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FINAL TECHNICAL REPORT

Georgia Tech Project B-587 Under IPA Agreement 2142040

STOCHASTIC DETECTION MODELS FOR TREATED AND UNTREATED GROUND STRUCTURES

By

B. J. Cown

April 1985

Prepared for

U. S. Army Corp. of Engineers Waterways Experiment Station Environmental Laboratory Vicksburg, Mississippi 39180

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SECTION I

INTRODUCTION

The research work summarized in this report is oriented toward the development of mathematical and computer models for estimating the detectability of Army ground installations, such as aircraft hangars and other fixed structures, by incoming threat missiles or aircraft. It is recognized that a rigorous, detailed analysis of ground target detection in clutter environments is beyond the scope of the work described herein. Accordingly, the primary goal of this effort is to develop relatively simple radar detection models that permit valid comparisons of detectability versus range for treated and untreated structures located in specified clutter environments. In particular, reasonably accurate and simple radar detection models are sought for estimating the probability of detection versus range as a function of specified radar, target, and background characteristics.

This report is organized as follows. Mathematical models are presented and discussed in Section II. A model that is valid for analyzing stochastic near-field multiple-scattering situations involving complex targets located in clutter environments is summarized therein for possible future applications, along with simplified models intended for use during the current research effort. Estimated values of the radar cross section (RCS) and probability of detection (p.o.d) for selected ground targets are presented and discussed in Section III. Concluding remarks and recommendations are presented in Section IV. A list of references is included in Section V.

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SECTION II

MATHEMATICAL MODELS

A. Introduction

We consider the general situation depicted in Figure 1, where a missile equipped with a microwave or millimeter wave radar approaches a ground target situated in clutter. We wish to determine the probability of detection versus range of the target by the missile seeker as a function of the operating characteristics of the radar, the electromagnetic scattering properties of the target and clutter, and atmospheric attenuation effects.

The backscattered electric field arriving at the seeker receiving antenna is a stochastic variable since the scattering by the target plus clutter, as well as atmospheric propagation fluctuations, are stochastic processes. Accordingly, the signal induced in subsequent coherent or incoherent detectors is a stochastic variable. The manner in which the input signal is processed in the receiver has an important effect on the stochastic characteristics of the output signal and, hence, on the probability of detection of a given target in clutter. The stochastic properties of the processed signal are a function of both the stochastic properties of the backscattered field at the receiver antenna and the particular signal processing scheme employed in the receiver.

The analysis of the probability of detection can be divided into two major parts: (1) analysis of the stochastic complex voltage induced in the seeker antenna terminals, and (2) analysis of the effects of signal processing on the stochastic properties of the output signal in the video stage where decisions are made electronically to determine whether or not a target has been detected. Square law detection is assumed throughout this report.

An analysis of the stochastic electric field scattered toward the receiving antenna and the induced voltage is presented first, followed by an analysis of the effects of signal processing on the stochastic characteristics of the output signal. In particular, expressions for the probability of detection are derived and discussed for selected types of signal processing schemes. Some preliminary computations of radar cross section and probability of detection are presented and discussed in Section III for selected ground targets.

B. The Scattered Field and the Induced Antenna Voltage

An analysis is sought that is valid for stochastic near-field scattering by multiple, interacting scattering elements illuminated by a microwave or millimeter wave seeker antenna. A stochastic analysis is required because it is impossible to know the exact geometrical, physical, and electromagnetic scattering properties of the targets and their surroundings. A formulation that is valid for near-field scattering phenomena is required for evaluation of detection and tracking performance for missile-target distances that are small compared to $(2 L^2/\lambda)$, where L is the largest characteristic dimension of a target and λ is the wavelength of the seeker radar. Finally, it is necessary to include the effects of multiple scattering, to at least first order, to obtain valid estimates of scattering by closely-spaced scattering elements, and by focusing geometries such as corner reflectors formed by two or more scattering elements.

Let \overline{E}^{s} $(\theta, \phi | \theta_{0}, \phi_{0})$ denote the stochastic scattered near-field electric field at (R, θ, ϕ) due to illumination of targets plus clutter by a seeker radar located (instantaneously) at $(R, \theta_{0}, \phi_{0})$. Taking first order multiple scattering into account, \overline{E}^{s} $(\theta, \phi | \theta_{0}, \phi_{0})$ may be expressed as

$$\bar{\mathbf{E}}^{\mathbf{S}} (\theta, \phi | \theta_{o}, \phi_{o}) = \sum_{n} \bar{\mathbf{E}}_{n}^{\mathbf{S}} (\theta, \phi | \theta_{o}, \phi_{o})$$

$$+ \sum_{m} \sum_{n} \bar{\mathbf{E}}_{mn}^{\mathbf{S}} (\theta, \phi | \theta_{o}, \phi_{o}) ,$$
(1)

where \overline{E}_m^s is the field scattered by scattering element n due to illumination by the seeker antenna field and \overline{E}_{mn}^s is the field scattered by scattering element m due to illumination by scattered field of the nth scattering element. The scattering elements may be whole targets, portions of complex targets, large patches of clutter or small patches of clutter, depending on the level of modeling detail chosen for the analysis. Greater detail generally permits greater accuracy, but the number of scattering elements N rapidly becomes too large to handle as modeling fineness is increased. Engineering judgments have to be made by the analyst in order to select a compromise level of modeling detail that yields both reasonable accuracy and tolerable complexity. This is by no means an easy task, and we do not attempt to provide comprehensive guidelines in this report.

 \overline{F}^a in Equation (4) is the SAF for the seeker antenna, i.e., $\overline{F}^a(\theta^*, \phi^*)$ = $|\overline{R}_n| \exp \left[jk |\overline{R}_n| \right] \overline{E}^a(\theta^*, \phi^*)$, where \overline{E}^a is the far-field electric field of the seeker antenna. We emphasize that Equations (1) through (5) are valid for <u>near field</u> scattering analyses, and they are also valid for the usual far-field scattering analyses obtained in the limit of all $|\overline{R}_n|$, $|\overline{R}_m| > 2 L^2/\lambda$. \overline{S}_n and \overline{S}_m in Equations (4) and (5) are the Plane Wave scattering dyadics for scattering elements n and m, respectively [1-3]. Expressions for \overline{S} are available for flat rectangular plates, triangular plates, discs, cylinders, spheres, and some other simple shapes that can be used as basic scattering elements for modeling complex targets and clutter patches.

The stochastic complex voltage V induced in the terminals of the seeker antenna is given by the expression $\begin{bmatrix} 1 \end{bmatrix}$

$$V(R,\theta_{0},\phi_{0}) = \iint_{\Omega'} \left[\overline{F}^{a}(\theta-\beta, \phi-\alpha) \cdot \overline{F}^{s}(\theta,\phi) \\ \cdot \exp\left[-j\overline{k}(\theta,\phi) \cdot \overline{R}(\theta_{0},\phi_{0}) \right] \right] d\Omega , \qquad (6)$$

where β , α are elevation and azimuth pointing directions of the seeker antenna. The vector function $\overline{F}^{s}(\theta, \phi)$ is computed as

$$\overline{F}^{s}(\theta,\phi) = \int \int \left\{ \overline{\overline{\tau}}(\theta,\phi|\xi,\eta) \cdot \left[\sum_{n} \overline{F}^{s}_{n}(\xi,\eta) + \sum_{m} \sum_{n} \overline{F}^{s}_{mn}(\xi,\eta) \right] \sup_{\substack{(7) \\ (7)}} \right\},$$

where $\overline{\tau}$ is the angular spectrum representation of the polarization transformation matrix \overline{T} described in Reference 1.

The stochastic behavior of $V(R,\theta_0,\phi_0)$ is thus determined by the deterministic and stochastic variables appearing in Equations (4), (5), and (6), as well as by atmospheric attenuation effects that are not included in the analysis at this point. The complex V is subsequently either envelope detected or square-law detected in the receiver. Accordingly, it is the amplitude V of V (V = |V|) or the power S = V² which are of primary interest since the probability of detection is a function of the stochastic properties of V or S, depending on whether the receiver employs envelope or square law signal detection.

$$X = \sum_{p=1}^{P} X_{p}, \text{ and}$$
(8)
$$Y = \sum_{p=1}^{P} Y_{p} .$$
(9)

 X_p and Y_p are obtained from Equations (4), (5), and (6) by taking the real and imaginary parts of the integrated sums in Equation (6) and grouping the terms as just described. Hence, X and Y are each just the sum of weakly dependent random variables, and if the number of voltages is > 7, the p.d.f. for X and the p.d.f. for Y will both be Gaussian [10] to good approximation. The power signal S will therefore follow the Nakagami power p.d.f..

The derivation of the Nakagami p.d.f. involves the use of the joint probability density of X and Y. The joint p.d.f. for two Gaussian random variables X and Y is known to be [10],

$$F(X,Y) = \frac{1}{2\pi\sigma_1\sigma_2\gamma} \left[\exp\left[-\frac{(X-\langle X \rangle)^2}{2\gamma^2\sigma^2} - \frac{(Y-\langle Y \rangle)^2}{2\gamma^2\sigma^2}\right] \right] \left[\exp\left[-\frac{2\rho(X-\langle X \rangle)(Y-\langle Y \rangle)}{2\gamma^2\sigma_1\sigma_2}\right] \right], \quad (10)$$

where <X> and <Y> are the average values of X and Y, and σ^1 and σ^2 are the standard deviations of X and Y, respectively. ρ is the correlation coefficient for X and Y, and $\gamma = \sqrt{1-\rho^2}$.

The Nakagami p.d.f. F(S) for the signal power S in the video stage is obtained as follows. First, the joint p.d.f. for X and Y is expressed in polar coordinates by replacing X and Y by

$$X = \sqrt{S} \cos (\phi) \text{ and}$$

$$Y = \sqrt{S} \sin (\phi),$$

where $S = X^2 + Y^2$, and integrated over the annular ring of thickness dS

where ${}^{<}S_{r}^{>}$, ${}^{<}S_{d}^{>}$, and ${}^{<}S_{c}^{>}$ denote the statistical average values of the receiver noise power, the diffuse return power from the target, and the clutter return power, consecutively.

The average value $\langle S \rangle$ of S and its standard deviation A_S for the approximate Nakagami distribution of Equation (12) may be deduced from results presented in Reference 9, to wit:

$$A_{s} = \left[\langle S_{0} \rangle^{2} + 2 \langle S_{0} \rangle \langle S_{s} \rangle \right]^{\frac{1}{2}} .$$
 (14)

These two expressions will be needed in the computation of the probability of detection for N pulses.

2. The p.d.f. for N Integrated Pulses

If successive pulses are statistically independent, then the density of the sum of N integrated pulses equals the convolution of the p.d.f.'s for the successive pulses. Thus, for the first two pulses we have, for $S = S_1 + S_2$,

$$f_{12}(S) = \int_{0}^{\infty} f_{1}(S-S_{2}) f_{2}(S_{2})dS_{2}, \qquad (15)$$

where f_1 and f_2 denote the p.d.f.'s for the first pulse and the second pulse, respectively. This process can be continued. Setting $S = (S_1 + S_2)$ + S₃, we have

$$f_{123}(S) = \int_{0}^{\infty} f_{12} (S-S_3) f_3(S_3) dS_3$$
, (16)

and so forth. Hence, the p.d.f. for N pulses can be computed as a multiple convolution, provided that the pulses are statistically independent. It is assumed herein that the return pulses are statistically independent. The cumulative probability distribution, c.p.d., for N integrated pulses is obtained by integrating the p.d.f. for N integrated pulses, The parameter S_t is the power threshold level. The next step is to set S_t to achieve a specified probability of false alarm $P_{f.a.}$ when the signal power is due to "noise" alone. The noise floor, denoted as $\langle S \rangle$ noise, must be chosen by the analyst. The noise floor may be set equal to the receiver average noise power $\langle S_r \rangle$. However, a noise floor equal to $\langle S_r \rangle + \langle S_c \rangle$ can be selected in order to reduce the problem of false target detections caused by the clutter return $\langle S_c \rangle$. The threshold S_t will have to be adjusted as a function of range in order to maintain a constant $P_{f.a.}$ versus range since $\langle S_c \rangle$ increases with increasing range.

Let X_t denote the value of X that yields the specified $P_{f,a}$. Then we have

$$S_{t} = \xi_{i} N \langle S_{l} \rangle_{noise} - \sqrt{N\xi_{i}} X_{t}, \qquad (24)$$

which enables use of Equations (23) and (22) for estimating probability of detection.

3. Statistical Averaging of N Pulses

The signal/noise ratio in the low pass filter in the video stage can be obtained by forming the ratio of the "DC" power component of the signal to the "AC" component (the average fluctuating part of the signal) in order to discriminate more effectively against clutter and glint at the expense of reduced signal/noise ratio. We note that the original pulse signals and ratioed signals. We thus write the signal U in terms of S and η as

$$v = \frac{s}{\eta}$$
 (25)

We note explicitly that S and η are the sum of N random variables,

$$S = \frac{1}{N} \sum_{n=1}^{N} S_n$$
, and (26)

$$\Pi = \frac{1}{N} \sum_{n=1}^{N} \left[(S_n - S)^2 \right]^{\frac{1}{2}}, \qquad (27)$$

$$f(U) = \frac{\sqrt{\pi}}{2\pi a_{s}a_{\eta}\sqrt{1-r^{2}}} \exp \left[-\frac{b}{a_{s}^{2}} < s^{2} - \frac{b}{a_{\eta}^{2}} < \eta^{2}\right]$$

$$\cdot \left[\frac{2d < \eta^{2}}{\sigma_{\eta}^{2}} + \frac{2d < s^{2}}{\sigma_{s}^{2}}\right] U$$

$$\left[\frac{b}{\sigma_{s}^{2}} - \frac{u^{2} - \left[\frac{2br}{a_{s}a_{\eta}}\right]}{\sigma_{\eta}^{2}} + \frac{u^{2} - \left[\frac{2br}{a_{s}a_{\eta}}\right]}\right] 3/2$$

$$\exp\left\{ \begin{bmatrix} \frac{2b < \eta >}{2} + \begin{bmatrix} \frac{2b < S >}{2} \\ a_{g} \end{bmatrix}^{U} \\ 4\begin{bmatrix} \frac{b}{2} & U^{2} - \frac{2 - br}{a_{g}} & U + \frac{b}{a_{g}^{2}} \\ a_{g} & s & n & a_{\eta} \end{bmatrix} \right\},$$
(30)

where $b = \frac{1}{2(1-r^2)}$

We can calculate the probability of detection, or more precisely we can calculate the probability P that U exceeds some specified value U_0 , as

$$P(U > U_0) = \int_{U_0} f(U) dU. \qquad (31)$$

The parameters appearing in f(U) are defined as follows, along with other quantities needed for their evaluation

$$\langle S \rangle = \frac{1}{N} \sum_{n=1}^{N} \langle S_n \rangle$$
, (32)

$$A_{g}^{2} = \frac{1}{N} \sum_{n=1}^{N} a_{gn}^{2} , \qquad (33)$$

SECTION III

ESTIMATED RCS AND POD FOR SELECTED TARGETS

A. Introduction

Estimates of the Radar Cross Section (RCS) and the Probability of Detection p.o.d. for three selected ground targets are presented and discussed in this section. The three targets are depicted in Figures 2, 3, and 4. Figure 2 shows an aircraft hangar, Figure 3 shows a periodic structure, and Figure 4 shows a "box on a frustrum". The RCS of each target is computed for selected angles of arrival of a threat missile based on plane wave incidence (i.e., far-field analysis) computed as a function of range based on the simplified statistical analysis for pulse integration discussed in Section II-C (Equations (13, (14), (22), (23), and (24)).

B. RCS Estimates

1. Hangar

The hangar and ground form a dihedral at the ends of the hangar, as indicated in Figure 2. The hangar and ground regions are comprised of the five scattering elements shown in the figure.

The field reflection coefficients for scattering elements 1, 3, and 4 are $\Gamma_1 = F_3 = \Gamma_4 = 0.32$, hence the power reflection coefficients are $\Gamma_1^2 = \Gamma_3^2 = \Gamma_4^2 = 0.1$. Scattering element 2 is the hangar door. It has field reflection coefficient $\Gamma_2 = 1.0$ for a metal door and $\Gamma_2 = 0.1$ for an absorber covered door. For region 5, we will use measured values of σ_0 or γ_0 to compute the RCS.

Case 1 - Missile coming in along radial defined by $\phi_0 = 180^{\circ}, \theta_0 = 45^{\circ}$.

The RCS of this composite dihedral is estimated as

$$\sigma = \frac{4\pi}{\lambda^2} \left\{ \left[\sqrt{2} \mathbf{A}_3 \mathbf{\Gamma}_3 \mathbf{\Gamma}_4 \right]^2 + \left[\sqrt{2} \mathbf{A}_2 \mathbf{\Gamma}_1 \mathbf{\Gamma}_2 \right]^2 \right\},$$
(40)

where

A₃ =
$$\pi$$
 (a₁² - a₂²),
A₂ = π a₂²,
 $\Gamma_1 = \Gamma_3 = \Gamma_4 = 0.32$, and



Figure 3. Sketch of the periodic structure target.

where Γ_2 is 1.0 for metal and 0.1 for RAM.

The radii a_1 and a_2 are assumed to be $a_1=4.5m$ and $a_2=3.5m$, respectively.

We obtain the intermediate expression,

$$\sigma = \frac{1590 \text{ m}^4}{\lambda^2} , \qquad (41)$$

for the dihedral having a metal hangar door, where λ is the radar wavelength in meters. Thus, σ_x and σ_{ka} for X-band (10 GHz) and Ka-band (35 GHz), respectively, are

$$\sigma_{\rm x} = 1.77 \text{ x } 10^6 \text{ m}^2, \text{ and}$$
 (42)
 $\sigma_{\rm ka} = 2.15 \text{ x } 10^7 \text{ m}^2,$

for the dihedral with a metal hangar door.

For the RAM-covered door, $\Gamma_2=0.1$, hence the RAM reduces the reflection coefficient of the door by 20 dB. We obtain the intermediate expression,

$$\sigma = \frac{6\ 6.12\ m^4}{\lambda^2} , \qquad (43)$$

which leads to the results

$$\sigma_{\rm x} = 0.73 \ {\rm x} \ 10^5 \ {\rm m}^2$$
, and
 $\sigma_{\rm ka} = 0.87 \ {\rm x} \ 10^6 \ {\rm m}^2$, (44)

for the dihedral with RAM-covered door.

<u>Case 2</u> - Missile coming in along radial defined by $\phi_0=90^\circ$, $\theta_0=45^\circ$

For this case, the hangar appears as a horizontally-oriented cylinder. We assume that the dirt layer is "rough" enough at X-band and Ka-band to allow the assumption of diffuse scattering. Hence, we estimate σ_x and σ_{ka} as Case 3 - Missile at $\theta_0 = 0^\circ$.

The projected area is 2 La1, hence we obtain

$$\sigma_{\rm x} = 77.8 \,{\rm m}^2$$
, and
 $\sigma_{\rm ka} = 243 \,{\rm m}^2$. (51)

2. Periodic Structure

We consider two cases with large RCS. These two cases occur for a missile coming in along the radials defined by ($\phi_0 = 180^\circ$, $\theta_0 = 45^\circ$) and by $\theta_0 = 0^\circ$.

<u>Case 1.</u> Missile coming in along ($\phi_0 = 180^\circ$, $\theta_0 = 45^\circ$) the major contributors to the RCS are the slanted faces of length ℓ and width W having RCS σ_1 . A power reflection coefficient of $\Gamma^2 = 0.1$ for perpendicular incidence is assumed.

The total coherent RCS for a periodic structure of this type with M (even) ridges in the radar footprint is computed as

$$\sigma = \mathbf{M}^2 \sigma_1 \Gamma^2 \quad . \tag{52}$$

for

 $2\left(\frac{\Delta \mathbf{R}}{\lambda}\right)$ = even integer or zero, and

 $\sigma = 0 \tag{53}$

for $2\Delta R/\lambda$ odd integer. If the power is scattered completely noncoherently due to surface roughness and/or "random" spacing of the ridges, the total RCS can be estimated as

$$\sigma = M^2 \sigma_1 \Gamma^2 \qquad (54)$$

for M even or odd.

The RCS of one face is given by the formula, assuming a rough surface,

 $\sigma_1 \simeq \sigma_0 (\ell \cdot w) \quad . \tag{55}$

3. Box on a Frustrum

<u>Case 1.</u> Missile coming in along $\theta^{\circ} = 45^{\circ}$, $\phi = 180^{\circ}$, 270° , 90° or 0° . The worst case situation for this target occurs for a missile coming in along the radials defined by $\theta_{0} = 45^{\circ}$, for the indicated ϕ directions. The total RCS is the sum of the frustrum plus corner reflector contributions. However, the corner reflector is the main contributor for this elevation angle. Thus, the "worst case" estimate for this target is computed from the formula for the RCS of a composite dihedral,

$$\sigma \simeq \frac{16\pi (\ell, h_1)^2}{\lambda^2} \Gamma_1^2 \Gamma_2^2, \qquad (60)$$

where $\Gamma_1^2 = 1.0$ for the metal box and $\Gamma_2^2 = 0.1$ for the top of the frustrum. Assuming that $\ell_1 = 3.05$ meters and $h_1 = 1.83$ meters, we obtain the RCS values

$$\sigma_{\mathbf{x}} \simeq 1.74 \text{ x } 10^5 \text{ m}^2, \text{ and}$$

 $\sigma_{\mathbf{k}a} \simeq 2.07 \text{ x } 10^6 \text{ m}^2.$
(61)

for 10 GHz and 35 GHz, respectively.

Case 2. Missile at $\theta = 0^{\circ}$.

the metal box is the dominant contributor for this case. The flat metallic surface of the top of the box produces a large specular return. Hence, the RCS is

$$\sigma \simeq \frac{4\pi \,\ell_1^2}{\lambda^2} , \qquad (62)$$

which gives, for $\ell_1 = 10$ meters, the results

$$\sigma_{\rm X} \simeq 1.39 \text{ x } 10^6 \text{ m}^2, \text{ and}$$

 $\sigma_{\rm ka} \simeq 1.66 \text{ x } 10^7 \text{ m}^2, \qquad (63)$

for 10 GHz and 35 GHz, respectively.

The target is assumed to be within the footprint of the radar antenna mainbeam and to have constant RCS values σ_8 and σ_d during the engagement. The clutter RCS is a function of range. The clutter RCS for a pulse radar with clutter cell limited by pulse length is

$$\sigma_{\mathbf{c}} \simeq \gamma_{\mathbf{0}} (\mathbf{R} \Delta \theta) (\Delta \phi) \mathbf{F}(\theta_{\mathbf{0}}) \cos(\theta_{\mathbf{0}}), \qquad (69)$$

where

 γ_0 = reflectivity of the clutter patch, θ_a = 3-dB beamwidth of antenna beam, τ = transmitted pulsewidth, and c = speed of light, and where

$$f(\theta_{o}) = \begin{cases} R \text{ for } R < c\tau/\sin(\theta_{o}) \\ \frac{c\tau}{\sin(\theta_{o})} \end{cases}$$

We assume an incoming mmW seeker operating at 35 GHz and having the following characteristics:

$$P_{t} = 2 \text{ watts}, \\ \tau = 0.10\mu \text{ seconds}, \\ P.R.F. = 50 \text{ KHz}, \\ D = .163 \text{ meters}, \\ \xi_{a} = 0.6, \\ \theta_{a}, a = 3.6^{\circ}, \\ T_{o} = 290 \text{ $^{\circ}\text{Kelvin}, $} \\ B = 200 \text{ MHz}, \text{ and} \\ NF = 10. \\ \end{cases}$$

The number of pulses integrated, N, and the probability of false alarm are variables. The number of pulses N returned to the receiver as the antenna scans through its beamwidth is

$$N_{b} = \frac{\theta_{a}}{\theta_{scan}} (P.R.F.) , \qquad (70)$$





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SECTION IV CONCLUDING REMARKS AND RECOMMENDATIONS

The results of the research work presented herein provide a means for making rapid assessments of the probability of detection (p.o.d.) for untreated and treated ground structures in specified clutter environments. A computer program is supplied via a separate transmittal to compute the p.o.d. versus range for specified radar, target, and background characteristics with the probability of false alarm, $P_{f.a.}$, as a parameter.

The simplified analysis presented in this report permits WES to obtain rapid assessments of vulnerability to threat missiles and to evaluate the effectiveness of radar camouflage materials.

Improvements in the absolute accuracy and realism of the present relatively simple model described herein can be achieved based on the more general analytical models discussed in Section II. It is recommended that work be initiated in the following areas as part of additional to develop the more sophisticated p.o.d. models needed by WES.

- Develop a comprehensive electromagnetic scattering model for multiple rough-surface target elements located in the near-field of each other and valid for near-field separations between seeker and targets. The model would employ the stochastic SAF nearfield scattering analysis for interacting target elements described in Section II.
- 2. Implement the Nakagami p.d.f. as a computer subroutine in order to obtain a more accurate stochastic description of the singlepulse statistics, and also implement the convolution subroutine to obtain the probability of detection for N integrated pulses that follow the Nakagami p.d.f. This should be done for several detection schemes including those described in Section II.
- 3. Modify the p.o.d. algorithm to compute p.o.d., probability of acquisition (p.o.a.), and angular tracking error versus range for specified initial conditions of the missile and specified missile flight dynamics, based on the results of Tasks 1 and 2.

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SECTION V

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