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Work type : RES
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Prime #: Contract entity: GTRC

Subprojects ? : N CFDA:
Main project #: PE #:

Project unit: MECH ENGR Unit code: 02.010.126
Project director(s):
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Sponsor/division names: NAVY / NAVAL RESEARCH LAB, ORLANDO
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Title: EXTENSION OF SHIP FOR NON-AXISYMMETRIC APPLICATIONS

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N/A
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* MOD #P00002 PROVIDES A NO-COST EXTENSION THROUGH JUNE 30, 1992.

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 06/29/92

Project No. E-25-M75 Center No. 10/24-6-R7147-0A0

Project Director FERRI A A School/Lab MECH ENGR

Sponsor NAVY/NAVAL RESEARCH LAB, ORLANDO

Contract/Grant No. N62190-91-M-0227 Contract Entity GTRC

Prime Contract No. ____

Title EXTENSION OF SHIP FOR NON-AXISYMMETRIC APPLICATIONS

Effective Completion Date 920630 (Performance) 920630 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	Y	
Final Report of Inventions and/or Subcontracts	N	
Government Property Inventory & Related Certificate	N	
Classified Material Certificate	N	
Release and Assignment	N	
Other ____	N	

Comments ____

Subproject Under Main Project No. ____

Continues Project No. ____

Distribution Required:

Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Managment	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other ____	N
____	N

SHIP* Users Manual

*Simplified-Helmholtz-Integral Program

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Contents

Introduction

The SHIP computer program was developed by Peter Rogers in 1972[1,2] and was one of the earliest boundary element codes for acoustic applications. Although the original SHIP code was limited to right circular cylinders and rings undergoing axisymmetric excitation, the SHIP program was useful because of its accuracy and computational efficiency. The general purpose boundary element computer code CHIEF[3,4] is similar to SHIP in that it is based on the surface Helmholtz integral equation. However, there are several important differences. The CHIEF computer code discretizes an object's wetted surface into a number of surface patches or elements that form a two-dimensional grid on the surface. The pressure and surface normal velocity are both assumed to be constant over each element, thus the length and width of the element should be small with respect to an acoustic wavelength. (Other boundary element codes make use of higher-order boundary elements and can be "larger" than the CHIEF elements.) In contrast, the SHIP formulation discretizes the wetted surface into circumferential *bands*. The surface pressure and the surface normal velocity are assumed to be constant *across each band but they can* vary *circumferentially*. However, since the original SHIP code was restricted to axisymmetric excitation, this capability was never used. Another difference between the CHIEF-type computer program and the SHIP computer program is that the CHIEF code can accommodate very generally shaped objects while, as mentioned above, the SHIP code is restricted to axisymmetric right circular cylinders and rings.

Currently, all boundary element computer codes for acoustic applications are burdened by increasing computational effort as the frequency or wavenumber is increased. The increased computational effort is driven by the need to use boundary elements that are small with respect to an acoustic wavelength. For a given frequency, the number of surface boundary elements in a CHIEF-type formulation grows with the square of the object's size. For a given object, the number of boundary elements grows as the square of increasing frequency. For the SHIP formulation, the number of bands grows *linearly* with object size and *linearly* with increasing frequency.

The major shortcoming of the original SHIP computer code is its restriction to right-circular cylinders and rings and its limitation to axisymmetric radiation and scattering problems. This manual describes the revised SHIP computer code that, although still limited to right-circular cylinders and rings, has been upgraded to accommodate non-axisymmetric excitation. The operation of the computer code is presented first followed by sample runs for radiation and scattering from rigid cylinders and rings. The general theory and the computer code are presented in appendices placed at the end of this manual.

User Inputs and Subroutine Calls

The input to the SHIP computer code includes input to specify the cylinder or ring exterior dimensions, input determining the boundary-element mesh, input specifying the type and frequency of the excitation, input which controls the output quantities, and input related to accuracy control. SHIP reads its input from device number 4 from a file that must be named "sh.inp". For both rings and cylinders, the object geometry and surface quantities are described using a cylindrical coordinate system (r,θ,z) with the z-axis being the axis of symmetry. The far-field pressures are described using a spherical coordinate system (R,θ,ϕ). (See Figures A.2 and C.1 for precise coordinate system definitions).

GENERAL INPUT PARAMETERS:

FKS	- starting value of wavenumber $k=2\pi/\lambda$
NF	- number of frequencies
DELFK	- wavenumber increment
RHOC	- density * soundspeed
NHARM	- highest harmonic included in the analysis (≤ 10)
ISYM	- 1, velocity and/or incident wave symmetric about z=0
	- -1, velocity and/or incident wave antisymmetric about z=0
	- 0, no symmetry
IRHEVEN	- 1, velocity and incident wave even with respect to $\theta=0$
	- 0, velocity and incident wave not even with respect to $\theta=0$

GEOMETRY AND MESH-RELATED INPUT:

H	- half-height of cylinder or ring
RIN	- inner radius of ring (=0 for a cylinder)
ROUT	- outer radius of ring or cylinder
JMAXH	- half the number of bands on the inner and outer sides (≤ 40)
IMAX	- number of bands on the top and bottom surfaces (≤ 20)

PARAMETERS RELATED TO ACCURACY:
The coefficient matrix elements in [DM] and [GM] are given by an integral from zero to infinity. (See Appendices A and B) These integrals are calculated in SHIP by breaking the integral up into small intervals. Over each interval, the integration is performed using gaussian quadrature of order NQD1. The contribution from each interval is summed until the contribution from the last interval is less than EPS times the value of the integral up to that point. The maximum number of intervals is specified by the user as KMAX. Some of the integrands in the infinite integrals are themselves described by integrals that can not be evaluated in closed form. These integrals are calculated in SHIP using a Gaussian quadrature procedure of order NQD2.

NQD1	- Gaussian quadrature order for integral from 0 to infinity
NQD2	- Gaussian quadrature order for inner integral

KMAX - maximum number of intervals into which the infinite integral is broken up $KMAX \leq 8000/NQD1$

EPS - convergence parameter for infinite integrals

PARAMETERS FOR OUTPUT OF FAR-FIELD PRESSURES:

NPTS - number of far-field directions (≤ 100)

THETA - vector of θ values for far-field directions (deg)

PHI - vector of ϕ values for far-field directions (deg)

PARAMETERS FOR DEFINITION OF INCIDENT WAVE:

AINC - amplitude of incident plane wave (M/m^2)

PSI - angle that the incident wave direction makes with the z-axis (deg)

PARAMETERS FOR OUTPUT OF SURFACE PRESSURES AND VELOCITIES:

THS - starting value of θ for printing of surface pressures and velocities (deg)

NTH - number of θ points at which surface pressures and velocities are evaluated

DTH - θ increment for printing of surface pressures and velocities (deg)

PARAMETERS FOR SELECTION OF RIGID AND PRESSURE-RELEASE BANDS:

MXD - 1, the cylinder or ring has some pressure-release bands

JMXT - vector containing 1's and 0's; 0 indicates a pressure-release impedance condition and 1 indicates a rigid surface impedance condition

PRINT OUTPUT CONTROL PARAMETERS:

The SHIP program prints results to device 15 (fort.15). Optional information can be obtained by setting print parameters IPRO, IPRP, IPRM, IPRD to one (1). Optional output is printed to device 3 (fort.3).

IPRO - 1, print comments from original SHIP code (0 for no print)

IPRM - 1, print coefficient matrices [DM] and [GM] (0 for no print)

IPRP - 1, print check of incident pressures (0 for no print)

IPRD - 1, print diagnostic information for convergence of far-field pressures and convergence of matrix elements (0 for no print)

SURFACE VELOCITY INPUT:

Currently, the SHIP program can except surface normal velocity input in two ways. If IRIGID=1, the object is assumed to be rigid and stationary; i.e., all surface normal velocities are prescribed to be zero. The only exception occurs when MXD=1, in which case only the non-pressure release bands have zero surface normal velocity. IF IRIGID=0, the user must specify the velocity for each harmonic of each band beginning on the line directly below IRIGID. If |ISYM|=1, velocities for only the first LCMAXH bands are required; if IRHEVEN=1, only the cosine harmonics are required.

IRIGID	- =1, all surface normal velocities are assumed to be zero
	- =0, specify each velocity harmonic for each band
VEL(I,J)	- surface normal velocity for Ith band; $1 \leq J \leq (NHARM+1)$- cosine harmonics, $NHARM+1 < J \leq (2*NHARM+1)$- sine harmonics VEL(I,J) is read only if IRIGID=0

CALCULATED VARIABLES:

The following variables are calculated by SHIP and used extensively throughout the program.

LCMAX	- total number of bands; ($\leq$200)
LCMAXH	- half the total number of bands
PR	- IMAXx1 vector of radial midpoints for top and bottom bands
PZ	- JMAXx1 vector of z-coordinates for each band on sides
PLZ	- vector of z-coordinates of lower edge of bands on sides
DELR	- radial thickness of bands on top and bottom = (ROUT-RIN)/IMAX
DELZ	- height of bands on side = 2*H/JMAX
NRUN	- total number of sine and cosine harmonics = 2*NHARM+1
NBESL	- starting point for downward recursion of Bessel functions

Input File Format

The input in the file "sh.inp" must have the following order:

```
RIN
ROUT
H
FKS, DELFK, NF
RHOC
ISYM
IRHEVEN
JMAXH
IMAX
AINC
PSI
NHARM
NTH
THS
DTH
EPS
KMAX
NQD1
NQD2
NPTS
     THETA(1), PHI(1)                  |
     THETA(2), PHI(2)                  |
     ......                            |    Omit if NPTS=0
                                       |
     THETA(NPTS), PHI(NPTS)            |
IRIGID
     VEL(1,1)                          |
     VEL(1,2)                          |
     VEL(1,3)                          |
     VEL(1,4)                          |    Omit if IRIGID=1
     .......                           |
                                       |
     VEL(LCMAX,2*NHARM+1)              |
MXD
     JMXT(1)                           |
     JMXT(2)                           |    Omit if MXD=0
     ......                            |
     JMXT(LCMAXH)                      |
IPRO,IPRM,IPRP,IPRD
```

Description of Subroutines

SHIP92

This is the main calling program for the SHIP computer code. SHIP92 reads all input data, converts all angles from degrees to radians, calculates the total number of bands, LCMAX, based on IMAX, JMAX and whether the object is a ring or a cylinder. The program then calls the subroutine that calculates surface pressures, SOLRIN, and the subroutine that calculates far-field pressures, FARFLD, at each wavenumber of interest.

SOLRIN

This subroutine is the main program for calculating surface pressures. The subroutine handles symmetry, and prints results. SOLRIN calls a variety of other subroutines as summarized below:

MESH	generates the radial and axial coordinates for each band
PLWAVE	calculates the cosine harmonics for the incident wave if AINC>0
GAUSS	generates Gauss points and weights
INITIAL	initializes the Bessel-function matrices and vector FKFAST
ERING	assembles matrices [DM] and [GM]
ODS	decomposes coefficient matrix [DM]
ZRAD	calculates radiation impedance if IRIGID=0

CTBM

Generates the unique elements of the sub-matrices [$G^{tt}{}_{m}$], [$G^{tb}{}_{m}$], [$M^{tb}{}_{m}$]
Whenever possible, only the independent elements are calculated.

CSSM

Generates the unique elements of the sub-matrices [$G^{ss}{}_{m}$], [$G^{si}{}_{m}$], [$G^{is}{}_{m}$], [$G^{ii}{}_{m}$] [$M^{ss}{}_{m}$], [$M^{si}{}_{m}$], [$M^{is}{}_{m}$], [$M^{ii}{}_{m}$].

CSTM

Generates the unique elements of the sub-matrices [$G^{st}{}_{m}$], [$G^{it}{}_{m}$], [$M^{st}{}_{m}$], [$M^{it}{}_{m}$].

CTSM

Generates the unique elements of the sub-matrices [$G^{ts}{}_{m}$], [$G^{ti}{}_{m}$], [$M^{ts}{}_{m}$], [$M^{ti}{}_{m}$].

BESJ_N

Calculates the Bessel function $J_n(x)$ using downward recursion and double-precision arithematic. When x is sufficiently large, asymptotic forms of $J_n(x)$ are used from function DASYMPJ.

FARFLD

This is the main subroutine for calculating the far-field pressure in any direction.

Subroutine Hierarchy

Given here, is a list of how the different subroutines participate in the SHIP computer program. Subroutines listed to the right and below the name of a particular subroutine or function called by it.

- SHIP92
  - SOLRIN
    - MESH
    - PLWAVE
    - GAUSS
      - ROOT
        - RECUR
    - INITIAL
    - ERING
      - CTBM
        - BESJ_N
          - DASYMPJ
      - CSSM
        - BESJ_N
          - DASYMPJ
      - CSTM
        - BESJ_N
          - DASYMPJ
      - CTSM
        - BESJ_N
          - DASYMPJ
    - ODS
    - ZRAD
  - FARFLD
    - BESJ_N
      - DASYMPJ

SAMPLE RUN 1
Scattering From a Rigid Cylinder

The problem considered here is scattering of a normally-incident plane wave by a rigid, right-circular ring. Plane wave scattering is an excellent benchmark problem for SHIP because it requires the use of several harmonics. The frequency of the incident wave corresponds to a wavenumber ka=2. The mesh for this problem consists of 20 bands along the outside, and 8 bands on the top and bottom (IMAX=8, JMAXH=10, LCMAX=36) The outer radius is 1 meter and the inner is 0.5 meter. The total ring height is 2 meters.

Input file:

```
0.0            ! RIN, inner radius
1.0            ! ROUT, outer radius
1.0            ! H, half-height of cylinder
2.0,1.0,1      ! FKS, DELFK, NF
1.5E6          ! RHOC, density*(sound speed)
1              ! ISYM, -1 for antisymmetry, 1 for symmetry, 0 for neither
1              ! IRHEVEN, +1=even forcing with respect to theta, 0=odd
10             ! JMAXH, half the number of bands along height
8              ! IMAX, number of bands on top and bottom
1.0            ! AINC, amplitude of incident wave
90.0           ! PSI, angle of plane wave direction with z-axis in degrees
4              ! NHARM, largest harmonic number included in analysis
37             ! NTH, number of angles for print of surface pressures and velocities
0.0            ! THS, start angle for print of surface quantities (deg)
5              ! DTH, increment angle for print of surface quantities (deg)
0.001          ! EPS, convergence parameter for evaluation of coefficient matrices
300            ! KMAX, maximum number of intervals for integration
16             ! NQD1, number of Gauss points
8              ! NQD2, number of Gauss points
19             ! NPTS, number of far-field points
180.0,90.0     ! THETA(1), PHI(1) (deg)
170.0,90.0     ! THETA(2), PHI(2) (deg)
160.0,90.0          .
150.0,90.0          .
140.0,90.0          .
130.0,90.0          .
120.0,90.0
110.0,90.0
100.0,90.0
90.0,90.0
80.0,90.0
70.0,90.0
60.0,90.0
50.0,90.0
40.0,90.0
30.0,90.0
20.0,90.0
10.0,90.0
00.0,90.0      ! THETA(19), PHI(19) (deg)
1              ! IRIGID
0              ! MXD, =1 if any bands have pressure release impedance condition
0,0,0,1        ! IPRO,IPRM,IPRP,IPRD print options
```

Excerpts from output file:

```
CYLINDER TRANSDUCER

WAVENUMBER, K=      2.000000
KA=                 2.000000
FLUID IMPEDANCE, RHOC=    1500000.

OUTER RADIUS=     1.000000     M
INNER RADIUS=   0.0000000E+00 M
CYLINDER HALF-HEIGHT=    1.000000     M
BANDS ON TOP AND BOTTOM, IMAX=           8
BANDS ON SIDE SURFACES, JMAX=           20
TOTAL NUMBER OF BANDS, LCMAX=           36

INCIDENT PRESSURE AMPLITUDE=    1.000000     PA
PLANE WAVE ANGLE WITH Z-AXIS=    90.00000     DEG

SYMMETRY CONDITION, ISYM=              1
RIGHT-HAND SIDE EVENESS, IRHEVEN=             1
NUMBER OF HARMONICS, NHARM=            4

CONVERGENCE PARAMETER, EPS=   1.0000000E-03
MAXIMUM NUMBER OF INTERVALS, KMAX=          300
NUMBER OF GAUSS POINTS, NQD1=           16
NUMBER OF GAUSS POINTS, NQD2=            8
HIGHEST HARMONIC, NBESL=          10
PRESSURE-RELEASE PARAMETER, MXD=            0
```

*** PRESSURES AT SELECTED LOCATIONS ***

BAND NUMBER 1

THETA (DEG)	REAL	IMAGINARY	MAGNITUDE	PHASE (DEG)
0.000	1.2204	-0.29849	1.2564	-13.744
5.000	1.2205	-0.29803	1.2563	-13.723
10.000	1.2206	-0.29664	1.2561	-13.660
15.000	1.2208	-0.29433	1.2558	-13.555
20.000	1.2210	-0.29112	1.2553	-13.410
25.000	1.2214	-0.28702	1.2546	-13.225
30.000	1.2217	-0.28205	1.2538	-13.000
35.000	1.2221	-0.27624	1.2529	-12.737
40.000	1.2224	-0.26963	1.2518	-12.438
45.000	1.2228	-0.26225	1.2506	-12.105
50.000	1.2231	-0.25416	1.2492	-11.739
55.000	1.2234	-0.24540	1.2477	-11.343
60.000	1.2236	-0.23604	1.2461	-10.919
65.000	1.2237	-0.22616	1.2444	-10.471
70.000	1.2238	-0.21581	1.2427	-10.001
75.000	1.2238	-0.20509	1.2409	-9.5137
80.000	1.2237	-0.19408	1.2390	-9.0121
85.000	1.2236	-0.18287	1.2372	-8.5004
90.000	1.2233	-0.17155	1.2353	-7.9828
95.000	1.2230	-0.16022	1.2335	-7.4635
100.000	1.2227	-0.14897	1.2317	-6.9469
105.000	1.2222	-0.13790	1.2300	-6.4373
110.000	1.2217	-0.12709	1.2283	-5.9391
115.000	1.2212	-0.11664	1.2267	-5.4563
120.000	1.2205	-0.10663	1.2252	-4.9930

125.000	1.2199	-0.97139E-01	1.2237	-4.5529
130.000	1.2192	-0.88236E-01	1.2223	-4.1396
135.000	1.2184	-0.79991E-01	1.2211	-3.7561
140.000	1.2177	-0.72462E-01	1.2199	-3.4055
145.000	1.2170	-0.65701E-01	1.2188	-3.0902
150.000	1.2163	-0.59755E-01	1.2178	-2.8125
155.000	1.2157	-0.54661E-01	1.2169	-2.5744
160.000	1.2152	-0.50451E-01	1.2162	-2.3774
165.000	1.2147	-0.47151E-01	1.2156	-2.2229
170.000	1.2144	-0.44780E-01	1.2152	-2.1118
175.000	1.2142	-0.43352E-01	1.2150	-2.0448
180.000	1.2141	-0.42875E-01	1.2149	-2.0225

BAND NUMBER 2

0.000	1.1590	-0.52940	1.2741	-24.550
5.000	1.1595	-0.52816	1.2741	-24.490
10.000	1.1610	-0.52445	1.2739	-24.310
15.000	1.1634	-0.51827	1.2737	-24.011
20.000	1.1668	-0.50963	1.2732	-23.594
25.000	1.1709	-0.49855	1.2727	-23.063
30.000	1.1757	-0.48504	1.2719	-22.418
35.000	1.1810	-0.46915	1.2708	-21.665
40.000	1.1867	-0.45094	1.2695	-20.807
45.000	1.1925	-0.43046	1.2678	-19.849
50.000	1.1983	-0.40783	1.2658	-18.796
55.000	1.2038	-0.38315	1.2633	-17.655
60.000	1.2090	-0.35658	1.2605	-16.432
65.000	1.2136	-0.32828	1.2572	-15.136
70.000	1.2175	-0.29847	1.2535	-13.774
75.000	1.2205	-0.26736	1.2494	-12.356
80.000	1.2225	-0.23520	1.2449	-10.891
85.000	1.2234	-0.20228	1.2400	-9.3884
90.000	1.2232	-0.16887	1.2348	-7.8601
95.000	1.2219	-0.13527	1.2294	-6.3171
100.000	1.2195	-0.10179	1.2237	-4.7712
105.000	1.2160	-0.68721E-01	1.2179	-3.2346
110.000	1.2115	-0.36371E-01	1.2121	-1.7196
115.000	1.2062	-0.50274E-02	1.2062	-0.23880
120.000	1.2002	0.25038E-01	1.2005	1.1951
125.000	1.1937	0.53571E-01	1.1949	2.5695
130.000	1.1869	0.80336E-01	1.1896	3.8723
135.000	1.1799	0.10512	1.1845	5.0916
140.000	1.1729	0.12775	1.1798	6.2161
145.000	1.1662	0.14805	1.1756	7.2351
150.000	1.1600	0.16590	1.1718	8.1392
155.000	1.1544	0.18117	1.1685	8.9196
160.000	1.1496	0.19379	1.1658	9.5687
165.000	1.1457	0.20366	1.1636	10.080
170.000	1.1428	0.21076	1.1621	10:449
175.000	1.1410	0.21503	1.1611	10.672
180.000	1.1404	0.21645	1.1608	10.747

....

BAND NUMBER 8

0.000	-0.16235	-0.66129	0.68093	-103.79
5.000	-0.15615	-0.66820	0.68620	-103.15
10.000	-0.13740	-0.68828	0.70186	-101.29
15.000	-0.10566	-0.71961	0.72733	-98.353
20.000	-0.60331E-01	-0.75913	0.76152	-94.544

25.000	-0.89069E-03	-0.80286	0.80286	-90.064
30.000	0.72892E-01	-0.84614	0.84928	-85.076
35.000	0.16070	-0.88399	0.89848	-79.697
40.000	0.26144	-0.91139	0.94815	-73.994
45.000	0.37312	-0.92368	0.99620	-68.004
50.000	0.49273	-0.91683	1.0408	-61.745
55.000	0.61632	-0.88774	1.0807	-55.229
60.000	0.73918	-0.83446	1.1148	-48.465
65.000	0.85604	-0.75633	1.1423	-41.461
70.000	0.96142	-0.65403	1.1628	-34.226
75.000	1.0501	-0.52956	1.1760	-26.762
80.000	1.1173	-0.38613	1.1821	-19.065
85.000	1.1593	-0.22796	1.1815	-11.124
90.000	1.1737	-0.60045E-01	1.1753	-2.9286
95.000	1.1595	0.11217	1.1649	5.5256
100.000	1.1171	0.28309	1.1524	14.221
105.000	1.0485	0.44734	1.1400	23.105
110.000	0.95721	0.60005	1.1297	32.082
115.000	0.84751	0.73712	1.1232	41.015
120.000	0.72453	0.85548	1.1211	49.738
125.000	0.59372	0.95321	1.1230	58.083
130.000	0.46047	1.0296	1.1279	65.904
135.000	0.32979	1.0852	1.1342	73.095
140.000	0.20600	1.1215	1.1402	79.592
145.000	0.92594E-01	1.1412	1.1449	85.361
150.000	-0.78561E-02	1.1474	1.1474	90.392
155.000	-0.93667E-01	1.1439	1.1477	94.681
160.000	-0.16393	1.1345	1.1463	98.222
165.000	-0.21832	1.1228	1.1438	101.00
170.000	-0.25691	1.1119	1.1412	103.01
175.000	-0.27991	1.1044	1.1393	104.22
180.000	-0.28755	1.1017	1.1386	104.63

BAND NUMBER 9

0.000	-0.62668	-0.22035	0.66429	-160.63
5.000	-0.62014	-0.23448	0.66298	-159.29
10.000	-0.60002	-0.27579	0.66037	-155.32
15.000	-0.56505	-0.34107	0.66001	-148.88
20.000	-0.51334	-0.42521	0.66658	-140.36
25.000	-0.44288	-0.52155	0.68422	-130.34
30.000	-0.35194	-0.62229	0.71492	-119.49
35.000	-0.23966	-0.71904	0.75793	-108.43
40.000	-0.10647	-0.80333	0.81036	-97.550
45.000	0.45588E-01	-0.86723	0.86842	-86.991
50.000	0.21265	-0.90380	0.92848	-76.760
55.000	0.38906	-0.90762	0.98749	-66.797
60.000	0.56763	-0.87510	1.0431	-57.031
65.000	0.74006	-0.80472	1.0933	-47.397
70.000	0.89751	-0.69718	1.1365	-37.840
75.000	1.0313	-0.55527	1.1713	-28.298
80.000	1.1337	-0.38378	1.1969	-18.702
85.000	1.1983	-0.18914	1.2132	-8.9693
90.000	1.2211	0.20976E-01	1.2213	0.98413
95.000	1.2002	0.23814	1.2236	11.223
100.000	1.1364	0.45370	1.2237	21.763
105.000	1.0333	0.65934	1.2257	32.542
110.000	0.89619	0.84754	1.2335	43.402
115.000	0.73251	1.0121	1.2494	54.104
120.000	0.55058	1.1484	1.2736	64.385
125.000	0.35921	1.2537	1.3042	74.012
130.000	0.16689	1.3274	1.3378	82.834

135.000	-0.18697E-01	1.3705	1.3707	90.782
140.000	-0.19123	1.3862	1.3993	97.855
145.000	-0.34598	1.3789	1.4217	104.08
150.000	-0.47988	1.3543	1.4368	109.51
155.000	-0.59148	1.3186	1.4452	114.16
160.000	-0.68058	1.2784	1.4482	118.03
165.000	-0.74793	1.2396	1.4478	121.10
170.000	-0.79473	1.2078	1.4458	123.34
175.000	-0.82220	1.1870	1.4439	124.71
180.000	-0.83125	1.1797	1.4432	125.17

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BAND NUMBER 18

0.000	-0.98997	0.59795	1.1565	148.87
5.000	-0.98214	0.57722	1.1392	149.56
10.000	-0.95808	0.51648	1.0884	151.67
15.000	-0.91618	0.41995	1.0078	155.37
20.000	-0.85411	0.29437	0.90342	160.98
25.000	-0.76933	0.14853	0.78354	169.07
30.000	-0.65969	-0.72414E-02	0.65973	-179.37
35.000	-0.52407	-0.16178	0.54847	-162.84
40.000	-0.36290	-0.30381	0.47328	-140.07
45.000	-0.17866	-0.42262	0.45883	-112.92
50.000	0.23965E-01	-0.50880	0.50937	-87.303
55.000	0.23810	-0.55481	0.60374	-66.773
60.000	0.45494	-0.55542	0.71796	-50.680
65.000	0.66431	-0.50811	0.83635	-37.411
70.000	0.85536	-0.41312	0.94990	-25.779
75.000	1.0174	-0.27346	1.0535	-15.044
80.000	1.1408	-0.94700E-01	1.1448	-4.7452
85.000	1.2179	0.11544	1.2233	5.4148
90.000	1.2432	0.34758	1.2909	15.620
95.000	1.2147	0.59130	1.3510	25.956
100.000	1.1333	0.83585	1.4082	36.411
105.000	1.0029	1.0708	1.4671	46.875
110.000	0.83031	1.2866	1.5312	57.163
115.000	0.62444	1.4753	1.6020	67.059
120.000	0.39556	1.6311	1.6784	76.368
125.000	0.15448	1.7503	1.7571	84.956
130.000	-0.88258E-01	1.8319	1.8340	92.758
135.000	-0.32312	1.8770	1.9046	99.768
140.000	-0.54215	1.8892	1.9655	106.01
145.000	-0.73933	1.8741	2.0147	111.53
150.000	-0.91068	1.8384	2.0516	116.35
155.000	-1.0541	1.7899	2.0772	120.50
160.000	-1.1692	1.7365	2.0935	123.95
165.000	-1.2566	1.6859	2.1027	126.70
170.000	-1.3176	1.6445	2.1072	128.70
175.000	-1.3535	1.6175	2.1091	129.92
180.000	-1.3653	1.6082	2.1096	130.33

STATIONARY RIGID SURFACE- ALL VELOCITIES ZERO

FAR-FIELD PRESSURES

THETA (DEG) PHI (DEG) REAL IMAGINARY

180.000	90.000	0.53831	-0.74776
170.000	90.000	0.52414	-0.74084
160.000	90.000	0.48120	-0.72071
150.000	90.000	0.40846	-0.68923
140.000	90.000	0.30529	-0.64942
130.000	90.000	0.17306	-0.60536
120.000	90.000	0.17127E-01	-0.56187
110.000	90.000	-0.15132	-0.52414
100.000	90.000	-0.31451	-0.49716
90.000	90.000	-0.44927	-0.48504
80.000	90.000	-0.53056	-0.49036
70.000	90.000	-0.53711	-0.51361
60.000	90.000	-0.45786	-0.55289
50.000	90.000	-0.29722	-0.60395
40.000	90.000	-0.77142E-01	-0.66060
30.000	90.000	0.16523	-0.71553
20.000	90.000	0.38456	-0.76133
10.000	90.000	0.53729	-0.79168
0.000	90.000	0.59201	-0.80230

THETA (DEG)	PHI (DEG)	MAGNITUDE	PHASE (DEG)
180.000	90.000	0.92137	-54.250
170.000	90.000	0.90751	-54.721
160.000	90.000	0.86659	-56.270
150.000	90.000	0.80117	-59.347
140.000	90.000	0.71760	-64.822
130.000	90.000	0.62961	-74.046
120.000	90.000	0.56213	-88.254
110.000	90.000	0.54555	-106.10
100.000	90.000	0.58829	-122.32
90.000	90.000	0.66114	-132.81
80.000	90.000	0.72246	-137.26
70.000	90.000	0.74316	-136.28
60.000	90.000	0.71787	-129.63
50.000	90.000	0.67313	-116.20
40.000	90.000	0.66509	-96.661
30.000	90.000	0.73436	-76.997
20.000	90.000	0.85294	-63.201
10.000	90.000	0.95678	-55.837
0.000	90.000	0.99707	-53.577

Comparison with CHIEF computer program:

The SHIP results from this sample run as well as other SHIP runs using fewer harmonics were examined for convergence and were compared extensively to results from the CHIEF computer code. The CHIEF surface mesh places 36 subdivisions around each SHIP band. Figure 1 shows the convergence of surface pressures in band 18 (the band nearest to the midplane) with increasing NHARM=NH. It is seen that the surface pressures converge to the CHIEF result for surface pressure with approximately 4 circumferential harmonics. In general it was found for both cylinders and rings that approximately ka+2 harmonics were required to produce an accurate representation of surface pressures for the case of normal-incidence plane wave scattering.

Figure 2 shows the convergence of the far-field pressures in the z=0 plane with increasing NH=NHARM. It is seen that approximately 3 harmonics are required for an accurate prediction of far-field scattered pressure. In general it was found for both cylinders and rings that approximately ka+1 harmonics were required for accurate far-field pressures for the case of normally-incident plane-wave scattering. The convergence of both the real and imaginary parts of the far-field pressure in the backscatter direction is demonstrated in Figure 3. Again, note that the solution is well converged at NHARM=3. Figure 4 shows the real and imaginary parts of the far-field pressures for SHIP with NHARM=4 and CHIEF. The agreement is excellent.

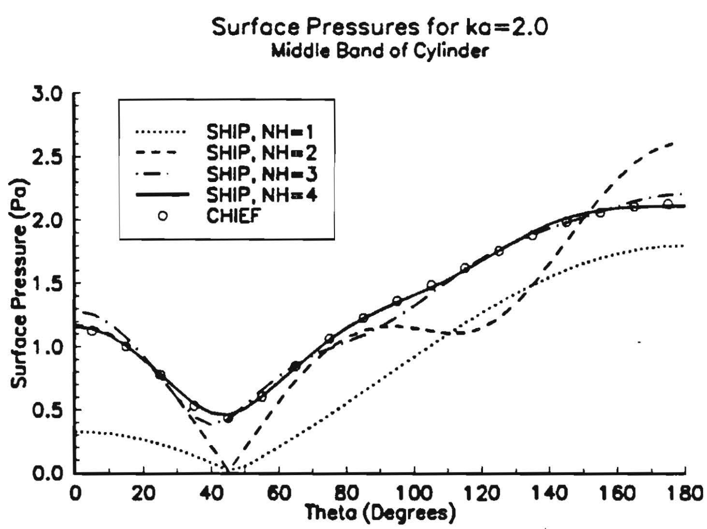

Figure 1

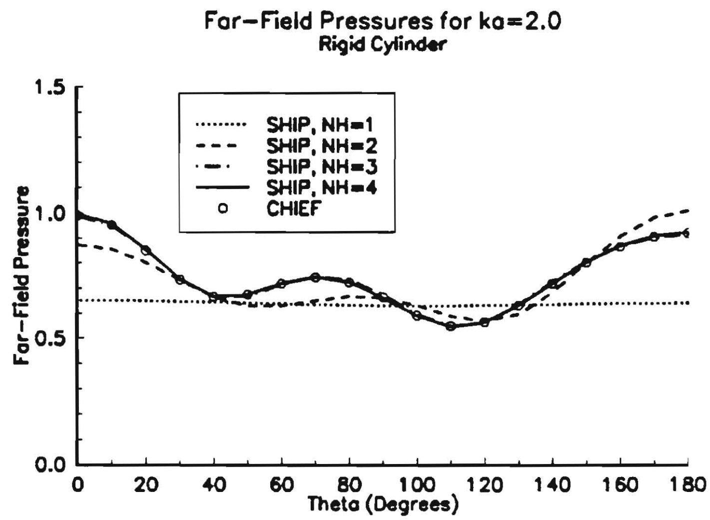

Figure 2

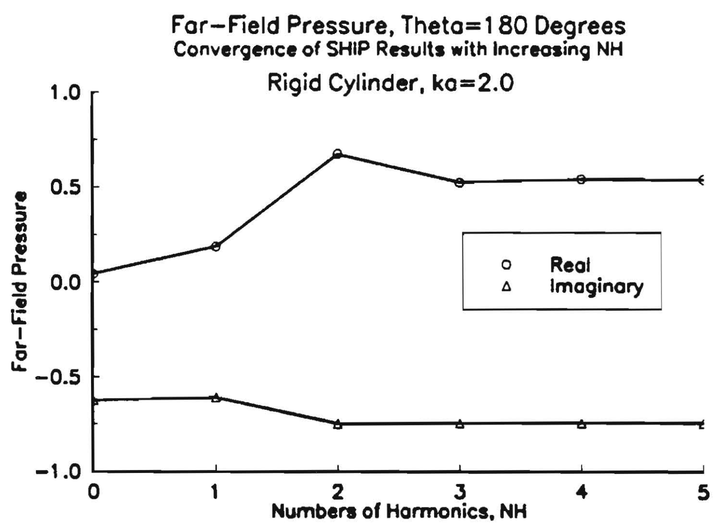

Figure 3

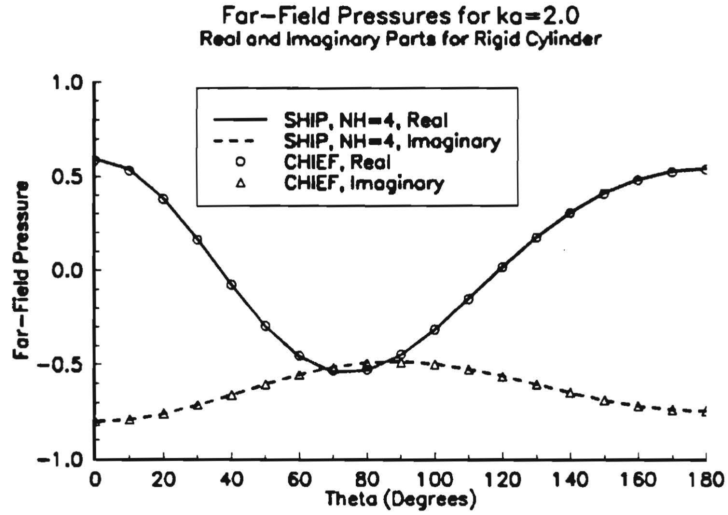
Far–Field Pressures for ka=2.0
Real and Imaginary Parts for Rigid Cylinder
SHIP, NH=4, Real
SHIP, NH=4, Imaginary
CHIEF, Real
CHIEF, Imaginary
1.0
0.5
0.0
−0.5
−1.0
Far–Field Pressure
0
20
40
60
80
100
120
140
160
180
Theta (Degrees)

Figure 4

SAMPLE RUN 2
Radiation From a Cylinder

Here, we consider the problem of a rigid, right-circular cylinder that is oscillating in a direction transverse to its axis of symmetry. The frequency of oscillation corresponds to a wavenumber ka=2. The mesh consists of 20 bands along the outside cylindrical surface and 8 bands on the top and bottom.

Input file:

```
0.0           ! RIN, inner radius
1.0           ! ROUT, outer radius
1.0           ! H, half-height of cylinder
2.0,1.0,1     ! FKS, DELFK, NF
1.5E6         ! RHOC, density*(sound speed)
1             ! ISYM, -1 for antisymmetry, 1 for symmetry, 0 for neither
1             ! IRHEVEN, +1=even forcing with respect to theta, 0=odd
10            ! JMAXH, half the number of bands along height
8             ! IMAX, number of bands on top and bottom
0.0           ! AINC, amplitude of incident wave
90.0          ! PSI, angle of plane wave direction with z-axis in degrees
1             ! NHARM, largest harmonic number included in analysis
37            ! NTH, number of angles for print of surface pressures and velocities
0.0           ! THS, start angle for print of surface quantities (deg)
5             ! DTH, increment angle for print of surface quantities (deg)
0.001         ! EPS, convergence parameter for evaluation of coefficient matrices
300           ! KMAX, maximum number of intervals for integration
16            ! NQD1, number of Gauss points
8             ! NQD2, number of Gauss points
19            ! NPTS, number of far-field points
180.0,90.0    ! THETA(1), PHI(1) (deg)
170.0,90.0    ! THETA(2), PHI(2) (deg)
160.0,90.0          .
150.0,90.0          .
140.0,90.0          .
130.0,90.0          .
120.0,90.0
110.0,90.0
100.0,90.0
90.0,90.0
80.0,90.0
70.0,90.0
60.0,90.0
50.0,90.0
40.0,90.0
30.0,90.0
20.0,90.0
10.0,90.0
00.0,90.0     ! THETA(19), PHI(19) (deg)
0             ! IRIGID
0.0,0.0       ! VEL(1,1),   band 1,  harmonic 0  (first band on top surface)
0.0,0.0       ! VEL(1,2),   band 1,  harmonic 1  (Note real and imaginary parts)
0.0,0.0       ! VEL(2,1),   band 2,  harmonic 0
0.0,0.0       ! VEL(2,2),   band 2,  harmonic 1
0.0,0.0       ! VEL(3,1)              .
0.0,0.0       ! VEL(3,2)              .
0.0,0.0       ! VEL(4,1)              .
0.0,0.0       ! VEL(4,2)              .
0.0,0.0       ! VEL(5,1)
```

```
0.0,0.0       ! VEL(5,2)
0.0,0.0       ! VEL(6,1)
0.0,0.0       ! VEL(6,2)
0.0,0.0       ! VEL(7,1)
0.0,0.0       ! VEL(7,2)
0.0,0.0       ! VEL(8,1),   band 8,  harmonic 0 (last band on top surface)
0.0,0.0       ! VEL(8,2),   band 8,  harmonic 1
0.0,0.0       ! VEL(9,1),   band 9,  harmonic 0 (first band on side surface)
1.0E-06,0.0 ! VEL(9,2),   band 9,  harmonic 1
0.0,0.0       ! VEL(10,1),  band 10, harmonic 0
1.0E-06,0.0 ! VEL(10,2),  band 10, harmonic 1
0.0,0.0       ! VEL(11,1)            .
1.0E-06,0.0 ! VEL(11,2)            .
0.0,0.0       ! VEL(12,1)            .
1.0E-06,0.0 ! VEL(12,2)            .
0.0,0.0       ! VEL(13,1)
1.0E-06,0.0 ! VEL(13,2)
0.0,0.0       ! VEL(14,1)
1.0E-06,0.0 ! VEL(14,2)
0.0,0.0       ! VEL(15,1)
1.0E-06,0.0 ! VEL(15,2)
0.0,0.0       ! VEL(16,1)
1.0E-06,0.0 ! VEL(16,2)
0.0,0.0       ! VEL(17,1)
1.0E-06,0.0 ! VEL(17,2)
0.0,0.0       ! VEL(18,1),  band 18, harmonic 0 (last band on side surface)
1.0E-06,0.0 ! VEL(18,2),  band 18, harmonic 1
0             ! MXD, =1 if any bands have pressure release impedance condition
0,0,0,1       ! IPRO,IPRM,IPRP,IPRD print options
```

Excerpts from output file:

```
CYLINDER TRANSDUCER

WAVENUMBER, K=     2.000000
KA=                2.000000
FLUID IMPEDANCE, RHOC=    1500000.

OUTER RADIUS=     1.000000     M
INNER RADIUS=   0.0000000E+00 M
CYLINDER HALF-HEIGHT=     1.000000     M
BANDS ON TOP AND BOTTOM, IMAX=             8
BANDS ON SIDE SURFACES, JMAX=             20
TOTAL NUMBER OF BANDS, LCMAX=             36

INCIDENT PRESSURE AMPLITUDE=   0.0000000E+00 PA
PLANE WAVE ANGLE WITH Z-AXIS=    90.00000     DEG

SYMMETRY CONDITION, ISYM=             1
RIGHT-HAND SIDE EVENESS, IRHEVEN=             1
NUMBER OF HARMONICS, NHARM=             1

CONVERGENCE PARAMETER, EPS=   1.0000000E-03
MAXIMUM NUMBER OF INTERVALS, KMAX=           300
NUMBER OF GAUSS POINTS, NQD1=             16
NUMBER OF GAUSS POINTS, NQD2=              8
HIGHEST HARMONIC, NBESL=             7
PRESSURE-RELEASE PARAMETER, MXD=             0

COMPLEX RADIATION IMPEDANCE, RADIMP=  (6.1208243E-13,12.00000)
NORMALIZED BY RHOC*(TOTAL SURFACE AREA)
```

*** PRESSURES AT SELECTED LOCATIONS ***

BAND NUMBER 1

THETA (DEG)	REAL	IMAGINARY	MAGNITUDE	PHASE (DEG)
0.000	0.35020E-01	-0.33764E-01	0.48646E-01	-43.954
5.000	0.34887E-01	-0.33635E-01	0.48461E-01	-43.954
10.000	0.34488E-01	-0.33251E-01	0.47907E-01	-43.954
15.000	0.33827E-01	-0.32613E-01	0.46988E-01	-43.954
20.000	0.32908E-01	-0.31728E-01	0.45712E-01	-43.954
25.000	0.31739E-01	-0.30601E-01	0.44088E-01	-43.954
30.000	0.30328E-01	-0.29240E-01	0.42129E-01	-43.954
35.000	0.28687E-01	-0.27658E-01	0.39848E-01	-43.954
40.000	0.26827E-01	-0.25865E-01	0.37265E-01	-43.954
45.000	0.24763E-01	-0.23875E-01	0.34398E-01	-43.954
50.000	0.22511E-01	-0.21703E-01	0.31269E-01	-43.954
55.000	0.20087E-01	-0.19366E-01	0.27902E-01	-43.954
60.000	0.17510E-01	-0.16882E-01	0.24323E-01	-43.954
65.000	0.14800E-01	-0.14269E-01	0.20559E-01	-43.954
70.000	0.11978E-01	-0.11548E-01	0.16638E-01	-43.954
75.000	0.90639E-02	-0.87388E-02	0.12590E-01	-43.954
80.000	0.60812E-02	-0.58630E-02	0.84473E-02	-43.954
85.000	0.30522E-02	-0.29427E-02	0.42398E-02	-43.954
90.000	-0.15308E-08	0.14759E-08	0.21264E-08	136.05
95.000	-0.30522E-02	0.29427E-02	0.42398E-02	136.05
100.000	-0.60812E-02	0.58631E-02	0.84473E-02	136.05

105.000	-0.90639E-02	0.87388E-02	0.12590E-01	136.05
110.000	-0.11978E-01	0.11548E-01	0.16638E-01	136.05
115.000	-0.14800E-01	0.14269E-01	0.20559E-01	136.05
120.000	-0.17510E-01	0.16882E-01	0.24323E-01	136.05
125.000	-0.20087E-01	0.19366E-01	0.27902E-01	136.05
130.000	-0.22511E-01	0.21703E-01	0.31269E-01	136.05
135.000	-0.24763E-01	0.23875E-01	0.34398E-01	136.05
140.000	-0.26827E-01	0.25865E-01	0.37265E-01	136.05
145.000	-0.28687E-01	0.27658E-01	0.39848E-01	136.05
150.000	-0.30328E-01	0.29240E-01	0.42129E-01	136.05
155.000	-0.31739E-01	0.30601E-01	0.44088E-01	136.05
160.000	-0.32908E-01	0.31728E-01	0.45712E-01	136.05
165.000	-0.33827E-01	0.32613E-01	0.46988E-01	136.05
170.000	-0.34488E-01	0.33251E-01	0.47907E-01	136.05
175.000	-0.34887E-01	0.33635E-01	0.48461E-01	136.05
180.000	-0.35020E-01	0.33764E-01	0.48646E-01	136.05

....

BAND NUMBER 8

0.000	0.58224	-0.20304	0.61663	-19.225
5.000	0.58002	-0.20227	0.61428	-19.225
10.000	0.57339	-0.19996	0.60726	-19.225
15.000	0.56240	-0.19613	0.59561	-19.225
20.000	0.54712	-0.19080	0.57944	-19.225
25.000	0.52769	-0.18402	0.55885	-19.225
30.000	0.50423	-0.17584	0.53401	-19.225
35.000	0.47694	-0.16632	0.50511	-19.225
40.000	0.44602	-0.15554	0.47236	-19.225
45.000	0.41170	-0.14357	0.43602	-19.225
50.000	0.37425	-0.13051	0.39636	-19.225
55.000	0.33396	-0.11646	0.35368	-19.225
60.000	0.29112	-0.10152	0.30831	-19.225
65.000	0.24606	-0.85810E-01	0.26060	-19.225
70.000	0.19914	-0.69445E-01	0.21090	-19.225
75.000	0.15069	-0.52552E-01	0.15959	-19.225
80.000	0.10110	-0.35258E-01	0.10708	-19.225
85.000	0.50745E-01	-0.17696E-01	0.53742E-01	-19.225
90.000	-0.25450E-07	0.88753E-08	0.26954E-07	160.77
95.000	-0.50745E-01	0.17697E-01	0.53743E-01	160.77
100.000	-0.10110	0.35258E-01	0.10708	160.77
105.000	-0.15069	0.52552E-01	0.15959	160.77
110.000	-0.19914	0.69445E-01	0.21090	160.77
115.000	-0.24606	0.85810E-01	0.26060	160.77
120.000	-0.29112	0.10152	0.30831	160.77
125.000	-0.33396	0.11646	0.35368	160.77
130.000	-0.37425	0.13051	0.39636	160.77
135.000	-0.41170	0.14357	0.43602	160.77
140.000	-0.44602	0.15554	0.47236	160.77
145.000	-0.47694	0.16632	0.50511	160.77
150.000	-0.50423	0.17584	0.53401	160.77
155.000	-0.52769	0.18402	0.55885	160.77
160.000	-0.54712	0.19080	0.57944	160.77
165.000	-0.56240	0.19613	0.59561	160.77
170.000	-0.57339	0.19996	0.60726	160.77
175.000	-0.58002	0.20227	0.61428	160.77
180.000	-0.58224	0.20304	0.61663	160.77

BAND NUMBER 9

0.000	0.84411	0.24397	0.87866	16.121

5.000	0.84090	0.24304	0.87532	16.121
10.000	0.83129	0.24027	0.86531	16.121
15.000	0.81535	0.23566	0.84872	16.121
20.000	0.79321	0.22926	0.82567	16.121
25.000	0.76503	0.22111	0.79634	16.121
30.000	0.73102	0.21129	0.76095	16.121
35.000	0.69146	0.19985	0.71976	16.121
40.000	0.64663	0.18689	0.67310	16.121
45.000	0.59688	0.17251	0.62131	16.121
50.000	0.54259	0.15682	0.56479	16.121
55.000	0.48416	0.13994	0.50398	16.121
60.000	0.42206	0.12199	0.43933	16.121
65.000	0.35674	0.10311	0.37134	16.121
70.000	0.28870	0.83444E-01	0.30052	16.121
75.000	0.21847	0.63145E-01	0.22741	16.121
80.000	0.14658	0.42365E-01	0.15258	16.121
85.000	0.73569E-01	0.21264E-01	0.76581E-01	16.121
90.000	-0.36897E-07	-0.10664E-07	0.38408E-07	-163.88
95.000	-0.73569E-01	-0.21264E-01	0.76581E-01	-163.88
100.000	-0.14658	-0.42365E-01	0.15258	-163.88
105.000	-0.21847	-0.63145E-01	0.22742	-163.88
110.000	-0.28870	-0.83444E-01	0.30052	-163.88
115.000	-0.35674	-0.10311	0.37134	-163.88
120.000	-0.42206	-0.12199	0.43933	-163.88
125.000	-0.48416	-0.13994	0.50398	-163.88
130.000	-0.54259	-0.15682	0.56479	-163.88
135.000	-0.59688	-0.17251	0.62131	-163.88
140.000	-0.64663	-0.18689	0.67310	-163.88
145.000	-0.69146	-0.19985	0.71976	-163.88
150.000	-0.73102	-0.21129	0.76095	-163.88
155.000	-0.76503	-0.22111	0.79634	-163.88
160.000	-0.79321	-0.22926	0.82567	-163.88
165.000	-0.81535	-0.23566	0.84872	-163.88
170.000	-0.83129	-0.24027	0.86531	-163.88
175.000	-0.84090	-0.24304	0.87532	-163.88
180.000	-0.84411	-0.24397	0.87866	-163.88

....

BAND NUMBER 18

0.000	1.7034	1.0380	1.9948	31.356
5.000	1.6969	1.0340	1.9872	31.356
10.000	1.6775	1.0222	1.9644	31.356
15.000	1.6454	1.0026	1.9268	31.356
20.000	1.6007	0.97537	1.8745	31.356
25.000	1.5438	0.94072	1.8079	31.356
30.000	1.4752	0.89891	1.7275	31.356
35.000	1.3954	0.85025	1.6340	31.356
40.000	1.3049	0.79513	1.5281	31.356
45.000	1.2045	0.73396	1.4105	31.356
50.000	1.0949	0.66719	1.2822	31.356
55.000	0.97705	0.59535	1.1441	31.356
60.000	0.85171	0.51898	0.99738	31.356
65.000	0.71990	0.43866	0.84302	31.356
70.000	0.58261	0.35501	0.68225	31.356
75.000	0.44088	0.26865	0.51628	31.356
80.000	0.29580	0.18024	0.34639	31.356
85.000	0.14846	0.90465E-01	0.17385	31.356
90.000	-0.74459E-07	-0.45371E-07	0.87193E-07	-148.64
95.000	-0.14846	-0.90465E-01	0.17385	-148.64
100.000	-0.29580	-0.18024	0.34639	-148.64

105.000	-0.44088	-0.26865	0.51628	-148.64
110.000	-0.58261	-0.35501	0.68225	-148.64
115.000	-0.71990	-0.43867	0.84302	-148.64
120.000	-0.85171	-0.51898	0.99738	-148.64
125.000	-0.97705	-0.59535	1.1441	-148.64
130.000	-1.0949	-0.66719	1.2822	-148.64
135.000	-1.2045	-0.73396	1.4105	-148.64
140.000	-1.3049	-0.79513	1.5281	-148.64
145.000	-1.3954	-0.85026	1.6340	-148.64
150.000	-1.4752	-0.89891	1.7275	-148.64
155.000	-1.5438	-0.94072	1.8079	-148.64
160.000	-1.6007	-0.97537	1.8745	-148.64
165.000	-1.6454	-1.0026	1.9268	-148.64
170.000	-1.6775	-1.0222	1.9644	-148.64
175.000	-1.6969	-1.0340	1.9872	-148.64
180.000	-1.7034	-1.0380	1.9948	-148.64

*** VELOCITIES (X10**6) AT SELECTED LOCATIONS ***

THETA (DEG)	REAL	IMAGINARY	MAGNITUDE	PHASE (DEG)
BAND NUMBER 1				
0.000	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
5.000	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00

... (all velocities on top surface, bands 1-8, are zero) ...

THETA (DEG)	REAL	IMAGINARY	MAGNITUDE	PHASE (DEG)
BAND NUMBER 9				
0.000	1.0000	0.00000E+00	1.0000	0.00000E+00
5.000	0.99619	0.00000E+00	0.99619	0.00000E+00
10.000	0.98481	0.00000E+00	0.98481	0.00000E+00
15.000	0.96593	0.00000E+00	0.96593	0.00000E+00
20.000	0.93969	0.00000E+00	0.93969	0.00000E+00
25.000	0.90631	0.00000E+00	0.90631	0.00000E+00
30.000	0.86603	0.00000E+00	0.86603	0.00000E+00
35.000	0.81915	0.00000E+00	0.81915	0.00000E+00
40.000	0.76604	0.00000E+00	0.76604	0.00000E+00
45.000	0.70711	0.00000E+00	0.70711	0.00000E+00
50.000	0.64279	0.00000E+00	0.64279	0.00000E+00
55.000	0.57358	0.00000E+00	0.57358	0.00000E+00
60.000	0.50000	0.00000E+00	0.50000	0.00000E+00
65.000	0.42262	0.00000E+00	0.42262	0.00000E+00
70.000	0.34202	0.00000E+00	0.34202	0.00000E+00
75.000	0.25882	0.00000E+00	0.25882	0.00000E+00
80.000	0.17365	0.00000E+00	0.17365	0.00000E+00
85.000	0.87156E-01	0.00000E+00	0.87156E-01	0.00000E+00
90.000	-0.43711E-07	0.00000E+00	0.43711E-07	180.00
95.000	-0.87156E-01	0.00000E+00	0.87156E-01	180.00
100.000	-0.17365	0.00000E+00	0.17365	180.00
105.000	-0.25882	0.00000E+00	0.25882	180.00
110.000	-0.34202	0.00000E+00	0.34202	180.00
115.000	-0.42262	0.00000E+00	0.42262	180.00
120.000	-0.50000	0.00000E+00	0.50000	180.00
125.000	-0.57358	0.00000E+00	0.57358	180.00
130.000	-0.64279	0.00000E+00	0.64279	180.00
135.000	-0.70711	0.00000E+00	0.70711	180.00
140.000	-0.76604	0.00000E+00	0.76604	180.00
145.000	-0.81915	0.00000E+00	0.81915	180.00

150.000	-0.86603	0.00000E+00	0.86603	180.00
155.000	-0.90631	0.00000E+00	0.90631	180.00
160.000	-0.93969	0.00000E+00	0.93969	180.00
165.000	-0.96593	0.00000E+00	0.96593	180.00
170.000	-0.98481	0.00000E+00	0.98481	180.00
175.000	-0.99619	0.00000E+00	0.99619	180.00
180.000	-1.0000	0.00000E+00	1.0000	180.00

... (velocities for bands 10-18 are the same as those for band 9) ...

FAR-FIELD PRESSURES

THETA (DEG)	PHI (DEG)	REAL	IMAGINARY
180.000	90.000	1.6175	0.18303
170.000	90.000	1.5930	0.18025
160.000	90.000	1.5200	0.17199
150.000	90.000	1.4008	0.15851
140.000	90.000	1.2391	0.14021
130.000	90.000	1.0397	0.11765
120.000	90.000	0.80876	0.91514E-01
110.000	90.000	0.55323	0.62599E-01
100.000	90.000	0.28088	0.31783E-01
90.000	90.000	0.70704E-07	0.80004E-08
80.000	90.000	-0.28088	-0.31783E-01
70.000	90.000	-0.55323	-0.62599E-01
60.000	90.000	-0.80876	-0.91514E-01
50.000	90.000	-1.0397	-0.11765
40.000	90.000	-1.2391	-0.14021
30.000	90.000	-1.4008	-0.15851
20.000	90.000	-1.5200	-0.17199
10.000	90.000	-1.5930	-0.18025
0.000	90.000	-1.6175	-0.18303

THETA (DEG)	PHI (DEG)	MAGNITUDE	PHASE (DEG)
180.000	90.000	1.6279	6.4557
170.000	90.000	1.6031	6.4557
160.000	90.000	1.5297	6.4557
150.000	90.000	1.4098	6.4557
140.000	90.000	1.2470	6.4557
130.000	90.000	1.0464	6.4557
120.000	90.000	0.81393	6.4557
110.000	90.000	0.55676	6.4557
100.000	90.000	0.28267	6.4557
90.000	90.000	0.71156E-07	6.4557
80.000	90.000	0.28267	-173.54
70.000	90.000	0.55676	-173.54
60.000	90.000	0.81393	-173.54
50.000	90.000	1.0464	-173.54
40.000	90.000	1.2470	-173.54
30.000	90.000	1.4098	-173.54
20.000	90.000	1.5297	-173.54
10.000	90.000	1.6031	-173.54
0.000	90.000	1.6279	-173.54

Comparison with CHIEF computer program:

The results above are compared graphically with CHIEF results in Figures 5 and 6. Figure 5 shows a comparison of the real and imaginary parts of the surface pressures in band 18 (the band closest to the midplane). Figure 6 compares the real and imaginary parts of the far-field pressure. Again, the CHIEF model can be viewed as dividing each SHIP band into 36 circumferential elements.

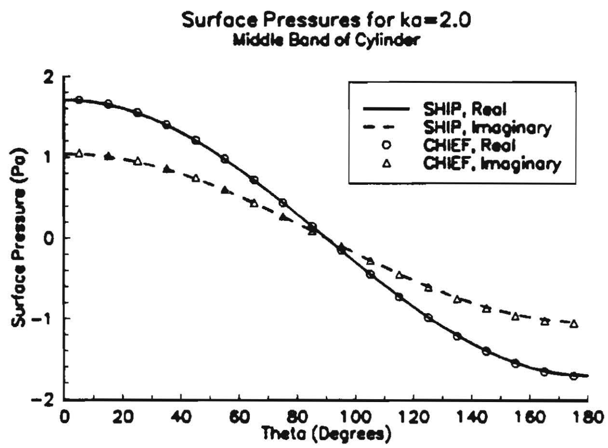

Figure 5

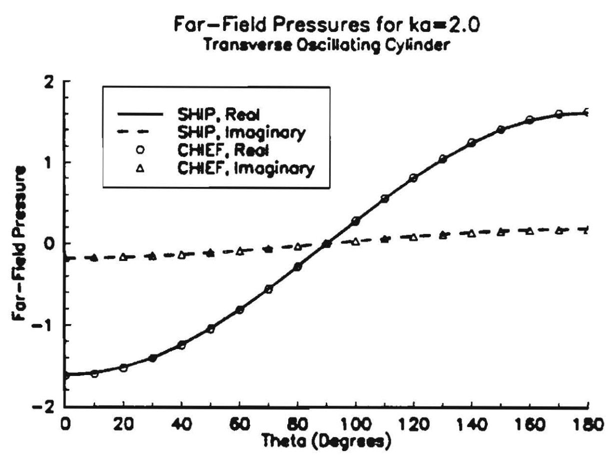

Figure 6

SAMPLE RUN 3
Scattering From a Pressure-Release Cylinder

The problem considered here is that of scattering from a cylinder with pressure-release surface boundary conditions. The plane wave is normally incident, and has a driving frequency corresponding to a wavenumber ka=2.

Input file:

```
0.0            ! RIN, inner radius
1.0            ! ROUT, outer radius
1.0            ! H, half-height of cylinder
2.0,1.0,1      ! FKS, DELFK, NF
1.5E6          ! RHOC, density*(sound speed)
1              ! ISYM, -1 for antisymmetry, 1 for symmetry, 0 for neither
1              ! IRHEVEN, +1=even forcing with respect to theta, 0=odd
10             ! JMAXH, half the number of bands along height
8              ! IMAX, number of bands on top and bottom
1.0            ! AINC, amplitude of incident wave
90.0           ! PSI, angle of plane wave direction with z-axis in degrees
4              ! NHARM, largest harmonic number included in analysis
37             ! NTH, number of angles for print of surface pressures and velocities
0.0            ! THS, start angle for print of surface quantities (deg)
5              ! DTH, increment angle for print of surface quantities (deg)
0.001          ! EPS, convergence parameter for evaluation of coefficient matrices
300            ! KMAX, maximum number of intervals for integration
16             ! NQD1, number of Gauss points
8              ! NQD2, number of Gauss points
19             ! NPTS, number of far-field points
180.0,90.0     ! THETA(1), PHI(1) (deg)
170.0,90.0     ! THETA(2), PHI(2) (deg)
160.0,90.0          .
150.0,90.0          .
140.0,90.0          .
130.0,90.0          .
120.0,90.0
110.0,90.0
100.0,90.0
90.0,90.0
80.0,90.0
70.0,90.0
60.0,90.0
50.0,90.0
40.0,90.0
30.0,90.0
20.0,90.0
10.0,90.0
00.0,90.0      ! THETA(19), PHI(19) (deg)
1              ! IRIGID
1              ! MXD, =1 if any bands have pressure release impedance condition
0              ! JMXT(1)  (zero indicates a pressure-release condition)
0              ! JMXT(2)
0              ! JMXT(3)
0              ! JMXT(4)
0              ! JMXT(5)
0              ! JMXT(6)
0              ! JMXT(7)
0              ! JMXT(8)
0              ! JMXT(9)
```

```
0            ! JMXT(10)
0            ! JMXT(11)
0            ! JMXT(12)
0            ! JMXT(13)
0            ! JMXT(14)
0            ! JMXT(15)
0            ! JMXT(16)
0            ! JMXT(17)
0            ! JMXT(18)
0,0,0,1      ! IPRO,IPRM,IPRP,IPRD print options
```

Excerpts from output file:

```
CYLINDER TRANSDUCER

WAVENUMBER, K=     2.000000
KA=                2.000000
FLUID IMPEDANCE, RHOC=    1500000.

OUTER RADIUS=    1.000000     M
INNER RADIUS=   0.0000000E+00 M
CYLINDER HALF-HEIGHT=    1.000000     M
BANDS ON TOP AND BOTTOM, IMAX=            8
BANDS ON SIDE SURFACES, JMAX=            20
TOTAL NUMBER OF BANDS, LCMAX=            36

INCIDENT PRESSURE AMPLITUDE=    1.000000     PA
PLANE WAVE ANGLE WITH Z-AXIS=    90.00000     DEG

SYMMETRY CONDITION, ISYM=              1
RIGHT-HAND SIDE EVENESS, IRHEVEN=              1
NUMBER OF HARMONICS, NHARM=              4

CONVERGENCE PARAMETER, EPS=   1.0000000E-03
MAXIMUM NUMBER OF INTERVALS, KMAX=          300
NUMBER OF GAUSS POINTS, NQD1=            16
NUMBER OF GAUSS POINTS, NQD2=             8
HIGHEST HARMONIC, NBESL=            10
PRESSURE-RELEASE PARAMETER, MXD=              1
```

*** PRESSURES AT SELECTED LOCATIONS ***

THETA (DEG)	REAL	IMAGINARY	MAGNITUDE	PHASE (DEG)
BAND NUMBER	1 (PRESSURE-RELEASE BAND)			
0.000	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
5.000	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
	....(all pressures are zero)....			
175.000	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
180.000	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00

*** VELOCITIES (X10**6) AT SELECTED LOCATIONS ***

THETA (DEG)	REAL	IMAGINARY	MAGNITUDE	PHASE (DEG)
BAND NUMBER	1 (PRESSURE-RELEASE BAND)			
0.000	-0.20794	0.31917	0.38094	123.08
5.000	-0.20813	0.31912	0.38100	123.11
10.000	-0.20869	0.31897	0.38117	123.19
15.000	-0.20961	0.31872	0.38147	123.33
20.000	-0.21090	0.31834	0.38187	123.52
25.000	-0.21253	0.31785	0.38236	123.77
30.000	-0.21448	0.31723	0.38293	124.06
35.000	-0.21675	0.31647	0.38358	124.41
40.000	-0.21930	0.31556	0.38428	124.80
45.000	-0.22212	0.31451	0.38504	125.23

50.000	-0.22518	0.31332	0.38584	125.70
55.000	-0.22845	0.31199	0.38668	126.21
60.000	-0.23190	0.31053	0.38756	126.75
65.000	-0.23552	0.30895	0.38849	127.32
70.000	-0.23927	0.30728	0.38945	127.91
75.000	-0.24312	0.30553	0.39046	128.51
80.000	-0.24704	0.30373	0.39151	129.12
85.000	-0.25101	0.30188	0.39260	129.74
90.000	-0.25501	0.30001	0.39374	130.37
95.000	-0.25900	0.29813	0.39492	130.98
100.000	-0.26295	0.29625	0.39612	131.59
105.000	-0.26685	0.29439	0.39733	132.19
110.000	-0.27065	0.29256	0.39855	132.77
115.000	-0.27435	0.29076	0.39976	133.34
120.000	-0.27790	0.28899	0.40093	133.88
125.000	-0.28129	0.28728	0.40206	134.40
130.000	-0.28449	0.28563	0.40313	134.89
135.000	-0.28747	0.28404	0.40413	135.34
140.000	-0.29021	0.28255	0.40504	135.77
145.000	-0.29269	0.28116	0.40585	136.15
150.000	-0.29489	0.27989	0.40657	136.49
155.000	-0.29678	0.27877	0.40718	136.79
160.000	-0.29836	0.27781	0.40767	137.04
165.000	-0.29960	0.27705	0.40806	137.24
170.000	-0.30049	0.27649	0.40834	137.38
175.000	-0.30103	0.27615	0.40850	137.47
180.000	-0.30121	0.27603	0.40856	137.50

BAND NUMBER 2 (PRESSURE-RELEASE BAND)

0.000	-0.10035	0.31973	0.33510	107.43
5.000	-0.10088	0.31972	0.33525	107.51
10.000	-0.10247	0.31968	0.33570	107.77
15.000	-0.10511	0.31961	0.33645	108.20
20.000	-0.10878	0.31948	0.33749	108.80
25.000	-0.11346	0.31925	0.33882	109.57
30.000	-0.11913	0.31889	0.34042	110.48
35.000	-0.12575	0.31836	0.34230	111.55
40.000	-0.13328	0.31760	0.34443	112.77
45.000	-0.14167	0.31655	0.34681	114.11
50.000	-0.15084	0.31518	0.34941	115.58
55.000	-0.16074	0.31341	0.35223	117.15
60.000	-0.17128	0.31121	0.35523	118.83
65.000	-0.18237	0.30853	0.35840	120.59
70.000	-0.19392	0.30533	0.36171	122.42
75.000	-0.20582	0.30160	0.36514	124.31
80.000	-0.21797	0.29732	0.36866	126.25
85.000	-0.23025	0.29249	0.37224	128.21
90.000	-0.24255	0.28714	0.37587	130.19
95.000	-0.25476	0.28130	0.37951	132.17
100.000	-0.26676	0.27502	0.38314	134.13
105.000	-0.27846	0.26837	0.38673	136.06
110.000	-0.28975	0.26143	0.39026	137.94
115.000	-0.30055	0.25429	0.39370	139.77
120.000	-0.31078	0.24706	0.39701	141.52
125.000	-0.32036	0.23984	0.40019	143.18
130.000	-0.32923	0.23275	0.40319	144.74
135.000	-0.33735	0.22590	0.40600	146.19
140.000	-0.34468	0.21942	0.40859	147.52
145.000	-0.35119	0.21340	0.41094	148.71
150.000	-0.35685	0.20796	0.41302	149.77
155.000	-0.36165	0.20318	0.41482	150.67

160.000	-0.36559	0.19916	0.41632	151.42
165.000	-0.36865	0.19595	0.41749	152.01
170.000	-0.37084	0.19362	0.41834	152.43
175.000	-0.37215	0.19221	0.41886	152.69
180.000	-0.37259	0.19173	0.41903	152.77

....

BAND NUMBER 8 (PRESSURE-RELEASE BAND)

0.000	0.28004	-0.49622E-01	0.28440	-10.048
5.000	0.28620	-0.47168E-01	0.29006	-9.3586
10.000	0.30398	-0.39370E-01	0.30652	-7.3796
15.000	0.33129	-0.25005E-01	0.33223	-4.3163
20.000	0.36482	-0.22957E-02	0.36483	-0.36054
25.000	0.40031	0.30724E-01	0.40148	4.3889
30.000	0.43283	0.75780E-01	0.43942	9.9306
35.000	0.45722	0.13391	0.47642	16.324
40.000	0.46841	0.20504	0.51132	23.641
45.000	0.46190	0.28775	0.54420	31.922
50.000	0.43405	0.37904	0.57626	41.130
55.000	0.38245	0.47445	0.60940	51.128
60.000	0.30609	0.56823	0.64543	61.690
65.000	0.20554	0.65376	0.68531	72.547
70.000	0.82942E-01	0.72407	0.72880	83.465
75.000	-0.58071E-01	0.77243	0.77461	94.299
80.000	-0.21257	0.79301	0.82100	105.01
85.000	-0.37460	0.78141	0.86656	115.61
90.000	-0.53757	0.73512	0.91070	126.18
95.000	-0.69462	0.65382	0.95393	136.73
100.000	-0.83910	0.53943	0.99753	147.26
105.000	-0.96495	0.39598	1.0430	157.69
110.000	-1.0671	0.22932	1.0915	167.87
115.000	-1.1419	0.46545E-01	1.1429	177.67
120.000	-1.1872	-0.14451	1.1960	-173.06
125.000	-1.2025	-0.33600	1.2486	-164.39
130.000	-1.1892	-0.52064	1.2982	-156.36
135.000	-1.1502	-0.69224	1.3425	-148.96
140.000	-1.0901	-0.84609	1.3799	-142.18
145.000	-1.0144	-0.97918	1.4099	-136.01
150.000	-0.92957	-1.0902	1.4327	-130.45
155.000	-0.84256	-1.1792	1.4493	-125.55
160.000	-0.76013	-1.2476	1.4609	-121.35
165.000	-0.68856	-1.2975	1.4688	-117.96
170.000	-0.63327	-1.3309	1.4739	-115.45
175.000	-0.59836	-1.3501	1.4768	-113.90
180.000	-0.58643	-1.3563	1.4777	-113.38

BAND NUMBER 9 (PRESSURE-RELEASE BAND)

0.000	0.14200	-0.27119	0.30612	-62.363
5.000	0.15586	-0.27132	0.31290	-60.125
10.000	0.19606	-0.27049	0.33407	-54.063
15.000	0.25868	-0.26528	0.37052	-45.722
20.000	0.33743	-0.25064	0.42033	-36.605
25.000	0.42421	-0.22069	0.47818	-27.486
30.000	0.50969	-0.16980	0.53723	-18.425
35.000	0.58401	-0.93607E-01	0.59147	-9.1061
40.000	0.63752	0.99359E-02	0.63760	0.89289
45.000	0.66155	0.13987	0.67618	11.938
50.000	0.64908	0.29183	0.71167	24.209
55.000	0.59532	0.45800	0.75111	37.572
60.000	0.49811	0.62759	0.80124	51.561

65.000	0.35822	0.78764	0.86527	65.544
70.000	0.17933	0.92414	0.94138	79.018
75.000	-0.32096E-01	1.0233	1.0238	91.796
80.000	-0.26720	1.0732	1.1059	103.98
85.000	-0.51523	1.0643	1.1825	115.83
90.000	-0.76422	0.99149	1.2518	127.62
95.000	-1.0018	0.85376	1.3162	139.56
100.000	-1.2159	0.65491	1.3811	151.69
105.000	-1.3957	0.40310	1.4528	163.89
110.000	-1.5322	0.11014	1.5361	175.89
115.000	-1.6188	-0.20961	1.6323	-172.62
120.000	-1.6520	-0.54051	1.7382	-161.88
125.000	-1.6315	-0.86707	1.8476	-152.01
130.000	-1.5602	-1.1753	1.9533	-143.01
135.000	-1.4439	-1.4539	2.0491	-134.80
140.000	-1.2916	-1.6948	2.1308	-127.31
145.000	-1.1139	-1.8938	2.1971	-120.46
150.000	-0.92348	-2.0505	2.2488	-114.25
155.000	-0.73329	-2.1675	2.2882	-108.69
160.000	-0.55631	-2.2501	2.3179	-103.89
165.000	-0.40453	-2.3047	2.3399	-99.955
170.000	-0.28819	-2.3378	2.3555	-97.027
175.000	-0.21510	-2.3552	2.3650	-95.218
180.000	-0.19018	-2.3605	2.3682	-94.606

....

BAND NUMBER 18 (PRESSURE-RELEASE BAND)

0.000	-0.18186	-0.62488E-01	0.19230	-161.04
5.000	-0.17315	-0.65809E-01	0.18523	-159.19
10.000	-0.14777	-0.74915E-01	0.16568	-153.12
15.000	-0.10797	-0.87364E-01	0.13888	-141.02
20.000	-0.57245E-01	-0.99488E-01	0.11478	-119.92
25.000	-0.17377E-03	-0.10694	0.10694	-90.093
30.000	0.57998E-01	-0.10537	0.12028	-61.172
35.000	0.11173	-0.91126E-01	0.14417	-39.201
40.000	0.15559	-0.61873E-01	0.16745	-21.686
45.000	0.18474	-0.17127E-01	0.18553	-5.2967
50.000	0.19523	0.41485E-01	0.19959	11.996
55.000	0.18439	0.11019	0.21480	30.861
60.000	0.15103	0.18329	0.23749	50.512
65.000	0.95573E-01	0.25368	0.27108	69.356
70.000	0.20099E-01	0.31349	0.31413	86.332
75.000	-0.71784E-01	0.35492	0.36211	101.43
80.000	-0.17514	0.37105	0.41030	115.27
85.000	-0.28400	0.35659	0.45586	128.53
90.000	-0.39176	0.30854	0.49867	141.78
95.000	-0.49162	0.22649	0.54128	155.26
100.000	-0.57702	0.11280	0.58795	168.94
105.000	-0.64212	-0.27635E-01	0.64271	-177.54
110.000	-0.68216	-0.18782	0.70754	-164.61
115.000	-0.69385	-0.35933	0.78137	-152.62
120.000	-0.67558	-0.53314	0.86061	-141.72
125.000	-0.62760	-0.70047	0.94050	-131.86
130.000	-0.55200	-0.85362	1.0166	-122.89
135.000	-0.45262	-0.98664	1.0855	-114.64
140.000	-0.33487	-1.0958	1.1458	-106.99
145.000	-0.20535	-1.1796	1.1974	-99.875
150.000	-0.71488E-01	-1.2393	1.2413	-93.301
155.000	0.58911E-01	-1.2776	1.2790	-87.360
160.000	0.17818	-1.2990	1.3111	-82.189

165.000	0.27925	-1.3084	1.3379	-77.952
170.000	0.35610	-1.3109	1.3584	-74.802
175.000	0.40413	-1.3105	1.3714	-72.861
180.000	0.42047	-1.3100	1.3759	-72.205

FAR-FIELD PRESSURES

THETA (DEG)	PHI (DEG)	REAL	IMAGINARY
180.000	90.000	0.26529	0.83938
170.000	90.000	0.28031	0.83390
160.000	90.000	0.32456	0.81634
150.000	90.000	0.39544	0.78350
140.000	90.000	0.48815	0.73046
130.000	90.000	0.59493	0.65127
120.000	90.000	0.70394	0.53978
110.000	90.000	0.79835	0.39073
100.000	90.000	0.85644	0.20085
90.000	90.000	0.85346	-0.30025E-01
80.000	90.000	0.76583	-0.29809
70.000	90.000	0.57715	-0.59508
60.000	90.000	0.28487	-0.90830
50.000	90.000	-0.94719E-01	-1.2212
40.000	90.000	-0.52513	-1.5150
30.000	90.000	-0.95341	-1.7697
20.000	90.000	-1.3189	-1.9670
10.000	90.000	-1.5650	-2.0920
0.000	90.000	-1.6519	-2.1348

THETA (DEG)	PHI (DEG)	MAGNITUDE	PHASE (DEG)
180.000	90.000	0.88030	72.461
170.000	90.000	0.87975	71.420
160.000	90.000	0.87849	68.318
150.000	90.000	0.87763	63.219
140.000	90.000	0.87856	56.246
130.000	90.000	0.88210	47.588
120.000	90.000	0.88707	37.481
110.000	90.000	0.88884	26.078
100.000	90.000	0.87968	13.198
90.000	90.000	0.85399	-2.0149
80.000	90.000	0.82180	-21.268
70.000	90.000	0.82899	-45.877
60.000	90.000	0.95192	-72.587
50.000	90.000	1.2249	-94.435
40.000	90.000	1.6034	-109.12
30.000	90.000	2.0102	-118.31
20.000	90.000	2.3682	-123.84
10.000	90.000	2.6126	-126.80
0.000	90.000	2.6993	-127.73

Comparison with CHIEF computer code:

The SHIP results are compared graphically to CHIEF results below. The CHIEF model consists of 36 subdivisions per SHIP band. Figure 7 compares surface velocities in band 18 (the band closest to the midplane) while Figure 8 compares the far-field pressure predictions.

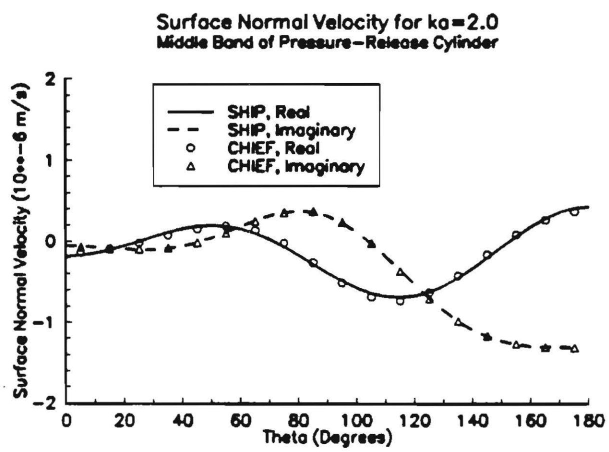

Figure 7

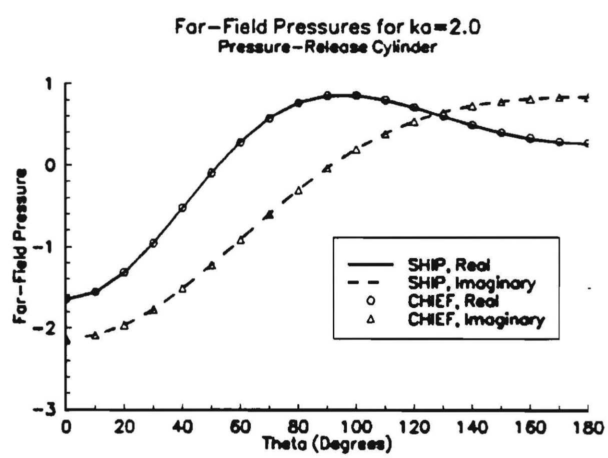

Figure 8

SAMPLE RUN 4
Scattering From a Rigid Ring

The problem consists of normal-incidence scattering of a plane wave on a rigid ring. The ring has outer radius 1 meter, inner radius 0.5 meter, has 4 bands on the top and bottom surfaces and has 20 bands along the inside and outside surfaces. (IMAX=4, JMAX=20, LCMAX=48) The frequency of the plane wave corresponds to a wavenumber ka=2.

Input file:

```
0.5           ! RIN, inner radius
1.0           ! ROUT, outer radius
1.0           ! H, half-height of cylinder
2.0,1.0,1     ! FKS, DELFK, NF
1.5E6         ! RHOC, density*(sound speed)
1             ! ISYM, -1 for antisymmetry, 1 for symmetry, 0 for neither
1             ! IRHEVEN, +1=even forcing with respect to theta, 0=odd
10            ! JMAXH, half the number of bands along height
4             ! IMAX, number of bands on top and bottom
1.0           ! AINC, amplitude of incident wave
90.0          ! PSI, angle of plane wave direction with z-axis in degrees
4             ! NHARM, largest harmonic number included in analysis
37            ! NTH, number of angles for print of surface pressures and velocities
0.0           ! THS, start angle for print of surface quantities (deg)
5             ! DTH, increment angle for print of surface quantities (deg)
0.001         ! EPS, convergence parameter for evaluation of coefficient matrices
300           ! KMAX, maximum number of intervals for integration
16            ! NQD1, number of Gauss points
8             ! NQD2, number of Gauss points
19            ! NPTS, number of far-field points
180.0,90.0    ! THETA(1), PHI(1) (deg)
170.0,90.0    ! THETA(2), PHI(2) (deg)
160.0,90.0          .
150.0,90.0          .
140.0,90.0          .
130.0,90.0          .
120.0,90.0
110.0,90.0
100.0,90.0
90.0,90.0
80.0,90.0
70.0,90.0
60.0,90.0
50.0,90.0
40.0,90.0
30.0,90.0
20.0,90.0
10.0,90.0
00.0,90.0     ! THETA(19), PHI(19) (deg)
1             ! IRIGID
0             ! MXD, =1 if any bands have pressure release impedance condition
0,0,0,1       ! IPRO,IPRM,IPRP,IPRD print options
```

Excerpts from output file:

```
RING TRANSDUCER

WAVENUMBER, K=     2.000000
KA=                2.000000
FLUID IMPEDANCE, RHOC=    1500000.

OUTER RADIUS=    1.000000     M
INNER RADIUS=   0.5000000     M
CYLINDER HALF-HEIGHT=    1.000000     M
BANDS ON TOP AND BOTTOM, IMAX=            4
BANDS ON SIDE SURFACES, JMAX=           20
TOTAL NUMBER OF BANDS, LCMAX=           48

INCIDENT PRESSURE AMPLITUDE=    1.000000     PA
PLANE WAVE ANGLE WITH Z-AXIS=    90.00000     DEG

SYMMETRY CONDITION, ISYM=           1
RIGHT-HAND SIDE EVENESS, IRHEVEN=           1
NUMBER OF HARMONICS, NHARM=           4

CONVERGENCE PARAMETER, EPS=   1.0000000E-03
MAXIMUM NUMBER OF INTERVALS, KMAX=          300
NUMBER OF GAUSS POINTS, NQD1=           16
NUMBER OF GAUSS POINTS, NQD2=            8
HIGHEST HARMONIC, NBESL=          10
PRESSURE-RELEASE PARAMETER, MXD=           0
```

*** PRESSURES AT SELECTED LOCATIONS ***

THETA (DEG)	REAL	IMAGINARY	MAGNITUDE	PHASE (DEG)
BAND NUMBER	1			
0.000	-0.90819	-0.31794	0.96224	-160.71
5.000	-0.90821	-0.31775	0.96219	-160.72
10.000	-0.90826	-0.31718	0.96205	-160.75
15.000	-0.90836	-0.31625	0.96183	-160.80
20.000	-0.90850	-0.31495	0.96154	-160.88
25.000	-0.90869	-0.31332	0.96120	-160.98
30.000	-0.90895	-0.31137	0.96081	-161.09
35.000	-0.90928	-0.30913	0.96039	-161.22
40.000	-0.90968	-0.30662	0.95996	-161.37
45.000	-0.91014	-0.30387	0.95952	-161.54
50.000	-0.91065	-0.30091	0.95908	-161.71
55.000	-0.91121	-0.29778	0.95863	-161.90
60.000	-0.91179	-0.29450	0.95817	-162.10
65.000	-0.91237	-0.29109	0.95768	-162.30
70.000	-0.91292	-0.28760	0.95715	-162.51
75.000	-0.91341	-0.28404	0.95656	-162.73
80.000	-0.91384	-0.28043	0.95590	-162.94
85.000	-0.91417	-0.27681	0.95516	-163.15
90.000	-0.91439	-0.27319	0.95433	-163.37
95.000	-0.91450	-0.26959	0.95341	-163.57
100.000	-0.91451	-0.26604	0.95242	-163.78
105.000	-0.91441	-0.26255	0.95136	-163.98
110.000	-0.91423	-0.25914	0.95025	-164.17
115.000	-0.91398	-0.25583	0.94911	-164.36

120.000	-0.91369	-0.25264	0.94798	-164.54
125.000	-0.91338	-0.24960	0.94687	-164.72
130.000	-0.91307	-0.24672	0.94581	-164.88
135.000	-0.91278	-0.24402	0.94483	-165.03
140.000	-0.91252	-0.24153	0.94394	-165.17
145.000	-0.91230	-0.23928	0.94316	-165.30
150.000	-0.91213	-0.23727	0.94248	-165.42
155.000	-0.91200	-0.23553	0.94192	-165.52
160.000	-0.91191	-0.23408	0.94147	-165.60
165.000	-0.91185	-0.23294	0.94113	-165.67
170.000	-0.91181	-0.23211	0.94089	-165.72
175.000	-0.91179	-0.23161	0.94075	-165.75
180.000	-0.91179	-0.23145	0.94070	-165.76

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BAND NUMBER 9

0.000	0.91792E-01	-0.25794	0.27378	-70.411
5.000	0.92370E-01	-0.25702	0.27311	-70.232
10.000	0.94084E-01	-0.25425	0.27110	-69.694
15.000	0.96879E-01	-0.24964	0.26778	-68.790
20.000	0.10067	-0.24314	0.26316	-67.509
25.000	0.10532	-0.23475	0.25730	-65.837
30.000	0.11070	-0.22445	0.25026	-63.748
35.000	0.11662	-0.21221	0.24214	-61.210
40.000	0.12289	-0.19805	0.23307	-58.181
45.000	0.12930	-0.18197	0.22323	-54.603
50.000	0.13565	-0.16402	0.21285	-50.407
55.000	0.14173	-0.14426	0.20223	-45.508
60.000	0.14732	-0.12280	0.19179	-39.813
65.000	0.15223	-0.99756E-01	0.18201	-33.236
70.000	0.15630	-0.75297E-01	0.17349	-25.722
75.000	0.15937	-0.49614E-01	0.16692	-17.292
80.000	0.16133	-0.22929E-01	0.16295	-8.0892
85.000	0.16208	0.45100E-02	0.16214	1.5939
90.000	0.16157	0.32438E-01	0.16479	11.352
95.000	0.15979	0.60576E-01	0.17089	20.762
100.000	0.15676	0.88638E-01	0.18009	29.485
105.000	0.15255	0.11634	0.19185	37.329
110.000	0.14726	0.14340	0.20555	44.240
115.000	0.14101	0.16956	0.22053	50.253
120.000	0.13397	0.19458	0.23624	55.452
125.000	0.12633	0.21822	0.25215	59.933
130.000	0.11830	0.24030	0.26784	63.789
135.000	0.11010	0.26065	0.28295	67.101
140.000	0.10196	0.27913	0.29716	69.935
145.000	0.94104E-01	0.29562	0.31023	72.342
150.000	0.86769E-01	0.31003	0.32195	74.365
155.000	0.80160E-01	0.32231	0.33213	76.034
160.000	0.74469E-01	0.33240	0.34064	77.372
165.000	0.69860E-01	0.34028	0.34738	78.398
170.000	0.66469E-01	0.34591	0.35224	79.123
175.000	0.64394E-01	0.34930	0.35519	79.555
180.000	0.63695E-01	0.35043	0.35617	79.698

BAND NUMBER 10

0.000	0.24538	-0.30629	0.39246	-51.301
5.000	0.24644	-0.30522	0.39230	-51.082
10.000	0.24961	-0.30199	0.39179	-50.425
15.000	0.25478	-0.29654	0.39096	-49.332

20.000	0.26181	-0.28881	0.38981	-47.807
25.000	0.27050	-0.27867	0.38837	-45.853
30.000	0.28059	-0.26603	0.38666	-43.474
35.000	0.29178	-0.25077	0.38474	-40.678
40.000	0.30373	-0.23280	0.38269	-37.469
45.000	0.31607	-0.21206	0.38062	-33.858
50.000	0.32840	-0.18851	0.37866	-29.857
55.000	0.34033	-0.16219	0.37700	-25.481
60.000	0.35144	-0.13318	0.37583	-20.754
65.000	0.36136	-0.10163	0.37539	-15.709
70.000	0.36974	-0.67770E-01	0.37590	-10.386
75.000	0.37627	-0.31874E-01	0.37762	-4.8420
80.000	0.38069	0.57124E-02	0.38073	0.85969
85.000	0.38280	0.44594E-01	0.38539	6.6447
90.000	0.38250	0.84337E-01	0.39169	12.434
95.000	0.37975	0.12448	0.39963	18.149
100.000	0.37458	0.16455	0.40913	23.715
105.000	0.36713	0.20407	0.42004	29.068
110.000	0.35759	0.24260	0.43212	34.154
115.000	0.34623	0.27971	0.44510	38.933
120.000	0.33338	0.31501	0.45866	43.377
125.000	0.31940	0.34817	0.47248	47.468
130.000	0.30471	0.37892	0.48624	51.196
135.000	0.28973	0.40704	0.49962	54.557
140.000	0.27490	0.43235	0.51234	57.551
145.000	0.26064	0.45475	0.52414	60.181
150.000	0.24735	0.47416	0.53480	62.450
155.000	0.23543	0.49055	0.54412	64.363
160.000	0.22518	0.50392	0.55195	65.922
165.000	0.21691	0.51429	0.55816	67.131
170.000	0.21084	0.52167	0.56267	67.994
175.000	0.20712	0.52609	0.56539	68.510
180.000	0.20588	0.52756	0.56631	68.682

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BAND NUMBER 24

0.000	-0.92724	0.50049	1.0537	151.64
5.000	-0.91943	0.47989	1.0371	152.44
10.000	-0.89541	0.41953	0.98882	154.90
15.000	-0.85358	0.32362	0.91287	159.24
20.000	-0.79161	0.19889	0.81621	165.90
25.000	-0.70698	0.54117E-01	0.70904	175.62
30.000	-0.59753	-0.10039	0.60591	-170.46
35.000	-0.46216	-0.25351	0.52712	-151.25
40.000	-0.30130	-0.39397	0.49598	-127.41
45.000	-0.11744	-0.51110	0.52441	-102.94
50.000	0.84751E-01	-0.59551	0.60151	-81.900
55.000	0.29839	-0.63968	0.70585	-64.993
60.000	0.51468	-0.63844	0.82006	-51.126
65.000	0.72345	-0.58925	0.93305	-39.163
70.000	0.91386	-0.49239	1.0381	-28.316
75.000	1.0753	-0.35090	1.1311	-18.074
80.000	1.1980	-0.17036	1.2101	-8.0934
85.000	1.2744	0.41503E-01	1.2750	1.8653
90.000	1.2991	0.27530	1.3279	11.965
95.000	1.2699	0.52061	1.3725	22.291
100.000	1.1878	0.76665	1.4138	32.839
105.000	1.0568	1.0030	1.4570	43.502
110.000	0.88370	1.2201	1.5065	54.085
115.000	0.67729	1.4101	1.5643	64.345

120.000	0.44789	1.5671	1.6298	74.049
125.000	0.20632	1.6874	1.6999	83.029
130.000	-0.36887E-01	1.7699	1.7703	91.194
135.000	-0.27219	1.8159	1.8362	98.525
140.000	-0.49164	1.8290	1.8939	105.05
145.000	-0.68921	1.8146	1.9411	110.80
150.000	-0.86091	1.7795	1.9768	115.82
155.000	-1.0047	1.7316	2.0019	120.12
160.000	-1.1200	1.6787	2.0180	123.71
165.000	-1.2076	1.6284	2.0273	126.56
170.000	-1.2688	1.5872	2.0320	128.64
175.000	-1.3048	1.5604	2.0340	129.90
180.000	-1.3166	1.5511	2.0346	130.33

STATIONARY RIGID SURFACE- ALL VELOCITIES ZERO

FAR-FIELD PRESSURES

THETA (DEG)	PHI (DEG)	REAL	IMAGINARY
180.000	90.000	0.34784	-0.78251
170.000	90.000	0.33311	-0.77574
160.000	90.000	0.28849	-0.75605
150.000	90.000	0.21295	-0.72527
140.000	90.000	0.10587	-0.68636
130.000	90.000	-0.31328E-01	-0.64333
120.000	90.000	-0.19324	-0.60092
110.000	90.000	-0.36862	-0.56424
100.000	90.000	-0.53958	-0.53819
90.000	90.000	-0.68281	-0.52684
80.000	90.000	-0.77307	-0.53272
70.000	90.000	-0.78885	-0.55632
60.000	90.000	-0.71878	-0.59574
50.000	90.000	-0.56691	-0.64676
40.000	90.000	-0.35480	-0.70325
30.000	90.000	-0.11918	-0.75796
20.000	90.000	0.95009E-01	-0.80356
10.000	90.000	0.24451	-0.83375
0.000	90.000	0.29813	-0.84432

THETA (DEG)	PHI (DEG)	MAGNITUDE	PHASE (DEG)
180.000	90.000	0.85634	-66.034
170.000	90.000	0.84424	-66.761
160.000	90.000	0.80922	-69.115
150.000	90.000	0.75588	-73.637
140.000	90.000	0.69448	-81.231
130.000	90.000	0.64409	-92.788
120.000	90.000	0.63123	-107.83
110.000	90.000	0.67398	-123.16
100.000	90.000	0.76210	-135.07
90.000	90.000	0.86243	-142.35
80.000	90.000	0.93884	-145.43
70.000	90.000	0.96528	-144.81
60.000	90.000	0.93357	-140.35
50.000	90.000	0.86005	-131.24
40.000	90.000	0.78769	-116.77

30.000	90.000	0.76727	-98.936
20.000	90.000	0.80916	-83.257
10.000	90.000	0.86887	-73.656
0.000	90.000	0.89541	-70.552

Comparison with CHIEF computer code:

These results, plus the result of other SHIP runs with zero through three harmonics, (NH=NHARM=0,4) were compared to the results from the CHIEF computer code. The CHIEF model for the ring is equivalent to placing 36 subdivisions circumferentially around each SHIP band. Figure 9 shows a comparison of SHIP surface pressures in band 24 (the band closest to the midplane) and CHIEF surface pressures. As for the case of scattering from a cylinder, it is seen here that convergence of surface quantities is observed when NH≥ka+2. Figure 10 shows the comparison of far-field pressures in the z=0 plane.

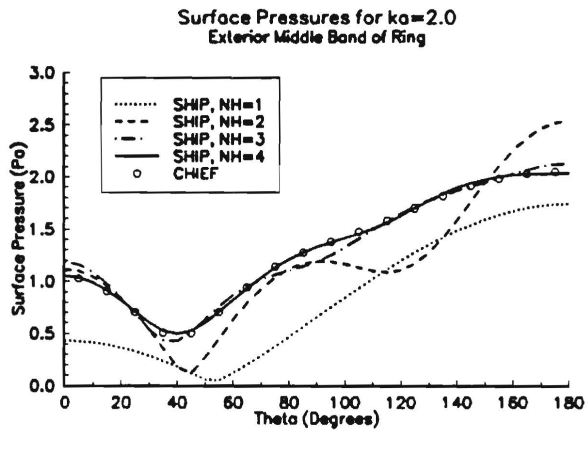

Figure 9

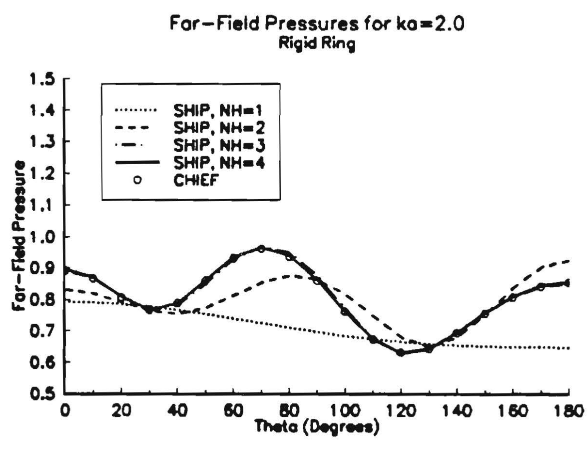

Figure 10

References

1. P.H. Rogers, "SHIP (Simplified-Helmholtz-Integral Program): A Fast Computer Program for Calculating the Acoustic Radiation and Radiation Impedance for Free-Flooded-Ring and Finite-Circular-Cylinder Sources," NRL Report 7240, June 12, 1972.

2. P.H. Rogers, and J.F. Zalesak, "Coefficients for Calculating Radiation Impedances and Far-Field Pressures for Free-Flooded Ring Transducers," NRL Report 7749, June, 1974.

3. H. Schenck, "Improved Integral Formulation for Acoustic Radiation Problems," Journal of the Acoustical Society of America, Vol. 44, No. 1, December 1967, pp. 41-58.

4. G.W. Benthien, D. Barach, and D. Gillette, "CHIEF Users Manual," Naval Ocean Systems Center Technical Document 970, Revision 1, September 1988.

Appendix A: Theory

The SHIP program is a boundary element code that is restricted to axisymmetric objects. The starting point for the numerical technique is the Helmholtz Integral Equation

$$\epsilon\phi(\vec{R}) = \int_S \phi(\vec{R}_o)\frac{\partial g(\vec{R}_o|\vec{R})}{\partial n_o} - \frac{\partial\phi(\vec{R}_o)}{\partial n_o} g(\vec{R}_o|\vec{R})\,dS_o + \phi_{inc}(\vec{R}) \qquad (A.1)$$

where

$$\epsilon = \begin{cases} 0 & \vec{R} \text{ interior to S} \\ \tfrac{1}{2} & \vec{R} \text{ on S} \\ 1 & \vec{R} \text{ exterior to S} \end{cases} \qquad (A.2)$$

The free-space Green's Function $g(R_o|R)$ (assuming a time dependence of $e^{i\omega t}$) is given by:

$$g(\vec{R}_o|\vec{R}) = \frac{e^{-ik|\vec{R}_o - \vec{R}|}}{4\pi|\vec{R}_o - \vec{R}|} \qquad (A.3)$$

The SHIP computer program differs from other boundary element programs in that it makes use of a free-space Green's function expressed in cylindrical coordinates- $R=(r, z, \theta)$

$$g(r_o,z_o,\theta_o|r,z,\theta) = \frac{\epsilon_m}{4\pi}\sum_{m=0}^{\infty} F_m(r_o,z_o|r,z)\cos(m(\theta_o - \theta)) \qquad (A.4)$$

where

$$\epsilon_m = \begin{cases} 1 & m=0 \\ 2 & m>0 \end{cases} \qquad (A.5)$$

$$F_m(r_o,z_o|r,z) = \int_0^{\infty} J_m(\ell r_o)\,J_m(\ell r)\,\mu^{-1} e^{-\mu|z_o - z|}\,\ell\,d\ell \qquad (A.6)$$

and

$$\mu = \sqrt{\ell^2 - k^2} \qquad (A.7)$$

The surface normal velocity is related to the gradient of the velocity potential as

$$v = -\frac{\partial \phi}{\partial n} \tag{A.8}$$

Using (A.8) in (A.1) and considering R to be a point on the surface S, $R=(r_s,\ z_s,\ \theta_s)$ the following expression is obtained:

$$\begin{aligned}
\frac{1}{2}\phi(r_s,z_s,\theta_s) = & \int_0^{2\pi}\int_{-h}^{h} g(b,z_o,\theta_o \mid r_s,z_s,\theta_s)\ v(b,z_o,\theta_o) b\, dz_o\, d\theta_o \\
& + \int_0^{2\pi}\int_b^{a} g(r_o,h,\theta_o \mid r_s,z_s,\theta_s)\ v(r_o,h,\theta_o) r_o\, dr_o\, d\theta_o \\
& + \int_0^{2\pi}\int_b^{a} g(r_o,-h,\theta_o \mid r_s,z_s,\theta_s)\ v(r_o,-h,\theta_o) r_o\, dr_o\, d\theta_o \\
& + \int_0^{2\pi}\int_{-h}^{h} g(a,z_o,\theta_o \mid r_s,z_s,\theta_s)\ v(a,z_o,\theta_o) a\, dz_o\, d\theta_o \\
& - \int_0^{2\pi}\int_{-h}^{h} \left[\frac{\partial g(r_o,z_o,\theta_o \mid r_s,z_s,\theta_s)}{\partial r_o}\right]_{r_o=b} \phi(b,z_o,\theta_o) b\, dz_o\, d\theta_o \\
& + \int_0^{2\pi}\int_b^{a} \left[\frac{\partial g(r_o,z_o,\theta_o \mid r_s,z_s,\theta_s)}{\partial z_o}\right]_{z_o=h} \phi(r_o,h,\theta_o) r_o\, dr_o\, d\theta_o \\
& - \int_0^{2\pi}\int_b^{a} \left[\frac{\partial g(r_o,z_o,\theta_o \mid r_s,z_s,\theta_s)}{\partial z_o}\right]_{z_o=-h} \phi(r_o,-h,\theta_o) r_o\, dr_o\, d\theta_o \\
& + \int_0^{2\pi}\int_{-h}^{h} \left[\frac{\partial g(r_o,z_o,\theta_o \mid r_s,z_s,\theta_s)}{\partial r_o}\right]_{r_o=a} \phi(a,z_o,\theta_o) a\, dz_o\, d\theta_o \\
& + \phi_{inc}(r_s,z_s,\theta_s)
\end{aligned} \tag{A.9}$$

In equation (A.9) the outer radius of the ring and the inner radius of the ring are replaced with "a" and "b", respectively, see Figure A.1. In the case of a right circular cylinder, the equation is still valid after one substitutes in $b=0$.

Both the velocity potential and the surface normal velocity may be expanded in Fourier series as follows

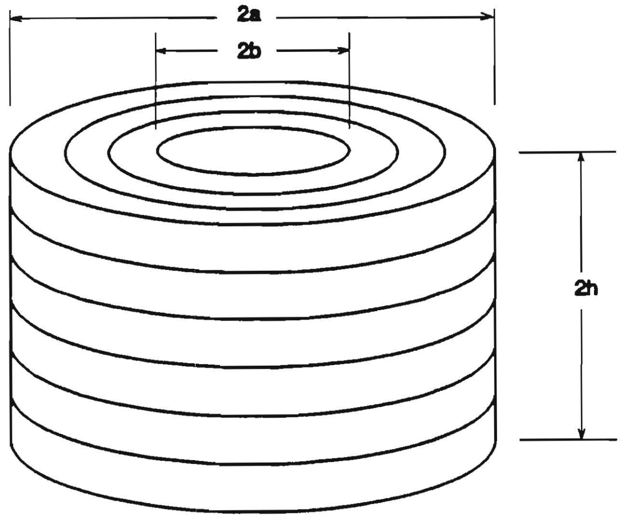

Figure A.1 Ring Transducer Geometry; IMAX=3 and JMAX=6

$$\phi(r,z,\theta) = \sum_{m=0}^{\infty} \phi_m^c(r,z)\cos m\theta + \phi_m^s(r,z)\sin m\theta \tag{A.10}$$

$$v(r,z,\theta) = \sum_{m=0}^{\infty} v_m^c(r,z)\cos m\theta + v_m^s(r,z)\sin m\theta \tag{A.11}$$

The incident pressure $\phi_{inc}(R)$ is given by

$$\phi_{inc}(\overline{R}) = \frac{p_o}{i\rho ck} e^{-ik\hat{N}\cdot\overline{R}} \tag{A.12}$$

where N is a unit vector indicating the direction of the plane wave, see Figure A.1. For scattering problems, the $\theta=0$ coordinate direction is defined by the body's axis of symmetry (the z axis) and the vector N. In the case of normal incidence ($\phi_p=90^o$), $\theta=0$ is the forward-scattering direction; in the general case ($0<\phi_p<90^o$) the component of N perpendicular to the z axis defines the $\theta=0$ direction; finally, the end-incidence case ($\phi_p=0$) is the axisymmetric case where the solution is constant with respect to θ.

Like ϕ and v, the plane wave excitation can be expressed in cylindrical coordinates as follows:

$$\phi_{inc}(r,z,\theta) = \frac{1}{i\rho ck} \sum_{m=0}^{\infty} P_m(r,z) \cos m\theta \tag{A.13}$$

where

$$P_m(r,z) = p_o e^{-ik\cos\phi_p z} \epsilon_m (-i)^m J_m(k\sin\phi_p r) \tag{A.14}$$

Note that the incident pressure is assumed to be even with respect to $\theta=0$. In the special cases $\phi_p=0$ and $\phi_p=90^o$, P_m can be written more simply as

$$\phi_p = 0:$$

$$P_o(r, z) = p_o e^{-ikz}$$

$$P_m(r, z) = 0 \qquad m>0$$

$$\phi_p = 90^o:$$

$$P_m(r, z) = p_o \epsilon_m (-i)^m J_m(kr)$$

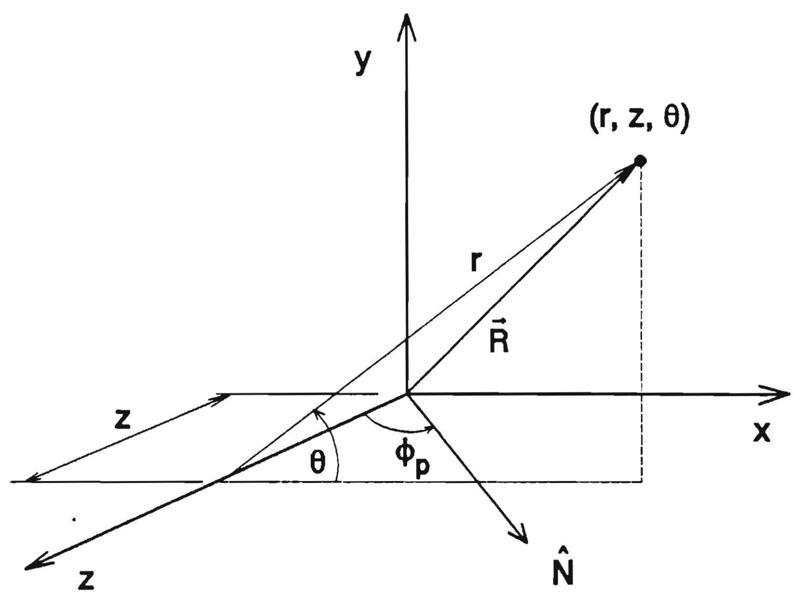

Figure A.2. Definition of cylindrical coordinates and incident pressure.

The next steps are straightforward but cumbersome. The Fourier expansions equations (A.10), (A.11) and (A.13) are substituted into (A.9) and the integrations with respect to θ_o are carried out exactly. The harmonic balance procedure is then used to equate the coefficients of $\cos(m\theta_s)$ and $\sin(m\theta_s)$. This results in two equations for each harmonic m - one pertaining to the cosine and one to the sine component of the integral equation. The equation for the cosine coefficients for the m^{th} harmonic is given by:

$$
\begin{aligned}
\phi_m^c(r_s,z_s) = & \int_{-h}^{h} F_m(b,z_o \,|\, r_s,z_s)\; v_m^c(b,z_o) b\,dz_o \\
& + \int_{b}^{a} F_m(r_o,h \,|\, r_s,z_s)\; v_m^c(r_o,h) r_o\,dr_o \\
& + \int_{b}^{a} F_m(r_o,-h \,|\, r_s,z_s)\; v_m^c(r_o,-h) r_o\,dr_o \\
& + \int_{-h}^{h} F_m(a,z_o \,|\, r_s,z_s)\; v_m^c(a,z_o) a\,dz_o \\
& - \int_{-h}^{h} \left[\frac{\partial F_m(r_o,z_o, \,|\, r_s,z_s)}{\partial r_o} \right]_{r_o=b} \phi(b,z_o) b\,dz_o \\
& + \int_{b}^{a} \left[\frac{\partial F_m(r_o,z_o \,|\, r_s,z_s)}{\partial z_o} \right]_{z_o=h} \phi(r_o,h) r_o\,dr_o \\
& - \int_{b}^{a} \left[\frac{\partial F_m(r_o,z_o \,|\, r_s,z_s)}{\partial z_o} \right]_{z_o=-h} \phi(r_o,-h) r_o\,dr_o \\
& + \int_{-h}^{h} \left[\frac{\partial F_m(r_o,z_o \,|\, r_s,z_s)}{\partial r_o} \right]_{r_o=a} \phi(a,z_o) a\,dz_o \\
& + \frac{2\,P_m(r_s,z_s)}{i\rho ck}
\end{aligned}
\qquad (A.15)
$$

The corresponding equation for the sine component is the same, except that the superscript "c" is replaced with "s" and there is no incident pressure term.

The surface of the ring (cylinder) is divided into IMAX bands on the top and bottom and JMAX bands on the side surfaces. (See Figure A.1) The bands on the top and bottom surfaces have width D_t = (a-b)/IMAX wide, and the bands on the sides are D_s = 2h/JMAX wide, where h is the cylinder half-height. The centers of the top and bottom bands have radial coordinates

$$r_i = (i-0.5)\,D_t, \quad i = 1, \ldots \text{IMAX}, \qquad (A.16)$$

and the centers of the side bands have z coordinates

$$z_j = h - (j-0.5)D_s\,, \quad j = 1, \ldots \text{JMAX}. \tag{A.17}$$

In each band, the quantities $\phi_m{}^c$, $v_m{}^c$, $\phi_m{}^s$, $v_m{}^s$, and P_m are assumed to be constant. Finally, the surface point (r_s, z_s) is moved over the surface and the integral equation is evaluated once for (r_s, z_s) placed midway along each band.

For (r_s, z_s) located within the bands on the inside surface of a ring, the following equations are obtained:

$$\begin{aligned}
\tfrac{1}{2}\,\phi^c_m(b,z_i) &= \frac{1}{i\rho ck} P_m(b,r_i) \\
&+ \sum_{j=1}^{\text{IMAX}} \left[G^{it}_{m_{ij}} v^c_m(r_j,h) + G^{ib}_{m_{ij}} v^c_m(r_j,-h) + M^{it}_{m_{ij}} \phi^c_m(r_j,h) + M^{ib}_{m_{ij}} \phi^c_m(r_j,-h) \right] \\
&+ \sum_{j=1}^{\text{JMAX}} \left[G^{is}_{m_{ij}} v^c_m(a,z_j) + G^{ii}_{m_{ij}} v^c_m(b,r_j) + M^{is}_{m_{ij}} \phi^c_m(a,z_j) + M^{ii}_{m_{ij}} \phi^c_m(b,z_j) \right] \\
& \qquad i = 1, \text{JMAX} \quad ; \quad m = 0, \text{NH}
\end{aligned} \tag{A.18}$$

For (r_s, z_s) located within the bands on the top surface of a ring, the following equations are obtained:

$$\begin{aligned}
\tfrac{1}{2}\,\phi^c_m(r_i,h) &= \frac{1}{i\rho ck} P_m(r_i,h) \\
&+ \sum_{j=1}^{\text{IMAX}} \left[G^{tt}_{m_{ij}} v^c_m(r_j,h) + G^{tb}_{m_{ij}} v^c_m(r_j,-h) + M^{tt}_{m_{ij}} \phi^c_m(r_j,h) + M^{tb}_{m_{ij}} \phi^c_m(r_j,-h) \right] \\
&+ \sum_{j=1}^{\text{JMAX}} \left[G^{ts}_{m_{ij}} v^c_m(a,z_j) + G^{ti}_{m_{ij}} v^c_m(b,r_j) + M^{ts}_{m_{ij}} \phi^c_m(a,z_j) + M^{ti}_{m_{ij}} \phi^c_m(b,z_j) \right] \\
& \qquad i = 1, \text{IMAX} \quad ; \quad m = 0, \text{NH}
\end{aligned} \tag{A.19}$$

For (r_s, z_s) located within the bands on the outside surface of a ring, the following equations are obtained:

$$\tfrac{1}{2}\,\phi^c_m(a,z_i) = \frac{1}{i\rho c k} P_m(a,z_i)$$

$$+ \sum_{j=1}^{IMAX} \left[G^{st}_{m_{ij}} v^c_m(r_j,h) + G^{sb}_{m_{ij}} v^c_m(r_j,-h) + M^{st}_{m_{ij}} \phi^c_m(r_j,h) + M^{sb}_{m_{ij}} \phi^c_m(r_j,-h) \right]$$

$$+ \sum_{j=1}^{JMAX} \left[G^{ss}_{m_{ij}} v^c_m(a,z_j) + G^{si}_{m_{ij}} v^c_m(b,r_j) + M^{ss}_{m_{ij}} \phi^c_m(a,z_j) + M^{si}_{m_{ij}} \phi^c_m(b,z_j) \right]$$

$$i = 1, JMAX \quad ; \quad m = 0, NH$$

(A.20)

For (r_s, z_s) located within the bands on the bottom surface of a ring, the following equations are obtained:

$$\tfrac{1}{2}\,\phi^c_m(r_i,-h) = \frac{1}{i\rho c k} P_m(r_i,-h)$$

$$+ \sum_{j=1}^{IMAX} \left[G^{bt}_{m_{ij}} v^c_m(r_j,h) + G^{bb}_{m_{ij}} v^c_m(r_j,-h) + M^{bt}_{m_{ij}} \phi^c_m(r_j,h) + M^{bb}_{m_{ij}} \phi^c_m(r_j,-h) \right]$$

$$+ \sum_{j=1}^{JMAX} \left[G^{bs}_{m_{ij}} v^c_m(a,z_j) + G^{bi}_{m_{ij}} v^c_m(b,r_j) + M^{bs}_{m_{ij}} \phi^c_m(a,z_j) + M^{bi}_{m_{ij}} \phi^c_m(b,z_j) \right]$$

$$i = 1, IMAX \quad ; \quad m = 0, NH$$

(A.21)

The matrix elements G and M in the equations above are fully described in Appendix B. Note the all together, there are 2*(IMAX+JMAX)*(NH+1) scalar equations given above. There are additional equations that arise due to the sine terms. Those equations are identical to those above except that the superscript "c" is replaced with "s" and the subscript m varies between 1 and NH rather than 0 to NH. The equations for a cylinder can be obtained from the above relations by eliminating any terms having a superscript "i".

For a ring, the total number of bands over the surface is LCMAX = 2*(IMAX+JMAX); for a cylinder, LCMAX=2*IMAX+JMAX. The unknown velocity potentials ϕ and the given surface normal velocities can be organized in the following way into (LCMAX)x1 vectors:

Ring:

$$\underline{\phi_m^c} = [\ \phi_m^c(b, z_{JMAXH}),\ \phi_m^c(b, z_{JMAX-1}),\ \phi_m^c(b, z_{JMAX-2}), \ldots,\ \phi_m^c(b, z_1),$$

$$\phi_m^c(r_1, h), \ldots,\ \phi_m^c(r_{IMAX}, h),\ \phi_m^c(a, z_1), \ldots,\ \phi_m^c(a, z_{JMAX}),$$

$$\phi_m^c(r_{IMAX}, -h), \ldots,\ \phi_m^c(r_1, -h),\ \phi_m^c(b, z_{JMAX}), \ldots,\ \phi_m^c(b, z_{JMAXH+1})]^T$$

(A.22)

where JMAXH=JMAX/2. All other vector quantities for a ring-type transducer are organized in the same way. Note that the bands are numbered starting at the ring half- plane, going along the inner surface to the top surface, then along the top surface to the outside surface, then along the outer surface to the bottom surface, then along the bottom surface to the inside surface, and finally along the inside surface to the ring half-plane. This numbering scheme is displayed in Figure A.3 on a cross-sectional view of the ring having IMAX=3 and JMAX=6 shown in Figure A.1.

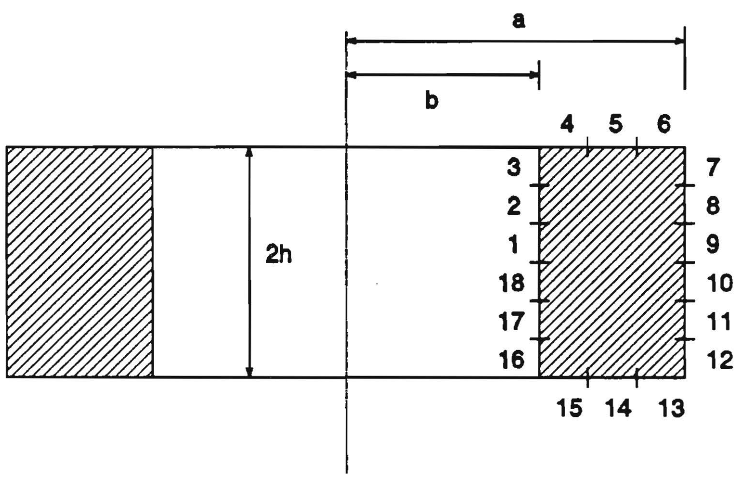

Figure A.3. Ring transducer, cross-sectional view showing band numbering

Cylinder:

$$\underline{\phi_m^c} = [\ \phi_m^c(r_1, h), \ldots,\ \phi_m^c(r_{IMAX}, h),\ \phi_m^c(a, z_1), \ldots,\ \phi_m^c(a, z_{JMAX}),$$

$$\phi_m^c(r_{IMAX}, -h), \ldots,\ \phi_m^c(r_1, -h)]^T$$

(A.23)

All other vector quantities for a cylinder-type transducer are organized in the same way. Note that the bands are numbered starting at the cylinder axis of symmetry on the top surface, going along the top surface to the outside surface, then along the outer surface to the bottom surface, then finally along the bottom surface to the cylinder axis of symmetry. This numbering scheme is displayed in Figure A.4 on a cross-sectional view of a cylinder having IMAX=3 and JMAX=6.

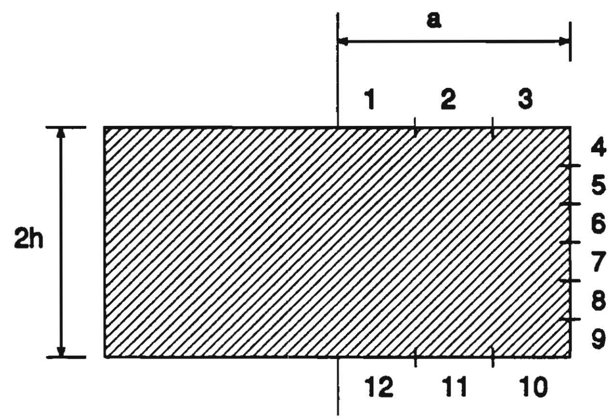

Figure A.4. Cylinder transducer, cross-sectional view showing band numbering

Equations (A.18) through (A.21) can written succinctly in matrix-vector form using the numbering scheme shown above for either ring or cylinder transducers. For every harmonic m, the following matrix system needs to be solved:

$$\left[\frac{1}{2}[I] - [M_m]\right] \underline{\phi^c_m} = [G_m]\, \underline{v^c_m} + \underline{P_m} \tag{A.24a}$$

$$\left[\frac{1}{2}[I] - [M_m]\right] \underline{\phi^s_m} = [G_m]\, \underline{v^s_m} \tag{A.24b}$$

where $\underline{\phi^c_m}$, $\underline{\phi^s_m}$, $\underline{v^c_m}$, $\underline{v^s_m}$, $\underline{P_m}$ are LCMAXx1 vectors

Fortunately, the coefficient matrix for $\underline{\phi}^c{}_m$ and $\underline{\phi}^s{}_m$ is the same. Thus, for each harmonic m, the solution of both systems can be accomplished using a single matrix decomposition followed by two back-substitutions. Once $\underline{\phi}^c{}_m$ and $\underline{\phi}^s{}_m$ are determined for m = 1,NH the pressure and velocity at any angle θ and can be determined from equations (A.10) and (A.11).

Appendix B: Definition of Matrix Elements

As discussed in Appendix A, the Surface Helmholtz Integral Equation is broken into integrals over a finite number bands on the surface. The bands on the top and bottom surfaces are fully characterized by the outer radius "a", the inner radius "b", and the number of bands IMAX. The mid-radius of each band is given by:

$$r_j = (j-0.5)\,D_t, \qquad j = 1,\ldots,\text{IMAX}$$

where D_t=(a-b)/IMAX. The inner and outer radii of the j^{th} band are given, respectively, by r_j^L and r_j^U:

$$r_j^L = r_j - \frac{D_t}{2} \;;\; r_j^U = r_j + \frac{D_t}{2}$$

Similarly, the bands on the cylindrical outside surface (and, for the case of a ring, the inside surface) are fully characterized by the half-height H and the number of bands along the side JMAX. The z coordinates of the midplane of each band is given by:

$$z_j = h - (j-0.5)D_s, \qquad j=1,\ldots,\text{JMAX}$$

where the band height D_s=2h/JMAX. The lower and upper edges of the j^{th} band is given by:

$$z_j^L = z_j - \frac{D_s}{2} \;;\; z_j^U = z_j + \frac{D_s}{2}$$

The r and z coordinates defined above are used in the definitions for the following functions. These functions are used extensively in the expressions for the elements of the pressure and velocity coefficient matrices.

$$f_{jm}(\ell) \equiv \int_{r_j^L}^{r_j^U} J_m(\ell r_o) r_o dr_o \tag{B.1}$$

$$e_j(\mu) \equiv \int_{r_j^L}^{r_j^U} e^{-\mu|z_o-h|} dz_o = \frac{1}{\mu}[e^{-\mu(h-z_j^U)} - e^{-\mu(h-z_j^L)}] \tag{B.2}$$

$$g_{ij}(\mu) \equiv \int_{r_j^L}^{r_j^U} e^{-\mu|z_o - z_i|} dz_o$$

$$= \begin{cases} \frac{1}{\mu}[e^{-\mu(h-z_j^U)} - e^{-\mu(h-z_j^L)}] & i<j \\ \frac{2}{\mu}[1 - e^{-\mu D_s/2}] & i=j \\ \frac{1}{\mu}[e^{-\mu(z_j^L - z_i)} - e^{-\mu(z_j^U - z_i)}] & i>j \end{cases} \qquad \text{(B.3)}$$

Note that:

$g_{ij}(\mu) = g_{ji}(\mu)$ for $i,j=1,\text{JMAX}$

$g_{ij}(\mu) = g_{km}(\mu)$ whenever $|i-j| = |k-m|$

Pressure coefficient matrices:

$$M_{m_{ij}}^{tt} = 0 \qquad \text{(B.4)}$$

$$M_{m_{ij}}^{tb} = -½ \int_0^\infty J_m(\ell r_i) e^{-2\mu h} \ell f_{jm}(\ell) d\ell \qquad \text{(B.5)}$$

$$M_{m_{ij}}^{ts} = \frac{a}{2} \int_0^\infty J_m(\ell r_i)[\frac{m}{a} J_m(\ell a) - \ell J_{m+1}(\ell a)]\mu^{-1} \ell e_j(\mu) d\ell \qquad \text{(B.6)}$$

$$M_{m_{ij}}^{ti} = -\frac{b}{2} \int_0^\infty J_m(\ell r_i)[\frac{m}{b} J_m(\ell b) - \ell J_{m+1}(\ell b)]\mu^{-1} \ell e_j(\mu) d\ell \qquad \text{(B.7)}$$

$$M^{st}_{m_{ij}} = -\tfrac{1}{2}\int_0^\infty J_m(\ell a)e^{-\mu|h-z_i|}\ell f_{jm}(\ell)d\ell \tag{B.8}$$

$$M^{it}_{m_{ij}} = -\tfrac{1}{2}\int_0^\infty J_m(\ell b)e^{-\mu|h-z_i|}\ell f_{jm}(\ell)d\ell \tag{B.9}$$

$$M^{ss}_{m_{ij}} = \frac{a}{2}\int_0^\infty J_m(\ell a)[\frac{m}{a}J_m(\ell a) - \ell J_{m+1}(\ell a)]\mu^{-1}\ell g_{ij}(\mu)d\ell \tag{B.10}$$

$$M^{is}_{m_{ij}} = \frac{a}{2}\int_0^\infty J_m(\ell b)[\frac{m}{a}J_m(\ell a) - \ell J_{m+1}(\ell a)]\mu^{-1}\ell g_{ij}(\mu)d\ell \tag{B.11}$$

$$M^{si}_{m_{ij}} = -\frac{b}{2}\int_0^\infty J_m(\ell a)[\frac{m}{b}J_m(\ell b) - \ell J_{m+1}(\ell b)]\mu^{-1}\ell g_{ij}(\mu)d\ell \tag{B.12}$$

$$M^{ii}_{m_{ij}} = -\frac{b}{2}\int_0^\infty J_m(\ell b)[\frac{m}{b}J_m(\ell b) - \ell J_{m+1}(\ell b)]\mu^{-1}\ell g_{ij}(\mu)d\ell \tag{B.13}$$

Velocity Coefficient matrices:

$$G^{tt}_{m_{ij}} = \tfrac{1}{2}\int_0^\infty J_m(\ell r_i)\mu^{-1}\ell f_{jm}(\ell)d\ell \tag{B.14}$$

$$G^{tb}_{m_{ij}} = \tfrac{1}{2}\int_0^\infty J_m(\ell r_i)\mu^{-1}e^{-2\mu h}\ell f_{jm}(\ell)d\ell \tag{B.15}$$

$$G^{ts}_{m_{ij}} = \frac{a}{2}\int_0^\infty J_m(\ell r_i)J_m(\ell a)\mu^{-1}\ell e_j(\mu)d\ell \tag{B.16}$$

$$G^{ti}_{m_{ij}} = \frac{b}{2}\int_0^\infty J_m(\ell r_i)\, J_m(\ell b)\mu^{-1}\ell e_j(\mu)\, d\ell \qquad \text{(B.17)}$$

$$G^{st}_{m_{ij}} = ½\int_0^\infty J_m(\ell a)\mu^{-1} e^{-\mu|h-z_i|}\ell f_{jm}(\ell)\, d\ell \qquad \text{(B.18)}$$

$$G^{it}_{m_{ij}} = ½\int_0^\infty J_m(\ell b)\mu^{-1} e^{-\mu|h-z_i|}\ell f_{jm}(\ell)\, d\ell \qquad \text{(B.19)}$$

$$G^{ss}_{m_{ij}} = \frac{a}{2}\int_0^\infty J_m(\ell a)^2\mu^{-1}\ell g_{ij}(\mu)\, d\ell \qquad \text{(B.20)}$$

$$G^{is}_{m_{ij}} = \frac{a}{2}\int_0^\infty J_m(\ell b) J_m(\ell a)\mu^{-1}\ell g_{ij}(\mu)\, d\ell \qquad \text{(B.21)}$$

$$G^{si}_{m_{ij}} = \frac{b}{2}\int_0^\infty J_m(\ell a)\, J_m(\ell b)\mu^{-1}\ell g_{ij}(\mu)\, d\ell \qquad \text{(B.22)}$$

$$G^{ii}_{m_{ij}} = \frac{b}{2}\int_0^\infty J_m(\ell b)^2\mu^{-1}\ell g_{ij}(\mu)\, d\ell \qquad \text{(B.23)}$$

These matrix elements are not all distinct. The following relations apply:

$$\left.\begin{aligned} G^{tt}_{m_{ij}} &= G^{bb}_{m_{ij}} \\ M^{tt}_{m_{ij}} &= M^{bb}_{m_{ij}} \end{aligned}\right\} \quad i,j = 1, \text{IMAX} \; ; \quad m = 0, \text{NH} \qquad \text{(B.24)}$$

$$\left.\begin{aligned} G^{tb}_{m_{ij}} &= G^{bt}_{m_{ij}} \\ M^{tb}_{m_{ij}} &= M^{bt}_{m_{ij}} \end{aligned}\right\} \quad i,j = 1, IMAX \; ; \quad m = 0, NH \tag{B.25}$$

$$\left.\begin{aligned} G^{ts}_{m_{ij}} &= G^{bs}_{m_{i,\,JMAX+1-j}} \\ M^{ts}_{m_{ij}} &= M^{bs}_{m_{i,\,JMAX+1-j}} \end{aligned}\right\} \quad i = 1, IMAX \; ; \quad j = 1, JMAX \; ; \quad m = 0, NH \tag{B.26}$$

$$\left.\begin{aligned} G^{st}_{m_{ij}} &= G^{sb}_{m_{JMAX+1-i,j}} \\ M^{st}_{m_{ij}} &= M^{sb}_{m_{JMAX+1-i,j}} \end{aligned}\right\} \quad i = 1, JMAX \; ; \quad j = 1, IMAX \; ; \quad m = 0, NH \tag{B.27}$$

$$\left.\begin{aligned} G^{pp}_{m_{ij}} &= G^{pp}_{m_{ji}} \\ M^{pp}_{m_{ij}} &= M^{pp}_{m_{ji}} \end{aligned}\right\} \quad i,j = 1, JMAX \; ; \quad m = 0, NH \tag{B.28}$$

$$\left.\begin{aligned} G^{pq}_{m_{ij}} &= G^{pq}_{m_{kn}} \\ M^{pq}_{m_{ij}} &= M^{pq}_{m_{kn}} \end{aligned}\right\} \quad i,j,k,n = 1, JMAX \; ; \quad |i-j| = |k-n| \; ; \quad m = 0, NH \tag{B.29}$$

where p and q in (B.28) and (B.29) can be either "i" or "s". Note that relations (B.28) and (B.29) follow directly from the properties of g_{ij}.

Evaluation of Matrix Elements:

The matrix elements given above are computed by breaking the integral from zero to infinity into many small intervals and using Gaussian quadrature over each "small" segment. The calculation is terminated when the contribution from the last segment does not significantly affect the result based on the previous segments.

Note that the quantity $\mu=\text{sqrt}(\ell^2-k^2)$ is pure imaginary for $\ell<k$. Thus, the matrix elements are complex quantities. Furthermore, the imaginary portion of the matrix elements depends solely on the integral from 0 to k; the real portion depends on the integration over the entire range $0\leq\ell<\infty$. It may also be noted that many of the integrands above are singular at $\ell=k$. Accuracy is maintained by using a change of variables procedure described below for a particular example.

Consider the sub-matrix G_m^{tb}:

$$G^{tb}_{m_{ij}} = G_1 + G_2 \tag{B.30}$$

where

$$G_1 = ½\int_0^k J_m(\ell r_i)\mu^{-1}e^{-2\mu h}\ell f_{jm}(\ell)d\ell \tag{B.31}$$

$$G_2 = ½\int_k^\infty J_m(\ell r_i)\mu^{-1}e^{-2\mu h}\ell f_{jm}(\ell)d\ell \tag{B.32}$$

For the range $0\leq\ell<k$, μ is pure imaginary so the quantity μ is replaced with $i\gamma$. To remove the singularity a $\ell=k$, the variable v is introduced:

$$v = \sqrt{k-\ell} \quad ; \quad \ell = k - v^2$$

$$d\ell = -2\,v\,dv$$

Note that

$$\gamma = \sqrt{k^2 - \ell^2} = \sqrt{k-\ell} * \sqrt{k+\ell} = v\sqrt{2k - v^2}$$

Changing variables from ℓ to v, the real and imaginary parts of G_1 are given by:

$$\mathrm{Real}(G_1) = -\int_0^{\sqrt{k}} J_m((k-v^2)r_i)\sin(2hv\sqrt{2k-v^2})f_{jm}(k-v^2)\frac{k-v^2}{\sqrt{2k-v^2}}dv$$

(B.33)

$$\mathrm{Imag}(G_1) = -\int_0^{\sqrt{k}} J_m((k-v^2)r_i)\cos(2hv\sqrt{2k-v^2})f_{jm}(k-v^2)\frac{k-v^2}{\sqrt{2k-v^2}}dv$$

(B.34)

The integral G_2 is also singular at $\ell = k$ so a similar change of variables is used to maintain accuracy during Gaussian integration. The variable u is defined as

$$u = \sqrt{\ell - k}$$

Thus

$$\ell = k + u^2 \; ; \quad d\ell = 2u\,du \; ; \quad \mu = \sqrt{\ell^2 - k^2} = u\sqrt{2k + u^2}$$

The integral G_2 is pure real and is given by

$$G_2 = \int_0^{\infty} J_m((k+u^2)r_i)\, e^{-2hu\sqrt{2k+u^2}} f_{jm}(k+u^2)\frac{k+u^2}{\sqrt{2k+u^2}}du$$

(B.35)

The complete answer can be written explicitly as

$$G^{tb}_{m_{ij}} = (\mathrm{Real}(G_1) + G_2) + i\,\mathrm{Imag}(G_1) \qquad \text{(B.36)}$$

All other matrix elements are calculated using this same procedure.

Appendix C: Far-Field Calculation

The far-field calculations are performed using the asymptotic expressions for the Green's Function and the normal derivative of the Green's Function:

$$g(\vec{R} \mid \vec{R}_o) = \frac{1}{4\pi} \frac{e^{-ikR}}{R} e^{ik\hat{R}\cdot\vec{R}_o} \tag{C.1}$$

$$\frac{\partial g(\vec{R} \mid \vec{R}_o)}{\partial n_o} = \frac{ik}{4\pi} \hat{R}\cdot\hat{n}_o \frac{e^{-ikR}}{R} e^{ik\hat{R}\cdot\vec{R}_o} \tag{C.2}$$

In these expressions, R_o is a vector on the surface and R is a vector describing an exterior point a distance R away from the origin. Equations (C.1) and (C.2) become more accurate as $R \rightarrow \infty$. These expressions can be substituted into (A.1) to yield the far-field asymptotic form of the exterior Helmholtz Integral Equation. When the surface is discretized as discussed in Appendix A and the Fourier expansions for surface velocity and surface velocity potential are substituted into (A.1), the integral equations can be evaluated in "closed-form."

The far-field directions are described using spherical coordinates as shown in Figure C.1. Thus:

$$\hat{R} = \sin\phi \cos\theta\, \hat{i} + \sin\phi \sin\theta\, \hat{j} + \cos\phi\, \hat{k} \tag{C.3}$$

The quantities R_o and n_o are expressed in cylindrical coordinates:

$$\vec{R}_o = r_o \cos\theta_o \hat{i} + r_o \sin\theta_o \hat{j} + z_o \hat{k} \tag{C.4}$$

$$\hat{n}_o = \begin{cases} \hat{k} & ;\ \vec{R}_o \text{ on top} \\ \cos\theta_o \hat{i} + \sin\theta_o \hat{j} & ;\ \vec{R}_o \text{ on outside} \\ -\cos\theta_o \hat{i} - \sin\theta_o \hat{j} & ;\ \vec{R}_o \text{ on inside} \\ -\hat{k} & ;\ \vec{R}_o \text{ on bottom} \end{cases} \tag{C.5}$$

The far-field pressure in each of the far-field directions (i=1,NPTS) is normalized according to

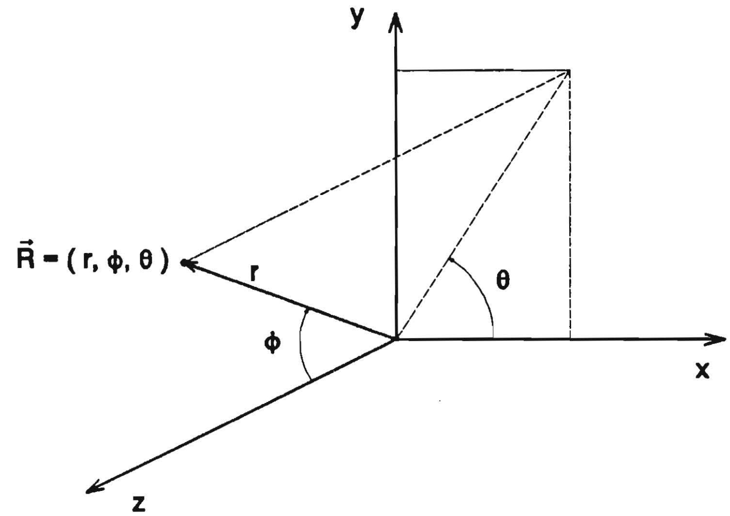

Figure C.1. Spherical coordinates

$$p_{ff}(\vec{R}_i) = \frac{e^{-ikR}}{R} \tilde{p}_{ff}(\vec{R}_i) \tag{C.6}$$

where the "i" in the exponential term is $\sqrt{-1}$, and should not be confused with the far-field direction index. The quantity $\tilde{p}_{ff}$ stays finite as $R \rightarrow \infty$, and is given by

$$\tilde{p}_{ff}(\vec{R}_i) = \sum_{m=0}^{NH} \cos(m\,\theta_i)\, \tilde{p}^c_{ff_m}(\vec{R}_i) + \sum_{m=1}^{NH} \sin(m\,\theta_i)\, \tilde{p}^s_{ff_m}(\vec{R}_i) \tag{C.7}$$

where

$$\tilde{p}^c_{ff_m}(\vec{R}_i) = \sum_{j=1}^{IMAX} \left[G^t_{m_{i,j}} v^c_m(r_j, h) + G^t_{m_{i,j}} v^c_m(r_j, -h) + M^t_{m_{i,j}} \phi^c_m(r_j, h) + M^b_{m_{i,j}} \phi^c_m(r_j, -h) \right]$$
$$+ \sum_{j=1}^{JMAX} \left[G^s_{m_{i,j}} v^c_m(a, z_j) + G^i_{m_{i,j}} v^c_m(b, z_j) + M^s_{m_{i,j}} \phi^c_m(a, z_j) + M^i_{m_{i,j}} \phi^c_m(b, z_j) \right] \tag{C.8}$$

and

$$\tilde{p}^{s}_{ff_m}(\overline{R}_i) = \sum_{j=1}^{IMAX}\left[G^{t}_{m_{i,j}}v^{s}_{m}(r_j,h) + G^{t}_{m_{i,j}}v^{s}_{m}(r_j,-h) + M^{t}_{m_{i,j}}\phi^{s}_{m}(r_j,h) + M^{b}_{m_{i,j}}\phi^{s}_{m}(r_j,-h)\right]$$

$$+ \sum_{j=1}^{JMAX}\left[G^{s}_{m_{i,j}}v^{s}_{m}(a,z_j) + G^{i}_{m_{i,j}}v^{s}_{m}(b,z_j) + M^{s}_{m_{i,j}}\phi^{s}_{m}(a,z_j) + M^{i}_{m_{i,j}}\phi^{s}_{m}(b,z_j)\right]$$

(C.9)

Pressure coefficients:

The coefficients in equations (C.8) and (C.9) are given below. To avoid confusion, all i's that do not occur as subscripts or superscripts are the imaginary number $\sqrt{-1}$; i's that occur as subscripts designate the far-field direction R_i; i's that occur in superscripts designate that the quantity pertains to the inner surface of a ring.

$$M^{t}_{m_{ij}} = \frac{k\cos(\phi_i)\, i^{(m+1)}}{2} e^{ikh\cos\phi_i} f_{jm}(k\sin\phi_i) \qquad \text{(C.10)}$$

$$M^{b}_{m_{ij}} = -\frac{k\cos(\phi_i)\, i^{(m+1)}}{2} e^{-ikh\cos\phi_i} f_{jm}(k\sin\phi_i) \qquad \text{(C.11)}$$

$$M^{s}_{m_{ij}} = \frac{ika\sin\phi_i}{4}\left[i^{m+1}J_{m+1}(ka\sin\phi_i) + i^{m-1}J_{m-1}(ka\sin\phi_i)\right]$$
$$\text{x}\left[\frac{1}{ik\cos\phi_i}\left[e^{ik\cos\phi_i\Delta z} - 1\right]e^{ikz_j^L\cos\phi_i}\right] \qquad \text{(C.12)}$$

$$M^{i}_{m_{ij}} = -\frac{ikb\sin\phi_i}{4}\left[i^{m+1}J_{m+1}(kb\sin\phi_i) + i^{m-1}J_{m-1}(kb\sin\phi_i)\right]$$
$$\text{x}\left[\frac{1}{ik\cos\phi_i}\left[e^{ik\cos\phi_i\Delta z} - 1\right]e^{ikz_j^L\cos\phi_i}\right] \qquad \text{(C.13)}$$

In (C.10) and (C.11), $f_{jm}(\ell)$ is as given in Appendix B (Eq. B.1):

$$f_{jm}(\ell) \equiv \int_{r_j^L}^{r_j^U} J_m(\ell r_o) r_o dr_o \qquad \text{(C.14)}$$

When $\cos(\phi_i)=0$, the bracketed term on the second line of the equations (C.12) and (C.13) above is given by:

$$\left[\Delta z\right] \qquad \text{(C.15)}$$

If m=0, the first bracketed term on the first line of the equations (C.12) and (C.13) above is more concisely given by:

$$\left[2iJ_1(kr\sin\phi_i)\right] \qquad \text{(C.16)}$$

where r=a for (C.12) and r=b for (C.13).

Velocity Coefficients:

The coefficients in equations (C.8) and (C.9) are given below.

$$G^t_{m_{ij}} = \frac{k\rho c\, i^{(m+1)}}{2} e^{ikh\cos\phi_i} f_{jm}(k\sin\phi_i) \qquad \text{(C.17)}$$

$$G^b_{m_{ij}} = \frac{k\rho c\, i^{(m+1)}}{2} e^{-ikh\cos\phi_i} f_{jm}(k\sin\phi_i) \qquad \text{(C.18)}$$

$$G^s_{m_{ij}} = \frac{a}{2} k\rho c\, i^{(m+1)} J_m(ka\sin\phi_i) \times \left[\frac{1}{ik\cos\phi_i} \left[e^{ik\cos\phi_i \Delta z} - 1 \right] e^{ikz_j^L \cos\phi_i} \right] \qquad \text{(C.19)}$$

$$G^i_{m_{ij}} = \frac{b}{2} k\rho c\, i^{(m+1)} J_m(kb \sin\phi_i)$$
$$\times \left[\frac{1}{ik\cos\phi_i} \left[e^{ik\cos\phi_i \Delta z} - 1 \right] e^{ikz_j^L \cos\phi_i} \right] \qquad (C.20)$$

As for the pressure coefficients, when $\cos(\phi_i)=0$, the bracketed term on the second line of the equations (C.19) and (C.20) above is given by $[\Delta z]$.

Appendix D: SHIP Fortran Listing

```
C***********************************************************************
      PROGRAM SHIP92
C***********************************************************************

        COMMON/BANDS/JMAX,JMAXH,IMAX,LCMAX,LCMAXH
        COMMON/HARM/NHARM,NRUN,MH
        COMMON/INTEG/NQD1,NQD2,NBESL,EPS,KMAX
        COMMON/SYM/ISYM,IRHEVEN,IRIGID
        COMMON/GEOM/H,RIN,ROUT
        COMMON/CONST/PI,RHOC
        COMMON/FAR/NPTS,THETA,PHI
        COMMON/PLANE/AINC,PSI
        COMMON/PREL/JMXT,MXD
        COMMON/SURFQ/VEL,ANS
        COMMON/FREQ/FK,FKFAST(8000)
        COMMON/NOPR/IPRO,IPRM,IPRP,IPRD
        COMMON/SURFPV/NTH,THS,DTH

        DIMENSION VEL(200,21),ANS(200,21),JMXT(200),THETA(100),PHI(100)
        COMPLEX VEL,ANS
        CHARACTER*1 BEL
        PARAMETER (BEL = CHAR(7))
        DATA PI/3.141592654/


C***********************************************************************
C               GLOSSARY OF VARIABLES
C***********************************************************************
C  THIS GLOSSARY GIVES THE NAME AND DESCRIPTION OF EACH VARIABLE
C  WHICH IS IN A COMMON BLOCK.  THE VARIABLES ARE GROUPED BY WHICH
C  COMMON BLOCK THEY ARE IN.
C
C  GEOM
C    H            HALF-HEIGHT OF RING OR CYLINDER
C    RIN          INNER RADIUS
C    ROUT         OUTER RADIUS
C
C  CONST
C    PI
C    RHOC         DENSITY * SOUNDSPEED
C
C  BANDS
C    JMAX         # OF BANDS ON SIDE (LIMITED TO 80 OR LESS)
C    JMAXH        HALF THE # OF BANDS ON THE SIDE
C    IMAX         # OF BANDS ON THE TOP AND BOTTOM (20 OR LESS)
C    LCMAX        TOTAL NUMBER OF BANDS (200 OR LESS)
C    LCMAXH       HALF THE TOTAL NUMBER OF BANDS
C
C  COORDS
C    PR           R-COORDINATE OF MIDPOINT OF BAND ON TOP OR BOT
C    PZ           Z-COORDINATE OF MIDPOINT OF BAND ON SIDES
C    PLZ          Z-COORDINATE OF LOWER EDGE OF BAND ON SIDES
C    DELR         DELTA R FOR BANDS ON TOP AND BOTTOM
C    DELZ         DELTA Z FOR BANDS ON SIDES
C
C  HARM
C    NHARM        HIGHEST HARMONIC INCLUDED IN THE ANALYSIS
C                 NHARM MUST BE LESS THAN OR EQUAL TO TEN
C    NRUN         2*NHARM+1:  NUMBER OF SINE AND COSINE TERMS
```

Appendix D: SHIP Fortran Listing

```
C***********************************************************************
      PROGRAM SHIP92
C***********************************************************************

        COMMON/BANDS/JMAX,JMAXH,IMAX,LCMAX,LCMAXH
        COMMON/HARM/NHARM,NRUN,MH
        COMMON/INTEG/NQD1,NQD2,NBESL,EPS,KMAX
        COMMON/SYM/ISYM,IRHEVEN,IRIGID
        COMMON/GEOM/H,RIN,ROUT
        COMMON/CONST/PI,RHOC
        COMMON/FAR/NPTS,THETA,PHI
        COMMON/PLANE/AINC,PSI
        COMMON/PREL/JMXT,MXD
        COMMON/SURFQ/VEL,ANS
        COMMON/FREQ/FK,FKFAST(8000)
        COMMON/NOPR/IPRO,IPRM,IPRP,IPRD
        COMMON/SURFPV/NTH,THS,DTH

        DIMENSION VEL(200,21),ANS(200,21),JMXT(200),THETA(100),PHI(100)
        COMPLEX VEL,ANS
        CHARACTER*1 BEL
        PARAMETER (BEL = CHAR(7))
        DATA PI/3.141592654/


C**************************************************************************
C               GLOSSARY OF VARIABLES
C**************************************************************************
C  THIS GLOSSARY GIVES THE NAME AND DESCRIPTION OF EACH VARIABLE
C  WHICH IS IN A COMMON BLOCK.  THE VARIABLES ARE GROUPED BY WHICH
C  COMMON BLOCK THEY ARE IN.
C
C  GEOM
C    H            HALF-HEIGHT OF RING OR CYLINDER
C    RIN          INNER RADIUS
C    ROUT         OUTER RADIUS
C
C  CONST
C    PI
C    RHOC         DENSITY * SOUNDSPEED
C
C  BANDS
C    JMAX         # OF BANDS ON SIDE (LIMITED TO 80 OR LESS)
C    JMAXH        HALF THE # OF BANDS ON THE SIDE
C    IMAX         # OF BANDS ON THE TOP AND BOTTOM (20 OR LESS)
C    LCMAX        TOTAL NUMBER OF BANDS (200 OR LESS)
C    LCMAXH       HALF THE TOTAL NUMBER OF BANDS
C
C  COORDS
C    PR           R-COORDINATE OF MIDPOINT OF BAND ON TOP OR BOT
C    PZ           Z-COORDINATE OF MIDPOINT OF BAND ON SIDES
C    PLZ          Z-COORDINATE OF LOWER EDGE OF BAND ON SIDES
C    DELR         DELTA R FOR BANDS ON TOP AND BOTTOM
C    DELZ         DELTA Z FOR BANDS ON SIDES
C
C  HARM
C    NHARM        HIGHEST HARMONIC INCLUDED IN THE ANALYSIS
C                 NHARM MUST BE LESS THAN OR EQUAL TO TEN
C    NRUN         2*NHARM+1:  NUMBER OF SINE AND COSINE TERMS
```

```
C    MH             HARMONIC NUMBER
C
C  INTEG
C    NQD1           GAUSSIAN QUADRATURE ORDER FOR INTEGRAL OVER L
C    NQD2           ORDER OF GAUSSIAN QUADRATURE FOR INNER INTEGRATION
C    KMAX           NUMBER OF INTERVALS FROM ZERO TO INFINITY BEFORE
C                   GIVING UP ON THE INTEGRAL
C                   KMAX*NQD1<8000
C    NBESL          STARTING POINT FOR DOWNWARD RECURSION OF BESSEL FUNCTS
C                   SHOULD BE AT LEAST 5 GREATER THAN NHARM
C    EPS            CONVERGENCE PARAMETER FOR INTEGRALS.  INTEGRATION IS
C                   HALTED WHEN CONTRIBUTION TO THE INTEGRAL FROM THE
C                   LAST INTERVAL IS <= EPS*(PREVIOUS INTEGRAL VALUE).
C
C  SYM
C    ISYM           =1 FOR SYMMETRY ABOUT Z=0
C                   =-1 FOR ANTISYMMETRY
C                   =0 FOR NO SYMMETRY
C    IRHEVEN        =1 EXCITATION AND SOLUTION EVEN ABOUT THETA=0
C                   =0 EXCITATION AND SOLUTION ODD ABOUT THETA=0
C    IRIGID         =1 ALL SURFACE VELOCITIES ASSUMED TO BE ZERO
C                      (UNLESS MXD=1)
C                   =0 VELOCITIES PRESCRIBED BY USER
C
C  FAR
C    NPTS           NUMBER OF FAR-FIELD POINTS (LESS THAN OR = TO 100)
C    THETA          VECTOR OF THETA VALUES FOR FAR-FIELD DIRECTIONS (RAD)
C    PHI            VECTOR OF PHI VALUES FOR FAR-FIELD DIRECTIONS (RAD)
C
C  PLANE
C    AINC           AMPLITUDE OF THE INCIDENT PLANE WAVE IN N/M**2
C    PSI            ANGLE OF THE INC WAVE DIRECTION WITH THE Z AXIS (RAD)
C
C  PREL
C    MXD            =1 IF THERE ARE ANY PRESSURE-RELEASE BANDS
C                   (IF MXD=1, ISYM MUST BE +1 or -1)
C    JMXT           INTEGER VECTOR OF LENGTH LCMAXH
C                   JMXT(I)=0:  BAND I IS PRESSURE RELEASE
C                   JMXT(I)=1:  BAND I IS NOT PRESSURE RELEASE
C
C  COEFM
C    DM             COEFFICIENT MATRIX FOR SURFACE PRESSURES
C    GM             COEFFICIENT MATRIX FOR SURFACE VELOCITIES
C
C  SURFQ
C    VEL            MATRIX OF SURFACE VELOCITIES, VEL(I,J) WHERE I
C                   REFERS TO THE BAND NUMBER AND J REFERS TO THE HARMONIC
C                   NUMBER (0<J<NHARM+1 ARE COSINE COMPONENTS AND
C                   NHARM<J<NRUN+1 ARE SINE COMPONENTS)
C    ANS            MATRIX OF SURFACE PRESSURES, ANS(I,J) WHERE I AND
C                   J HAVE THE SAME MEANINGS FOR ANS AS FOR VEL
C
C  FREQ
C    FK             WAVENUMBER
C    FKFAST         VECTOR OF INTEGRATION LIMITS FOR INTEGRAL FROM
C                   ZERO TO INFINITY.
C
C  SURFPV
C    NTH            NUMBER OF CIRCUMFERENTIAL POINTS AROUND EACH BAND AT
C                   WHICH SURFACE PRESS. AND VELOCITIES ARE CALCULATED
C    THS            STARTING VALUE OF THETA FOR PRINTING OF SURFACE
C                   QUANTITIES
```

```
C    DTH          THETA INCREMENT FOR PRINTING OF SURFACE QUANTITIES
C
C  GSPTS
C    TK1          GAUSS POINTS FOR INTEGRAL OVER L, -1<TK1<1
C    WK1          WEIGHTS FOR GAUSSIAN INTEGRATION OVER L
C    TK2          GAUSS POINTS FOR INTEGRAL OVER RO, -1<TK1<1
C    WK2          WEIGHTS FOR GAUSSIAN INTEGRATION OVER RO
C
C  NOPR
C    IPRO         =1 FOR PRINTING OF PROBLEM DATA FROM OLD SHIP CODE
C    IPRM         =1 FOR PRINTING OF COEF. MATRICES FOR ANS AND VEL
C    IPRP         =1 FOR PRINTING OF CHECK OF INCIDENT PRESSURES
C    IPRD         =1 FOR PRINTING OF DIAGNOSTIC INFORMATION
C
C  CTB
C    TTV         G SUP TT
C    TBM         M SUP TB
C    TBV         G SUP TB
C
C  CTS
C    TSM        M SUP TS
C    TSV        G SUP TS
C    TIM        M SUP TI
C    TIV        G SUP TI
C
C  CST
C    STM        M SUP ST
C    STV        G SUP ST
C    ITM        M SUP IT
C    ITV        G SUP IT
C
C  CSS
C    SSM        M SUP SS
C    SSV        G SUP SS
C    SIM        M SUP SI
C    SIV        G SUP SI
C    ISM        M SUP IS
C    ISV        G SUP IS
C    IIM        M SUP II
C    IIV        G SUP II
C
C  BESSL
C    JBESM      MATRIX CONTAINING INT(J_SUB_M(L*RO)*RO*DRO)
C               IT HAS THE FORM JBESM(I,J,K) WHERE
C                  I DESIGNATES THE PARTICULAR GAUSS POINT IN QUESTION
C                  J DESIGNATES THE BAND NUMBER ON THE TOP OR BOT
C                  K DESIGNATES THE HARMONIC NUMBER
C    JBESR      MATRIX CONTAINING J_SUB_M(L*R_SUB_I)
C               IT HAS THE FORM JBESR(I,J,K) WHERE
C                  I DESIGNATES THE PARTICULAR GAUSS POINT IN QUESTION
C                  J DESIGNATES THE BAND NUMBER ON THE TOP OR BOT
C                  K DESIGNATES THE HARMONIC NUMBER
C    JBESA      MATRIX CONTAINING J_SUB_M(L*ROUT)
C               IT HAS THE FORM JBESA(I,K) WHERE
C                  I DESIGNATES THE PARTICULAR GAUSS POINT IN QUESTION
C                  K DESIGNATES THE HARMONIC NUMBER
C    JBESB      MATRIX CONTAINING J_SUB_M(L*RIN)
C               IT HAS THE FORM JBESB(I,K) WHERE
C                  I DESIGNATES THE PARTICULAR GAUSS POINT IN QUESTION
C                  K DESIGNATES THE HARMONIC NUMBER
C
C
```

```
C***************************************************************************
      OPEN(4,FILE='sh.inp',STATUS='old')

      WRITE(6,*) ' READING DATA.... '

      READ(4,*) RIN
      READ(4,*) ROUT
      READ(4,*) H
      READ(4,*) FKS,DELFK,NF
      READ(4,*) RHOC
      READ(4,*) ISYM
      READ(4,*) IRHEVEN

      READ(4,*) JMAXH
      READ(4,*) IMAX
      IF(RIN.EQ.0.0) THEN
         LCMAXH = IMAX + JMAXH
      ELSE
         LCMAXH = 2*JMAXH + IMAX
      END IF
      LCMAX = 2*LCMAXH
      JMAX = 2*JMAXH

      READ(4,*) AINC
      READ(4,*) PSI
      PSI=PSI*PI/180.0

      READ(4,*) NHARM
      NRUN=2*NHARM+1
      READ(4,*) NTH
      READ(4,*) THS
      READ(4,*) DTH
      THS = THS*PI/180.0
      DTH = DTH*PI/180.0

      READ(4,*) EPS
      READ(4,*) KMAX
      READ(4,*) NQD1
      READ(4,*) NQD2

      READ(4,*) NPTS
      DO 150 I=1,NPTS
         READ(4,*) THETA(I),PHI(I)
         THETA(I) = THETA(I)*PI/180.0
         PHI(I) = PHI(I)*PI/180.0
 150  CONTINUE

      READ(4,*) IRIGID
      IF(IRIGID.EQ.1) THEN
         DO I=1,LCMAX
            DO J=1,NRUN
               VEL(I,J) = CMPLX(0.0,0.0)
            END DO
         END DO
      ELSE
         LCM=LCMAX
         IF(ABS(ISYM).EQ.1) LCM=LCMAXH
         NR=NRUN
         IF(IRHEVEN.EQ.1) NR=NHARM+1
         DO I=1,LCM
            DO J=1,NR
```

```
              READ(4,*) VR, VI
              VEL(I,J)=CMPLX(VR,VI)
           END DO
        END DO
     END IF

     READ(4,*) MXD
     IF(MXD.EQ.1) THEN
          DO 20 I=1,LCMAXH
               READ(4,*) JMXT(I)
 20       CONTINUE
     END IF

     READ(4,*) IPRO,IPRM,IPRP,IPRD


C  NBESL IS THE STARTING POINT FOR DOWNWARD RECURSION ON BESSEL FUNCTIONS.
C  IT SHOULD BE AT LEAST 5 GREATER THAN THE LARGEST ORDER OF BESSEL
C  FUNCTION USED BY THE PROGRAM-- NHARM+1.

     NBESL = NHARM + 1 + 5


     DO 900 I=1,NF
        FK = FKS + (I-1)*DELFK
        CALL SOLRIN
        IF(NPTS.GT.0) CALL FARFLD
 900 CONTINUE

      TYPE *, BEL, BEL, BEL

     STOP
     END
```

```
C*****************************************************************
        SUBROUTINE SOLRIN
C*****************************************************************

        COMPLEX V,RRCC,RADIMP,ANS,VEL,DM,GM,VEL1,VEL2
        COMPLEX VV,AA
        COMPLEX PINC1(200),PINC(200,21)
        COMPLEX ANCS(200),ANCA(200),ANSS(200),ANSA(200),DM1(200,200)
        COMPLEX TVECT(200)
        DIMENSION BODS(200),IPIVTR(200)
        DIMENSION ANS(200,21),V(200),VEL(200,21),GM(200,200),
     1   DM(200,200),VEL1(200),PZ(80),PLZ(80),PR(20),
     2   VEL2(200),JMXT(200)
        DIMENSION TK1(100),WK1(100),CK(100)
        DIMENSION TK2(100),WK2(100)
        DIMENSION THETA(100),PHI(100)

        COMMON/BANDS/JMAX,JMAXH,IMAX,LCMAX,LCMAXH
        COMMON/COORDS/PR,PZ,PLZ,DELR,DELZ
        COMMON/HARM/NHARM,NRUN,MH
        COMMON/INTEG/NQD1,NQD2,NBESL,EPS,KMAX
        COMMON/SYM/ISYM,IRHEVEN,IRIGID
        COMMON/GEOM/H,RIN,ROUT
        COMMON/CONST/PI,RHOC
        COMMON/FAR/NPTS,THETA,PHI
        COMMON/PLANE/AINC,PSI
        COMMON/PREL/JMXT,MXD
        COMMON/COEFM/DM,GM
        COMMON/SURFQ/VEL,ANS
        COMMON/FREQ/FK,FKFAST(8000)
        COMMON/NOPR/IPRO,IPRM,IPRP,IPRD
        COMMON/SURFPV/NTH,THS,DTH
        COMMON/GSPTS/TK1,WK1,TK2,WK2
C
        RRCC = FK * CMPLX(0.0, RHOC)
C
C CALL TO MESH CREATES VECTORS PR, PZ, PLZ AND TERMS DELZ AND DELR

        WRITE(6,*) ' GENERATING MESH.... '
        CALL MESH

C
C  PRINT OUTPUT TO FILE

        IF(IPRO.EQ.1) THEN
  88       IF(RIN.NE.0.) WRITE(15,744)
 744       FORMAT (//,45X,15HRING TRANSDUCER//)
           IF(RIN.EQ.0.) WRITE(15,746)
 746       FORMAT(//,43X,19HCYLINDER TRANSDUCER//)
           WRITE(15,701) RIN,ROUT,H,FK,NQD1,IMAX,JMAX
 701       FORMAT(3X,5HRIN =,F7.3,3X,6HROUT =,F7.3,3X,3HH =,F7.3,3X,
     &     4HFK =,F7.3,/,3X,6HNQD1 =,I3,3X,6HIMAX =,I3,3X,6HJMAX =,
     &     I3,//)
        END IF

C
C  CALCULATE INCIDENT PRESSURE VECTOR (PROVIDED THAT AINC GT .0)
C
         IF(AINC.GT.0.0) THEN
            WRITE(6,*) ' CALCULATING INCIDENT PRESSURES....'
            CALL PLWAVE(PINC)
         ELSE
```

```
           DO 4060 I=1,LCMAX
              DO 4060 J=1,NHARM+1
                  PINC(I,J) = CMPLX(0.0,0.0)
 4060      CONTINUE
        END IF

C  CREATE GAUSS QUADRATURE POINTS

       WRITE(6,*) ' GENERATING GAUSS POINTS AND WEIGHTS.... '
       CALL GAUSS( NQD1, TK1, WK1, CK)
       CALL GAUSS( NQD2, TK2, WK2, CK)

C  INITIALIZE MATRICES AND PARAMETERS NEEDED FOR INTEGRATION SUBROUTINES

       CALL INITIAL

C  BEGIN HARMONIC LOOP
C    MH = THE HARMONIC NUMBER;  MH=0,1,2,3,...,NHARM
C    IH = THE INDEX PERTAINING TO COSINE COMPONENTS
C    IS = THE INDEX PERTAINING TO SINE COMPONENTS

       DO 1000 IH=1,NHARM+1
       MH=IH-1
       IS=IH+NHARM

C  CALCULATE MATRIX ELEMENTS USING SUBROUTINE ERING

       WRITE(6,*) ' BEGINNING SOLUTION FOR HARMONIC ',MH,' .... '
       CALL ERING

       IF(IPRM.EQ.1) THEN

       WRITE(3,*)
       WRITE(3,*)
       WRITE(3,*) ' HARMONIC NUMBER: ',MH
       WRITE(3,*)
       WRITE(3,*) '   ROW   COL                DM',
    &     '                          GM '
       WRITE(3,*)
       DO 9011 I=1,LCMAXH
       DO 9011 J=1,LCMAX
          WRITE(3,'(3X,I3,3X,I3,5X,2(G12.5,2X),3X,2(G12.5,2X))')
    &        I,J,DM(I,J),GM(I,J)
 9011  CONTINUE

       END IF

C
C  INITIALIZE ANCS (SOLUTION FOR COSINE COMPONENT,  SYMMETRIC    PART)
C             ANCA (   "      "  COSINE      "    ANTI-SYMMETRIC   " )
C             ANSS (   "      "   SINE       "       SYMMETRIC     " )
C             ANSA (   "      "   SINE       "    ANTISYMMETRIC    " )

C  FORM PRESSURE COEF MATRIX FOR SYMMETRIC SUBCASE, DM1

       DO 14 J=1,LCMAXH
          ANCS(J) = CMPLX(0.,0.)
          ANCA(J) = CMPLX(0.,0.)
          ANSA(J) = CMPLX(0.,0.)
          ANSA(J) = CMPLX(0.,0.)
          DO 14 I=1,LCMAXH
             DM1(I,J) = DM(I,J) + DM(I, LCMAX + 1 -J)
```

```
                GM(I,J) = GM(I,J) + GM(I, LCMAX + 1 -J)
   14     CONTINUE

C   ISYM=0 IS THE GENERAL CASE OF NO SYMMETRY OR ANTI-SYMMETRY WITH
C   RESPECT TO THE Z=0 PLANE

          IF(ISYM.EQ.0) GO TO 202

C   TAKE CARE OF SYMMETRY.  ISYM=1 MEANS THAT V AND PINC ARE SYM WRT Z=0
C                           ISYM=-1 "      "    "   "    "     " ANTI-SYM "
C                           ISYM=0 GENERAL CASE
C   IF ISYM= +1 OR -1, ONLY THE UPPER HALF OF VEL NEED BE SPECIFIED

          DO 203 II = 1, LCMAXH
             VEL(LCMAX + 1 - II,IH) = ISYM * VEL(II,IH)
  203     CONTINUE

  202     CONTINUE

          WRITE(6,*) '   TREATING SYMMETRIC PART.... '

C   CALCULATE VELOCITY FOR SYMMETRIC PART

          IF(ISYM.EQ.-1) GO TO 99
          DO 7 J=1,LCMAXH
             VEL1(J) = .5*(VEL(J,IH) + VEL(LCMAX + 1 - J,IH) )
             PINC1(J) = .5*(PINC(J,IH) + PINC(LCMAX + 1 - J,IH) )
             IF((IH.EQ.1).OR.(IRHEVEN.EQ.1)) GO TO 7
             VEL2(J) = .5*(VEL(J,IS)+VEL(LCMAX+1-J,IS))
    7     CONTINUE

C   TAKE CARE OF PRESSURE-RELEASE SECTIONS

          IF(MXD.NE.1) GO TO 806
          DO 805 J=1,LCMAXH
             IF(JMXT(J).NE.0 ) GO TO 805
             DO 807 I=1,LCMAXH
                DM1(I,J) = - GM(I,J)
                GM(I,J) = CMPLX(0.0,0.0)
  807        CONTINUE
  805     CONTINUE

  806     CONTINUE

C   CALCULATE [G]{VEL1} + {PINC1}

          DO 246 I=1,LCMAXH
             V(I) = PINC1(I)
             DO 246 J=1,LCMAXH
                V(I) = V(I) + GM(I,J)*VEL1(J)
  246     CONTINUE

C   SOLVE SYSTEM OF EQUATIONS

          WRITE(6,*) '     SOLVING MATRIX EQUATION.... '
          CALL ODS(1,DM1,LCMAXH,V,ANCS,TVECT,BODS,IPIVTR)

C SOLVE AGAIN IF IRHEVEN=0

         IF((IRHEVEN.EQ.0).AND.(IH.GT.1)) THEN

            DO 5246 I=1,LCMAXH
```

```
            V(I) = CMPLX(0.0,0.0)
            DO 5246 J=1,LCMAXH
               V(I) = V(I) + GM(I,J)*VEL2(J)
 5246    CONTINUE

         CALL ODS(2,DM1,LCMAXH,V,ANSS,TVECT,BODS,IPIVTR)

      END IF

C
      IF(ISYM.EQ.1) GO TO 98

   99 CONTINUE

      WRITE(6,*) '   TREATING ANTISYMMETRIC PART.... '

C  FORM PRESSURE COEF MATRIX FOR ANTISYMMETRIC PART

      DO 15 I=1,LCMAXH
         DO 15 J=1,LCMAXH
            DM1(I,J) = DM(I,J) - DM(I,LCMAX+1-J)
            GM(I,J) = GM(I,J) - GM(I,LCMAX+1-J) - GM(I,LCMAX+1-J)
   15 CONTINUE

C  CALCULATE VELOCITY FOR ANTISYMMETRIC PART

      DO 8 J=1,LCMAXH
         VEL1(J) = .5*(VEL(J,IH) - VEL(LCMAX + 1 - J,IH) )
         PINC1(J) = .5*(PINC(J,IH) - PINC(LCMAX + 1 - J,IH) )
         IF((IH.EQ.1).OR.(IRHEVEN.EQ.1)) GO TO 8
         VEL2(J) = .5*(VEL(J,IS) - VEL(LCMAX+1-J,IS))
    8 CONTINUE

C  CALCULATE [G]{VEL1} + {PINC1}

      DO 248 I=1,LCMAXH
         V(I) = PINC1(I)
         DO 248 J=1,LCMAXH
            V(I) = V(I) + GM(I,J)*VEL1(J)
  248 CONTINUE

C  SOLVE SYSTEM OF EQUATIONS

       WRITE(6,*) '     SOLVING MATRIX EQUATION.... '
       CALL ODS(1,DM1,LCMAXH,V,ANCA,TVECT,BODS,IPIVTR)
C
      IF ((IRHEVEN.EQ.0).AND.(IH.GT.1)) THEN

         DO 5248 I=1,LCMAXH
            V(I) = CMPLX(0.0,0.0)
            DO 5248 J=1,LCMAXH
               V(I) = V(I) + GM(I,J)*VEL2(J)
 5248    CONTINUE

         CALL ODS(2,DM1,LCMAXH,V,ANSA,TVECT,BODS,IPIVTR)

      END IF

C  ADD SYMMETRIC AND ANTISYMMETRIC PARTS

   98 DO 10 J=1,LCMAXH
         ANS(J,IH) = ANCS(J) + ANCA(J)
```

```
            ANS(LCMAX + 1 - J,IH) = ANCS(J) - ANCA(J)
   10    CONTINUE

         IF((IRHEVEN.EQ.0).AND.(IH.GT.1)) THEN
         DO 5010 J=1,LCMAXH
            ANS(J,IS) = ANSS(J) + ANSA(J)
            ANS(LCMAX+1-J,IS) = ANSS(J) - ANSA(J)
 5010    CONTINUE
         END IF
C
C
C  ADJUST FOR PRESSURE-RELEASE SECTIONS

         IF(MXD.NE.1) GO TO 809

         DO 808 J=1,LCMAXH
            IF(JMXT(J).NE.0 ) GO TO 808
            VEL(J,IH) = ANS(J,IH)
            VEL(LCMAX+1-J,IH) = ANS(J,IH)
            ANS(J,IH) = CMPLX(0.,0.)
            ANS(LCMAX+1-J,IH) = CMPLX(0.,0.)
  808    CONTINUE

         IF(IRHEVEN.EQ.0) THEN
         DO 1808 J=1,LCMAXH
            IF(JMXT(J).NE.0 ) GO TO 1808
            VEL(J,IS) = ANS(J,IS)
            VEL(LCMAX+1-J,IS) = ANS(J,IS)
            ANS(J,IS) = CMPLX(0.,0.)
            ANS(LCMAX+1-J,IS) = CMPLX(0.,0.)
 1808    CONTINUE
         END IF

  809    CONTINUE

 1000    CONTINUE

C  CALCULATE RADIATION IMPEDANCE UNLESS SURFACE VELOCITY=0

         IF(IRIGID.NE.1) THEN
            WRITE(6,*) ' CALCULATING RADIATION IMPEDANCE.... '
            CALL ZRAD(RADIMP)
         END IF

C  CONVERT VELOCITY POTENTIAL TO PRESSURE

         DO 401 J=1,LCMAX
            DO 401 I=1,2*NHARM+1
               ANS(J,I) = RRCC*ANS(J,I)
  401    CONTINUE

C
C   PRINT INPUT DATA FOR FILE CONTAINING SURFACE PRESSURES AND VELOCITIES
C
           WRITE(6,*) ' WRITING SURFACE PRESSURES AND VELOCITIES... '
           WRITE(15,*)
           IF(RIN.NE.0.0) THEN
              WRITE(15,*) ' RING TRANSDUCER '
           ELSE
              WRITE(15,*) ' CYLINDER TRANSDUCER '
           END IF
           WRITE(15,*)
```

```
         WRITE(15,*) ' WAVENUMBER, K= ',FK
         WRITE(15,*) ' KA=             ',FK*ROUT
         WRITE(15,*) ' FLUID IMPEDANCE, RHOC= ',RHOC
         WRITE(15,*)
         WRITE(15,*) ' OUTER RADIUS= ',ROUT,' M'
         WRITE(15,*) ' INNER RADIUS= ',RIN,' M'
         WRITE(15,*) ' CYLINDER HALF-HEIGHT= ',H,' M'
         WRITE(15,*) ' BANDS ON TOP AND BOTTOM, IMAX= ',IMAX
         WRITE(15,*) ' BANDS ON SIDE SURFACES, JMAX= ',JMAX
         WRITE(15,*) ' TOTAL NUMBER OF BANDS, LCMAX= ',LCMAX
         WRITE(15,*)
         WRITE(15,*) ' INCIDENT PRESSURE AMPLITUDE= ',AINC,' PA'
         WRITE(15,*) ' PLANE WAVE ANGLE WITH Z-AXIS= ',PSI*180./PI,
     &        ' DEG'
         WRITE(15,*)
         WRITE(15,*) ' SYMMETRY CONDITION, ISYM= ',ISYM
         WRITE(15,*) ' RIGHT-HAND SIDE EVENESS, IRHEVEN= ',IRHEVEN
         WRITE(15,*) ' NUMBER OF HARMONICS, NHARM= ',NHARM
         WRITE(15,*)
         WRITE(15,*) ' CONVERGENCE PARAMETER, EPS= ',EPS
         WRITE(15,*) ' MAXIMUM NUMBER OF INTERVALS, KMAX= ',KMAX
         WRITE(15,*) ' NUMBER OF GAUSS POINTS, NQD1= ',NQD1
         WRITE(15,*) ' NUMBER OF GAUSS POINTS, NQD2= ',NQD2
         WRITE(15,*) ' HIGHEST HARMONIC, NBESL= ',NBESL
         WRITE(15,*) ' PRESSURE-RELEASE PARAMETER, MXD= ',MXD
         WRITE(15,*)
         IF(IRIGID.NE.1) THEN
            WRITE(15,*) ' COMPLEX RADIATION IMPEDANCE, RADIMP= ',
     &                 RADIMP
            WRITE(15,*) ' NORMALIZED BY RHOC*(TOTAL SURFACE AREA)'
         END IF
         WRITE(15,*)
         WRITE(15,*)
C
C  PUT THE SERIES TOGETHER

       WRITE(15,*) '            *** PRESSURES AT ',
     $ 'SELECTED LOCATIONS ***'
       WRITE(15,*)
       WRITE(15,*) 'THETA (DEG)     REAL       IMAGINARY ',
     &    '      MAGNITUDE     PHASE (DEG) '
       LCM=LCMAX
       IF(ABS(ISYM).EQ.1) LCM=LCMAXH
       DO 542 J=1,LCM
         WRITE(15,*)
         IF((MXD.EQ.1).AND.(JMXT(J).EQ.0)) THEN
           WRITE(15,*) '  BAND NUMBER',J,' (PRESSURE-RELEASE BAND) '
         ELSE
           WRITE(15,*) '  BAND NUMBER',J
         END IF
         WRITE(15,*)

         DO 542 K=1,NTH
           TH=THS+DTH*(K-1)
           AA=ANS(J,1)
           DO 541 I=1,NHARM
             II=I+1
             IS=IH+NHARM
             AA=AA+ANS(J,I+1)*COS(I*TH)+
     $         ANS(J,IS+1)*SIN(I*TH)
  541      CONTINUE
```

```
          PMAG=CABS(AA)
          IF(PMAG.NE.0.0) THEN
            ANGD = ATAN2D(AIMAG(AA),REAL(AA))
          ELSE
            ANGD = 0.0
          END IF
          THD=TH*180.0/PI
          WRITE(15,'(2X,F8.3,2(4X,G12.5,2X,G12.5))') THD,AA,PMAG,ANGD
 542      CONTINUE

      IF((IRIGID.EQ.1).AND.(MXD.EQ.0)) THEN
      WRITE(15,*)
      WRITE(15,*)
      WRITE(15,*) ' STATIONARY RIGID SURFACE- ALL VELOCITIES ZERO'
      WRITE(15,*)
      WRITE(15,*)
      ELSE
      WRITE(15,*)
      WRITE(15,*)
      WRITE(15,*) '             *** VELOCITIES (X10**6) AT ',
     $     'SELECTED LOCATIONS ***'
      WRITE(15,*)
      WRITE(15,*) 'THETA (DEG)     REAL       IMAGINARY ',
     &  '     MAGNITUDE     PHASE (DEG) '
      DO 9542 J=1,LCM
        WRITE(15,*)
        IF((MXD.EQ.1).AND.(JMXT(J).EQ.0)) THEN
          WRITE(15,*) '  BAND NUMBER',J,' (PRESSURE-RELEASE BAND) '
        ELSE
          WRITE(15,*) '  BAND NUMBER',J
        END IF
        WRITE(15,*)

        DO 9542 K=1,NTH
          TH=THS+DTH*(K-1)
          VV=VEL(J,1)
          DO 9541 I=1,NHARM
            II=I+1
            IS=IH+NHARM
            VV=VV+VEL(J,I+1)*COS(I*TH)+
     $        VEL(J,IS+1)*SIN(I*TH)
9541      CONTINUE

          VV=VV*1.0E06
          VMAG=CABS(VV)
          IF(VMAG.NE.0.0) THEN
            ANGD = ATAN2D(AIMAG(VV),REAL(VV))
          ELSE
            ANGD = 0.0
          END IF
          THD=TH*180.0/PI
          WRITE(15,'(2X,F8.3,2(4X,G12.5,2X,G12.5))') THD,VV,VMAG,ANGD
9542  CONTINUE
      END IF

      RETURN
      END
```

```
C*****************************************************************
      SUBROUTINE MESH
C*****************************************************************

      DIMENSION PZ(80),PLZ(80),PR(20)

      COMMON/BANDS/JMAX,JMAXH,IMAX,LCMAX,LCMAXH
      COMMON/COORDS/PR,PZ,PLZ,DELR,DELZ
      COMMON/INTEG/NQD1,NQD2,NBESL,EPS,KMAX
      COMMON/GEOM/H,RIN,ROUT
      COMMON/CONST/PI,RHOC
      COMMON/FREQ/FK,FKFAST(8000)

      DELZ = H/JMAXH
      DELR = (ROUT-RIN)/IMAX

      DO 1 J=1,JMAXH
         KC = JMAX + 1 - J
         PZ(J) = H + .5*DELZ - J*DELZ
         PZ(KC) = -PZ(J)
         PLZ(J) = PZ(J)-.5*DELZ
         PLZ(KC)=PZ(KC) - .5*DELZ
    1 CONTINUE

      DO 2 I=1,IMAX
         PR(I) = RIN - .5*DELR + I*DELR
    2 CONTINUE

      RETURN
      END
```

```
C*********************************************************************
      SUBROUTINE PLWAVE(PINC)
C*********************************************************************

      COMPLEX VV,AA,JWRHO
      COMPLEX PAMP,CII,PINC(200,21)
      REAL*8 FX
      REAL*8 BJJD(0:21),BJPD(0:21)
      DIMENSION  PZ(80),PLZ(80),PR(20)

      COMMON/BANDS/JMAX,JMAXH,IMAX,LCMAX,LCMAXH
      COMMON/COORDS/PR,PZ,PLZ,DELR,DELZ
      COMMON/HARM/NHARM,NRUN,MH
      COMMON/INTEG/NQD1,NQD2,NBESL,EPS,KMAX
      COMMON/SYM/ISYM,IRHEVEN,IRIGID
      COMMON/GEOM/H,RIN,ROUT
      COMMON/CONST/PI,RHOC
      COMMON/PLANE/AINC,PSI
      COMMON/FREQ/FK,FKFAST(8000)
      COMMON/SURFPV/NTH,THS,DTH
      COMMON/NOPR/IPRO,IPRM,IPRP,IPRD

      JWRHO=CMPLX(0.0,FK*RHOC)
      ARGZ=FK*COS(PSI)*H
      PAMP=AINC*CMPLX(COS(ARGZ),-SIN(ARGZ))

      IRING=1
      IF(RIN.EQ.0) IRING=0
      DO 4000 I=1,IMAX
         IT=I+IRING*JMAXH
         IB=2*IMAX+JMAX+IRING*JMAXH-I+1
         FX=DBLE(FK*SIN(PSI)*PR(I))
         CALL BESJ_N(NBESL,FX,BJJD,BJPD)
         DO 4010 J=1,NHARM+1
            CII=CMPLX(0.0,-1.0)**(J-1)
            PINC(IT,J)=PAMP*CII*BJJD(J-1)
            PINC(IB,J)=CONJG(PAMP)*CII*BJJD(J-1)
 4010    CONTINUE
 4000 CONTINUE

      FX=DBLE(FK*SIN(PSI)*ROUT)
      CALL BESJ_N(NBESL,FX,BJJD,BJPD)
      DO 4020 I=1,JMAXH
         IT=IMAX+IRING*JMAXH+I
         IB=IMAX+IRING*JMAXH+JMAX-I+1
         ARGZ=FK*COS(PSI)*PZ(I)
         PAMP=AINC*CMPLX(COS(ARGZ),-SIN(ARGZ))
         DO 4030 J=1,NHARM+1
            CII=CMPLX(0.0,-1.0)**(J-1)
            PINC(IT,J)=PAMP*CII*BJJD(J-1)
            PINC(IB,J)=CONJG(PAMP)*CII*BJJD(J-1)
 4030    CONTINUE
 4020 CONTINUE

      IF(IRING.EQ.1) THEN
         FX=DBLE(FK*SIN(PSI)*RIN)
         CALL BESJ_N(NBESL,FX,BJJD,BJPD)
         DO 4040 I=1,JMAXH
            IT=JMAXH+1-I
            IB=2*IMAX+JMAXH+JMAX+I
            ARGZ=FK*COS(PSI)*PZ(I)
            PAMP=AINC*CMPLX(COS(ARGZ),-SIN(ARGZ))
```

```
          DO 4050 J=1,NHARM+1
             CII=CMPLX(0.0,-1.0)**(J-1)
             PINC(IT,J)=PAMP*CII*BJJD(J-1)
             PINC(IB,J)=CONJG(PAMP)*CII*BJJD(J-1)
 4050     CONTINUE
 4040  CONTINUE
       END IF

C
C  ACCOUNT FOR EPS_SUB_M IN THE EXPANSION FOR PLANE WAVE
C  ALSO TURN INCIDENT PRESSURE INTO INCIDENT POTENTIAL PHI_INC
C
       DO 4060 J=1,NHARM+1
          EJ=2.0
          IF(J.EQ.1) EJ=1.0
          DO 4070 I=1,LCMAX
             PINC(I,J)=EJ*PINC(I,J)/JWRHO
 4070     CONTINUE
 4060  CONTINUE
C
       IF(IPRP.EQ.0) RETURN

C  COMPARE FINITE FOURIER SUM WITH EXACT INCIDENT PLANE WAVE PRESSURE:

       WRITE(3,*)
       WRITE(3,*)
       WRITE(3,*) ' INCIDENT PRESSURE FOR PSI=',PSI,' RAD'
       WRITE(3,*)
       WRITE(3,*) '    THETA (DEG)   PINC (FROM HARMONICS)',
     &     '          PINC (FROM EXP(-IKX)) '
       WRITE(3,*)

       IF(IRING.EQ.1) THEN
          DO 5543 J=1,JMAXH
             WRITE(3,*) ' BAND # ',J
             DO 5542 K=1,NTH
                TH=THS+DTH*(K-1)
                ARG=SIN(PSI)*RIN*COS(TH)+COS(PSI)*PZ(JMAXH+1-J)
                ARG=ARG*FK
                VV=AINC*CMPLX(COS(ARG),-SIN(ARG))
                AA=PINC(J,1)*JWRHO
                DO 5541 I=1,NHARM
                   AA=AA+PINC(J,I+1)*COS(I*TH)*JWRHO
 5541           CONTINUE
                WRITE(3,9012) TH*180.0/PI,AA,VV
 5542        CONTINUE
 5543     CONTINUE
       END IF

       DO 6543 J=1,IMAX
          JT=J+IRING*JMAXH
          WRITE(3,*) ' BAND # ',JT
          DO 6542 K=1,NTH
             TH=THS+DTH*(K-1)
             ARG=SIN(PSI)*PR(J)*COS(TH)+COS(PSI)*H
             ARG=ARG*FK
             VV=AINC*CMPLX(COS(ARG),-SIN(ARG))
             AA=PINC(JT,1)*JWRHO
             DO 6541 I=1,NHARM
                AA=AA+PINC(JT,I+1)*COS(I*TH)*JWRHO
 6541        CONTINUE
             WRITE(3,9012) TH*180.0/PI,AA,VV
```

```
6542       CONTINUE
6543    CONTINUE

        DO 7543 J=1,JMAX
           JO=J+IMAX+IRING*JMAXH
           WRITE(3,*) ' BAND # ',JO
           DO 7542 K=1,NTH
              TH=THS+DTH*(K-1)
              ARG=SIN(PSI)*ROUT*COS(TH)+COS(PSI)*PZ(J)
              ARG=ARG*FK
              VV=AINC*CMPLX(COS(ARG),-SIN(ARG))
              AA=PINC(JO,1)*JWRHO
              DO 7541 I=1,NHARM
                 AA=AA+PINC(JO,I+1)*COS(I*TH)*JWRHO
7541          CONTINUE
              WRITE(3,9012) TH*180.0/PI,AA,VV
7542       CONTINUE
7543    CONTINUE

        DO 8543 J=1,IMAX
           JB=J+IMAX+JMAX+IRING*JMAXH
           WRITE(3,*) ' BAND # ',JB
           DO 8542 K=1,NTH
              TH=THS+DTH*(K-1)
              ARG=SIN(PSI)*PR(IMAX-J+1)*COS(TH)+COS(PSI)*(-H)
              ARG=ARG*FK
              VV=AINC*CMPLX(COS(ARG),-SIN(ARG))
              AA=PINC(JB,1)*JWRHO
              DO 8541 I=1,NHARM
                 AA=AA+PINC(JB,I+1)*COS(I*TH)*JWRHO
8541          CONTINUE
              WRITE(3,9012) TH*180.0/PI,AA,VV
8542       CONTINUE
8543    CONTINUE

        DO 9143 J=1,IRING*JMAXH
           JI=JMAXH+2*IMAX+JMAX+J
           WRITE(3,*) ' BAND # ',JI
           DO 9142 K=1,NTH
              TH=THS+DTH*(K-1)
              ARG=SIN(PSI)*RIN*COS(TH)+COS(PSI)*(-PZ(J))
              ARG=ARG*FK
              VV=AINC*CMPLX(COS(ARG),-SIN(ARG))
              AA=PINC(JI,1)*JWRHO
              DO 9141 I=1,NHARM
                 AA=AA+PINC(JI,I+1)*COS(I*TH)*JWRHO
9141          CONTINUE
              WRITE(3,9012) TH*180.0/PI,AA,VV
9142       CONTINUE
9143    CONTINUE

9012    FORMAT(3X,F10.3,5X,2(G12.5,2X),3X,2(G12.5,2X))

        RETURN
        END
```

```
C**********************************************************************
      SUBROUTINE ZRAD(RADIMP)
C**********************************************************************

      COMPLEX SUMJ,RADIMP,ANS,VEL
      DIMENSION ANS(200,21),VEL(200,21),
     1  PZ(80),PLZ(80),PR(20),AREA(200)

      COMMON/BANDS/JMAX,JMAXH,IMAX,LCMAX,LCMAXH
      COMMON/COORDS/PR,PZ,PLZ,DELR,DELZ
      COMMON/HARM/NHARM,NRUN,MH
      COMMON/GEOM/H,RIN,ROUT
      COMMON/CONST/PI,RHOC
      COMMON/SURFQ/VEL,ANS
      COMMON/FREQ/FK,FKFAST(8000)
      COMMON/NOPR/IPRO,IPRM,IPRP,IPRD
      COMMON/SURFPV/NTH,THS,DTH

      SUMJ = CMPLX(0.0,0.0)
      DO 537 J=1,JMAXH
         IF(RIN.EQ.0.) GO TO 600
         AREA(J) = DELZ*RIN
         AREA(JMAXH+IMAX+J) = DELZ*ROUT
         GO TO 537
  600    AREA(IMAX+J) = ROUT*DELZ
  537 CONTINUE

      JRING = JMAXH
      IF(RIN.EQ.0.) JRING=0
      DO 538 J=1,IMAX
         AREA(JRING+J)= PR(J)*DELR
  538 CONTINUE

      DO 539 J=1,LCMAXH
         AREA(LCMAX + 1 - J) = AREA(J)
  539 CONTINUE

      DO 401 J=1,LCMAX
         SUMJ= SUMJ + CONJG(VEL(J,1))*ANS(J,1)*AREA(J)
         DO 402 I=1,NHARM
            IS=I+NHARM
            SUMJ=SUMJ+0.5+(CONJG(VEL(J,I+1))*ANS(J,I+1)+
     $         CONJG(VEL(J,IS+1))*ANS(J,IS+1))*AREA(J)
  402    CONTINUE
  401 CONTINUE

      A1 = RIN+ROUT
      AREAT =A1*( H + H + ROUT - RIN)
      SUMJ = FK*(0.0,1.0)*SUMJ/AREAT
      RADIMP=SUMJ

      IF(IPRO.EQ.1) THEN
         WRITE(3,*)
         WRITE(3,506) RADIMP
         WRITE(3,*)
  506    FORMAT(/' THE COMPLEX RADIATION IMPEDANCE IN UNITS ',
     1     'OF RHO*C*A IS (',1PE10.3,' , ',E10.3,')')
      END IF

      RETURN
      END
```

```
C*****************************************************************
      SUBROUTINE INITIAL
C*****************************************************************

      DIMENSION TK1(100),WK1(100),TK2(100),WK2(100)
      DIMENSION PR(20),PZ(80),PLZ(80)
C
C DIMENSIONS CORRESPOND TO:
C      JBESR=JBESR(KMAX*NQD1,IMAX,NHARM)
C      JBESM=JBESM(KMAX*NQD1,IMAX,NHARM)
C      JBESA=JBESA(KMAX*NQD1,NHARM+1)
C      JBESB=JBESB(KMAX*NQD1,NHARM+1)

      REAL JBESR(8000,20,0:11),JBESM(8000,20,0:11)
      REAL JBESA(8000,0:11),JBESB(8000,0:11)

      COMMON/BANDS/JMAX,JMAXH,IMAX,LCMAX,LCMAXH
      COMMON/COORDS/PR,PZ,PLZ,DELR,DELZ
      COMMON/HARM/NHARM,NRUN,MH
      COMMON/INTEG/NQD1,NQD2,NBESL,EPS,KMAX
      COMMON/CONST/PI,RHOC
      COMMON/GEOM/H,RIN,ROUT
      COMMON/FREQ/FK,FKFAST(8000)
      COMMON/GSPTS/TK1,WK1,TK2,WK2
      COMMON/BESSL/JBESR,JBESM,JBESA,JBESB
      COMMON/NOPR/IPRO,IPRM,IPRP,IPRD
C
C  INITIALIZE MATRICES THAT WILL HOLD BESSEL FUNCTIONS:
C
      DO 1000 KL=1,(KMAX+2)*NQD1
         DO 1000 KI=0,NHARM+1
            JBESA(KL,KI) = 0.0
            JBESB(KL,KI) = 0.0
            DO 1100 JR=1,IMAX
               JBESM(KL,JR,KI) = 0.0
               JBESR(KL,JR,KI) = 0.0
 1100       CONTINUE
 1000 CONTINUE

C INITIALIZE ZEROTH HARMONIC OF ALL MATRICES TO -1.0E+06.
C THIS FLAG IS USED TO DETERMINE WHETHER THAT PARTICULAR QUANTITY
C HAS BEEN CALCULATED BEFORE OR NOT.  IT PREVENTS ANY BESSEL FUNCTION
C FROM BEING EVALUATED MORE THAN ONCE.

      DO 2000 KL=1,(KMAX+2)*NQD1
         JBESA(KL,0) = -1.0E+06
         JBESB(KL,0) = -1.0E+06
         DO 2100 JR=1,IMAX
            JBESM(KL,JR,0) = -1.0E+06
            JBESR(KL,JR,0) = -1.0E+06
 2100    CONTINUE
 2000 CONTINUE

      DUMIN=0.001
      FKFAST(1) = 0.0
      FKFAST(2) = SQRT( 2.0*PI/ROUT - FK )
      DO 3000 K=3,KMAX+1
         U = FKFAST(K-1)
         DELU = PI/(2.0*ROUT*U)
         IF(DELU.LT.DUMIN) DELU = DUMIN
         FKFAST(K) = U + DELU
 3000 CONTINUE
```

```
C PRINT SOME INFO ON THE STEP SIZES:

      IF(IPRD.EQ.0) RETURN
      WRITE(3,*)
      WRITE(3,*) ' SMALLEST STEP SIZE= ',DELU
      WRITE(3,*)
      WRITE(3,*) '  STEP # K          DELU              FKFAST(K)'
      WRITE(3,*)
      KSTEP=KMAX/10
      DO 4000 K=1,10
         DELU= FKFAST(K+1)-FKFAST(K)
         WRITE(3,100) K,DELU,FKFAST(K)
 4000 CONTINUE
      DO 5000 K=20,KMAX,KSTEP
         DELU= FKFAST(K+1)-FKFAST(K)
         WRITE(3,100) K,DELU,FKFAST(K)
 5000 CONTINUE
      WRITE(3,*)
      WRITE(3,*)

  100 FORMAT(5X,I4,8X,F10.5,10X,F10.5)

      RETURN
      END
```

```
C*******************************************************************
      SUBROUTINE CTBM
C*******************************************************************

      DIMENSION TK1(100),WK1(100),TK2(100),WK2(100)
      DIMENSION TTV(20,20),TBM(20,20),TBV(20,20)
      DIMENSION PR(20),PZ(80),PLZ(80)
      REAL*8    BJ(0:21),BJPRIME(0:21)
      REAL*8    RD
      REAL JBESR(8000,20,0:11),JBESM(8000,20,0:11)
      REAL JBESA(8000,0:11),JBESB(8000,0:11)
      COMPLEX TTV,TBM,TBV

      COMMON/BANDS/JMAX,JMAXH,IMAX,LCMAX,LCMAXH
      COMMON/COORDS/PR,PZ,PLZ,DELR,DELZ
      COMMON/HARM/NHARM,NRUN,MH
      COMMON/INTEG/NQD1,NQD2,NBESL,EPS,KMAX
      COMMON/GEOM/H,RIN,ROUT
      COMMON/CONST/PI,RHOC
      COMMON/FREQ/FK,FKFAST(8000)
      COMMON/NOPR/IPRO,IPRM,IPRP,IPRD
      COMMON/CTB/TTV,TBM,TBV
      COMMON/GSPTS/TK1,WK1,TK2,WK2
      COMMON/BESSL/JBESR,JBESM,JBESA,JBESB

      HD=0.5*DELR

      DO 7 JR=1,IMAX

         R2 = PR(JR)
         RL = R2 - HD
         RU = R2 + HD
         C1=(RU-RL)/2.
         C2=(RU+RL)/2.

         DO 7 IR=1,IMAX

            RRI = PR(IR)

            TBVSUMR=0.0
            TTVSUMR=0.0
            TBMSUMR=0.0

            TBVSUMI=0.0
            TTVSUMI=0.0
            TBMSUMI=0.0

            KL=0
            K = 0
 31         K = K + 1

               IF (K .EQ. 1) THEN
                  RA=0.0
                  RB=SQRT( FK )
               ELSE
                  RA=FKFAST(K-1)
                  RB=FKFAST(K)
               ENDIF

               CC1= (RB-RA)/2.0
               CC2=(RB+RA)/2.0
```

```
C  INITIALIZE TERMS FOR SUMMATION IN EACH INTERVAL

                TBVSR=0.0
                TTVSR=0.0
                TBMSR=0.0
                TBVSI=0.0
                TTVSI=0.0
                TBMSI=0.0

                DO 20 J=1, NQD1

                   KL = KL + 1

                   V = CC1*TK1(J)+CC2
                   V2 = V*V

                   IF ( K .EQ. 1) THEN
                      RLX = FK - V2
                   ELSE
                      RLX = FK + V2
                   ENDIF

                   IF(JBESM(KL,JR,0).LT.-999999) THEN
                      JBESM(KL,JR,0)=0.0
                      DO 10 I=1,NQD2
                         R0=C1*TK2(I)+C2
                         RX=RLX*R0
                         RD=DBLE(RX)
                         CALL BESJ_N(NBESL, RD, BJ, BJPRIME)
                         DO 5 KI=0,NHARM
                            BESLM=BJ(KI)
                            SUM1=WK2(I)*BESLM*R0*C1
                            JBESM(KL,JR,KI)= JBESM(KL,JR,KI) + SUM1
  5                      CONTINUE
 10                   CONTINUE
                   END IF

                   IF(JBESR(KL,IR,0).LT.-999999) THEN
                      RRX1=RLX*PR(IR)
                      RD=DBLE(RRX1)
                      CALL BESJ_N(NBESL, RD, BJ, BJPRIME)
                      DO 45 KI=0,NHARM
                         JBESR(KL,IR,KI) = BJ(KI)
 45                   CONTINUE
                   END IF

                   S1 = JBESM(KL,JR,MH)
                   BESL1=JBESR(KL,IR,MH)

C  CALCULATE THE IMAG. PART OF THE INTEGRAL WHEN L LESS THAN FK

                   TBVI=0.0
                   TTVI=0.0
                   TBMI=0.0

                   IF ( K .EQ. 1) THEN
                      VAR1=SQRT( 2.0*FK-V2 )
                      VAR2=2.0*H*V*VAR1
                      VAR3=WK1(J)*BESL1*S1*2*RLX
                      SI=SIN( VAR2 )
                      CO=COS( VAR2 )
C  REAL PART
```

```
                  TBVR=-VAR3*SI/VAR1
                  TTVR=0.0
                  TBMR=-VAR3*CO*V
C  IMAGINARY PART

                  TBVI=-VAR3*CO/VAR1
                  TTVI=-VAR3/VAR1
                  TBMI=VAR3*SI*V
                  GO TO 15
               END IF

               VAR4=SQRT( V2+2.0*FK )
               VAR5=WK1(J)*BESL1*S1*2*RLX
               EX=1.0/EXP( 2.0*H*V*VAR4 )
               TBVR=VAR5*EX/VAR4
               TTVR=VAR5/VAR4
               TBMR=-VAR5*EX*V

 15            TBVSR=TBVSR+TBVR
               TTVSR=TTVSR+TTVR
               TBMSR=TBMSR+TBMR
               TBVSI=TBVSI+TBVI
               TTVSI=TTVSI+TTVI
               TBMSI=TBMSI+TBMI

 20         CONTINUE

C  ADD CONTRIBUTION FROM KTH INTERVAL

            TBVSR=TBVSR*CC1
            TTVSR=TTVSR*CC1
            TBMSR=TBMSR*CC1
            TBVSI=TBVSI*CC1
            TTVSI=TTVSI*CC1
            TBMSI=TBMSI*CC1

            TBVSUMR=TBVSUMR+TBVSR
            TTVSUMR=TTVSUMR+TTVSR
            TBMSUMR=TBMSUMR+TBMSR
            TBVSUMI=TBVSUMI+TBVSI
            TTVSUMI=TTVSUMI+TTVSI
            TBMSUMI=TBMSUMI+TBMSI

C  CHECK FOR CONVERGENCE OF INTEGRAL

            E1=ABS(TBVSR/TBVSUMR)
            E2=ABS(TTVSR/TTVSUMR)
            E3=ABS(TBMSR/TBMSUMR)
            IF(K.GT.KMAX) THEN
               WRITE(3,*)
               WRITE(3,*) ' POOR CONVERGENCE FOR CTBM, MH= ',MH
               WRITE(3,*) ' IR = ', IR, '   JR = ',JR
               IF(E1.GT.EPS) WRITE(3,*) ' GTB: ',E1,TBVSUMR
               IF(E2.GT.EPS) WRITE(3,*) ' GTT: ',E2,TTVSUMR
               IF(E3.GT.EPS) WRITE(3,*) ' MTB: ',E3,TBMSUMR
               GO TO 35
            END IF

         IF((E1.GT.EPS).OR.(E2.GT.EPS).OR.(E3.GT.EPS))
     &         GO TO 31

 35      TBV(IR,JR)= CMPLX( TBVSUMR, TBVSUMI)/2.0
```

```
          TTV(IR,JR)= CMPLX( TTVSUMR, TTVSUMI)/2.0
          TBM(IR,JR)= CMPLX( TBMSUMR, TBMSUMI)/2.0

          IF(IPRD.EQ.1) WRITE(3,*) ' CTBM: IR,JR,K ',IR,JR,K
          IF(IPRD.EQ.1) WRITE(3,*)
     &             ' E1,E2,E3=',E1,E2,E3

 7    CONTINUE

      RETURN
      END
```

```
C***********************************************************************
      SUBROUTINE CSSM
C**********************************************************************

      DIMENSION TK1(100),WK1(100),TK2(100),WK2(100)
      DIMENSION SSM(80,80),SSV(80,80),SIM(80,80),SIV(80,80)
      DIMENSION ISM(80,80),ISV(80,80),IIM(80,80),IIV(80,80)
      DIMENSION PR(20),PZ(80),PLZ(80)
      REAL*8    BJ(0:21),BJPRIME(0:21)
      REAL*8    RD
      REAL JBESR(8000,20,0:11),JBESM(8000,20,0:11)
      REAL JBESA(8000,0:11),JBESB(8000,0:11)
      COMPLEX   SSM,SSV,SIM,SIV,ISM,ISV,IIM,IIV

      REAL ISVSUMR,ISVSUMI,ISVSR,ISVSI,ISVR,ISVI
      REAL IIVSUMR,IIVSUMI,IIVSR,IIVSI,IIVR,IIVI
      REAL ISMSUMR,ISMSUMI,ISMSR,ISMSI,ISMR,ISMI
      REAL IIMSUMR,IIMSUMI,IIMSR,IIMSI,IIMR,IIMI

      COMMON/BANDS/JMAX,JMAXH,IMAX,LCMAX,LCMAXH
      COMMON/COORDS/PR,PZ,PLZ,DELR,DELZ
      COMMON/HARM/NHARM,NRUN,MH
      COMMON/INTEG/NQD1,NQD2,NBESL,EPS,KMAX
      COMMON/SYM/ISYM,IRHEVEN,IRIGID
      COMMON/GEOM/H,RIN,ROUT
      COMMON/CONST/PI,RHOC
      COMMON/FREQ/FK,FKFAST(8000)
      COMMON/NOPR/IPRO,IPRM,IPRP,IPRD
      COMMON/SURFPV/NTH,THS,DTH
      COMMON/GSPTS/TK1,WK1,TK2,WK2
      COMMON/BESSL/JBESR,JBESM,JBESA,JBESB
      COMMON/CSS/SSM,SSV,SIM,SIV,ISM,ISV,IIM,IIV

      A=ROUT
      B=RIN

      DO 7 JR=1,JMAX
         ZL = PLZ(JR)
         ZU = ZL + DELZ
         IR=1
         ZI = PZ(IR)

         SSVSUMR=0.0
         SSMSUMR=0.0
         SSVSUMI=0.0
         SSMSUMI=0.0

         ISVSUMR=0.0
         IIVSUMR=0.0
         ISMSUMR=0.0
         SIMSUMR=0.0
         IIMSUMR=0.0
         ISVSUMI=0.0
         IIVSUMI=0.0
         ISMSUMI=0.0
         SIMSUMI=0.0
         IIMSUMI=0.0

         KL = 0
         K = 0
 31      K = K + 1
```

```
            IF (K .EQ. 1) THEN
               RA=0.0
               RB=SQRT( FK )
            ELSE
               RA=FKFAST(K-1)
               RB=FKFAST(K)
            ENDIF

            CC1= (RB-RA)/2.0
            CC2=(RB+RA)/2.0

C  INITIALIZE

            SSVSR=0.0
            SSMSR=0.0
            SSVSI=0.0
            SSMSI=0.0

            ISVSR=0.0
            IIVSR=0.0
            ISMSR=0.0
            SIMSR=0.0
            IIMSR=0.0
            ISVSI=0.0
            IIVSI=0.0
            ISMSI=0.0
            SIMSI=0.0
            IIMSI=0.0

            DO 20 J=1, NQD1

               KL = KL + 1

               V = CC1*TK1(J)+CC2
               V2 = V*V

               IF ( K .EQ. 1) THEN
                  RLX = FK - V2
               ELSE
                  RLX = FK + V2
               ENDIF

               IF(JBESA(KL,0).LT.-999999) THEN
                  RLA=RLX*ROUT
                  RD=DBLE(RLA)
                  CALL BESJ_N( NBESL, RD, BJ, BJPRIME)
                  DO 80 KI=0,NHARM+1
                     JBESA(KL,KI) = BJ(KI)
 80               CONTINUE
               END IF

               BESLMA=JBESA(KL,MH)
               BESLMA1=JBESA(KL,MH+1)

               IF ( RIN .NE. 0.0 ) THEN
                  IF(JBESB(KL,0).LT.-999999) THEN
                     RLB=RLX*RIN
                     RD=DBLE(RLB)
                     CALL BESJ_N( NBESL, RD, BJ, BJPRIME)
                     DO 90 KI=0,NHARM+1
                        JBESB(KL,KI) = BJ(KI)
 90                  CONTINUE
```

```
            END IF
            BESLMB=JBESB(KL,MH)
            BESLMB1=JBESB(KL,MH+1)
         END IF

         SSVI=0.0
         SSMI=0.0

         ISVI=0.0
         IIVI=0.0
         ISMI=0.0
         SIMI=0.0
         IIMI=0.0

         IF ( K .EQ. 1 ) GO TO 12

         U = V*SQRT( V2 + 2.0*FK )
         IF (IR .GT. JR) THEN
            ANG1=U*(ZL-ZI)
            ANG2=U*(ZU-ZI)
            EXI= 1./EXP(ANG1) - 1.0/EXP(ANG2)
         ELSEIF ( IR .LT. JR) THEN
            ANG1=U*(ZI-ZU )
            ANG2=U*( ZI-ZL )
            EXI= 1.0/EXP(ANG1) - 1.0/EXP(ANG2)
         ELSEIF ( IR .EQ. JR ) THEN
            ANG1=U*( ZI - ZL )
            ANG2=U*( ZU - ZI )
            EXI= 2.0-1.0/EXP(ANG1) - 1.0/EXP(ANG2)
         ENDIF

         VAR3= 2.0*WK1(J)*EXI*RLX/( V*(V2+2.0*FK) )
         VAR4=(MH*BESLMA/A - RLX*BESLMA1 )
         SSVR=VAR3*BESLMA*BESLMA*A
         SSMR=VAR3*VAR4*BESLMA*A
         IF ( RIN .EQ. 0.0 ) GO TO 15
         VAR5=(MH*BESLMB/B - RLX*BESLMB1 )
         ISVR=VAR3*BESLMA*BESLMB*A
         IIVR=VAR3*BESLMB*BESLMB*B
         ISMR=VAR3*VAR4*BESLMB*A
         SIMR=-VAR3*VAR5*BESLMA*B
         IIMR=-VAR3*VAR5*BESLMB*B

         GO TO 15

C  CALCULATE THE IMAG. AND REAL PART OF THE INTEGRAL WHEN L LESS THAN FK

 12      U =V*SQRT( 2.0*FK - V2 )
         IF ( IR .GT. JR ) THEN
            ANG1=U*(ZL-ZI)
            ANG2=U*(ZU-ZI)
            SI=COS(ANG1)-COS(ANG2)
            CI=SIN(ANG2)-SIN(ANG1)
         ELSEIF ( IR .LT. JR ) THEN
            ANG1=U*(ZI-ZU )
            ANG2=U*( ZI-ZL )
            SI=COS(ANG1)-COS(ANG2)
            CI=SIN(ANG2)-SIN(ANG1)
         ELSEIF ( IR .EQ. JR ) THEN
            ANG1=U*(ZI-ZL)
            ANG2=U*( ZU - ZI )
            SI=2.0- COS(ANG1)-COS(ANG2)
```

```
         CI=SIN(ANG1) + SIN(ANG2)
      ENDIF

      VAR0=2.0*WK1(J)*RLX/( V*(2.0*FK - V2))
      VAR1=RLX*BESLMA1-MH*BESLMA/A
      SSVR=-( VAR0*SI*BESLMA*BESLMA*A )
      SSVI=-( VAR0*CI*BESLMA*BESLMA*A )
      SSMR= VAR0*SI*VAR1*BESLMA*A
      SSMI= VAR0*CI*VAR1*BESLMA*A
      IF ( RIN .EQ. 0.0 ) GO TO 15
      VAR2=RLX*BESLMB1-MH*BESLMB/B
      ISVR=-(VAR0*SI*BESLMA*BESLMB*A)
      ISVI=-(VAR0*CI*BESLMA*BESLMB*A)
      IIVR=-(VAR0*SI*BESLMB*BESLMB*B)
      IIVI=-(VAR0*CI*BESLMB*BESLMB*B)
      ISMR=VAR0*SI*VAR1*BESLMB*A
      ISMI=VAR0*CI*VAR1*BESLMB*A
      SIMR=-(VAR0*SI*VAR2*BESLMA*B)
      SIMI=-(VAR0*CI*VAR2*BESLMA*B)
      IIMR=-(VAR0*SI*VAR2*BESLMB*B)
      IIMI=-(VAR0*CI*VAR2*BESLMB*B)

C  SUMMATION TO CALCULATE CONTRIBUTION FOR KTH INTERVAL

 15   SSVSR=SSVSR+SSVR
      SSMSR=SSMSR+SSMR

      SSVSI=SSVSI+SSVI
      SSMSI=SSMSI+SSMI

      IF ( RIN .EQ. 0.0 ) GO TO 20

      ISVSR=ISVSR+ISVR
      IIVSR=IIVSR+IIVR
      ISMSR=ISMSR+ISMR
      SIMSR=SIMSR+SIMR
      IIMSR=IIMSR+IIMR

      ISVSI=ISVSI+ISVI
      IIVSI=IIVSI+IIVI
      ISMSI=ISMSI+ISMI
      SIMSI=SIMSI+SIMI
      IIMSI=IIMSI+IIMI

  20   CONTINUE

C  ADD CONTRIBUTION FROM THE KTH INTERVAL

       SSVSR=SSVSR*CC1
       SSMSR=SSMSR*CC1
       SSVSI=SSVSI*CC1
       SSMSI=SSMSI*CC1

       SSVSUMR=SSVSUMR+SSVSR
       SSMSUMR=SSMSUMR+SSMSR
       SSVSUMI=SSVSUMI+SSVSI
       SSMSUMI=SSMSUMI+SSMSI

       IF (RIN .EQ. 0.0) GO TO 30

       ISVSR=ISVSR*CC1
       IIVSR=IIVSR*CC1
```

```
          ISMSR=ISMSR*CC1
          SIMSR=SIMSR*CC1
          IIMSR=IIMSR*CC1
          ISVSI=ISVSI*CC1
          IIVSI=IIVSI*CC1
          ISMSI=ISMSI*CC1
          SIMSI=SIMSI*CC1
          IIMSI=IIMSI*CC1

          ISVSUMR=ISVSUMR+ISVSR
          IIVSUMR=IIVSUMR+IIVSR
          ISMSUMR=ISMSUMR+ISMSR
          SIMSUMR=SIMSUMR+SIMSR
          IIMSUMR=IIMSUMR+IIMSR
          ISVSUMI=ISVSUMI+ISVSI
          IIVSUMI=IIVSUMI+IIVSI
          ISMSUMI=ISMSUMI+ISMSI
          SIMSUMI=SIMSUMI+SIMSI
          IIMSUMI=IIMSUMI+IIMSI

 30       CONTINUE

C  CHECK FOR CONVERGENCE OF INTEGRAL

          E1=ABS(SSVSR/SSVSUMR)
          E2=ABS(SSMSR/SSMSUMR)
          E3=0.0
          E4=0.0
          E5=0.0
          E6=0.0
          E7=0.0
          IF(RIN.NE.0.0) THEN
             E3=ABS(ISVSR/ISVSUMR)
             E4=ABS(ISMSR/ISMSUMR)
             E5=ABS(IIVSR/IIVSUMR)
             E6=ABS(IIMSR/IIMSUMR)
             E7=ABS(SIMSR/SIMSUMR)
          END IF
          IF(K.GT.KMAX) THEN
             WRITE(3,*)
             WRITE(3,*) ' POOR CONVERGENCE FOR CSSM, MH= ',MH
             WRITE(3,*) ' IR = ', IR, '   JR = ',JR
             IF(E1.GT.EPS) WRITE(3,*) ' GSS: ',E1,SSVSUMR
             IF(E2.GT.EPS) WRITE(3,*) ' MSS: ',E2,SSMSUMR
             IF(E3.GT.EPS) WRITE(3,*) ' GIS: ',E3,ISVSUMR
             IF(E4.GT.EPS) WRITE(3,*) ' MIS: ',E4,ISMSUMR
             IF(E5.GT.EPS) WRITE(3,*) ' GII: ',E5,IIVSUMR
             IF(E6.GT.EPS) WRITE(3,*) ' MII: ',E6,IIMSUMR
             IF(E7.GT.EPS) WRITE(3,*) ' MSI: ',E7,SIMSUMR
             GO TO 35
          END IF

        IF((E1.GT.EPS).OR.(E2.GT.EPS).OR.(E3.GT.EPS)
     &     .OR.(E4.GT.EPS).OR.(E5.GT.EPS).OR.(E6.GT.EPS)
     &     .OR.(E7.GT.EPS)) GO TO 31

 35     SSV(IR,JR)= CMPLX( SSVSUMR, SSVSUMI)/2.0
        SSM(IR,JR)= CMPLX( SSMSUMR, SSMSUMI)/2.0
        IF(IPRD.EQ.1) WRITE(3,*) ' CSSM: JR,K ',JR,K
        IF(IPRD.EQ.1) WRITE(3,*)
     &           ' E1,E2,E3,E4,E5,E6,E7=',E1,E2,E3,E4,E5,E6,E7
```

```
          IF ( RIN .EQ. 0.0 ) GO TO 7

C FOR THE RING

          ISV(IR,JR)=CMPLX( ISVSUMR, ISVSUMI)/2.0
          SIV(IR,JR)=ISV(IR,JR)*B/A
          IIV(IR,JR)=CMPLX( IIVSUMR, IIVSUMI)/2.0
          ISM(IR,JR)=CMPLX( ISMSUMR, ISMSUMI)/2.0
          SIM(IR,JR)=CMPLX( SIMSUMR, SIMSUMI)/2.0
          IIM(IR,JR)=CMPLX( IIMSUMR, IIMSUMI)/2.0

  7     CONTINUE

        DO 66 J=2,JMAX
           IF(J.GT.JMAXH) GO TO 66
           SSM(J,1)=SSM(1,J)
           SSV(J,1)=SSV(1,J)
           ISM(J,1)=ISM(1,J)
           SIM(J,1)=SIM(1,J)
           ISV(J,1)=ISV(1,J)
           IIM(J,1)=IIM(1,J)
           IIV(J,1)=IIV(1,J)
           SIV(J,1)=SIV(1,J)
   66   CONTINUE

        JMAXM1 = JMAX - 1

        DO 122 J=1,JMAXM1
           JILT = JMAX-J+1
           DO 11 M=2,JILT
              IF(M.GT.JMAXH) GO TO 601
              SSM(M,M+J-1) = SSM(1,J)
              SSV(M,M+J-1) = SSV(1,J)
              ISM(M,M+J-1) = ISM(1,J)
              ISV(M,M+J-1) = ISV(1,J)
              IIM(M,M+J-1) = IIM(1,J)
              IIV(M,M+J-1) = IIV(1,J)
              SIM(M,M+J-1) = SIM(1,J)
              SIV(M,M+J-1) = SIV(1,J)
  601         CONTINUE
              IF(M+J-1.GT.JMAXH) GO TO 11
              SSM(M+J-1,M) = SSM(1,J)
              SSV(M+J-1,M) = SSV(1,J)
              ISM(M+J-1,M) = ISM(1,J)
              ISV(M+J-1,M) = ISV(1,J)
              IIM(M+J-1,M) = IIM(1,J)
              IIV(M+J-1,M) = IIV(1,J)
              SIM(M+J-1,M) = SIM(1,J)
              SIV(M+J-1,M) = SIV(1,J)
   11      CONTINUE

  122   CONTINUE

        RETURN
        END
```

```
C***********************************************************************
      SUBROUTINE CSTM
C*********************************************************************

      DIMENSION TK1(100),WK1(100),TK2(100),WK2(100)
      DIMENSION STM(80,20),STV(80,20),ITM(80,20),ITV(80,20)
      DIMENSION PR(20),PZ(80),PLZ(80)
      REAL*8    BJ(0:21),BJPRIME(0:21)
      REAL*8    RD
      REAL JBESR(8000,20,0:11),JBESM(8000,20,0:11)
      REAL JBESA(8000,0:11),JBESB(8000,0:11)
      COMPLEX STM,STV,ITM,ITV
      REAL   ITVSUMR,ITVSUMI,ITVSR,ITVSI,ITVR,ITVI
      REAL   ITMSUMR,ITMSUMI,ITMSR,ITMSI,ITMR,ITMI

      COMMON/BANDS/JMAX,JMAXH,IMAX,LCMAX,LCMAXH
      COMMON/COORDS/PR,PZ,PLZ,DELR,DELZ
      COMMON/HARM/NHARM,NRUN,MH
      COMMON/INTEG/NQD1,NQD2,NBESL,EPS,KMAX
      COMMON/GEOM/H,RIN,ROUT
      COMMON/CONST/PI,RHOC
      COMMON/NOPR/IPRO,IPRM,IPRP,IPRD
      COMMON/FREQ/FK,FKFAST(8000)
      COMMON/GSPTS/TK1,WK1,TK2,WK2
      COMMON/BESSL/JBESR,JBESM,JBESA,JBESB
      COMMON/CST/STM,STV,ITM,ITV

      A=ROUT
      B=RIN
      HD=0.5*DELR

      DO 7 JR=1,IMAX
         R2 = PR(JR)
         RL = R2 - HD
         RU = R2 + HD
         C1=(RU-RL)/2.
         C2=(RU+RL)/2.

         DO 7 IR=1,JMAX

            ARGP= H-PZ(IR)

            STVSUMR=0.0
            STMSUMR=0.0
            STVSUMI=0.0
            STMSUMI=0.0

            ITVSUMR=0.0
            ITVSUMI=0.0
            ITMSUMR=0.0
            ITMSUMI=0.0

            KL = 0
            K = 0
 31         K = K + 1

               IF (K .EQ. 1) THEN
                  RA=0.0
                  RB=SQRT( FK )
               ELSE
                  RA=FKFAST(K-1)
                  RB=FKFAST(K)
```

```
      ENDIF
      CC1= (RB-RA)/2.0
      CC2=(RB+RA)/2.0

C  INITIALIZE

      STVSR=0.0
      STMSR=0.0
      STVSI=0.0
      STMSI=0.0

      ITVSR=0.0
      ITVSI=0.0
      ITMSR=0.0
      ITMSI=0.0

      DO 20 J=1, NQD1

         KL = KL + 1

         V =CC1*TK1(J)+CC2
         V2 =V*V
         IF ( K .EQ. 1 ) THEN
            RLX =FK-V2
         ELSE
            RLX =FK+V2
         ENDIF

         IF(JBESM(KL,JR,0).LT.-999999.0) THEN
            JBESM(KL,JR,0)=0.0
            DO 10 I=1,NQD2
               R0=C1*TK2(I)+C2
               RX=RLX*R0
               RD=DBLE(RX)
               CALL BESJ_N(NBESL, RD, BJ, BJPRIME)
               DO 5 KI=0,NHARM
                  BESLM=BJ(KI)
                  SUM1=WK2(I)*BESLM*R0*C1
                  JBESM(KL,JR,KI)= JBESM(KL,JR,KI) + SUM1
 5             CONTINUE
10          CONTINUE
         END IF
         S1 = JBESM(KL,JR,MH)


         IF(JBESA(KL,0).LT.-999999) THEN
            RLA=RLX*ROUT
            RD=DBLE(RLA)
            CALL BESJ_N( NBESL, RD, BJ, BJPRIME)
            DO 80 KI=0,NHARM+1
               JBESA(KL,KI) = BJ(KI)
80          CONTINUE
         END IF
         BESLMA=JBESA(KL,MH)

         IF ( RIN .NE. 0.0 ) THEN
            IF(JBESB(KL,0).LT.-999999) THEN
               RLB=RLX*RIN
               RD=DBLE(RLB)
               CALL BESJ_N( NBESL, RD, BJ, BJPRIME)
               DO 90 KI=0,NHARM+1
                  JBESB(KL,KI) = BJ(KI)
```

```
 90                          CONTINUE
                          END IF
                          BESLMB=JBESB(KL,MH)
                       ENDIF

C  CALCULATE THE IMAG. PART OF THE INTEGRAL WHEN L LESS THAN FK

                       STVI=0.0
                       STMI=0.0
                       ITVI=0.0
                       ITMI=0.0

                       IF ( K .EQ. 1 ) THEN
                          VAR1=SQRT( FK*2.0 - V2 )
                          VAR2=V*VAR1*ARGP
                          VAR3=2.0*WK1(J)*S1*RLX
                          SI=SIN( VAR2 )
                          CO=COS( VAR2 )
                          STVR=-VAR3*BESLMA*SI/VAR1
                          STVI=-VAR3*BESLMA*CO/VAR1
                          STMR=-VAR3*BESLMA*CO*V
                          STMI= VAR3*BESLMA*SI*V
                          IF (RIN .EQ. 0.0) GO TO 15
                          ITVR=-VAR3*BESLMB*SI/VAR1
                          ITVI=-VAR3*BESLMB*CO/VAR1
                          ITMR=-VAR3*BESLMB*CO*V
                          ITMI= VAR3*BESLMB*SI*V
                          GO TO 15
                       END IF

                       VAR4=SQRT( FK*2.0 + V2 )
                       EX=1.0/ EXP( V*VAR4*ARGP )
                       VAR5=2.0*WK1(J)*S1*EX*RLX
                       STVR=VAR5*BESLMA/VAR4
                       STMR=-VAR5*BESLMA*V
                       IF ( RIN .EQ. 0.0 ) GO TO 15
                       ITVR=VAR5*BESLMB/VAR4
                       ITMR=-VAR5*BESLMB*V

 15                    STVSR=STVSR+STVR
                       STMSR=STMSR+STMR
                       STVSI=STVSI+STVI
                       STMSI=STMSI+STMI

                       IF ( RIN .EQ. 0.0 ) GO TO 20

                       ITVSR=ITVSR+ITVR
                       ITVSI=ITVSI+ITVI
                       ITMSR=ITMSR+ITMR
                       ITMSI=ITMSI+ITMI

 20                 CONTINUE

C  ADD CONTRIBUTION FROM KTH INTERVAL

                    STVSR=STVSR*CC1
                    STMSR=STMSR*CC1
                    STVSI=STVSI*CC1
                    STMSI=STMSI*CC1

                    STVSUMR=STVSUMR+STVSR
                    STMSUMR=STMSUMR+STMSR
```

```
                 STVSUMI=STVSUMI+STVSI
                 STMSUMI=STMSUMI+STMSI

                 IF ( RIN .EQ. 0.0 ) GO TO 30

                 ITVSR=ITVSR*CC1
                 ITVSI=ITVSI*CC1
                 ITMSR=ITMSR*CC1
                 ITMSI=ITMSI*CC1

                 ITVSUMR=ITVSUMR+ITVSR
                 ITVSUMI=ITVSUMI+ITVSI
                 ITMSUMR=ITMSUMR+ITMSR
                 ITMSUMI=ITMSUMI+ITMSI

 30              CONTINUE

C   CHECK FOR CONVERGENCE OF INTEGRAL

                 E1=ABS(STVSR/STVSUMR)
                 E2=ABS(STMSR/STMSUMR)
                 E3=0.0
                 E4=0.0
                 IF(RIN.NE.0.0) THEN
                    E3=ABS(ITVSR/ITVSUMR)
                    E4=ABS(ITMSR/ITMSUMR)
                 END IF
                 IF(K.GT.KMAX) THEN
                    WRITE(3,*)
                    WRITE(3,*) ' POOR CONVERGENCE FOR CSTM, MH= ',MH
                    WRITE(3,*) ' IR = ', IR, '    JR = ',JR
                    IF(E1.GT.EPS) WRITE(3,*) ' GST: ',E1,STVSUMR
                    IF(E2.GT.EPS) WRITE(3,*) ' MST: ',E2,STMSUMR
                    IF(E3.GT.EPS) WRITE(3,*) ' GIT: ',E3,ITVSUMR
                    IF(E4.GT.EPS) WRITE(3,*) ' MIT: ',E4,ITMSUMR
                    GO TO 35
                 END IF

              IF((E1.GT.EPS).OR.(E2.GT.EPS).OR.(E3.GT.EPS)
     &           .OR.(E4.GT.EPS)) GO TO 31

 35           STV(IR,JR)= CMPLX( STVSUMR, STVSUMI)/2.0
              STM(IR,JR)= CMPLX( STMSUMR, STMSUMI)/2.0
              IF(IPRD.EQ.1) WRITE(3,*) ' CSTM: IR,JR,K ',IR,JR,K
              IF(IPRD.EQ.1) WRITE(3,*)
     &               ' E1,E2,E3,E4=',E1,E2,E3,E4

              IF ( RIN .EQ. 0.0 ) GO TO 7
              ITV(IR,JR)= CMPLX( ITVSUMR, ITVSUMI)/2.0
              ITM(IR,JR)= CMPLX( ITMSUMR, ITMSUMI)/2.0

 7      CONTINUE

        RETURN
        END
```

```
C***********************************************************************
      SUBROUTINE CTSM
C*********************************************************************

      DIMENSION TK1(100),WK1(100),TK2(100),WK2(100)
      DIMENSION TSM(20,80),TSV(20,80),TIM(20,80),TIV(20,80)
      DIMENSION PR(20),PZ(80),PLZ(80)
      REAL*8    BJ(0:21),BJPRIME(0:21)
      REAL*8    RD
      REAL JBESR(8000,20,0:11),JBESM(8000,20,0:11)
      REAL JBESA(8000,0:11),JBESB(8000,0:11)
      COMPLEX   TSM,TSV,TIM,TIV

      COMMON/BANDS/JMAX,JMAXH,IMAX,LCMAX,LCMAXH
      COMMON/COORDS/PR,PZ,PLZ,DELR,DELZ
      COMMON/HARM/NHARM,NRUN,MH
      COMMON/INTEG/NQD1,NQD2,NBESL,EPS,KMAX
      COMMON/GEOM/H,RIN,ROUT
      COMMON/CONST/PI,RHOC
      COMMON/FREQ/FK,FKFAST(8000)
      COMMON/NOPR/IPRO,IPRM,IPRP,IPRD
      COMMON/GSPTS/TK1,WK1,TK2,WK2
      COMMON/BESSL/JBESR,JBESM,JBESA,JBESB
      COMMON/CTS/TSM,TSV,TIM,TIV

      A=ROUT
      B=RIN

      DO 7 JR=1,JMAX

         ZL = PLZ(JR)
         ZU = ZL + DELZ

         DO 7 IR=1,IMAX

            RI = PR(IR)

            TSVSUMR=0.0
            TSMSUMR=0.0
            TIVSUMR=0.0
            TIMSUMR=0.0

            TSVSUMI=0.0
            TSMSUMI=0.0
            TIVSUMI=0.0
            TIMSUMI=0.0

            KL = 0
            K = 0
 31         K = K + 1

               IF (K .EQ. 1) THEN
                  RA=0.0
                  RB=SQRT( FK )
               ELSE
                  RA=FKFAST(K-1)
                  RB=FKFAST(K)
               ENDIF
               CC1= (RB-RA)/2.0
               CC2=(RB+RA)/2.0

C  INITIALIZE
```

```
      TSVSR=0.0
      TSMSR=0.0
      TIVSR=0.0
      TIMSR=0.0

      TSVSI=0.0
      TSMSI=0.0
      TIVSI=0.0
      TIMSI=0.0

      DO 20 J=1, NQD1

         KL = KL + 1

         V =CC1*TK1(J)+CC2
         V2 =V*V
         IF ( K .EQ. 1) THEN
            RLX =FK - V2
         ELSE
            RLX =FK + V2
         ENDIF

         IF(JBESR(KL,IR,0).LT.-999999) THEN
            RRX1=RLX*PR(IR)
            RD=DBLE(RRX1)
            CALL BESJ_N(NBESL, RD, BJ, BJPRIME)
            DO 45 KI=0,NHARM
               JBESR(KL,IR,KI) = BJ(KI)
45          CONTINUE
         END IF
         BESLRI=JBESR(KL,IR,MH)

         IF(JBESA(KL,0).LT.-999999) THEN
            RLA=RLX*ROUT
            RD=DBLE(RLA)
            CALL BESJ_N( NBESL, RD, BJ, BJPRIME)
            DO 80 KI=0,NHARM+1
               JBESA(KL,KI) = BJ(KI)
80          CONTINUE
         END IF
         BESLMA=JBESA(KL,MH)
         BESLMA1=JBESA(KL,MH+1)

         IF ( RIN .NE. 0.0 ) THEN
            IF(JBESB(KL,0).LT.-999999) THEN
               RLB=RLX*RIN
               RD=DBLE(RLB)
               CALL BESJ_N( NBESL, RD, BJ, BJPRIME)
               DO 90 KI=0,NHARM+1
                  JBESB(KL,KI) = BJ(KI)
90             CONTINUE
            END IF
            BESLMB=JBESB(KL,MH)
            BESLMB1=JBESB(KL,MH+1)
         END IF

         TSVI=0.0
         TSMI=0.0

         TIVI=0.0
         TIMI=0.0
```

```
              IF ( K .EQ. 1 ) GO TO 12

              U=V*SQRT( V2 + 2.0*FK )
              ANG1=U*(H-ZU)
              ANG2=U*(H-ZL)
              EXI= 1./EXP(ANG1) - 1.0/EXP(ANG2)
              DIN= V*(V2+2.0*FK)
              VAR3= 2.0*WK1(J)*BESLRI*EXI*RLX/DIN
              TSVR=VAR3*BESLMA*A
              TSMR=VAR3*( MH*BESLMA/A - RLX*BESLMA1 )*A
              IF (RIN .EQ. 0.0)  GO TO 15
              TIVR=VAR3*BESLMB*B
              TIMR=-VAR3*( MH*BESLMB/B - RLX*BESLMB1 )*B
              GO TO 15

C  CALCULATE THE IMAG. AND REAL PART OF THE INTEGRAL WHEN L LESS THAN FK

 12           U=V*SQRT( 2.0*FK - V2 )
              ANG1=U*(H-ZU)
              ANG2=U*(H-ZL)
              SI=COS(ANG2)-COS(ANG1)
              CI=SIN(ANG1)-SIN(ANG2)
              VAR0=2*WK1(J)*BESLRI*RLX/( V*(2.0*FK-V2) )
              VAR1=( MH*BESLMA/A - RLX*BESLMA1 )*A
              TSVR= VAR0*SI*BESLMA*A
              TSVI= VAR0*CI*BESLMA*A
              TSMR= VAR0*SI*VAR1
              TSMI= VAR0*CI*VAR1
              IF ( RIN .EQ. 0.0 )  GO TO 15
              VAR2=( MH*BESLMB/B - RLX*BESLMB1 )*B
              TIVR=VAR0*SI*BESLMB*B
              TIVI=VAR0*CI*BESLMB*B
              TIMR=-(VAR0*SI*VAR2)
              TIMI=-(VAR0*CI*VAR2)

 15           TSVSR=TSVSR+TSVR
              TSMSR=TSMSR+TSMR
              TSVSI=TSVSI+TSVI
              TSMSI=TSMSI+TSMI

              IF ( RIN .EQ. 0.0 ) GO TO 20

              TIVSR=TIVSR+TIVR
              TIVSI=TIVSI+TIVI
              TIMSR=TIMSR+TIMR
              TIMSI=TIMSI+TIMI

 20         CONTINUE

C  ADD CONTRIBUTION FROM THE KTH INTERVAL

            TSVSR=TSVSR*CC1
            TSMSR=TSMSR*CC1
            TSVSI=TSVSI*CC1
            TSMSI=TSMSI*CC1

            TSVSUMR=TSVSUMR+TSVSR
            TSMSUMR=TSMSUMR+TSMSR
            TSVSUMI=TSVSUMI+TSVSI
            TSMSUMI=TSMSUMI+TSMSI
```

```
            IF (RIN .EQ. 0.0 ) GO TO 30

            TIVSR=TIVSR*CC1
            TIVSI=TIVSI*CC1
            TIMSR=TIMSR*CC1
            TIMSI=TIMSI*CC1

            TIVSUMR=TIVSUMR+TIVSR
            TIVSUMI=TIVSUMI+TIVSI
            TIMSUMR=TIMSUMR+TIMSR
            TIMSUMI=TIMSUMI+TIMSI

 30         CONTINUE

C  CHECK FOR CONVERGENCE OF INTEGRAL

            E1=ABS(TSVSR/TSVSUMR)
            E2=ABS(TSMSR/TSMSUMR)
            E3=0.0
            E4=0.0
            IF(RIN.NE.0.0) THEN
               E3=ABS(TIVSR/TIVSUMR)
               E4=ABS(TIMSR/TIMSUMR)
            END IF
            IF(K.GT.KMAX) THEN
               WRITE(3,*)
               WRITE(3,*) ' POOR CONVERGENCE FOR CTSM, MH= ',MH
               WRITE(3,*) ' IR = ', IR, '   JR = ',JR
               IF(E1.GT.EPS) WRITE(3,*) ' GTS: ',E1,TSVSUMR
               IF(E2.GT.EPS) WRITE(3,*) ' MTS: ',E2,TSMSUMR
               IF(E3.GT.EPS) WRITE(3,*) ' GTI: ',E3,TIVSUMR
               IF(E4.GT.EPS) WRITE(3,*) ' MTI: ',E4,TIMSUMR
               GO TO 35
            END IF

         IF((E1.GT.EPS).OR.(E2.GT.EPS).OR.(E3.GT.EPS)
     &      .OR.(E4.GT.EPS)) GO TO 31

 35      TSV(IR,JR)= CMPLX( TSVSUMR, TSVSUMI)/2.0
         TSM(IR,JR)= CMPLX( TSMSUMR, TSMSUMI)/2.0
         IF(IPRD.EQ.1) WRITE(3,*) ' CTSM: IR,JR,K ',IR,JR,K
         IF(IPRD.EQ.1) WRITE(3,*)
     &         ' E1,E2,E3,E4=',E1,E2,E3,E4

         IF (RIN .EQ. 0.0 ) GO TO 7

         TIV(IR,JR)=CMPLX( TIVSUMR, TIVSUMI )/2.0
         TIM(IR,JR)=CMPLX( TIMSUMR, TIMSUMI )/2.0

 7    CONTINUE

      RETURN
      END
```

```
C***********************************************************************
       SUBROUTINE GAUSS(N, XD, WD, CD)
C***********************************************************************

      IMPLICIT REAL*8(A-H,O-Z)
      PARAMETER (NQD1=200)
C     COMPUTES GAUSSIAN QUADRATURE ROOTS AND WEIGHTS
      DIMENSION X(NQD1),W(NQD1),C(NQD1)
      REAL*4  XD(N),WD(N),CD(N)

      RN=N
      CC=2.0
      DO 10 J=2,N
      RJ=J
      C(J)=(RJ-1.0)*(RJ-1.0)/((2.0*RJ-1.0)*(2.0*RJ-3.0))
10    CC=CC*C(J)

      LN=INT(0.5*RN)
      IF(MOD(N,2).EQ.1) LN=LN+1

      DO 50 I=1,LN

      IF(I.EQ.1) THEN
        XT=1.0-2.78/(4.0+RN*RN)
        GO TO 25
      END IF
      IF(I.EQ.2) THEN
        RATIO=4.1*(1.0+0.06*(RN-8.0)/RN)
        XT=XT-RATIO*(1.0-XT)
        GO TO 25
      END IF
      IF(I.EQ.3) THEN
        RATIO=1.67*(1.0+0.22*(RN-8.0)/RN)
        XT=XT-RATIO*(X(1)-XT)
        GO TO 25
      END IF

      XT=3.0*(X(I-1)-X(I-2))+X(I-3)

25    CALL ROOT(XT,N,DPN,PN1,C)
      X(I)=XT
      W(I)=CC/(DPN*PN1)

50    CONTINUE

      DO 60 I=1,LN
      K=N-I+1
      X(K)=-X(I)
60    W(K)=W(I)

C      CHANGE THE DOUBLE PRECISION X W C TO SINGLE PRECISION

      DO 70 I=1,N
      XD(I)=X(I)
      WD(I)=W(I)
      CD(I)=C(I)
  70  CONTINUE

      RETURN
      END
```

```
C***********************************************************************
      SUBROUTINE RECUR(PN,DPN,PN1,XT,N,C)
C***********************************************************************

      IMPLICIT REAL*8(A-H,O-Z)

C     USED BY SUBROUTINE ROOT

      DIMENSION C(N)

      P1=1.0
      P=XT
      DP1=0.0
      DP=1.0

      DO 10 J=2,N

      Q=XT*P-C(J)*P1
      DQ=XT*DP+P-C(J)*DP1
      P1=P
      P=Q
      DP1=DP
      DP=DQ
10    CONTINUE

      PN=P
      DPN=DP
      PN1=P1

      RETURN
      END
```

```
C**********************************************************************
      SUBROUTINE ROOT(XT,N,DPN,PN1,C)
C**********************************************************************

      IMPLICIT REAL*8(A-H,O-Z)

C     USED BY SUBROUTINE GAUSS

      DIMENSION C(N)

      DO 10 I=1,5

      CALL RECUR(P,DP,PN1,XT,N,C)

      D=P/DP
      IF(ABS(D).LT.1.0E-9) THEN
        DPN=DP
        RETURN
      END IF
      XT=XT-D
      DPN=DP
10    CONTINUE

      RETURN
      END
```

```
C*********************************************************************
      FUNCTION DASYMPJ(NU,Z)
C*********************************************************************

C  FUNCTION CALCULATES AN ASYMPTOTIC VALUE FOR THE BESSEL FUNCTION
C                     J_NU(Z)
C  ASYMPTOTIC EXPRESSION TAKEN FROM ABRAMOWITZ AND STEGUN

      IMPLICIT REAL*8 (A-H, O-Z)
      REAL*8  P,Q,Z,MU,X,DPI

      IF((NU.LT.0).OR.(NU.GT.20)) THEN
         WRITE(6,*) ' ERROR CALLED FROM FUNCTION ASYMPJ:     '
         WRITE(6,*) ' ORDER OF BESSEL FUNCTION, NU= ',NU
         STOP
      END IF

      DPI=3.141592654D0

C  THE ORDER OF THE APPROXIMATION CAN BE VARIED BY COMMENTING OUT THE
C  ADDITIONAL LINES FOR P AND Q.

 10   X  = Z - (NU/2.0D0+0.25D0)*DPI
      MU = 4.0D0*NU*NU
      P  = 1.0D0
    &    - (MU-1.D0)*(MU-9.D0)/128.D0/(Z*Z)
    &    + (MU-1.D0)*(MU-9.D0)*(MU-25.D0)*(MU-49.D0)/98304.D0/(Z*Z*Z*Z)
      Q  = 0.0D0
    &       + (MU-1.D0)/8.D0/Z
    &       - (MU-1.D0)*(MU-9.D0)*(MU-25.D0)/3072.D0/(Z*Z*Z)

      DASYMPJ = DSQRT(2.D0/DPI/Z) * (P*DCOS(X) - Q*DSIN(X))

      RETURN
      END
```

```
C*********************************************************************
      SUBROUTINE BESJ_N(N, X, BJJ, BJPRIME)
C*********************************************************************

C EVALUATE THE BESSEL FUNCTIONS J<N>(X) AND THEIR FIRST DERIVATIVES
C   FOR ANY REAL X AND N <= N.
C THIS PROGRAM USES DOWNWARD RECURSION BASED ON THE IDENTITY:
C       J<0>(X) + 2 * SUM(EVEN N)[J<N>(X)] = 1
C
C XMIN IS THE SMALLEST NUMBER CONSIDERED TO BE NON-ZERO.
C
      IMPLICIT REAL*8 (A-H, O-Z)
      REAL*8 BIGNO,BIGNI,XMIN
      PARAMETER (BIGNO=1.D10,BIGNI=1.D-10, XMIN=1.D-30)
      REAL*8 BJJ(0:21), BJPRIME(0:21), X
C
C SKIP COMPUTATIONS WHEN X IS VERY SMALL
C
      IF(ABS(X) .LE. XMIN) THEN
         DO 5 I = 0, N
C
C           INITIAL ALL VALUES TO ZERO
C
            BJJ(I) = 0.D+0
            BJPRIME(I) = 0.D+0
 5       CONTINUE
C
* ONLY J<0>(0) AND J'<1>(0) ARE NONZERO
C
         BJJ(0) = 1.D+0
         IF (N .GT. 0) BJPRIME(1) = 0.5D+0
         RETURN
      ENDIF

C  NU IS THE HIGHEST ORDER BESSEL FUNCTION ACTUALLY NEEDED BY SHIP
C  RECALL THAT N=NBESL= NHARM+1+5
C  (NOTE THAT SHIP DOES NOT REQUIRE BJPRIME)
C  FIRST CHECK THAT J_NU(X) IS IN THE ASYMPTOTIC RANGE:

      NU = N-5
      IF((NU.LT.3).AND.(X.LT.2.0D0)) GO TO 8
      IF((NU.LT.6).AND.(X.LT.(3.0D0*NU-7.0D0))) GO TO 8
      IF((NU.LT.10).AND.(X.LT.(4.5D0*NU-15.0D0))) GO TO 8
      IF((NU.LE.20).AND.(X.LT.(7.0D0*NU-40.0D0))) GO TO 8

      DO 9 I = 0, NU
         BJJ(I) = DASYMPJ(I,X)
         BJPRIME(I) = 0.0D0
 9    CONTINUE
      RETURN

 8    CONTINUE

C
* REGULAR RECURSION SCHEME STARTS HERE.
C
      XPOS = DABS(X)
      TOX=2.D+0/XPOS
C
* USE DOWNWARD RECURRENCE STARTING FROM AN EVEN VALUE OF M.
* SELECT OF VALUE OF M THAT IS SUFFICIENTLY LARGE TO GIVE J<M>(X) = 0.
*   THIS ESTIMATE FOR M IS BASED ON THE ASYMPTOTIC EXPANSION OF J<N>(X)
```

```
*   FOR LARGE N AND X FIXED.
* THE ARRAY FORM OF THE ALGORITHM REQUIRES THAT M EXCEED N.
C
      M = 2 * MAX((INT(XPOS + LOG(XPOS/XMIN))/2), N/2 + 2)
C
      JSUM = 0.D+0

* JSUM WILL ALTERNATE BETWEEN 0 AND 1; WHEN IT IS 1, WE ACCUMULATE IN
*   SUM THE EVEN TERMS IN J<0> + 2 * [SUM(EVEN N) J<N>] = 1
      SUM = 0.D+0
      BJ_P = 0.D+0
      BJ = 1.D+0

* START THE DOWNWARD RECURRENCE BUT DON'T SAVE THE HIGHER ORDERS
*   NOTE THAT M IS EVEN
      DO 10 I = M, N + 2, -1

* BJ_M = J<I-1>(X)
         BJ_M = I * TOX * BJ - BJ_P
         BJ_P = BJ
         BJ = BJ_M

* RENORMALIZE TO PREVENT OVERFLOWS
         IF (ABS(BJ) .GT. BIGNO) THEN
            BJ = BJ * BIGNI
            BJ_P = BJ_P * BIGNI
            SUM = SUM * BIGNI
         ENDIF

* ACCUMULATE THE SUM.
         SUM = SUM + JSUM*BJ

* CHANGE JSUM FROM 0 TO 1 OR VICE VERSA
         JSUM = 1 - JSUM
 10   CONTINUE

* SAVE THE LAST RECURSION VALUE TO OBTAIN J<1>(X) WHEN N = 0
      IF (N .EQ. 0) BJ_1 = BJ

* PERFORM RECURSIONS FOR THE ORDERS TO BE SAVED
      DO 20 I = N + 1, 1, -1
* BJ_M = J<I-1>(X)
          BJ_M = I * TOX * BJ - BJ_P
          BJ_P = BJ
          BJ = BJ_M
* SAVE VALUE
          BJJ(I - 1) = BJ

* RENORMALIZE TO PREVENT OVERFLOWS
          IF (ABS(BJ) .GT. BIGNO) THEN
             BJ = BJ * BIGNI
             BJ_P = BJ_P * BIGNI
             SUM = SUM * BIGNI
* FACTOR PREVIOUSLY SAVED VALUES
             DO 30 K = N, I - 1, -1
                BJJ(K) = BJJ(K) * BIGNI
 30          CONTINUE
          ENDIF

* ACCUMULATE THE SUM.
          SUM = SUM + JSUM*BJ
```

```
* CHANGE JSUM FROM 0 TO 1 OR VICE VERSA
         JSUM = 1 - JSUM
 20    CONTINUE

       SUM = 2. * SUM - BJ

       BJ_M = BJJ(0)/SUM
       BJJ(0) = BJ_M

* CASE WHERE N = 0 IS DONE SEPARATELY
       IF (N .EQ. 0) THEN
          BJPRIME(0) = - BJ_1/SUM
          IF(X.LT.0.) BJPRIME(0) = - BJPRIME(0)
          RETURN
       ENDIF

*
       DO 40 I = 0, N - 1
          BJ = BJJ(I + 1)/SUM
          BJJ(I + 1) = BJ
* EVALUATE DERIVATIVES
          BJPRIME(I) = - BJ + I * BJ_M/XPOS
          BJ_M = BJ
 40    CONTINUE

       BJPRIME(N) = BJJ(N - 1) - N * BJJ(N)/XPOS
C
C       CHANGE SIGN FOR ODD ORDER IF X IS NEGATIVE
C
       IF(X.LT.0.) THEN
          DO 25 I=1, N, 2
             BJJ(I) = - BJJ(I)
             BJPRIME(I - 1) = - BJPRIME(I)
 25       CONTINUE
C
       ENDIF
C
       RETURN
       END
```

```
C*****************************************************************
      SUBROUTINE ERING
C*****************************************************************

      COMPLEX GM,ISM,SIM,ITM,TIM,IIM,SSM,TSM,STM,TBM,DM,
     & ISV,SIV,ITV,TIV,IIV,SSV,TSV,STV,TBV,TTV
      DIMENSION GM(200,200),DM(200,200),SSM(80,80),SSV(80,80),
     & TSM(20,80),TBM(20,20),STM(80,20),TTV(20,20),TSV(20,80),
     & TBV(20,20),STV(80,20),ISM(80,80),ITM(80,20),SIM(80,80),
     & TIM(20,80),IIM(80,80),ISV(80,80),SIV(80,80),ITV(80,20),
     & TIV(20,80),IIV(80,80),PLZ(80),PR(20),PZ(80)
      DIMENSION TK1(100),WK1(100),TK2(100),WK2(100)

      COMMON/BANDS/JMAX,JMAXH,IMAX,LCMAX,LCMAXH
      COMMON/COORDS/PR,PZ,PLZ,DELR,DELZ
      COMMON/HARM/NHARM,NRUN,MH
      COMMON/INTEG/NQD1,NQD2,NBESL,EPS,KMAX
      COMMON/GEOM/H,RIN,ROUT
      COMMON/CONST/PI,RHOC
      COMMON/COEFM/DM,GM
      COMMON/FREQ/FK,FKFAST(8000)
      COMMON/GSPTS/TK1,WK1,TK2,WK2
      COMMON/CTB/TTV,TBM,TBV
      COMMON/CTS/TSM,TSV,TIM,TIV
      COMMON/CST/STM,STV,ITM,ITV
      COMMON/CSS/SSM,SSV,SIM,SIV,ISM,ISV,IIM,IIV

C  CALL SUBROUTINES TO GET MATRIX ELEMENTS

      WRITE(6,*) '    CALCULATING MTT,MTB,GTT,GTB.....'
      CALL CTBM
      WRITE(6,*) '    CALCULATING MSS,MIS,MSI,MII,GSS,GIS,GSI,GII.....'
      CALL CSSM
      WRITE(6,*) '    CALCULATING MST,MIT,GST,GIT.....'
      CALL CSTM
      WRITE(6,*) '    CALCULATING MTS,MTI,GTS,GTI.....'
      CALL CTSM

C  ASSEMBLE GM AND DM

      DO 6000 I=1,LCMAXH
      DO 6000 J=1,LCMAX
        GM(I,J) = CMPLX(0.0,0.0)
        DM(I,J) = CMPLX(0.0,0.0)
 6000 CONTINUE

C  FOR A RING

      IF(RIN.EQ.0.) GO TO 80
      DO 64 I=1,IMAX
      DO 64 J= 1,IMAX
        IP=JMAXH+I
        IQ=JMAXH+J
        GM(IP,IQ)=TTV(I,J)
   64 CONTINUE

      DO 31  I=1,JMAXH
         KITE = JMAXH + 1 - I
         DO 32 J=1,JMAXH
            KETE=JMAXH+1-J
            DM(I,J)  = IIM(KITE,KETE)
            GM(I,J)  = IIV(KITE,KETE)
```

```
          KATE=LCMAX+1-J
          KOTE=JMAXH+J
          GM(I,KATE) = IIV(KITE,KOTE)
          DM(I,KATE) = IIM(KITE,KOTE)
32      CONTINUE
        DO 33 J=1,IMAX
          JITE=JMAXH+J
          GM(I,JITE) = ITV(KITE,J)
          DM(I,JITE) = ITM(KITE,J)
33      CONTINUE
        DO 34 J=1,JMAX
          JITE=JMAXH+IMAX+J
          GM(I,JITE)= ISV(KITE,J)
          DM(I,JITE)= ISM(KITE,J)
34      CONTINUE
        DO 35 J=1,IMAX
          JITE=JMAXH+LCMAXH+J
          KOTE=JMAX+1-KITE
          KETE=IMAX+1-J
          GM(I,JITE)= ITV(KOTE,KETE)
          DM(I,JITE)= ITM(KOTE,KETE)
35      CONTINUE
31    CONTINUE

      DO 36 I=1,IMAX
        KOOT = JMAXH+I
        DO 37 J=1,JMAXH
          KITE = JMAXH + 1 -J
          GM(KOOT,J)= TIV(I,KITE)
          DM(KOOT,J)= TIM(I,KITE)
          KETE=LCMAX+1-J
          KOTE=JMAXH+J
          GM(KOOT,KETE)= TIV(I,KOTE)
          DM(KOOT,KETE)= TIM(I,KOTE)
37      CONTINUE
        DO 38 J=1,JMAX
          KETE=JMAXH+IMAX+J
          GM(KOOT,KETE)= TSV(I,J)
          DM(KOOT,KETE)= TSM(I,J)
38      CONTINUE
        DO 39 J=1,IMAX
          KETE=LCMAXH+JMAXH+J
          KOTE=IMAX+1-J
          GM(KOOT,KETE)= TBV(I,KOTE)
          DM(KOOT,KETE)= TBM(I,KOTE)
39      CONTINUE
36    CONTINUE

      DO 40 I=1,JMAXH
        KOOT = JMAXH + IMAX + I
        DO 41 J=1,JMAXH
          KITE = JMAXH+1-J
          GM(KOOT,J) = SIV(I,KITE)
          DM(KOOT,J) = SIM(I,KITE)
          KETE=LCMAX+1-J
          KOTE=JMAXH+J
          DM(KOOT,KETE)= SIM(I,KOTE)
          GM(KOOT,KETE)= SIV(I,KOTE)
41      CONTINUE
        DO 42 J=1,IMAX
          JITE=JMAXH+LCMAXH+J
          KETE=JMAX+1-I
```

```
            KOTE=IMAX+1-J
            KATE=JMAXH+J
            DM(KOOT,JITE)=STM(KETE,KOTE)
            GM(KOOT,JITE)=STV(KETE,KOTE)
            GM(KOOT,KATE)= STV(I,J)
            DM(KOOT,KATE)= STM(I,J)
   42     CONTINUE
          DO 43 J=1,JMAX
            KETE=JMAXH+IMAX+J
            GM(KOOT,KETE) = SSV(I,J)
            DM(KOOT,KETE) = SSM(I,J)
   43     CONTINUE
   40   CONTINUE

        GO TO 990

C  FOR A CYLINDER

   80   DO 84 I=1,IMAX
        DO 84 J= 1,IMAX
          GM(I,J) = TTV(I,J)
   84   CONTINUE
        DO 71 I=1,IMAX
          DO 72 J=1,JMAX
            KOTE=IMAX+J
            GM(I,KOTE) = TSV(I,J)
            DM(I,KOTE) = TSM(I,J)
   72     CONTINUE
          DO 71 J=1,IMAX
            KOTE=JMAX+IMAX+J
            GM(I,JMAX + IMAX + J) = TBV(I,IMAX + 1 - J)
            DM(I,JMAX + IMAX + J) = TBM(I,IMAX + 1 - J)
   71   CONTINUE
        DO 73 I=1,JMAXH
          K= IMAX + I
          DO 74 J=1,IMAX
            GM(K,J) = STV(I,J)
            DM(K,J) = STM(I,J)
   74     CONTINUE
          DO 75 J=1,JMAX
            GM(K,IMAX + J) = SSV(I,J)
            DM(K,IMAX + J) = SSM(I,J)
   75     CONTINUE
          DO 73 J=1,IMAX
            GM(K,JMAX+IMAX+J)= + STV(JMAX +1-I,IMAX + 1 -J)
            DM(K,JMAX+IMAX+J)= + STM(JMAX +1-I,IMAX + 1 -J)
   73   CONTINUE

  990   DO 1000 I=1,LCMAXH
          DO 1000 J=1,LCMAX
            DM(I,J)=-DM(I,J)
            IF(I.EQ.J) DM(I,J)=DM(I,J)+CMPLX(0.5,0.0)
 1000   CONTINUE

        RETURN
        END
```

```
C**********************************************************************
      SUBROUTINE FARFLD
C**********************************************************************

      COMMON/INTEG/NQD1,NQD2,NBESL,EPS,KMAX
      COMMON/GSPTS/TK1,WK1,TK2,WK2
      COMMON/BANDS/JMAX,JMAXH,IMAX,LCMAX,LCMAXH
      COMMON/COORDS/PR,PZ,PLZ,DELR,DELZ
      COMMON/HARM/NHARM,NRUN,MH
      COMMON/SYM/ISYM,IRHEVEN,IRIGID
      COMMON/GEOM/H,RIN,ROUT
      COMMON/CONST/PI,RHOC
      COMMON/FAR/NPTS,THETA,PHI
      COMMON/PLANE/AINC,PSI
      COMMON/SURFQ/VEL,ANS
      COMMON/FREQ/FK,FKFAST(8000)
      COMMON/NOPR/IPRO,IPRM,IPRP,IPRD
      COMMON/COEFM/DM,GM

      DIMENSION RO(200)
      DIMENSION TK1(100),WK1(100),TK2(100),WK2(100)
      DIMENSION THETA(100),PHI(100)
      DIMENSION BESM(1:20,0:11)
      REAL*8    BJJ(0:21),BJPRIME(0:21)
      REAL*8    JMKAS(0:21),JMKBS(0:21)
      REAL*8    RRX,Z
      COMPLEX CEP,CEM,EM1,COEF,ENL,TERM,ADDC,ADDS
      COMPLEX PMTC,PMBC,PMOC,PMIC,PMTS,PMBS,PMOS,PMIS
      COMPLEX VMTC,VMBC,VMOC,VMIC,VMTS,VMBS,VMOS,VMIS
      COMPLEX FFTB,FFOI,FFPH(100,21),FFP(100)
      COMPLEX CI,SUM,PP,PM,CA,CB
      COMPLEX PTC(20),PBC(20),VTC(20),VBC(20)
      COMPLEX PTS(20),PBS(20),VTS(20),VBS(20)
      COMPLEX POC(80),PIC(80),VOC(80),VIC(80)
      COMPLEX POS(80),PIS(80),VOS(80),VIS(80)
      COMPLEX ANS(200,21),VEL(200,21)
      COMPLEX DM(200,200),GM(200,200)
      DIMENSION PZ(80),PLZ(80),PR(20)
      DATA ALICE/8.685889638/

      WRITE(6,*) ' CALCULATING FAR-FIELD PRESSURES.... '

C INITIALIZE VECTORS

      DO 2001 I=1,IMAX
         PTC(I)=CMPLX(0.0,0.0)
         PBC(I)=CMPLX(0.0,0.0)
         VTC(I)=CMPLX(0.0,0.0)
         VBC(I)=CMPLX(0.0,0.0)
C
         PTS(I)=CMPLX(0.0,0.0)
         PBS(I)=CMPLX(0.0,0.0)
         VTS(I)=CMPLX(0.0,0.0)
         VBS(I)=CMPLX(0.0,0.0)
 2001 CONTINUE

      DO 2002 I=1,JMAX
         POC(I)=CMPLX(0.0,0.0)
         PIC(I)=CMPLX(0.0,0.0)
         VOC(I)=CMPLX(0.0,0.0)
         VIC(I)=CMPLX(0.0,0.0)
C
```

```
          POS(I)=CMPLX(0.0,0.0)
          PIS(I)=CMPLX(0.0,0.0)
          VOS(I)=CMPLX(0.0,0.0)
          VIS(I)=CMPLX(0.0,0.0)
 2002  CONTINUE

       DO 2003 I=1,NPTS
          FFP(I) = CMPLX(0.0,0.0)
 2003  CONTINUE

       IF(IPRO.EQ.1) THEN
          WRITE(3,43)
   43     FORMAT(///,13X,28HFARFIELD PATTERN AT INFINITY,//,10X,5HANGLE,
     &       6X,2HDB,9X,9HMAGNITUDE,5X,5HANGLE,/)
       END IF
C
       CI = (0.,1.)
       C1=DELR/2.

C  START LOOP ON NUMBER OF FIELD POINTS, IFFP=1,NPTS

       DO 1000 IFFP=1,NPTS
          COST=COS(THETA(IFFP))
          SINT=SIN(THETA(IFFP))
          COSP=COS(PHI(IFFP))
          SINP=SIN(PHI(IFFP))

C  EVALUATE ALL BESSEL FUNCTIONS NEEDED FOR FAR-FIELD CALCULATIONS

          Z=DBLE(FK*ROUT*SINP)
          CALL BESJ_N( NBESL, Z, JMKAS, BJPRIME)
          IF(RIN.GT.0.0) THEN
             Z=DBLE(FK*RIN*SINP)
             CALL BESJ_N( NBESL, Z, JMKBS, BJPRIME)
          END IF

C  DEFINE CONSTANTS NEEDED FOR VELOCITY AND PRESSURE SUMMATIONS

          CEP=CEXP(CI*FK*H*COSP)
          CEM=CONJG(CEP)
          EM1=CEXP(CI*FK*COSP*DELZ) - CMPLX(1.0,0.0)

C  NOW EVALUATE ALL BESSEL FUNCTIONS NEEDED IN INT(J_M*RO*DRO)

          DO 100 J=1,IMAX
             C2 = PR(J)
             DO 90 K=0,NHARM
                BESM(J,K)=0.0
 90          CONTINUE
             DO 10 I=1,NQD2
                RO(I)=C1*TK2(I)+C2
                RX=FK*SINP*RO(I)
                RRX=DBLE(RX)
                CALL BESJ_N(NBESL, RRX, BJJ, BJPRIME)
                DO 80 K=0,NHARM
                   BESLM=BJJ(K)
                   BESM(J,K)= BESM(J,K) + WK2(I)*BESLM*RO(I)*C1
 80             CONTINUE
 10          CONTINUE
 100      CONTINUE

C  PBC AND VBC REFER TO THE BOTTOM SURFACE FOR THE COSINE COMPONENT
```

```
C  PBS AND VBS REFER TO THE BOTTOM SURFACE  "   "   SINE    "
C  PTC AND VTC REFER TO THE TOP SURFACE     "   "  COSINE   "
C  PTS AND VTS REFER TO THE TOP SURFACE     "   "   SINE    "
C
          DO 987 IH=1,NHARM+1
             MH=IH-1
             IS=IH+NHARM
             CMT=COS(MH*THETA(IFFP))
             SMT=SIN(MH*THETA(IFFP))

C
C  FOR CONVENIENCE, PLACE PRESSURES AND VELOCITIES FOR EACH REGION
C  IN SEPARATE VECTORS.  THE SUFFIX "C" DESIGNATES A COSINE COMPONENT
C  WHILE AN "S" DESIGNATES A SINE COMPONENT.
C
             JMAXT = JMAXH
             IF(RIN.EQ.0.) JMAXH = 0
             DO 2 J=1,IMAX
                PBC(J) = ANS(LCMAX - JMAXH + 1 - J, IH)
                VBC(J) = VEL(LCMAX - JMAXH + 1 - J, IH)
                VTC(J) = VEL(JMAXH + J, IH)
                PTC(J) = ANS(JMAXH + J, IH)
                IF((IH.EQ.1).OR.(IRHEVEN.EQ.1)) GO TO 2
                PBS(J) = ANS(LCMAX - JMAXH + 1 - J, IS)
                VBS(J) = VEL(LCMAX - JMAXH + 1 - J, IS)
                VTS(J) = VEL(JMAXH + J, IS)
                PTS(J) = ANS(JMAXH + J, IS)
    2        CONTINUE

C POC AND VOC REFER TO THE OUTSIDE SURFACE FOR THE COSINE COMPONENT
C POS AND VOS REFER TO THE OUTSIDE SURFACE FOR THE SINE COMPONENT
C
             DO 3 J=1,JMAXT
                POC(J)= ANS(IMAX+JMAXH+J, IH)
                VOC(J)= VEL(IMAX+JMAXH+J, IH)
                VOC(JMAX +1-J) = VEL( LCMAX +1 -JMAXH-IMAX - J, IH)
                POC(JMAX +1-J) = ANS( LCMAX +1 -JMAXH-IMAX - J, IH)
                IF((IH.EQ.1).OR.(IRHEVEN.EQ.1)) GO TO 3
                POS(J)= ANS(IMAX+JMAXH+J, IS)
                VOS(J)= VEL(IMAX+JMAXH+J, IS)
                VOS(JMAX +1-J) = VEL( LCMAX +1 -JMAXH-IMAX - J, IS)
                POS(JMAX +1-J) = ANS( LCMAX +1 -JMAXH-IMAX - J, IS)
    3        CONTINUE
C
             JMAXH = JMAXT
             IF(RIN.NE.0.) THEN

C  PIC AND VIC REFER TO THE INSIDE SURFACE OF A RING COSINE COMPONENT
C  PIS AND VIS REFER TO THE INSIDE SURFACE OF A RING SINE COMPONENT

                DO 965 J=1,JMAXH
                   PIC(J) = ANS(JMAXH+1-J, IH)
                   VIC(J) = VEL(JMAXH+1-J, IH)
                   VIC(JMAX+1 - J) = VEL(LCMAX -JMAXH + J, IH)
                   PIC(JMAX+1 - J) = ANS(LCMAX -JMAXH + J, IH)
                   IF((IH.EQ.1).OR.(IRHEVEN.EQ.1)) GO TO 965
                   PIS(J) = ANS(JMAXH+1-J, IS)
                   VIS(J) = VEL(JMAXH+1-J, IS)
                   VIS(JMAX+1 - J) = VEL(LCMAX -JMAXH + J, IS)
                   PIS(JMAX+1 - J) = ANS(LCMAX -JMAXH + J, IS)
  965           CONTINUE
```

```
            ELSE
C  PIC, PIS, VIC, VIS ARE ZERO FOR A CYLINDER
               DO 969 J=1,JMAX
                  VIC(J) = CMPLX(0.0,0.0)
                  PIC(J) = CMPLX(0.0,0.0)
                  IF((IH.EQ.1).OR.(IRHEVEN.EQ.1)) GO TO 969
                  VIS(J) = CMPLX(0.0,0.0)
                  PIS(J) = CMPLX(0.0,0.0)
  969          CONTINUE
            END IF
C  SUM CONTRIBUTIONS FROM THE TOP AND BOTTOM:
            SUM=CMPLX(0.0,0.0)
            COEF = CI**(MH+1)*FK/2.
            DO 5 K=1,IMAX
               PMTC = CEP*COEF*COSP*BESM(K,MH)*CMT
               PMTS = CEP*COEF*COSP*BESM(K,MH)*SMT
               PMBC = -CEM*COEF*COSP*BESM(K,MH)*CMT
               PMBS = -CEM*COEF*COSP*BESM(K,MH)*SMT
               VMTC = RHOC*CEP*BESM(K,MH)*COEF*CMT
               VMTS = RHOC*CEP*BESM(K,MH)*COEF*SMT
               VMBC = RHOC*CEM*BESM(K,MH)*COEF*CMT
               VMBS = RHOC*CEM*BESM(K,MH)*COEF*SMT
               ADDC = PMTC*PTC(K)+PMBC*PBC(K)+VMTC*VTC(K)+VMBC*VBC(K)
               ADDS = PMTS*PTS(K)+PMBS*PBS(K)+VMTS*VTS(K)+VMBS*VBS(K)
               SUM = SUM + ADDC + ADDS
    5       CONTINUE
            FFTB = SUM
C
C  CALCULATE TERMS NEEDED FOR CONTRIBUTIONS FROM INSIDE AND OUTSIDE
C
            PP=CI**(MH+1)
            PM=CI**(MH-1)
            IF(MH.GT.0) THEN
               CA = PP*JMKAS(MH+1) + PM*JMKAS(MH-1)
               CB = PP*JMKBS(MH+1) + PM*JMKBS(MH-1)
            ELSE
               CA = 2.0*CI*JMKAS(1)
               CB = 2.0*CI*JMKBS(1)
            END IF
C
C  CALCULATE CONTRIBUTION FROM INSIDE AND OUTSIDE
            SUM = CMPLX(0.0,0.0)
            DO 37 K=1,JMAX
               IF(ABS(COSP).LT.1.0E-4) THEN
                  PMOC=CI*FK*ROUT*SINP*DELZ*CA*CMT/4.0
                  PMOS=CI*FK*ROUT*SINP*DELZ*CA*SMT/4.0
                  PMIC=-CI*FK*RIN*SINP*DELZ*CB*CMT/4.0
                  PMIS=-CI*FK*RIN*SINP*DELZ*CB*SMT/4.0
                  VMOC=(CI**(MH+1))*FK*CMT*RHOC*ROUT*JMKAS(MH)*DELZ/2.0
                  VMOS=(CI**(MH+1))*FK*SMT*RHOC*ROUT*JMKAS(MH)*DELZ/2.0
                  VMIC=(CI**(MH+1))*FK*CMT*RHOC*RIN*JMKBS(MH)*DELZ/2.0
                  VMIS=(CI**(MH+1))*FK*SMT*RHOC*RIN*JMKBS(MH)*DELZ/2.0
               ELSE
                  ENL= CEXP(CI*FK*COSP*PLZ(K))
                  TERM=ENL*EM1/COSP
                  PMOC=ROUT*SINP*TERM*CA*CMT/4.0
```

```
                  PMOS=ROUT*SINP*TERM*CA*SMT/4.0
                  PMIC=-RIN*SINP*TERM*CB*CMT/4.0
                  PMIS=-RIN*SINP*TERM*CB*SMT/4.0
                  VMOC=(CI**MH)*CMT*RHOC*ROUT*JMKAS(MH)*TERM/2.0
                  VMOS=(CI**MH)*SMT*RHOC*ROUT*JMKAS(MH)*TERM/2.0
                  VMIC=(CI**MH)*CMT*RHOC*RIN*JMKBS(MH)*TERM/2.0
                  VMIS=(CI**MH)*SMT*RHOC*RIN*JMKBS(MH)*TERM/2.0
               END IF

               ADDC = PMOC*POC(K) + PMIC*PIC(K)
     &              + VMOC*VOC(K) + VMIC*VIC(K)
               ADDS = PMOS*POS(K) + PMIS*PIS(K)
     &              + VMOS*VOS(K) + VMIS*VIS(K)
               SUM = SUM + ADDC + ADDS
   37       CONTINUE
            FFOI = SUM
C
C  ADD CONTRIBUTIONS FROM TOP/BOTTOM AND OUTSIDE/INSIDE
C
            FFPH(IFFP,IH) = FFTB + FFOI

 987     CONTINUE

         IF(IPRD.EQ.1) THEN
            WRITE(3,*)
            WRITE(3,*) ' CONVERGENCE TREND FOR FFP NUMBER',IFFP
            WRITE(3,*)
         END IF
         DO 950 I=1,NHARM+1
            FFP(IFFP) = FFP(IFFP) + FFPH(IFFP,I)
            IF(IPRD.EQ.1) WRITE(3,*) ' MH, ¦FFPH(MH)¦, ¦FFP(MH)¦= ',
     &         I-1,CABS(FFPH(IFFP,I)),CABS(FFP(IFFP))
 950     CONTINUE

 1000  CONTINUE

C  PRINT FAR-FIELD PRESSURES USING REAL AND IMAGINARY PARTS:

       WRITE(15,*)
       WRITE(15,*)
       WRITE(15,*) ' FAR-FIELD PRESSURES '
       WRITE(15,*)
       WRITE(15,*) ' THETA (DEG)      PHI (DEG)            REAL ',
     &     '      IMAGINARY'
       WRITE(15,*)

       DO 2000 I=1,NPTS
          WRITE(15,'(2X,F8.3,8X,F8.3,10X,G12.5,5X,G12.5)')
     &      THETA(I)*180.0/PI,PHI(I)*180.0/PI,FFP(I)
2000   CONTINUE

C  PRINT FAR-FIELD PRESSURES USING MAGNITUDE AND PHASE:

       WRITE(15,*)
       WRITE(15,*)
       WRITE(15,*) ' THETA (DEG)      PHI (DEG)         MAGNITUDE ',
     &     '      PHASE (DEG)'
       WRITE(15,*)
       DO 3000 I=1,NPTS
          TH=THETA(I)*180.0/PI
          PH=PHI(I)*180.0/PI
          PMAG=CABS(FFP(I))
```

```
          IF(PMAG.NE.0.0) THEN
              ANGD = ATAN2D(AIMAG(FFP(I)),REAL(FFP(I)))
          ELSE
              ANGD = 0.0
          END IF
          WRITE(15,'(2X,F8.3,8X,F8.3,10X,G12.5,5X,G12.5)')
     &       TH,PH,PMAG,ANGD
 3000   CONTINUE
C
        RETURN
        END
```

```
C**********************************************************************
      SUBROUTINE ODS(IOPT,CMTX,NROWS,CRHS,CSOLN,TVECT,B,IPIVTR)
C**********************************************************************

C     PERFORMS HOUSEHOLDER DECOMPOSITION AND BACK SUBSTITUTIONS

C     IOPT=0 THEN DECOMPOSE
C     IOPT=1 THEN DECOMPOSE AND SOLVE
C     IOPT=2 THEN SOLVE

      COMPLEX CMTX(200,200),TVECT(200),CRHS(200),CSOLN(NROWS)
      DIMENSION B(200),IPIVTR(200)
      COMPLEX TSC, ARG
      DIMENSION TZ(2)
      EQUIVALENCE (TZ(1),TSC)

      NCOLS=NROWS

      IF(IOPT.NE.2) THEN
         DO 1 I=1,NROWS
1        IPIVTR(I)=I
      ELSE
         GO TO 999
      END IF

      DO 100 I=1,NCOLS

         PIVMAX=-1.0
         DO 105 J=I,NROWS
            TSC=CMTX(J,I)
            TEMP=ABS(TZ(1))+ABS(TZ(2))
            IF(PIVMAX.LT.TEMP) THEN
               PIVMAX=TEMP
               NPIVOT=J
            END IF
105      CONTINUE
         IF(NPIVOT.NE.I) THEN
            NTEMP=IPIVTR(I)
            IPIVTR(I)=IPIVTR(NPIVOT)
            IPIVTR(NPIVOT)=NTEMP
            DO 115 J=1,NCOLS
               TSC=CMTX(I,J)
               CMTX(I,J)=CMTX(NPIVOT,J)
115         CMTX(NPIVOT,J)=TSC
         END IF

         SUM=0.0
         DO 200 J=I,NROWS
            TSC=CMTX(J,I)
200      SUM=SUM+TZ(1)*TZ(1)+TZ(2)*TZ(2)
         SUM=SQRT(SUM)
         TEMP=CABS(CMTX(I,I))
         IF(TEMP.EQ.0.0) THEN
            ARG=CMPLX(1.0,1.0)
         ELSE
            ARG=CMTX(I,I)/TEMP
         END IF
         B(I)=1.0/(SUM*(TEMP+SUM))
         TVECT(I)=CMTX(I,I)+SUM*ARG
         CMTX(I,I)=-SUM*ARG

         II=I+1
```

```
         IF(II.GT.NCOLS) GO TO 100

         DO 300 J=II,NCOLS
            ARG=CONJG(TVECT(I))*CMTX(I,J)
            DO 350 K=II,NROWS
350         ARG=ARG+CONJG(CMTX(K,I))*CMTX(K,J)
            ARG=ARG*B(I)
            CMTX(I,J)=CMTX(I,J)-ARG*TVECT(I)
            DO 375 K=II,NROWS
375         CMTX(K,J)=CMTX(K,J)-ARG*CMTX(K,I)
300      CONTINUE

100   CONTINUE

999   IF(IOPT.EQ.0) RETURN

      DO 500 I=1,NCOLS

         K=IPIVTR(I)
         ARG=CONJG(TVECT(I))*CRHS(K)

         II=I+1
         IF(II.GT.NROWS) GO TO 601

         DO 600 J=II,NROWS
            K=IPIVTR(J)
600      ARG=ARG+CONJG(CMTX(J,I))*CRHS(K)

601      ARG=ARG*B(I)
         K=IPIVTR(I)
         CRHS(K)=CRHS(K)-ARG*TVECT(I)
         IF(II.GT.NROWS) GO TO 500
         DO 700 J=II,NROWS
            K=IPIVTR(J)
700      CRHS(K)=CRHS(K)-ARG*CMTX(J,I)

500   CONTINUE

      L=NCOLS
      K=IPIVTR(L)
      CSOLN(L)=CRHS(K)/CMTX(L,L)
      N=NCOLS-1
      NN=N

      DO 800 I=1,NN

         JJ=NCOLS-I+1
         K=IPIVTR(N)
         ARG=CMPLX(0.0,0.0)
         DO 900 J=JJ,NCOLS
900      ARG=ARG+CSOLN(J)*CMTX(N,J)
         TSC=CRHS(K)-ARG
         CSOLN(N)=TSC/CMTX(N,N)

800   N=N-1

      RETURN
      END
```