frac{d\Delta P_f}{dt} = B\,\frac{d\Delta P_f}{dt} \qquad (22)$$

where $\underline{B}$ is a constant for a given filtration. The $\underline{R}$ described by Equation (22) applies to a small but measurable section of the forming mat immediately above the retaining screen.

For a relatively dilute slurry, $\underline{C}$ may be assumed independent of $\underline{t}$. Integration of Equation (22) will then yield an equation describing the average specific filtration resistance, $<\underline{R}>$, for the whole mat,

$$<R> = B \frac{\Delta P_f}{t} \tag{23}$$

Problems in obtaining accurate values for $d\Delta P_f/dt$ have necessitated use of Equation (23) for calculation of filtration resistance values (26). Recently, these problems have been resolved through development of an accurate numerical procedure for differentiation (described in a later section).

The major factors influencing specific filtration resistance are pressure and fiber dimensions. It may be observed from Equations (22) and (23) that filtration resistance increases with $d\Delta P_f/dt$ and $\Delta P_f/t$ and, therefore, pressure and time are important parameters in the filtration analysis. Further mathematical resolution of filtration resistance into its component parts (described below) reveals that fiber specific surface and mat porosity are also important factors. Filtration resistance increases with the square of specific surface, and decreases with increasing porosity.

SPECIFIC SURFACE AND SPECIFIC VOLUME

The most successful mathematical relationship to describe the creeping permeation of an incompressible porous bed as a function of certain physical properties of the material composing the bed is, again, the Darcy equation [Equation (19)]. The Darcy proportionality factor, $\underline{K}$, is not only related to the filtration resistance but is dependent on the structure of the porous medium.

This dependence of $\underline{K}$ on the physical properties of the porous medium was found by Kozeny (36) to be a function of the porosity, ε, and the specific surface per unit volume, S_v. Kozeny's relationship was expanded by Carman (37) resulting in the Kozeny-Carman equation.

$$K = \frac{\varepsilon^3}{k\, S_v^2\, (1-\varepsilon)^2} \tag{24}$$

The Kozeny factor, k, in Equation (24) was unfortunately found not to be a constant as originally believed, but also dependent on the porosity at porosities greater than about 0.8.

In a study of air flow through fibrous materials, Davies (38) obtained an empirical correlation to describe the dependence of k on ε. Later, Ingmanson, *et al.* (39) discovered that k was also dependent on fiber orientation. Since mats formed during papermaking processes usually have fibers oriented with their axis perpendicular to flow, the empirical correlation developed by Davies was slightly modified to Equation (25) for solid circular cylindrical fibers.

$$k = \frac{k_1 \varepsilon^3}{(1-\varepsilon)^{1/2}} \left[1 + k_2(1-\varepsilon)^3\right] \tag{25}$$

where $k_1 = 3.5$ and $k_2 = 57$.

Substitution of Equation (20) into the modified Darcy equation used to describe filtration resistance [Equation (22)] yields an expression relating the constant rate filtration terms, B and $d\Delta P_f/dt$, to K.

$$d\Delta P_f/dt = (BcK)^{-1} \tag{26}$$

Substitution of Equation (25) into Equation (24) yields an expression for K in terms of ε and S_v. By definition, $\varepsilon = 1-vc$ and $S_v = S_W/v$, where v is the specific volume (volume denied to flow) and S_W is the specific surface per gram of fiber (surface to mass ratio). Incorporation of these substitutions into Equation (26) results in:

$$\frac{d\Delta P_f}{dt} = \frac{k_1\ S_W^2\ c^{1/2}}{B\ v^{1/2}}\left[1 + k_2\ v^3\ c^3\right] \qquad (27)$$

Equation (27) can be solved for v and S_W from constant rate filtration data and compressibility data with the aid of additional information describing the pressure relationship of either S_W or v.

Alternatively, Equation (27) may be integrated and rectified (rearranged to the form of a linear equation) with the assumptions that v and S_W remain constant with respect to the integration, and the average mat density, c_{avg}, is of the form (40)

$$c_{avg} = (1-N/2)^2\ M\Delta P_f^N = (1-N/2)^2\ c \qquad (28)$$

This integration results in Equation (29) which can be solved for $<v>$ and $<S_W>$ (average values of v and S_W) through linear interpretation of a plot of $\Delta P_f/(c^{1/2}t)$ vs. c^3.

$$\frac{\Delta P_f}{c^{1/2}t} = \frac{3.5(1-N/2)<S_W>^2}{B<v>^{1/2}}\left[1 + 57<v>^3\ (1-N/2)^6 c^3\right] \qquad (29)$$

Linear interpretation of Equation (29) and other forms of the modified Darcy equation (31) has been the accepted procedure for determining $<v>$ and $<S_W>$ at the Institute. The development of this procedure has enabled clarification of the relative effects of beating (29) and cooking (15,16) on the hydrodynamic properties of pulps as well as contributed to the basic understanding of water removal from fiber mats (31). Equation (29) appears to be in widespread use throughout the paper industry and currently represents the best available method of determining $<S_W>$ and $<v>$.

Alternative procedures for the hydrodynamic evaluation of specific surface and swollen volume as functions of pressure have been presented (41,42). These procedures, however, involve an invalid assumption. The procedures and assumption are discussed in the section on Feasibility of Determining S_w and v as Functions of Pressure.

EXPERIMENTAL PROCEDURES

ISOLATION OF FIBER POPULATIONS

A two way fiber classification scheme was developed; it is capable of separating a large amount of wood pulp fiber into earlywood and latewood fiber populations of varying length. A bleached southern pine kraft pulp was first separated into earlywood and latewood fiber populations with a Jacquelin apparatus. The earlywood and latewood populations were then subdivided into smaller populations of varying fiber length distribution using a Bauer-McNett classifier. The resulting fiber populations were morphologically homogeneous, making them a desirable raw material for hydrodynamic evaluation.

RAW MATERIAL

A 27-year old medium dense loblolly pine was obtained from a natural even-aged stand in Union Camp's experimental forest in Effingham County, Georgia. The tree was 9.1 inches dbh (diameter at breast height), 81-feet high, and cut into 16 5-foot bolts.

The bottom 5 bolts were longitudinally cut (on a sawmill circular saw) into three sections as shown in Fig. 6 in order to increase the relative percentage of juvenile wood in the sample.

The three sections of each 5-foot bolt were then further divided (by sawing with an 8-inch circular saw) as shown in Fig. 7.

PREPARATION OF PULP

The chips obtained from the rail portion of the fifth 5-foot bolt were used in a preliminary cooking investigation to determine applicability of selected cooking conditions (3).

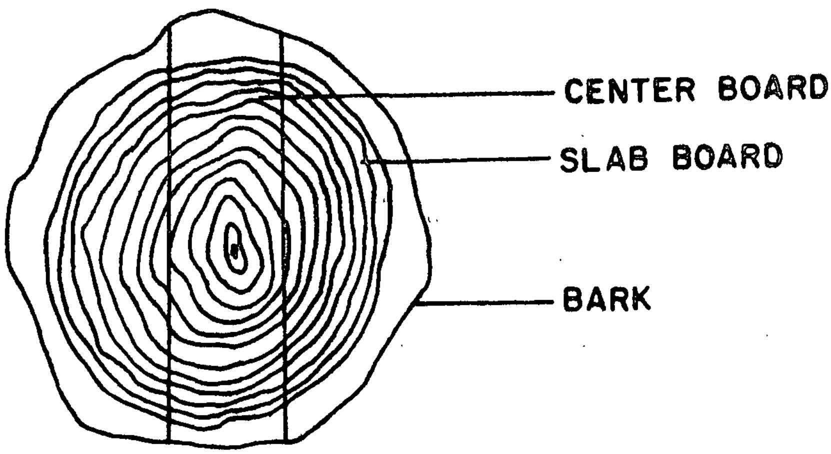

Figure 6. Cross Section of 5-Foot Loblolly Pine
Bolt Depicting First Sawing Pattern

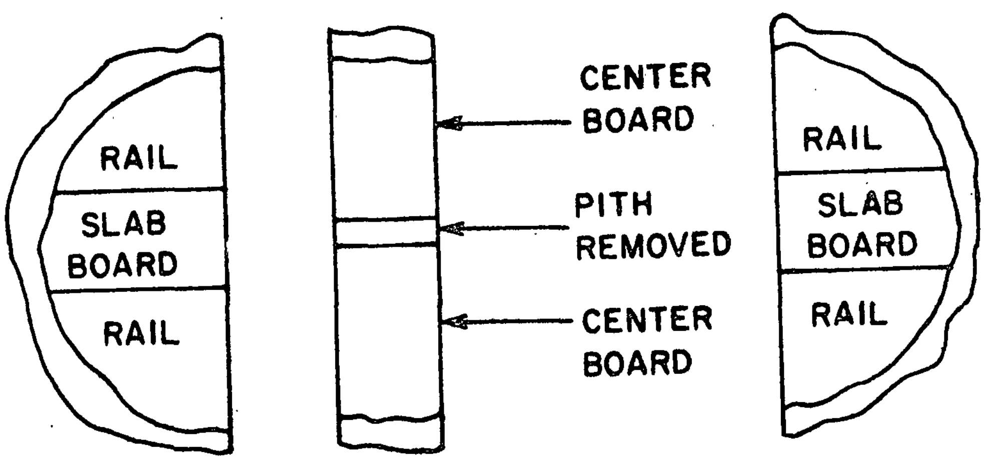

Figure 7. Cross Section of 5-Foot Loblolly Pine
Bolt Depicting Second Sawing Pattern

Four conventional kraft cooks were then performed in a batch digester, each using 900 g (o.d. basis) of chips selected from the center and slab boards. After cooking, the chips were washed and disintegrated with hot then cold water in a pulp washer, and dewatered without fines retention in a laundry centrifuge. The resulting pulp was screened on a 0.009-inch slot pulsating screen and screened yield determined. A representative sample of the screened pulp was removed and kappa number determined according to TAPPI Standard Method T 236 m-60. A summary of the pulping data is presented in Table I.

TABLE I

PULPING DATA

Active alkali (as Na_2O)	= 20.6%
Sulfidity (as Na_2O)	= 25.0%
Liquor to wood ratio	= 4.0:1 ml/g
Maximum temperature	= 170°C
Time to maximum temperature	= 1.5 hr
Total cooking time	= 4.0 hr
pH at end of cook	= 12.9
Yield	= 46%
Screened rejects	= negligible
Kappa number of screened pulp	= 19.8

For reasons previously discussed, bleaching was necessary. A CEDED bleaching sequence based on prior experience (IPC unpublished work) was employed. The bleaching conditions are given in Table II.

EARLYWOOD AND LATEWOOD SEPARATION

The bleached pulp was first separated into earlywood and latewood fiber populations using the method originally developed by Jacquelin (18).

An electric stirrer was used to gently disintegrate 100 g (o.d. basis) of the bleached kraft pulp with 10 liters of distilled water. The pulp slurry (1.0% consistency) was poured into one bowl of the Jacquelin apparatus (schematically shown in Fig. 1) and rotated for 12 hours at 36 rpm. The rotating agitation causes the thick-walled latewood fibers to felt into balls while the more flexible thin-walled earlywood fibers remain in the field fraction. The resulting floc and field fractions were then isolated as follows.

TABLE II

BLEACHING CONDITIONS

First Stage — Chlorination	
Chlorine, %	6.0
Consistency, %	3.1
Temperature, °C	25
Time, min	60
Residual Cl_2, % of applied	16.2
pH	1.8
Second Stage — Alkaline Extraction	
NaOH, %	2.5
Consistency, %	10
Temperature, °C	60
Time, min	60
pH	11.3
Permanganate number	6.0
Third Stage — Chlorine Dioxide	
ClO_2, %	1.0
Consistency, %	10
Temperature, °C	70
Time, min	165
Residual ClO_2, %	2.9
Fourth Stage — Alkaline Extraction	
NaOH, %	1.8
Consistency, %	10
Temperature, °C	60
Time, min	60
pH	11.4
Fifth Stage — Chlorine Dioxide	
ClO_2, %	0.6
Consistency, %	10
Temperature, °C	70
Time, min	240
pH	4.6

Standard brightness as received = 89.4%

Yield = 97%

Three hundred grams of previously rotated pulp (3 bowls of 100 g each represent 1 run) were poured into a stainless steel tank containing 300 liters of filtered tap water at 3-5°C. The flocs readily sank to the bottom, but with gentle agitation the field fibers remained suspended and could be siphoned onto a muslin-covered wash box. The tank was refilled with cold water and the siphoning repeated until the amount of field fibers in suspension were depleted to the point of negligible recovery. The tank was again refilled and the temperature raised above 15°C. With the addition of heat, the solubility of air in the water decreased and small bubbles were formed. These bubbles became trapped in the flocs and carried them to the surface. Without agitation, the field fibers remained at the bottom of the tank. The floc fibers were easily skimmed from the water surface, leaving the remaining field fibers to be siphoned onto the wash box.

The isolated floc fraction was washed to remove adhering field fibers using the following method developed by Chang (43).

Ony liter of fiber flocs was poured into a 4-liter stainless steel beaker which had been drilled with 2.5 mm diameter holes at about 1-cm spacing. Raising and lowering the beaker in a large tank of filtered tap water diluted the free fibers and caused them to flow outward through the holes, which were too small for the flocs. The flocs were removed from the perforated beaker when the amount of free fibers being removed became negligible. After all of the flocs had been washed, the free fibers were siphoned onto a muslin-covered wash box and added to the field fraction.

Each fraction was then redispersed and reagitated to increase the degree of separation.

This procedure enabled separation of 30 g (o.d. basis) of pulp into earlywood and latewood fiber populations each day.

FIBER LENGTH SEPARATION

A Bauer-McNett classifier was used to separate floc and field pulp fractions into fiber populations of varying length. The screen sizes (10, 20, and 65 mesh) were experimentally determined to yield an optimum amount of fiber retained (weight basis) as well as to maximize differences in mean fiber length. An unidentified equipment change, detected by percent retained data, resulted in two groups of on-65 mesh fibers for latewood. The separation, based on TAPPI Standard Method T 233 su-64 was as follows.

Ten grams (o.d. basis) of the respective pulp fraction were briefly agitated (approximately 5 seconds) in a British disintegrator with 2 liters of water. The resultant pulp slurry was then added to the first tank of the Bauer-McNett classifier and classified for 15.0 minutes. Water temperature was 3-5°C. After classification, the separated fibers were flushed from their respective screens onto muslin-covered wash boxes. This method permits 30 g of pulp to be separated each hour.

FIBER ANALYSIS

Standard 1.6 g handsheets wet pressed to 50 psig for 15.0 minutes made from representative floc and field fractions were used to monitor the degree of latewood-earlywood separation. Representative sections of the handsheets were coated with a 60:40 mixture of Au/Pd and viewed with a JSM-U3 scanning electron microscope to obtain a qualitative indication of the degree of separation. No attempt was made to quantify the degree of separation by fiber counting.

Fiber lengths were determined on samples of not less than 1000 fibers with The Institute of Paper Chemistry semiautomatic fiber length recorder according to the method of Illvessalo-Pfaffli and Alfthan (44).

Fiber width and cell wall thickness were measured at 210X with a filar micrometer on samples of not less than 100 fibers. Fibers were wet mounted on glass slides in a mixture of water and glycerin and covered lightly with a cover glass to insure that fibers, particularly earlywood, were not flattened.

The number of fibers per gram was determined by directly weighing air dry samples of 100-300 fibers on a quartz balance accurate to 0.5×10^{-6} g and compensating for moisture content.

Percentage whole fibers were determined by counting the number of whole and broken fibers in representative samples of not less than 200 fibers of pulp fractions. The fibers were mounted on glass slides in mineral oil and viewed at 35X.

HYDRODYNAMIC MEASUREMENTS

The hydrodynamic properties of the isolated fiber populations were measured according to the procedures of wet mat compressibility, constant rate filtration, and multiple pressure tap permeation.

WET MAT COMPRESSIBILITY

Apparent wet mat density as a function of pressure was determined for each isolated pulp fraction with the equipment and procedure developed by Ingmanson and Andrews (29).

A schematic of the equipment is shown in Fig. 3. Prior to each run, the micrometer was zeroed with the piston placed in the empty cylinder. The piston was then removed and the septum flooded with water from the reservoir. A representative sample of a pulp fraction slurried in a British disintegrator at about 0.5% consistency was poured into the tube, stirred thoroughly with a glass stirring rod, and allowed to settle until the fiber level was below that of the overflow level. The piston and cover were inserted, and the piston was gently lowered by hand until it rested on the loosely formed fiber mat, at which point it was released and the foot of the dial micrometer was placed on top of the piston rod. The mass of the piston and piston arm corrected for the buoyancy force of the water formed the first weight.

Fifteen minutes after the micrometer was positioned, the micrometer was read to obtain the mat thickness. A brass weight similar to the one shown in Fig. 3 was placed on the weight support and after 15 minutes the micrometer was read again. This procedure was repeated with five additional weights of increasing mass.

After the seventh micrometer reading was recorded the water was drained out of the tube, piston and cover removed, and the cylinder assembly detached from the septum. The pad was quantitatively removed from the septum, placed in a tared weighing bottle, dried overnight at 105°C in a forced air oven, and weighed.

Apparent wet mat density, $\underline{c}$, was calculated as a function of pressure from the pad weight, $\underline{W'}$, cross-sectional area of the tube, $\underline{A}$, and measured pad thicknesses, $\underline{L}$, at various pressures using the following equation:

$$c = \frac{W'}{AL} \tag{30}$$

An identical procedure was used in a second compressibility apparatus. The mass and water displacement of the new piston head and shaft, however, was greater than that of the original equipment, and although the same brass weights were used, actual compacting pressures differed slightly. Pad thickness was measured with a cathetometer.

CONSTANT RATE FILTRATION

The filtration resistance of the separated fiber populations was quantified with the aid of a research model constant rate filtration apparatus schematically shown in Fig. 4 using the procedure developed by Ingmanson and Whitney (31).

Deaerated pulp at about 0.01% consistency was agitated in the feed tank and permitted to flow through the flow control valve into the filtration tube. In the filtration tube the pulp slurry was maintained at a constant head while the pulp fibers were collected by filtration on the septum. A constant filtration rate was monitored with the rotameter and was maintained by varying the speed of the gear pump. During the run a plot of time (t, seconds) vs. pressure drop (P_f, cm H_2O) was recorded with a strip chart recorder.

The information obtained from the experiment was used with Equations (22) and (23) to respectively obtain local and average specific filtration resistance values as a function of pressure for the samples.

PERMEATION

A multiple pressure tap permeation procedure was adapted from the experimental techniques developed by Chang and Han (45). A schematic of the equipment is shown in Fig. 8.

Deaerated pulp was placed into the feed tank and diluted with filtered, freshly distilled water to approximately 0.01% consistency. A thick mat was formed by slow filtration (1 cm/sec flow rate) and conditioned at 60.0 cm H_2O overall pressure drop for 30 minutes by permeation with filtered, freshly distilled water. The permeation was then stopped and the mat allowed to expand freely for 15 minutes. The process was repeated until the mat thickness at 60.0 cm H_2O remained the same (approximately 5-7 cycles). At this point, the pressure was measured at different levels in the mat with a pressure transducer indicator.

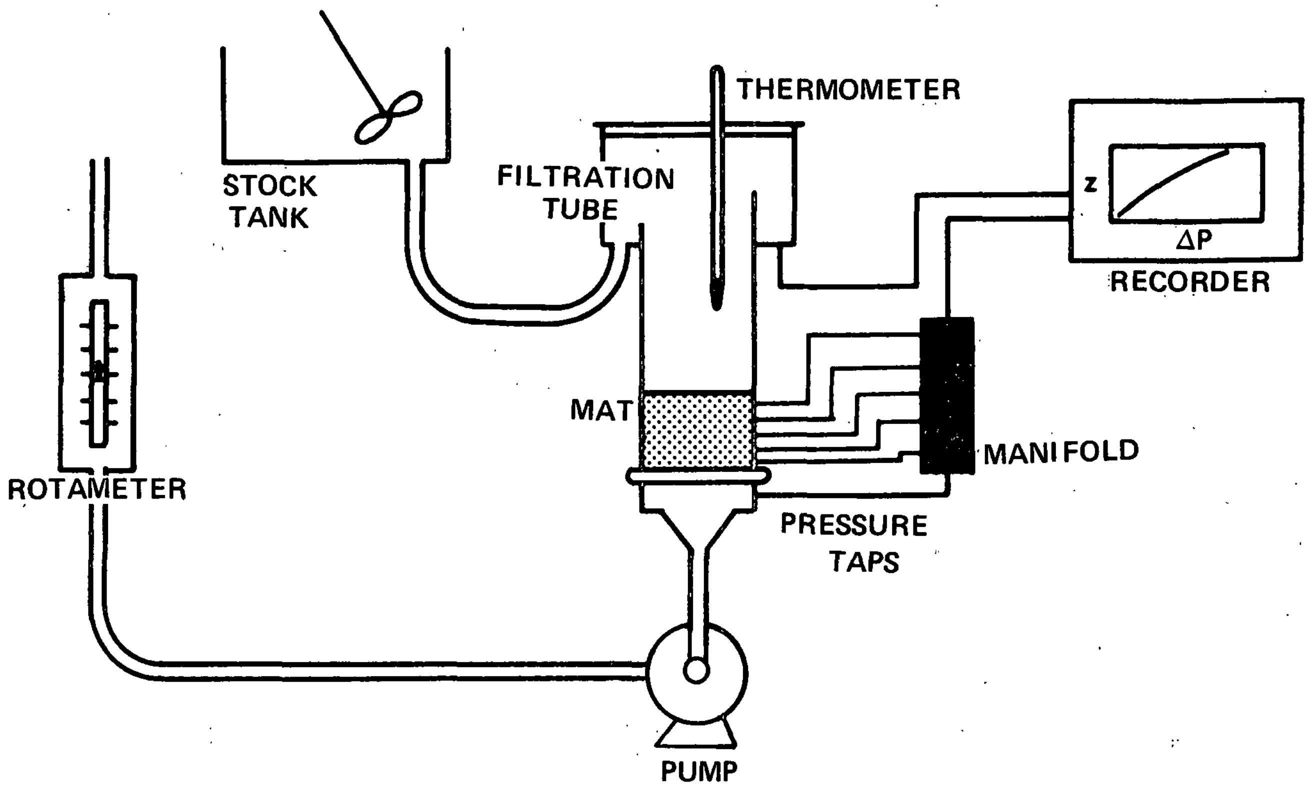

Figure 8. Permeation Apparatus

Latewood and whole pulp fiber populations with fiber lengths greater than 2.7 mm would not develop sufficient fluid drag forces to attain 60.0 cm H_2O overall pressure drop without the formation of mats of excessive thickness or permeation velocities which exceeded the limits of laminar flow. To decrease the porosity (and thus increase fluid drag force) a permeable piston with a static load was applied to these fiber mats.

After permeation the mat was permitted to expand freely for 45 minutes. A permeable piston was then placed on top of the mat, and the mat density determined as a function of static load. From this point the compressibility procedure was identical to that described earlier except a cathetometer instead of a dial micrometer was used to measure mat thickness.

RESULTS AND DISCUSSION

MEASURED AND CALCULATED MORPHOLOGICAL PROPERTIES

FIBER SEPARATION AND DIMENSIONS

The two way fiber classification scheme developed for this work and presented in the Experimental Procedures section resulted in a series of earlywood and latewood fiber populations of varying length.

Figure 9 is a comparison of scanning electron micrographs of handsheets composed of field and floc fractions of bleached loblolly pine after two separations with the Jacquelin apparatus. The micrographs show the field fraction is composed of predominantly earlywood fibers which easily collapse into flat "ribbons," thus forming relatively dense mats with few openings for water to pass through. In contrast, the floc fraction is composed of predominantly latewood fibers which form mats of high porosity.

Jacquelin (18) attributed this morphological separation to relative fiber stiffness. Observations made during this study suggest there is a critical degree of relative stiffness required to achieve morphological separation. Unbleached fibers yielded field and floc fractions which contained no apparent morphological separation. Only after additional lignin had been removed through bleaching did separation into earlywood and latewood occur.

Table III summarizes the results of the Jacquelin separation and also shows the degree of variability encountered in the system. An attempt was made to compare the variability obtained during this study with that obtained by Jacquelin (18,46-48); however, data of this type does not appear available in the general literature. Apparently, this study represents one of the first attempts to statistically quantify separation data.

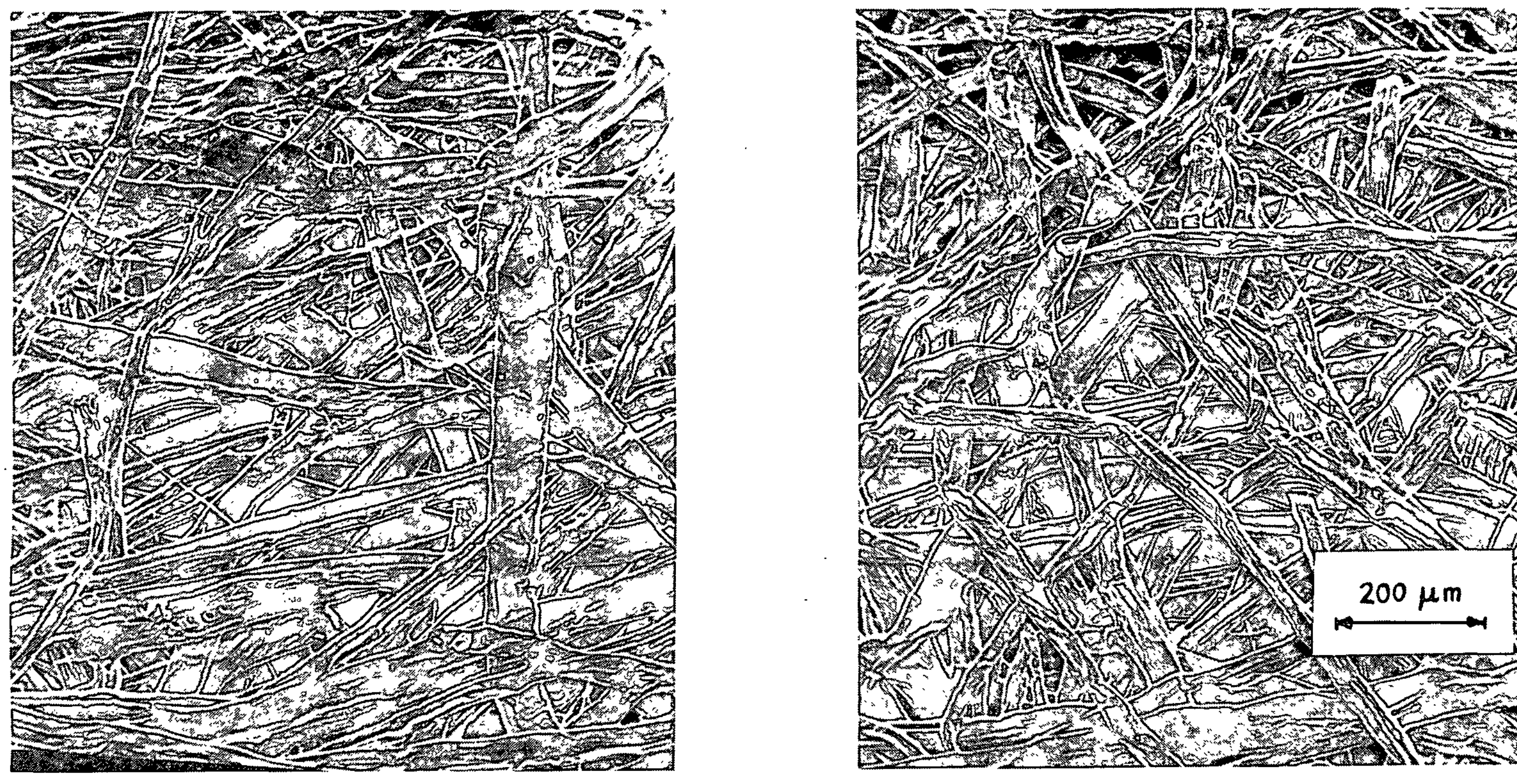

Figure 9. Scanning Electron Micrographs of Handsheets Composed of Field (Left) and Floc Fractions of Bleached Loblolly Pine After Two Separations by the Jacquelin Apparatus

TABLE III

QUANTITATIVE RESULTS OF JACQUELIN SEPARATION

Pulp Fraction	Quantity Separated, g	Relative Percent
First Separation		
Floc	9,340.4	57.7 ± 3.0[a]
Field	6,844.0	42.3 ± 3.0
Total	16,184.4	100.0%
Second Separation[b]		
Floc→floc	4,418.5	38.1 ± 3.1
Floc→field[c]	2,471.2	21.1 ± 3.0
Field→floc[d]	2,716.5	17.1 ± 2.1
Field±field	3,990.8	25.2 ± 2.1
Total	13,597.0	101.5%

[a] 95% Confidence limits.

[b] Quantity separated and relative percentages are not proportional since isolation of floc-floc fractions was terminated after a sufficient quantity had been separated.

[c] Field fibers resulting from floc fraction.

[d] Floc fibers resulting from field fraction.

The table shows that significantly higher percentages of fiber were retained in the floc and floc-floc fractions. This result is in qualitative agreement with the unusually high amounts of latewood fiber observed from the cross sections of the unprocessed logs.

The quantitative results of the Bauer-McNett separation are given in Table IV, along with the mean fiber length, $\underline{L_f}$, of each population. In addition, the variability of results is reported.

TABLE IV

QUANTITATIVE RESULTS OF BAUER-McNETT CLASSIFICATION

Water Temperature = 3-5°C (Normal Winter Temperature)

Pulp Fraction	Screen Size	Amount Retained, g	Percent Retained	Mean Fiber Length, $\underline{L_f}$, mm
Field-field (400 separations)	10 mesh	1592.9	39.9 ± 2.5[a]	3.94 ± 0.04[a]
	20 mesh	1412.1	35.4 ± 2.4	3.05 ± 0.03
	65 mesh	586.1	14.7 ± 0.9	1.63 ± 0.02
	Fines[b]	399.7	10.0 ± 1.4	
Floc-floc (450 separations)	10 mesh	1558.6	35.3 ± 3.5	4.13 ± 0.06
	20 mesh	1256.5	28.4 ± 9.0	2.98 ± 0.03
	65 mesh(I)	1028.0	23.3	2.07 ± 0.04
	65 mesh(II)	385.5	8.7	1.74 ± 0.03
	Fines[b]	197.9	4.5 ± 2.0	
Whole pulp (150 separations)	10 mesh	531.7	36.3 ± 0.7	3.88 ± 0.03
	20 mesh	607.1	41.4 ± 2.4	2.76 ± 0.04
	65 mesh	300.3	20.5 ± 1.9	1.49 ± 0.01
	Fines[b]	25.9	1.8 ± 3.9	

[a]Arithmetic mean ± 95% confidence limits.

[b]Determined by difference.

Fines loss for field-field was found to be significantly higher than for other fractions. This higher loss was anticipated since ray cells and other parenchymal tissue fragments would concentrate in this fraction.

The $\underline{L_f}$ values for fibers retained on 10, 20, and 65 mesh screens are significantly different as graphically depicted on the histograms of Fig. 10-12. The histograms also show that the lengths of the fibers retained on the various mesh screens follow an approximately normal distribution function, thus supporting the choice to use arithmetic means in Table IV to describe mean fiber length. In addition, the histograms reveal that fibers retained on the 65 mesh screen

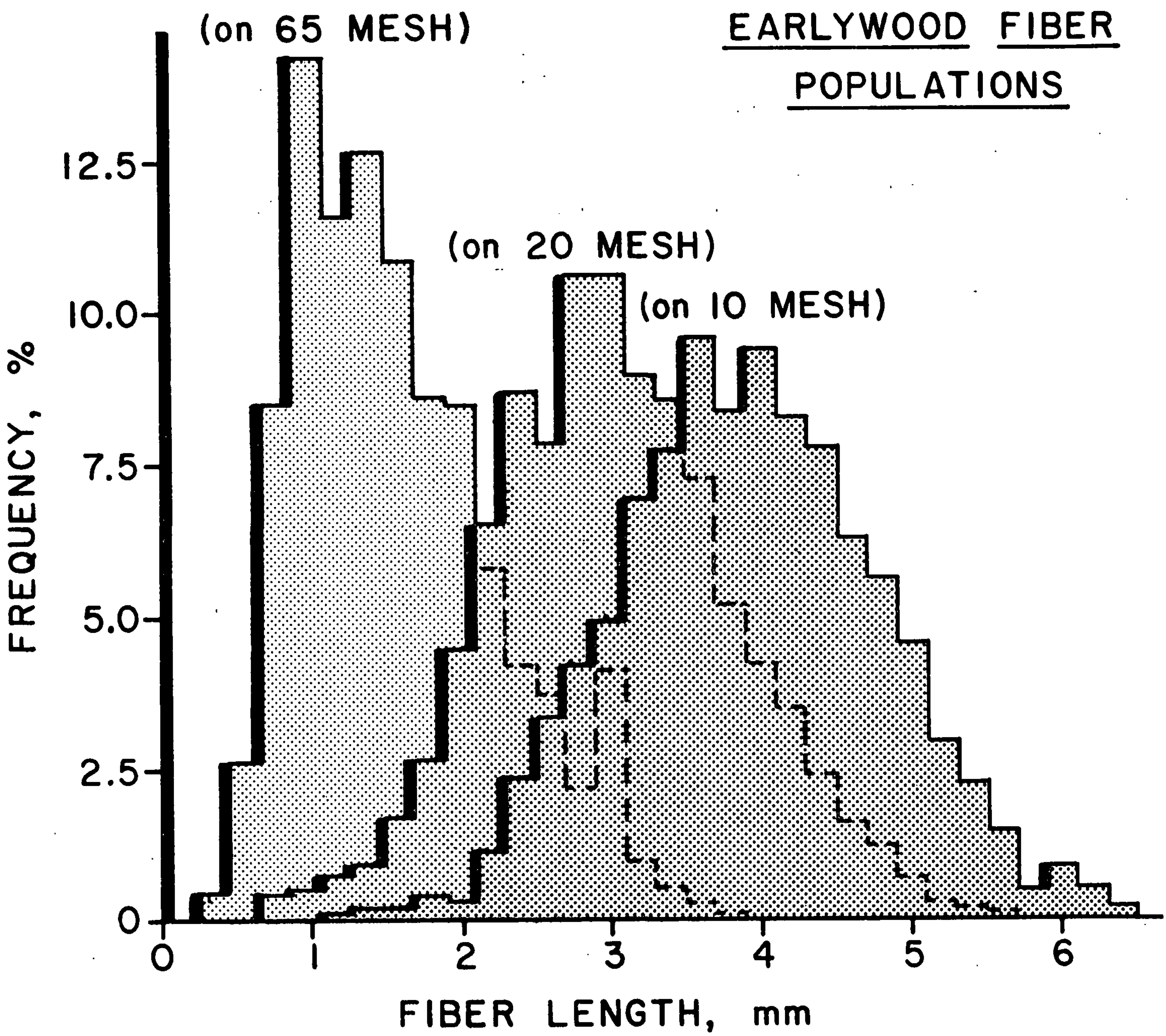

Figure 10. Fiber Length Distribution for Field Fraction Retained on 10, 20, and 65-Mesh Screens

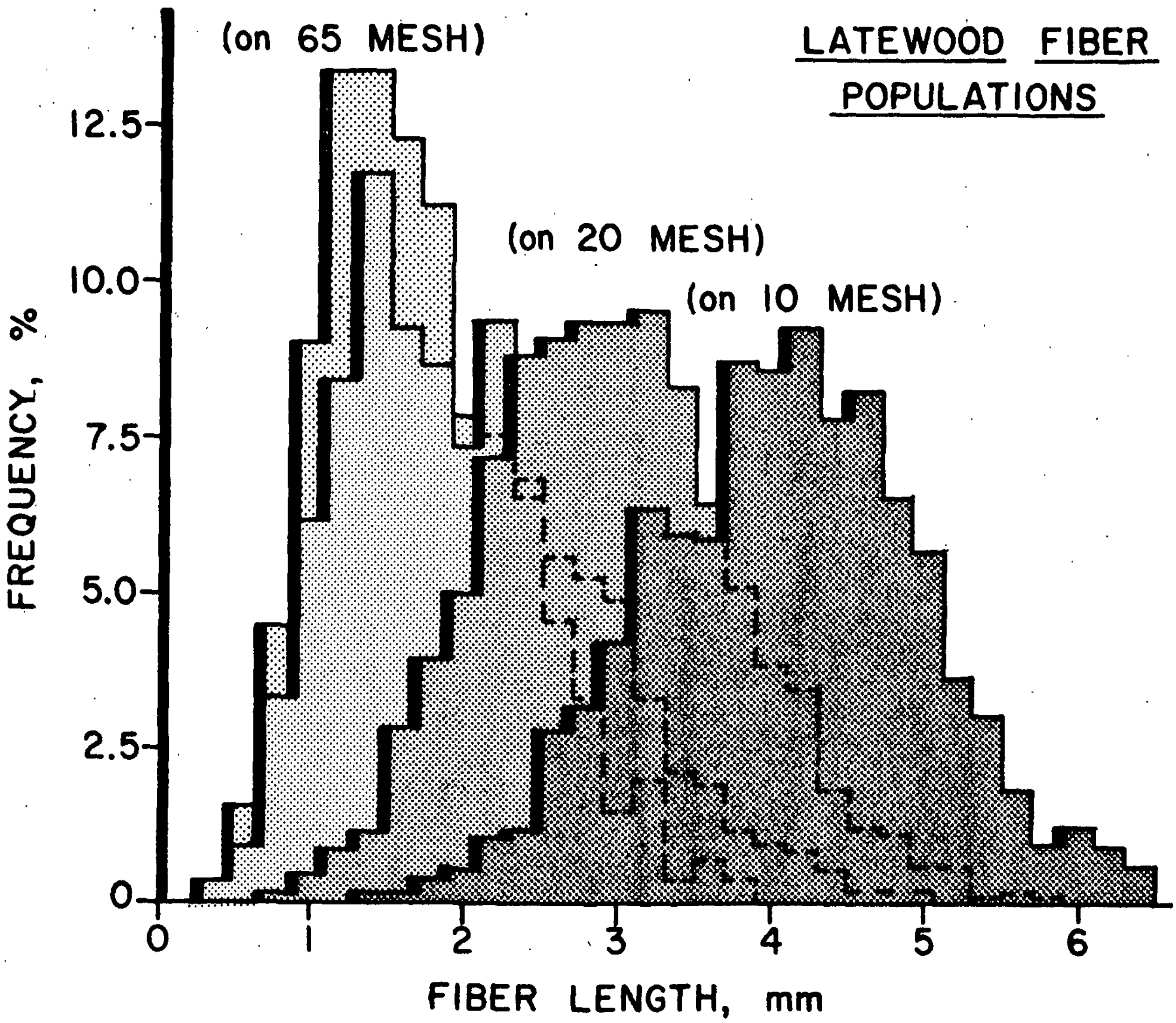

Figure 11. Fiber Length Distribution for Floc Fraction Retained on 10, 20, and 65-Mesh Screens. Double Peak for 65-Mesh Screen Described in Text

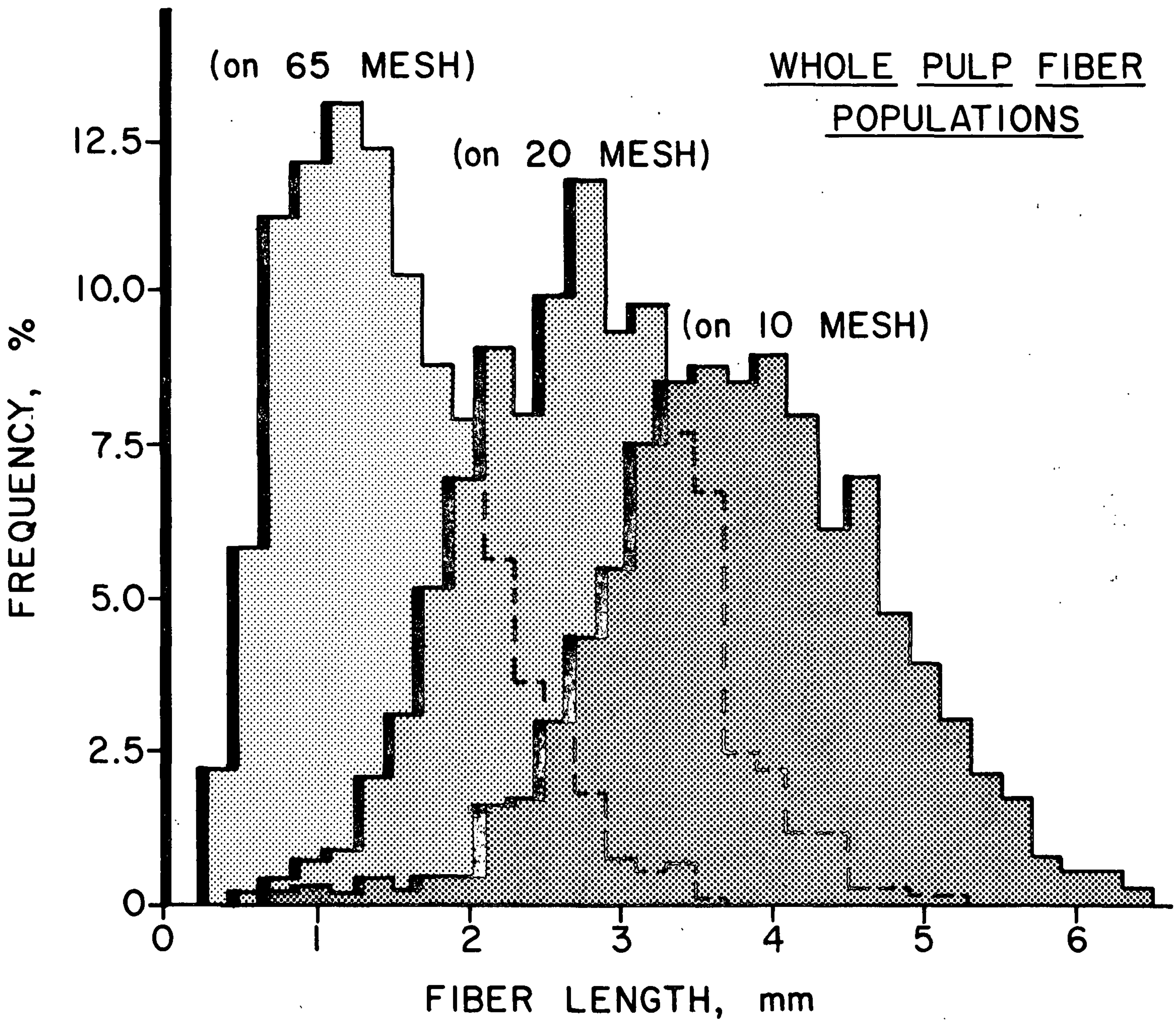

Figure 12. Fiber Length Distribution for Whole Pulp Retained on 10, 20, and 65-Mesh Screens

have a narrower length distribution; this is reflected in the 95% confidence limits.

Figure 13 is a comparison of scanning electron micrographs of handsheets composed of latewood fibers with respective mean fiber lengths of 1.74 and 4.13 mm. As previously reported by Dinwoodie (49), the micrographs show porosity increases with increasing fiber length. These qualitative results can be extrapolated to wet fiber mats; i.e., mats composed of short fibers should exhibit lower porosity than mats of long fibers of comparable wall fraction because short fibers, which are also thinner, pack tighter.

The measured morphological properties of the isolated fiber populations are summarized in Table V.

An additional indication of the separation into earlywood and latewood is given by the data on wall fraction, which was calculated from fiber diameters, d_f, and wall thicknesses, WT. The wall fraction of 30-32% for earlywood compared with 62-68% for latewood fiber fractions (resulting in fibers that are readily and less readily flattened as shown in Fig. 9) defines two distinct sets of fibers. Wall fraction data are in agreement with that calculated from previously reported fiber width and thickness measurements for loblolly pine (2). Earlywood and latewood are further subdivided in Table V according to L_f. The range in L_f of 1.6-3.9 mm for earlywood is about comparable with that of 1.7-4.1 mm for latewood and also agrees with values previously reported for loblolly pine (2,3).

It is virtually impossible to chip, pulp, bleach, and isolate wood fibers without imparting some physical damage to the fibers. However, great care was taken during fiber isolation procedures to minimize the degree of damage. Table V shows that in both sets 50 to over 80% of the fibers were whole. Therefore, it was expected that the isolated fiber populations show similar variation in

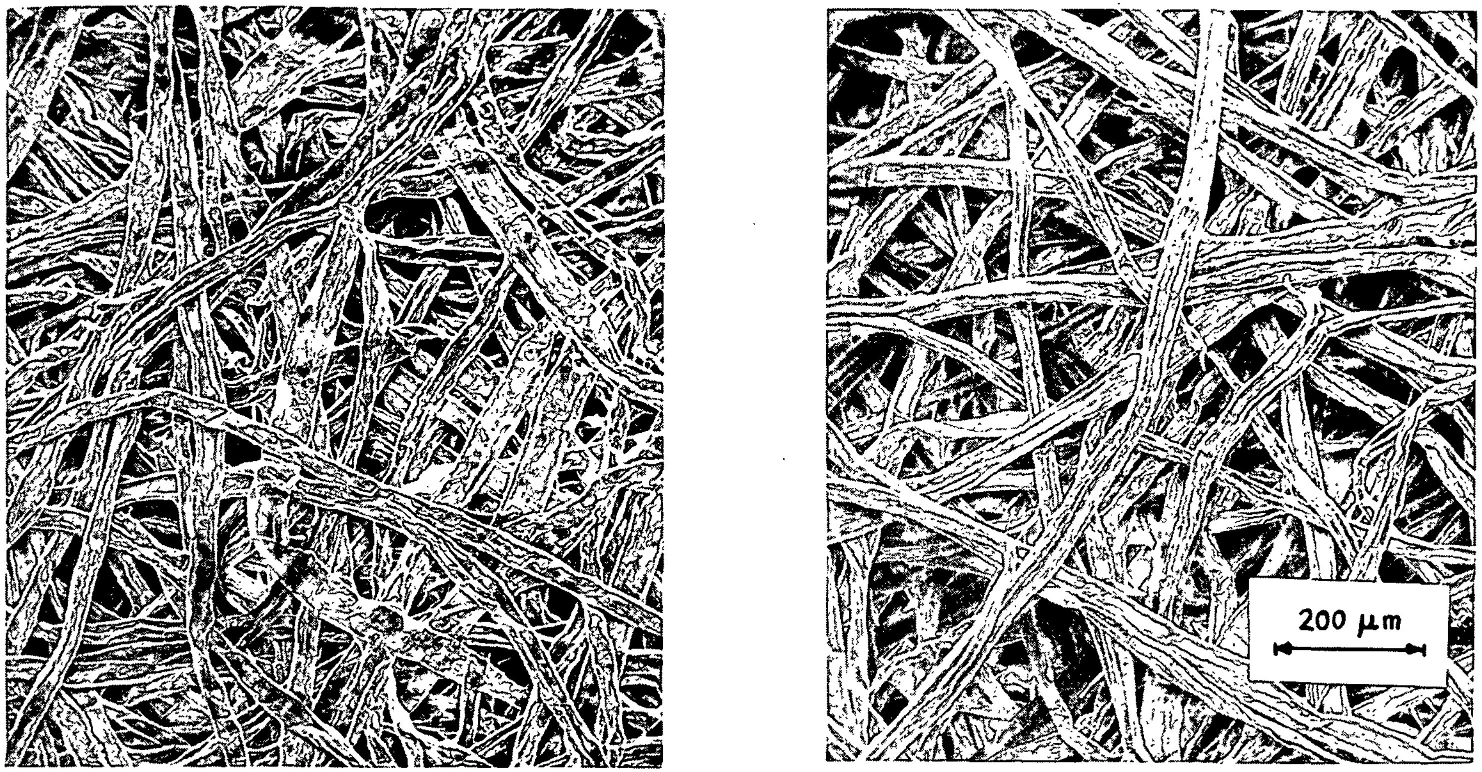

Figure 13. Comparison of Scanning Electron Micrographs of Handsheets Composed of Predominantly Latewood Fibers with Average Fiber Lengths of 1.74 mm (Left) and 4.13 mm

TABLE V

FIBER DIMENSIONS[a]

Mean Fiber Length, L_f, mm	Fiber Diameter, $d_f \times 10^2$, mm	Cell Wall Thickness, WT $\times 10^2$ mm	Wall Fraction[b], %	Whole Fibers, %	No. Fibers[f] per Gram, $\times 10^{-5}$ g^{-1}: n_f	n_{fG}
		Earlywood				
1.63 ± 0.02[c]	3.99 ± 0.25	0.59 ± 0.05	30	51	21.46	35.96
3.05 ± 0.03[d]	4.50 ± 0.22	0.73 ± 0.07	32	80	13.66	14.03
3.94 ± 0.04[e]	4.96 ± 0.23	0.80 ± 0.07	32	89	9.40	8.99
		Latewood				
1.74 ± 0.03[c]	3.24 ± 0.19	1.00 ± 0.05	62	50	22.73	22.77
2.07 ± 0.04[c]	3.41 ± 0.17	1.13 ± 0.07	66	59	16.98	16.53
2.98 ± 0.03[d]	3.57 ± 0.20	1.18 ± 0.06	66	73	11.12	10.52
4.13 ± 0.06[e]	3.70 ± 0.17	1.26 ± 0.05	68	85	6.85	6.96
		Whole Pulp				
1.49 ± 0.31[c]						
2.76 ± 0.75[d]						
3.88 ± 0.77[e]						

[a]Arithmetic mean ± 95% confidence limits where applicable.

[b]Calculated from fiber width and cell wall thickness.

[c,d,e]On 65, 20, and 10-mesh screens, respectively.

[f]n_f = experimental, n_{fG} = calculated.

fiber dimensions to that previously described for the tree. The increases in fiber width and cell wall thickness observed for earlywood and latewood fiber populations correspond to the increases in mean fiber length as expected.

In summary, the major variations in fiber characteristics and in the wall fraction for between-sets and in mean fiber length for within-sets of earlywood and latewood. However, within-sets, increases in fiber width and wall thickness accompany increases in mean fiber length. For convenience these joint trends will generally be referred to and indexed in terms of mean fiber length.

NUMBER OF FIBERS PER GRAM

The measured number of fibers per gram, $\underline{n_f}$, decreases as $\underline{L_f}$, $\underline{d_f}$, and $\underline{WT}$ increase for earlywood and latewood fiber populations, and the values observed agree with calculated values of 10.3-13.3 and 6.9-10.0 x 10^5, respectively, reported for earlywood and latewood fibers isolated from southern pine (50). Changes in $\underline{n_f}$ were also predicted by assuming a circular cylindrical model to calculate the volume of the fiber wall, $\underline{V_W}$:

$$V_W = \pi L_f \left[\left(\frac{d_f}{2}\right)^2 - \left(\frac{d_f}{2} - WT\right)^2\right] \qquad (31)$$

The number of fibers per gram can then be calculated geometrically, $\underline{n_{fG}}$, using Equation (32) with an assumption of a fiber wall density, $\underline{\rho_W}$, for bleached loblolly pine earlywood and latewood:

$$n_{fG} = 1/(V_W \ \rho_W) \qquad (32)$$

Unfortunately, values for $\underline{\rho_W}$ are difficult to obtain experimentally and do not appear available in the general literature. However, calculated values for $\underline{\rho_W}$ of 0.27 and 0.36 for earlywood and latewood, respectively, were obtained to allow

$\underline{n}_{fG}$ to agree closely with $\underline{n}_f$ in Table V. They are in the range of 0.29 and 0.63, respectively, reported for extracted swollen earlywood and latewood (51).

CALCULATED MOMENTS OF INERTIA

Moment of inertia, $\underline{I}$, has been shown in a previous section to be an important fiber property influencing the behavior of wet mat compressibility [Equation (18)]. Since $\underline{I}$ is a function of fiber dimensions, it may be calculated from the data in Table V with the assumption of an appropriate fiber model. Thus, $\underline{I}$ might be expected to correlate with compressibility data to give some insight of the effects of morphology on compressibility.

Choice of an appropriate model for calculation of $\underline{I}$ is critical because $\underline{I}$ is also a strong function of cross-sectional shape. For this reason two fiber models are presented in Fig. 14: the upper model represents the fiber cross section as circular with the lumen uncollapsed; the lower model represents the fiber cross section as flattened into a rectangular shape with the lumen completely collapsed. Both models are based on experimentally measured fiber dimensions.

Although the models are a gross oversimplification of the real system, they enable calculation of trends in relative $\underline{I}$-values for fibers in the collapsed and uncollapsed state.

Assuming the fiber wall is homogeneous and of uniform mass distribution, moments of inertia for the circular, $\underline{I}_C$, and flattened, $\underline{I}_F$, cross sections may be obtained from Equations (33) and (34) which have been derived from the elementary principles of mechanics:

$$I_C = \pi \frac{r_o^4 - r_i^4}{4} \tag{33}$$

where $\underline{r_o} = \underline{d_f}/2$ and $\underline{r_i} = (\underline{d_f}/2) - (\underline{WT})$,

$$I_F = 2/3\ d_f\ (WT)^3 \tag{34}$$

Calculated values for $\underline{I_C}$ and $\underline{I_F}$ are presented in Table VI and are related to fiber length in Fig. 15.

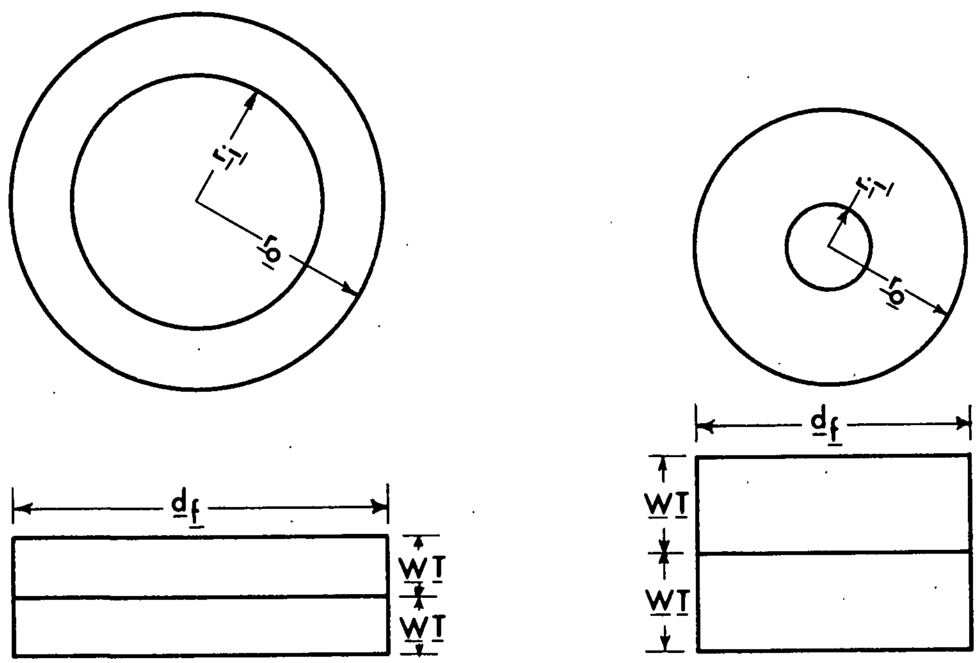

Figure 14. Scale Drawings (1000X) of Assumed Fiber Cross Sections for Calculation of $\underline{I}$ Comparing 3.94 mm Earlywood (Left) and 4.13 mm Latewood

Both $\underline{I_C}$ and $\underline{I_F}$ increase with increasing $\underline{L_f}$ in expected agreement with corresponding increases in $\underline{d_f}$ and $\underline{WT}$. $\underline{I_C}$ is greater in all cases than $\underline{I_F}$ because the wall material is distributed further from the central axis of the fiber when the fiber is not collapsed.

It is interesting to note that $\underline{I_C}$ for earlywood is about 2 to 2.5 times larger than that of latewood and that this trend is reversed for $\underline{I_F}$. Logically, the trends of the flattened fiber model ($\underline{I_F}$) appear more reasonable; especially since in bending studies of southern pine pulp fibers under water, latewood was shown to be five times stiffer than earlywood (52). This stiffness or flexural rigidity is a measure of the product, EI.

TABLE VI

CALCULATED MOMENTS OF INERTIA

L_f, mm	$d_f \times 10^2$, mm	$WT \times 10^2$, mm	$I_C{}^a \times 10^8$, mm^4	$I_F{}^b \times 10^8$, mm^4
		Earlywood		
1.63	3.99	0.59	9.4	0.6
3.05	4.50	0.73	15.9	1.2
3.94	4.96	0.80	23.5	1.7
		Latewood		
1.74	3.24	1.00	5.3	2.2
2.07	3.41	1.13	6.6	3.3
2.98	3.57	1.18	7.9	3.9
4.13	3.70	1.26	9.1	4.9

[a]Calculated from circular cross-sectional model.
[b]Calculated from flattened cross-sectional model.

Data for the elastic moduli of wet southern pine pulp are not available, but in general elastic moduli of latewood are greater than earlywood and E increases with increasing fiber length (10-14,23-25). However, there is no evidence to indicate that E is sufficiently low to compensate for the high moment values associated with the circular model. Therefore, on the basis of these studies (10-14, 23,25), it appears that the flattened fiber model may be more applicable to an understanding of fiber bending, whereas the circular model is more representative of the experimentally measured fiber dimensions.

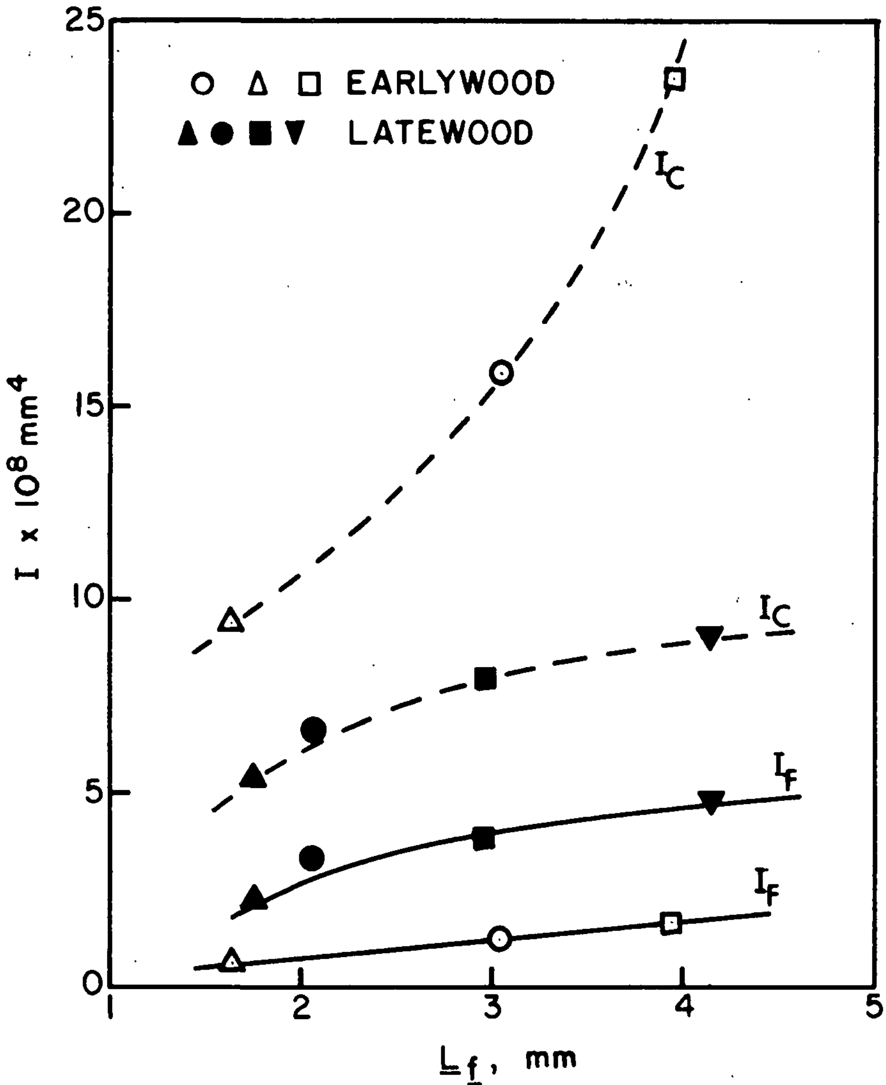

Figure 15. Relationship Between Fiber Length and Moments of Inertia for Circular, I_C, and Flattened, I_F, Models

CALCULATED SURFACE AND VOLUME TO MASS RATIOS

The information obtained in Table V can also be used to calculate an approximate surface to mass ratio, S_{WG}, and volume to mass ratio, v_G, for a fiber population assuming a circular cylindrical model to describe fiber shape as before. Since surface and volume to mass ratios are important hydrodynamic properties [see Equation (29)], calculations of this type will indicate the manner in which the morphological factors may be expected to influence hydrodynamic properties.

The model equations developed for $\underline{S_{WG}}$ and $\underline{v_G}$ are represented by Equations (35) and (36), respectively.

$$S_{WG} = \pi\, d_f\, L_f\, n_f \tag{35}$$

$$v_G = \pi\, d_f^{\,2}\, L_f\, n_f/4 \tag{36}$$

In Equation (36) $\underline{v_G}$ represents the total volume of the fiber-wall volume plus lumen volume.

$\underline{S_{WG}}$ and $\underline{v_G}$ values for earlywood and latewood fiber populations are presented in Table VII.

TABLE VII

GEOMETRIC SURFACE- AND VOLUME-TO-MASS RATIOS

Mean Fiber Length, mm	S_{WG}, cm^2/g	v_G, cm^3/g
	Earlywood	
1.63	4385 (7347)[a]	4.37 (7.33)[a]
3.05	5890	6.63
3.94	5771	7.16
	Latewood	
1.74	4026	3.26
2.07	3766	3.21
2.98	3716	3.32
4.13	3290	3.04

[a] Calculated using $\underline{n_{fG}}$.

As mean fiber length increases, $\underline{S_{WG}}$ decreases for the latewood fiber populations. A similar trend is expected for earlywood, but is not observed because $\underline{S_{WG}}$ for the 1.63 mm fiber population appears abnormally low. Experimental inaccuracy in the measurement of $\underline{d_f}$, $\underline{L_f}$, or $\underline{n_f}$ could conceivably account for this abnormality. Calculation of a value for S_{WG} which is greater than 5890 cm^2/g for this fiber population, through systematic variation of each of these fiber properties, reveals that experimental error could have reasonably occurred only in determination of n_f. The value in parenthesis uses n_{fG} in Equation (35) and demonstrates the expected trend. Also in Table VII, earlywood fibers have higher S_{WG} values than latewood.

Values of v_G for earlywood fiber populations are also higher than those for latewood but are not within the range of 1 to 4 cm^3/g usually observed by measuring volume to mass ratios by filtration analysis (32).

The abnormally high value for earlywood may be a result of the change in swollen volume of the fiber with pressure. Fiber measurements were made on fibers with essentially circular cross sections, and the model reflects this cross-sectional shape in calculation of v_G. Earlywood fibers, however, have thin cell walls and are more likely to collapse under pressure. Collapse of the fibers would cause expulsion of water from the lumen and subsequently lower the volume to mass ratio. On the other hand, thick-walled latewood fibers probably retain much of their cross-sectional shape under pressure.

Fiber collapse may cause a slight increase in surface to mass ratio as the fiber changes from circular to elliptical in cross section.

FIBER MORPHOLOGY AND WET MAT COMPRESSIBILITY

COMPRESSIBILITY CONSTANTS M AND N

Initial compressibility constants for the various morphological fiber fractions described in Table V were obtained from linear regression of log-log plots of wet mat density, c, vs. static load, P_f, using the log form of Equation (2) in which the c_o term is neglected:

$$\log c = \log M + N \log P_f \tag{37}$$

The data and resulting best fit curves (solid line) for earlywood and latewood fiber fractions are presented in Fig. 16 and 17, respectively. The original data are presented in Appendix I, and applicability of Equation (37) to the data may be visually confirmed from these figures. Calculated values for the initial slopes, N_1, and intercepts M_1, are tabulated in Table VIII and are within the range of values previously reported by Han (9).

TABLE VIII

VALUES FOR COMPRESSIBILITY CONSTANTS N AND M

L_f, mm	L_f/d_f	N_1 [a]	$M_1 \times 10^3$, [a] c.g.s. units [b]	N [c]	$M \times 10^3$, [c] c.g.s. units [b]
			Earlywood		
1.63	40.9	0.361 ± 0.005 [d]	2.02 ± 0.01 [d]	0.373	1.79 ± 0.01 [d]
3.05	67.8	0.390 ± 0.010	1.43 ± 0.02	0.373	1.70 ± 0.02
3.94	79.4	0.376 ± 0.015	1.61 ± 0.03	0.373	1.66 ± 0.03
			Latewood		
1.74	53.7	0.373 ± 0.017	1.65 ± 0.03	0.373	1.64 ± 0.03
2.07	60.7	0.376 ± 0.018	1.55 ± 0.03	0.373	1.60 ± 0.03
2.98	83.5	0.363 ± 0.013	1.73 ± 0.02	0.373	1.56 ± 0.02
4.13	111.6	0.366 ± 0.019	1.69 ± 0.03	0.373	1.57 ± 0.03

[a] Linear regression of individual fiber populations.

[b] $(g/cm^3)/(dynes/cm^2)^N$.

[c] Multiple linear regression of total earlywood and latewood data, confidence limit negligible (0.0003) due to large sample size.

[d] 95% confidence limits.

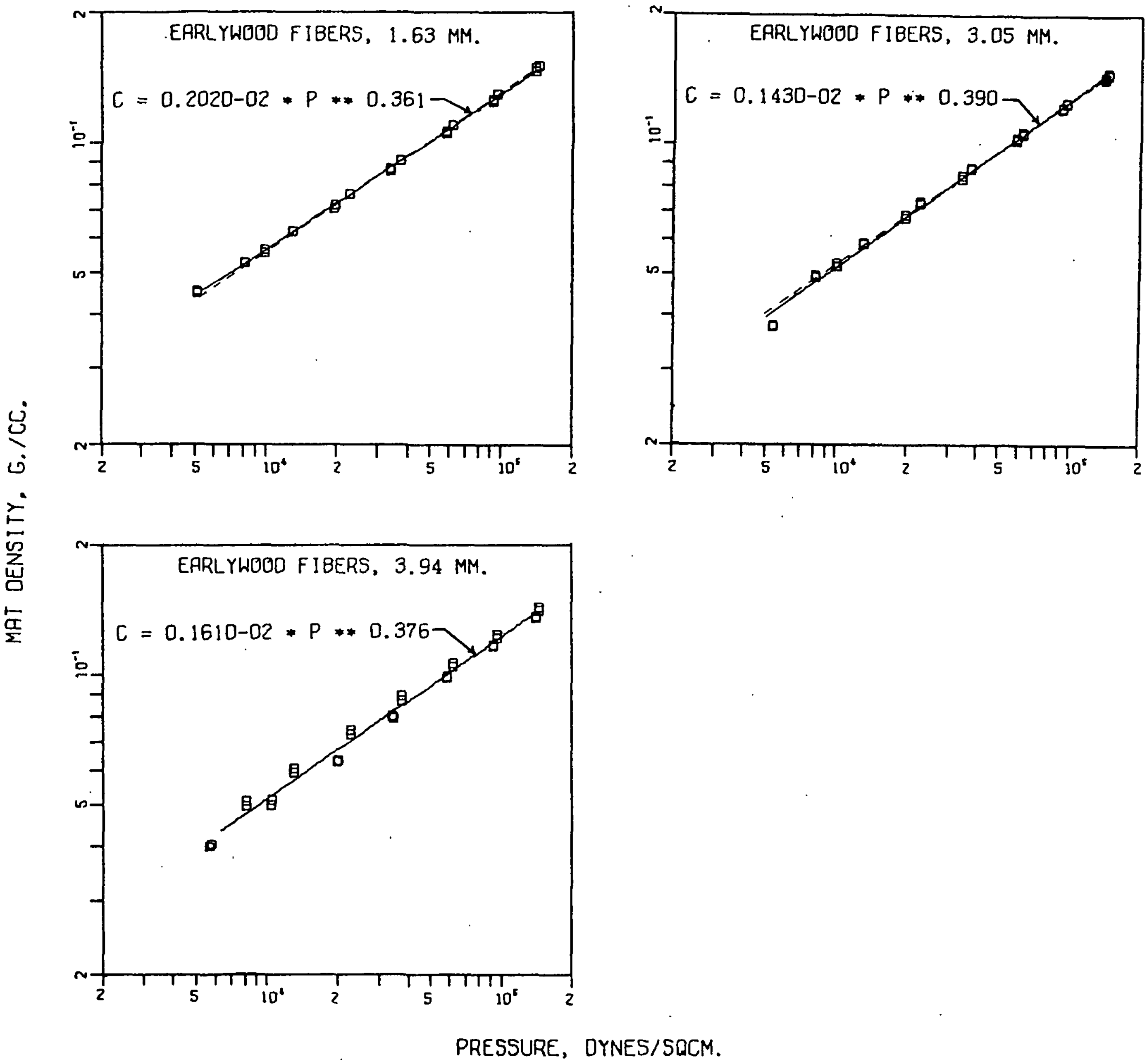

Figure 16. Compressibility Data and Resulting Best Fit Curves for Earlywood Fiber Fractions; Solid Line (Labelled) Corresponds to $\underline{c} = \underline{M_1 P^{N_1}_{f}}$, Broken Line Corresponds to $\underline{c} = \underline{MP^{N}_{f}}$

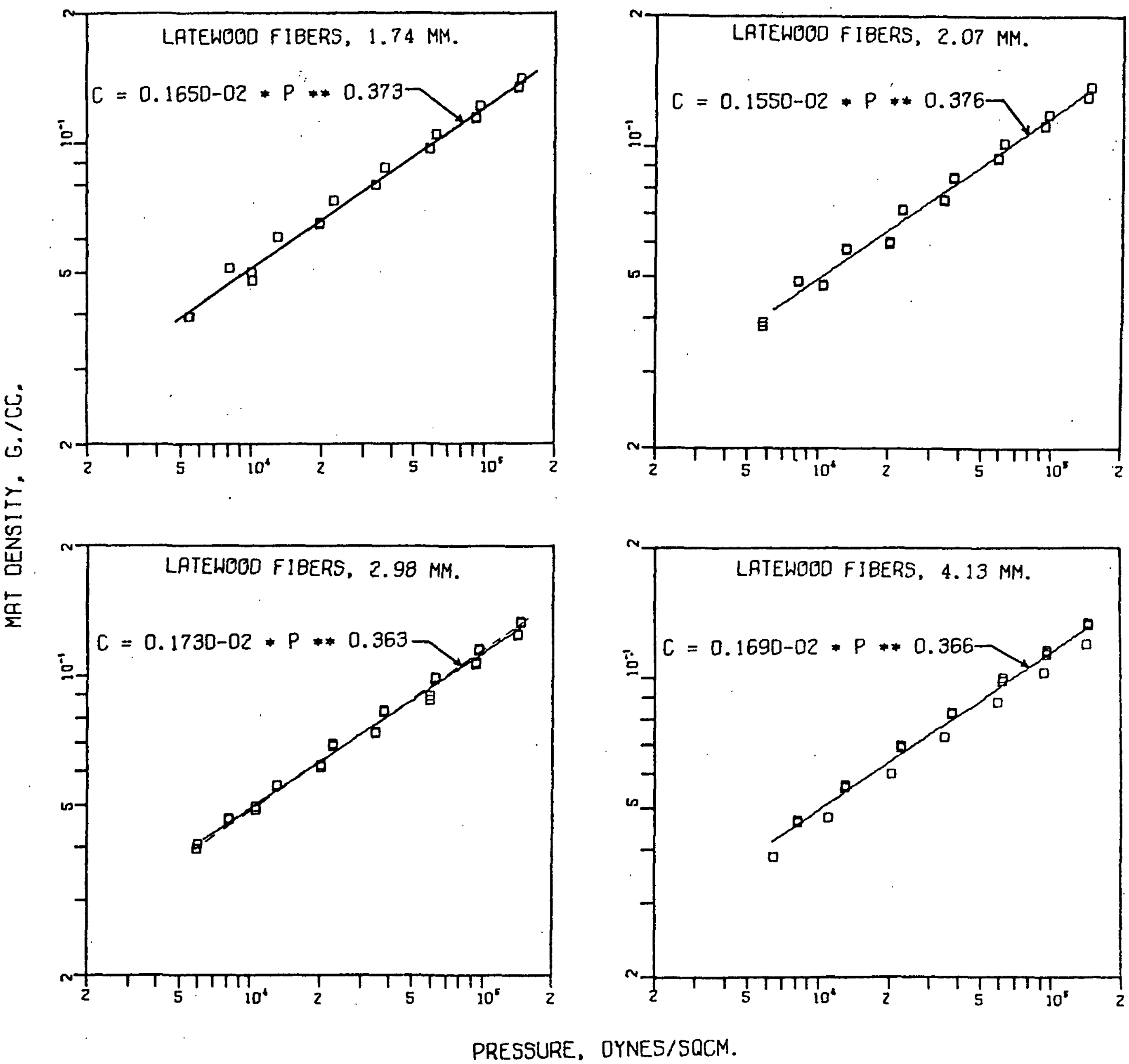

Figure 17. Compressibility Data and Resulting Best Fit Curves for Latewood Fiber Fractions (as in Fig. 16)

Although the $\underline{M}_1$ values were determined statistically, they are sensitive to the exact positioning of the slope (or $\underline{N}_1$ value), and normal variability in the experimental data results in uncertainty over what slope should be used. This uncertainty is manifested by the lack of correlation of $\underline{M}_1$ with fiber dimensions. Assuming the compressibility model, Equation (18), is applicable to wood pulp fiber systems, a more tenable result would be an observed correlation between $\underline{M}_1$ and $\underline{L}_f$ in Table VIII [since $\underline{M}$ in Equation (18) is a function of $\underline{d}_f$, $\underline{I}$, and $\underline{E}$, and increases in each of these fiber properties correspond to increases in $\underline{L}_f$, $\underline{M}$ should also correlate to $\underline{L}_f$]. Also on the basis of Equation (18), $\underline{N}$ would be expected to remain essentially constant with respect to changing fiber dimensions. This contention is supported by previous work in which beating and high consistency refining did not influence $\underline{N}$ values appreciably (9).

Since $\underline{N}_1$ does not correlate with fiber dimensions (Table VIII) and in view of Equation (18), $\underline{N}$ values for earlywood and latewood were assumed constant to facilitate further data analysis. Incorporation of the logs of the earlywood and latewood mat density vs. pressure data of Appendix I into a simultaneous linear multiple regression analysis in which the same value of $\underline{N}$ is assumed for every group (53,54), gave an overall adjusted value of $\underline{N}$ equal to 0.373 as well as an individual value of $\underline{M}$ for each fiber population. These values correponded to the weighted averages computed for separate independent regression runs. The results of this regression analysis are reported as $\underline{N}$ and $\underline{M}$ in Table VIII. Visual confirmation of the applicability of $\underline{N}$ and $\underline{M}$ to the experimental data is provided by the broken lines in Fig. 16 and 17. On the whole, respective values for $\underline{N}$ and $\underline{M}$ fit the data extremely well and in most cases the broken lines superimpose the solid lines calculated from $\underline{N}_1$ and $\underline{M}_1$.

COMPRESSIBILITY

Further evidence that $\underline{N}$ is essentially constant for the fiber populations results from defining compressibility as the change in mat density with pressure, i.e., $d\underline{c}/d\underline{P_f}$, which represents the slope at a given pressure for plots of $\underline{c}$ vs. $\underline{P_f}$.

$$\frac{dc}{dP_f} = \frac{N}{P_f} M P_f^N = \frac{N}{P_f} c \qquad (38)$$

From Equation (38) for a given pressure, $d\underline{c}/d\underline{P_f}$ is expected to be proportional to $\underline{c}$ and $\underline{M}$ if $\underline{N}$ is essentially constant. Table IX shows $d\underline{c}/d\underline{P_f}$ remains practically constant at $\underline{P_f}$ = 10 and 90 cm H_2O for earlywood and latewood fiber populations regardless of whether $\underline{N}_1$ and $\underline{M}_1$ or $\underline{N}$ and $\underline{M}$ are used in the calculation. However, only when $\underline{N}$ is constant for the fiber populations is $d\underline{c}/d\underline{P_f}$ proportional to $\underline{M}$ at a given pressure. Hence, this mean value of $\underline{N}$ is believed more generally appliable to the experimental data than the individual initial values. Since for any two fiber fractions studied it would not necessarily be established that $\underline{N}$ can be treated as constant, $d\underline{c}/d\underline{P_f}$ provides a more reliable indication of relative compressibility than the coefficient "$\underline{M}$."

From Table IX it is observed at $\underline{P_f}$ = 10 cm H_2O that the compressibility of earlywood fibers is greater than the compressibility of latewood. Also as fiber length increases, compressibility decreases for both earlywood and latewood fiber populations. At $\underline{P_f}$ = 90 cm H_2O the compressibility of the mats has greatly diminished relative to $\underline{P_f}$ = 10 cm H_2O; but differences between earlywood, latewood, and fiber lengths are still present albeit they are not as great.

TABLE IX

COMPRESSIBILITY VALUES, $d\underline{c}/d\underline{P}_f$, FOR MATS OF EARLYWOOD AND LATEWOOD FIBER FRACTIONS AT $\underline{P}_f$ = 10 AND 90 cm H_2O

	$\underline{P}_f$ = 10 cm H_2O		$\underline{P}_f$ = 90 cm H_2O	
Mean Fiber Length, mm	$(d\underline{c}/d\underline{P}_f)_1$[a] x 10^7 g/cm-dyne	$(d\underline{c}/d\underline{P}_f)$[b] x 10^7 g/cm-dyne	$(d\underline{c}/d\underline{P}_f)_1$[a] x 10^7 g/cm-dyne	$(d\underline{c}/d\underline{P}_f)$[b] x 10^7 g/cm-dyne
		Earlywood		
1.63	21.0	21.0	5.0	5.3
3.05	20.5	19.9	5.4	5.0
3.94	19.6	19.5	4.9	4.9
		Latewood		
1.74	19.3	19.2	4.9	4.8
2.07	18.9	18.8	4.8	4.7
2.98	18.1	18.3	4.4	4.6
4.13	18.3	18.4	4.5	4.6

[a]Calculated using $\underline{N}_1$ and $\underline{M}_1$.
[b]Claculated using $\underline{N}$ and $\underline{M}$.

WET MAT DENSITY

The systematic correlation of wet mat density and the major morphological variations represented by differences in wall fraction of earlywood compared with latewood and mean fiber length is shown in the three dimensional diagram of Fig. 18. For clarification of trends, tabulated values are presented in Table X and agree with results found earlier for first compression of loblolly pine latewood (5). Mat densities and pressures are interpolated values based on $\underline{N}$ and $\underline{M}$ as in Table VIII. The computer program used to develop the three-dimensional plot is given in Appendix II.

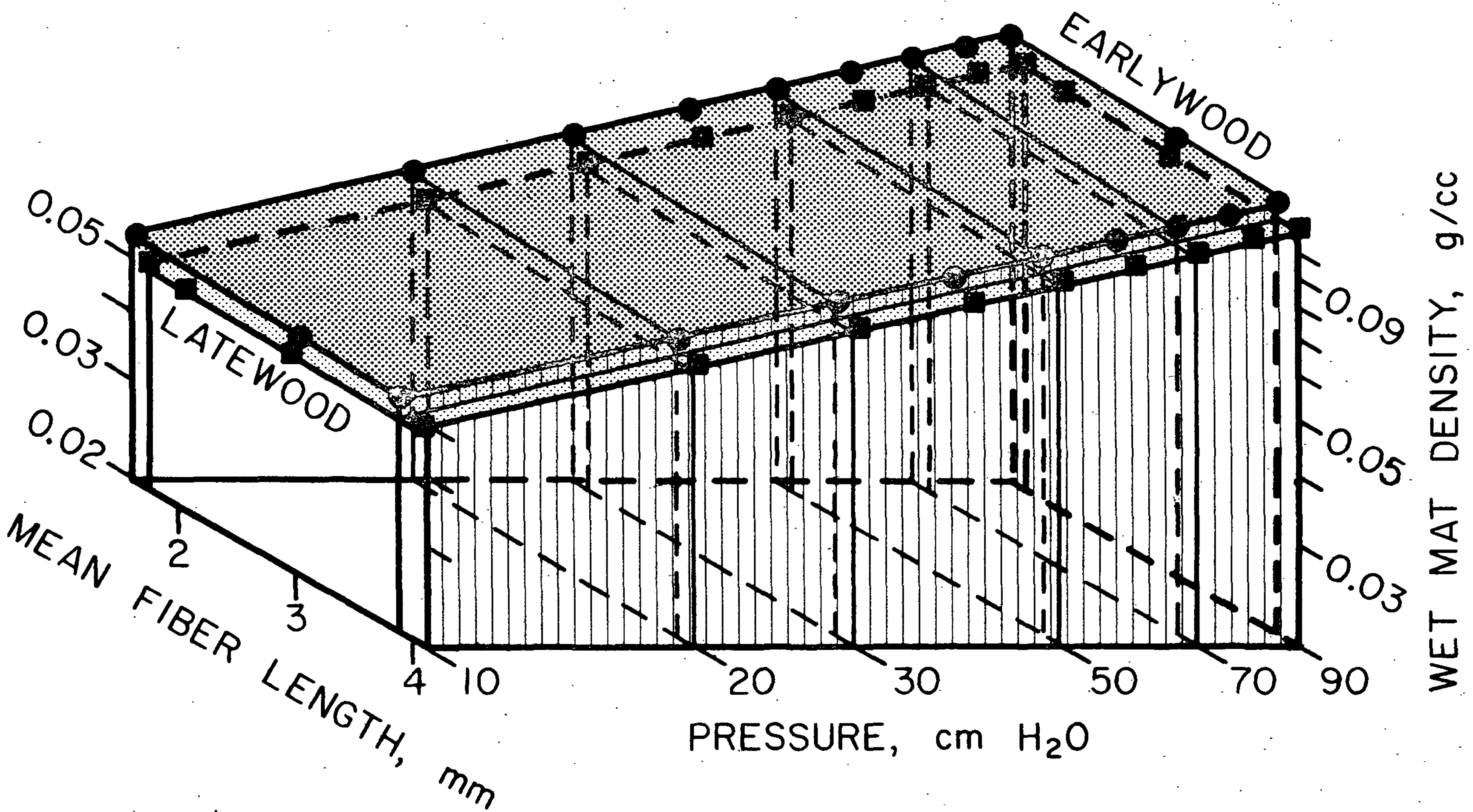

Figure 18. Log-Log Plots of Interpolated Wet Mat Density vs. Pressure with Mean Fiber Length of Earlywood and Latewood Plotted Linearly on the z-Axis

TABLE X

INTERPOLATED WET MAT DENSITY,
G/CC

Pressure, cm H_2O	Earlywood Fiber Populations[a]			Latewood Fiber Populations[a]			
	1.63 mm	3.05 mm	3.94 mm	1.74 mm	2.07 mm	2.98 mm	4.13 mm
10	0.0552	0.0524	0.0512	0.0505	0.0493	0.0481	0.0484
20	0.0714	0.0679	0.0663	0.0655	0.0639	0.0623	0.0627
30	0.0831	0.0789	0.0771	0.0761	0.0743	0.0724	0.0729
40	0.0925	0.0879	0.0858	0.0848	0.0827	0.0864	0.0812
50	0.1006	0.0955	0.0933	0.0921	0.0899	0.0876	0.0882
60	0.1076	0.1022	0.0998	0.0986	0.0962	0.0938	0.0944
70	0.1140	0.1083	0.1057	0.1044	0.1019	0.0994	0.1000
80	0.1198	0.1138	0.1111	0.1098	0.1071	0.1044	0.1051
90	0.1252	0.1189	0.1161	0.1147	0.1119	0.1091	0.1098

[a]Mean fiber lengths.

The figure, although complex in appearance, is not difficult to analyze if one imagines it represents a box. Concentrating on the base of the box, the two horizontal lines (one in back and one in front) represent the pressure axis with pressure increasing from left to right as indicated. The two shorter lines forming the sides of the base are the fiber length axis with mean fiber length increasing from back to front. Wet mat density is represented by vertical distances from the base. Notice that wet mat density and pressure axes are logarithmic whereas the fiber length axis is linear.

Separation of the two planes is associated with wall fraction differences of earlywood compared to latewood (Table V), and reflects the ability of the more easily collapsible earlywood fibers to form mats of higher density. The planes are parallel because of the single value for _N_.

The trend of decreasing mat densities with increases in mean fiber length is shown by the tilting of the planes. Note that at all pressures in Fig. 18 or Table X, interpolated mat density is greater for smaller fibers, for which there are more fibers per gram (Table V). Probably greater density arises because smaller fibers cause a decrease in segment length, $\underline{L}_s$, and thus increase the number of contacts, $\underline{n}_c$, in the mat [Equation (6)].

The morphologically ranked $\underline{M}$ values may be visualized as resulting from the extrapolation of the two planes in Fig. 18 to log $\underline{P}_f$ = 0 cm H_2O. But, if $\underline{N}$ had not been smoothed, the calculated $\underline{M}$ values would have been appreciably different as indicated in Table VIII.

CORRELATION OF $\underline{M}$ AND FIBER DIMENSIONS

Latewood fibers with their characteristic high wall fraction and near circular cross section closely resemble the shape of the more intensively studied circular synthetic fiber systems. Therefore, a comparison of the mat density response to latewood fiber shape (length to diameter ratio, $\underline{L}_f/\underline{d}_f$) with that of a synthetic fiber system may indicate, at least partially, the extent to which the two systems are similar. Such a comparison is made in Fig. 19 between latewood fiber populations at $\underline{P}_f$ = 50 and 90 cm H_2O and Nylon at $\underline{P}_f$ = 50 and 100 cm H_2O.

The figure shows that the mat density response to $\underline{L}_f/\underline{d}_f$ for the two fiber systems is similar. Toward low axis ratios the curves for both latewood and Nylon fibers at comparable pressures begin to rise rapidly, while at high axis ratios the mat density appears to remain constant.

With $\underline{N}$ constant for the fiber populations in this study, the effects of pressure may be eliminated from Fig. 19 by plotting fiber dimensions vs. compressibility constant $\underline{M}$. Figure 20 compares correlations of $\underline{M}$ with $\underline{L}_f/\underline{d}_f$ as well

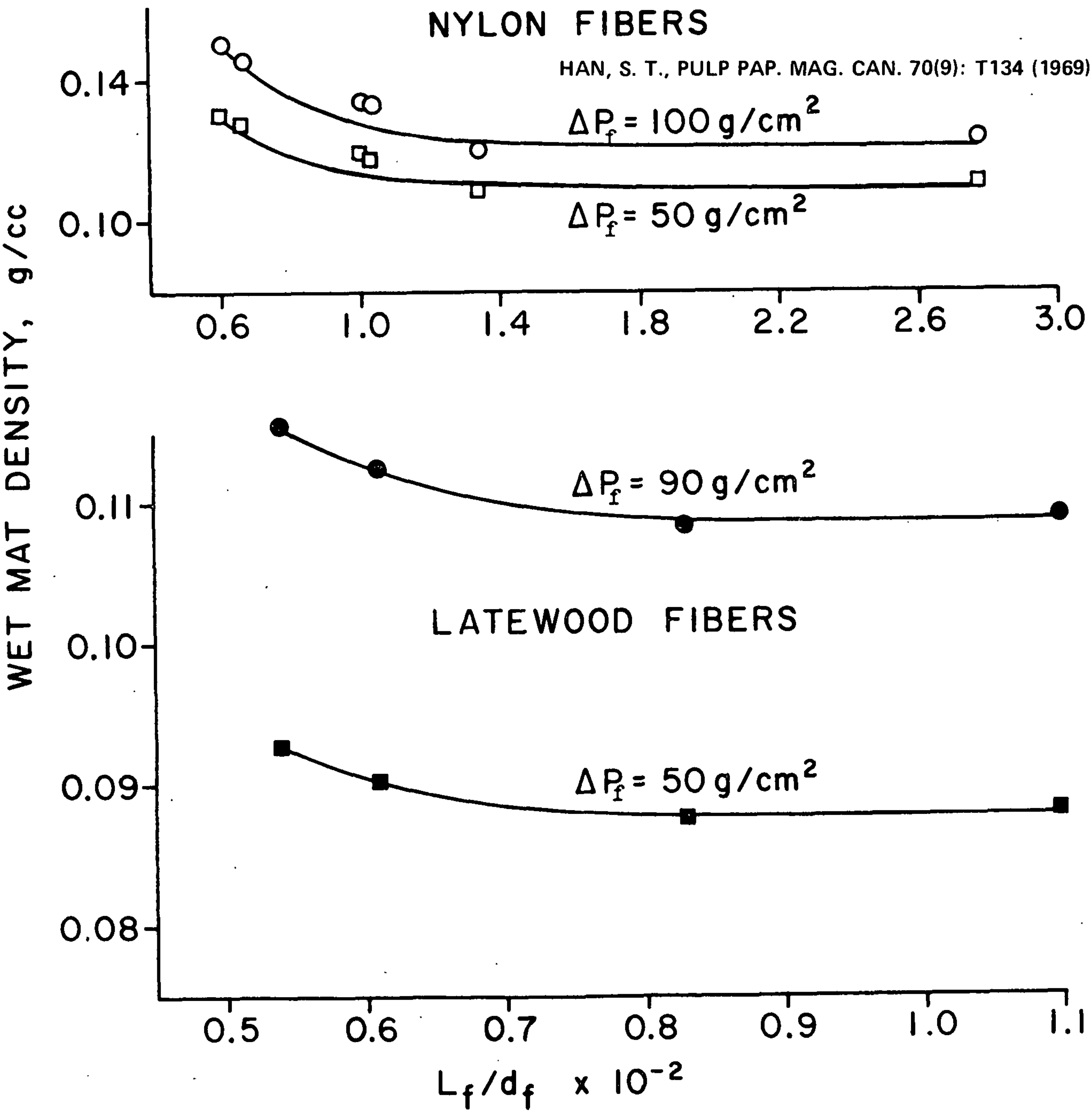

Figure 19. Comparison of Latewood to Nylon Fibers

as mean fiber length, $\underline{L}_f$, for earlywood and latewood fiber populations; and as anticipated the shape of the $\underline{M}$ vs. $\underline{L}_f/\underline{d}_f$ curves are similar to that already discussed for Fig. 19. However, in Fig. 20, notice the comparable correlation of $\underline{M}$ and $\underline{L}_f$. This is believed to result from the greater complexities of the natural fiber system in which variations in fiber diameter are biologically linked to changes in fiber length (Table V).

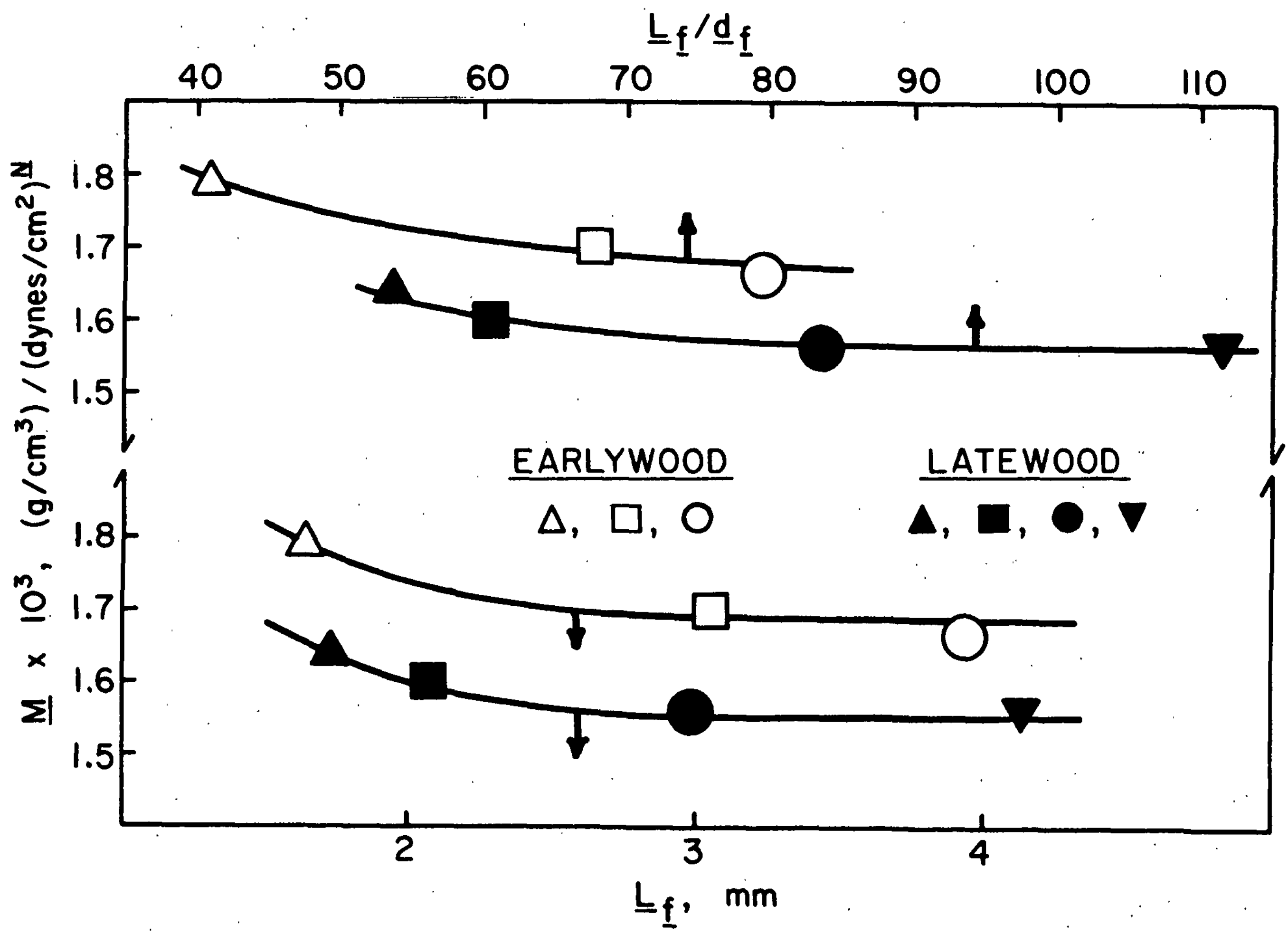

Figure 20. Compressibility Constant, $\underline{M}$, vs. Fiber Length to Fiber Diameter Ratio, $\underline{L}_f/\underline{d}_f$, (Upper Curves) and Fiber Length, $\underline{L}_f$, (Lower Curves)

As noted in development of Equation (18), the distance between points of contact in a fiber mat (segment length) is an important property of the mechanism of compressibility (9). The flat region of the curves in Fig. 20 corresponds to a condition in which the fiber length is sufficiently long so as to insure constant segment length. However, below a certain point, segment length will decrease with decreasing fiber length (6). Lower segment lengths cause higher mat density (9), and in this case the higher values of M observed at low L_f.

The separation of the curves for earlywood and latewood corresponds to differences in wall fraction, 32 *vs*. 66%, respectively. Using Equation (18) as a guide, this separation could be due to differences in d_f, I, and/or E, since:

$$M \propto \left[\frac{d_f{}^4}{E\ I}\right]^{1/3} \tag{39}$$

Attempts to correlate M with d_f and $(d_f{}^4/I)^{1/3}$ failed to give meaningful results in agreement with studies by Jones (5) in which wet mat compression response was found to be independent of fiber diameter.

In view of the lack of correlation with d_f and $(d_f{}^4/I)^{1/3}$ in compressibility of wood pulp, it appears from Equation (39) that fluxural rigidity (EI) is the prominant fiber property contributing to the separation of the two curves in Fig. 20. In order to fully interpret fluxural rigidity in terms of fiber dimensions, the moduli as well as the moment of the fibers must be known.

Wood pulp fibers are anisotropic, and the mechanism of bending, which is complex, involves more than a single modulus of the fiber wall. However, from the literature previously cited it is reasonable to expect that the elastic moduli for fiber bending would behave similarly to I_F in Fig. 15. On this basis and Equation (39), M would be expected to correlate with $(1/E\ I_F)^{1/3}$.

Since modulus values were not measured for the fiber populations studied, and cannot be quantified from the literature, a plot of $\underline{M}$ vs. $(1/\underline{I}_F)^{1/3}$ is presented in Fig. 21.

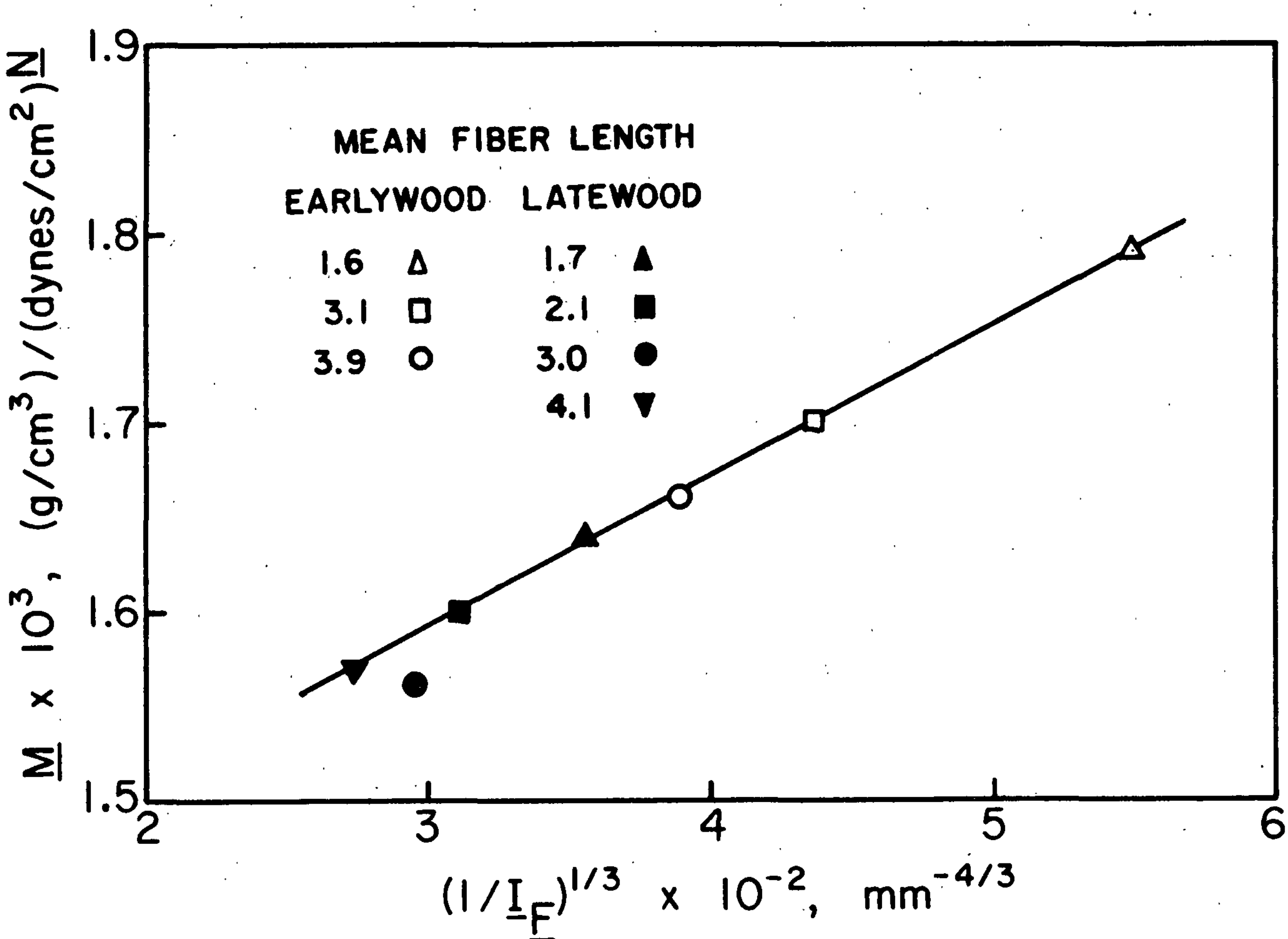

Figure 21. Plot of $\underline{M}$ vs. $(1/\underline{I}_F)^{1/3}$ for Earlywood and Latewood Fiber Fractions

Amazingly, this figure reveals a straight line essentially connecting the seven fiber populations studied.*

---

* A constant area model based on the circular fiber model was also used to calculate $\underline{I}_F$. When plotted with $\underline{M}$ a similar curve with a slight increase in the scatter of the data was obtained.

In view of the simplicity of the models involved, this result is quite significant; first, it supports bending as the dominant mechanism in compressibility of wood pulp; and second, it suggests that the wood pulp fiber is essentially flattened prior to bending. Since $\underline{I}_F$ is predominantly a function of wall thickness [Equation (34)], it appears in a wood pulp fiber system that the relative compressibility of a wood pulp fiber mat is primarily influenced by fiber bending which in turn is governed by the thickness of the fiber walls, and to a lesser extent by the fiber diameter. These results do not imply, however, that modulus is not sharing at least an equally important role in the mechanism of wood pulp compressibility.

Not that decreases in $\underline{M}$ in Fig. 21 also relate to an increase in mean fiber length (pyramids in circles) or earlywood and latewood (open and closed symbols, respectively). The relationship of $\underline{M}$ with these morphological factors corresponds to observations on the increase in relative elastic modulus of earlywood and latewood in twenty successive growth rings of loblolly pine (13). Furthermore, this modulus for latewood was higher than that for earlywood by an amount about equal to the increase associated with growth. Growth, in turn, is accompanied by an increase in fiber length, with some trends toward increase in fiber width and wall thickness, particularly during the early decades of growth (21). Thus, the trend of changes in $\underline{M}$ with changes in mean fiber length of earlywood and latewood tends to parallel previously reported changes in elastic modulus of earlywood and latewood in successive growth rings.

The observed correlation of $\underline{M}$ with $(1/\underline{I}_F)^{1/3}$ also corresponds to high correlations of apparent axial modulus with fibril angle and fiber length (11,23,24). As longer fibers are formed during successive years of tree growth, fibril angle with respect to cell axis decreases [Equation (1)]; fiber axial modulus increases, and, on the basis of this study, $\underline{M}$ decreases.

Differences in M between the earlywood and latewood fiber fractions is also supported by elastic modulus data (10-12,14). Relatively large differences were observed between the axial modulus of latewood compared with earlywood and also in the collapse force of wet fibers. Differences in the transverse modulus between earlywood and latewood were not as great.

It may be interesting to notice that these differences between earlywood and latewood are visually supported by Fig. 22 which compares actual oven-dry samples of mats of comparable weight and mean fiber length previously subjected to compressibility experiments. The earlywood fiber mat is of significantly higher density than would be expected from Fig. 20. This is probably a result of the irreversible collapse and subsequent conformability of the earlywood fibers and subsequently less springback due to lower fiber stiffness.

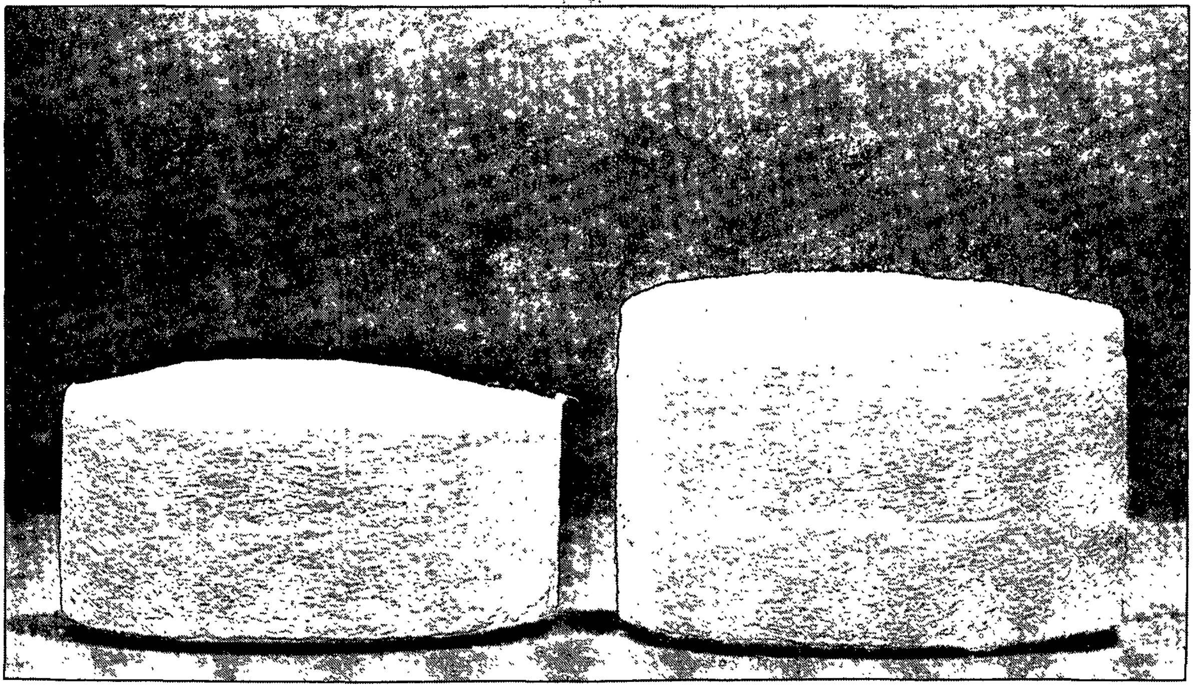

Figure 22. Comparison of 3.98 mm Earlywood (Left) and 4.13 mm Latewood Fiber Mats of Comparable Weight After Compressibility Experiments. Pad Diameters are Approximately 3 Inches

MATHEMATICAL PROCEDURE FOR DETERMINATION OF DERIVATIVES FROM CURVES OF EXPERIMENTAL DATA

During the course of data analysis, it is often desirable to fit a smooth curve as closely as possible to a set of experimental points. This is hand drawn (with a conventional French curve or similar aid). However, unless the curve is linear, the mathematical expression describing it is unknown.

One method of obtaining a mathematical expression to fit the data has been to use conventional statistical procedures. These procedures, based on least squares analysis, force the best linear, quadratic, cubic, or quartic expression to the experimental points. Unfortunately, plots of the experimental points and curves of the best fitting mathematical relationships are not always in close agreement over the entire range of data.

A review of the literature revealed that an equation of the form:

$$y = b_0 + b_1x^m + b_2x^n \tag{40}$$

has the capability of accurately fitting a wide variety of regularly curved plots of experimental data (55).

By combining the least squares approach used in conventional statistics with the versatile curve-fitting properties of Equation (40), a computerized method of accurately describing experimental data points mathematically was developed. This accurate mathematical description was easily differentiated, yielding a derivative of a curved plot of experimental data. Derivatives of this form were essential in solving many of the hydrodynamic equations.

LEAST SQUARES SOLUTION OF $y = \underline{b_0} + \underline{b_1x^m} + \underline{b_2x^n}$

The conventional least squares approach was used to fit experimental data to the form of Equation (40).

Let $\underline{y_i}$ = dependent variable and $\underline{f_i}$ = approximation to the dependent variable $= \underline{b_0} + \underline{b_1 x_i^m} + \underline{b_2 x_i^n}$. Then the square of the difference is:

$$S = \sum \left[y_i - f_i \right]^2$$

$$= \sum \Big[y_i^2 - 2b_0 y_i - 2b_1 x_i^m y_i - 2b_2 x_i^n yi + b_0^2 + 2b_0 b_1 x_i^m + 2b_0 b_2 x_i^n + b_1^2 x_i^{2m} + 2b_1 b_2 x_i^m x_i^n + b_2^2 x_i^{2n} \Big] \qquad (41)$$

The conditions of a best fitting curve are achieved when $\underline{S}$ is minimized. This minimum occurs when the partial derivatives of $\underline{S}$ with respect to the five unknowns ($\underline{b_0}$, $\underline{b_1}$, $\underline{b_2}$, $\underline{m}$, and $\underline{n}$) in Equation (41) are zero. The result is the respective normalized Equations (42a) through (42e).

$$\sum y_i = Nb_0 + b_1 \sum x_i^m + b_2 \sum x_i^n \qquad (42a)$$

$$\sum x_i^m y_i = b_0 \sum x_i^m + b_1 \sum x_i^{2m} + b_2 \sum x_i^m x_i^n \qquad (42b)$$

$$\sum x_i^n y_i = b_0 \sum x_i^n + b_1 \sum x_i^m x_i^n + b_2 \sum x_i^{2n} \qquad (42c)$$

$$\sum x_i^m y_i \ln(x_i) = b_0 \sum x_i^m \ln(x_i) + b_1 \sum x_i^{2m} \ln(x_i) + b_2 \sum x_i^m x_i^n \ln(x_i) \qquad (42d)$$

$$\sum x_i^n y_i \ln(x_i) = b_0 \sum x_i^n \ln(x_i) + b_1 \sum x_i^m x_i^n \ln(x_i) + b_2 \sum x_i^{2n} \ln(x_i) \qquad (42e)$$

To obtain values of $\underline{b_0}$, $\underline{b_1}$, $\underline{b_2}$, $\underline{m}$, and $\underline{n}$, this nonlinear system of normalized equations was solved simultaneously. The procedure used has been presented by Nelson (56), and the mathematical method employed is based on the truncation of a Taylor's expression. The procedure is detailed below.

Let the five normal equations be respectively represented by Equations (43a) through (43e):

$$F(b_0, b_1, b_2, m, n) = 0 \qquad (43a)$$

$$G(b_0, b_1, b_2, m, n) = 0 \qquad (43b)$$

$$H(b_0, b_1, b_2, m, n) = 0 \qquad (43c)$$

$$I(b_0, b_1, b_2, m, n) = 0 \tag{43d}$$

$$J(b_0, b_1, b_2, m, n) = 0 \tag{43e}$$

Then let

$$f_1 = \frac{\partial F}{\partial b_0} = N$$

$$f_2 = \frac{\partial F}{\partial b_1} = \sum x_i^m$$

$$f_3 = \frac{\partial F}{\partial b_2} = \sum x_i^n$$

$$f_4 = \frac{\partial F}{\partial m} = b_1 \sum x_i^m \ln(x_i)$$

$$f_5 = \frac{\partial F}{\partial n} = b_2 \sum x_i^n \ln(x_i)$$

$$g_1 = \frac{\partial G}{\partial b_0} = \sum x_i^m$$

$$g_2 = \frac{\partial G}{\partial b_1} = \sum x_i^{2m}$$

$$g_3 = \frac{\partial G}{\partial b_2} = \sum x_i^m x_i^n$$

$$g_4 = \frac{\partial G}{\partial m} = b_0 \sum x_i^m \ln(x_i) + 2b_1 \sum x_i^{2m} \ln(x_i) + b_2 \sum x_i^m x_i^n \ln(x_i) - \sum x_i^m y_i \ln(x_i)$$

$$g_5 = \frac{\partial G}{\partial n} = b_2 \sum x_i^m x_i^n \ln(x_i)$$

$$h_1 = \frac{\partial H}{\partial b_0} = \sum x_i^n$$

$$h_2 = \frac{\partial H}{\partial b_1} = \sum x_i^m x_i^n$$

$$h_3 = \frac{\partial H}{\partial b_2} = \sum x_i^{2n}$$

$$h_4 = \frac{\partial H}{\partial m} = \sum x_i^m x_i^n \ln(x_i)$$

$$h_5 = \frac{\partial H}{\partial n} = b_0 \sum x_i^n \ln(x_i) + b_1 \sum x_i^m x_i^n \ln(x_i) +$$

$$2b_2 \sum x_i^{2n} \ln(x_i) - \sum x_i^n y_i \ln(x_i)$$

$$i_1 = \frac{\partial I}{\partial b_0} = \sum x_i^m \ln(x_i)$$

$$i_2 = \frac{\partial I}{\partial b_1} = \sum x_i^{2m} \ln(x_i)$$

$$i_3 = \frac{\partial I}{\partial b_2} = \sum x_i^m x_i^n \ln(x_i)$$

$$i_4 = \frac{\partial I}{\partial m} = b_0 \sum x_i^m [\ln(x_i)]^2 + 2b_1 \sum x_i^{2m} [\ln(x_i)]^2 +$$

$$b_2 \sum x_i^m x_i^n [\ln(x_i)]^2 - \sum x_i^m y_i [\ln(x_i)]^2$$

$$i_5 = \frac{\partial I}{\partial n} = b_2 \sum x_i^m x_i^n [\ln(x_i)]^2$$

$$j_1 = \frac{\partial J}{\partial b_0} = \sum x_i^n \ln(x_i)$$

$$j_2 = \frac{\partial J}{\partial b_1} = \sum x_i^m x_i^n \ln(x_i)$$

$$j_3 = \frac{\partial J}{\partial b_2} = \sum x_i^{2n} \ln(x_i)$$

$$j_4 = \frac{\partial J}{\partial m} = b_1 \sum x_i^m x_i^n [\ln(x_i)]^2$$

$$j_5 = \frac{\partial J}{\partial n} = b_0 \sum x_i^n [\ln(x_i)]^2 + b_1 \sum x_i^m x_i^n [\ln(x_i)]^2 +$$

$$2b_2 \sum x_i^{2n} [\ln(x_i)]^2 - \sum x_i^n y_i [\ln(x_i)]^2$$

and let the displacements be defined as:

$$\delta b_0 = b_0 - \overline{b_0}$$

$$\delta b_1 = b_1 - \overline{b_1}$$

$$\delta b_2 = b_2 - \overline{b_2}$$

$$\delta m = m - \overline{m}$$

$$\delta n = n - \overline{n}$$

where $\overline{b_0}$, $\overline{b_1}$, $\overline{b_2}$, $\overline{m}$, and $\overline{n}$ represent arbitrary first estimates of b_0, b_1, b_2, m, and n, respectively.

A truncation of Taylor's theorem can then be applied to yield the following linearized equations:

$$f_1\delta b_0 + f_2\delta b_1 + f_3\delta b_2 + f_4\delta m + f_5\delta n = -F(\overline{b_0},\overline{b_1},\overline{b_2},\overline{m},\overline{n}) \tag{44a}$$

$$g_1\delta b_0 + g_2\delta b_1 + g_3\delta b_2 + g_4\delta m + g_5\delta n = -G(\overline{b_0},\overline{b_1},\overline{b_2},\overline{m},\overline{n}) \tag{44b}$$

$$h_1\delta b_0 + h_2\delta b_1 + h_3\delta b_2 + h_4\delta m + h_5\delta n = -H(\overline{b_0},\overline{b_1},\overline{b_2},\overline{m},\overline{n}) \tag{44c}$$

$$i_1\delta b_0 + i_2\delta b_1 + i_3\delta b_2 + i_4\delta m + i_5\delta n = -I(\overline{b_0},\overline{b_1},\overline{b_2},\overline{m},\overline{n}) \tag{44d}$$

$$j_1\delta b_0 + j_2\delta b_1 + j_3\delta b_2 + j_4\delta m + j_5\delta n = -J(\overline{b_0},\overline{b_1},\overline{b_2},\overline{m},\overline{n}) \tag{44e}$$

Using Gaussian elimination, Equations (44a) through (44e) can be solved simultaneously for b_0, b_1, b_2, m and n. The displacements are then calculated, respectively added to the initial estimates ($\overline{b_0}$, $\overline{b_1}$, $\overline{b_2}$, $\overline{m}$, and $\overline{n}$) to form a revised estimate, and the equations resolved. Values for b_0, b_1, b_2, m, and n are obtained when the displacements become negligible.

COMPUTER PROGRAM

This numerical procedure has been incorporated into the computer program used to calculate local specific filtration resistance (next section) and is presented in its entirety in Appendix III. As written, the program requires the user to estimate initial values of $\underline{b}_0$, $\underline{b}_1$, $\underline{b}_2$, $\underline{m}$, and $\underline{n}$. Initial estimates of $\underline{b}_0$, $\underline{b}_1$, and $\underline{b}_2$ are relatively unimportant and may be assigned a value of 1.0. Initial estimates of $\underline{m}$ and $\underline{n}$, however, are important. The number of iterations, and subsequently the time required for program execution, is greatly reduced if estimates of $\underline{m}$ and $\underline{n}$ are close to solution values.

More than one combination of values for $\underline{b}_0$, $\underline{b}_1$, $\underline{b}_2$, $\underline{m}$, and $\underline{n}$ will satisfy the conditions for a solution. Therefore, the user must be cautious in using the accurate mathematical description of his data derived from this program in the development of theories.

If desired, output is computed which can be plotted. A plotting program used to plot the output is listed in Appendix IV.

The use of the program is exemplified in the sections on filtration resistance.

FIBER MORPHOLOGY AND SPECIFIC FILTRATION RESISTANCE

Specific filtration resistance obtained from constant rate filtration experiments may be calculated for either average, $\langle \underline{R} \rangle$, or local, $\underline{R}$, values by using Equations (23) or (22), respectively. The physical difference between these two values is that $\langle \underline{R} \rangle$ represents an average filtration resistance for the whole mat during formation, whereas $\underline{R}$ represents the filtraiton resistance of a small but measurable section immediately above the septum. Since there is a pressure gradient throughout the mat (caused by the frictional force of the water as it flows around the fibers), $\langle \underline{R} \rangle$ may be considered related to this gradient, whereas $\underline{R}$ is more closely related to the actual pressure drop measured.

Values for $\langle \underline{R} \rangle$ and $\underline{R}$ were calculated for various pressures from averaged pressure *vs*. time data using the computer program listed in Appendix V; the output for earlywood, latewood, and whole pulp fiber populations is presented in Appendix VI. Values are lower than those reported by other researchers for unbeaten softwood pulps (16,29) due to the lack of fines and fibrillations in the fiber systems.

AVERAGE SPECIFIC FILTRATION RESISTANCE

The systematic correlation of average specific filtration resistance and the major morphological variations represented by differences in wall fraction of earlywood compared with latewood and mean fiber length is shown in Fig. 23. Values are based on at least triplicate runs for each of the fiber fractions and the 95% confidence limits were ±2 to 8.5% of the means ($\Delta \underline{P}_{\underline{f}}$ = 50 cm H_2O) which ranked from 6.80 x 10^6 cm/g for the longest latewood fibers to 30.48 x 10^6 cm/g for the shortest earlywood (Appendix VI). It is evident from the figure that at a given pressure, populations of thin-walled earlywood fibers (top plane) attain higher $\langle \underline{R} \rangle$ values at pressure drops between 10 and 90 cm H_2O than do the populations of thick-walled latewood fibers of comparable length, and, $\langle \underline{R} \rangle$ increases as fiber length decreases for either earlywood or latewood fiber populations. Values for whole pulp data which are not displayed but may be obtained from Appendix VI fall between the earlywood and latewood planes, and analysis has shown that the higher percentage of latewood observed in the whole pulp (Table III) is reflected in the proximity of the whole pulp values to those of latewood.

These changes in average filtration resistance with fiber morphology may best be explained by qualitative analysis of the geometry of the fibers. Slight rearrangement of Equation (29) results in an expression for $\langle \underline{R} \rangle$ in terms of its components, average specific surface, $\langle \underline{S}_{\underline{W}} \rangle$, and average specific volume, $\langle \underline{v} \rangle$, as well as mat density, $\underline{c}$:

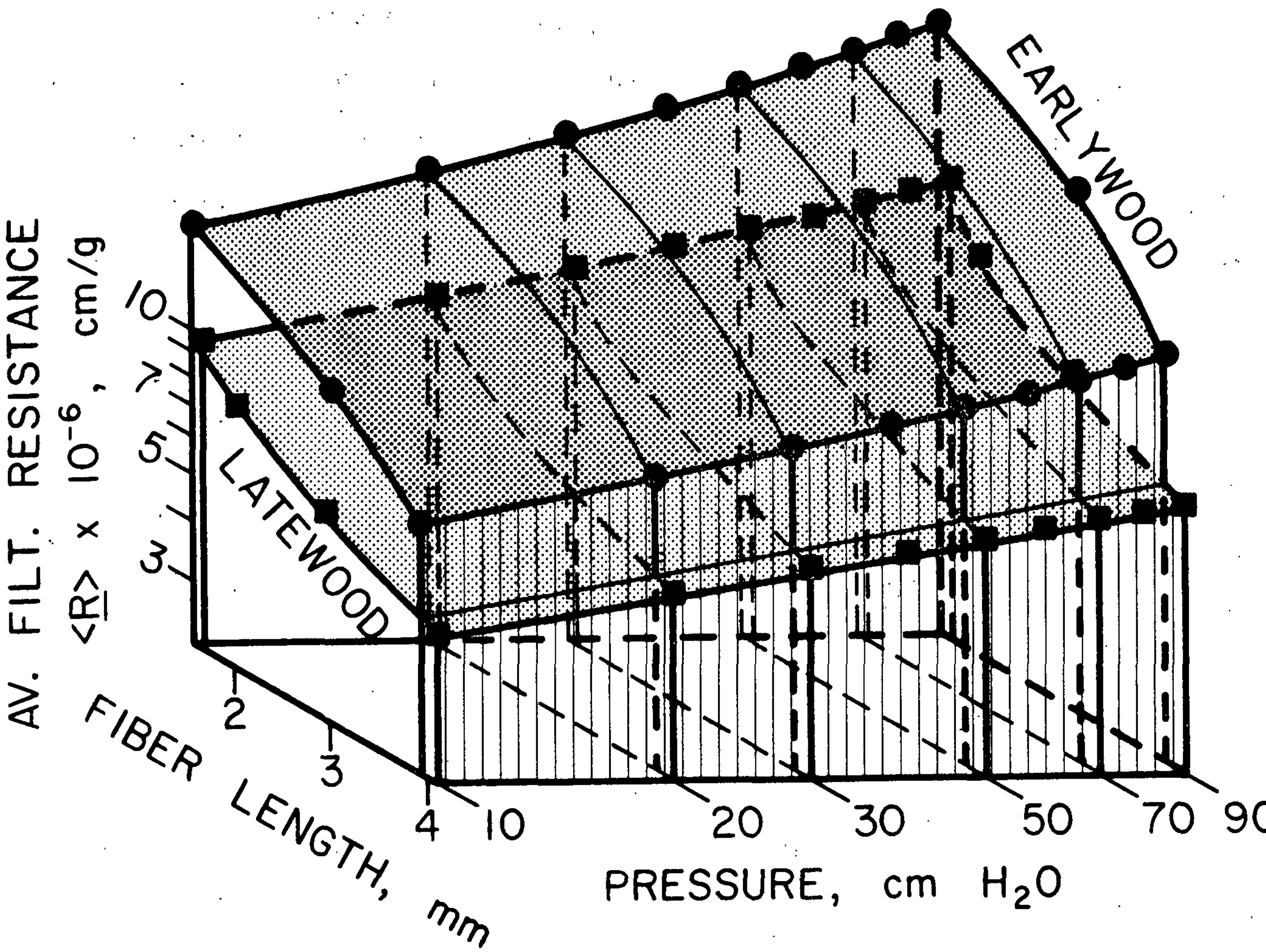

Figure 23. Log-log Plots of Average Specific Filtration Resistance vs. Pressure with Mean Fiber Length of Earlywood and Latewood Plotted Linearly on the z-Axis

$$\langle R\rangle = \langle S_W\rangle^2 \left\{\frac{3.5\ (1-N/2)\ c^{1/2}}{\langle v\rangle^{1/2}}\ [1 + 57\ \langle v\rangle^3\ (1-N/2)^6 c^3]\right\} \tag{45}$$

From Equation (45), $\langle \underline{R}\rangle$ is observed to be proportional to $\langle \underline{S_W}\rangle^2$ (a point discussed in detail in a later section). Table VII has already shown that the geometric surface to mass ratio, $\underline{S_{WG}}$, increases significantly as fiber length decreases. The corresponding increase in $\langle \underline{R}\rangle$ is in accord with this finding.

The data presented in Table V show that the thinner-walled earlywood fiber populations are composed of fibers of greater width and generally lower mass than latewood fibers of comparable length. Thus, higher geometric surface to mass ratios are expected for the earlywood fiber populations, and on this basis so are the higher values for $\langle R \rangle$. Qualitatively, this comparison may be visualized from Fig. 24 which compares identically formed handsheets of longest and shortest earlywood and latewood fiber fractions. By observation, fiber surface area per unit of mat exposed to fluid flow is highest for the 1.63 mm earlywood and lowest for the 4.13 mm latewood fibers.

The increase in $\langle R \rangle$ with increasing pressure drop also varies significantly, the greatest percent increase being for the smallest earlywood fibers. This variation in $\langle R \rangle$ with pressure is about comparable, on a percentage change basis, to those trends observed for wet mat density. However, the change in $\langle R \rangle$ with fiber length is much greater than the change found for c. This means that in a comparison of mats of constant density and varying fiber morphology, as in Table XI, about four times the filtration resistance for the shortest earlywood fibers occurs at a lower pressure compared to that of the longest latewood fibers.

LOCAL SPECIFIC FILTRATION RESISTANCE

Accurate calculation of local specific filtration resistance, R, results from the mathematical procedure for determining the derivative, dP_f/dt, using Equation (40). The precision with which Equation (40) fits the data is exemplified in the average pressure *vs.* time plots for earlywood, latewood, and whole pulp fiber populations shown in Fig. 25, 26, and 27, respectively. The empirical constants, b_0, b_1, b_2, m, and n, which were statistically derived for each set of data are summarized in Table XII.

The effects of fiber morphological variation and pressure drop on R are shown in Fig. 28. Trends are similar to those previously described for average filtration resistance (Fig. 23); i.e., for a given pressure, thin-walled earlywood fibers (top plane) attain higher R values at pressure drops between 10 and

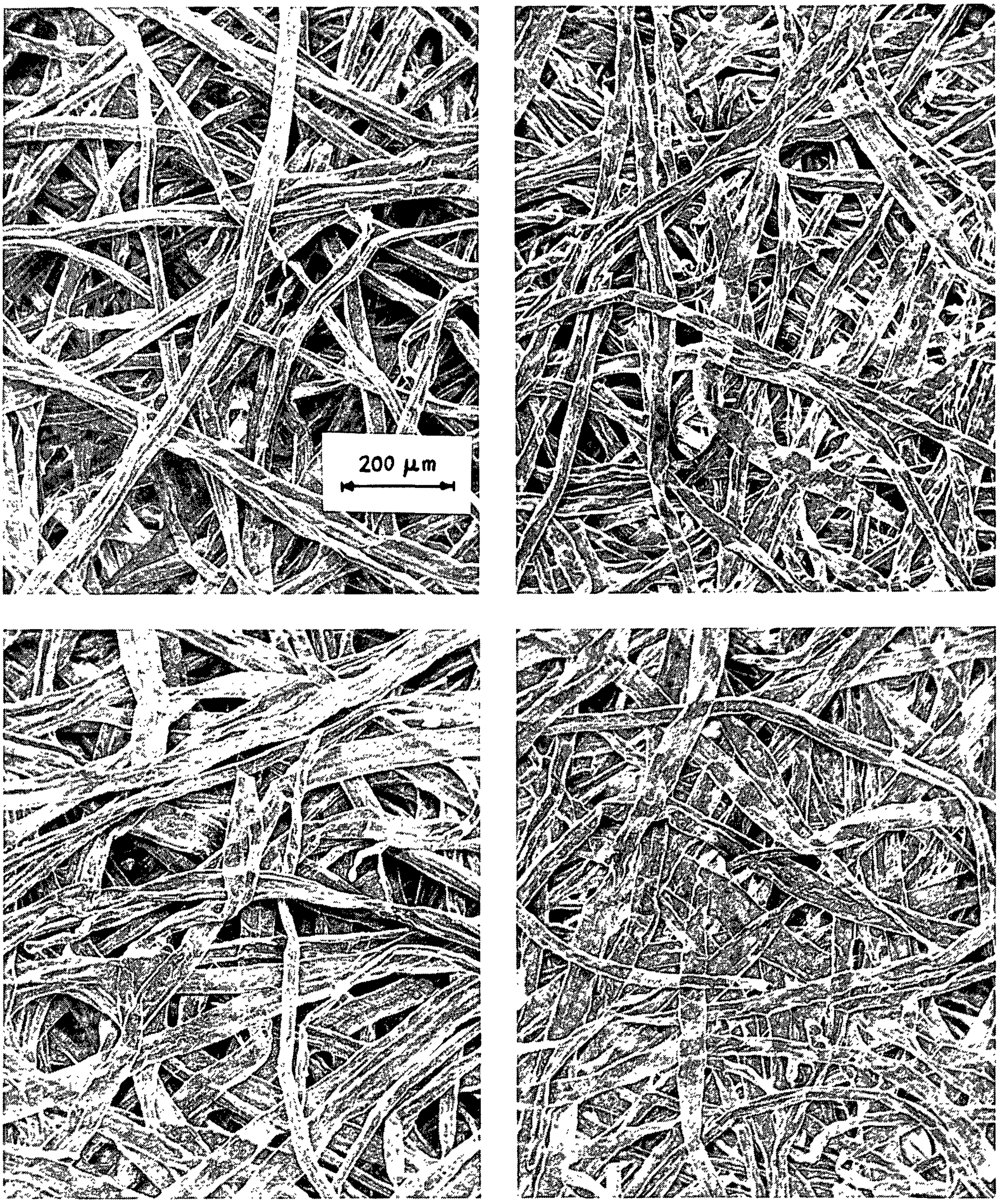

Figure 24. Scanning Electron Micrographs of 1.6 g Handsheets Wet Pressed to 50 psig of 4.13 mm Latewood (Upper Left), 1.74 mm Latewood (Upper Right), 3.94 mm Earlywood (Lower Left), and 1.63 mm Earlywood Fibers (Lower Right)

90 cm H_2O than do populations of thick-walled latewood fibers of comparable length, and $\underline{R}$ increases as fiber length decreases for both fiber populations. Once again, whole pulp data (Appendix VI) falls between the earlywood and latewood planes.

TABLE XI

COMPARISON OF FIBER MORPHOLOGY AND FILTRATION RESISTANCE AT CONSTANT MAT DENSITY ($\underline{c}$ = 0.10 g/cm^3)

Mean Fiber Length, mm	ΔP_f[a], cm H_2O	$\langle R \rangle \times 10^{-6}$, cm/g	$\langle R \rangle / \Delta P_f$, cm^3/(g-dynes)
		Earlywood	
1.63	49.1	30.2	627
3.05	56.2	21.4	388
3.94	60.2	13.1	222
		Latewood	
1.74	61.5	17.1	284
2.07	66.3	13.5	208
2.98	70.7	10.3	149
4.13	70.1	7.4	108
		Whole Pulp	
1.49	63.8	22.7	363
2.76	74.7	13.7	187
3.88	71.0	9.4	135
Not class.	74.3	18.0	247

[a]Calculated from $c = M\Delta P_f^N$.

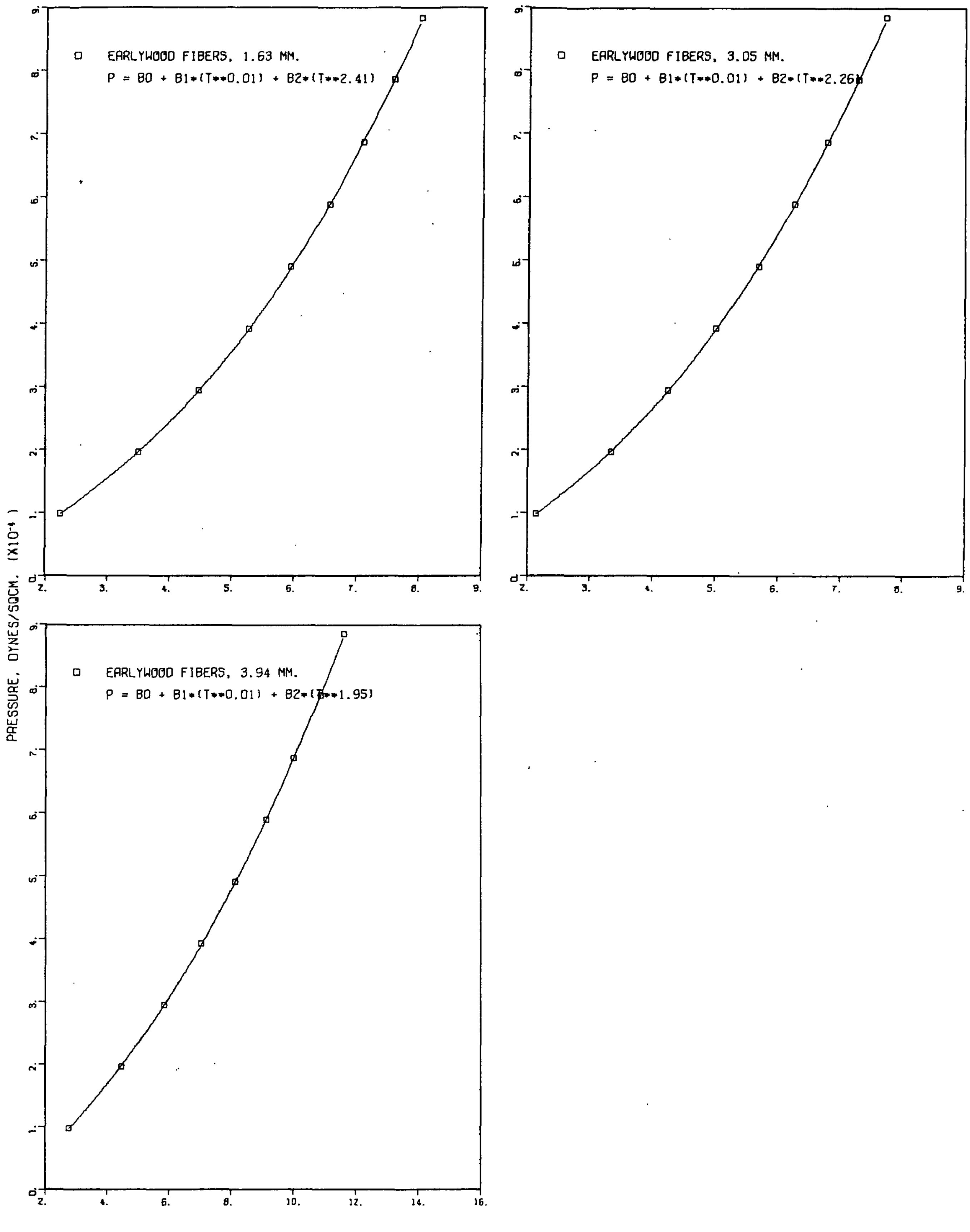

Figure 25. Averaged Pressure, $\underline{\Delta P_f}$, vs. Time, $\underline{t}$, Data for Earlywood Fiber Populations Fitted with a Curve of the Form

$$\underline{\Delta P_f} = \underline{b_0} + \underline{b_1}\underline{t}^{\underline{m}} + \underline{b_2}\underline{t}^{\underline{n}}$$

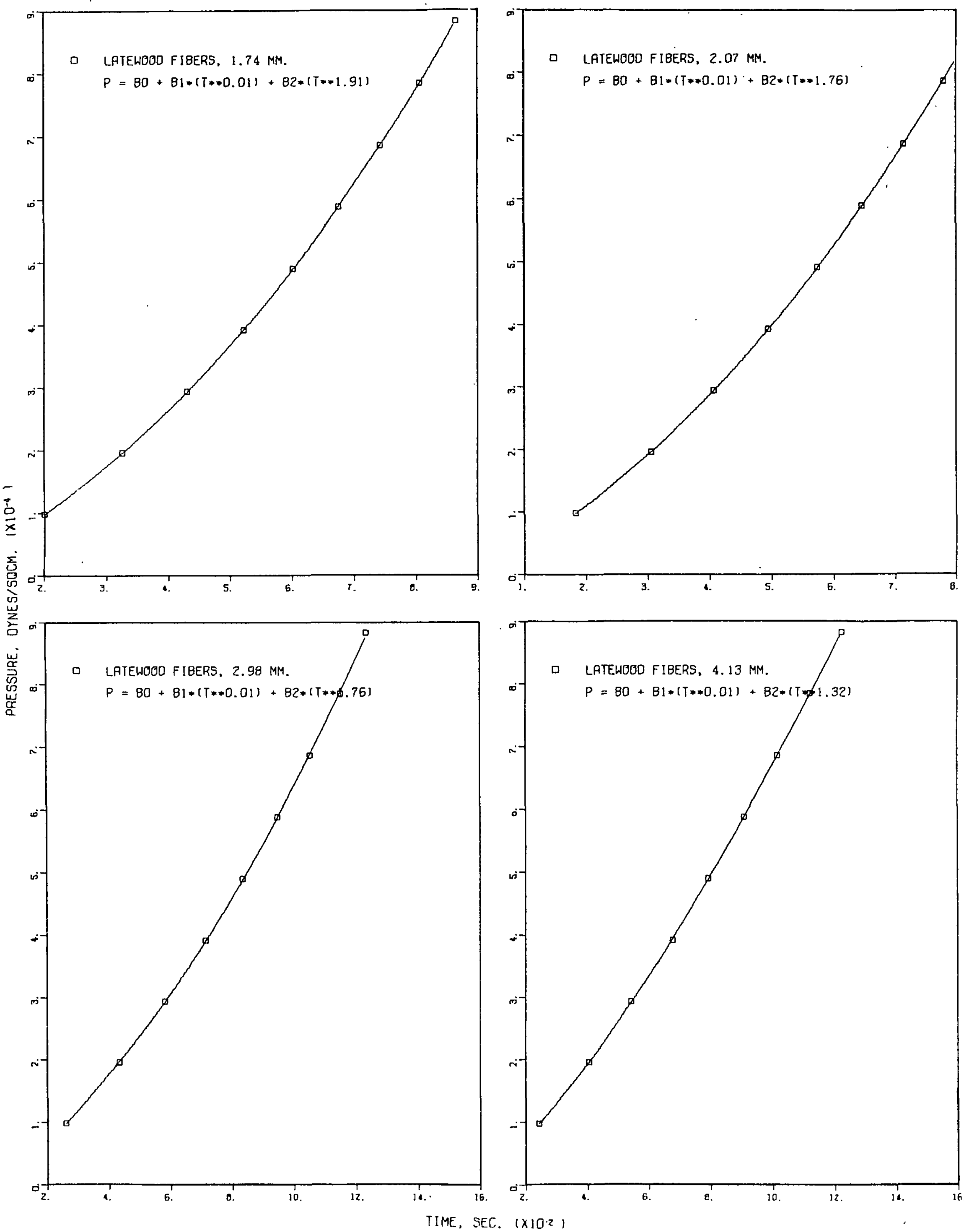

Figure 26. Averaged Pressure, ΔP_f, vs. Time, t, Data for Latewood Fiber Populations Fitted with a Curve of the Form

$$\Delta P_f = b_0 + b_1 t^m + b_2 t^n$$

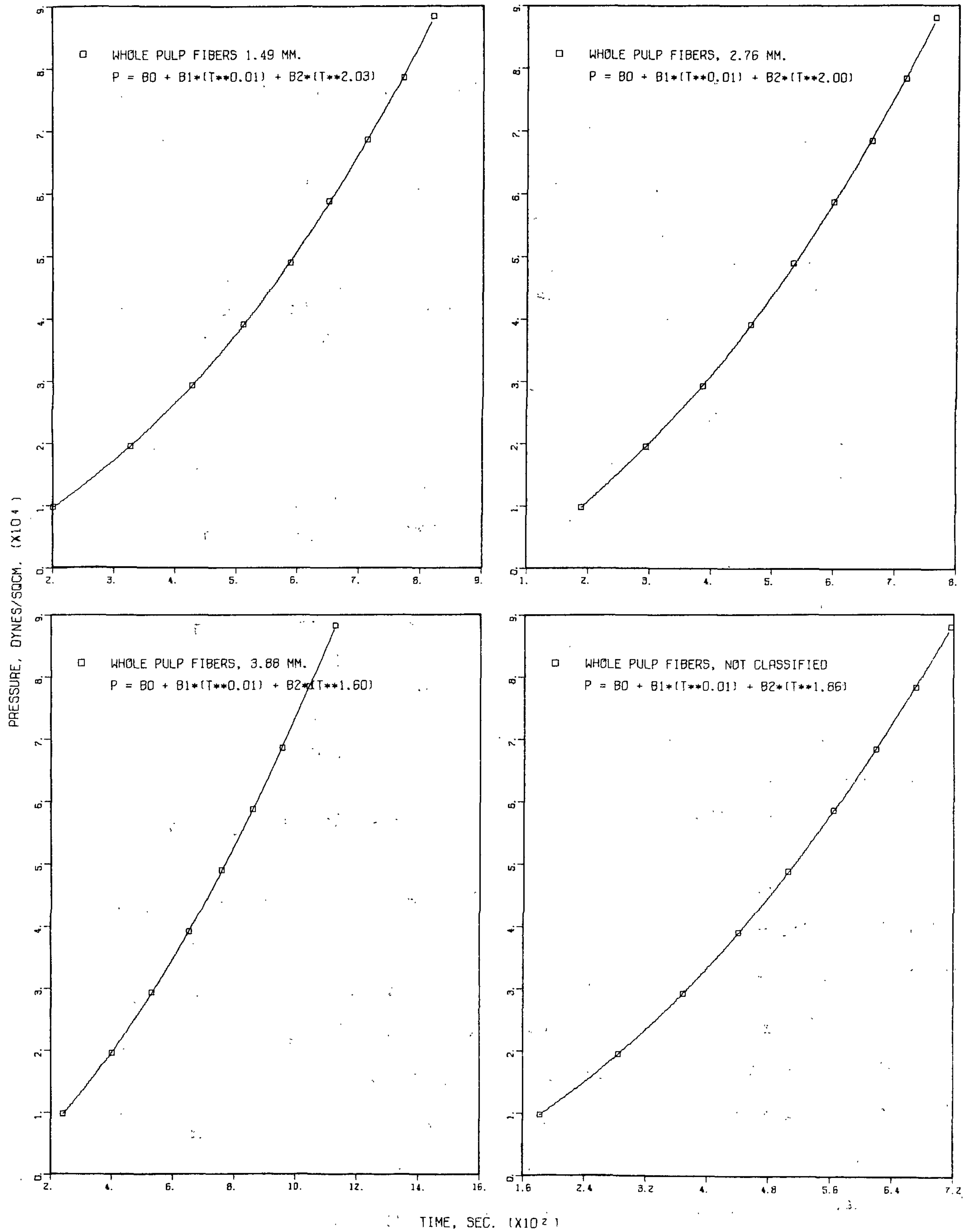

Figure 27. Averaged Pressure, ΔP_f, vs. Time, t, Data for Whole Pulp Fiber Populations Fitted with a Curve of the Form

$$\Delta P_f = b_0 + b_1 t^m + b_2 t^n$$

TABLE XII

EMPIRICAL CONSTANTS USED IN FILTRATION ANALYSIS[a]

Fiber Length	b_0	b_1	b_2	m	n
		Earlywood			
1.63	-777440	742610	0.0074	0.01	2.41
3.05	-641980	613810	0.0219	0.01	2.26
3.94	-637350	607420	0.0758	0.01	1.95
		Latewood			
1.74	-561620	537580	0.1896	0.01	1.91
2.07	-394920	379010	0.5700	0.01	1.76
2.98	-584210	557390	0.2644	0.01	1.76
4.13	132160	-126080	7.6378	0.01	1.32
		Whole Pulp			
1.49	-511760	490500	0.0927	0.01	2.03
2.76	-789060	753910	0.1245	0.01	2.00
3.88	-230480	220940	1.0664	0.01	1.60
Not class.	-408810	391560	0.3787	0.01	1.86

[a] $\Delta P_f = b_0 + b_1 t^m + b_2 t^n$, where ΔP_f = pressure drop and t = time.

Although variations in R with morphology are similar to those previously described for $\langle R \rangle$, variations in R with pressure are significantly different. Comparison of Fig. 23 and 28 reveal that at all pressures in the range studied, R is greater than $\langle R \rangle$, and the difference between these filtration resistance values increases with increasing pressure. Both figures show the increase in filtration resistance with pressure is greater for earlywood (in comparison to latewood fibers of comparable length) and short fibers (in comparison within sets), with the greatest increase being for the 1.63-mm earlywood fiber population;

however, these increases are much more pronounced in values for R where pressure effects are localized and not averaged throughout the mat.

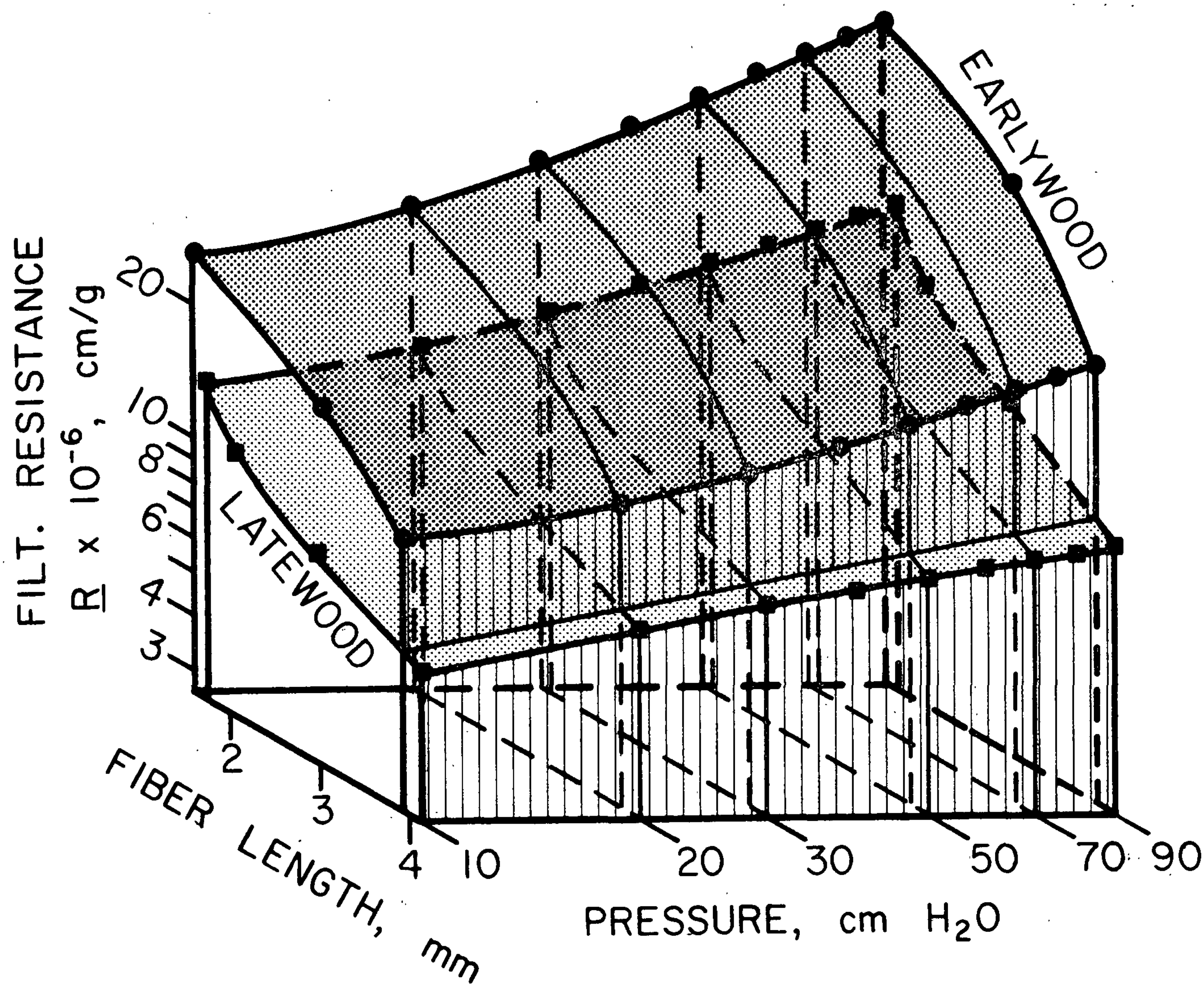

Figure 28. Log-log Plots of Local Specific Filtration Resistance vs. Pressure with Mean Fiber Length of Earlywood and Latewood Plotted Linearly on the z-Axis

FEASIBILITY OF DETERMINING S_W AND v AS FUNCTIONS OF PRESSURE

In the review section on hydrodynamic evaluation of pulp, Equation (29) was developed; it is generally used to calculate the average hydrodynamic specific surface and swollen volume. The solution procedure involves rectification (linear interpretation) of a curved plot of $\Delta P_f/(c^{1/2}t)$ vs. c^3. However, this procedure has been questioned (57), and as an alternative, Meyer (41) presented a procedure for determining $<v>$ and $<S_W>$ from Equation (29) which recognizes the physical phenomena of fiber deswelling with pressure. When the plot of $\Delta P_f/(c^{1/2}t)$ vs. c^3 was analyzed over narrow ranges of c^3 with the aid of Lagrange interpolation, values of $<v>$ were observed to decrease with increasing pressure whereas $<S_W>$ slightly increased (41,58).

As a second alternative, Meyer (42) solved Equation (19) in terms of pressure changes within a permeated mat, thereby establishing a means of eliminating the objectionable integration process in which S_W and v must be assumed constant. Equation (46) is the resulting equation in rectified form:

$$F = S_W^2\, v^{-1/2}\, (1 + 57\, v^3\, X) \qquad (46)$$

where F and X represent complex functions of experimentally measureable quantities involving pressure and mat density (45). A second equation was obtained by some authors (45,59) by taking the derivative of F with respect to X. In doing so it was assumed that S_W and v were insensitive to pressure over very narrow ranges. Accordingly, dS_W/dX and dv/dX were taken as zero and the resulting equations solved with the aid of quadratic smoothing. It was later recognized by the authors that the method was invalid (60), and the work was discontinued.

The advantage of these procedures was that they recognized the pressure dependence of S_W and v brought about through fiber conformability and deswelling. Their major drawback, however, was the inherent assumptions which cannot be justified but are necessary for solution. This section briefly describes refinements developed to improve these calculation procedures, analyzes the inherent assumptions involved in their calculation, and thus presents the basis for rejecting them in favor of adopting the method presented by Ingmanson and Andrews (40) for determining average values of specific surface and volume.

REFINEMENT OF CALCULATION PROCEDURES

As mentioned above, the curved plots of $\Delta \underline{P}_{\underline{f}}/(\underline{c}^{1/2}\underline{t})$ vs. $\underline{c}^3$ from Equation (29) and $\underline{F}$ vs. $\underline{X}$ from Equation (46) were analyzed using Lagrange interpolation and quadratic smoothing to determine values of specific surface and swollen volume as functions of pressure. Both of these procedures, however, are cumbersome. Prior to the realization that these procedures do not yield meaningful results, they were refined using the procedure for numerical differentiation presented earlier. Through computerization of these refined solution procedures for Equations (29) and (46), values of specific surface and swollen volume were obtained as functions of pressure which were in qualitative agreement with those presented previously (41,45,58,59). The computer programs describing the calculation procedure are presented with the results in Appendices VII and VIII, respectively.

ANALYSIS OF ASSUMPTION IN DETERMINING $\underline{S}_{\underline{W}}$ AND $\underline{v}$ AS FUNCTIONS OF PRESSURE

The above methods of analyzing filtration and permeation data were once considered the best procedures for determining $\underline{S}_{\underline{W}}$ and $\underline{v}$ for wood pulp fiber systems. But recently, Grace (61) and Nelson (53) have criticized the procedure involving constant rate filtration on the basis that it involved an unjustifiable assumption which had no physical significance. The assumption results from solving one basic equation containing two unknowns without additional information. The permeation procedure which also solves one equation with two unknowns is subject to a similar assumption. The mathematical argument leading to the exposure of the filtration assumption is presented in Appendix IX.

Clearly then, in order to establish the dependence of $\underline{S}_{\underline{W}}$ and $\underline{v}$ on pressure, a second independent relationship is needed. Such a relationship is not now available. Therefore, the procedure presented by Ingmanson and Andrews (40) which determines a single average value for surface and swollen volume from constant rate filtration and compressibility data was adopted for this study.

As already mentioned, this procedure involves drawing the best straight line through a curved plot of $\Delta P_f/(c^{1/2}t)$ vs. c^3 as shown in Fig. 29. Values for $\langle S_W \rangle$ were observed to be relatively insensitive to the exact positioning of this line. Variations observed were insignificant compared with changes in $\langle S_W \rangle$ between the fiber populations isolated. However, values for $\langle v \rangle$ were observed to be very sensitive to positioning of the best fit line.

To ensure unbiased results, linear regression was used to position the best straight line through the data for values of c^3 corresponding to ΔP_f = 50 to 90 cm H_2O (Fig. 29). This pressure range corresponds to the approximately linear portion of the data.

Changes in $\langle S_W \rangle$ and $\langle v \rangle$ with wall fraction and fiber length using this calculation procedure are presented in the next section.

FIBER MORPHOLOGY AND AVERAGE SPECIFIC SURFACE AND VOLUME

Average specific surface, $\langle S_W \rangle$, and average specific volume, $\langle v \rangle$, were determined by graphical solution of Equation (29) involving linear regression of a plot of $\Delta P_f/(c^{1/2}t)$ vs. c^3 from ΔP_f = 50 to 90 cm H_2O, utilizing filtration resistance and compressibility data. Details of the solution procedure are presented in the computer program of Appendix V. Results are included in Appendix VI and Table XIII.

Values for $\langle S_W \rangle$ are slightly lower than those reported previously for unbeaten bleached southern pine kraft (31). The difference is attributed to the lack of

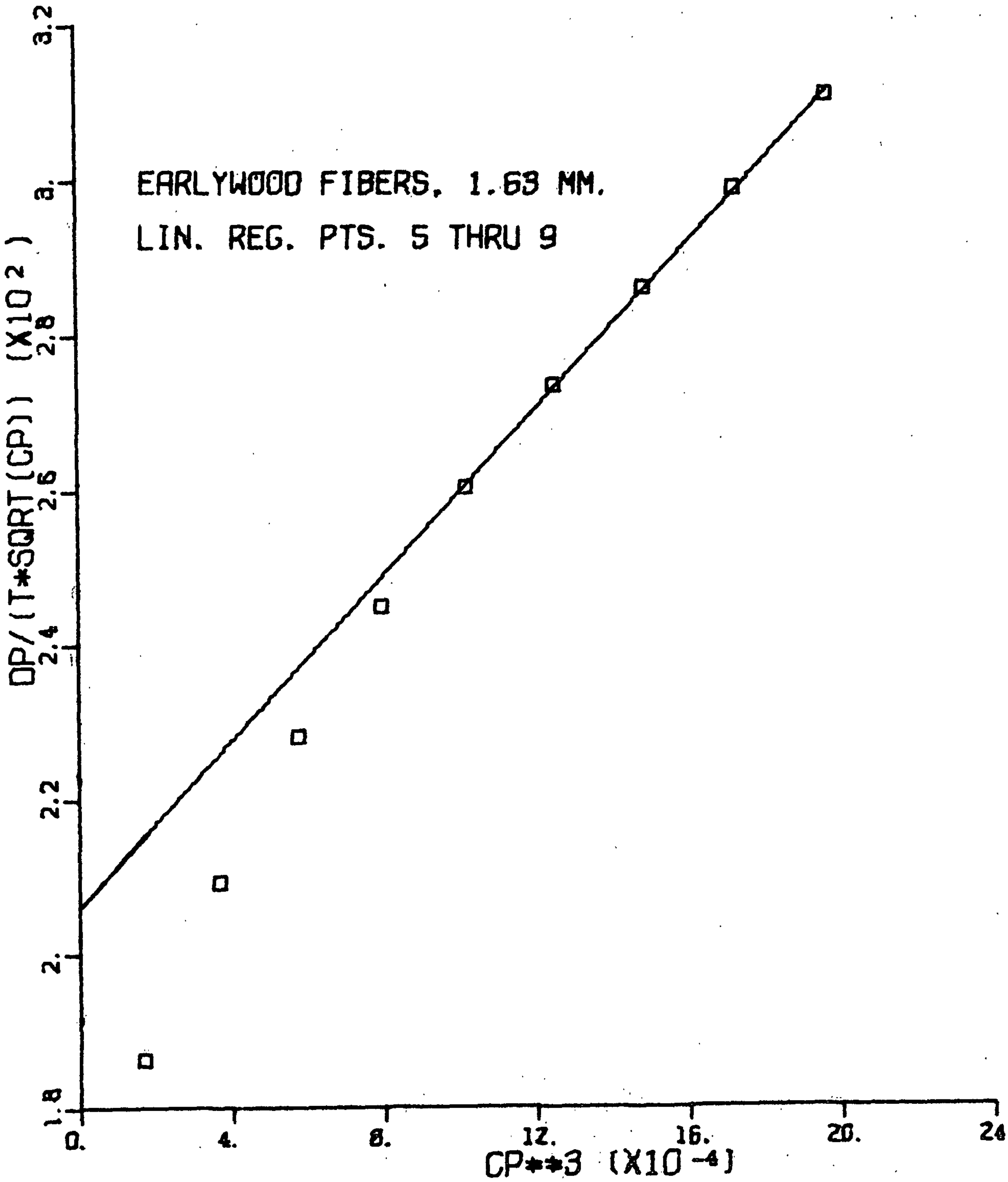

Figure 29. Experimental Plot of $\Delta \underline{P}_{\underline{f}}/(\underline{c}^{1/2}\underline{t})$ vs. $\underline{c}^3$

fines material in these highly classified pulp fiber populations. Values for $<\underline{v}>$ are within the usual range of 1 to 4 cm^3/g observed for filtration data (32).

TABLE XIII

CORRELATION OF MORPHOLOGICAL VARIATION WITH AVERAGE SPECIFIC SURFACE, $<\underline{S}_W>$, AND AVERAGE SPECIFIC VOLUME, $<\underline{v}>$

Mean Fiber Length, mm	$<\underline{v}>$, cm^3/g	τ_1 [b]	τ_2	$<\underline{R}>/<\underline{S}_W>^2$ [b]	$<\underline{S}_W> \times 10^{-3}$ cm^2/g	$\underline{S}_{WG} \times 10^{-3}$ cm^2/g
			Earlywood			
1.63	2.50 ± 0.03[a]	0.57	0.15	0.72	6.50 ± 0.09[a]	4.39 (7.35)[c]
3.05	2.59 ± 0.02	0.55	0.13	0.68	5.44 ± 0.13	5.89
3.94	2.30 ± 0.04	0.57	0.09	0.66	4.28 ± 0.13	5.71
			Latewood			
1.74	2.27 ± 0.05	0.57	0.09	0.66	4.89 ± 0.21	4.03
2.07	2.19 ± 0.06	0.58	0.07	0.65	4.34 ± 0.11	3.77
2.98	2.11 ± 0.19	0.58	0.06	0.64	3.78 ± 0.04	3.72
4.13	1.66 ± 0.25	0.66	0.03	0.69	3.14 ± 0.09	3.29
			Whole Pulp			
1.49	2.48 ± 0.03	0.55	0.10	0.65	5.62 ± 0.01	--[d]
2.76	2.47 ± 0.05	0.53	0.09	0.62	4.34 ± 0.01	--
3.88	2.08 ± 0.05	0.58	0.06	0.64	3.61 ± 0.06	--
Not class.	2.52 ± 0.03	0.53	0.09	0.62	5.00 ± 0.05	--

[a] 95% Confidence limits.

[b] $$\frac{<\underline{R}>}{<\underline{S}_W>^2} = \frac{3.5(1-N/2)\underline{c}^{1/2}}{<\underline{v}>^{1/2}}\left[1 + 57 <\underline{v}>^3(1-N/2)^6\underline{c}^3\right] = \tau_1 + \tau_2.$$

[c] From Table VII.

[d] Values for $\underline{d}_f$ and $\underline{n}_f$ not available.

AVERAGE SPECIFIC SURFACE

A log-log plot of average specific filtration resistance, <$\underline{R}$>, vs. average specific surface, <$\underline{S}_W$>, is presented for earlywood, latewood, and whole pulp fiber populations in Fig. 30. The figure shows that there is an essentially linear relationship between log <$\underline{R}$> and log <$\underline{S}_W$> which agrees with results presented previously (26). The slope of the line connecting the data equals 2.0 and implies that in Equation (45) the sum of the terms containing $\underline{c}$ and <$\underline{v}$> does not vary significantly for the various fiber fractions.

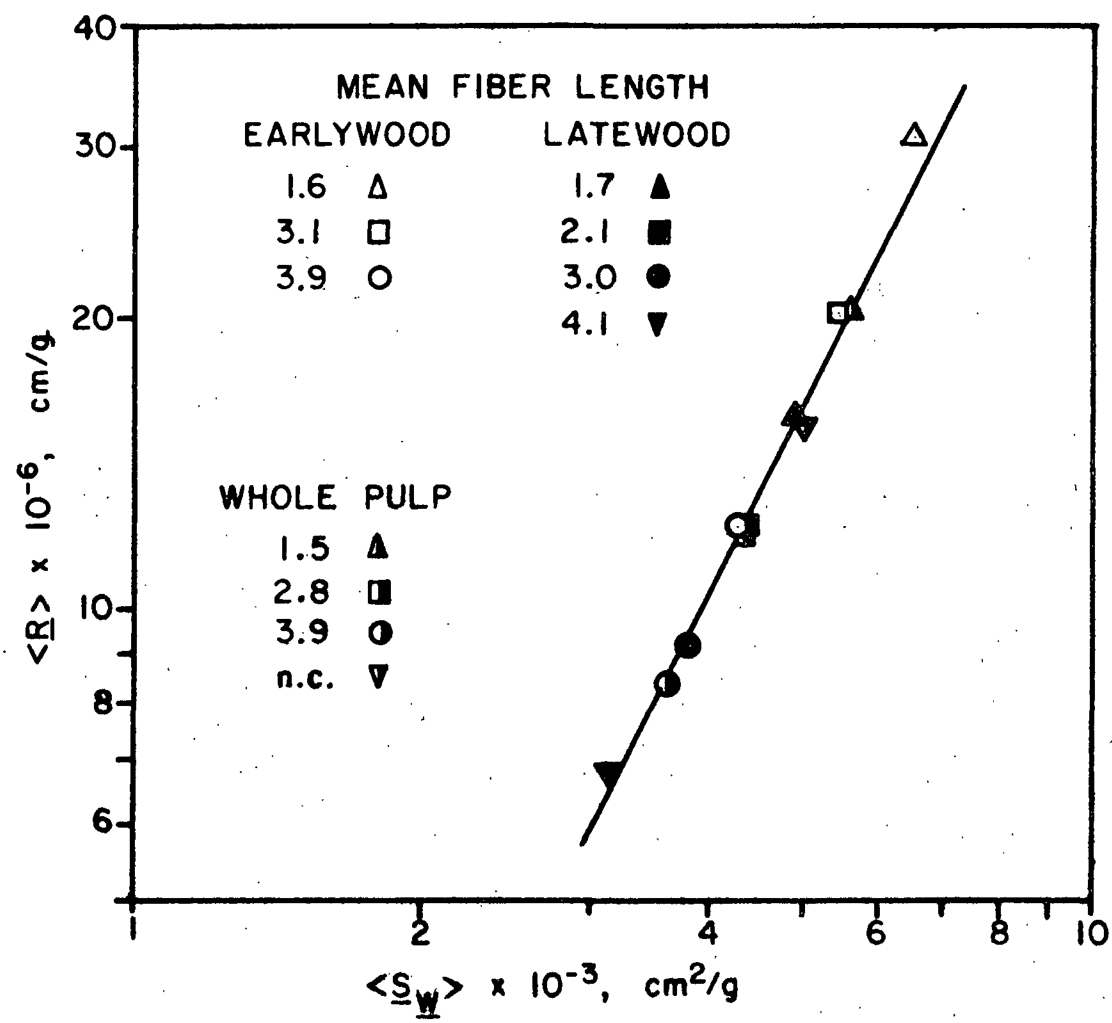

Figure 30. Plot of Average Specific Filtration Resistance, <$\underline{R}$>, vs. Average Specific Surface, <$\underline{S}_W$>, with Slope of Line = 2.0 at $\Delta \underline{P}_f$ = 50 cm H_2O

The values for each term, τ_1 and τ_2 are shown in Table XIII. Although the second term has a clear trend, its value is significantly less than that of the other term, which varies relatively little. Consequently, whatever trend there may be in the sum of the terms, $\langle \underline{R} \rangle / \langle \underline{S}_W \rangle^2$, it is insufficient to cause the slope in Fig. 30 to vary significantly from a value of 2.

Hence, for the various fiber fractions under study, $\langle \underline{R} \rangle$ is essentially a reflection of $\langle \underline{S}_W \rangle^2$ and is relatively insensitive to changes in mat density and specific volume. Therefore, variation in $\langle \underline{S}_W \rangle$ with fiber morphology will be relatively the same as those previously described for $\langle \underline{R} \rangle$.

The tendency of the longest latewood fibers (4.13 mm fraction) to resist collapse, as shown in Fig. 24, indicates that aqueous suspensions of these fibers compared with those in other fractions probably would be closest in behavior to fibers with circular cross section. This is supported by the similarity in the corresponding hydrodynamic, $\langle \underline{S}_W \rangle$, and geometric, $\underline{S}_{WG}$, specific surface data in Table XIII. Changes in morphology, such as in more flattened cross section (Fig. 24) and significantly more broken fibers with open ends (Table V) for the shortest compared with the longest fibers, are believed to result in the greater differences between the corresponding values for $\langle \underline{S}_W \rangle$ and $\underline{S}_{WG}$ presented in Table XIII.

Thus, the latewood fraction with the greatest mean fiber length, which is probably nearest in behavior to the circular fibers, has a hydrodynamic specific surface closest to the calculated geometric specific surface.

AVERAGE SPECIFIC VOLUME

Table XIV compares the average specific volume, $\langle \underline{v} \rangle$, obtained from filtration resistance data with the geometric volume, $\underline{v}_G$, calculated from the fiber dimensions using a circular cylindrical model (Table VII).

TABLE XIV

COMPARISON OF SWOLLEN VOLUMES

L_f, mm	$\langle v \rangle$, cm^3/g	v_G, cm^3/g	$v_G/\langle v \rangle$	Rectangular Volume[b], cm^3/g
		Earlywood		
1.63	2.50	(7.33)[a]	2.9	(1.38)[a]
3.05	2.59	6.63	2.6	1.37
3.94	2.30	7.16	3.1	1.47
		Latewood		
1.74	2.27	3.26	1.4	1.28
2.07	2.19	3.21	1.5	1.35
2.98	2.11	3.32	1.6	1.40
4.13	1.66	3.04	1.8	1.32

[a]Based on n_{fG}.

[b]Rectangular volume = $2(WT)\, d_f\, L_f\, n_f$.

The values for $\langle v \rangle$ are considerably lower than those for v_G, with the ratios of the two volumes equal to about 3 for earlywood and 1.6 for latewood. The large differences observed between $\langle v \rangle$ and v_G are in part a reflection of the inability of the circular model to account for a decrease in swollen volume due to collapse of the fiber lumen. However, the collapse of the lumen is probably not complete since calculated rectangular volumes in which the fiber is assumed completely collapsed, as in the lower portion of Fig. 14, are lower than corresponding values of $\langle v \rangle$.

The average specific volume, $\langle v \rangle$, of fibers presented in Table XIII is in essence the volume of dry fibers plus their associated immobilized water per gram of dry fiber (31). By subtracting an assumed constant inverse pycnometric density of 0.62 cc/g for dry fibers from the specific volume (29), the increase in <v> in Table XIII can be observed to correspond to an increase in immobilized water of 1.04 to 1.97 cm^3 water per g fiber. Further assuming there is no increase in fiber swelling as found when fibers are beaten (29), it is concluded that most of this increase arises from more water being immobilized in the interstitial regions of the fiber mat as a reflection of changes in fiber morphology, particularly decreases in fiber length (accompanied by decreases in fiber width and wall thickness) and wall fraction.

Specific volume data may also be converted to pulp consistency [consistency = mass dry fibers/(mass of dry fibers + water)]. For specific volume of 1.66 to 2.59 cc/g, the apparent fiber consistency is approximately 49 to 33%, respectively. A consistency of about 33% would be a more credible maximum since fibers at this consistency are to a great extent considered dewatered. This discrepancy is probably a reflection of the limitations of the assumptions made in deriving Equation (29). Nevertheless, a relatively high consistency suggests that the immobile water is not in a free flowing form, but physically entrapped in the micropores and lumens.

CONCLUSIONS

Mechanical separation of unbeaten, bleached loblolly pine kraft pulp resulted in fiber populations of predominantly unbroken fibers which could be characterized by wall fraction and mean fiber length. Furthermore, variation in fiber length within the set of earlywood (low wall fraction) and latewood (high wall fraction) fiber populations correlated with wall thickness, fiber diameter, and number of fibers per gram in a manner similar to that within a tree.

Static compressibility data show the relationship between mat density, $\underline{c}$, and pressure, $\underline{P_f}$, followed the usual power function $\underline{c} = \underline{M}\ \underline{P_f}^{\underline{N}}$ for $\underline{P_f}$ of about 10 to 150 cm H_2O. In this expression the compressibility constant $\underline{N}$ was found to equal 0.373 for earlywood and latewood fiber populations and the compressibility constant $\underline{M}$ correlated with fiber length for all fiber fractions. Compressibility, $d\underline{c}/d\underline{P_f}$, was greatest for shortest earlywood fibers and least for longest latewood fibers. $\underline{M}$ was found to be linearly related to $(1/\underline{I_F})^{1/3}$, where $\underline{I_F}$ represents the moment of inertia for a flattened fiber model, in agreement with the simple compressibility model originally developed by Wilder. The linearity of the relationship supports bending as the dominent mechanism in compressibility of wood pulp, and suggests that the wood pulp fiber is essentially flattened prior to bending. Since $\underline{I_F}$ was defined as $2/3\ (\underline{WT})^3\ \underline{d_f}$, where $\underline{WT}$ is wall thickness and $\underline{d_f}$ fiber diameter, it appears that the wood pulp fiber dimensions influencing compressibility are primarily wall thickness and to a lesser degree fiber diameter. Trends in $\underline{M}$ with changes in mean fiber length of earlywood and latewood fibers tend to follow previously reported changes in elastic modulus of earlywood and latewood in successive growth rings, and also in elastic moduli and fibril angle.

Average specific filtration resistance, $\langle\underline{R}\rangle$, obtained from constant rate filtration data at a given pressure drop, $\Delta\underline{P_f}$, in the range 10 to 90 cm H_2O

also correlated with wall fraction and fiber length. Percentage change in $\langle R \rangle$ with pressure was comparable to that trend observed for c; however, change in $\langle R \rangle$ with L_f was much greater. Smallest earlywood fibers had highest $\langle R \rangle$ values and these increased most with increases in ΔP_f compared with other fiber fractions.

Local specific filtration resistance, R, was calculated from pressure *vs*. time data obtained from constant rate filtration using a newly developed statistical procedure for determining derivatives. Changes in R with fiber wall fraction and length were similar to those found for $\langle R \rangle$, but R values were much higher and increased significantly more with pressure.

The square of average hydrodynamic specific surface, $\langle S_W \rangle^2$, is proportional to $\langle R \rangle$, and this relationship was comparatively insensitive to changes in c and average specific volume, $\langle v \rangle$. Calculated geometric specific surface is closest to $\langle S_W \rangle$ for latewood with greatest fiber length, probably because these fibers most closely approximate circular fibers. Values for $\langle v \rangle$ were considerably lower than calculated values, v_G, using a circular cylindrical model, indicating that the fibers collapse under fluid drag forces. The ratios of the two corresponding volumes was almost 3 for earlywood and about 1.6 for latewood. Data for $\langle v \rangle$ indicate immobilized water varies from 1.04 to 1.97 cc/g with morphological changes. Apparently most of this water is within the fiber walls or uncollapsed lumina and not elsewhere.

IMPLICATIONS OF RESULTS

The results of this study imply that alteration of fiber properties in the chip supply by inclusion of more juvenile wood, whole-tree chips, and/or low or high density logs may alter the compressibility and filtration resistance properties of the unbeaten pulp.

Compressibility and filtration resistance are important parameters in processes involving water flow through pulp fiber mats; and, the experimental conditions used closely approximate conditions of displacement bleaching and washing. Since the displacement process also involves diffusion which was not studied, little can be concluded about the effects of fiber properties on washing efficiency, however, the results of this study do apply to the gross transport of fluid through the mat.

From the text it is apparent that morphological variation which relates to growth within a tree can result in relatively large changes in filtration resistance at 5-10% consistency, which is the practical range encountered in displacement bleaching and washing. As a result, if more juvenile wood were included in a chip supply, filtration resistance would tend to increase, whereas if more mature wood were included, filtration resistance would tend to decrease. A similar argument is valid for low density (more earlywood) vs. high density (more latewood) wood sources.

However, a displacement bleaching process is more precisely a permeation, so that Equations (22) and (23) cannot be used in their present form to relate filtration resistance to the displacement processes. But these equations may be rearranged into the form applicable to permeation:

$$\frac{\langle R \rangle}{\Delta P_f} = \frac{\mu W'}{A^2} \frac{1}{dV/dt} \qquad (47)$$

which defines the volumetric flow rate ($d\underline{V}/d\underline{t}$) in terms of the cross-sectional area, $\underline{A}$, of the bed; fluid viscosity, μ; total mass, $\underline{W}'$, of the fibers in bed; total pressure drop, $\Delta\underline{P}_{\underline{f}}$, across the bed; and average filtration resistance, $\langle\underline{R}\rangle$, (31). For displacement bleaching or washing equipment operating at constant consistency, μ, $\underline{W}'$, and $\underline{A}^2$ are constant so that in Table XI the sixfold change for $\langle\underline{R}\rangle/\Delta\underline{P}_{\underline{f}}$ arising from fiber length and wall fraction variation, results in an inversely proportional change in ($d\underline{V}/d\underline{t}$). Similarly, if a constant flow rate is maintained by changing the drainage area, $\underline{A}$, and since the square of this is proportional to $\langle\underline{R}\rangle/\Delta\underline{P}_{\underline{f}}$, there would be a two- to threefold change in $\underline{A}$ arising from morphological variation.

Thus, if more juvenile wood is utilized, it probably would be necessary in displacement bleaching and washing at constant average consistency to use a lower maximum pressure between the washer head and screen to avoid significant thickening of the mat. In existing equipment the increased filtration resistance, even at lower pressure, would be expected to result in a slower flow of liquor through the pulp mat. For new equipment assuming constant flow rate is maintained, the increased filtration resistance could imply a need for more drainage area at increase in capital cost. When utilizing more mature wood the converse would tend to apply.

A similar argument is applicable to a discussion of high density *vs.* low density wood.

SUGGESTIONS FOR FUTURE WORK

Understanding the effects of fiber morphology on process parameters, such as the hydrodynamic properties of pulp, necessitates use of two major approaches to solving problems: the theoretical approach of "predicting and proving" and the experimental approach of "learning and discovery."

In the theoretical area there remains the definite desire to be able to mathematically predict the changes in hydrodynamic surface and volume of wood pulp resulting from changes in fluid stress using filtration and permeation experiments. Attempts in this area have failed the rigors of being mathematically justifiable.

In the experimental area it would be desirable to extend the correlative aspects of wet mat compressibility and fiber morphology by quantifying the modulus values descriptive of wet fiber collapse and bending. Also, the implications of the research described in this dissertation concerning displacement bleaching and washing would be greatly enhanced with quantitative information correlating rates of diffusion with fiber properties.

LIST OF SYMBOLS

A	= area, empirical constant
B	= filtration constant; empirical constant
b_0, b_1, b_2	= empirical constants
C	= stock consistency, empirical constant
c	= mat density
c_{avg}	= average mat density
c_L	= initial mat density
c_o	= initial mat density
d_f	= fiber diameter
E	= apparent modulus
I	= moment of inertia
I_C	= moment of inertia for circular fiber cross section
I_F	= moment of inertia for flattened fiber cross section
K	= Darcy permeability coefficient
K_n	= constant depending on load distribution
k	= Kozeny factor
k_1, k_2	= constants
L	= pad thickness
L_f	= fiber length
L_n	= free span between two supports
L_s	= segment length
$L_{s,o}$	= initial segment length
M, N	= compressibility constants
M_1, N_1	= initial compressibility constants
m, n	= empirical constants
n	= number of layers in mat

n_c = number of contacts per layer

$n_{c,o}$ = initial number of contacts per layer

n_f = number fibers per gram

n_{fG} = calculated number fibers per gram

P_f = static pressure

P_n = magnitude of total load in bending

ΔP_f = fluid pressure drop

ΔP_L = total pressure drop across the mat

ΔP_z = pressure drop at distance z from top of mat

q = volumetric flow rate

R = local specific filtration resistance

$\langle R \rangle$ = average specific filtration resistance

r_o = outside fiber diameter

r_i = inside fiber diameter

S_v = specific surface per unit volume

$\langle S_v \rangle$ = average specific surface per unit volume

S_W = specific surface per unit mass

$\langle S_W \rangle$ = geometric surface to mass ratio

t = time

U = superficial fluid velocity

V = filtrate volume

V_W = volume of fiber wall

v = specific volume

v_G = geometric volume to mass ratio

$\langle v \rangle$ = average specific volume

W = mass of fibers per unit area

W' = mass of fibers in bed

WT = wall thickness

$\underline{x}$,$\underline{y}$	= defined variables
$\underline{z}$	= mat thickness; distance from top of mat
α,β	= constants associated with load distribution
δ	= deflection
ε	= porosity
θ	= $\underline{S}_2$ fibril angle
μ	= fluid viscosity
$\rho_{\underline{f}}$	= fiber density
$\rho_{\underline{W}}$	= fiber wall density
τ_1	= $3.5 (1 - N/2)\ \underline{c}^{1/2}/\langle\underline{v}\rangle^{1/2}$
τ_2	= $\tau_1\ 57\ \langle\underline{v}\rangle^3\ (1 - N/2)^6\ \underline{c}^3$

ACKNOWLEDGMENTS

Sincere appreciation is expressed to the many people who assisted in the completion of this thesis.

The author is especially grateful for the encouragement and advice of Dr. Gordon A. Nicholls, Thesis Chairman and to Mr. N. L. Chang, Mr. S. T. Han, and Mr. H. Meyer, members of his Advisory Committee.

Special acknowledgments for their advice and assistance are also extended to Mr. B. D. Andrews, Dr. G. A. Baum, Dr. T. M. Grace, Dr. R. W. Nelson, and Mr. V. J. Van Drunen.

The assistance of the following people is also appreciated: Mr. H. Bailey of the Union Camp Corporation for the selection, harvesting, and delivery of a complete loblolly pine; Mr. M. C. Filz, Jr. and Mr. P. F. Van Rossum, whose skillful construction of much of the equipment used in this study greatly contributed to the success of this investigation; Mr. J. D. Hankey, Mrs. H. M. Kaustinen, and Dr. R. A. Parham for their assistance in microscopy; Mr. J. J. Bachhuber and Mr. J. O. Church for their assistance in computer programming; Mr. D. E. Beyer, Mr. C. H. Schabo, and Mr. F. R. Sweeney for their assistance in photographing and reducing the many figures; and Mr. H. J. Grady, Mr. O. C. Kuehl, and Mr. J. Tierney for their assistance on special problems.

LITERATURE CITED

1. Einspahr, D. W., Tappi 47(4):180(1964).

2. Einspahr, D. W., van Buijtenen, J. P., and Peckham, J. R., Silvae Genet. 18(3):57(1969).

3. Barefoot, A. C., Hitchings, R. G., Ellwood, E. L., Wilson, E. H. The relationship between loblolly pine fiber morphology and kraft paper properties. North Carolina Agr. Expt. Sta. Tech. Bull. No. 202, Dec., 1970. 89 p.

4. van Buijtenen, J. P., Alexander, S. D., Einspahr, D. W., Ferrie, A. E., Hart, T., Kellog, R. M., Porterfield, R. L., and Zobel, B. J., Tappi 58(9): 129(1975).

5. Jones, R. L. An investigation of the effect of fiber structural properties on the compression response of fibrous beds. Doctoral Dissertation, Appleton, Wisconsin, The Institute of Paper Chemistry, 1962. Tappi 46(1):20(1963).

6. Elias, T. C. An investigation of the compression response of ideal unbonded fibrous structures by direct observation. Doctoral Dissertation, Appleton, Wisconsin, The Institute of Paper Chemistry, 1965. Tappi 50(3):125(1967).

7. Bliesner, W. C. A study of the porous structure of fibrous sheets using permeability techniques. Doctoral Dissertation, Appleton, Wisconsin, The Institute of Paper Chemistry, 1963. Tappi 47(7):392(1964).

8. Labrecque, R. P. An investigation of the effects of fiber cross-sectional shape on the resistance to the flow of fluids through fiber mats. Doctoral Dissertation, Appleton, Wisconsin, The Institute of Paper Chemistry, 1967. Tappi 51(1):8(1968).

9. Han, S. T., Pulp Paper Mag. Can. 70(9):T134(1969).

10. Hardacker, K. W. The physics and chemistry of wood pulp fibers. TAPPI/STAP No. 8:201(1970).

11. Alexander, S. D., Marton, R., and McGovern, S. D., Tappi 51(6):277(1968).

12. Leopold, B. and Thorpe, J. L., Tappi 51(7):304(1968).

13. Yiannos, P. N. and Taylor, D. L., Tappi 50(1):40(1967).

14. Hartler, N. and Nyren, J. TAPPI Paper Physics Seminar, "The physics and chemistry of wood pulp fiber." p. 76. Appleton, Wisconsin, May, 1969. Tappi 53(5):820(1970).

15. Gren, U. B., Svensk Papperstid. 75(19):785(1972).

16. Andrews, B. D. and Nicholls, G. A., Appita 27(6):411(1974).

17. Gullichsen, J., Personal communication.

18. Jacquelin, G., Tech. Rech. Pepet. 7:22(1966).

19. Chang, N. L. and Meyer, H., Unpublished work, Appleton, Wisconsin, The Institute of Paper Chemistry, 1967.

20. Nicholls, G. A. and Jamieson, R. G., Appita 29(5):339(1976).

21. Wardrop, A. B. and Dadswell, H. E., Holzforschung 7(2/3):33(1953).

22. McMillin, C. W., Wood Sci. Technol. 2(3):166(1968).

23. Panshin, A. J. and deZeeuw, C. Textbook of wood technology. Vol. I. Chap. 7. New York, McGraw Hill Book Company, 1970.

24. Mark, R. E. Cell wall mechanics of tracheids. Chap. 7. p. 168-170. New Haven and London, Yale Univ. Press, 1967.

25. Cowdry, D. R. and Preston, R. D. In Cote's The cellular ultrastructure of woody plants. Syracuse Univ. Press, 1965.

26. Han, S. T. Hydrodynamic evaluation of fiber surface area and swollen volume. A critical review. Appleton, Wisconsin, The Institute of Paper Chemistry, 1967.

27. Han, S. T. The status of the sheet-forming process. A critical review. Appleton, Wisconsin, The Institute of Paper Chemistry, 1965.

28. Kesler, R. B., Unpublished work, Appleton, Wisconsin, The Institute of Paper Chemistry, 1969.

29. Ingmanson, W. L. and Andrews, B. D., Tappi 42(1):29(1959).

30. Campbell, W. B., Pulp Paper Mag. Can. 48(3):103(1947).

31. Ingmanson, W. L. and Whitney, R. P., Tappi 37(11):523(1954).

32. Ingmanson, W. L., Tappi 35(10):439(1952).

33. Wilder, H. D. The compression creep properties of wet pulp mats. Doctoral Dissertation, Appleton, Wisconsin, The Institute of Paper Chemistry, 1960. Tappi 43(8):715(1960).

34. Ingmanson, W. L., Chem. Eng. Progr. 49(11):577(1953).

35. Whitney, R. P., Ingmanson, W. L., and Han, S. T., Tappi 38(3):157(1955).

36. Kozeny, J., Sitzber. Akad. Wiss. Wein, Math. Naturw. Kl., 136(Abt. IIa):271 (1927).

37. Carman, P. C., Trans. Inst. Chem. Eng. (London) 15:150(1937).

38. Davies, C. N., Proc. Inst. Mech. Engr. (London) 1B:185(1952).

39. Ingmanson, W. L., Andrews, B. D., and Johnson, R. C., Tappi 42(10):840(1959).

40. Ingmanson, W. L. and Andrews, B. D., Tappi 46(3):150(1963).

41. Meyer, H., Tappi 52(9):1716(1969).

42. Meyer, H., see Ref. 9.

43. Chang, N. L., Unpublished work, Appleton, Wisconsin, The Institute of Paper Chemistry, 1967.

44. Illvessalo-Pfaffli, M. S. and Alfthan, G. V., Paperi Puu 39(11):509(1957).

45. Chang, N. L. and Han, S. T., Unpublished work, Appleton, Wisconsin, The Institute of Paper Chemistry, 1969.

46. Jacquelin, G. Cohesion of papermaking fibrous networks in aqueous medium. ATIP Rev. 20:4-153(1966).

47. Jacquelin, G. Cohesion of fibrous networks in suspension. Influence of physico-chemical factors. C.T.P. Document No. 322, 1968.

48. Jacquelin, G. Cohesion of fibrous networks in suspension — the influence of physico-chemical factors. ATIP Rev. 23:5-293(1969).

49. Dinwoodie, J. M., Tappi 48(8):440(1968).

50. Horn, R. A. and Coens, C. L., Tappi 53(11):2120(1970).

51. Koch, P. Utilization of the southern pines. Chap. 7. p. 235-264. U.S. Dept. Agri. Forest Serv., 1972.

52. Seborg, C. O. and Simmonds, F. A., Paper Trade J. 113(17):49(Oct. 23, 1941).

53. Nelson, R. W., Personal communication, 1976.

54. Draper, N. R. and Smith, H. Applied regression analysis. p. 134. New York, Wiley, 1966.

55. Jensen, C. E. Matchcurve-3: Multiple-component and multidimensional mathematical models for natural resource studies. USDA Forest Serv. Res. Paper INT-146, Dec., 1973.

56. Nelson, R. W., Personal communication, 1975.

57. Meyer, H., Tappi 45(4):292(1962).

58. Chang, N. L., Tappi 52(5):923(1969).

59. Chang, N. L. and Han, S. T., Unpublished work, Appleton, Wisconsin, The Institute of Paper Chemistry, 1969.

60. Chang, N. L., Personal communication, 1976.

61. Grace, T. M., Personal communication, 1976.

62. Onagi, S. and Sasaguri, K., Tappi 44(12):874(1961).

APPENDIX I

ORIGINAL MAT DENSITY DATA

MAT DENSITY VS. PRESSURE FOR EARLYWOOD FIBERS

Earlywood — 1.63 mm

P_f, dynes/cm^2	c, g/cm^3
0.8150000D 04	0.5279918D-01
0.1310000D 05	0.6238218D-01
0.2290000D 05	0.7638610D-01
0.3770000D 05	0.9142408D-01
0.6230000D 05	0.1102416D 00
0.9640000D 05	0.1296117D 00
0.1458000D 06	0.1510779D 00
0.5169705D 04	0.4478318D-01
0.9960845D 04	0.5571480D-01
0.1968604D 05	0.7080314D-01
0.3433145D 05	0.8616340D-01
0.5881047D 05	0.1058376D 00
0.9278004D 05	0.1248414D 00
0.1418500D 06	0.1470713D 00
0.5156455D 04	0.4529720D-01
0.9948472D 04	0.5655541D-01
0.1981460D 05	0.7184901D-01
0.3432347D 05	0.8729677D-01
0.5880528D 05	0.1064986D 00
0.9277478D 05	0.1261844D 00
0.1418447D 06	0.1493995D 00

Earlywood — 3.05 mm

P_f, dynes, cm^2	c, g/cm^3
0.8150000D 04	0.4927090D-01
0.1310000D 05	0.5878155D-01
0.2290000D 05	0.7314638D-01
0.3770000D 05	0.8805862D-01
0.6230000D 05	0.1061714D 00
0.9640000D 05	0.1249350D 00
0.1458000D 06	0.1453122D 00
0.8150000D 04	0.4959801D-01
0.1310000D 05	0.5923686D-01
0.2290000D 05	0.7371080D-01
0.3770000D 05	0.8883135D-01
0.6230000D 05	0.1072566D 00
0.9640000D 05	0.1257618D 00
0.1458000D 06	0.1467571D 00
0.5431371D 04	0.3764577D-01
0.1012575D 05	0.5207957D-01
0.1981124D 05	0.6779796D-01
0.3443178D 05	0.8378922D-01
0.5889488D 05	0.1026239D 00
0.9285028D 05	0.1222349D 00
0.1419107D 06	0.1438070D 00
0.5419669D 04	0.3785901D-01
0.1010973D 05	0.5309019D-01
0.1980044D 05	0.6874349D-01
0.3442341D 05	0.8478941D-01
0.5888843D 05	0.1036400D 00
0.9284809D 05	0.1217777D 00
0.1419099D 06	0.1425072D 00

Earlywood — 3.94 mm

P_f, dynes/cm^2	c, g/cm^3
0.8150000D 04	0.5071700D-01
0.1310000D 05	0.6049166D-01
0.2290000D 05	0.7437724D-01
0.3770000D 05	0.8902608D-01
0.6230000D 05	0.1063683D 00
0.9640000D 05	0.1237433D 00
0.1458000D 06	0.1432001D 00
0.8150000D 04	0.4946601D-01
0.1310000D 05	0.5904757D-01
0.2290000D 05	0.7274701D-01
0.3770000D 05	0.8706051D-01
0.6230000D 05	0.1042859D 00
0.9640000D 05	0.1214908D 00
0.1458000D 06	0.1403827D 00
0.5866204D 04	0.4026533D-01
0.1054850D 05	0.5100782D-01
0.2019556D 05	0.6324006D-01
0.3475199D 05	0.7906226D-01
0.5915559D 05	0.9898162D-01
0.9307910D 05	0.1168853D 00
0.1421138D 06	0.1359893D 00
0.5758152D 04	0.3989485D-01
0.1046561D 05	0.4969367D-01
0.2011198D 05	0.6267705D-01
0.3466943D 05	0.7997459D-01
0.5909394D 05	0.9838845D-01
0.9302544D 05	0.1160990D 00
0.1420649D 06	0.1356771D 00

MAT DENSITY VS. PRESSURE FOR LATEWOOD FIBERS

Latewood — 1.74 mm

P_f, dynes/cm^2	c, g/cm^3
0.8150000D 04	0.5125400D-01
0.1310000D 05	0.6088509D-01
0.2290000D 05	0.7382238D-01
0.3770000D 05	0.8787152D-01
0.6230000D 05	0.1049588D 00
0.9640000D 05	0.1222571D 00
0.1458000D 06	0.1416354D 00
0.5496903D 04	0.3963738D-01
0.1025403D 05	0.4803851D-01
0.1989331D 05	0.6550495D-01
0.3450451D 05	0.8019796D-01
0.5895846D 05	0.9753138D-01
0.9290722D 05	0.1152214D 00
0.1419612D 06	0.1348582D 00
0.5479884D 04	0.3969656D-01
0.1021458D 05	0.5016990D-01
0.1988484D 05	0.6513779D-01
0.3449640D 05	0.7993317D-01
0.5895147D 05	0.9723933D-01
0.9290153D 05	0.1147378D 00
0.1419547D 06	0.1351301D 00

Latewood — 1.07 mm

P_f, dynes/cm^2	c, g/cm^3
0.8150000D 04	0.4889348D-01
0.1310000D 05	0.5818156D-01
0.2290000D 05	0.7141021D-01
0.3770000D 05	0.8522575D-01
0.6230000D 05	0.1021106D 00
0.9640000D 05	0.1189648D 00
0.1458000D 06	0.1373969D 00
0.8150000D 04	0.4876813D-01
0.1310000D 05	0.5780539D-01
0.2290000D 05	0.7182054D-01
0.3770000D 05	0.8554811D-01
0.6230000D 05	0.1021113D 00
0.9640000D 05	0.1188058D 00
0.1458000D 06	0.1376319D 00
0.5802130D 04	0.3914425D-01
0.1051726D 05	0.4796642D-01
0.2015838D 05	0.6033726D-01
0.3471849D 05	0.7564638D-01
0.5912985D 05	0.9420443D-01
0.9305376D 05	0.1119744D 00
0.1420905D 06	0.1305948D 00
0.5786817D 04	0.3838779D-01
0.1049347D 05	0.4766313D-01
0.2014054D 05	0.5970369D-01
0.3469563D 05	0.7587985D-01
0.5911032D 05	0.9453260D-01
0.9303695D 05	0.1123154D 00
0.1420767D 06	0.1306368D 00

Latewood — 2.98 mm

P_f, dynes/cm^2	c, g/cm^3
0.8150000D 04	0.4637585D-01
0.1310000D 05	0.5574181D-01
0.2290000D 05	0.6956378D-01
0.3770000D 05	0.8334080D-01
0.6230000D 05	0.9933061D-01
0.9640000D 05	0.1155684D 00
0.1458000D 06	0.1338240D 00
0.8150000D 04	0.4662525D-01
0.1310000D 05	0.5573423D-01
0.2290000D 05	0.6896866D-01
0.3770000D 05	0.8282932D-01
0.6230000D 05	0.9917476D-01
0.9640000D 05	0.1153688D 00
0.1458000D 06	0.1335343D 00
0.5994565D 04	0.3949222D-01
0.1068206D 05	0.4896003D-01
0.2030512D 05	0.6139605D-01
0.3487053D 05	0.7407593D-01
0.5927271D 05	0.9020933D-01
0.9317516D 05	0.1079842D 00
0.1421981D 06	0.1258577D 00
0.6009660D 04	0.4075155D-01
0.1070905D 05	0.4976180D-01
0.2033461D 05	0.6195928D-01
0.3490386D 05	0.7400566D-01
0.5931532D 05	0.8818320D-01
0.9320791D 05	0.1064529D 00
0.1422229D 06	0.1251793D 00

Latewood — 4.13 mm

P_f, dynes/cm^2	c, g/cm^3
0.8150000D 04	0.4661566D-01
0.1310000D 05	0.5637759D-01
0.2290000D 05	0.6965699D-01
0.3770000D 05	0.8337050D-01
0.6230000D 05	0.9975207D-01
0.9640000D 05	0.1159120D 00
0.1458000D 06	0.1340585D 00
0.8150000D 04	0.4668012D-01
0.1310000D 05	0.5576747D-01
0.2290000D 05	0.6889082D-01
0.3770000D 05	0.8264765D-01
0.6230000D 05	0.9837012D-01
0.9640000D 05	0.1146331D 00
0.1458000D 06	0.1325234D 00
0.8150000D 04	0.4603597D-01
0.1310000D 05	0.5601656D-01
0.2290000D 05	0.6899832D-01
0.3770000D 05	0.8268338D-01
0.6230000D 05	0.9848966D-01
0.9640000D 05	0.1134732D 00
0.1458000D 06	0.1329610D 00
0.6463832D 04	0.3840769D-01
0.1113351D 05	0.4728675D-01
0.2070911D 05	0.5984335D-01
0.3522975D 05	0.7284598D-01
0.5960092D 05	0.8772408D-01
0.9348173D 05	0.1028131D 00
0.1424740D 06	0.1196364D 00

MAT DENSITY VS. PRESSURE FOR WHOLE PULP FIBERS

Whole Pulp — 1.49 mm

P_f, dynes/cm^2	c, g/cm^3
0.8150000D 04	0.4869344D-01
0.1310000D 05	0.5783450D-01
0.2290000D 05	0.7101295D-01
0.3770000D 05	0.8548899D-01
0.6230000D 05	0.1029136D 00
0.9640000D 05	0.1205749D 00
0.1458000D 06	0.1403368D 00
0.5492390D 04	0.3761313D-01
0.1021763D 05	0.4816187D-01
0.1986255D 05	0.6595798D-01
0.3447869D 05	0.8069782D-01
0.5893322D 05	0.9915962D-01
0.9288573D 05	0.1170050D 00
0.1419423D 06	0.1370937D 00
0.5485009D 04	0.3752867D-01
0.1021212D 05	0.4799393D-01
0.1986058D 05	0.6538658D-01
0.3447734D 05	0.7993203D-01
0.5893465D 05	0.9744221D-01
0.9288647D 05	0.1151867D 00
0.1419423D 06	0.1353157D 00

Whole Pulp — 2.76 mm

P_f, dynes/cm^2	c, g/cm^3
0.8150000D 04	0.4676894D-01
0.1310000D 05	0.5591713D-01
0.2290000D 05	0.6967646D-01
0.3770000D 05	0.8348040D-01
0.6230000D 05	0.1004957D 00
0.9640000D 05	0.1175662D 00
0.1458000D 06	0.1364895D 00
0.5971225D 04	0.3783484D-01
0.1065706D 05	0.4710987D-01
0.2027248D 05	0.5991390D-01
0.3483545D 05	0.7295845D-01
0.5922985D 05	0.9079601D-01
0.9313734D 05	0.1087654D 00
0.1421610D 06	0.1278166D 00
0.5922493D 04	0.3732386D-01
0.1061038D 05	0.4661501D-01
0.2023923D 05	0.5866763D-01
0.3481037D 05	0.7097254D-01
0.5919366D 05	0.9076442D-01
0.9310808D 05	0.1082059D 00
0.1421368D 06	0.1266825D 00

Whole Pulp — 3.88 mm

P_f, dynes/cm^2	c, g/cm^3
0.8150000D 04	0.4704740D-01
0.1310000D 05	0.5626533D-01
0.2290000D 05	0.6950423D-01
0.3770000D 05	0.8327794D-01
0.6230000D 05	0.9996938D-01
0.9640000D 05	0.1164388D 00
0.1458000D 06	0.1351842D 00
0.6080014D 04	0.4387227D-01
0.1078113D 05	0.5316521D-01
0.2040875D 05	0.6543423D-01
0.3497533D 05	0.7766837D-01
0.5938074D 05	0.9231913D-01
0.9328111D 05	0.1084817D 00
0.1422869D 06	0.1277465D 00
0.6159665D 04	0.3982958D-01
0.1084314D 05	0.4893220D-01
0.2045712D 05	0.6079975D-01
0.3501379D 05	0.7269041D-01
0.5941154D 05	0.8687168D-01
0.9330871D 05	0.1020787D 00
0.1423108D 06	0.1203493D 00

Whole Pulp — Not Class.

P_f, dynes/cm^2	c, g/cm^3
0.8150000D 04	0.4562335D-01
0.1310000D 05	0.5494349D-01
0.2290000D 05	0.6836177D-01
0.3770000D 05	0.8247910D-01
0.6230000D 05	0.9896725D-01
0.9640000D 05	0.1157578D 00
0.1458000D 06	0.1343930D 00
0.5697347D 04	0.3671692D-01
0.1042189D 05	0.4510315D-01
0.2007172D 05	0.5713932D-01
0.3462116D 05	0.7503564D-01
0.5905279D 05	0.9234183D-01
0.9298851D 05	0.1092652D 00
0.1420306D 06	0.1284662D 00
0.5660418D 04	0.3738418D-01
0.1039292D 05	0.4567838D-01
0.2004159D 05	0.5839888D-01
0.3460246D 05	0.7576348D-01
0.5903935D 05	0.9269678D-01
0.9297538D 05	0.1101282D 00
0.1420200D 06	0.1291505D 00
0.5631177D 04	0.3784716D-01
0.1036408D 05	0.4644513D-01
0.1999431D 05	0.6183906D-01
0.3457902D 05	0.7760351D-01
0.5901768D 05	0.9536743D-01
0.9295732D 05	0.1130746D 00
0.1420037D 06	0.1328685D 00

Variability in the mat density *vs*. pressure data for the whole pulp fiber populations was found to be abnormally high. This variation has been attributed to difficulties in eliminating vibrations from the enviornment during some of the experiments. For this reason, compressibility data for whole pulp has been eliminated from the body of this report. However, Table XV and Fig. 31 show that $\underline{N} = 0.373$ is generally applicable to the data.

TABLE XV

VALUES FOR COMPRESSIBILITY CONSTANTS $\underline{N}$ AND $\underline{M}$[a] FOR WHOLE PULP

$\underline{L_f}$, mm	$\underline{N_1}$[a]	$\underline{M_1} \times 10^3$,[a] c.g.s. units[b]	$\underline{N}$	$\underline{M} \times 10^3$, c.g.s. units[b]
1.49	0.389	1.37	0.373	1.62
2.76	0.384	1.37	0.373	1.53
3.88	0.350	1.99	0.373	1.56
Not class.	0.392	1.27	0.373	1.53

[a]As described in text.

[b]$(g/cm^3)/(dynes/cm^2)^{\underline{N}}$.

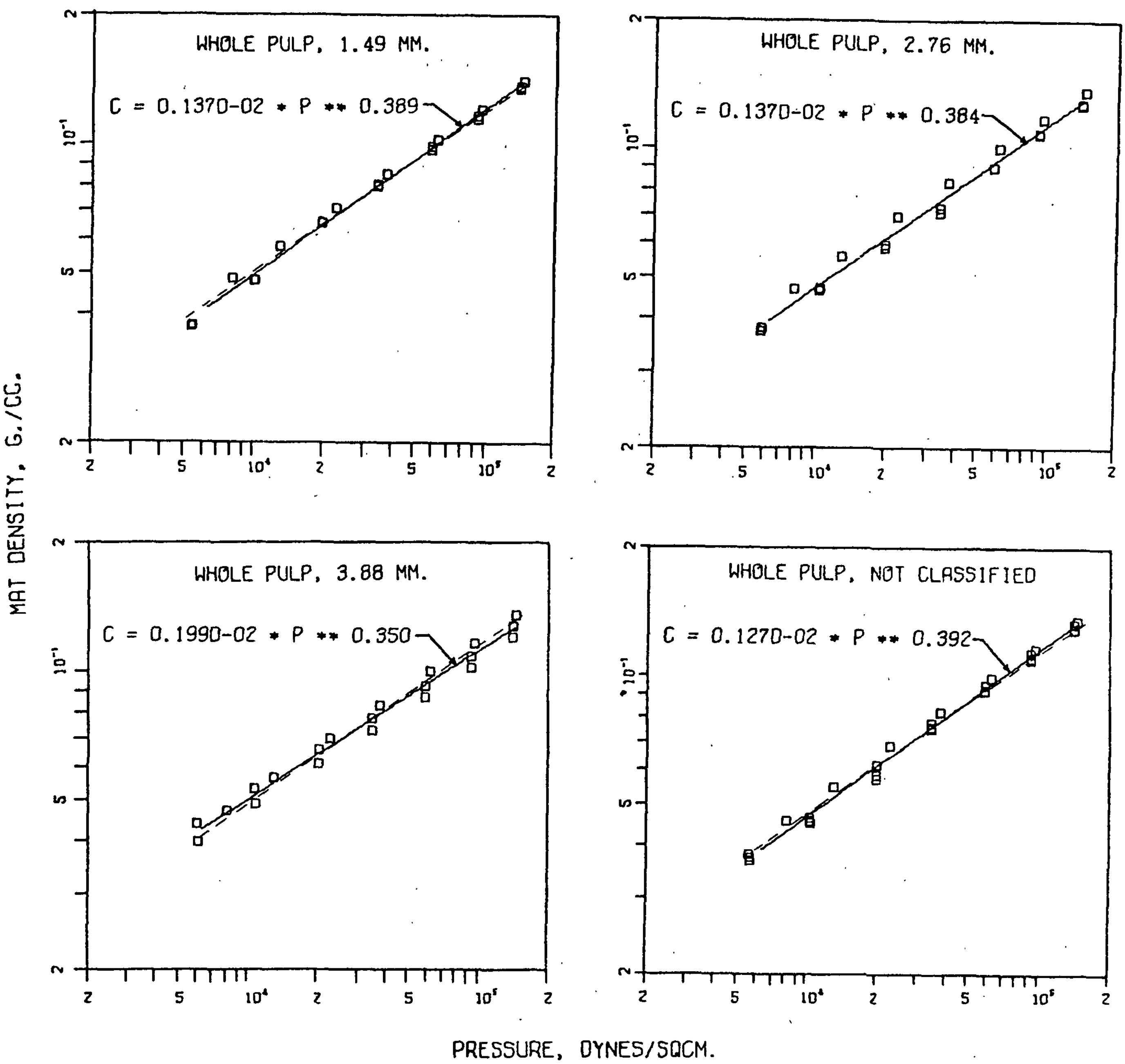

Figure 31. Compressibility Data and Resulting Best Fit Curves for Whole Pulp Fiber Fractions (as in Fig. 16)

APPENDIX II

THREE-DIMENSIONAL PLOTTING PROGRAM

```
/JOB GO,TIME=20
C   LE DISK=(1,PLOTS,NREC=5000),RSIZ=80,VOL=SYSFL1,DISP=(OLD,DELETE)
C
C                    XYZPLT === 3-DIMENSIONAL PLOTTING
C
C                          ANTHONY P. BINOTTO
C                            JUNE 1, 1975
C
C      THIS PROGRAM WILL PLOT ANY SET OF NUMBERS ON X-Y COORDINATES
C      UTILIZING RETZKE'S PLOTTING PROGRAM.
C      THE USER MUST ENTER THE FOLLOWING CARDS IN ORDER.
C                /ID
C                /INCLUDE ABPLOT
C      DATA CARD 1.  LABEL FOR X AXIS
C      DATA CARD 2.  LABEL FOR Y AXIS
C      DATA CARD 3.  TITLE OF PLOT
C      DATA CARD 4.  N,NX,NY,LT,IEQ (IN FORMAT 5I3)
C          WHERE...
C               N = TOTAL NUMBER OF X,Y-PAIRS
C              NX = 1 IF X IS PLOTTED AS LINEAR
C                   2 IF X IS PLOTTED AS LOG10(X)
C                  -1 OR -2 IF NEW SET OF AXIS IS NOT MADE
C              NY = SAME AS FOR NX EXCEPT MINUS HAS NO EFFECT
C              LT = -1 IF SYMBOLS ONLY ARE PLOTTED FOR EACH DATA POINT
C                    0 IF A LINE ONLY CONNECTS EACH DATA POINT
C                   +1 IF BOTH LINE AND SYMBOLS ARE PLOTTED
C             IEQ = INTERGER EQUIVALENT OF PLOTTING SYMBOL (1-16)
C      DATA CARD 5-N.  X AND Y COORDINATES IN 2E15.7 FORMAT
C                /END
C
C
      DIMENSION X(900),Y(900)
      COMMON LX(7),LY(7),ID(13),SVAL(4),FIRSTX,XSCALE,FIRSTY,YSCALE
      COMMON FIRSTZ,ZSCALE,LZ(7),XPRIME,YPRIME,ZANGLE
9006  FORMAT (E11.4,44X,E11.4)
      ID(13) = 0
      FIRSTX = 1.
      XSCALE = 7./1.
      FIRSTY = 2.
      YSCALE = 8.5/2.
      FIRSTZ = 0.
      ZSCALE = 1.
      ZANGLE = 330.
      IN = 5
C     REWIND IN
      CALL GET(LX,24,IN)
      CALL GET(LY,24,IN)
      CALL GET(LZ,24,IN)
      LX(7) = IVERR(LX,1,24)
      LY(7) = IVERR(LY,1,24)
      LZ(7) = IVERR(LZ,1,24)
    1 READ (IN,9004)(ID(I),I=1,12),Z
      IF (ICOMP(ID,1,249,4,1)) 2,3,2
    2 READ (IN,9005) N,NX,NY,LT,IEQ
      READ (IN,9006)(X(I),Y(I),I=1,N)
      XPRIME = Z * COS(ZANGLE/57.2958)
      YPRIME = Z * SIN(ZANGLE/57.2958)
      CALL GRAPH (X,Y,NX,NY,N,LT,IEQ)
      GO TO 1
    3 CALL FINAL
      CALL EXIT
 9004 FORMAT (12A4,F6.2)
 9005 FORMAT (5I3)
      END
C
C
      SUBROUTINE GRAPH (X,Y,NX,NY,N,LT,IEQ)
      DIMENSION X(1),Y(1),ISMBL(16)
      COMMON LX(7),LY(7),ID(13),SVAL(4),FIRSTX,XSCALE,FIRSTY,YSCALE
      COMMON FIRSTZ,ZSCALE,LZ(7),XPRIME,YPRIME,ZANGLE
      ISMBL(1)=170
      ISMBL(2)=183
      ISMBL(3)=175
      ISMBL(4)=176
      ISMBL(5)=181
      ISMBL(6)=182
      ISMBL(7)=185
      ISMBL(8)=171
      ISMBL(9)=172
      ISMBL(10)=173
      ISMBL(11)=174
      ISMBL(12)=177
      ISMBL(13)=178
      ISMBL(14)=179
      ISMBL(15)=180
      ISMBL(16)=184
      IF(ID(13)) 13,1,2
    1 CALL ITLZ
      CALL OPT(1,4)
      CALL PLOT(0.0,-11.0,-3)
      CALL PLOT(0.0,1.5,-3)
    2 IF(NX) 10,13,3
    3 ID(13)=1
      CALL PLOT(10.75,-1.5,3)
      CALL PLOT(10.75,9.0,2)
      CALL PLOT(12.0,0.75,-3)
      CALL AXIS (0.0,0.0,LZ,-LZ(7),4.0,ZANGLE,FIRSTZ,ZSCALE)
      CALL AXLOG(0.0,0.0,LX,-LX(7),7.0,0.0,FIRSTX,XSCALE)
      CALL AXLOG(0.0,0.0,LY,LY(7),8.5,90.0,FIRSTY,YSCALE)
   10 YPAGE=8.5-(ID(13)*0.30)
      IF(LT) 11,12,11
   11 CALL DRAW(0.5,YPAGE+0.08,0.0,ISMBL(IEQ),1.0,0.0,0.35)
   12 CALL SYMBOL(1.0,YPAGE,0.14,ID,0.0,48)
      ID(13)=ID(13)+1
      MX=IABS(NX)
      MY=IABS(NY)
      X(N+1)=FIRSTX
      X(N+2)=XSCALE
      Y(N+1)=FIRSTY
      Y(N+2)=YSCALE
      CALL PLOT (XPRIME,YPRIME,-3)
      CALL LILOG(X,MX,Y,MY,N,1,LT,IEQ)
      XPRIME = -1.* XPRIME
      YPRIME = -1.* YPRIME
      CALL PLOT (XPRIME,YPRIME,-3)
   13 RETURN
      END
```

APPENDIX III

COMPUTER PROGRAM USED TO CALCULATE AN EQUATION OF THE FORM

$$\underline{y} = \underline{b}_0 + \underline{b}_1\underline{x}^{\underline{m}} + \underline{b}_2\underline{x}^{\underline{n}}$$

```
/JOB GO,TIME=20
/FILE DISK=(2,CRVFIT,NREC=9000),RSIZ=80,VOL=SYSFL1,DISP=(NEW,KEEP)
C
C              CRVFIT === CURVE FITTING
C
C                 ANTHONY P. BINOTTO
C                  AUGUST 25, 1975
C
C       THIS PROGRAM IS DESIGNED TO CALCULATE THE BEST EQUATION
C       OF THE FORM...
C                  Y = B0 + B1*(X**M) + B2*(X**N)
C       TO ANY SET OF DATA.  THE PROCEDURE INVOLVES MINIMIZING THE
C       SUM OF SQUARES.
C       *** WARNING***  THE RESULTING EQUATION WILL ONLY BE ONE OF
C       MANY COMBINATIONS OF B0,B1,B2,M,N WHICH GIVE A ''BEST''
C       FIT TO THE DATA.  THE FINAL EQUATION IS THEREFORE DEPENDENT
C       ON THE INITIAL ESTIMATE OF M AND N.
C
C       THE USER MUST ENTER THE FOLLOWING CARDS IN ORDER.
C                    /ID ********
C                    /INCLUDE CRVFIT
C          DATA CARD 1.  TITLE FOR DATA SET 1
C          DATA CARD 2.  NUMBER OF X,Y-PAIRS IN I3 FORMAT
C          DATA CARD 3+  X,Y-PAIRS OF DATA IN 2E15.7 FORMAT
C          REPEAT DATA CARDS 1 THROUGH 3+ FOR SUBSEQUENT DATA SETS
C                    999999999999999
C                    /END
C
C
      DIMENSION ITITLE(20),X(200),Y(200),A(5,6)
      DOUBLE PRECISION X,Y,B0,B1,B2,M,N,A
      DOUBLE PRECISION DELB0,DELB1,DELB2,DELM,DELN
C**** IOUT == OUTPUT STORED ON DISK OR TAPE FOR PLOTTING.
      IOUT = 2
      NCOUNT = 0.0
      WRITE (IOUT,300)
      WRITE (IOUT,200)
300   FORMAT ('CP**3')
200   FORMAT ('DP/T(SQRT(CP))')
100   READ (5,2) ITITLE
      IF(ICOMP(ITITLE,1,249,4,1)) 1,14,1
1     READ (5,3) NPTS
      READ (5,4) (X(I),Y(I),I=1,NPTS)
      WRITE (IOUT,2) ITITLE
      WRITE (IOUT,5) NPTS
      WRITE (IOUT,4) (X(I),Y(I),I=1,NPTS)
      WRITE (6,6) ITITLE
2     FORMAT (20A4)
3     FORMAT (I3)
4     FORMAT (2E15.7)
5     FORMAT (I3,'  1   1 -1   3')
6     FORMAT ('0',20A4,//)
C**** ESTIMATE VALUES OF B0,B1,B2,M,N
      B0 = 200.0
      B1 = 1000.0
      B2 = 1000.0
      M = 0.120
      N = 0.500
      TEST1 = 0.0
7     CALL SUM (X,Y,NPTS,B0,B1,B2,M,N,A)
      CALL SIMEQN (A,5,6)
      DELB0 = A(1,6) / A(1,1)
      DELB1 = A(2,6) / A(2,2)
      DELB2 = A(3,6) / A(3,3)
      DELM = A(4,6) / A(4,4)
      DELN = A(5,6) / A(5,5)
C**** DATA FIT MAY BE IMPROVED AT THE EXPENSE OF CALCULATION TIME
C**** BY REDUCING THE VALUE OF ''TEST''.
      TEST = 10.
      TEST2 = DELB0 + DELB1 + DELB2 + DELM + DELN
      DIFF = TEST2 - TEST1
      IF (ABS(DIFF)-TEST) 12,12,8
8     TEST1 = TEST2
      NCOUNT = NCOUNT + 1
      B0 = B0 + DELB0
      B1 = B1 + DELB1
      B2 = B2 + DELB2
      IF (NCOUNT-100) 7,7,9
9     NCOUNT = 0
      WRITE (6,10) DIFF,B0,B1,B2,M,N,NCOUNT
      WRITE (6,11)  DELB0,DELB1,DELB2,DELM,DELN
10    FORMAT ('0',6E15.4,I5)
11    FORMAT (' ',15X,5E15.4)
      M = M + DELM
      N = N + DELN
      GO TO 7
12    WRITE (6,13)B0,B1,B2,M,N,NCOUNT
13    FORMAT ('0 B0 = ',E12.4,5X,'B1 = ',E12.4,5X,'B2 = ',E12.4,5X,
     *'M = ',E12.4,5X,'N = ',E12.4,I5,///)
      CALL EQN (B0,B1,B2,M,N,IOUT,X,Y)
      GO TO 100
14    WRITE (IOUT,9999)
9999  FORMAT ('9999999999999999999')
      CALL EXIT
      END
C
C     SUBROUTINE EQN === GENERATION OF PLOTTING POINTS
C
C     THIS SUBROUTINE GENERATES DATA POINTS FROM THE CALCULATED
C     EQUATION TO BE USED IN PLOTTING PROGRAM ''ABPLOT''.
      SUBROUTINE EQN (B0,B1,B2,M,N,IOUT,X,Y)
      DIMENSION X(200),Y(200)
      DOUBLE PRECISION X,Y,DPTS,XFIRST,XLAST,XINCR,B0,B1,B2,M,N
      NPTS = 100
      XFIRST = X(1)
      XLAST = X(9)
      DPTS = NPTS
      XINCR = (XLAST - XFIRST) / DPTS
      WRITE (IOUT,1) M,N
      WRITE (IOUT,2) NPTS
1     FORMAT ('Y = B0 + B1*(X**',F4.2,') + B2*(X**',F4.2,')')
2     FORMAT (I3,' -1 -1  0')
      X(1) = XFIRST
      DO 3 I=1,NPTS
      Y(I) = B0 + B1*(X(I)**M) + B2*(X(I)**N)
      X(I+1) = X(I) + XINCR
3     WRITE (IOUT,4) X(I),Y(I)
4     FORMAT (2E15.7)
      RETURN
      END
```

APPENDIX III (Continued)

```
C
C       SUBROUTINE SUM === CALCULATION OF SUM OF SQUARES
C
        SUBROUTINE SUM (X,Y,NPTS,B0,B1,B2,M,N,A)
        DIMENSION X(20),Y(20),A(5,6)
        DOUBLE PRECISION X,Y,B0,B1,B2,M,N,A,PTS,SXMYR2
        DOUBLE PRECISION SY,SXM,SXN,SXMY,SX2M,SXMXN,SXNY,SX2N,SXMYR,SXMR
        DOUBLE PRECISION SX2MR,SXMXNR,SXNYR,SXNR,SXMR2,SX2MR2,SXMNR2
        DOUBLE PRECISION SXNR2,SX2NR2,SXNYR2,SX2NR,XN,XM,X2M,X2N,R,R2
        PTS = NPTS
        SY = 0.0
        SXM = 0.0
        SXN = 0.0
        SXMY = 0.0
        SX2M = 0.0
        SXMXN = 0.0
        SXNY = 0.0
        SX2N = 0.0
        SXMYR = 0.0
        SXMR = 0.0
        SX2MR = 0.0
        SXMXNR = 0.0
        SXNYR = 0.0
        SXNR = 0.0
        SX2NR = 0.0
        SXMR2 = 0.0
        SX2MR2 = 0.0
        SXMNR2 = 0.0
        SXMYR2 = 0.0
        SXNR2 = 0.0
        SX2NR2 = 0.0
        SXNYR2 = 0.0
        DO 1 I=1,NPTS
        XM = X(I) ** M
        XN = X(I) ** N
        X2M = XM ** 2
        X2N = XN ** 2
        R = DLOG(X(I))
        R2 = R ** 2
        SY = SY + Y(I)
        SXM = SXM + XM
        SXN = SXN + XN
        SXMY = SXMY + (XM * Y(I))
        SX2M = SX2M + X2M
        SXMXN = SXMXN + (XM * XN)
        SXNY = SXNY + (XN * Y(I))
        SX2N = SX2N + X2N
        SXMYR = SXMYR + (XM * Y(I) * R)
        SXMR = SXMR + (XM * R)
        SX2MR = SX2MR + (X2M * R)
        SXMXNR = SXMXNR + (XM * XN * R)
        SXNYR = SXNYR + (XN * Y(I) * R)
        SXNR = SXNR + (XN * R)
        SX2NR = SX2NR + (X2N * R)
        SXMR2 = SXMR2 + (XM * R2)
        SX2MR2 = SX2MR2 + (X2M * R2)
        SXMNR2 = SXMNR2 + (XM * XN * R2)
        SXMYR2 = SXMYR2 + (XM * Y(I) * R2)
        SXNR2 = SXNR2 + (XN * R2)
        SX2NR2 = SX2NR2 + (X2N * R2)
1       SXNYR2 = SXNYR2 + (XN * Y(I) * R2)
        A(1,1) = PTS
        A(1,2) = SXM
        A(1,3) = SXN
        A(1,4) = B1 * SXMR
        A(1,5) = B2 * SXNR
        A(1,6) = -(PTS*B0 + B1*SXM + B2*SXN - SY)
        A(2,1) = SXM
        A(2,2) = SX2M
        A(2,3) = SXMXN
        A(2,4) = B0*SXMR + 2.0*B1*SX2MR + B2*SXMXNR - SXMYR
        A(2,5) = B2*SXMXNR
        A(2,6) = -(B0*SXM + B1*SX2M + B2*SXMXN - SXMY)
        A(3,1) = SXN
        A(3,2) = SXMXN
        A(3,3) = SX2N
        A(3,4) = SXMXNR
        A(3,5) = B0*SXNR + B1*SXMXNR + 2.0*B2*SX2NR - SXNYR
        A(3,6) = -(B0*SXN + B1*SXMXN + B2*SX2N - SXNY)
        A(4,1) = SXMR
        A(4,2) = SX2MR
        A(4,3) = SXMXNR
        A(4,4) = B0*SXMR2 + 2.0*B1*SX2MR2 + B2*SXMNR2 - SXMYR2
        A(4,5) = B2*SXMNR2
        A(4,6) = -(B0*SXMR + B1*SX2MR + B2*SXMXNR - SXMYR)
        A(5,1) = SXNR
        A(5,2) = SXMXNR
        A(5,3) = SX2NR
        A(5,4) = B1*SXMNR2
        A(5,5) = B0*SXNR2 + B1*SXMNR2 + 2.0*B2*SX2NR2 - SXNYR2
        A(5,6) = -(B0*SXNR + B1*SXMXNR + B2*SX2NR - SXNYR)
        RETURN
        END
C
C       SUBROUTINE SIMEQN === SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS
C
      SUBROUTINE SIMEQN (A,NR,NC)
      DIMENSION A(5,6),R(100)
      DOUBLE PRECISION A,R,D,T
      IF(NR - NC) 4,4,3
    3 NCT = NC
      GO TO 5
    4 NCT = NR
    5 K = 1
      D = 1.
    7 IF(A(K,K)) 12,8,12
    8 DO 9 I = K,NR
      IF(A(I,K)) 10,9,10
    9 CONTINUE
      GO TO 17
   10 DO 11 J = 1,NC
      T = A(I,J)
      A(I,J) = A(K,J)
   11 A(K,J) = T
   12 DO 16 I = 1,NR
      IF(I - K) 13,16,13
   13 DO 14 J = 1,NC
   14 R(J) = (A(I,J)*A(K,K) - A(I,K)*A(K,J))/D
      DO 15 J = 1,NC
   15 A(I,J) = R(J)
   16 CONTINUE
      D = A(K,K)
      K = K + 1
      IF(K - NCT) 7,7,17
   17 RETURN
      END
/DATA
```

APPENDIX IV

TWO-DIMENSIONAL PLOTTING PROGRAM

```
/JOB GO,TIME=20
/FILE DISK=(1,CRVFIT,NREC=9000),RSIZ=80,VOL=SYSFL1,DISP=(OLD,DELETE)
C
C                        ABPLOT === PLOTTING PROGRAM
C
C                              ANTHONY P. BINOTTO
C                                JUNE 1, 1975
C
C       THIS PROGRAM WILL PLOT ANY SET OF NUMBERS ON X-Y COORDINATES
C       UTILIZING RETZKE'S PLOTTING PROGRAM.
C       THE USER MUST ENTER THE FOLLOWING CARDS IN ORDER.
C                  /ID
C                  /INCLUDE ABPLOT
C       DATA CARD 1.  LABEL FOR X AXIS
C       DATA CARD 2.  LABEL FOR Y AXIS
C       DATA CARD 3.  TITLE OF PLOT
C       DATA CARD 4.  N,NX,NY,LT,IEQ (IN FORMAT 5I3)
C             WHERE...
C                    N = TOTAL NUMBER OF X,Y-PAIRS
C                   NX = 1 IF X IS PLOTTED AS LINEAR
C                        2 IF X IS PLOTTED AS LOG10(X)
C                       -1 OR -2 IF NEW SET OF AXIS IS NOT MADE
C                   NY = SAME AS FOR NX EXCEPT MINUS HAS NO EFFECT
C                   LT = -1 IF SYMBOLS ONLY ARE PLOTTED FOR EACH DATA POINT
C                         0 IF A LINE ONLY CONNECTS EACH DATA POINT
C                        +1 IF BOTH LINE AND SYMBOLS ARE PLOTTED
C                  IEQ = INTERGER EQUIVALENT OF PLOTTING SYMBOL (1-16)
C       DATA CARD 5-N.  X AND Y COORDINATES IN 2E15.7 FORMAT
C                  /END
C
C
      DIMENSION X(900),Y(900)
      COMMON LX(7),LY(7),ID(13),SVAL(4)
      ID(13) = 0
      IN = 1
      REWIND IN
      CALL GET(LX,24,IN)
      CALL GET(LY,24,IN)
      LX(7) = IVERR(LX,1,24)
      LY(7) = IVERR(LY,1,24)
1     READ (IN,2)(ID(I),I=1,12)
2     FORMAT (12A4)
      IF (ICOMP(ID,1,249,4,1)) 3,6,3
3     READ (IN,4) N,NX,NY,LT,IEQ
4     FORMAT (5I3)
      READ (IN,5)(X(I),Y(I),I=1,N)
5     FORMAT (2E15.7)
      CALL GRAPH (X,Y,NX,NY,N,LT,IEQ)
      GO TO 1
6     CALL FINAL
      CALL EXIT
      END
C
C     SUBROUTINE GRAPH - WRITTEN BY JAMES R RETZKE.  30 OCTOBER 1972
C
      SUBROUTINE GRAPH (X,Y,NX,NY,N,LT,IEQ)
      DIMENSION X(1),Y(1),ISMBL(16)
      COMMON LX(7),LY(7),ID(13),SVAL(4)
      ISMBL(1)=170
      ISMBL(2)=183
      ISMBL(3)=175
      ISMBL(4)=176
      ISMBL(5)=181
      ISMBL(6)=182
      ISMBL(7)=185
      ISMBL(8)=171
      ISMBL(9)=172
      ISMBL(10)=173
      ISMBL(11)=174
      ISMBL(12)=177
      ISMBL(13)=178
      ISMBL(14)=179
      ISMBL(15)=180
      ISMBL(16)=184
      IF(ID(13))100,10,11
10    CALL ITLZ
      CALL OPT(1,4)
      CALL PLOT(0.0,-11.0,-3)
      CALL PLOT(0.0,1.5,-3)
11    IF(NX)8,100,7
7     ID(13)=1
      CALL PLOT(7.75,-1.5,3)
      CALL PLOT(7.75,9.0,2)
      CALL PLOT(8.5,0.0,-3)
      IF(NX-1)100,1,2
1     CALL SCALE(X,7.0,N,1)
      CALL AXIS(0.0,0.0,LX,-LX(7),7.0,0.0,X(N+1),X(N+2))
      GO TO 3
2     CALL SCALOG(X,7.0,N,1)
      CALL AXLOG(0.0,0.0,LX,-LX(7),7.0,0.0,X(N+1),X(N+2))
3     IF(NY-1)100,4,5
4     CALL SCALE(Y,7.0,N,1)
      CALL AXIS(0.0,0.0,LY,LY(7),7.0,90.0,Y(N+1),Y(N+2))
      GO TO 6
5     CALL SCALOG(Y,7.0,N,1)
      CALL AXLOG(0.0,0.0,LY,LY(7),7.0,90.0,Y(N+1),Y(N+2))
6     SVAL(1)=X(N+1)
      SVAL(2)=X(N+2)
      SVAL(3)=Y(N+1)
      SVAL(4)=Y(N+2)
8     YPAGE=7.2-(ID(13)*0.35)
      IF(LT)45,44,45
45    CALL DRAW(0.5,YPAGE+0.08,0.0,ISMBL(IEQ),1.0,0.0,0.35)
44    CALL SYMBOL(1.0,YPAGE,0.14,ID,0.0,48)
      ID(13)=ID(13)+1
      MX=IABS(NX)
      MY=IABS(NY)
      X(N+1)=SVAL(1)
      X(N+2)=SVAL(2)
      Y(N+1)=SVAL(3)
      Y(N+2)=SVAL(4)
      CALL LILOG(X,MX,Y,MY,N,1,LT,IEQ)
100   RETURN
      END
/DATA
```

APPENDIX V

COMPUTER PROGRAM FOR THE ANALYSIS OF CONSTANT RATE FILTRATION DATA

```
/JOB GO,TIME=99
/FILE DISK=(1,NREC=10000),RSIZ=128
/FILE DISK=(2,PLOTI,NREC=3000),RSIZ=80,VOL=SYSFL1,DISP=(NEW,DELETE)
/FILE DISK=(3,PLOTJ,NREC=3000),RSIZ=80,VOL=SYSFL1,DISP=(NEW,KEEP)
/FILE DISK=(4,PRTOUT,NREC=1000),RSIZ=128,VOL=SYSFL1,DISP=(NEW,DELETE)
C
C                 CRFILT === CONSTANT RATE FILTRATION
C
C
C                            ANTHONY P. BINOTTO
C                            NOVEMBER 13,1974
C
C
C     THIS PROGRAM IS DESIGNED TO CALCULATE ...
C     1.   AVERAGE SPECIFIC FILTRATION RESISTANCE
C     2.   LOCAL SPECIFIC FILTRATION RESISTANCE
C     3.   WET MAT DENSITY
C     4.   COMPRESSIBILITY CONSTANTS M AND N
C     5.   AVERAGE SPECIFIC SURFACE
C     6.   AVERAGE SPECIFIC VOLUME
C     FROM DATA OBTAINED THROUGH CONSTANT RATE FILTRATION AND WET MAT COM-
C     PRESSIBILITY EXPERIMENTS.  FOR ADDITIONAL INFORMATION REFER TO THE
C     AUTHOR'S THESIS.
C
C     THE USER'S CARD DECK SHOULD CONTAIN THE FOLLOWING CARDS IN ORDER...
C                 1.   /ID XXXXXXXX
C                 2.   /INCLUDE CRFILT
C                 3.   /INCLUDE CMPRSS
C                 4.   /INCLUDE FILTCV
C                 5.   /INCLUDE AVEVS
C                 6.   /DATA
C                      A.   TITLE IN A4 FORMAT (ALL OTHER DATA IN 9F8.0)
C                      B.   CHART READINGS FROM FILT. EXP., NOTE THAT
C                           CHTSPD = 2 IN/MIN.  CHANGE IF NECESSARY.
C                      C.   FILT. PAD WT. IN GRAMS, FLOW RATE IN CC./SEC.,
C                           TEMP. IN DEG. C, AND EQUIV. VOLUME
C                      D.   COMP. PAD WT. AND PAD THICK. IN INCHES FOR WTS.
C                           1 THRU 7 IN 9F8.0.
C                 7.   99999999999
C                 8.   /END
C
C
      DIMENSION DP(20),CP(20),CP3(20),T(20),Y(20),CHTRD(20),LOCR(20)
      DIMENSION DP2(20),SCONC(20),PDTHCK(20),DPDT(20),DPTCP(20)
      DIMENSION WPRESS(20),ITITLE(20),AVER(20)
      DOUBLE PRECISION DP,CP,M,N,CP3,T,Y,DP2,SCONC,PDTHCK,MU,WR,UO,TEMP
      DOUBLE PRECISION EQVOL,A,B0,B1,B2,EXPM,EXPN,DPDT,C1,B,AVESV,AVESW
      DOUBLE PRECISION AVER,LOCR,WCP,DPTCP,AVEV
      COMMON ITITLE,K,IOUT,JOUT,KOUT
      COMMON M,N,MU,UO,A,WR,EQVOL
C***  SAVE IOUT FOR PLOTS OF P VS T
      IOUT = 2
C***  SAVE JOUT FOR PLOTS OF DP/(T*SQRT(CP)) VS. CP**3
      JOUT = 3
C***  SAVE LOUT FOR PUNCHED OUTPUT
      LOUT = 7
C***  SAVE MOUT FOR PRINTED OUTPUT
      MOUT = 1
      J = 1
      K = 9
      CHTSPD = 2.
      A = 45.51
      WRITE (IOUT,9037)
      WRITE (IOUT,9038)
      WRITE (JOUT,9052)
      WRITE (JOUT,9053)
      WRITE (LOUT,9046)
      WRITE (LOUT,9052)
      WRITE (LOUT,9048)
    1 READ (5,9009) ITITLE
      IF(ICOMP(ITITLE,1,249,4,1)) 2,8,2
    2 READ(5,9010) (CHTRD(I),I=J,K)
      READ (5,9010) WR,UO,TEMP,EQVOL
      READ (5,9010) WCP,(PDTHCK(I),I=1,7)
C***  CALCULATION OF VISCOSITY (MU) FROM WATER TEMPERATURE
      IF (TEMP - 20.) 3,3,4
    3 ETA=(1301./(998.333+8.1855*(TEMP-20.)+.00585*((TEMP-20.)**2)))
      MU = 10. ** (ETA - 3.30233)
      GO TO 5
    4 ETA=(1.3272*(20.-TEMP)-.001053*((TEMP-20.)**2))/(TEMP+105.)
      MU = (10. ** ETA) * .01002
    5 WRITE (MOUT,9011)
      WRITE (MOUT,9012)
      WRITE (MOUT,9012)
```

```
      WRITE (MOUT,9013) ITITLE
      WRITE (MOUT,9012)
      WRITE (MOUT,9012)
      WRITE (MOUT,9014)
      WRITE (MOUT,9015)
      WRITE (MOUT,9016)
      WRITE (MOUT,9017)
      WRITE (MOUT,9018)
      WRITE (MOUT,9019)
      C1 = WR / EQVOL
      C1P = C1 * 100.
      DO 6 I=J,K
      T(I) = (CHTRD(I) * 60.D0) / CHTSPD
      WPRESS(I) = (I) * 10
    6 DP(I) = WPRESS(I) * 980.665D0
C***  CALCULATION OF COMPRESSIBILITY CONSTANTS IN SUBROUTINE CMPRSS
      CALL CMPRSS (WCP,PDTHCK,N,M,CP,DP2,DP,T,SCONC,CP3,DPTCP)
      WRITE (MOUT,9020)(WPRESS(I),CHTRD(I),T(I),DP2(I),PDTHCK(I),
     * SCONC(I),I=J,7)
      WRITE (MOUT,9120) (WPRESS(I),CHTRD(I),T(I),I=8,K)
      WRITE (MOUT,9021) WR,EQVOL,C1P
      WRITE (MOUT,9022) UO,MU,WCP
C***  CALCULATION OF AVERAGE AND LOCAL FILTRATION RESISTANCES
      B = (A**2) / (MU * (UO**2) * C1)
      CALL CRVFIT (T,DP,B0,B1,B2,EXPM,EXPN,J,K)
      DO 7 I=J,K
      DPDT(I) = EXPM*B1*(T(I)**(EXPM-1.))+ EXPN*B2*(T(I)**(EXPN-1.))
      AVER(I) = B * DP(I) / T(I)
    7 LOCR(I) = B * DPDT(I)
C***  CALCULATION OF AVERAGE VALUES FOR V, SW, AND SV FROM SUB. AVEVS
      CALL AVEVS (DPTCP,CP3,B,N,AVEV,AVESW,AVESV,JOUT,ITITLE)
      WRITE (MOUT,9044)
      WRITE (MOUT,9023)
      WRITE (MOUT,9024)
      WRITE (MOUT,9025)
      WRITE (MOUT,9026)
      WRITE (MOUT,9027)
      WRITE (MOUT,9028)
      WRITE (MOUT,9029) (WPRESS(I),DP(I),AVER(I),LOCR(I),CP(I),DPDT(I),
     *I=J,K)
      WRITE (MOUT,9044)
      WRITE (MOUT,9030)
      WRITE (MOUT,9031)
      WRITE (MOUT,9032)
      WRITE (MOUT,9033)
      WRITE (MOUT,9034) M,B0,EXPM
      WRITE (MOUT,9035) N,B1,EXPN
      WRITE (MOUT,9036) B2
      WRITE (MOUT,9044)
      WRITE (MOUT,9139)
      WRITE (MOUT,9140)
      WRITE (MOUT,9141) AVEV
      WRITE (MOUT,9142) AVESW
      WRITE (MOUT,9143) AVESV
      WRITE (MOUT,9044)
      WRITE (LOUT,9009) ITITLE
      WRITE (LOUT,9050)
      WRITE (LOUT,9045)(WPRESS(I),AVER(I),LOCR(I),CP(I),I=J,K)
      GO TO 1
    8 WRITE (IOUT,9051)
      WRITE (JOUT,9051)
      WRITE (LOUT,9051)
      WRITE (MOUT,9051)
      END FILE 1
      END FILE 2
      END FILE 3
      END FILE 4
      CALL EXIT
 9009 FORMAT (20A4)
 9010 FORMAT (9F8.0)
 9011 FORMAT (1H1,16X,'*****************************************************
     ****************************************')
 9012 FORMAT (' ',16X,'*',85X,'*')
 9013 FORMAT (' ',16X,'*',3X,20A4,2X,'*')
 9014 FORMAT (' ',16X,'****************************************************
     ****************************************')
 9015 FORMAT ('0','ORIGINAL DATA')
 9016 FORMAT (' ','-------- ----',//)
 9017 FORMAT ('0',10X,'FILTRATION EXPERIMENT',31X,'COMPRESSIBILITY EXPE
     1RIMENT',/)
 9018 FORMAT(' ',10X,'PRESSURE',7X,'CHART',9X,'TIME',20X,'PRESSURE',10X,
     1'PAD THICK.',8X,'SOLIDS CONC.')
 9019 FORMAT(' ',10X,'CM. H2O',8X,'READING',7X,'SEC.',20X,'DYNES/SQCM.'
     1,7X,'INCHES',12X,'G./CC.',//)
 9020 FORMAT ('0',12X,F4.0,8X,F6.2,7X,F8.2,16X,F10.0,10X,F8.4,11X,F6.4)
 9120 FORMAT ('0',12X,F4.0,8X,F6.2,7X,F8.2)
 9021 FORMAT ('0','FILT. PAD WT. =',F7.4,' G.',5X,'EQ. VOL. =',F9.2,
     1' CC.',7X,'CONSISTENCY = ',F5.3,' PERCENT')
```

Appendix V (Continued)

```
9022  FORMAT ('0','FLOW RATE =',F5.1,' CC./SEC.',5X,'H2O VISC. =',F8.6,
     1' POISE',5X,'COMP. PAD WT. =',F7.4,' G.',///)
9023 FORMAT ('0','FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS
     1 FUNCTIONS OF PRESSURE')
9024 FORMAT (' ','---------- ---------- --- --- ------ ------------ --
     1 --------- -- ---------',//)
9025  FORMAT ('0',44X,'FILTRATION RESISTANCE',15X,'MAT')
9026  FORMAT (' ',16X,'PRESSURE',18X,'AVERAGE',12X,'LOCAL',12X,
     * 'DENSITY')
9027  FORMAT ('0',19X,'P',24X,'(R)',16X,'R',16X,'C',14X,'DP/DT')
9028  FORMAT ('0',11X,'CM. H2O',3X,'DYNES/SQCM.',10X,'CM./G.',12X,
     * 'CM./G.',12X,'G./CC.',//)
9029 FORMAT ('0',12X,F4.0,8X,F6.0,10X,E10.4,8X,E10.4,10X,F6.4,10X,F6.2)
9030 FORMAT ('0','EXPERIMENTAL EMPIRICAL CONSTANTS')
9031 FORMAT (' ','------------ --------- ---------')
9032 FORMAT ('0',10X,'C = M * (P ** N)',20X,'P = B0 + B1*(T**EXPM) + B2
     * *(T**EXPN)')
9033 FORMAT ('0',10X,'WHERE...',28X,'WHERE...')
9034 FORMAT ('0',15X,'M = ',E10.4,22X,'B0 = ',E12.5,5X,'EXPM = ',E12.5)
9035 FORMAT ('0',15X,'N = ',F6.4,26X,'B1 = ',E12.5,5X,'EXPN = ',E12.5)
9036 FORMAT ('0',51X,'B2 = ',E12.5)
9037 FORMAT ('TIME, SEC.')
9038 FORMAT ('PRESSURE, DYNES/SQCM.')
9139 FORMAT ('0','AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE')
9140 FORMAT (' ','------- ------ --- -------- ------ --- -------')
9141 FORMAT ('0','AVERAGE SPECIFIC VOLUME, (V)  = ',F5.2)
9142 FORMAT ('0','AVERAGE SPECIFIC SURFACE, (SW)  = ',F6.0)
9143 FORMAT ('0','AVERAGE SPECIFIC SURFACE, (SV)  = ',F6.0)
9044 FORMAT ('0',///)
9045 FORMAT (4E11.4)
9046 FORMAT ('PRESSURE, CM. H2O')
9048 FORMAT ('FIBER LENGTH, MM.')
9050 FORMAT ('  9  2  2  2  3')
9051 FORMAT ('9999999999999999999999999999999999999')
9052 FORMAT ('CP**3')
9053 FORMAT ('DP/(T*SQRT(CP))')
     END
C
C              FILTCV ==== CURVE FITTING
C
      SUBROUTINE CRVFIT(X,Y,B0,B1,B2,M,N,J,K)
      DIMENSION ITITLE(20),A(5,6),X(20),Y(20)
      DOUBLE PRECISION X,Y,B0,B1,B2,M,N,A,ORIGM,ORIGN,DPTS
      DOUBLE PRECISION DELB0,DELB1,DELB2,DELM,DELN
      COMMON ITITLE,NPTS,IOUT,JOUT,KOUT
C****  IOUT === OUTPUT STORED ON DISK OR TAPE FOR PLOTTING
      WRITE (IOUT,9001) ITITLE
      WRITE (IOUT,9002) NPTS
      WRITE (IOUT,9003) (X(I),Y(I),I=1,NPTS)
9001  FORMAT (20A4)
9002  FORMAT (I3,'  1  1 -1  3')
9003  FORMAT (2E15.7)
      DPTS = 9.00
C****  INITIAL VALUES OF B0, B1, AND M ESTIMATED BY 'SUBROUTINE XM'
      M = .01
      N = 1.00
      B0 = 27000.
      B1 = 10000.
      B2 = 75000.
      TEST = 1000.
C***   DATA FIT MAY BE IMPROVED AT THE EXPENSE OF CALCULATION TIME
C***   BY DECREASING THE VALUE OF ''TEST''.
    1 MCOUNT = 0
      NCOUNT = 0
      ORIGM = M
      ORIGN = N
      TEST1 = 0.0
    2 CALL SUM (X,Y,NPTS,B0,B1,B2,M,N,A,J,K)
      CALL SIMEQ (A,5,6)
      DELB0 = A(1,6) / A(1,1)
      DELB1 = A(2,6) / A(2,2)
      DELB2 = A(3,6) / A(3,3)
      DELM = A(4,6) / A(4,4)
      DELN = A(5,6) / A(5,5)
      TEST2 = DABS(DELB0) + DABS(DELB1) + DABS(DELB2) + DABS(DELM) +
     * DABS(DELN)
      DIFF = TEST2 - TEST1
      IF (ABS(DIFF)-TEST) 6,6,3
    3 TEST1 = TEST2
      NCOUNT = NCOUNT + 1
      B0 = B0 + DELB0
      B1 = B1 + DELB1
      B2 = B2 + DELB2
C     IF (NCOUNT-100) 2,2,4
C   4 NCOUNT = 0
      M = M + DELM
      N = N + DELN
      MCOUNT = MCOUNT + 1
```

```
      IF (MCOUNT - 99) 2,2,5
    5 M = ORIGM
      N = ORIGN + 0.01
      GO TO 1
    6 CALL EQN (B0,B1,B2,M,N,IOUT,X,J,K)
      RETURN
      END
C
C     SUBROUTINE EQN === GENERATION OF PLOTTING POINTS
C
C     THIS SUBROUTINE GENERATES DATA POINTS FROM THE CALCULATED
C     EQUATION TO BE USED IN PLOTTING PROGRAM ''ABPLOT''.
      SUBROUTINE EQN (B0,B1,B2,M,N,IOUT,X,J,K)
      DIMENSION X(20)
      DOUBLE PRECISION X,DPTS,XINCR,B0,B1,B2,M,N,P,R
      NPTS = 200
      DPTS = NPTS
      XINCR = (X(K) - X(J)) / DPTS
      WRITE (IOUT,9002) M,N
      WRITE (IOUT,9003) NPTS
      P = X(J)
      DO 1 I=1,NPTS
      R = B0 + B1 * (P ** M) + B2 * (P ** N)
      WRITE (IOUT,9004) P,R
    1 P = P + XINCR
      RETURN
 9002 FORMAT ('P = B0 + B1*(T**',F4.2,') + B2*(T**',F4.2,')')
 9003 FORMAT (I3,' -1 -1  0')
 9004 FORMAT (2D15.7)
      END
C
C     SUBROUTINE SUM === CALCULATION OF SUM OF SQUARES
C
      SUBROUTINE SUM (X,Y,NPTS,B0,B1,B2,M,N,A,J,K)
      DIMENSION X(12),Y(12),A(5,6)
      DOUBLE PRECISION X,Y,B0,B1,B2,M,N,A,PTS,SXMYR2
      DOUBLE PRECISION SY,SXM,SXN,SXMY,SX2M,SXMXN,SXNY,SX2N,SXMYR,SXMR
      DOUBLE PRECISION SX2MR,SXMXNR,SXNYR,SXNR,SXMR2,SX2MR2,SXMNR2
      DOUBLE PRECISION SXNR2,SX2NR2,SXNYR2,SX2NR,XN,XM,X2M,X2N,R,R2
      PTS = NPTS
      SY = 0.0
      SXM = 0.0
      SXN = 0.0
      SXMY = 0.0
      SX2M = 0.0
      SXMXN = 0.0
      SXNY = 0.0
      SX2N = 0.0
      SXMYR = 0.0
      SXMR = 0.0
      SX2MR = 0.0
      SXMXNR = 0.0
      SXNYR = 0.0
      SXNR = 0.0
      SX2NR = 0.0
      SXMR2 = 0.0
      SX2MR2 = 0.0
      SXMNR2 = 0.0
      SXMYR2 = 0.0
      SXNR2 = 0.0
      SX2NR2 = 0.0
      SXNYR2 = 0.0
      DO 1 I=J,K
      XM = X(I) ** M
      XN = X(I) ** N
      X2M = XM ** 2
      X2N = XN ** 2
      R = DLOG(X(I))
      R2 = R ** 2
      SY = SY + Y(I)
      SXM = SXM + XM
      SXN = SXN + XN
      SXMY = SXMY + (XM * Y(I))
      SX2M = SX2M + X2M
      SXMXN = SXMXN + (XM * XN)
      SXNY = SXNY + (XN * Y(I))
      SX2N = SX2N + X2N
      SXMYR = SXMYR + (XM * Y(I) * R)
      SXMR = SXMR + (XM * R)
      SX2MR = SX2MR + (X2M * R)
      SXMXNR = SXMXNR + (XM * XN * R)
      SXNYR = SXNYR + (XN * Y(I) * R)
      SXNR = SXNR + (XN * R)
      SX2NR = SX2NR + (X2N * R)
      SXMR2 = SXMR2 + (XM * R2)
      SX2MR2 = SX2MR2 + (X2M * R2)
      SXMNR2 = SXMNR2 + (XM * XN * R2)
      SXMYR2 = SXMYR2 + (XM * Y(I) * R2)
```

Appendix V (Continued)

```
      SXNR2 = SXNR2 + (XN * R2)
      SX2NR2 = SX2NR2 + (X2N * R2)
    1 SXNYR2 = SXNYR2 + (XN * Y(I) * R2)
      A(1,1) = PTS
      A(1,2) = SXM
      A(1,3) = SXN
      A(1,4) = B1 * SXMR
      A(1,5) = B2 * SXNR
      A(1,6) = -(PTS*B0 + B1*SXM + B2*SXN - SY)
      A(2,1) = SXM
      A(2,2) = SX2M
      A(2,3) = SXMXN
      A(2,4) = B0*SXMR + 2.0*B1*SX2MR + B2*SXMXNR - SXMYR
      A(2,5) = B2*SXMXNR
      A(2,6) = -(B0*SXM + B1*SX2M + B2*SXMXN - SXMY)
      A(3,1) = SXN
      A(3,2) = SXMXN
      A(3,3) = SX2N
      A(3,4) = SXMXNR
      A(3,5) = B0*SXNR + B1*SXMXNR + 2.0*B2*SX2NR - SXNYR
      A(3,6) = -(B0*SXN + B1*SXMXN + B2*SX2N - SXNY)
      A(4,1) = SXMR
      A(4,2) = SX2MR
      A(4,3) = SXMXNR
      A(4,4) = B0*SXMR2 + 2.0*B1*SX2MR2 + B2*SXMNR2 - SXMYR2
      A(4,5) = B2*SXMNR2
      A(4,6) = -(B0*SXMR + B1*SX2MR + B2*SXMXNR - SXMYR)
      A(5,1) = SXNR
      A(5,2) = SXMXNR
      A(5,3) = SX2NR
      A(5,4) = B1*SXMNR2
      A(5,5) = B0*SXNR2 + B1*SXMNR2 + 2.0*B2*SX2NR2 - SXNYR2
      A(5,6) = -(B0*SXNR + B1*SXMXNR + B2*SX2NR - SXNYR)
      RETURN
      END
C
C       SUBROUTINE SIMEQN === SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS
C
      SUBROUTINE SIMEQ (A,NR,NC)
      DIMENSION A(5,6),R(100)
      DOUBLE PRECISION A,R,D,T
      IF(NR - NC) 2,2,1
    1 NCT = NC
      GO TO 3
    2 NCT = NR
    3 K = 1
      D = 1.
    4 IF(A(K,K)) 9,5,9
    5 DO 6 I = K,NR
      IF(A(I,K)) 7,6,7
    6 CONTINUE
      GO TO 14
    7 DO 8 J = 1,NC
      T = A(I,J)
      A(I,J) = A(K,J)
    8 A(K,J) = T
    9 DO 13 I = 1,NR
      IF(I - K) 10,13,10
   10 DO 11 J = 1,NC
   11 R(J) = (A(I,J)*A(K,K) - A(I,K)*A(K,J))/D
      DO 12 J = 1,NC
   12 A(I,J) = R(J)
   13 CONTINUE
      D = A(K,K)
      K = K + 1
      IF(K - NCT) 4,4,14
   14 RETURN
      END
C
C  SUBROUTINE EXPONM = ESTIMATION OF INITIAL VALUE OF EXPONENT ''M''
C
        SUBROUTINE EXPONM (X,Y,NPTS,BOMIN,B1MIN,MMIN,G,H,MAXM)
        DIMENSION X(12),Y(12)
        DOUBLE PRECISION X,XM,Y,M,PTS,SUMXM,SUMX2M,SUMY,SUMXMY,SMIN
        DOUBLE PRECISION S,B0,B1,MMIN,BOMIN,B1MIN,MAXM
        INTEGER G,H
        PTS = NPTS
        M = MMIN
        UNIT = .0001
        SMIN = 16.0D70
        J = 1
    6   SUMXM = 0.
        SUMX2M = 0.
        SUMY = 0.
        SUMXMY = 0.
        DO 7 I=G,H
        XM = X(I) ** M
        SUMXM = SUMXM + XM
```

```
      SUMX2M = SUMX2M + (XM**2)
      SUMY = SUMY + Y(I)
7     SUMXMY = SUMXMY + (XM * Y(I))
      Q1 = SUMXMY - (SUMXM * SUMY)/PTS
      Q2 = SUMX2M - (SUMXM ** 2)/PTS
      B1 = Q1 / Q2
      B0 = (SUMY - B1 * SUMXM) / PTS
      S = DSQRT(B0**2 + B1**2)
      IF (S - SMIN) 8,9,9
8     SMIN = S
      B0MIN = B0
      B1MIN = B1
      MMIN = M
9     J = J + 1
      M = M + UNIT
      IF (M - MAXM) 6,6,11
11    RETURN
      END
C              SUBROUTINE CMPRSS === COMPRESSIBILITY
C
C                     ANTHONY P. BINOTTO
C                        JUNE 16, 1975
C
C         THIS SUBROUTINE IS USED TO CALCULATE THE MAT SOLIDS
C         CONCENTRATION FROM MEASUREMENTS OF PAD THICKNESS AND THE
C         O.D. WT. OF THE MAT.  THE PROCEDURE INVOLVES A LEAST SQUARES
C         APPROACH TO COMPUTING COMPRESSIBILITY CONSTANTS M AND N.
C
C
      SUBROUTINE CMPRSS (W,PDTHCK,N,M,CP,DP2,DP,T,MATCON,CP3,DPTCP)
      DIMENSION PDTHCK(20),MATCON(20),LMC(20),LDP2(20),DP2(20)
      DIMENSION DP(20),CP(20),T(20),CP3(20),DPTCP(20)
      DOUBLE PRECISION MATCON,LMC,LDP2,SUMY,SUMY2,SUMX,SUMX2,SUMXY,Q1
      DOUBLE PRECISION Q2,N,M,SUMM,DP2,CP,CP3,DPTCP,DP,PDTHCK,T,W
      AREA = 45.54D0
      NPTS = 7
      DP2(1) = 8150.D0
      DP2(2) = 13100.D0
      DP2(3) = 22900.D0
      DP2(4) = 37700.D0
      DP2(5) = 62300.D0
      DP2(6) = 96400.D0
      DP2(7) = 145800.D0
      DO 1 I=1,7
      MATCON(I) = W / (AREA * PDTHCK(I) * 2.54)
      LMC(I) = DLOG(MATCON(I))
1     LDP2(I) = DLOG(DP2(I))
      CALL LINREG (LDP2,LMC,NPTS,M,N,1,7)
      M = DEXP(M)
      DO 2 I=1,9
      CP(I) = M * DP(I) ** N
      CP3(I) = CP(I) ** 3
2     DPTCP(I) = DP(I) / (T(I) * DSQRT(CP(I)))
      RETURN
      END
C               LINREG === LINEAR REGRESSION
C                     ANTHONY P. BINOTTO
C                        JUNE 16, 1975
C
C         THIS SUBROUTINE IS USED TO CALCULATE THE STRAIGHT LINE
C         RELATIONSHIP BETWEEN PAIRS OF X,Y-DATA
C
      SUBROUTINE LINREG (X,Y,NPTS,INTCPT,SLOPE,J,K)
      DIMENSION X(20),Y(20)
      DOUBLE PRECISION X,Y,SUMX,SUMX2,SUMY,SUMXY,SLOPE,INTCPT,Q1,Q2,PTS
      DOUBLE PRECISION STDERR,XBAR,YBAR,SYP2,S2X
      PTS = NPTS
      SUMY = 0.D0
      SUMX = 0.D0
      SUMX2 = 0.D0
      SUMXY = 0.D0
      SYP2 = 0.D0
      DO 1 I=J,K
      SUMY = SUMY + Y(I)
      SUMX = SUMX + X(I)
      SUMX2 = SUMX2 + (X(I) ** 2)
1     SUMXY = SUMXY + (X(I) * Y(I))
      Q1 = SUMXY - (SUMX * SUMY)/PTS
      Q2 = SUMX2 - (SUMX ** 2)/PTS
      SLOPE = Q1/Q2
      INTCPT= (SUMY - SLOPE * SUMX) / PTS
      YBAR = SUMY / PTS
      XBAR = SUMX / PTS
C***  CALCULATION OF STATISTICAL QUANTITIES USED TO COMPUTE C.L.
      DO 2 I=J,K
2     SYP2 = SYP2 + (Y(I) - (YBAR + SLOPE*(X(I) - XBAR))) ** 2
      STDERR = SYP2 / (PTS - 2.D0)
      S2X = (SUMX2 - (SUMX**2)/PTS)/(PTS - 1.D0)
      RETURN
      END
```

Appendix V (Continued)

```
C           AVEVS === CALCULATION OF (V), (SW), AND (SV)
      SUBROUTINE AVEVS (DPTCP,CP3,B,N,AVEV,AVESW,AVESV,JOUT,ITITLE)
      DIMENSION DPTCP(20),CP3(20),ITITLE(20)
      DOUBLE PRECISION DPTCP,CP3,B,N,NOVER2,AVEV,AVESW,AVESV,SLOPE
      DOUBLE PRECISION INTCPT
      WRITE (JOUT,9001) ITITLE
      WRITE (JOUT,9002)
      WRITE (JOUT,9003) (CP3(I),DPTCP(I),I=1,9)
C***  J IS THE INITIAL POINT OF LINEAR INTERPRETATION OF THE DPTCP
C***  VS. CP3 PLOT.  K IS THE FINAL POINT.
      J = 5
      K = 9
      NPTS = K - J + 1
      WRITE (JOUT,9004) J,K
      WRITE (JOUT,9005)
      NOVER2 = 1.D0 - N / 2.D0
      CALL LINREG (CP3,DPTCP,NPTS,INTCPT,SLOPE,J,K)
      AVEV = ((SLOPE/INTCPT) / (57.D0*(NOVER2**6))) ** (1.D0/3.D0)
      DPTCP(1) = INTCPT + SLOPE * CP3(1)
      DPTCP(K) = INTCPT + SLOPE * CP3(K)
      AVESW = DSQRT((B*DSQRT(AVEV)*INTCPT) / (3.5D0*NOVER2))
      AVESV = AVESW / AVEV
      WRITE (JOUT,9003) CP3(1),DPTCP(1),CP3(K),DPTCP(K)
 9001 FORMAT (20A4)
 9002 FORMAT ('   9  1   1 -1   3')
 9003 FORMAT (2E15.7)
 9004 FORMAT ('LIN. REG. PTS. ',I1,' THRU ',I1)
 9005 FORMAT ('   2 -1 -1   0')
      RETURN
      END
/DATA
         FIELD FRACTION RETAINED ON 65 MESH     FIBER LENGTH = 1.63 MM.
  7.4633 11.6833 14.9100 17.5500 19.8000 21.8667 23.6967 25.2900 26.7633
  6.7979 78.5667 26.0000 65132.2
  6.1767  1.0338  0.8665  0.7039  0.5848  0.4851  0.4124  0.3536       F-65AVE
         FIELD FRACTION RETAINED ON 20 MESH     FIBER LENGTH = 3.05 MM.
  7.1475 11.1175 14.1275 16.6725 18.9375 20.8100 22.5775 24.2050 25.6825
  9.7035 78.2499 23.1250 62375.0
  7.2896  1.2830  1.0754  0.8736  0.7258  0.6021  0.5118  0.4388       F-20AVE
         FIELD FRACTION RETAINED ON 10 MESH     FIBER LENGTH = 3.94 MM.
  9.2700 14.9000 19.5067 23.4800 27.1100 30.4233 33.3600 36.1167 38.6800
 16.8514 81.1000 24.4000 97743.3
 11.2539  2.0327  1.7037  1.3841  1.1498  0.9538  0.8109  0.6952       F-10AVE
         FLOC FRACTION RETAINED ON 65 MESH     FIBER LENGTH = 1.74 MM.
  6.6767 10.8633 14.3600 17.4400 20.0733 22.5233 24.7666 26.8633 28.8267
 11.2212 84.9000 20.1667 75825.4
  8.8940  1.6190  1.3570  1.1024  0.9158  0.7597  0.6458  0.5537       B-65IIAV
         FLOC FRACTION RETAINED ON 65 MESH     FIBER LENGTH = 2.07 MM.
  6.1280 10.1960 13.5600 16.5120 19.1300 21.5700 23.8380 26.0020 27.9060
 15.3695 87.3800 23.5000 74956.9
 10.9453  2.0490  1.7174  1.3952  1.1590  0.9615  0.8174  0.7008       B-65IAVE
         FLOC FRACTION RETAINED ON 20 MESH     FIBER LENGTH = 2.98 MM.
  8.6375 14.4900 19.3725 23.7925 27.8100 31.5525 35.0875 38.3225 41.1600
 20.8284 87.6000 23.7000110658.8
 13.3006  2.5497  2.1370  1.7361  1.4422  1.1964  1.0171  0.8720       B-20AVE
         FLOC FRACTION RETAINED ON 10 MESH     FIBER LENGTH = 4.13 MM.
  8.1100 13.5100 18.1433 22.5133 26.3867 30.1700 33.8033 37.2933 40.7133
 30.3815 89.9666 26.8000111436.7
 11.5314  2.2036  1.8469  1.5004  1.2465  1.0340  0.8790  0.7536       B-10AVE
           WHOLE PULP RETAINED ON 65 MESH     FIBER LENGTH = 1.49 MM.
6.71     10.93    14.28    17.08    19.62    21.73    23.84    25.77    27.42     W-65-1F
9.8641   77.3     21.9     66882.94                                             W-65-1F
  8.5246  1.5735  1.3188  1.0714  0.8900  0.7383  0.6277  0.5381      W-65AVE
           WHOLE PULP RETAINED ON 20 MESH     FIBER LENGTH = 2.76 MM.
6.29     9.82     12.90    15.50    17.76    19.97    22.00    23.80    25.39     W-20-1F
17.1414  77.3     21.4     61389.57                                             W-20-1F
 13.9570  2.7308  2.2888  1.8594  1.5447  1.2814  1.0893  0.9339      W-20AVE
           WHOLE PULP RETAINED ON 10 MESH     FIBER LENGTH = 3.88 MM.
8.00     13.40    17.75    21.77    25.33    28.72    31.95    34.86    37.65     W-10-1F
24.8004  77.3     21.3     90146.69                                             W-10-1F
 17.9365  3.4449  2.8874  2.3457  1.9486  1.6165  1.3742  1.1782      W-10AVE
                          WHOLE PULP -- NOT CLASSIFIED
6.07     9.50     12.30    14.69    16.83    18.80    20.65    22.37    23.88     W-NC-AF
13.5742  77.3     22.5     58602.66                                             W-NC-AF
 10.9475  2.1378  1.7918  1.4557  1.2093  1.0032  0.8528  0.7311      W-NCAVE
9999999999999999999
```

APPENDIX VI

OUTPUT FROM PROGRAM CRFILT USING AVERAGED PRESSURE VS. TIME DATA FROM CONSTANT RATE FILTRATION, AND AVERAGED COMPRESSIBILITY DATA

Calculated values are presented for:

Average specific filtration resistance;

Local specific filtration resistance;

Apparent wet mat density;

Average specific surface; and

Average specific volume.

Appendix VI (Continued)

FIELD FRACTION RETAINED ON 65 MESH   FIBER LENGTH = 1.63 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	7.46	223.90	8150.	1.0338	0.0517
20.	11.68	350.50	13100.	0.8665	0.0616
30.	14.91	447.30	22900.	0.7039	0.0759
40.	17.55	526.50	37700.	0.5848	0.0913
50.	19.80	594.00	62300.	0.4851	0.1101
60.	21.87	656.00	96400.	0.4124	0.1295
70.	23.70	710.90	145800.	0.3536	0.1510
80.	25.29	758.70			
90.	26.76	802.90			

FILT. PAD WT. = 6.7979 G.   EQ. VOL. = 65132.20 CC.   CONSISTENCY = 0.010 PERCENT

FLOW RATE = 78.6 CC./SEC.   H2O VISC. =0.008705 POISE   COMP. PAD WT. = 6.1767 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE P		FILTRATION RESISTANCE AVERAGE (R)	LOCAL R	MAT DENSITY C	DP/DT
CM. H2O	DYNES/SQCM.	CM./G.	CM./G.	G./CC.	
10.	9807.	0.16170 08	0.26080 08	0.0553	70.63
20.	19613.	0.20660 08	0.32980 08	0.0716	89.32
30.	29420.	0.24290 08	0.41300 08	0.0833	111.82
40.	39227.	0.27510 08	0.49280 08	0.0927	133.44
50.	49033.	0.30480 08	0.56730 08	0.1007	153.63
60.	58840.	0.33120 08	0.64030 08	0.1078	173.39
70.	68647.	0.35660 08	0.70810 08	0.1141	191.75
80.	78453.	0.38190 08	0.76940 08	0.1199	208.35
90.	88260.	0.40590 08	0.82780 08	0.1253	224.17

EXPERIMENTAL EMPIRICAL CONSTANTS

C = M * (P ** N)
WHERE...
M = 0.18120-02
N = 0.3720

P = B0 + B1*(T**EXPM) + B2 *(T**EXPN)
WHERE...
B0 = -0.777440 06   EXPM = 0.10002D-01
B1 = 0.742610 06   EXPN = 0.240540 01
B2 = 0.73725D-02

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME, (V) = 2.50
AVERAGE SPECIFIC SURFACE, (SW) = 6500.
AVERAGE SPECIFIC SURFACE, (SV) = 2602.

Appendix VI (Continued)

FIELD FRACTION RETAINED ON 20 MESH FIBER LENGTH = 3.05 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	7.15	214.42	8150.	1.2830	0.0491
20.	11.12	333.52	13100.	1.0754	0.0586
30.	14.13	423.82	22900.	0.8736	0.0721
40.	16.67	500.17	37700.	0.7258	0.0868
50.	18.94	568.13	62300.	0.6021	0.1047
60.	20.81	624.30	96400.	0.5118	0.1231
70.	22.58	677.32	145800.	0.4388	0.1436
80.	24.20	726.15			
90.	25.68	770.47			

FILT. PAD WT. = 9.7035 G. EQ. VOL. = 62375.00 CC. CONSISTENCY = 0.016 PERCENT
FLOW RATE = 78.2 CC./SEC. H2O VISC. =0.009299 POISE COMP. PAD WT. = 7.2896 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE		FILTRATION RESISTANCE AVERAGE	FILTRATION RESISTANCE LOCAL	MAT DENSITY	
P		(R)	R	C	DP/DT
CM. H2O	DYNES/SQCM.	CM./G.	CM./G.	G./CC.	
10.	9807.	0.1069D 08	0.1721D 08	0.0526	73.58
20.	19613.	0.1375D 08	0.2227D 08	0.0681	95.24
30.	29420.	0.1623D 08	0.2756D 08	0.0792	117.85
40.	39227.	0.1834D 08	0.3258D 08	0.0881	139.35
50.	49033.	0.2018D 08	0.3737D 08	0.0958	159.82
60.	58840.	0.2204D 08	0.4151D 08	0.1025	177.53
70.	68647.	0.2370D 08	0.4555D 08	0.1085	194.82
80.	78453.	0.2526D 08	0.4938D 08	0.1140	211.17
90.	88260.	0.2679D 08	0.5293D 08	0.1192	226.34

EXPERIMENTAL EMPIRICAL CONSTANTS

C = M * (P ** N)
WHERE...
M = 0.1723D-02
N = 0.3720

P = B0 + B1*(T**EXPM) + B2 *(T**EXPN)
WHERE...
B0 = -0.64198D 06 EXPM = 0.10000D-01
B1 = 0.61381D 06 EXPN = 0.22618D 01
B2 = 0.21936D-01

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME, (V) = 2.59
AVERAGE SPECIFIC SURFACE, (SW) = 5438.
AVERAGE SPECIFIC SURFACE, (SV) = 2099.

Appendix VI (Continued)

FIELD FRACTION RETAINED ON 10 MESH   FIBER LENGTH = 3.94 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT

PRESSURE CM. H2O	CHART READING	TIME SEC.
10.	9.27	278.10
20.	14.90	447.00
30.	19.51	585.20
40.	23.48	704.40
50.	27.11	813.30
60.	30.42	912.70
70.	33.36	1000.80
80.	36.12	1083.50
90.	38.68	1160.40

COMPRESSIBILITY EXPERIMENT

PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
8150.	2.0327	0.0479
13100.	1.7037	0.0571
22900.	1.3841	0.0703
37700.	1.1498	0.0846
62300.	0.9538	0.1020
96400.	0.8109	0.1200
145800.	0.6952	0.1399

FILT. PAD WT. =16.8514 G.   EQ. VOL. = 97743.30 CC.   CONSISTENCY = 0.017 PERCENT
FLOW RATE = 81.1 CC./SEC.   H2O VISC. =0.009028 POISE   COMP. PAD WT. =11.2539 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE P CM. H2O	PRESSURE P DYNES/SQCM.	FILTRATION RESISTANCE AVERAGE (R) CM./G.	FILTRATION RESISTANCE LOCAL R CM./G.	MAT DENSITY C G./CC.	DP/DT
10.	9807.	0.7134D 07	0.1110D 08	0.0513	54.87
20.	19613.	0.8877D 07	0.1303D 08	0.0664	64.38
30.	29420.	0.1017D 08	0.1530D 08	0.0772	75.62
40.	39227.	0.1127D 08	0.1745D 08	0.0859	86.25
50.	49033.	0.1220D 08	0.1949D 08	0.0933	96.35
60.	58840.	0.1304D 08	0.2140D 08	0.0999	105.76
70.	68647.	0.1388D 08	0.2310D 08	0.1057	114.20
80.	78453.	0.1465D 08	0.2472D 08	0.1111	122.18
90.	88260.	0.1539D 08	0.2623D 08	0.1161	129.63

EXPERIMENTAL EMPIRICAL CONSTANTS

C = M * (P ** N)
WHERE...
M = 0.1679D-02
N = 0.3720

P = B0 + B1*(T**EXPM) + B2 *(T**EXPN)
WHERE...
B0 = -0.63735D 06   EXPM = 0.10006D-01
B1 = 0.60742D 06   EXPN = 0.19538D 01
B2 = 0.75838D-01

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME, (V) = 2.30
AVERAGE SPECIFIC SURFACE, (SW) = 4277.
AVERAGE SPECIFIC SURFACE, (SV) = 1856.

Appendix VI (Continued)

FLOC FRACTION RETAINED ON 65 MESH FIBER LENGTH = 1.74 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT

PRESSURE CM. H2O	CHART READING	TIME SEC.
10.	6.68	200.30
20.	10.86	325.90
30.	14.36	430.80
40.	17.44	523.20
50.	20.07	602.20
60.	22.52	675.70
70.	24.77	743.00
80.	26.86	805.90
90.	28.83	864.80

COMPRESSIBILITY EXPERIMENT

PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
8150.	1.6190	0.0475
13100.	1.3570	0.0567
22900.	1.1024	0.0697
37700.	0.9158	0.0840
62300.	0.7597	0.1012
96400.	0.6458	0.1191
145800.	0.5537	0.1389

FILT. PAD WT. =11.2212 G. EQ. VOL. = 75825.40 CC. CONSISTENCY = 0.015 PERCENT

FLOW RATE = 84.9 CC./SEC. H2O VISC. =0.009979 POISE COMP. PAD WT. = 8.8940 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE		FILTRATION RESISTANCE AVERAGE	LOCAL	MAT DENSITY	
P		(R)	R	C	DP/DT
CM. H2O	DYNES/SQCM.	CM./G.	CM./G.	G./CC.	
10.	9807.	0.9526D 07	0.1403D 08	0.0509	72.08
20.	19613.	0.1171D 08	0.1664D 08	0.0658	85.51
30.	29420.	0.1329D 08	0.1962D 08	0.0766	100.85
40.	39227.	0.1459D 08	0.2245D 08	0.0852	115.37
50.	49033.	0.1584D 08	0.2493D 08	0.0926	128.13
60.	58840.	0.1694D 08	0.2727D 08	0.0991	140.14
70.	68647.	0.1798D 08	0.2942D 08	0.1049	151.19
80.	78453.	0.1894D 08	0.3143D 08	0.1103	161.54
90.	88260.	0.1986D 08	0.3332D 08	0.1152	171.24

EXPERIMENTAL EMPIRICAL CONSTANTS

C = M * (P ** N)

WHERE...

M = 0.1666D-02

N = 0.3720

P = B0 + B1*(T**EXPM) + B2 *(T**EXPN)

WHERE...

B0 = -0.56162D 06 EXPM = 0.99979D-02

B1 = 0.53758D 06 EXPN = 0.19052D 01

B2 = 0.18961D 00

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME, (V) = 2.27

AVERAGE SPECIFIC SURFACE, (SW) = 4891.

AVERAGE SPECIFIC SURFACE, (SV) = 2158.

Appendix VI (Continued)

FLOC FRACTION RETAINED ON 65 MESH   FIBER LENGTH = 2.07 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT PRESSURE CM. H2O	CHART READING	TIME SEC.	COMPRESSIBILITY EXPERIMENT PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	6.13	183.84	8150.	2.0490	0.0462
20.	10.20	305.88	13100.	1.7174	0.0551
30.	13.56	406.80	22900.	1.3952	0.0678
40.	16.51	495.36	37700.	1.1590	0.0816
50.	19.13	573.90	62300.	0.9615	0.0984
60.	21.57	647.10	96400.	0.8174	0.1158
70.	23.84	715.14	145800.	0.7008	0.1350
80.	26.00	780.06			
90.	27.91	837.18			

FILT. PAD WT. =15.3695 G.   EQ. VOL. = 74956.90 CC.   CONSISTENCY = 0.021 PERCENT

FLOW RATE = 87.4 CC./SEC.   H2O VISC. =0.009218 POISE   COMP. PAD WT. =10.9453 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE P CM. H2O	DYNES/SQCM.	FILTRATION RESISTANCE AVERAGE (R) CM./G.	LOCAL R CM./G.	MAT DENSITY C G./CC.	DP/DT
10.	9807.	0.7656D 07	0.1054D 08	0.0495	73.46
20.	19613.	0.9203D 07	0.1280D 08	0.0640	89.15
30.	29420.	0.1038D 08	0.1496D 08	0.0744	104.21
40.	39227.	0.1137D 08	0.1688D 08	0.0829	117.59
50.	49033.	0.1226D 08	0.1857D 08	0.0900	129.37
60.	58840.	0.1305D 08	0.2012D 08	0.0963	140.19
70.	68647.	0.1378D 08	0.2155D 08	0.1020	150.12
80.	78453.	0.1443D 08	0.2289D 08	0.1072	159.46
90.	88260.	0.1513D 08	0.2405D 08	0.1120	167.57

EXPERIMENTAL EMPIRICAL CONSTANTS

C = M * (P ** N)
WHERE...
M = 0.1620D-02
N = 0.3720

P = B0 + B1*(T**EXPM) + B2 *(T**EXPN)
WHERE...
B0 = -0.39492D 06   EXPM = 0.99969D-02
B1 = 0.37901D 06   EXPN = 0.17557D 01
B2 = 0.57300D 00

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME, (V) = 2.19
AVERAGE SPECIFIC SURFACE, (SW) = 4344.
AVERAGE SPECIFIC SURFACE, (SV) = 1983.

FLOC FRACTION RETAINED ON 20 MESH   FIBER LENGTH = 2.98 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT PRESSURE CM. H2O	CHART READING	TIME SEC.	COMPRESSIBILITY EXPERIMENT PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	8.64	259.12	8150.	2.5497	0.0451
20.	14.49	434.70	13100.	2.1370	0.0538
30.	19.37	581.17	22900.	1.7361	0.0662
40.	23.79	713.77	37700.	1.4422	0.0797
50.	27.81	834.30	62300.	1.1964	0.0961
60.	31.55	946.57	96400.	1.0171	0.1131
70.	35.09	1052.62	145800.	0.8720	0.1319
80.	38.32	1149.67			
90.	41.16	1234.80			

FILT. PAD WT. =20.8284 G.   EQ. VOL. =110658.80 CC.   CONSISTENCY = 0.019 PERCENT

FLOW RATE = 87.6 CC./SEC.   H2O VISC. =0.009175 POISE   COMP. PAD WT. =13.3006 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE P CM. H2O	DYNES/SQCM.	FILTRATION RESISTANCE AVERAGE (R) CM./G.	LOCAL R CM./G.	MAT DENSITY C G./CC.	DP/DT
10.	9807.	0.59150 07	0.85550 07	0.0483	54.74
20.	19613.	0.70520 07	0.95400 07	0.0625	61.04
30.	29420.	0.79120 07	0.10840 08	0.0727	69.36
40.	39227.	0.85890 07	0.12110 08	0.0809	77.49
50.	49033.	0.91860 07	0.13290 08	0.0879	85.01
60.	58840.	0.97150 07	0.14380 08	0.0941	92.02
70.	68647.	0.10190 08	0.15410 08	0.0996	98.60
80.	78453.	0.10670 08	0.16340 08	0.1047	104.58
90.	88260.	0.11170 08	0.17160 08	0.1094	109.78

EXPERIMENTAL EMPIRICAL CONSTANTS

C = M * (P ** N)
WHERE...
M = 0.1582D-02
N = 0.3720

P = B0 + B1*(T**EXPM) + B2 *(T**EXPN)
WHERE...
B0 = -0.58421D 06   EXPM = 0.10009D-01
B1 = 0.55739D 06   EXPN = 0.17610D 01
B2 = 0.26444D 00

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME, (V) = 2.11
AVERAGE SPECIFIC SURFACE, (SW) = 3779.
AVERAGE SPECIFIC SURFACE, (SV) = 1790.

Appendix VI (Continued)

FLOC FRACTION RETAINED ON 10 MESH   FIBER LENGTH = 4.13 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT

PRESSURE CM. H2O	CHART READING	TIME SEC.
10.	8.11	243.30
20.	13.51	405.30
30.	18.14	544.30
40.	22.51	675.40
50.	26.39	791.60
60.	30.17	905.10
70.	33.80	1014.10
80.	37.29	1118.80
90.	40.71	1221.40

COMPRESSIBILITY EXPERIMENT

PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
8150.	2.2036	0.0452
13100.	1.8469	0.0540
22900.	1.5004	0.0664
37700.	1.2465	0.0800
62300.	1.0340	0.0964
96400.	0.8790	0.1134
145800.	0.7536	0.1323

FILT. PAD WT. =30.3815 G.   EQ. VOL. =111436.70 CC.   CONSISTENCY = 0.027 PERCENT

FLOW RATE = 90.0 CC./SEC.   H2O VISC. =0.008551 POISE   COMP. PAD WT. =11.5314 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE P CM. H2O	DYNES/SQCM.	FILTRATION RESISTANCE AVERAGE (R) CM./G.	LOCAL R CM./G.	MAT DENSITY C G./CC.	DP/DT
10.	9807.	0.4424D 07	0.5870D 07	0.0485	53.48
20.	19613.	0.5312D 07	0.7261D 07	0.0627	66.16
30.	29420.	0.5933D 07	0.8111D 07	0.0729	73.89
40.	39227.	0.6375D 07	0.8764D 07	0.0812	79.85
50.	49033.	0.6799D 07	0.9266D 07	0.0882	84.42
60.	58840.	0.7135D 07	0.9705D 07	0.0944	88.42
70.	68647.	0.7430D 07	0.1009D 08	0.1000	91.93
80.	78453.	0.7697D 07	0.1043D 08	0.1050	95.04
90.	88260.	0.7931D 07	0.1074D 08	0.1098	97.89

EXPERIMENTAL EMPIRICAL CONSTANTS

C = M * (P ** N)
WHERE...
M = 0.1587D-02
N = 0.3720

P = B0 + B1*(T**EXPM) + B2 *(T**EXPN)
WHERE...
B0 = 0.13216D 06   EXPM = 0.99997D-02
B1 = -0.12608D 06   EXPN = 0.13213D 01
B2 = 0.76378D 01

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME, (V) = 1.66
AVERAGE SPECIFIC SURFACE, (SW) = 3139.
AVERAGE SPECIFIC SURFACE, (SV) = 1895.

Appendix VI (Continued)

WHOLE PULP RETAINED ON 65 MESH   FIBER LENGTH = 1.49 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT PRESSURE CM. H2O	CHART READING	TIME SEC.	COMPRESSIBILITY EXPERIMENT PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	6.71	201.30	8150.	1.5735	0.0468
20.	10.93	327.90	13100.	1.3188	0.0559
30.	14.28	428.40	22900.	1.0714	0.0688
40.	17.08	512.40	37700.	0.8900	0.0828
50.	19.62	588.60	62300.	0.7383	0.0998
60.	21.73	651.90	96400.	0.6277	0.1174
70.	23.84	715.20	145800.	0.5381	0.1370
80.	25.77	773.10			
90.	27.42	822.60			

FILT. PAD WT. = 9.8641 G.   EQ. VOL. = 66882.94 CC.   CONSISTENCY = 0.015 PERCENT

FLOW RATE = 77.3 CC./SEC.   H2O VISC. =0.009571 POISE   COMP. PAD WT. = 8.5246 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE P CM. H2O	DYNES/SQCM.	FILTRATION RESISTANCE AVERAGE (R) CM./G.	LOCAL R CM./G.	MAT DENSITY C G./CC.	DP/DT
10.	9807.	0.1196D 08	0.1703D 08	0.0502	69.36
20.	19613.	0.1469D 08	0.2159D 08	0.0649	87.93
30.	29420.	0.1686D 08	0.2628D 08	0.0755	107.03
40.	39227.	0.1880D 08	0.3050D 08	0.0840	124.20
50.	49033.	0.2046D 08	0.3446D 08	0.0913	140.35
60.	58840.	0.2216D 08	0.3782D 08	0.0977	154.03
70.	68647.	0.2357D 08	0.4123D 08	0.1035	167.91
80.	78453.	0.2492D 08	0.4438D 08	0.1088	180.73
90.	88260.	0.2635D 08	0.4709D 08	0.1136	191.78

EXPERIMENTAL EMPIRICAL CONSTANTS

C = M * (P ** N)
WHERE...
M = 0.1643D-02
N = 0.3720

P = B0 + B1*(T**EXPM) + B2 *(T**EXPN)
WHERE...
B0 = -0.51176D 06   EXPM = 0.99991D-02
B1 = 0.49050D 06   EXPN = 0.20271D 01
B2 = 0.92674D-01

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME, (V) = 2.48
AVERAGE SPECIFIC SURFACE, (SW) = 5616.
AVERAGE SPECIFIC SURFACE, (SV) = 2261.

Appendix VI (Continued)

WHOLE PULP RETAINED ON 20 MESH FIBER LENGTH = 2.76 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	6.29	188.70	8150.	2.7308	0.0442
20.	9.82	294.60	13100.	2.2888	0.0527
30.	12.90	387.00	22900.	1.8594	0.0649
40.	15.50	465.00	37700.	1.5447	0.0781
50.	17.76	532.80	62300.	1.2814	0.0942
60.	19.97	599.10	96400.	1.0893	0.1108
70.	22.00	660.00	145800.	0.9339	0.1292
80.	23.80	714.00			
90.	25.39	761.70			

FILT. PAD WT. =17.1414 G. EQ. VOL. = 61389.57 CC. CONSISTENCY = 0.028 PERCENT

FLOW RATE = 77.3 CC./SEC. H2O VISC. =0.009686 POISE COMP. PAD WT. =13.9570 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE P CM. H2O	DYNES/SQCM.	FILTRATION RESISTANCE AVERAGE (R) CM./G.	LOCAL R CM./G.	MAT DENSITY C G./CC.	DP/DT
10.	9807.	0.6660D 07	0.1135D 08	0.0473	88.59
20.	19613.	0.8532D 07	0.1277D 08	0.0613	99.62
30.	29420.	0.9743D 07	0.1486D 08	0.0712	115.91
40.	39227.	0.1081D 08	0.1687D 08	0.0793	131.63
50.	49033.	0.1179D 08	0.1873D 08	0.0861	146.11
60.	58840.	0.1259D 08	0.2060D 08	0.0922	160.73
70.	68647.	0.1333D 08	0.2236D 08	0.0976	174.45
80.	78453.	0.1408D 08	0.2394D 08	0.1026	186.79
90.	88260.	0.1485D 08	0.2535D 08	0.1072	197.79

EXPERIMENTAL EMPIRICAL CONSTANTS

C = M * (P ** N)

WHERE...

M = 0.1550D-02

N = 0.3720

P = B0 + B1*(T**EXPM) + B2 *(T**EXPN)

WHERE...

B0 = -0.78906D 06 EXPM = 0.99968D-02

B1 = 0.75391D 06 EXPN = 0.19982D 01

B2 = 0.12453D 00

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME, (V) = 2.47

AVERAGE SPECIFIC SURFACE, (SW) = 4372.

AVERAGE SPECIFIC SURFACE, (SV) = 1769.

Appendix VI (Continued)

WHOLE PULP RETAINED ON 10 MESH   FIBER LENGTH = 3.88 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT

PRESSURE CM. H2O	CHART READING	TIME SEC.
10.	8.00	240.00
20.	13.40	402.00
30.	17.75	532.50
40.	21.77	653.10
50.	25.33	759.90
60.	28.72	861.60
70.	31.95	958.50
80.	34.86	1045.80
90.	37.65	1129.50

COMPRESSIBILITY EXPERIMENT

PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
8150.	3.4449	0.0450
13100.	2.8874	0.0537
22900.	2.3457	0.0661
37700.	1.9486	0.0796
62300.	1.6165	0.0959
96400.	1.3742	0.1128
145800.	1.1782	0.1316

FILT. PAD WT. =24.8004 G.   EQ. VOL. = 90146.69 CC.   CONSISTENCY =.0.028 PERCENT

FLOW RATE = 77.3 CC./SEC.   H2O VISC. =0.009709 POISE   COMP. PAD WT. =17.9365 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE P CM. H2O	PRESSURE P DYNES/SQCM.	FILTRATION RESISTANCE AVERAGE (R) CM./G.	FILTRATION RESISTANCE LOCAL R CM./G.	MAT DENSITY C G./CC.	DP/DT
10.	9807.	0.5302D 07	0.7184D 07	0.0482	55.36
20.	19613.	0.6331D 07	0.8824D 07	0.0624	68.00
30.	29420.	0.7159D 07	0.1012D 08	0.0726	78.00
40.	39227.	0.7794D 07	0.1126D 08	0.0808	86.77
50.	49033.	0.8373D 07	0.1222D 08	0.0877	94.17
60.	58840.	0.8862D 07	0.1310D 08	0.0939	100.92
70.	68647.	0.9294D 07	0.1390D 08	0.0994	107.13
80.	78453.	0.9735D 07	0.1460D 08	0.1045	112.54
90.	88260.	0.1014D 08	0.1526D 08	0.1092	117.58

EXPERIMENTAL EMPIRICAL CONSTANTS

C = M * (P ** N)

WHERE...

M = 0.1579D-02

N = 0.3720

P = B0 + B1*(T**EXPM) + B2 *(T**EXPN)

WHERE...

B0 = -0.23048D 06   EXPM = 0.10019D-01

B1 = 0.22094D 06   EXPN = 0.15996D 01

B2 = 0.10664D 01

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME, (V) = 2.08

AVERAGE SPECIFIC SURFACE, (SW) = 3608.

AVERAGE SPECIFIC SURFACE, (SV) = 1732.

Appendix VI (Continued)

WHOLE PULP -- NOT CLASSIFIED

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	6.07	182.10	8150.	2.1378	0.0443
20.	9.50	285.00	13100.	1.7918	0.0528
30.	12.30	369.00	22900.	1.4557	0.0650
40.	14.69	440.70	37700.	1.2093	0.0783
50.	16.83	504.90	62300.	1.0032	0.0943
60.	18.80	564.00	96400.	0.8528	0.1110
70.	20.65	619.50	145800.	0.7311	0.1295
80.	22.37	671.10			
90.	23.88	716.40			

FILT. PAD WT. =13.5742 G.   EQ. VOL. = 58602.66 CC.   CONSISTENCY = 0.023 PERCENT

FLOW RATE = 77.3 CC./SEC.   H2O VISC. =0.009436 POISE   COMP. PAD WT. =10.9475 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE P		FILTRATION RESISTANCE AVERAGE (R)	FILTRATION RESISTANCE LOCAL R	MAT DENSITY C	DP/DT
CM. H2O	DYNES/SQCM.	CM./G.	CM./G.	G./CC.	
10.	9807.	0.8540D 07	0.1355D 08	0.0474	85.43
20.	19613.	0.1091D 08	0.1696D 08	0.0614	106.92
30.	29420.	0.1264D 08	0.2009D 08	0.0714	126.69
40.	39227.	0.1412D 08	0.2283D 08	0.0794	143.98
50.	49033.	0.1540D 08	0.2530D 08	0.0863	159.53
60.	58840.	0.1654D 08	0.2757D 08	0.0924	173.82
70.	68647.	0.1757D 08	0.2969D 08	0.0978	187.20
80.	78453.	0.1854D 08	0.3165D 08	0.1028	199.58
90.	88260.	0.1954D 08	0.3337D 08	0.1074	210.39

EXPERIMENTAL EMPIRICAL CONSTANTS

C = M * (P ** N)
WHERE...
M = 0.1553D-02
N = 0.3720

P = B0 + B1*(T**EXPM) + B2 *(T**EXPN)
WHERE...
B0 = -0.40881D 06   EXPM = 0.10001D-01
B1 = 0.39156D 06   EXPN = 0.18625D 01
B2 = 0.37869D 00

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME, (V) = 2.52
AVERAGE SPECIFIC SURFACE, (SW) = 5003.
AVERAGE SPECIFIC SURFACE, (SV) = 1985.

99999999999999999999999999999999999999

APPENDIX VII

HISTORICAL PROGRAM AND OUTPUT FOR CALCULATION OF $\langle S_W \rangle$ AND $\langle v \rangle$ AS FUNCTIONS OF PRESSURE FROM FILTRATION RESISTANCE AND COMPRESSIBILITY DATA

```
/JOB GO,TIME=50
/FILE DISK=(1,IOUT,NREC=5000),RSIZ=80,VOL=SYSFL1,DISP=(NEW,DELETE)
/FILE DISK=(2,JOUT,NREC=5000),RSIZ=80,VOL=SYSFL1,DISP=(NEW,DELETE)
/FILE DISK=(3,KOUT,NREC=5000),RSIZ=80,VOL=SYSFL1,DISP=(NEW,DELETE)
C
C            CRFILP === CONSTANT RATE FILTRATION (PRESSURE)
C
C
C                          ANTHONY P. BINOTTO
C                          NOVEMBER 13,1974
C
C        THIS PROGRAM IS DESIGNED TO CALCULATE THE AVERAGE FIBER
C        SPECIFIC VOLUME, V, AND SPECIFIC SURFACE, SW, FROM CONSTANT
C        RATE FILTRATION EXPERIMENTS AND COMPRESSIBILITY EXPERIMENTS.
C        THE SPECIFIC VOLUME AND SPECIFIC SURFACE ARE ALSO
C        CALCULATED AS FUNCTIONS OF WET MAT PRESSURE BY EMPLOYING
C        THE FOLLOWING TECHNIQUE...
C        VALUES OF CP**3 VS. DP/(T*SQRT(CP)) ARE PLOTTED.  THE CURVED
C        PLOT WHICH USUALLY RESULTS IS FITTED EXACTLY WITH AN EQUATION
C        OF THE FORM...   Y = B0 + B1*(X**M) + B2*(X**N).   THIS EQUATION
C        IS USED TO GENERATE A LARGE NUMBER OF DATA POINTS.  THE DATA
C        POINTS ARE THEN TAKEN THREE AT A TIME AND FITTED WITH A BEST
C        STRAIGHT LINE USING LINEAR REGRESSION.  THE SLOPE AND INTERCEP
C        OF THE STRAIGHT LINE ARE USED TO CALCULATE V AND SW FROM THE
C        MODIFIED DARCY RELATIONSHIP...
C        DP/(T*SQRT(CP)) = A*((SW**2)/SQRT(V)) * (1 + B*(V**3)*CP**3))
C        WHERE...
C        A = (3.5*(1-N/2)*(W/EQVOL)*U0*MU)
C        B = 57.0*((1-N/2)**6)
C
C        OFF-LINE PLOTS OF ....
C            1.  CP**3 VS. DP/(T*SQRT(CP))
C            2.  V VS. PRESSURE
C            3.  SW VS. PRESSURE
C        MAY BE OBTAINED BY KEEPING DATA FILES AND USING PLOTTING
C        PROGRAM ''ABPLOT''.
C
C        THE USER MUST SUPPLY THE FOLLOWING DATA
C             1.  DATA CARD 1 -- ANY TITLE.  CENTERING ON THE CARD
C                 WILL ENSURE CENTERING ON OUTPUT.
C             2.  DATA CARD 2 -- CHART READINGS FROM THE CONSTANT
C                 RATE FILTRATION EXPERIMENT. CHART SPEED SHOULD BE
C                 2 INCHES PER MINUTE.  FORMAT (9F8.0).
C             3.  DATA CARD 3 -- THE FOLLOWING INFORMATION IN ORDER
C                 USEING FORMAT (9F8.0).
C                   A.  PAD WT. FROM FILTRATION EXPERIMENT IN GRAMS
C                   B.  FILTRATION FLOW RATE IN CC./SEC.
C                   C.  WATER TEMPERATURE IN DEG. C
C                   D.  FILTRATION EQUIVALENT VOLUME IN CC.
C            4.DATA CARD 4 -- PAD WT. FROM THE COMPRESSIBILITY
C               EXPERIMENT IN GRAMS, AND PAD THICKNESSES IN INCHES FROM
C               FOR THE RESPECTIVE INCREMENTS IN PRESSURE.  FORMAT(9F8.0
C
C
C
C        THE USERS CARD DECK SHOULD CONTAIN THE FOLLOWING CARDS
C                   /ID XXXXXXXX,XXX
C                   /INCLUDE CRFILP
C                   /INCLUDE CMPRSS
C                   /INCLUDE PRESSR
C                   /INCLUDE SUBCRV
C                   /INCLUDE LINREG
C                   /INCLUDE PLOTVS
C                   /DATA
C                   ****************
C                    USERS DATA DECK
C                   ****************
C                   9999999999999999
C                   /END
C
      DIMENSION DP(20),CP(20),CP2(20),CP3(20),T(20),Y(20),CHTRD(20)
      DIMENSION R(20),DP2(20),SCONC(20),PDTHCK(20),DPTCP(20)
      DIMENSION WPRESS(20),ITITLE(20),V(20),SW(20),SV(20)
      DOUBLE PRECISION DP,CP,CP2,CP3,T,Y,V,SW,SV,DP2,SCONC,PDTHCK
      DOUBLE PRECISION M,N,MU,WR,U0,TEMP,EQVOL,A
      COMMON ITITLE,NPTS,IOUT,JOUT,KOUT
      COMMON M,N,MU,U0,A,WR,EQVOL
      IOUT = 1
      JOUT = 2
      KOUT = 3
      LOUT = 1
      CHTSPD = 2.
      NPTS = 9
      PTS = NPTS
```

Appendix I (Continued)

```
      A = 45.43
      AC = 45.54
      DP2(3) = 9036.78
      DP2(4) = 13998.48
      DP2(5) = 23805.57
      DP2(6) = 38506.50
      DP2(7) = 63033.89
      DP2(8) = 97014.77
      DP2(9) = 146112.62
      WRITE (IOUT,30)
      WRITE (IOUT,31)
      WRITE (JOUT,45)
      WRITE (KOUT,45)
      WRITE (JOUT,46)
      WRITE (KOUT,47)
1     READ (5,2) ITITLE
      IF(ICOMP(ITITLE,1,249,4,1))3,99,3
2     FORMAT (20A4)
3     READ(5,4) (CHTRD(I),I=1,9)
      READ (5,4) WR,UO,TEMP,EQVOL
      READ (5,4) WCP,(PDTHCK(I),I=3,9)
4     FORMAT (9F8.0)
      IF (TEMP - 20.) 5,5,6
5     ETA=(1301./(998.333+8.1855*(TEMP-20.)+.00585*((TEMP-20.)**2)))
      MU = (10. ** ETA) * .01
      GO TO 7
6     ETA=(1.3272*(20.-TEMP)-.001053*((TEMP-20.)**2))/(TEMP+105.)
      MU = (10. ** ETA) * .01002
7     WRITE (6,8)
8     FORMAT (1H1,16X,'*****************************************************
     *****************************************')
      WRITE (6,9)
      WRITE (6,9)
      WRITE (6,10) ITITLE
      WRITE (6,9)
      WRITE (6,9)
9     FORMAT (' ',16X,'*',85X,'*')
10    FORMAT (' ',16X,'*',3X,20A4,2X,'*')
      WRITE (6,11)
11    FORMAT (' ',16X,'*****************************************************
     *****************************************')
      WRITE (6,12)
      WRITE (6,13)
12    FORMAT ('0','ORIGINAL DATA')
13    FORMAT (' ','-------- ----',//)
      WRITE (6,14)
14    FORMAT ('0',10X,'FILTRATION EXPERIMENT',31X,'COMPRESSIBILITY EXPE
     1RIMENT',/)
      WRITE (6,15)
      WRITE (6,16)
15    FORMAT(' ',10X,'PRESSURE',7X,'CHART',9X,'TIME',20X,'PRESSURE',10X
     1'PAD THICK.',8X,'SOLIDS CONC.')
16    FORMAT(' ',10X,'CM. H2O',8X,'READING',7X,'SEC.',20X,'DYNES/SQCM.'
     1,7X,'INCHES',12X,'G./CC.',//)
      C1 = WR / EQVOL
      C1P = C1 * 100.
      DO 17 I=3,9
17    SCONC(I) = WCP / (AC* PDTHCK(I) * 2.54)
      DO 18 I=1,9
      T(I) = (CHTRD(I) * 60.) / CHTSPD
      WPRESS(I) = I * 10
      DP(I) = WPRESS(I) * 980.665
      R(I) = (DP(I) * A**2)/(MU * T(I) * (UO**2) * C1)
18    WRITE (6,19) WPRESS(I),CHTRD(I),T(I),DP2(I),PDTHCK(I),SCONC(I)
19    FORMAT ('0',12X,F4.0,8X,F6.2,7X,F8.2,16X,F10.0,10X,F8.4,11X,F6.4)
      WRITE (6,20) WR,EQVOL,C1P
      WRITE (6,21) UO,MU,WCP
20    FORMAT ('0','FILT. PAD WT. =',F7.4,' G.',5X,'EQ. VOL. =',F9.2,
     1' CC.',7X,'CONSISTENCY = ',F5.3,' PERCENT')
21    FORMAT ('0','FLOW RATE =',F5.1,' CC./SEC.',5X,'H2O VISC. =',F8.6,
     1' POISE',5X,'COMP. PAD WT. =',F7.4,' G.',///)
C***** LEAST SQUARES APPROACH TO CALCULATING COMPRESSIBILITY
        CALL CMPRSS (WCP,PDTHCK,AC,N,M,CP,CP2,CP3,Y,DP2,DP,T)
      WRITE (6,22)
      WRITE (6,23)
22    FORMAT ('0','FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS
     1 FUNCTIONS OF PRESSURE')
23    FORMAT (' ','---------- ---------- --- --- ------ ------------- --
     1 -------- -- --------',//)
      WRITE (6,24)
      WRITE (6,25)
      WRITE (6,26)
24    FORMAT ('0',36X,'FILTRATION',5X,'MAT SOLIDS')
25    FORMAT (' ',15X,'PRESSURE',13X,'RESISTANCE',5X,'CONCENTRATION',
     15X,'DP/(T*SQRT(CP))',5X,'CP**3',/)
26    FORMAT ('0',10X,'CM. H2O',4X,'DYNES/SQCM.',6X,'CM./G.',10X,
     1'G./CC.',//)
      DO 27 I=1,9
```

```
27    WRITE (6,28) WPRESS(I),DP(I),R(I),CP(I),Y(I),CP3(I)
28    FORMAT ('0',12X,F4.0,4X,F10.0,6X,E9.4,7X,F8.4,10X,F9.2,6X,E12.4)
      WRITE (6,29) M,N
29    FORMAT ('0',25X,'M =',E10.4,10X,'N =',F6.4,//)
C***** CALCULATION OF V, SW, AND SV
 30   FORMAT (' CP**3')
 31   FORMAT ('DP/(T*SQRT(CP))')
      CALL PRESSR (CP3,Y,V,SW,SV)
      WRITE (6,32)
      WRITE (6,33)
      WRITE (6,34)
      WRITE (6,35)
32    FORMAT ('0','HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS
     1 OF PRESSURE')
33    FORMAT (' ','------------ -------- ------ --- ------- -- ---------
     1 -- --------',//)
34    FORMAT (' ',15X,'PRESSURE',11X,'V',13X,'SW',16X,'SV')
35    FORMAT (' ',15X,'CM. H2O',10X,'CC./G.',8X,'SQCM./G.',9X
     1,'SQCM./CC.',//)
      DO 36 I=1,9
36    WRITE (6,37) WPRESS(I),V(I),SW(I),SV(I)
37    FORMAT ('0',18X,F3.0,11X,F5.2,7X,F10.2,7X,F10.2)
      WRITE (6,38)
38    FORMAT ('0',///)
      WRITE (LOUT,2) ITITLE
      WRITE (LOUT,39)(WPRESS(I),DP(I),R(I),CP(I),V(I),SW(I),SV(I),I=1,9)
39    FORMAT (7E11.4)
      CALL AVE (PTS,V,SW,SV,VBAR,SWBAR,SVBAR)
      WRITE (6,40)
      WRITE (6,41)
40    FORMAT ('0','AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE')
41    FORMAT (' ','------- ------ --- -------- ------ --- -------',//)
      WRITE (6,42) VBAR
      WRITE (6,43) SWBAR
      WRITE (6,44) SVBAR
42    FORMAT ('0',5X,'AVERAGE SPECIFIC VOLUME = ',F5.2,' CC./G.')
43    FORMAT ('0',5X,'AVERAGE SPECIFIC SURFACE, SW = ',F9.2,' SQCM./G.')
44    FORMAT ('0',5X,'AVERAGE SPECIFIC SURFACE, SV = ',F9.2,' SQCM./CC.
     * ',///)
      WRITE (6,38)
      GO TO 1
99    WRITE (IOUT,48)
      END FILE IOUT
      WRITE (JOUT,48)
      END FILE JOUT
      WRITE (KOUT,48)
      END FILE KOUT
45    FORMAT ('PRESSURE, CM. H2O')
46    FORMAT ('SPEC. VOLUME, CC./G.')
47    FORMAT ('SPEC. SURFACE, SQCM./G.')
48    FORMAT ('9999999999999999999999999999999999999999')
      CALL EXIT
      END
       SUBROUTINE AVE (PTS,V,SW,SV,VBAR,SWBAR,SVBAR)
       DIMENSION V(20),SW(20),SV(20)
       DOUBLE PRECISION V,SW,SV
       NPTS = PTS
       SUMV = 0.
       SUMSW = 0.
       SUMSV = 0.
       DO 1 I=1,NPTS
       SUMV = SUMV + V(I)
       SUMSW = SUMSW + SW(I)
1      SUMSV = SUMSV + SV(I)
       VBAR = SUMV / PTS
       SWBAR = SUMSW / PTS
       SVBAR = SUMSV / PTS
       RETURN
       END
C                SUBROUTINE CMPRSS === COMPRESSIBILITY
C
C                         ANTHONY P. BINOTTO
C                           JUNE 16, 1975
C
C          THIS SUBROUTINE IS USED TO CALCULATE THE MAT SOLIDS
C          CONCENTRATION FROM MEASUREMENTS OF PAD THICKNESS AND THE
C          O.D. WT. OF THE MAT.  THE PROCEDURE INVOLVES A LEAST SQUARES
C          APPROACH TO COMPUTING COMPRESSIBILITY CONSTANTS M AND N.
C
C
       SUBROUTINE CMPRSS (W,PDTHCK,AREA,N,M,CP,CP2,CP3,DPTCP,DP2,DP,T)
       DIMENSION PDTHCK(20),MATCON(20),LMC(20),LDP2(20),DP2(20),CP2(20)
       DIMENSION DP(20),CP(20),T(20),CP3(20),DPTCP(20)
       DOUBLE PRECISION MATCON,LMC,LDP2,SUMY,SUMY2,SUMX,SUMX2,SUMXY,Q1
       DOUBLE PRECISION Q2,N,M,SUMM,CP2,CP,CP3,DPTCP,DP,PDTHCK,DP2,T
       NPTS = 7
       PTS = NPTS
       SUMY = 0.
```

Appendix VII (Continued)

```
      SUMY2 = 0.
      SUMX = 0.
      SUMX2 = 0.
      SUMXY = 0.
      SUMM = 0.
      DO 1 I=3,9
      MATCON(I) = W / (AREA * PDTHCK(I) * 2.54)
      LMC(I) = DLOG(MATCON(I))
      LDP2(I) = DLOG(DP2(I))
      SUMY = SUMY + LMC(I)
      SUMY2 = SUMY2 + (LMC(I) ** 2)
      SUMX = SUMX + LDP2(I)
      SUMX2 = SUMX2 + (LDP2(I) ** 2)
1     SUMXY = SUMXY + (LMC(I) * LDP2(I))
      Q1 = ( PTS * SUMXY) - (SUMX * SUMY)
      Q2 = ( PTS * SUMX2) - (SUMX ** 2)
      N = Q1 / Q2
      DO 2 I=3,9
2     SUMM = SUMM + (MATCON(I) / (DP2(I) ** N))
      M = SUMM / NPTS
      DO 3 I=1,9
      CP(I) = M * DP(I) ** N
      CP2(I) = M * DP2(I) ** N
      CP3(I) = CP(I) ** 3
3     DPTCP(I) = DP(I) / (T(I) * SQRT(CP(I)))
      RETURN
      END
C          SUBROUTINE PRESSR === V,SW,SV AS FUNCTIONS OF PRESSURE
C
C                        ANTHONY P. BINOTTO
C                          JUNE 17,1975
C
C         THIS SUBROUTINE CALCULATES VALUES OF V, SW, AND SV AS FUNCTION
C         OF PRESSURE FROM FILTRATION RESISTANCE AND COMPRESSIBILITY
C         DATA.
C         THE PROCEDURE INVOLVES FITTING THE BEST SMOOTH CURVE USING
C         NON-LINEAR REGRESSION (NLREG) TO ANALYSE THE PLOT OF CP**3 VS.
C         DP/(T*SQRT(CP)).  THE DERIVITIVES (SLOPES) AND INTERCEPT
C         ARE CALCULATED FROM THE REGRESSION EQUATIONS AND ARE USED TO
C         SOLVE THE MODIFIED DARCY EQUATION.
C
      SUBROUTINE PRESSR (CP3,DPTCP,V,SW,SV)
      DIMENSION CP3(20),DPTCP(20),V(20),SW(20),SV(20),ITITLE(20)
      DIMENSION X1(3),Y1(3),X(500),Y(500),DP(500),WPRESS(500)
      DOUBLE PRECISION CP3,DPTCP,SLOPE,INTCPT,CONST1,CONST2,EQ1
      DOUBLE PRECISION EQ2,V,SW,SV,N,CONSIS,MU,UO,A,WR,EQVOL,M
      DOUBLE PRECISION B0,B1,B2,EXPM,EXPN,X,Y,DP,X1,Y1
      COMMON ITITLE,NPTS,IOUT,JOUT,KOUT
      COMMON M,V,MU,UO,A,WR,EQVOL
      DPTS = 300.0
      DO 1 I=1,NPTS
      X(I) = CP3(I)
1     Y(I) = DPTCP(I)
      CONSIS = WR / EQVOL
      CONST1 = ((1 - N/2.)* 3.5 * CONSIS * UO * MU) / (A**2)
      CONST2 = 57. * ((1 - N/2.)**6)
      CALL CRVFIT (X,Y,B0,B1,B2,EXPM,EXPN,DPTS)
      XFIRST = CP3(1)
      XLAST = CP3(NPTS)
      XINCR = (XLAST - XFIRST) / DPTS
      DO 3 I=1,NPTS
      X1(1)= CP3(I) - XINCR
      X1(2)= CP3(I)
      X1(3)= CP3(I) + XINCR
      DO 2 J=1,3
2     Y1(J)= B0 + B1*(X1(J)**EXPM) + B2*(X1(J)**EXPN)
      CALL LINREG (X1,Y1,3,INTCPT,SLOPE)
      EQ1 = INTCPT / CONST1
      EQ2 = DABS(SLOPE) / (CONST1 * CONST2)
      V(I) = (EQ2/EQ1) ** .333333
      SW(I) = DSQRT((INTCPT * DSQRT(V(I))) / (CONST1 * UO))
      SV(I) = SW(I) / V(I)
      DP(I) =((X1(2)**.333333)/M)**(1/N)
3     WPRESS(I) = DP(I) / 980.665
      WRITE (JOUT,4) ITITLE
      WRITE (KOUT,4) ITITLE
      WRITE (JOUT,5) NPTS
      WRITE (KOUT,5) NPTS
      WRITE (JOUT,6) (WPRESS(I),V(I),I=1,NPTS)
      WRITE (KOUT,6) (WPRESS(I),SW(I),I=1,NPTS)
4     FORMAT (20A4)
5     FORMAT (I3,'  2  2 -1  3')
6     FORMAT (2E15.7)
      CALL PLOTVS (X,Y,DPTS,CONST1,CONST2)
      RETURN
      END
```

```
C                 CRVFIT === CURVE FITTING
C
C                   ANTHONY P. BINOTTO
C                    AUGUST 25, 1975
C
C       THIS PROGRAM IS DESIGNED TO CALCULATE THE BEST EQUATION
C       OF THE FORM...
C                     Y = B0 + B1*(X**M) + B2*(X**N)
C       TO ANY SET OF DATA.  THE PROCEDURE INVOLVES MINIMIZING THE
C       SUM OF SQUARES.
C       *** WARNING***  THE RESULTING EQUATION WILL ONLY BE ONE OF
C       MANY COMBINATIONS OF B0,B1,B2,M,N WHICH GIVE A ''BEST''
C       FIT TO THE DATA.  THE FINAL EQUATION IS THEREFORE DEPENDENT
C       ON THE INITIAL ESTIMATE OF M AND N.
C
        SUBROUTINE CRVFIT (X,Y,B0,B1,B2,M,N,DPTS)
        DIMENSION ITITLE(20),A(5,6),X(500),Y(500)
        DOUBLE PRECISION X,Y,B0,B1,B2,M,N,A
        DOUBLE PRECISION DELB0,DELB1,DELB2,DELM,DELN
        COMMON ITITLE,NPTS,IOUT,JOUT,KOUT
C****   IOUT == OUTPUT STORED ON DISK OR TAPE FOR PLOTTING.
        NCOUNT = 0.0
        WRITE (IOUT,1) ITITLE
        WRITE (IOUT,2) NPTS
        WRITE (IOUT,3) (X(I),Y(I),I=1,NPTS)
1       FORMAT (20A4)
2       FORMAT (I3,'  1   1 -1   3')
3       FORMAT (2E15.7)
C****   ESTIMATE VALUES OF B0,B1,B2,M,N
        B0 = 200.0
        B1 = 1000.0
        B2 = 1000.0
        M = 0.120
        N = 0.500
        TEST1 = 0.0
4       CALL SUM (X,Y,NPTS,B0,B1,B2,M,N,A)
        CALL SIMEQN (A,5,6)
        DELB0 = A(1,6) / A(1,1)
        DELB1 = A(2,6) / A(2,2)
        DELB2 = A(3,6) / A(3,3)
        DELM = A(4,6) / A(4,4)
        DELN = A(5,6) / A(5,5)
C****   DATA FIT MAY BE IMPROVED AT THE EXPENSE OF CALCULATION TIME
C****   BY REDUCING THE VALUE OF ''TEST''.
        TEST = 10.
        TEST2 = DELB0 + DELB1 + DELB2 + DELM + DELN
        DIFF = TEST2 - TEST1
        IF (ABS(DIFF)-TEST) 7,7,5
5       TEST1 = TEST2
        NCOUNT = NCOUNT + 1
        B0 = B0 + DELB0
        B1 = B1 + DELB1
        B2 = B2 + DELB2
        IF (NCOUNT-100) 4,4,6
6       NCOUNT = 0
        M = M + DELM
        N = N + DELN
        GO TO 5
7       CALL EQN (B0,B1,B2,M,N,IOUT,X,Y,DPTS)
        RETURN
        END

        SUBROUTINE EQN (B0,B1,B2,M,N,IOUT,X,Y,DPTS)
C***    THIS SUBROUTINE GENERATES DATA POINTS FROM THE CALCULATED
C***    EQUATION TO BE USED IN PLOTTING PROGRAM ''ABPLOT''.
        DIMENSION X(500),Y(500)
        DOUBLE PRECISION X,Y,DPTS,XFIRST,XLAST,XINCR,B0,B1,B2,M,N
        LPTS = DPTS
        XFIRST = X(1)
        XLAST = X(9)
        DPTS = LPTS
        XINCR = (XLAST - XFIRST) / DPTS
        WRITE (IOUT,1)
        WRITE (IOUT,2) LPTS
1       FORMAT ('Y = B0 + B1*(X**M) + B2*(X**N)')
2       FORMAT (I3,' -1 -1  0')
        X(1) = XFIRST
        DO 3 I=1,LPTS
        Y(I) = B0 + B1*(X(I)**M) + B2*(X(I)**N)
        X(I+1) = X(I) + XINCR
3       WRITE (IOUT,4) X(I),Y(I)
4       FORMAT (2E15.7)
        RETURN
        END
        SUBROUTINE SUM (X,Y,NPTS,B0,B1,B2,M,N,A)
        DIMENSION A(5,6),X(500),Y(500)
        DOUBLE PRECISION X,Y,B0,B1,B2,M,N,A,PTS,SXMYR2
        DOUBLE PRECISION SY,SXM,SXN,SXMY,SX2M,SXMXN,SXNY,SX2N,SXMYR,SXMR
```

Appendix VII (Continued)

```
      DOUBLE PRECISION SX2MR,SXMXNR,SXNYR,SXNR,SXMR2,SX2MR2,SXMNR2
      DOUBLE PRECISION SXNR2,SX2NR2,SXNYR2,SX2NR,XN,XM,X2M,X2N,R,R2
      PTS = NPTS
      SY = 0.0
      SXM = 0.0
      SXN = 0.0
      SXMY = 0.0
      SX2M = 0.0
      SXMXN = 0.0
      SXNY = 0.0
      SX2N = 0.0
      SXMYR = 0.0
      SXMR = 0.0
      SX2MR = 0.0
      SXMXNR = 0.0
      SXNYR = 0.0
      SXNR = 0.0
      SX2NR = 0.0
      SXMR2 = 0.0
      SX2MR2 = 0.0
      SXMNR2 = 0.0
      SXMYR2 = 0.0
      SXNR2 = 0.0
      SX2NR2 = 0.0
      SXNYR2 = 0.0
      DO 1 I=1,NPTS
      XM = X(I) ** M
      XN = X(I) ** N
      X2M = XM ** 2
      X2N = XN ** 2
      R = DLOG(X(I))
      R2 = R ** 2
      SY = SY + Y(I)
      SXM = SXM + XM
      SXN = SXN + XN
      SXMY = SXMY + (XM * Y(I))
      SX2M = SX2M + X2M
      SXMXN = SXMXN + (XM * XN)
      SXNY = SXNY + (XN * Y(I))
      SX2N = SX2N + X2N
      SXMYR = SXMYR + (XM * Y(I) * R)
      SXMR = SXMR + (XM * R)
      SX2MR = SX2MR + (X2M * R)
      SXMXNR = SXMXNR + (XM * XN * R)
      SXNYR = SXNYR + (XN * Y(I) * R)
      SXNR = SXNR + (XN * R)
      SX2NR = SX2NR + (X2N * R)
      SXMR2 = SXMR2 + (XM * R2)
      SX2MR2 = SX2MR2 + (X2M * R2)
      SXMNR2 = SXMNR2 + (XM * XN * R2)
      SXMYR2 = SXMYR2 + (XM * Y(I) * R2)
      SXNR2 = SXNR2 + (XN * R2)
      SX2NR2 = SX2NR2 + (X2N * R2)
1     SXNYR2 = SXNYR2 + (XN * Y(I) * R2)
      A(1,1) = PTS
      A(1,2) = SXM
      A(1,3) = SXN
      A(1,4) = B1 * SXMR
      A(1,5) = B2 * SXNR
      A(1,6) = -(PTS*B0 + B1*SXM + B2*SXN - SY)
      A(2,1) = SXM
      A(2,2) = SX2M
      A(2,3) = SXMXN
      A(2,4) = B0*SXMR + 2.0*B1*SX2MR + B2*SXMXNR - SXMYR
      A(2,5) = B2*SXMXNR
      A(2,6) = -(B0*SXM + B1*SX2M + B2*SXMXN - SXMY)
      A(3,1) = SXN
      A(3,2) = SXMXN
      A(3,3) = SX2N
      A(3,4) = SXMXNR
      A(3,5) = B0*SXNR + B1*SXMXNR + 2.0*B2*SX2NR - SXNYR
      A(3,6) = -(B0*SXN + B1*SXMXN + B2*SX2N - SXNY)
      A(4,1) = SXMR
      A(4,2) = SX2MR
      A(4,3) = SXMXNR
      A(4,4) = B0*SXMR2 + 2.0*B1*SX2MR2 + B2*SXMNR2 - SXMYR2
      A(4,5) = B2*SXMNR2
      A(4,6) = -(B0*SXMR + B1*SX2MR + B2*SXMXNR - SXMYR)
      A(5,1) = SXNR
      A(5,2) = SXMXNR
      A(5,3) = SX2NR
      A(5,4) = B1*SXMNR2
      A(5,5) = B0*SXNR2 + B1*SXMNR2 + 2.0*B2*SX2NR2 - SXNYR2
      A(5,6) = -(B0*SXNR + B1*SXMXNR + B2*SX2NR - SXNYR)
      RETURN
      END
```

```
C          SIMEQN === SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS
C
      SUBROUTINE SIMEQN (A,NR,NC)
C******        SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS
C******               MATRIX OF COEFFICIENTS = A
C******               NR = NUMBER OF ROWS IN A
C******               NC = NUMBER OF COLUMNS IN A
      DIMENSION A(5,6),R(100)
      DOUBLE PRECISION A,R,D,T
      IF(NR - NC) 4,4,3
    3 NCT = NC
      GO TO 5
    4 NCT = NR
    5 K = 1
      D = 1.
    7 IF(A(K,K)) 12,8,12
C******     DIAGONAL=0, FIND A ROW WITH A NON-ZERO ELEMENT
C******     AND INTERCHANGE THE ROWS
    8 DO 9 I = K,NR
      IF(A(I,K)) 10,9,10
    9 CONTINUE
C******    IF THERE IS NO NON-ZERO ELEMENT, PROBLEM IS COMPLETE
      GO TO 17
C******    INTERCHANGE ROW I AND ROW K
   10 DO 11 J = 1,NC
      T = A(I,J)
      A(I,J) = A(K,J)
   11 A(K,J) = T
C******     CORRECT THE SYSTEM OF EQUATIONS FOR ROW K
   12 DO 16 I = 1,NR
      IF(I - K) 13,16,13
   13 DO 14 J = 1,NC
   14 R(J) = (A(I,J)*A(K,K) - A(I,K)*A(K,J))/D
      DO 15 J = 1,NC
   15 A(I,J) = R(J)
   16 CONTINUE
      D = A(K,K)
      K = K + 1
      IF(K - NCT) 7,7,17
   17 RETURN
      END

C                   LINREG === LINEAR REGRESSION
C                          ANTHONY P. BINOTTO
C                             JUNE 16, 1975
C
C          THIS SUBROUTINE IS USED TO CALCULATE THE STRAIGHT LINE
C          RELATIONSHIP BETWEEN PAIRS OF X,Y-DATA
C
       SUBROUTINE LINREG (X,Y,NPTS,INTCPT,SLOPE)
       DIMENSION X(10),Y(10)
       DOUBLE PRECISION X,Y,LY,SUMX,SUMX2,SUMY,SUMY2,SUMXY,SLOPE,INTCPT
       DOUBLE PRECISION Q1,Q2,PTS
       PTS = NPTS
       SUMY = 0.
       SUMX = 0.
       SUMX2 = 0.
       SUMXY = 0.
       SUMINT = 0.
       DO 1 I=1,NPTS
       SUMY = SUMY + Y(I)
       SUMX = SUMX + X(I)
       SUMX2 = SUMX2 + (X(I) ** 2)
1      SUMXY = SUMXY + (X(I) * Y(I))
       Q1 = SUMXY - (SUMX * SUMY)/PTS
       Q2 = SUMX2 - (SUMX ** 2)/PTS
       SLOPE = Q1/Q2
       INTCPT= (SUMY - SLOPE * SUMX) / PTS
       RETURN
       END

C          PLOTVS === PLOTTING POINTS FOR V AND SW
C
       SUBROUTINE PLOTVS (X,Y,DPTS,CONST1,CONST2)
       DIMENSION ITITLE(20),X(500),Y(500),X1(3),Y1(3)
       DIMENSION V(300),SW(300),SV(300),DP(500),WPRESS(500)
       DOUBLE PRECISION M,N,MU,UO,A,WR,EQVOL,X,Y,X1,Y1,V,SW,SV,DP
       DOUBLE PRECISION WPRESS,CONST1,CONST2,SLOPE,INTCPT
       COMMON ITITLE,NPTS,IOUT,JOUT,KOUT
       COMMON M,N,MU,UO,A,WR,EQVOL
       NCOUNT = 0
       LPTS = DPTS/3.0
       WRITE (JOUT,1)
       WRITE (KOUT,1)
       WRITE (JOUT,2) LPTS
       WRITE (KOUT,2) LPTS
1      FORMAT ('         ')
2      FORMAT (I3,' -2 -2  0')
```

Appendix VII (Continued)

```
      I = 1
      J = 1
      K = 1
3     X1(1) = X(J)
      X1(2) = X(J+1)
      X1(3) = X(J+2)
      Y1(1) = Y(J)
      Y1(2) = Y(J+1)
      Y1(3) = Y(J+2)
      CALL LINREG (X1,Y1,3,INTCPT,SLOPE)
      EQ1 = INTCPT/CONST1
      EQ2 = DABS(SLOPE)/(CONST1*CONST2)
      V(K) = (EQ2/EQ1) ** .333333
      SW(K) = DSQRT((INTCPT*DSQRT(V(K)))/(CONST1*UO))
      SV(K) = SW(K)/V(K)
      DP(K) = ((X1(2)**.333333)/M) ** (1/N)
      WPRESS(K) = DP(K) / 980.665
      WRITE (JOUT,4) WPRESS(K),V(K)
      WRITE (KOUT,4) WPRESS(K),SW(K)
4     FORMAT (2E15.7)
      J = J + 3
      K = K + 1
      NCOUNT = NCOUNT + 1
      IF (NCOUNT - LPTS) 3,5,5
5     RETURN
      END
/DATA
        FIELD FRACTION RETAINED ON 65 MESH      FIBER LENGTH = 1.63 MM.
  7.4633 11.6833 14.9100 17.5500 19.8000 21.8667 23.6967 25.2900 26.7633
  6.7979 78.5667 26.0000 65132.2
  6.1767  1.0338  0.8665  0.7039  0.5848  0.4851  0.4124  0.3536      F-65AVE
        FIELD FRACTION RETAINED ON 20 MESH      FIBER LENGTH = 3.05 MM.
  7.1475 11.1175 14.1275 16.6725 18.9375 20.8100 22.5775 24.2050 25.6825
  9.7035 78.2499 23.1250 62375.0
  7.2896  1.2830  1.0754  0.8736  0.7258  0.6021  0.5118  0.4388      F-20AVE
        FIELD FRACTION RETAINED ON 10 MESH      FIBER LENGTH = 3.94 MM.
  9.2700 14.9000 19.5067 23.4800 27.1100 30.4233 33.3600 36.1167 38.6800
 16.8514 81.1000 24.4000 97743.3
 11.2539  2.0327  1.7037  1.3841  1.1498  0.9538  0.8109  0.6952      F-10AVE
        FLOC FRACTION RETAINED ON 65 MESH      FIBER LENGTH = 1.74 MM.
  6.6767 10.8633 14.3600 17.4400 20.0733 22.5233 24.7666 26.8633 28.8267
 11.2212 84.9000 20.1667 75825.4
  8.8940  1.6190  1.3570  1.1024  0.9158  0.7597  0.6458  0.5537      B-65IIAV
        FLOC FRACTION RETAINED ON 65 MESH      FIBER LENGTH = 2.07 MM.
  6.1280 10.1960 13.5600 16.5120 19.1300 21.5700 23.8380 26.0020 27.9060
 15.3695 87.3800 23.5000 74956.9
 10.9453  2.0490  1.7174  1.3952  1.1590  0.9615  0.8174  0.7008      B-65IAVE
        FLOC FRACTION RETAINED ON 20 MESH      FIBER LENGTH = 2.98 MM.
  8.6375 14.4900 19.3725 23.7925 27.8100 31.5525 35.0875 38.3225 41.1600
 20.8284 87.6000 23.7000110658.8
 13.3006  2.5497  2.1370  1.7361  1.4422  1.1964  1.0171  0.8720      B-20AVE
        FLOC FRACTION RETAINED ON 10 MESH      FIBER LENGTH = 4.13 MM.
  8.1100 13.5100 18.1433 22.5133 26.3867 30.1700 33.8033 37.2933 40.7133
 30.3815 89.9666 26.8000111436.7
 11.5314  2.2036  1.8469  1.5004  1.2465  1.0340  0.8790  0.7536      B-10AVE
          WHOLE PULP RETAINED ON 65 MESH      FIBER LENGTH = 1.49 MM.
6.71    10.93    14.28    17.08    19.62    21.73    23.84    25.77    27.42      W-65-1F
9.8641  77.3     21.9     66882.94                                             W-65-1F
  8.5246  1.5735  1.3188  1.0714  0.8900  0.7383  0.6277  0.5381      W-65AVE
          WHOLE PULP RETAINED ON 20 MESH      FIBER LENGTH = 2.76 MM.
6.29     9.82    12.90    15.50    17.76    19.97    22.00    23.80    25.39      W-20-1F
17.1414 77.3     21.4     61389.57                                             W-20-1F
 13.9570  2.7308  2.2888  1.8594  1.5447  1.2814  1.0893  0.9339      W-20AVE
          WHOLE PULP RETAINED ON 10 MESH      FIBER LENGTH = 3.88 MM.
8.00    13.40    17.75    21.77    25.33    28.72    31.95    34.86    37.65      W-10-1F
24.8004 77.3     21.3     90146.69                                             W-10-1F
 17.9365  3.4449  2.8874  2.3457  1.9486  1.6165  1.3742  1.1782      W-10AVE
                   WHOLE PULP -- NOT CLASSIFIED
6.07     9.50    12.30    14.69    16.83    18.80    20.65    22.37    23.88      W-NC-AF
13.5742 77.3     22.5     58602.66                                             W-NC-AF
 10.9475  2.1378  1.7918  1.4557  1.2093  1.0032  0.8528  0.7311      W-NCAVE
99999999999999999999

/END CARD READ, JOB TERMINATED
```

Appendix VII (Continued)

FIELD FRACTION RETAINED ON 65 MESH   FIBER LENGTH = 1.63 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	7.46	223.90	0.	0.0	0.0
20.	11.68	350.50	0.	0.0	0.0
30.	14.91	447.30	9037.	1.0338	0.0517
40.	17.55	526.50	13998.	0.8665	0.0616
50.	19.80	594.00	23806.	0.7039	0.0759
60.	21.87	656.00	38507.	0.5848	0.0913
70.	23.70	710.90	63034.	0.4851	0.1101
80.	25.29	758.70	97015.	0.4124	0.1295
90.	26.76	802.90	146113.	0.3536	0.1510

FILT. PAD WT. = 6.7979 G.   EQ. VOL. = 65132.20 CC.   CONSISTENCY = 0.010 PERCENT

FLOW RATE = 78.6 CC./SEC.   H2O VISC. =0.008705 POISE   COMP. PAD WT. = 6.1767 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE		FILTRATION RESISTANCE	MAT SOLIDS CONCENTRATION	DP/(T*SQRT(CP))	CP**3
CM. H2O	DYNES/SQCM.	CM./G.	G./CC.		
10.	9807.	.1612E 08	0.0537	189.05	0.1547D-03
20.	19613.	.2059E 08	0.0701	211.38	0.3442D-03
30.	29420.	.2420E 08	0.0819	229.80	0.5497D-03
40.	39227.	.2742E 08	0.0915	246.30	0.7662D-03
50.	49033.	.3038E 08	0.0997	261.42	0.9913D-03
60.	58840.	.3301E 08	0.1070	274.26	0.1223D-02
70.	68647.	.3553E 08	0.1135	286.64	0.1462D-02
80.	78453.	.3805E 08	0.1195	299.16	0.1705D-02
90.	88260.	.4045E 08	0.1250	310.90	0.1954D-02

M =0.1563D-02   N =0.3848

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
10.	3.80	6502.79	1712.43
20.	3.30	6467.64	1958.15
30.	3.03	6478.16	2140.98
40.	2.84	6502.60	2293.49
50.	2.69	6532.48	2427.39
60.	2.58	6564.57	2548.41
70.	2.48	6597.43	2659.87
80.	2.40	6630.34	2763.86
90.	2.33	6662.94	2861.80

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME = 2.83 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 6548.77 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 2374.04 SQCM./CC.

Appendix VII (Continued)

FIELD FRACTION RETAINED ON 20 MESH   FIBER LENGTH = 3.05 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	7.15	214.42	0.	0.0	0.0
20.	11.12	333.52	0.	0.0	0.0
30.	14.13	423.82	9037.	1.2830	0.0491
40.	16.67	500.17	13998.	1.0754	0.0586
50.	18.94	568.13	23806.	0.8736	0.0721
60.	20.81	624.30	38507.	0.7258	0.0868
70.	22.58	677.32	63034.	0.6021	0.1047
80.	24.20	726.15	97015.	0.5118	0.1231
90.	25.68	770.47	146113.	0.4388	0.1436

FILT. PAD WT. = 9.7035 G.   EQ. VOL. = 62375.00 CC.   CONSISTENCY = 0.016 PERCENT

FLOW RATE = 78.2 CC./SEC.   H2O VISC. =0.009299 POISE   COMP. PAD WT. = 7.2896 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE		FILTRATION RESISTANCE	MAT SOLIDS CONCENTRATION	DP/(T*SQRT(CP))	CP**3
CM. H2O	DYNES/SQCM.	CM./G.	G./CC.		
10.	9807.	.1066E 08	0.0510	202.43	0.1330D-03
20.	19613.	.1370E 08	0.0666	227.79	0.2960D-03
30.	29420.	.1617E 08	0.0779	248.71	0.4727D-03
40.	39227.	.1827E 08	0.0870	265.87	0.6588D-03
50.	49033.	.2011E 08	0.0948	280.29	0.8524D-03
60.	58840.	.2196E 08	0.1017	295.53	0.1052D-02
70.	68647.	.2362E 08	0.1079	308.51	0.1257D-02
80.	78453.	.2517E 08	0.1136	320.53	0.1466D-02
90.	88260.	.2669E 08	0.1189	332.24	0.1680D-02

M =0.1486D-02   N =0.3848

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
10.	4.14	5435.06	1312.66
20.	3.52	5412.05	1538.66
30.	3.18	5424.29	1703.87
40.	2.96	5446.19	1840.14
50.	2.79	5471.41	1958.76
60.	2.66	5497.68	2065.24
70.	2.55	5524.06	2162.72
80.	2.46	5550.12	2253.19
90.	2.38	5575.64	2338.02

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME = 2.96 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 5481.83 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 1908.14 SQCM./CC.

Appendix VII (Continued)

FIELD FRACTION RETAINED ON 10 MESH   FIBER LENGTH = 3.94 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	9.27	278.10	0.	0.0	0.0
20.	14.90	447.00	0.	0.0	0.0
30.	19.51	585.20	9037.	2.0327	0.0479
40.	23.48	704.40	13998.	1.7037	0.0571
50.	27.11	813.30	23806.	1.3841	0.0703
60.	30.42	912.70	38507.	1.1498	0.0846
70.	33.36	1000.80	63034.	0.9538	0.1020
80.	36.12	1083.50	97015.	0.8109	0.1200
90.	38.68	1160.40	146113.	0.6952	0.1399

FILT. PAD WT. =16.8514 G.   EQ. VOL. = 97743.30 CC.   CONSISTENCY = 0.017 PERCENT

FLOW RATE = 81.1 CC./SEC.   H2O VISC. =0.009028 POISE   COMP. PAD WT. =11.2539 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	FILTRATION RESISTANCE CM./G.	MAT SOLIDS CONCENTRATION G./CC.	DP/(T*SQRT(CP))	CP**3
10.	9807.	.7109E 07	0.0497	158.11	0.12300-03
20.	19613.	.8846E 07	0.0649	172.18	0.27390-03
30.	29420.	.1014E 08	0.0759	182.47	0.43740-03
40.	39227.	.1123E 08	0.0848	191.24	0.60960-03
50.	49033.	.1215E 08	0.0924	198.34	0.78880-03
60.	58840.	.1300E 08	0.0991	204.78	0.97350-03
70.	68647.	.1383E 08	0.1052	211.51	0.11630-02
80.	78453.	.1460E 08	0.1107	217.61	0.13570-02
90.	88260.	.1533E 08	0.1158	223.47	0.15550-02

M =0.1448D-02   N =0.3848

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
10.	3.74	4451.23	1189.68
20.	3.12	4376.03	1401.97
30.	2.81	4347.00	1547.32
40.	2.61	4333.72	1662.24
50.	2.46	4327.91	1759.22
60.	2.35	4326.22	1844.20
70.	2.25	4327.04	1920.47
80.	2.18	4329.48	1990.12
90.	2.11	4333.01	2054.51

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME = 2.62 CC./G.

AVERAGE SPECIFIC SURFACE, SW = 4350.18 SQCM./G.

AVERAGE SPECIFIC SURFACE, SV = 1707.75 SQCM./CC.

Appendix VII (Continued)

FLOC FRACTION RETAINED ON 65 MESH   FIBER LENGTH = 1.74 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	6.68	200.30	0.	0.0	0.0
20.	10.86	325.90	0.	0.0	0.0
30.	14.36	430.80	9037.	1.6190	0.0475
40.	17.44	523.20	13998.	1.3570	0.0567
50.	20.07	602.20	23806.	1.1024	0.0697
60.	22.52	675.70	38507.	0.9158	0.0840
70.	24.77	743.00	63034.	0.7597	0.1012
80.	26.86	805.90	97015.	0.6458	0.1191
90.	28.83	864.80	146113.	0.5537	0.1389

FILT. PAD WT. =11.2212 G.   EQ. VOL. = 75825.40 CC.   CONSISTENCY = 0.015 PERCENT
FLOW RATE = 84.9 CC./SEC.   H2O VISC. =0.009979 POISE   COMP. PAD WT. = 8.8940 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE		FILTRATION RESISTANCE	MAT SOLIDS CONCENTRATION	DP/(T*SQRT(CP))	CP**3
CM. H2O	DYNES/SQCM.	CM./G.	G./CC.		
10.	9807.	.9493E 07	0.0494	220.39	0.1202D-03
20.	19613.	.1167E 08	0.0644	237.08	0.2676D-03
30.	29420.	.1324E 08	0.0753	248.84	0.4273D-03
40.	39227.	.1454E 08	0.0841	258.48	0.5956D-03
50.	49033.	.1579E 08	0.0917	268.91	0.7706D-03
60.	58840.	.1688E 08	0.0983	277.68	0.9511D-03
70.	68647.	.1791E 08	0.1044	286.01	0.1136D-02
80.	78453.	.1887E 08	0.1099	293.71	0.1326D-02
90.	88260.	.1979E 08	0.1149	301.02	0.1519D-02

M =0.1437D-02   N =0.3848

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
10.	3.45	5106.59	1482.25
20.	2.97	5014.75	1689.16
30.	2.72	4976.48	1832.29
40.	2.55	4957.42	1946.43
50.	2.42	4947.78	2043.47
60.	2.32	4943.51	2129.04
70.	2.24	4942.61	2206.27
80.	2.17	4943.93	2277.14
90.	2.11	4946.80	2342.96

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME = 2.55 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 4975.54 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 1994.33 SQCM./CC.

Appendix VII (Continued)

FLOC FRACTION RETAINED ON 65 MESH   FIBER LENGTH = 2.07 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	6.13	183.84	0.	0.0	0.0
20.	10.20	305.88	0.	0.0	0.0
30.	13.56	406.80	9037.	2.0490	0.0462
40.	16.51	495.36	13998.	1.7174	0.0551
50.	19.13	573.90	23806.	1.3952	0.0678
60.	21.57	647.10	38507.	1.1590	0.0816
70.	23.84	715.14	63034.	0.9615	0.0984
80.	26.00	780.06	97015.	0.8174	0.1158
90.	27.91	837.18	146113.	0.7008	0.1350

FILT. PAD WT. =15.3695 G.   EQ. VOL. = 74956.90 CC.   CONSISTENCY = 0.021 PERCENT

FLOW RATE = 87.4 CC./SEC.   H2O VISC. =0.009218 POISE   COMP. PAD WT. =10.9453 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE		FILTRATION RESISTANCE	MAT SOLIDS CONCENTRATION	DP/(T*SQRT(CP))	CP**3
CM. H2O	DYNES/SQCM.	CM./G.	G./CC.		
10.	9807.	.7629E 07	0.0480	243.50	0.11050-03
20.	19613.	.9171E 07	0.0627	256.16	0.24600-03
30.	29420.	.1034E 08	0.0732	267.24	0.39280-03
40.	39227.	.1133E 08	0.0818	276.86	0.54760-03
50.	49033.	.1222E 08	0.0891	286.16	0.70840-03
60.	58840.	.1300E 08	0.0956	294.05	0.87440-03
70.	68647.	.1373E 08	0.1015	301.35	0.10450-02
80.	78453.	.1438E 08	0.1068	307.73	0.12190-02
90.	88260.	.1508E 08	0.1118	315.34	0.13960-02

M =0.13970-02   N =0.3848

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
10.	3.18	4560.27	1432.07
20.	2.81	4474.97	1592.44
30.	2.60	4434.63	1705.33
40.	2.46	4411.70	1796.01
50.	2.35	4397.75	1873.42
60.	2.26	4389.13	1941.86
70.	2.19	4383.98	2003.75
80.	2.13	4381.20	2060.63
90.	2.07	4380.12	2113.52

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME = 2.45 CC./G.

AVERAGE SPECIFIC SURFACE, SW = 4423.75 SQCM./G.

AVERAGE SPECIFIC SURFACE, SV = 1835.45 SQCM./CC.

Appendix VII (Continued)

FLOC FRACTION RETAINED ON 20 MESH FIBER LENGTH = 2.98 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	8.64	259.12	0.	0.0	0.0
20.	14.49	434.70	0.	0.0	0.0
30.	19.37	581.17	9037.	2.5497	0.0451
40.	23.79	713.77	13998.	2.1370	0.0538
50.	27.81	834.30	23806.	1.7361	0.0662
60.	31.55	946.57	38507.	1.4422	0.0797
70.	35.09	1052.62	63034.	1.1964	0.0961
80.	38.32	1149.67	97015.	1.0171	0.1131
90.	41.16	1234.80	146113.	0.8720	0.1319

FILT. PAD WT. =20.8284 G. EQ. VOL. =110658.80 CC. CONSISTENCY = 0.019 PERCENT
FLOW RATE = 87.6 CC./SEC. H2O VISC. =0.009175 POISE COMP. PAD WT. =13.3006 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE		FILTRATION RESISTANCE	MAT SOLIDS CONCENTRATION	DP/(T*SQRT(CP))	CP**3
CM. H2O	DYNES/SQCM.	CM./G.	G./CC.		
10.	9807.	.5894E 07	0.0469	174.82	0.10290-03
20.	19613.	.7027E 07	0.0612	182.40	0.22910-03
30.	29420.	.7884E 07	0.0715	189.28	0.36590-03
40.	39227.	.8559E 07	0.0799	194.43	0.51000-03
50.	49033.	.9153E 07	0.0871	199.19	0.65980-03
60.	58840.	.9681E 07	0.0934	203.41	0.81440-03
70.	68647.	.1016E 08	0.0991	207.17	0.97300-03
80.	78453.	.1063E 08	0.1043	211.28	0.11350-02
90.	88260.	.1113E 08	0.1092	216.35	0.13000-02

M =0.13640-02 N =0.3848

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
10.	3.07	4014.35	1309.66
20.	2.67	3922.46	1468.05
30.	2.46	3876.80	1575.46
40.	2.32	3848.76	1659.86
50.	2.21	3829.84	1730.78
60.	2.13	3816.39	1792.71
70.	2.06	3806.53	1848.16
80.	2.00	3799.18	1898.68
90.	1.95	3793.67	1945.31

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME = 2.32 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 3856.44 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 1692.07 SQCM./CC.

Appendix VII (Continued)

FLOC FRACTION RETAINED ON 10 MESH · FIBER LENGTH = 4.13 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT / COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	8.11	243.30	0.	0.0	0.0
20.	13.51	405.30	0.	0.0	0.0
30.	18.14	544.30	9037.	2.2036	0.0452
40.	22.51	675.40	13998.	1.8469	0.0540
50.	26.39	791.60	23806.	1.5004	0.0664
60.	30.17	905.10	38507.	1.2465	0.0800
70.	33.80	1014.10	63034.	1.0340	0.0964
80.	37.29	1118.80	97015.	0.8790	0.1134
90.	40.71	1221.40	146113.	0.7536	0.1323

FILT. PAD WT. =30.3815 G.   EQ. VOL. =111436.70 CC.   CONSISTENCY = 0.027 PERCENT
FLOW RATE = 90.0 CC./SEC.   H2O VISC. =0.008551 POISE   COMP. PAD WT. =11.5314 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	FILTRATION RESISTANCE CM./G.	MAT SOLIDS CONCENTRATION G./CC.	DP/(T*SQRT(CP))	CP**3
10.	9807.	.4409E 07	0.0470	185.90	0.1039D-03
20.	19613.	.5293E 07	0.0614	195.32	0.2313D-03
30.	29420.	.5912E 07	0.0717	201.79	0.3694D-03
40.	39227.	.6352E 07	0.0801	205.15	0.5148D-03
50.	49033.	.6775E 07	0.0873	209.60	0.6661D-03
60.	58840.	.7110E 07	0.0937	212.40	0.8222D-03
70.	68647.	.7404E 07	0.0994	214.70	0.9823D-03
80.	78453.	.7670E 07	0.1046	216.77	0.1146D-02
90.	88260.	.7904E 07	0.1095	218.37	0.1313D-02

M =0.1369D-02   N =0.3848

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
10.	3.44	3535.82	1027.34
20.	2.65	3396.39	1279.68
30.	2.28	3317.14	1457.44
40.	2.04	3261.71	1599.67
50.	1.87	3219.07	1720.40
60.	1.74	3184.41	1826.46
70.	1.64	3155.20	1921.76
80.	1.56	3129.94	2008.78
90.	1.49	3107.67	2089.17

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME = 2.08 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 3256.37 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 1658.97 SQCM./CC.

WHOLE PULP RETAINED ON 65 MESH FIBER LENGTH = 1.49 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	6.71	201.30	0.	0.0	0.0
20.	10.93	327.90	0.	0.0	0.0
30.	14.28	428.40	9037.	1.5735	0.0468
40.	17.08	512.40	13998.	1.3188	0.0559
50.	19.62	588.60	23806.	1.0714	0.0688
60.	21.73	651.90	38507.	0.8900	0.0828
70.	23.84	715.20	63034.	0.7383	0.0998
80.	25.77	773.10	97015.	0.6277	0.1174
90.	27.42	822.60	146113.	0.5381	0.1370

FILT. PAD WT. = 9.8641 G. EQ. VOL. = 66882.94 CC. CONSISTENCY = 0.015 PERCENT

FLOW RATE = 77.3 CC./SEC. H2O VISC. =0.009571 POISE COMP. PAD WT. = 8.5246 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	FILTRATION RESISTANCE CM./G.	MAT SOLIDS CONCENTRATION G./CC.	DP/(T*SQRT(CP))	CP**3
10.	9807.	.1192E 08	0.0487	220.82	0.1153D-03
20.	19613.	.1464E 08	0.0635	237.28	0.2566D-03
30.	29420.	.1680E 08	0.0743	251.97	0.4098D-03
40.	39227.	.1873E 08	0.0830	265.76	0.5713D-03
50.	49033.	.2038E 08	0.0904	277.04	0.7391D-03
60.	58840.	.2209E 08	0.0970	289.82	0.9123D-03
70.	68647.	.2349E 08	0.1029	299.19	0.1090D-02
80.	78453.	.2483E 08	0.1083	308.30	0.1272D-02
90.	88260.	.2625E 08	0.1134	318.67	0.1457D-02

M =0.1417D-02 N =0.3848

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
10.	3.53	5763.89	1630.83
20.	3.16	5694.86	1799.60
30.	2.94	5670.55	1927.92
40.	2.78	5663.76	2035.35
50.	2.66	5665.67	2129.70
60.	2.56	5672.48	2214.94
70.	2.48	5682.27	2293.37
80.	2.41	5693.93	2366.47
90.	2.34	5706.82	2435.28

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME = 2.76 CC./G.

AVERAGE SPECIFIC SURFACE, SW = 5690.46 SQCM./G.

AVERAGE SPECIFIC SURFACE, SV = 2092.60 SQCM./CC.

Appendix VII (Continued)

WHOLE PULP RETAINED ON 20 MESH   FIBER LENGTH = 2.76 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	6.29	188.70	0.	0.0	0.0
20.	9.82	294.60	0.	0.0	0.0
30.	12.90	387.00	9037.	2.7308	0.0442
40.	15.50	465.00	13998.	2.2888	0.0527
50.	17.76	532.80	23806.	1.8594	0.0649
60.	19.97	599.10	38507.	1.5447	0.0781
70.	22.00	660.00	63034.	1.2814	0.0942
80.	23.80	714.00	97015.	1.0893	0.1108
90.	25.39	761.70	146113.	0.9339	0.1292

FILT. PAD WT. =17.1414 G.   EQ. VOL. = 61389.57 CC.   CONSISTENCY = 0.028 PERCENT

FLOW RATE = 77.3 CC./SEC.   H2O VISC. =0.009686 POISE   COMP. PAD WT. =13.9570 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE		FILTRATION RESISTANCE	MAT SOLIDS CONCENTRATION	DP/(T*SQRT(CP))	CP**3
CM. H2O	DYNES/SQCM.	CM./G.	G./CC.		
10.	9807.	.6637E 07	0.0459	242.53	0.9680D-04
20.	19613.	.8502E 07	0.0600	271.91	0.2155D-03
30.	29420.	.9709E 07	0.0701	287.18	0.3441D-03
40.	39227.	.1077E 08	0.0783	301.51	0.4796D-03
50.	49033.	.1175E 08	0.0853	315.11	0.6206D-03
60.	58840.	.1254E 08	0.0915	324.69	0.7660D-03
70.	68647.	.1328E 08	0.0971	333.81	0.9151D-03
80.	78453.	.1403E 08	0.1022	343.69	0.1068D-02
90.	88260.	.1480E 08	0.1069	354.32	0.1223D-02

M =0.1337D-02   N =0.3848

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
10.	4.45	4509.75	1013.47
20.	3.56	4448.18	1250.23
30.	3.13	4423.50	1413.98
40.	2.86	4411.48	1543.45
50.	2.67	4405.47	1652.39
60.	2.52	4402.81	1747.45
70.	2.40	4402.19	1832.38
80.	2.31	4402.90	1909.56
90.	2.22	4404.52	1980.58

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME = 2.90 CC./G.

AVERAGE SPECIFIC SURFACE, SW = 4423.42 SQCM./G.

AVERAGE SPECIFIC SURFACE, SV = 1593.72 SQCM./CC.

Appendix VII (Continued)

WHOLE PULP RETAINED ON 10 MESH   FIBER LENGTH = 3.88 MM.

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	8.00	240.00	0.	0.0	0.0
20.	13.40	402.00	0.	0.0	0.0
30.	17.75	532.50	9037.	3.4449	0.0450
40.	21.77	653.10	13998.	2.8874	0.0537
50.	25.33	759.90	23806.	2.3457	0.0661
60.	28.72	861.60	38507.	1.9486	0.0796
70.	31.95	958.50	63034.	1.6165	0.0959
80.	34.86	1045.80	97015.	1.3742	0.1128
90.	37.65	1129.50	146113.	1.1782	0.1316

FILT. PAD WT. =24.8004 G.   EQ. VOL. = 90146.69 CC.   CONSISTENCY = 0.028 PERCENT

FLOW RATE = 77.3 CC./SEC.   H2O VISC. =0.009709 POISE   COMP. PAD WT. =17.9365 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE		FILTRATION RESISTANCE	MAT SOLIDS CONCENTRATION	DP/(T*SQRT(CP))	CP**3
CM. H2O	DYNES/SQCM.	CM./G.	G./CC.		
10.	9807.	.5284E 07	0.0468	188.93	0.10230-03
20.	19613.	.6309E 07	0.0611	197.42	0.22780-03
30.	29420.	.7144E 07	0.0714	206.78	0.36380-03
40.	39227.	.7767E 07	0.0797	212.69	0.50710-03
50.	49033.	.8344E 07	0.0869	218.90	0.65600-03
60.	58840.	.8831E 07	0.0932	223.69	0.80970-03
70.	68647.	.9261E 07	0.0989	227.73	0.96740-03
80.	78453.	.9700E 07	0.1041	232.49	0.11290-02
90.	88260.	.1010E 08	0.1089	236.74	0.12930-02

M =0.1362D-02   N =0.3848

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
10.	3.38	3860.99	1142.04
20.	2.83	3759.68	1328.08
30.	2.56	3710.86	1451.94
40.	2.38	3681.19	1547.97
50.	2.25	3661.16	1627.80
60.	2.15	3646.82	1696.89
70.	2.07	3636.16	1758.28
80.	2.00	3628.06	1813.83
90.	1.94	3621.82	1864.79

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME = 2.39 CC./G.

AVERAGE SPECIFIC SURFACE, SW = 3689.64 SQCM./G.

AVERAGE SPECIFIC SURFACE, SV = 1581.29 SQCM./CC.

Appendix VII (Continued)

```
*************************************************************************
*                                                                       *
*                   WHOLE PULP -- NOT CLASSIFIED                        *
*                                                                       *
*************************************************************************
```

ORIGINAL DATA

FILTRATION EXPERIMENT			COMPRESSIBILITY EXPERIMENT		
PRESSURE CM. H2O	CHART READING	TIME SEC.	PRESSURE DYNES/SQCM.	PAD THICK. INCHES	SOLIDS CONC. G./CC.
10.	6.07	182.10	0.	0.0	0.0
20.	9.50	285.00	0.	0.0	0.0
30.	12.30	369.00	9037.	2.1378	0.0443
40.	14.69	440.70	13998.	1.7918	0.0528
50.	16.83	504.90	23806.	1.4557	0.0650
60.	18.80	564.00	38507.	1.2093	0.0783
70.	20.65	619.50	63034.	1.0032	0.0943
80.	22.37	671.10	97015.	0.8528	0.1110
90.	23.88	716.40	146113.	0.7311	0.1295

FILT. PAD WT. =13.5742 G. EQ. VOL. = 58602.66 CC. CONSISTENCY = 0.023 PERCENT

FLOW RATE = 77.3 CC./SEC. H2O VISC. =0.009436 POISE COMP. PAD WT. =10.9475 G.

FILTRATION RESISTANCE AND MAT SOLIDS CONCENTRATION AS FUNCTIONS OF PRESSURE

PRESSURE		FILTRATION RESISTANCE	MAT SOLIDS CONCENTRATION	DP/(T*SQRT(CP))	CP**3
CM. H2O	DYNES/SQCM.	CM./G.	G./CC.		
10.	9807.	.8510E 07	0.0460	251.08	0.9736D-04
20.	19613.	.1088E 08	0.0601	280.79	0.2167D-03
30.	29420.	.1260E 08	0.0702	300.90	0.3461D-03
40.	39227.	.1407E 08	0.0784	317.83	0.4824D-03
50.	49033.	.1535E 08	0.0855	332.20	0.6242D-03
60.	58840.	.1649E 08	0.0917	344.57	0.7704D-03
70.	68647.	.1751E 08	0.0973	355.29	0.9204D-03
80.	78453.	.1847E 08	0.1024	365.32	0.1074D-02
90.	88260.	.1947E 08	0.1072	376.37	0.1230D-02

M =0.1339D-02 N =0.3848

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
10.	4.60	5116.50	1111.73
20.	3.68	5067.98	1375.69
30.	3.24	5053.32	1559.10
40.	2.96	5049.62	1704.65
50.	2.76	5050.81	1827.50
60.	2.61	5054.55	1934.96
70.	2.49	5059.72	2031.21
80.	2.39	5065.75	2118.84
90.	2.31	5072.29	2199.63

AVERAGE VALUES FOR SPECIFIC VOLUME AND SURFACE

AVERAGE SPECIFIC VOLUME = 3.01 CC./G.

AVERAGE SPECIFIC SURFACE, SW = 5065.61 SQCM./G.

AVERAGE SPECIFIC SURFACE, SV = 1762.59 SQCM./CC.

APPENDIX VIII

HISTORICAL PROGRAM AND OUTPUT FOR CALCULATION OF $\underline{S_W}$ AND $\underline{v}$ AS FUNCTIONS OF PRESSURE FROM MULTIPLE PRESSURE TAP PERMEATION

```
/JOB GO,TIME=99
/FILE DISK=(1,NREC=20000),RSIZ=128
/FILE DISK=(2,IOUT,NREC=4000),RSIZ=80,VOL=SYSFL1,DISP=(NEW,DELETE)
/FILE DISK=(3,KOUT,NREC=4000),RSIZ=80,VOL=SYSFL1,DISP=(NEW,DELETE)
/FILE DISK=(4,MOUT,NREC=5000),RSIZ=128,VOL=SYSFL1,DISP=(NEW,DELETE)
C
C               PRMEAT === PERMEATION DATA ANALYSIS
C
C                         ANTHONY P. BINOTTO
C                          JANUARY 16, 1975
C
C           THIS PROGRAM IS DESIGNED TO CALCULATE THE HYDRODYNAMIC FIBER
C           SPECIFIC SURFACE AND SPECIFIC VOLUME AS FUNCTIONS OF PRESSURE
C           USEING THE PERMEABILITY EQUATION DEVELOPED BY H. MEYER.
C
C          THE PROGRAM IS DIVIDED INTO SEVERAL SUBROUTINES WHICH CALCULATE
C          THE VARIOUS PARAMETERS NEEDED TO SOLVE THE EQUATION.  THE
C          FUNCTIONS OF EACH SUBROUTINE ARE EXPLAINED IN THEIR RESPECTIVE
C          COMMENT STATEMENTS.
C
C          THE USER MUST SUPPLY THE FOLLOWING CARDS.
C                            /ID XXXXXXXX,XXX
C                            /INCLUDE PRMEAT
C                            /INCLUDE SOLCON
C                            /INCLUDE DATFIT
C                            /INCLUDE FVSX
C                            /INCLUDE CURVE
C                            /DATA
C              DATA CARD 1 -- ANY TITLE, CENTERING ON THE CARD WILL
C                              ENSURE CENTERING ON OUTPUT.
C              DATA CARD 2 -- THE FOLLOWING DATA IN 8F10.0 FORMAT.
C                              1ST.  O.D. WT. OF MAT, G.
C                              2ND.  WET MAT THICKNESS, CM.
C                              3RD.  PERMEATION VELOCITY, CM./SEC.
C                              4TH.  WATER TEMP., DEG. C.
C              DATA CARD 3 -- PRESSURE DROPS FOR TAPS 1 THRU 8
C              DATA CARD 4 -- PRESSURE DROPS FOR TAPS 9 THRU 12
C              DATA CARD 5 -- MAT THICK. FROM COMPRESSIBILITY EXP.
C              DATA CARD 6 -- INITIAL WATER LEVEL FOR COMP. EXPERIMENT
C                            /END
C
C
      DIMENSION DPZ(12),PDTHCK(12),DP2(8),MATCON(8),V(12),SW(12),SV(12)
      DIMENSION CMH2O(8),Z(12),H2ODPZ(12),ITITLE(20),OH2ODP(12)
      DIMENSION ODPZ(12)
      DOUBLE PRECISION N,M,MATCON,EXPM,EXPN,V,DPL,DPZ,L,Z,A,B,C,H2OLVL
      DOUBLE PRECISION MU,U,H2ODPZ,H2ODPL,B2,W,PDTHCK,AREA,DP2,B0,B1
      DOUBLE PRECISION SW,SV
      COMMON MU,U,M,N,L,DPL,A,B,C,B0,B1,B2,EXPM,EXPN
C***  K = 12, LAST TAP TO BE ANALYZED
      K = 12
C***  SAVE FILE ''IOUT'' TO OBTAIN PLOTS OF ''CP**3 VS. F''
      IOUT = 2
C***  SAVE FILE ''JOUT'' TO OBTAIN PUNCHED OUTPUT OF ''P,V,SW,SV''
      JOUT = 1
C***  SAVE FILE ''KOUT'' TO OBTAIN PLOTS OF ''DPZ/DPL VS. Z/L''
      KOUT = 3
C***  SAVE FILE ''LOUT'' TO OBTAIN PLOTS OF ''SOLIDS CONC. VS. PRESS.''
      LOUT = 1
C***  SAVE FILE ''MOUT'' TO OBTAIN PRINTED OUTPUT OF DATA AND RESULTS
      MOUT = 6
      WRITE (IOUT,9008)
      WRITE (IOUT,9009)
      WRITE (JOUT,9006)
      WRITE (JOUT,9007)
      WRITE (KOUT,9014)
      WRITE (KOUT,9015)
      WRITE (LOUT,9011)
      WRITE (LOUT,9012)
    1 READ(5,9019) ITITLE
      IF (ICOMP(ITITLE,1,249,4,1)) 2,7,2
    2 READ(5,9120) W,L,U,T,J,LIN,EXPN,INIT
      NPTS = K - J + 1
      READ(5,9020) (OH2ODP(I),I=1,12)
      READ(5,9020) (PDTHCK(I), I=1,8)
      WRITE (JOUT,9019) ITITLE
      WRITE (JOUT,9005) NPTS
      WRITE (IOUT,9019) ITITLE
      WRITE (KOUT,9019) ITITLE
      WRITE (KOUT,9016)
      WRITE (LOUT,9019) ITITLE
      WRITE (LOUT,9013)
      READ (5,9020) H2OLVL,DPLO
      H2ODPL = 60.
      DPL = H2ODPL * 980.638
```

Appendix VIII (Continued)

```
      DO 3 I=1,12
      ODPZ(I) = OH2ODP(I) * 980.638
    3 DPZ(I) = OH2ODP(I) * 980.638
      AREA = 45.58
      BW = W / (L * AREA)
      IF (T-20) 4,4,5
    4 ETA = (1301./(998.333+8.1855*(T-20.)+.00585*((T-20.)**2)))
      MU = 10. ** (ETA - 3.30233)
      GO TO 6
    5 ETA = (1.3272*(20.-T)-.001053*((T-20.)**2))/(T+105.)
      MU = (10. ** ETA) * .01002
    6 CALL SOLCON (W,POTHCK,AREA,N,M,MATCON,DP2,CMH2O,H2OLVL,LOUT,DPZ,
     *L,DPLO)
      CALL DATFIT (DPZ,Z,INIT,K,KOUT)
      CALL FVSX (DPZ,Z,V,SW,SV,J,LIN,K,IOUT,DPLO,B0,B2)
      DO 8 I=1,12
    8 H2ODPZ(I) = DPZ(I) / 980.638
      WRITE (MOUT,9042)
      WRITE (MOUT,9021)
      WRITE (MOUT,9022)
      WRITE (MOUT,9022)
      WRITE (MOUT,9023) ITITLE
      WRITE (MOUT,9022)
      WRITE (MOUT,9022)
      WRITE (MOUT,9024)
      WRITE (MOUT,9025)
      WRITE (MOUT,9026)
      WRITE (MOUT,9027) BW
      WRITE (MOUT,9028) W
      WRITE (MOUT,9029) L
      WRITE (MOUT,9030) H2ODPL
      WRITE (MOUT,9031) U
      WRITE (MOUT,9032) T
      WRITE (MOUT,9033) MU
      WRITE (MOUT,9034)
      WRITE (MOUT,9035)
      WRITE (MOUT,9036)
      WRITE (MOUT,9037)
      WRITE (MOUT,9038)
      WRITE (MOUT,9039)
      WRITE (MOUT,9040) (I,Z(I),OH2ODP(I),ODPZ(I),CMH2O(I),DP2(I),
     * POTHCK(I),MATCON(I),I=1,8)
      WRITE (MOUT,9041) (I,Z(I),OH2ODP(I),ODPZ(I),I=9,12)
      WRITE (MOUT,9042)
      WRITE (MOUT,9043)
      WRITE (MOUT,9044)
      WRITE (MOUT,9045) M
      WRITE (MOUT,9046) N
      WRITE (MOUT,9042)
      WRITE (MOUT,9047)
      WRITE (MOUT,9048)
      WRITE (MOUT,9049)
      WRITE (MOUT,9050)
      WRITE (MOUT,9051) A,B0,EXPM
      WRITE (MOUT,9052) B,B1,EXPN
      WRITE (MOUT,9053) C,B2
      WRITE (MOUT,9042)
      IF (LIN-12) 10,9,7
 9    WRITE (MOUT,9059)
      WRITE (MOUT,9060)
      WRITE (MOUT,9061)DPLO,H2ODPL
      WRITE (MOUT,9062) V(K)
      WRITE (MOUT,9063) SW(K)
      WRITE (MOUT,9064) SV(K)
      WRITE (MOUT,9042)
      GO TO 1
 10   WRITE (MOUT,9054)
      WRITE (MOUT,9055)
      WRITE (MOUT,9056)
      WRITE (MOUT,9057)
      WRITE (MOUT,9058) (H2ODPZ(I),V(I),SW(I),SV(I),I=J,K)
      WRITE (MOUT,9042)
      WRITE (JOUT,9018) (H2ODPZ(I),V(I),SW(I),SV(I),I=J,K)
      GO TO 1
    7 WRITE (IOUT,9017)
      WRITE (JOUT,9017)
      WRITE (KOUT,9017)
      WRITE (LOUT,9017)
      WRITE (MOUT,9017)
      END FILE IOUT
C     END FILE JOUT
      END FILE KOUT
      END FILE LOUT
      END FILE MOUT
      STOP
```

Appendix VIII (Continued)

```
9005   FORMAT (I3,'   2   2 -1   3')
9006   FORMAT ('PRESSURE, CM. H2O')
9007   FORMAT ('SPEC. VOLUME, CC./G.')
9008   FORMAT ('(CL*(DPZ/DPL)**N)**3')
9009   FORMAT ('F')
9011   FORMAT ('PRESSURE, DYNES/SQCM.')
9012   FORMAT ('SOLIDS CONC., G./CC.')
9013   FORMAT ('   7   2   2 -1   3')
9014   FORMAT ('Z/L')
9015   FORMAT ('DPZ/DPL')
9016   FORMAT ('  12   1   1 -1   3')
9017   FORMAT ('99999999999999')
9018   FORMAT (4E15.7)
9019   FORMAT (20A4)
9020   FORMAT (8F10.0)
9120   FORMAT (4F10.0,I2,8X,I2,8X,F10.0,I2)
9021   FORMAT (1H1,16X,'****************************************************
      ***************************************')
9022   FORMAT (' ',16X,'*',85X,'*')
9023   FORMAT (' ',16X,'*',3X,20A4,2X,'*')
9024   FORMAT (' ',16X,'****************************************************
      ***************************************',//)
9025   FORMAT ('0','ORIGINAL DATA')
9026   FORMAT (' ','-------- ----',//)
9027   FORMAT ('0',10X,'MAT SOLIDS CONC. ',12X,'=',F10.4,' G./CC.')
9028   FORMAT ('0',10X,'O.D. MAT WT., W',14X,'=',F10.4,' G.')
9029   FORMAT ('0',10X,'MAT THICKNESS, L',13X,'=',F8.2,' CM.')
9030   FORMAT ('0',10X,'TOTAL PRESSURE DROP, DPL',5X,'=',F8.2,
      *' CM. H2O')
9031   FORMAT ('0',10X,'PERMEATION VELOCITY, U',7X,'=',F12.6,
      *' CM./SEC.')
9032   FORMAT ('0',10X,'WATER TEMPERATURE, T',9X,'=',F7.1,' DEG. C.')
9033   FORMAT ('0',10X,'WATER VISCOSITY, MU',10X,'=',F12.6,' POISES',//)
9034   FORMAT ('0',10X,'PERMEATION EXPERIMENT',35X,'COMPRESSIBILITY EXPE
      *RIMENT')
9035   FORMAT ('0',26X,'Z',19X,'DPZ')
9036   FORMAT ('0',10X,'PRESSURE         DISTANCE',62X,'MAT       MAT SOLIDS')
9037   FORMAT (' ',12X,'TAP',8X,'FROM TOP',6X,'PRESSURE TAP READING',14X,
      * 'PRESSURE',11X,'THICKNESS CONCENTRATION')
9038   FORMAT (' ',11X,'NUMBER',7X,'OF MAT')
9039   FORMAT (' ',25X,'CM.',8X,'CM. H2O',4X,'DYNES/SQCM.',8X,'CM. H2O',
      * 3X,'DYNES/SQCM.',6X,'CM.',7X,'G./CC.',//)
9040   FORMAT ('0',13X,I2,8X,F7.4,7X,F6.2,4X,F9.2,10X,F6.2,6X,F7.0,6X,
      * F6.4,6X,F6.4)
9041   FORMAT ('0',13X,I2,8X,F7.4,7X,F6.2,4X,F9.2)
9042   FORMAT ('0',///)
9043   FORMAT ('0','COMPRESSIBILITY CONSTANTS')
9044   FORMAT (' ','---------------- ---------',//)
9045   FORMAT ('0',10X,'M = ',E11.4)
9046   FORMAT ('0',10X,'N = ',F7.4)
9047   FORMAT ('0','EXPERIMENTAL EMPIRICAL CONSTANTS')
9048   FORMAT (' ','------------ --------- ---------',//)
9049   FORMAT ('0',10X,'DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)',5X,'F = B
      *0 + B1*(X**EXPM) + B2*(X**EXPN)')
9050   FORMAT ('0',10X,'WHERE...',34X,'WHERE...')
9051   FORMAT ('0',15X,'A = ',F9.5,29X,'B0 = ',E12.5,5X,'EXPM = ',E12.5)
9052   FORMAT ('0',15X,'B = ',F9.5,29X,'B1 = ',E12.5,5X,'EXPN = ',E12.5)
9053   FORMAT ('0',15X,'C = ',F9.5,29X,'B2 = ',E12.5)
9054   FORMAT ('0','HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTION
      *S OF PRESSURE')
9055   FORMAT (' ','------------ -------- ------ --- ------- -- --------
      *- -- --------',//)
9056   FORMAT (' ',15X,'PRESSURE',11X,'V',13X,'SW',16X,'SV')
9057   FORMAT (' ',15X,'CM. H2O',10X,'CC./G.',8X,'SQCM./G.',9X,
      *'SQCM./CC.',//)
9058   FORMAT ('0',15X,F6.2,11X,F5.2,7X,F10.2,7X,F10.2)
9059   FORMAT ('0','HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE')
9060   FORMAT (' ','------------ -------- ------ --- -------',//)
9061   FORMAT ('0',10X,'STATIC LOAD = ',F6.2,' CM. H2O',5X,
      * 'FLUID PRESSURE DROP = ',F5.2,' CM. H2O')
9062   FORMAT ('0',10X,'AVERAGE SPECIFIC VOLUME, V = ',F4.2,' CC./G.')
9063   FORMAT ('0',10X,'AVERAGE SPECIFIC SURFACE, SW = ',F7.2,
      * ' SQCM./G.')
9064   FORMAT ('0',10X,'AVERAGE SPECIFIC SURFACE, SV = ',F7.2,
      * ' SQCM./CC.')
       END
```

Appendix VIII (Continued)

DATFIT === DATA FITTING

```
C
C
C
C      THIS SUBROUTINE IS USED TO FIT EXPERIMENTAL VALUES OF DPZ/DPL
C      VS. Z/L TO THE EXPRESSION.
C            DPZ/DPL = A * (Z/L)**B * EXP(C * (Z/L))
C      THE COEFFICIENTS A,B,AND C ARE DETERMINED BY MULTIPLE REGRESSION.
C
C
       SUBROUTINE DATFIT (DPZ,Z,J,K,KOUT)
       DIMENSION DPZ(12),Z(12),A1(3,4),X2(12),Y(12),DLNY(12)
       DOUBLE PRECISION Y,SUMY,X1,SUMX1,SSQX1,X2,SUMX2,SSQX2,SX1X2,A1
       DOUBLE PRECISION SUMX1Y,SUMX2Y,DLNY,DPZ,DPL,Z,L,A,B,C,MU,U,N,M
       COMMON MU,U,M,N,L,DPL,A,B,C
       SUMY = 0.
       SUMX1 = 0.
       SSQX1 = 0.
       SUMX2 = 0.
       SSQX2 = 0.
       SX1X2 = 0.
       SUMX1Y = 0.
       SUMX2Y = 0.
       COUNT = 0.0
       DO 3 I=1,12
       Z(I) = L - (.3084 + (12 - I) * .3810)
       X2(I) = Z(I)/L
       IF (Z(I)) 1,1,2
     1 DPZ(I) = 0.0
     2 Y(I) = DPZ(I)/DPL
     3 WRITE (KOUT,9005) X2(I),Y(I)
       DO 5 I=J,K
       IF (Z(I)) 5,5,4
     4 DLNY(I) = DLOG(Y(I))
       SUMY = SUMY + DLNY(I)
       X1 = DLOG(X2(I))
       SUMX1 = SUMX1 + X1
       SSQX1 = SSQX1 + (X1**2)
       SUMX2 = SUMX2 + X2(I)
       SSQX2 = SSQX2 + (X2(I)**2)
       SX1X2 = SX1X2 + (X1 * X2(I))
       SUMX1Y = SUMX1Y + (X1 * DLNY(I))
       SUMX2Y = SUMX2Y + (X2(I) * DLNY(I))
       COUNT = COUNT + 1.
     5 CONTINUE
       A1(1,1) = COUNT
       A1(1,2) = SUMX1
       A1(1,3) = SUMX2
       A1(1,4) = SUMY
       A1(2,1) = A1(1,2)
       A1(2,2) = SSQX1
       A1(2,3) = SX1X2
       A1(2,4) = SUMX1Y
       A1(3,1) = A1(1,3)
       A1(3,2) = A1(2,3)
       A1(3,3) = SSQX2
       A1(3,4) = SUMX2Y
       CALL SIMEQN (A1,3,4)
       A = DEXP(A1(1,4)/A1(1,1))
       B = A1(2,4) / A1(2,2)
       C = A1(3,4) / A1(3,3)
C***   PLOTTING DATA FOR ''DPZ/DPL VS. Z/L''
       WRITE (KOUT,9006)
       WRITE (KOUT,9007)
       ZL = X2(J)
       ZLINCR = (X2(K) - X2(J))/ 200.
       DO 6 I=1,200
       DPZDPL = A * (ZL**B) * EXP(C*ZL)
       WRITE (KOUT,9005) ZL,DPZDPL
     6 ZL = ZL + ZLINCR
C***   DATA SMOOTHING
       DO 7 I=J,K
     7 DPZ(I) = DPL * ((A * (Z(I)/L)**B) * DEXP(C*Z(I)/L))
       RETURN
  9005 FORMAT (2E15.7)
  9006 FORMAT ('DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)')
  9007 FORMAT ('200 -1 -1  0')
       END
```

SIMEQN === SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS

```
C
C
      SUBROUTINE SIMEQN (A,NR,NC)
      DIMENSION A(3,4),R(100)
      DOUBLE PRECISION A,R,D,T
      IF(NR - NC) 2,2,1
    1 NCT = NC
      GO TO 3
    2 NCT = NR
    3 K = 1
      D = 1.
    4 IF(A(K,K)) 9,5,9
    5 DO 6 I = K,NR
      IF(A(I,K)) 7,6,7
    6 CONTINUE
      GO TO 14
    7 DO 8 J = 1,NC
      T = A(I,J)
      A(I,J) = A(K,J)
    8 A(K,J) = T
    9 DO 13 I = 1,NR
      IF(I - K) 10,13,10
   10 DO 11 J = 1,NC
   11 R(J) = (A(I,J)*A(K,K) - A(I,K)*A(K,J))/D
      DO 12 J = 1,NC
   12 A(I,J) = R(J)
   13 CONTINUE
      D = A(K,K)
      K = K + 1
      IF(K - NCT) 4,4,14
   14 RETURN
      END
```

Appendix VIII (Continued)

```
C              SOLCON === SOLIDS CONCENTRATION
C
C
C         THIS SUBROUTINE IS USED TO CALCULATE THE MAT SOLIDS
C         CONCENTRATION FROM MEASUREMENTS OF PAD THICKNESS AND THE
C         O.D. WT. OF THE MAT.  THE PROCEDURE INVOLVES A LEAST SQUARES
C         APPROACH TO COMPUTING COMPRESSIBILITY CONSTANTS M AND N.
C
      SUBROUTINE SOLCON (W,PDTHCK,AREA,N,M,MATCON,DP2,CMH2O,H2OLVL,
     *LOUT,DPZ,L)
      DIMENSION PDTHCK(12),DP2(8),MATCON(8),LMC(8),LDP2(8),CMH2O(8)
      DIMENSION DPZ(12)
      DOUBLE PRECISION MATCON,LMC,LDP2,SUMY,SUMY2,SUMX,SUMX2,SUMXY,Q1,L
      DOUBLE PRECISION Q2,N,M,SUMM,W,PDTHCK,AREA,DP2,H2OLVL,DPZ,CO,DPZO
      PTS = 7.0
      NPTS = 8
      CALL PISTON (H2OLVL,PDTHCK,DP2)
      SUMY = 0.
      SUMY2 = 0.
      SUMX = 0.
      SUMX2 = 0.
      SUMXY = 0.
      SUMM = 0.
      MATCON(1) = W / (PDTHCK(1) * AREA)
      DO 1 I=2,NPTS
      CMH2O(I) = DP2(I) / 980.638
      MATCON(I) = W / (PDTHCK(I) * AREA)
      LMC(I) = DLOG(MATCON(I))
      LDP2(I) = DLOG(DP2(I))
      SUMY = SUMY + LMC(I)
      SUMY2 = SUMY2 + (LMC(I) ** 2)
      SUMX = SUMX + LDP2(I)
      SUMX2 = SUMX2 + (LDP2(I) ** 2)
    1 SUMXY = SUMXY + (LMC(I) * LDP2(I))
      Q1 = ( PTS * SUMXY) - (SUMX * SUMY)
      Q2 = ( PTS * SUMX2) - (SUMX ** 2)
      N = Q1/Q2
      DO 2 I=2,NPTS
    2 SUMM = SUMM + (MATCON(I) / (DP2(I) ** N))
      M = SUMM / PTS
C***  GENERATION OF PLOTTING DATA FOR ''SOLIDS CONC. VS. PRESSURE''
      WRITE (LOUT,9006) (DP2(I),MATCON(I),I=2,NPTS)
      WRITE (LOUT,9007)
      WRITE (LOUT,9008)
      XINCR = (DP2(8) - DP2(2))/100.
      X = DP2(2)
      DO 5 I=1,100
      Y = M * X ** N
      WRITE (LOUT,9006) X,Y
    5 X = X + XINCR
      RETURN
 9006 FORMAT (2E15.7)
 9007 FORMAT ('C = M * DELTAP ** N')
 9008 FORMAT ('100 -2 -2  0')
      END
      SUBROUTINE PISTON (H2OLVL,PDTHCK,DP2)
      DIMENSION DP2(8),DISSUB(8),VOLDIS(8),PDTHCK(12)
      DOUBLE PRECISION H2OLVL,DP2,PDTHCK
C***  CUMULATIVE WEIGHT IN GRAMS
      DP2(2) = 345.90
      DP2(3) = 572.58
      DP2(4) = 1027.99
      DP2(5) = 1710.66
      DP2(6) = 2849.66
      DP2(7) = 4428.66
      DP2(8) = 6708.66
C***  CONVERT WEIGHT INTO PRESSURE
      DO 1 I=2,8
      DISSUB(I) = H2OLVL - PDTHCK(I)
      VOLDIS(I) = 10.**(.025164 * DISSUB(I) + 1.47138)
    1 DP2(I) = ((DP2(I) - VOLDIS(I)) / 45.58) * 980.665
      DP2(1) = 0.0
      RETURN
     END
```

Appendix VIII (Continued)

```
C          FVSX === FITS BEST CURVES TO PLOTS OF ''CP**3 VS. F''
C
      SUBROUTINE FVSX (DPZ,Z,V,SW,SV,J,LIN,K,IOUT,DPLO)
      DIMENSION DPZ(12),Z(12),F(12),X(12),SV(12),F1(12),V(12),SW(12)
      DOUBLE PRECISION DPL,DPZ,L,Z,B,C,N,MU,U,F,X,K1,K2,A,M,Q,CL
      DOUBLE PRECISION B0,B1,B2,EXPM,EXPN,F1,SLOPE,INTCPT,V,SW,SV
      COMMON MU,U,M,N,L,DPL,A,B,C,B0,B1,B2,EXPM,EXPN
      K1 = 3.5
      K2 = 57.
      LPTS = LIN - J + 1
      NPTS = K - J + 1
      WRITE (IOUT,9017) NPTS
      IF (DPLO) 3,3,1
    1 BOYNCY = 10.**(.025164 * (28. - L) + 1.47138)
      DPLO = ((DPLO - BOYNCY)/45.58)*980.665
    3 CL = M * (DPL + DPLO) ** N
      Q = DPL/(MU * L * U * K1 * (CL**1.5))
      DO 4 I=J,K
      X(I) = (CL**3) * ((DPZ(I)/DPL)**(3*N))
      F(I) = Q * (C + (B*L/Z(I)))*((DPZ(I)/DPL)**(1-1.5*N))
    4 WRITE (IOUT,9018) X(I),F(I)
      DPLO = DPLO/980.638
      IF (LPTS - 1) 9,9,5
C***  CALCULATION OF SLOPE AND INTERCEPT FOR THE INITIAL PORTION OF THE
C***  ''CP**3 VS. F'' PLOT.  THIS PORTION IS USUALLY LINEAR AT LOW DPZ.
    5 CALL LINREG (X,F,LPTS,INTCPT,SLOPE,J,LIN)
      B0 = INTCPT
      B2 = SLOPE
      WRITE (IOUT,9019) INTCPT,SLOPE
      WRITE (IOUT,9020)
      XLINCR = (X(LIN) - X(J))/10.
      XL = X(J)
      DO 7 I=1,2
      FL = INTCPT + SLOPE * XL
      WRITE (IOUT,9018) XL,FL
    7 XL = X(LIN) + XLINCR
      DO 8 I=J,LIN
      V(I) = (SLOPE/(INTCPT*K2)) ** .333333
      SW(I) = DSQRT(INTCPT * DSQRT(V(I)))
    8 SV(I) = SW(I) / V(I)
      IF (K - LIN) 15,15,9
    9 CALL CRVFIT (X,F,B0,B1,B2,EXPM,EXPN,LIN,K,IOUT)
C***  CALCULATION OF SLOPE AND INTERCEPT USING PLOT OF ''CP**3 VS. F''
      IF (LPTS - 1) 11,11,10
   10 LIN = LIN + 1
   11 DO 14 I=LIN,K
      F1(I) = B0 + B1*(X(I)**EXPM) + B2*(X(I)**EXPN)
      SLOPE = EXPM*B1*(X(I)**(EXPM-1.)) + EXPN*B2*(X(I)**(EXPN-1.))
      INTCPT = F1(I) - SLOPE * X(I)
      IF (INTCPT) 14,14,12
   12 IF (SLOPE) 14,14,13
   13 V(I) = (SLOPE/(INTCPT*K2)) ** 0.333333
      SW(I) = DSQRT(INTCPT * DSQRT(V(I)))
      SV(I) = SW(I) / V(I)
   14 CONTINUE
   15 WRITE (1,9016) (I,V(I),SW(I),I=J,K)
      RETURN
 9016 FORMAT (' ',I5,5X,F6.2,5X,F10.2)
 9017 FORMAT (I3,'  1  1 -1  3')
 9018 FORMAT (2E15.7)
 9019 FORMAT ('F = ',E7.2,' + ',E7.2,' * CP3')
 9020 FORMAT ('  2 -1 -1  0')
     END
C                LINREG === LINEAR REGRESSION
C                      ANTHONY P. BINOTTO
C                         JUNE 16, 1975
C
C          THIS SUBROUTINE IS USED TO CALCULATE THE STRAIGHT LINE
C          RELATIONSHIP BETWEEN PAIRS OF X,Y-DATA
C
      SUBROUTINE LINREG (X,Y,NPTS,INTCPT,SLOPE,J,LIN)
      DIMENSION X(12),Y(12)
      DOUBLE PRECISION X,Y,SUMX,SUMX2,SUMY,SUMXY,SLOPE,INTCPT,Q1,Q2,PTS
      PTS = NPTS
      SUMY = 0.
      SUMX = 0.
      SUMX2 = 0.
      SUMXY = 0.
      DO 1 I=J,LIN
      SUMY = SUMY + Y(I)
      SUMX = SUMX + X(I)
      SUMX2 = SUMX2 + (X(I) ** 2)
    1 SUMXY = SUMXY + (X(I) * Y(I))
      Q1 = SUMXY - (SUMX * SUMY)/PTS
      Q2 = SUMX2 - (SUMX ** 2)/PTS
      SLOPE = Q1/Q2
      INTCPT= (SUMY - SLOPE * SUMX) / PTS
      RETURN
     END
C
C
C
```

Appendix VIII (Continued)

```
                CURVE ==== CURVE FITTING
      SUBROUTINE CRVFIT(X,Y,B0,B1,B2,M,N,J,K,IOUT)
      DIMENSION X(12),Y(12),A(5,6)
      DOUBLE PRECISION X,Y,B0,B1,B2,M,N,A,ORIGM,ORIGN
      DOUBLE PRECISION DELB0,DELB1,DELB2,DELM,DELN
      NPTS = K - J + 1
C**** INITIAL VALUES OF B0, B1, AND M ESTIMATED BY 'SUBROUTINE XM'
      CALL EXPONM (X,Y,NPTS,B0,B1,M,J,K)
      B2 = 1000.
C     N = 0.48
C***  DATA FIT MAY BE IMPROVED AT THE EXPENSE OF CALCULATION TIME
C***  BY DECREASING THE VALUE OF ''TEST''.
      TEST = .10E06
    1 MCOUNT = 0
      NCOUNT = 0
      ORIGM = M
      ORIGN = N
      TEST1 = 0.0
    2 CALL SUM (X,Y,NPTS,B0,B1,B2,M,N,A,J,K)
      CALL SIMEQ (A,5,6)
      DELB0 = A(1,6) / A(1,1)
      DELB1 = A(2,6) / A(2,2)
      DELB2 = A(3,6) / A(3,3)
      DELM = A(4,6) / A(4,4)
      DELN = A(5,6) / A(5,5)
      TEST2 = DELB0 + DELB1 + DELB2 + DELM + DELN
      DIFF = TEST2 - TEST1
      IF (ABS(DIFF)-TEST) 6,6,3
    3 TEST1 = TEST2
      NCOUNT = NCOUNT + 1
      B0 = B0 + DELB0
      B1 = B1 + DELB1
      B2 = B2 + DELB2
      IF (NCOUNT-100) 2,2,4
    4 NCOUNT = 0
      M = M + DELM
      N = N + DELN
      MCOUNT = MCOUNT + 1
      IF (MCOUNT - 04) 2,2,5
    5 M = ORIGM
      N = ORIGN + 0.01
      GO TO 1
    6 CALL EQN (B0,B1,B2,M,N,IOUT,X,J,K)
      RETURN
     END
C
C     SUBROUTINE EQN === GENERATION OF PLOTTING POINTS
C
C     THIS SUBROUTINE GENERATES DATA POINTS FROM THE CALCULATED
C     EQUATION TO BE USED IN PLOTTING PROGRAM ''ABPLOT''.
      SUBROUTINE EQN (B0,B1,B2,M,N,IOUT,X,J,K)
      DIMENSION X(12)
      DOUBLE PRECISION X,DPTS,XINCR,B0,B1,B2,M,N,P,R
      NPTS = 200
      DPTS = NPTS\
      XINCR = (X(K) - X(J)) / DPTS
      WRITE (IOUT,9002) M,N
      WRITE (IOUT,9003) NPTS
      P = X(J)
      DO 1 I=1,NPTS
      R = B0 + B1 * (P ** M) + B2 * (P ** N)
      WRITE (IOUT,9004) P,R
    1 P = P + XINCR
      RETURN
 9002 FORMAT ('F = B0 + B1*(CP3**',F4.2,') + B2*(CP3**',F4.2,')')
 9003 FORMAT (I3,' -1 -1  0')
 9004 FORMAT (2D15.7)
     END
C
C     SUBROUTINE SUM === CALCULATION OF SUM OF SQUARES
C
      SUBROUTINE SUM (X,Y,NPTS,B0,B1,B2,M,N,A,J,K)
      DIMENSION X(12),Y(12),A(5,6)
      DOUBLE PRECISION X,Y,B0,B1,B2,M,N,A,PTS,SXMYR2
      DOUBLE PRECISION SY,SXM,SXN,SXMY,SX2M,SXMXN,SXNY,SX2N,SXMYR,SXMR
      DOUBLE PRECISION SX2MR,SXMXNR,SXNYR,SXNR,SXMR2,SX2MR2,SXMNR2
      DOUBLE PRECISION SXNR2,SX2NR2,SXNYR2,SX2NR,XN,XM,X2M,X2N,R,R2
      PTS = NPTS
      SY = 0.0
      SXM = 0.0
      SXN = 0.0
      SXMY = 0.0
      SX2M = 0.0
      SXMXN = 0.0
      SXNY = 0.0
      SX2N = 0.0
      SXMYR = 0.0
      SXMR = 0.0
      SX2MR = 0.0
      SXMXNR = 0.0
      SXNYR = 0.0
```

```
      SXNR = 0.0
      SX2NR = 0.0
      SXMR2 = 0.0
      SX2MR2 = 0.0
      SXMNR2 = 0.0
      SXMYR2 = 0.0
      SXNR2 = 0.0
      SX2NR2 = 0.0
      SXNYR2 = 0.0
      DO 1 I=J,K
      XM = X(I) ** M
      XN = X(I) ** N
      X2M = XM ** 2
      X2N = XN ** 2
      R = DLOG(X(I))
      R2 = R ** 2
      SY = SY + Y(I)
      SXM = SXM + XM
      SXN = SXN + XN
      SXMY = SXMY + (XM * Y(I))
      SX2M = SX2M + X2M
      SXMXN = SXMXN + (XM * XN)
      SXNY = SXNY + (XN * Y(I))
      SX2N = SX2N + X2N
      SXMYR = SXMYR + (XM * Y(I) * R)
      SXMR = SXMR + (XM * R)
      SX2MR = SX2MR + (X2M * R)
      SXMXNR = SXMXNR + (XM * XN * R)
      SXNYR = SXNYR + (XN * Y(I) * R)
      SXNR = SXNR + (XN * R)
      SX2NR = SX2NR + (X2N * R)
      SXMR2 = SXMR2 + (XM * R2)
      SX2MR2 = SX2MR2 + (X2M * R2)
      SXMNR2 = SXMNR2 + (XM * XN * R2)
      SXMYR2 = SXMYR2 + (XM * Y(I) * R2)
      SXNR2 = SXNR2 + (XN * R2)
      SX2NR2 = SX2NR2 + (X2N * R2)
    1 SXNYR2 = SXNYR2 + (XN * Y(I) * R2)
      A(1,1) = PTS
      A(1,2) = SXM
      A(1,3) = SXN
      A(1,4) = B1 * SXMR
      A(1,5) = B2 * SXNR
      A(1,6) = -(PTS*B0 + B1*SXM + B2*SXN - SY)
      A(2,1) = SXM
      A(2,2) = SX2M
      A(2,3) = SXMXN
      A(2,4) = B0*SXMR + 2.0*B1*SX2MR + B2*SXMXNR - SXMYR
      A(2,5) = B2*SXMXNR
      A(2,6) = -(B0*SXM + B1*SX2M + B2*SXMXN - SXMY)
      A(3,1) = SXN
      A(3,2) = SXMXN
      A(3,3) = SX2N
      A(3,4) = SXMXNR
      A(3,5) = B0*SXNR + B1*SXMXNR + 2.0*B2*SX2NR - SXNYR
      A(3,6) = -(B0*SXN + B1*SXMXN + B2*SX2N - SXNY)
      A(4,1) = SXMR
      A(4,2) = SX2MR
      A(4,3) = SXMXNR
      A(4,4) = B0*SXMR2 + 2.0*B1*SX2MR2 + B2*SXMNR2 - SXMYR2
      A(4,5) = B2*SXMNR2
      A(4,6) = -(B0*SXMR + B1*SX2MR + B2*SXMXNR - SXMYR)
      A(5,1) = SXNR
      A(5,2) = SXMXNR
      A(5,3) = SX2NR
      A(5,4) = B1*SXMNR2
      A(5,5) = B0*SXNR2 + B1*SXMNR2 + 2.0*B2*SX2NR2 - SXNYR2
      A(5,6) = -(B0*SXNR + B1*SXMXNR + B2*SX2NR - SXNYR)
      RETURN
      END
C
C      SUBROUTINE SIMEQN === SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS
C
      SUBROUTINE SIMEQ (A,NR,NC)
      DIMENSION A(5,6),R(100)
      DOUBLE PRECISION A,R,D,T
      IF(NR - NC) 2,2,1
    1 NCT = NC
      GO TO 3
    2 NCT = NR
    3 K = 1
      D = 1.
    4 IF(A(K,K)) 9,5,9
    5 DO 6 I = K,NR
      IF(A(I,K)) 7,6,7
    6 CONTINUE
      GO TO 14
    7 DO 8 J = 1,NC
      T = A(I,J)
      A(I,J) = A(K,J)
    8 A(K,J) = T
    9 DO 13 I = 1,NR
      IF(I - K) 10,13,10
   10 DO 11 J = 1,NC
   11 R(J) = (A(I,J)*A(K,K) - A(I,K)*A(K,J))/D
      DO 12 J = 1,NC
   12 A(I,J) = R(J)
   13 CONTINUE
      D = A(K,K)
      K = K + 1
      IF(K - NCT) 4,4,14
   14 RETURN
      END
```

```
C
C  SUBROUTINE EXPONM = ESTIMATION OF INITIAL VALUE OF EXPONENT ''M''
C
      SUBROUTINE EXPONM (X,Y,NPTS,B0MIN,B1MIN,MMIN,G,H)
      DIMENSION X(12),Y(12)
      DOUBLE PRECISION X,XM,Y,M,PTS,SUMXM,SUMX2M,SUMY,SUMXMY,SMIN
      DOUBLE PRECISION S,B0,B1,MMIN,B0MIN,B1MIN,MAXM
      INTEGER G,H
      PTS = NPTS
      M = 0.10
      MAXM = 0.2
      UNIT = .0001
      SMIN = 16.0D70
      J = 1
6     SUMXM = 0.
      SUMX2M = 0.
      SUMY = 0.
      SUMXMY = 0.
      DO 7 I=G,H
      XM = X(I) ** M
      SUMXM = SUMXM + XM
      SUMX2M = SUMX2M + (XM**2)
      SUMY = SUMY + Y(I)
7     SUMXMY = SUMXMY + (XM * Y(I))
      Q1 = SUMXMY - (SUMXM * SUMY)/PTS
      Q2 = SUMX2M - (SUMXM ** 2)/PTS
      B1 = Q1 / Q2
      B0 = (SUMY - B1 * SUMXM) / PTS
      S = DSQRT(B0**2 + B1**2)
      IF (S - SMIN) 8,9,9
8     SMIN = S
      B0MIN = B0
      B1MIN = B1
      MMIN = M
9     J = J + 1
      M = M + UNIT
      IF (M - MAXM) 6,6,11
11    RETURN
      END
```

Appendix VIII (Continued)

```
*******************************************************************************
*                                                                             *
*   FIELD CN 65 MESH, RUN 1                                                   *
*                                                                             *
*******************************************************************************
```

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0666 G./CC.
O.D. MAT WT., W	=	12.1858 G.
MAT THICKNESS, L	=	4.01 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	0.577000 CM./SEC.
WATER TEMPERATURE, T	=	29.0 DEG. C.
WATER VISCOSITY, MU	=	0.008149 POISES

PERMEATION EXPERIMENT | COMPRESSIBILITY EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DPZ PRESSURE TAP READING DYNES/SQCM.	PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
1	-0.4844	0.0	0.0	0.0	0.	6.6000	0.0405
2	-0.1034	0.0	0.0	6.24	6116.	4.2450	0.0630
3	0.2776	0.25	245.16	11.17	10956.	3.7750	0.0708
4	0.6586	1.05	1029.67	21.12	20713.	3.2600	0.0820
5	1.0396	2.40	2353.53	36.06	35366.	2.8400	0.0941
6	1.4206	4.70	4609.00	61.02	59839.	2.4450	0.1093
7	1.8016	7.60	7452.85	95.64	93784.	2.1200	0.1261
8	2.1826	11.60	11375.40	145.63	142813.	1.8350	0.1457
9	2.5636	17.00	16670.84				
10	2.9446	24.20	23731.43				
11	3.3256	34.00	33341.69				
12	3.7066	46.00	45109.34				

COMPRESSIBILITY CONSTANTS

M = 0.6001D-02
N = 0.2657

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.21597
B = 1.51946
C = 1.50437

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.22525D 09 EXPM = 0.13540D 00
B1 = -0.18538D 10 EXPN = 0.22802D 00
B2 = 0.27234D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
1.06	3.73	6171.22	1655.23
2.46	3.73	6171.22	1655.23
4.55	3.45	6232.32	1805.10
7.53	3.41	6254.89	1834.55
11.63	3.26	6352.42	1946.98
17.13	3.07	6509.63	2117.54
24.38	2.87	6710.52	2336.10
33.84	2.67	6942.40	2597.83
46.02	2.48	7195.85	2900.40

Appendix VIII (Continued)

FIELD ON 65 MESH, RUN 2

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0660 G./CC.
O.D. MAT WT., W	=	12.7468 G.
MAT THICKNESS, L	=	4.24 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	0.657800 CM./SEC.
WATER TEMPERATURE, T	=	30.5 DEG. C.
WATER VISCOSITY, MU	=	0.007892 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DPZ PRESSURE TAP READING DYNES/SQCM.
1	-0.2594	0.0	0.0
2	0.1216	0.20	196.13
3	0.5026	0.95	931.61
4	0.8836	2.00	1961.28
5	1.2646	3.50	3432.23
6	1.6456	5.80	5687.70
7	2.0266	8.85	8678.64
8	2.4076	13.10	12846.36
9	2.7886	18.90	18534.05
10	3.1696	25.70	25202.39
11	3.5506	35.00	34322.32
12	3.9316	48.00	47070.62

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	7.1700	0.0390
6.32	6200.	4.5800	0.0611
11.25	11037.	4.0350	0.0693
21.20	20791.	3.4500	0.0811
36.14	35444.	3.0000	0.0932
61.10	59914.	2.5550	0.1095
95.71	93858.	2.2100	0.1265
145.71	142887.	1.9050	0.1468

COMPRESSIBILITY CONSTANTS

M = 0.51790-02
N = 0.2789

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.09231
B = 0.95777
C = 2.42722

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.31888D 09
B1 = -0.38826D 10
B2 = 0.47647D 10
EXPM = 0.14130D 00
EXPN = 0.19925D 00

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
2.05	3.78	5631.70	1491.72
3.59	3.78	5631.70	1491.72
5.74	3.78	5631.70	1491.72
8.71	3.66	5701.94	1555.82
12.78	3.58	5763.63	1610.02
18.30	3.40	5918.38	1739.24
25.73	3.18	6151.47	1936.19
35.67	2.93	6445.86	2198.40
48.92	2.69	6785.97	2525.78

Appendix VIII (Continued)

```
****************************************************************************
*                                                                          *
*   FIELD ON 20 MESH, RUN 1                                                *
*                                                                          *
****************************************************************************
```

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0643 G./CC.
O.D. MAT WT., W	=	11.9946 G.
MAT THICKNESS, L	=	4.09 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	0.678000 CM./SEC.
WATER TEMPERATURE, T	=	30.7 DEG. C.
WATER VISCOSITY, MU	=	0.007858 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DPZ PRESSURE TAP READING DYNES/SQCM.
1	-0.4094	0.0	0.0
2	-0.0284	0.0	0.0
3	0.3526	0.20	196.13
4	0.7336	0.85	833.54
5	1.1146	1.95	1912.24
6	1.4956	3.60	3530.30
7	1.8766	6.20	6079.95
8	2.2576	8.84	8668.84
9	2.6386	14.00	13728.93
10	3.0196	22.00	21574.03
11	3.4006	31.50	30890.09
12	3.7816	47.25	46335.14

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	6.7050	0.0392
6.50	6373.	4.0450	0.0651
11.44	11221.	3.5850	0.0734
21.40	20986.	3.0700	0.0857
36.35	35647.	2.6700	0.0986
61.31	60127.	2.2900	0.1149
95.94	94079.	1.9850	0.1326
145.94	143115.	1.7250	0.1526

COMPRESSIBILITY CONSTANTS

M = 0.5717D-02
N = 0.2741

EXPERIMENTAL EMPIRICAL CONSTANTS

```
DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
     A =   0.14953
     B =   1.59952
     C =   1.86714

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
     B0 =  0.12403D 09      EXPM =  0.12960D 00
     B1 = -0.80945D 09      EXPN =  0.26458D 00
     B2 =  0.16158D 10
```

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
1.87	4.07	5392.15	1324.78
3.55	4.01	5415.08	1350.56
6.08	3.77	5531.47	1467.93
9.72	3.47	5719.77	1647.09
14.84	3.17	5957.17	1878.25
21.92	2.89	6226.88	2157.66
31.54	2.62	6517.37	2484.29
44.48	2.39	6820.89	2858.60

Appendix VIII (Continued)

FIELD ON 20 MESH, RUN 2

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0596 G./CC.
O.D. MAT WT., W	=	14.8896 G.
MAT THICKNESS, L	=	5.48 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	0.748700 CM./SEC.
WATER TEMPERATURE, T	=	28.5 DEG. C.
WATER VISCOSITY, MU	=	0.008237 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DYNES/SQCM.
1	0.9856	0.85	833.54
2	1.3666	1.70	1667.08
3	1.7476	2.80	2745.79
4	2.1286	4.30	4216.74
5	2.5096	6.30	6178.02
6	2.8906	9.00	8825.74
7	3.2716	12.80	12552.16
8	3.6526	17.00	16670.84
9	4.0336	22.90	22456.60
10	4.4146	29.30	28732.68
11	4.7956	38.50	37754.56
12	5.1766	49.00	48051.26

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	6.1900	0.0528
6.48	6350.	5.6150	0.0582
11.40	11180.	4.8800	0.0669
21.34	20928.	4.1350	0.0790
36.28	35577.	3.5750	0.0914
61.23	60046.	3.0650	0.1066
95.84	93987.	2.6500	0.1233
145.84	143016.	2.3050	0.1417

COMPRESSIBILITY CONSTANTS

M = 0.46980-02
N = 0.2850

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.16667
B = 1.61452
C = 1.78481

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.11232D 09 EXPM = 0.13030D 00
B1 = -0.89447D 09 EXPN = 0.22766D 00
B2 = 0.14177D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
2.79	3.99	5007.86	1255.44
4.34	3.94	5019.81	1273.56
6.40	3.79	5070.69	1336.68
9.11	3.60	5156.75	1432.07
12.59	3.39	5271.04	1554.36
17.03	3.18	5407.06	1700.99
22.62	2.97	5559.48	1870.68
29.62	2.77	5724.07	2062.88
38.33	2.59	5897.58	2277.52
49.09	2.42	6077.50	2514.82

Appendix VIII (Continued)

FIELD CN 10 MESH, RUN 1

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0616 G./CC.
O.D. MAT WT., W	=	14.0474 G.
MAT THICKNESS, L	=	5.00 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	1.546900 CM./SEC.
WATER TEMPERATURE, T	=	30.0 DEG. C.
WATER VISCOSITY, MU	=	0.007976 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DPZ PRESSURE TAP READING DYNES/SQCM.
1	0.5056	1.10	1078.70
2	0.8866	2.10	2059.34
3	1.2676	3.20	3138.04
4	1.6486	4.95	4854.16
5	2.0296	7.10	6962.53
6	2.4106	10.00	9806.38
7	2.7916	14.10	13826.99
8	3.1726	18.80	18435.98
9	3.5536	24.20	23731.43
10	3.9346	31.50	30890.09
11	4.3156	40.00	39225.52
12	4.6966	51.00	50012.53

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	8.9750	0.0343
6.12	5997.	5.5950	0.0551
11.02	10811.	4.8550	0.0635
20.95	20545.	4.1300	0.0746
35.88	35183.	3.5950	0.0857
60.82	59642.	3.1000	0.0994
95.42	93574.	2.6850	0.1148
145.41	142593.	2.3350	0.1320

COMPRESSIBILITY CONSTANTS

M = 0.49860-02
N = 0.2738

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.09581
B = 0.83485
C = 2.41167

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.17458D 09   EXPM = 0.13900D 00
B1 = -0.27309D 10   EXPN = 0.17953D 00
B2 = 0.31040D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
3.36	4.01	3623.44	903.85
5.03	4.01	3623.44	903.85
7.19	4.01	3623.44	903.85
9.98	3.94	3656.44	929.14
13.55	3.91	3666.70	936.92
18.12	3.80	3723.66	978.71
23.93	3.64	3823.06	1050.78
31.31	3.44	3958.47	1151.36
40.64	3.22	4123.04	1279.68
52.40	3.00	4310.38	1435.60

Appendix VIII (Continued)

FIELD ON 10 MESH, RUN 2

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0603 G./CC.
O.D. MAT WT., W	=	14.7078 G.
MAT THICKNESS, L	=	5.35 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	1.385200 CM./SEC.
WATER TEMPERATURE, T	=	27.0 DEG. C.
WATER VISCOSITY, MU	=	0.008513 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DYNES/SQCM.
1	0.8506	2.00	1961.28
2	1.2316	3.10	3039.98
3	1.6126	4.50	4412.87
4	1.9936	6.19	6070.14
5	2.3746	8.40	8237.36
6	2.7556	11.50	11277.34
7	3.1366	15.00	14709.57
8	3.5176	19.20	18828.24
9	3.8986	24.30	23829.49
10	4.2796	31.50	30890.09
11	4.6606	40.00	39225.52
12	5.0416	51.00	50012.53

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	9.4100	0.0343
6.31	6184.	5.7800	0.0558
11.22	11003.	5.0050	0.0645
21.15	20741.	4.2250	0.0764
36.08	35386.	3.7000	0.0872
61.03	59848.	3.1800	0.1015
95.64	93785.	2.7650	0.1167
145.63	142809.	2.4050	0.1342

COMPRESSIBILITY CONSTANTS

M = 0.4899D-02
N = 0.2769

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.08271
B = 0.71294
C = 2.51176

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.16405D 09 EXPM = 0.14070D 00
B1 = -0.27029D 10 EXPN = 0.17955D 00
B2 = 0.30543D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
4.50	4.05	3446.34	850.43
6.26	4.05	3446.34	850.43
8.48	4.05	3446.34	850.43
11.28	3.92	3500.03	892.98
14.79	3.87	3521.22	908.92
19.19	3.76	3580.96	952.10
24.69	3.60	3675.90	1020.53
31.56	3.41	3801.23	1113.31
40.11	3.21	3951.75	1230.11
50.73	3.01	4122.49	1371.02

Appendix VIII (Continued)

FIELD ON 10 MESH, RUN 3

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0613 G./CC.
O.D. MAT WT., W	=	14.2630 G.
MAT THICKNESS, L	=	5.10 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	1.607500 CM./SEC.
WATER TEMPERATURE, T	=	30.0 DEG. C.
WATER VISCOSITY, MU	=	0.007976 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DYNES/SQCM.
1	0.6056	1.20	1176.77
2	0.9866	2.10	2059.34
3	1.3676	3.60	3530.30
4	1.7486	5.35	5246.41
5	2.1296	8.65	8482.52
6	2.5106	10.90	10688.95
7	2.8916	14.20	13925.05
8	3.2726	18.90	18534.05
9	3.6536	24.50	24025.63
10	4.0346	32.00	31380.41
11	4.4156	40.00	39225.52
12	4.7966	51.00	50012.53

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	9.4350	0.0332
6.56	6436.	5.7600	0.0543
11.49	11268.	5.0150	0.0624
21.43	21015.	4.1900	0.0747
36.37	35667.	3.6400	0.0860
61.33	60138.	3.1150	0.1005
95.94	94082.	2.7000	0.1159
145.94	143111.	2.3300	0.1343

COMPRESSIBILITY CONSTANTS

M = 0.4181D-02
N = 0.2902

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.14870
B = 1.06670
C = 1.92432

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.11947D 09 EXPM = 0.13790D 00
B1 = -0.17846D 10 EXPN = 0.17951D 00
B2 = 0.20468D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
3.67	3.46	3854.32	1115.33
5.50	3.46	3854.32	1115.33
7.84	3.46	3854.32	1115.33
10.78	3.38	3875.91	1145.11
14.47	3.33	3895.52	1170.18
19.06	3.23	3939.75	1221.06
24.75	3.10	4006.39	1294.20
31.77	2.95	4092.42	1387.89
40.38	2.79	4194.74	1501.32
50.91	2.64	4310.49	1634.22

Appendix VIII (Continued)

FLOC ON 65 MESH -- II, RUN 2

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0662 G./CC.
O.D. MAT WT., W	=	15.0664 G.
MAT THICKNESS, L	=	4.99 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	1.102300 CM./SEC.
WATER TEMPERATURE, T	=	28.5 DEG. C.
WATER VISCOSITY, MU	=	0.008237 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DPZ PRESSURE TAP READING DYNES/SQCM.
1	0.4906	1.45	1421.92
2	0.8716	2.60	2549.66
3	1.2526	4.30	4216.74
4	1.6336	6.30	6178.02
5	2.0146	8.90	8727.68
6	2.3956	12.80	12552.16
7	2.7766	16.50	16180.52
8	3.1576	21.30	20887.57
9	3.5386	27.10	26575.28
10	3.9196	34.00	33341.69
11	4.3006	42.50	41677.11
12	4.6816	52.00	50993.17

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	7.8400	0.0422
6.33	6205.	5.4150	0.0610
11.26	11041.	4.8500	0.0682
21.20	20792.	4.2200	0.0783
36.14	35440.	3.7150	0.0890
61.09	59906.	3.2150	0.1028
95.70	93846.	2.8150	0.1174
145.69	142869.	2.4450	0.1352

COMPRESSIBILITY CONSTANTS

M = 0.65150-02
N = 0.2524

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.16012
B = 0.91156
C = 1.89057

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.29920D 09 EXPM = 0.15020D 00
B1 = -0.64203D 10 EXPN = 0.17968D 00
B2 = 0.69346D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
4.38	3.45	4018.66	1165.80
6.45	3.45	4018.66	1165.80
9.02	3.45	4018.66	1165.80
12.20	3.45	4018.66	1165.80
16.12	3.45	4018.66	1165.80
20.94	3.45	4018.66	1165.80
26.84	3.42	4042.17	1181.55
34.04	3.36	4088.00	1215.73
42.79	3.26	4174.05	1279.42
53.41	3.13	4295.21	1370.65

Appendix VIII (Continued)

FLOC ON 65 MESH -- I, RUN 1

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0645 G./CC.
O.D. MAT WT., W	=	15.3764 G.
MAT THICKNESS, L	=	5.23 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	1.455900 CM./SEC.
WATER TEMPERATURE, T	=	29.0 DEG. C.
WATER VISCOSITY, MU	=	0.008149 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DYNES/SQCM.
1	0.7306	1.70	1667.08
2	1.1116	2.95	2892.88
3	1.4926	4.60	4510.93
4	1.8736	6.85	6717.37
5	2.2546	9.55	9365.09
6	2.6356	13.00	12748.29
7	3.0166	17.10	16768.90
8	3.3976	22.00	21574.03
9	3.7786	27.80	27261.72
10	4.1596	35.00	34322.32
11	4.5406	43.00	42167.43
12	4.9216	52.00	50993.17

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	8.3300	0.0405
6.40	6280.	5.8250	0.0579
11.34	11116.	5.2250	0.0646
21.28	20864.	4.5250	0.0746
36.21	35512.	3.9800	0.0848
61.16	59977.	3.4500	0.0978
95.77	93916.	3.0250	0.1115
145.76	142940.	2.6300	0.1283

COMPRESSIBILITY CONSTANTS

M = 0.61010-02
N = 0.2537

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.20397
B = 1.13078
C = 1.63461

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.19962D 09
B1 = -0.33888D 10
B2 = 0.37553D 10
EXPM = 0.14310D 00
EXPN = 0.17959D 00

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
4.73	3.49	3749.18	1075.37
6.89	3.49	3749.18	1075.37
9.56	3.49	3749.18	1075.37
12.85	3.49	3749.18	1075.37
16.86	3.45	3765.31	1090.35
21.73	3.44	3771.32	1095.50
27.60	3.38	3806.41	1124.68
34.66	3.29	3867.70	1174.54
43.11	3.18	3951.67	1243.06
53.20	3.05	4054.74	1328.91

Appendix VIII (Continued)

FLOC ON 65 MESH -- I, RUN 2

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0637 G./CC.
O.D. MAT WT., W	=	14.7122 G.
MAT THICKNESS, L	=	5.06 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	1.516600 CM./SEC.
WATER TEMPERATURE, T	=	27.0 DEG. C.
WATER VISCOSITY, MU	=	0.008513 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DPZ PRESSURE TAP READING DYNES/SQCM.
1	0.5656	3.45	3383.20
2	0.9466	4.75	4658.03
3	1.3276	6.55	6423.18
4	1.7086	8.60	8433.48
5	2.0896	11.40	11179.27
6	2.4706	15.00	14709.57
7	2.8516	18.90	18534.05
8	3.2326	23.20	22750.80
9	3.6136	28.70	28144.30
10	3.9946	35.00	34322.32
11	4.3756	43.00	42167.43
12	4.7566	52.00	50993.17

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	8.3700	0.0366
6.36	6237.	5.7000	0.0566
11.29	11067.	5.0450	0.0640
21.23	20816.	4.3750	0.0738
36.16	35462.	3.8300	0.0843
61.11	59928.	3.3200	0.0972
95.72	93864.	2.8800	0.1121
145.71	142889.	2.5150	0.1283

COMPRESSIBILITY CONSTANTS

M = 0.56710-02
N = 0.2601

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.10827
B = 0.41938
C = 2.26711

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.221730 09 EXPM = 0.145900 00
B1 = -0.417810 10 EXPN = 0.179630 00
B2 = 0.458650 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
8.85	4.01	3175.58	791.86
11.42	4.01	3175.58	791.86
14.53	4.01	3175.58	791.86
18.30	4.01	3175.58	791.86
22.87	4.01	3176.75	791.58
28.42	3.96	3204.76	808.38
35.15	3.86	3270.50	847.59
43.31	3.71	3370.73	908.44
53.19	3.53	3500.95	990.50

Appendix VIII (Continued)

```
****************************************************************************************
*                                                                                      *
*   FLOC ON 20 MESH, RUN 1                                                             *
*                                                                                      *
****************************************************************************************
```

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0793 G./CC.
O.D. MAT WT., W	=	14.7360 G.
MAT THICKNESS, L	=	4.07 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	1.789300 CM./SEC.
WATER TEMPERATURE, T	=	30.0 DEG. C.
WATER VISCOSITY, MU	=	0.007976 POISES

PERMEATION EXPERIMENT — COMPRESSIBILITY EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DPZ PRESSURE TAP READING DYNES/SQCM.	PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
1	-0.4244	0.20	196.13	0.0	0.	7.4650	0.0433
2	-0.0434	0.60	588.38	6.33	6203.	5.3200	0.0608
3	0.3376	4.20	4118.68	11.26	11042.	4.7950	0.0674
4	0.7186	7.25	7109.63	21.21	20797.	4.2300	0.0764
5	1.0996	10.40	10198.63	36.15	35449.	3.7650	0.0859
6	1.4806	14.60	14317.31	61.10	59918.	3.3050	0.0978
7	1.8616	19.00	18632.12	95.71	93856.	2.8850	0.1121
8	2.2426	24.00	23535.31	145.70	142880.	2.5100	0.1288
9	2.6236	30.50	29909.46				
10	3.0046	37.00	36283.60				
11	3.3856	44.50	43638.39				
12	3.7666	53.00	51973.81				

COMPRESSIBILITY CONSTANTS

M = 0.7440D-02
N = 0.2366

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.26628
B = 0.58727
C = 1.36890

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.56848D 07 EXPM = 0.0
B1 = 0.0 EXPN = 0.20000D 00
B2 = 0.85128D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE

STATIC LOAD = 19.96 CM. H2O FLUID PRESSURE DROP = 60.00 CM. H2O
AVERAGE SPECIFIC VOLUME, V = 2.97 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 3130.75 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 1053.15 SQCM./CC.

Appendix VIII (Continued)

FLOC ON 20 MESH, RUN 2

ORIGINAL DATA

MAT SOLIDS CONC. = 0.0812 G./CC.
O.D. MAT WT., W = 16.0273 G.
MAT THICKNESS, L = 4.33 CM.
TOTAL PRESSURE DROP, DPL = 60.00 CM. H2O
PERMEATION VELOCITY, U = 1.556900 CM./SEC.
WATER TEMPERATURE, T = 30.0 DEG. C.
WATER VISCOSITY, MU = 0.007976 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DPZ PRESSURE TAP READING DYNES/SQCM.
1	-0.1694	0.15	147.10
2	0.2116	3.10	3039.98
3	0.5926	6.20	6079.95
4	0.9736	9.42	9237.61
5	1.3546	13.40	13140.55
6	1.7356	17.90	17553.41
7	2.1166	22.10	21672.09
8	2.4976	27.20	26673.35
9	2.8786	33.50	32851.37
10	3.2596	39.00	38244.88
11	3.6406	46.50	45599.66
12	4.0216	54.00	52954.45

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	8.0150	0.0439
6.41	6283.	5.7800	0.0608
11.35	11128.	5.3100	0.0662
21.30	20883.	4.7000	0.0748
36.24	35535.	4.2000	0.0837
61.19	60001.	3.6700	0.0958
95.79	93939.	3.2200	0.1092
145.78	142960.	2.8000	0.1256

COMPRESSIBILITY CONSTANTS

M = 0.77360-02
N = 0.2309

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.31888
B = 0.63027
C = 1.19225

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.66982D 07 EXPM = 0.0
B1 = 0.0 EXPN = 0.20000D 00
B2 = 0.69701D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE

STATIC LOAD = 19.99 CM. H2O FLUID PRESSURE DROP = 60.00 CM. H2O
AVERAGE SPECIFIC VOLUME, V = 2.86 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 3366.86 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 1175.54 SQCM./CC.

Appendix VIII (Continued)

FLOC ON 10 MESH, RUN 1

ORIGINAL DATA

MAT SOLIDS CONC. = 0.0963 G./CC.
O.D. MAT WT., W = 16.5439 G.
MAT THICKNESS, L = 3.77 CM.
TOTAL PRESSURE DROP, DPL = 60.00 CM. H2O
PERMEATION VELOCITY, U = 1.870200 CM./SEC.
WATER TEMPERATURE, T = 30.0 DEG. C.
WATER VISCOSITY, MU = 0.007976 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DPZ PRESSURE TAP READING DYNES/SQCM.
1	-0.7294	0.40	392.26
2	-0.3484	1.00	980.64
3	0.0326	2.50	2451.59
4	0.4136	6.85	6717.37
5	0.7946	12.00	11767.65
6	1.1756	17.20	16866.97
7	1.5566	22.00	21574.03
8	1.9376	27.50	26967.54
9	2.3186	34.00	33341.69
10	2.6996	40.00	39225.52
11	3.0806	47.50	46580.30
12	3.4616	54.50	53444.77

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	8.2200	0.0442
6.39	6268.	6.1050	0.0595
11.31	11086.	5.2650	0.0689
21.25	20842.	4.6650	0.0775
36.20	35495.	4.2150	0.0861
61.15	59966.	3.7650	0.0964
95.76	93906.	3.3500	0.1083
145.75	142929.	2.9600	0.1226

COMPRESSIBILITY CONSTANTS

M = 0.8458D-02
N = 0.2231

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.56425
B = 0.75014
C = 0.58976

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.78651D 07 EXPM = 0.0
B1 = 0.0 EXPN = 0.10000D 01
B2 = 0.18065D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE

STATIC LOAD = 54.55 CM. H2O FLUID PRESSURE DROP = 60.00 CM. H2O
AVERAGE SPECIFIC VOLUME, V = 1.59 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 3149.86 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 1979.41 SQCM./CC.

Appendix VIII (Continued)

```
****************************************************************************************
*                                                                                      *
*   FLOC ON 10 MESH, RUN 2                                                             *
*                                                                                      *
****************************************************************************************
```

ORIGINAL DATA

MAT SOLIDS CONC. = 0.0897 G./CC.
O.D. MAT WT., W = 18.3970 G.
MAT THICKNESS, L = 4.50 CM.
TOTAL PRESSURE DROP, DPL = 60.00 CM. H2O
PERMEATION VELOCITY, U = 1.839900 CM./SEC.
WATER TEMPERATURE, T = 32.0 DEG. C.
WATER VISCOSITY, MU = 0.007647 POISES

PERMEATION EXPERIMENT

	Z	DPZ	
PRESSURE TAP NUMBER	DISTANCE FROM TOP OF MAT	PRESSURE TAP READING	
	CM.	CM. H2O	DYNES/SQCM.
1	0.0006	2.50	2451.59
2	0.3816	5.95	5834.79
3	0.7626	9.60	9414.12
4	1.1436	14.00	13728.93
5	1.5246	17.60	17259.22
6	1.9056	23.20	22750.80
7	2.2866	27.60	27065.60
8	2.6676	33.00	32361.05
9	3.0486	38.00	37264.24
10	3.4296	43.50	42657.75
11	3.8106	50.00	49031.89
12	4.1916	56.00	54915.72

COMPRESSIBILITY EXPERIMENT

PRESSURE		MAT THICKNESS	MAT SOLIDS CONCENTRATION
CM. H2O	DYNES/SQCM.	CM.	G./CC.
0.0	0.	8.9850	0.0449
6.43	6304.	6.9450	0.0581
11.33	11113.	5.9550	0.0678
21.28	20868.	5.3450	0.0755
36.22	35517.	4.8100	0.0839
61.16	59980.	4.2450	0.0951
95.77	93914.	3.7550	0.1075
145.75	142929.	3.2650	0.1236

COMPRESSIBILITY CONSTANTS

M = 0.76460-02
N = 0.2313

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.48837
B = 0.68145
C = 0.76601

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.67684D 07   EXPM = 0.0
B1 = 0.0   EXPN = 0.20000D 00
B2 = 0.30605D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE

STATIC LOAD = 40.04 CM. H2O   FLUID PRESSURE DROP = 60.00 CM. H2O
AVERAGE SPECIFIC VOLUME, V = 1.99 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 3091.69 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 1550.19 SQCM./CC.

Appendix VIII (Continued)

WHOLE PULP ON 65 MESH, RUN 1

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0642 G./CC.
O.D. MAT WT., W	=	13.0303 G.
MAT THICKNESS, L	=	4.45 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	1.011400 CM./SEC.
WATER TEMPERATURE, T	=	29.0 DEG. C.
WATER VISCOSITY, MU	=	0.008149 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DYNES/SQCM.
1	-0.0494	0.20	196.13
2	0.3316	0.80	784.51
3	0.7126	1.80	1765.15
4	1.0936	3.30	3236.10
5	1.4746	5.30	5197.38
6	1.8556	8.10	7943.16
7	2.2366	11.80	11571.52
8	2.6176	16.50	16180.52
9	2.9986	22.00	21574.03
10	3.3796	29.00	28438.50
11	3.7606	38.50	37754.56
12	4.1416	49.50	48541.57

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	7.2350	0.0395
6.28	6156.	4.7950	0.0596
11.21	10991.	4.2400	0.0674
21.15	20743.	3.6600	0.0781
36.09	35394.	3.1950	0.0895
61.05	59863.	2.7550	0.1038
95.66	93805.	2.3900	0.1196
145.65	142832.	2.0750	0.1378

COMPRESSIBILITY CONSTANTS

M = 0.5714D-02
N = 0.2653

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.12232
B = 0.92842
C = 2.15303

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.29737D 09 EXPM = 0.14510D 00
B1 = -0.54785D 10 EXPN = 0.17962D 00
B2 = 0.60558D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
3.39	3.66	4494.34	1228.17
5.37	3.66	4494.34	1228.17
8.00	3.66	4494.34	1228.17
11.43	3.66	4494.34	1228.17
15.91	3.57	4551.36	1275.76
21.70	3.49	4607.00	1319.63
29.16	3.35	4725.42	1411.07
38.72	3.17	4896.97	1546.20
50.93	2.97	5111.00	1723.08

Appendix VIII (Continued)

WHOLE PULP ON 65 MESH, RUN 2

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0647 G./CC.
O.D. MAT WT., W	=	12.9967 G.
MAT THICKNESS, L	=	4.40 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	1.131500 CM./SEC.
WATER TEMPERATURE, T	=	34.0 DEG. C.
WATER VISCOSITY, MU	=	0.007340 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DYNES/SQCM.
1	-0.0944	0.50	490.32
2	0.2866	1.15	1127.73
3	0.6676	2.30	2255.47
4	1.0486	3.80	3726.42
5	1.4296	5.90	5785.76
6	1.8106	8.75	8580.58
7	2.1916	12.00	11767.65
8	2.5726	16.80	16474.70
9	2.9536	22.30	21868.21
10	3.3346	29.50	28928.82
11	3.7156	39.00	38244.88
12	4.0966	50.00	49031.89

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	7.1650	0.0398
6.38	6259.	4.7750	0.0597
11.32	11098.	4.2300	0.0674
21.27	20856.	3.6600	0.0779
36.21	35508.	3.1850	0.0895
61.16	59979.	2.7250	0.1046
95.78	93922.	2.3500	0.1213
145.77	142951.	2.0250	0.1408

COMPRESSIBILITY CONSTANTS

M = 0.5292D-02
N = 0.2732

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.09132
B = 0.63930
C = 2.45745

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.28751D 09 EXPM = 0.14570D 00
B1 = -0.54521D 10 EXPN = 0.17962D 00
B2 = 0.60246D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE AS FUNCTIONS OF PRESSURE

PRESSURE CM. H2O	V CC./G.	SW SQCM./G.	SV SQCM./CC.
3.93	3.72	4359.78	1173.39
5.92	3.72	4359.78	1173.39
8.52	3.72	4359.78	1173.39
11.91	3.72	4359.78	1173.39
16.32	3.66	4398.83	1200.59
22.05	3.57	4469.58	1253.19
29.47	3.40	4605.95	1353.84
39.06	3.20	4798.34	1500.51
51.42	2.98	5035.90	1692.60

Appendix VIII (Continued)

WHOLE PULP ON 20 MESH, RUN 1

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0745 G./CC.
O.D. MAT WT., W	=	13.2351 G.
MAT THICKNESS, L	=	3.90 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	1.567100 CM./SEC.
WATER TEMPERATURE, T	=	32.0 DEG. C.
WATER VISCOSITY, MU	=	0.007647 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DYNES/SQCM.
1	-0.5994	0.0	0.0
2	-0.2184	0.0	0.0
3	0.1626	2.30	2255.47
4	0.5436	4.65	4559.96
5	0.9246	7.60	7452.85
6	1.3056	11.50	11277.34
7	1.6866	15.50	15199.89
8	2.0676	20.50	20103.07
9	2.4486	26.00	25496.59
10	2.8296	33.00	32361.05
11	3.2106	42.00	41186.79
12	3.5916	51.50	50502.85

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	7.1250	0.0408
6.30	6173.	4.9200	0.0590
11.23	11013.	4.4250	0.0656
21.18	20771.	3.9000	0.0745
36.12	35423.	3.4400	0.0844
61.07	59892.	2.9900	0.0971
95.69	93833.	2.6050	0.1115
145.68	142859.	2.2750	0.1276

COMPRESSIBILITY CONSTANTS

M = 0.67290D-02
N = 0.2447

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.18217
B = 0.52321
C = 1.75901

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.65434D 07 EXPM = 0.0
B1 = 0.0 EXPN = 0.20000D 00
B2 = 0.14564D 11

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE

STATIC LOAD = 19.93 CM. H2O FLUID PRESSURE DROP = 60.00 CM. H2O
AVERAGE SPECIFIC VOLUME, V = 3.39 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 3471.66 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 1023.29 SQCM./CC.

Appendix VIII (Continued)

WHOLE PULP ON 20 MESH, RUN 2

ORIGINAL DATA

MAT SOLIDS CONC. = 0.0758 G./CC.
O.D. MAT WT., W = 13.5194 G.
MAT THICKNESS, L = 3.91 CM.
TOTAL PRESSURE DROP, DPL = 60.00 CM. H2O
PERMEATION VELOCITY, U = 1.425600 CM./SEC.
WATER TEMPERATURE, T = 29.0 DEG. C.
WATER VISCOSITY, MU = 0.008149 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DYNES/SQCM.
1	-0.5844	0.10	98.06
2	-0.2034	0.35	343.22
3	0.1776	2.85	2794.82
4	0.5586	5.20	5099.32
5	0.9396	8.20	8041.23
6	1.3206	12.10	11865.71
7	1.7016	16.20	15886.33
8	2.0826	21.40	20985.64
9	2.4636	26.90	26379.15
10	2.8446	34.00	33341.69
11	3.2256	42.00	41186.79
12	3.6066	52.00	50993.17

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	7.5700	0.0392
6.33	6211.	5.1150	0.0580
11.27	11051.	4.6000	0.0645
21.22	20805.	4.0150	0.0739
36.16	35457.	3.5450	0.0837
61.11	59925.	3.0700	0.0966
95.72	93866.	2.6750	0.1109
145.71	142890.	2.3150	0.1281

COMPRESSIBILITY CONSTANTS

M = 0.62260-02
N = 0.2513

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.17763
B = 0.46357
C = 1.79133

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.71134D 07 EXPM = 0.0
B1 = 0.0 EXPN = 0.20000D 00
B2 = 0.15913D 11

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE

STATIC LOAD = 14.96 CM. H2O FLUID PRESSURE DROP = 60.00 CM. H2O
AVERAGE SPECIFIC VOLUME, V = 3.40 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 3621.22 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 1065.60 SQCM./CC.

WHOLE PULP ON 10 MESH, RUN 1

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0838 G./CC.
O.D. MAT WT., W	=	14.3837 G.
MAT THICKNESS, L	=	3.76 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	1.880300 CM./SEC.
WATER TEMPERATURE, T	=	29.0 DEG. C.
WATER VISCOSITY, MU	=	0.008149 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DYNES/SQCM.
1	-0.7344	0.0	0.0
2	-0.3534	0.0	0.0
3	0.0276	1.60	1569.02
4	0.4086	6.20	6079.95
5	0.7896	10.00	9806.38
6	1.1706	14.90	14611.50
7	1.5516	19.00	18632.12
8	1.9326	24.50	24025.63
9	2.3136	31.00	30399.77
10	2.6946	37.00	36283.60
11	3.0756	44.00	43148.07
12	3.4566	53.00	51973.81

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	7.2850	0.0433
6.38	6259.	5.2650	0.0599
11.32	11104.	4.8100	0.0656
21.28	20864.	4.2650	0.0740
36.22	35518.	3.8150	0.0827
61.17	59989.	3.3450	0.0943
95.79	93931.	2.9500	0.1070
145.79	142962.	2.6600	0.1186

COMPRESSIBILITY CONSTANTS

M = 0.84160D-02
N = 0.2209

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.37018
B = 0.62821
C = 1.00625

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.62169D 07   EXPM = 0.0
B1 = 0.0   EXPN = 0.20000D 00
B2 = 0.69349D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE

STATIC LOAD = 29.91 CM. H2O   FLUID PRESSURE DROP = 60.00 CM. H2O
AVERAGE SPECIFIC VOLUME, V = 2.69 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 3194.62 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 1185.47 SQCM./CC.

Appendix VIII (Continued)

```
*******************************************************************************
*                                                                             *
*   WHOLE PULP ON 10 MESH, RUN 2                                              *
*                                                                             *
*******************************************************************************
```

ORIGINAL DATA

MAT SOLIDS CONC. = 0.0818 G./CC.
O.D. MAT WT., W = 17.9218 G.
MAT THICKNESS, L = 4.80 CM.
TOTAL PRESSURE DROP, DPL = 60.00 CM. H2O
PERMEATION VELOCITY, U = 1.486200 CM./SEC.
WATER TEMPERATURE, T = 28.0 DEG. C.
WATER VISCOSITY, MU = 0.008328 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DPZ PRESSURE TAP READING DYNES/SQCM.
1	0.3056	4.00	3922.55
2	0.6866	6.95	6815.43
3	1.0676	11.80	11571.52
4	1.4486	15.80	15494.08
5	1.8296	19.20	18828.24
6	2.2106	23.20	22750.80
7	2.5916	27.60	27065.60
8	2.9726	32.00	31380.41
9	3.3536	37.00	36283.60
10	3.7346	42.00	41186.79
11	4.1156	48.00	47070.62
12	4.4966	55.00	53935.09

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	8.7000	0.0452
6.44	6314.	6.5300	0.0602
11.38	11162.	6.0850	0.0646
21.33	20920.	5.4950	0.0716
36.27	35568.	4.9300	0.0798
61.21	60029.	4.3250	0.0909
95.82	93961.	3.7850	0.1039
145.80	142977.	3.2800	0.1199

COMPRESSIBILITY CONSTANTS

M = 0.83830-02
N = 0.2194

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.53488
B = 0.78143
C = 0.61615

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.82792D 07   EXPM = 0.0
B1 = 0.0   EXPN = 0.20000D 00
B2 = 0.42856D 10

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE

STATIC LOAD = 20.06 CM. H2O   FLUID PRESSURE DROP = 60.00 CM. H2O
AVERAGE SPECIFIC VOLUME, V = 2.09 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 3458.12 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 1657.52 SQCM./CC.

Appendix VIII (Continued)

```
*****************************************************************************
*                                                                           *
*   WHOLE PULP -- NOT CLASSIFIED, RUN 1                                     *
*                                                                           *
*****************************************************************************
```

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0762 G./CC.
O.D. MAT WT., W	=	16.7617 G.
MAT THICKNESS, L	=	4.82 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H20
PERMEATION VELOCITY, U	=	1.223600 CM./SEC.
WATER TEMPERATURE, T	=	37.5 DEG. C.
WATER VISCOSITY, MU	=	0.006849 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H20	 PRESSURE TAP READING DYNES/SQCM.
1	0.3256	3.60	3530.30
2	0.7066	6.00	5883.82
3	1.0876	8.70	8531.55
4	1.4686	11.80	11571.52
5	1.8496	15.50	15199.89
6	2.2306	19.40	19024.37
7	2.6116	23.50	23044.99
8	2.9926	27.50	26967.54
9	3.3736	33.00	32361.05
10	3.7546	39.50	38735.20
11	4.1356	47.00	46089.98
12	4.5166	54.00	52954.45

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H20	PRESSURE DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	8.5200	0.0432
6.40	6280.	6.2000	0.0593
11.34	11125.	5.7250	0.0642
21.29	20880.	5.1100	0.0720
36.22	35524.	4.5100	0.0815
61.17	59984.	3.9100	0.0941
95.77	93916.	3.3900	0.1085
145.75	142933.	2.9150	0.1262

COMPRESSIBILITY CONSTANTS

M = 0.6820D-02

N = 0.2411

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)

WHERE...

A = 0.26383

B = 0.59202

C = 1.36708

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)

WHERE...

B0 = 0.87342D 07 EXPM = 0.0

B1 = 0.0 EXPN = 0.20000D 00

B2 = 0.14373D 11

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE

STATIC LOAD = 20.07 CM. H20 FLUID PRESSURE DROP = 60.00 CM. H20

AVERAGE SPECIFIC VOLUME, V = 3.07 CC./G.

AVERAGE SPECIFIC SURFACE, SW = 3911.25 SQCM./G.

AVERAGE SPECIFIC SURFACE, SV = 1274.98 SQCM./CC.

Appendix VIII (Continued)

WHOLE PULP -- NOT CLASSIFIED , RUN 2

ORIGINAL DATA

MAT SOLIDS CONC.	=	0.0737 G./CC.
O.D. MAT WT., W	=	16.5539 G.
MAT THICKNESS, L	=	4.92 CM.
TOTAL PRESSURE DROP, DPL	=	60.00 CM. H2O
PERMEATION VELOCITY, U	=	1.102500 CM./SEC.
WATER TEMPERATURE, T	=	32.0 DEG. C.
WATER VISCOSITY, MU	=	0.007647 POISES

PERMEATION EXPERIMENT

PRESSURE TAP NUMBER	Z DISTANCE FROM TOP OF MAT CM.	DPZ PRESSURE TAP READING CM. H2O	DPZ PRESSURE TAP READING DYNES/SQCM.
1	0.4256	3.45	3383.20
2	0.8066	5.90	5785.76
3	1.1876	8.82	8649.22
4	1.5686	12.00	11767.65
5	1.9496	15.10	14807.63
6	2.3306	18.50	18141.80
7	2.7116	22.20	21770.16
8	3.0926	27.10	26575.28
9	3.4736	33.00	32361.05
10	3.8546	38.00	37264.24
11	4.2356	46.00	45109.34
12	4.6166	53.00	51973.81

COMPRESSIBILITY EXPERIMENT

PRESSURE CM. H2O	PRESSURE DYNES/SQCM.	MAT THICKNESS CM.	MAT SOLIDS CONCENTRATION G./CC.
0.0	0.	8.6950	0.0418
6.43	6302.	6.3050	0.0576
11.37	11146.	5.8100	0.0625
21.31	20897.	5.1400	0.0707
36.24	35539.	4.4950	0.0808
61.18	59999.	3.8800	0.0936
95.79	93931.	3.3550	0.1083
145.77	142948.	2.8900	0.1257

COMPRESSIBILITY CONSTANTS

M = 0.60660D-02
N = 0.2511

EXPERIMENTAL EMPIRICAL CONSTANTS

DPZ/DPL = A * ((Z/L)**B) * EXP(C*Z/L)
WHERE...
A = 0.30793
B = 0.72668
C = 1.16871

F = B0 + B1*(X**EXPM) + B2*(X**EXPN)
WHERE...
B0 = 0.10606D 00 EXPM = 0.0
B1 = 0.0 EXPN = 0.20000D 00
B2 = 0.11561D 11

HYDRODYNAMIC SPECIFIC VOLUME AND SURFACE

STATIC LOAD = 20.08 CM. H2O FLUID PRESSURE DROP = 60.00 CM. H2O
AVERAGE SPECIFIC VOLUME, V = 2.67 CC./G.
AVERAGE SPECIFIC SURFACE, SW = 4164.54 SQCM./G.
AVERAGE SPECIFIC SURFACE, SV = 1557.31 SQCM./CC.

APPENDIX IX

ANALYSIS OF INHERENT ASSUMPTION IN DETERMINING S_W AND v AS FUNCTIONS OF PRESSURE SIMULTANEOUSLY

During the analysis of average specific surface, $\langle S_W \rangle$, and average specific volume, $\langle v \rangle$, from constant rate filtration and wet mat compressibility data, Equations (29) and (46) are used; these are of the general form:

$$Y = k_1 \langle S_W \rangle^2 \langle v \rangle^{-1/2} + k_2 \langle S_W \rangle^2 \langle v \rangle^{2.5} X \tag{48}$$

where $\langle S_W \rangle = S_W$ and $\langle v \rangle = v$ in Equation (46) and where:

	Equation (29)	Equation (46)
k_1 =	$3.5(1-N/2)$	1
k_2 =	$k_1\ 57(1-N/2)^6$	57
X =	c^3	$[c_L(\Delta P_z/\Delta P_L)^N]^3$
Y =	$\dfrac{\Delta P}{c^{1/2} t}$	$\dfrac{\Delta P_L (C + BL/z)}{3.5 \mu L U c_L^{1.5}} (\Delta P_z/\Delta P_L)^{1-1.5N}$

Equation (31) is in the form of a straight line; therefore, a plot of Y vs. X should be linear assuming $\langle S_W \rangle$ and $\langle v \rangle$ independent of X. That is:

$$\text{slope} = \frac{dY}{dX} = k_2 \langle S_W \rangle^2 \langle v \rangle^{2.5} \tag{49}$$

$$\text{intercept} = Y - X \frac{dY}{dX} = k_1 \langle S_W \rangle^2 \langle v \rangle^{-1/2} \tag{50}$$

In actuality, a plot of Y vs. X is not linear, and the nonlinearity has been attributed to the dependence of $\langle S_W \rangle$ and $\langle v \rangle$ on X (41); such that:

$$\frac{dY}{dX} = Q + k_2 \langle S_W \rangle^2 \langle v \rangle^{2.5} \tag{51}$$

$$Y - X\,\frac{dY}{dX} = k_1 \, \langle S_W \rangle^2 \, \langle v \rangle^{-1/2} - X\,Q \tag{52}$$

where Q represents those terms containing $d\langle S_W \rangle/dX$ and $d\langle v \rangle/dX$. Previously it was believed that if Q was assumed negligible then $\langle S_W \rangle$ and $\langle v \rangle$ may be determined as functions of X, and subsequently pressure, by dividing a plot of Y vs. X into segments and applying Equations (51) and (52) to each segment. This procedure was further improved by direct calculation of dY/dX using the numerical procedure discussed in the text.

But, Grace (61) and Nelson (53) have demonstrated this method of analysis is valid only if Q is exactly equal to zero. Furthermore, Nelson (53) has shown by analysis of this method of data reduction for Equation (29) that it is equivalent to a further (and unstated) assumption represented by Equation (53).

$$\frac{k_1}{k_2 \langle v \rangle^3} = c^3 \left[\frac{\dfrac{\Delta P_f}{c^3}\,\dfrac{dc^3}{d\Delta P_f}}{1 - \dfrac{\Delta P_f}{6c^3}\,\dfrac{dc^3}{d\Delta P_f} - \dfrac{\Delta P_f}{t}\,\dfrac{dt}{d\Delta P_f}} \right] \tag{53}$$

Equation (53) represents the implied definition of $\langle v \rangle$. Since Equation (53) presently has no physical significance, it must be rejected as a plausable assumption; and therefore Equation (29) may only be solved for a singular value of $\langle S_W \rangle$ and $\langle v \rangle$. The argument pertaining to Equation (46) is believed similar.