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(Hot Surface Drying of Paper)
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HOT SURFACE DRYING OF PAPER

PRELIMINARY EXPERIMENTS MEASURING THERMAL CONDUCTIVITY BY A NONSTEADY-STATE TECHNIQUE

SUMMARY

Project Report 2, Project 1901, reviews the literature on the techniques of measuring thermal conductivity. From this review, a non-steady-state method devised by Krischer and Esdorn (1) was selected as the most promising for our purpose, that of measuring thermal conductivities of wet paper. This particular method puts to use one solution of the Fourier equation by observing the necessary boundary conditions under experimental conditions. For further details of the solution, refer to Project Report 2 and the literature cited therein.

Equipment, essentially as described by Krischer and Esdorn, was constructed. Some modifications in the design of the heating elements were included to simplify their fabrication, but basically the unit was the same as described. Temperature measurements were taken by thermocouples made from Chromel and Alumel wire 0.002 inch in diameter following the technique of Ulmanen (2).

The material under test was lightly beaten, bleached sulfite pulp formed into pads of about 150 mg./cm.² For the preliminary tests, the pads of pulp were air dried.

From the beginning of the test program, the measurement of thermal conductivity was not reproducible. The electric power supply to the heaters was altered in several ways in attempts to obtain a steady power input. Finally, an A.C. circuit with voltage stabilizer and transformers was used that did give a satisfactory electrical input. Several thermocouple arrangements were also investigated, both in regard to the thermocouples and the measuring instruments. Despite this, the calculated values for k still varied as much as fourfold.

The net result, then, is that this technique has not been successful in measuring the thermal conductivity of paper.

APPARATUS

The equipment built and used in this work for measuring the thermal conductivity is similar to the apparatus described by Krischer and Esdorn (1).

The heating elements were fabricated from 0.001-inch stainless steel shimstock with a heating area of 10 centimeters by 10 centimeters. Opposite edges of the heater were fitted with copper bus bars 10 centimeters long, 1 centimeter wide, and several millimeters thick. In order to attach the heavy bus bars to the stainless steel foil, slots were milled in the bars; the slots were then filled with solder, and the edge of the foil inserted. With the somewhat irregular edge formed by the solder, the precise area of the heating element is not 100 square centimeters. An error of perhaps one-half per cent maximum is inherent in the design of the heaters, but this error is constant. Four heating

units were prepared that were then connected in series by fastening heavy copper leads between the bus bars of adjacent units.

In operation, the series of heater units are sandwiched throughout a stack of eight identical pads of the test substance, as shown in Fig. 1. The entire test zone is placed under a slight compressive load (several pounds per square inch) to insure good thermal contact between the heaters and the test material. The whole setup is then housed in a small closed box to reduce heat losses.

Thermocouples to measure temperature rise during a test are located within the pile. One thermocouple is placed between the two pads of test material forming the central plane of the stack. Two other thermocouples are located between the heating elements and the test substance on either side of the central plane of the entire stack. The thermocouple junctions are placed no nearer than 4 centimeters from the edges of the heaters in the plane of the heaters. The thermocouples are connected to a Minneapolis-Honeywell multipoint recording potentiometer.

Several power sources have been tried. Lead-acid storage batteries were first used to supply the necessary current to the heating elements, but because of the relatively high amperage requirements, the batteries were unable to supply a constant power output. Alternating current through a voltage regulator and a transformer was also tried. This second procedure gave a constant energy input throughout the course of an experiment. The heat generated is determined by measuring the voltage drop across and the current flowing through a heating element.

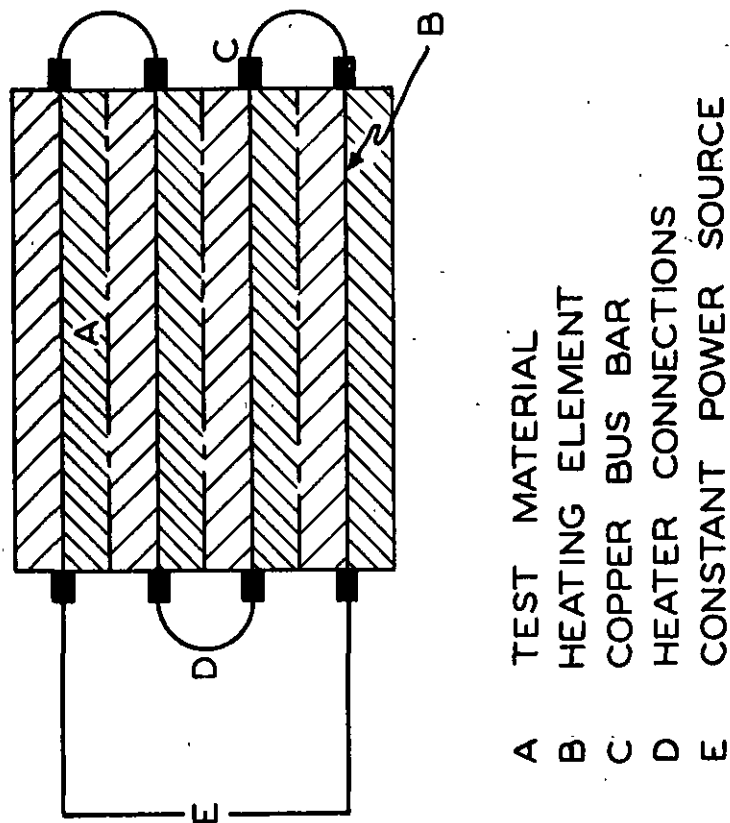


Fig. 1. Schematic Diagram of Thermal Conductivity Measuring Apparatus

An assumption is made that the copper bus bars and leads are of sufficiently low resistance that their power consumption is negligible compared with the stainless steel foil.

EXPERIMENTAL

The initial work was planned to test the apparatus by measuring and checking the conductivity of a dry sheet or pad of lightly beaten bleached sulfite pulp against the values reported in the literature for dry pulp. Sheets with a basis weight of approximately 150 milligrams per square centimeter were formed in a sheet-forming apparatus. These sheets (eight) were uniformly wet pressed to about 100 centimeters of mercury for one minute, and then trimmed to the necessary size for the conductivity apparatus (10 cm. by 10 cm.). The eight sheets were then stacked, placed under a slight compressive load, and air dried to their equilibrium point. Only one set of eight sheets was prepared, and these same sheets were used in all of the experiments.

Thermocouples were prepared in the same manner as described by Ulmanen (2). Chromel and Alumel wire, 0.002 inch in diameter, were joined with silver solder. As two of the thermocouples in the system were to be in direct contact with the metal heating elements, the thermocouples were given a thin coating of "Formvar" to electrically insulate them from the heaters.

The sheets of pulp, heating elements, and thermocouples were then stacked as shown in Fig. 1. In the first runs (1-21), a storage battery was used as the source of power. It was soon detected that the power output was not holding constant through an experiment. This led to a number of trials in which the two-volt cells were hooked up to form a 3-cell 2-volt battery, two 6-volt batteries in parallel, a 6-volt

The differential thermocouple was connected to a "Sargent" single pen recording millivoltmeter.

RESULTS

The calculated values for the thermal conductivity, k , of a dry sheet of paper are recorded in Table I. Han and Ulmanen (3) estimated k from their data to be from 0.00018 to 0.00020 cal./cm.² x °C./cm. for essentially the same dry pulp.

It is obvious that some factor or factors are not being controlled in this present work, as the values of k vary from 0.00010 to 0.00044 cal./cm.² x sec. x °C./cm. Experimental attempts to eliminate the uncontrolled variables have been unsuccessful. The first suspect was the power supply. After a number of attempts to obtain steady D.C. power to the heaters, the system was switched to A.C. This did give a constant power input. A second suspect was the temperature measurement. Three different potentiometers were used at various times--a Honeywell recording potentiometer, a Sargent recording potentiometer, and a student potentiometer-galvanometer arrangement. Though a difference in the generated emf was detected (runs 22-26) when two of these measuring circuits were used concurrently, neither device gave a constant value of k for a pad of dry paper.

Neither refinements in the power supply nor in the measurement of the pad temperature had an effect on the reproducibility of the k value. Unfortunately, time has not permitted further investigation into the causes of the nonreproducibility of k . Because the thermal

battery in parallel with a battery charger, and other combinations.

In most cases, the power would hold constant to only within 10% for an experiment.

Eventually (runs 22-33), an A.C. source of current was used.

The line current was run through a voltage stabilizer, then through a variable transformer (Varic) and finally a step-down transformer (46:1).

This arrangement gave a constant power output.

The power supplied to the heating elements was measured (runs 1-7) by means of a D.C. millivoltmeter and a D.C. milliammeter, with appropriate shunts or series resistances. After run 7, the milliammeter was inadvertently damaged and had to be returned to the manufacturer for repair. Runs 8-9 made use of a 15-amp. D.C. ammeter. Runs 10-17 used a borrowed D.C. Polyranger. Runs 18-21 used the original equipment again. The runs using an A.C. power supply necessitated A.C. instrumentation, so an A.C. ammeter together with an A.C. voltmeter (Heath kit) were used. It was recognized that these instruments did not have the accuracy that the D.C. millivolt and milliammeters had, but as there was already considerable nonreproducibility in the experiments, this was ignored for the time being.

Beginning with run no. 18, a differential thermocouple was also employed to measure the temperature drop across the pad of test material. A differential thermocouple is actually two normal thermocouples with the two Chromel (or Alumel) wires connected together. The other wires (Alumel or Chromel) are then connected to the measuring millivoltmeter. The reading of this thermocouple is simply the difference in emf between the two junctions and will not indicate the actual temperature. The

TABLE I (Continued)

Run	Voltage, volts	Amperage, amps.	$\frac{k \times 10^5}{\text{cal.}/(\text{cm}^2)(\text{sec.})}(\text{C./cm.})$	
			Sargent	Honeywell
			($\Delta T.C.$)	(Normal T.C.)
22 ^{3,4}	0.9	8.4	18	14
23 ^{3,4}	0.75	6.8	25	16
24 ^{3,4}	0.59	6.2	35	20
25 ^{3,4}	0.94	8.2	27	17
26 ^{3,4}	0.94	8.2	27	15
27 ^{3,5}	0.92	8.25	14	
28 ^{3,5}	1.15	10.4	15	
29 ^{3,5}	0.75	6.70	28	
30 ^{3,5}	0.86	7.75	20	
31 ^{3,6}	1.05	8.30	27	
32 ^{3,6}	0.85	6.65	23	
33 ^{3,6}	0.72	5.00	44	

³ A.C. Power supply

⁴ Use of two (2) Δt measuring devices, i.e., a normal pair of T.C.'s and a differential T.C.

⁵ Only a differential T.C. used, its values recorded by the Sargent recorder.

⁶ T.C. emf Measured with student potentiometer.

TABLE I

Run	Voltage, volts	Amperage, amps.	$\frac{k}{\text{cal.}} \times 10^5, (\text{sec.}) (^\circ\text{C./cm.})$
1 ¹	0.78	4.45	20
2 ¹	1.32	5.28	13
3 ¹	1.55	6.10	14
4 ¹	0.63	1.82	11
5 ¹	0.68	2.32	10
6 ¹	0.86	5.47	21
7 ¹	1.17	7.85	22
8 ¹	0.68	4.30	23
9 ¹	0.78	4.90	20
10 ¹	1.78	13.3	25
11 ¹	1.34	8.91	15
12 ¹	0.65	4.29	15
13 ¹	0.99	6.54	13
14 ¹	1.30	8.66	14
15 ¹	0.61	3.58	12
16 ¹	0.88	5.74	11
17 ¹	1.66	10.95	17
18 ^{1,2}	--	--	--
19 ^{1,2}	--	--	--
20 ^{1,2}	--	--	--
21 ^{1,2}	--	--	--

¹ D.C. Power source; batteries plus charger in several combinations; various voltmeters and ammeters and arrangements of shunts for these meters.

² A large convergence of the t_o and t_s was noted throw these runs out!

conductivity of a dry sheet could not be measured, no effort was made to make measurements on pads of wet pulp.

LITERATURE CITED

1. Krischer, O., and Esdorn, H., VDI Forschungsh., Beil. Forsch. Gebiete. Ingenieurw. 321, no. 450:28-39(1955).
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3. Han, S. T., and Ulmanen, T., Tappi 41, no. 4:185-189(1958).

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HOT SURFACE DRYING OF PAPER METHODS OF MEASURING THERMAL CONDUCTIVITIES OF MOIST PAPER

SUMMARY

The determination of the thermal conductivity of moist paper is of importance to the paper industry. For example, a knowledge of it will enable a more rigorous design of the drier sections of paper machines.

A number of schemes have been developed for measuring thermal conductivities. In almost all instances, however, the methods are only good for dry materials. Several techniques have been worked out that claim success on moist materials. Of these, some have rather apparent faults and it is doubtful that accurate measurements could be made. One method does seem to be satisfactory, and it is the method that has been selected for the work to be done at the Institute.

The thermal conductivity that will be measured is in fact only an apparent thermal conductivity. The heat that is transferred across a moist porous pad is the sum of three heats being moved by three driving forces. There is the heat flowing due to true thermal conduction and two types of thermal convection (liquid and vapor).

A technique that isolates the true thermal conduction from the other forms of heat transfer is presented in this report. This

method has not been reduced to practice, and for that reason in part was not selected as the means to be used for the measurements.

The procedure that appears best suited for measuring the thermal conductivities of paper is the one developed by Krischer and Esdorn. It is described in some detail in the body of this report.

Several other methods that were investigated are also presented along with the reasons for their rejection.

INTRODUCTION

The study of hot surface drying in the Chemical Engineering Department of the Institute had its start with the work of Dreshfield (1, 2). His was a study of the moisture movement within a pad of paper as it dried. Han and Ulmanen (3) expanded on Dreshfield's work by measuring the temperature gradient as well as the moisture movement within the paper pad as it dried.

From this work, estimates of the apparent thermal conductivity of moist paper have been made. However, due to the ever changing moisture contents and the necessarily transient state of heat flow, these are only over-all estimates of thermal conductivity.

A more exact measurement of the thermal conductivity is a logical step in the drying study program. For this reason, a survey of methods of measuring the thermal conductivity has been made in an attempt to find or devise a technique suitable for our use. There are a great number of procedures for measuring the thermal conductivity

of a material. In general, they fall into two major categories: the steady state or the nonsteady state.

The steady state technique usually involves the determination of the thermal conductivity, k , from this equation:

$$q = kA \left(\frac{dt}{dx} \right). \quad (1)$$

The heat flowing (q), the area normal to its flow (A), and the temperature change with respect to sample thickness ($\frac{dt}{dx}$) can all be measured independently, which then allows the calculation of k .

In this method, it is imperative that a state of equilibrium be reached before the measurements are made. This often means a minimum of 24 hours even for materials of high thermal conductivity, and upwards to 400 hours in some cases.

The nonsteady state techniques all involve the solutions to the Fourier equation,

$$\frac{\partial t}{\partial \theta} = \alpha \left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right), \quad (2)$$

and the observation of several boundary conditions. The resulting solution of Equation (2) determines the necessary measurements for calculating the thermal conductivity of the test specimen.

An advantage of the nonsteady state technique is that the entire test usually requires only a few minutes of time as compared with the steady state procedure.

Upon examining the methods that have been developed for measuring thermal conductivity, it appears that none is entirely

satisfactory for use with moist porous materials. This is because there is moisture movement within the test specimen when a temperature difference is impressed across the specimen. This leads to the conclusion that what probably will be measured in any thermal conductivity study is actually only an apparent thermal conductivity. That is, the heat movement across the test specimen will be by a combination of conduction and convection.

DESCRIPTION OF THERMAL CONDUCTIVITY MEASURING TECHNIQUES

The means of measuring thermal conductivity generally fall into one of two categories, that is, either the steady state or the nonsteady state procedure.

Of the steady state methods, the classical procedure is the use of a guarded hot plate apparatus, which in itself has a number of variations. One of these variations is described in this report (the Fluker (5, 6) method). A modified steady state method as outlined by Daane (7) is also presented.

Of the nonsteady state methods, two are reported. The first, a method developed by Beatty, Armstrong, and Schoenborn (8), has been used on dry materials but not on moist, porous materials. A second one, the Krischer-Esdorn (9) procedure, has been used for dry materials as well as moist, porous materials. This second nonsteady state procedure appears to be the method best suited for our requirements.

GUARDED HOT PLATE METHOD (4)

The classical method of measuring the thermal conductivity of a slab of material is by a steady state technique known commonly as the guarded hot plate method. This method has slowly evolved over the years and at the present time is the standard method of measuring thermal conductivity of homogeneous solids.

This technique simply requires that the necessary measurements be taken so that k in Equation (1) may be calculated.

$$q = kA \left(\frac{dt}{dx} \right). \quad (1)$$

This, of course, is carried out under steady state conditions.

A vast number of variations of this method have been developed and tried. These methods vary as to means of measuring the heat flow, the heat source, etc. The accepted standard apparatus is described by the American Society for Testing Materials (1945).

The name, "guarded hot plate," is derived from the use of two heat sources. The two sources are physically arranged so that one surrounds the other. The central source is the testing unit. To insure that the heat flowing from this central source is unidirectional and all of it passes through the test specimen, the outer heat source is used. The outer source is kept at the same temperature as the inner source, and as a result, no heat from the test zone has a tendency to escape as there is no temperature differential to drive it from the test zone. The heat generated in the central zone is very carefully measured and the outer source "guards" against the escape of any of

this heat from the testing unit.

FLUKER METHOD (STEADY STATE) (5, 6)

A method developed at the Texas Engineering Experiment Station for measuring the thermal properties of soil makes use of the so-called "heat meter" principle. This device requires that the heat passing through the test substance must also pass through a material of known thermal properties. The amount of heat flowing may be determined by measuring the temperature drop across the known material. Only the thermal conductivity of the test material is left as an unknown, and it may be calculated from a knowledge of the temperature drop through the test zone.

This apparatus, as shown in Fig. 1,

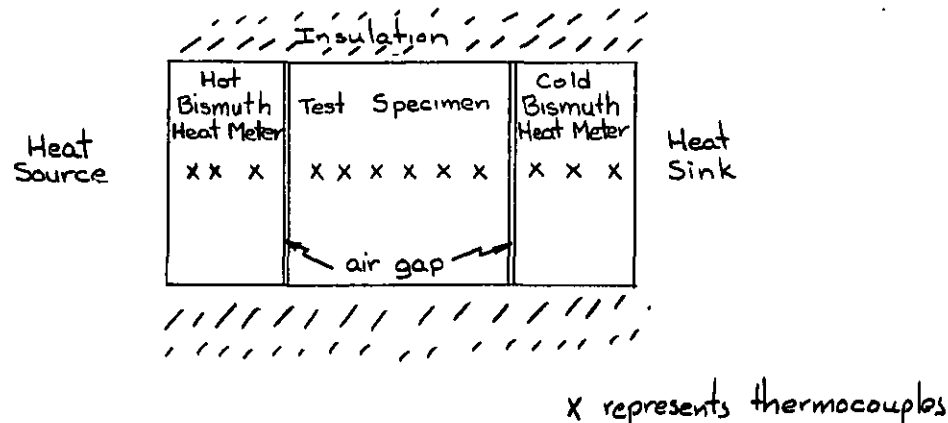


Fig. 1.

is operated under steady state conditions. Heat supplied at the heat source travels through the bismuth heat meter into and through the test specimen, through the second heat meter, and finally into the

heat sink. The heat flowing can be calculated through a knowledge of the thermal properties of bismuth and the measured temperatures in the bismuth heat meters,

$$q = kA\left(\frac{dt}{dx}\right), \quad (1)$$

where q is the heat flowing, k is thermal conductivity of bismuth, A is the cross-sectional area normal to the heat flow, and dt/dx is temperature gradient.

For the operation of this system with two heat meters, measurements are made of the heat flow in both meters until the difference is no larger than 5%. An average q is then assumed to be the heat passing through the test specimen. The k of the test specimen was determined by

$$k = \frac{q_{\text{average}}}{A \left(\frac{dt}{dx}\right)}. \quad (1a)$$

In actual operation, with moist soil as the test specimen, it is found that equilibrium is not reached until after 150 to 400 hours have elapsed.

Actually, this technique for measuring the thermal conductivity of moist, porous materials measures only an apparent thermal conductivity. However, in the course of just one test, a number of moisture levels can be taken into account. This is accomplished by having a number of thermocouples located along the axis of the test specimen at measured intervals. The temperature differentials between these thermocouples give a number of temperature gradients, (dt/dx) . After the test, the sample is dissected and the actual

moisture in each segment determined by weight loss on drying.

The original uniform moisture distribution changes when the sample is being subjected to the flow of heat along its axis. Greater amounts of moisture accumulate at the colder end. This redistribution of moisture is to a large extent the reason for the slow approach to a state of equilibrium.

This particular apparatus employs bismuth for the heat meters. This was done strictly for simplicity of design, i.e., bismuth has a low melting point and therefore is easy to cast to the desired shape in the laboratory. Any metal whose thermal properties are known may be used as a heat meter.

DAANE METHOD (MODIFIED STEADY STATE) (7)

The transfer of heat across a bed or pad of fibers containing moisture can be visualized as being caused by three separate driving forces. These heat quantities moved by each driving force are additive, and their sum equals the total heat transferred per unit area through the system being studied. The first of these is the heat transferred by true conduction,

$$q_1 = k \frac{\partial t}{\partial x}, \quad (3)$$

the driving force being the change of temperature with distance and k being the true thermal conductivity.

The second quantity of heat flowing is that caused by water vapor actually moving within the pad of fibers. This can be represented

by

$$q_2 = \beta \frac{\partial P_v}{\partial x} \quad (4)$$

The driving force in this case is the change of the vapor pressure with distance through the pad. And β is the proper conductivity for this situation.

Finally, the third quantity of heat flow is that caused by the movement of liquid water within the pad,

$$q_3 = \phi \frac{\partial^2 w}{\partial x^2} \quad (5)$$

In this case the quantity of heat is a function of the rate of change of moisture content with distance and the conductivity is ϕ .

The total heat flow through the pad is the sum of the three:

$$q_T = q_1 + q_2 + q_3 = k \frac{\partial t}{\partial x} + \beta \frac{\partial P_v}{\partial x} + \phi \frac{\partial^2 w}{\partial x^2} \quad (6)$$

The determination of the true thermal conductivity, k , is possible if q_2 and q_3 can be made small. A means by which this can be accomplished is described as follows.

In an apparatus as shown in Figure 2 and at the initial time ($\theta = 0$), the entire system (metal, paper) is at a constant temperature (t_a). At this time, the heat source begins to generate a heat impulse that alternately rises above t_a and falls below t_a by an amount, $|\Delta t_o|$. This cycling heat should follow a sine wave as shown in Figure 3.

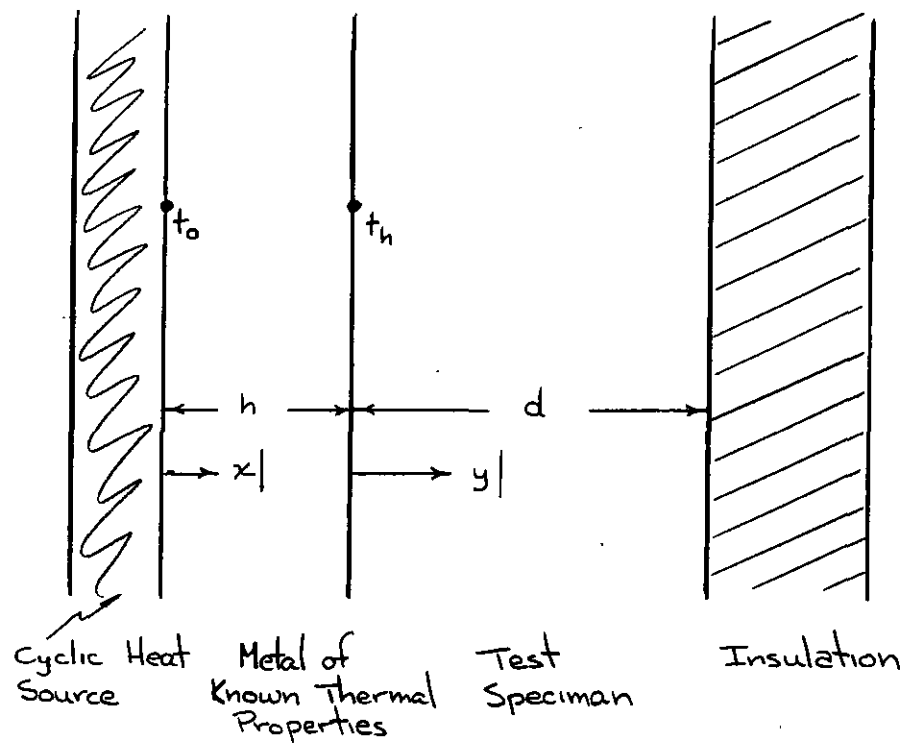


Fig. 2.

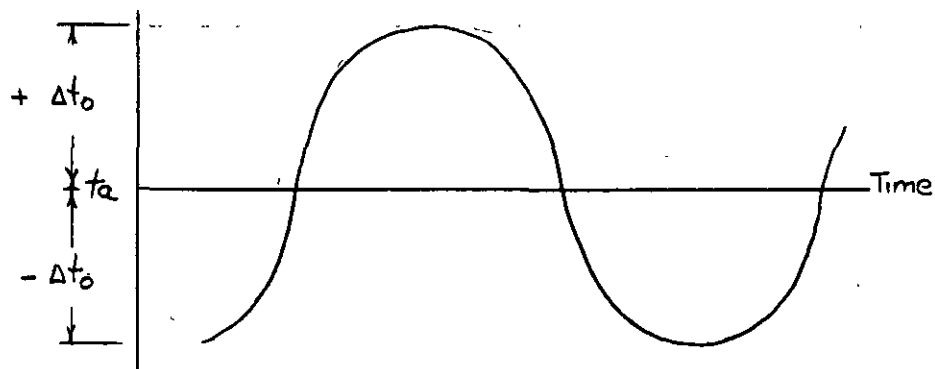


Fig. 3.

The temperature at $x = 0$ at any time, θ , is, then

$$t_o = t_a + |\Delta t_o| \sin \omega \theta \quad (7)$$

where ω is the heat cycling frequency in radians per second, θ is the time in seconds, and $|\Delta t_o|$ is the temperature amplitude in degrees.

For unidirectional heat flow, the following Fourier equation holds:

$$\frac{\partial t}{\partial \theta} = \alpha \frac{\partial^2 t}{\partial x^2} \quad (8)$$

One solution to this is

$$\Delta t = A e^{-\gamma x} \sin(\omega \theta - \gamma x) + B e^{\gamma x} (\sin \omega \theta + \gamma x) \quad (9)$$

where

$$\gamma = \sqrt{\frac{\omega}{\alpha}} = \sqrt{\frac{\omega c \rho}{k}} \quad (10)$$

c is specific heat and ρ is density. The temperature difference, Δt , at any point, x , in the metal is also given by Equation (11):

$$\Delta t = t - t_a. \quad (11)$$

If thermocouples are located at $x = 0$ and $x = h$, Δt_o and Δt_h can be measured directly. Then, at $x = 0$ and from Equation (9),

$$\Delta t_o = (A + B) \sin \omega \theta \quad (12)$$

or

$$|\Delta t_o| = A + B \quad (12a)$$

at $x = h$, and $\theta = 0$, and setting $\gamma h = R$,

$$\Delta t_h = -A e^{-R} \sin R + B e^R \sin R \quad (13)$$

$$= (A e^{-R} - B e^R) \sin R \quad (13a)$$

at $\underline{x} = \underline{h}$, and $\theta = \pi/2$.

$$\Delta t_h = Ae^{-R} \cos R + Be^R \cos R \quad (14)$$

$$= (Ae^{-R} + Be^R) \cos R \quad (14a)$$

It is also apparent (similar to Equation 7) that

$$t_h = t_a + \Delta t_h \sin (\omega\theta - \psi) \quad (15)$$

where ψ represents the lag or delay of the heat flowing from $\underline{x} = 0$ to $\underline{x} = \underline{h}$.

From this we get

$$\Delta t_h = |\Delta t_h| \sin (\omega\theta - \psi). \quad (15a)$$

Then when $\theta = 0$,

$$\Delta t_h = |\Delta t_h| \sin \psi \quad (15b)$$

and when $\theta = \pi/2$.

$$\Delta t_h = |\Delta t_h| \cos \psi. \quad (15c)$$

Substituting in Equations (13a) and (14a),

$$|\Delta t_h| \sin \psi = (Ae^{-R} - Be^R) \sin R \quad (13b)$$

$$|\Delta t_h| \cos \psi = (Ae^{-R} + Be^R) \cos R. \quad (14b)$$

Squaring and adding Equations 13b and 14b,

$$|\Delta t_h|^2 = A^2 e^{-2R} + 2AB(\cos R - \sin R) + B^2 e^{2R}. \quad (16)$$

Equation 16 together with Equation 12a make it possible to solve for the two constants, \underline{A} and \underline{B} .

Within the metal (see Fig. 2), all the heat is transferred by conduction,

$$q = k \frac{\partial t}{\partial x}; \quad (3)$$

since \underline{t}_a is constant,

$$q = k \frac{\partial \Delta t}{\partial x}; \quad (3a)$$

then substituting in Equation (9) and differentiating,

$$q_h = k \left\{ -A \sqrt{e}^{-R} [\sin(\omega\theta - R) + \cos(\omega\theta - R)] + B \sqrt{e}^R [\sin(\omega\theta + R) + \cos(\omega\theta + R)] \right\}. \quad (17)$$

In Equation 15, \underline{R} and \underline{k} are known from the metal used, \underline{A} and \underline{B} have been solved for from Equations (12a) and (16), and ω and $|\Delta \underline{t}_o|$ and $|\Delta \underline{t}_h|$ can be measured. As a result, \underline{q}_h can be calculated. As \underline{q}_h is the heat at the interface of the metal and fiber mat leaving the metal, it is also the heat entering the bed of fibers at $\underline{y} = 0$ (see Fig. 2).

By measuring the temperature in the sheet, $\partial \underline{t} / \partial \underline{y}$ can be determined at $\underline{y} = 0$. Then by rearranging Equation (3),

$$k = \frac{q_h}{\left(\frac{\partial t}{\partial y} \right)_0} \quad (3b)$$

where the \underline{k} is now the thermal conductivity of the bed of paper. As it stands [see Equation (6)], it represents only the apparent thermal conductivity. However, if the second and third terms in Equation (6) can be made negligible, then the \underline{k} will approach the true thermal conductivity of the bed of fibers.

The second term of Equation (6), or the term represented by Equation (4), is the quantity of heat moving due to the difference in

vapor pressures within the pad. As $\underline{t_a}$ is lowered, this quantity of heat will become less significant with respect to the total flow of heat across the pad.

The third term of Equation (6), or the term represented by Equation (5), is the quantity of heat moving in the form of liquid water due to a temperature driving force. In this case, due to the mass of the water involved, the inertia, or resistance to motion, can keep the liquid water virtually stationary if the direction of heat flow is changed often enough. The number of heating cycles per unit time (ω) must be adjusted to make this true.

To determine the limits within which a true thermal conductivity can be measured, a series of exploratory experiments must be made. If the apparent thermal conductivities at several $\underline{t_a}$'s and ω 's appear as shown in Fig. (4), then at $\omega \geq \omega^*$, the third term is negligible and also the second term is not dependent on ω . Cross plotting at $\omega \geq \omega^*$, a correlation as shown in Fig. (5) should be obtained.

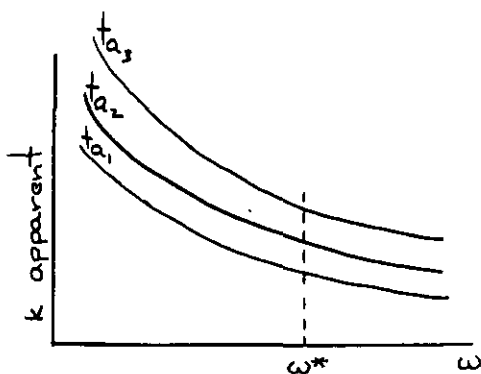


Fig. 4.

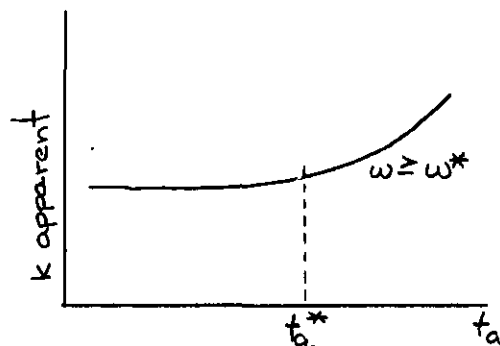


Fig. 5.

In Fig. 5, when $\underline{t}_a \leq \underline{t}_a^*$, the second term of Equation (6) is also negligible and the value of $\underline{k}_{\text{apparent}}$ may be assumed to be the true thermal conductivity.

In the event that the initial correlation of $\underline{k}_{\text{apparent}}$ at several ω 's and \underline{t}_a 's does not appear as shown in Fig. 4 but rather looks like Fig. 6, then ω is still too small to make the third term of Equation (6) negligible, provided, of course, that \underline{t}_{a1} is less than some estimated value at which the third term becomes negligible.

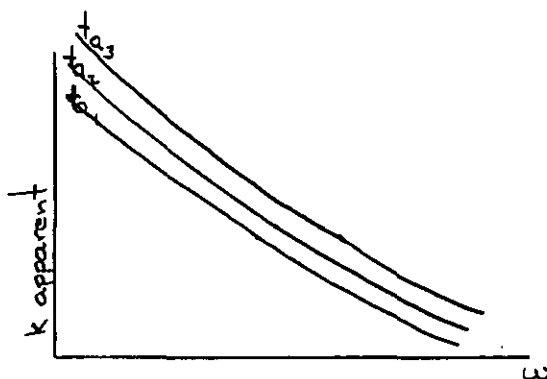


Fig. 6.

BEATTY, ARMSTRONG, AND SCHOENBORN METHOD (NONSTEADY STATE) (8)

Heat transfer through a material by conduction can be described by the Fourier equation. When the transfer of heat can be shown (or assumed) to be along only one axis of a three-dimensional system, the Fourier equation takes the following form:

$$\frac{\partial t}{\partial \theta} = \alpha \frac{\partial^2 t}{\partial x^2} \quad (8)$$

The solutions to this differential are many and varied, each depending

on the particular boundary conditions to be met.

Beatty, Armstrong, and Schoenborn have shown in their method developed at North Carolina State College a means by which the thermal conductivity, thermal diffusivity, and heat capacity of a test specimen can be measured through a particular solution to Equation (8). The system is shown in Fig. 7. Heat is supplied from the heat source which is maintained at a constant temperature, T_0 . This supply of heat begins at time zero ($\theta = 0$), prior to which the test specimen and the copper block are at a constant temperature, t_0 .

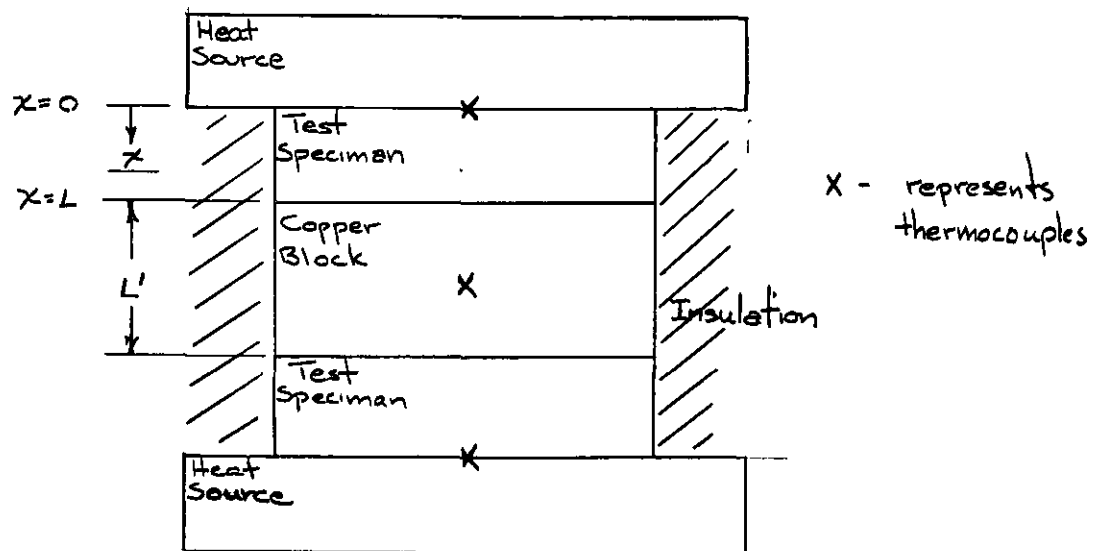


Fig. 7.

When the test begins, the heat source is placed in intimate contact with the test specimen, causing heat to be conducted through the specimen into the copper. By measuring the temperature rise within

the copper block (assumed to be uniform throughout) and knowing the thermal properties of copper, the heat transferred may be calculated.

The boundary conditions expressed mathematically are:

$$t(x, 0) = t_o \quad (\text{for all values of } x) \quad (18)$$

$$t(0, \theta) = T_o \quad (\text{for } \theta > 0) \quad (19)$$

$$kA \frac{\partial t}{\partial x} = w'c' \frac{\partial t'}{\partial \theta} \quad \text{at } x = L; \theta > 0 \quad (20)$$

where t is temperature at time, θ , t_o is initial temperature, T_o is hot surface temperature, t' is the temperature of the copper block, x is distance normal to hot surface in test specimen, k is thermal conductivity of test specimen, A is area of contact between test specimen and copper, w' is weight of copper, and c' is specific heat of copper.

Equation (20) may be rewritten

$$\left(\frac{\alpha H}{L}\right) \left(\frac{\partial t}{\partial x}\right) + \frac{\partial t'}{\partial \theta} = 0, \quad \text{at } x = L \quad (21)$$

where

$$H = \frac{wc}{w'c'} = \frac{L\rho c}{L'\rho'c'} \quad (22)$$

The term, α , is the thermal diffusivity of the test specimen, L and L' are defined by Fig. 7, ρ and c are the density and specific heat of the test specimen, respectively.

With the assumption that at $x = L$, $t = t'$, Equation (21) becomes

$$\left(\frac{\alpha H}{L}\right) \left(\frac{\partial t}{\partial x}\right) + \frac{\partial t}{\partial \theta} = 0 \quad \text{at } x = L. \quad (21a)$$

With the above boundary conditions, one general solution to Equation (8) is

$$\frac{T_o - t}{T_o - t_o} = \sum_{n=1}^{\infty} \frac{2(\lambda_n^2 + H^2) e^{-\left(\frac{\alpha \lambda_n^2}{L^2}\right)\theta}}{\lambda_n(\lambda_n^2 + H^2 + H)} \sin \frac{\lambda_n x}{L} \quad (23)$$

where λ_n for $n = 1, 2, 3, \dots$ are the positive roots of

$$H = \lambda \tan \lambda. \quad (24)$$

Substituting $\lambda_n = \sqrt{\frac{Z_n H}{L}}$ in Equation 23 and rearranging,

$$\frac{T_o - t}{T_o - t_o} = \sum_{n=1}^{\infty} \left[\frac{2(Z_n + H)}{Z_n + H + 1} \right] \left[\frac{1}{\sqrt{Z_n H}} \sin \frac{\sqrt{Z_n H}}{L} x \right] \left[e^{-\left(\frac{\alpha Z_n H}{L^2}\right)\theta} \right] \quad (25)$$

As θ increases, the terms after the first term become less significant. Considering only the first term and taking the natural logarithm of both sides,

$$\ln \frac{T_o - t}{T_o - t_o} = -\frac{\alpha Z_1 H}{L^2} \theta + \ln \left[\frac{2(Z_1 + H)}{Z_1 + H + 1} \right] \left[\frac{1}{\sqrt{Z_1 H}} \right] \left[\sin \frac{\sqrt{Z_1 H}}{L} x \right] \quad (26)$$

The curve obtained by plotting $(T_o - t)/(T_o - t_o)$ vs. θ on semilogarithmic paper results in a straight line, the slope of this line being

$$\text{slope} = m = -\frac{\alpha Z H}{2.303 L^2} = -\frac{\alpha \lambda^2}{2.303 L^2} \quad (27)$$

Further substitution yields

$$m = -k \frac{Z}{(2.303)(L' \rho' c')(L)} \quad (28)$$

or

$$k = \frac{-(2.303)(L' \rho' c')(L)(m)}{Z} \quad (28a)$$

Equation (28a) shows that determining the thermal conductivity of the test specimen is dependent on the slope of the temperature-time

curve, the properties of the copper block, and \underline{Z} .

The value of \underline{Z} may be determined in several ways. If thermal diffusivity, α , is known, Equation (27) may be rearranged to give the quantity, \underline{ZH} :

$$\underline{ZH} = \frac{2.303L^2m}{\alpha} . \quad (27a)$$

\underline{Z} may then be determined from \underline{ZH} by referring to Equation (24) and

$$\lambda_n = \sqrt{\underline{Z}_n \underline{H}} . \quad (29)$$

A second way by which \underline{Z} may be determined is from Equations (24) and (29) directly if the heat capacity of the test specimen is known.

The third method of determining \underline{Z} requires no advance knowledge of the thermal properties. Referring to Equation (26), it can be shown that the intercept for the straight line plot on semilogarithmic paper for $\underline{x} = \underline{L}$ is given by

$$\text{intercept} = I = \left[\frac{2(\underline{Z} + \underline{H})}{\underline{Z} + \underline{H} + 1} \right] \left[\frac{1}{\sqrt{\underline{ZH}}} \right] \left[\sin \sqrt{\underline{ZH}} \right] . \quad (30)$$

Since \underline{Z} is itself a function of \underline{R} , it is possible to determine a value of \underline{Z} from Equation (30). The relationship between \underline{I} and \underline{Z} , and \underline{I} and \underline{H} can be shown graphically.

The thermal diffusivity (α) may be determined in much the same manner as described for the thermal conductivity. The apparatus is the same except that the copper block is removed and a temperature measurement is made between the two test specimens, i.e., at $\underline{x} = \underline{L}$.

Plotting the data in the same manner as previously described, the slope of the curve is now a direct measure of the thermal diffusivity

$$\alpha = - \frac{4L^2 m 2.303}{\pi^2} \quad (31)$$

KRISCHER-ESDORN METHOD (NONSTEADY STATE) (9)

The method herein described as the Krischer-Esdorn Method (named after the men at the Technical Institute of Darmstadt, Germany, who developed it) is similar to another procedure for thermal property measurements which was developed virtually simultaneously and independently by Clarke and Kingston (10, 11) in Melbourne, Australia. The essence of these procedures is as follows.

If, referring to Fig. 8, a constant supply of heat, $2q_o$, is produced in a metal foil, b , of zero mass and lying between two flat plates, a , of thickness, s , from time, $\theta = 0$, the temperature, t , will obey the following relation as long as there is no temperature increase at $x = s$.

$$t(x, \theta) = \frac{2}{\sqrt{\pi}} \frac{q_o}{\sqrt{kcp}} \sqrt{\theta} (e^{-\eta^2} - \eta \sqrt{\pi} \operatorname{erfc} \eta) \quad (32)$$

where

$$\eta = \frac{x}{2\sqrt{\alpha\theta}}, \quad \alpha = \frac{k}{cp}, \quad \operatorname{erfc} \eta = 1 - \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\eta^2} d\eta.$$

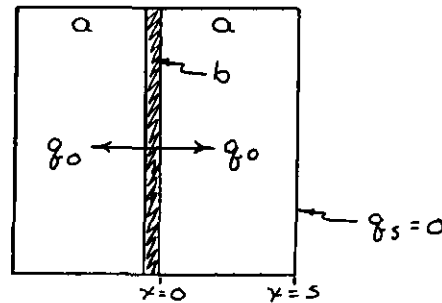


Fig. 8

for the plane, $x = 0$

$$t_{(0, \theta)} = \frac{2}{\sqrt{\pi}} \frac{q_0}{\sqrt{kcp}} \sqrt{\theta}. \quad (33)$$

The heat penetration modulus, \sqrt{kcp} , can then be determined from the slope of the curve of $t_{(0, \theta)}$ against $\sqrt{\theta}$ and a knowledge of the heat generated, q_0 .

The thermal conductivity, k , can be determined as follows. As the system shown in Fig. 8 continues to be operated, heat will begin to reach the plane, $x = s$. If heat transfer at this surface can be prevented, then after a period of time the temperature within the plates (a) will obey this equation:

$$t_{(x, \theta)} = \frac{q_0 s}{k} \left[\frac{1}{2} \left(\frac{x}{s} \right)^2 - \frac{x}{s} + G + \frac{\alpha \theta}{s^2} \right], \quad (34)$$

G being an arbitrary constant dependent on starting conditions.

The correlation between temperature and distance, x , and temperature and time, θ , will appear as shown in Fig. 9 and 10.

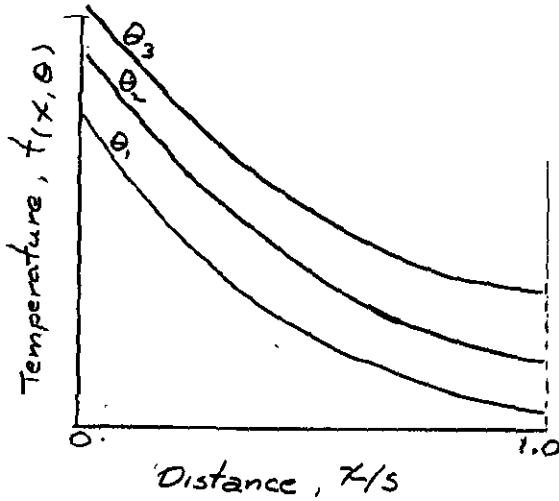


Fig. 9

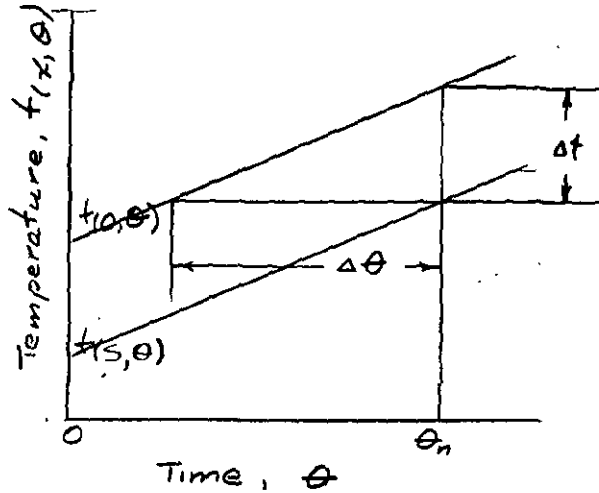


Fig. 10

For the difference, Δt , of the temperatures, $t_{(0, \theta)}$ and $t_{(s, \theta)}$, at time, θ_n , the following relation holds:

$$\Delta t = t_{(0, \theta_n)} - t_{(s, \theta_n)} = q_0 s / 2k. \quad (35)$$

From this, by rearrangement,

$$k = \frac{q_0 s}{2\Delta t}. \quad (35a)$$

The slope, $\frac{\partial t(x, \theta)}{\partial \theta}$, of the temperature versus time graph (Fig. 10) can be derived from Equation (34),

$$\frac{\partial t(x, \theta)}{\partial \theta} = \frac{q_0}{c p s}. \quad (36)$$

Since the slope remains constant for all values of x , cp can be computed,

$$c_p = \frac{q_o}{\left(\frac{\partial t(x, \theta)}{\partial \theta} \right) (s)} \quad (36a)$$

For the time difference, $\Delta\theta$ (Fig. 10) elapsing between the planes, $\underline{x} = 0$ and $\underline{x} = \underline{s}$ for the same arbitrary temperature, \underline{t}_n , to be achieved, the following relation holds from Equation (34),

$$\Delta\theta = s^2 / 2\alpha. \quad (37)$$

From this it is seen

$$\alpha = s^2 / 2\Delta\theta. \quad (37a)$$

It is to be noted that α , the thermal diffusivity, is found independently of the heat energy, \underline{q}_o .

The preceding establishes several boundary conditions that must be maintained in order that Equations (32-37a) are applicable. Among these are that no heat passes the plane, $\underline{x} = \underline{s}$, and that the flow of heat is strictly a unidirectional flow, normal to the source, \underline{b} . A third condition requires that the heat source have no mass.

The first of these conditions is accomplished by the design of the apparatus (Fig. 11). The second boundary condition can be met by dimensioning the system properly. That is, the ratio of sample thickness to sample diameter must be sufficiently low so that heat flow parallel to the heat source is kept negligible or below a specified maximum. The third boundary condition dealing with the mass of the heat source is solved by applying a correction factor and by building the heat source originally as small as possible.

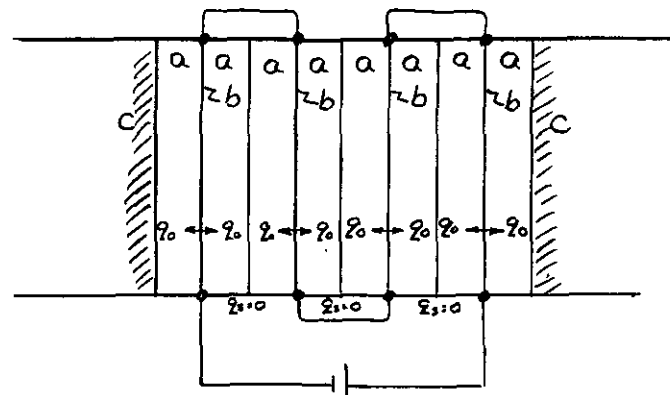


Fig. 11

- a. Test specimen (8 equal specimens)
- b. Heat source (4 equal sources)
- c. Insulation

The apparatus as shown in Fig. 11 consists of a sandwich of 8 similar test specimens arranged symmetrically about 4 heat elements. The heating elements are very thin metal sheets of high electrical resistance. Krischer and Esdorn used a material called Vacromium, 0.01 mm. thick (0.0004 inches). Clarke and Kingston used shim steel, the dimensions not given. Stainless steel shim stock is available in this country, 0.001 inch thick with an electrical resistivity high enough to give fairly satisfactory results. The four heating units should be as nearly identical as it is possible to make them. The metal foil heating units are connected together in series by strips of material of lower electrical resistance and greater thickness such as 0.005-inch copper foil. The four heating units, when the apparatus is in operation, are connected to a source of power, and an electrical current is passed through the circuit. Heat ($2q_0$) is generated in each heat source, and by symmetry, half of this heat (q_0) will travel

away from the heating unit through each of the adjacent test specimens.

Each test specimen receives a quantity of heat equal to q_0 at $x = 0$, and this heat then moves outward toward $x = s$. The test specimens are identical so the heat arriving at $x = s$ from one direction will be met by an equal quantity from the opposite direction and no heat will cross the $x = s$ boundary. Therefore, the first boundary condition is met. The testing zone in this apparatus is the central pair of test specimens, thermocouples being located at $x = 0$ and $x = s$. The outer sets of test specimens and heaters are aids in insuring equal division of heat within the test zone.

It is possible to show mathematically that with a set of eight test specimens, the conditions required to meet the first boundary condition will have less than a 0.5% error.

Equations (32-37a) are related to definite time periods in the test. Knowledge of these time limits are essential for the analysis of the data. The time limits can be found from a comparison of the theoretical temperature-time curve with the measured curve. When the measured slope deviates less than 0.5% from the theoretical case, the measurements properly belong to that portion of the test for which the theoretical curve was drawn.

As a preliminary estimate of the time involved for each portion of the test, calculations may be made. The expression for the analytical shape of the curve relating temperature to all periods of the experiment is:

$$t_{(x,\theta)} = \frac{q_0 s}{k} \left\{ \left[\frac{1}{2} \left(\frac{x}{s} \right)^2 - \frac{x}{s} + \frac{1}{3} + \frac{\alpha \theta}{s^2} \right] - \sum_{n=1}^{\infty} \frac{2}{(\pi n)^2} \cos(\pi n \frac{x}{s}) e^{-(\pi n)^2 \frac{\alpha \theta}{s^2}} \right\}. \quad (38)$$

The slope of the realistic curve can be calculated from Equation (38).

At startup, when $x = 0$, the slope becomes

$$\frac{d\bar{t}_{(0,\theta)}}{d\theta} = \left(\frac{q_0 s}{k} \right) \left(\frac{\alpha}{s^2} \right) \left[1 + \sum_{n=1}^{\infty} 2e^{-(\pi n)^2 \frac{\alpha \theta}{s^2}} \right] \quad (39)$$

the bar over the \bar{t} indicating the realistic situation.

The slope of the ideal curve at starting conditions can be calculated from Equation 33:

$$\frac{dt_{(0,\theta)}}{d\theta} = \frac{q_0 s \sqrt{\alpha}}{k \sqrt{\pi s^2 \theta}}. \quad (40)$$

From an earlier definition, that relating the slope of the ideal curve to the realistic curve,

$$(1 + \epsilon) \frac{dt_{(0,\theta)}}{d\theta} = \frac{d\bar{t}_{(0,\theta)}}{d\theta}, \quad (41)$$

and stating that the slopes shall not differ by more than $\epsilon = 0.005$ during the test time, θ_{M_1} , the following equation is arrived at:

$$1.005 \left(\frac{q_0 s}{k} \right) \left(\frac{1}{\sqrt{\pi}} \right) \sqrt{\frac{\alpha}{s^2 \theta_{M_1}}} = \left(\frac{q_0 s}{k} \right) \left(\frac{\alpha}{s^2} \right) \left[1 + \sum_{n=1}^{\infty} 2e^{-(\pi n)^2 \frac{\alpha \theta_{M_1}}{s^2}} \right] \quad (42)$$

This can be reduced to show that

$$\frac{\alpha \theta_{M_1}}{s^2} \cong \frac{1}{6} \quad (\text{see Appendix I}) \quad (43)$$

or by rearranging

$$\theta_{M_1} = \frac{s^2}{6\alpha} = \theta_u; \quad (44)$$

that is, the maximum time for the first part of the test is equal to

$s^2/6\alpha$, or θ_u .

All other characteristic times will be multiples of θ_u .

After a period of operation and when heat begins to arrive at $\underline{x} = \underline{s}$, the slope of the ideal temperature-time curve will be [from Equation (34)]

$$\frac{dt(0, \theta)}{d\theta} = \left(\frac{q_o s}{k}\right) \left(\frac{\alpha}{s^2}\right). \quad (45)$$

The slope of the realistic curve has been shown as

$$\frac{d\bar{t}(0, \theta)}{d\theta} = \left(\frac{q_o s}{k}\right) \left(\frac{\alpha}{s^2}\right) \left[1 + \sum_{n=1}^{\infty} 2e^{-(n\pi)^2 \frac{\alpha \theta}{s^2}}\right]. \quad (39)$$

Setting the slope equal to each other in accordance with Equation (41), the following results:

$$\epsilon = \sum_{n=1}^{\infty} 2e^{-(n\pi)^2 \frac{\alpha \theta_{M_2}}{s^2}} = 0.005. \quad (46)$$

The solution to this gives

$$\frac{\alpha \theta_{M_2}}{s^2} = \frac{3.5}{6} \quad (47)$$

or

$$\theta_{M_2} = \frac{3.5}{6} \frac{s^2}{\alpha} = 3.5 \theta_u. \quad (48)$$

That is to say, this time must elapse before the second period of measurements may begin. If this portion of the test can be conducted within a period of time equal to $1.5\theta_u$, the total time elapsed for a complete test will be $5\theta_u$.

The relationship between sample thickness and width must be established. The thickness of the sample must be selected according to the thermal properties of the material and the time limits of the

measurements so that the heat losses do not exceed some predetermined value. The solution to this is found from the following general heat transfer equation for heat conduction in a cylinder:

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} = \frac{1}{\alpha} \frac{dt}{d\theta} \quad (49)$$

The boundary conditions are that the heat source has no mass, and at a radial distance, r , from the axis of the cylinder, the temperature is t . The use of this equation comes about by imagining that a cylinder of radius, R , is contained within the test specimen (see Fig. 12). At time, $\theta = 0$, the temperature is t_A . Further, it is assumed that the cylinder transfers heat to its environment from $\theta = 0$ at a constant temperature, t_u .

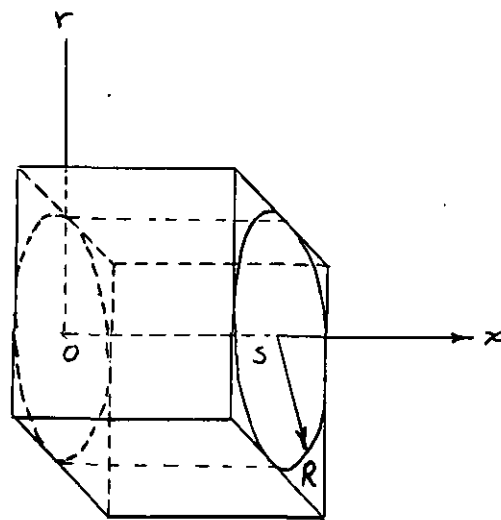


Fig. 12

The boundary conditions, stated mathematically, are:

$$-k\left(\frac{\partial t}{\partial r}\right)_{r=R} = -ht(R, \theta) \quad (49a)$$

$$\left(\frac{\partial t}{\partial r}\right)_{r=0} = 0 \quad (49b)$$

$$t_{(r,0)} = t_A \quad (49c)$$

The solution to Equation (49) is, then:

$$t_{(r,t)} = \sum_{n=1}^{\infty} c_n J_0(\beta_n r) e^{-\beta_n^2 \alpha t} \quad (50)$$

This equation (50) may further be reduced to finally show a relationship between the quotient of plate thickness and plate length, ($\underline{s}/2\underline{R}$) to the Nusselt Number ($\underline{h}2\underline{R}/\underline{k}$).

This relationship can be plotted with the reciprocal of the Nusselt Number as the ordinate and the sample thickness as the abscissa. The sample length ($2\underline{R}$) are parameters. This plot is shown as Fig. 11 of reference (9).

One of the original boundary conditions is that the heat source have no mass. A rather involved mathematical analysis can be made to show the influence of the mass of the heating foil on the system. The corrections arrived at for this deviation from the boundary conditions is only about 2% of the measured values.

DISCUSSION

The system of interest is a sheet of cellulose fibers containing water. The ratio of fiber to water is one of the more interesting variables to be investigated. It is therefore extremely important that in a given test the fiber-to-water ratio be known and, furthermore, that no change take place in the ratio during the course of a test.

It is apparent that it would be difficult to maintain a constant moisture content with several of the described procedures. For this reason, any method requiring a substantial period of time to make the measurements must be discarded. This eliminates any guarded hot plate method, such as the Fluker (5, 6) method.

As stated earlier, all the methods measure an apparent thermal conductivity rather than a true thermal conductivity. The reason for this was also pointed out. One technique was found that attempts a separation of the true thermal conductivity from the other modes of heat transfer. It would appear that the Daane (7) method would therefore be the logical selection as a procedure for this work. Were the details of this method more thoroughly worked out, this could very well be true. The perfection of the mechanics of this technique, however, might take a considerable length of time. Because of this, this method is being sidetracked for the present.

Thus, there remain, of the methods presented, the two non-steady state processes--the Beatty, Armstrong, and Schoenborn (8) method and the Krischer-Esdorn (9) method. Of these, the Krischer-Esdorn has been selected as the more suitable of the two.

The Beatty, et al., (8) technique appears on analysis to be more complicated mechanically to operate. For example, the heat source must at time zero be at a constant temperature, T_0 , and at the same time be in good thermal contact with the test specimen. In addition, the heat flow is measured by a "heat meter," receiving the heat from the test specimen. Any film resistance between the specimen and the

meter can seriously affect the results. And finally, the published mathematical analysis of this procedure has not been as completely worked out as has the Krischer-Esdorn (9) method.

On the other hand, the boundary conditions set forth in the Krischer-Esdorn (9) method appear rather simple to meet. The mathematics involved are presented in detail in the article describing this process. And the mechanical operations and measurement are straightforward. This technique has therefore been selected as the means of measuring the apparent thermal conductivity of sheets of moist paper.

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APPENDIX

To reduce

$$1.005\left(\frac{q_0 s}{k}\right)\left(\frac{1}{\sqrt{\pi}}\right)\sqrt{\frac{\alpha}{s^2 \theta_{M1}}} = \left(\frac{q_0 s}{k}\right)\left(\frac{\alpha}{s^2}\right)\left[1 + \sum_{n=1}^{\infty} 2e^{-\frac{(n\pi)^2 \alpha \theta_{M1}}{s^2}}\right] \quad (42)$$

rearranging

$$\sqrt{\pi} \frac{1.005}{\sqrt{\frac{\alpha \theta_{M1}}{s^2}}} = 1 + \sum_{n=1}^{\infty} 2e^{-\frac{(n\pi)^2 \alpha \theta_{M1}}{s^2}} \quad (42a)$$

This resolves into the solution of the group:

$$\sum_{n=1}^{\infty} 2e^{-\frac{(n\pi)^2 \alpha \theta_{M1}}{s^2}}$$

An expansion of $e^{-\frac{(n\pi)^2 \alpha \theta_{M1}}{s^2}}$ or at $n = 1$, $e^{-\frac{\pi^2 \alpha \theta_{M1}}{s^2}}$ can be shown to converge rapidly if $\frac{\pi^2 \alpha \theta_{M1}}{s^2} \leq 0.1$, therefore making the following true::

$$e^{-\frac{\pi^2 \alpha \theta_{M1}}{s^2}} \approx 1 - \frac{\pi^2 \alpha \theta_{M1}}{s^2}$$

However, if $\frac{\pi^2 \alpha \theta_{M1}}{s^2} > 0.1$. To get around this, write

$$e^{-\frac{\pi^2 \alpha \theta_{M1}}{s^2}} = e^{-Z_1} e^{Z_1 - \frac{\pi^2 \alpha \theta_{M1}}{s^2}}$$

and find the Z_1 , such that $Z_1 - \frac{\pi^2 \alpha \theta_{M1}}{s^2} = 0.1$ for $\frac{\pi^2 \alpha \theta_{M1}}{s^2} = 1/6$

$$Z = 0.1 + \frac{\pi^2 \alpha \theta_{M1}}{6}$$

$$Z_1 = 1.75;$$

then the first term, i.e., $n = 1$ in $\sum_{n=1}^{\infty} e^{-\frac{(n\pi)^2 \alpha \theta_{M1}}{s^2}}$ which is equal to

$$e^{-Z_1} e^{Z_1 - \frac{\pi^2 \alpha \theta_{M1}}{s^2}} \text{ becomes}$$

$$e^{-\frac{\pi^2 \alpha \theta_{M1}}{s^2}} \approx e^{-1.75} \left[1 + \frac{1.75 - \frac{\pi^2 \alpha \theta_{M1}}{s^2}}{1.75}\right]$$

$$\approx e^{-1.75} \left[2.75 - \frac{\pi^2 \alpha \theta_{M1}}{s^2}\right]$$

The other terms can be written with their appropriate Z_n 's. The summation of these terms gives

$$\sum_{n=1}^{\infty} e^{-(n\pi)^2 \frac{\alpha\theta}{s^2}}$$

For the first several values of n , Z_n becomes

n	Z_n	$= 0.1 + \frac{(n\pi)^2}{6}$
1	1.75	
2	6.67	
3	14.90	
4	26.50	
.	.	.

The summation becomes

$$\begin{aligned} \sum_{n=1}^{\infty} e^{-(n\pi)^2 \frac{\alpha\theta}{s^2}} &= e^{-1.75} \left[2.75 - \pi^2 \frac{\alpha\theta}{s^2} \right] \\ &+ e^{-6.67} \left[7.67 - 4\pi^2 \frac{\alpha\theta}{s^2} \right] \\ &+ e^{-14.9} \left[15.9 - 9\pi^2 \frac{\alpha\theta}{s^2} \right] \\ &+ e^{-26.5} \left[27.5 - 16\pi^2 \frac{\alpha\theta}{s^2} \right] \\ &+ \dots \end{aligned}$$

It is apparent that after $n = 1$, the terms become negligible.

Therefore, it can be said that

$$\sum_{n=1}^{\infty} e^{-(n\pi)^2 \frac{\alpha\theta}{s^2}} = e^{-1.75} \left[2.75 - \pi^2 \frac{\alpha\theta}{s^2} \right].$$

Placing this solution for the summation into Equation (42a) and letting

$$\frac{\alpha\theta}{s^2} = \frac{1}{6}, \text{ we find}$$

$$\frac{1.005}{\sqrt{\pi}} = \sqrt{1/6} + 2\sqrt{1/6} [e^{-1.75}(2.75 - \frac{\pi^2}{6})]$$

$$0.565 = 0.41 + 0.82[0.174(2.75 - 1.65)]$$

$$0.565 = 0.41 + 0.157 = 0.567.$$

Check

This is trial-and-error type solution; our trial was that $\alpha\theta/s^2 \cong 1/6$.

An alternate and more general solution is as follows:

To reduce the form,

$$1.005\left(\frac{q_0 s}{k}\right)\left(\frac{1}{\sqrt{\pi}}\right)\sqrt{\frac{\alpha}{s^2\theta_{M1}}} = \left(\frac{q_0 s}{k}\right)\left(\frac{\alpha}{s^2}\right) \left[1 + \sum_{n=1}^{\infty} 2e^{-(n\pi)^2 \frac{\alpha\theta_{M1}}{s^2}}\right]$$

rearranged this becomes for $n = 1$,

$$1.005\sqrt{\frac{s^2}{\alpha\theta\pi}} = 1 + 2e^{-\pi^2 \frac{\alpha\theta}{s^2}}$$

Let $\frac{s^2}{\alpha\theta} = \beta^2$; then

$$\beta = \frac{\sqrt{\pi}}{1.005} \left[1 + 2e^{-\left(\frac{\pi}{\beta}\right)^2}\right]$$

This may then be solved graphically. To speed the solution, write

$$f(\beta) = \frac{\pi}{1.005} \left[1 + 2e^{-\left(\frac{\pi}{\beta}\right)^2}\right] - \beta.$$

Taking the derivative of $f(\beta)$ with respect to β gives the slope of the curve

$$\frac{df(\beta)}{d\beta} = \frac{4}{1.005} \frac{\pi^{3/2}}{\beta^3} e^{-\left(\frac{\pi}{\beta}\right)^2} - 1$$

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Knowing the slope at a value of δ will help select the next trial for a trial-and-error solution. This type of solution requires that values of the summation group found in Equation (42) at n greater than 1 be negligible.

PROJECT REPORT FORM

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PROJECT NO. 1901
COOPERATOR Institute
REPORT NO. One
DATE July 3, 1956
NOTE BOOK 1461 and 1498
PAGE 6-160 TO 6-134
SIGNED *Tapio Ulmanen*
Tapio Ulmanen

STUDY OF HOT SURFACE DRYING OF PAPER

INTRODUCTION

The purpose of this study was to complete the study made by Arthur Dreshfield, Jr. Especially interesting was to get a more detailed picture of both moisture and temperature distribution inside a paper sheet being dried on a hot surface.

Since the time of Dreshfield's thesis work, practically no new ideas have been presented in the literature, and the literature study made by Dreshfield still applies. Therefore, the author of this work refers only to Dreshfield's publications.

DESCRIPTION OF TECHNIQUES USED

I. Drying Apparatus

The main part of the apparatus used in this study was the same as that used by Dreshfield. However, the hot water circulation system was totally rebuilt, and the clamping device was reconstructed in order to prevent, as much as possible, the reduction of the areal size of the samples caused by shrinkage phenomenon.

The total elimination of shrinkage can hardly be carried out using the existing devices, even if shrinkage could be reduced to a great extent.

Concerning the hot water flow in the heating system and the air flow in the ventilation system, careful calculations were made in order to

see if they were sufficiently high, and no appreciable temperature or R.H. change took place during the evaporation period.

II. The Fixed General Drying Conditions

Hot water temperature was fixed at 194°F., and the rate of flow of ventilating air was 150 ft.³/min. Dry bulb temperature of the air was 100°F. and wet bulb temperature, 78°F.

III. Temperature Distribution Measurements

For this purpose, 0.002-inch diameter chromel and alumel Hoskins Manufacturing Company thermocouple wires were used. Thermocouples were prepared using silver solder as a medium to form the junction. The thermocouples were always placed between laminates so that the wire ends came out from the space between laminates on the opposite long edges of the test sheets. Thus any short circuits between wires inside the sheet were prevented. A Minneapolis-Honeywell Recording Potentiometer was used to record the temperatures at any desired sheet level in the course of the drying run.

Because this existing potentiometer was calibrated against copper-constantan thermocouples, careful calibrations must be carried out. By using an accurate water thermostat and BNS-calibrated thermometer, these calibrations were made and calibration curves worked out. It was also noticed that by connecting the thermocouples directly to the instrument best results were obtained. During the calibration procedure, junctions were soaked in the water. This could be done without any special insulation procedures, which fact was noticed when comparing a silicone resin insulated thermocouple with a similar thermocouple without any insulation.

When measuring the temperature between the hot surface and the

sample, thermocouple wires were coated with Dow Chemical Company's silicone resin. In addition, soon after the junction points both of the wires were led under the sample. This was done simply by cutting a tiny slice on the bottom surface of the sheet and the wires were immersed in this slice. At the edges where they came out of the sample they were fixed with a plastic cement. The sensitivity of these thermocouples was very good, and even if the wires used were not the standardized ones, an entire agreement existed between all prepared thermocouples, provided the junctions were properly made.

IV. Moisture Distribution Determination

The recording apparatus assembly for radioactive materials built and used by Dreshfield was accepted for this work too. In general, when equipped with a new photomultiplier tube, Tracerlab's scintillation detector worked very satisfactorily, as did the Autoscaler also. When a long enough warming-up period was allowed, 5 to 6 hours, the radioactive standard which was used to control the condition of the recording apparatus gave very reproducible figures. This control procedure was repeated before and after every run. During the entire run, the so-called background counting rate was about 4500 counts per minute, the thallium sources emitted a recorded radiation of about 200,000 counts per minute, and the strontium sources a counting rate of 400,000 counts per minute. When the background counting rate varied only in the limits of a few hundred, the effect of this variation was negligible. The gain setting of the scintillation detector was so adjusted that the background counting rate remained relatively low, but the recorded counting rates of radioactive sources were high enough without preparing the sources stronger than necessary. When increasing the strength of

the sources, more binding substances per source must be used, which very soon causes a dangerous resistance moment through the area where such source will be placed. Gain setting between Gain 12 and Gain 4.5 was used, giving the figures presented above.

The limitations of the techniques developed by Dreshfield are discussed later. Perhaps a chart recording instead of the existing photographic way would be less laborious when picking up the time differences.

The recorded figures were not always applicable to be converted directly to moisture differences since the shrinkage taking place during the drying, being not totally eliminated in horizontal areas and not at all in vertical direction, must be taken into consideration. This multi-directional shrinkage influences the results as follows. At first the areal shrinkage increases the basis weight of the laminate under investigation, then causes an additional absorption of beta rays. The vertical shrinkage causes an increase in the distance between radioactive source and the scintillation detector and then reduces the rate of recorded radiation. Both these phenomena were so pronounced for certain time moments ahead that corrections must be made. The largest correction was to be made to the recordings at the end of each run, and the magnitude of this correction amounted to about 5%.

The magnitudes of both kinds of corrections over the whole range used were properly measured, and correction curves were plotted to be used when necessary.

Concerning the emitter isotopes used in this study, the radiation energy of the thallium radioactive isotope is too low to be used anywhere in this study. Therefore strontium-yttrium isotope was accepted to be used from the points where thallium seemed to be too weak. Thallium was used until the

the basis weights of 30 mg./cm.² and strontium over the rest of the range. The highest basis weight of a laminate was approximately the same as the sample basis weight or 150 mg./cm.², and one should note that this basis weight lies very close to the upper limit of the range where strontium can be used. This applies where the starting moisture content is about 200%. If only fairly dry paper sheets are tested, the basis weight range becomes naturally much wider in both cases.

The preparation of radioactive sources was made following the method of Dreshfield. Only when the isotope-Amberlite suspension and the resin (Penacolite) solution were mixed together, mixing took place in the opposite direction as described by Dreshfield. More finely divided suspensions were gained in this way.

Before any runs, the very extensive calibrations were carried out in order to create an accurate basis for moisture constant calculations. As shown later, one cannot expect reliable results by using only one single calibration curve for the quantities of moisture in paper, but several curves must be determined using o.d. basis weight as parameter. This applies for both of the used isotopes. During these calibrations, sources of varying strengths were used with the purpose of seeing whether the source strength variations have any influence on the slopes of calibration curves, and then what will be the greatest heterogeneity of the sources that can be allowed.

In general, the calibrations against the moisture content were carried out so that for one basis weight only one sample was used, and different moisture contents were obtained by evaporating water gradually away.

Always when a test point was reached, the sample was sealed in aluminum foil, and transmission was measured. Immediately after each transmission measurement, the amount of shrinkage was determined also. The calibrations were actually made in the same circumstances as the drying runs.

V. Procedure to Prepare Samples and the Drying Procedure

Weyerhaeuser bleached sulfite pulp beaten to 800 ml. S.-R. was used in this study. After careful consideration, a relatively high basis weight was chosen to be used as total basis weights of the test sheets throughout the entire investigation. Several factors caused this basis weight to be fixed at 150 mg./cm.² The most important factors which eliminated the use of still higher basis weight were as follows: At first, strontium-yttrium isotope would not have been used beyond this value. The sheet formation would have been suffering still more, as well as the proper wet pressing. Moreover, the thickness of a sheet as compared with its area would not have been too reasonable, and to maintain a good contact between the sample and the hot surface would have been still worse than it was now. Even now, quite a few runs were abandoned because of varying contact.

The size of the sample was limited by the existing drying equipment, and the size of 8 cm. x 10 cm. was accepted. This size is greater than the size used by Dreshfield. The author of this work would have liked to use still larger samples to avoid the edge effects.

The handsheets were made in the ordinary way, using the British sheet mold. The sheets were formed from deionized water in order to prevent any exchange of radioactive cations. As is known, the radioactive cations

are at first bound by the ion exchange resin (Amberlite) which is then fixed into a piece of teabag stock by means of Penacolate phenolic resin.

In order to form the desired laminates, the slurry which was disintegrated in the standard defibrator was poured from a bottle into a graduated cylinder. This simple procedure provided a sharp division, and the deviations in samples were merely total basis weight variations rather than the weight ratio deviations of the laminates. The total basis weight variations were caused by differences in raw pulp consistency and in the wet pressing effect, especially when using as thick sheets as here.

When the separately formed laminates were taken out from sheet mold, both thermocouple and the radioactive source were placed on the lower laminate. Before being placed on this laminate, the source was soaked in water for so long a period that practically no air was left in the source when it was taken to the use. A piece about 12 mm. x 12 mm. was cut and removed from the center of each source with the purpose of maintaining a better contact between laminates at that place and thus preventing as much as possible any delamination effects. The junctions at the thermocouples were placed inside this source opening. In general, the thermocouple was put at first on the surface of the lower laminate so that it crossed the laminate in the direction perpendicular to the long edge of the final sample. Then the radioactive source and the upper laminate were placed, and finally a roller was used to remove the air which was entrapped between the laminates.

✓ The so-formed sandwich was put into the Williams plane press and pressed very carefully during 40 seconds to the moisture content of about 280%. This fairly slight wet pressing was accepted especially to prevent ✓

again as much as possible the shrinkage during the drying as well as a too extensive hardening of the samples.

The procedure described above differs in some respects from that used by Dreshfield. At first the aluminum foil recommended by Dreshfield to be used as a bottom layer at the sample was abandoned. The investigations showed namely that such an aluminum foil causes considerable resistance to heat flow, and, moreover, it tends to loosen toward the end of the drying run. The Noble and Wood rotary sheet press could not be used either, because it caused a very dangerous unevenness at the sheet as well as an increasing delamination tendency. No starch was used either.

After the wet pressing, the samples of the final size were cut with a sharp razor blade. This sheet, 8 cm. x 10 cm., was always removed from the center area of the circular sandwich, and so the radioactive source remained symmetrically in the middle of the sheet. The edges of these test sheets were then sealed with "901" cement produced by American Phenolic Corporation.

A sheet so prepared was then exposed to a warm air stream in order to condition it for the drying treatment. The expected starting moisture content of each sample was 206%. After this conditioning treatment, the samples were sealed in aluminum foil. Each sample was given a drying treatment on the same day it was prepared. The purpose of this rule was to make the time shorter for the possible isotope cation migration.

When all the other preparations for a run were made, the sample was taken, still being sealed in aluminum foil, weighed properly, and placed in the front of the duct opening. The thermocouple wires were led and

connected to the potentiometer. Any metallic connections before the potentiometer were not allowed. Thereafter the aluminum foil was removed and the sample was placed on the hot surface and clamped properly. Simultaneously, the starting buttons of the recording apparatus were pushed on.

After each run, the final moisture contents of both the laminates were determined gravimetrically as well as the exact basis weight of the upper laminate. The calculations based on recorded values were made as follows:

The temperature recordings picked up from the chart of the potentiometer were corrected, using the calibration data determined earlier. These corrected temperatures were then used when plotting the temperature distribution curves. For some few samples a small correction was made due to a deviation on the starting moisture content of that kind of sample. When needed, this correction was carried out so that the constant rate period of such sample was extended to continue as much further as would have been needed to evaporate that amount of water which was less than the expected value.

With the purpose of determining the moisture distribution through the sheet at any time, the calibration curves were used. The method of calculating the moisture content was as follows: The moisture content of the upper laminate was determined gravimetrically before and after each run as well as the basis weight too. These figures were then taken as the basis for the calculations, and the starting moisture content of the upper laminate formed the fixed starting point. Primarily, the time intervals for 10,000 recorded impulses or so-called counts were recorded during each run. Using this value,

the transmission at any stage of drying was calculated directly as a ratio between the time interval at the starting moment and the same at any desired moment. These relative values, corrected if needed, were then used to calculate moisture content differences, using the following equation:

$$\Delta m = \frac{\log \frac{N_o}{N_n}}{a}$$

where Δm = moisture content difference (mg./cm.²)

N_o = time interval for 10,000 counts at starting moments

N_n = the same at any time

a = the slope of calibration curve at desired point

The whole system was fixed when the starting moisture contents were known and used as starting points.

The moisture contents at several drying intervals were then calculated and the corresponding curves for each run were plotted.

Using this method, only ratios were used instead of any absolute transmission or counting rate values.

It was possible to check the accuracy of this method, too, by comparing the calculated moisture figure with the figure determined gravimetrically at the end of each run. If deviation occurred, correction was made using the corresponding gravimetric value. This was needed very seldom, and it shows that the migration of the radioactive cations did not take place to any dangerous extent.

VI. Errors Involved and Limitations of the Existing Methods

When considering the results of this work presented in the next

section, one has to keep in his mind the following remarks:

The author of this work, when carrying out his work for Diploma in Finland, noticed that the areal shrinkage of a sheet during the drying treatment was to be prevented. This was important for two reasons. At first, the more even drying rate resulted as well as the better capillary characteristics. On the normally managed drying machines, practically no shrinkage is allowed to take place. Using the existing apparatus, the clamping system was impossible to be constructed so that pretension enough and a constant tension in general could have been given the samples. This was unfavorable not only because of the corrections which were to be made but because of the slightly variable contact between a sample and the hot surface. This is actually well demonstrated by the temperature curves showing the course of temperature at the bottom surface of the sample. Not only the tension of the sample must be under control, but the pressure of the sample against the hot surface, too. This can be done simultaneously with tension control.

Concerning the moisture determinations, the use of teabag stock sources is perhaps not the final solution when using this tracer method. This method itself is a very useful one indeed, and when made properly by means of good devices, gives a relatively good accuracy. Two remarks are to be made against those teabag sheet sources. At first they do not always provide an ideal contact between laminates, but in certain circumstances even cause a delamination effect. When they do, it depends upon their location which apparently determines how a sheet behaves mechanically. The opening, cut into the source, helped very much, but in certain cases the source sheet apparently disturbed the drying.

Another thing is the migration of the radioactivity from the source. Apparently the ion exchange method is not reliable enough, because the ion exchange reaction is very fast and very sensitive too. This was actually shown already by Dreshfield; but because the error so created remained relatively small, the existing method was accepted. However, a method using some kind of metallic wire net as a frame on which the tracer would be precipitated and fixed in a suitable way should provide a safer operation. The ratio between the area of this net and the area of the sample should be only small enough. At the same time, the size of the samples should be increased when using a basis weight like 150 mg./cm.²

The temperature measurements were relatively easy to be carried out and the accuracy apparently very good as well as the sensitivity of the thermocouples. One thing which is worthy of mention, however, is the fact that by using these tiny junctions, the point temperature only is recorded. In order to get the best results, a series of thermocouples should be used simultaneously or several duplicate runs should be made.

In spite of the facts here presented, the purpose of this project was apparently reached with satisfactory accuracy.

RESULTS

I. Number of Runs

The total number of 42 drying runs were made when collecting the necessary data. Twenty of these runs were totally accepted, and some runs were totally abandoned.

II. Calibrations

Concerning the use of radioactive isotopes when determining a mass

per unit area, the general theory is well known. However, this kind of radiation does not follow the theory so well that, by using the derived equation, values of moisture contents could be directly calculated using experimental transmission data. Therefore, the proper calibration must always be done.

The shape and the general course of the calibration curves are seen in Figs. 1, 2, 3, and 4. There the transmission vs. mass per unit area has been plotted. The curves for two extreme source strengths used are presented. One notes that depending upon the basis weight of the pulp sheets, different slope values for these transmission curves are received. On the other hand, the curves are so nearly linear that two slope values for each curve are enough to cover the whole range in each case. One sees also that the rate of radiation per unit of area influences the slope values too to some extent. This is especially the case when using thallium isotope.

III. Temperature Distribution

Recorded and corrected values were plotted at first in the form of Fig. 5 where the distance from the open surface is used as parameter and temperatures vs. time have been plotted. Using this diagram, Fig. 6 was prepared. They already give a fairly clear picture of the course of the temperature distribution development. As seen later, these temperature plots agree well with the corresponding moisture plots. In Table I the temperature data which have been used when plotting the curves are presented.

IV. Moisture Distribution

The corrected moisture content figures which are presented in Table II have been the basis when plotting Fig. 7. It shows the accumulative

moisture content vs. distance from the open surface at the several stages of drying. By measuring the slopes of these curves, Fig. 8 was worked out.

There the moisture distribution is presented again at several stages of drying.

With the purpose of giving some general picture of the course of the drying runs, Fig. 9 is presented. It shows how the moisture content of three different laminates have been changing during corresponding drying runs. These curves, as said earlier, were prepared for each accepted run, and Fig. 7 was actually worked out by using the moisture values taken from these plots.

V. Discussion

Concerning the calibration part of this work, the main conclusion which is to be drawn is that careful calibrations must be made before any ordinary measurements. It turned out that the basis weight of sheet has an influence when trying to determine the amount of water in the paper. It is evident too that the thallium isotope is more suitable to be used for moisture determinations than strontium-yttrium until the basis weight of about 25 mg./cm.² is reached. This range actually covers a big variety of papers and boards. The only limitation when using thallium is its relatively short half-life. It was seen that when the rate of radiation per unit source area is changing, small changes on slopes of the transmission curves must be expected. Practically, this means that when a source is getting weaker by time, new calibrations become necessary in order to get continuously reliable results. In that respect, the strontium-yttrium isotope is more suitable with its longer half-life. What is said here of water applies also to the basis weight

determinations, and thallium, when having the steeper slope of the calibration curve, should give a better accuracy on its special area.

The results received by moisture and temperature measurements agree fairly well principally with conclusions drawn by Dreshfield. He, however, did not measure the temperature distributions and used only expected values derived on the basis of McCready's and his own work. As said before, in the present study, the agreement between the moisture and temperature distribution is evident when thinking what kind of physical picture they give of the course of drying.

One can see the following things: The so-called constant rate period takes place during about the first 12-14 minutes. Quite strictly speaking, the rate of evaporation during this period is apparently not constant because the temperature inside the sheet is increasing slightly toward the end of this period. The average moisture content of the sheet is 110-115% at the end of this period. Thereafter, the so-called falling rate period starts practically simultaneously through the sheet. The main phenomenon which starts this period can be seen in temperature distribution figures. It is evident that the border layer of the continuous water network inside the sheet begins to migrate toward the inner parts of the sheet leaving at this moment the hot surface side. There builds up a considerable heat flow resistance which can be seen from a big temperature gradient which is formed at the same moment.

By the time the border layer continuously migrates away from the surfaces and because there is still under the sheet free water to be evaporated, this means increasing area of the steeper temperature gradients. Because the thermal exposure mainly takes place on only one side of the

sheet, the nonsymmetric distribution curves result in the course of continuing drying. Simultaneously, if one follows the moisture conditions inside of the sheet, he notes that during the earlier stages of drying a large area of even moisture distribution remains over the distance from 10-20% to 70%. This area then gets narrower with continued drying, and the center point of this area moves gradually toward the open surface. This area includes the maximum moisture content layer too. This process continues until the development of the gradients covers the whole distance between the surfaces. Then, too, the main part of water has been evaporated, and because the heat demand goes down gradually, an even temperature distribution results through the sheet.

On the basis of this picture, one is able to expect what would happen if the sheet had been heated on both sides, which is the case on the ordinary paper machine. The material conclusion is that instead of the nonsymmetric distribution curves, symmetric ones would have been received.

What is the picture of water migration which can be got on the basis of this work? Primarily, the capillary forces are dominant on the other side and during the whole so-called constant rate period when causing the water to migrate toward the surfaces. It is logical, too, that the water which is vaporized at the hot surface (or anywhere inside the sheet) begins to move upward in the form of practically saturated steam. When no great amounts of air are mixed in this vapor and because the temperature distribution at this stage is very even, the condensation begins to take place very soon after the vapor has left the hot surface. No sharp condensation area can be expected to exist in the inner region of the sheet. Naturally big amounts of vapor go through without condensing, but how much

its amounts deviate from that amount which should correspond to equilibrium conditions through the sheet is impossible to say.

Later on, the contact to hot surfaces becomes worse, and the vapor is partly superheated and mixed with air. Then perhaps the condensation takes place over a narrower region, but the rate of condensation must decrease too.

From the viewpoint of practice, it is interesting to realize that when the constant rate period is over, the main resistant for heat flow begins to be built up in the sheet itself; and even if the contact between the hot surface and the sheet can be improved in certain cases to some extent, this helps only fairly slightly. More heat should be transported directly inside the sheet if a more pronounced effect is wanted. This applies especially to the sheets with high basis weight, but certainly to all kinds of papers.

TABLE I
DATA OF TEMPERATURES AT DIFFERENT SHEET LEVELS AFTER
DIFFERENT PERIODS OF DRYING

Weight fraction, %	Temperature, °C.								
	0	5	10	15	20	25	30	35	45
2.9	29	51.5	52	53	51	50.5	51	51	50
5.1	28.5	52.5	54	53	52	57	51	51	50
8.0	30	54	55	54	53	52.5	52	51.5	50.5
10.1	27.5	55.5	55.5	55	54	54	52.5	51.5	51
15.2	33	59	60	58.5	58	57	56	54.5	53
20	30.5	59.5	60	58.5	56	55.5	54.5	53	52.5
21.5	28	60	61	59.5	57	56.5	55.5	54.5	53
33.3	33.5	66.5	67	65	63	60.5	58	55.5	56
33.4	30.5	61.5	62	63.5	62.5	58.5	55	55.5	56
44.4	30.5	68.5	68.5	68	65.5	63	60	57.5	59.5
46	30.5	66.5	67	67.5	65.5	63.5	61.5	57.5	58
53	35	71	71.5	69.5	67	64.5	62.5	59.5	64.5
64	30.5	72.5	73.5	72	69	66.5	63	63	68
74	28.5	78	78.5	75	70.5	67	65	66	71
80	31.5	79	79.5	78	71.5	71.5	73.5	78	79
84	28.5	81	81	77.5					
94	30.5	82	82	78.5	79.5	81.5	82	82	83
97	30.5	81.5	82.5	80	80	81.5	82	82	83
100	30	84	84.5	85.5	86	86	86	86	87

TABLE II

DATA OF MOISTURE CONTENTS OF DIFFERENT SHEET WEIGHT FRACTIONS
 AFTER DIFFERENT PERIODS OF DRYING

Weight fraction, %	Moisture content (mg. water/cm. ²)								
	0 (206%)	5	10	15	20	25	30	35	45
2.9	8.9	6.9	3.2	0.8	0.7	0.6	0.6	0.6	0.2
5.1	15.1	12.2	7.8	3.6	2.1	2	2	1.2	0.5
8.0	25	18	11	4.6	2.8	2.8	2.5	2	1.6
10.1	31	26	18.5	11.2	6.7	5.2	4	3	3
15.2	46	36	25	15	11	9.3	8	7.5	5.3
20	62	51	38	27	21	16.5	14.5	12	6.5
21.5	65.5	54	40	29	22	17	14.5	12	7.3
25.2	78	64	50	35	26	21	20	14	9
33.3	102	81	57	41	31	24	21	21	13
33.4	103	83	62	44	30	23	19	15	9
44.4	137	112	82	56	42	33	28	24	10
46	142	120	93	67	51	41	34	29	18
53	164	138	107	79	62	51	42	35	21
59	182	151	113	85	67	53	44	34	18
64	198	168	133	102	80	64	52	40	21
74	229	192	150	114	91	79	61	40	20
80	247	201	154	123	98	79	62	40	20
84	259	210	161	124	96	79	56	39	20
88	269	227	182						
97	294	245	190	152	115	82	62	44	18
100	309								

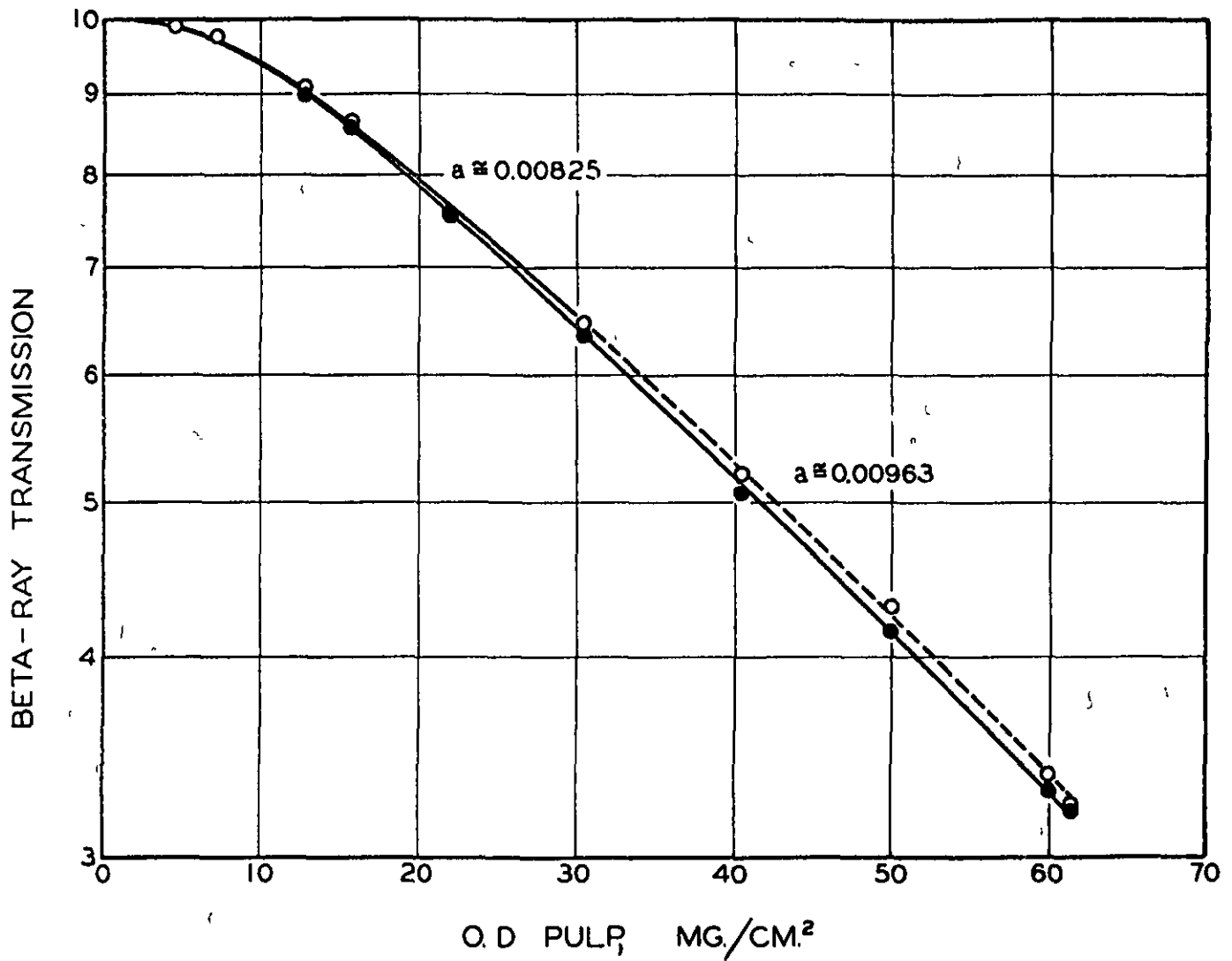


Fig. 1. Thallium source

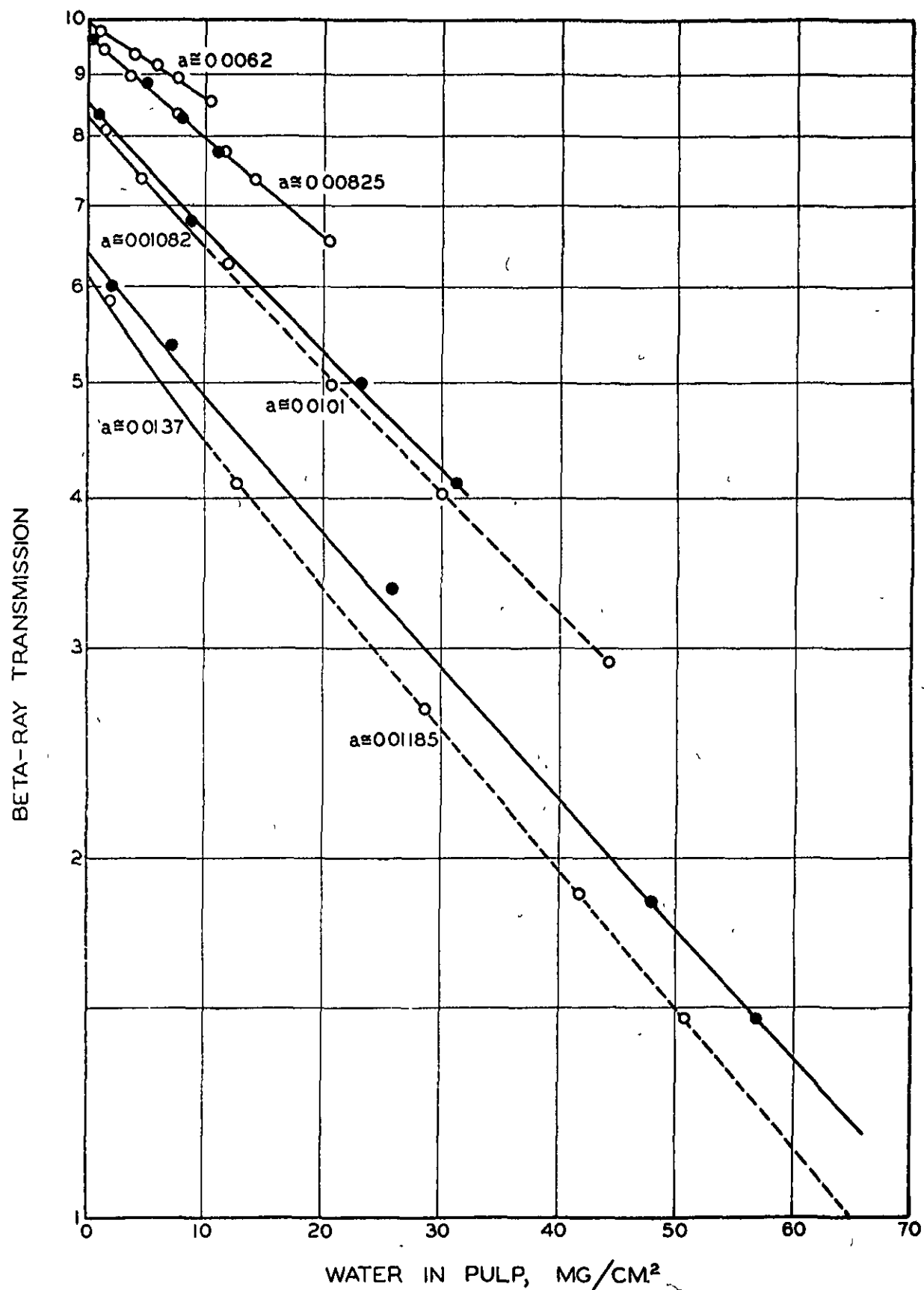


Fig. 2. Thallium source

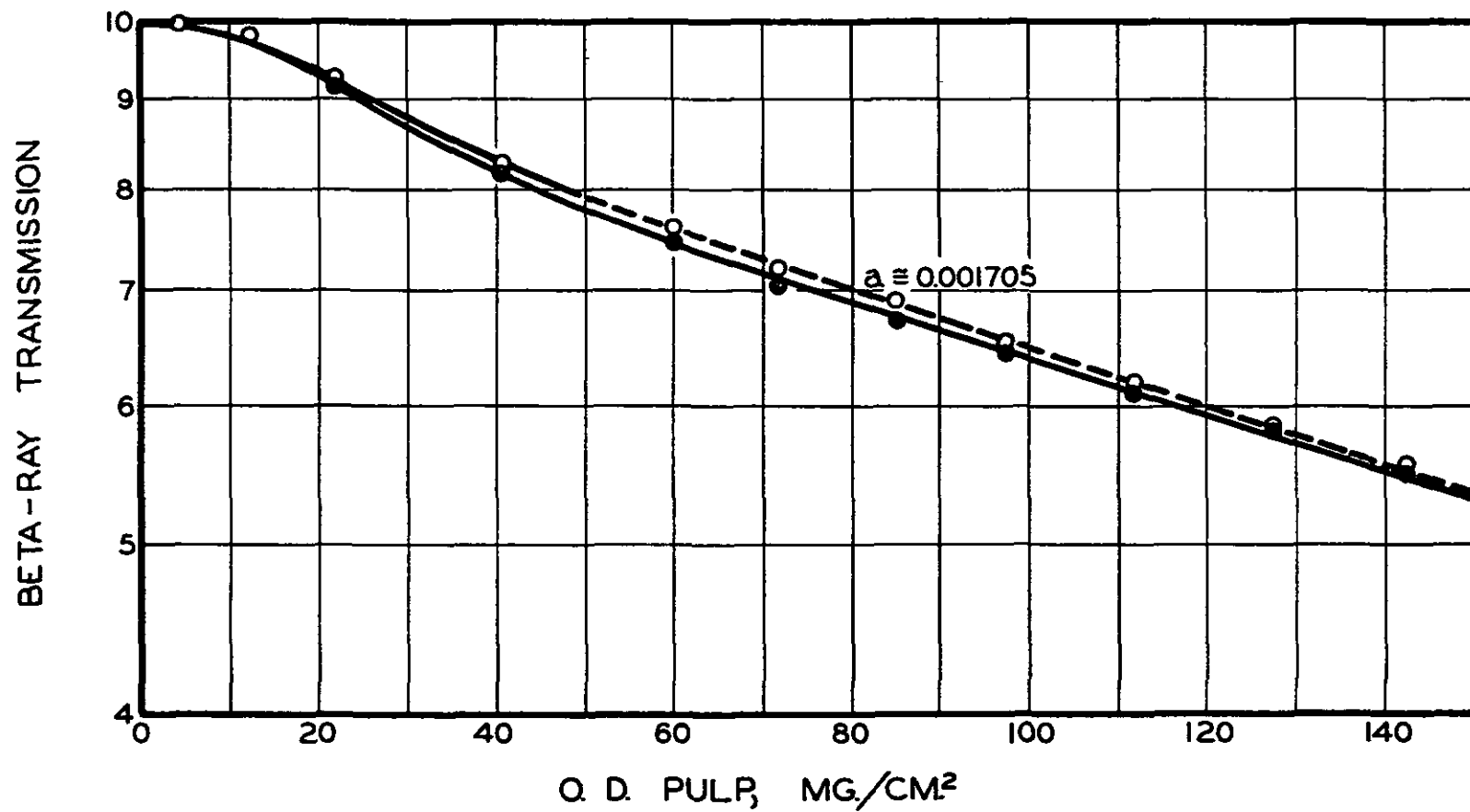


Fig. 3. Strontium source

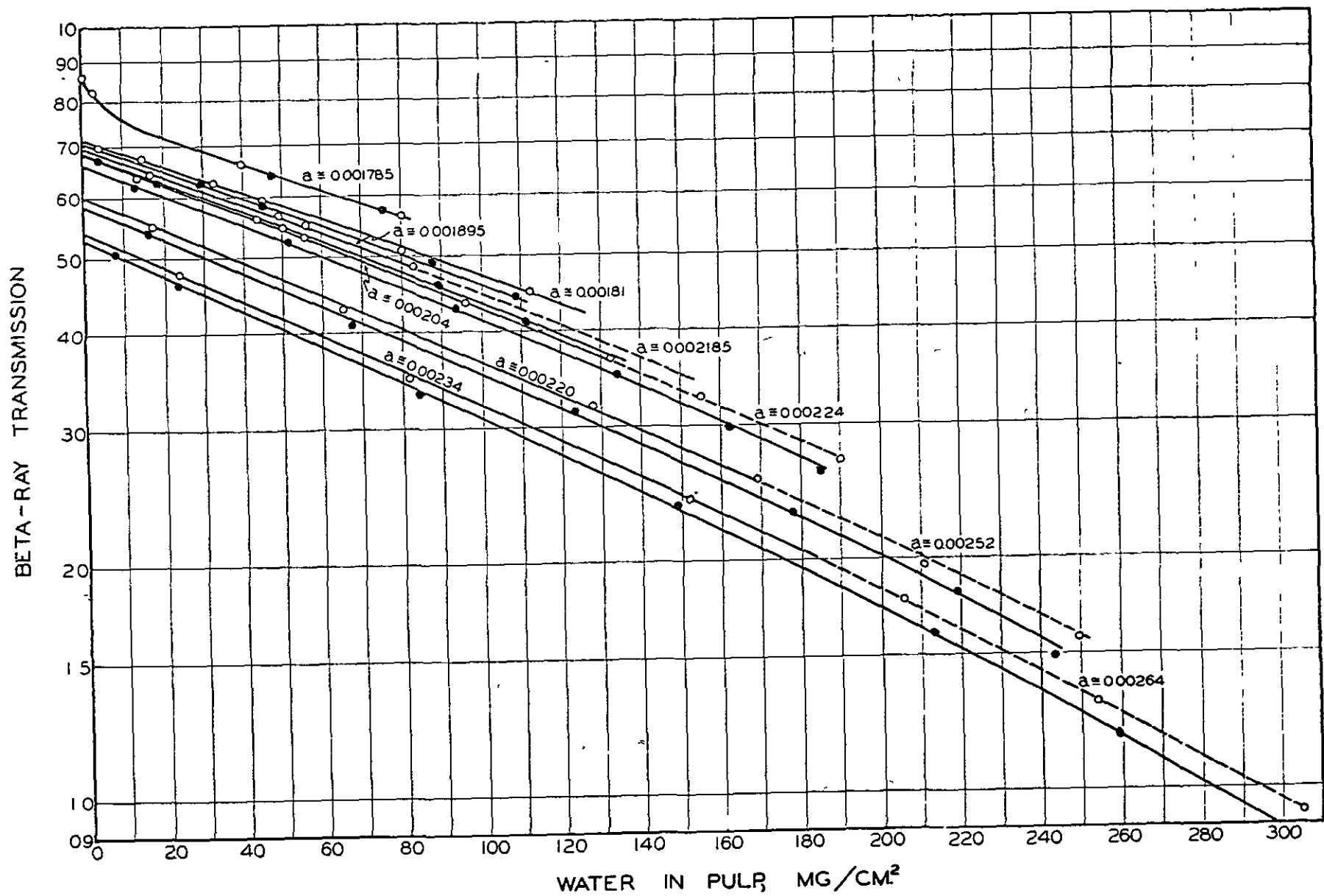


Fig. 4. Strontium source

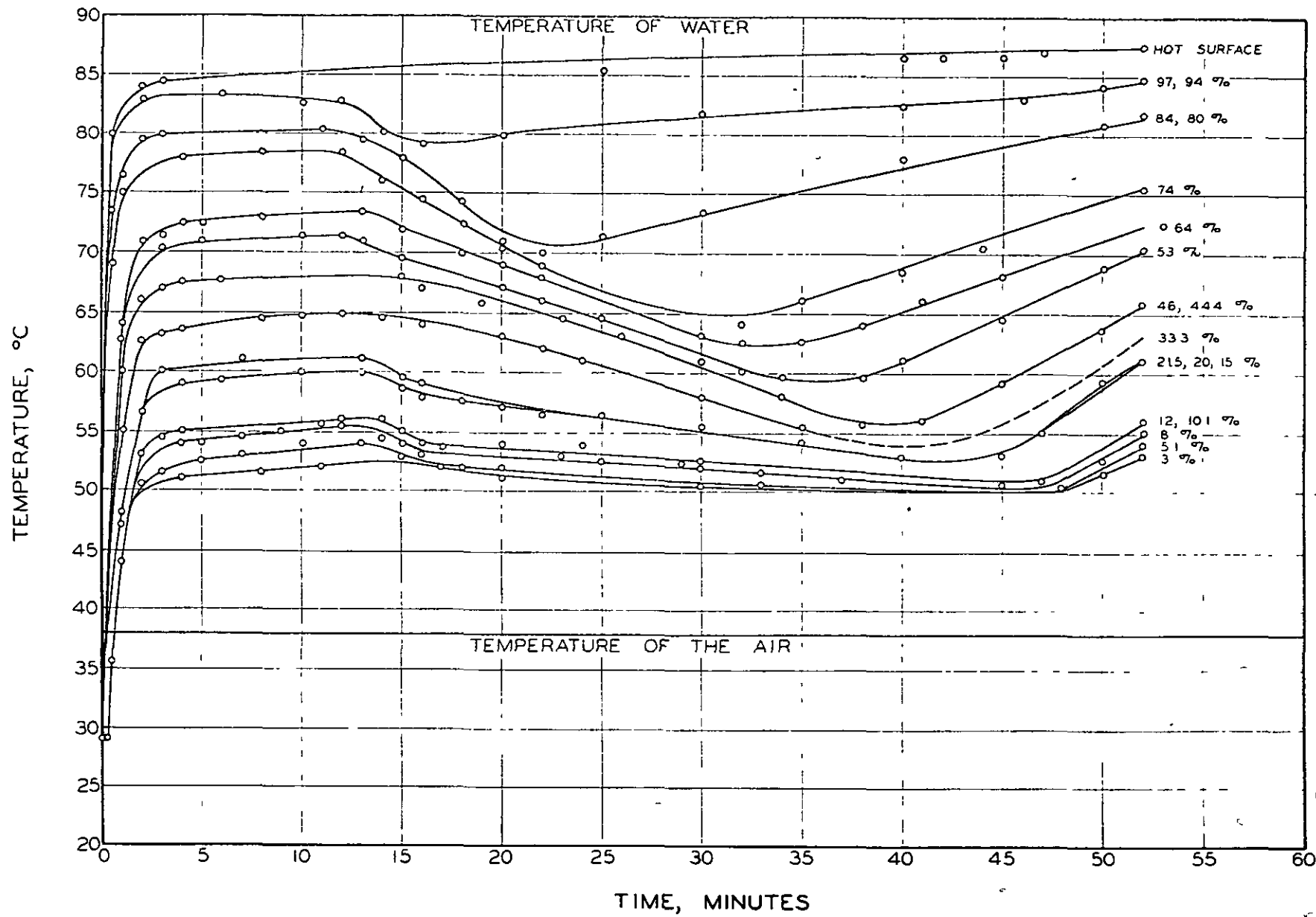


Fig. 5. Temperature inside sheet

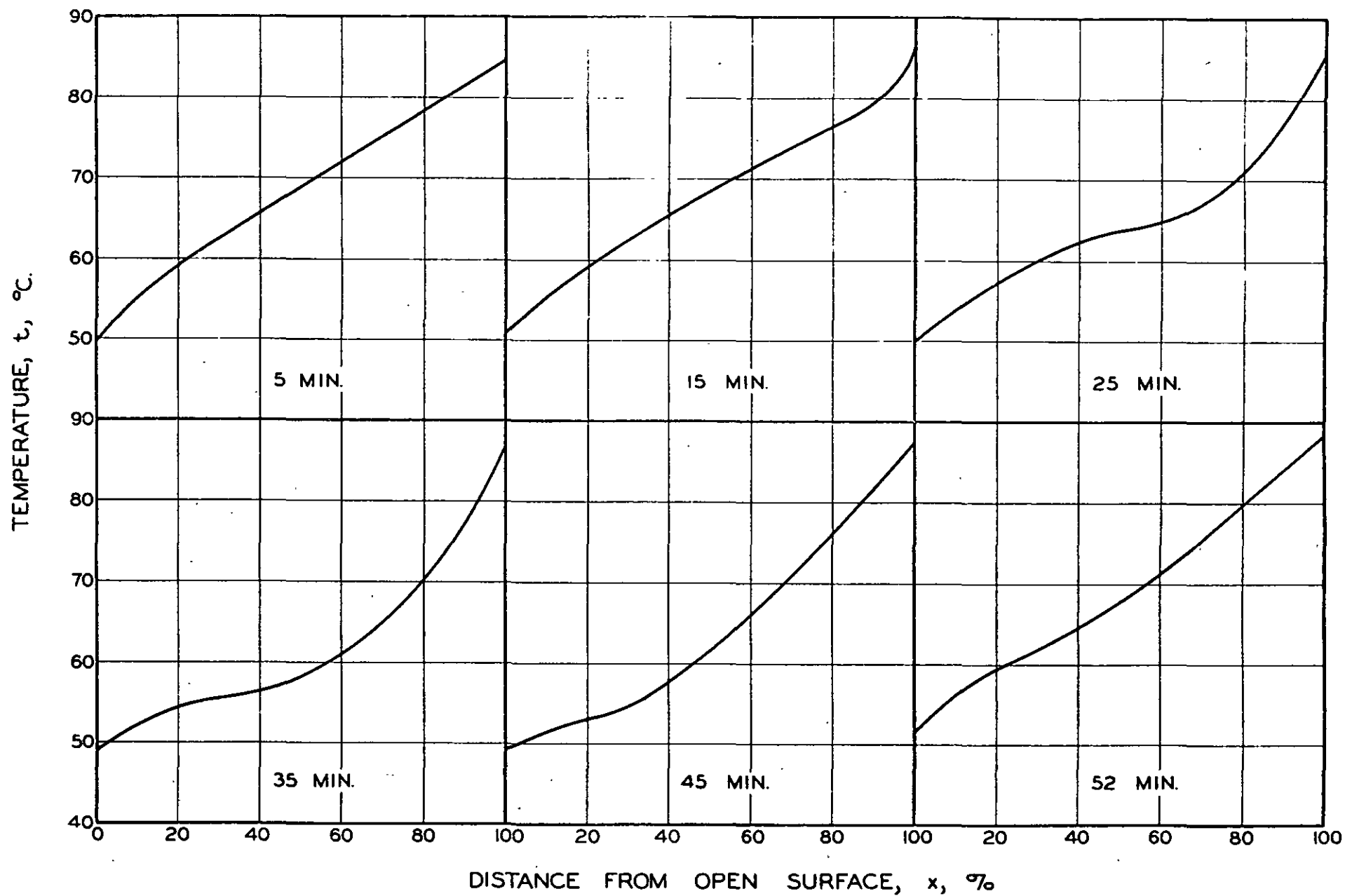


Fig. 6. Temperature distribution through the sheet

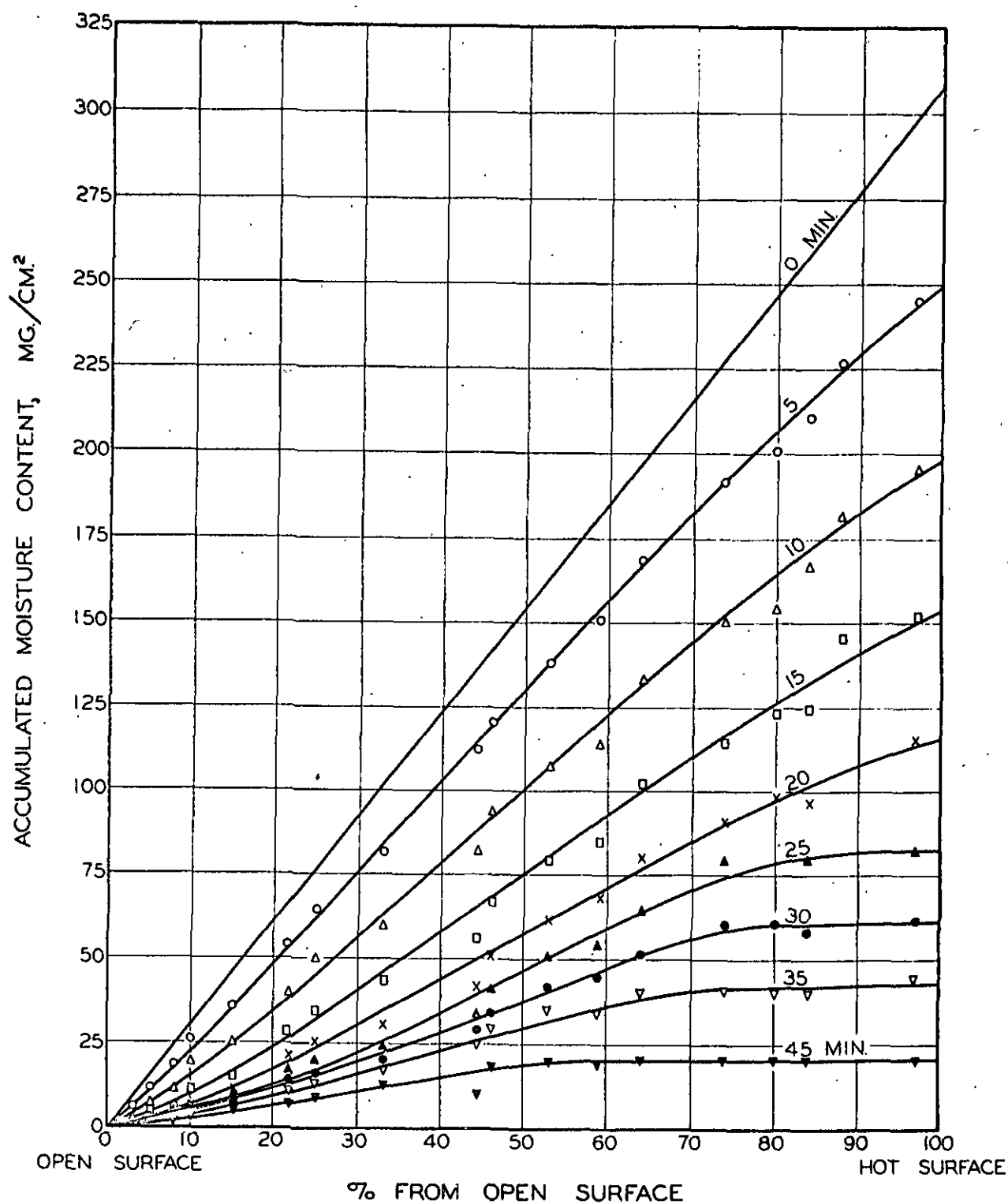


Fig. 7. Accumulative moisture of sheet at different times

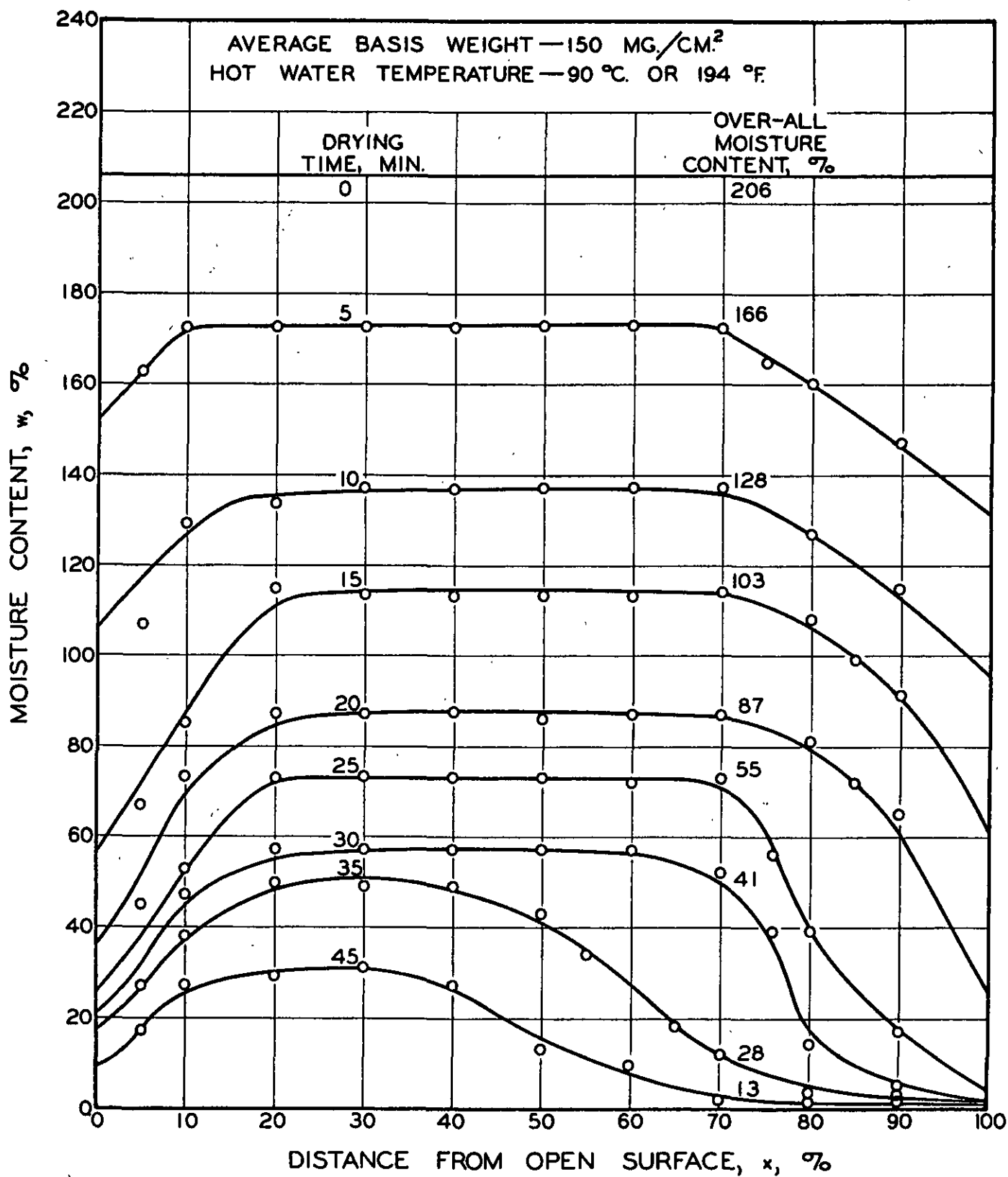


Fig. 8. Moisture distribution curves

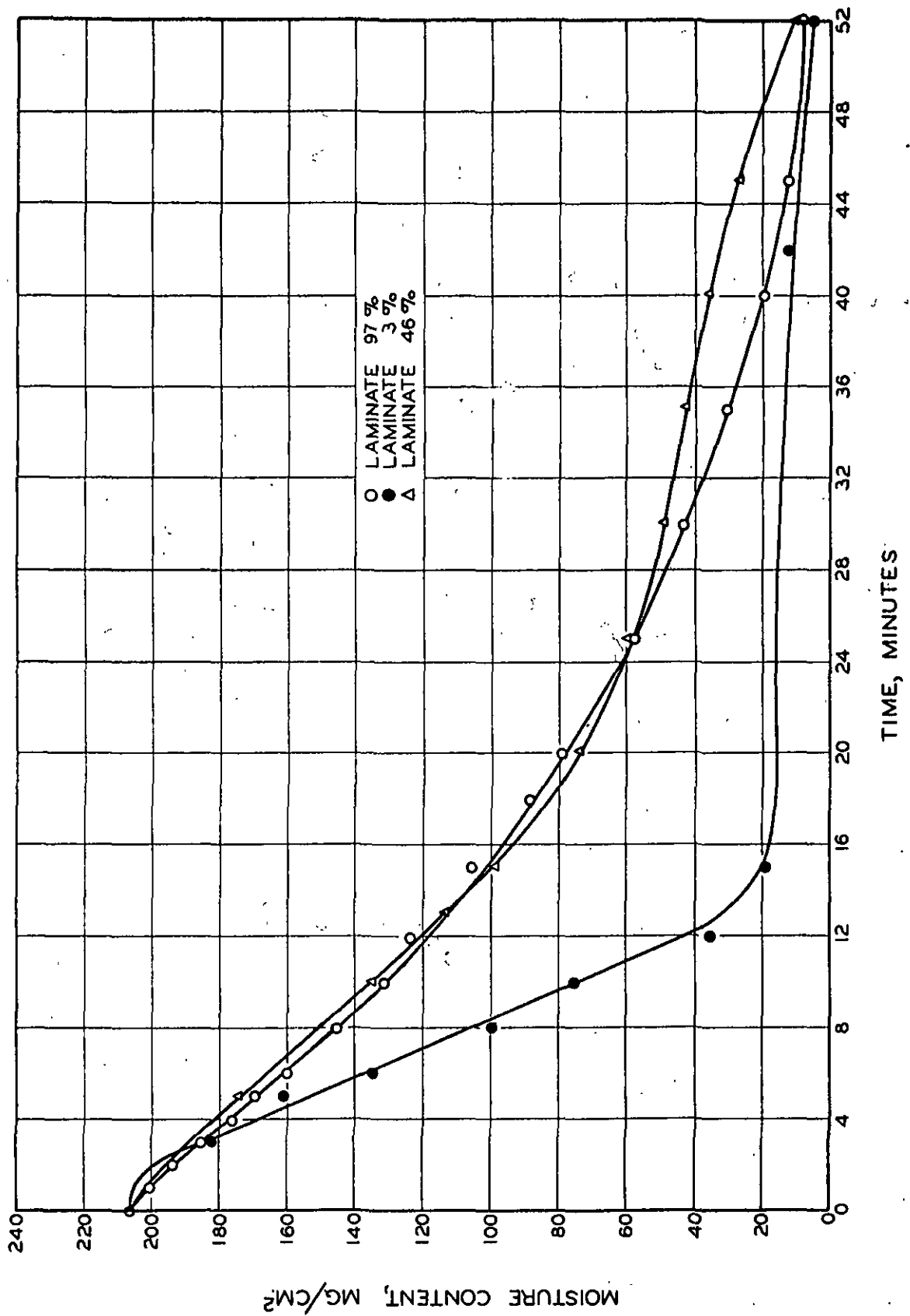


Fig. 9. Change of moisture content for 3 drying runs