Competition in Supply Chain with Service Contributions

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

SUMMARY

Much of the literature on supply chain management (SCM) has overlooked the importance of nonprice factors to consumer demand and does not include it in its model[1]. Even when a nonprice factor such as service is being considered, it is often studied only at the seller/retailer-customer level, and rarely at the manufacturer-customer level. The literature overlooks the fact that manufacturer, as well, can influence demand through interacting with customers via nonprice factors. In reality, for example in a PC market, the demand for, say, IBM PCs does not depend solely on price, but on IBM's level of technical customer supports as well. This research is being conducted to fill in this gap by studying the influence of both price and services in a dynamic supply chain environment.

To capture both horizontal strategic interaction and vertical strategic interaction between firms in a supply chain, this report studies a case where there are two manufacturers producing competing products and selling them through a common retailer. The two manufacturers must decide on the wholesale price and the level of service they plan to provide to the consumer. We assume that the two manufacturers have equal bargaining power and makes their decisions simultaneously. We assume, as a base case, that the manufacturers have more bargaining power than the retailer. Thus, the manufacturers have the power to set wholesale prices and service levels before the retailer sets the retail prices. The retailer, having less power, makes his decisions (on retail price and ordering quantity of

[1]Examples of nonprice factors are customer supports, service quality, advertising, etc.

each product) after observing the actions of the manufacturers. Each firm are assumed to optimize only its own profit (uncoordinated). The consumer demand depends on two factors: (1) retail price, and (2) service level provided by the manufacturer. All firms in the industry are assumed to behave as if they have perfect knowledge of the demand and the cost structures within the industry.

We extend the study on this basic model in three directions. First, we explore the role of bargaining power in supply chain strategic interactions; particularly, we investigate the strategic behavior of firms when the retailer possesses more bargaining power than that assumed in the base case. We derive and compare equilibrium solutions for the supply chain under three different scenarios (e.g., Manufacturer Stackelberg, Retailer Stackelberg, and Vertical Nash). We found that it is more beneficial to consumers when there is no dominant player(s) in vertical strategic interaction. Furthermore, when one manufacturer has economic advantage in providing service, the retailer will act to separate market segment by selling products with low service at low price and selling products with high service at high price.

We then extend the framework to study multi-period model. In this model, demand also depends on the past period retail prices and service levels, as well as current prices and service levels. This assumption captures the "learning through repeated transactions" behavior of demand. We investigate how the "learning" behavior by consumers would affect the strategic behavior of firms over multiple periods. Game-theoretic approaches and dynamic system and control theory are used as tools to model the problem. We found that if demand is only sensitive to price in its learning process, the company with any type of cost advantage will gain more profit and capture a larger market base than its competitor. The retailer will sell both products at the same retail price but the firm with cost advantage will be able to support more service to its customers.

We also compare the results from our model to those obtained from a myopic model. In a myopic model, the firms only care about their profits in the current period and ignore any future effects their behavior might cause over time. We found that myopic firms are not capable to cope with the learning consumers. Their markets shrink and they earn less profit over time. On the other hand, our model, with think-ahead firms, can prevent this phenomenon from happening. Firms plan their actions to take advantage of the learning behavior of demand. The service levels and prices are chosen such that the firms are rewarded by the consumers. Thus, markets keep growing for both products while firms can keep earning more profits.

Finally, we examine a single period problem with stochastic demand. When demand is uncertain, the retailer faces a newsvendor-type problem. In our model, the newsvendor must manage two competing products against a price-dependent demand. The retailer must decide how much to order from the two manufacturers and at what price each product should be sold in order to maximize his profit. This problem has not been examined in literature. We investigate this problem and analyze how uncertainty can impact strategic interactions among firms in the supply chain, and compared the results to the deterministic case. We derive an expression for the newsvendor's optimal retail prices. We also derive a sufficient condition such that there exists a unique solution for the retail prices. Next, we extend the model to include the two manufacturers. We provide an algorithm to search for the equilibrium wholesale price and service level, given that the manufacturers know the retailer's reaction function. Some numerical examples are provided.

CHAPTER I

INTRODUCTION

1.1 Background and Motivation

Over the past decade, supply chain planning has gained significant importance, due primarily to advances in information technology such as Enterprise Resource Planning (ERP), distributed client-server networks, and the Internet. As an example, with the advent of the Internet, several auction sites have emerged, allowing consumers to bid on their desired item. Consumers have more access to information about products, including price and quality of products as well as level (or quality) of customer service manufacturers provide to their customers. This phenomenon has created a paradigm shift for both retailers and manufacturers. Retailers (or more appropriately, e-tailers) are now more inclined to compete on price (see Keeney(1999) [39], Brynjolfsson *et al.*(2000) [5], and Dewan *et al.* (2003) [19] for more details on online competition).

On the other hand, manufacturers, rather than competing solely on price, have focused more on using services and/or the quality of their products to build brand loyalty. As an example, IBM and Dell are famous for their customer support. These reputations give them an edge over their competitors when customers decide which brand to buy. When customers buy a new computer, they not only consider the hardware, but also the software that comes with it. Customer service is also one of the attributes that influences the customers' decision. This is because customer support can "help the customers obtain maximum value from their purchases" (Goffin (1999)[27]). Another example is in electronics appliances such as washer/dryer machines, refrigerators, etc. In the washer market, Maytag and GE are competing to sell their appliances through common retailers such as Sears or Bestbuy. The major concern for end consumers is not only low price but also the service that comes with the appliance. Therefore, it is important for both GE and Maytag to provide good service in order to maintain loyal customers and lure customers from their competitors.

These examples show efforts by manufacturers to distinguish themselves from their competitors through nonprice factors such as services. In both markets mentioned above, the manufacturers interact directly with the end consumers to create demand for their products. This emphasizes the importance of services on demand, which has been largely overlooked by academics. However, the return on investment in services usually has a decreasing return to scale; the next dollar invested in service produces a lesser increment in service than the last dollar invested. In other words, it costs more to provide the next unit of service than the last one[1]. Therefore, it is important to find the optimal level of service (or quality) that can be achieved for any given demand level.

Aside from emphasizing service and quality, competitive pressures have also forced manufacturing and service organizations to streamline their supply chain operations, reduce system operating costs, while improving speed and reliability. All of these must be done through channel coordination and channel selection.

Different objectives of channel members, however, can create conflicts within a channel. As a result, its members often fail to reach the system-wide optimal pricing decisions. For example, in a market with a monopolist or a group of oligopolists, the manufacturers have

[1]See Chapter 2 for literature review with similar assumption

more negotiation power than the retailers and are therefore able to sell their product with a premium above the competitive price. Many studies focus on vertical coordination among channel members through various transfer pricing schemes or formal agreements such as contracts to achieve maximum channel profit. Many of these studies consider only one manufacturer (monopoly) and its channel intermediaries, the analysis of competition and cooperation is confined only to vertical interaction, and involves only one product.

Little attention has been given by researchers to a larger segment of most consumer goods and electronics markets in which retailers sell multiple (often competing) brands at the same location. This latter channel structure represents numerous markets including those consisting of specialty stores (e.g. consumer electronics, sporting goods, and automobile parts, to name a few), department stores, supermarkets, and convenience stores. This research analyzes the channel structure with multiple-brand vendors (common retailer). Specifically, we analyze a market structure in which there are two competing manufacturers and one common retailer. We also include nonprice factor in the study of the competition.

In the majority of the existing literature in economics and operations research, when a nonprice factor such as service is considered, it is often studied only at the retail level. Only a few studies explore the fact that manufacturers can influence the demand through nonprice factors as well. Our goal is to investigate the above channel structure with price and nonprice factors considered together. Figure 1 shows the channel structure studied in this research.

Our study integrates pricing and service/support decisions into one model. Traditionally, decision on levels (quality) of customer support, wholesale prices, and production have all been determined by separate divisions within the same manufacturing firm. The same is true for a retailer who must make decisions on retail price and ordering quantity to the manufacturers or wholesalers. We plan to study these decisions within a single framework to see how certain parameters affect the optimal solution. Our research would impact the way firms operate and make their decisions regarding competition and supply chain coordination.

One of the main features lacking in the existing literature is the study of channel structure in the multi-period setting with learning demand[2]. Also lacking in the literature is the study on how bargaining power affects behavior of firms in a supply chain. We are interested in the behavior of retailers and manufacturers when faced with learning demand. We plan to investigate how learning demand and bargaining power can affect service levels, retail price, wholesale price, and the profit of each firms. We believe this is an important contribution that has not been explored in the existing literature.

In our model, we define service as the following:

DEFINITION 1.1. *Service is any action which the provider takes to enhance the experience of the customer while he/she is consuming the product. Hence, the customer's willingness to pay for that product increases. Examples of services include post-sale customer support, improved quality, etc.*

Service may increase the value of the product to the customer because it results in informational asymmetries that favor the firm, or possibly because it generates benefits that customers weigh against search costs when deciding where to shop (for alternatives).

[2] This simply means that the customer demand in the current period depends on prices and service levels in the past period(s).

1.2 Problem Description

The model considered in this report has two manufacturers and one retailer. The manufacturers sell physical products to the retailer and provide services directly to the consumer. The consumer demand for each product is sensitive to both retail price and service level provided by the manufacturer. Figure 1 shows the schematic presentation of this supply chain. We assume that the two manufacturers have equal bargaining power. This translates to simultaneous moves by both of them. We also assume, as a base case, that the manufacturers have more bargaining power than the retailer. Thus, they have the power to set wholesale prices and service levels before the retailer sets the retail prices. Chronologically, within each period, events happen in the following order (see Figure 2):

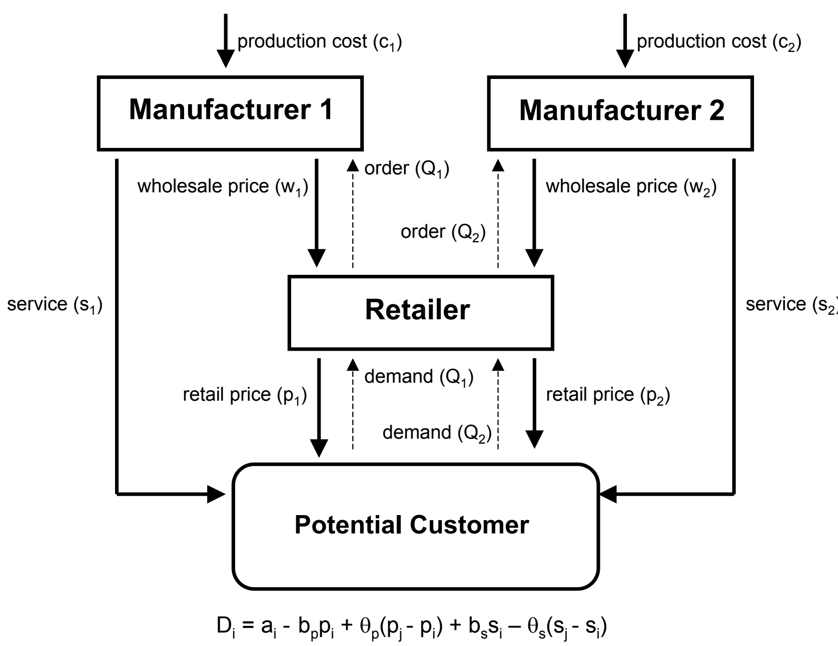

Figure 1: Main Model.

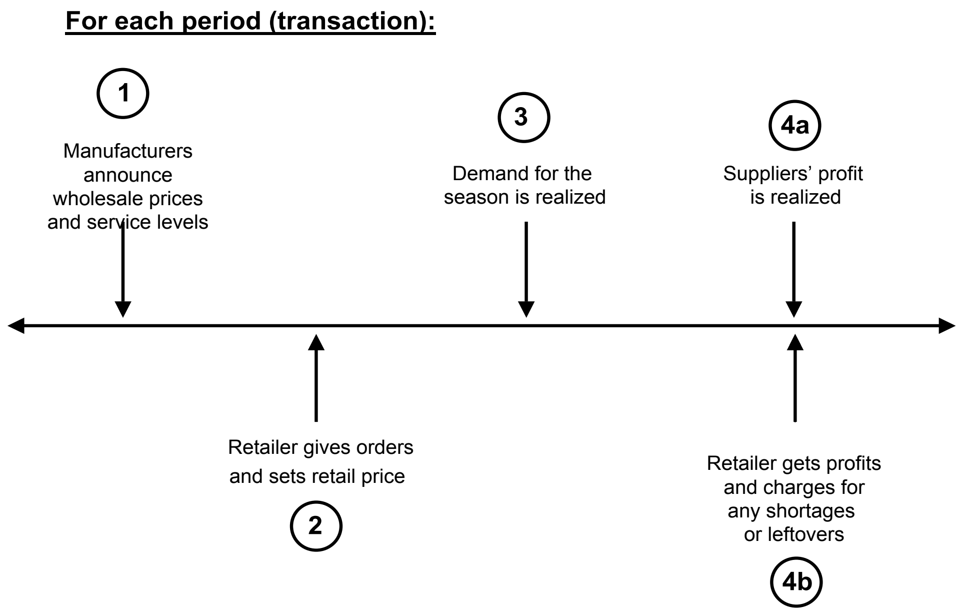

Figure 2: Timeline of events within each transaction.

Step 1. The manufacturers simultaneously announce wholesale prices to be offered to the retailer and service levels to be offered to consumers.

Step 2. In response to the manufacturers' announcement, the retailer decides the retail price and ordering quantity of each product that would maximize his expected profit. The retailer's ordering quantities become incoming demands for each manufacturer.

Step 3. The consumer demand for each product is realized.

Step 4. The shortage cost or disposal cost for each product is charged to the retailer, depending on the demand and the stocking level. The manufacturers realize their profit in this transaction.

We extend the study on this basic model in three directions (see Figure 3). First, we explore the role of bargaining power in supply chain strategic interactions; particularly, we investigate the strategic behavior of firms when the retailer possesses more bargaining power than that assumed in the base case. We derive and compare equilibrium solutions for the supply chain under three different bargaining power assumptions (e.g., Manufacturer Stackelberg, Retailer Stackelberg, and Vertical Nash). The results from this investigation will shed some light on how retail prices and service levels are influenced by bargaining power. Details of our investigation are given in Chapter 3.

We then extend the problem to multiple periods. In the multiple period model, demand also depends on past period retail prices and past service levels. This is to capture the "learning through repeated transactions" behavior of demand. For example, in PC industry, consumers upgrade their PCs every 3-5 years. Thus, it is possible that customers gain more experience on price and service every time they upgrade their products. The price they paid and the service they received during their last experience will influence their next upgrading decisions. We investigate how "learning" behavior by consumers would affect the strategic behavior of firms over multiple periods. Game-theoretical approaches and dynamic system and control theory are used as tools to model the problem. Details on this part of the research are given in Chapter 4.

Lastly, we modify our model to capture demand uncertainty. When demand is stochastic, the retailer faces a newsvendor-type problem. In the classic newsvendor problem, there is only one product, with a given fixed price and uncertain demand. In our model, however, the newsvendor must manage two competing products against a price-dependent demand. The retailer must decide how much to order from the two manufacturers and at what price each product should be sold. We investigate this problem and analyze how uncertainty can impact strategic interactions among firms in the supply chain, as compared to the deterministic case. Chapter 5 provides details of our investigation on the uncertain demand case. The next section gives guidelines and details of each chapter within this report.

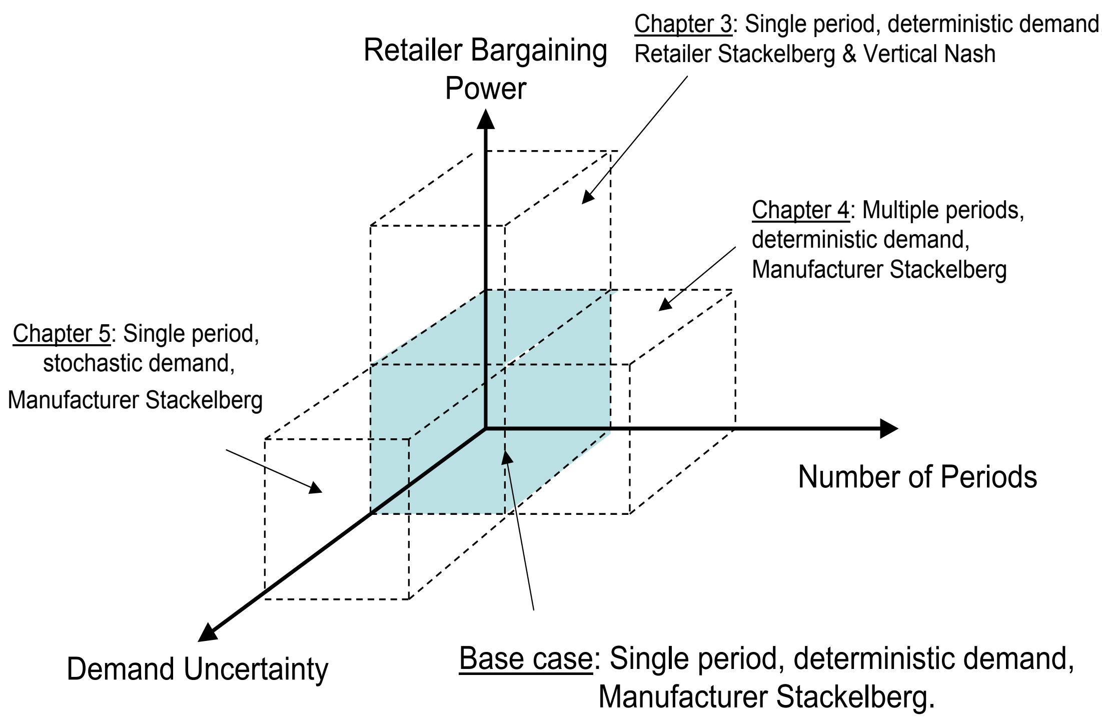

Figure 3: Research Direction.

1.3 Outline and Research Contribution of this report

This report examines a supply chain structure with two competing manufacturers and a common retailer. The demand depends on prices and service levels. In Chapter 2 we review the relevant literature, and compare it to the problem we investigate in this thesis.

Next we investigate the role of bargaining power in Chapter 3. Namely, we consider the problem with single period setting and with deterministic demand. We derive the expressions for the equilibrium retail prices, wholesale prices, and service levels under different bargaining power assumptions. We then compare results from the three scenarios and derive some insights on the strategic interactions among firms in the supply chain. We then study the influence of each parameter in the model on the equilibrium solution by using sensitivity analysis.

In Chapter 4 we extend the basic case over multiple periods. We study the behavior of each firm (one retailer and two competing manufacturers) over time when faced with "learning" demand. We assume that demand for each product in any given period is affected by two types of components: (1) the difference in prices and service levels between the two products in the previous period, and (2) the amount of investment by each manufacturer at the beginning of each period to expand the market base of its product (or brand). We approach the problem by introducing a new methodology for game-based decision making in multiple periods by combining a game-theoretic approach with concepts from dynamic systems and control theory. Finally, we derive economic and managerial insights using the knowledge from dynamic systems and control theory.

In Chapter 5 we consider the first part of the stochastic problem by studying the two-product newsvendor problem with price-dependent demand. We derive an expression for the optimal retail price. We also derive a sufficient condition such that there exists a unique solution for the retail price and for the order quantity for both products. We find that the hazard rate of the demand distribution is crucial for the existence of a unique solution. We also give a set of conditions that must be satisfied by the optimal solution. Next, we extend the model to include the two manufacturers. We then formally describe the model using mathematical expressions. The results from the investigation of two-product newsvendor are used to determine the retailer's reaction function. We then propose an algorithm to provide the manufacturers with the equilibrium wholesale price and service level, given that they know the retailer's reaction function. Finally, some numerical examples are shown.

Chapter 6 concludes with our contributions and gives some possible extensions to our thesis for future studies.

CHAPTER II

LITERATURE REVIEW

2.1 Introduction

There are many existing studies on supply chain management. They range from economics and marketing to operations research and management. We will concentrate on those that are related to our model. As shown in Chapter 1, our research spans three different directions. Each direction requires different combinations of knowledge from various relevant fields. In the single-period deterministic problem considered in Chapter 3, game theory and optimization are used to investigate the problem. Chapter 4 requires game theory and dynamic systems and optimal control to model multi-period problem with demand learning. Lastly, Chapter 5 studies the supply chain with stochastic demand. The model requires knowledge from both game theory and operations research. In this chapter, related literature for each direction will be reviewed and addressed.

We categorize literature into groups according to the three research directions of this thesis. To systematically review existing literature, it is very helpful to consider the following five factors:

(1) Channel structure: Number of Supplier(s) and Retailer(s). The structure of the channel being studied is the most important factor. Early studies focus on one retailer and one manufacturer (two-stage supply chain) or two competing retailers.

Recent studies include other situations such as one supplier-multiple retailers, multiple suppliers-single retailer, or multiple suppliers-multiple retailers. We assume one retailer and two suppliers in our model.

(2) Number of products: Single versus Multiple. Most models in the literature assume a single product; some assume two or more competing (heterogeneous, substitutable) products. Our model assumes two competing products, each being produced by a different manufacturer.

(3) Decision attribute(s): Price versus Nonprice. In most literature, firms compete only on price. However, nonprice factors such as service, advertisement and quality level can also influence the demand for the product. In the literature, there are differences in how various types of nonprice factors are defined and used in different models. For example, So (2000) [80] uses a delivery time guarantee, Iyer (1998) [32] uses locational differentiation among retailers, while Hall and Porteus (2000) [28] use customer service capacity. In our model, we define our nonprice factor to be the level (or quality) of service. This is similar to the definition used in McGahan and Ghemawat (1994) [58].

(4) Demand: Deterministic versus Stochastic. Most of the marketing and economic literature assumes that the demand is deterministic while most of the literature in operations assumes a stochastic demand. In this thesis, we investigate the problem with both deterministic and stochastic demand assumptions. The literature on the deterministic case provides good background to our stochastic model and should be studied carefully.

(5) Number of periods: Single versus Multiple. Most marketing and microeconomic models assume a single period and focus on how each player interacts and on critical factors that influence their decision. The multi-period model, on the other hand, is used to capture the dynamic and "learning" aspect of the model. Our model in Chapter 4 is multi-period, but results from the single-period problem studied in Chapter 3 are the basic building blocks for the multi-period model.

Different combinations of the above 5 factors give rise to various situations. Some have been extensively studied, while some have been overlooked and not yet explored. Table 2.1 shows some of the literature categorized according to the assumption used in the main model on various factors (e.g., demand, number of periods, attributes, number of products, and channel structure). In the table, our problem is compared to the rest of the literature.

The marketing literature often focuses on the coordination of pricing decisions in a single period, without production and inventory considerations. The operations literature, on the other hand, has traditionally been focused on coordinating production and inventory decisions, assuming that price and, hence, demand are given. The problems that are most heavily studied are within the EOQ setting where both the supplier and the retailer face a fixed production/ordering cost and a linear inventory holding cost.

Other studies in marketing and management literature focus on decentralized supply chain and try to find mechanisms to coordinate the actions among various players to achieve the optimal (centralized) solution. In a decentralized supply chain, closely inter-related business activities are often performed by multiple firms with conflicting objectives. When the decisions of these firms are uncoordinated, the supply chain as a whole encounters a significant loss of efficiency. Coordination of activities among different firms offers numerous

Structure	Attributes	# Prod.	# Periods	Representing Paper(s)
1 Retailer, 1 Manu.	Price & Service	1	1	Jeuland & Shugan (1983)
1 Retailer, 1 Manu.	Price	1	1	Moorthy (1987)
1 Retailer, 2 Manu.	Price	1	1	Choi (1991)
1 Retailer, 1 Manu.	Price & Power	1	1	Ertek and Griffin (2002)
2 Firms	Price & Service	1	1	McGahan and Ghemawat (1994)
N Firms	Price & Service	1	1	So (2000)
2 Retailers, 1 Supplier	Price & Service	1	1	Tsay and Agrawal (2000)
1 Supplier, 2 Retailers	Price & location	1	1	Iyer (1998)
1 Retailer, 2 Supplier	**Price & Service**	**2**	**1**	**Chapter 3**
2 Firms	Price & Advertising	2	∞	Vilcassim et. al. (1999)
2 Firms	Advertising	2	$t > 1$	Fruchter and Kalish (1997)
2 Firms	Advertising	2	∞	Chintagunta (1993)
2 Firms	Price & Brand	2	∞	Chintagunta and Rao (1996)
1 Firm	Price & Inventory	1	$t > 1$	Federgruen and Heching (1999)
1 Newsvendor	Price	1	$t > 1$	Pertuzzi and Dada (1999)
1 Newsvendor	Price	1	$t > 1$	Petruzzi and Dada (2002)
1 Retailer, 1 Supplier	Price	1	$t > 1$	Chen et al. (2000)
1 Retailer, 2 Supplier	**Price & Service**	**2**	$t > 1$	**Chapter 4**
1 Newsvendor	Price	1	1	Silver et al. (1998)
1 Newsvendor	Price & Quantity	1	1	Lau and Lau (1988)
1 Newsvendor	Price & Quantity	1	1	Dana and Petruzzi (2001)
1 Newsvendor	Price & Quantity	2	1	Khouja et al. (1996)
1 Newsvendor	Price	2	1	Li et al. (1991)
1 Newsvendor	Price	$m > 1$	1	Lau and Lau (1995)
N Newsvendor	Price	1	1	Lippman and McCardle (1997)
1 Newsvendor, 1 Supplier	Price	1	1	Lariviere and Porteus (2001)
1 Newsvendor, N suppliers	Price	1	2	Petruzzi and Dada (2001)
1 Firm	Price & Inventory	1	1	Johnson and Montgomery (1974)
1 Newsvendor, 2 suppliers	**Price & Service**	**2**	1	**Chapter 5**

Table 1: Summary table of the existing literature

challenges and opportunities both for academic and real-world applications. Therefore, there exist many studies in economics , marketing, and operations management addressing coordination among firms in supply chain. Examples of reviews of advances in this problem can be found in Thomas and Griffin (1996) [84], Simchi-Levi *et al.* (1999) [79], Tayur *et al.* (1999) [82], and Cachon (2001) [10].

Our model combines marketing and operations approach to study both pricing and ordering decisions faced by firms in the supply chain. We investigate strategic interactions among firms in the supply chain using combinations of game theory, dynamic systems and control theory, and operations research. The coordination issue is not the focus of this thesis[1].

[1]Possible extensions of this thesis are be given and discussed in Chapter 6

In the following sections of this chapter, we review some of the works related to our problem. In Section 2.2 we review literature in which models assume a deterministic demand. These models are mostly from the marketing or economics literature and are used to demonstrate the coordination of channel through price or other payment schemes. The literature provides a good basis for understanding how players in each stage of a supply chain interact with one another. Literature on nonprice competition is also discussed in this section since our model assumes that demand is sensitive to both price and service[2].

Section 2.3 reviews literature in which multiple-period models are analyzed, especially those that involve "learning" through repeated transactions. There are two streams of research on "learning." The first group refers to "learning" as a process of updating information on demand distribution (e.g., Petruzzi and Dada (2001) [68] and Cachon and Porteus (1999) [8]). Another stream of research imbeds "learning" into demand function as part of demand modelling (e.g., Vilcassim *et al.* (1999) [91]). We follow this latter approach in our analysis of the multi-period problem in Chapter 4.

Section 2.4 reviews literature involving the newsvendor model and its extensions. Our model defined in Chapter 5 differs from existing models because of additional assumptions on (1) price-dependent demand, and (2) multiple products being sold by the newsvendor. The existing literature assumes and studies models with only one of these assumptions at a time, but never assumes both in the same model. Our work in Chapter 5 is an important extension to existing literature on the newsvendor model.

[2]The deterministic version of our model can be seen in Chapter 3.

2.2 Deterministic Demand

The literature with deterministic demand can be separated into two main groups. In the first group, demand is assumed to depend solely on price. In the second group, demand may depend on other attribute(s) such as service or quality.

2.2.1 Price-Sensitive Demand Models

Studies on horizontal competition between two or more producers (or sellers) can be traced back to classic economic models such as Cournot, Bertrand, and Stackelberg competition. Reviews on these models and their variants can be found in Tirole (2000) [87]. However, these studies are primarily based on a single-echelon environment. Early studies on a multi-echelon environment can be found in Jeuland and Shugan (1983) [35], McGuire and Staelin (1983) [59], Moorthy (1987) [62], and Ingene and Parry (1995) [31]. Most of these studies consider only the case with a single manufacturer and a single retailer. They have focused on vertical coordination among channel members through measures such as transfer pricing schemes or formal agreements. Particularly, Jeuland and Shugan (1983) show that the supplier can use a quantity discount schedule to induce the retailer to choose the channel-optimal retail price. Moorthy (1987) shows that channel coordination can also be achieved through a simple two-part tariff: the supplier sells the product at his own (marginal) production cost and charges the retailer a fixed side-payment.

Lee and Staelin (1997) [51] examine strategic pricing policies in uncoordinated supply chain (no vertical integration or two-part tariff). Using a game theoretic approach, the paper shows that the question of using linear or nonlinear demand functions is not as critical as whether the demand functions imply vertical strategic substitute (VSS) or vertical strategic complement (VSC). They also show that it is not always beneficial to use the knowledge of the competitors' reactions.

Kim and Staelin (1999) [42] examine a single period profit maximizing game with two manufacturers and two retailers. The manufacturers must decide on how large a side payment to give to each retailer, and retailers decide on how much of this side payment to use to promote the manufacturer's product. The authors derive the optimal solutions and perform sensitivity analysis on the results. They find that if consumers become more sensitive to differences in merchandizing activity between brands within a store, the retailers' profits increase and the manufacturers' profits decrease.

Weng (1995) [93] studies both pricing and production/ordering decisions. Weng considers a system with a single manufacturer and multiple, identical retailers and shows that channel coordination can be achieved by using a quantity discount policy. Chen *et al.* (2001) extend Weng's model to non-identical retailers. Both papers consider a static model (i.e., stationary demand) with concave cost function, which is different from our dynamic model.

Choi (1991) [15] examines a channel structure with two competing manufacturers and one common retailer that sells both manufacturers' products. The study includes a one-period problem with deterministic, price-sensitive demand, and three noncooperative games of different power structures between the two manufacturers and the retailer, i.e., two Stackelberg games and one Nash game.

Tsay and Agrawal (2000) [90] study a distribution system in which a manufacturer supplies a common product to two independent retailers. The demand is deterministic and depends on both the retail prices and retail services. They show that the intensity of competition with respect to each competitive dimension plays a key role, as does the degree of cooperation between the retailers.

Zhao and Wang (2002) [98] study the multiple-period problem of a system with one supplier and one retailer facing a deterministic, price-sensitive and time-dependent demand over a finite horizon (one selling season). They study the case where the manufacturer is the Stackelberg leader and give the retailer a price schedule for every period in the coming season. Dynamic programming techniques are used to obtain some key results and to show that there exists a wholesale price schedule that can lead to the channel-optimal solution.

Ertek and Griffin (2002) [22] study decentralized pricing-production decisions in a supplier-buyer channel with EOQ cost structure. They examine the impact of power structure on price, sensitivity of market price, and profits in a single-product, two-stage supply chain with one supplier and one retailer. They consider the cases where each of the player (supplier/retailer) has a dominant bargaining power. However, they do not address coordination issues.

2.2.2 Service/Quality-Sensitive Demand Models

Nonprice factors include services, quality, negotiation power, etc. However, the majority of the studies mentioned above have considered price or product quantity as the only dimension of competition. Some researchers have recognized this weakness and developed models containing additional attribute(s) that may influence consumer demand. Early research to include attributes such as product quality and service can be found in economics literature such as Spence (1975) [81] and Dixit (1979) [20]. In marketing literature, Jeuland and Shugan (1983) include nonprice variable such as quality and services in their model with the profit function as a linear function of service amount. Our model, on the other hand, has the profit function as a nonlinear function to service amount due to the decreasing returns of providing service. Moorthy (1988) [63] examines a competition in duopoly through both price and quality. Our model considers both horizontal and vertical relationships. Recently, there have been many studies that incorporate nonprice factors in a model to analyze its impact on competition or channel coordination.

Iyer (1998) [32] examines a channel with one manufacturer and two retailers who compete on both price and non-price factors. He assumes that consumers are heterogeneous in spatial locations (as in the spatial models of horizontal differentiation) and in their willingness to pay for retail services (as in the model of vertical differentiation). In the model, the manufacturer is the Stackelberg leader, while the two retailers move simultaneously. The paper shows that in markets with small locational differentiation and substantial diversity in consumer willingness to pay, the manufacturer's problem is not just to align retailer interests, but also to use a channel contract to induce the optimal level of retail differentiation.

McGahan and Ghemawat (1994) [58] study a single-transaction, game-theoretic model in which duopolists attempt to retain old customers through service and attract new customers through price. They use a two-stage model in which the duopolists simultaneously commit to expenditures on customer service in the first stage and then (simultaneously) name their prices in the second stage. The paper shows that large firms are likely to exhibit greater customer retention rates than their smaller rivals in equilibrium.

Hall and Porteus (2000) [28] study a finite multiple-period problem in which two firms compete by investing in capacity that is used to provide goods or services to their customers.

They assume that there is a fixed total market of customers whose demands for the goods or service are random. The paper obtains results for both single-period and finite-horizon problems.

So (2000) [80] studies the problem where several heterogeneous service firms use delivery time guarantees to compete for customers in the marketplace. Demand is deterministic and is assumed to be sensitive to both price and delivery time guarantees. The objective of the firm is to maximize profit by selecting the optimal price and time guarantee. He finds that the high capacity firms provide better time guarantees, while firms with lower operating costs offer lower price, and the differentiation becomes more acute as demands become more time-sensitive.

2.3 Multiple-Period Models

Existing studies on multiple-period models can be separated into two groups. Studies in the first group are mostly from the industrial engineering and operations research community; they focus on production and/or inventory management by a single firm. The second group is mostly from the marketing and management community; they concentrate on competition and interactions among firms through either price or nonprice factor(s) over time. Our research in Chapter 4 fits into the second group. Literature in each group is reviewed here.

2.3.1 Productions and Supply Chain Management

Early literature in this group include Thomas (1970) [83], who considers the joint pricing-production decision in a discrete-time (multi-period) setting. Federgruen and Heching

(1999) [23] consider pricing-production models with concave revenue functions. Particularly, they examine a multiple-period, single-item problem, in which a firm faces uncertain price-dependent demand. The paper addresses the simultaneous determination of pricing and inventory replenishment strategies for such a firm in both finite and infinite horizon models, with the objective of maximizing total expected discounted profit or its time average value.

There is a rich collection of literature on supply chain coordination with stochastic demand. Several mechanisms have been identified to coordinate manufacturer-retailer channels. They include the inventory buyback/return policy (Pasternack (1985) [67] and the quantity flexibility policy (Tsay (1999) [89]) for models without pricing decisions, and with revenue sharing contracts (Cachon and Lariviere (2000) [9]) and with a two-part tariff (Weng (1997) [94]) for systems with pricing and production decisions. The coordination mechanisms serve as means to share risk among firms in a channel in order to resolve incentive incompatibilities due to uncertainties. For an up-to-date, comprehensive review on this line of research, the reader is referred to Cachon (2001) [10].

2.3.2 Multi-period Dynamic Competition

The majority of studies in this group are from the marketing and management community. Marketing literature models demand as diffusion of acceptance with adoption rate/sales rate and focus on consumer adoption process of a new product. The current research on diffusion models originated with the Bass model (Bass (1969) [3]), which does not include price. Robinson and Lakhani (1975) [73] were the first to incorporate the variable of price into the Bass model. In recent work, the cost experience curve has been introduced on the production side; hence there are learning effects on both demand and cost. Most applications deal with durable goods where each adopter represents one unit of sales. In most cases, repeated sales have been ignored. However, Jeuland and Dolan (1982) [34] and Mahajan *et al.* (1983) [57]) included repeated purchases in their models. Dockner (1985) [21] generalizes the Robinson-Lakhani model to a duopoly. Dockner applies a game-theoretic approach to find a Nash Equilibrium. However, this group of literature focuses only on price as the main decision variable. It also does not consider the role of retailer in the supply chain during dynamic competition.

There is another parallel stream of research in economics and marketing that is not based on Bass's diffusion model. Demand is assumed to be derived from aggregate (retailer level) scanner data. Our model follows this approach which is common in microeconomics (see Tirole (2000) [87] and Shy (2000) [76]). Both price and nonprice variable(s) can be included in the model. Hotelling (1929) [30] was the first to introduce a formal model of product differentiation through price and location. Gabszewicz and Thisse (1979) [26] and Cohen and Whang (1997) [17] develop models where customers' preference for products can be strictly ordered (for example, quality - the higher, the better). Other studies such as Chintagunta (1993) [13] examine the sensitivity of equilibrium profits in advertising game in a duopolistic market. Chintagunta and Rao (1996) [14] consider pricing strategies in a dynamic duopoly. Fruchter and Kalish (1997) [24] investigate dynamic competition through advertisement between two firms.

It is only recently that "learning" through repeated transactions has been integrated into multi-period models. There are two streams of research on "learning." Petruzzi and Dada (2001, 2002) [69], [70] and Cachon and Porteus (1999) [8] are among the studies in the first group which regard "learning" as a process of updating information on demand distribution. Petruzzi and Dada (2001) analyze the problem of determining inventory and pricing decisions in a two-period retail setting when an opportunity to refine information about uncertain demand is available. In particular, they determine the optimal stocking and pricing policies over time when a given market parameter of the demand process, though fixed, is initially unknown. Petruzzi and Dada (2002) [70] extend the problem by considering a multiple period problem. The authors use dynamic programming to formulate their model.

Another stream of research embeds "learning" into the demand function as part of demand modelling. Vilcassim *et al.* (1999) [91] use this approach in their analysis of price and advertising competition among firms in a given product market. Firm (or brand) level demand functions account for the contemporaneous and carry-over effects of these marketing activities, and also allow for the effects of competitor actions. This approach enables them to quantify both the direction and magnitude of competitive reactions, and also to identify the form of market conduct that generates the particular pattern of interaction. We follow this latter approach of "learning" in our analysis of multi-period problem in Chapter 4.

2.4 Newsvendor Model

The classical newsvendor problem assumes that the selling price is given and that the demand is independent of the product's price. This assumption is used in many extensions of the newsvendor problem. However, recently many researchers start to address the newsvendor problem with price-dependent demand. Our model in Chapter 5 follows this trend. In this section, we first review some of the extensions on the classical newsvendor that still assume price-demand independence. Newsvendor models with joint price-ordering decision are then discussed.

2.4.1 Price-Independent Demand Model

Numerous extensions to the classical newsvendor model have recently been proposed in the literature (see Khouja (1999) [41] for extensive reviews). In particular, Lau and Lau (1988) [45] introduce a price-sensitive demand model under two objectives to maximize expected profit to and maximize the probability of achieving a target level of profit. Parlar (1988) [66] characterizes a duopoly of two newsvendors who become competitors because their products are partially substitutable (i.e., when either of the firms' stock is out, a fixed fraction of the excess demand transfers to the other). Lippman and McCardle (1997) [55] generalize this by considering various scenarios where the realized aggregate demand is initially split between the firms as a function of their inventory levels and more substitutable patterns. In their model, the two newsvendors sell perishable goods and choose quantities to be sold at a predetermined market price. Both papers (Parlar (1988) and Lippman and McCardle (1997)) examine the existence and uniqueness properties of Nash solutions. However, explicit computation of the equilibria turns out to be nontrivial in both settings.

Lau and Lau (1988) [46] study a two-product newsvendor problem under the objective of maximizing the probability of achieving a profit target. Li *et al.* (1990) [53] present an analytical solution of the problem for the case of independent and uniformly distributed demands. Lau and Lau (1991) [47] present an analytical solution of the problem for the case of independent and exponentially distributed demands. The problem of a multi-product newsvendor problem with capacity constraints is examined in Lau and Lau (1995) [48].

Khouja *et al.* (1996) [40] examine a single-period newsvendor problem with two substitutable products. However, as in the classic newsvendor problem, they assume that the price is given. Since many local optimal solutions may exist, the authors use Monte Carlo simulation to identify the optimal solution (no analytical solution is given).

Lariviere and Porteus (2001) [43] study a simple supply chain contract in which a manufacturer sells to a retailer facing a (standard) newsvendor problem with wholesale price as the lone contract parameter. They study a single-period model with the manufacturer as a Stackelberg leader. The "optimal" contract is created such that it maximizes the manufacturer's profit subject to assuring retailer acceptance (the retailer has opportunity cost to compare with the expected profit).

Petruzzi and Dada (2001) [69] study a two-period newsvendor problem with the possibility of refining information about the uncertain demand. The newsvendor uses the demand information he gets in the first period to help him plan for his actions in the second period.

2.4.2 Joint Price-Ordering Decisions

Research in joint pricing-ordering decisions was first formulated by Whitin (1955) [95]. He incorporates pricing into the classic Economic Order Quantity (EOQ) model by assuming that the demand rate depends linearly on the price. Major extensions to Whitin's model include Porteus (1985) [71], who considers investment to reduce setup cost, and Cohen (1977) [16], who models perishable products. The decisions in all these models are static in nature but the analysis on these models provide the basis for research on newsvendor-type model.

Although most newsvendor models assume that price is given, there have been, in recent years, many papers addressing the newsvendor problem with price-sensitive demand. Lau and Lau (1988) [45] is one of the earliest papers that study the problem with newsvendors facing such demand. In their model, the newsvendor simultaneously decides the price and order quantity. However, they use an alternative objective of maximizing the probability of achieving the target profit level. Petruzzi and Dada (1999) [68] also examine an extension of the classical newsvendor problem in which ordering quantity and selling price are set simultaneously because the demand faced by the newsvendor is price-sensitive. They first study a single-period version of the problem and then extend it to a multiple-period one. Using the change-of-variable method, they find the condition that would guarantee the existence of a unique solution. Chapter 5 of this thesis extends their work to the case of a two-product newsvendor facing price-sensitive demand.

Dana and Petruzzi (2001) [18] examine a firm's price and inventory policy when it faces uncertain demand that depends on both price and inventory level. The authors extend the classic newsvendor model by assuming that consumers choose between visiting the firm and consuming an exogenous outside option. The paper investigates both the case in which the firm's price is exogenous and the case in which price may be chosen optimally. The paper shows that the firm holds more inventory, provides a higher fill rate, attracts more customers, and earns higher profits when it internalizes the effect of its inventory on demand.

CHAPTER III

SINGLE-PERIOD DETERMINISTIC DEMAND

3.1 Introduction

With the current dynamic and competitive environment, product manufacturers must compete with more complicated strategies than simply lowering their price. Non-price factors such as service have become more important in affecting a consumer's decision to buy a product. In this research, service is defined as any action that the manufacturer takes to "help the customers obtain maximum value from their purchases" (Goffin 1999, [27]). Example of services include post-sale customer support, product advertising, improved product quality, product delivery, etc.

There are quite a few successful firms that have focused on service and quality of their products in building brand loyalty. For example, IBM and HP are both famous for their customer support. This reputation gives them an edge over their competitors. Another example can be seen in consumer electronics such as digital cameras. Nikon and Canon are competing to sell their products through common retailers such as Ritz Camera or BestBuy. One of the major concerns for end customers is not only how low the price is, but also how good the service he or she expects to receive that comes with the product. In both of these examples, the manufacturers interact directly with the end consumers through the service channel.

The impact of the manufacturer's service quality to consumers' buying decisions is likely to influence the strategic interactions between a manufacturer and his retailer regarding pricing and ordering quantity. Moreover, competitive pressure from other manufacturers and their interactions with the retailer are issues that a manufacturer must also consider in its decision processes. Our research focuses on the supply chain depicted in Figure 4, where each manufacturer provides services directly to the customers and the retailer sells competing products to end consumers[1]. To the best of our knowledge, very few studies have considered all these issues of price and service interactions, manufacturers' competition and supply chain's channel coordination simultaneously; even though most consumer goods and electronics products are sold by retailers who sell multiple competing brands at the same location (See Chapter 2 for literature reviews). This research will make significant contributions in this important research area.

In order to study the role of service in competition between two manufacturers in this supply chain we need to make assumptions regarding vertical strategic interactions between manufacturers and the retailer. In general, in a market with a monopolist or a group of oligopolists the manufacturers would possess more bargaining power than the retailer and would be able to sell their product with some premium above the competitive price. On the other hand, if the retailer possesses more negotiation power, it can bring down the manufacturer's profit and absorb the majority of the profit to itself. The reason we need to consider these cases is because the retailer that deals with a number of competing products is often a large retailer that can influence the market substantially. It has been reported that the bargaining power has transferred from manufacturers to retailers in some retailing

[1]This research does not study the impact of retailer's service to customers due to its potential "conflict" with the service provided by the manufacturers. Issues about possible differentiation between services from the retailer and those from the manufacturers is not the focus of our study here.

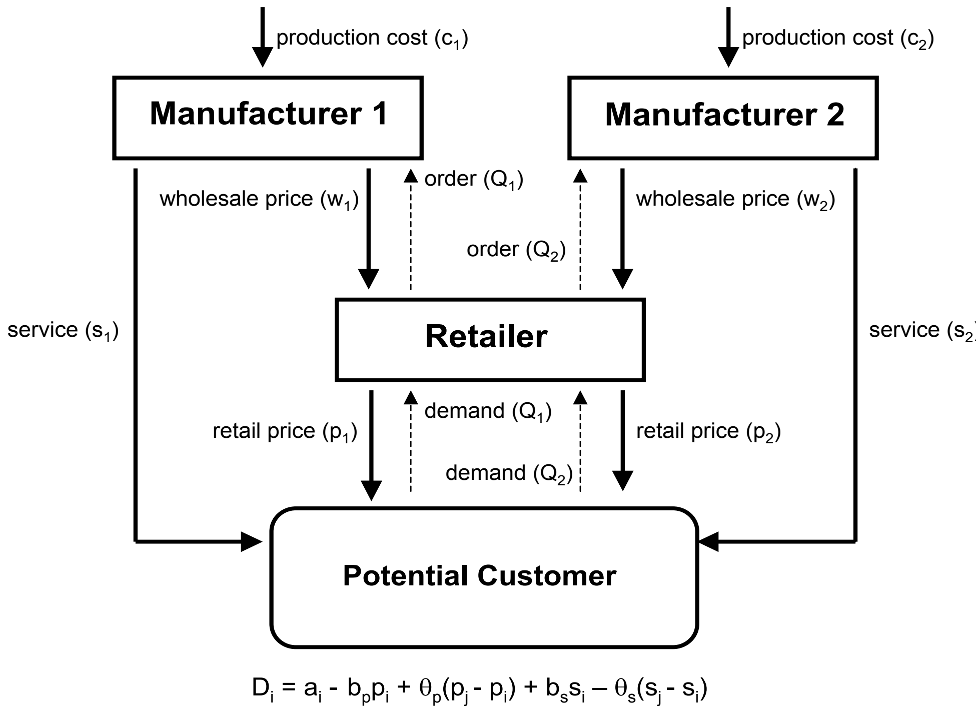

Figure 4: Schematic illustration of the supply chain.

markets (see Messinger and Narasimhan (1995) [60] and Yoruk and Radosevic (2000) [97]). Thus, it is important to study how bargaining power can affect the supply chain equilibrium solution. To achieve this goal, the following three scenarios are examined: Manufacturer Stackelberg (MS), Retailer Stackelberg (RS), and Vertical Nash (VN).

In this chapter we apply a game-theoretic approach to derive equilibrium solutions for prices (and ordering quantities), service levels, and profits for each channel member. The derivations are benchmarked with results obtained in the literature (e.g., Choi 1991) without service factors. Our research concludes that consumers receive more service when every channel member possesses equal bargaining power (e.g., Vertical Nash). An interesting but less intuitive result shows that as market base of one product increases, the competitor also benefits but at a smaller amount than the manufacturer of the original product. Furthermore, when one manufacturer has some economic advantage in providing service, the retailer will take an active role to separate market segments by selling the product with low service at a low price and selling the product with high service at a high price.

In Section 3.2 a basic model of noncooperative games is developed. Demand function, cost structure for each firm, and vertical strategic interactions are also specified in this section. Section 3.3 presents derivations and comparisons of analytical equilibrium solutions (e.g., prices, services, and profits) under three scenarios using the game-theoretic approach. Section 3.4 performs analysis on the results and sensitivity analysis on key parameters by examining their influences on the equilibrium solution. The last section summarizes major findings and delineates several possible extensions to this research.

3.2 Model

In our supply chain structure, there are two manufacturers producing different but substitutable products. Both of these manufacturers sell their products to a common retailer, who in turn sells the products to the end consumer. We assume that there is only one retailer in the area. In other words, we assume that the distance between each retailer is so large that there is no competition among retailers. This may be a strong assumption for some markets. However, it allows us to focus on the competition between the two manufacturers. We also assume that consumer demand for each product is sensitive to two factors: (1) retail price, (2) service provided by the manufacturer. Notice that only services that are provided by the competing manufacturers are considered. Effectively, we ignore the effect of the services provided by the retailer to the customer demand for each product. We can think of this as the retailer providing the same level of service to both products; the only difference to the customer's perception (other than price) is the services provided by the manufacturer.

We assume also that the investment in services has a decreasing return to scale. Namely, the next dollar invested by the manufacturer returns less service than the last dollar invested, i.e., it is harder (and costs more) to provide the next unit of service than the last one. This can be reflected in the quadratic form of the cost of providing services. The same quadratic equation is also used in Tsay and Agrawal (2000).

In this section, the mathematical model of the supply chain depicted in Figure 1 is defined. In our model, we assume that all activity occurs within a single period. There are two manufacturers, indexed by $i \in \{1, 2\}$, and one retailer. Each manufacturer produces one product, also indicated by the same index as its producer, and also provides service directly to consumers. The retailer carries the products of both manufacturers and faces a deterministic consumer demand that is influenced by both the retail prices and the manufacturer's service of both products. Each manufacturer must decide on his product's wholesale price and level of service to be provided to consumers, while the retailer controls the retail price of both products.

3.2.1 Demand Function

Our model represents a generalization of the model found in Choi (1991). Given this structure, we next specify the consumer demand function and cost structure for each firm. In defining the demand function, we follow the approach by McGuire and Staelin (1983)

[59]. This approach uses a set of basic characteristics of the type of demand of each product, e.g., downward sloping in its own price, increasing with respect to the competitor's price, and then specifies an analytically feasible function (e.g., linear) that captures these desired characteristics. An alternative approach would derive specific (nonlinear) functions facing the retailer. Typically, this latter approach requires making explicit assumptions concerning consumer tastes, or the existence of a few types of market segments. Examples of this latter approach can be seen in Lal and Matutes (1994) [44].

As pointed out by Lee and Staelin (1997) [50] and Choi (1991), although a liner demand functions do not have good forecasting properties (possibility for negative quantities), they outperform multiplicative and exponential demand functions for analysis of the primary interests such as category pricing or product line pricing. For our model, we make the following assumptions regarding demand functions:

ASSUMPTION 1. *The demand structure is symmetric between the two products. Demand for one product is decreasing in its own retail price and increasing in the competitor retail price. On the other hand, it is increasing in its own service and decreasing in the competitor service.*

ASSUMPTION 2. *Product i has market base a_i and production cost c_i. Market base a_i measures the size of product i's market. It is the demand of product i faced by the retailer when both products are priced at zero but the manufacturers offer no service.*

ASSUMPTION 3. *Decreasing product retail price or increasing service level will trigger two phenomena. First, a group of customers will decide to switch from the other product. Second, a group of customers who otherwise would not have bought either product will purchase at this lower price or higher service. The opposite happens when price is increased* *or service level is decreased.*

From Assumption 1 to 3, the demand for product i, which is the same as the retailer ordering quantity, can be expressed as:

$$Q_i(p_i, p_j, s_i, s_j) \quad = \quad a_i - (b_p + \theta_p)p_i + \theta_p p_j + (b_s + \theta_s)s_i - \theta_s s_j \tag{1}$$

where $a_i > 0$, $b_p > 0$, $\theta_p > 0$, $b_s > 0$, $\theta_s > 0$, $i = 1, 2$, and $j = 3 - i$.

Here, a_i is a non-negative constant. It can be thought of as a "market base" (Tsay and Agrawal 2000) as defined in Assumption 2. We assume that a_i is large enough so that Q_i will always be non-negative. We can think of $(b_p + \theta_p)$ as the measure of the responsiveness of each manufacturer's market demand to its own price. As specified in Assumption 3, when the price of product i is decreased by one unit, the product will gain $b_p + \theta_p$ more customers. Amongst these customers, θ_p of them are switching from the competitor's product while b_p of them are the direct result of a larger market demand due to the smaller price. In other words, b_p of them would not buy the product otherwise. A similar explanation can be used for service-related parameters b_s and θ_s.

Note that we can rearrange the terms in Equation 1 to the following form:

$$Q_i(p_i, p_j, s_i, s_j) \quad = \quad a_i - b_p p_i + \theta_p(p_j - p_i) + b_s s_i - \theta_s(s_j - s_i). \tag{2}$$

This is similar to the demand function used in Tsay and Agrawal (2000) [90], except their model was used to study a system with one manufacturer and two competing retailers in their study.

Just as in Choi (1996) and Tsay and Agrawal (2000), the functional representation of the mean demand given here has the property that, for a fixed set of retailer's and manufacturers' actions, the total market size does not change with variation in θ_p, θ_s. This can be seen through a comparison with the alternative form

$$Q_i(p_i, p_j, s_i, s_j) = a_i - b_p p_i + \theta_p p_j + b_s s_i - \theta_s s_j.$$

Here, increasing $\theta_p(\theta_s)$ would increase (decrease) the total demand. This is difficult to "rationalize economically as well as to reconcile with the aspiration of using these parameters to represent competitive industry" (Tsay and Agrawal 2000).

3.2.2 Cost Structure

In our model, the manufacturers can influence the demand by setting the wholesale prices and the service levels. On the other hand, the retailer can independently influence the (retail) price of each product. We do not assume any collusion or cooperation among firms. Each channel member has the same goal: to maximize his own profit. This leads us to the following assumption:

ASSUMPTION 4. *All channel members try to maximize their own profit and behave as if they have perfect information of the demand and the cost structures of other channel members.*

The state of information specified in Assumption 4 is typical in analytical modelling, although it overstates the information climate of the real world. From the model and Assumption 4, the retailer's objective is to maximize its profit function, which can be described by the following equation:

$$\Pi_R = \sum_{i=1}^{2} (p_i - w_i) Q_i(p_i, p_j, s_i, s_j) \tag{3}$$

where $Q_i(p_i, p_j, s_i, s_j)$ is as specified in Equation 2.

To specify each manufacturer's profit function, we note that manufacturers carry two types of cost: production cost and service cost. The latter includes the cost of providing service to customers. This may include the total wage of employees in the service department, the cost of training these employees, or the cost of hiring outsiders to provide customer service. Just as in Tsay and Agrawal (2000) [90], we assume diminishing returns of service. This is specified in the next assumption.

ASSUMPTION 5. *Cost of providing service has a decreasing-return property; the next dollar invested would produce less unit of service than the last dollar - i.e., it becomes more expensive to provide the next unit of service.*

This diminishing return of service can be captured by the quadratic form of service cost. In our model we assume that the cost of providing s_i units of service is $\eta_i {s_i}^2/2$. This function is also used in Tsay and Agrawal (2000) [90]. Thus, the manufacturers' profit function can be written as:

$$\Pi_{M_i} = (w_i - c_i)Q_i(p_i, p_j, s_i, s_j) - \frac{\eta_i s_i^2}{2} \qquad ;\text{i} = 1,\ 2 \tag{4}$$

where η_i is the service cost coefficient of manufacturer i.

3.2.3 Strategic Interactions

Note that so far we have not made any assumptions regarding the bargaining power possessed by each channel member. The assumption regarding bargaining power possessed by each firm can influence how the pricing game is solved in our model. Depending on the situation in any particular industry, the bargaining power of retailers and manufacturers can vary significantly. In the last few decades there are widely accepted notion that retailers are gaining "power" over the manufacturers. However, the validity of the notion that retailers are gaining power at the expense of the manufacturers is being questioned and studied by researchers in recent years(Ailawadi *et al.* 1995 [1], Messinger and Narasimhan 1995 [60], Kim and Staelin 1999 [42]).

Following the notions in Choi (1991), variation in bargaining power in a particular supply chain can create one of the following three scenarios:

1) Manufacturer Stackelberg (MS): The manufacturers have more bargaining power than the retailer and thus are the Stackelberg leader.

2) Retailer Stackelberg (RS): The retailer has more bargaining power than the manufacturers and is the Stackelberg leader.

3) Vertical Nash (VN): Every firm in the system has equal bargaining power.

In modelling the problem, the level of bargaining power possessed by each firm (as compared to the other firms) is translated into whether the firm is a leader or a follower. In the game-theoretical approach, the firm with more bargaining power has the first-mover advantage (Stackelberg leader). The firm with less power would have to respond to the leader's decisions. For example, in the Manufacturer Stackelberg game, both manufacturers simultaneously select wholesale prices and service levels in the first step. The retailer observes the decisions made by the manufacturers and makes his response to those decisions in the second step (by choosing retail prices). In the Retailer Stackelberg game the events take place in reverse, while every firm moves simultaneously in the Vertical Nash game. In this research, we analyze our model with all three scenarios of different power structures. We are interested to see the effect of bargaining power on the results.

3.3 Analytical Results (Manufacturer Stackelberg, Retailer Stackelberg, Vertical Nash)

To analyze our model, we follow a game-theoretical approach. The leader in each scenario makes his decisions to maximize his own profit, conditioned on the follower's response function. The problem can be solved backwards. We begin by first solving for the reaction function of the follower of the game, given that he has observed the leader's decisions. For example, in Manufacturer Stackelberg, the retailer reaction function is derived first, given that the retailer has observed the decisions made by the manufacturers (on wholesale prices and service levels). Then, each manufacturer solves his problem given that he knows how the retailer would react to his decisions.

3.3.1 Manufacturer Stackelberg

3.3.1.1 Retailer Reaction Function

The retailer in this game must choose retail prices p_1^* and p_2^* to maximize his equilibrium profit. That is,

$$p_i^* \in arg \max_{p_i} \Pi_R(p_i, p_j^* | w_1, w_2, s_1, s_2) \tag{5}$$

where $\Pi_R(p_i, p_j | w_1, w_2, s_1, s_2)$ denotes the profit to the retailer at this stage when he sets retail prices p_i, p_j, given earlier decisions by the manufacturers are w_1, w_2, s_1, s_2. The first order condition can be shown as

$$\begin{aligned} 0 = \frac{\partial \Pi_R}{\partial p_i} \quad &= \quad a_i - 2b_p p_i + \theta_p(p_j - 2p_i) + b_s s_i - \theta_s(s_j - s_i) + w_i b_p + w_i \theta_p \\ &\quad +p_j \theta_p - w_j \theta_p \end{aligned} \tag{6}$$

where $i \in \{1,2\}$ and $j = 3 - i$. To check the optimality, we check the Hessian matrix:

$$\begin{aligned} \frac{\partial \Pi_R^2}{\partial p_i^2} &= -2b_p - 2\theta_p \\ \frac{\partial \Pi_R^2}{\partial p_i \partial p_j} = \frac{\partial \Pi_R^2}{\partial p_j \partial p_i} &= 2\theta_p \end{aligned}$$

Assuming that $b_p > 0$ and $\theta_p > 0$, we have a negative definite Hessian. Therefore, the p_1 and p_2 calculated above are the optimal reaction functions for the retailer.

Using the first and second order optimality conditions above, we have the following expression for the retailer's reaction function

$$p_i^* = \frac{w_i}{2} + \frac{(b_p+\theta_p)a_i + \theta_p a_j}{2b_p(b_p+2\theta_p)} - \frac{\theta_s(s_j - s_i)}{2(b_p+2\theta_p)} + \frac{(b_p+\theta_p)b_s s_i + \theta_p b_s s_j}{2b_p(b_p+2\theta_p)} \tag{7}$$

where $i \in \{1,2\}$ and $j = 3 - i$. From equation (7) and (1), we can also obtain the demand quantities for products 1 and 2 as

$$Q_i^* = \frac{a_i}{2} - \frac{(b_p+\theta_p)}{2} w_i + \frac{\theta_p}{2} w_j + \frac{(b_s+\theta_s)}{2} s_i - \frac{\theta_s}{2} s_j \tag{8}$$

where $i = 1,2$ and $j = 3 - i$. We can see that the equilibrium quantities p_i^* and Q_i^* for each product are linear functions of the wholesale prices and service levels by the manufacturers, and the market bases (a_1 and a_2).

3.3.1.2 Manufacturers Decisions

Using the retailer's reaction function, we can derive each manufacturer's optimal wholesale price and service level. This is carried out by maximizing each manufacturer's profit shown in Equation (4), given the retailer reaction function. The manufacturer i chooses the wholesale prices w_i^* and service level s_i^* to maximize his own individual profit. Recall that the manufacturers move simultaneously. Thus, a Nash Equilibrium exists between them. That is,

$$w_i^* \in arg\max_{w_i} \Pi_{M_i}(w_i, w_j^*, s_i^*, s_j^*) \tag{9}$$

and

$$s_i^* \in arg\max_{s_i} \Pi_{M_i}(w_i^*, w_j^*, s_i, s_j^*) \tag{10}$$

where $\Pi_{M_i}(w_i, w_j, s_i, s_j)$ is the profit of manufacturer i at this stage when manufacturers set their wholesale prices at w_i, w_j and service levels at s_i, s_j. To find the optimal wholesale price, w_i, we first look at the first order condition.

$$\begin{aligned} 0 = \frac{\partial \Pi_{M_i}}{\partial w_i} \quad &= \quad a_i - b_p \left[w_i + \frac{(b_p + \theta_p)a_i + \theta_p a_j}{2b_p(b_p + 2\theta_p)} - \frac{\theta_s(s_j - s_i)}{2(b_p + 2\theta_p)} + \frac{(b_p + \theta_p)b_s s_i + \theta_p b_s s_j}{2b_p(b_p + 2\theta_p)} \right] \\ &\quad + \theta_p \left[\frac{a_j - a_i}{2(b_p + 2\theta_p)} + \frac{w_j - 2w_i}{2} + \frac{(2\theta_s + b_s)(s_j - s_i)}{2(b_p + 2\theta_p)} \right] \\ &\quad + b_s s_i - \theta_s(s_j - s_i) + \frac{c_i b_p}{2} + \frac{c_i \theta_p}{2} \end{aligned}$$

$$0 = \frac{\partial \Pi_{M_i}}{\partial s_i} \quad = \quad (w_i - c_i) \left[-\frac{b_p \theta_s}{2(b_p + 2\theta_p)} - \frac{b_p(b_p + \theta_p)b_s}{2b_p(b_p + 2\theta_p)} - \frac{\theta_p(b_s + 2\theta_s)}{2(b_p + 2\theta_p)} + b_s + \theta_s \right] - \eta_i s_i$$

The second order condition is given below to check the optimality:

$$\begin{aligned} \frac{\partial \Pi_{M_i}^2}{\partial w_i^2} &= -b_p - \theta_p \\ \frac{\partial \Pi_{M_i}^2}{\partial w_i \partial s_i} &= \frac{b_s + \theta_s}{2} \\ \frac{\partial \Pi_{M_i}^2}{\partial s_i^2} &= -\eta_i \end{aligned}$$

Assuming that $b_p > 0$ and $\theta_p > 0$, we have a negative definite Hessian. Therefore, the w_i and s_i calculated above are the optimal reaction functions for the manufacturer i.

The detailed derivation of these expressions may be found in Appendix A. The following proposition gives the actual closed form solution of wholesale price and service level.

PROPOSITION 3.1. *The manufacturer's equilibrium wholesale price and service level are:*

$$w_i^* = \frac{2\eta_i A_j}{A_1A_2 - B_1B_2}\Big[a_i + D_j a_j + (E_i + F_i D_j)c_i + (F_j + E_j D_j)c_j\Big] \tag{11}$$

$$\begin{aligned} s_i^* &= (b_s + \theta_s)\Big\{\frac{A_j}{A_1A_2 - B_1B_2}\Big[a_i + D_j a_j + (F_j + E_j D_j)c_j\Big] \\ &\quad + \Big[\frac{A_j(E_i + F_i D_j)}{A_1A_2 - B_1B_2} - \frac{1}{2\eta_i}\Big] c_i\Big\} \end{aligned} \tag{12}$$

where $i = 1, 2$ *and* $j = 3 - i$ *and* $A_i = 4\eta_i(b_p + \theta_p) + (b_s + \theta_s)^2, B_i = 2\eta_i\theta_p - \theta_s(b_s + \theta_s)\left(\frac{b_p - b_s + 2\theta_p}{b_p + 2\theta_p}\right), D_i = \frac{B_i}{A_i}, E_i = (b_p + \theta_p) - \frac{(b_s+\theta_s)^2}{2\eta_i}, F_i = \frac{\theta_s(b_s+\theta_s)}{2\eta_i} - \frac{\theta_p b_s(b_s+\theta_s)}{2\eta_i(b_p+2\theta_p)}$.

Proof: See Appendix A.

Note that if service is not taken into account or is assumed to be zero, equation (7), (8), and (11) reduce to

$$p_i^{NS*} = \frac{w_i}{2} + \frac{(b_p + \theta_p)a_i + \theta_p a_j}{2b_p(b_p + 2\theta_p)} \tag{13}$$

$$Q_i^{NS*} = \frac{a_i}{2} - \frac{(b_p + \theta_p)w_i}{2} + \frac{\theta_p w_j}{2} \quad \text{and} \tag{14}$$

$$w_i^{NS*} = \frac{1}{4(b_p + \theta_p)^2 - \theta_p^2}\Big[2(b_p + \theta_p)a_i + \theta_p a_j + 2(b_p + \theta_p)^2 c_i + \theta_p(b_p + \theta_p)c_j\Big] \tag{15}$$

for $i \in \{1, 2\}$ and $j = 3 - i$. These are the results derived by Choi (1991). Choi (1991) defines a linear duopoly demand function as $Q_i = a - bp_i + \gamma p_j$ where $b = b_p + \theta_p$ and

$\gamma = \theta_p$. His model does not take into account the service provided by manufacturers and assumes that the two products have equal market base ($a_1 = a_2 = a$). Thus, his model is a special case of our model. Comparing equations (8) and (14), we can see that $Q_i{}^*$ (demand

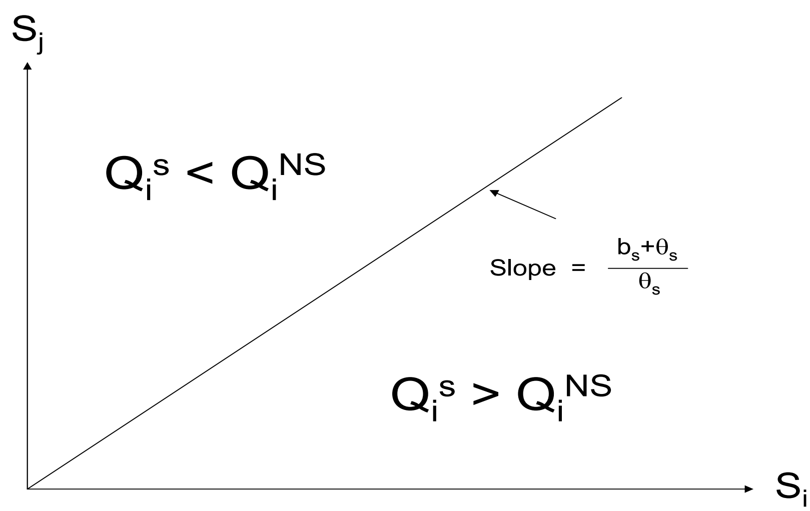

Figure 5: Regions where $Q_i{}^*$ and Q_i^{NS*} are compared.

of product i when both manufacturers provide service) will be greater than Q_i^{NS*} (demand of product i when no service is provided) if

$$\frac{s_i}{s_j} \geq \frac{\theta_s}{(b_s + 2\theta_s)}. \tag{16}$$

Figure 5 shows the regions where $Q_i{}^*$ and Q_i^{NS*} are compared. Thus, when manufacturer i provides its service $s_i \geq \frac{\theta_s}{(b_s+2\theta_s)} s_j$, product i can capture a bigger market than its competitor.

Now, comparing equations (7) and (13), $p_i{}^*$ (retail price of product i when both manufacturers provide service) will be greater than p_i^{NS*} (retail price of product i when no

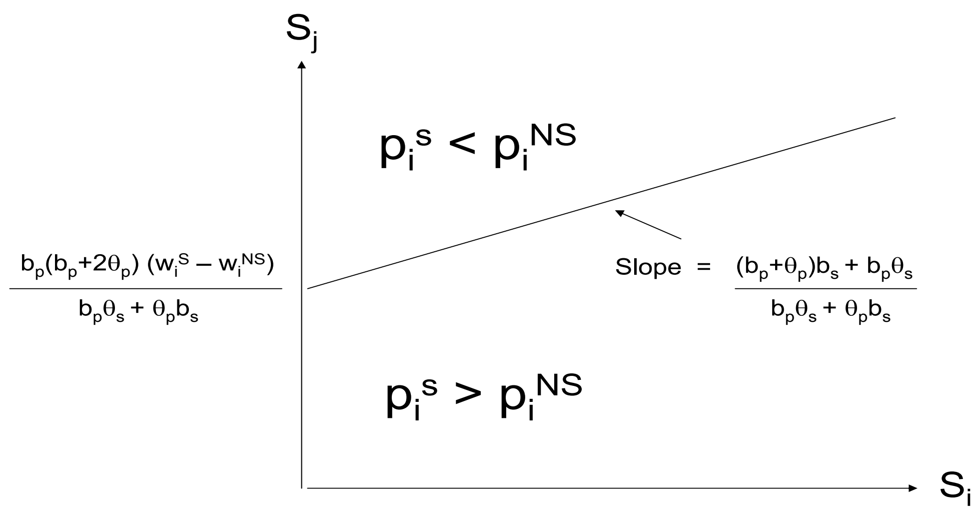

Figure 6: Regions where ${p_i}^*$ and p_i^{NS*} are compared.

service is provided) if the following condition is satisfied

$$w_i^* - w_i^{NS*} \quad \geq \quad \frac{[b_p\theta_s + \theta_p b_p]s_j - [(b_p+\theta_p)b_s + b_p\theta_s]s_i}{b_p(b_p+2\theta_p)}. \tag{17}$$

In other words, if

$$s_j \quad \leq \quad \frac{b_p(b_p+2\theta_p)(w_i^* - w_i^{NS*})}{b_p\theta_s + \theta_p b_p} + \frac{[(b_p+\theta_p)b_s + b_p\theta_s]}{b_p\theta_s + \theta_p b_p}s_i, \tag{18}$$

then ${p_i}^*$ will be greater than p_i^{NS*}. Figure 6 shows the regions where ${p_i}^*$ and p_i^{NS*} are compared.

3.3.2 Retailer Stackelberg

The Retailer Stackelberg scenario arises in markets where retailers' sizes are large compared to their suppliers. For example, large retailers like Walmart and Target can influence each product's sales by lowering price. Because of their sizes, the retailers can maintain their margin on sales while squeezing profit from their suppliers. The suppliers are mostly concerned with receiving orders from the retail giants. Similar game-theoretic framework as applied in the Manufacturer Stackelberg case is implemented to solve this problem; i.e., the problem is solved backwards. First, the suppliers' problem is solved to derive the response function conditional on the retail prices chosen by the retailer. The retailer problem is then solved given that the retailer knows how the manufacturers would react to the retail prices he sets.

3.3.2.1 Manufacturers Reaction Functions

Each manufacturer is trying to maximize his own profit

$$\Pi_{M_i} = (w_i - c_i)Q_i - \frac{\eta_i s_i^2}{2}$$

for $i \in \{1, 2\}$. To cope with competition, manufacturer i chooses equilibrium wholesale price w_i and service level s_i. That is, for each manufacturer i

$$w_i^* \in arg \max_{w_i} \Pi_{M_i}(w_i, w_j^*, s_i^*, s_j^* | p_1, p_2) \tag{19}$$

and

$$s_i^* \in arg \max_{s_i} \Pi_{M_i}(w_i^*, w_j^*, s_i, s_j^* | p_1, p_2) \tag{20}$$

where $\Pi_{M_i}(w_1, w_2, s_1, s_2 | p_1, p_2)$ is the profit to manufacturer i at this stage when manufacturers set wholesale prices w_1, w_2 and service levels s_1, s_2, given earlier decisions on retail price p_1, p_2 by the retailer.

The first order conditions are

$$0 = \frac{\partial \Pi_{M_i}}{\partial w_i} = Q_i + (w_i - c_i)(-b_p - \theta_p)$$

$$0 = \frac{\partial \Pi_{M_i}}{\partial s_i} = (w_i - c_i)(b_s + \theta_s) - \eta_i s_i$$

The second order condition is given below to check for optimality

$$\begin{aligned}
\frac{\partial \Pi^2_{M_i}}{\partial w_i^2} &= -b_p - \theta_p \\
\frac{\partial \Pi^2_{M_i}}{\partial w_i \partial s_i} &= b_s + \theta_s \\
\frac{\partial \Pi^2_{M_i}}{\partial s_i^2} &= -\eta_i
\end{aligned}$$

The second order condition shows that we have a negative definite Hessian. Therefore, the w_i and s_i calculated above are the optimal reaction functions for the manufacturer i.Using the first and second order conditions above, the response wholesale price and service level for each manufacture can be derived and are given in the next proposition.

PROPOSITION 3.2. *The manufacturer's response function given retail prices p_i and p_j are:*

$$w_i^* = \frac{\eta_i H_j}{H_1 H_2 - K^2}\Big[a_i - L_j a_j - (\theta_p L_j + G)p_i + (G L_j + \theta_p)p_j + (M_i - L_j N_i)c_i\Big]. \tag{21}$$

$$s_i^* = \frac{H_j(b_s + \theta_s)}{H_1 H_2 - K^2}\Big[a_i - L_j a_j - (\theta_p L_j + G)p_i + (G L_j + \theta_p)p_j\Big] \tag{22}$$

where $G = b_p + \theta_p, H_i = \eta_i(b_p + \theta_p) - (b_s + \theta_s)^2, K = \theta_s(b_s + \theta_s), L_i = \frac{K}{H_i}, M_i = \frac{H_i}{\eta_i} = (b_p + \theta_p) - \frac{(b_s+\theta_s)^2}{\eta_i}, N_i = \frac{K}{\eta_i} = \frac{\theta_s(b_s+\theta_s)}{\eta_i}$,

Proof: See Appendix A.

We can see that the optimal service responses for both manufacturers do not depend on the production cost. Even for the optimal wholesale price responses, the manufacturers do not need to know the production cost of their competitors. Retail prices can be easily observed in the market. The value of market bases can be estimated by the manufacturers by conducting a market survey.

3.3.2.2 Retailer Decision

Having the information about the reaction functions of manufacturers, the retailer would then use them to maximize his profit

$$\Pi_R = (p_1 - w_1(p_1, p_2))Q_1(p_1, p_2) + (p_2 - w_2(p_1, p_2))Q_2(p_1, p_2). \tag{23}$$

The retailer in this game must choose retail prices p_1^* and p_2^* to maximize his equilibrium profit. That is,

$$p_i^* \in arg\max_{p_i} \Pi_R(p_i, p_j^*) \tag{24}$$

where $\Pi_R(p_1, p_2)$ denotes the profit to the retailer at this stage when he set retail prices p_1, p_2. The first order condition can be shown as

$$\begin{aligned} 0 = \frac{\partial \Pi_R}{\partial p_i} &= \left(1 - \frac{\partial w_i(p_i, p_j)}{\partial p_i}\right) Q_i(p_i, p_j) + (p_i - w_i(p_i, p_j))\frac{\partial Q_i(p_i, p_j)}{\partial p_i} \\ &+ \left(-\frac{\partial w_j(p_i, p_j)}{\partial p_i}\right) Q_j(p_i, p_j) + (p_j - w_j(p_i, p_j))\frac{\partial Q_j(p_i, p_j)}{\partial p_i} \end{aligned} \tag{25}$$

where

$$\frac{\partial w_i(p_i, p_j)}{\partial p_i} = \frac{2\eta_i H_j}{H_i H_j - K^2}(\theta_p L_j - G) \tag{26}$$

$$\frac{\partial w_j(p_i,p_j)}{\partial p_i} = \frac{2\eta_j H_i}{H_iH_j - K^2}(GL_i - \theta_p) \tag{27}$$
$$\frac{\partial w_i(p_i,p_j)}{\partial p_j} = \frac{2\eta_i H_j}{H_iH_j - K^2}(GL_j - \theta_p) \tag{28}$$
$$\frac{\partial w_j(p_i,p_j)}{\partial p_j} = \frac{2\eta_j H_i}{H_iH_j - K^2}(\theta_p L_i - G) \tag{29}$$

To check for optimality, we check the Hessian matrix:

$$\begin{aligned}
\frac{\partial \Pi_R^2}{\partial p_i^2} &= -2b_p - 2\theta_p \\
\frac{\partial \Pi_R^2}{\partial p_i \partial p_j} = \frac{\partial \Pi_R^2}{\partial p_j \partial p_i} &= 2\theta_p
\end{aligned}$$

Assuming that $b_p > 0$ and $\theta_p > 0$, we have a negative definite Hessian. Therefore, the p_1 and p_2 calculated above are the optimal reaction functions for the retailer.

Using the first and second order optimization conditions, the equilibrium retail prices can be derived and are given in the following proposition.

PROPOSITION 3.3. *In the Retailer Stackelberg case, the equilibrium retail price p_1^* and p_2^* chosen by the retailer are*

$$\begin{aligned}
p_1^* &= \frac{(X_2U_1 - YV_1)a_1 + (YV_2 - X_2U_2)a_2 + (X_2\rho_1 - Y\sigma_1)Wc_1 + (Y\rho_2 - X_2\sigma_2)Wc_2}{X_1X_2 - Y^2} \\
p_2^* &= \frac{(YU_1 - X_1V_1)a_1 + (X_1V_2 - YU_2)a_2 + (Y\rho_1 - X_1\sigma_1)Wc_1 + (X_1\rho_2 - Y\sigma_2)Wc_2}{X_1X_2 - Y^2}.
\end{aligned}$$

Proof: See Appendix A.

These equations show linear relationship between retail price and market bases and production costs. When $b_s = \theta_s = 0$, the expressions are reduced to the results given in Choi (1991).

3.3.3 Vertical Nash

The Vertical Nash model is studied as a benchmark to both the Manufacturer Stackelberg and Retailer Stackelberg cases. In this model, every firm has equal bargaining power and thus makes his decisions simultaneously. This scenario arises in a market in which there are relatively small to medium-sized manufacturers and retailers. In this market it is reasonable to assume that a manufacturer may not know the competitor's wholesale price but can observe its retail price. Since a manufacturer cannot dominate the market over the retailer, his price decision is conditioned on how the retailer prices the product. On the other hand, the retailer must also condition its retail price decisions on the wholesale price.

Again, game-theoretic framework is employed to derive the reaction function of each firm in the supply chain. Fortunately, the reaction functions for the retailer and the manufacturers were already derived in the Manufacturer Stackelberg game and the Retailer Stackelberg game respectively. From the Manufacturer Stackelberg game, the retailer reaction function for given wholesale prices w_1, w_2 and service levels s_1, s_2 is given in Equation 7 as

$$p_i^* = \frac{w_i^*}{2} + \frac{(b_p + \theta_p)a_i + \theta_p a_j}{2b_p(b_p + 2\theta_p)} - \frac{\theta_s(s_j^* - s_i^*)}{2(b_p + 2\theta_p)} + \frac{(b_p + \theta_p)b_s s_i^* + \theta_p b_s s_j^*}{2b_p(b_p + 2\theta_p)}$$

where $i \in \{1, 2\}$ and $j = 3 - i$. From the Retailer Stackelberg game, the manufacturers reaction function for given retail prices p_1, p_2 are given in Equations 121 and 22 as

$$w_i^* = \frac{\eta_i H_j}{H_1 H_2 - K^2}\Big[a_i - L_j a_j - (\theta_p L_j + G)p_i + (G L_j + \theta_p)p_j + (M_i - L_j N_i)c_i\Big]$$

$$s_i^* = \frac{H_j(b_s + \theta_s)}{H_1 H_2 - K^2}\Big[a_i - L_j a_j - (\theta_p L_j + G)p_i + (G L_j + \theta_p)p_j\Big]$$

for wholesale price and service level respectively. H_i, K, L_i, M_i, N_i, and G for $i = 1, 2$ and $j = 3 - i$ are defined as in the Retailer Stackelberg game. Solving the above equations simultaneously yields the Nash equilibrium solution. The equilibrium retail prices can be derived and are given in the following proposition.

PROPOSITION 3.4. *In Vertical Nash case, the equilibrium retail price p_1^* and p_2^* chosen by the retailer are*

$$p_1 = \frac{(\gamma_2\kappa_1 + \lambda_1\kappa_2)a_1 + (\gamma_2\nu_1 + \lambda_1\nu_2)a_2 + \gamma_2\psi_1 c_1 + \lambda_1\psi_2 c_2}{\gamma_1\gamma_2 - \lambda_1\lambda_2} \tag{30}$$

$$p_2 = \frac{(\gamma_1\kappa_2 + \lambda_2\kappa_1)a_1 + (\gamma_1\nu_2 + \lambda_2\nu_1)a_2 + \gamma_1\psi_2 c_1 + \lambda_2\psi_1 c_2}{\gamma_1\gamma_2 - \lambda_1\lambda_2} \tag{31}$$

where κ_i, λ_i, ν_i and ψ_i for $i = 1, 2$ are constants.

Proof: See Appendix A.

3.3.4 Comparison of Results

In this section, we compare the results from the three different scenarios to focus on the effect of power structure on prices, service levels, and profits of each channel member. However, when the two manufacturers are not identical (in production cost or market base), it is difficult to compare the results from different scenarios since there will be a market leader and a follower. In order to separate the effects of different power structures from the effects of cost differences, we assume identical manufacturers (same market base, production cost and service cost coefficient). This assumption simplifies the results given previously by setting $a_i = a_j = a$, $c_i = c_j = c$, and $\eta_i = \eta_j = \eta$. The following theorem summarizes the results with the identical manufacturers assumption.

THEOREM 3.1. *When the two manufacturers are identical (same market base, production cost and service cost coefficient), the retail price, wholesale price, service level, demand quantity, and profit can be calculated as shown in Table 3.1*

	Scenario		
	MS	VN	RS
Manufacturers			
Wholesale Price	$\frac{2\eta}{A-B}(a+(E+F)c)$	$\frac{\eta}{\Phi}\left(a+\frac{2(H+K)+b_s(b_s+\theta_s)}{\eta}c\right)$	$\frac{\eta}{\Psi}\left(a+\frac{2(H+K)+\eta b_p}{\eta}c\right)$
Service Level	$\frac{(b_s+\theta_s)}{A-B}(a+\Gamma c)$	$\frac{b_s+\theta_s}{\Phi}(a-b_pc)$	$\frac{b_s+\theta_s}{\Psi}(a-b_pc)$
Profit	$\frac{\eta}{A-B}\Upsilon(a+\Gamma c)$	$\frac{\eta[H+\eta(b_p+\theta_p)]}{2\Phi^2}(a-b_pc)^2$	$\frac{\eta[H+\eta(b_p+\theta_p)]}{2\Psi^2}(a-b_pc)^2$
Retailer			
Retail Price	$\frac{1}{b_p}\left(\Lambda+\frac{2\eta b_p}{A-B}\right)a+\left(\Theta+\frac{2\eta(E+F)}{A-B}\right)c$	$\frac{\Phi-(H+K)}{b_p\Phi}a+\frac{(H+K)}{\Phi}c$	$\frac{H+K+2\eta b_p}{b_p\Psi}a+\frac{H+K}{\Psi}c$
Profit	$\frac{2}{b_p}(\Lambda a-b_p\Theta c)^2$	$\frac{2}{b_p}\left[\frac{\eta(b_p+\theta_p)}{\Phi}(a-b_pc)\right]^2$	$\frac{\eta(b_p+\theta_p)}{b_p\Psi}(a-b_pc)^2$
Demand	$\Lambda a-b_p\Theta c$	$\frac{\eta(b_p+\theta_p)}{\Phi}(a-b_pc)$	$\frac{\eta(b_p+\theta_p)}{\Psi}(a-b_pc)$

Note:

$\Gamma = E+F-\frac{A-B}{2\eta}$, $\quad \Lambda = \frac{A-B-2\eta b_p+b_s(b_s+\theta_s)}{2(A-B)}$

$\Theta = \frac{\eta(E+F)}{A-B}-\frac{b_s(b_s+\theta_s)}{2b_p}\left(\frac{E+F}{A-B}-\frac{1}{2\eta}\right)$, $\quad \Phi = 2(H+K)+\eta b_p+b_s(b_s+\theta_s)$

$\Psi = 2(H+K+\eta b_p)$, $\quad \Upsilon = 2(\Lambda a+b_p\Theta c)-\frac{(b_s+\theta_s)^2}{2(A-B)}(a+\Gamma c)$

Table 2: Comparison of results from three scenarios

Proof: See Appendix A.

When $b_s = \theta_s = 0$, the results given in Table 3.1 reduce to the results given by Choi (1991) in which competition in service is not taken into account. The results in Table 3.1 show that the equilibrium wholesale and retail price, service level, and demand quantity are a linear function of both market base and production cost. By comparing the results from each scenario, we have the following proposition.

COROLLARY 3.1. *When the two manufacturers are identical (same market base, production cost and service cost coefficient) and* $\eta b_p > b_s(b_s+\theta_s)$ *and* $a > b_pc$, $s^{MS} < s^{RS} < s^{VN}$.

Proof: See Appendix A.

This proposition states that when the manufacturers possess the most bargaining power, consumers receive the least benefit from service. The proposition shows that the consumers are better off when there is no dominant power between the retailer and manufacturers. This is reflected in higher service levels and greater demand quantity in the VN scenario as compared to those in MS and RS.

We next compares other quantities among the three scenarios. We find that the results of comparison depend on the value of b_s and θ_s. When b_s and θ_s are greater than zero, the results from Manufacturer Stackelberg can vary, depending on the values of the parameters. Thus, they can not be compared to the results from the other two cases. However, when $b_s = \theta_s = 0$, the results from all three games in Table 3.1 can be simplified and compared. The following corollary states these findings.

COROLLARY 3.2. *When the two manufacturers are identical (same market base, production cost and service cost coefficient) and $\eta b_p > b_s(b_s + \theta_s)$ and $a > b_p c$, we have the following results*

	(a) If b_s and $\theta_s > 0$	*(b) If b_s and $\theta_s = 0$*
1	*N/A*	$p^{MS} < p^{VN} < p^{RS}$
2	$Q^{RS} < Q^{VN}$	$Q^{MS}, Q^{RS} < Q^{VN}$
3	$w^{VN} > w^{RS}$	$w^{MS} > w^{VN} > w^{RS}$
4	$\Pi_M^{VN} > \Pi_M^{RS}$	$\Pi_M^{MS} > \Pi_M^{VN} > \Pi_M^{RS}$
5	$\Pi_R^{VN} < \Pi_R^{RS}$	$\Pi_R^{MS} < \Pi_R^{VN} < \Pi_R^{RS}$

Proof: See Appendix A.

Part (b) of Corollary 3.2 is similar to the results given by Choi (1991) and Lee and Staelin (1997) [50]. Their models do not include the service component (i.e., $b_s = \theta_s = 0$ in their models). Thus, Proposition 3.2 provides more general results than those in existing literature.

Note that when b_s and $\theta_s > 0$, it is not possible to compare the results from the Manufacturer Stackelberg case with the other two cases. This is because the values of b_s and θ_s can influence the nature of competition. When b_s and θ_s are significant larger than b_p and θ_p, the two manufacturers will focus on service competition. On the other hand, if b_p and θ_p are significant larger than b_s and θ_s, manufacturers will concentrate on price competition. Thus, the relative amount of price and service level in the Manufacturer Stackelberg case as compared to the other two cases can vary.

Note also that we can not compare the retail price among the three cases. This is also due to the nature of competition in the industry. When b_p and θ_p are significant larger than b_s and θ_s, the result will be close to that given in part (b) (i.e., $p^{MS} < p^{VN} < p^{RS}$). However, if b_s and θ_s are significant larger than b_p and θ_p, the manufacturers will focus on service competition. In this case, no definite statement can be concluded from the comparison of the retail price between the three cases.

3.4 Numerical Studies

In this section, we use numerical approach to studies the behavior of firms when facing changing environment. We follow existing literature (e.g., Tsay and Agrawal (2000) [90] and Vilcassin *et al.* (1999) [91]) in defining the range of some parameters[2]. We explore how retail prices, wholesale prices, service levels, and profits are affected by changes in industry conditions, i.e., a_i, c_i, η_i, b_p, θ_p, b_s and θ_s. Changes in a_i, c_i, and η_i reflect changes from

[2]The range of parameters we use in this section can be found in Appendix A.

individual company. On the other hand, variations in b_p, θ_p, b_s and θ_s reflect the dynamic environment of the whole industry. This is because the degree to which prices or services affect brand loyalty can change over time due to continuing competition. Thus, the results in this section can help us understand the sensitivity of our results to either firm-specific or industry-wide changes.

3.4.1 Individual Manufacturer Change

From the results in the last section, we found that, regardless of power structure, as market base of manufacturer i increases, the firm can sell its product at a higher price and with larger quantity. This brings in more revenue and makes it affordable for the manufacturer to provide more services. This in turn creates even more demand for the product and increases profit. We also found out that as the market base of product i increases, manufacturer j's profit also increases. This might be counterintuitive but can be explained as follows: the increase in competitor's profit is due to the factor θ_p in demand function which reflects the fact that some fixed proportion of customers will switch from product i to product j due to competition. However, the increase in price, service level and profit of product j will be smaller than those of product i. This result is summarized in the next proposition.

OBSERVATION 3.1. *Regardless of power structure, an increase in market base in one company benefits its competitor as well but at a lesser extent. Namely,*

a) $0 < \frac{\partial w_j}{\partial a_i} < \frac{\partial w_i}{\partial a_i}$

b) $0 < \frac{\partial s_j}{\partial a_i} < \frac{\partial s_i}{\partial a_i}$

c) $0 < \frac{\partial p_j}{\partial a_i} < \frac{\partial p_i}{\partial a_i}$

d) $0 < \frac{\partial Q_j}{\partial a_i} < \frac{\partial Q_i}{\partial a_i}$

e) $0 < \frac{\partial \Pi_{M_j}}{\partial a_i} < \frac{\partial \Pi_{M_i}}{\partial a_i}$

f) $0 < \frac{\partial \Pi_R}{\partial a_i}$

Similar phenomenon also occurs when c_i increases, except that now the increase has adverse effect on demand quantity, service level and profit of product i. The result shows that firm i will sell its product at a higher price and provide less service. This brings the firm less profit. We found out that as c_i increases, p_j also increases. However, this increase in p_j is at a smaller magnitude than the increase in p_i. As s_i decreases due to a higher c_i, s_j increases. Thus, demand and profit for product j increase while those for product i decrease. Note that the retailer is also hurt if the production cost of one of the manufacturers increases; this is because of the decrease in total demand due to a higher price. The next proposition states this result.

OBSERVATION 3.2. *Regardless of power structure, an increase in production cost in one company decreases its profit while increases its competitor's profit, but at a lesser extent.*

a) $0 < \frac{\partial w_j}{\partial c_i} < \frac{\partial w_i}{\partial c_i}$

b) $0 < \frac{\partial s_j}{\partial c_i} < -\frac{\partial s_i}{\partial c_i}$

c) $0 < \frac{\partial p_j}{\partial c_i} < \frac{\partial p_i}{\partial c_i}$

d) $0 < \frac{\partial Q_j}{\partial c_i} < -\frac{\partial Q_i}{\partial c_i}$

e) $0 < \frac{\partial \Pi_{M_j}}{\partial c_i} < -\frac{\partial \Pi_{M_i}}{\partial c_i}$

f) $0 > \frac{\partial \Pi_R}{\partial c_i}$

We also found that when manufacturer i has an advantage on service cost coefficient (i.e., $\eta_i < \eta_j$), it will provide more service, and sell the product at lower wholesale price.

However, its product will have a higher retail price. This leads us to the following result.

OBSERVATION 3.3. *When $\eta_i < \eta_j$, the retailer will act as a market segmenter and sell the product with high service at high (retail) price and sell the product with low service at low (retail) price. Namely, $s_i > s_j$ and $p_i > p_j$ even though $w_i < w_j$.*

When manufacturer i has an advantage on service cost coefficient (i.e., $\eta_i < \eta_j$), he can sell at a lower price since his service cost is less than that of his competitor. However, the retailer will sell product i at a higher retail price. The retailer makes up for the smaller profit from the low service product by a bigger profit from the higher service product. This result emphasizes the role of the retailer as an intermediary. The consumers can not enjoy better service and lower price offered by manufacturer i due to the existence of the retailer. In order to receive high service offered by manufacturer i, they must pay a higher price.

3.4.2 Industry Change

To study the influence of b_p, θ_p, b_s and θ_s, we assume that the two manufacturers are identical with the same market base (a_i), production cost (c_i), and service cost coefficient (η_i). Tables 3.2 to 3.4 show the results from changes in both the individual and industry parameters. The effect can be either monotonic increasing (as indicated by a + sign) or decreasing ($-$ sign), or neither ($+/-$ sign). It can be linear (as indicated by subscript l), nonlinear with a convex (cx), or a concave (cc) characteristic, or neither (nl). The results from this analysis give the next observation.

OBSERVATION 3.4. *The characteristic of changes in prices, service levels and profits from variations in b_p, and θ_p does not depend on the power structure assumption. On the other hand, the characteristic of changes due to service-related parameters (b_s and θ_s)*

depends on the power structure assumption and on the values of other parameters.

We next examine in details the results from Table 3.2 to 3.4 on the sensitivity of prices, service levels, and profits to changes of each parameter.

Manufacturer Stackelberg							
	a_i	c_i	η_i	b_p	θ_p	b_s	θ_s
Wholesale Price	$+_l$	$+_l$	$+_{cc}$	$-_{cx}$	$-_{cx}$	$-_{cc}$	$-_{nl}$
Service Level	$+_l$	$-_l$	$-_{cx}$	$-_{cx}$	$-_{cx}$	$+_{cc}$	$-_{cx}$
Profit	$+_{cx}$	$-_{cx}$	$+/-$	$-_{cx}$	$-_{cx}$	$+/-$	$-_{cc}$
Retail Price	$+_l$	$+_l$	$+_{cc}$	$-_{cx}$	$-_{cx}$	$+/-$	$-_{cx}$
Profit	$+_{cx}$	$-_{cx}$	$-_{cx}$	$-_{cx}$	$+_{cc}$	$+_{nl}$	$+/-$
Demand	$+_l$	$-_l$	$-_{cx}$	$-_{cx}$	$+_{cc}$	$+_{nl}$	$+/-$

Table 3: Sensitivity Analysis with Increases in Specific Parameters for Manufacturer Stackelberg

Retailer Stackelberg							
	a_i	c_i	η_i	b_p	θ_p	b_s	θ_s
Wholesale Price	$+_l$	$+_l$	$+_{cc}$	$-_{cx}$	$-_{cx}$	$+_{cx}$	$+/-$
Service Level	$+_l$	$-_l$	$+/-$	$-_{cx}$	$-_{cx}$	$+_{cx}$	$+/-$
Profit	$+_{cx}$	$-_{cx}$	$-_{cx}$	$-_{cx}$	$-_{cx}$	$+_{nl}$	$+/-$
Retail Price	$+_l$	$+_l$	$+_{cx}$	$-_{cx}$	$-_{cx}$	$+_{nl}$	$+/-$
Profit	$+_{cx}$	$-_{cx}$	$-_{cx}$	$-_{cx}$	$+_{cc}$	$+_{nl}$	$+/-$
Demand	$+_l$	$-_l$	$-_{cx}$	$-_{cx}$	$+_{cc}$	$+/-$	$+/-$

Table 4: Sensitivity Analysis with Increases in Specific Parameters for Retailer Stackelberg Case

Vertical Nash							
	a_i	c_i	η_i	b_p	θ_p	b_s	θ_s
Wholesale Price	$+_l$	$+_l$	$+_{cc}$	$-_{cx}$	$-_{cx}$	$+_{cx}$	$+_{cx}$
Service Level	$+_l$	$-_l$	$-_{cx}$	$-_{cx}$	$-_{cx}$	$+_{cx}$	$+_{cx}$
Profit	$+_{cx}$	$-_{cx}$	$+_{cc}$	$-_{cx}$	$-_{cx}$	$+/-$	$-_{cc}$
Retail Price	$+_l$	$+_l$	$-_{cx}$	$-_{cx}$	$-_{cx}$	$+_{cx}$	$+_{cx}$
Profit	$+_{cx}$	$-_{cx}$	$-_{cx}$	$-_{cx}$	$+_{cc}$	$+_{cx}$	$+_{cx}$
Demand	$+_l$	$-_l$	$+_{cc}$	$-_{cx}$	$+_{cc}$	$+_{cx}$	$+_{cx}$

Table 5: Sensitivity Analysis with Increases in Specific Parameters for Vertical Nash Case

3.4.2.1 Market Sensitivity to Own Price (b_p)

Recall that b_p is the number of new customers the product attracts as its price is decreased by 1 unit. This is different from the number of a competitor's customers who switch products due to a price difference, θ_p. As can be seen in Tables 3.2 to 3.4, as price sensitivity increases, the manufacturers tend to concentrate on lowering price and provide less service. However, it turns out that demand decreases due to less service. Therefore, the profits for the retailer and manufacturers decrease.

3.4.2.2 Market Sensitivity to Price Difference (θ_p)

We can see that as θ_p increases, the two manufacturers tend to concentrate on competing to lower the price. This allows them to decrease the service level since it is not the key competing factor. As the two manufacturers compete in price, they also induce a gain in demand quantity. The retailer's profit improves as it enjoys more sales. However, the manufacturers' profit decreases due to the competition to cut price.

3.4.2.3 Market Sensitivity to Own Service (b_s)

When the value of b_s is less than the value of b_p, the two manufacturers will concentrate on lowering retail price. This is because consumer demand is more sensitive to price than service. This leads to a decreasing retail price and a smaller service level. When b_s is higher than b_p, the two manufacturers will begin to compete to provide a higher service level. This allows the retailer to charge a higher retail price as demand is more sensitive to service changes than to price changes. However, since the cost of providing the next unit of service increases by a power of 2, it is not economical to keep increasing the service level. Therefore, when the service reaches a certain level the two competitors must again switch to competing to offer the lower price. This leads to a decreasing retail price again when b_s is large. Demand quantities for both products and the retailer's profit increase when this phenomenon occurs. The manufacturers' profit first increases due to higher revenue. However, it later decreases when the cost of providing services gets too high, as they can not make up for the investment in service from sales to the retailer.

3.4.2.4 Market Sensitivity to Service Difference (θ_s)

We next examine the effects of varying the value of θ_s, the market sensitivity to difference in service, on the optimal solution, given that the two manufacturers are symmetrical. It turns out that as θ_s increases, the level of service decreases. This phenomenon may be explained as follows: since the cost of providing service increases by a power of 2, it is not economical to invest in service when the market is so sensitive to the competitor's service. Therefore, the service level lowers when θ_s increases. The two manufacturers switch to competing to lower the price. This is reflected in the low price for both products when θ_s is high. The demand for both products first increases because of price competition. However, it then decreases once the level of service decreases beyond a certain level. The profits of the retailer and manufacturers express the same property. Namely, they decrease as the market sensitivity to competitor's service increases.

3.5 Conclusions

Our primary objective is to highlight the importance of service from manufacturers in the interactions between two competing manufacturers and their common retailer, facing end consumers who are sensitive to both retail price and manufacturer service. We also explore the role of the retailer and its bargaining power by examining the supply chain over three different scenarios. Using the game-theoretic approach, our analysis found a number of insights into the economic behavior of firms, which could serve as the basis for empirical study in the future.

In this chapter, we derive expressions for equilibrium retail and wholesale prices, service levels, profits, and demand quantity for each product. We then analyze the results and give some insights on the influence of each parameter. Our results show that it is more beneficial to consumers when there is no dominant player(s) in vertical strategic interaction. In such case, the consumers receive more manufacturer service and can buy the product at a lower price. A counterintuitive result shows that as market base of one product increases, the competitor also benefits but at a smaller magnitude. Furthermore, when one manufacturer has an economic advantage in providing service, the retailer will act to separate market segment by selling products with low service at a low price and selling products with high service at a high price.

Our results, however, are based upon simplistic assumptions about the demand function. Thus, there are possible extensions to improve our model. First, different or more general forms of the demand function can be used to analyze the problem. Another possibility is to consider the problem with demand uncertainty. In that case, the problem faced by the retailer will be a two-product newsvendor problem with price-dependent demand (we consider this stochastic demand assumption in Chapter 5).

Our model assumes a decreasing return in providing service to the consumers. An alternative assumption is to assume economies-of-scale in providing service. Similar alternatives can also be applied to the production cost function. Our model assumes linear production cost (fixed per-unit production cost). A more general production function can be used to reflect scale economies.

We can extend the model over multiple periods to specifically study temporal dynamics in the supply chain. The learning effect can be then examined. The new model can analyze how firms and consumers can make use of their experiences and learn over repeated transactions. We explore this multiple period model in Chapter 4.

Furthermore, the retailer in our model enjoys regional monopolistic advantage. An alternative is to build a model with two or more competing retailers. Other possibilities may include the situation where one manufacturer owns and controls a retailer as a “company store” and competes with regular channel. A more general model can also be built to have the service components both from the manufacturers and from the retailer. Another possible extension is to examine various mechanisms to coordinate the supply chain, such as vertical integration or two-part tariffs.

CHAPTER IV

LEARNING IN SUPPLY CHAIN WITH REPEATED TRANSACTIONS AND SERVICE CONTRIBUTIONS

4.1 Introduction

Chapter 3 studies the situation where two manufacturers compete through one common retailer. We examine the problem with a single period model. However, in reality, the interactions between the manufacturers, retailer, and consumers can occur repeatedly over multiple periods or product generations. For example, in the PC industry, consumers upgrade their PCs every 3-5 years (see [2]). The industry has also seen its product life cycle decrease in recent years ([4], [52]). Thus, customers gain more experience on price and service every time they upgrade products. The price they paid and the service they received during their last experience will influence their next upgrading decisions. In addition, with the advent of the Internet, information on prices and service reputation of many manufacturers has been made available to consumers on many websites.

In this chapter, we study the inter-temporal behavior of the manufacturers and retailer in the supply chain introduced in Chapter 3. Each period can be thought of as one selling season or a span over one product generation. Thus each period in our model can span over one quarter, 6 months, or 2 years, depending on the nature of the product being considered.

We are interested in studying the behavior of each firm (one retailer and two competing manufacturers) over time when faced with "learning" demand. Namely, we assume that demand for each product in any given period is affected by two types of components: (1) the difference in prices and services between the two products in the previous period, and (2) the amount of investment by each manufacturer between each period to expand the market base of its product (or brand). This assumption on the behavior of consumers demand reflects the fact that consumers have learned from the experience they had with the service provided by each manufacturer and the price they paid for the product. They also are influenced by the investment by each manufacturer to expand its product's market base (i.e., promotions, advertising campaigns, etc.).

Thus, within each period, a manufacturer has to make decisions on wholesale price, service level and amount of investment to expand its market base for that period. The decision on the amount of investment is taken at the very beginning of each period. The decisions on the wholesale price and service level are taken by each manufacturer after the market has been influenced by the investment. Finally, the retailer makes its decision on the retail price of both products at the end of each period. The decision cycle is repeated over time in this order. Note that we concentrate on the Manufacturer Stackelberg model in this chapter[1].

In the past, some studies in literature have addressed the pricing in a multi-period setting. From economic literature, there is stream of research that addresses the issue of competing firms over multiple periods (see [87] and [76]). However, most studies in existing literature model problems as a simple repeated game over multiple periods, with

[1] The study on the Retailer Stackelberg and Vertical Nash is possible in the future and is discussed in Chapter 6.

no demand learning. To our knowledge, no one has considered the situation with both learning consumer demand and competing products sold through one common retailer.

There are also some studies in operations literature that study decision-making over multiple periods. However, the majority of them only consider a single manufacturer. Issues related to competition and gaming in pricing decisions among different firms have only been addressed recently. For example, Cohen and Whang (1997) [17] study a set of strategic choices facing manufacturers as they design the joint product/service bundle for a product which may require after-sales service. The price and service quality/price are characterized by an equilibrium to a sequential game. Vilcassim *et al.* (1999) [91] formulate a game-theoretic model of firm interaction to analyze the dynamic price and advertising competition among firms in a given product market. However, their model does not include a retailer. Their study uses an econometric model to estimate demand and competitive interaction parameters and derive some managerial implications for competitive interactions.

In this chapter, we approach the problem by introducing a new methodology for game-based decision making in multiple (transactions) periods using dynamic systems and control theory. By applying this new methodology, we answer the following questions:

i. How do the manufacturers make their pricing decisions over time?

ii. How are the prices and service levels in the second period influenced by those in the first period?

iii. How does the whole supply chain behave over time? What indication(s) is there for us to learn about the firms' inter-temporal behavior?

In Section 4.2, the notations are defined and the model is described. The analysis of the model using game theory and dynamic systems and control theory is presented in Section 4.3 and 4.4, respectively. Numerical examples are given in the Section 4.5 to represent possible real world cases. The comparison of results from our model and a myopic model is presented in Section 4.6. Finally, in Section 4.7 we give final remarks for this chapter.

4.2 Model

4.2.1 Notations

In multiple period model, each decision variable has a subscript t to indicate period being considered. This is in addition to subscript i which indicates the manufacturer (or product) associated with the variable.

Let $i = 1, 2$ be the index for manufacturer/product.

$t = 1, 2, ..., N$ is the index for transaction (or period or cycle or season).

$\Pi_{R,t}$ = Retailer's total profit in the t^{th} transaction.

$\Pi_{M_i,t}$ = Manufacturer i's total profit in the t^{th} transaction.

$p_{i,t}$ = Retail price of product i in the t^{th} transaction.

$s_{i,t}$ = The amount of service provided by supplier i to the consumer in the t^{th} transaction.

$w_{i,t}$ = Wholesale price of product i to the retailer in the t^{th} transaction.

$Q_{i,t}$ = Demand for product i in the t^{th} transaction.

$a_{i,t}$ = Market base for product i in the t^{th} transaction.

$I_{i,t}$ = Investment from manufacturer i to expand its market base at the beginning of the t^{th} transaction.

4.2.2 Supply Chain Descriptions

We study the same supply chain structure introduced in the previous chapter. In addition we extend the problem over multiple periods, *i.e.* there are two suppliers (manufacturers). Each supplier manufactures one product. The two products are sold competitively to end consumers through one common retailer. Just as in Chapter 3, the demand for each product in each period depends on two factors: (1) the difference in retail prices between the two competing products, and (2) the difference in level of service provided by the product's manufacturer and its competitor.

Thus, within each period, the demand for each product can be expressed as:

$$Q_{i,t} = a_{i,t} - (b_p + \theta_p)p_{i,t} + \theta_p p_{j,t} + (b_s + \theta_s)s_{i,t} - \theta_s s_{j,t} \tag{32}$$

where the definition and range of b_p, θ_p, b_s, and θ_s are the same as is defined in Chapter 3. Notice that now the variables have two subscripts: one to indicate the manufacturer and the other to indicate time.

In this chapter, we concentrate on the Manufacturer Stackelberg model. We assume that the suppliers in oligopolistic markets are able to establish a supplier-driven channel [2]. In each transaction the manufacturer decides the level of service. In this model, we use the same definition of service as defined in Chapter 1. We assume that both manufacturers are Stackelberg leaders of the supply chain. They simultaneously announce the value of their decision variables before any transaction occurs. In our model, each manufacturer announces (simultaneously) the wholesale price and the service level. After that the retailer

[2] The studies on the other two models introduced in Chapter 3 (Retailer Stackelberg and Vertical Nash) are possible in the future.

reacts to the announcement by deciding what the retail price of each product should be.

However, the multi-period model studied in this chapter also takes into account the inter-temporal influence of retail prices and services on consumer demand in the next period. This is a result of the "learning" behavior of consumers. This "learning" behavior is reflected in the increase or decrease in the size of each product's market base over time (indicated by $a_{i,t}$ in the Equation 32 above. Particularly, each product's market base is affected by two inter-temporal factors: (1) the difference in retail price from the previous period, and (2) the difference in level of service provided by the manufacturers in the previous period.

In addition to these two inter-temporal factors, the manufacturers can influence the size of their product's market base by making some investment to expand its market base (*i.e.*, through advertising campaigns, improved business infrastructure, alliance formation, promotions, etc.) at the beginning of each period. This action taken by the manufacturers also affects the size of $a_{i,t}$.

Figure 7 shows the timeline of events within each period. Within each period, the overall pricing and ordering decisions in the channel follow the following sequence:

Step 1. Manufacturers simultaneously choose the level of investment to influence their market base for this period.

Step 2. The market base ($a_{i,t}$) for each product is updated according to the influence from the following factors: (1) the difference in retail price from the previous period, (2) the difference in level of service provided by the manufacturers in the previous period, and (3) the amount of investment each manufacturer make at the beginning of the

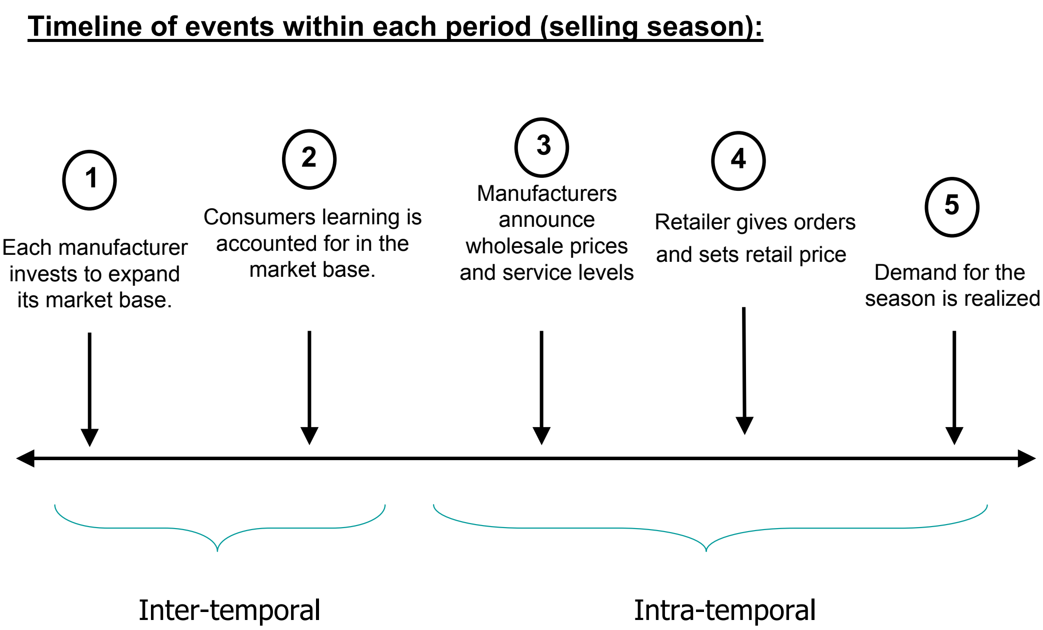

Figure 7: Timeline of events within each transaction.

period to influence the size of its product's market base. The information about the market base is revealed before the next step.

Step 3. Manufacturers simultaneously choose their wholesale price to be offered to the retailer and the service levels to be offered to end consumers. Each manufacturer makes their decisions so as to maximize its own profit.

Step 4. In response to manufacturers' actions, the retailer decides the retail price of both products so as to maximize his expected profit.

Step 5. The consumer demand for each product is realized. The profit for every firm in the supply chain is realized.

We assume that each manufacturer has complete information about its competitor and the retailer's cost parameters and also the consumer's demand responsiveness to the retail price. Therefore, considering the problem from Step 3 to Step 5, for given wholesale prices chosen in Step 3, the manufacturer knows the retailer's response in Step 4 and, hence, their own profit in Step 5. Each manufacturer will take this into account so as to choose the wholesale price and service level to maximize his own profit. Similar reasoning also applies when we consider the problem faced by the manufacturers in Step 1. The manufacturers can anticipate the market reaction (through the size of market base) in Step 2 when making their decisions on the amount of investment in Step 1. Furthermore, the manufacturers in Step 1 can also take into account their own best anticipated courses of action in Step 3 and the retailer reactions in Step 4 to maximize their individual profit to be realized in Step 5. Figure 8 give a schematic representation of the supply chain being studied in this chapter.

4.2.3 Learning Demand Function

As mentioned briefly in the description of the events that occur within each period, the market base ($a_{i,t}$) for each product in any period is influenced by the following factors: (1) the difference in retail price from the previous period, (2) the difference in level of service provided by the manufacturers in the previous period, and (3) the amount of investment each manufacturer make at the beginning of the period to influence the size of his product's market base. We also make the following assumptions about the consumer behavior regarding prices and services they received.

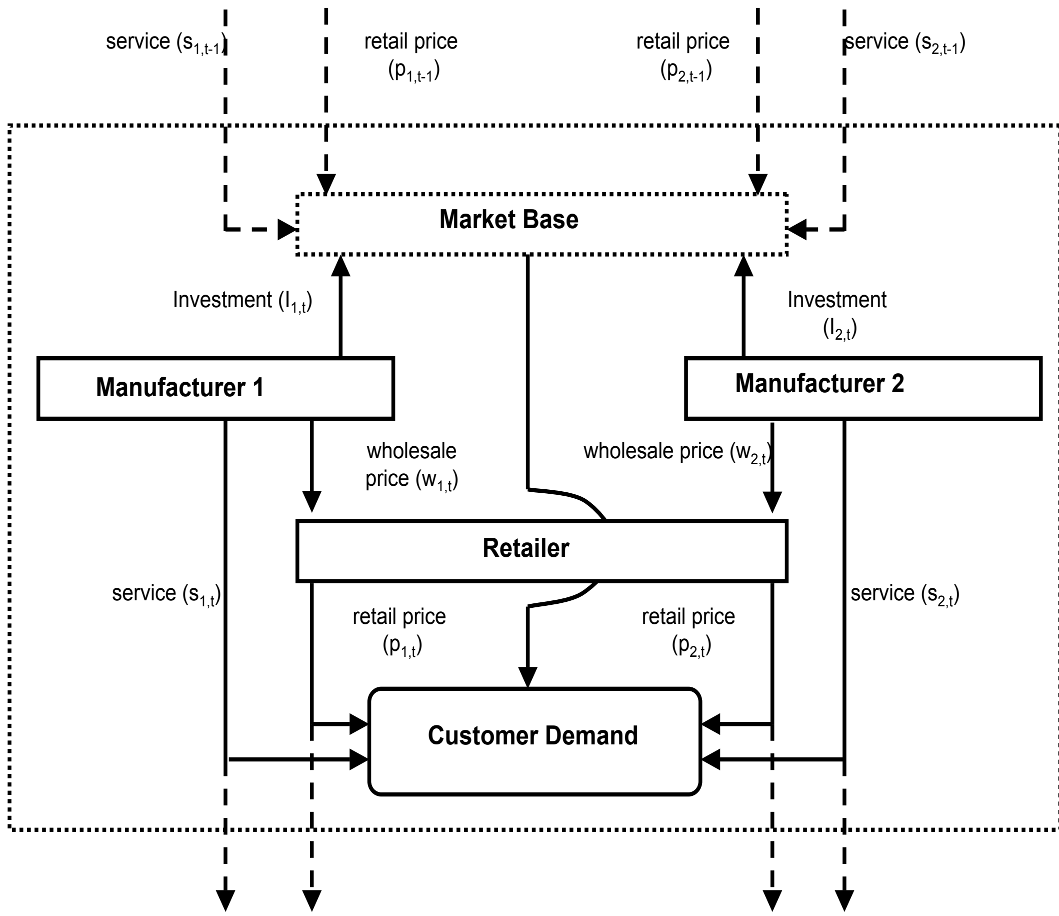

Figure 8: Supply Chain System.

ASSUMPTION 1. *Consumers have the memory of past transactions only for the last period.*

This assumption simply states that consumer's memory on history of past transactions can go back to only one period. This assumption simplifies our model on the learning part of consumers.

ASSUMPTION 2. *In learning process, consumers memorize and value the differences* *in retail prices and service levels from the last period.*

We make this assumption to emphasize the net effect of the differences in retail prices and services on demand. In other words, consumers only care about the relative differences in the retail prices and service levels between the two products. Information on past prices and service reputation has been made available recently to consumers on many websites on the Internet.

ASSUMPTION 3. *The investment by one manufacturer ($I_{i,t}$) does not directly affect the market base of the other product within the same period ($a_{j,t}$).*

This assumption separates the direct effect of investment by one manufacturer from the action by another manufacturer. However, through strategic movement by the two manufacturers, it is possible that an indirect effect exists. Namely, an increase in investment by manufacturer i can induce more investment by manufacturer j. Our analysis of the model considers this indirect influence through game-theoretic framework.

ASSUMPTION 4. *The investment in market base by a manufacturer has a decreasing return-to-scale.*

This assumption is used to capture the fact that the manufacturers can not keep investing their money to expand their market base. The assumption is characterized by the square root of $I_{i,t}$ in Equation (34).

With the assumption above, the exact market base equation is given below:

$$a_{i,t+1} = a_{i,t} - \gamma(p_{i,t} - p_{j,t}) + \sigma(s_{i,t} - s_{j,t}) + \beta\sqrt{I_{i,t+1}} \tag{33}$$

Equation (33) can be rewritten algebraically as

$$\begin{bmatrix} a_{1,t+1} \\ a_{2,t+1} \end{bmatrix} = \begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix} + \begin{bmatrix} -\gamma & \gamma \\ \gamma & -\gamma \end{bmatrix} \begin{bmatrix} p_{1,t} \\ p_{2,t} \end{bmatrix} + \begin{bmatrix} \sigma & -\sigma \\ -\sigma & \sigma \end{bmatrix} \begin{bmatrix} s_{1,t} \\ s_{2,t} \end{bmatrix} + \begin{bmatrix} \beta\sqrt{I_{1,t+1}} \\ \beta\sqrt{I_{2,t+1}} \end{bmatrix} \tag{34}$$

Equations (33) and (34) reflect the "learning" by consumers about the experience they had gained before making their buying decisions within this period (before Step 3-5 begins).

4.2.4 Manufacturers' and Retailer's Profit Functions

As in Chapter 3, Manufacturer i's profit within each period is the revenue minus cost. However, in this chapter we introduce $I_{i,t}$ as part of the cost to influence the market base in period t. Therefore, the manufacturer's profit is slightly modified in this chapter.

$$\Pi_{M_i,t} = (w_{i,t} - c_i)Q_{i,t} - \frac{\eta_i s_{i,t}^2}{2} - I_{i,t} \tag{35}$$

where $i = 1, 2$ and η_i is the service cost coefficient of manufacturer i. The retailer's profit within each period is still the same as in Chapter 3. Namely,

$$\Pi_{R,t} = \sum_{i=1}^{2} (p_{i,t} - w_{i,t})Q_{i,t} \tag{36}$$

where $Q_{i,t}$ is as specified in Equation (32).

However, in this chapter we make the following assumption about the objective of each firm in the supply chain when making its decisions within each period t.

ASSUMPTION 5. *In any period t, both the manufacturers and the retailer are maximizing their own "moving" two-period profit when making their decision on either prices* *or service levels.*

This assumption states that both manufacturers and the retailer have a "one-period look-ahead" behavior. This means that in any period t, each firm will try to maximize the sum of profits in period t and $t+1$. Vilcassim *et al.* (1999) [91] also assume this framework in their analysis on firms competing on both price and advertisement[3]. This two-period optimization assumption is in contrast to the alternative "myopic" assumption in which firms only care for current period profit when making their decisions. It is also in contrast to another alternative model in which firms try to maximize their profits over all N periods (i.e., until the end of a finite time horizon). The difficulty in that framework is tractability of the closed-form solution.

The firms in our model try to maximize two-period profits. The question in our framework is whether the moving two-period solution provides a reasonable approximation to the behavior of firms in the real world. To address this question, we refer to results from an empirical study by Vilcassim *et al.* (1999) [91]. They found that the relative effect of current period actions on demand two periods in the future "ranged from around 18% to 9%, while the effect three periods into the future was at most around 8%." Hence, the moving two-period model can be treated as a reasonable approximation to real profit maximizing behavior of firms.

Thus, in any period, the manufacturers must ask how the decision they makes on investment ($I_{i,t}$), will affect the market base in the same period t. They must also ask how the decision on wholesale price and service level in period t would induce the reaction by

[3]Their study uses econometric model to estimate the demand and competitive interaction parameters. Some parameters in our study rely on their study to get an estimation on the range of value.

the retailer within period t, and also the market base in period $t+1$. The market base ($a_{i,t}$) would in turn influence the decisions by every firm in period $t+1$. The retailer must also ask a similar question. Specifically, the retailer must think ahead how his reaction to wholesale prices and service levels in period t would affect both products' market bases (and his own profit) in period $t+1$.

The game-theoretic approach to analyze the problem must then take this fact into account. Note that the analysis on this situation requires more than just a simple repeated game framework, but a combination of game theory and dynamic systems control. The game concepts are employed to analyze strategic interactions among firms in the supply chain. Equilibrium can then be derived. Dynamic systems and control theory concepts are employed to analyze the evolving equilibrium of the supply chain over time.

4.3 Analysis of the Model Part I: Equilibrium

The equilibrium concept used in our analysis is the subgame-perfect equilibrium. Using a game-theoretic framework, the problem is solved backwards. Note that the problem must be analyzed with a two-period framework, according to the way firms set their objectives as described earlier. However, even though firms maximize their profits over two periods (t and $t+1$), the decision process is carried out every period by each firm until the end of time horizon.

Thus, to make a decision for period t, we begin by considering the $t+1^{st}$ period problem. Once the reaction functions in the $t+1^{st}$ period are derived, the decision problems by each firm in the t^{th} period are then derived and analyzed. The methodology in calculating (re)action functions in both periods is similar. First, the reaction function (on retail price) by the retailer must be derived. Then the equilibrium wholesale price and service level given by each manufacturer are derived. Finally, the amount of investment (to induce market base) made by each manufacturer must be calculated. The only difference between the calculations in both periods is that when performing the calculations in the $t+1^{st}$ period, we assume that firms already have information on the value of prices and service levels in the t^{th} period and they are trying to maximize only the profit in the $t+1^{st}$ period. The calculations for values in the t^{th} period are carried out with the assumption that firms know the reaction functions in the $t+1^{st}$ and they all try to maximize their individual profits over two periods.

4.3.1 Second (Next) Period Analysis

We solve the problem by first separating the problem into two phrases. The first one can be called an $inter-temporal$ subproblem. This is the subproblem where the decision variables involve some variables from the previous period. This subproblem covers the Step 1 and Step 2 defined in Figure 7. The other subproblem is an $intra-temporal$ subproblem. This is the subproblem in which all the parameters and variables are the results of decisions made within the period. This subproblem covers the Step 3 to Step 5 in Figure 7. We solve the problem by working backwards. Thus, we solve the intra-temporal subproblem first. Then we can consider solving the inter-temporal part of the problem.

4.3.1.1 Intra-Temporal Subproblem

The $intra-temporal$ subproblem in period $t+1$ is the same as the problem we already studied in Chapter 3. This is because by the time the retailer made decision on retail prices in Step 5 and the manufacturers make their decisions on wholesale prices and service levels in Step 3, the market base parameters ($a_{i,t+1}$ for $i = 1, 2$) have already taken into account the investment made by the manufacturers in Step 1 of period $t+1$ and any inter-temporal effects from the previous period. Thus, the objective function of each manufacturer right before the start of Step 3 does not include the investment ($I_{i,t+1}$). Therefore, the results of studies on the Manufacturer Stackelberg model in Chapter 3 can be applied to the $intra-temporal$ subproblem here. The manufacturer's equilibrium wholesale price and service level are

$$\begin{bmatrix} w_{1,t+1} \\ w_{2,t+1} \end{bmatrix} = \begin{bmatrix} \varphi_1 & \varphi_1 D_2 \\ \varphi_2 D_1 & \varphi 2 \end{bmatrix} \begin{bmatrix} a_{1,t+1} \\ a_{2,t+1} \end{bmatrix} + \begin{bmatrix} \varphi_1 n_{11} & \varphi_1 n_{12} \\ \varphi_2 n_{21} & \varphi_2 n_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\begin{bmatrix} s_{1,t+1} \\ s_{2,t+1} \end{bmatrix} = \begin{bmatrix} l_1 & l_1 D_2 \\ l_2 D_1 & l_2 \end{bmatrix} \begin{bmatrix} a_{1,t+1} \\ a_{2,t+1} \end{bmatrix} + \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \tag{37}$$

where $i, j \in \{1, 2\}$, $j \neq i$ and

$$\begin{array}{ll}
A_i = 4\eta_i(b_p + \theta_p) + (b_s + \theta_s)^2 & B_i = 2\eta_i\theta_p - \theta_s(b_s + \theta_s)\left(\frac{b_p - b_s + 2\theta_p}{b_p + 2\theta_p}\right) \\
D_i = \frac{B_i}{A_i} & \\
E_i = (b_p + \theta_p) - \frac{(b_s+\theta_s)^2}{2\eta_i} & F_i = \frac{\theta_s(b_s+\theta_s)}{2\eta_i} - \frac{\theta_p b_s(b_s+\theta_s)}{2\eta_i(b_p+2\theta_p)} \\
n_{11} = E_1 + F_1 D_2 & n_{12} = F_2 + E_2 D_2 \\
n_{21} = F_1 + E_1 D_1 & n_{22} = E_2 + F_2 D_1 \\
\varphi_i = \frac{2\eta_i A_j}{A_1 A_2 - B_1 B_2} & l_i = \varphi_i \frac{(b_s+\theta_s)}{2\eta_i} \\
m_{11} = l_1(E_1 + F_1 D_2 - \frac{1}{\varphi_1}) & m_{12} = l_1(F_2 + E_2 D_2) \\
m_{21} = l_2(F_1 + E_1 D_1) & m_{22} = l_2(E_2 + F_2 D_1 - \frac{1}{\varphi_2}).
\end{array}$$

With the results shown above we can calculate for the expression of the retail price ($p_{i,t}$ for $i = 1, 2$) and demand quantity ($Q_{i,t}$ for $i = 1, 2$). That is

$$\begin{bmatrix} p_{1,t+1} \\ p_{2,t+1} \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} a_{1,t+1} \\ a_{2,t+1} \end{bmatrix} + \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \tag{38}$$

$$\begin{bmatrix} Q_{1,t+1} \\ Q_{2,t+1} \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} a_{1,t+1} \\ a_{2,t+1} \end{bmatrix} + \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \tag{39}$$

where the definition of t_{ij}, y_{ij}, g_{ij}, and h_{ij} for $i, j \in \{1, 2\}$ are given in Appendix B.

Note that the market bases ($a_{i,t+1}$ for $i = 1, 2$) in Equation (37) to (39) are the market bases after the "learning" effect by the consumer (shown in Equation (34)) has taken place earlier in the period. In the next section, the amount of investment ($I_{i,t+1}$) each manufacturer should invest to influence the market base will be derived.

4.3.1.2 Inter-Temporal Decisions

We next consider the decisions by the two manufacturers on the investment ($I_{i,t+1}$). The objective function of the manufacturers at this stage is as shown in Equation (35). The manufacturer i must choose the investment $I^*_{i,t+1}$ to maximize its equilibrium profit. Let $\boldsymbol{p_t} = [p_{1,t}, p_{2,t}]$, $\boldsymbol{w_t} = [w_{1,t}, w_{2,t}]$, $\boldsymbol{s_t} = [s_{1,t}, s_{2,t}]$, $\boldsymbol{I_t} = [I_{1,t}, I_{2,t}]$. The investment $I^*_{i,t+1}$ at equilibrium can be expressed as

$$I^*_{i,t+1} \in arg \max_{I_{i,t+1}} \Pi_{M_i,t+1}(I_{i,t+1}, I^*_{j,t+1} | \boldsymbol{p_t}, \boldsymbol{w_t}, \boldsymbol{s_t}, \boldsymbol{I_t}) \tag{40}$$

where $\Pi_{M_i,t+1}(I_{i,t+1}, I^*_{j,t+1} | \boldsymbol{p_t}, \boldsymbol{w_t}, \boldsymbol{s_t}, \boldsymbol{I_t})$ denotes the profit to manufacturer i at this stage, given earlier decisions on retail price $\boldsymbol{p_t}$, wholesale price $\boldsymbol{w_t}$, service levels $\boldsymbol{s_t}$, and Investment

$\boldsymbol{I_t}$. Using Equation 34, the first order conditions can be shown as

$$\begin{aligned} 0 &= \Big\{2\eta_i Q_{i,t+1}K_j + (w_{i,t+1} - c_i)\Big[\frac{1}{2} - (b_p+\theta_p)\eta_i K_j + \theta_p\eta_j D_i K_i + \frac{(b_s+\theta_s)^2}{2}K_j \\ &\quad -\frac{\theta_s}{2}(b_s+\theta_s)D_i K_i\Big] - \eta_i(b_s+\theta_s)K_j s_{i,t+1}\Big\}\frac{\beta}{2\sqrt{I_{i,t+1}}} - 1 \end{aligned} \tag{41}$$

where $i, j \in \{1, 2\}$ and $j \neq i$.

Given the first order condition in Equation 41 and the results from Equation 37, 37, and 39, one can derive the following linear relationship between the square root of the investment amount and the market bases, retail prices and service levels from previous period t.

$$\begin{aligned} \begin{bmatrix} \sqrt{I_{1,t+1}} \\ \sqrt{I_{2,t+1}} \end{bmatrix} &= \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}\begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix} + \begin{bmatrix} (\delta_{12}-\delta_{11})\gamma & -(\delta_{12}-\delta_{11})\gamma \\ (\delta_{22}-\delta_{21})\gamma & -(\delta_{22}-\delta_{21})\gamma \end{bmatrix}\begin{bmatrix} p_{1,t} \\ p_{2,t} \end{bmatrix} \\ &\quad + \begin{bmatrix} -(\delta_{12}-\delta_{11})\sigma & (\delta_{12}-\delta_{11})\sigma \\ -(\delta_{22}-\delta_{21})\sigma & (\delta_{22}-\delta_{21})\sigma \end{bmatrix}\begin{bmatrix} s_{1,t} \\ s_{2,t} \end{bmatrix} + \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \end{aligned} \tag{42}$$

where the definition of δ_{ij}, and Δ_i for $i, j \in \{1, 2\}$ are given in Appendix B.

Finally, the market base in period $t+1$ ($a_{i,t+1}$), which is the market base after influence from the investment $I_{1,t+1}$ and $I_{2,t+1}$ has been taken into account, can be derived by substituting Equation (42) into Equation (33). As a result, the market base in period $t+1$ can be expressed as a function of the retail prices, service levels, and market bases in period t as

$$\begin{aligned} \begin{bmatrix} a_{1,t+1} \\ a_{2,t+1} \end{bmatrix} &= \begin{bmatrix} (\delta_{12}-\delta_{11})\gamma\beta - \gamma & -(\delta_{12}-\delta_{11})\gamma\beta + \gamma \\ (\delta_{22}-\delta_{21})\gamma\beta + \gamma & -(\delta_{22}-\delta_{21})\gamma\beta - \gamma \end{bmatrix}\begin{bmatrix} p_{1,t} \\ p_{2,t} \end{bmatrix} \\ &\quad + \begin{bmatrix} -(\delta_{12}-\delta_{11})\sigma\beta + \sigma & (\delta_{12}-\delta_{11})\sigma\beta - \sigma \\ -(\delta_{22}-\delta_{21})\sigma\beta - \sigma & (\delta_{22}-\delta_{21})\sigma\beta + \sigma \end{bmatrix}\begin{bmatrix} s_{1,t} \\ s_{2,t} \end{bmatrix} \end{aligned}$$

$$+\begin{bmatrix} (\beta\delta_{11}+1) & \beta\delta_{12} \\ \beta\delta_{21} & (\beta\delta_{22}+1) \end{bmatrix}\begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix}+\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \tag{43}$$

4.3.2 First (Current) Period Analysis

After knowing how each firm will behave in period $t+1$ given the information on decision in period t, we next consider the decisions faced by each firm in period t.

4.3.2.1 Intra-Temporal Subproblem

Retailer Reaction Function

The retailer's decision on the retail prices of both products in period t can now be examined. Note that the objective of the retailer is now to maximize the profit over two periods. Specifically, its objective function is

$$\Pi_{R,t}(\boldsymbol{p_t}|\boldsymbol{w_t},\boldsymbol{s_t},\boldsymbol{I_t}) = \sum_{\tau=t}^{t+1}\sum_{i=1}^{2}(p_{i,\tau}-w_{i,\tau})Q_{i,\tau} \tag{44}$$

where $\Pi_{R,t}(\boldsymbol{p_t}|\boldsymbol{w_t},\boldsymbol{s_t},\boldsymbol{I_t})$ is the sum of retailer's profit in period t and $t+1$ when the retail price in period t is $\boldsymbol{p_t}=[p_{1,t},p_{2,t}]$. The retailer must find the retail price $p^*_{1,t}$ and $p^*_{2,t}$ to maximize its profit. That is,

$$p^*_{i,t} \in arg\max_{p_{i,t}} \Pi_{R,t}(p_{i,t},p^*_{j,t}|\boldsymbol{w_t},\boldsymbol{s_t},\boldsymbol{I_t}) \tag{45}$$

where $\Pi_{R,t}(p_{i,t},p_{j,t}|\boldsymbol{w_t},\boldsymbol{s_t},\boldsymbol{I_t})$ is the retailer profit when the retail price is $p_{i,t}$ and $p_{j,t}$, given earlier decisions on wholesale prices $\boldsymbol{w_t}$, service levels $\boldsymbol{s_t}$, and investment $\boldsymbol{I_t}$. The first order condition for Equation (44) is used to find $p^*_{i,t}$ ($i\in\{1,2\}$).

$$\frac{\partial \Pi_{R,t}}{\partial p_{i,t}} = Q_{i,t}+(p_{i,t}-w_{i,t})\frac{\partial Q_{i,t}}{\partial p_{i,t}}+(p_{j,t}-w_{j,t})\frac{\partial Q_{j,t}}{\partial p_{i,t}}$$

$$+\Big(\frac{\partial p_{i,t+1}}{\partial p_{i,t}} - \frac{\partial w_{i,t+1}}{\partial p_{i,t}}\Big)Q_{i,t+1} + (p_{i,t+1} - w_{i,t+1})\frac{\partial Q_{i,t+1}}{\partial p_{i,t}}$$
$$+\Big(\frac{\partial p_{j,t+1}}{\partial p_{i,t}} - \frac{\partial w_{j,t+1}}{\partial p_{i,t}}\Big)Q_{j,t+1} + (p_{j,t+1} - w_{j,t+1})\frac{\partial Q_{j,t+1}}{\partial p_{i,t}}$$

Using the first order condition above, the retailer's reaction function to wholesale prices and service levels in period t can be derived as

$$\begin{bmatrix} p_{1,t} \\ p_{2,t} \end{bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \begin{bmatrix} s_{1,t} \\ s_{2,t} \end{bmatrix} + \begin{bmatrix} \zeta_{11} & \zeta_{12} \\ \zeta_{21} & \zeta_{22} \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} + \begin{bmatrix} \Upsilon_1 \\ \Upsilon_2 \end{bmatrix}. \tag{46}$$

where the expressions for ψ_{ij}, ζ_{ij}, and Υ_i for $i, j \in \{1, 2\}$ and $j \neq i$ are given in Appendix B.

Manufacturers Decision Process

The retailer's reaction function in Equation (46) gives the manufacturers information on how their decisions will affect the retail prices and their profits. The manufacturers then use this information to set wholesale prices and service levels to maximize their individual profits over two periods (t and $t + 1$). Each manufacturer's objective at this stage can be expressed as

$$\Pi_{M_i,t} = \sum_{\tau=t}^{t+1} \Big[(w_{i,\tau} - c_i)Q_{i,\tau} - \frac{\eta_i s_{i,\tau}^2}{2} - I_{i,\tau}\Big] \tag{47}$$

for $i, j \in \{1, 2\}$ and $j \neq i$. Note that in Equation (47), $I_{i,t}$ is a constant. This is because when the manufacturers make their decisions on the wholesale prices and service levels, the decisions on $I_{1,t}$ and $I_{2,t}$ have already been made. Each manufacturer must choose the wholesale price and service level to maximize its own objective. That is,

$$w_{i,t}^* \in arg \max_{w_{i,t}} \Pi_{M_i,t}(w_{i,t}, w_{j,t}^*, s_{i,t}^*, s_{j,t}^* | \boldsymbol{I_t}) \tag{48}$$

and

$$s_{i,t}^* \in arg \max_{s_{i,t}} \Pi_{M_i,t}(w_{i,t}^*, w_{j,t}^*, s_{i,t}, s_{j,t}^* | \boldsymbol{I_t}) \tag{49}$$

where $\Pi_{M_i,t}(\boldsymbol{w_t}, \boldsymbol{s_t} | \boldsymbol{I_t})$ is the profit of manufacturer i at this stage when manufacturers set wholesale prices at $\boldsymbol{w_t}$ and service levels at $\boldsymbol{s_t}$, given earlier decisions on investment $\boldsymbol{I_t}$. First order conditions for each $i = 1, 2$ can be derived as follows:

$$\begin{aligned}
0 \quad &= \quad \frac{\partial \Pi_{M_i,t}}{\partial w_{i,t}} = Q_{i,t} + (w_{i,t} - c_i)\frac{\partial Q_{i,t}}{\partial w_{i,t}} + \frac{\partial w_{i,t+1}}{\partial w_{i,t}} Q_{i,t+1} \\
& \quad\quad +(w_{i,t+1} - c_i)\frac{\partial Q_{i,t+1}}{\partial w_{i,t}} - \eta_i s_{i,t+1}\frac{\partial s_{i,t+1}}{\partial w_{i,t}} - \frac{\partial I_{i,t+1}}{\partial w_{i,t}} \\
0 \quad &= \quad \frac{\partial \Pi_{M_i,t}}{\partial s_{i,t}} = (w_{i,t} - c_i)\frac{\partial Q_{i,t}}{\partial s_{i,t}} - \eta_i s_{i,t} + \frac{\partial w_{i,t+1}}{\partial s_{i,t}} Q_{i,t+1} \\
& \quad\quad +(w_{i,t+1} - c_i)\frac{\partial Q_{i,t+1}}{\partial s_{i,t}} - \eta_i s_{i,t+1}\frac{\partial s_{i,t+1}}{\partial s_{i,t}} - \frac{\partial I_{i,t+1}}{\partial s_{i,t}}
\end{aligned}$$

Solving the first order conditions above, the expression for $w_{i,t}$ and $s_{i,t}$ can be derived as linear functions of $a_{i,t}$ and c_i (for $i \in \{1, 2\}$).

$$\begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} = \begin{bmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{bmatrix} \begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix} + \begin{bmatrix} \nu_{11} & \nu_{12} \\ \nu_{21} & \nu_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \tag{50}$$

$$\begin{bmatrix} s_{1,t} \\ s_{2,t} \end{bmatrix} = \begin{bmatrix} \vartheta_{11} & \vartheta_{12} \\ \vartheta_{21} & \vartheta_{22} \end{bmatrix} \begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix} + \begin{bmatrix} \varsigma_{11} & \varsigma_{12} \\ \varsigma_{21} & \varsigma_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \tag{51}$$

The expressions for $\vartheta_{ij}, \varsigma_{ij}$, κ_{ij}, and ν_{ij} for $i \in \{1, 2\}$ and $j = 3 - i$ are given in Appendix B.

4.3.2.2 Inter-Temporal Subproblem

Continuing working backwards, the next step is for the manufacturers to analyze their decisions on the level of investment $I_{i,t}$. Manufacturer i's objective at this stage can be expressed as

$$\Pi_{M_i,t} \quad = \quad \sum_{\tau=t}^{t+1} \delta^{\tau-t}\Big[(w_{i,\tau}-c_i)Q_{i,\tau} - \frac{\eta_i s_{i,\tau}^2}{2} - I_{i,\tau}\Big] \tag{52}$$

where $i \in \{1,2\}$ and δ is discount factor. For simplicity, we assume that δ is 1 from now on. Here $I_{i,t}$ ($i \in \{1,2\}$) are decision variables. Each manufacturer must choose the investment $I_{i,t}$ to maximize its equilibrium profit. That is,

$$I_{i,t}^* \in arg \max_{I_{i,t}} \Pi_{M_i,t}(I_{i,t}, I_{j,t}^*) \tag{53}$$

where $\Pi_{M_i,t}(I_{1,t}, I_{2,t})$ is manufacturer i's profit over two periods when manufacturer 1 and 2 invest $I_{1,t}$ and $I_{2,t}$, respectively. The first order condition from Equation (52) with respect to the investment $I_{i,t}$ is then stated as follows

$$\begin{aligned} 0 \quad = \quad & (w_{i,t}-c_i)\frac{\partial Q_{i,t}}{\partial I_{i,t}} + \frac{\partial w_{i,t}}{\partial I_{i,t}}Q_{i,t} - \eta_i s_{i,t}\frac{\partial s_{i,t}}{\partial I_{i,t}} + (w_{i,t+1}-c_i)\frac{\partial Q_{i,t+1}}{\partial I_{i,t}} \\ & + \frac{\partial w_{i,t+1}}{\partial I_{i,t}}Q_{i,t+1} - \eta_i s_{i,t+1}\frac{\partial s_{i,t+1}}{\partial I_{i,t}} - \frac{\partial I_{i,t+1}}{\partial I_{i,t}} - 1 \end{aligned} \tag{54}$$

Solving the first order condition in Equation (54), the investment $I_{i,t}^*$ must satisfy the following relationship

$$\begin{bmatrix} \sqrt{I_{1,t}^*} \\ \sqrt{I_{2,t}^*} \end{bmatrix} = \begin{bmatrix} \varpi_{11} & \varpi_{12} \\ \varpi_{21} & \varpi_{22} \end{bmatrix} \begin{bmatrix} a_{1,t-1} \\ a_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}. \tag{55}$$

where the expressions for ϖ_{ij}, and ε_{ij} for $i,j \in \{1,2\}$ and $j \neq i$ are given in Appendix B.

Using the fact that the market base in the t^{th} period is

$$a_{i,t} \quad = \quad a_{i,t-1} - \gamma(p_{i,t-1} - p_{j,t-1}) + \sigma(s_{i,t-1} - s_{j,t-1}) + \beta\sqrt{I_{i,t}} \tag{56}$$

We can derive the system equation for the market base as

$$\begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix} = \begin{bmatrix} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{bmatrix} \begin{bmatrix} a_{1,t-1} \\ a_{2,t-1} \end{bmatrix} + \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \tag{57}$$

where the expressions for χ_{ij}, and ω_{ij} for $i, j \in \{1, 2\}$ and $j \neq i$ are given in Appendix B.

The result in Equation (57) is based on the assumption that $a_{i,0} = a_i$ where a_i is the initial market base for product i, and $p_{i,0} = s_{i,0} = 0$ for $i \in \{1, 2\}$.

4.4 Analysis of The Model Part II: Dynamic Systems

Equation (57) governs the dynamics of market bases and production cost over time. Alternatively, we can write it in the following form:

$$\begin{bmatrix} a_{1,t} \\ a_{2,t} \\ c_{1,t} \\ c_{2,t} \end{bmatrix} = \begin{bmatrix} \chi_{11} & \chi_{12} & \omega_{11} & \omega_{12} \\ \chi_{21} & \chi_{22} & \omega_{21} & \omega_{22} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{1,t-1} \\ a_{2,t-1} \\ c_{1,t-1} \\ c_{2,t-1} \end{bmatrix}. \tag{58}$$

or

$$\mathbf{\Phi(t) = M\Phi(t-1)} \tag{59}$$

Equation (58) and (59) both represent a homogeneous dynamic system. Note that the production cost $c_{1,t}$ and $c_{2,t}$ does not change over time. From the system equation above, a "*system equilibrium*" point can be defined in the following definition.

DEFINITION 4.1. *A vector $\overline{\mathbf{\Phi}}$ is an equilibrium point of a dynamic system if it has the* *property that once the system state vector is equal to $\overline{\mathbf{\Phi}}$ it remains equal to $\overline{\mathbf{\Phi}}$ for all future time.*

Thus, with the assumption that $\mathbf{\Phi}(\mathbf{0}) \neq \mathbf{0}$, a system equilibrium point must satisfy the condition

$$\overline{\mathbf{\Phi}} = \mathrm{M}\overline{\mathbf{\Phi}} \tag{60}$$

Equation (60) is useful to find the system equilibrium when one exists. From dynamic system theory, the existence of system equilibrium depends on the value of the dominant eigenvalue of M. Specifically, the following lemma is derived from dynamic system theory.

LEMMA 4.1. *(Luenberger [1979])Long term behavior of the market base sizes is determined by the dominant eigenvalue of* $\boldsymbol{M}$*. Subdominant eigenvalues of* $\boldsymbol{M}$ *determine how quickly the market bases converge or diverge.*

Lemma 4.1 follows directly from dynamic system theory (see Luenberger (1979) [56] for details) and the structure of homogeneous dynamic system of market bases stated in Equation (58) and (59). The dominant eigenvalue of $\mathbf{M}$ is the eigenvalue with the largest absolute value. Subdominant eigenvalues of $\mathbf{M}$ refer to all other eigenvalues of $\mathbf{M}$. From Lemma 4.1, we can analyze the dynamic behavior of the whole supply chain (*i.e.*, retail prices, wholesale prices and service levels) through the dynamic behavior of market bases governed by Equation (58) and (59). This is because from Section 4.3 we can express any such variables in any period as a function of market bases and production costs in that period. From the special structure of $\mathbf{M}$, one can find the dominant eigenvalue directly from the component of $\mathbf{M}$.

THEOREM 4.1. *Let λ_D be the dominant eigenvalue of **M**, then λ_D equals*

(a) 1, if $1 > |\chi_{11}|,|\chi_{22}|$

(b) χ_{11}, if $|\chi_{11}| > 1,|\chi_{22}|$

(c) χ_{22}, if $|\chi_{22}| > 1,|\chi_{11}|$

where χ_{11}, χ_{22} are as defined in Equation (58).

Proof:

Eigenvalues of $\mathbf{M}$ are scalars λ such that $\mathbf{M} - \lambda\mathbf{I}$ is singular. This implies one must solve for λ that satisfies $det(\mathbf{M} - \lambda\mathbf{I}) = 0$. However, from the definition of $\mathbf{M}$ in Equation (58), eigenvalues of $\mathbf{M}$ are $1, \chi_{11}$, and χ_{22} (with 1 having algebraic multiplicity of 2 in this case). Thus, the dominant eigenvalue must be the biggest of these three numbers. $\square$

The following theorem states the condition that governs the convergent or divergent behavior of the system.

THEOREM 4.2. *If the dominant eigenvalue of **M** equals to 1, the market bases of both products converge to a constant over time. Otherwise, the market bases diverge. If the dominant eigenvalue equals to 1, the system can converge or diverge.*

Proof: See Appendix B.

Theorem 4.2 states the dynamic behavior of the market bases of both products over time. Although the theorem states only the behavior of market bases, other quantities such as wholesale prices, retail prices, service levels, and demand quantity also follow the behavior of market bases. If the market bases converge, these variables will converge as well. Likewise, if the market bases diverge, they will also diverge.

Even when we know exactly whether the market bases will converge or diverge from the eigenvalue of **M**, the dynamic period-by-period behavior of the market bases (and prices and service levels) can vary. For example, there can be some oscillation in the market bases' sizes before each of them converge to a value. Alternatively, the market base can smoothly increase or decrease to a value over time. In the first case, the leader-follower roles can be alternating between the two products. Namely, the two products can switch the market leader-follower role[4] during the oscillation and before they reach the final convergent value. The following theorem states the conditions that govern the period-by-period behavior of the market bases.

THEOREM 4.3. *The dynamic behavior of market bases can be predicted by the value of its dominant eigenvalue as follows:*

(i) If every eigenvalue of $\boldsymbol{M}$ is positive, the dynamic pattern of market bases is a geometric sequence of the form r^k, which (increasingly) diverge if $r > 1$ and converge if $r \leq 1$. No oscillation occurs in this case.

(ii) If there is at least one eigenvalue that is negative, the response is an alternating geometric sequence of the form $(-1)^k|r|^k$. If $|r| < 1$, market base sizes will converge with decreasing oscillations. If $|r| > 1$, market base sizes diverge with increasing oscillations.

Proof: See Appendix B.

Theorem 4.3 characterizes the period-by-period behavior of the system variables. When

[4] A firm holds market leadership if it has bigger market base than its competition. On the other hand, the firm will be called the market follower if it has a smaller market base.

the dominant eigenvalue is negative, the market leader-follower roles between the two manufacturers can be alternating every period due to the oscillation in the system variables (i.e., market bases, prices, service levels). When the dominant eigenvalue is real and positive, it is still possible that the two manufacturers switch their market leadership. However, this switching can only occur once since there will be no oscillation in the system variables. Figure 9 shows the situation when all eigenvalues are positive. The dominant eigenvalue of **M** is 1.000, while the subdominant eigenvalue equals 0.0588. The system smoothly converges to the system equilibrium predicted by Equation (60). Figure 10 presents a situation

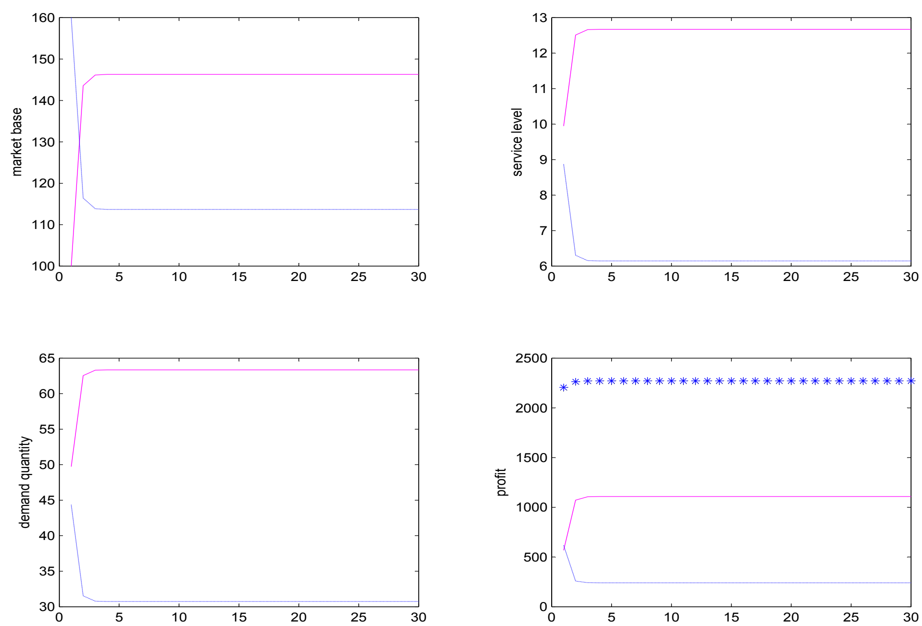

Figure 9: The evolution of equilibrium market bases when $\beta = 0.0, \gamma = 2.8, \sigma = 2.8$, $b_p = b_s = 2, \theta_p = \theta_s = 1.1, a_{1,0} = 100, a_{2,0} = 160, c_1 = 5, c_2 = 15, \eta_1 = \eta_2 = 5$ (Manufacturer 1: Red, Manufacturer 2: Blue, Retailer: *).

when the dominant eigenvalue is greater than one (1.0046 in this case). The subdominant eigenvalue is 0.0595. In this case, the system does not converge to a system equilibrium. However, it has a smooth behavior with no oscillation over time.

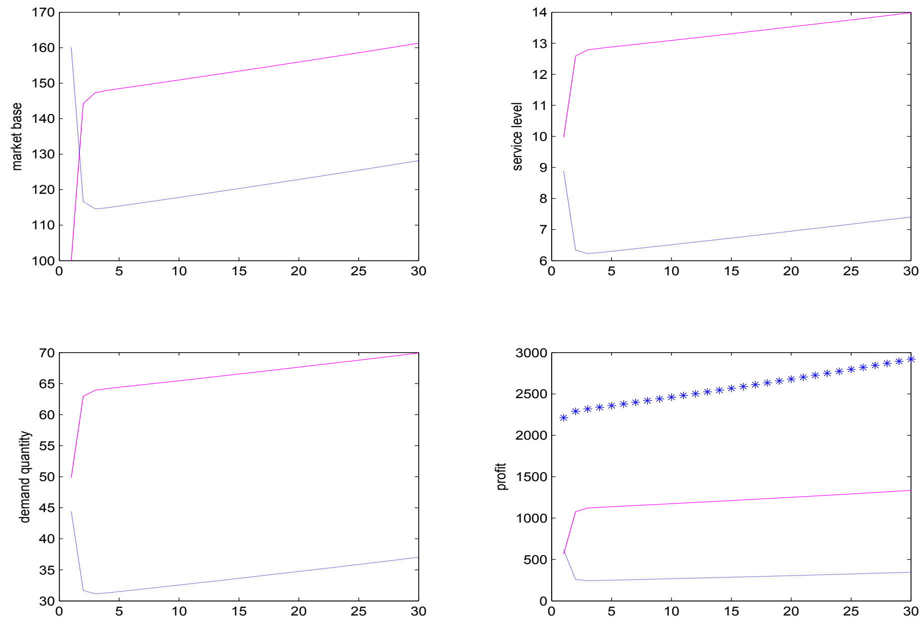

Figure 10: The evolution of equilibrium market bases when $\beta = 0.3, \gamma = 2.8, \sigma = 2.8$, $b_p = b_s = 2, \theta_p = \theta_s = 1.1, a_{1,0} = 100, a_{2,0} = 160, c_1 = 5, c_2 = 15, \eta_1 = \eta_2 = 5$ (Manufacturer 1: Red, Manufacturer 2: Blue, Retailer: *).

Figure 11 presents an example of a situation when the dominant eigenvalue is positive and equal to 1 but the subdominant eigenvalue is negative and greater than -1. In this example, the dominant eigenvalue of $\mathbf{M}$ is 1, while the subdominant eigenvalue equals -0.5634. Figure 11 shows that after some oscillation, the system settles down to the system equilibrium (which follows Equation (60)).

Figure 12 shows the situation when the dominant eigenvalue is positive and greater than 1 but the subdominant eigenvalue is negative and greater than -1. In this example, the dominant eigenvalue of $\mathbf{M}$ is 1.0046, while the subdominant eigenvalue equals -0.5671. As can be seen from the figure, the system oscillates during the first few periods before it settles on a smoother increasing behavior. Note that Manufacturer 1 starts off being a market leader but ends up by being a market follower. Detailed discussions on this behavior

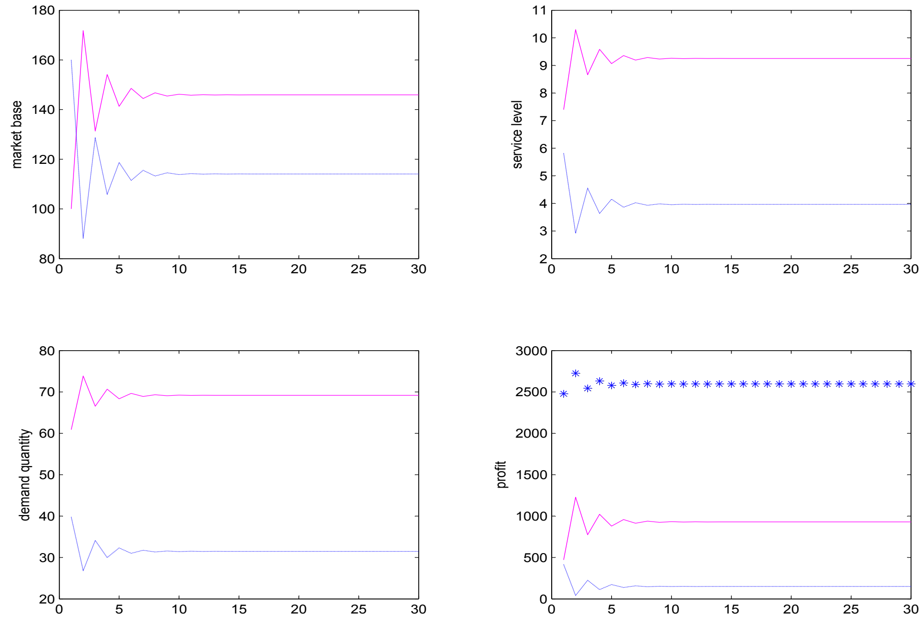

Figure 11: The evolution of equilibrium market bases when $\beta = 0.0, \gamma = 3.8, \sigma = 2.8$, $b_p = b_s = 2, \theta_p = \theta_s = 1.1, a_{1,0} = 100, a_{2,0} = 160, c_1 = 5, c_2 = 15, \eta_1 = \eta_2 = 5$ (Manufacturer 1: Red, Manufacturer 2: Blue, Retailer: *).

of the two manufacturers will be presented in Section 4.5. In the next section, numerical examples from several scenarios are presented. Some observations and managerial insights are then provided.

4.5 Numerical Studies on Special Cases

In this section, we show numerical examples of possible real cases. We follow existing literature (e.g., Tsay and Agrawal (2000) [90] and Vilcassin *et al.* (1999) [91]) in defining the range of some parameters. The range of parameters we use in this section can be found in Appendix B.

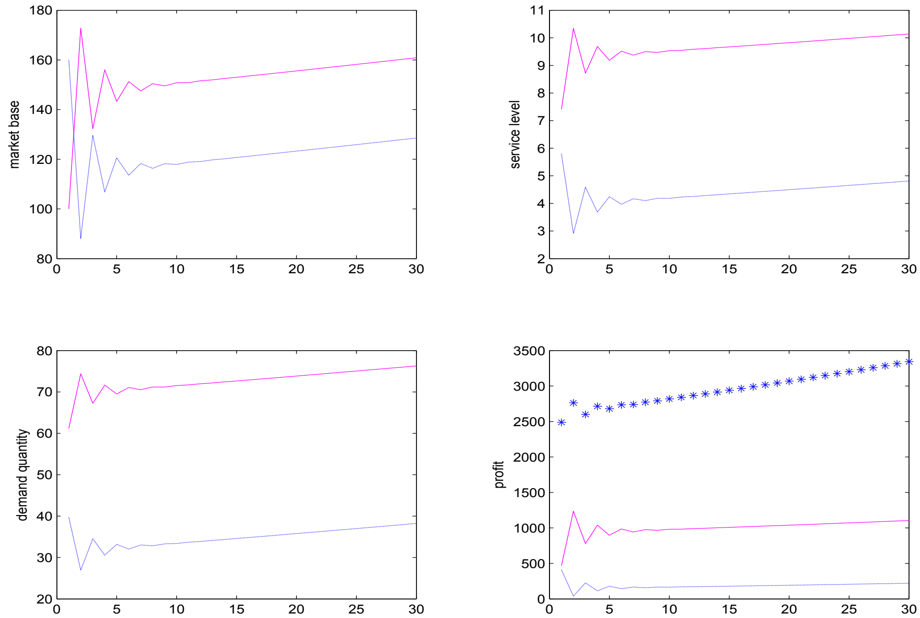

Figure 12: The evolution of equilibrium market bases when $\beta = 0.3, \gamma = 3.8, \sigma = 2.8$, $b_p = b_s = 2, \theta_p = \theta_s = 1.1, a_{1,0} = 100, a_{2,0} = 160, c_1 = 5, c_2 = 15, \eta_1 = \eta_2 = 5$ (Manufacturer 1: Red, Manufacturer 2: Blue, Retailer: *).

4.5.1 No Oscillation

From Equation (58) and (59) and Theorem 4.3, the occurrence of oscillation behavior of market depends on the parameter values. With numerical studies, we observe a range of parameters such that the system smoothly moves to the system equilibrium as defined in Equation (60). The following observation gives the market conditions such that oscillation in market behavior would not occur.

OBSERVATION 4.1. *The oscillation behavior of the supply chain system will not occur if all the following conditions hold:*

(a) $\gamma \leq \max(b_p, \theta_p)$

(b) $\sigma \leq \max(b_s, \theta_s)$

(c) $0.5\sigma \leq \gamma \leq 1.5\sigma$.

This observation states that if values of both γ and σ are not very different from each other and not far different from b_p, θ_p, b_s, and θ_s, market base evolution over time will be smooth (no oscillations). Part (a) and (b) is reasonable and valid in most situations since demand should be more sensitive to current price (service) than last period price (service). To understand part (c), we should examine the case when this condition is not satisfied. If demand is a lot more sensitive to last period price than last period service ($\gamma >> \sigma$), a two-period profit-maximizing manufacturer may sell product at a low price in period t and plan to overprice in period $t+1$. However, when period $t+1$ is reached, the manufacturer will find that it has a smaller market base in period $t+1$ due to overpricing. It then would have to underprice again in period $t+2$ in order to regain the market base loss due to overpricing in period $t+1$. This phenomenon would repeat itself overtime and cause oscillation in market bases, prices, and service levels. A similar situation can occur when demand is more sensitive to last period service than the last period price. Thus, in a situation where demand sensitivities to prices and service levels are not far different from each other, there will be no oscillation in the system.

Now, consider the situations given in Figure 11 to 12. The only difference in parameter values between these situations is the β value. When $\beta = 0$, the investment $I_{i,t}$ will not affect the market base for product i in period t for $i = 1, 2$. Namely,

$$a_{i,t} = a_{i,t-1} - \gamma(p_{i,t-1} - p_{j,t-1}) + \sigma(s_{i,t-1} - s_{j,t-1}) \quad (61)$$

for $i, j \in \{1, 2\}$ and $j \neq i$. This is the case when consumers are not sensitive to the investment made by the manufacturers in current period. Thus, it is not beneficial to the manufacturers to invest in any market expansion activities ($I_{i,t} = 0$ for $i \in 1,2$). On the other hand, if $\beta > 0$, the consumers are sensitive to the market investment made by the manufacturers. Thus, it is always beneficial for the manufacturers to invest some money for market expansion activities in this case. The following observation captures both scenarios.

OBSERVATION 4.2. *The value of consumer sensitivity to market expansion investment (β) determines whether the system is convergent or divergent:*

(i) When $\beta = 0$ there will be no investment to expand market bases in any period and the system will converge.

(ii) When $\beta > 0$ the manufacturers will keep investing in expanding the market bases and the system will diverge.

Figure 9 and 11 shows the situation when $\beta = 0$. They show that the system finally becomes stable. When $\beta > 0$, both manufacturers will keep investing to expand their market bases. In that case, the market base will keep growing as shown in Figure 10 and 12.

From now on, we assume that $\beta > 0$ so that the investment to expand market base by manufacturers will not be zero ($I_{i,t} > 0$ for $i = 1,2$). We also assume the validity of conditions given in Observation 4.1 on the range of γ and σ. This is to prevent oscillations in market bases over time.

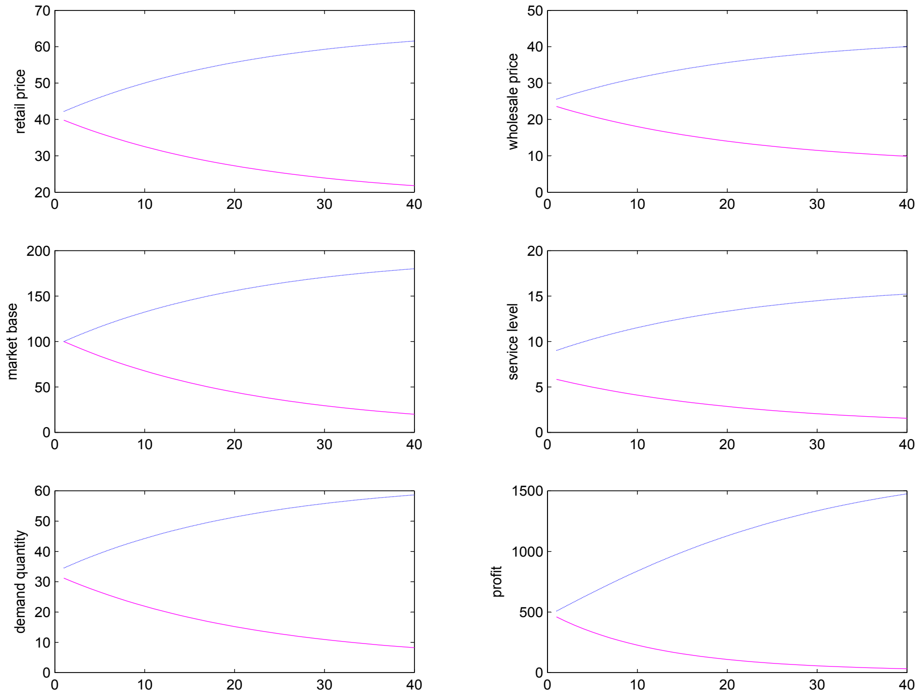

Figure 13: The evolution of equilibrium market bases when $\beta = 0.0, \gamma = 0.6, \sigma = 1.8$, $b_p = b_s = 2, \theta_p = \theta_s = 1.1, a_1 = 100, a_2 = 100, c_1 = 5,\ c_2 = 5, \eta_1 = 7,\ \eta_2 = 5$ (Manufacturer 1: Red, Manufacturer 2: Blue).

4.5.2 Service-Emphasized Market

Figure 13 shows a typical case where demand is more sensitive to last period service than last period price in the learning process[5]. In this case, we find that the firm with service cost advantage will be the winner over time. This result assures the importance of service component in competitions over repeated transactions.

[5]It is unrealistic to consider market with service only sensitivity and ignore the price component all together (i.e., $\beta = \gamma = 0$). At least that is not the product type we are concentrating on with our model.

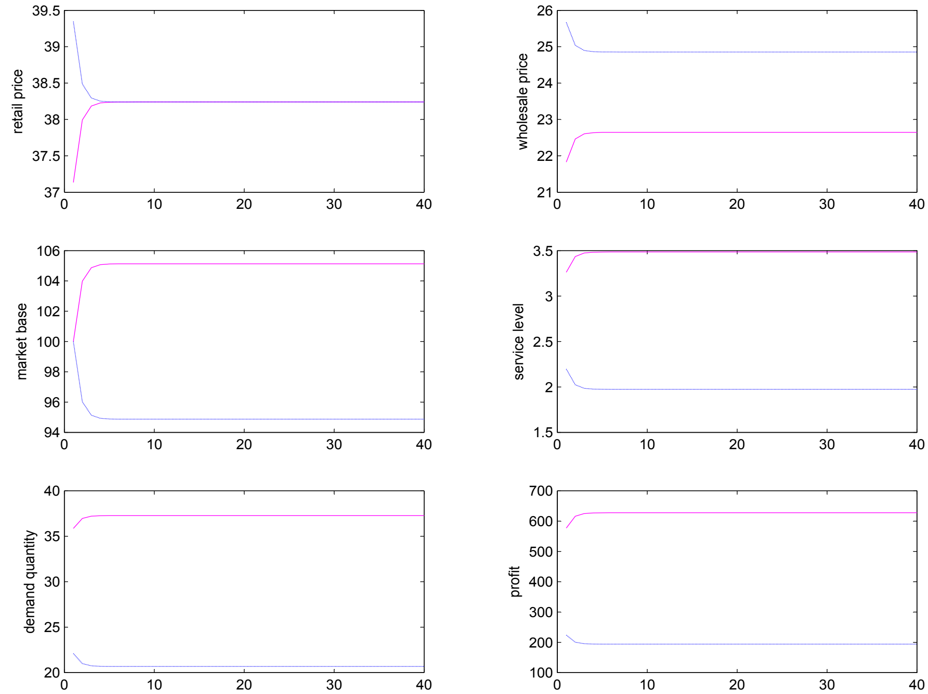

Figure 14: The evolution of equilibrium market bases when $\beta = 0.0, \gamma = 1.8, \sigma = 0.0$, $b_p = b_s = 2, \theta_p = \theta_s = 1.1, a_1 = 100, a_2 = 100, c_1 = 5$, $c_2 = 15, \eta_1 = 5$, $\eta_2 = 5$ (Manufacturer 1: Red, Manufacturer 2: Blue).

4.5.3 Price Sensitive Market

In this example, we investigate the result of a special case where consumers only care about prices from previous period in their learning process. Namely, $\beta = \sigma = 0$ while $\gamma > 0$. Since $\beta = 0$, the system will finally converge to a system equilibrium. In this case, the final retail price of both products will be the same. However, the company with the cost advantage (either production cost or service cost advantage) can afford to sell its product cheaper while providing more service to consumers. This leads to an equilibrium in which the company with the cost advantage gets more demand for its product and earns greater profit. The retailer will sell both products at the same price. The following observation states this result.

OBSERVATION 4.3. *Given that demand is only sensitive to price in its learning process (i.e., $\beta = \sigma > 0$ and $\gamma > 0$), the company with any type of cost advantage will gain more profit and capture a larger market base than its competitor. The retailer will sell both products at the same retail price but the firm with cost advantage will be able to support more service to its customers.*

This situation emphasizes the role of the retailer as a middle man who can control the consumer demand through retail price setting. Since demand is not sensitive to service from the last period, the role of the middle man is highlighted in this case. Figure 14 shows the system dynamics in a typical price-sensitive market.

4.5.4 Identical Manufacturers

In this example, we consider the situation when the two manufacturers are identical in product and service cost. Namely, $c_1 = c_2 = c$ and $\eta_1 = \eta_2 = \eta$. We observe that no matter how different the initial value the manufacturers have for market bases, both products will be sold at the same price with the same level of service provided to the consumers. An example of this scenario is shown in Figure 15.

OBSERVATION 4.4. *If all the costs are the same, the two manufacturers will converge to the same market size and sell their products at the same price, while providing equal level of service to consumers. This happens even though the two products may start with different market bases initially.*

This observation emphasizes the importance of production and service cost components

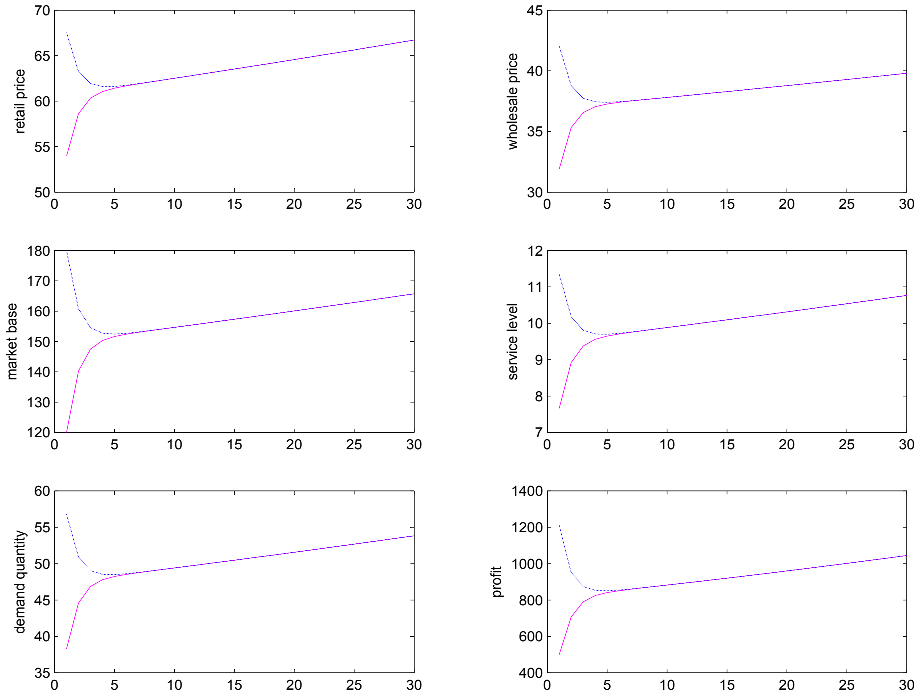

Figure 15: The evolution of equilibrium market bases when $\beta = 0.3, \gamma = 2, \sigma = 2$, $b_p = b_s = 2, \theta_p = \theta_s = 1.1, a_1 = 120, a_2 = 80, c_1 = c_2 = 15, \eta_1 = \eta_2 = 5$ (Manufacturer 1: Red, Manufacturer 2: Blue, Retailer: *).

in competition between the two manufacturers over repeated transactions. If the two manufacturers possess similar underlying production and service capability, initial advantage by either company on the market base vanishes over time.

4.5.5 Production Cost Leader vs. Service Cost Leader

In this example, we study competition between two manufacturers that possess different advantages. One manufacturer, company 1, possesses superior production technology and thus has a lower production cost. The other, company 2, is more efficient in providing service and thus has a smaller service cost coefficient. Thus, $c_1 < c_2$ and $\eta_1 > \eta_2$. We are interested in investigating the extent to which each advantage can help a company to compete with the other.

The following observation states that the company with service cost advantage will always win in the long-run over the company with production cost advantage, no matter how big the production cost advantage or how small the service cost advantage.

OBSERVATION 4.5. *Given that demand is equally sensitive to both price and service level (i.e., $b_p = b_s$, $\theta_p = \theta_s$, $\gamma = \sigma > 0$) and $\beta > 0$, the company with service cost advantage may earn less profit and capture smaller market base in the beginning. However, it will finally gain more profit and capture larger market base than its smaller-production cost competitor. This happens no matter how big the production cost advantage company 1 has over company 2, or how small the service cost advantage company 2 has over company 1.*

This observation emphasizes the importance of service component in competition over the long-run. Figure 16 shows a typical situation in the competition between a production cost leader and a service cost leader. Notice that at the beginning, the service cost leader may have a smaller demand and earn less profit. However, as it keeps increasing service levels to consumers, it can finally win more customers and earn bigger profit than its production cost leader competitor. Note also that the production cost leader company has a bigger initial market base but that still does not change the end result.

4.6 Comparisons with Myopic Model

In this section we compare our results with myopic models. In the model, the two manufacturers and the retailer just try to optimize their single-period profits. We study and compare the results from this myopic model to those from our model. We perform this comparison under two different assumptions on demand. In the first case, demand is

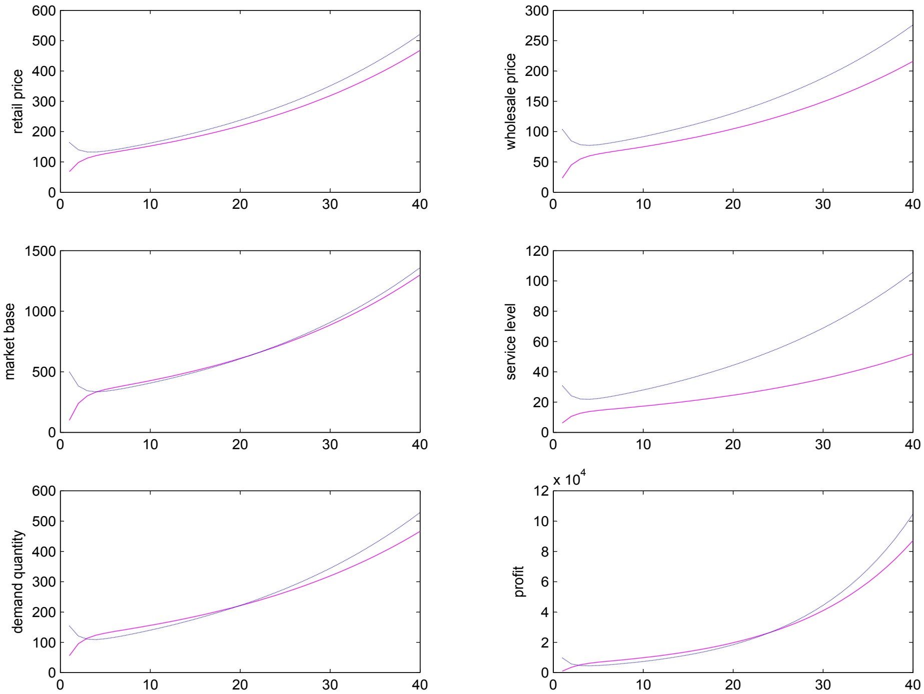

Figure 16: The evolution of equilibrium market bases when $\beta = 0.9, \gamma = 1.8, \sigma = 1.8$, $b_p = b_s = 2, \theta_p = \theta_s = 1.1, a_1 = 100, a_2 = 500, c_1 = 3$, $c_2 = 25, \eta_1 = 9$, $\eta_2 = 5$ (Manufacturer 1: Red, Manufacturer 2: Blue).

memoryless. Consumers in this case do not learn from past experience and only concern about prices and services in current period. For our model, this is a special case when $\beta = \gamma = \sigma = 0$. In the second case, consumers learn from past experience. Thus, demand for product i will depend on prices and service in both the previous and the current periods. To begin our comparison, we first state the equilibrium decisions made by myopic firms.

4.6.1 Myopic Decision Model

Myopic firms optimize profit in the current period only. In comparison to our model, it is as if firms are in the second stage of the two-period profit-optimizing model studied in Section 4.3.1. Therefore, the results from Section 4.3.1 can be applied here. Thus, for any time period t, manufacturers' investment to expand market base within current period can be calculated by

$$\begin{bmatrix} \sqrt{I_{1,t}} \\ \sqrt{I_{2,t}} \end{bmatrix} = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} \begin{bmatrix} a_{1,t-1} \\ a_{2,t-1} \end{bmatrix} + \begin{bmatrix} (\delta_{12}-\delta_{11})\gamma & -(\delta_{12}-\delta_{11})\gamma \\ (\delta_{22}-\delta_{21})\gamma & -(\delta_{22}-\delta_{21})\gamma \end{bmatrix} \begin{bmatrix} p_{1,t-1} \\ p_{2,t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} -(\delta_{12}-\delta_{11})\sigma & (\delta_{12}-\delta_{11})\sigma \\ -(\delta_{22}-\delta_{21})\sigma & (\delta_{22}-\delta_{21})\sigma \end{bmatrix} \begin{bmatrix} s_{1,t-1} \\ s_{2,t-1} \end{bmatrix} + \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}$$

Also, from Section 4.3.1 the wholesale prices, retail prices, service levels, and demand quantities for both products can be calculated by

$$\begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} = \begin{bmatrix} \varphi_1 & \varphi_1 D_2 \\ \varphi_2 D_1 & \varphi 2 \end{bmatrix} \begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix} + \begin{bmatrix} \varphi_1 n_{11} & \varphi_1 n_{12} \\ \varphi_2 n_{21} & \varphi_2 n_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\begin{bmatrix} s_{1,t} \\ s_{2,t} \end{bmatrix} = \begin{bmatrix} l_1 & l_1 D_2 \\ l_2 D_1 & l_2 \end{bmatrix} \begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix} + \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\begin{bmatrix} p_{1,t} \\ p_{2,t} \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix} + \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\begin{bmatrix} Q_{1,t} \\ Q_{2,t} \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix} + \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.$$

All the parameters are as defined previously in Section 4.3.1. We now compare the numerical results from our model and the myopic model.

4.6.2 Myopic Firms with Memoryless Demand

Figure 17 compares the results from our model and those from a model with myopic firms. The market bases are the same for both models and do not change over time since there is no learning demand. The manufacturers do not have to invest since demand is not affected by their investments ($\beta = 0$). It can be seen that the manufacturers' profits are higher in our model. Service levels and prices are also higher in our model, even though demand is smaller. Thus, the manufacturers in our model concentrate on the higher end of the market (high service, high price), where as the manufacturers in myopic model focus on the lower end (low service, low price). This is an important insight for firms in a market where the learning effect from consumers is small. High-end consumers are willing to pay more for higher service level and firms can earn more profits focusing on this group of consumers.

4.6.3 Myopic Firms with Learning Demand

Figure 18 shows the comparison when consumers learn from the past period. This figure shows the major difference between the results from our model and those from a myopic model. In a myopic model, the firms only care about their profits in the current period and ignore any future effects their behavior might cause over time. Thus, they are not capable to cope with the learning consumers. Their markets shrink and they earn less profit over time. On the other hand, our model, with think-ahead firms, can prevent this phenomenon from happening. They plan their actions to take advantage of the learning behavior of demand. The service levels and prices are chosen such that the firms are rewarded by the consumers. Thus, markets keep growing for both products while firms can keep earning more profits.

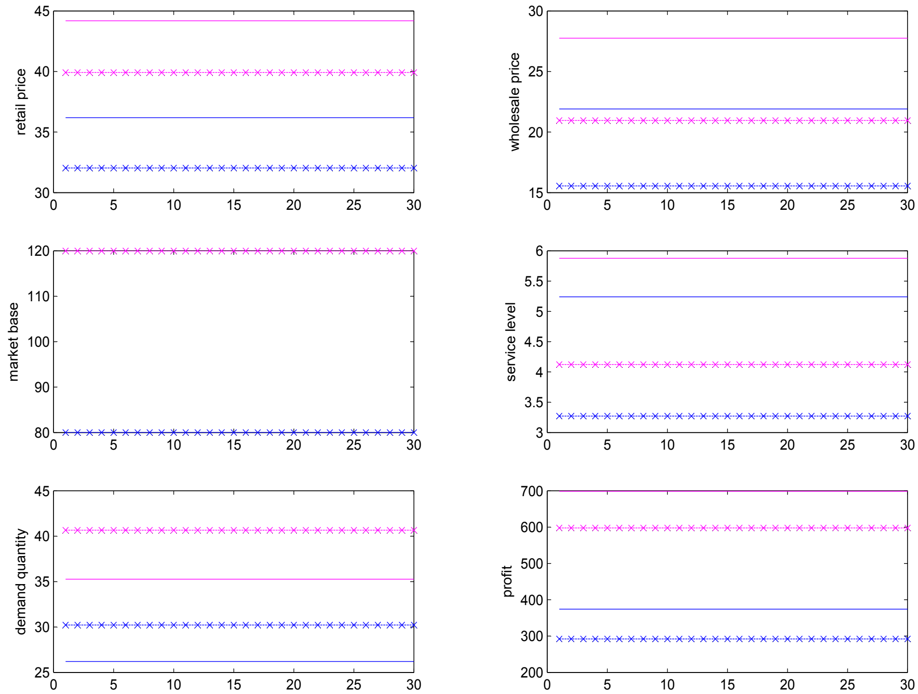

Figure 17: Comparison between Myopic and Two-period profit optimizing model. $\beta = 0, \gamma = 0, \sigma = 0$, $b_p = b_s = 2, \theta_p = \theta_s = 1.1, a_1 = 120, a_2 = 80, c_1 = 5, c_2 = 5, \eta_1 = 6, \eta_2 = 5$ (Myopic: xxx, Two-Period: –, Manufacturer 1: Red, Manufacturer 2: Blue).

4.7 Final Remarks

To develop our multi-period model, we apply both game theory and dynamic systems and control theory to characterize our model. We assume that firms use a moving two-period profit-maximizing strategy. Demand is assumed to have a "learning" capability. Information on the previous period prices and services, as well as manufacturers' investment to expand market bases, can influence market base of each product in current period. Using concepts from dynamic systems and control with numerical studies on some special cases, some managerial insights are obtained.

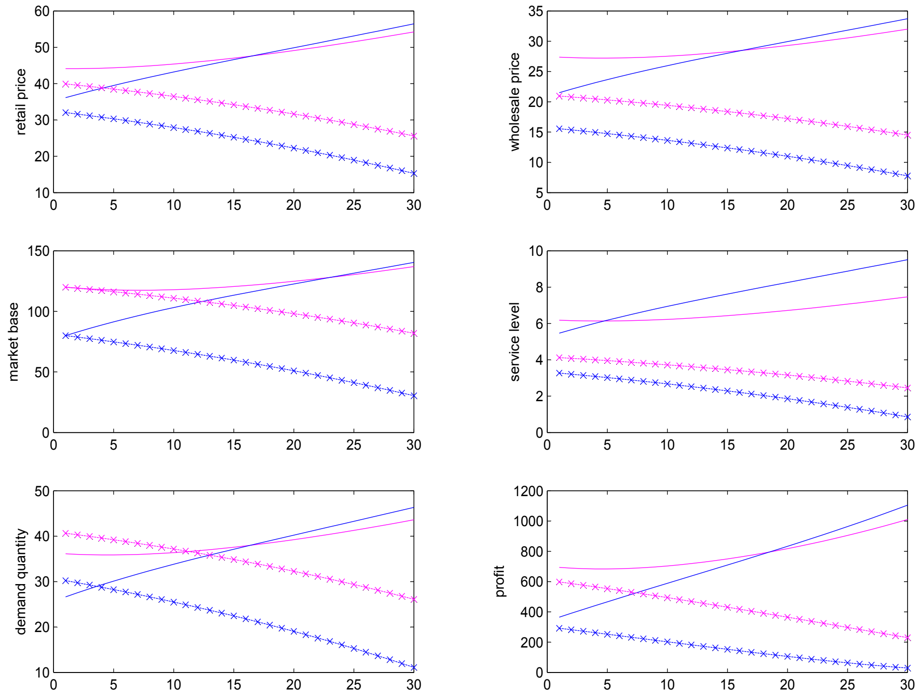

Figure 18: Comparison between Myopic and Two-period profit optimizing model. $\beta = 0.5, \gamma = 0.4, \sigma = 0.3, b_p = b_s = 2, \theta_p = \theta_s = 1.1, a_1 = 120, a_2 = 80, c_1 = 5, c_2 = 5, \eta_1 = 6, \eta_2 = 5$ (Myopic: xxx, Two-Period: –, Manufacturer 1: Red, Manufacturer 2: Blue).

We find that if demand is only sensitive to price in the learning process, the manufacturer with any type of cost advantage will gain more profit and capture a larger market base than its competitor. The retailer will sell both products at the same retail price but the firm with cost advantage will be able to support more service to its customers. Also, if all the costs are the same between two identical manufacturers, they will possess equal market size and sell their products to the same group of customers even though they may initially start with different market bases.

Our main finding is that if demand is equally sensitive to both price and service level, the manufacturer with service cost advantage may earn less profit and capture a smaller market base in the beginning. However, it will finally gain more profit and capture a larger market base than its smaller-production cost competitor. This happens no matter how big the production cost advantage its competitor has, or how small the service cost advantage the manufacturer has over its competitor.

We realize that our assumption on constant unit production cost over time may not be realistic. Other alternatives such as economy-of-scale production cost or decreasing return-to-scale production cost can be explored in the future. These assumptions will affect the pricing behavior of both products over time. In our case, since unit production cost is constant, a firm can increase service levels and keep charging a higher price without worrying much about production cost. Thus, retail price can keep increasing as long as service can make up for the price increase. Other assumptions on production cost are likely to yield different results.

CHAPTER V

COMPETITION IN SUPPLY CHAIN WITH UNCERTAIN DEMAND

5.1 Introduction

In Chapter 3 we studied the supply chain with two manufacturers producing different but substitutable products. These manufacturers sell their products to a common retailer, who in turn, sell the products to the end consumer. We assumed in Chapter 3 that consumer demand for each product is deterministic and is sensitive to two factors: (1) retail price, (2) service provided by the manufacturer. In this chapter, the single-period problem is extended to the case where demand is stochastic. Figure 19 shows the supply chain that will be studied in this chapter.

Only the manufacturers Stackelberg case is considered here. The manufacturers try to maximize their own profits and simultaneously announce wholesale prices and service levels at the beginning of the period. The retailer must decide how much to order from each manufacturer and what the retail price of each product should be. The order quantities from the retailer become demands for the manufacturers to fill. We assume that the manufacturers' production process and the retailer's procurement process have zero lead-time. At the retailer level, products are put into inventory. Demand for each product is then satisfied from the retailer's inventory. At the end of the period, the retailer will either

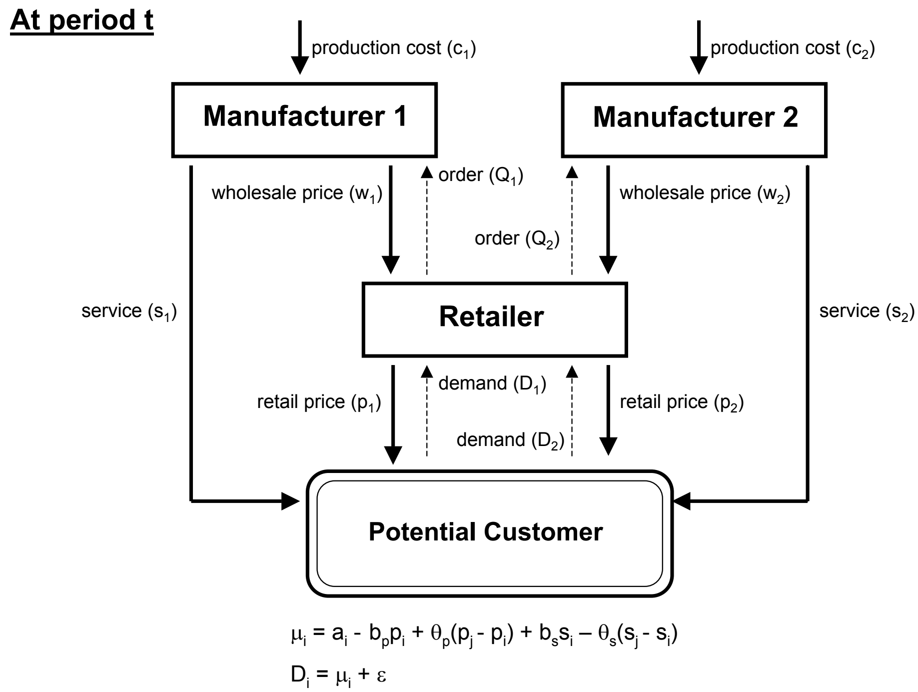

Figure 19: Stochastic Model.

obtain a salvage value for any leftover inventory, or be charged a shortage cost for each unit of unfulfilled demand (i.e., there is no backlogging).

In summary, for each transaction, the overall pricing and ordering decisions in the channel follow the following sequence:

Step 1. Manufacturers simultaneously announce wholesale prices to be offered to the retailer and service levels to be offered to consumers.

Step 2. In response to the manufacturers' announcements, the retailer decides the retail price and ordering quantity of each product that would maximize his expected profit. Retailer's ordering quantities become incoming demands for each manufacturer.

Step 3. Consumer demand for each product is realized.

Step 4. Shortage cost or disposal cost for each product is charged to the retailer, depending on the demand and the stocking level. The manufacturers realize their profit in this transaction.

Figure 20 shows the timeline of events in this model.

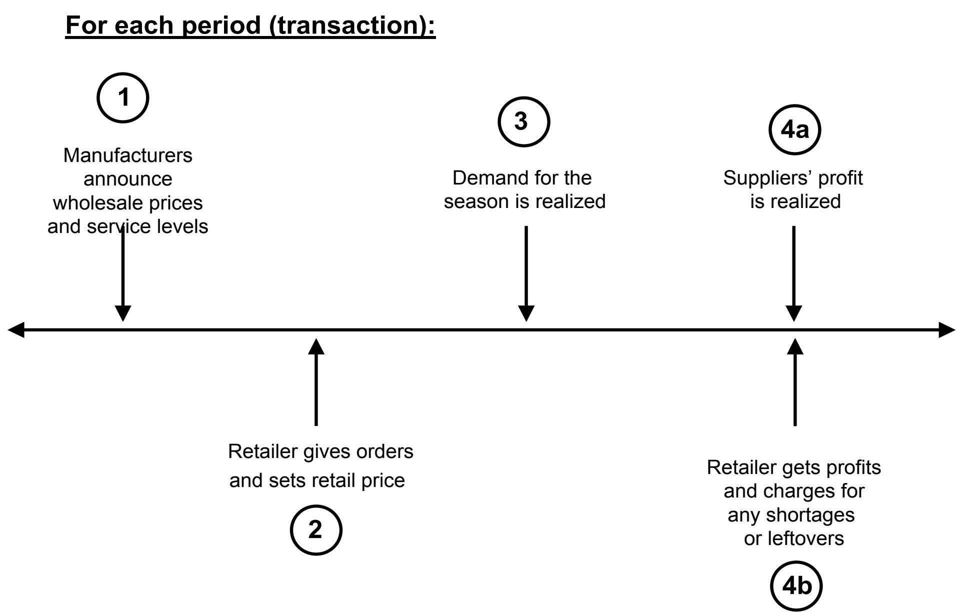

Figure 20: Timeline of events within each transaction.

We assume that each manufacturer has complete information about its competitor and the retailer's cost parameters and also the consumer's demand responsiveness to the retail price. Therefore, for given wholesale prices chosen in Step 1, each manufacturer can anticipate the retailer's response in Step 2. Each manufacturer will take this into account so as to choose the wholesale price and service level in Step 1 to maximize its own profit in Step 4.

We also assume that the retailer, though not knowing the exact demand for each product for a given retail price, knows its distribution. Since demand for each product is stochastic and is price- and service-sensitive, the retailer is facing a two-product newsvendor problem with a joint price-ordering decision in each period. This means the retailer has to take into account the demand sensitivity to the retail price when he determines the retail price and the order quantity for each product[1].

In our model, the consumer demand is sensitive to prices and to service provided by the manufacturers. However, from the retailer's point-of-view, demand is only sensitive to retail price. This is because we are assuming that manufacturers posses more bargaining power and announce their wholesale price and service level before the retailer makes its decision about the retail price and order quantity. The retailer can only react by choosing the retail price and ordering quantity for each product. It is as if he is facing a newsvendor-type problem. In order to study the model, it is important to first study how the retailer would solve his problem.

The two-product newsvendor problem with price-dependent demand distribution will be examined first because it is the problem faced by the retailer in our model. The notations to be used in this chapter will be given. In Section 5.2, the decision faced by every firm in the supply chain (two manufacturers and one retailer) will be analyzed. Specifically, their profit functions will be defined. The demand function will also be specified.

[1]Note that as opposed to the traditional newsvendor model where the retail price is fixed, the retail price of each product is a decision variable of the retailer. This problem was first addressed by Lau and Lau (1988) [45] for a single product newsvendor.

In Section 5.3, the standard newsvendor model and its key results will be presented. Some key literature on the newsvendor problem with price-dependent demand distribution will also be briefly reviewed. A new newsvendor model will be introduced and studied. Conditions that would guarantee the existence of a unique optimal solution is presented. These conditions can be viewed as generalizations of the results in existing literature.

Finally, numerical examples is provided in Section 5.5.

5.2 Supply Chain Model

The supply chain in this case is similar to the one defined in Chapter 3. However, with stochastic demand, the amount of the order the retailer places to each manufacturer is not necessarily the same as the actual demand size. There are some costs associated with the uncertainty in demand. Thus, the retailer's profit is defined slightly differently from the one given in the deterministic case. It is important to define some new variables as well as redefine some of the old variables.

5.2.1 Notations

Let $i = 1, 2$ be the index for the manufacturer/product.

TR_R = Retailer's total revenue.

TR_{M_i} = Manufacturer i's total revenue.

TC_R = Retailer's total cost.

TC_{M_i} = Manufacturer i's total cost.

Π_R = Retailer's profit.

Π_{M_i} = Manufacturer i's profit.

p_i = Retailer's selling price of product i.

w_i = Manufacturer i's selling price to the retailer.

Q_i = Amount bought from manufacturer i by the retailer.

D_i = Total demand of product i by end consumers.

s_i = Service level for product i provided by manufacturer i.

c_i = Per unit production cost of product i as faced by manufacturer i.

g_i = Retailer's unit salvage value of product i's leftover at the end of the season. Note that if this quantity is negative, it can be regarded as the holding cost or the disposal cost.

b_i = Retailer's oppportunity cost of product i. This cost is charged when there is not enough inventory to satisfy the demand.

5.2.2 Retailer's Profit Function

The retailer has two types of revenue for each product. The first type is the revenue received from the sale of each product within the normal transaction (at price p_i). The amount sold is the minimum of Q_i and D_i. The other income is the salvage value of each product, g_i, obtained at the very end of the sale season if there are leftovers, i.e., if $Q_i > D_i$. Note that if g_i is negative, then it stands for the cost of getting rid of the product (disposal cost). Therefore, the retailer's revenue can be expressed as

$$TR_R = \sum_{i=1}^{2} \left\{ p_i min(Q_i, D_i) + g_i(Q_i - D_i)^+ \right\}. \tag{62}$$

There are also two types of cost faced by the retailer. The first one is the cost of acquiring each unit of the product, w_i. This is the same as the wholesale price charged by each manufacturer. The second type of cost is the underage cost, b_i. This cost is charged when there is not enough inventory to meet the realized demand. The retailer's total cost can then be expressed as

$$TC_R = \sum_{i=1}^{2} \left\{ w_i Q_i + b_i (D_i - Q_i)^+ \right\}. \tag{63}$$

From (62) and (63), we can write the retailer's profit function as

$$\Pi_R = \sum_{i=1}^{2} \left\{ p_i min(Q_i, D_i) + g_i (Q_i - D_i)^+ - [w_i Q_i + b_i (D_i - Q_i)^+] \right\}. \tag{64}$$

5.2.3 Manufacturer's Profit Function

We assume that each manufacturer uses a per-unit charge ($w_i Q_i$) for the product sold to the retailer. Therefore, $TR_{M_i} = w_i Q_i$. The types of cost faced by each manufacturer are the cost of producing each unit of product and the costs of providing the service. As done in previous chapters, we make the assumption that the cost of providing service has a decreasing-return-to-scale property. Namely, the cost of providing s_i unit of service is $\frac{\eta_i {s_i}^2}{2}$. This cost function reflects the assumption that it is getting more expensive to provide the next unit of service. Therefore, each manufacturer's profit function can be expressed as

$$\Pi_{M_i} = (w_i - c_i) Q_i - \frac{\eta_i {s_i}^2}{2}. \tag{65}$$

5.2.4 Demand Function

The expected demand can be expressed by

$$\mu_i(p_i, p_j, s_i, s_j) = a_i - b_p p_i + \theta_p (p_j - p_i) + b_s s_i - \theta_s (s_j - s_i) \tag{66}$$

where $a_i, b_p > 0$ and $b_s, b_u, \theta_p, \theta_s, \theta_u \geq 0$, and are defined as in Chapter 3. We can rewrite the above equations in the matrix form as

$$\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} -(b_p + \theta_p) & \theta_p \\ \theta_p & -(b_p + \theta_p) \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \tag{67}$$

$$+ \begin{bmatrix} (b_s + \theta_s) & -\theta_s \\ -\theta_s & (b_s + \theta_s) \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}. \tag{68}$$

In short,

$$\boldsymbol{\mu}(\boldsymbol{p}, \boldsymbol{s}) = \boldsymbol{A} + \boldsymbol{B_p p} + \boldsymbol{B_s s}. \tag{69}$$

We define ϵ_i as a random variable for product i where $i \in 1, 2$. It is defined on the range $[A_i, B_i]$ and has zero expectation. We further assume that ϵ_i is the same for every period. Let $F_i(.)$ represent the cumulative distribution function of ϵ_i, and $f_i(.)$ be the probability density function. Therefore, $\epsilon_i \sim F_i(0, \sigma_i^2)$. With this definition of ϵ_i, we can express the demand function as

$$\begin{aligned} \boldsymbol{D}(\boldsymbol{p}, \boldsymbol{s}, \boldsymbol{\epsilon}) &= \boldsymbol{\mu}(\boldsymbol{p}, \boldsymbol{s}) + \boldsymbol{\epsilon} \\ &= \boldsymbol{A} + \boldsymbol{B_p p} + \boldsymbol{B_s s} + \boldsymbol{\epsilon}, \end{aligned} \tag{70}$$

where $\boldsymbol{\epsilon} = [\epsilon_1 \quad \epsilon_2]^T$.

Note that in the case of a single period problem with deterministic demand and no learning, Equation (69) reduces to

$$\boldsymbol{D}(\boldsymbol{p}, \boldsymbol{s}) = \boldsymbol{A} + \boldsymbol{B_p p} + \boldsymbol{B_s s}. \tag{71}$$

This describes the case studied previously in Chapter 3.

5.3 Two-Product Newsvendor Problem

In our model, the retailer must calculate four decision variables: the order quantities and the retail prices for both products. The retailer is facing the two-product newsvendor problem with a price-dependent demand distribution. This model requires more special treatment than the classical newsvendor model where the demand is independent of the given fixed retail price. The retail price will affect the mean of the demand distribution.

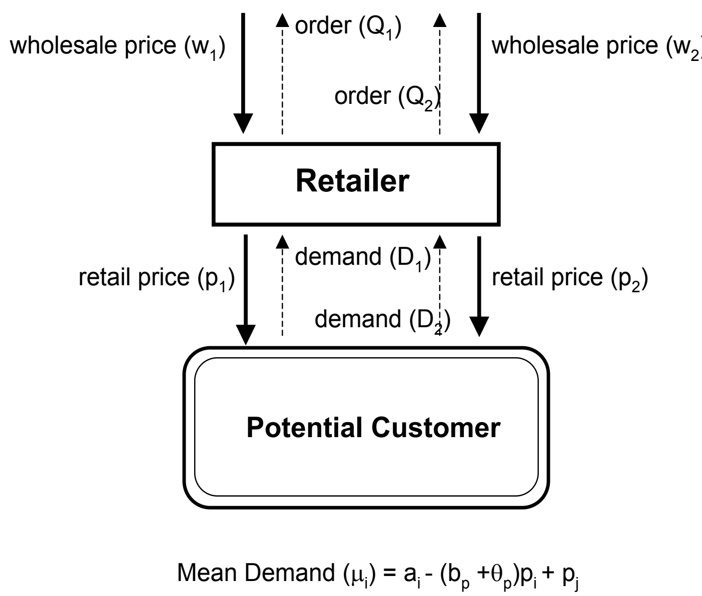

Figure 21: Two-Product Newsvendor Problem .

Although many studies have been developed on the newsvendor problem over the past decade, none have addressed the problem faced by the retailer in our model. The price sensitive version of the newsvendor problem was first formulated by Whitin (1955)[95]. In his model, selling price and stocking quantity are decided simultaneously. Whitin used the newsvendor model where the probability distribution of demand depends on the unit selling price, and where price is a decision variable rather than an external parameter. Mills (1959)[61] refined the formulation by explicitly specifying mean demand as a function of selling price.

The problem of the newsvendor with price-dependent demand was later studied by Lau and Lau (1988)[45] and Petruzzi and Dada (1999). Lau and Lau (1988) considered the problem where the demand distribution is normal with expected demand linearly related to price, and has a constant standard deviation. Petruzzi and Dada (1999) investigated an extension to the problem. Particularly, they studied the problem with both the linear demand function (additive form) and the constant elasticity demand function (multiplicative form). They also gave conditions such that a unique optimal solution exists. However, both Lau and Lau (1988) and Petruzzi and Dada (1999) only considered the case of a single product newsvendor.

5.3.1 Classical (Standard) Newsvendor Model

First, we review the classical newsvendor model where the retail price is given. Let c_u denote the underage cost associated with each demand that cannot be met, and c_o denote the overage cost associated with each newspaper that is not sold. The retailer (newsvendor), facing the uncertain demand D, has to decide the order quantity Q that minimizes his cost. That is, the retailer seeks to minimize expected cost

$$\mathrm{Min} \quad \mathrm{C(Q) = c_u E(D - Q)^+ + c_o E(Q - D)^+}$$

where X^+ denotes the function $max(0, X)$. It is well known that the optimal order quantity Q^* is implicitly determined by the equation (see Silver *et al.* (1998)[78] p. 386)

$$Q^* = F^{-1}\left(\frac{c_u}{c_u + c_o}\right) \tag{72}$$

It can be shown that this optimal solution can also be obtained if one tries to maximize the profit instead of trying to minimize the cost. Let p be the per-unit selling price, w be the per-unit cost of acquiring inventory, g be the per-unit salvage value, b be the per-unit shortage cost, and demand be distributed over the range $[A, B]$ with cumulative distribution function $F(\cdot)$. As a result, we can write the expected profit as

$$E[\Pi] = pEmin[D, Q] - wQ + gE(Q - D)^+ - bE(D - Q)^+. \tag{73}$$

The (expected) marginal cost is given by

$$\begin{aligned} w + b\frac{d}{dQ}E(D - Q)^+ &= w + b\frac{d}{dQ}\left[\int_Q^B xdF(x) - Q(1 - F(Q))\right] \\ &= w + b[-Qf(Q) - (1 - F(Q)) + Qf(Q)] \\ &= w - b(1 - F(Q)). \end{aligned}$$

The (expected) marginal revenue is given by

$$\begin{aligned} \frac{d}{dQ}[pEmin[D, Q] + gE(Q - D)^+] &= p\frac{d}{dQ}\left[\int_A^Q xdF(x) + Q(1 - F(Q))\right] \\ &\quad + g\frac{d}{dQ}\left[QF(Q) - \int_A^Q xdF(x)\right] \\ &= p[Qf(Q) + (1 - F(Q)) - Qf(Q)] \\ &\quad + g[F(Q) + Qf(Q) - Qf(Q)] \\ &= p(1 - F(Q)) + gF(Q) \end{aligned}$$

Equating marginal revenue with marginal cost yields the optimal inventory level, Q^*, as

$$Q^* = F^{-1}\left(\frac{p+b-w}{p+b-g}\right). \tag{74}$$

Now, if we let $p+b-w = c_u$ and $w-g = c_o$, the optimal solution given above will be the same as the one shown in Equation 72.

In the next section, basic theory for the two-product newsvendor problem with a price-dependent demand distribution will be developed.

5.3.2 Model Development

Let the demand of product i be distributed with mean $\mu_i(p_1, p_2)$ and a constant standard deviation σ_i, where $\mu_i(p_1, p_2)$ can be expressed as

$$\mu_i(p_1, p_2) = a_i - b_p p_i + \theta_p(p_j - p_i). \tag{75}$$

Here, the definitions of a_i, b_p, and θ_p are as defined in previous chapters. Namely, a_i can be thought of as a "market base" [90] of product i. As defined in Chapter 3, b_p can be thought of as the measure of the responsiveness of each manufacturer's market demand to his own price, and θ_p is the measure of the sensitivity of the market to the price difference between the two products (*loyalty*).

Let ϵ_i be a random variable defined on the range $[A_i, B_i]$. We also assume that ϵ_i is the same for every period and has zero expectation. Let $F_i(.)$ represent the cumulative distribution function of ϵ_i, and $f_i(.)$ be its probability density function. Therefore, $\epsilon_i \sim F_i(0, \sigma_i^2)$. With the definition of $\mu_i(p_1, p_2)$ and ϵ_i given, we can express the demand for product i as

$$D_i(p_1, p_2) = \mu_i(p_1, p_2) + \epsilon_i. \tag{76}$$

We note that there are four different scenarios for the retailer's profit, as shown in Figure 22.

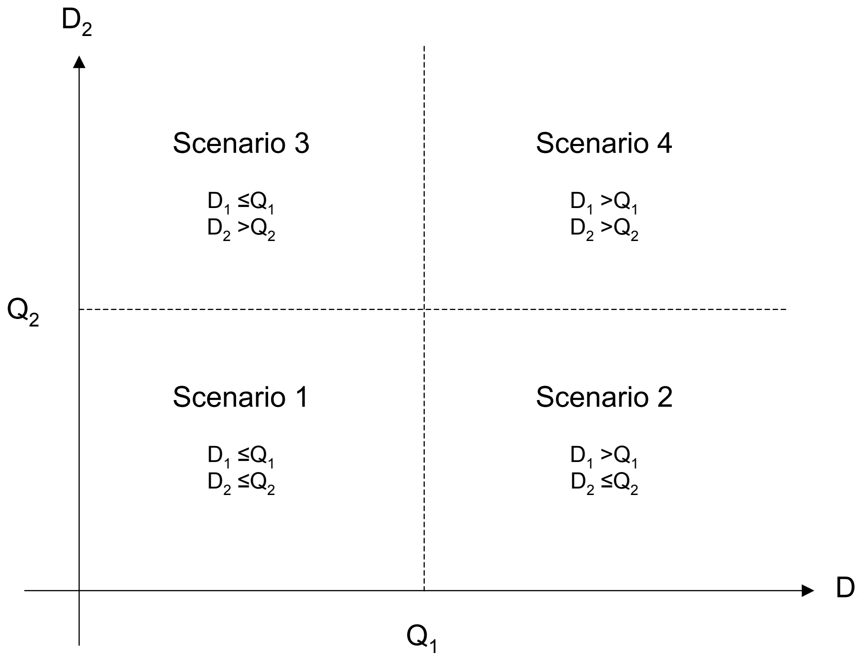

Figure 22: Quantity and Demand.

(1) If $D_1 \leq Q_1$ and $D_2 \leq Q_2$:

$$\begin{aligned} \Pi(Q_1, Q_2, p_1, p_2) &= p_1 D_1(p_1, p_2, \epsilon_1) - w_1 Q_1 + g_1[Q_1 - D_1(p_1, p_2, \epsilon_1)] \\ &+ p_2 D_2(p_1, p_2, \epsilon_2) - w_2 Q_2 + g_2[Q_2 - D_2(p_1, p_2, \epsilon_2)] \end{aligned} \tag{77}$$

(2) If $D_1 > Q_1$ and $D_2 \leq Q_2$:

$$\begin{aligned} \Pi(Q_1, Q_2, p_1, p_2) &= p_1 Q_1 - w_1 Q_1 - b_1[D_1(p_1, p_2, \epsilon_1) - Q_1] \\ &+ p_2 D_2(p_1, p_2, \epsilon_2) - w_2 Q_2 + g_2[Q_2 - D_2(p_1, p_2, \epsilon_2)] \end{aligned} \tag{78}$$

(3) If $D_1 \leq Q_1$ and $D_2 > Q_2$:

$$\begin{aligned} \Pi(Q_1, Q_2, p_1, p_2) &= p_1 D_1(p_1, p_2, \epsilon_1) - w_1 Q_1 + g_1[Q_1 - D_1(p_1, p_2, \epsilon_1)] \\ &+ p_2 Q_2 - w_2 Q_2 - b_2[D_2(p_1, p_2, \epsilon_2) - Q_2] \end{aligned} \tag{79}$$

(4) If $D_1 > Q_1$ and $D_2 > Q_2$:

$$\begin{aligned} \Pi(Q_1, Q_2, p_1, p_2) &= p_1 Q_1 - w_1 Q_1 - b_1[D_1(p_1, p_2, \epsilon_1) - Q_1] \\ &+ p_2 Q_2 - w_2 Q_2 - b_2[D_2(p_1, p_2, \epsilon_2) - Q_2] \end{aligned} \tag{80}$$

Now, consistent with the approach by Thowsen (1975)[86] and Petruzzi and Dada (1999)[68], we define a new decision variable:

$$z_i = Q_i - \mu_i(p_1, p_2) \tag{81}$$

where $\mu_i(p_1, p_2)$ is the expected demand for product i. This transformation of decision variables provides an alternative interpretation of the order quantity (Petruzzi and Dada (1999)): If z_i is larger than the realized value of ϵ_i, then there are leftovers. If z_i is smaller than the realized value of ϵ_i, then the shortage cost is applied. Therefore, we can say that the retailer will not select a value of z_i lower than A_i or greater than B_i because he is sure to lose money. Thus,

$$A_i \leq z_i \leq B_i. \tag{82}$$

One can think of z_i as the *Risky Order Quantity (ROQ)* since it is the amount ordered above the expected demand[2]. Using (81), we can rewrite the conditions in (77) - (80) as:

[2]It can also be interpreted as a surrogate for safety stock [68], since safety stock is defined as the deviation of stocking quantity from expected demand (i.e., safety stock $\equiv Q_i - E[D_i(p_1, p_2, \epsilon)]$.

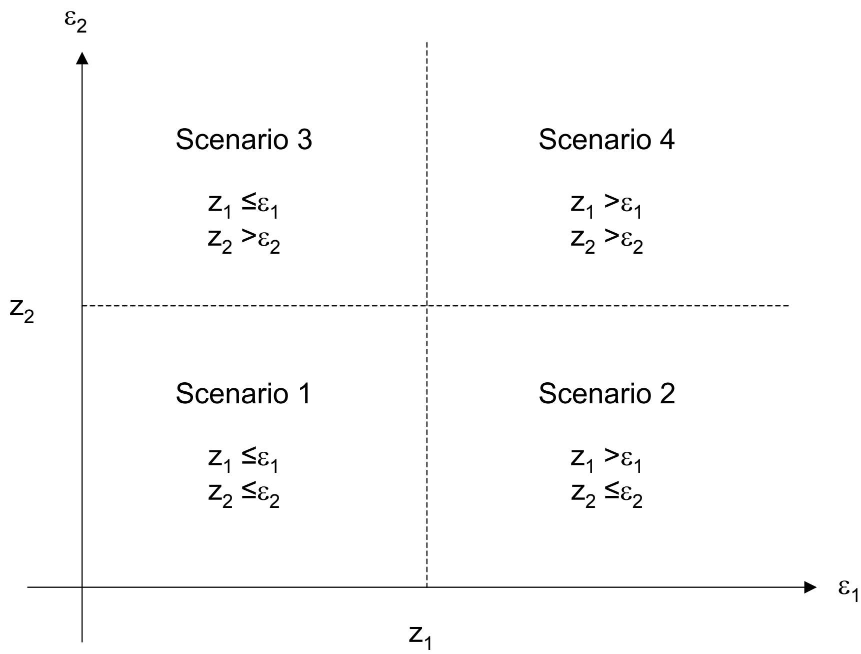

Figure 23: z and e.

(1) If $\epsilon_1 \leq z_1$ and $\epsilon_2 \leq z_2$:

$$\begin{aligned} \Pi(z_1, z_2, p_1, p_2) &= p_1[\mu_1(p_1, p_2) + \epsilon_1] - w_1[\mu_1(p_1, p_2) + z_1] + g_1[z_1 - \epsilon_1] \\ &+ p_2[\mu_2(p_1, p_2) + \epsilon_2] - w_2[\mu_2(p_1, p_2) + z_2] + g_2[z_2 - \epsilon_2] \end{aligned} \tag{83}$$

(2) If $\epsilon_1 > z_1$ and $\epsilon_2 \leq z_2$:

$$\begin{aligned} \Pi(z_1, z_2, p_1, p_2) &= p_1[\mu_1(p_1, p_2) + z_1] - w_1[\mu_1(p_1, p_2) + z_1] - b_1[z_1 - \epsilon_1] \\ &+ p_2[\mu_2(p_1, p_2) + \epsilon_2] - w_2[\mu_2(p_1, p_2) + z_2] + g_2[z_2 - \epsilon_2] \end{aligned} \tag{84}$$

(3) If $\epsilon_1 \leq z_1$ and $\epsilon_2 > z_2$:

$$\begin{aligned} \Pi(z_1, z_2, p_1, p_2) &= p_1[\mu_1(p_1, p_2) + \epsilon_1] - w_1[\mu_1(p_1, p_2) + z_1] + g_1[z_1 - \epsilon_1] \\ &+ p_2[\mu_2(p_1, p_2) + z_2] - w_2[\mu_2(p_1, p_2) + z_2] - b_2[z_2 - \epsilon_2] \end{aligned} \tag{85}$$

(4) If $\epsilon_1 > z_1$ and $\epsilon_2 > z_2$:

$$\begin{aligned}
\Pi(z_1, z_2, p_1, p_2) &= p_1[\mu_1(p_1, p_2) + z_1] - w_1[\mu_1(p_1, p_2) + z_1] - b_1[z_1 - \epsilon_1] \\
&+ p_2[\mu_2(p_1, p_2) + z_2] - w_2[\mu_2(p_1, p_2) + z_2] - b_2[z_2 - \epsilon_2] \qquad (86)
\end{aligned}$$

Using (81) and (83) - (86), we can write the retailer's expected profit as:

$$\begin{aligned}
E[\Pi(z_1, z_2, p_1, p_2)] &= \sum_{i=1}^{2} \left\{ \int_{A_i}^{z_i} \{p_i[\mu_i(p_1, p_2) + x_i] + g_i[z_i - x_i]\} f_i(x_i) dx_i \right. \\
&- w_i[\mu_i(p_1, p_2) + z_i] \\
&+ \left. \int_{z_i}^{B_i} \{p_i[\mu_i(p_1, p_2) + z_i] - b_i[x_i - z_i]\} f_i(x_i) dx_i \right\} \qquad (87)
\end{aligned}$$

We can rearrange the expression (87) into a combination of the riskless profit and the loss due to the risk created by demand uncertainty. This can be accomplished by first defining

$$\begin{aligned}
\underline{\Theta}_i(z_i) &= \int_{A_i}^{z_i} (z_i - x_i) f_i(x_i) dx_i = \text{Expected leftover of } z_i. \\
\overline{\Theta}_i(z_i) &= \int_{z_i}^{B_i} (x_i - z_i) f_i(x_i) dx_i = \text{Expected shortage of } z_i.
\end{aligned}$$

For each i in (87), by adding and subtracting $\int_{z_i}^{B_i} p_i x_i f_i(x_i) dx_i$ and using the fact that the mean of the distribution F(.) is zero, we obtain the following lemma.

LEMMA 5.1. *The expected profit that the retailer receives for each product i is*

$$E[\Pi_i(z_1, z_2, p_1, p_2)] = (p_i - w_i)\mu_i(p_1, p_2) - (w_i - g_i)\underline{\Theta}_i(z_i) - (p_i + b_i - w_i)\overline{\Theta}_i(z_i) \qquad (88)$$

.

Proof: See Appendix C.

The right side of Equation 88 can be separated into two parts. The first term, $(p_i - w_i)\mu_i(p_1, p_2)$ represents the riskless profit function, the profit for a given price when there is no uncertainty in the demand of product i. The last two terms together represent the loss function which takes into account the loss due to overage or underage cost of product i created by the uncertainty in demand (Mills (1959)[61], Silver and Peterson (1985)[77], and Petruzzi and Dada (2001)[69]). From (88), we can rewrite (87) as

$$E[\Pi(z_1, z_2, p_1, p_2)] = \sum_{i=1}^{2} \left\{(p_i - w_i)\mu_i(p_1, p_2) - (w_i - g_i)\underline{\Theta}_i(z_i) - (p_i + b_i - w_i)\overline{\Theta}_i(z_i)\right\} . \tag{89}$$

Now, in order to maximize the retailer's expected profit, we calculate the first and second partial derivatives of $E[\Pi(z_1, z_2, p_1, p_2)]$ with respect to z_1, z_2 and p_1, p_2. Note that $\partial\underline{\Theta}_i(z_i)/\partial z_i = F_i(z_i)$ and $\partial\overline{\Theta}_i(z_i)/\partial z_i = -(1 - F_i(z_i))$. The first order derivatives are

$$\frac{\partial E[\Pi(z_1, z_2, p_1, p_2)]}{\partial z_1} = -(w_1 - g_1) + (p_1 + b_1 - g_1)(1 - F_1(z_1)) \tag{90}$$

$$\frac{\partial E[\Pi(z_1, z_2, p_1, p_2)]}{\partial z_2} = -(w_2 - g_2) + (p_2 + b_2 - g_2)(1 - F_2(z_2)) \tag{91}$$

$$\frac{\partial E[\Pi(z_1, z_2, p_1, p_2)]}{\partial p_1} = a_1 - 2(b_p + \theta_p)p_1 + 2\theta_p p_2 + (b_p + \theta_p)w_1 - \theta_p w_2 - \overline{\Theta}_1(z_1) \tag{92}$$

$$\frac{\partial E[\Pi(z_1, z_2, p_1, p_2)]}{\partial p_2} = a_2 - 2(b_p + \theta_p)p_2 + 2\theta_p p_1 + (b_p + \theta_p)w_2 - \theta_p w_1 - \overline{\Theta}_2(z_2) \tag{93}$$

The second order derivatives are

$$\frac{\partial^2 E[\Pi(z_1, z_2, p_1, p_2)]}{(\partial z_1)^2} = -(p_1 + b_1 - g_1)f_1(z_1), \tag{94}$$

$$\frac{\partial^2 E[\Pi(z_1, z_2, p_1, p_2)]}{(\partial z_2)^2} = -(p_2 + b_2 - g_2) f_2(z_2), \quad \text{and} \tag{95}$$

$$\frac{\partial^2 E[\Pi(z_1, z_2, p_1, p_2)]}{(\partial p_1)^2} = -2(b_p + \theta_p) = \frac{\partial^2 E[\Pi(z_1, z_2, p_1, p_2)]}{(\partial p_2)^2}. \tag{96}$$

Notice from (96) that, for any given z_1 and z_2, $E[\Pi(z_1, z_2, p_1, p_2)]$ is concave in both p_1 and p_2. Therefore, we can first solve for p_1 and p_2 as functions of z_1 and z_2 using (92) and (93). Then, we can substitute these functions into $E[\Pi(z_1, z_2, p_1, p_2)]$ and optimize with respect to z_1 and z_2. Similarly, since (94) and (95) imply that for any given p_1 and p_2, $E[\Pi(z_1, z_2, p_1, p_2)]$ is concave in both z_1 and z_2, so we can optimize z_1 and z_2 for the given p_1 and p_2 using (90) and (91), and then search for the values of p_1 and p_2 that maximize $E[\Pi(z_1, z_2, p_1, p_2)]$. Both procedures yield the same answer (see [69]), but only the first method will be used here. The following proposition gives the expressions for optimal retail price as a function of z_1 and z_2.

PROPOSITION 5.1. *Given z_1 and z_2, the optimal p_i can be calculated by:*

$$p_i{}^* = p_i(z_1, z_2) = p_i^0 - \alpha\overline{\Theta}_i(z_i) - \beta\overline{\Theta}_j(z_j) \tag{97}$$

where $\alpha = \frac{b_p+\theta_p}{2b_p(b_p+2\theta_p)}$, $\beta = \frac{\theta_p}{2b_p(b_p+2\theta_p)}$, $p_i^0 = \frac{(b_p+\theta_p)\xi_i+\theta_p\xi_j}{2b_p(b_p+2\theta_p)}$ *and* $\xi_i = a_i + (b_p + \theta_p)w_i - \theta_p w_j$.

Proof: See Appendix C.

The optimal prices shown in proposition 5.1 can be separated into two parts. The first part, p_i^0, is the riskless price. It is the price that maximizes the riskless part of the Equation (88). The optimal prices, $p_1{}^*$ and $p_2{}^*$, are less than the optimal riskless price due to the risk created by the demand uncertainty for each product. Both Mills (1959) and Petruzzi and Dada (1999) give similar observations for the single product case.

We can now use (97) by substituting them into the profit function to optimize $E[\Pi(z_1, z_2)]$. Attention needs to be paid to the shape of $E[\Pi(z_1, z_2)]$ since the function might have multiple optimal points, or the point that satisfy the first order condition might not be the global optimal, or no optimal point might exist. These scenarios might occur depending on the parameters of the problem. For example, suppose the manufacturers are symmetrical and demand is uniformly distributed. Let b_p =0.8, θ_p =0.2, $w_1 = w_2 = 5$, $g_1 = g_2 = 3$, $b_1 = b_2 = 4$, $a_1 = a_2 = 2$, $B_1 = B_2 = 8$. In such a case, Figure 24 shows that no unique maximum point exists. On the other hand, with the same demand distribution, suppose we let b_p =0.4, θ_p =0.6, $w_1 = w_2 = 5$, $g_1 = g_2 = 3$, $b_1 = b_2 = 4$, $a_1 = a_2 = 20$, $B_1 = B_2 = 2$. Then, we can see from Figure 25 that there exists a unique solution.

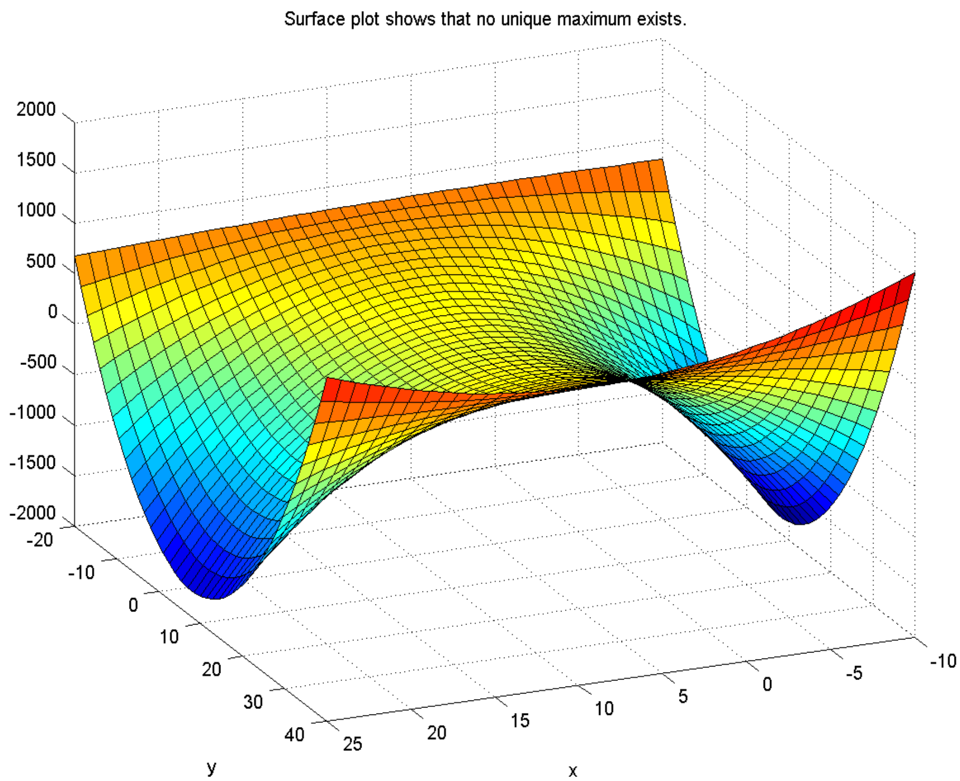

Figure 24: Surface plot shows that no unique maximum exists.

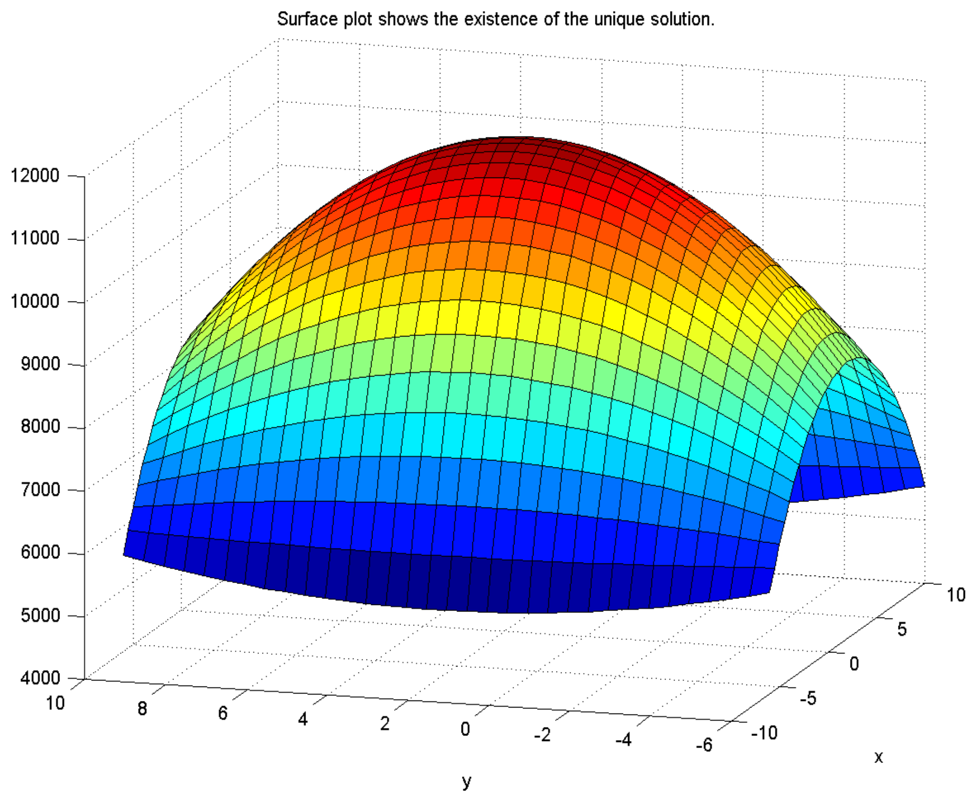

Figure 25: Surface plot shows that a unique maximum point exists.

Therefore, we have to analyze the shape of the function and find the sufficient condition for the existence of a unique solution. The following proposition gives such a condition.

PROPOSITION 5.2. *For ${p_1}^*$ and ${p_2}^*$ given in Proposition 5.1, the newsvendor's optimal ordering quantity for each product i, ${Q_i}^*$, equals $\mu_i({p_1}^*, {p_2}^*) + {z_i}^*$, where each of ${z_1}^*$ and ${z_2}^*$ can be determined according to the following:*

(i) For any arbitrary distribution function $F_i(\cdot)$, ${z_i}^*$ can be found by an exhaustive search over the region $[A_i, B_i]$.

(ii) Let $\phi_i(\cdot) = \frac{f_i(\cdot)}{1-F_i(\cdot)}$ be the hazard rate of the distribution $F_i(\cdot)$. If, for each i,

$$2\phi(z_i)^2 + \frac{d\phi(z_i)}{dz_i} \quad > \quad 0 \tag{98}$$

for $z_i \in [A_i, B_i]$, then the largest $z_i \in [A_i, B_i]$ that satisfies the first order optimality condition is ${z_i}^*$.

(iii) If condition (ii) is met and

$$2b_p(b_p + 2\theta_p)(b_i - w_i) + (b_p + \theta_p)(\xi_i + A_i) + \theta_p(\xi_j - \overline{\Theta}_j(z_j)) > 0$$

for each $i = 1, 2$ and $j = 3 - i$, then $({z_1}^*, {z_2}^*)$ is the unique point in the space $[A_1, B_1] \times [A_2, B_2]$ that satisfies the first-order optimality condition.

(iv) The optimal ${z_i}^*$ must satisfy the following equations:

$$(w_1 - g_1) \quad = \quad \left\{ \frac{(b_p + \theta_p)(\xi_1 - \overline{\Theta}_1(z_1)) + \theta_p(\xi_2 - \overline{\Theta}_2(z_2))}{2[(b_p + \theta_p)^2 - \theta_p^2]} + b_1 - g_1 \right\} (1 - F_1(z_1))$$

and

$$(w_2 - g_2) \quad = \quad \left\{ \frac{\theta_p(\xi_1 - \overline{\Theta}_1(z_1)) + (b_p + \theta_p)(\xi_2 - \overline{\Theta}_2(z_2))}{2[(b_p + \theta_p)^2 - \theta_p^2]} + b_2 - g_2 \right\} (1 - F_2(z_2))$$

where $\xi_i = a_i + (b_p + \theta_p)w_i - \theta_p w_j$.

Proof: See Appendix C.

The above results are generalizations of the results by Petruzzi and Dada (2001)[68]. The differences arise from: (a) the fact that the retailer is carrying two competitive products while Petruzzi and Dada (2001) studied the case with only a single product, and (b) the way we define our demand function (when $\theta_p = 0$, our results reduce to their results). Condition (iii) guarantees that $E[\Pi(\boldsymbol{z}, \boldsymbol{p}(\boldsymbol{z}))]$ is unimodal, provided that $2\phi(z_i)^2 + d\phi(z_i)/dz_i > 0$.

It turns out that all Increasing Failure Rate (IFR) distributions (see Shaked and Shanthikumar (1994)[75]) such as the normal or uniform distribution would satisfy condition (ii). Note that condition (ii), $2\phi(z_i)^2 + d\phi(z_i)/dz_i > 0$, is a necessary condition but not a sufficient condition. Therefore, having a demand with a IFR distribution is not sufficient for a unique solution to exist. Only when condition (iii) is also satisfied would there exist a unique solution.

However, for a general IFR distribution, it is difficult to obtain the closed form solution for condition (iv) in Proposition 5.2. Therefore, in the next section, we will focus on uniformly distributed demand.

5.3.2.1 *EXAMPLE: Uniformly Distributed Demand*

Since the uniform distribution has the IFR property, we can use it in our initial investigation. For $i \in 1, 2$, ϵ_i is distributed over the range $[-B_i, B_i]$ because it is now uniformly distributed and must have a mean of zero. As a result, $f_i(.) = \frac{1}{2B_i}$ and $F_i(x) = \frac{x+B_i}{2B_i}$. The failure (or hazard) rate is calculated to be

$$\phi_i(x) \quad = \quad \frac{f_i(x)}{1-F_i(x)} = \frac{1/2B_i}{1-(x+B_i)/(2B_i)} = \frac{1}{B_i - x}. \tag{99}$$

The expected shortage at z_i, $\overline{\Theta}_i(z_i)$, and the expected leftover at z_i, $\underline{\Theta}_i(z_i)$, can be calculated as (the details of the derivations of the following expressions can be found in Appendix C):

$$\overline{\Theta}_i(z_i)) = \frac{(B_i - z_i)^2}{4B_i} \tag{100}$$

$$\underline{\Theta}_i(z_i)) = \frac{(B_i + z_i)^2}{4B_i} \tag{101}$$

We know that $-B_i \leq z_i \leq B_i$ from (82); (100) shows that the expected shortage of z_i is monotonically decreasing in the range $[-B_i, B_i]$, which is consistent with our intuition. Figure 26 shows such a property for the case where $B_i = 2$.

Using condition (iv) in proposition 5.2, we find that the optimal solution $({z_1}^*, {z_2}^*)$ must satisfy the following condition when both products have uniformly distributed demand.

LEMMA 5.2. *Assuming that both products have uniformly distributed demand, the following condition must be satisfied for each $i \in \{1, 2\}$ and $j = 3 - i$ at the optimal solution:*

$$\begin{aligned} 2b_p(b_p + 2\theta_p)(w_i - g_i) \quad &= \quad \frac{[(b_p + \theta_p)\xi_i + \theta_p \xi_j + 2b_p(b_p + 2\theta_p)(b_i - g_i)]}{2B_i}(B_i - z_i) \\ &- \quad \frac{(b_p + \theta_p)(B_i - z_i)^3}{8B_i^2} - \frac{\theta_p(B_i - z_i)(B_j - z_j)^2}{8B_iB_j} \end{aligned} \tag{102}$$

Proof: See Appendix C.

In the special case where both products have the same parameters, i.e., $g_1 = g_2 = g, b_1 = b_2 = b, B_1 = B_2 = B, \xi_1 = \xi_2 = \xi$, and $w_1 = w_2 = w$, we can further simplify the above lemma. One can think of this as having identical manufacturers, i.e. the two

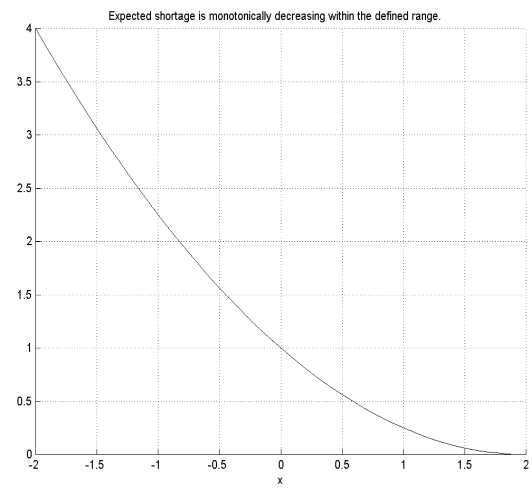

Figure 26: Expected shortage is monotonically decreasing within the defined range.

manufacturers possess the same production technology and produce similar products. In this case, the newsvendor would choose the same decision variable for both products, i.e., $z_1 = z_2 = z$. Therefore, the conditions in Lemma 5.2 can be reduced to:

$$2b_p(w-g) \quad = \quad [\xi + 2b_p(b-g)]\frac{B-z}{2B} - \frac{(B-z)^3}{8B^2} \tag{103}$$

5.4 The Solutions

Now, to analyze the whole supply chain, we work backwards to find the equilibrium solution. First, we find the retailer's reaction function, given the information about the suppliers' action and the demand from the last period. Namely, we express demands Q_1, Q_2 and retail prices p_1, p_2 as a function of wholesale prices w_1, w_2, and service levels s_1, s_2. We calculate the retailer's reaction function using the results from Section 5.3, where we studied the two-product newsvendor problem with a price-dependent demand distribution. We then find the suppliers' optimal action given that they know how the retailer is going to react to their moves.

5.4.1 Retailer's Reaction Function

In Section 5.3 we provide conditions that guarantee the existence of a unique optimal solution. The results can now be applied to find the retailer's reaction function. Since the existence of a unique optimal solution for the retailer is important in gaming if we are to use the concept of pure strategy, we assume that condition (*iii*) in proposition 5.2 is valid from now on.

Note that in this chapter we write μ_i for $i \in \{1, 2\}$ as a function of p_1, p_2, s_1, s_2, as opposed to the expression in Equation (75). This is because, from the point of view of the newsvendor (retailer), s_1 and s_2 are already known by the time of his decision; he only needs to take into account his action (p_1, p_2) on the mean of demand distribution. Thus, the market base defined in Equation (75) equals to $a_i + b_s s_i - \theta_s(s_j - s_i)$ where $i, j \in \{1, 2\}$ and $j \neq i$. To find the retailer's reaction function, we must express z_1 and z_2 in terms of w_1, w_2, s_1 and s_2 using condition (*iv*) given in Proposition 5.2. Namely, we have to simultaneously solve

$$(w_1 - g_1) \quad = \quad \left\{ \frac{(b_p + \theta_p)(\xi_1 - \overline{\Theta}_1(z_1)) + \theta_p(\xi_2 - \overline{\Theta}_2(z_2))}{2[(b_p + \theta_p)^2 - \theta_p^2]} + b_1 - g_1 \right\} (1 - F_1(z_1))$$

and

$$(w_2 - g_2) = \left\{ \frac{\theta_p(\xi_1 - \overline{\Theta}_1(z_1)) + (b_p + \theta_p)(\xi_2 - \overline{\Theta}_2(z_2))}{2[(b_p + \theta_p)^2 - \theta_p^2]} + b_2 - g_2 \right\} (1 - F_2(z_2))$$

where $\xi_i = a_i + (b_p + \theta_p)w_i - \theta_p w_j$. It turns out that such closed forms of ${z_1}^*$ and ${z_2}^*$ can not be easily obtained. However, we can get the values of ${z_1}^*$ and ${z_2}^*$ for any given w_1, w_2, s_1 and s_2 by using numerical analysis. This is done by plotting the "reaction surface" of ${z_1}^*$ and ${z_2}^*$ as a function of w_1, w_2, s_1 and s_2.

In general, given such "reaction surfaces" ${z_1}^*$ and ${z_2}^*$ from the retailer, we can find the corresponding optimal ${p_1}^*$ and ${p_2}^*$ by using the result from Proposition 5.1. The optimal order quantity for each product can then be obtained using Proposition 5.1 and 5.2. The retailer's optimal ordering quantity for each product i can then be calculated to be

$${Q_i}^*(w, s) = \mu_i({p_1}^*, {p_2}^*) + {z_i}^*(w_1, w_2, s_1, s_2) \tag{104}$$

5.4.2 Manufacturers' Problem

Each manufacturer i faces the demand distribution given in (104) and (**??**). His profit function can then be expressed as

$$\Pi_{M_i} = (w_i - c_i)Q_i(w_1, w_2, s_1, s_2) - \frac{\eta_i {s_i}^2}{2}. \tag{105}$$

We assume that both manufacturers have complete access to the information of their competitor and make their moves simultaneously. The Nash Equilibrium will be chosen by each manufacturer, after taking into account the other's decision when choosing wholesale price and service level to maximize his profit. That is, $({w_1}^*, {w_2}^*, {s_1}^*, {s_2}^*)$ is a Nash equilibrium if, for each $i = 1, 2$,

$$\begin{aligned} {w_i}^* &= arg\max_{w_i} \Pi_{M_i}(w_1, {w_2}^*, {s_1}^*, {s_2}^*) \quad \text{and,} & (106) \\ {s_i}^* &= arg\max_{s_i} \Pi_{M_i}({w_1}^*, {w_2}^*, s_1, {s_2}^*). & (107) \end{aligned}$$

Normally, if the first-order condition approach is valid, the optimal wholesale price and service level for each product can be obtained by solving the two sets of first-order optimality conditions simultaneously. Namely, we have for each product i,

$$\begin{aligned} \frac{\partial \Pi_{M_i}}{\partial w_i} = 0 &= Q_i(w_1, w_2, s_1, s_2) + (w_i - c_i)\frac{\partial Q_i(w_1, w_2, s_1, s_2)}{\partial w_i} & (108) \\ \frac{\partial \Pi_{M_i}}{\partial s_i} = 0 &= -\eta_i s_i + (w_i - c_i)\frac{\partial Q_i(w_1, w_2, s_1, s_2)}{\partial s_i} & (109) \end{aligned}$$

To gain the main insights of our model, we assume here that there exists a unique solution set of ${w_1}^*, {w_2}^*, {s_1}^*$, and ${s_2}^*$ that satisfies the first order condition specified above.

5.4.3 Equilibrium Search Algorithm

We can see from the previous section that the retailer's reaction function can not be easily obtained as a closed form function of w_i and s_i for $i = 1, 2$. Therefore, we propose the following simple algorithm for the manufacturers to calculate their optimal wholesale price and service level.

ALGORITHM 5.1. *For a single period problem and a given IFR demand distribution,*

(Step 0) Set W_i^L and S_i^L to be 0 and W_i^U and S_i^U to be some positive numbers such that $W_i^L << W_i^U$ and $S_i^L << S_i^U$.

(Step 1) For each i, discretize values of w_i and s_i for $W_i^L < w_i < W_i^U$ and $S_i^L < s_i < S_i^U$ into N points.

(Step 2) *Calculates the value of* z_1 *and* z_2 *that satisfy the following equations for each combination of* w_1, w_2, s_1, s_2.

(i) $2b_p(b_p + 2\theta_p)(b_i - w_i) + (b_p + \theta_p)(\xi_i + A_i) + \theta_p(\xi_j - \overline{\Theta}_j(z_j)) > 0$

(ii) $(w_i - g_i) = \left\{ \frac{(b_p+\theta_p)(\xi_i - \overline{\Theta}_i(z_i)) + \theta_p(\xi_j - \overline{\Theta}_j(z_j))}{2[(b_p+\theta_p)^2 - \theta_p^2]} + b_i - g_i \right\} (1 - F_i(z_i))$

(iii) $A_i \leq z_i \leq B_i$.

(Step 3) *From the surface of* ${z_1}^*$ *and* ${z_2}^*$ *obtained in Step 1, use the results from Proposition 5.3 and 5.2 to find the corresponding surface of* ${p_1}^*$, ${p_2}^*$, ${Q_1}^*$, *and* ${Q_2}^*$.

(Step 4) *Using* ${Q_1}^*$ *and* ${Q_2}^*$ *obtained in the previous step, each manufacturer can calculate his profit(*Π_{M_i}*) by using Equation (105).*

(Step 5) *From the profit surface (*Π_{M_i}*) obtained in the last step, find the optimal* ${w_i}^*$ *and* ${s_i}^*$ *that maximize the profit, where*

$$\begin{aligned} {w_i}^* &= arg \max_{w_i} \Pi_{M_i}(w_i, {w_j}^*, {s_i}^*, {s_j}^*) \quad and, \\ {s_i}^* &= arg \max_{s_i} \Pi_{M_i}({w_i}^*, {w_j}^*, s_i, {s_j}^*). \end{aligned}$$

(Step 6) *If for each* Π_{M_i}, $\frac{\partial \Pi_{M_i}}{\partial w_i}\Big|_{{w_i}^*}$, $\frac{\partial \Pi_{M_i}}{\partial w_j}\Big|_{{w_j}^*}$, $\frac{\partial \Pi_{M_i}}{\partial s_i}\Big|_{{s_i}^*}$, *and* $\frac{\partial \Pi_{M_i}}{\partial s_j}\Big|_{{s_j}^*}$ *are all less than* δ, *Stop. Else, for each* i, *set*

$$\begin{aligned} W_i^L &= {w_i}^* - \frac{W_i^U - W_i^L}{N}, \\ W_i^U &= {w_i}^* + \frac{W_i^U - W_i^L}{N}, \\ S_i^L &= {s_i}^* - \frac{S_i^U - S_i^L}{N}, \\ S_i^U &= {s_i}^* + \frac{S_i^U - S_i^L}{N} \end{aligned}$$

and go to Step (1).

In this algorithm, we start from a wide range of wholesale prices and service levels and narrow down to find the optimal solution. We stop when the slope of the profit surface is less than δ, where δ can be chosen to be a very small number (ideally, we want to stop at the point where the slope is zero).

5.5 Identical Duopolists with Uniformly Distributed Demand

In this section, we give a numerical example and study the effects of each parameter on the optimal solution. We focus on the case where demand is uniformly distributed. We also assume that the two manufacturers are identical. This means that they both possess the same technology and skills and have the same market base ($a_1 = a_2$). In Section 5.3 we developed some calculations for the uniformly distributed demand. In this section, we will use the calculations in examples involving identical manufacturers. We know from Lemma 5.2 that, for identical duopolists, the following equation must be satisfied

$$2b_p(w-g) \;=\; [\xi + 2b_p(b-g)]\frac{B-z}{2B} - \frac{(B-z)^3}{8B^2}. \tag{110}$$

However, we know from Proposition 5.1 and equation (**??**) that, for identical duopolists, ξ^t in the above equation can be written as

$$\xi \;=\; a + b_p w + b_s s.$$

Therefore, we can rewrite equation (110) as

$$2b_p(w-g) \;=\; [a + b_p w + b_s s + 2b_p(b-g)]\frac{B-z}{2B} - \frac{(B-z)^3}{8B^2} \tag{111}$$

where we define $u^0 = 0$.

Let $b_p = 0.7, \theta_p = 0.2, b_s = 0.8, \theta_s = 0.2, a_1 = a_2 = 40, b_1 = b_2 = 2, g_1 = g_2 = 4, c_1 = c_2 = 5, \eta_1 = \eta_2 = 2$, and $B_1 = B_2 = 15$. From section 5.3, we know that deriving a closed form solution for ${z_1}^*$ and ${z_2}^*$ for a general demand distribution is very difficult. In Lemma 5.2, the necessary condition for the optimal solution to uniformly distributed demand is given. In such a case, solving for the closed form solution of z_i is still not a simple task. Even in the special case where the two manufacturers are identical, we have the following two conditions that must be satisfied simultaneously

$$1) \qquad 2b_p(w-g) = [a + b_p(w + 2b - 2g) + b_s s]\frac{B-z}{2B} - \frac{(B-z)^3}{8B^2} \tag{112}$$

$$2) \qquad -B \leq z \leq B \tag{113}$$

where the first condition follows from Lemma 5.2 and the second condition follows from the definition of z. Note that since the two manufacturers are symmetrical, their wholesale price and service level will be equal. Therefore, $w_1 = w_2 = w$ and $s_1 = s_2 = s$. Also, $z_1 = z_2 = z$, which can be derived using Equations 112 and 113. This scenario gives an example of the "reaction surface" for the case of identical manufacturers with uniformly distributed demand.

Figure 27 gives the surface of the optimal z^* for each value of w and s obtained by using Equation (103). The surfaces for p^* and Q^* are derived from the optimal z^* and are given in Figures 28 and 29, respectively.

Figure 28 shows that at any wholesale price, the higher the level of service the manufacturers provide, the higher the price the retailer can charge. This shows that services add value to the customers who are more willing to pay a higher price for the product with a higher level of service. Figure 28 also shows that, at any level of service, the higher the wholesale price, the higher the retail price. This is due to double marginalization. Therefore, the retail price is highest when the manufacturers set a high wholesale price and also provide a high level of service. The opposite is true when the retail price is lowest.

Figure 29 shows that the ordering quantity is inversely related to the wholesale price and is directly related to the level of service. We see that the ordering quantity is lowest when the wholesale price is at its highest and the level of service is at its lowest. The opposite happens when the price is low and the level of service is high. This behavior is consistent with our intuition.

From the figures, we can compare and see the common relationship between z^*, p^*, and Q^*. When the price p^* is high (in other words, when w and s are high), z^* and Q^* are low.

Using Algorithm 5.1 defined previously, we can obtain the manufacturers' profit. Figure 30 shows the surface of the the manufacturers profit. The values of w^* and s^* turn out to be 26.9744 and 12.5256, respectively. The corresponding optimal manufacturer profit is 295.9627. Using Equation (103), we calculate z to be -0.5444. Using Propositions 5.1 and 5.2, we calculate the optimal retail price, p^*, and optimal ordering quantity, Q^*, to be 40.3396 and 17.0384 respectively. The corresponding expected retailer's profit is 358.5509[3]. The following table compares these results with those obtained from the deterministic demand model (with the same parameters) in Chapter 3.

As can be seen from the table, to cope with uncertainty, the retailer must order a larger quantity to avoid shortage cost. At the same time, the retail and the wholesale prices are chosen to be higher to extract more revenue from each unit sold. However, the

[3]From an experiment with 50 runs

Parameter	Deterministic	Stochastic
Wholesale Price ($w_1 = w_2$)	23.2439	26.9744
Service Level ($s_1 = s_2$)	2.7366	12.5256
Manufacturer Profit ($\Pi_{M_1} = \Pi_{M_2}$)	266.1563	295.9627
Retail Price ($p_1 = p_2$)	41.7571	40.3396
Ordering Quantity ($Q_1 = Q_2$)	14.9993	17.0384
Retailer Profit (Π_R)	479.8361	358.5509 (expected)

Table 6: Comparing results from the deterministic demand case and stochastic demand case when both manufacturers are symmetrical.

manufacturers have to provide more services to attract more potential customers. The manufacturers earn higher profit in the stochastic case while the retailer earns less. This is because the retailer has to order more to avoid a shortage penalty, while being exposed to the possibility of being able to sell less than the expected value. The fact that manufacturers have more power (first mover advantage) also gives them the advantage over the retailer when there is uncertainty in demand. Moreover, manufacturers have control over service levels, which can influence consumer demand as well.

5.6 Final Remarks

In this chapter, we first study the two-product newsvendor problem with price-dependent demand. We find that there can exist many optimal solutions within the defined range. Therefore, the first and second optimality condition would not give the unique solution. We provide the conditions such that a unique optimal solution exists, and give some sample calculations for the uniformly distributed demand case.

We then consider the whole supply chain by including the two manufacturers into the model. We focused on the Manufacturer Stackelberg case. With the model in place, we used the results from Section 5.3 to find the retailer's reaction function. We then proposed an algorithm for the manufacturers to find the equilibrium wholesale price and service level, given that they can anticipate the retailer's reaction function. Finally, a numerical example is provided for the case where the two manufacturers are symmetrical and the demand is uniformly distributed.

In our model, we assume that there is no delivery time lag, that the leftover from one period cannot be carried over to the next period, and that the excess demand is lost. The extension to relax these assumptions is possible in the future.

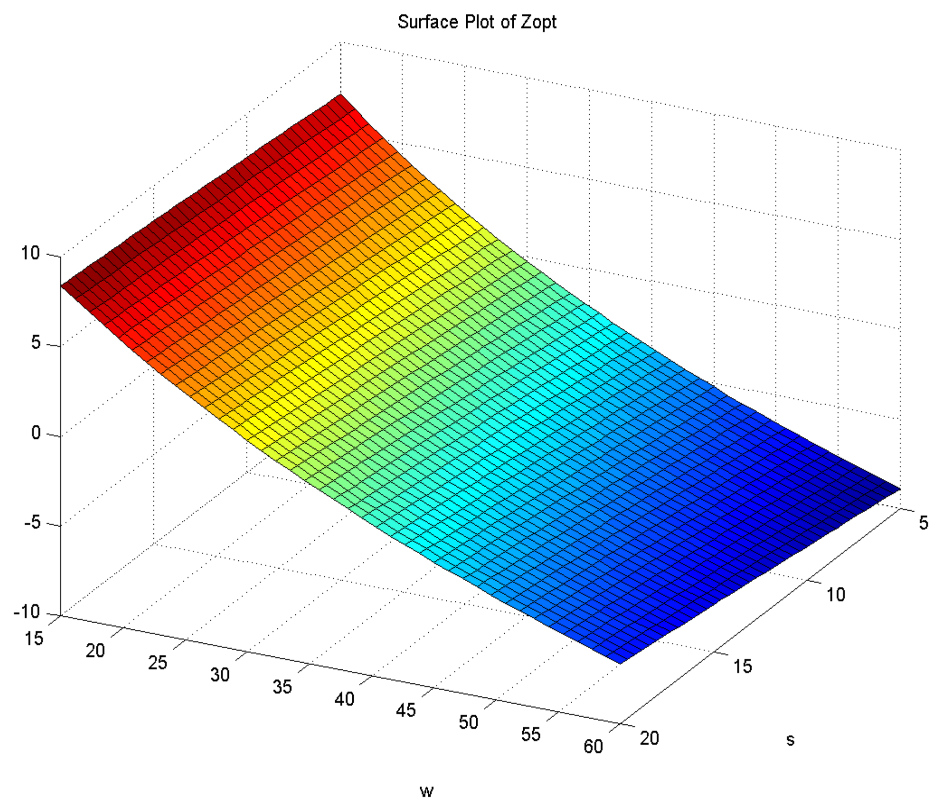

Figure 27: Optimal z as a function of w and s in the first period for the case of identical duopolists and uniformly distributed demand.

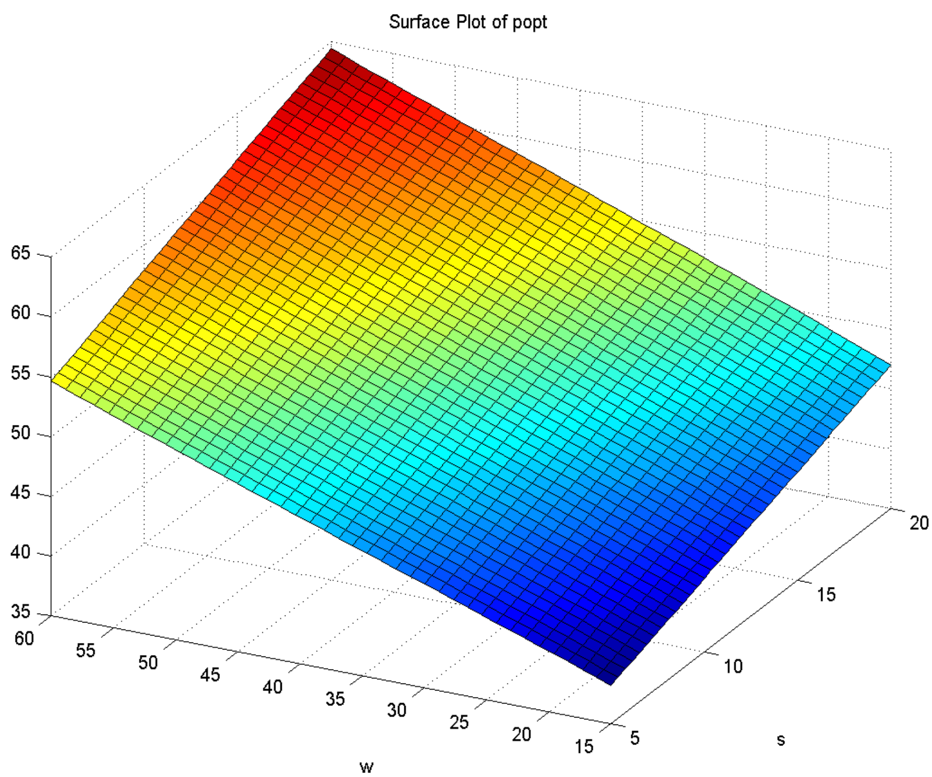

Figure 28: Optimal p as a function of w and s in the first period for the case of identical duopolists and uniformly distributed demand.

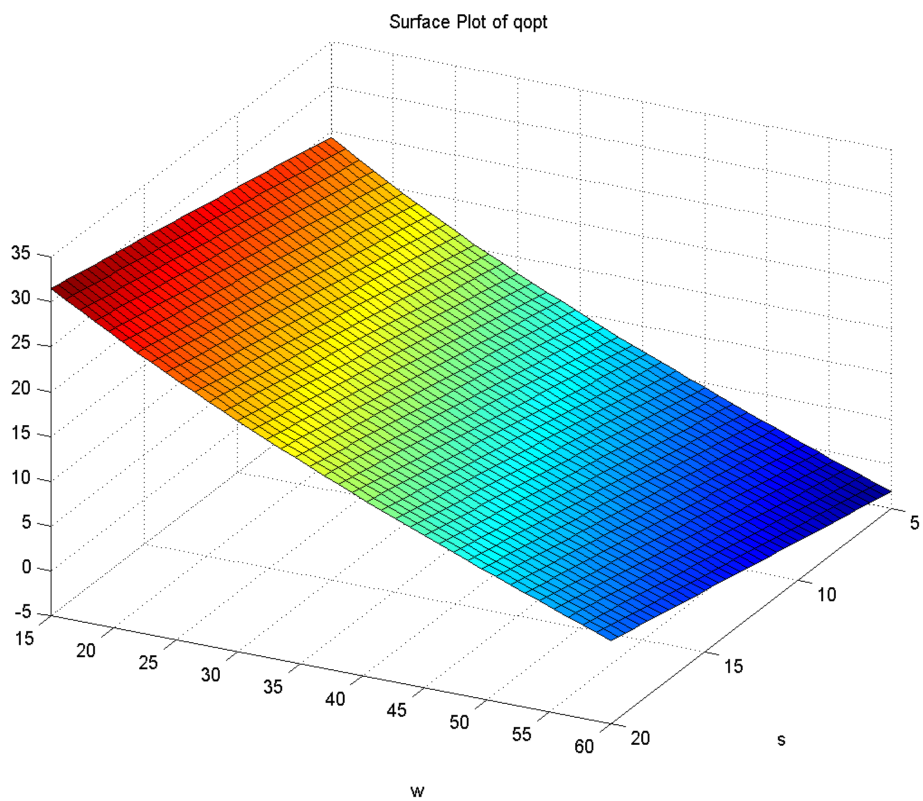

Figure 29: Optimal Q as a function of w and s in the first period for the case of identical duopolists and uniformly distributed demand.

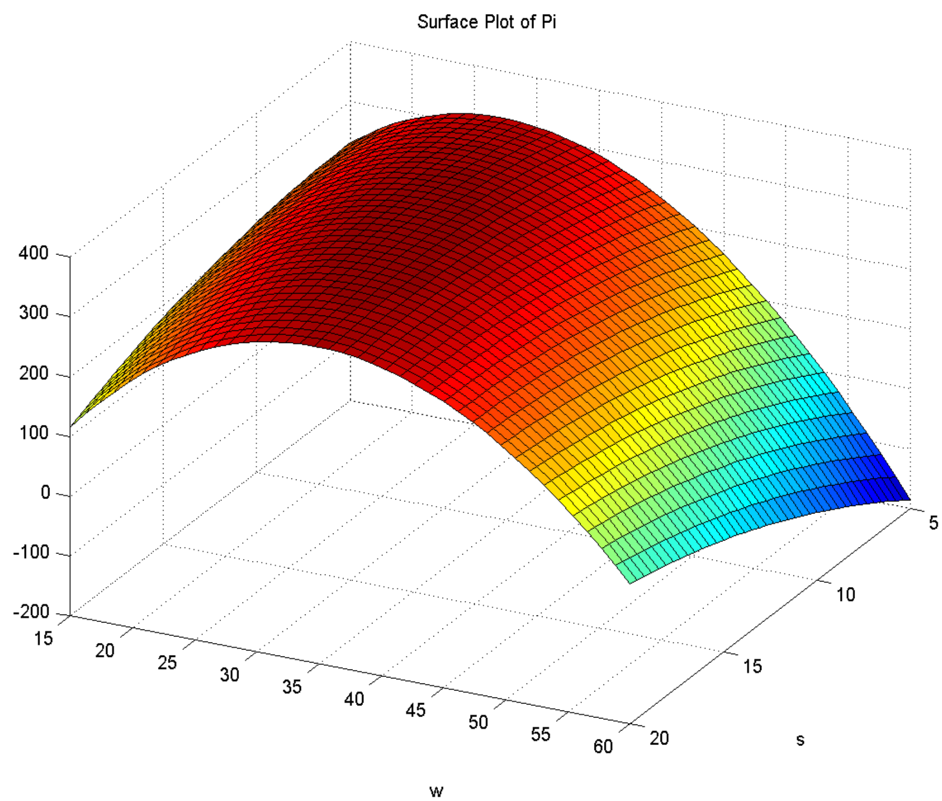

Figure 30: Optimal manufacturer profit as a function of w and s in the first period for the case of identical duopolists and uniformly distributed demand.

CHAPTER VI

SUMMARY AND FUTURE RESEARCH

6.1 Summary

This thesis aims at developing and analyzing models using techniques from various fields to gain some insights in the area of supply chain management. More specifically, we focus on a two-stage supply chain with two manufacturers and one common retailer. Following an introduction in Chapter 1 and a literature review in Chapter 2, we presented an analysis of the deterministic demand case in Chapter 3.

In Chapter 3, we extended the model from existing literature by including service from the manufacturers to consumers. We studied how different assumptions on bargaining power between retailer and manufacturers influence their strategic interactions at equilibrium. We also investigate how parameters associate with the supply chain (such as market base, market sensitivities, and production cost) can affect the equilibrium solution. We analyze the effects through parameters such as the retail price, wholesale prices, service levels, retailer's ordering quantities, and profits. We found that it is more beneficial to consumers when there is no dominant player(s) in vertical strategic interaction. In such case, the consumers receive more manufacturer service and can buy product at a lower price. A counterintuitive result shows that as the market base of one product increases, the competitor also benefits but at a lesser extent. Furthermore, when one manufacturer has economic advantage in providing service, the retailer will act to separate market segment by selling the product with low service at a low price and selling the product with high service at a high price.

Chapter 4 studies the model over multiple periods with demand learning. The learning process assumed in demand function is to capture how experience from past interactions influences customer demand in the future. Information on the previous period prices and services, as well as manufacturers' investment, can influence the market size of each product in the current period. We apply both game theory and dynamic systems and control theory to characterize our model. We assume that firms use a moving two-period profit-maximizing strategy. Using concepts from dynamic systems and control theory with numerical studies on some special cases, managerial insights are obtained.

We find that if all the costs are the same between two identical manufacturers, they will eventually possess equal market size and sell their products to the same group of customers even though they may start with different market bases initially. Our main finding is if demand is equally sensitive to both price and service level, the company with service cost advantage may earn less profit and capture a smaller market base in the beginning. However, it will finally gain more profit and capture a larger market base than its smaller-production cost competitor. This happens no matter how big the production cost advantage its competitor has, or how small the service cost advantage the company has over its competitor.

In Chapter 5, we first study the two-product newsvendor problem with price-dependent demand. We find that there can exist many optimal solutions within the defined range. Therefore, the first and second optimality condition would not give the unique solution. We provide the conditions such that a unique optimal solution exists, and give some sample calculations for the uniformly distributed demand case.

We then consider the whole supply chain by including the two manufacturers into the model. We focused on the Manufacturer Stackelberg case. With the model in place, we used the results from Section 5.3 to find the retailer's reaction function. We then proposed an algorithm for the manufacturers to find the equilibrium wholesale price and service level, given that they can anticipate the retailer's reaction function. Finally, a numerical example is provided for the case where the two manufacturers are symmetrical and the demand is uniformly distributed.

6.2 Future Research Plan

One possible extension is to compare our model to the supply chain with a centralized planner - a single firm with the capability to produce both products to meet the uncertain consumer demand. Explorations on different payment schemes to induce the system-optimal solutions (e.g., two-part tariff, or other type of payment contract) can be carried out. Expected contribution here will be an important inclusion to the existing literature on channel coordination (see Cachon (2001) [10] for reviews).

Another interesting extension would be to study the n-product newsvendor model with price-dependent demand. This would generalize the results from Chapter 5. This extension would contribute to the literature on the newsvendor model.

It is also possible to investigate and compare our results to those from models with other forms of demand function. In our model, we used linear additive demand function. Some of the possible alternatives are to use exponential demand functions. One particular interesting question is whether these changes would have any major influence to the results we derived.

APPENDIX A: APPENDIX FOR CHAPTER 3

A.1 Proof to Proposition 3.1 (Supplier Stackelberg)

I first study the equilibrium in this game. In this Supplier Stackelberg game, each supplier first simultaneously announces his price and service level. The retailer observe the prices and service levels and then decides the prices he is going to charge for each product.

Consumer demand for product i, q_i, is

$$Q_i(p_i, p_j, s_i, s_j) = a_i - b_p p_i + \theta_p(p_j - p_i) + b_s s_i - \theta_s(s_j - s_i)$$

Here, b_s and b_p measure the responsiveness of each manufacturer's market demand to its own price and service, respectively. On the other hand, θ_s and θ_p measure the *loyalty* of the market. Namely, when the price of product i is decreased by one unit, it will gain $b_p+\theta_p$ more customers. Among these customers, θ_p of them are switching from its competitor's product while b_p of them are the direct result of bigger market demand due to smaller price.

Supplier i's profit function is

$$\Pi_{M_i} = (w_i - c_i)Q_i - \frac{\eta_i s_i^2}{2}$$

for $i \in \{1, 2\}$. Note that the quadratic function is used here to reflect the diminishing return on investment in providing services.

Retailer's profit is

$$\Pi_R = (p_1 - w_1)Q_1 + (p_2 - w_2)Q_2$$

To solve this problem, we work backwards in time (a standard approach in solving Stackelberg game). We first look at the retailer's reaction function after he has the information about prices and service levels announced by the suppliers. The retailer's profit function can be expressed as:

$$\begin{aligned}\Pi_R &= (p_1 - w_1)[a_1 - b_p p_1 + \theta_p(p_2 - p_1) + b_s s_1 - \theta_s(s_2 - s_1)] \\ &\quad +(p_2 - w_2)[a_2 - b_p p_2 + \theta_p(p_1 - p_2) + b_s s_2 - \theta_s(s_1 - s_2)].\end{aligned}$$

Retailer's Problem

We first find the retailer's reaction function for product i:

$$\begin{aligned}0 = \frac{\partial \Pi_R}{\partial p_i} &= a_i - 2b_p p_i + \theta_p(p_j - 2p_i) + b_s s_i - \theta_s(s_j - s_i) + w_i b_p + w_i \theta_p \\ &\quad +p_j\theta_p - w_j\theta_p \\ 2b_p p_i + 2\theta_p p_i &= a_i + \theta_p p_j + b_s s_i - \theta_s(s_j - s_i) + w_i(b_p + \theta_p) + \theta_p(p_j - w_j)\end{aligned}$$

$$p_i = \frac{a_i + b_s s_i - \theta_s(s_j - s_i) + w_i(b_p + \theta_p) + \theta_p(2p_j - w_j)}{2(b_p + \theta_p)}.$$

To check the optimality, we check the Hessian matrix:

$$\begin{aligned}\frac{\partial \Pi_R^2}{\partial p_i^2} &= -2b_p - 2\theta_p \\ \frac{\partial \Pi_R^2}{\partial p_j \partial p_i} = \frac{\partial \Pi_R^2}{\partial p_i \partial p_j} &= 2\theta_p \\ \frac{\partial \Pi_R^2}{\partial p_j^2} &= -2b_p - 2\theta_p.\end{aligned}$$

Assuming that $b_p \geq 0$ and $\theta_p \geq 0$, we have a negative definite Hessian. Therefore, the p_1 and p_2 calculated above are the optimal reaction functions for the retailer. Solving for p_i^* and p_j^* by plugging p_i into p_j, we have

$$p_i^* \quad = \quad \frac{w_i}{2} + \frac{(b_p+\theta_p)a_i + \theta_p a_j}{2b_p(b_p+2\theta_p)} - \frac{\theta_s(s_j - s_i)}{2(b_p+2\theta_p)} + \frac{(b_p+\theta_p)b_s s_i + \theta_p b_s s_j}{2b_p(b_p+2\theta_p)}. \tag{114}$$

This is the reaction function of the retailer given that he has observe w_1, w_2, s_1 and s_2.

Suppliers' Problem

The suppliers in this game move simultaneously. They simultaneously announce w_i and s_i, their prices and level of services they are going to invest, respectively. Knowing the reaction function from the retailer, they calculate the optimal w_i and s_i.

Supplier i faces the following demand function:

$$Q_i = a_i - b_p p_i + \theta_p(p_j - p_i) + b_s s_i - \theta_s(s_j - s_i). \tag{115}$$

From the retailer's reaction function, we know that:

$$p_j^* - p_i^* \quad = \quad \frac{a_j - a_i}{2(b_p+2\theta_p)} + \frac{(w_j - w_i)}{2} + \frac{(2\theta_s + b_s)(s_i - s_j)}{2(b_p+2\theta_p)}. \tag{116}$$

Substituting (116) and (114) into (115), we have

$$\begin{aligned}
Q_i \quad &= \quad a_i - b_p\Big[\frac{w_i}{2} + \frac{(b_p+\theta_p)a_i + \theta_p a_j}{2b_p(b_p+2\theta_p)} - \frac{\theta_s(s_j - s_i)}{2(b_p+2\theta_p)} + \frac{(b_p+\theta_p)b_s s_i + \theta_p b_s s_j}{2b_p(b_p+2\theta_p)}\Big] \\
&\quad +\theta_p\Big[\frac{a_j - a_i}{2(b_p+2\theta_p)} + \frac{w_j - w_i}{2} + \frac{(2\theta_s + b_s)(s_j - s_i)}{2(b_p+2\theta_p)}\Big] + b_s s_i + \theta_s(s_j - s_i) \\
\Pi_{M_i} \quad &= \quad (w_1 i - c_i)\Big\{a_i - b_p\Big[\frac{w_i}{2} + \frac{(b_p+\theta_p)a_i + \theta_p a_j}{2b_p(b_p+2\theta_p)} - \frac{\theta_s(s_j - s_i)}{2(b_p+2\theta_p)} \\
&\quad + \frac{(b_p+\theta_p)b_s s_i + \theta_p b_s s_j}{2b_p(b_p+2\theta_p)}\Big] + \theta_p\Big[\frac{a_j - a_i}{2(b_p+2\theta_p)} + \frac{w_j - w_i}{2} + \frac{(2\theta_s + b_s)(s_j - s_i)}{2(b_p+2\theta_p)}\Big] \\
&\quad + b_s s_i - \theta_s(s_j - s_i)\Big\} - \frac{\eta_i s_i^2}{2}.
\end{aligned}$$

To find the optimal wholesale price, w_i, we first look at the first order condition.

$$\begin{aligned} 0 = \frac{\partial \Pi_{M_i}}{\partial w_i} &= a_i - b_p \left[w_i + \frac{(b_p+\theta_p)a_i + \theta_p a_j}{2b_p(b_p+2\theta_p)} - \frac{\theta_s(s_j - s_i)}{2(b_p+2\theta_p)} + \frac{(b_p+\theta_p)b_s s_i + \theta_p b_s s_j}{2b_p(b_p+2\theta_p)} \right] \\ &+\theta_p \left[\frac{a_j - a_i}{2(b_p+2\theta_p)} + \frac{w_j - 2w_i}{2} + \frac{(2\theta_s + b_s)(s_j - s_i)}{2(b_p+2\theta_p)} \right] \\ &+ b_s s_i - \theta_s(s_j - s_i) + \frac{c_i b_p}{2} + \frac{c_i \theta_p}{2} \end{aligned}$$

From this condition, we have

$$w_i = \frac{1}{2(b_p+\theta_p)} \left\{ \theta_p w_j + a_i + c_i(b_p+\theta_p) + (b_s+\theta_s)s_i + \left(\frac{\theta_p b_s}{(b_p+2\theta_p)} - \theta_s \right) s_j \right\} \tag{117}$$

To find the optimal level of service, we also find the first order condition.

$$0 = \frac{\partial \Pi_{M_i}}{\partial s_i} = (w_i - c_i) \left[-\frac{b_p\theta_s}{2(b_p+2\theta_p)} - \frac{b_p(b_p+\theta_p)b_s}{2b_p(b_p+2\theta_p)} - \frac{\theta_p(b_s+2\theta_s)}{2(b_p+2\theta_p)} + b_s + \theta_s \right] - \eta_i s_i$$

From this first order condition, we have

$$s_i^* = \frac{(w_i - c_i)(b_s+\theta_s)}{2\eta_i} \tag{118}$$

Substitute (118) into (117), we have

$$\begin{aligned} w_i^* &= \frac{2\eta_i}{4\eta_i(b_p+\theta_p) + (b_s+\theta_s)^2} \Big\{ a_i + \Big(\frac{2\eta_i(b_p+\theta_p) - (b_s+\theta_s)^2}{2\eta_i} \Big) c_i \\ &+ \Big(\frac{\theta_s(b_p+2\theta_p) - \theta_p b_s}{2\eta_j(b_p+2\theta_p)} \Big) (b_s+\theta_s) c_j \\ &+ \Big(\theta_p + \frac{[\theta_p b_s - \theta_s(b_p+2\theta_p)](b_s+\theta_s)}{2\eta_j(b_p+2\theta_p)} \Big) w_j \Big\} \end{aligned} \tag{119}$$

Let

$$\begin{aligned} A_i &= 4\eta_i(b_p+\theta_p) + (b_s+\theta_s)^2 \\ B_i &= 2\eta_i\theta_p - \theta_s(b_s+\theta_s)\left(\frac{b_p - b_s + 2\theta_p}{b_p+2\theta_p} \right) \end{aligned}$$

$$
\begin{aligned}
D_i &= \frac{B_i}{A_i} \\
E_i &= (b_p + \theta_p) - \frac{(b_s + \theta_s)^2}{2\eta_i} \\
F_i &= \frac{\theta_s(b_s + \theta_s)}{2\eta_i} - \frac{\theta_p b_s (b_s + \theta_s)}{2\eta_i (b_p + 2\theta_p)}
\end{aligned}
$$

We can now rewrite (119) as

$$
w_i^* = \frac{2\eta_i}{A_i} \left[a_i + E_i c_i + F_j c_j + (\frac{B_j}{2\eta_j}) w_j \right]
$$

Using this equation, w_i^* becomes:

$$
w_i^* = \frac{2\eta_i A_j}{A_i A_j - B_i B_j} \Big[(a_i + D_j a_j) + (E_i + F_i D_j) c_i + (F_j + E_j D_j) c_j \Big]
$$

Substitute (120) into (118), we have

$$
\begin{aligned}
s_i^* = (b_s + \theta_s) \Big\{ & \frac{A_j}{A_i A_j - B_i B_j} \Big[(a_i + D_j a_j) + (F_j + E_j D_j) c_j \Big] \\
& + \Big[\frac{A_j (E_i + F_i D_j)}{A_i A_j - B_i B_j} - \frac{1}{2\eta_i} \Big] c_i \Big\}
\end{aligned}
$$

These w_i^*, and s_i^* constitute the Nash equilibrium and take into account to retailer's reaction function.

A.2 Proof to Proposition 3.2 and 3.3 (Retailer Stackelberg)

In this situation, we assume that the retailer has more power in the relationship with the its suppliers. This higher power is reflected in its earlier move than the suppliers. Particularly, the retailer first announce the margin (and retail price) it desires. The suppliers then take this information and decide their optimal wholesale price and service level.

Let m_i be the margin of product i enjoyed by the retailer. Namely,

$$p_i = w_i + m_i.$$

Suppliers' Problem

Since the retailer moves first in this game, we need to calculate for the suppliers'reaction function. Note that the suppliers move simultaneously. Therefore, we need to calculate the Nash equilibrium between them. The profit function for supplier i can be expressed as:

$$\begin{aligned}
\Pi_{M_i} &= (w_i - c_i)Q_i - \frac{\eta_i s_i^2}{2}; \text{where} \\
Q_i &= a - b_p p_i + \theta_p(p_j - p_i) + b_s s_i - \theta_s(s_j - s_i)
\end{aligned}$$

To find the suppliers' reaction function, we need to find the first order condition which can be expressed as:

$$0 = \frac{\partial \Pi_{M_i}}{\partial w_i} = Q_i + (w_i - c_i)\frac{\partial Q_i}{\partial p_i}\frac{\partial p_i}{\partial w_i}$$

where

$$\frac{\partial Q_i}{\partial p_i} = -b_p - \theta_p$$

$$\frac{\partial p_i}{\partial w_i} = 1$$

Therefore,

$$0 = a_i - (b_p + \theta_p)p_i + \theta_p p_j + (b_s + \theta_s)s_i - \theta_s s_j - (b_p + \theta_p)(w_i - c_i)$$

To find the optimal level of service for supplier i, we also find the first order condition:

$$\begin{aligned} 0 = \frac{\partial \Pi_{M_i}}{\partial s_i} &= (w_i - c_i)\frac{\partial Q_i}{\partial s_i} - 2\eta_i s_i \\ 0 &= (w_i - c_i)(b_s - \theta_s) - 2\eta_i s_i \end{aligned}$$

Therefore,

$$s_i^* = \frac{(w_i - c_i)(b_s - \theta_s)}{2\eta_i}$$

From this equation, we can derive w_i^* to be

$$\begin{aligned} w_i^* = & \frac{2\eta_i}{2\eta_i(b_p + \theta_p) - (b_s + \theta_s)^2}\left\{a_i - (b_p + \theta_p)p_i - \theta_p p_j + \left[(b_p + \theta_p) - \frac{(b_s - \theta_s)^2}{2\eta_i}\right]c_i \right. \\ & \left. -\frac{\theta_s(b_s - \theta_s)}{2\eta_2}c_j - \frac{\theta_s(b_s - \theta_s)}{2\eta_j}w_j\right\} \end{aligned} \tag{120}$$

Let

$$\begin{aligned} G &= b_p + \theta_p \\ H_i &= 2\eta_i(b_p + \theta_p) + (b_s + \theta_s)^2 \\ K &= \theta_s(b_s + \theta_s) \\ L_i &= \frac{K}{H_i} \\ M_i &= \frac{H_i}{\eta_i} = (b_p + \theta_p) - \frac{(b_s + \theta_s)^2}{\eta_i} \\ N_i &= \frac{K}{2\eta_i} = \frac{\theta_s(b_s + \theta_s)}{\eta_i} \end{aligned}$$

Using the above notation, Equation (120) can be expressed as

$$w_i^* \;=\; \frac{2\eta_i H_j}{H_i H_j - K^2}\Big[a_i - L_j a_j - (\theta_p L_j + G)p_i + (GL_j + \theta_p)p_j + (M_i - L_j N_i)c_i\Big].$$

The corresponding equilibrium service level can be calculated to be

$$s_i^* \;=\; \frac{H_j(b_s + \theta_s)}{H_i H_j - K^2}\Big[a_i - L_j a_j - (\theta_p L_j + G)p_i + (GL_j + \theta_p)p_j\Big]$$

Retailer's Problem

Retailer is the Stackelberg leader in this problem. He makes decision about p_1 and p_2 after observing w_1, w_2, s_1 and s_2. His profit function can be expressed as:

$$\Pi_R \;=\; (p_1 - w_1(p_1,p_2))Q_1(p_1,p_2) + (p_2 - w_2(p_1,p_2))Q_2(p_1,p_2)$$

To calculate for his optimal actions, we need to use the first order condition:

$$\begin{aligned} 0 = \frac{\partial \Pi_R}{\partial p_i} \;=\; & \left(1 - \frac{\partial w_i(p_1,p_2)}{\partial p_i}\right) Q_i(p_1,p_2) + (p_i - w_i(p_1,p_2))\frac{\partial Q_i(p_1,p_2)}{\partial p_1} \\ & + \left(-\frac{\partial w_j(p_1,p_2)}{\partial p_i}\right) Q_j(p_1,p_2) + (p_j - w_j(p_1,p_2))\frac{\partial Q_j(p_1,p_2)}{\partial p_i} \end{aligned}$$

where

$$\begin{aligned} \frac{\partial w_i(p_1,p_2)}{\partial p_i} &= \frac{2\eta_i H_j}{H_i H_j - K^2}(\theta_p L_j - G) \\ \frac{\partial w_j(p_1,p_2)}{\partial p_i} &= \frac{2\eta_j H_i}{H_i H_j - K^2}(GL_i - \theta_p) \\ \frac{\partial w_i(p_1,p_2)}{\partial p_j} &= \frac{2\eta_i H_j}{H_i H_j - K^2}(GL_j - \theta_p) \\ \frac{\partial w_j(p_1,p_2)}{\partial p_j} &= \frac{2\eta_j H_i}{H_i H_j - K^2}(\theta_p L_i - G) \end{aligned}$$

Using the above conditions, we can calculate p_1^* and p_2^* to be

$$p_1^* \;=\; \frac{(X_2U_1 - YV_1)a_1 + (YV_2 - X_2U_2)a_2 + (X_2\rho_1 - Y\sigma_1)Wc_1 + (Y\rho_2 - X_2\sigma_2)Wc_2}{X_1X_2 - Y^2}$$

$$p_2^* = \frac{(YU_1 - X_1V_1)a_1 + (X_1V_2 - YU_2)a_2 + (Y\rho_1 - X_1\sigma_1)Wc_1 + (X_1\rho_2 - Y\sigma_2)Wc_2}{X_1X_2 - Y^2}.$$

where

$$\begin{aligned}
W &= H_1H_2 - K^2 \\
U_1 &= \xi_1\rho_1 + \omega_1\gamma_1 + \xi_2L_1\sigma_2 + \psi_2\phi_2 \\
U_2 &= \xi_2\sigma_2 + \omega_1\phi_1 + \xi_1L_2\rho_1 + \psi_2\gamma_2 \\
V_1 &= \psi_1\gamma_1 + \xi_1\sigma_1 + \omega_2\phi_2 + \xi_2L_1\rho_2 \\
V_2 &= \psi_1\phi_1 + \xi_2\rho_2 + \omega_2\gamma_2 + \xi_1L_2\sigma_1 \\
X_1 &= 2(\omega_1\rho_1 + \psi_2\sigma_2) \\
X_2 &= 2(\omega_2\rho_2 + \psi_1\sigma_1) \\
Y &= \psi_1\rho_1 + \psi_2\rho_2 + \omega_1\sigma_1 + \omega_2\sigma_2 \\
\xi_i &= \eta_iH_j \\
\omega_i &= H_iH_j - K^2 + \eta_iH_j(G + \theta_pL_j) \\
\psi_i &= \eta_iH_j(GL_j + \theta_p) \\
\gamma_i &= \eta_iH_jG \\
\phi_i &= \eta_iKG \\
\rho_i &= \eta_iG(H_jG + \theta_pK) \\
\sigma_i &= \eta_iG(GK + \theta_pH_j)
\end{aligned}$$

A.3 Proof to Proposition 3.4 (Vertical Nash)

Game-theoretic framework is employed to derive the reaction function of each firm in the supply chain. Fortunately, the reaction functions for the retailer and the manufacturers were already derived in the Manufacturer Stackelberg game and the Retailer Stackelberg game respectively. From the Manufacturer Stackelberg game, the retailer's reaction function for any given wholesale prices w_1, w_2 and service levels s_1, s_2 is provided in Equation (7) as

$$p_i^* = \frac{w_i^*}{2} + \frac{(b_p+\theta_p)a_i + \theta_p a_j}{2b_p(b_p+2\theta_p)} - \frac{\theta_s(s_j^* - s_i^*)}{2(b_p+2\theta_p)} + \frac{(b_p+\theta_p)b_s s_i^* + \theta_p b_s s_j^*}{2b_p(b_p+2\theta_p)}$$

where $i \in \{1,2\}$ and $j = 3-i$. From the Retailer Stackelberg game, the manufacturers reaction function for given retail prices p_1, p_2 are given in Equations 121 and 22 as

$$w_i^* = \frac{\eta_i H_j}{H_1 H_2 - K^2}\Big[a_i - L_j a_j - (\theta_p L_j + G)p_i + (GL_j + \theta_p)p_j + (M_i - L_j N_i)c_i\Big]$$

$$s_i^* = \frac{H_j(b_s+\theta_s)}{H_1 H_2 - K^2}\Big[a_i - L_j a_j - (\theta_p L_j + G)p_i + (GL_j + \theta_p)p_j\Big]$$

for wholesale price and service level respectively. H_i, K, L_i, M_i, N_i, and G for $i = 1,2$ and $j = 3-i$ are defined as in the Retailer Stackelberg game. Solving the above equations simultaneously yields the Nash equilibrium solution. The final expressions for the retail prices are

$$p_1 = \frac{(\gamma_2\kappa_1 + \lambda_1\kappa_2)a_1 + (\gamma_2\nu_1 + \lambda_1\nu_2)a_2 + \gamma_2\psi_1 c_1 + \lambda_1\psi_2 c_2}{\gamma_1\gamma_2 - \lambda_1\lambda_2}$$

$$p_2 = \frac{(\gamma_1\kappa_2 + \lambda_2\kappa_1)a_1 + (\gamma_1\nu_2 + \lambda_2\nu_1)a_2 + \gamma_1\psi_2 c_1 + \lambda_2\psi_1 c_2}{\gamma_1\gamma_2 - \lambda_1\lambda_2}$$

where

$$\gamma_1 = 2b_p(b_p+2\theta_p)W + \eta_1 H_2(\theta_p L_2 + G)b_p(b_p+2\theta_p) + \vartheta_1 H_2(b_s+\theta_s)(\theta_p L_2 + G)$$

$$
\begin{aligned}
& -\vartheta_2 H_1(GL_1+\theta_p) \\
\gamma_2 &= 2b_p(b_p+2\theta_p)W+\eta_2 H_1(\theta_p L_1+G)b_p(b_p+2\theta_p)+\vartheta_1 H_1(b_s+\theta_s)(\theta_p L_1+G) \\
& -\vartheta_2 H_2(GL_2+\theta_p) \\
\kappa_1 &= \eta_1 H_2 b_p(b_p+2\theta_p)+(b_p+\theta_p)W+\vartheta_1 H_2(b_s+\theta_s)-\vartheta_2 H_1 L_1 \\
\kappa_2 &= -\eta_2 H_1 L_1 b_p(b_p+2\theta_p)+\theta_p W-\vartheta_1 H_1 L_1(b_s+\theta_s)+\vartheta_2 H_2 \\
\nu_1 &= -\eta_1 H_2 L_2 b_p(b_p+2\theta_p)+\theta_p W-\vartheta_1 H_2 L_2(b_s+\theta_s)+\vartheta_2 H_1 \\
\nu_2 &= \eta_2 H_1 b_p(b_p+2\theta_p)+(b_p+\theta_p)W+\vartheta_1 H_1(b_s+\theta_s)-\vartheta_2 H_2 L_2 \\
\psi_1 &= \eta_1 b_p(b_p+2\theta_p)H_2(U_1-L_2V_1) \\
\psi_2 &= \eta_2 b_p(b_p+2\theta_p)H_1(U_2-L_1V_2) \\
\lambda_1 &= \eta_1 H_2(GL_1+\theta_p)b_p(b_p+2\theta_p)+\vartheta_1 H_2(b_s+\theta_s)(GL_2+\theta_p)-\vartheta_2 H_1(\theta_p L_1+G) \\
\lambda_2 &= \eta_2 H_1(GL_1+\theta_p)b_p(b_p+2\theta_p)+\vartheta_1 H_1(b_s+\theta_s)(GL_1+\theta_p)-\vartheta_2 H_2(\theta_p L_2+G) \\
\vartheta_1 &= \theta_s b_p+b_s(b_p+\theta_p) \\
\vartheta_2 &= (\theta_p b_s-b_p\theta_s)(b_s+\theta_s)
\end{aligned}
$$

A.4 Proof to Theorem 3.1

When the two manufacturers are identical, the wholesale prices, retail prices, and service levels for the two manufacturers will also be identical (i.e., they are going after the same market).

Manufacturer Stackelberg

From Section 3.3.1, if we let $a_1 = a_2 = a$, $c_1 = c_2 = c$, $\eta_1 = \eta_2 = \eta$, we will have $H_1 = H_2 = H$, $L_1 = L_2 = L$, $M_1 = M_2 = M$, and $N_1 = N_2 = N$. The consumer demand and retailer reaction function now become

$$\begin{aligned}
Q^{MS} &= \frac{a}{2} - \frac{b_p w^{MS}}{2} + \frac{b_s s^{MS}}{2} \\
p^{MS} &= \frac{a}{2b_p} + \frac{w^{MS}}{2} + \frac{b_s s^{MS}}{2b_p}.
\end{aligned}$$

The decision on equilibrium wholesale price and service level by each manufacturer can also be calculated to be

$$\begin{aligned}
w^{MS} &= \frac{2\eta}{A-B}[a + (E+F)c] \\
s^{MS} &= \frac{(b_s+\theta_s)}{A-B}a + (b_s+\theta_s)\Big[\frac{E+F}{A-B} - \frac{1}{2\eta}\Big],
\end{aligned}$$

respectively. Using these expressions, the retail price and demand quantity can be calculated as a linear function of a and c as follows

$$\begin{aligned}
Q^{MS} &= \Big[\frac{1}{2} - \frac{b_p\eta}{A-B} + \frac{b_s(b_s+\theta_s)}{2(A-B)}\Big]a - \Big[\frac{b_p\eta(E+F)}{A-B} - \frac{b_s(b_s+\theta_s)}{2(A-B)}\Big(\frac{E+F}{A-B} - \frac{1}{2\eta}\Big)\Big]c \\
p^{MS} &= \Big[\frac{A-B+2b_p\eta + b_s(b_s+\theta_s)}{2b_s(A-B)}\Big]a - \Big[\frac{\eta(E+F)}{A-B} + \frac{b_s(b_s+\theta_s)}{2b_p}\Big(\frac{E+F}{A-B} - \frac{1}{2\eta}\Big)\Big]c
\end{aligned}$$

In this case, $p^{MS} - w^{MS} = \frac{Q^{MS}}{b_p}$. Therefore, the retailer profit can by calculated as

$$\Pi_R^{MS} = 2(p^{MS} - w^{MS})Q^{MS}$$

$$= \frac{2}{b_p}(Q^{MS})^2$$

The manufacturer's profit can be calculated as

$$\begin{aligned}
\Pi_M^{MS} &= (w^{MS} - c)Q^{MS} - \frac{\eta(s^{MS})^2}{2} \\
&= \frac{2\eta}{A-B}\Big[a + (E + F - \frac{A-B}{2\eta})c\Big](Q^{MS}) - \frac{\eta(b_s+\theta_s)^2}{2(A-B)^2}\Big[a + (E + F + \frac{A-B}{2\eta})c\Big]^2
\end{aligned}$$

Retailer Stackelberg

With identical manufacturers, each manufacturer reaction functions on wholesale price and service level are

$$\begin{aligned}
w^{RS} &= \frac{\eta}{H+K}[a - b_p p^{RS}] + c \\
s^{RS} &= \frac{(b_s+\theta_s)}{H+K}[a - b_p p^{RS}]
\end{aligned}$$

The equilibrium retail price can be also be simplified as shown below

$$p^{RS} = \frac{H + K + 2\eta b_p}{2b_p(H + K + \eta b_p)}a + \frac{H+K}{2(H + K + \eta b_p)}c.$$

With this expression for retail price, the wholesale price and service level can be expressed as

$$\begin{aligned}
w^{RS} &= \frac{\eta}{2(H + K + \eta b_p)}a + \frac{2(H+K) + \eta b_p}{2(H + K + \eta b_p)}c, \\
s^{RS} &= \frac{b_s + \theta_s}{2(H + K + \eta b_p)}[a - b_p c].
\end{aligned}$$

Demand quantity can be calculated to be

$$Q^{RS} = \frac{\eta(b_p + \theta_p)}{2(H + K + \eta b_p)}[a - b_p c].$$

The manufacturer margin, $w^{RS} - c$, can be derived to be $\frac{\eta}{2(H+K+\eta b_p)}[a - b_p c]$. Thus, the manufacturer's profit can be calculated to be

$$\begin{aligned} \Pi_M^{RS} &= (w^{RS} - c)Q^{RS} - \frac{\eta(s^{RS})^2}{2} \\ &= \frac{\eta(\eta(b_p + \theta_p) + H)}{8(H + K + \eta b_p)^2}[a - b_p c]^2. \end{aligned}$$

The retailer's profit can be derived as

$$\begin{aligned} \Pi_R^{RS} &= 2(p^{RS} - w^{RS})Q^{RS} \\ &= \frac{\eta(b_p + \theta_p)}{2b_p(H + K + \eta b_p)}[a - b_p c]^2. \end{aligned}$$

Vertical Nash

For the Vertical Nash case, we can use the reaction function from the previous two cases. These functions are

$$\begin{aligned} p^{VN} &= \frac{a}{2b_p} + \frac{w^{MS}}{2} + \frac{b_s s^{MS}}{2b_p}, \\ w^{VN} &= \frac{\eta}{H + K}[a - b_p p^{RS}] + c, \\ s^{VN} &= \frac{(b_s + \theta_s)}{H + K}[a - b_p p^{RS}]. \end{aligned}$$

To obtain the equilibrium solution, the three equations above are solved simultaneously. The equilibrium retail price can be calculated to be

$$p^{VN} = \left[\frac{H + K + \eta b_p + b_s(b_s + \theta_s)}{b_p(2(H + K) + \eta b_p + b_s(b_s + \theta_s))}\right]a + \left[\frac{H + K}{2(H + K) + \eta b_p + b_s(b_s + \theta_s)}\right]c.$$

The equilibrium wholesale price and service level are

$$\begin{aligned} w^{VN} &= \left[\frac{\eta}{2(H + K) + \eta b_p + b_s(b_s + \theta_s)}\right]a + \left[\frac{2(H + K) + b_s(b_s + \theta_s)}{2(H + K) + \eta b_p + b_s(b_s + \theta_s)}\right]c, \\ s^{VN} &= \frac{b_s + \theta_s}{2(H + K) + \eta b_p + b_s(b_s + \theta_s)}(a - b_p c). \end{aligned}$$

The consumer demand can be derived to be

$$\begin{aligned}
Q^{VN} &= a - b_p p^{VN} + b_s s^{VN} \\
&= \frac{\eta(b_s + \theta_s)}{2(H+K) + \eta b_p + b_s(b_s + \theta_s)}(a - b_p c).
\end{aligned}$$

The manufacturer profit margin, $w^{VN} - c$ is

$$w^{VN} - c = \frac{\eta}{2(H+K) + \eta b_p + b_s(b_s + \theta_s)}(a - b_p c).$$

Thus, the manufacturer profit can be calculated to be

$$\Pi_M^{VN} = \frac{\eta[\eta(b_s + \theta_s) + H]}{2[2(H+K) + \eta b_p + b_s(b_s + \theta_s)]}(a - b_p c)^2.$$

Using the above equations for equilibrium retail price, wholesale price and consumer demand, the retailer profit are

$$\Pi_R^{VN} = \frac{2(Q^{VN})^2}{b_p} = \frac{2}{b_p}\Big[\frac{\eta(b_p + \theta_p)}{2(H+K) + \eta b_p + b_s(b_s + \theta_s)}(a - b_p c)\Big]^2.$$

A.5 Proof to Corollary 3.1

From Table 3.1, we can see that if $\eta b_p > b_s(b_s + \theta_s)$, $\Phi < \Psi$. Therefore,

$$\begin{aligned} \frac{b_s+\theta_s}{\Phi}(a - b_p c) &> \frac{b_s+\theta_s}{\Psi}(a - b_p c). \\ s^{VN} &> s^{RS}. \end{aligned}$$

For the service provided in the Manufacturer Stackelberg case,

$$\begin{aligned} s^{MS} &= \frac{(b_s+\theta_s)}{H-K}a + (b_s+\theta_s)\Big[\frac{M+N}{H-K} - \frac{1}{2\eta}\Big] \\ &= \frac{(b_s+\theta_s)}{H-K}\Big[a - (b_p + \frac{(b_s+\theta_s)^2}{\eta})c\Big]. \end{aligned}$$

From the definition given in Chapter 3, we have

$$\begin{aligned} H - K &= 2\eta(2b_p + \theta_p) + (b_s+\theta_s)^2 + \frac{\theta_s(b_s+\theta_s)(b_p + 2\theta_p - b_s)}{(b_p + 2\theta_p)} \\ &> 2\eta(b_p + \theta_p) + 2\eta b_p \\ &> \Psi \end{aligned}$$

Therefore,

$$\begin{aligned} s^{MS} &= \frac{(b_s+\theta_s)}{H-K}\Big[a - (b_p + \frac{(b_s+\theta_s)^2}{\eta})c\Big] \\ &< \frac{(b_s+\theta_s)}{\Psi}\Big[a - (b_p + \frac{(b_s+\theta_s)^2}{\eta})c\Big] \\ &< \frac{(b_s+\theta_s)}{\Psi}(a - b_p c) \\ &< s^{RS} \end{aligned}$$

Therefore, $s^{MS} < s^{RS} < s^{VN}$. $\square$

A.6 Proof to Corollary 3.2

The case when $b_s = \theta_s = 0$ is trivial and will not be given here. We are focus on the case when $b_s, \theta_s > 0$. We know that if $\eta b_p > b_s(b_s + \theta_s)$, $\Phi < \Psi$. Thus, from the results given in Theorem 3.1, it follows immediately that

$$\begin{aligned} Q^{VN} &> Q^{RS} \\ \Pi_M^{VN} &< \Pi_M^{RS} \end{aligned}$$

To prove that $\Pi_R^{VN} < \Pi_R^{RS}$, we first note that

$$\begin{aligned} X^2 &> \{X - (\eta b_p - b_s(b_s + \theta_s))\}\{X + (\eta b_p - b_s(b_s + \theta_s))\} \\ \{2\eta(b_p + \theta_p) + \eta b_p - b_s(b_s + \theta_s)\}^2 &> \{2\eta(b_p + \theta_p)\}\{2\eta(b_p + \theta_p) + 2\eta b_p - 2b_s(b_s + \theta_s)\} \\ \{2(H + K) + \eta b_p + b_s(b_s + \theta_s)\}^2 &> \{2\eta(b_p + \theta_p)\}\{2(H + K) + 2\eta b_p\} \\ \frac{1}{2(H + K) + 2\eta b_p} &> \frac{2\eta(b_p + \theta_p)}{2(H + K) + \eta b_p + b_s(b_s + \theta_s)} \\ \frac{1}{\Psi} &> \frac{2\eta(b_p + \theta_p)}{\Phi^2} \\ \frac{\eta(b_p + \theta_p)}{b_p \Psi}(a - b_p c)^2 &> \frac{2\eta(b_p + \theta_p)}{\Phi} \frac{\eta(b_p + \theta_p)}{b_p \Phi}(a - b_p c)^2 \\ \frac{H + K + 2\eta b_p}{b_p \Psi} a + \frac{H + K}{\Psi} c &> \frac{\Phi - (H + K)}{b_p \Phi} a + \frac{(H + K)}{\Phi} c \\ \Pi_R^{RS} &> \Pi_R^{VN} \end{aligned}$$

To show that $w^{VN} > w^{RS}$, we begin with the assumption that $a > b_p c$ and $\eta b_p > b_s(b_s + \theta_s)$. Therefore,

$$\begin{aligned} 0 &< \Big(\frac{\eta b_p - b_s(b_s + \theta_s)}{\Phi \Psi}\Big)\Big(\frac{\eta(a - b_p c)}{\Phi}\Big) \\ 0 &< \eta\Big(\frac{\eta b_p - b_s(b_s + \theta_s)}{\Phi \Psi}\Big) a + \Big(\frac{\eta b_p - b_s(b_s + \theta_s)}{\Psi}\Big)\Big(\frac{\Phi - \eta b_p}{\Phi} - 1\Big) c \\ 0 &< \eta\Big(\frac{\eta b_p - b_s(b_s + \theta_s)}{\Phi \Psi}\Big)\Big(a + \frac{\Phi - \eta b_p}{\eta} c\Big) - \Big(\frac{\eta b_p - b_s(b_s + \theta_s)}{\Psi}\Big) c \end{aligned}$$

$$0 < w^{VN} - w^{RS}$$

Therefore, $w^{VN} > w^{RS}$.

A.7 Range of Parameters Used in Numerical Studies

Parameter	Range
b_p	$\{0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5\}$
θ_p	$\{0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5\}$
b_s	$\{0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5\}$
θ_s	$\{0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5\}$
a_i	$\{40, 60, 80, 100, 120\}$
c_i	$\{2, 4, 6, 8, 10\}$
η_i	$\{2, 4, 6, 8, 10\}$

The range of these parameters are based on related literature such as Tsay and Agrawal (2000) [90] and Vilcassin *et al.* (1999) [91] (see Chapter 2 for the review of these papers). Figure 31 to 37 show how changes in each parameter affect the equilibrium solution.

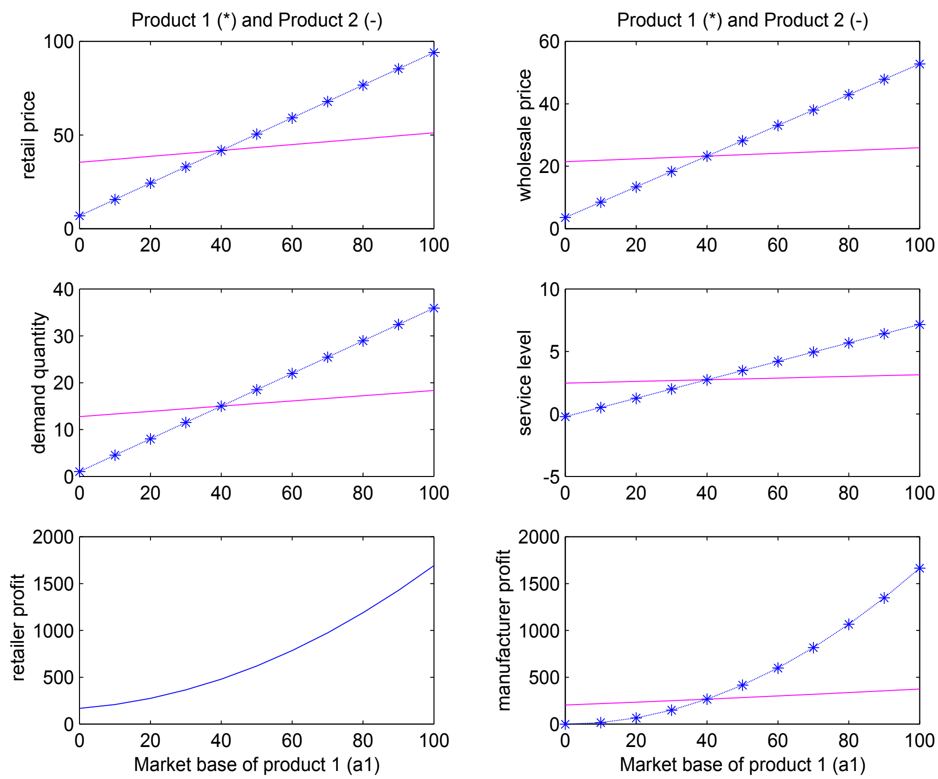

Figure 31: Analysis of the supply chain when a_1 is changing.

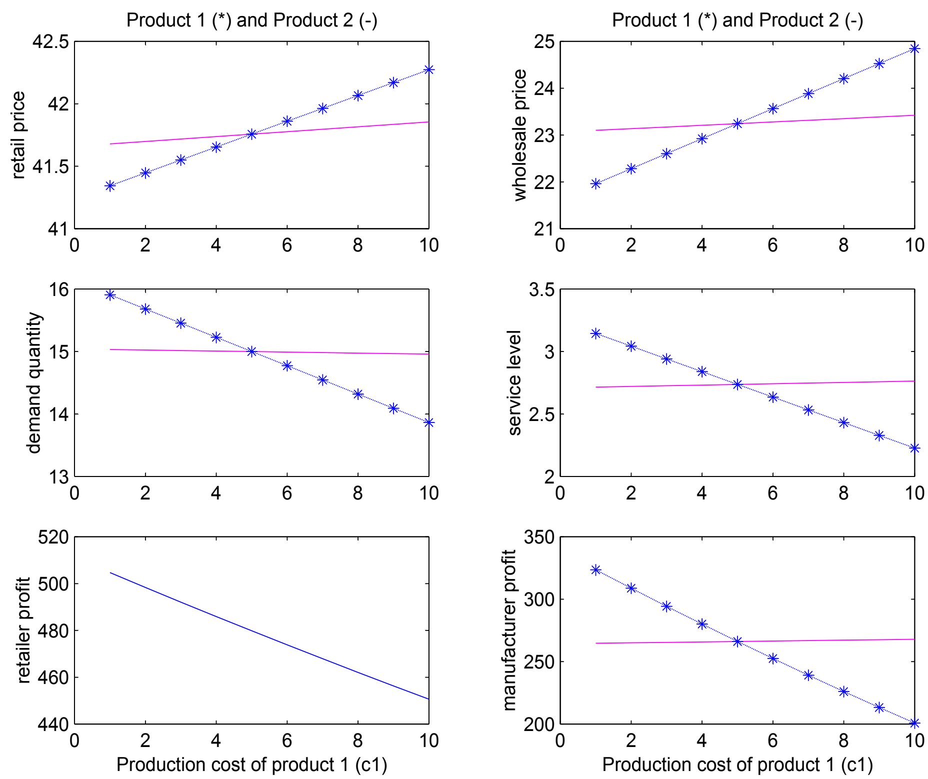

Figure 32: Analysis of the supply chain when c_1 is changing.

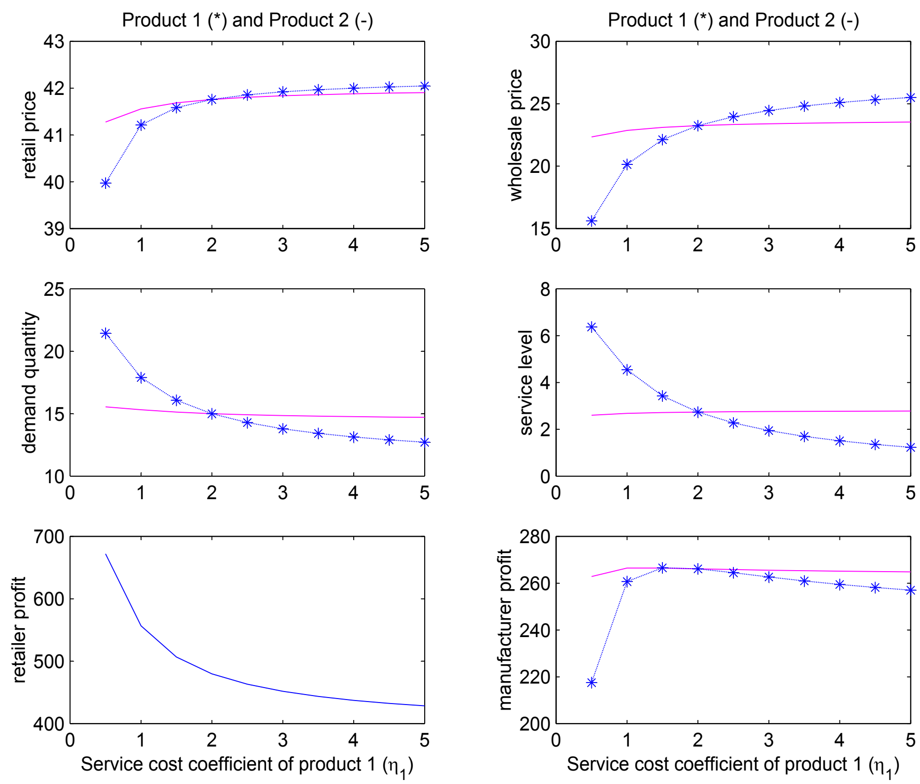

Figure 33: Analysis of the supply chain when η_1 is changing.

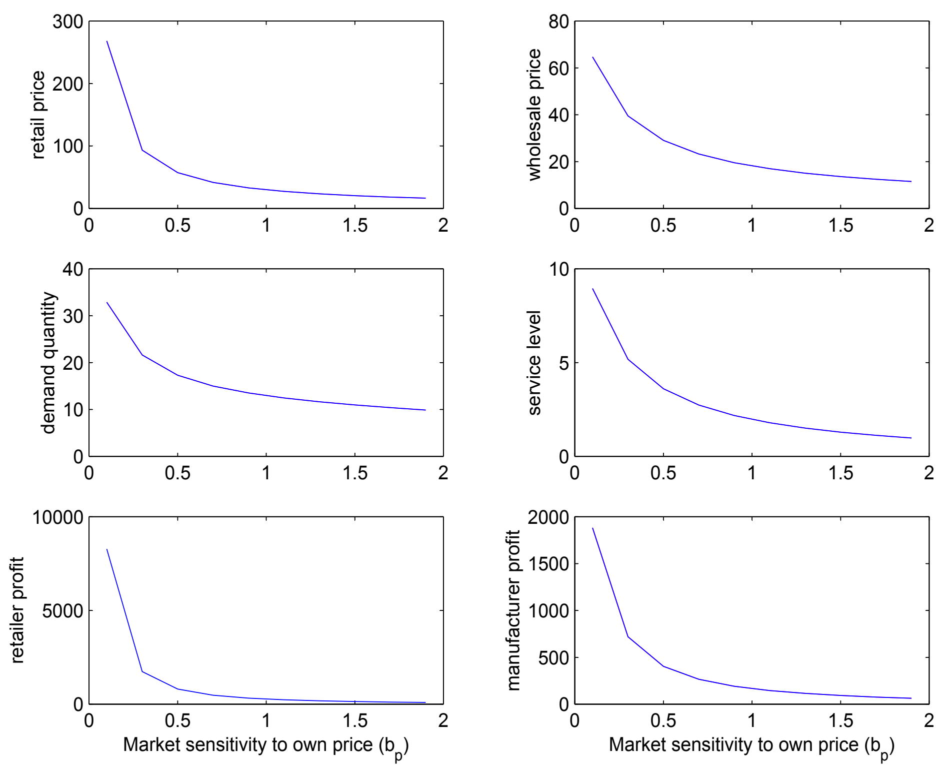

Figure 34: Analysis of the supply chain when b_p is changing.

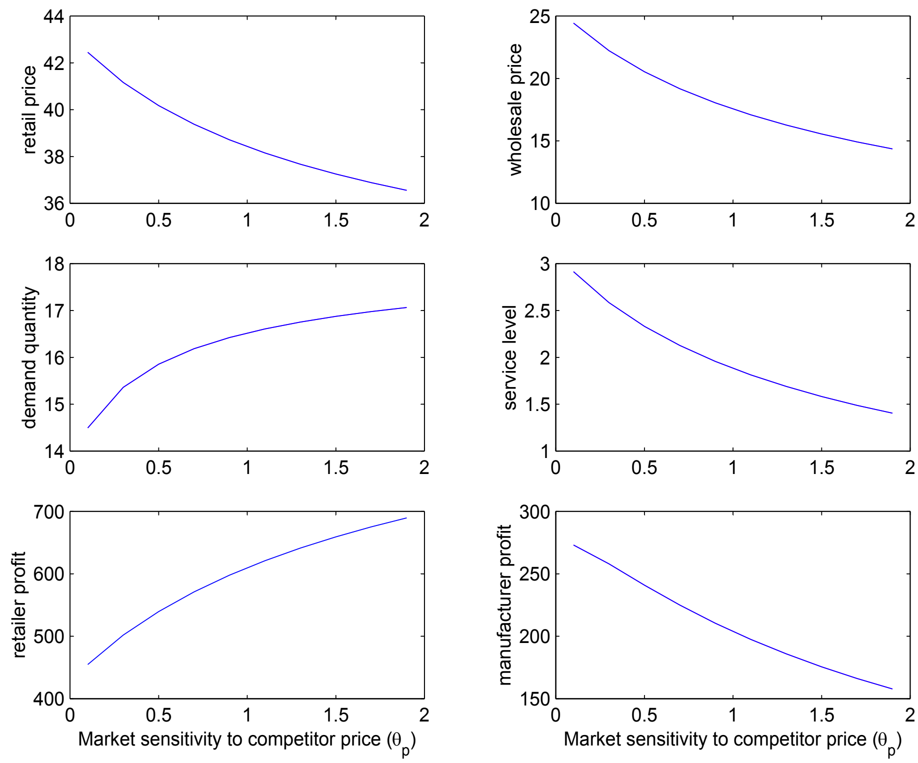

Figure 35: Analysis of the supply chain when θ_p is changing.

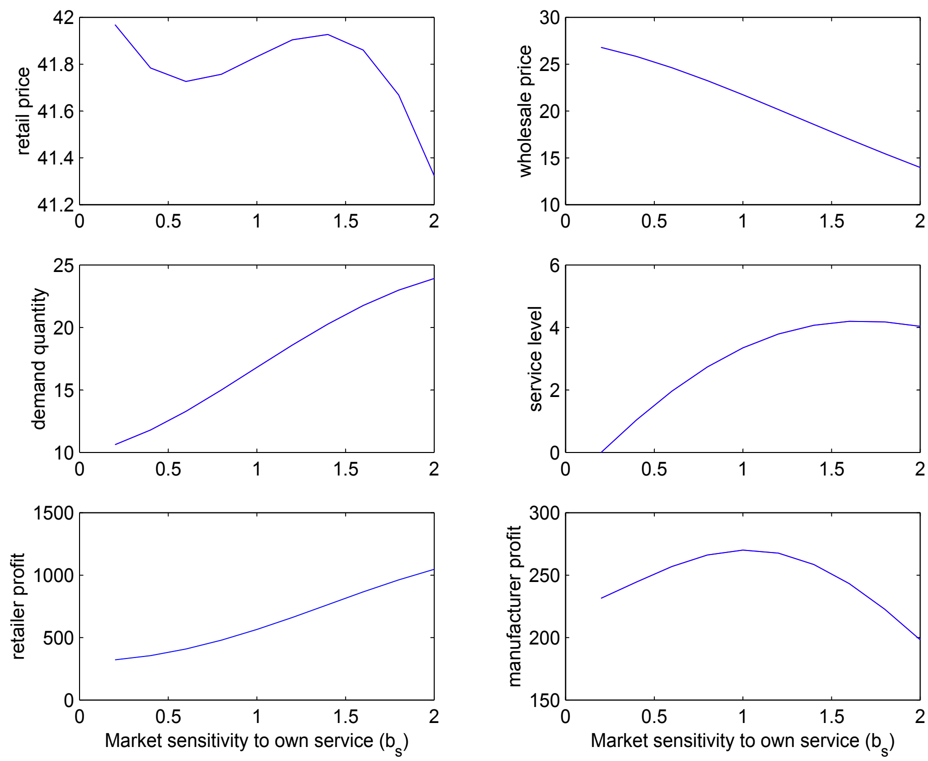

Figure 36: Analysis of the supply chain when b_s is changing.

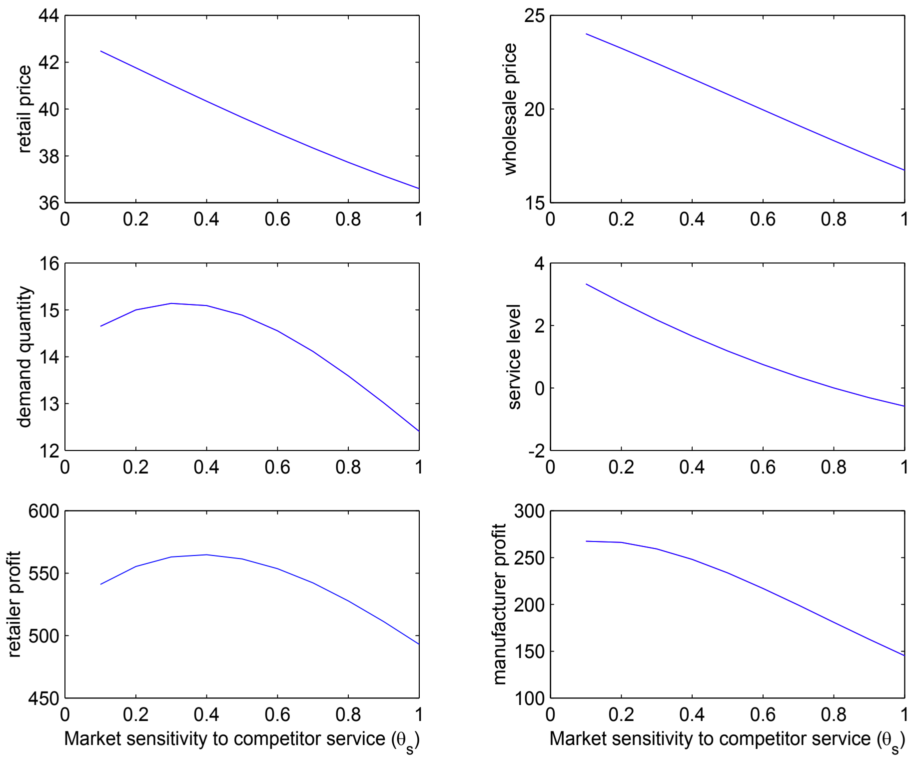

Figure 37: Analysis of the supply chain when θ_s is changing.

APPENDIX B: APPENDIX FOR CHAPTER 4

B.1 Parameters Specifications for Equation (38) and (39)

$G = \frac{\theta_s b_p + b_s(b_p+\theta_p)}{2b_p(b_p+2\theta_p)}$, $\quad H = \frac{\theta_p b_s - \theta_s b_p}{2b_p(b_p+2\theta_p)}$

$t_{11} = \frac{\varphi_1}{2} + \frac{(b_p+\theta_p)}{2b_p(b_p+2\theta_p)} + Gl_1 + Hl_2 D_1$, $\quad t_{12} = \frac{\varphi_1 D_2}{2} + \frac{\theta_p}{2b_p(b_p+2\theta_p)} + Gl_1 D_2 + Hl_2$

$t_{21} = \frac{\varphi_2 D_1}{2} + \frac{\theta_p}{2b_p(b_p+2\theta_p)} + Hl_1 + Gl_2 D_1$, $\quad t_{22} = \frac{\varphi_2}{2} + \frac{(b_p+\theta_p)}{2b_p(b_p+2\theta_p)} + Hl_1 D_2 + Gl_2$

$y_{11} = \varphi_1 \frac{E_1+F_1D_2}{2} + Gm_{11} + Hm_{21}$, $\quad y_{12} = \varphi_1 \frac{F_2+E_2D_2}{2} + Gm_{12} + Hm_{22}$

$y_{21} = \varphi_2 \frac{F_1+E_1D_1}{2} + Hm_{11} + Gm_{21}$, $\quad y_{22} = \varphi_2 \frac{E_2+F_2D_1}{2} + Hm_{12} + Gm_{22}$

$g_{11} = \frac{1}{2} - \frac{b_p+\theta_p}{2}\varphi_1 + \frac{\theta_p}{2}\varphi_2 D_1 + \frac{b_s+\theta_s}{2} l_1 - \frac{\theta_s}{2} l_2 D_1$,

$g_{12} = -\frac{b_p+\theta_p}{2}\varphi_1 D_2 + \frac{\theta_p}{2}\varphi_2 + \frac{b_s+\theta_s}{2} l_1 D_2 - \frac{\theta_s}{2} l_2$,

$g_{21} = -\frac{b_p+\theta_p}{2}\varphi_2 D_1 + \frac{\theta_p}{2}\varphi_1 + \frac{b_s+\theta_s}{2} l_2 D_1 - \frac{\theta_s}{2} l_1$,

$g_{22} = \frac{1}{2} - \frac{b_p+\theta_p}{2}\varphi_2 + \frac{\theta_p}{2}\varphi_1 D_2 + \frac{b_s+\theta_s}{2} l_2 - \frac{\theta_s}{2} l_1 D_2$,

$h_{11} = -\frac{b_p+\theta_p}{2}\varphi_1(E_1+F_1D_2) + \frac{\theta_p}{2}\varphi_2(F_1+E_1D_1) + \frac{b_s+\theta_s}{2} m_{11} - \frac{\theta_s}{2} m_{21}$

$h_{12} = -\frac{b_p+\theta_p}{2}\varphi_1(F_2+E_2D_2) + \frac{\theta_p}{2}\varphi_2(E_2+F_2D_1) + \frac{b_s+\theta_s}{2} m_{12} - \frac{\theta_s}{2} m_{22}$

$h_{21} = -\frac{b_p+\theta_p}{2}\varphi_2(F_1+E_1D_1) + \frac{\theta_p}{2}\varphi_1(E_1+F_1D_2) + \frac{b_s+\theta_s}{2} m_{21} - \frac{\theta_s}{2} m_{11}$

$h_{22} = -\frac{b_p+\theta_p}{2}\varphi_2(E_2+F_2D_1) + \frac{\theta_p}{2}\varphi_1(F_2+E_2D_2) + \frac{b_s+\theta_s}{2} m_{22} - \frac{\theta_s}{2} m_{12}$

B.2 Parameters Specifications for Equation (42)

$$\alpha_{i,t} = a_{i,t} - \gamma(p_{i,t} - p_{j,t}) + \sigma(s_{i,t} - s_{j,t})$$

$$\phi_i = (E_i + F_i D_j)c_i + (F_j + E_j D_j)c_j$$

$$\mu_i = \frac{A_i(b_s+\theta_s)}{A_1A_2 - B_1B_2}$$

$$\lambda_i = \left[\frac{A_i(b_s+\theta_s)(F_i+E_iD_i)}{A_1A_2-B_1B_2}\right]c_i + \left[\frac{A_i(E_j+F_jD_i)}{A_1A_2-B_1B_2} - \frac{1}{2\eta_j}\right](b_s+\theta_s)c_j$$

$$\tau_i = \frac{1}{2}\Big[(b_s+\theta_s)\mu_j - (b_p+\theta_p)\varphi_i + \theta_p\varphi_j D_i - \theta_s\mu_i D_i\Big]$$

$$\upsilon_i = \frac{1}{2}\Big[(b_s+\theta_s)\mu_j D_j - (b_p+\theta_p)\varphi_i D_j + \theta_p\varphi_j - \theta_s\mu_i\Big]$$

$$\varrho_i = -\frac{(b_p+\theta_p)\varphi_i}{2}\phi_i + \frac{\theta_p\varphi_j}{2}\phi_j - \frac{\theta_s}{2}\lambda_i + \frac{(b_s+\theta_s)}{2}\lambda_j$$

$$\rho_i = (\tfrac{1}{2} + \tau_i)\alpha_{i,t} + \upsilon_i\alpha_{j,t} + \varrho_i$$

$$\Lambda_i = \varphi_i\rho_i + \xi_i\varphi_i(\alpha_{i,t} + Dj\alpha_{j,t} + \phi_i) - \eta_i\mu_j^2(\alpha_{i,t} + Dj\alpha_{j,t}) - \eta_i\mu_j\lambda_j - \xi_1 c_1$$

$$r_i = \varphi_i(0.5 + 2\tau_i) - \eta_i\mu_j^2$$

$$\Psi_i = \varphi_i\upsilon_i + \varphi_i(0.5+\tau_i)D_j - \eta_i\mu_j^2 D_j$$

$$\delta_{11} = \frac{\beta(2-\beta^2r_2)(0.5\varphi_1+r_1)+\beta^3\Psi_1\Psi_2}{(2-\beta^2r_1)(2-\beta^2r_2)-\beta^4\Psi_1\Psi_2}, \quad \delta_{12} = \frac{\beta(2-\beta^2r_2)\Psi_1+\beta^3\Psi_1(0.5\varphi_2+r_2)}{(2-\beta^2r_1)(2-\beta^2r_2)-\beta^4\Psi_1\Psi_2}$$

$$\delta_{21} = \frac{\beta(2-\beta^2r_1)\Psi_2+\beta^3\Psi_2(0.5\varphi_1+r_1)}{(2-\beta^2r_1)(2-\beta^2r_2)-\beta^4\Psi_1\Psi_2}, \quad \delta_{22} = \frac{\beta(2-\beta^2r_1)(0.5\varphi_2+r_2)+\beta^3\Psi_1\Psi_2}{(2-\beta^2r_1)(2-\beta^2r_2)-\beta^4\Psi_1\Psi_2}$$

$$\Delta_1 = \frac{\beta(2-\beta^2r_2)\Omega_1+\beta^3\Psi_1\Omega_2}{(2-\beta^2r_1)(2-\beta^2r_2)-\beta^4\Psi_1\Psi_2}, \qquad \Omega_1 = \varphi_1\varrho_1 + (0.5+\tau_1)\varphi_1\phi_1 - \eta_1\mu_2\lambda_2 - (0.5+\tau_1)c_1$$

$$\Delta_2 = \frac{\beta^3r_2\Omega_1+\beta(2-\beta^2r_1)\Omega_2}{(2-\beta^2r_1)(2-\beta^2r_2)-\beta^4\Psi_1\Psi_2}, \qquad \Omega_2 = \varphi_2\varrho_2 + (0.5+\tau_2)\varphi_2\phi_2 - \eta_2\mu_1\lambda_1 - (0.5+\tau_2)c_2$$

$$\widehat{\nu} = \widehat{\nu_1}^2 - \widehat{\nu_2}^2$$

$$\widehat{\alpha_1} = \beta(\delta_{12} - \delta_{11} - \tfrac{1}{\beta}), \qquad \widehat{\alpha_2} = \beta(\delta_{22} - \delta_{21} + \tfrac{1}{\beta})$$

B.3 Parameters Specifications for Equation (46)

$$\psi_{11} = \widehat{\nu}(\widehat{\nu_2}\widehat{\vartheta}_3 - \widehat{\nu_3}\widehat{\vartheta}_2), \qquad \psi_{12} = \widehat{\nu}(\widehat{\nu_2}\widehat{\vartheta}_4 - \widehat{\nu_4}\widehat{\vartheta}_2)$$

$$\psi_{21} = \widehat{\nu}(\widehat{\nu_1}\widehat{\vartheta}_3 - \widehat{\nu_3}\widehat{\vartheta}_1), \qquad \psi_{22} = \widehat{\nu}(\widehat{\nu_1}\widehat{\vartheta}_4 - \widehat{\nu_4}\widehat{\vartheta}_1)$$

$$\zeta_{11} = -\widehat{\nu}(\theta_p\widehat{\nu_2} + (b_p+\theta_p)\widehat{\vartheta}_2), \qquad \zeta_{12} = \widehat{\nu}((b_p+\theta_p)\widehat{\nu_2} + \theta_p\widehat{\vartheta}_2)$$

$$\zeta_{21} = \widehat{\nu}(\theta_p\widehat{\nu_1} + (b_p+\theta_p)\widehat{\vartheta}_1), \qquad \zeta_{22} = -\widehat{\nu}((b_p+\theta_p)\widehat{\nu_1} + \theta_p\widehat{\vartheta}_1)$$

$$\Upsilon_1 = \widehat{\nu}(\widehat{\nu_2}\widehat{\vartheta}_5 + \widehat{\nu_5}\widehat{\vartheta}_2), \qquad \Upsilon_2 = \widehat{\nu}(\widehat{\nu_5}\widehat{\vartheta}_1 + \widehat{\nu_1}\widehat{\vartheta}_5)$$

$$\widehat{\kappa_1} = t_{11}\gamma\widehat{\alpha_1} + t_{12}\gamma\widehat{\alpha_2}, \qquad \widehat{\kappa_2} = \varphi_1\gamma\widehat{\alpha_1} + \varphi_1 D_2\gamma\widehat{\alpha_2}$$

$$\widehat{\kappa_3} = g_{11}\gamma\widehat{\alpha_1} + g_{12}\gamma\widehat{\alpha_2}, \qquad \widehat{\kappa_4} = t_{21}\gamma\widehat{\alpha_1} + t_{22}\gamma\widehat{\alpha_2}$$

$$\widehat{\kappa_5} = \varphi_2 D_1\gamma\widehat{\alpha_1} + \varphi_2\gamma\widehat{\alpha_2}, \qquad \widehat{\kappa_6} = g_{21}\gamma\widehat{\alpha_1} + g_{22}\gamma\widehat{\alpha_2}$$

$$\widehat{\mu_1} = -t_{11}\sigma\widehat{\alpha_1} - t_{12}\sigma\widehat{\alpha_2}, \qquad \widehat{\mu_2} = -\varphi_1\sigma\widehat{\alpha_1} - \varphi_1 D_2\sigma\widehat{\alpha_2}$$

$$\widehat{\mu_3} = -g_{11}\sigma\widehat{\alpha_1} - g_{12}\sigma\widehat{\alpha_2}, \qquad \widehat{\mu_4} = -t_{21}\sigma\widehat{\alpha_1} - t_{22}\sigma\widehat{\alpha_2}$$

$$\widehat{\mu_5} = -\varphi_2 D_1\sigma\widehat{\alpha_1} - \varphi_2\sigma\widehat{\alpha_2}, \qquad \widehat{\mu_6} = -g_{21}\sigma\widehat{\alpha_1} - g_{22}\sigma\widehat{\alpha_2}$$

$$\widehat{\phi}_1 = (\beta\delta_{11}+1)a_{1,t-1} + \beta\delta_{12}a_{2,t-1} + \beta\Delta_1, \qquad \widehat{\phi}_2 = (\beta\delta_{22}+1)a_{2,t-1} + \beta\delta_{21}a_{1,t-1} + \beta\Delta_1$$

$$\widehat{\psi_1} = g_{11}\widehat{\phi}_1 + g_{12}\widehat{\phi}_2 + j_{11}c_1 + j_{12}c_2, \qquad \widehat{\psi_2} = g_{21}\widehat{\phi}_1 + g_{22}\widehat{\phi}_2 + j_{21}c_1 + j_{22}c_2$$

$$\widehat{\nu_1} = -2(b_p+\theta_p) + 2\widehat{\kappa_3}(\widehat{\kappa_1} - \widehat{\kappa_2}) + 2\widehat{\kappa_6}(\widehat{\kappa_4} - \widehat{\kappa_5}), \qquad \widehat{\nu_2} = 2\theta_p - 2\widehat{\kappa_3}(\widehat{\kappa_1} - \widehat{\kappa_2}) - 2\widehat{\kappa_6}(\widehat{\kappa_4} - \widehat{\kappa_5})$$

$$\widehat{\nu_3} = (b_s+\theta_s) + 2\widehat{\mu_3}(\widehat{\kappa_1} - \widehat{\kappa_2}) + 2\widehat{\mu_6}(\widehat{\kappa_4} - \widehat{\kappa_5}), \qquad \widehat{\nu_4} = -\theta_s - 2\widehat{\mu_3}(\widehat{\kappa_1} - \widehat{\kappa_2}) - 2\widehat{\mu_6}(\widehat{\kappa_4} - \widehat{\kappa_5})$$

$$\widehat{\nu_5} = a_{1,t} + \widehat{\psi_1}(\widehat{\kappa_1} - \widehat{\kappa_2}) + \widehat{\kappa_3}\widehat{\tau_1} + \widehat{\psi_2}(\widehat{\kappa_4} - \widehat{\kappa_5}) + \widehat{\kappa_6}\widehat{\tau_2}$$

$$\widehat{\vartheta}_1 = 2\theta_p - 2\widehat{\kappa_3}(\widehat{\kappa_1} - \widehat{\kappa_2}) - 2\widehat{\kappa_6}(\widehat{\kappa_4} - \widehat{\kappa_5}),$$

$$\widehat{\vartheta}_2 = -2(b_p+\theta_p) + 2\widehat{\kappa_3}(\widehat{\kappa_1} - \widehat{\kappa_2}) + 2\widehat{\kappa_6}(\widehat{\kappa_4} - \widehat{\kappa_5}),$$

$$\widehat{\vartheta}_3 = -\theta_s - 2\widehat{\mu_3}(\widehat{\kappa_1} - \widehat{\kappa_2}) - 2\widehat{\mu_6}(\widehat{\kappa_4} - \widehat{\kappa_5}),$$

$$\widehat{\vartheta}_4 = (b_s+\theta_s) + 2\widehat{\mu_3}(\widehat{\kappa_1} - \widehat{\kappa_2}) + 2\widehat{\mu_6}(\widehat{\kappa_4} - \widehat{\kappa_5}),$$

$$\widehat{\vartheta}_5 = a_{2,t} - \widehat{\psi_1}(\widehat{\kappa_1} - \widehat{\kappa_2}) - \widehat{\kappa_3}\widehat{\tau_1} - \widehat{\psi_2}(\widehat{\kappa_4} - \widehat{\kappa_5}) - \widehat{\kappa_6}\widehat{\tau_2}$$

B.4 Parameters Specifications for Equation (50) and (51)

$$\kappa_{11} = \frac{\widetilde{\nu_{51}}\widetilde{\rho_4} - \widetilde{\phi_{51}}\widetilde{\rho_1}}{\widetilde{\rho_1}\widetilde{\rho_2} - \widetilde{\rho_3}\widetilde{\rho_4}}, \quad \kappa_{12} = \frac{\widetilde{\nu_{52}}\widetilde{\rho_4} - \widetilde{\phi_{52}}\widetilde{\rho_1}}{\widetilde{\rho_1}\widetilde{\rho_2} - \widetilde{\rho_3}\widetilde{\rho_4}}$$

$$\kappa_{21} = \frac{\widetilde{\nu_{51}}\widetilde{\rho_2} - \widetilde{\phi_{51}}\widetilde{\rho_3}}{\widetilde{\rho_3}\widetilde{\rho_4} - \widetilde{\rho_1}\widetilde{\rho_2}}, \quad \kappa_{22} = \frac{\widetilde{\nu_{52}}\widetilde{\rho_2} - \widetilde{\phi_{52}}\widetilde{\rho_3}}{\widetilde{\rho_3}\widetilde{\rho_4} - \widetilde{\rho_1}\widetilde{\rho_2}}$$

$$\nu_{11} = \frac{\widetilde{\nu_{53}}\widetilde{\rho_4} - \widetilde{\phi_{53}}\widetilde{\rho_1}}{\widetilde{\rho_1}\widetilde{\rho_2} - \widetilde{\rho_3}\widetilde{\rho_4}}, \quad \nu_{12} = \frac{\widetilde{\nu_{54}}\widetilde{\rho_4} - \widetilde{\phi_{54}}\widetilde{\rho_1}}{\widetilde{\rho_1}\widetilde{\rho_2} - \widetilde{\rho_3}\widetilde{\rho_4}}$$

$$\nu_{21} = \frac{\widetilde{\nu_{53}}\widetilde{\rho_2} - \widetilde{\phi_{53}}\widetilde{\rho_3}}{\widetilde{\rho_3}\widetilde{\rho_4} - \widetilde{\rho_1}\widetilde{\rho_2}}, \quad \nu_{22} = \frac{\widetilde{\nu_{54}}\widetilde{\rho_2} - \widetilde{\phi_{54}}\widetilde{\rho_3}}{\widetilde{\rho_3}\widetilde{\rho_4} - \widetilde{\rho_1}\widetilde{\rho_2}}$$

$$\vartheta_{11} = \frac{\widetilde{\lambda_{51}}\widetilde{\rho_8} - \widetilde{\theta_{51}}\widetilde{\rho_5}}{\widetilde{\rho_5}\widetilde{\rho_6} - \widetilde{\rho_7}\widetilde{\rho_8}}, \quad \vartheta_{12} = \frac{\widetilde{\lambda_{52}}\widetilde{\rho_8} - \widetilde{\theta_{52}}\widetilde{\rho_5}}{\widetilde{\rho_5}\widetilde{\rho_6} - \widetilde{\rho_7}\widetilde{\rho_8}}$$

$$\vartheta_{21} = \frac{\widetilde{\xi_{51}}\widetilde{\rho_{12}} - \widetilde{\varphi_{51}}\widetilde{\rho_9}}{\widetilde{\rho_9}\widetilde{\rho_{10}} - \widetilde{\rho_{11}}\widetilde{\rho_{12}}}, \quad \vartheta_{22} = \frac{\widetilde{\xi_{52}}\widetilde{\rho_{12}} - \widetilde{\varphi_{52}}\widetilde{\rho_9}}{\widetilde{\rho_9}\widetilde{\rho_{10}} - \widetilde{\rho_{11}}\widetilde{\rho_{12}}}$$

$$\varsigma_{11} = \frac{\widetilde{\lambda_{53}}\widetilde{\rho_8} - \widetilde{\theta_{53}}\widetilde{\rho_5}}{\widetilde{\rho_5}\widetilde{\rho_6} - \widetilde{\rho_7}\widetilde{\rho_8}}, \quad \varsigma_{12} = \frac{\widetilde{\lambda_{54}}\widetilde{\rho_8} - \widetilde{\theta_{52}}\widetilde{\rho_5}}{\widetilde{\rho_5}\widetilde{\rho_6} - \widetilde{\rho_7}\widetilde{\rho_8}}$$

$$\varsigma_{21} = \frac{\widetilde{\xi_{53}}\widetilde{\rho_{12}} - \widetilde{\varphi_{53}}\widetilde{\rho_9}}{\widetilde{\rho_9}\widetilde{\rho_{10}} - \widetilde{\rho_{11}}\widetilde{\rho_{12}}}, \quad \varsigma_{22} = \frac{\widetilde{\xi_{54}}\widetilde{\rho_{12}} - \widetilde{\varphi_{54}}\widetilde{\rho_9}}{\widetilde{\rho_9}\widetilde{\rho_{10}} - \widetilde{\rho_{11}}\widetilde{\rho_{12}}}$$

where,

$$\widetilde{\nu_{51}} = (\widetilde{\delta_{51}}\widetilde{\alpha_1} - \tilde{\delta_1}\widetilde{\alpha_{51}})(\tilde{\beta_2}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_2}) - (\tilde{\delta_2}\widetilde{\alpha_1} - \tilde{\delta_1}\widetilde{\alpha_2})(\widetilde{\beta_{51}}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_{51}})$$

$$\widetilde{\nu_{52}} = (\widetilde{\delta_{52}}\widetilde{\alpha_1} - \tilde{\delta_1}\widetilde{\alpha_{52}})(\tilde{\beta_2}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_2}) - (\tilde{\delta_2}\widetilde{\alpha_1} - \tilde{\delta_1}\widetilde{\alpha_2})(\widetilde{\beta_{52}}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_{52}})$$

$$\widetilde{\nu_{53}} = (\widetilde{\delta_{53}}\widetilde{\alpha_1} - \tilde{\delta_1}\widetilde{\alpha_{53}})(\tilde{\beta_2}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_2}) - (\tilde{\delta_2}\widetilde{\alpha_1} - \tilde{\delta_1}\widetilde{\alpha_2})(\widetilde{\beta_{53}}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_{53}})$$

$$\widetilde{\nu_{54}} = (\widetilde{\delta_{54}}\widetilde{\alpha_1} - \tilde{\delta_1}\widetilde{\alpha_{54}})(\tilde{\beta_2}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_2}) - (\tilde{\delta_2}\widetilde{\alpha_1} - \tilde{\delta_1}\widetilde{\alpha_2})(\widetilde{\beta_{54}}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_{54}})$$

$$\widetilde{\phi_{51}} = (\widetilde{\gamma_{51}}\widetilde{\alpha_1} - \tilde{\gamma_1}\widetilde{\alpha_{51}})(\tilde{\beta_2}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_2}) - (\tilde{\gamma_2}\widetilde{\alpha_1} - \tilde{\gamma_1}\widetilde{\alpha_2})(\widetilde{\beta_{51}}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_{51}})$$

$$\widetilde{\phi_{52}} = (\widetilde{\gamma_{52}}\widetilde{\alpha_1} - \tilde{\gamma_1}\widetilde{\alpha_{52}})(\tilde{\beta_2}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_2}) - (\tilde{\gamma_2}\widetilde{\alpha_1} - \tilde{\gamma_1}\widetilde{\alpha_2})(\widetilde{\beta_{52}}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_{52}})$$

$$\widetilde{\phi_{53}} = (\widetilde{\gamma_{53}}\widetilde{\alpha_1} - \tilde{\gamma_1}\widetilde{\alpha_{53}})(\tilde{\beta_2}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_2}) - (\tilde{\gamma_2}\widetilde{\alpha_1} - \tilde{\gamma_1}\widetilde{\alpha_2})(\widetilde{\beta_{53}}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_{53}})$$

$$\widetilde{\phi_{54}} = (\widetilde{\gamma_{54}}\widetilde{\alpha_1} - \tilde{\gamma_1}\widetilde{\alpha_{54}})(\tilde{\beta_2}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_2}) - (\tilde{\gamma_2}\widetilde{\alpha_1} - \tilde{\gamma_1}\widetilde{\alpha_2})(\widetilde{\beta_{54}}\widetilde{\alpha_1} - \tilde{\beta_1}\widetilde{\alpha_{54}})$$

$$\widetilde{\xi_{51}} = (\widetilde{\delta_{51}}\widetilde{\alpha_4} - \tilde{\delta_4}\widetilde{\alpha_{51}})(\tilde{\beta_2}\widetilde{\alpha_4} - \tilde{\beta_4}\widetilde{\alpha_2}) - (\tilde{\delta_2}\widetilde{\alpha_4} - \tilde{\delta_4}\widetilde{\alpha_2})(\widetilde{\beta_{51}}\widetilde{\alpha_4} - \tilde{\beta_4}\widetilde{\alpha_{51}})$$

$$\widetilde{\xi_{52}} = (\widetilde{\delta_{52}}\widetilde{\alpha_4} - \tilde{\delta_4}\widetilde{\alpha_{52}})(\tilde{\beta_2}\widetilde{\alpha_4} - \tilde{\beta_4}\widetilde{\alpha_2}) - (\tilde{\delta_2}\widetilde{\alpha_4} - \tilde{\delta_4}\widetilde{\alpha_2})(\widetilde{\beta_{52}}\widetilde{\alpha_4} - \tilde{\beta_4}\widetilde{\alpha_{52}})$$

$$\widetilde{\xi_{53}} = (\widetilde{\delta_{53}}\widetilde{\alpha_4} - \tilde{\delta_4}\widetilde{\alpha_{53}})(\tilde{\beta_2}\widetilde{\alpha_4} - \tilde{\beta_4}\widetilde{\alpha_2}) - (\tilde{\delta_2}\widetilde{\alpha_4} - \tilde{\delta_4}\widetilde{\alpha_2})(\widetilde{\beta_{53}}\widetilde{\alpha_4} - \tilde{\beta_4}\widetilde{\alpha_{53}})$$

$$\widetilde{\xi_{54}} = (\widetilde{\delta_{54}}\widetilde{\alpha_4} - \tilde{\delta_4}\widetilde{\alpha_{54}})(\tilde{\beta_2}\widetilde{\alpha_4} - \tilde{\beta_4}\widetilde{\alpha_2}) - (\tilde{\delta_2}\widetilde{\alpha_4} - \tilde{\delta_4}\widetilde{\alpha_2})(\widetilde{\beta_{54}}\widetilde{\alpha_4} - \tilde{\beta_4}\widetilde{\alpha_{54}})$$

$$\widetilde{\varphi_{51}} = (\widetilde{\gamma_{51}}\widetilde{\alpha_4} - \widetilde{\gamma_4}\widetilde{\alpha_{51}})(\widetilde{\beta_2}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_2}) - (\widetilde{\gamma_2}\widetilde{\alpha_4} - \widetilde{\gamma_4}\widetilde{\alpha_2})(\widetilde{\beta_{51}}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_{51}})$$

$$\widetilde{\varphi_{52}} = (\widetilde{\gamma_{52}}\widetilde{\alpha_4} - \widetilde{\gamma_4}\widetilde{\alpha_{52}})(\widetilde{\beta_2}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_2}) - (\widetilde{\gamma_2}\widetilde{\alpha_4} - \widetilde{\gamma_4}\widetilde{\alpha_2})(\widetilde{\beta_{52}}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_{52}})$$

$$\widetilde{\varphi_{53}} = (\widetilde{\gamma_{53}}\widetilde{\alpha_4} - \widetilde{\gamma_4}\widetilde{\alpha_{53}})(\widetilde{\beta_2}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_2}) - (\widetilde{\gamma_2}\widetilde{\alpha_4} - \widetilde{\gamma_4}\widetilde{\alpha_2})(\widetilde{\beta_{53}}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_{53}})$$

$$\widetilde{\varphi_{54}} = (\widetilde{\gamma_{54}}\widetilde{\alpha_4} - \widetilde{\gamma_4}\widetilde{\alpha_{54}})(\widetilde{\beta_2}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_2}) - (\widetilde{\gamma_2}\widetilde{\alpha_4} - \widetilde{\gamma_4}\widetilde{\alpha_2})(\widetilde{\beta_{54}}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_{54}})$$

$$\widetilde{\lambda_{51}} = (\widetilde{\delta_{51}}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_{51}})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4}) - (\widetilde{\delta_4}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_4})(\widetilde{\beta_{51}}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_{51}})$$

$$\widetilde{\lambda_{52}} = (\widetilde{\delta_{52}}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_{52}})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4}) - (\widetilde{\delta_4}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_4})(\widetilde{\beta_{52}}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_{52}})$$

$$\widetilde{\lambda_{53}} = (\widetilde{\delta_{53}}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_{53}})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4}) - (\widetilde{\delta_4}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_4})(\widetilde{\beta_{53}}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_{53}})$$

$$\widetilde{\lambda_{54}} = (\widetilde{\delta_{54}}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_{54}})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4}) - (\widetilde{\delta_4}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_4})(\widetilde{\beta_{54}}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_{54}})$$

$$\widetilde{\theta_{51}} = (\widetilde{\gamma_{51}}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_{51}})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4}) - (\widetilde{\gamma_4}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_4})(\widetilde{\beta_{51}}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_{51}})$$

$$\widetilde{\theta_{52}} = (\widetilde{\gamma_{52}}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_{52}})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4}) - (\widetilde{\gamma_4}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_4})(\widetilde{\beta_{52}}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_{52}})$$

$$\widetilde{\theta_{53}} = (\widetilde{\gamma_{53}}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_{53}})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4}) - (\widetilde{\gamma_4}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_4})(\widetilde{\beta_{53}}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_{53}})$$

$$\widetilde{\theta_{54}} = (\widetilde{\gamma_{54}}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_{54}})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4}) - (\widetilde{\gamma_4}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_4})(\widetilde{\beta_{54}}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_{54}})$$

$$\widetilde{\rho_1} = (\widetilde{\delta_4}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_4})(\widetilde{\beta_2}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_2}) - (\widetilde{\delta_2}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_2})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4})$$

$$\widetilde{\rho_2} = (\widetilde{\gamma_3}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_3})(\widetilde{\beta_2}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_2}) - (\widetilde{\gamma_2}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_2})(\widetilde{\beta_3}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_3})$$

$$\widetilde{\rho_3} = (\widetilde{\delta_3}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_3})(\widetilde{\beta_2}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_2}) - (\widetilde{\delta_2}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_2})(\widetilde{\beta_3}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_3})$$

$$\widetilde{\rho_4} = (\widetilde{\gamma_4}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_4})(\widetilde{\beta_2}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_2}) - (\widetilde{\gamma_2}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_2})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4})$$

$$\widetilde{\rho_5} = (\widetilde{\delta_3}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_3})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4}) - (\widetilde{\delta_4}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_4})(\widetilde{\beta_3}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_3})$$

$$\widetilde{\rho_6} = (\widetilde{\gamma_2}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_2})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4}) - (\widetilde{\gamma_4}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_4})(\widetilde{\beta_2}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_2})$$

$$\widetilde{\rho_7} = (\widetilde{\delta_2}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_2})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4}) - (\widetilde{\delta_4}\widetilde{\alpha_1} - \widetilde{\delta_1}\widetilde{\alpha_4})(\widetilde{\beta_2}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_2})$$

$$\widetilde{\rho_8} = (\widetilde{\gamma_3}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_3})(\widetilde{\beta_4}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_4}) - (\widetilde{\gamma_4}\widetilde{\alpha_1} - \widetilde{\gamma_1}\widetilde{\alpha_4})(\widetilde{\beta_3}\widetilde{\alpha_1} - \widetilde{\beta_1}\widetilde{\alpha_3})$$

$$\widetilde{\rho_9} = (\widetilde{\delta_3}\widetilde{\alpha_4} - \widetilde{\delta_4}\widetilde{\alpha_3})(\widetilde{\beta_2}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_2}) - (\widetilde{\delta_2}\widetilde{\alpha_4} - \widetilde{\delta_4}\widetilde{\alpha_2})(\widetilde{\beta_3}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_3})$$

$$\widetilde{\rho_{10}} = (\widetilde{\gamma_1}\widetilde{\alpha_4} - \widetilde{\gamma_4}\widetilde{\alpha_1})(\widetilde{\beta_2}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_2}) - (\widetilde{\gamma_2}\widetilde{\alpha_4} - \widetilde{\gamma_4}\widetilde{\alpha_2})(\widetilde{\beta_1}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_1})$$

$$\widetilde{\rho_{11}} = (\widetilde{\delta_1}\widetilde{\alpha_4} - \widetilde{\delta_4}\widetilde{\alpha_1})(\widetilde{\beta_2}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_2}) - (\widetilde{\delta_2}\widetilde{\alpha_4} - \widetilde{\delta_4}\widetilde{\alpha_2})(\widetilde{\beta_1}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_1})$$

$$\widetilde{\rho_{12}} = (\widetilde{\gamma_3}\widetilde{\alpha_4} - \widetilde{\gamma_4}\widetilde{\alpha_3})(\widetilde{\beta_4}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_2}) - (\widetilde{\gamma_2}\widetilde{\alpha_4} - \widetilde{\gamma_4}\widetilde{\alpha_2})(\widetilde{\beta_3}\widetilde{\alpha_4} - \widetilde{\beta_4}\widetilde{\alpha_3})$$

$$\widetilde{\alpha_1} = \widetilde{\eta_{11}} + \widetilde{\kappa_{11}}\widetilde{\tau_{11}} + \widetilde{\psi_{11}}[\widetilde{\mu_{11}} - (\widetilde{\kappa_{11}}/4\eta_1)(b_s + \theta_s)^2] - 2(\delta_{12} - \delta_{11})^2(\gamma\widetilde{\vartheta}_1 - \sigma)\gamma\widetilde{\varrho_1}$$

$$\widetilde{\alpha_2} = \widetilde{\eta_{12}} + \widetilde{\kappa_{11}}\widetilde{\tau_{12}} + \widetilde{\psi_{12}}[\widetilde{\mu_{11}} - (\widetilde{\kappa_{11}}/4\eta_1)(b_s + \theta_s)^2] - 2(\delta_{12} - \delta_{11})^2(\gamma\widetilde{\vartheta}_2 - \sigma)\gamma\widetilde{\varrho_1}$$

$$\widetilde{\alpha_3} = 2\widetilde{\pi_{11}} + \widetilde{\kappa_{11}}\widetilde{\mu_{11}} + \widetilde{\kappa_{11}}[\widetilde{\mu_{11}} - (\widetilde{\kappa_{11}}/4\eta_1)(b_s + \theta_s)^2] - 2(\delta_{12} - \delta_{11})^2\gamma^2\widetilde{\varrho_1}^2$$

$$\widetilde{\alpha_4} = \widetilde{\pi_{12}} + \widetilde{\kappa_{11}}\widetilde{\mu_{12}} + \widetilde{\kappa_{12}}[\widetilde{\mu_{11}} - (\widetilde{\kappa_{11}}/4\eta_1)(b_s + \theta_s)^2] - 2(\delta_{12} - \delta_{11})^2\gamma^2\widetilde{\varrho_1}\widetilde{\varrho_2}$$

$$\widetilde{\alpha_{51}} = \widetilde{\upsilon_{11}} + \widetilde{\kappa_{11}}\widehat{\sigma_{11}} + \widehat{\alpha_{11}}[\widetilde{\mu_{11}} - (\widetilde{\kappa_{11}}/4\eta_1)(b_s + \theta_s)^2]$$

$$\widetilde{\alpha_{52}} = \widetilde{\upsilon_{12}} + \widetilde{\kappa_{11}}\widehat{\sigma_{12}} + \widehat{\alpha_{12}}[\widetilde{\mu_{11}} - (\widetilde{\kappa_{11}}/4\eta_1)(b_s + \theta_s)^2]$$

$$\widetilde{\alpha_{53}} = \widetilde{\omega_{11}} - \widehat{\pi_{11}} + \widetilde{\kappa_{11}}\widehat{\theta_{11}} + (\widehat{\omega_{11}} - 1)[\widetilde{\mu_{11}} - (\widetilde{\kappa_{11}}/4\eta_1)(b_s + \theta_s)^2]$$

$$\widetilde{\alpha_{54}} = \widetilde{\omega_{12}} + \widetilde{\kappa_{11}}\widehat{\theta_{12}} + \widehat{\omega_{12}}[\widetilde{\mu_{11}} - (\widetilde{\kappa_{11}}/4\eta_1)(b_s + \theta_s)^2]$$

$$\widetilde{\beta}_1 = -\eta_1 + \widetilde{\psi_{11}}\widetilde{\tau_{11}} + \widetilde{\psi_{11}}[\widetilde{\tau_{11}} - (\widetilde{\psi_{11}}/4\eta_1)(b_s + \theta_s)^2] - 2(\delta_{12} - \delta_{11})^2(\gamma\widetilde{\vartheta}_1 - \sigma)^2$$

$$\widetilde{\beta}_2 = \widetilde{\psi_{11}}\widetilde{\tau_{12}} + \widetilde{\psi_{12}}[\widetilde{\tau_{11}} - (\widetilde{\psi_{11}}/4\eta_1)(b_s + \theta_s)^2] - 2(\delta_{12} - \delta_{11})^2(\gamma\widetilde{\vartheta}_1 - \sigma)(\gamma\widetilde{\vartheta}_2 - \sigma)$$

$$\widetilde{\beta}_3 = \widetilde{\eta_{11}} + \widetilde{\psi_{11}}\widetilde{\mu_{11}} + \widetilde{\kappa_{11}}[\widetilde{\tau_{11}} - (\widetilde{\psi_{11}}/4\eta_1)(b_s + \theta_s)^2] - 2(\delta_{12} - \delta_{11})^2(\gamma\widetilde{\vartheta}_1 - \sigma)\gamma\widetilde{\varrho_1}$$

$$\widetilde{\beta}_4 = \widetilde{\psi_{11}}\widetilde{\mu_{12}} + \widetilde{\kappa_{12}}[\widetilde{\tau_{11}} - (\widetilde{\psi_{11}}/4\eta_1)(b_s + \theta_s)^2] - 2(\delta_{12} - \delta_{11})^2(\gamma\widetilde{\vartheta}_1 - \sigma)\gamma\widetilde{\varrho_2}$$

$$\widetilde{\beta_{51}} = \widetilde{\psi_{11}}\widehat{\sigma_{11}} + \widehat{\alpha_{11}}[\widetilde{\tau 11} - (\widetilde{\psi_{11}}/4\eta_1)(b_s + \theta_s)^2]$$

$$\widetilde{\beta_{52}} = \widetilde{\psi_{11}}\widehat{\sigma_{12}} + \widehat{\alpha_{12}}[\widetilde{\tau_{11}} - (\widetilde{\psi_{11}}/4\eta_1)(b_s + \theta_s)^2]$$

$$\widetilde{\beta_{53}} = -\widehat{\eta_{11}} + \widetilde{\psi_{11}}\widehat{\theta_{11}} + (\widehat{\omega_{11}} - 1)[\widetilde{\tau_{11}} - (\widetilde{\psi_{11}}/4\eta_1)(b_s + \theta_s)^2]$$

$$\widetilde{\beta_{54}} = \widetilde{\psi_{11}}\widehat{\theta_{12}} + \widehat{\omega_{12}}[\widetilde{\tau_{11}} - (\widetilde{\psi_{11}}/4\eta_1)(b_s + \theta_s)^2]$$

$$\widetilde{\gamma}_1 = \widetilde{\eta_{21}} + \widetilde{\kappa_{22}}\widetilde{\tau_{21}} + \widetilde{\psi_{21}}[\widetilde{\mu_{22}} - (\widetilde{\kappa_{22}}/4\eta_2)(b_s + \theta_s)^2] - 2(\delta_{22} - \delta_{21})^2(\gamma\widetilde{\vartheta}_1 - \sigma)\gamma\widetilde{\varrho_2}$$

$$\widetilde{\gamma}_2 = \widetilde{\eta_{22}} + \widetilde{\kappa_{22}}\widetilde{\tau_{22}} + \widetilde{\psi_{22}}[\widetilde{\mu_{22}} - (\widetilde{\kappa_{22}}/4\eta_2)(b_s + \theta_s)^2] - 2(\delta_{22} - \delta_{21})^2(\gamma\widetilde{\vartheta}_2 - \sigma)\gamma\widetilde{\varrho_2}$$

$$\widetilde{\gamma}_3 = \widetilde{\pi_{21}} + \widetilde{\kappa_{22}}\widetilde{\mu_{21}} + \widetilde{\kappa_{21}}[\widetilde{\mu_{22}} - (\widetilde{\kappa_{22}}/4\eta_2)(b_s + \theta_s)^2] - 2(\delta_{22} - \delta_{21})^2\gamma^2\widetilde{\varrho_1}\widetilde{\varrho_2}$$

$$\widetilde{\gamma}_4 = 2\widetilde{\pi_{22}} + \widetilde{\kappa_{22}}\widetilde{\mu_{22}} + \widetilde{\kappa_{22}}[\widetilde{\mu_{22}} - (\widetilde{\kappa_{22}}/4\eta_2)(b_s + \theta_s)^2] - 2(\delta_{22} - \delta_{21})^2\gamma^2\widetilde{\varrho_2}^2$$

$$\widetilde{\gamma_{51}} = \widetilde{\upsilon_{21}} + \widetilde{\kappa_{22}}\widehat{\sigma_{21}} + \widehat{\alpha_{21}}[\widetilde{\mu_{22}} - (\widetilde{\kappa_{22}}/4\eta_2)(b_s + \theta_s)^2]$$

$$\widetilde{\gamma_{52}} = \widetilde{\upsilon_{22}} + \widetilde{\kappa_{22}}\widehat{\sigma_{22}} + \widehat{\alpha_{22}}[\widetilde{\mu_{22}} - (\widetilde{\kappa_{22}}/4\eta_2)(b_s + \theta_s)^2]$$

$$\widetilde{\gamma_{53}} = \widetilde{\omega_{21}} + \widetilde{\kappa_{22}}\widehat{\theta_{21}} + \widehat{\omega_{21}}[\widetilde{\mu_{22}} - (\widetilde{\kappa_{22}}/4\eta_2)(b_s + \theta_s)^2]$$

$$\widetilde{\gamma_{54}} = \widetilde{\omega_{22}} - \widehat{\pi_{22}} + \widetilde{\kappa_{22}}\widehat{\theta_{22}} + (\widehat{\omega_{22}} - 1)[\widetilde{\mu_{22}} - (\widetilde{\kappa_{22}}/4\eta_2)(b_s + \theta_s)^2]$$

$$\widetilde{\delta_1} = \widetilde{\psi_{22}}\widetilde{\tau_{21}} + \widetilde{\psi_{21}}[\widetilde{\tau_{22}} - (\widetilde{\kappa_{22}}/4\eta_2)(b_s+\theta_s)^2] - 2(\delta_{22}-\delta_{21})^2(\gamma\widetilde{\vartheta}_1 - \sigma)(\gamma\widetilde{\vartheta}_2 - \sigma)$$

$$\widetilde{\delta_2} = -\eta_1 + \widetilde{\psi_{22}}\widetilde{\tau_{22}} + \widetilde{\psi_{22}}[\widetilde{\tau_{22}} - (\widetilde{\kappa_{22}}/4\eta_2)(b_s+\theta_s)^2] - 2(\delta_{22}-\delta_{21})^2(\gamma\widetilde{\vartheta}_2 - \sigma)^2$$

$$\widetilde{\delta_3} = \widetilde{\psi_{22}}\widetilde{\mu_{21}} + \widetilde{\kappa_{21}}[\widetilde{\tau_{22}} - (\widetilde{\kappa_{22}}/4\eta_2)(b_s+\theta_s)^2] - 2(\delta_{22}-\delta_{21})^2(\gamma\widetilde{\vartheta}_2 + \sigma)\gamma\widetilde{\varrho_1}$$

$$\widetilde{\delta_4} = \widetilde{\eta_{22}} + \widetilde{\psi_{22}}\widetilde{\mu_{22}} + \widetilde{\kappa_{22}}[\widetilde{\tau_{22}} - (\widetilde{\kappa_{22}}/4\eta_2)(b_s+\theta_s)^2] - 2(\delta_{22}-\delta_{21})^2(\gamma\widetilde{\vartheta}_2 + \sigma)\gamma\widetilde{\varrho_2}$$

$$\widetilde{\delta_{51}} = \widetilde{\psi_{22}}\widehat{\sigma_{21}} + \widehat{\alpha_{21}}[\widetilde{\tau 22} - (\widetilde{\psi_{22}}/4\eta_2)(b_s+\theta_s)^2]$$

$$\widetilde{\delta_{52}} = \widetilde{\psi_{22}}\widehat{\sigma_{22}} + \widehat{\alpha_{22}}[\widetilde{\tau_{22}} - (\widetilde{\psi_{22}}/4\eta_2)(b_s+\theta_s)^2]$$

$$\widetilde{\delta_{53}} = \widetilde{\psi_{22}}\widehat{\theta_{21}} + \widehat{\omega_{21}}[\widetilde{\tau_{22}} - (\widetilde{\psi_{22}}/4\eta_2)(b_s+\theta_s)^2]$$

$$\widetilde{\delta_{54}} = -\widehat{\eta_{22}} + \widetilde{\psi_{22}}\widehat{\theta_{22}} + (\widehat{\omega_{22}} - 1)[\widetilde{\tau_{22}} - (\widetilde{\psi_{22}}/4\eta_2)(b_s+\theta_s)^2]$$

$$\widetilde{\eta_{11}} = -(b_p+\theta_p)\widehat{\nu}(\widehat{\vartheta}_3\widehat{\nu_2} - \widehat{\vartheta}_2\widehat{\nu_3}) - \theta_p\widehat{\nu}(\widehat{\vartheta}_3\widehat{\nu_1} - \widehat{\vartheta}_1\widehat{\nu_3}) + (b_s+\theta_s)$$

$$\widetilde{\eta_{12}} = -(b_p+\theta_p)\widehat{\nu}(\widehat{\vartheta}_4\widehat{\nu_2} - \widehat{\vartheta}_2\widehat{\nu_4}) - \theta_p\widehat{\nu}(\widehat{\vartheta}_4\widehat{\nu_1} - \widehat{\vartheta}_1\widehat{\nu_4}) - \theta_s$$

$$\widetilde{\eta_{21}} = \theta_p\widehat{\nu}(\widehat{\vartheta}_3\widehat{\nu_2} - \widehat{\vartheta}_2\widehat{\nu_3}) + (b_p+\theta_p)\widehat{\nu}(\widehat{\vartheta}_3\widehat{\nu_1} - \widehat{\vartheta}_1\widehat{\nu_3}) - \theta_s$$

$$\widetilde{\eta_{22}} = \theta_p\widehat{\nu}(\widehat{\vartheta}_4\widehat{\nu_2} - \widehat{\vartheta}_2\widehat{\nu_4}) + (b_p+\theta_p)\widehat{\nu}(\widehat{\vartheta}_4\widehat{\nu_1} - \widehat{\vartheta}_1\widehat{\nu_4}) + (b_s+\theta_s)$$

$$\widetilde{\pi_{11}} = (b_p+\theta_p)\widehat{\nu}(\theta_p\widehat{\nu_2} + \widehat{\vartheta}_2(b_p+\theta_p)) + \theta_p\widehat{\nu}(\theta_p\widehat{\nu_1} + \widehat{\vartheta}_1(b_p+\theta_p))$$

$$\widetilde{\pi_{12}} = -(b_p+\theta_p)\widehat{\nu}((b_p+\theta_p)\widehat{\nu_2} + \widehat{\vartheta}_2\theta_p) - \theta_p\widehat{\nu}((b_p+\theta_p)\widehat{\nu_1} + \widehat{\vartheta}_1\theta_p)$$

$$\widetilde{\pi_{21}} = -\theta_p\widehat{\nu}(\theta_p\widehat{\nu_2} + \widehat{\vartheta}_2(b_p+\theta_p)) - (b_p+\theta_p)\widehat{\nu}(\theta_p\widehat{\nu_1} + \widehat{\vartheta}_1(b_p+\theta_p))$$

$$\widetilde{\pi_{22}} = \theta_p\widehat{\nu}((b_p+\theta_p)\widehat{\nu_2} + \widehat{\vartheta}_2\theta_p) + (b_p+\theta_p)\widehat{\nu}((b_p+\theta_p)\widehat{\nu_1} + \widehat{\vartheta}_1\theta_p)$$

$$\widetilde{\chi_{11}} = \varphi_1\gamma(\widehat{\alpha_1} + D_2\widehat{\alpha_2}) = -\widetilde{\chi_{12}}$$

$$\widetilde{\chi_{21}} = \varphi_2\gamma(D_1\widehat{\alpha_1} + \widehat{\alpha_2}) = -\widetilde{\chi_{22}}$$

$$\widetilde{\varpi_{11}} = \varphi_1\sigma(\widehat{\alpha_1} + D_2\widehat{\alpha_2}) = -\widetilde{\varpi_{12}}$$

$$\widetilde{\varpi_{21}} = \varphi_2\sigma(D_1\widehat{\alpha_1} + \widehat{\alpha_2}) = -\widetilde{\varpi_{22}}$$

$$\widetilde{\psi_{11}} = \widetilde{\chi_{11}}\widehat{\nu}(\widehat{\vartheta}_3\widehat{\nu_2} - \widehat{\vartheta}_2\widehat{\nu_3}) - \widetilde{\chi_{12}}\widehat{\nu}(\widehat{\vartheta}_3\widehat{\nu_1} - \widehat{\vartheta}_1\widehat{\nu_3}) + \widetilde{\varpi_{11}}$$

$$\widetilde{\psi_{12}} = \widetilde{\chi_{11}}\widehat{\nu}(\widehat{\vartheta}_4\widehat{\nu_2} - \widehat{\vartheta}_2\widehat{\nu_4}) - \widetilde{\chi_{12}}\widehat{\nu}(\widehat{\vartheta}_4\widehat{\nu_1} - \widehat{\vartheta}_1\widehat{\nu_4}) + \widetilde{\varpi_{12}}$$

$$\widetilde{\psi_{21}} = \widetilde{\chi_{21}}\widehat{\nu}(\widehat{\vartheta}_3\widehat{\nu_2} - \widehat{\vartheta}_2\widehat{\nu_3}) - \widetilde{\chi_{22}}\widehat{\nu}(\widehat{\vartheta}_3\widehat{\nu_1} - \widehat{\vartheta}_1\widehat{\nu_3}) + \widetilde{\varpi_{21}}$$

$$\widetilde{\psi_{22}} = \widetilde{\chi_{21}}\widehat{\nu}(\widehat{\vartheta}_4\widehat{\nu_2} - \widehat{\vartheta}_2\widehat{\nu_4}) - \widetilde{\chi_{22}}\widehat{\nu}(\widehat{\vartheta}_4\widehat{\nu_1} - \widehat{\vartheta}_1\widehat{\nu_4}) + \widetilde{\varpi_{22}}$$

$$\widetilde{\kappa_{11}} = -\widetilde{\chi_{11}}\widehat{\nu}(\theta_p\widehat{\nu_2} + \widehat{\vartheta}_2(b_p + \theta_p)) + \widetilde{\chi_{12}}\widehat{\nu}(\theta_p\widehat{\nu_1} + \widehat{\vartheta}_1(b_p + \theta_p))$$

$$\widetilde{\kappa_{12}} = \widetilde{\chi_{11}}\widehat{\nu}((b_p + \theta_p)\widehat{\nu_2} + \widehat{\vartheta}_2\theta_p) - \widetilde{\chi_{12}}\widehat{\nu}((b_p + \theta_p)\widehat{\nu_1} + \widehat{\vartheta}_1\theta_p)$$

$$\widetilde{\kappa_{21}} = -\widetilde{\chi_{21}}\widehat{\nu}(\theta_p\widehat{\nu_2} + \widehat{\vartheta}_2(b_p + \theta_p)) - \widetilde{\chi_{22}}\widehat{\nu}(\theta_p\widehat{\nu_1} + \widehat{\vartheta}_1(b_p + \theta_p))$$

$$\widetilde{\kappa_{22}} = \widetilde{\chi_{21}}\widehat{\nu}((b_p + \theta_p)\widehat{\nu_2} + \widehat{\vartheta}_2\theta_p) + \widetilde{\chi_{22}}\widehat{\nu}((b_p + \theta_p)\widehat{\nu_1} + \widehat{\vartheta}_1\theta_p)$$

$$\widetilde{\tau_{11}} = \widehat{\kappa_3}\widehat{\nu}(\widehat{\vartheta}_3\widehat{\nu_2} - \widehat{\vartheta}_2\widehat{\nu_3}) + \widehat{\kappa_3}\widehat{\nu}(\widehat{\vartheta}_3\widehat{\nu_1} - \widehat{\vartheta}_1\widehat{\nu_3}) + \widehat{\mu_3}$$

$$\widetilde{\tau_{12}} = \widehat{\kappa_3}\widehat{\nu}(\widehat{\vartheta}_4\widehat{\nu_2} - \widehat{\vartheta}_2\widehat{\nu_4}) + \widehat{\kappa_3}\widehat{\nu}(\widehat{\vartheta}_4\widehat{\nu_1} - \widehat{\vartheta}_1\widehat{\nu_4}) - \widehat{\mu_3}$$

$$\widetilde{\tau_{21}} = \widehat{\kappa_6}\widehat{\nu}(\widehat{\vartheta}_3\widehat{\nu_2} - \widehat{\vartheta}_2\widehat{\nu_3}) + \widehat{\kappa_6}\widehat{\nu}(\widehat{\vartheta}_3\widehat{\nu_1} - \widehat{\vartheta}_1\widehat{\nu_3}) + \widehat{\mu_6}$$

$$\widetilde{\tau_{22}} = \widehat{\kappa_6}\widehat{\nu}(\widehat{\vartheta}_4\widehat{\nu_2} - \widehat{\vartheta}_2\widehat{\nu_4}) + \widehat{\kappa_6}\widehat{\nu}(\widehat{\vartheta}_4\widehat{\nu_1} - \widehat{\vartheta}_1\widehat{\nu_4}) - \widehat{\mu_6}$$

$$\widetilde{\mu_{11}} = -\widehat{\kappa_3}\widehat{\nu}(\theta_p\widehat{\nu_2} + \widehat{\vartheta}_2(b_p + \theta_p)) + \widehat{\kappa_3}\widehat{\nu}(\theta_p\widehat{\nu_1} + \widehat{\vartheta}_1(b_p + \theta_p))$$

$$\widetilde{\mu_{12}} = \widehat{\kappa_3}\widehat{\nu}((b_p + \theta_p)\widehat{\nu_2} + \widehat{\vartheta}_2\theta_p) - \widehat{\kappa_3}\widehat{\nu}((b_p + \theta_p)\widehat{\nu_1} + \widehat{\vartheta}_1\theta_p)$$

$$\widetilde{\mu_{21}} = -\widehat{\kappa_6}\widehat{\nu}(\theta_p\widehat{\nu_2} + \widehat{\vartheta}_2(b_p + \theta_p)) - \widehat{\kappa_6}\widehat{\nu}(\theta_p\widehat{\nu_1} + \widehat{\vartheta}_1(b_p + \theta_p))$$

$$\widetilde{\mu_{22}} = \widehat{\kappa_6}\widehat{\nu}((b_p + \theta_p)\widehat{\nu_2} + \widehat{\vartheta}_2\theta_p) + \widehat{\kappa_6}\widehat{\nu}((b_p + \theta_p)\widehat{\nu_1} + \widehat{\vartheta}_1\theta_p)$$

$$\widetilde{\vartheta}_1 = 2b_p\widehat{\nu}(\widehat{\nu_3} - \widehat{\vartheta}_3), \quad \widetilde{\vartheta}_2 = 2b_p\widehat{\nu}(\widehat{\nu_4} - \widehat{\vartheta}_4)$$

$$\widetilde{\vartheta}_3 = 2b_p\widehat{\nu}(\widehat{\nu_5} - \widehat{\vartheta}_5), \quad \widetilde{\varrho}_1 = -\widetilde{\varrho}_2 = 2b_p\widehat{\nu}(b_p + 2\theta_p)$$

B.5 Proof to Theorem 4.2

The system given in Equation 59 is

$$\mathbf{\Phi(t)} = \mathbf{M\Phi(t-1)}. \tag{121}$$

Let $\mathbf{P}$ be the modal matrix of $\mathbf{M}$. That is, $\mathbf{P}$ is the 4x4 matrix whose 4 columns are the eigenvectors of $\mathbf{M}$. For a given $\mathbf{\Phi(t)}$, we define a vector $\mathbf{z(t)}$ by

$$\mathbf{\Phi(t)} = \mathbf{Pz(t)}. \tag{122}$$

This transformation follows from the fact that any vector $\mathbf{\Phi(t)}$ can be written as a linear combination of its eigenvectors. That is $\mathbf{\Phi(t)}$ can be expressed as

$$\mathbf{\Phi(t)} = z_1(t)\mathbf{e_1} + \mathrm{z}_2(\mathrm{t})\mathbf{e_2} + \mathrm{z}_3(\mathrm{t})\mathbf{e_3} + \mathrm{z}_4(\mathrm{t})\mathbf{e_4}. \tag{123}$$

where $z_i(t), i \in 1,2,3,4$ are scalars. Using the fact that $\mathbf{Me_i} = \lambda_\mathbf{i}\mathbf{e_i}$ (where λ_i is an eigenvalue of $\mathbf{M}$), multiplying the equation above by matrix $\mathbf{M}$ yields

$$\begin{aligned} \mathbf{\Phi(t+1)} &= \mathbf{M\Phi(t)} && (124)\\ &= \lambda_1 z_1(t)\mathbf{e_1} + \lambda_2\mathrm{z}_2(\mathrm{t})\mathbf{e_2} + \lambda_3\mathrm{z}_3(\mathrm{t})\mathbf{e_3} + \lambda_4\mathrm{z}_4(\mathrm{t})\mathbf{e_4}. && (125) \end{aligned}$$

In this new transformation, the original system in Equation 59 can be represented as

$$\mathbf{Pz(t+1)} = \mathbf{MPz(t)}. \tag{126}$$

or, equivalently,

$$\mathbf{z(t+1)} = \mathbf{P}^{-1}\mathbf{MPz(t)}. \tag{127}$$

This defines a new system that is related to the original system by a change of variable. The new system matrix $\mathbf{P}^{-1}\mathbf{MP}$ is equal to $\mathbf{\Lambda}$, where $\mathbf{\Lambda}$ is the diagonal matrix with the eigenvalues of $\mathbf{M}$ on the diagonal. Thus, when written out in detail, Equation 127 becomes

$$\begin{bmatrix} z_1(t+1) \\ z_2(t+1) \\ z_3(t+1) \\ z_4(t+1) \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \end{bmatrix} \tag{128}$$

which explicitly displays the diagonal form obtained by the change of variable.

The state-transition matrix of a constant coefficient discrete-time system at period k is $\mathbf{M}^k$. The system matrix can be calculated by first converting $\mathbf{M}$ to diagonal form as

$$\mathbf{M} = \mathbf{P\Lambda P}^{-1}. \tag{129}$$

which provides a representation of $\mathbf{M}$ in terms of its eigenvalues and eigenvectors. It then follows that for any $k \geq 0$

$$\mathbf{M^k} = \mathbf{P\Lambda^k P}^{-1}. \tag{130}$$

Therefore, calculation of $\mathbf{M^k}$ is transferred to the calculation of Λ^k. Since Λ is diagonal, one finds immediately that

$$\Lambda^k = \begin{bmatrix} \lambda_1^k & 0 & 0 & 0 \\ 0 & \lambda_2^k & 0 & 0 \\ 0 & 0 & \lambda_3^k & 0 \\ 0 & 0 & 0 & \lambda_4^k \end{bmatrix} \tag{131}$$

One can see immediately that if the magnitude of dominant eigenvalue is less than 1 (i.e., $|\lambda_i| < 1$ for all i), as k increases, Λ^k tends toward zero. This corresponds to the system converging over time. On the other hand, if there is at least one eigenvalue with magnitude greater than one ($|\lambda_j| > 1$ for some j), the system of market bases evolution will increases geometrically towards infinity. This corresponds to a divergent system.

B.6 Proof to Theorem 4.3

Let λ be the dominant eigenvalue of a discrete time system. It is possible to express λ in the form

$$\lambda = re^{i\theta} = r(\cos\theta + i\sin\theta). \tag{132}$$

The characteristic response due to this eigenvalue is

$$\lambda^k = r^k e^{ik\theta} = r^k(\cos k\theta + i\sin k\theta). \tag{133}$$

The coefficient that multiplies the associated eigenvector varies according to this characteristic pattern. From the above equation, one can see that if λ is real and positive, the response pattern is the geometric sequence r^k, which increases if $r > 1$ and decreases if $r < 1$. No oscillation will occur with positive eigenvalue since r^k remains positive for any k. However, if λ is negative, the response will be an alternating geometric sequence since r^k switches sign for every step.

If λ is complex, it will appear with its complex conjugate. The real response due to both eigenvalues is of the form $r^k(A\cos k\theta + iB\sin k\theta)$. If $\theta \neq 0$, the expression within the parentheses will change sign a k changes. However, the exact pattern of variation will not be perfectly regular. In our problem, we assume that λ is not complex. Therefore, this irregular oscillation case is excluded from our analysis.

B.7 Range of Parameters Used in Numerical Studies

Parameter	Range
b_p	$\{0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5\}$
θ_p	$\{0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5\}$
b_s	$\{0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5\}$
θ_s	$\{0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5\}$
a_i	$\{40, 60, 80, 100, 120\}$
c_i	$\{2, 4, 6, 8, 10\}$
η_i	$\{2, 4, 6, 8, 10\}$
γ	$\{0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0\}$
σ	$\{0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0\}$
β	$\{0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0\}$

The range of these parameters are based on related literature such as Tsay and Agrawal (2000) [90] and Vilcassin *et al.* (1999) [91] (see Chapter 2 for the review of these papers).

APPENDIX C: APPENDIX FOR CHAPTER 5

C.1 Proof to Lemma 5.1

For each i in (87), adding and subtracting $\int_{z_i}^{B_i} p_i x_i f_i(x_i) dx_i$ to get:

$$\begin{aligned}
E[\Pi_i(\boldsymbol{z}, \boldsymbol{p})] &= \int_{A_i}^{z_i} p_i[\mu_i(p_1,p_2) + x_i] f_i(x_i) dx_i + g_i \underline{\Theta}_i(z_i) + \int_{z_i}^{B_i} p_i[\mu_i(p_1,p_2) + z_i] \\
&\quad - b_i \overline{\Theta}_i(z_i) - w_i[\mu_i(p_1,p_2) + z_i] + \int_{z_i}^{B_i} p_i x_i f_i(x_i) dx_i - \int_{z_i}^{B_i} p_i x_i f_i(x_i) dx_i \\
&= \int_{A_i}^{B_i} p_i[\mu_i(p_1,p_2) + x_i] f_i(x_i) dx_i + \int_{z_i}^{B_i} [z_i - x_i] f_i(x_i) dx_i \\
&\quad - g_i \underline{\Theta}_i(z_i) - b_i \overline{\Theta}_i(z_i) - w_i[\mu_i(p_1,p_2) + z_i] \\
&= p_i \mu_i(p_1,p_2) - p_i \overline{\Theta}_i(z_i) + g_i \underline{\Theta}_i(z_i) - b_i \overline{\Theta}_i(z_i) - w_i \mu_i(p_1,p_2) - w_i z_i \\
&= (p_i - w_i)\mu_i(p_1,p_2) + g_i \underline{\Theta}_i(z_i) - (p_i + b_i)\overline{\Theta}_i(z_i) - w_i z_i
\end{aligned}$$

Now, the last term, $-w_i z_i$, can be expressed as:

$$\begin{aligned}
-w_i z_i = w_i(0 - z_i) &= \int_{A_i}^{B_i} w_i(x_i - z_i) f_i(x_i) dx_i \\
&= w_i \int_{A_i}^{z_i} (x_i - z_i) f_i(x_i) dx_i + w_i \int_{z_i}^{B_i} (x_i - z_i) f_i(x_i) dx_i \\
&= -w_i \underline{\Theta}_i(z_i) + w_i \overline{\Theta}_i(z_i)
\end{aligned}$$

Therefore, for each i in (87)

$$E[\Pi_i(\boldsymbol{z}, \boldsymbol{p})] = (p_i - w_i)\mu_i(p_1,p_2) - (w_i - g_i)\underline{\Theta}_i(z_i) - (p_i + b_i - w_i)\overline{\Theta}_i(z_i)$$

$\square$

C.2 Proof to Proposition 5.1

To solve for p_1 and p_2 in terms of z_1 and z_2, we set the first order derivatives in (92) and (93) to zero:

$$
\begin{aligned}
0 &= a_1 - 2(b_p + \theta_p)p_1 + 2\theta_p p_2 + (b_p + \theta_p)w_1 - \theta_p w_2 - \overline{\Theta}_1(z_1) \\
&= \xi_1 - 2(b_p + \theta_p)p_1 + 2\theta_p p_2 - \overline{\Theta}_1(z_1) \\
{p_1}^*(p_2) &= \frac{\xi_1 + 2\theta_p p_2 - \overline{\Theta}_1(z_1)}{2(b_p + \theta_p)}
\end{aligned}
$$

where $\xi_i = a_i + (b_p + \theta_p)w_i - \theta_p w_j$. Set (93) to zero and substitute the above expression of ${p_1}^*$ into it.

$$
\begin{aligned}
0 &= a_2 - 2(b_p + \theta_p)p_2 + 2\theta_p p_1 + (b_p + \theta_p)w_2 - \theta_p w_1 - \overline{\Theta}_2(z_2) \\
&= \xi_2 - 2(b_p + \theta_p)p_2 - \overline{\Theta}_2(z_2) + \frac{2\theta_p \left[\xi_1 + 2\theta_p p_2 - \overline{\Theta}_1(z_1)\right]}{2(b_p + \theta_p)} \\
\left[2[(b_p + \theta_p)^2 - \theta_p^2]\right] p_2 &= \theta_p(\xi_1 - \overline{\Theta}_1(z_1)) + (b_p + \theta_p)(\xi_2 - \overline{\Theta}_2(z_2)) \\
{p_2}^* &= \frac{\theta_p(\xi_1 - \overline{\Theta}_1(z_1)) + (b_p + \theta_p)(\xi_2 - \overline{\Theta}_2(z_2))}{2[(b_p + \theta_p)^2 - \theta_p^2]}
\end{aligned}
$$

Substituting the expression of ${p_2}^*$ into the optimal ${p_1}^*(p_2)$. We get

$$
{p_1}^* = \frac{\xi_1 - \overline{\Theta}_1(z_1) + 2\theta_p \left[\frac{\theta_p(\xi_1 - \overline{\Theta}_1(z_1)) + (b_p + \theta_p)(\xi_2 - \overline{\Theta}_2(z_2))}{2[(b_p + \theta_p)^2 - \theta_p^2]}\right]}{2(b_p + \theta_p)} \tag{134}
$$

With some algebra rearrangement, we finally get

$$
{p_1}^* = \frac{(b_p + \theta_p)(\xi_1 - \overline{\Theta}_1(z_1)) + \theta_p(\xi_2 - \overline{\Theta}_2(z_2))}{2[(b_p + \theta_p)^2 - \theta_p^2]}
$$

□

C.3 Proof to Proposition 5.2

The proof provided here is a more generalized version of the one given in Petruzzi and Dada (1999)[68]. In their work, they assume that the newsvendor carries only one item. Our model assume that the newsvendor carries two competitive products with a demand function defined differently from the one assumed in their work. To prove proposition 5.2, we note that from (90), (91) and proposition 5.1 we can get the following equations:

$$\begin{aligned}\frac{\partial E[\Pi(\boldsymbol{z},\boldsymbol{p}(\boldsymbol{z}))]}{\partial z_1} &= \left\{\frac{(b_p+\theta_p)(\xi_1-\overline{\Theta}_1(z_1))+\theta_p(\xi_2-\overline{\Theta}_2(z_2))}{2[(b_p+\theta_p)^2-\theta_p^2]}+b_1-g_1\right\}(1-F_1(z_1)) \\ &\quad -(w_1-g_1) \qquad (135)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial E[\Pi(\boldsymbol{z},\boldsymbol{p}(\boldsymbol{z}))]}{\partial z_2} &= \left\{\frac{\theta_p(\xi_1-\overline{\Theta}_1(z_1))+(b_p+\theta_p)(\xi_2-\overline{\Theta}_2(z_2))}{2[(b_p+\theta_p)^2-\theta_p^2]}+b_2-g_2\right\}(1-F_2(z_2)) \\ &\quad -(w_2-g_2) \qquad (136)\end{aligned}$$

We need to find the values of z_1 and z_2 such that the first order optimality conditions above are satisfied. If we let $\Gamma_i(\boldsymbol{z}) = \frac{\partial E[\Pi(\boldsymbol{z},\boldsymbol{p}(\boldsymbol{z}))]}{\partial z_i}$, we then need to find the zeros of $\Gamma_i(\boldsymbol{z})$. We first approach this task by checking $d\Gamma_i(\boldsymbol{z})/dz_j$,

$$\frac{d\Gamma_i(\boldsymbol{z})}{dz_j} = \frac{\theta_p(1-F_1(z_1))(1-F_2(z_2))}{2b_p(b_p+2\theta_p)} \qquad (137)$$

From (137), you can see that $\frac{d\Gamma_i(\boldsymbol{z})}{dz_j}$ is monotone and non-negative with its value equal zero when $z_1 = B_1$ or $z_2 = B_2$. Therefore, there exists only one value of z_j for each value of z_i that $\Gamma_i(\boldsymbol{z})$ is zero. Thus, we only need to worry about the value(s) of z_i that generate(s) zero(s) of $\Gamma_i(\boldsymbol{z})$.

$$\frac{d\Gamma_i(\boldsymbol{z})}{dz_i} = \frac{(b_p+\theta_p)\xi_i+\theta_p\xi_j+2(b_i-g_i)[(b_p+\theta_p)^2-\theta_p^2]}{2[(b_p+\theta_p)^2-\theta_p^2]}(-f_i(z_i))$$

$$
\begin{aligned}
& -\frac{(b_p+\theta_p)\overline{\Theta}_i(z_i)}{2[(b_p+\theta_p)^2-\theta_p^2]}(-f_i(z_i)) + \frac{(b_p+\theta_p)(1-F_i(z_i))^2}{2[(b_p+\theta_p)^2-\theta_p^2]} \\
& -\frac{\theta_p\overline{\Theta}_j(z_j)}{2[(b_p+\theta_p)^2-\theta_p^2]}(-f_i(z_i)) \qquad (138)
\end{aligned}
$$

With some rearrangement,

$$
\begin{aligned}
\frac{d\Gamma_i(\boldsymbol{z})}{dz_i} &= -\frac{f_i(z_i)}{2[(b_p+\theta_p)^2-\theta_p^2]}\Big\{(b_p+\theta_p)\xi_i+\theta_p\xi_j+2(b_i-g_i)[(b_p+\theta_p)^2-\theta_p^2] \\
&\quad -(b_p+\theta_p)\overline{\Theta}_i(z_i)-\theta_p\overline{\Theta}_j(z_j)-\frac{(b_p+\theta_p)(1-F_i(z_i))}{\phi_i(z_i)}\Big\} \qquad (139)
\end{aligned}
$$

where $\phi_i(x)=\frac{f(.)}{1-F(.)}$ denotes the hazard rate.

$$
\begin{aligned}
\frac{d^2\Gamma_i(\boldsymbol{z})}{d{z_i}^2} &= -\frac{f_i(z_i)}{2[(b_p+\theta_p)^2-\theta_p^2]}\left\{(b_p+\theta_p)(1-F_i(z_i))+\frac{(b_p+\theta_p)}{\phi_i(z_i)}f_i(z_i)\right. \\
&\quad \left.+\frac{(b_p+\theta_p)(1-F_i(z_i))[d\phi_i(z_i)/dz_i]}{\phi_i(z_i)^2}\right\}+\left[\frac{d\Gamma_i(\boldsymbol{z})/dz_i}{f_i(z_i)}\right]\frac{df_i(z_i)}{dz_i} \qquad (140)
\end{aligned}
$$

By letting $\frac{d\Gamma_i(\boldsymbol{z})}{dz_i}=0$, we have:

$$
\begin{aligned}
\left.\frac{d^2\Gamma_i(\boldsymbol{z})}{d{z_i}^2}\right|_{\frac{d\Gamma_i(\boldsymbol{z})}{dz_i}=0} &= -\frac{(b_p+\theta_p)f_i(z_i)}{2[(b_p+\theta_p)^2-\theta_p^2](\phi_i(z_i))^2} * \\
&\quad \left\{(1-F_i(z_i))(\phi_i(z_i))^2+f_i(z_i)\phi_i(z_i)+(1-F_i(z_i))\frac{d\phi_i(z_i)}{dz_i}\right\} \\
&= -\frac{(b_p+\theta_p)f_i(z_i)(1-F_i(z_i))}{2[(b_p+\theta_p)^2-\theta_p^2](\phi_i(z_i))^2}\left\{2(\phi_i(z_i))^2+\frac{d\phi_i(z_i)}{dz_i}\right\} \qquad (141)
\end{aligned}
$$

Therefore, if $F_i(.)$ is a distribution satisfying the condition $2(\phi_i(z_i))^2+\frac{d\phi_i(z_i)}{dz_i}>0$, it follows that $\Gamma_i(\boldsymbol{z})$ is either monotone or unimodal in z_i. In either case, it implies that $\Gamma_i(\boldsymbol{z})=\frac{\partial E[\Pi(\boldsymbol{z},\boldsymbol{p}(\boldsymbol{z}))]}{\partial z_i}$ has at most two roots. Furthermore, we can see that $\Gamma_1(B_1,.)=-(w_1-g_1)<0$ and $\Gamma_2(.,B_2)=-(w_2-g_2)<0$. Therefore, if $\Gamma_i(\boldsymbol{z})$ has only one root, there is a change of sign for $\Gamma_i(\boldsymbol{z})$ from positive to negative. This corresponds to a local maximum for $E[\Pi(\boldsymbol{z},\boldsymbol{p}(\boldsymbol{z}))]$. On the other hand, if it has two roots, the larger of the two corresponds to a local maximum and the smaller of the two corresponds to a local minimum of $E[\Pi(\boldsymbol{z},\boldsymbol{p}(\boldsymbol{z}))]$.

In either case, $E[\Pi(\boldsymbol{z}, \boldsymbol{p}(\boldsymbol{z}))]$ has only one local maximum, identified either as the unique value of z_1 and z_2 that satisfy $\Gamma_i(\boldsymbol{z}) = 0$ for $i = 1, 2$, or as the largest value of z_i that satisfies $\Gamma_i(\boldsymbol{z}) = 0$ for $i = 1, 2$.

Now, since $E[\Pi(\boldsymbol{z}, \boldsymbol{p}(\boldsymbol{z}))]$ is unimodal if $\Gamma_i(\boldsymbol{z})$ has only one root for $i = 1, 2$, a sufficient condition for unimodality of $E[\Pi(\boldsymbol{z}, \boldsymbol{p}(\boldsymbol{z}))]$ is $\Gamma_1(A_1, .) > 0$ and $\Gamma_2(., A_2) > 0$ or, equivalently, $2[(b_p + \theta_p)^2 - \theta_p^2]\Gamma_1(A_1, z_2) > 0$ and $2[(b_p + \theta_p)^2 - \theta_p^2]\Gamma_2(z_1, A_2) > 0$. For example,

$$\begin{aligned} \Gamma_1(A_1, z_2) &= \left\{ \frac{(b_p + \theta_p)(\xi_1 - \overline{\Theta}_1(A_1)) + \theta_p(\xi_2 - \overline{\Theta}_2(z_2))}{2[(b_p + \theta_p)^2 - \theta_p^2]} + b_1 - g_1 \right\} (1 - F_1(A_1)) \\ &\quad -(w_1 - g_1) \end{aligned}$$

Using the fact that $F(A_i) = 0$, we get the following:

$$\begin{aligned} 2[(b_p + \theta_p)^2 - \theta_p^2]\Gamma_1(A_1, z_2) &= (b_p + \theta_p)(\xi_1 - \overline{\Theta}_1(A_1)) + \theta_p(\xi_2 - \overline{\Theta}_2(z_2)) \\ &\quad +2[(b_p + \theta_p)^2 - \theta_p^2](b_1 - g_1) - 2[(b_p + \theta_p)^2 - \theta_p^2](w_1 - g_1) \\ &= (b_p + \theta_p)(\xi_1 - \overline{\Theta}_1(A_1)) + \theta_p(\xi_2 - \overline{\Theta}_2(z_2)) \\ &\quad +2[(b_p + \theta_p)^2 - \theta_p^2](b_1 - w_1) \\ &= (b_p + \theta_p)(\xi_1 + A_1) + \theta_p(\xi_2 - \overline{\Theta}_2(z_2)) \\ &\quad +2b_p(b_p + 2\theta_p)(b_1 - w_1) \end{aligned}$$

where $\overline{\Theta}_1(A_1) = \mu_\epsilon - A_1 = -A_1$. The same demonstration can be done to prove the condition $2[(b_p + \theta_p)^2 - \theta_p^2]\Gamma_2(z_1, A_2) > 0$.

Now, assuming that the condition (iii) in the Proposition 5.2 is satisfied, the optimal z_1 and z_2 can be found by substituting the value of ${p_i}^*$ given in Proposition 5.1 into (90), (91) (This gives the same results as re-optimizing $E[\Pi(\boldsymbol{z}, \boldsymbol{p}(\boldsymbol{z}))]$. We then get the following:

$$(w_1 - g_1) \quad = \quad \left\{ \frac{(b_p + \theta_p)(\xi_1 - \overline{\Theta}_1(z_1)) + \theta_p(\xi_2 - \overline{\Theta}_2(z_2))}{2[(b_p + \theta_p)^2 - \theta_p^2]} + b_1 - g_1 \right\} (1 - F_1(z_1))$$

and

$$(w_2 - g_2) \quad = \quad \left\{ \frac{\theta_p(\xi_1 - \overline{\Theta}_1(z_1)) + (b_p + \theta_p)(\xi_2 - \overline{\Theta}_2(z_2))}{2[(b_p + \theta_p)^2 - \theta_p^2]} + b_2 - g_2 \right\} (1 - F_2(z_2))$$

Since we assume that condition (iii) of Proposition 5.2 is satisfied, there is one unique solution $({z_1}^*, {z_2}^*)$ in the space $[A_1, B_1] \times [A_2, B_2]$, which can be expressed as a function of ξ_1 and ξ_2. It is now straightforward to find the values of z_1 and z_2 such that the first order optimality conditions above are satisfied.

$\square$

C.4 Proof to Equation (100)

$$\begin{aligned}
\overline{\Theta}_i(z_i)) &= \int_{z_i}^{B_i} (u - z_i) f(u) du \\
&= \frac{1}{B_i - A_i} \left[\int_{z_i}^{B_i} u du - \int_{z_i}^{B_i} z_i du \right] \\
&= \frac{1}{B_i + B_i} \left[\frac{B_i^2 - {z_i}^2}{2} - (B_i z_i - {z_i}^2) \right] \\
&= \frac{(B_i - z_i)^2}{4B_i}
\end{aligned} \tag{142}$$

□

C.5 Proof to Lemma (5.2)

We know from equation (100) that

$$\overline{\Theta}_i(z_i)) = \frac{(B_i - z_i)^2}{4B_i}.$$

Since demand is uniformly distributed, therefore $F_i(z_i) = \frac{z_i + B_i}{2B_i}$ and $1 - F_i(z_i) = \frac{B_i - z_i}{2B_i}$.

From condition (iv) in Proposition 5.2, we have

$$(w_i - g_i) = \left\{ \frac{(b_p + \theta_p)(\xi_i - \overline{\Theta}_i(z_i)) + \theta_p(\xi_j - \overline{\Theta}_j(z_j))}{2[(b_p + \theta_p)^2 - \theta_p^2]} + b_i - g_i \right\} (1 - F_i(z_i)).$$

Plugging in $\overline{\Theta}_i(z_i))$ and $1 - F_i(z_i)$ into condition (iv) above, we get the expression for Lemma (5.2).

□

REFERENCES

[1] Ailawadi, K. L., Bolin, L., Farris, P. W., "Market Power and Performance: A Cross Industry Analysis of Manufacturers and Retailers," *Working Paper*, Darthmouth College, Hanover, NH.

[2] ACW Team, "Trim PC lifecycles to enhance IT Security," *Asia Computer Weekly*, Singapore, (Dec 8, 2003), page 1.

[3] Bass, F. M., "A New Product Growth Model for Consumer Durables," *Management Science*, 15, (Jan 1969), 215-227.

[4] Bayus, B. L., "An Analysis of Product Lifetimes in a Technologically Dynamic Industry," *Management Science*, 44(6), (June 1998), 763-775.

[5] Brynjolfsson, E., Smith, M. D., "Frictionless Commerce? A Comparison of Internet and Conventional Retailers," *Management Science*, 46(4), (April 2000), 563-586.

[6] Cachon, G. P., Lariviere, M. A. , "Capacity Allocation Using Past Sales: When to Turn-and-Earn," *Management Science*, 45(5), (May 1999), 685-703.

[7] Cachon, G. P., Zipkin, P. H. , "Competitive and Cooperative Inventory Policies in a Two-Stage Supply Chain," *Management Science*, 45(7), (July 1999), 936-953.

[8] Cachon, G. P., Porteus, E. , "Stalking Information: Bayesian Inventory Management with Unobserved Lost Sales," *Management Science*, 45(3), (March 1999), 436-454.

[9] Cachon, G. P., Lariviere, M., "Supply Chain Coordination with Revenue Sharing Contracts: Strngths and Limitations," *Working Paper, Wharton School, University of Pennsylvania*, (2000), Philadelphia, PA.

[10] Cachon, G. P., "Supply Chain Coordination With Contracts," *Working Paper, Wharton School, University of Pennsylvania*, (2001), Philadelphia, PA.

[11] Chen, F., Drezner, Z., Ryan, J. K., Simchi-Levi, D., "Quantifying the Bullwhip Effect in a Simple Supply Chain: The Impact of Forecasting, Lead Times, and Information," *Management Science*, 46(3), (March 2000), 436-443.

[12] Chen, F., Federgruen, A., Zheng, Y. S., "Coordination Mechanisms for a Distribution System With One Supplier and Multiple Retailers," *Management Science*, (47), (2001), 693-708.

[13] Chintagunta, P. K., "Investigating the Sensitivity of Equilibrium Profits to Advertising Dynamics and Competitive Effects," *Management Science*, 39(9), (Sept 1993), 1146-1162.

[14] Chintagunta, P. K., Rao, V. R., "Pricing Strategies in a Dynamic Duopoly: A Differential Game Model," *Management Science*, 42(11), (Nov 1996), 1501-1514.

[15] Choi, S. C. , "Price Competition in a Channel Structure with Common Retailer," *Marketing Science*, 10(4), (Fall 1991), 271-296.

[16] Cohen, M.A., "Joint Pricing and Ordering Policy For Exponentially Decaying Inventory With Known demand," *Naval Research Logistics Quarterly*, (24), (1977), 257-268.

[17] Cohen, M.A., Whang, S., "Competing in Product and Service: A Product Life-Cycle Model," *Management Science*, 43(4), (April 1997), 535-545.

[18] Dana, J. D., Petruzzi, N. C., "Note: The Newsvendor Model with Endogeneous Demand," *Management Science*, 47(11), (November 2001), 1488-1497.

[19] Dewan, R., Jing, B., Seidmann, A. , "Product Customization and Price Competition on the Internet," *Management Science*, 49(8), (August 2003), 1055-1071.

[20] Dixit, A. , "Quality and Quantity Competition," *Rev. Econom. Stud.*, 46, (1979), 587-599.

[21] Dockner, E. , "Optimal Pricing in Dynamic Duopoly Game Model," *Zeitschrift fur Operations Research*, Series B, 29, (1985), 1-16.

[22] Ertek, G., Griffin, P. M. , "Supplier- and Buyer-Driven Channels in a Two-Stage Supply Chain," *IIE Transactions*, 34(3), (2002), 691-700.

[23] Federgruen, A., Heching, A. , "Combined Pricing and Inventory Control Under Uncertainty," *Operations Research*, (), (1999), -.

[24] Fruchter, G. E., Kalish, S. , *Management Science*, 43(1), (Jan 1997), 54-63.

[25] Fudenberg, D., Tirole, J. , "Game Theory," *MIT Press*, Cambridge, MA, 2000.

[26] Gabszewicz, J., Thisse, J. F. , "Price Competition, Quality, and Income Disparities," *Journal Economic Theory*, 20, (1979), 340-359.

[27] Goffin, Keith, "Customer Support: A Cross-Industry Study of Distribution Channels and Strategies," *International Journal of Physical Distribution & Logistics Management*, 29(6), (1999), 374-397.

[28] Hall, J., Porteus, E., "Customer Service Competition in Capacitated Systems," *Manufacturing & Service Operations Management*, 2(2), (Spring 2000), 144-165.

[29] Heyman, D., Sobel, M. , "Stochastic Models in Operations Research: Volume I," *McGraw-Hill*, New York, 1984.

[30] Hotelling, H., "Stability in Competition," *Economic Journal*, 39, (1929), 41-57.

[31] Ingene, C. A., Parry, M. E., "Channel Coordination when Retailers Compete," *Marketing Science*, 17(4), (1998), 338-355.

[32] Iyer, G., "Coordinating Channels Under Price and Nonprice Competition," *Marketing Science*, 17(4), (1998), 338-355.

[33] Iyer, A. V., Ye, J. , "Assessing the Value of Information Sharing in a Promotional Retail Environment," *Manufacturing & Service Operations Management*, 2(2), (Spring 2000), 128-143.

[34] Jeuland, A., Dolan, R. J. , "An Aspect of New Product Planning: Dynamic Pricing," *Marketing Planning Models*, A. A. Zoltners (Ed.), Amsterdam: North-Holland, (1982), 1-21.

[35] Jeuland, A., Shugan, S. , "Managing Channel Profits," *Marketing Science*, 2, (1983), 239-272.

[36] Johnson, L. A., Montgomery, D. C., "Operations Research in Production Planning, Scheduling, and Inventory Control," *John Wiley & Sons*, New York, (1974).

[37] Kadiyala, V., Chintagunta, P., Vilcassim, N. , "Manufacturer-Retailer Channel Interactions and Implications for Channel Power: An Empirical Investigation of Pricing in a Local Market," *Marketing Science*, 19(2), (Spring 2000), 127-148.

[38] Kahn, J.A., "Inventories and the Volatility of Production," *American Economic Review*, 77(4), (1987), 667-679.

[39] Keeney, R. L., "The Value of Internet Commerce to the Customer," *Management Science*, 45(4), (April 1999), 533-543.

[40] Khouja, M., Mehrez, A., Rabinowitz, G. , "A Two-item Newsboy Problem with Substitutability," *Inter. J. Production Economics*, (44), (1996), 267-275.

[41] Khouja, M., "The Single-Period (News-Vendor) Problem: Literature Review and Suggestions for Future Research," *OMEGA*, (27), (1999), 537-553.

[42] Kim, S. Y., Staelin, R., "Manufacturer Allowances and Retailer Passthrough Rates in a Competitive Environment," *Marketing Science*, 18(1), (1999), 59-76.

[43] Lariviere M. A., Porteus, E. L., "Selling to the Newsvendor: An Analysis of Price-Only Contracts," *Manufacturing & Service Operations Management*, 3(4), (Fall 2001), 293-305.

[44] Lal, R., Matutes, C., "Retail Pricing and Advertising Strategies," *Journal Business*, 67(3), (1994), 345-370.

[45] Lau, A. H. ,Lau, H.S., "The Newsboy Problem with Price-Dependent Demand Distribution," *IIE Transactions*, 20(2), (June 1988), 168-175.

[46] Lau, A. H. ,Lau, H.S., "Maximizing the Probability of Achieving a Target Profit Level in a Two-Product Newsboy Problem," *Decision Sciences*, 19, (1988), 392-408.

[47] Lau, H. S., Lau, A. H. , "The Two-Product Newsboy Problem with Satisficing Objective and Independent Exponential Demamds," *IIE Transactions*, 23(1), (1991), 29-39.

[48] Lau, H. S., Lau, A. H. , "The Multi-product Multi-constraint Newsboy Problem: Applications, Formulation and Solution," *Journal of Operations Management*, 13, (1995), 153-162.

[49] Lau, A. H., Lau, H., "Reordering Strategies for a Newsboy-type Product," *European Journal of Operational Research*, 103(3), (1997), 557-572.

[50] Lee, E., Staelin, R., "Vertical Strategic Interaction: Implications for Channel Pricing Strategy," *Marketing Science*, 16(3), (1997), 185-207.

[51] Lee, H. L., Padmanabhan, V., Whang, S., "Information Distortion in a Supply Chain: The Bullwhip Effect," *Management Science*, 43(4), (April 1997), 546-558.

[52] Lee, H. L., Padmanabhan, V., Taylor, T. A., Whang, S., "Price Protection in the Personal Computer Industry," *Management Science*, 46(4), (April 2000), 467-482.

[53] Li, J., Lau, H.S., Lau, A. H. , "Some Analytical Results for a Two-Product Newsboy Problem," *Decision Sciences*, 21(3), (1990), 711-726.

[54] Li, J., Lau, H.S., Lau, A. H. , "A Two-Product Newsboy Problem with Satisficing Objective and Independent Exponential Demands," *IIE Transactions*, 23(1), (1991), 29-39.

[55] Lippman, S. A., McCardle, K. F. , "The Competitive Newsboy," *Operations Research*, 45(1), (Jan.-Feb. 1997), 54-65.

[56] Luenberger, D. G. , "Introduction to Dynamic Systems: Theory, Models & Applications," *John Wiley & Sons*, New York , (1979).

[57] Mahajan, V., Wind, J., Sharma, S. , "An Approach to Repeated-Purchase Diffusion Analysis," *Proceedings, American Marketing Association*, (1983).

[58] McGahan, A. M., Ghemawat, P. , "Competition to Retain Customers," *Marketing Science*, 13(2), (Spring 1994), 165-176.

[59] McGuire, T. W., Staelin, R. , "An Industry Equilibrium Analysis of Downstream Vertical Integration," *Marketing Science*, (2), (Spring 1983), 161-192.

[60] Messinger, P. R., Narasimhan, C., "Has Power Shifted in The Grocery Channel?," *Marketing Science*, (14), (1995), 189-223.

[61] Mills, E. S. , "Uncertainty and Price Theory," *Quarterly Journal of Economics*, (73), (1959), 116-130.

[62] Moorthy, K. S. , "Managing Channel Profits: Comments," *Marketing Science*, 6, (1987), 375-379.

[63] Moorthy, K. S. , "Product and Price Competition in a Duapoly," *Marketing Science*, 7(2), (Spring 1988), 141-168.

[64] Moorthy, K. S. , "Strategic Decentralization in Channels," *Marketing Science*, 7(4), (Fall 1988), 335-355.

[65] Nahmias, S. , "Production and Operations Analysis," *Irwin/McGraw-Hill*, International Edition, Singapore , (1997).

[66] Parlar, M., "Game Theoretic Analysis of the Substituable Product Inventory Problem with Random Demand," *Naval Research Logistics*, (35), (1988), 397-409.

[67] Pasternack, B. A., "Optimal Pricing and Return Policies for Perishable Commodities," *Management Science*, 4(2), (Spring 1985), 166-176.

[68] Petruzzi, N. C., Dada, M. , "Pricing and The Newsvendor Problem: A Review with Extensions," *Operations Research*, 47(2), (March-April 1999), 183-194.

[69] Petruzzi, N. C., Dada, M. , "Information and Inventory Recourse for a Two-Market, Price-Setting Retailer," *Manufacturing & Service Operations Management*, 3(3), (Summer 2001), 242-263.

[70] Petruzzi, N. C., Dada, M. , "Dynamic Pricing and Inventory Control with Learning," *Naval Research Logistics*, (49), (2002), 303-325.

[71] Porteus, E. L., "Investing in Reduced Setups in the EOQ Model," *Management Science*, (31), (1985), 998-1010.

[72] Raju, J. S., Srinivasan, V., Lal, R. , "The Effects of Brand Loyalty on Competitive Price Promotional Strategies," *Management Science*, 36(3), (1990), 276-304.

[73] Robinson, B., Lakhani, C., "Dynamic Price Model for New Product Planning," *Management Science*, 21, (June 1975), 1113-1122.

[74] Schmidt, G. M., Porteus, E. L. , "The Impact of an Integrated Marketing and Manufacturing Innovation," *Manufacturing & Service Operations Management*, 2(4), (Fall 2000), 317-336.

[75] Shaked, M., Shanthikumar, G. , "Stochastic Orders and Their Applications," *Academic Press*, San Diago, 1994.

[76] Shy, O. , "Industrial Orgainization: Theory and Applications," *MIT Press*, Cambridge, MA, 2000.

[77] Silver, E., Peterson, R. , "Decision Systems for Inventory Management and Production Planning," *John Weiley*, New York, 1985.

[78] Silver, E., Pyke, D. F., Peterson, R. , "Inventory Management and Production Planning and Scheduling," *John Wiley and Sons*, New York, 1998.

[79] Simchi-Levi, D., Kaminsky, P., Simchi-Levi, E., "Designing and Managing the Supply Chain," *McGraw Hill*, New York, (1999).

[80] So, K. C., "Price and Time Competition for Service Delivery," *Manufacturing & Service Operations Management*, 2(4), (Fall 2000), 329-409.

[81] Spence, A. M., "Monopoly, Quality, and Regulation," *Bell J. Econom.*, 6, 417-429.

[82] Tayur, S., Ganeshan, R., Magazine, M. , "Quantitative Models for Supply Chain Management," *Kluwer Academic Publishers*, Boston, (1999).

[83] Thomas, L. J., "Price-Production Decisions With Deterministic Demand," *Management Science*, (16), (1970), 747-760.

[84] Thomas, D. J., Griffin, P. M. , "Coordinated Supply Chain Management," *European Journal of Operational Research*, 94, (1996), 1-15.

[85] Thomas, D. J., "Committed Delivery Strategies for Supply Chain Management," *Ph.D. Thesis, School of Industrial and Systems Engineering, Georgia Tech*, (1999).

[86] Thowsen, G. T. , "A Dynamic, Nonstationary Inventory Problem For a Price/Quantity Setting Firm," *Naval Res. Logistics Quat.*, (33), 461-476.

[87] Tirole, J. , "The Theory of Industrial Organization," *MIT Press*, Cambridge, MA, 2000.

[88] Trivedi, M. , "Distribution Channels: An Extension of Exclusive Retailership," *Management Science*, 44(7), (1998), 896-909.

[89] Tsay, A., "Quantity-flexibility Contract and Supplier-Customer Incentives," *Management Science*, (45), (1999), 1339-1358.

[90] Tsay, A. A., Agrawal, N. , "Channel Dynamics Under Price and Service Competition," *Manufacturing & Service Operations Management*, 2(4), (Fall 2000), 372-391.

[91] Vilcassim, N. J., Kadiyali, V., Chintagunta, P. K., "Investigating Dynamic Multifirm Market Interactions in Price and Advertising," *Management Science*, 45(4), (April 1999), 499-518.

[92] Wang, Y., Gerchak, Y. , "Supply Chain Coordination When Demand Is Shelf-Space Dependent," *Manufacturing & Service Operations Management*, 3(1), (Winter 2001), 82-87.

[93] Weng, K., "Channel Coordination and Quantity Discounts," *Management Science*, (41), (1995), 1509-1522.

[94] Weng, K., "Pricing and Ordering Strategies in Manufacturing and Distribution Alliances," *IIE Transactions*, (29), (1997), 681-692.

[95] Whitin, T. M. , "Inventory Control and Price Theory," *Management Science*, (2), (1955), 61-68.

[96] Winkler, R. L., Roodman, G. M., Britney, R. R. , "The determination of partial moments," *Management Science*, 19(3), (1972), 290-296.

[97] Yoruk, D. E., Radosevic, S, "International Expansion and Buyer-Driven Commodity Chain: The Case of Tesco," *Working Paper, University College London(www.ssees.ac.uk/tesco.pdf)*, London, (2000).

[98] Zhao, W., Wang, Y. , "Coordination of Joint Pricing-Production Decisions in a Supply Chain," *IIE Transactions*, (34), (2002), 701-715.

Vita

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