RECONSTRUCTION AND ROBUST REDUCED-ORDER OBSERVATION OF FLEXIBLE VARIABLES

by

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Abstract

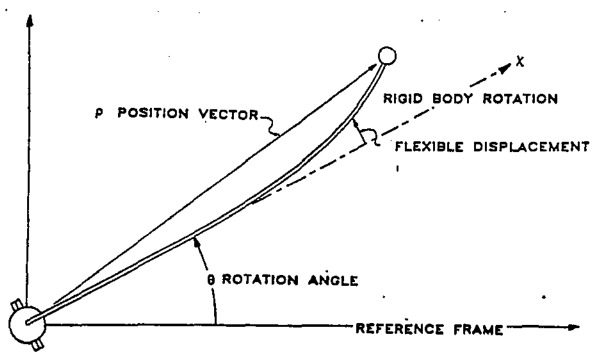
Most models intended for real-time control of distributed parameter systems such as flexible manipulators rely on N-modal approximation schemes[1]. Measurements made on flexible systems yield time varying quantities which are linear combinations of the system states. This paper discusses reconstruction and estimation of flexible variables from multiple strain measurements for use in state feedback control of flexible manipulators. Reconstruction is proposed for obtaining flexible mode amplitudes from the measurements, and estimation for the modal velocities. Reduced order observers are briefly reviewed, and then application to flexible manipulators is discussed. Design of the observer for estimation of the velocities is discussed with regard to robust implementation. The performance of the observer is examined experimentally for several specifications of the error dynamics.

Organization

The first section will present the general form of a truncated assumed mode model which has been investigated by many researchers [2,3,4]. The second section will discuss reconstruction of the flexible mode amplitudes from measurements. This will be followed by an application of reduced order observers to estimate modal velocities, and design for robust implementation. The last section will evaluate performance of the system.

Assumed Mode Model

Assumed modes is one approximate modal scheme which has repeatedly been applied to flexible manipulators[2,3,4]. The dynamics of the system is described by a rigid body rotation coupled with assumed vibratory modes, this is depicted in figure 1 for a one link arm.



Flexible Manipulator
Figure 1

Utilization of separable descriptions for the vibratory modes provides for the elimination of spatial dependence in the truncated model. Separability in this instance refers to describing the flexible deflections as a series modes which are products of two functions, one a function of a spatial variable, and the other a function of time. The position of the beam can then be noted as;

$$w(x,t)= x\theta(t)+\Sigma\phi_i(x)q_i(t) \text{ , for } i=1,2...n \qquad (1)$$

The distributed character of the flexible manipulator is taken into account via integral expressions over the spatial domain of the entire system in forming kinetic and potential energy expressions. The dynamic equations can be generated from Lagrange's equations applied to the energy expressions, and generalized forces.

$$\frac{d}{dt}\left|\frac{\partial KE}{\partial \dot{\xi}_i}\right| - \frac{\partial PE}{\partial \xi_i} = Q_i \qquad (2)$$

where the ξ_i are the coordinates, and Q_i are the generalized forces associated with each coordinate. The resulting dynamic system can linearized and organized into a linear state space model as;

$$\begin{vmatrix} \dot{\theta} \\ \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \ddot{\theta} \\ \ddot{q}_1 \\ \ddot{q}_2 \\ \vdots \end{vmatrix} = \begin{vmatrix} 0 & I \\ \hline -M^{-1}K & 0 \end{vmatrix} \begin{vmatrix} \theta \\ q_1 \\ q_2 \\ \vdots \\ \dot{\theta} \\ \dot{q}_1 \\ \dot{q}_2 \\ \vdots \end{vmatrix} + \begin{vmatrix} 0 \\ \hline M^{-1}Q \end{vmatrix} u \qquad (3)$$

where M and K are respectively mass and stiffness matrices resulting from the spatial integrations.

Measurement and Reconstruction

Joint angles, and joint rotational speeds can be measured directly as for rigid manipulators, however for state feedback control of manipulator flexibility it is desirable to make direct measurements of the modal variables. Three types of measurement are currently receiving attention for controlling flexibility in manipulators, optical measurement of end point position[5], optical measurement of

*This work was sponsored in part by NASA Grant No. MEA-8303539

To be presented at 1986 ASME Winter Annual Meeting, December 1986, Annaheim, CA

deflection[6], and measurement of strain on the link[7]. The measurement selected for this work is strain. Strain measurement has the following positive aspects:

- Measurement isolates beam variables from rigid motions.
- No restrictions on work envelope, or positioning.
- High compatibility with harsh industrial environments subjecting the sensors to process sprays of oils, solvents, and dispersed solids.
- Low cost sensor, and driving electronics with simple technology base.

Additionally the development presented here can be applied to optical measurements of deflection. Measurement zeroes observed in end point position measurements[5] may adversely effect application of reconstruction to this means of measurement.

Strain Relationships

The moment at any location along the beam is related to the curvature of the beam:

$$M = EI\frac{\partial^2 w(x,t)}{\partial x^2} \tag{4}$$

The stress of the fibers along the surface of the beam due to bending can be determined from the moment:

$$\sigma = \frac{Mc}{I} \tag{5}$$

The strain due to bending is then:

$$\varepsilon = \frac{\sigma}{E} \tag{6}$$

The strain can now be given in terms of the beam deflection $w(x,t)$:

$$\varepsilon(x,t) = c\frac{\partial^2 w(x,t)}{\partial x^2} \tag{7}$$

Assumed mode representation of the flexible deflections can be expressed by:

$$w(x,t) = \Sigma\phi_i(x)q_i(t) \tag{8}$$

The strain can then be represented in terms of the assumed modes as:

$$\varepsilon(x,t) = c\Sigma q_i(t)\frac{d^2\phi_i(x)}{dx^2} \tag{9}$$

This can be expanded to clearly show the contributions of each flexible mode to the measurement of strain at a location x=a on the beam.

$$\varepsilon(a,t) = c\left[\frac{d^2\phi_1}{dx^2}(a)q_1(t) + \frac{d^2\phi_2}{dx^2}(a)q_2(t) + \cdots \frac{d^2\phi_n}{dx^2}(a)q_n(t)\right] \tag{10}$$

For strain measurements at several locations a, b, ...,m this relationship can be presented in matrix form:

$$\begin{vmatrix} \varepsilon(a,t) \\ \varepsilon(b,t) \\ \vdots \\ \varepsilon(m,t) \\ \end{vmatrix} = \begin{vmatrix} c\frac{d^2\phi_1}{dx^2}(a) & c\frac{d^2\phi_2}{dx^2}(a) & \cdots c\frac{d^2\phi_n}{dx^2}(a) \\ c\frac{d^2\phi_1}{dx^2}(b) & c\frac{d^2\phi_2}{dx^2}(b) & \cdots c\frac{d^2\phi_n}{dx^2}(b) \\ \vdots & \vdots & \vdots \\ c\frac{d^2\phi_1}{dx^2}(m) & c\frac{d^2\phi_2}{dx^2}(m) & \cdots c\frac{d^2\phi_n}{dx^2}(m) \end{vmatrix} \begin{vmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_n \end{vmatrix} \tag{11}$$

The relationship depicted above relates the flexible variables to the strain measurements, and can be expressed as a variable transformation T^{-1}.

$$\varepsilon = T^{-1}q \tag{12}$$

The desired form of the transformation however is to "reconstruct" the flexible mode amplitudes from the strain measurements.

$$q = T\varepsilon \tag{13}$$

Inversion of the transformation T^{-1} may be difficult if the number of measurements is different than the number of modes to be identified. If there are more measurements than modes to be estimated, least squares may be applied. Based upon previous experimental results[8] it was decided to investigate a model based upon two assumed modes with reconstruction accomplished from two strain measurements. This case results in a square matrix T^{-1}. Sensor locations for this case can be selected which provide independent measurement of the two modes assuring that T exists.

Luenberger Reduced Order Observers

Direct measurement and reconstruction provides joint angle, joint velocity, and modal amplitude data for the controller. A reduced order observer can be designed to estimate the missing modal velocity amplitudes. The main advantage of a reduced order observer over full state estimation lies in computational savings, this translates into higher sampling frequencies during implementation.

The following paragraphs summarize the work of Luenberger[9], and Gopinath[10] reviewing the development of the equations which describe the behavior of reduced order observers. Assuming that reconstruction provides an accurate measurement of the flexible mode amplitudes, truncation of the model depicted by equation (8) to two modes has the familiar linear systems representation:

$$\dot{x} = Ax + Bu \tag{14}$$

$$y = Cx \tag{15}$$

$$u = Ky \tag{16}$$

where the unmeasured states correspond to the modal velocities. The system can be directly partitioned into measured and unmeasured states as follows;

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \end{vmatrix} = \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} B_1 \\ B_2 \end{vmatrix} |u| \tag{17}$$

$$C = \begin{vmatrix} I & 0 \\ 0 & 0 \end{vmatrix} \tag{18}$$

x_1 are the m measured states while x_2 are the unmeasured states to be estimated. Figure 2 presents a block diagram of a reduced order observer which is described by;

$$\dot{z} = A_{22}z - LC(x_2 - z) + A_{21}x_1 + B_2u \qquad (19)$$

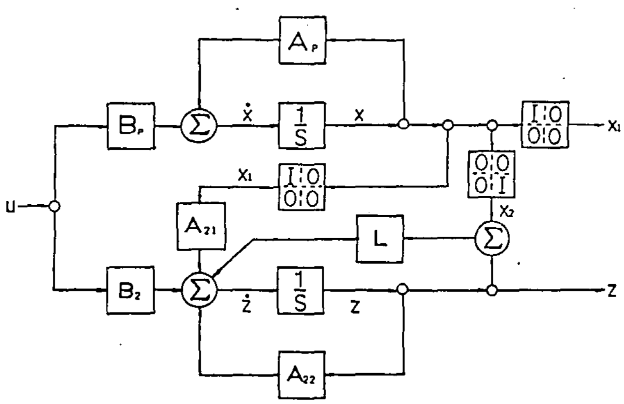


Figure 2. Reduced Order Observer

z is the estimate of the states x_2. The error dynamics for this system can be obtained by subtraction of equation 22 describing the unmeasured states from the estimation equation 19.

$$\dot{e}_2 = (A_{22} - LA_{12})e_2 \qquad (20a)$$

$$e_2 = x_2 - z \qquad (20b)$$

This estimate however depends directly upon measurement of the states to be determine. The dependence on measurements of x_2 can be eliminated via substitution of 24 with the following result;

$$\dot{z} = (A_{22} - LA_{12})z + L\dot{x}_1 - LA_{11}x_1 + (B_2 - LB_1)u \qquad (21)$$

This result although direct allows little insight, and may cause some confusion. The following derivation follows a more heuristic path, and provides more insight into the derivation process. To accomplish this, first cull the expressions for the unmeasured states from equation 17;

$$\dot{x}_2 = A_{22}x_2 + A_{21}x_1 + B_2u \qquad (22)$$

The quantity,

$$A_{21}x_1 + B_2u \qquad (23)$$

which appears in equation 22 can be considered as a known input as it contains only measured and computed quantities. The expressions for the measured states can be pulled out and reorganized as;

$$\dot{x}_1 - A_{11}x_1 - B_1u = A_{12}x_2 \qquad (24)$$

The terms to the left side of the equal sign;

$$\dot{x}_1 - A_{11}x_1 - B_1u \qquad (25)$$

contain only measured quantities, their derivatives, and the computed inputs u. Combining equation 22, and 24 results in an estimation equation;

$$\dot{z} = (A_{22}-LA_{12})z + A_{21}x_1 + L(\dot{x}_1-A_{11}x_1-B_1\ ) + B_2u \qquad (26)$$

Equation 26 provides an observation of the unmeasured states, based on state measurements, the time derivative of the measurements, and the inputs. Additionally, the measurement gain L appears to have the ability to specify the error dynamics. This equation is represented in block diagram form in figure 3.

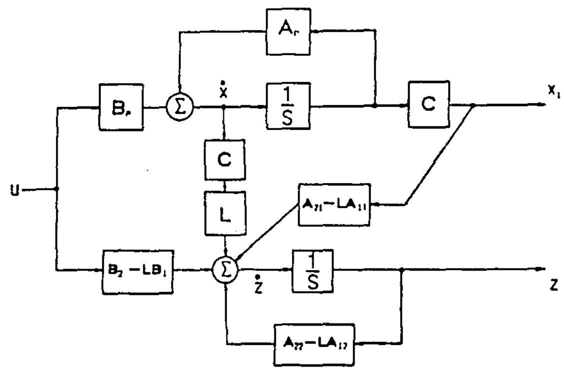


Figure 3. Observer Dependent on Measurement and Measurement Derivatives

<u>Adaptation for Implementation</u>

The resultant observation equation 26 meets the objective of controlling the rate at which the error converges, and eliminates the sensitivity to initial states as the process proceeds. The equation does, however, require the time derivative of the measured states. The time derivative of the measured states may be the variables it is desired to estimate. This is indeed the case for the flexible arm.

Figure 4 depicts an estimation system which does not require knowledge of the time derivative of the state measurements. This is accomplished by utilizing the following substitution.

$$L\dot{x}_1 = (A_{22} - LA_{12})Lx_1 \qquad (27)$$

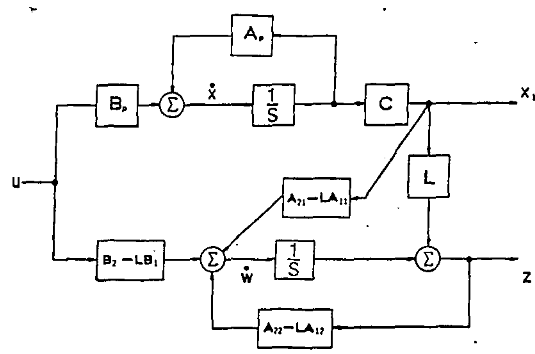


Figure 4. Observer Dependent on Measurement Only

Insertion of this result into the estimation equation 21 yields;

$$\dot{w} = (A_{22} - LA_{12})w + [(A_{21} - LA_{11}) + (A_{22} - LA_{12})L]x_1 + (B_2 - LB_1)u \qquad (28)$$

where,

$$z = w + Lx_1 \qquad (29)$$

The motivation for this substitution is more apparent by noting the adjustments made to figure 3 in deriving the observer shown in figure 4. This adjustment effectively pushes the time derivative of the measurement through the integration block.

Gopinath[10] showed that the error dynamics remain unchanged.

Application of Reduced Order Observers to Single Link Flexible Arms

This section describes the application of the general reduced order observer to the flexible manipulator. Reconstruction of the modal amplitudes is treated separately, and the following development considers these quantities as inputs for the estimation of the modal velocities.

Following the earlier partitioning scheme for measured, x_1, and unmeasured, x_2, states. The state vectors for the flexible manipulator can then be organized as;

$$x_1^T = [\theta, q_1, \dot{q}_2, .., q_n, \dot{\theta}] \qquad (30)$$

$$x_2^T = [\dot{q}_1, \dot{q}_2, .., \dot{q}_n] \qquad (31)$$

where the requirement is to form an estimate z of the modal velocities contained in the x_2 state vector. This form is directly compatible with the state space formulation derived in the dynamic modeling section. Conversion of the continuous estimation equations developed above to digital form appropriate for implementation in a micro-processor is accomplished by direct duality.

Specification of the Measurement Gain L

Selection of the measurement gain matrix L for the flexible arm system is not as direct as that implied by a casual glance at error dynamic equation 20. The estimation equation for the modal velocities of an nth mode series is depicted in equation 32.

$$\dot{w} = [F]_{nxn}w + [L']_{nxn+2}x_1 + [B']_{nx1}u \qquad (32a)$$

$$F = A_{22} - LA_{12} \qquad (32b)$$

$$L' = (A_{21} - LA_{11}) + (A_{22} - LA_{12})L \qquad (32c)$$

$$B' = B_2 - LB_1 \qquad (32d)$$

$$z = w + Lx_1 \qquad (32d)$$

Specification of estimator dynamic matrix F in equation 32a above results in n^2 equations. The measurement gain matrix L' however will have n^2+2n terms. Thus, specification of the error dynamics does not completely determine the elements of L. This will occur whenever more state measurements are made than states to be estimated. This allows significant freedom to the designer, and use of this freedom to improve system robustness will be discussed next.

Pole Placement and Robust Observers

The design freedom mentioned earlier can be used to increase the robustness of the observer system. By examining figure 5, a block diagram of the control implementation, it is apparent that the observer utilizes commanded torque as opposed to the actual torque. If the depicted system is broken at node A, which would correspond to the servo-amp for the motor turned off, the earlier discussion of poles for the combined observer/plant system does not apply. The poles are no longer separable, and the observer displays "closed loop" poles described by;

$$A_{22} - LA_{12} - (LB_1 - B_2)K_2 \qquad (33)$$

K_2 is a gain vector associated with modal velocities. These poles are not identified in the earlier discussion for observer design. Initial disturbances are readily available to this system via state measurement, and unstable poles quickly result in estimates which saturate the system. This results in an experimental system with a "hard start" behavior.

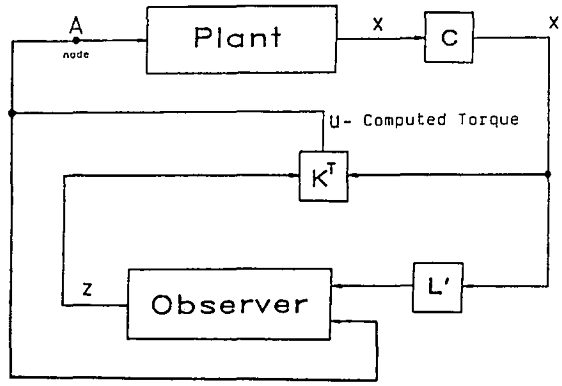


Figure 5. Controller Implementation

Problems of a similar nature were discussed by [11], and the reduction of sensitivity to this problem was termed an increase in robustness. The equations for the closed loop observer poles are combined with the equations for the observer dynamic matrix for determining the elements of the measurement gain matrix L.

Experimental Investigation

Real-time experiments were conducted to investigate modeling assumptions, and observer design performance. The major issues arising during implementation result from truncation of the modal series made to achieve a low order model, and hardware performance. Balas[12] considered the possibility of control "spill-over" into the higher neglected modes having deleterious effects. Also, the proximity of the flexible poles to the imaginary axis makes the system intolerant of unmodeled phase terms introduced by hardware[13].

Experimental Setup

This section describes the experimental apparatus used to measure the observer performance. The system consists of a four foot long flexible arm with payload, DC torque motor with servo-amp, signal conditioning with A/D conversion for data acquisition, 16 bit computer system for implementation of control algorithms, and D/A conversion for torque signal output.

The processor is equipped for hardware computation of floating point operations with a characteristic time for 32 bit multiplications of 19 micro-seconds. A torque motor is driven by a high internal gain DC servo-amp configured with a sense resistor on the motor output to act as a current source. The physical configuration of the flexible arm, torque motor, and sensors is represented in figure 6.

Control Algorithm

A linear quadratic steady state regulator with prescribed degree of stability[14] was designed and implemented for a two mode model. The controller design was executed for a sample and hold system. An optimal regulator design was selected with gains large enough to contrast the performance of the observer dynamic specifications. At low gains stability is hardly a problem, and at very large gains, component performance begins to cloud the observations.

Measured Performance

The first issue investigated was the impact of the controller cycle time. The reconstruction, observation, and control algorithm executed at roughly 178 Hz more than ten times the flexible frequency to be controlled, yet only twice the fourth modal frequency and four times the third. The first four clamped modes of the system are presented in table 1.

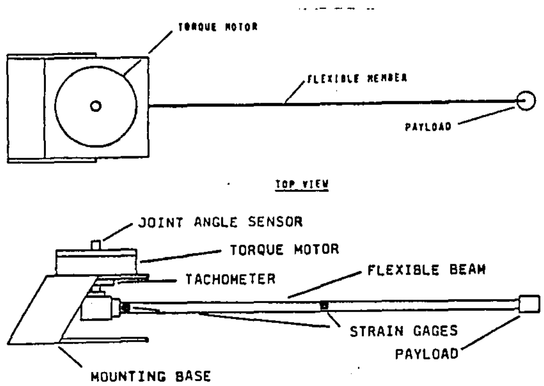


Figure 6. Experimental Configuration

Natural Frequencies(Hz)

Table 1

Mode	Measured	Calculated
1	2.08	2.096
2	13.92	13.989
3	41.38	41.524
4	81.18	81.225

The effect of the controller cycle time was examined by considering the step responses of a collocated controller (sensor and actuator at the same location) using joint angle, and joint velocity executing at 500Hz as shown in figure 7, and at the speed of observer/controller, 178Hz shown in figure 8. The

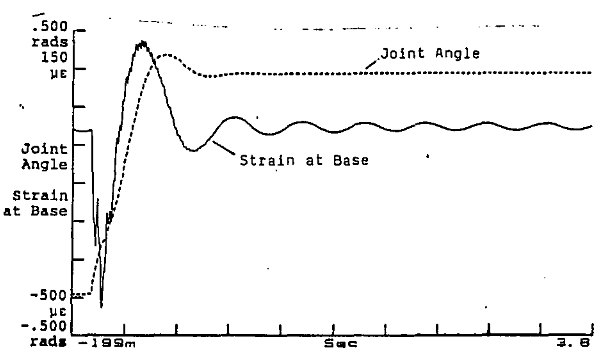


Figure 7. Collocated Controller, 500Hz

joint angle, and strain at the base of the beam were captured to characterize the time response. The gains utilized were the same as for the joint angle and joint velocity of the optimal regulator. The longer cycle time associated with the 178Hz controller resulted in a noticeable increase in the excitation of the third flexible mode. The amplitude of the flexible vibration is not as dramatic as the strain response.

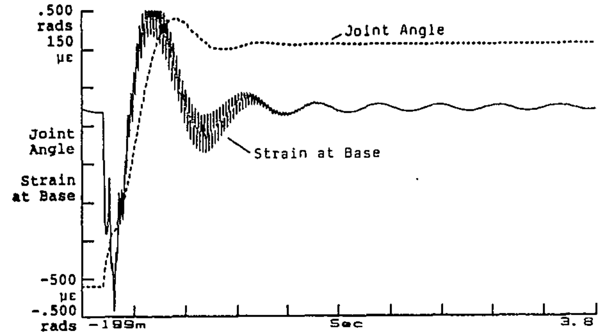


Figure 8. Collocated Controller, 178Hz

Next the observer/regulator combination was implemented with the discrete poles equivalent to negative real poles at two and a half times the flexible poles being examined. The relationship between the flexible modes is shown in table 2.

Table 2

Relationship between Flexible Modes and Observer Poles

Modal Frequency	Equivalent Continuous Pole Case 1(2.5x)	Case 2(5.0X)
2.08Hz	-5.2	-10.40
13.92Hz	-34.8	-69.60

The result for this observer is shown in figure 9.

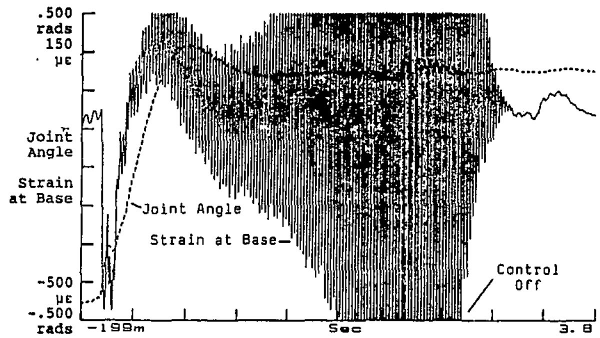


Figure 9. Observer Step Response, Poles 2.5 Times Faster than Dynamics

The controller was most sensitive to the modal velocity gains produced by the observer. The instabilities did not occur in the modes which were being treated by the truncated model, but in the modes truncated from the model. The fourth mode at 81Hz also had increased response although not apparent in the response. This is due to measurements and control torques aimed at the first two flexible modes "spilling" over into the higher untreated modes.

The measurement was repeated for an observer with poles at five times the flexible mode being treated. The relationship between the poles and the flexible modes is presented in table 2. The response for this observer/controller combination is shown in figure 10. This controller does a very good job controlling the first two flexible modes, reducing the amplitude of the strain and quickly damping the vibration. The untreated third mode however is still noticeably

excited, and the power spectrum indicated increased excitation of the fourth mode.
At higher gains, especially modal damping gains, even this observer yielded unstable results in the higher modes, even though the controlled modes were consistently well damped. The indicated trend is to push the observer poles farther and farther to the left,

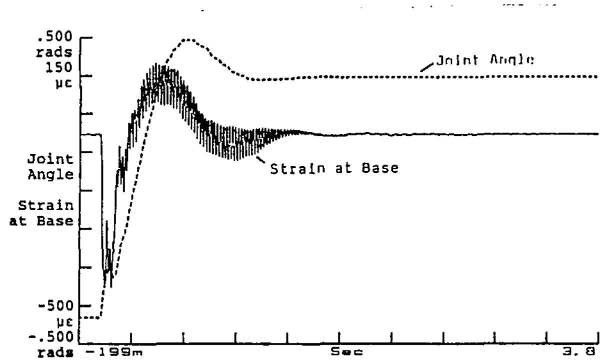


Figure 10. Observer Step Response, Poles 5.0 Times Faster than Dynamics

however, placing the faster observer pole equivalent to ten times the second mode's results in characteristic times for the observer approaching the cycle time of the controller. The response for this observer/controller combination is depicted in figure 11. This combination resulted in significant excitation of the third mode, and for the first time a dramatic response in the fourth mode. This is

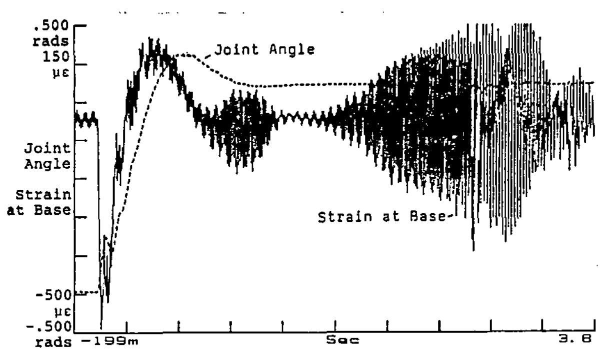


Figure 11. Observer Step Response, Poles 10 Times Faster than Dynamics

counter to the trend, and most likely represents a fatal combination of increased noise sensitivity as the observer poles are pushed father to the left, and aliasing resulting from the controller cycle rate.

Conclusions

Reconstruction and observation of flexible variables for use in controlling a single link flexible arm has been successfully demonstrated. The results indicate that the observer poles must be placed at least five times the flexible mode being estimated, and possibly faster for higher gains. Control spill-over was observed in several of the cases investigated, and this spill-over was aggravated by slow observers to the point of unstable responses for some designs. A dominant factor in the design of high performance observers/controllers for flexible systems appears to be the response of the higher modes. Future work might well focus on consideration of the higher modes in the design phase, if not the implementation. The application of passive damping[15], treating the neglected higher modes, may reduce the performance requirements of the observer/control system.

Bibliography

1. C. Kubrusly, and H. Moalebranche, "Sensors and Controllers Location in Distributed Systems - A Survey", 3rd IFAC Symposium of Control of Distibuted Parameter Systems, Toulouse, France June, 1982.
2. O. Neto, "Modal Analysis and Control of Flexible Manipulator Arms", PhD dissertation, School of Mech., MIT September 1974.
3. V. Sangveraphunsiri, "The Optimal Control and Design of a Flexible Manipulator Arm", PhD Thesis, Dept. of Mech., Georgia Tech., May 1984.
4. W. Book, "Recursive Langrangian Dynamics of Flexible Manipulator Arms via Transformation Matricies", Tech. Report, CMU-RI-TR-83-23, Carnegie-Mellon University, Dec. 1983
5. R. Cannon, and E. Schmitz,"Initial Experiments on the End-point Control of a One Link Experimental Manipulator", The Int. Journal of Robotics Research, The MIT Press, Cambridge, Mass., Vol. 3, No. 3, Fall 1984.
6. D. Wang, M. Vidyasagar, Dept. Elec. Eng., Univ. of Waterloo, Waterloo, Ontario, Canada - work in progress.
7. G. Hastings, and W. Book,"Experiments in the Control of a Flexible Robot Arm", Proc. SME Robots Nine, Detroit, MI, June 1985.
8. G. Hastings, and W. Book, "Verification of a Linear Dynamic Model for Flexible Manipulators", Proc. IEEE Robotics and Automation, Sanfransisco, CA, June 1986.
9. D. Luenberger,"Observing the State of a Linear System", IEEE Trans. on Mil. Electronics, April 1964.
10. B. Gopinath, "On the Control of Linear Multiple Input-Output Systems", Bell System Tech. Journal, Vol. 5, No. 1, March 1971.
11. J. Doyle, and G. Stein,"Robustness with Observers", IEEE Trans. Automatic Control, Vol. AC-24, No. 4, August 1979.
12. M. Balas,"Active Control of Flexible Systems", Journal of Optimization Theory and Applications, Vol. 25, No. 3, July 1978.
13. W. Book, S. Dickerson, G. Hastings, S. Cetinkunt, T. Alberts, "Combined Approaches to Lightweight Arm Utilization", Proceedings ASME WAM, Miami, Fla., Nov. 1985, PED-Vol.15.
14. B. Anderson, J. Moore, "Linear System Optimization with Prescribed Degree of Stability", Proc. IEEE, Vol 116, No. 12, Dec. 1969.
15. T. Alberts, G. Hastings, W. Book, S. Dickerson, "Experiments in Optimal Control of a Flexible Arm with Passive Damping", Fifth VPI&SU/AIAA Symposium on Dynamics and Control of Large Space Structures, Blacksburg, Va, June 12, 1985.