## AN APPROACH FOR THE ROBUST DESIGN OF AIR COOLED DATA CENTER SERVER CABINETS

A Thesis Presented to The Academic Faculty

by

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In Partial Fulfillment of the Requirements for the Degree Maser of Science in the School of Mechanical Engineering

Georgia Institute of Technology December 2005

# AN APPROACH FOR THE ROBUST DESIGN OF AIR COOLED DATA CENTER SERVER CABINETS

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### ACKNOWLEDGEMENTS

I thank my advisors Dr. Joshi and Dr. Allen for their continuous insight, and guidance throughout my graduate studies. I also greatly appreciate the support and encouragement of my committee members, Dr. Mistree and Dr. Garimella.

Special recognition is given to Jeff Rambo for his work in the development of POD based metamodeling, without which this work would not be possible. I would also like to thank all my friends and colleagues in the Systems Realization Laboratory and Microelectronics and Emerging Technologies Thermal Laboratory for providing me with a collaborative, congenial, and energetic work environment.

Lastly, the support of the Consortium for Energy Efficient Thermal Management (CEETHERM), a joint initiative between Georgia Institute of Technology and the University of Maryland, College Park is greatly appreciated.

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## NOMENCLATURE

### List of Abbreviations

## CRAC Computer Room Air Conditioner

- SQP Sequential Quadratic Programming
- CFM Cubic Foor per Minute
- POD Proper Orthogonal Decomposition
- ALP Adaptive Linear Programming
- CFD Computational Fluid Dynamics
- DSP Decision Support Problem
- RANS Reynolds Average Navier Stokes
- DOF Degree Of Freedom

### List of Symbols

- *a<sub>i</sub>* weighting factor
- $d_i^+, d_i^-$  deviation variables

- $g_i(\vec{x})$  inequality constraint function
- $h_i(\vec{x})$  equality constraint function
- *m* mass flow rate
- *m* number of observations/number of goals
- *n* degrees of freedom/number of design variables
- *p* number of inequality constraints
- *q* number of equality constraints
- *s* number of servers
- $\vec{u}(x)$  observed phenomena
- $\vec{x}$  design variables
- $x_{i,L,U}$  lower/upper bound of design variable  $x_i$
- $A(\vec{x})$  achievement function
- C coefficient matrix
- $F(\vec{u},\beta)$  flux function
- G flux goal vector
- G<sub>i</sub> design goal target

- Q heat generation rate
- R,R' covariance matrix
- T temperature
- $\vec{U}$  observation ensemble
- *V°* observation set
- W<sub>i</sub> goal weighing factor
- Z Archimedian objective function
- $\vec{\varphi}$  basis function
- Г control surface
- $\Omega, \partial \Omega$  system domain and boundary
- $k \varepsilon$  turbulent kinetic energy-turbulent dissipation

## Subscripts

- o ensemble average
- *r* reconstruction
- e error

### **SUMMARY**

The complex turbulent flow regimes encountered in many thermal-fluid engineering applications have proven resistant to the effective application of systematic design because of the computational expense of model evaluation and the inherent variability of turbulent systems. In this thesis the integration of the Proper Orthogonal Decomposition (POD) for reduced order modeling of turbulent convection with the application of robust design principles is proposed as a practical design approach. The POD has been used successfully to create low dimensional steady state flow models within a prescribed range of parameters. The underlying foundation of robust design is to determine superior solutions to design problems by minimizing the effects of variation on system performance, without eliminating their causes. The integration of these constructs utilizing the compromise Decision Support Problem (DSP) results in an efficient, effective robust design approach for complex turbulent convective systems.

The efficacy of the approach is illustrated through application to the configuration of data center server cabinets. Data centers are computing infrastructures that house large quantities of data processing equipment. The data processing equipment is stored in 2 m high enclosures known as cabinets. The demand for increased computational performance has led to very high power density cabinet design, with a single cabinet dissipating up to 30 kW. The computer servers are cooled by turbulent convection and have unsteady heat generation and cooling air flows, yielding substantial variability, yet require some of the most stringent operational requirements of any engineering system. Thermally efficient configurations that are insensitive to variations in operating conditions are determined through variation of the power load distribution and flow parameters, such as the rate of cooling air supplied.

This robust design approach is applied to three common data center server cabinet designs, in increasing levels of modeling detail and complexity. Results of the application of this approach to the example problems studied show that the resulting thermally efficient configurations are capable of dissipating up to a 50% greater heat load and a 60% decrease in the temperature variability using the same cooling infrastructure. These results are validated rigorously, including comparison of detailed Computational Fluid Dynamics (CDF) simulations with experimentally gathered temperature data of a mock server cabinet. Finally, with the approach validated, augmentations to the approach are considered for multi-scale design, extending the approaches domain of applicability.

### **CHAPTER 1**

#### **INTRODUCTION**

The principal objective set forth in this thesis is to:

Establish an approach for the design of data center server cabinets for efficient cooling, accounting for the inherent variability in both internal and external operating conditions, and enabling effective tradeoff between the goals of energy efficiency and reliability, with the potential for broader multi-scale thermal-fluid simulation based design applications.

The motivation for this research is the robust design of data center server cabinets. The complex turbulent flow regimes encountered in these cabinet systems, as well as many thermal-fluid engineering applications, has proven resistant to the effective application of systematic design. This is because the CFD models required for analysis are computationally expensive, making the application of iterative optimization algorithms extremely time consuming. Furthermore, turbulent flow regimes are inherently complex and unstable, and require significant simplifications and assumptions to be made in thier simulation [75]. The CFD models employed in simulation of engineering systems are based upon an average of the flow field, and thus the variance induced by turbulent perturbations such as eddies and vortices is not accounted for [47, 75]. Finally, as in any complex system design, multiple objectives must be considered in

a mathematically rigorous fashion that also accurately reflects the designer's preferences, particularly the tradeoffs between the stability and optimality of the solution.

Data centers are computing infrastructures housing large quantities of data processing equipment. Thermal management difficulties in data centers, caused by the rapidly increasing power densities of modern computational equipment, has lead to very high flow rates of cooling air, resulting in turbulent flow regimes with large variability in velocity magnitude. In data center server cabinets this variability is compounded by variable speed fans in the servers, Computer Room Air Conditioning (CRAC) units, and unsteady heat generation by the processors, yielding a highly variable problem. Compounding this problem, the reliability requirements of data centers are exceedingly high.

In this chapter an introduction to the work undertaken in this thesis is presented, organized as follows. In Section 1.1 the background and motivation for the work presented in this thesis is derived and explained. In Section 1.2 a review of the literature covering data center design, analysis, and optimization is presented. In Section 1.3 the core requirements of a data center server cabinet design approach are given, and a gap analysis between what is currently available and these requirements is addressed. In Section 1.4 the frame of reference of this thesis is given; the objective of increasing thermal efficiency while effectively coping with non-uniformity in airflow distribution. Section 1.5 addresses these core requirements with the three main constructs used to formulate this approach. In Section 1.6 the research questions and associated hypotheses,

derived from the core requirements, are presented. Section 1.7 and Section 1.9 give a roadmap of the validation strategy and organization of this thesis respectively.

#### **1.1 Background and Motivation**

#### **1.1.1 Introduction to Data Centers**

Data centers are large computing infrastructures that house vast quantities of data processing and storage equipment. These facilities have grown greatly in both size and power dissipation over the past decade, to as large as 50,000 m<sup>2</sup> dissipating hundreds of MW of power. Typical data centers today range from 100-10,000 m<sup>2</sup> and consume 100kW-10MW of power for operation of the data processing equipment and associated cooling hardware. The data processing equipment is stored in 2 m high enclosures known as cabinets. These cabinets are usually arranged in rows, creating a center layout as shown below in Figure 1.1.



Figure 1.1 - Typical medium size data center facility [48]

The demand for increased computational performance has lead to very high power density cabinet design, with a single cabinet dissipating up to 30 kW [90]. This increased computational density results in significantly increased performance and power consumption efficiency [24]; however, such large power dissipations result in unprecedented heat loads at both the cabinet and the facility level. Therefore, data centers require a dedicated cooling system for thermal management, where the state of the art consists of computer room air conditioning (CRAC) units that deliver cold air to the cabinets through perforated tiles placed over an under-floor plenum. Operation of this cooling hardware can constitute up to 50% of the power consumption of a data center, as discussed further in Section 1.1.4.

It is estimated that 2-10% of all power generated in the United States is used for computer, office and network use [85]. With the continuing trend of increasing demand for computation and communications processing on a large scale, data center operations will consume an even larger portion of power production on a national scale. It is therefore imperative that these centers be operated efficiently, as even a few percent increase in efficiency represents huge potential savings in power consumption.

#### **1.1.2** Data Center Equipment Thermal Management Challenges and Trends

Thermal management of today's high powered electronics components is already a challenge, easily visible by the use of large heat sinks, fans, and cooling enhancements attached to computer processors. This thermal management challenge is magnified when tens to hundreds of these processors are located within a single cabinet, such as the IBM blade server cabinet architecture, consuming as much as 30kW in less than a 1 m<sup>2</sup> footprint [87]. In order to create the same heat generation density in a human occupied room, an average auditorium would be filled with 100 people sitting on each chair.

Furthermore, computational equipment has much more stringent requirements on operating ambient conditions, requiring on average an inlet air dry bulb temperature of 20-25 °C with 40-50% relative humidity. In this manner, computer chips are considerably more demanding than people when it comes to thermal regulation. Finally, the computational equipment housed in data centers are used for mission-critical computing applications such as banking transactions, major website housing, and large scale simulations and scientific computational analyses. These processes require continual operation of the processing equipment, and hence a reliability of 99.9999%, the equivalent of 32 seconds of downtime per year, is required [48]. At present this reliability is accomplished through 24/7 monitoring of the equipment and the use of redundant backup systems. This high reliability requirement combined with the high computational density of the server cabinets act to create a very challenging thermal management problem.

#### **1.1.3 Data Center Cooling Approach**

Data centers house computation equipment that is almost exclusively air cooled. In order to dissipate the huge heating loads, a system of distributing the cooling air to the cabinets is employed. This system consists of an under floor plenum, commonly varying in depth from 1-4 feet (0.3-1.2 m), over a grid of floor tiles each measuring 2 ft by 2 ft (0.61 m by 0.61 m) each. CRAC units are essentially industrial air conditioning units, capable of providing up to 12,500 CFM (5.90 m<sup>3</sup>/s) of cold air per unit, which is pumped into the plenum. By replacing solid floor tiles with perforated units, a jet of cold air is emitted, enabling the distribution of cooling air within the room.

The resulting airflow patterns and distribution within the plenum from the placement of the perforated tiles and CRAC units are complex [76, 86, 88, 89, 115]. Exact prediction of the flow distribution is difficult even with CFD analyses, because of the turbulent flow conditions and blockages in the plenum from cabling and chilled water distribution lines [76, 80, 86-89, 115]. However, empirical measurement of the flow distribution within the center through the tiles is straightforward using a flow hood. This approach has lead to the development of the predominant data center cabinet layout; the hot aisle – cold aisle configuration, shown below in Figure 1.2 and Figure 1.3. However, the flow hood cannot be used to measure the flow rates through the individual servers.

In this configuration the cold air from the CRAC units is drawn through the perforated tiles to the computational equipment in the cabinets. The hot exhaust air is then pushed out and forced towards the ceiling of the room, where it is collected by the intake of the CRAC units and the cycle is repeated. The benefit of this configuration is the separation of the hot exhaust air from the cool inlet air, however, the complex recirculation in the upper portion of the room results in a degree of hot exhaust air being drawn into the racks in the cold aisle. Previous work on the study of this configuration and its cooling efficiency is described in Section 1.2.1.



Figure 1.2 - Data center hot aisle - cold aisle flow schematic: center perspective



Figure 1.3 - Date center hot aisle - cold aisle flow schematic: plenum perspective

The cabinets housing the processing equipment direct the cooling air through to the computers in two orientations, vertically or horizontally. These two configurations are shown below in Figure 1.4.



Figure 1.4 - Data center server cabinet designs (a) horizontal flow (b) vertical flow

In Figure 1.4 (a) the cold air is drawn in from the plenum through a perforated tile, and the air is distributed to the servers through a large perforated area in the front of the cabinet, and exhausted by the server fans through a similar perforated area in the rear. In Figure 1.4 (b) the cold air from the plenum is directly drawn into the cabinet through an opening in the bottom. This cabinet design sits directly over a floor tile space, but the tile is removed entirely to allow airflow. The server fans draw this air through the cabinet, and a fan at the top exhausts the hot air out the cabinet.
Both of these cabinet designs have been employed in commercial data centers, and both are investigated in this thesis. Each cabinet configuration has certain advantages regarding the distribution of cooling air to the servers it houses. The vertical flow oriented cabinet will never have any problems with hot air recirculation, and is directly fed cold air from the plenum. However, the horizontal flow cabinet has a significantly larger flow area, allowing a greater volume of cooling air to flow through it for a given flow velocity. For this reason, many of the highest density servers are designed for this style of cabinet. Note that although both architectures are investigated, the focus of this thesis is not upon the comparison of different cooling schemes effectiveness.

Ultimately, the heat extracted from the CRAC units must be removed. Most high capacity CRAC units operated using chilled water coils, where the cold water is supplied by an external chiller. In this manner, the chiller is responsible for rejection of the heat to the atmosphere and for a continuous supply of cooling water used to maintain the data center ambient temperature conditions, as dictated by the CRAC units to meet the demands of the processing equipment.

## **1.1.4 Thermal Efficiency Challenges Facing Data Center Design**

The cooling requirements of data centers represent up to 50% of the total energy consumption [111], corresponding to a significant cost and environmental impact. An example of an energy benchmarking survey, measuring the percentage of total power consumed by the Heating Ventilation and Air Conditioning (HVAC) equipment, compiled by Lawrence Berkeley National Labs is presented below in Figure 1.5.



Figure 1.5 - HVAC power consumption of 9 surveyed data centers [48]

From the figure above, it is evident that the amount of power consumed by the HVAC equipment is between 20% and 50%. Further breakdown of the best and worst case center configurations are shown below in Figure 1.6 [48].



Figure 1.6 - Energy consumption breakdown of best/worst case data center HVAC efficiencies [48]

The increase in computational density leads to the creation of hot spots and high thermal gradients, which lead to thermally inefficient configurations, such as presented in Figure 1.6 (a). Because of the complex airflow distribution, an efficient configuration for one heat load may not be efficient for another. This is demonstrated by the differences in efficiencies between Figure 1.6 (a) and (b), where the same data center is configured to dissipate different heat loads. Hence, the matching of the data center cooling approach with the total heat load is of great importance with respect to thermal efficiency.

A lifecycle mismatch exists between the data center infrastructure and the server cabinets housed within it. This is because new higher power computational equipment is added at an average of every 3 years, while the facility is overhauled at most every 25 years [16]. This lifecycle mismatch significantly increases data center cooling costs when older infrastructures are required to support excessive heat loads, resulting in poor thermal management. Therefore, the challenge is integrating this new equipment into the existing infrastructure, while maintaining reliable operations and high thermal efficiency. The breakdown of this problem and the foundations of the design approach created to address it are discussed in Section 1.5.

With the problem background, challenges, and general cooling approach described, a literature survey is completed. This provides information on what has been done before with respect to data center design and analysis, and server as the foundation to determine what further work needs to be done, and what can be leveraged into the proposed approach.

## **1.2 Foundations: Literature Review**

In order to provide a perspective on the work undertaken in this thesis regarding the robust design of data center server cabinets, a review of literature which documents analyses, design, and optimization approaches for the layout and operation of data centers. The thermal challenges surrounding data center design have only become of interest within the past few years. Hence no comprehensive approaches for effective data center design exist. The existing literature can be broadly classified into three groups: data center design, data center simulation, and data center optimization. Different groups have focused upon different aspects of data center design, with primary focus on flow distribution through the network of perforated tiles covering the plenum.

#### **1.2.1** Data Center Design

Previous development of design approaches for data centers are limited to analyses based upon experience, coarse experimental measurements, and simple correlations. Much data center design work is performed using ad hoc approaches, and simple measurements of the inlet air temperature to the cabinets, such as some work performed by Schmidt and coauthors [87].

The development of metrics for the more effective evaluation of data center layouts with respect to thermal efficiency have been pursued by Sharma, Bash, and Patel [97] in the form of Supply Heat Indices. Further similar work by Sharma and coauthors includes some limited CFD validation of these indices in data center layout [96]. Although the primary focus of the work is CFD analysis, Rambo and Joshi have developed metrics for center level efficiency based upon entropy and temperature gradient minimization [84]. Work by Shah and coauthors is in a similar vein, evaluating data center layouts with the metric of exergy maximization [92].

The flow distribution through the plenum to the network of perforated tiles is very important for center level analyses. Work by Kang and coauthors [43] has resulted in the development of a simple analytical correlation for the flow distribution through the perforated tile network, based upon the assumption of uniform plenum pressure, enabling resistance flow modeling [26], similar to traditional HVAC design procedures. However, this correlation's accuracy is questionable because of its assumptions, limited CFD validation, and lack of detailed understanding of the flow through perforated tiles. Based upon initial work upon the development of the analysis metrics and correlations, The American Society of Heating, Refrigeration, and Air-conditioning Engineers (ASHRAE) has published guidelines for data center layout and operations [7], based upon coarse room level considerations. Lastly, Lawrence Berkeley National Laboratory has released a series of case studies benchmarking data center energy usage [48].

Finally, a completely different approach is undertaken by those from the electrical engineering and computer science domain. There is interest in the dynamic allocation of computing workload in order to better distribute the heat load. This is investigated using the feedback from the temperature sensors within the cabinets and equipment, and approaches have been developed by Sharma and coauthors [98] and Boucher and coauthors [14] for center level management. An interesting approach is discussed by Patel and coauthors, [68], where the workload distribution on a global scale is suggested,

varying the load based upon regional climate and time of day in order to maximize potential energy efficiency.

#### **1.2.2 Data Center Simulation and Analysis**

Initial use of CFD to model the heat transfer to study air movement and temperature distribution in raised floor data centers has been performed by Patel and coauthors [70, 71]. These simulations are fairly coarse, using less than 600,000 nodes, and the cabinets are modeled as back boxes, with a uniform flow rate and heat generation rate. Different cabinet layouts are evaluated based upon the single metric of cabinet inlet air temperature. Rambo and Joshi have extended these CFD simulations, employing the RANS turbulence modeling approach, and extending the modeling domain to include dominant features within the rack in a multiscale modeling approach [78, 79]. Results of these simulations led to the development of the unit cell architecture, in which several different cabinet layouts and flow configurations are tested and evaluated using the developed metrics discussed in Section 1.2.1 [84]. Shrivastava and coauthors have performed CFD analyses of data centers with similar geometry to Rambo and Joshi's unit cell [99], however many flow configurations are evaluated, employing a combination of overhead diffusers and under floor plenum supply points. Iyengar and coauthors have also studied a similar geometry, however the plenum is removed, and only diffusers and layout geometric variables are considered for evaluation [40]. All layout configurations are evaluated based upon the sole objective of minimization of the inlet temperature to the cabinets, with the exception of Rambo and Joshi in which the minimization of entropy and temperature gradients are considered also [78, 79, 84].

Schmidt and coauthors have numerically and experimentally identified the flow through the perforated tiles as the main factor in flow mal-distribution that results in cabinet temperature variation [86]. Schmidt and coaurthors have also performed continuous studies on the distribution of CRAC unit supply air through the network of perforated tiles, using 2D and 3D CFD analyses as well as experimental validation using a flow hood to measure tile flow rates [88, 89]. The flow rates through the tiles are investigated with respect to tile positions, plenum depth, and the open area ratio of the perforated tiles. Radmehr and coauthors have investigated another significant source flow mal-distribution, the leakage of air around the tiles and out of the center using experimental measurements, using the results to correlate corrections to the prediction of a 2D CFD model [76]. Rambo and Joshi have also performed 2D CFD analysis of the flow through a row of perforated tiles, performing a detailed investigation of the correlation between predicted flow rate and the distance between the CRAC unit and perforated tile row [80]. VanGilder and Schmidt have performed 3D CFD analyses on the plenum predicting the effect on the uniformity of flow distribution through the perforated tile network as a function of plenum depth and percent open area of the perforated tiles for many different data center layouts, with some experimental validation [115].

Beyond the work in this thesis, and the cited work by Rambo and Joshi, little has been done on the analysis of data center server cabinets. Internal design analyses are performed by the electronic equipment manufacturer, but only now are the considerations of the cabinet mounting and flow being taken into consideration. This is discussed further in Section 7.3.2. Some analyses have been performed for cabinet enclosures in an outdoor environment, [33, 66, 120, 121]. In a similar manner, a detailed study of a data center server cabinet has been performed by Rambo and Joshi, using high fidelity 3D CFD to evaluate different power distribution profiles of a detailed fully populated cabinet model [81].

## **1.2.3** Data Center Optimization

Little work has been done on data center optimization. This is because of the complexity of the problem, its multiscale nature, and the computational expense of the CFD models employed. Patel and coauthors [12] has performed coarse CFD analyses of a unit cell architecture similar to Rambo and Joshi, varying three parameters: the room height, plenum depth, and cold aisle placement using a Design of Experiments (DOE) method to create a linear response model. This model is then used to find the optimal parameter values, and the configuration evaluated using CFD.

Shah and coauthors [93, 94] have taken a unique approach, based upon the global maximization of the data center exergy. This approach relies upon the decomposition of the center into sub systems, and this single metric is evaluated and optimized based upon global energy balances and mass fluxes. This approach is interesting, however its capability to generate physically effective solutions based upon global optimization of a single variable in a multi-scale problem remains to be strongly validated.

## 1.3 Approach Requirements and Gap Analysis

With the frame of reference and survey of available literature complete, the requirements for an effective, efficient data center server cabinet design approach are

established. There requirements are based upon the challenges regarding data center design that have not been addressed well or at all by the existing work on data centers. These requirements are listed below, and addressed in turn:

- Systematic approach
- Reduced order modeling
- Multi-scale cabinet level analysis
- Variability consideration
- Multi-objective tradeoffs
- Experimental validation

As discussed in Section 1.2, the past data center design and analysis work can be classified into three categories: design approaches, simulation and analysis work, and optimization work. In this section the leading work's attainment of the requirements set forth in Section 1.3 are investigated in Table 1.1. Analysis of Table 1.1 indicates that no previously developed method or approach meets all of the requirements set forth in Section 1.3. The closest single approach is that of Shah and coauthors [93, 94], in which the global exergy of the system is minimized. This objective is superior to the simpler goal of temperature minimization employed by other groups, and is similar (albeit more complex to implement) to Rambo and Joshi's temperature gradient minimization objective [78]. The downfall of this approach is its assumptions regarding the thermal

decomposition of the entire data center system, limited validation, and the complexity of its thermodynamic exergy modeling and implementation.

	Requirement					
Author	Systematic Approach	Reduced Order Modeling	Multi-Scale Analysis	Variability Consideration	Multi- Objective Tradeoffs	Experimental Validation
		Desi	gn Approaches	,		
Schmidt and coauthors [87]	×	×	×	×	×	$\checkmark$
Sharma, Bash, and Patel [97]	$\checkmark$	×	×	×	*	$\checkmark$
Kang and coauthors [43]	$\checkmark$	$\checkmark$	×	×	*	×
ASHRAE [7]	$\checkmark$	×	×	×	×	$\checkmark$
Boucher and coauthors [14]	$\checkmark$	×	$\checkmark$	$\checkmark$	×	×
		Simula	tion and Analy	sis		
Patel and coauthors [71]	×	×	×	×	×	✓
Rambo and Joshi [78, 79]	×	×	~	×	$\checkmark$	×
Shrivastava and coauthors [99]	×	×	×	×	×	×
Iyengar and coauthors [40]	×	×	×	×	*	×
Schmidt and coauthors [88]	×	×	×	×	×	$\checkmark$
VanGilder and Schmidt [115]	$\checkmark$	×	×	×	*	$\checkmark$
Optimization						
Patel and coauthors [12]	×	✓	×	×	×	×
Shah and coauthors [93, 94]	$\checkmark$	$\checkmark$	$\checkmark$	×	×	$\checkmark$

 Table 1.1 - Analysis of previous literature with respect to design approach requirements

✓ - Requirement met ► - Requirement not met

The approach developed in this thesis is described in Sections 1.4 and 1.5. It is built using some of the best practice approaches from the previous work. Rambo and Joshi's foundations for emphasis of data center cabinets are employed, as is Schmidt and co authors validation measurements using thermocouples and tile flow meters [89]. Lastly, optimization algorithms are used to search for the best solution, as employed by Shah and coauthors. However, the most important addition, that has not been explicitly considered, is the consideration of variability in the data center operational environment. Variability is somewhat considered by Boucher and coauthors [14] through their dymanic computing resource allocation approach, however this is an active approach over the passive (and much simpler) robust design approach utilized in this thesis. This variability consideration constitutes one of the most important features of the approach developed in this thesis. With the gap analysis complete, including aspects to be integrated into this thesis, the frame of reference and high level thoughts behind this approach developed is presented.

These requirements are all addressed through the work presented in this thesis. Some are directly tackled through the approach developed, others through its applications and broader potential. The details of each requirement, as well as its significance are now discussed.

## **1.3.1** A Systematic Approach

A lifecycle mismatch is present in data center operation. This is because data centers receive new high powered servers every 2 to 3 years [16], whereas the center infrastructure is only upgraded on the order of every 25 years. This means that the center must be reconfigured to handle the increased heat load quite frequently, and after a few iterations of the process the center is required to dissipate far greater loads than initially intended.

In order to maximize the effectiveness of existing data centers, an approach is needed that can enable the effective and efficient configuration of a data center and the cabinets housed within it. Existing work has focused primarily upon analysis, with almost no development of a design approach. The guidelines established are somewhat effective, but not systematic, and are simplistic for the complexity of the problem at hand. The only design approach under development is the exergy based analysis and optimization by Shah and coauthors [93, 94]. While innovative, and a commendable effort, the concept of exergy maximization is foreign to most of the data center community, who are more comfortable dealing with tradition design metrics such as flow rates and temperatures rather than more abstract quantities such as exergy. Furthermore, this approach does not consider the effects of variability, discussed further below in Section 1.3.4.

## 1.3.2 Reduced Order Modeling

The coarser numerical models used in much of the existing literature such as Patel and coauthors [70, 71] or Shrivastava and coauthors [99] that do not resolve details below the cabinet level consist of approximately  $3x10^6$  degrees of freedom (DOF). The more detailed multiscale simulations performed by Rambo and Joshi [79, 84] consist of more than  $10^7$  DOF. These large models are impractical for design use because their computational expense does not allow for extensive or detailed searching of the design space, as discussed by Shapiro [95].

In the analysis of a data center system, only the small part of the domain populated by the cabinet is used to asses the thermal performance of the system, representing a small portion of the total DOF of the system. However, because the flow dynamics within the cabinets is dependent upon the overall flow within the center, the overall center flow model cannot be dismissed. This motivates the development of model reduction techniques for turbulent convection, where some accuracy is traded for a large decrease in the number of DOF. Reduced order modeling techniques and meta-modeling techniques aim to extract the dominant characteristics of the system, trading a degree of accuracy for much greater computational speed.

There are many different meta-modeling approaches, each with specific strengths and weaknesses, as thoroughly documented by Simpson and coauthors [100]. Simple linear response surfaces using Design of Experiments (DOE) have been employed by Bhopte and coauthors [12] with limited success. Approaches such as krieging, multivariate adaptive regression splines, and other more advanced interpolation approaches offer superior approximations [100]; however, these methods also require a large number of data points for interpolation, a number which increases exponentially with the number of design variables [100]. For example, a simple full factorial design of five design variables using four factor levels would require 1024 runs, with an average evaluation time of 20 hours per run, a total of 20,480 hours or 2.33 years of computational time.

Although Design of Experiments and other techniques can be used to cut down on the number of experimental runs required, the computational expense of the CFD models is still far too high for use in iterative optimization algorithms. Therefore the use of reduced order models or meta-models is required. Resistor network type approaches have been used to effectively model heat transfer in electronics; however these approaches are most effective for conduction based heat transfer, or using constant convection coefficients. The complex flow distribution within a server cabinet or data center does not have many physical dividers or flow directing devices, and hence requires CFD analysis for accurate evaluation, as zonal methods as used for building HVAC evaluation and is too crude an approximation [26]. This makes the use of resistor network modeling infeasible for this design problem.

Many meta-modeling approaches are documented and evaluated by Simpson and coaurthors [100]. However in this thesis the Proper Orthogonal Decomposition (POD) based reduced order modeling approach is utilized. The key advantages of this approach are twofold. Firstly, the POD solution reconstructs the entire field solution, and is not just a black box response. The enables the analysis of the system as if it were a CFD solution, not just point temperature responses. Secondly, the POD solution is not dependent upon any interpolation techniques, which require the evaluation of many data points for accurate approximations, a number which increases exponentially with the number of design variables [100]. The development of the POD based modeling approach as an effective reduced order modeling tool is the focus of Jeff Rambo's PhD research, hence its use in this thesis as validation of its applicability. However, the POD modeling technique alone is not design oriented, hence leading to the development of the flux matching procedure, as documented in Section 3.1.3.

## 1.3.3 Multi-Scale Cabinet Level Analysis

Results of detailed data center CFD simulation work by Rambo and Joshi [78, 84] has shown that the modeling and consideration of the cabinet level at a greater resolution is important. The electronics thermal management community in the past has only focused upon the removal of heat generated by the chips and other components out of the server enclosure, and the unit is assumed to be provided with an adequate supply of cooling air. The cabinet enclosure system within a data center complicates this assumption, as the exhaust heat from one unit is drawn into another, and many different devices all may have different outputs and cooling requirements. Consideration of this multiscale cabinet level resolution is important because the heat dissipation at the chip and server length scales drives the data center scale temperature fields, yet airflow patterns from the data center provide cooling air from the server and chip length scales. In total, the data center system length scales spans four orders of magnitude, moving from the center, to the cabinet, to the server, to the chip levels, as shown below in Figure 1.7.



Figure 1.7 - Data center length scales

Beyond the chip level, the consideration of the wire bonding and interconnects, with features on the scale of 10 nm can be considered, creating a problem with a length scale that spans 10 orders of magnitude. The mesh required for accurate evaluation of all of these length scales is infeasible using commonly available computing hardware. The multiscale nature of data center thermal analysis provides another driver for the development of reduced order models.

Previous data center analyses have focused almost exclusively on the center level length scale, as shown in Section 1.2.2. As the multiscale analysis by Rambo [78, 84] has shown the importance of cabinet level considerations, the work in this thesis focuses upon the cabinet and server length scales, with some simple modeling of the chip length scale. How the cabinet model is parsed from the complete data center system is discussed in each of the studies presented. This combination of length scales has been chosen because of the lack of work in the area, as well as the potential for validation using the experimental mock blade server cabinet, described in Chapter 6. Furthermore, in many data center server cabinets, the load within a cabinet is not uniform. This is because different processing units are more efficient for different tasks [46], and hence the modular rack infrastructure enables the placement of different equipment within a cabinet, such as the configuration tested by APC [115] given below in Figure 1.8.



Figure 1.8 - Equipment variation with vertical cabinet position

In Figure 1.8 above the top 11 servers are Dell 1650's (a), the middle 14 Compaq 360DL's (b), and the bottom 7 IBM X-Series 345's (c). Each of these units has different heat outputs, flow rates, and cooling requirements. This situation is often encountered because of the cost of the complete population of a cabinet, and the multi-functional needs of most data centers. Hence, equipment is purchased in batches, and must be placed within the data center in the cabinet space available.

The detailed CFD simulation of the experimental cabinet requires 6e5 nodes, which is greater than many of the data center CFD analyses performed [12, 99, 115]. This indicates that the approach developed in this thesis would also be applicable to the larger data center length scale. Finally, considerations for the application of the developed approach for complete multiscale modeling and analysis is discussed in Chapter 7.

## 1.3.4 Variability Consideration

The airflow regimes encountered in data centers and the server cabinets they house is turbulent in nature, requiring the use of a more computationally expensive turbulence modeling approach for the CFD analysis. Furthermore, turbulent flow regimes are inherently unstable, and require significant simplifications and assumptions to be made in its simulation [75]. The CFD models employed in simulation of engineering systems are based upon an average of the flow field, and thus the variance induced by turbulent perturbations such as eddies and vortices is not accounted for [47, 75]. An example of this complexity is demonstrated in Figure 1.9 and explained below.



Figure 1.9 - (a) Average velocity and (b-d) individual frames of server exhaust PIV data

The vector fields (b)-(d) are single frame snapshots of the turbulent exhaust flow from a server rack, obtained using the PIV system in the Georgia Tech CEETHERM laboratory. Vector field (a) is a 100 frame average, which shows a comparatively smooth flow, compared with the eddies and vortices captured by the individual snapshots. Operational data centers contain variable flow output CRAC units, variable chilled water cooling loops, many fans, computer control algorithms, and variable heat loads from the processors. All of these sources, and countless others, all sum up to make a problem with a huge amount of internal and external variability. However, as discussed previously, the reliability and operational stability of these centers is of the utmost design concern. This inherent variability has not been quantitatively considered in any previous data center analyses.

## 1.3.5 Multi-Objective Tradeoffs

In any complex system design, multiple objectives must be considered in a mathematically rigorous fashion that accurately reflects a designer's preferences. There are many tradeoffs to consider in the design of a data center, such as the set point of the CRAC units versus the flow rate of the units, the power dissipated in a single cabinet, the operating temperature of the server, etc. In particular, the tradeoff between the objectives of finding the most invariant solution and optimal solution must be considered simultaneously, and the designer must be able to decide upon a final solution that meets or exceeds the requirements for both objectives.

The development of a single metric that quantifies the designer's preference for an efficient solution, in terms of rack inlet temperatures and temperature gradients, such as those presented in previous literature [7, 84, 97] is required for single objective problem formulations. However, in a complex system such as a data center, a more flexible approach is preferred, that enables the designer to place emphasis on specific goals that can be local to a cabinet or applied to the entire center. These metrics can still be employed to determine a global merit of a data center design; however with the detailed information available from cabinet scale solution, specific goals and constraint considerations are also required. Therefore, the approach developed needs to be founded upon a method that can effectively find solutions that tradeoff between multiple conflicting objectives. Further sensitivity analyses of the method to the designer's preferences are also required, because of the complexity of the system response.

## **1.3.6** Experimental Validation

Only some of the previous CFD analyses of data centers have been validated with experimental measurements [76, 88, 89, 115]. This past work is all focused on the predictive capability of CFD models to determine the airflow though the under floor plenum to the perforated tiles tested using a flow hood. This work has not investigated any type of thermal validation, or validation on the cabinet level. Any computer simulations require extensive validation. Much of that validation consists of modeling procedures and assumptions to ensure accuracy, however the strongest validation comes from comparison to analytical or experimental results. The experimental mock blade server cabinet servers as an excellent test bed for the evaluation of the CFD models capability to predict the chip temperatures, and the level of modeling detail required to obtain a reasonably accurate solution. This validation is performed in Chapter 6 of this thesis. Without this validation, only the trends in the CDM models would be analyzed, as there would be no way to ensure the model's accuracy. The requirements for effective, efficient data center server cabinet design are set forth in this section. The capability of previously employed approaches in existing literature towards attaining these requirements is investigated in the next section, in the form of a gap analysis.

## 1.4 Foundational Principals Underlying the Approach

In this thesis an approach for improving the energy efficiency of data centers using a combination of reduced order modeling and robust design methods is investigated. As made evident by the literature survey, this is not a trivial task, with many challenges hindering the development of an effective, efficient, all encompassing data center design approach. A complete breakdown of the requirements of a design approach is given below in Section 1.3; however it is also pertinent to give an overview of the driving concept behind the development of the approach.

Previous efforts in data center design strive for uniformity in the cooling airflow distribution, in an effort to make cabinet cooling location independent. It has been shown by many researchers that this goal borders on the impossible to achieve [76, 80, 88, 89, 115]. This makes data center design guidelines such as [43] highly questionable. Guidelines by ASHRAE [7] are slightly better as they are also based upon best practice center layouts, as established by benchmarking studies such as those by Lawrence Berkeley National Laboratories [48].

It has been demonstrated through the literature review in Section 1.2.2 that significant temperature and flow velocity gradients exist even in best practice data center

configurations. It is also argued that the often exercised goal of minimization of hot exhaust air recirculation does not lead to maximum efficiency, rather that some recirculation is necessary for most efficient operation [78, 93, 94]. The design principle utilized in this thesis is of a similar vein to these ideas, infused with the concept of robust design. If there is an inherent flow and temperature distribution within a data center, and the cabinets housed within it, while some efforts should be made towards improving the uniformity from a global energy efficiency standpoint, this non-uniformity does not necessarily hinder effective design. Guidelines such as those by ASRAE [7] can be utilized to make significant strides towards reduction of gradients and are easily physically implemented.

The principle underlying robust design, as discussed further in Section 2.2.1, is to minimize the effects of variation, without eliminating their causes. This is accomplished through the addition of a parameter design stage, where the design variables are tuned to meet the desired performance objectives while minimizing the effect of the variation. A similar goal is employed in the approach developed in this thesis: rather than wasting energy trying to excessively remove the sources of flow gradients, effective server configurations are found that make best use of the more and less efficient regions of the data center and server cabinets, creating an approach that is effective regardless of the overall data center layout.

#### **1.5** Approach Requirements, Constructs and Integration

Of the six requirements identified, four are directly tackled by the development of an approach for the robust design of data center server cabinets.

- Systematic approach An efficient yet mathematically rigorous approach to the design of data center server cabinets should be developed to enable the development of superior cooling solutions.
- Reduced order modeling The CFD models required to analyze the systems are impractical to use in iterative optimization algorithms, particularly with many variables.
- Variability consideration The turbulent flow regime is inherently unstable, this variability is not represented in CFD simulations.
- Multi-objective tradeoffs The multiple design objectives in a complex system should represent the designer's preferences accurately.

These requirements are addressed in this thesis through the application of three constructs: (1) The Proper Orthogonal Decomposition (POD), (2) robust design principles, and (3) the compromise Decision Support Problem (cDSP). The POD is a highly computationally efficient meta-modeling approach, enabling quick computation of turbulent convective simulations [37]. The principle of robust design is used to find solutions that are insensitive to changes in both internal and external operating conditions. This yields solutions that maintain their desired performance accounting for variability in both the system and inaccuracies in the model of the system [19]. The cDSP, a hybrid formulation of mathematical programming and goal programming, enables multi-objective solution finding through the specification of multiple goals, and thus is well suited to engineering applications [55]. The integration of these constructs

provides a mathematically rigorous systematic approach to data center server cabinet design, shown schematically below in Figure 1.10. Each of these constructs is explained in detail in Chapter 2 and Chapter 3.



Figure 1.10 - Requirements, constructs, and integration for a robust server cabinet design approach

The challenge in the application of robust design is the computation of the nonlinear numerical derivatives, required for determination of the system variance, that require many functional evaluations of computationally expensive CFD models. Simple response surface models are inadequate, as the non-linearity of the systems is not well represented by linear or quadratic approximations, as shown by the analyses in [12, 40, 99]. Krieging, multivariate adaptive regression splines, and other more advanced interpolation approaches offer superior approximations [100]; however, these methods also require a large number of data points for interpolation, a number which increases exponentially with the number of design variables [100]. Furthermore, the tradeoff between the objectives of finding the most invariant solution and optimal solution must be considered simultaneously, and the designer must be able to decide upon a final solution that meets or exceeds the requirements for both objectives. Therefore the integration of all three constructs is required in order to develop an approach for complex turbulent thermal-fluid systems.

The remaining two requirements are met through application of the developed approach. The validation requirement is met through the detailed CFD modeling of the experimental mock blade server cabinet, and comparison to the empirically obtained results, documented in Chapter 6. The multi-scale cabinet level analysis requirement is tackled through the application of this approach to the cabinet design example problems presented in this thesis in Chapters 4, 5, and 6. Further consideration of this requirement is met through discussion of the extension of the POD based modeling approach for multi-scale modeling and analysis in Chapter 7.

#### **1.6 Research Questions and Hypotheses**

The challenges, existing research shortcomings, requirements, and approach for the establishment of an approach for the design of data center server cabinets for efficient cooling have been described in Sections 1.1-1.5. The principal goal for this thesis can thus be summarized by the following principal research question and associated hypothesis:

# Primary Research Question:

How can data center server cabinets be configured for efficient cooling while allowing for variability of both internal and external operating conditions?

# Primary Research Hypothesis:

The construction and application of Proper Orthogonal Decomposition based reduced order models allow the application of the compromise DSP integrating robust design principles to determine the system parameters required to meet the desired performance constraints and objectives.

This primary research question is parsed into several secondary research questions, and associated hypotheses, addressing the overall requirements of the primary research question:

## Research Question 1:

How should data center server cabinets be configured for most efficient cooling, subject to physical constraints, to cope with increasing heat loads?

## Research Hypothesis 1:

The construction and application of Proper Orthogonal Decomposition based reduced order models allow the application of the compromise DSP integrating robust design principles to determine the system parameters required to meet the desired performance constraints and objectives. Research Question 2:

How can data center server operation cope with variability in heat loads and performance of cooling equipment?

## Research Hypothesis 2:

Data center server cabinets can be configured with the objectives of robustness type I (resistance to changes in external operating conditions) and robustness type II (resistance to changes in internal operating conditions) enabling reliable server operation during changing conditions of both workload and cooling systems.

## Research Question 3:

How can the results of the simulation based design be ensured to produce valid physical results?

# Research Hypothesis 3:

Validation of CFD and reduced order models, in both fluid dynamics and heat transfer through empirical testing can ensure their accuracy in real applications. Application of robust solutions compensates for a degree of model inaccuracy. In order to fully answer these research questions, two key research activities are required:

- 1. The development of an efficient, effective approach for data center server cabinet configuration design.
- 2. The application of this approach to major cabinet designs, and experimental validation of the results obtained by the approach.

Finally, the extension of this approach to the general domain of the design of complex, multi-scale thermal-fluid system design should be analyzed and considered.

The focus of the approach developed to answer these questions is to provide as much information to the designer who can then apply their preferences in the manner they see most appropriate. Therefore, the research questions posed are not focused upon providing the most efficient cabinet configuration, rather at helping the designer make a final decision, to develop a tool to provide *decision support*. The final decision regarding the server or data center configuration still rests with the designer and their understanding of the system under consideration.

# 1.6.1 Mapping Requirements to Research Questions, Hypotheses & Tasks

Mapping the identified major industrial requirements of data center server cabinet configuration problems to the posed research questions, hypotheses, and associated research tasks is shown in Table 1.2. Table 1.2 maps out the specific tasks documented in this thesis, and how they fit into the "bigger picture" set by the research questions and hypotheses.

		_		_	_	_
Table 1.2 - Mapping	requirements,	research	questions,	and re	search	tasks

	Mapping requirements, research questions, and research tasks				
Problem	The complex turbulent flow regimes encountered in many thermal-fluid engineering applications have proven resistant to the effective application of systematic design because of the computational expense of model evaluation and the inherent variability of turbulent systems. In this thesis the integration of the Proper Orthogonal Decomposition (POD) for reduced order modeling of turbulent convection with the application of robust design principles is proposed as a practical design approach. The POD has been used successfully to create low dimensional steady state flow models within a prescribed range of parameters. The underlying foundation of robust design is to determine superior solutions to design problems by minimizing the effects of variation on system performance, without eliminating their causes. The integration of these constructs utilizing the compromise Decision Support Problem (DSP) results in an efficient, effective robust design approach for complex turbulent convective systems. The efficacy of the approach is illustrated through application to the configuration of data center server cabinets. The computer servers are cooled by turbulent convection and have unsteady heat generation and cooling air flows, yielding substantial inherent variability, yet require some of the most stringent operational requirements of any engineering system. Through variation of the power load distribution and flow parameters, such as the rate of cooling air supplied, thermally efficient configurations that are insensitive to variations in operating conditions are determined.				
ary	Research Question:	How can data center server cabinets be configured for sustainable efficient cooling while allowing for variability of both internal and external operating conditions?			
Prim	Research Hypothesis:	The construction and application of Proper Orthogonal Decomposition based reduced order models allow the application of the compromise DSP integrating robust design principles to determine the system parameters required to meet the desired performance constraints and objectives.			
	Requirements	Research Questions	Hypotheses	Tasks	
Question 1	Data centers receive new high output servers every 2-3 years while infrastructure is upgraded every 25 years.	How should data center server cabinets be configured for most efficient cooling, subject to physical constraints, to cope with increasing heat loads?	The construction and application of Proper Orthogonal Decomposition based reduced order models allow the application of the compromise DSP integrating robust design principles to determine the system parameters required to meet the desired performance constraints and objectives.	2D cold aisle study Design variable selection POD model development Thermal model development & analysis Cold aisle characterization Design space mapping Validation	
Question 2	Data center environments are inherently unpredictable and variable, however redundancy and reliability is paramount.	How can data center server operation cope with variability in heat loads and performance of cooling equipment?	Data center server cabinets can be configured with the objectives of robustness type I (resistance to changes in external operating conditions) and robustness type II (resistance to changes in internal operating conditions) enabling reliable server operation during changing conditions of both workload and cooling systems.	2D cabinet investigation Model development Robust design application Design variable selection Results & analysis Pareto frontier development Robust vs. optimal solution investigation Validation	

		11 8 1		
Question 3	Computer models, CFD simulations, and optimizations must be grounded with strong validation, preferably empirically.	How can the results of the simulation based design be ensured to produce valid physical results?	Validation of CFD and reduced order models, in both fluid dynamics and heat transfer through empirical testing can ensure their accuracy in real applications. Application of robust solutions compensates for a degree of model inaccuracy.	CFD 3D model, complex development POD of 3D model Maximum cabinet heat load investigation Full cabinet goal tradeoff study Experimental cabinet data acquisition
Quest	empirically.		Application of robust solutions compensates for a degree of model inaccuracy.	study Experimental cabinet data acquisition Comparison of CFD & experimental model CFD model analysis Validation

Table 1.2 - Mapping requirements, research questions, and research tasks cont.

### 1.7 Validation Strategy

The validation and verification strategy for this thesis is based upon the validation square introduced by Pederson and coauthors [73], who state that validation of engineering research, defined as the justification of knowledge claims in a modeling context, has typically been anchored in formal, rigorous, quantitative validation based on logical induction and/or deduction. As long as engineering design is based primarily upon mathematical modeling, this approach works well. Engineering design methods, however, rely on subjective statements as well as mathematical modeling; thus, validation solely by means of logical induction or deduction is problematic. Pedersen et al. propose an alternative framework to validation of engineering design methods in which "knowledge validation becomes a process of building confidence in its usefulness with respect to purpose."

In this approach, the usefulness of a design method is associated with whether the method provides design solutions correctly (structural validity) and whether it provides

correct design solutions (performance validity). This validation process is represented in the Validation Square in Figure 1.11.



Figure 1.11 - The validation square [73]

The four quadrants of the validation square and how they are applied in this thesis are discussed below. Note that the primary motivation of this thesis is the personalization of robust design for the design of complex turbulent thermal-fluid systems, specifically data center server cabinets, not a general product/process design methodology. Hence the application of the validation square is used more as guidelines for the rigorous mathematical validation of the modeling tools and constructs employed.

A more detailed breakdown of the specific chapter sections that relate to the quadrants of the validation square is presented below in Table 1.3, explained in detail in the following sections.

Theoretical Structural Validity	Theoretical Performance Validity
Internal consistency of the design method	
Literature review to check internal	Ability to produce useful results beyond
consistency and validity of tools/methods	the chosen example problems
used inside method	
Relevant Sections	Relevant Sections
Section 2.1 – The POD	Section 2.1 – The POD
Section 2.2 – Robust Design	Section 2.2 – Robust Design
Methodologies	Methodologies
Section 2.3 – The cDSP	Section 2.3 – The cDSP
Section 3.1 – Flow Model Developemnt	Section 6.5.7 – Experimental Results vs.
Section 3.3 – Heat Transfer Model	CFD results
Development	Section 7.4.3 – Approach Broader Impact
Empirical Structural Validity	Empirical Performance Validity
Appropriateness of the chosen example	Ability to produce useful results for the
problems	chosen example problems
Intended example problems are within	Method is useful to define the problem,
tools/methods' valid ranges	model relationships, and solve the model
Example problems representative of	The solution reduces cost and/or time
intended applications	without sacrificing performance
Resulting data is capable of supporting	
conclusions	
Relevant Sections	Relevant Sections
Section 3.2 – Flow Model Validation	Section 4.5 – The cDSP for Energy
Section 3.4 – Heat Transfer Model	Efficient Cold Aisle Configuration
Validation	Section 5.5 – The cDSP for Energy
Section 4.5 – The cDSP for Energy	Efficient 2U Server Configuration
Efficient Cold Aisle Configuration	Section 5.6 - Cabinet Configuration for
	Increasing Heat Loads
	Section 5.7 – Pareto Frontier Development
	for the 2U Server
	Section 5.8 – Server Fan Configuration
	Investigation
	Section 6.4 – The cDSP for Energy
	Efficient Blade Cabinet Configuration
	Section 6.5 – The Experiemntal Cabinet
	Investigation

Table 1.3 - Complete validation road map

#### **1.7.1** Theoretical Structural Validity

Theoretical validity is the measure of the method's internal consistency, its logical soundness of its constructs, both of its individual components and the method as a whole. This involves searching and referencing literature related to each of the constructs employed in the design approach, as well as the internal consistency of their integration.

In this thesis theoretical structural validity is presented in Chapters 2, and 3. In Chapter 2 this involves describing the three primary constructs that are integrated to form the proposed design approach, their applicability to the data center problem, and what augmentations, if any, are required of each. In Chapter 3 the focus is upon the development of the modeling tools used in the investigation of the three example problems, hence the appropriateness of the foundations and representation of these modeling are investigated.

## **1.7.2 Empirical Structural Validity**

Empirical structure validity is a measure of how appropriate the examples used to test the design method or models. This involves the consideration of the example problems chosen for illustrating and verifying the performance of the individual components of the design approach. These components include physics modeling accuracy, reduced order modeling accuracy, and searching algorithm accuracy and convergence.

Empirical structural validity is established primarily in this thesis in Chapters 3 and 4. In Chapter 3 the validity of the primary models used is considered, and tested against known solutions. In Chapter 4 the validity of the solution finding algorithm is investigated against a graphically found solution and the Karesh-Kuhn-Tucker (KKT) optimality conditions [10] are tested.

#### **1.7.3 Empirical Performance Validity**

Empirical performance validity is a measure of the usefulness of the method to produce results for the chosen specific examples or problem under consideration. This involves using representative example problems to evaluate the outcome of the approach in terms of its usefulness. Metrics for usefulness should be related to the degree to which the method's purpose has been achieved. In the case of data center server cabinet design, the core metrics are increased energy efficiency and reduction of variability.

In this thesis empirical performance validity is explored in Chapters 4, 5, and 6. In these chapters the application of the devised approach is investigated for three different server cabinet designs, for models of increasing complexity and fidelity. Finally, in Chapter 6, particularly strong empirical performance validity is considered through experimental validation of the foundational CFD modeling method, its accuracy propagated through to the final solutions obtained using the method.

# 1.7.4 Theoretical Performance Validity

Theoretical performance validity is a measure of the ability of a method to produce useful results from application to problems outside of the example problems presented. This requires a "leap of faith" which is aided through the establishment of the validity of the work in Sections 1-3 of the validation square, building confidence in the approach developed. This includes showing that the problems are representative of a general class of problems and the approach is useful, inferring the general usefulness of the approach.

In this thesis, considerations of the theoretical performance validity are made in Chapter 7. In this chapter the viability of the approach for the robust design of multiscale complex thermal-fluid systems is discussed, and the limitations of the current approach, and further validation requirements presented. Because this thesis is focused upon a specific problem application, there is little focus upon this section of the validation square.

## 1.7.5 General Validation Considerations

There are three areas of the models and method developed that is subject to critical analysis using each of the first three quadrants of the validation square where applicable. These are:

- Process Representation
- Process Modeling
- Process of Solving Methods

The process representation is the validity and soundness of the representation of the system, the assumptions made and how the problem is partitioned to form the analysis. The process modeling is the selection of govorning equations and how they are integrated with the solution method. The process of solving the methods is the appropriateness and strength of the numerical algorithms or analytical techniques employed to solve the problem. Where applicable all three aspects are considered in each quadrant of the validation square.

# 1.8 Breakdown of Hypthesis Specific Validation

How the research hypotheses, presented in Section 1.6, map to the sections within the validation square are presented. Because of the similar application of the approach to problems with similar characteristics of differing complexity and cabinet architecture, the aspects of validation addressed for each research hypothesis is very similar, as every aspect of molding foundations and assumptions, to implementation must be investigated thoroughly.

# Hypothesis 1:

The construction and application of Proper Orthogonal Decomposition based reduced order models allow the application of the compromise DSP integrating robust design principles to determine the system parameters required to meet the desired performance constraints and objectives.

Relevant Sections of the Validation Square:
Theoretical Structural Validity	Theoretical Performance Validity	
Internal consistency of the design method Literature review to check internal consistency and validity of tools/methods used inside method	Ability to produce useful results beyond the chosen example problems	
Relevant Sections	Relevant Sections	
Section 2.1 – The POD	Section 2.1 – The POD	
Section 2.2 – Robust Design Methodologies	Section 2.2 – Robust Design Methodologies	
Section 2.3 – The cDSP	Section 2.3 – The cDSP	
Section 3.1 – Flow Model Developemnt		
Section 3.3 – Heat Transfer Model Development		
Empirical Structural Validity	Empirical Performance Validity	
Appropriateness of the chosen example problems	Ability to produce useful results for the chosen	
Intended example problems are within	example problems	
tools/methods' valid ranges	Method is useful to define the problem, model	
Example problems representative of intended	relationships, and solve the model	
applications	The solution reduces cost and/or time without	
Resulting data is capable of supporting conclusions	sacrificing performance	
Relevant Sections	Relevant Sections	
Section 3.2 – Flow Model Validation	Section 4.5 – The cDSP for Energy Efficient Cold	
Section 3.4 – Heat Transfer Model Validation	Aisle Configuration	
Section 4.5 – The cDSP for Energy Efficient Cold	Energy Efficient Cold Section 5.5 – The cDSP for Energy Efficient 2U	
Aisle Configuration	Server Configuration	
	Section 5.6 - Cabinet Configuration for Increasing	
	Heat Loads	
	Section 5.8 – Server Fan Configuration	
	Investigation	
	Section 6.4 – The cDSP for Energy Efficient Blade	
	Cabinet Configuration	
	Section 6.5 – The Experiemntal Cabinet	
	Investigation	

Table 1.4 - Validation square for hypothesis 1

## *Hypothesis 2:*

Data center server cabinets can be configured with the objectives of robustness type I (resistance to changes in external operating conditions) and robustness type II (resistance to changes in internal operating conditions) enabling reliable server operation during changing conditions of both workload and cooling systems.

Relevant Sections of the Validation Square:

Theoretical Structural Validity	Theoretical Performance Validity	
Internal consistency of the design method Literature review to check internal consistency and validity of tools/methods used inside method	Ability to produce useful results beyond the chosen example problems	
Relevant Sections	Relevant Sections	
Section 2.2 – Robust Design Methodologies	Section 2.1 – The POD	
Section 2.3 – The cDSP	Section 2.2 – Robust Design Methodologies	
	Section 2.3 – The cDSP	
	Section 6.5.7 – Experimental Results vs. CFD results	
	Section 7.4.3 – Approach Broader Impact	
Empirical Structural Validity	Empirical Performance Validity	
Appropriateness of the chosen example problems	Ability to produce useful results for the chosen	
Intended example problems are within	example problems	
tools/methods' valid ranges	Method is useful to define the problem, model	
Example problems representative of intended	relationships, and solve the model	
applications	The solution reduces cost and/or time without	
Resulting data is capable of supporting conclusions	sacrificing performance	
Relevant Sections	Relevant Sections	
	Section 4.5 – The cDSP for Energy Efficient Cold	
	Aisle Configuration	
	Section 5.5 – The cDSP for Energy Efficient 2U	
	Server Configuration	
	Section 5.6 – Cabinet Configuration for Increasing	
	Heat Loads	
	Section 5.7 – Pareto Frontier Development for the 2U Server	
	Section 5.8 – Server Fan Configuration	
	Investigation	
	Section 6.4 – The cDSP for Energy Efficient Blade Cabinet Configuration	

 Table 1.5 - Validation square for hypothesis 2

# Hypothesis 3:

Validation of CFD and reduced order models, in both fluid dynamics and heat transfer through empirical testing can ensure their accuracy in real applications. Application of robust solutions compensates for a degree of model inaccuracy.

Relevant Sections of the Validation Square:

Theoretical Structural Validity	Theoretical Performance Validity		
Internal consistency of the design method Literature review to check internal consistency and validity of tools/methods used inside method	Ability to produce useful results beyond the chosen example problems		
Relevant Sections	Relevant Sections		
Section 3.1 – Flow Model Developemnt	Section 2.1 – The POD		
Section 3.3 – Heat Transfer Model Development	Section 2.2 – Robust Design Methodologies		
	Section 2.3 – The cDSP		
	Section 6.5.7 – Experimental Results vs. CFD		
	results		
Empirical Structural Validity	Empirical Performance Validity		
Appropriateness of the chosen example problems	Ability to produce useful results for the chosen		
Intended example problems are within	example problems		
tools/methods' valid ranges	Method is useful to define the problem, model		
Example problems representative of intended	relationships, and solve the model		
applications	The solution reduces cost and/or time without		
Resulting data is capable of supporting conclusions	sacrificing performance		
Relevant Sections	Relevant Sections		
Section 3.2 – Flow Model Validation	Section 6.4 – The cDSP for Energy Efficient Blade		
Section 3.4 – Heat Transfer Model Validation	Cabinet Configuration		
	Section 6.5 – The Experiemntal Cabinet		
	Investigation		
	Section 6.5.7 – Experimental Results vs. CFD		
	results		

Table 1.6 - Validation square for hypothesis 3

### 1.9 Organization of the Work and Validation

The organization of this thesis is described with the purpose of achieving the principal goal set forth in the beginning of this chapter. Initially in Chpater 1 the objectives and motivation for the thesis are set forth, and quadrant (1) the theoretical structural validity is investigated. This is followed by the mathematical foundations and analysis models in Chatper 2, and quadrants (1) and (2) the empirical structural validity is investigated. These models are then applied in three case studies of increasing detail and complexity, in Chaptres 3, 4, and 5 respectively, building quadrant (3) the empirical performance validity of the approach. Finally, a critical review and future directions for the work presented in this thesis is discussed in Chapter 7, and quadrant (4) the

theoretical performance validity is investigated. How this process is laid out, as well as how each part and Chapter falls into the validation square is shown in Figure 1.12 and each chapter discussed in detail in turn.



Figure 1.12 - Thesis and validation roadmap

In Figure 1.12 the six requirements identified in Section 1.3 are numbered. The main three requirements, and how they are addressed by the three core constructs and integrated form the heart of this figure, and is identical to Figure 1.10. The systematic approach requirement is addressed through the integration of the other constructs with the compromise DSP template, the multi-scale requirement through the different example problems investigated, and the experimental validation through the experimental blade server cabinet. These requirements are tied into their appropriate sections of the thesis through the dashed boxes. Details on the objectives and material presented in each chapter are as follows:

- Chapter 1 The foundations for the development for an approach for the robust design of data center server cabinets is established, the research questions and hypotheses are presented, and the validation approach established.
- Chapter 2 The principal constructs integrated in the development of the approach are described mathematically, the Proper Orthogonal Decomposition, robust design principles, and the compromise Decision Support Problem. Analyses of the augmentations required to create the approach is presented.
- *Chapter 3* The first example application of the flow configuration of a horizontal flow cabinet in a cold aisle is presented. The characterization of the cold aisle model is completed. The validation of the search algorithm employed using the KKT conditions is performed. Sensitivity

to the amount of recirculation corresponding to the data center layout is performed.

- *Chapter 4* The first example application of the flow configuration of a horizontal flow cabinet in a cold aisle is presented. The characterization of the cold aisle model is completed. The validation of the search algorithm employed using the KKT conditions is performed. Sensitivity to the amount of recirculation corresponding to the data center layout is performed.
- Chapter 5 The application of the robust configuration approach to a vertical flow server cabinet is performed. The consideration of the power distribution, supply flow rates, and server fans used are investigated. The sensitivity of the system to the designer's preference for an optimal or robust solution is found using a Pareto Frontier.
- Chapter 6 The application of the robust configuration approach to a vertical flow blade server cabinet, modeled after the experimental mock blade server cabinet is investigated in a similar manner to Chapter 5. The sensitivity with respect to all three objectives is investigated with a multi-dimensional Pareto Frontier. The accuracy of the CFD models employed is validated against the experimental results and statistical analyses are performed to validate the selection of control variables used in the analyses.

*Chapter 7* – A critical review of the work is performed, and a summary of how thoroughly the research questions are answered is presented. A discussion of future extensions of the approach developed is presented, and the further validation requirements needed to meet the theoretical performance validity for multi-scale thermal-fluid systems is analyzed.

In the appendices in this thesis the work that supports and further validates the work presented in the main body, but does not fit directly into any chapter is presented. This consists of work performed by the author or colleagues that is referred to, but has not been published elsewhere for citation.

- Appendix A Perforated tile flow measurements of the Georgia Tech CEETHERM data center laboratory facility, taken by Charles Fraley.
- Appendix B D-optimal Design of Experiments approach and results for the generation of a quadratic response model of the perforated tile flow rates of the CEETHERM data center laboratory with respect to cold aisle position within the room.

## **CHAPTER 2**

# MATHEMATICAL TOOLS AND CONSTRUCTS

In this chapter the background, mathematical derivation, explanation, and application of the three core constructs used in the approach developed in this thesis are presented. The three constructs are the Proper Orthogonal Decomposition (POD) in Section 2.1, robust design methodologies in Section 2.2, and the compromise Decision Support Problem in Section 2.3. The requirements for augmenting these constructs into a robust server cabinet design approach are then discussed in Section 2.4, and the chapter synopsis and validation summary is presented in Section 2.5.

How this chapter falls into the overall structure of the thesis and the validation square is presented in Figure 2.1. This chapter builds upon the challenges identified in Chapter 1 through addressing them through the introduction of the three core constructs. This in turn addresses the theoretical performance and empirical performance validity, where the underlying principals and applicability of the constructs are investigated.



Figure 2.1 - Thesis and validation roadmap: Chapter 2

### 2.1 The Proper Orthogonal Decomposition

An emerging reduced order model development approach for turbulent flows is the Proper Orthogonal Decomposition (POD). This method has been used successfully to create low dimensional models of many types of turbulent flow problems [37], a summary of which is given in the following section.

The concept of the POD computation can be explained graphically. Given a set of multi-dimensional data, the aim of the POD is to accurately represent a complete data set in the most efficient manner possible. This is accomplished through finding the principal axes of the data set, representing the directions of minimum scatter. The orientation of these principal axes is found through orthogonal distance regression, derived in full in Section 2.1.5.

The focus of Jeff Rambo's PhD research is the use of the POD for development of reduced order models for steady state RANS based turbulent convective flows. A collaborative effort lead to the development of the flux matching procedure and modal coefficient interpolation flow models described in Section 3.1.3, however the fundamental decomposition concept and application was developed by Jeff Rambo. The reader is referred to [77, 82, 83] for a deeper mathematical background and derivation, however this section serves to provide a good introduction and enough detail for an understanding of its application in this thesis.

### 2.1.1 Past Applications

In many complex systems, there exist families of patterns for which it is possible to obtain a useful systematic characterization. This is often because the system is actually driven by only a small number of parameters; however the complexity of the system makes direct analysis of the system response to these driving parameters extremely difficult. These families of patterns are prolific in nature, and hence have a significant body of existing literature. Such examples include turbulent flows, [8, 11, 51, 101], image processing [35, 116], data compression, [6], human speech [118], and human face recognition [45]. The technique applied in all these applications is known as the Proper Orthogonal Decomposition (POD), but is also goes by other names such as the Karhunen-Loève decomposition [50], principal component analysis [42], and the Hotelling transform [28].

The POD was first proposed in 1901 [72], and then re-emerged in 1933 [38]. Its first application in the field of turbulence was by Theodorsen [112] and then Townsend [113], with further development and augmentations by Lumley [51]. These applications focused upon the identification and extraction of large-scale structures within turbulent boundary layers. Later work has applied the POD to turbulent flow related problems such as channel flows [9, 56, 119]. Previous work that relates to the creation of reduced order models have all focused upon using the Galerkin Projection of the system POD modes onto the governing equations, resulting in a set of coupled non-linear Ordinary Differential Equation's (ODE's) in time. The most relevant of these reconstructions have applied to the use of the reduced order models as a predictor for control schemes in

reactor flows and natural convection [53, 54, 64, 65, 103]. The geometry for all of these previous POD flow investigations has been either prototypical (such as flow around a cylinder), or simple geometry where inhomogeneous boundary conditions are easily homogenized by the inclusion of a source function in the decomposition. None of these previous applications have direct relevance for engineering design applications.

### 2.1.2 POD Fundamentals

The POD is similar to any modal decomposition, such as the Fourier series, where a system is decomposed into a series of fundamental modes and a linear approximation is obtained using the expansion theorem:

$$\vec{u}(x) = \sum_{i=1}^{\infty} a_i \vec{\varphi}_i(x) \tag{2.1}$$

Solution methods based on equation (2.1) are known as Galerkin or spectral methods, where  $\vec{u}(x)$  is the observed phenomenon, such as the flow field, and  $\vec{\varphi}_i$  are the basis functions and  $a_i$  are the weighting vectors. These basis functions span the entire domain  $\Omega$  bounded by system boundary  $\partial \Omega$ , not just finite elements. The uniqueness of the POD is that it is a stochastic tool, which uses principal component analysis to find the optimal linear bases for the modal decomposition presented in equation (2.1).

A series of system observations, which can be either numerical or experimentally gathered, is collected and mean centered. This mean centering changes the problem to the reconstruction of a perturbation from an average, the importance of which is explained graphically in Section 2.1.5. Furthermore, for flow applications this mean

centering helps homogenize the boundary conditions. This adds a source function into equation (2.1), where  $\vec{u}_o(x)$  is the ensemble average computed as the row-based average of  $\vec{u}(x)$ .

$$\vec{u}(x) = \vec{u}_o(x) + \sum_{i=1}^{\infty} a_i \vec{\varphi}_i(x)$$
 (2.2)

The empirical bases  $\varphi_i$  are found through maximizing the projection of the observations u(x) onto the basis functions, solving the following constrained variational problem through extremitizing the functional:

$$\left\langle \left| \left( \vec{u}, \vec{\phi} \right) \right|^2 \right\rangle - \lambda \left( \left\| \vec{\phi}^2 \right\| - 1 \right)$$
 (2.3)

Where  $\langle \cdot \rangle$  denotes ensemble averaging, (,) is the L<sub>2</sub> inner product, and  $\|.\|$  is the induced norm. The constraint term  $(\|\vec{\varphi}^2\|-1)$  is included to produce a normalized basis. Variational calculus can be applied to express the functional in equation (2.3) as the integral equation:

$$\int_{\Omega} R(x, x') \vec{\varphi}(x') dx' = \lambda \vec{\varphi}(x')$$
(2.4)

Where  $R(x, x') \equiv \langle u(x) \otimes u^*(x') \rangle$  is the correlation function. To compute R(x, x'), *m* system observations containing *n* DOF each are assembled into a matrix:

$$\vec{U} = \left\{ \vec{u}_1, \vec{u}_2, \dots, \vec{u}_m \right\} \in \mathbb{R}^{n \times m}$$
(2.5)

Then the cross-correlation tensor of the observations is taken:

$$R(x, x') = \left(\frac{1}{m}\right) \vec{U} \vec{U}^{T} \in \mathbb{R}^{n \times n}$$

$$(2.6)$$

The resulting eigenvectors of R(x,x') are the basis functions  $\phi_i$ , called POD modes, and the eigenvalues determine in decreasing magnitude the order of the modes. The eigenvalue spectrum is typically used as an 'energy criteria' where the magnitude of each eigenvalue determines what portion of the total energy of the system the corresponding eigenvector captures.

The basis produced by the POD can be proven to be the optimal linear decomposition, in the sense more energy is captured for a given number of modes than any other linear decomposition [37]. Therefore in general the first  $p \le m$  POD modes will better represent the system than the first p modes of any other linear decomposition. A graphical explanation of this argument is given in Section 2.1.5. The POD is able to create such a large reduction in the number of DOF in a system (up to a 10<sup>7</sup> reduction as presented in this thesis) because the eigenvalue spectrum exhibits a sharp decay, implying that only a few modes are needed to create an accurate system representation. An example eigenvalue spectrum is plotted in Figure 4.6 in Section 4.2.2. Finally, note that for 2-D or 3-D fluid flow, equation (2.4) becomes a vector-valued problem, however this does not impose any difficulties except that n becomes 2 or 3 times larger respectively.

### 2.1.3 The Method of Snapshots

The eigen-decomposition of R(x, x') defined in equation (2.6) is a limitation for large problems as current eigenvalue algorithms can only deal with matrices on the order of  $10^5$ . This means that the POD is not applicable to even medium size CFD problems, which includes the server cabinet simulations in this thesis. This problem is circumvented by the realization that:

$$span(\vec{u}_1, \vec{u}_2, ..., \vec{u}_m) = span(\vec{\varphi}_1, \vec{\varphi}_2, ..., \vec{\varphi}_m)$$
 (2.7)

If the observations  $u_i$  are linearly independent, which is almost always satisfied for the nonlinear Navier-Stokes equations used to solve the fluid flow problem, the POD basis can be expressed as a linear combination of the observations:

$$\vec{\varphi}_i(x) = \sum_{i=1}^{\infty} b_i \vec{u}_i(x) \tag{2.8}$$

The weight vector  $b_i$  in equation (2.8) are eigenvectors of the solution to:

$$R'b = \lambda b \tag{2.9}$$

Where  $R' = U^T U \in \mathbb{R}^{m \times m}$  and the *i*<sup>th</sup> eigenvector of equation (2.8) contains the weight coefficients to assemble the *i*<sup>th</sup> POD mode. This approach is known as the method of snapshots, developed by Sirovich [101, 102].

## 2.1.4 The POD Computation

The computation of either the standard POD or the method of snapshots resulting eigenvalue problem is efficiently solved using the Singular Value Decomposition (SVD):

$$A = L\Sigma V^T \tag{2.10}$$

Where  $A \in \mathbb{R}^{n \times m}$ ,  $L \in \mathbb{R}^{n \times m}$  are left singular vectors,  $V \in \mathbb{R}^{m \times m}$  are the right singular vectors, and  $\Sigma \in \mathbb{R}^{m \times m}$  is a diagonal matrix. It has be shown by [44] that:

$$AA^{T} = L\Sigma^{2}L^{T} \text{ and } A^{T}A = V\Sigma^{2}V^{T}$$

$$(2.11)$$

Therefore the computation of the POD modes is reduced to taking the SVD of *U*. This can be accomplished using a number of algorithms, such as those implemented by MATLAB [110]. Further information of the SVD is available in any good linear algebra textbook.

#### 2.1.5 Graphical Explanation & Validation of the POD

It is also possible to think of the POD modes simply as principal axes of the data contained in *U*, and thus the POD as a method of finding these axes [37], as performed in Principal Component Analysis. This is equivalent to the finding of a best-fit line or plane by minimizing perpendicular distances between the data points and regression line, commonly referred to as orthogonal distance regression. Once it is recognized that the best fit line or plane contains the centroid of the data, the sum of squared distances can be rewritten involving a Rayleigh quotient that uses the covariance matrix of the data (shifted by the centroid). The Rayleigh quotient is minimized and maximized by the eigenvectors of the covariance matrix that correspond to its smallest and largest eigen values [1]. The reader will recognize this approach as being identical to the POD approach given above, and once again in practice the vectors are computed using the Singular Value Decomposition (SVD).

Because of the difficulty in representation of data in multiple dimensions, this process is derived and shown graphically in two dimensions as follows. When performing a regression fit of a straight line to a set of data points in x and y it is common practice to minimize the sum of squares of the vertical distance between the data points and the regression line. However, this is not the only possible approach. It is also possible to minimize the horizontal distances from the points to the line, or the perpendicular distances from the data points to the line, as is done in the POD. The reason this is not commonly applied in regression is that, in general, the units of x and y may be different, and thus the angle of a line in the xy plane does not have an absolute significance. For instance, if x is time, and y is intensity, is no absolute weighting of the x errors in relation to y errors, and therefore there is no unique notion of perpendicular in the time-intensity plane. However, when x and y do have the same units (as in the case of the transport phenomena parameters modeled in this thesis), it is feasible to regress both xand y by minimizing the sum of squares of the perpendicular distances from the line. Note that the process given below is derived from [3], and is only applicable to a two dimensional problem, and is derived explicitly in order to validate the use of the SVD approach for multiple dimensions.

Given a set of two dimensional data in x and y where each vector contains n points the mean, or centroid, of the data is computed as:

$$X = \frac{1}{n} \sum_{i=1}^{n} x_i, \text{ and } Y = \frac{1}{n} \sum_{i=1}^{n} y_i$$
(2.12)

The data set is then mean centered, such that the point (0,0) becomes the centroid of the points:

$$x_i = x_i - X$$
, and  $y_i = y_i - Y$ , for  $i = 1, ..., n$  (2.13)

To find the principal axes, conceptually think of rotating the entire set of points about the origin through an angle  $\theta$ . This sends a point (*x*,*y*) to the point (*x'*,*y'*) where:

$$x' = x\cos(\theta) + y\sin(\theta) \tag{2.14}$$

$$y' = -x\sin(\theta) + y\cos(\theta) \tag{2.15}$$

For a fixed angle  $\theta$  the sum of the squared of the vertical heights of the *n* transformed data points is:

$$S = \sum_{i=1}^{n} y^{2}$$
(2.16)

In order to find the best fit, equation (2.16) is to be minimized. It is easiest to look at this as rotating the regression line so the perpendicular corresponds to the vertical. To find the minima of (2.16), the derivative with respect to  $\theta$  is taken and set equal to zero. The derivative of  $y'^2$  is:

$$2y'\frac{dy'}{d\theta} \tag{2.17}$$

The combination of equations (2.15)-(2.17) yields:

$$\frac{dS}{d\theta} = 2\sum_{i=1}^{n} \left[ \left( -x\sin(\theta) + y\cos(\theta) \right) \left( -x\cos(\theta) - y\sin(\theta) \right) \right]$$
(2.18)

Setting equation (2.18) to zero and dividing by 2, then expanding out the product and collecting terms into separate summations gives:

$$\left(\sum_{i=1}^{n} xy\right) \cdot \sin^{2}(\theta) + \left(\sum_{i=1}^{n} \left(x^{2} - y^{2}\right)\right) \cdot \sin(\theta) \cos(\theta) - \left(\sum_{i=1}^{n} xy\right) \cos^{2}(\theta) = 0 \quad (2.19)$$

Dividing through by  $\cos^2(\theta)$  yields a quadratic equation in  $\tan(\theta)$ :

$$\left(\sum_{i=1}^{n} xy\right) \cdot \tan^{2}(\theta) + \left(\sum_{i=1}^{n} \left(x^{2} - y^{2}\right)\right) \cdot \tan(\theta) - \left(\sum_{i=1}^{n} xy\right) = 0$$
(2.20)

Dividing equation (2.20) through by  $\sum_{i=1}^{n} xy$  gives:

$$\tan^{2}(\theta) + A \cdot \tan(\theta) - 1 = 0 \tag{2.21}$$

where, 
$$A = \frac{\sum_{i=1}^{n} (x^2 - y^2)}{\sum_{i=1}^{n} xy}$$
 (2.22)

Solving this quadratic for  $tan(\theta)$  is accomplished through the substitution of  $tan(\theta)$  for a variable *c*, thus rendering the solution to equation (2.21) possible through finding the roots of the polynomial  $c^2 + Ac - 1 = 0$ . These roots can be found using the quadratic formula:

$$c = -\frac{A}{2} \pm \frac{\sqrt{A^2 - 4}}{2} \tag{2.23}$$

This will yield two solutions, which correspond to the principal directions, i.e., the directions in which the scatter is maximum and minimum, in which the minimum is the desired final solution. The value *c* is the slope of the best-fit line, and thus  $\theta$  can be computed through taking the inverse tangent of *c*. If the data set were not mean centered, the principal axis found would simply be from the origin to the centroid of the data, and thus subtleties of the distribution of the data would be lost. Hence the importance of mean centering for this type of analysis.

To demonstrate the POD graphically, a two dimensional data set of n = 15 points was generated, with a random scattering of the points in  $x \in [0,10]$  and corresponding  $y = 0.75x \pm z$ , where z represents a random noise,  $z \in [0,2]$ . The data set used is given below in Table 2.1, where subscript *c* indicates the mean centered values from the centroid (*X*, *Y*), where X = 5.43 and Y = 3.83.

x	У	$x_c$	$y_c$
3.56	0.47	-1.86	-3.36
4.9	1.68	-0.44	-2.15
4.34	2.11	-1.08	-1.72
5.62	3.76	0.19	-0.07
6.16	4.49	0.73	0.65
1.13	2.85	-4.29	-0.94
8.98	7.21	3.55	3.38
7.54	4.80	2.11	0.96
7.91	5.82	2.47	1.98
8.14	6.59	2.71	2.76
6.70	3.33	1.26	-0.50
2.00	3.15	-3.42	-0.68
2.73	4.32	-2.70	0.49
6.26	5.17	0.83	1.33
5.36	1.67	-0.06	-2.16

Table 2.1 - Data set for POD example problem

The standard least squares fit is computed using the linear model:

$$y = \beta_1 x + \beta_0 \tag{2.24}$$

This translates into the matrix problem:

$$y = X\beta + \varepsilon \tag{2.25}$$

Where  $y \in \mathbb{R}^{n \times 1}$  is the vector of responses, in this case the *y* coordinates,  $\beta \in \mathbb{R}^{m \times 1}$ is the vector of coefficients of the linear model,  $X \in \mathbb{R}^{n \times m}$  is the matrix of data to be fit, in this case the *x* coordinates, and  $\varepsilon \in \mathbb{R}^{n \times 1}$  the vector of errors. The coefficient matrix  $\beta$ is computed as:

$$\beta = \left(X'X\right)^{-1}X'y \tag{2.26}$$

The original data points, least squares fit to the y, x, and orthogonal distance is shown below in Figure 2.2.



Figure 2.2 - Linear vs. orthogonal fitting of a best-fit line

In Figure 2.2 both axes have the same units with equal scaling. It is easily seen that a completely different fit is obtained if the data set is transposed, as shown by the different lines for the fit to x and y approaches. The orthogonal fit however makes best use of all of the available information, and thus creates a fit that is superior to either of these approaches. This is verified through the computation of the sums of the residuals, where the orthogonal fitting is always lower than either of the other regressions tested here. For the data presented in this example the sum of the residuals of the least squares fit to y is 18.13, whereas the sum of the residuals to the orthogonal fit is only 14.53. The difference in the computation of these residuals, and how the POD is accomplished in

general is shown below in Figure 2.3. Analysis of the residual lines in Figure 2.3, and the sum of the residuals, demonstrates the saying *'the shortest distance between two points is a straight line'* does indeed hold true, thus backing up the statement that the POD is the optimal linear representation of a system.



Figure 2.3 - y fit vs. orthogonal fit residual visualization

Using the POD approach to this same problem, the *x* and *y* coordinates are treated as two observations of *n* points, thus m = 2 and n = 15, and the SVD of  $U = \{x, y\} \in \mathbb{R}^{n \times m}$ is computed. Because *U* has a rank of 2 which is equal to *m*, The returned matrices  $L \in \mathbb{R}^{n \times m}$  and  $V \in \mathbb{R}^{m \times m}$  from equation (2.10) are the orthonormal bases for only the column space of *U* and the row space of *U* respectively, as opposed to the complete bases for all four fundamental subspaces [104].

The matrix V returned by the SVD is the rotational transformation matrix in order to rotate the data into alignment with the principle axis. Using the definition of a rotational transformation matrix [57], given in equation (2.27) below,  $\theta$  can be found through the inverse of the trigonometric function of any element in *R*.

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$
(2.27)

The computation of  $\theta$  yields 36.9 degrees when computed either using equation (2.27) and the SVD approach, or equation (2.21) and the direct 2D orthogonal fitting approach. This shows that the use of the SVD for the POD is a valid application.

The matrix *L* is the orthonormal bases, or POD modes, of the mean centered data set. This is the projection of the data onto the principal axes, and rescaled such that the norm is equal to 1. A comparison of a scatter plot of the *xy* data and the POD modes against each other reveals that the shape of the scatter is identical, however the POD mode plot is re-scaled and rotated about the angle  $\theta$ . The weight coefficients  $a_i$  in equation (2.2) can be found by projecting the POD modes onto the data set, to find a = [8.27, -2.71] for the reconstruction of the *x* data, and a = [6.22, 3.60] for the reconstructed data with the centroid added back in is shown below in Figure 2.4.



Figure 2.4 - POD reconstruction & original of example data set

The coincidence of every point in the data set shows that the POD reconstruction can exactly represent the original observed data for a simple system such as this. The discussion of the application of the POD to a more complex system, and how it must be extended in order to be useful as a flow modeling approach is discussed in the following section.

### 2.1.6 The Complementary POD Approach

As discussed in the previous section, the POD finds the principal axes of an ensemble of observations and projects the data onto them to obtain the orthonormal bases. If the data are not mean centered, the observation ensemble average point forms the direction of the principal axis. Using this idea, the concept of weighting different observations in the ensemble was utilized to influence the direction of the principal axis and the mean of the data, and hence the resulting POD modes [82, 83]. This can be used to localize the POD modes through adding the closest observations to the case to be reconstructed multiple times.

Although shown to be more accurate [83], a problem occurs when large weights are applied to a single observation. The eigenvalue spectrum asymptotically approaches  $\lambda = \{1, 0, 0, ...\}$  because a single observation is dominating the observation database. In other words, the POD subspace has been collapsed to a region near one observation and information in the POD subspace from other observations is lost. Therefore an approach is developed that creates both higher local accuracy as well as maintaining the more subtle system dynamics picked up by the complete computation of the higher order POD modes, called the Complementary POD (PODc) [82, 83], motivated by the concepts provided by Graham and Kverekidis [30] and Christensen *et al.* [21].

The POD procedure for application to turbulent flow modeling is improved by decomposing the POD basis into two orthogonal compliments:

$$\vec{\Phi} = \vec{\varphi}^{\perp} + \vec{\varphi}' \tag{2.28}$$

Where  $\vec{\varphi}^{\perp} \in \mathbb{R}^{n \times s}$  is a set of POD modes computed from only a selected subset of observations, and is constructed to be the orthogonal complement of the remaining set of modes,  $\vec{\varphi}' \in \mathbb{R}^{n \times m - s}$ . Letting  $\vec{u}^{\perp}$  denote the observations used to construct  $\vec{\varphi}^{\perp}$ , the reconstruction equation (2.2) is modified to:

$$\vec{u}^* = \langle \vec{u}^{\perp} \rangle + \sum_{i=1}^p a_i \vec{\varphi}_i$$
(2.29)

The two observations closest to and bounding the desired reconstruction form  $\vec{u}^{\perp}$ , which is further mean-centered:

$$\vec{u}^{\perp} = \vec{u}^{\perp} - \langle \vec{u}^{\perp} \rangle \tag{2.30}$$

The POD of the mean-centered set  $\vec{u}^{\perp}$  produces a single mode because the mean centering operation reduces the dimension by one. The orthogonal complement POD subspace is then defined as the resulting POD mode:

$$\vec{\varphi}^{\perp} = [\vec{\varphi}_1^{\perp}] \in \mathbb{R}^{n \times 2} \tag{2.31}$$

The remaining observations are then made orthogonal to  $\vec{\phi}^{\perp}$  and the POD is performed without mean-centering:

$$\vec{U}' = [\vec{u}'_1, \vec{u}'_2, ..., \vec{u}'_{m-2}], \vec{u}'_i \perp \varphi^{\perp}$$
(2.32)

$$b = SVD(\vec{U}'^{T}\vec{U}') \in \mathbb{R}^{m-2xm-2}$$
(2.33)

$$\vec{\varphi}' = \sum_{i=1}^{m-2} b_i \vec{u}'_i \in \mathbb{R}^{n \times m-2}$$
(2.34)

Equation (2.34) follows from the method of snapshots shown in equation (2.8). The full POD space is then assembled as in equation (2.28) using the definition of  $\vec{\phi}^{\perp}$  in equation (2.31). The eigenvalue spectra are of the two subspaces are combined, renormalized and sorted in order of descending magnitude. This modification to the POD attaches a more meaningful system reference point to  $\vec{\phi}^{\perp}$  while also generating a POD basis from the full ensemble to better capture the flow dynamics over the entire parameter ranges used to generate the observations. The only disadvantage to this approach over the standard POD is that the bases must be computed for every reconstruction, rather than re-used. However, because of the efficiency of the SVD algorithm, this computational time is trivial for even very large problems such as in Chapter 6.

### 2.1.7 Reynolds Average Navier-Stokes POD Approach

In order to apply the POD to the RANS equations a few deviations from the normal practice with of building models using the POD are taken. Once the POD basis is calculated, further analysis is often performed by taking the Galerkin projection of the governing equations onto the POD basis, resulting in a set of m coupled nonlinear equations. This approach is investigated fully in [82, 83], however, a summary of the difficulties is stated here. The first is that  $v_{eff}$ , the effective viscosity of the fluid is not a constant but a vector, which does not have a defined inverse, and this makes the Galerkin projection impossible. The second obstacle is that for non-homogeneous flows, such as those involved in almost any practical engineering problem, where the boundary pressure drives the flow field. The objective is to utilize the POD modes to create a model driven by design parameters such as inlet velocity. Finally, the computational expense of solving the Galerkin system is nearly as high as the original CFD computations, as computed by [82]. Therefore the approach taken is to develop methods based solely on the velocity field for RANS computations. These methods are discussed in Section 3.1.3.

Lastly a brief discussion of the physical modeling of the RANS equations is presented. When POD analyses for Direct Numerical Simulation (DNS) data are performed it is common to use upwards of 1000 modes in the model [9, 56, 119], while for laminar reconstructions only 10-20 modes are required [23, 30, 53, 64, 65, 103]. The RANS-based turbulence model essentially changes the governing equations to appear as laminar with a local strain rate dependent viscosity. The standard k- $\varepsilon$  model presented in equations (3.46)-(3.50) computes k and  $\varepsilon$  as functions of the local velocity gradients only. It is thus expected that the number of POD modes to represent a RANS based CFD computation will be on the order of a laminar flow reconstruction. This is important because as shown by equation (2.7), the number of observations in the ensemble determines the number of POD modes, and the creation of each observation constitutes the bulk of the computational work required for the application of this method.

#### 2.1.8 Flow Modeling Capability Example

A simple example designed to conceptually demonstrate the POD is presented in Section 2.1.5. This section serves to demonstrate the applicability of the POD to reconstruct a fluid flow field. The example problem geometry and fluid boundaries is shown below in Figure 2.5. This geometry is identical to the investigation in Chapter 4, and a complete description is available there. The geometry in Figure 2.5 shows a two dimensional representation of two server cabinets with four racks each on either side of a data center cold aisle. This cold aisle is the central flow path, and acts as the plenum to distribute the air to the individual server racks. The flow through this aisle comes primarily from the bottom, with continuity dictating the flow direction through the upper inlet yielding either an inflow or outflow depending upon the amount of air being drawn through the individual servers. At this time only the airflow within the cold aisle is considered.



Figure 2.5 - Problem geometry and flow boundaries

An observation ensemble was constructed from 48 observations, all combinations of different supply velocities and server outlet velocities using the commercial CFD program FLUENT v.6.1.22 [25]. The complete list of boundary conditions for all the observations, as well as how they were generated, is given in Table 4.1 in Section 4.2.1.

Below are the first 3 POD modes of the flow within a cold aisle section. The first captures the general upward velocity and entrance into racks, the second shows a dominantly left to right flow along the bottom portion of the cold aisle, and the third is a swirling type velocity field.



Figure 2.6 - First 3 POD modes of the cold aisle flow field [77]

To demonstrate the ability of the POD to create accurate, low dimension models, an arbitrary observation from the original set used to create the POD modes is reconstructed. The weight coefficients  $a_i$  are computed by projecting each POD mode onto the original observation, as was done in Section 2.1.5.



Figure 2.7 - Direct POD reconstruction of an observation with (a) one, (b) three, and (c) 5 modes [77]

As can be seen in Figure 2.7 and Figure 2.8 the error, computed as the norm of the difference between the reconstruction and original observation, is greatly reduced with the inclusion of additional modes, and tends to zero as the number of modes approaches the original number of DOF in the system. In Figure 2.8 u is the horizontal velocity component, v the vertical component, and P the pressure. The reconstruction with only 5 modes shows a small error, on the order of  $10^{-2}$ , which is acceptable for most engineering calculations. Ignoring the turbulence quantities, the exact solution contained 942 grid cells or 2826 DOF while the reconstruction contained 5 weight coefficients for each velocity component, for a total of 10 DOF. Thus, the POD has been shown to reduce the number of degrees of freedom by a factor of  $10^2$ .



Figure 2.8 - Reconstruction error for an observation [77]

This reconstruction has shown that the POD is effective at reconstruction of the more complex flow field data, over the simple example given in Section 2.1.5. Given that the reconstruction solutions are simply a linear combination of the POD modes, weight coefficients can be found to produce solutions that were not explicitly solved

through interpolation and error minimization techniques. The development of flow models based on these concepts is presented in Section 3.1.3.

### 2.2 Robust Design Methodologies

### 2.2.1 Robust Design Foundations and Principles

Robust design is an approach for the improvement of product and process quality through the reduction of their sensitivity to variations. However, this reduction in the effects of variability is sought without removing its sources [105]. Therefore, a robust design is a system that can be exposed to variations, either external environmental conditions or internal design specification and operation conditions, without suffering unacceptable performance degradation. The collection of design principles and methodologies known as robust design is founded on the philosophy of Genichi Taguchi, a Japanese industrial consultant. Taguchi stated product design is more cost effective approach to achieve robust, higher quality products than simply employing tight manufacturing tolerances and processes.

### 2.2.2 Type I & II Robust Design

The underlying principle of robust design is to determine superior solutions to design problems through minimizing the effects of variation, without eliminating their causes. There are two categories of robust design problems classified in [18]. Both simultaneously bring the mean system performance to a target and minimize performance variation; however, the sources of the variation are different [18, 19].

- *Type I* minimizing variations in performance caused by variations in noise factors (uncontrollable parameters)
- *Type II* minimizing variations in performance caused by variations in control factors (design variables)

The graphical representation of Type I robust design is shown below in Figure 2.9.



Figure 2.9 - Type I robust design diagram, modified from [19]

In robust design it is desirable to take advantage of interactions and nonlinear relationships between control factors and noise factors to reduce the influence of the noise factors on the response. This is shown graphically in Figure 2.9 above. Here the control factor settings are selected to minimize the sensitivity of the system response, plotted on the y axis, to variations in a noise factor, plotted on the x axis. In the above

problem, a control variable setting of x = a significantly reduces the amount of variability from the desired response.

In the thermo-fluid problems tackled in this thesis the effects of noise factors are much less significant than the control factors, as almost all aspects of the airflow and heat generation are under the designer's control. Therefore Type II robust design is the more applicable in this thesis. If the control factors are expected to fluctuate, as will occur given the arguments in Section 1.3.4, control factors settings are selected that minimize the sensitivity of the system response to control factor variation. As shown in Figure 2.10, this results in a compromise between mean performance, and performance variation.



Figure 2.10 - Type II robust design diagram, modified from [19]

In this figure the optimal solution considered is unlikely to produce a response that is close to the expected mean. The robust solution is insensitive to the same variation in the design variable, plotted on the x axis, and the variation in the system
response, plotted on the y axis. Traditional optimization techniques only bring the mean response to a target and do not consider the effects of the variation in the system parameters or control factors in the performance evaluation. Through accounting for variation, robust design techniques can produce results that are effective regardless of changing operating conditions, system parameters, assumptions and/or small inaccuracies made during the system modeling process. Throughout this thesis a "robust solution" is defined using the Type II concept of robustness, a solution that minimizes the variance of the solution , not the minimization of the signal to noise ratio as defined by Taguchi [105].

Type III robust design, robustness with respect to model uncertainty, as developed by Choi [20] is not applicable to the work in this thesis as all models used are deterministic, and hence upper and lower limits of the objective function cannot be found. Limitations of the Taguchi methodology in dealing with highly nonlinear problems [60] and the inefficient generation of orthogonal arrays [114] are addressed in this thesis through following the approach of Chen [18, 19] and the Robust Concept Exploration Method (RCEM). However, RCEM is not directly used in this work, rather the formulation of goals and constraints is applied, derived mathematically in the next section.

## 2.2.3 Application of Robust Design

The application of robust design in this thesis follows the approach taken by Chen and coauthors [19], through the formulation of a robust design problem as a multiobjective problem of bringing the mean to target and minimizing the variation of the response. In this approach both control and noise factors are considered as potential sources of variation, and constraints are modeled in a worst case scenario formulation to ensure feasibility. The flexibility in this approach is used to investigate the tradeoffs between robust and optimal solutions, as well as the sensitivity of the solutions found to changes in the designer's preferences. It is noted that the mathematical formulation of goals and constraints is described in Section 2.3, which is required for understanding of the application of robust design given in this section.

In finding the solution of a robust design problem the evaluation of the nominal value of the objective function is required as well as the variation of each response due to variation in each control and noise factor. This response, y, is a function of the control factors,  $\vec{x}$ , and noise factors,  $\vec{z}$ :

$$y = f(\vec{x}, \vec{z}) \tag{2.35}$$

This function *f* can be a simulation model, surrogate model, or physical system. The expected value,  $\mu_y$ , and variance,  $\sigma_y^2$  of the response must be computed for the response *y*. In this thesis as deterministic simulations are used, the value  $\mu_y$  is simply the output of equation (2.35), therefore:

$$\mu_y = f(\vec{x}, \vec{z}) \tag{2.36}$$

The variation is estimated using a first order Taylor series expansion [74]. This relates the variation in response,  $\sigma_y^2$ , to variation in each noise factor,  $\Delta z_i$  and control factor,  $\Delta x_i$ , as:

$$\sigma_{y}^{2} = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_{i}} \right) \Delta x_{i}^{2} + \sum_{i=1}^{m} \left( \frac{\partial f}{\partial z_{i}} \right) \Delta z_{i}^{2}$$
(2.37)

Where *n* is the number control factors, and *m* the number of noise factors. This form of the variance in equation (2.37) is known as the Second Order Reliability Method [74]. The use of absolute bounds of variability in control and noise parameters,  $\Delta x$  and  $\Delta z$ , can be substituted with statistical measures of standard deviation if available. The use of three standard deviations,  $3\sigma$ , meaning 99.8 % of the variability is accounted for, is proposed by Chen [19], and will produce superior results when accurate statistical data are available. The partial derivatives in (2.37) are computed with respect to all variables, which is equivalent to computing the response curvature. These partials can be computed using analytical expressions if available, or finite differencing techniques, such as the central difference [36] used in this thesis:

$$\frac{\partial f}{\partial x_i} = \frac{f(x - \delta x) - f(x + \delta x)}{2\delta x} + O(\delta x)^2$$
(2.38)

This approach is second order accurate, which is sufficient for the problems in this thesis, although more computationally intensive and accurate approaches are also applicable [13]. The use of the computation of this mean value and variance in the formulation of design goals is described next.

### 2.2.3.1 Formulation of Goals

As stated in the previous section, a robust solution is sought through obtaining a compromise solution between the point of minimum objective function value, the optimal

solution, and point of minimum response curvature, the least varient solution. These two points are identified on Figure 2.10. The tradeoff between these points is considered using a weighted sum preference aggregation technique, namely the Archimedean formulation of the compromise DSP. The mathematics of this approach are derived in Section 2.3.4. The exact tradeoff is subject to the designer's preference towards operational stability or the most efficient solutions available, however as shown in Figure 2.10, when the variability of the system is considered, this optimal solution may produce a mean response that is not as good as predicted.

## 2.2.3.2 Formulation of Constraints

Consideration of problem constraints incurs an added layer of complexity when a robust solution is sought over traditional optimization. This is because the variation of system response must be considered on top of the mean response value in a worst-case scenario, as given in equations (2.39) and (2.40).

$$g_{j}(\vec{x},\vec{z}) + \Delta g_{j} \le 0 \ j = 1,...,p$$
 (2.39)

Where *p* is the number of inequality constraints. The inequality constraint function,  $g_j$ , is a function of the same control and noise factors as the objective function *f*. Note that equation (2.39) is simplified from the form given in [19] as the mean of  $g_j$  is usually taken, however as the models used in this thesis are all deterministic, this step is not necessary. This constraint function value is added to the maximum response variation attainable though the variability of the control variables, given by  $\Delta g_j$ :

$$\Delta g_j = \sum_{i=1}^n \left| \frac{\partial g_j}{\partial x_i} \Delta x_i \right|, j = 1, \dots, p$$
(2.40)

The effects of this variance consideration are represented graphically in Figure 2.11. In the optimal solution point the variability inherent to the control variables means the solution violates the constraint since part of the area created by the variability in the control variables lies outside of the feasible region, despite having a feasible average value. The entire area surrounding robust solution point is fully inside the feasible region and hence is feasible even in the worst case variability scenario.



Figure 2.11 - Robust constraints application diagram

This worst case treatment of the constraints is appropriate as violation of a constraint is serious, resulting in the systems failure, and not simply being used to guide the designer's preference towards a more stable solution.

Equality constraints are computed using only the response values of the constraint function. This is because of the nature of an equality constraint, where the inclusion of variability in a worst case scenario does not make sense, as there is no way to ensure the constraint is always met, only that it will be met by the average conditions. Furthermore, because the variability is equal to either side of the constraint, preferential placement of the solution point to one side of the constraint does not make sense either. Hence, the implementation of equality constraints is unchanged from classical optimization approaches. The formulation and mathematics of the multi-objective robust compromise DSP is described in the next section.

## 2.3 The Compromise Decision Support Problem

In order to effectively trade off between multiple objectives, as well as the tradeoff between nominal and robust solutions, a method is required that allows flexibility in its formulation and the solutions found in order to best represent the designer's preferences. This enables the generation of families of multi-objective compromising solutions, for example, in the design of a supporting beam, the designer must seek a compromise between minimizing the mass of the beam, while simultaneously maximizing the beam's stiffness. The challenge is thus to identify design parameters for a solution that yields the preferred compromise between these conflicting requirements. The method used in this thesis, the Compromise DSP is described next.

#### 2.3.1 What is the Compromise DSP?

The compromise Decision Support Problem (DSP) [55] is a hybrid formulation of mathematical programming and goal programming well suited to engineering design applications. In its most general form, the conventional mathematical programming problem is formulated as:

Minimize:	$f(\vec{x})$ (2.41)	)
Subject to:	$\vec{g}(\vec{x}) \leq 0$	(2.42)
	$\vec{h}(\vec{x}) = 0$	(2.43)
	$\vec{x}_L \leq \vec{x} \leq \vec{x}_U$	(2.44)

Where  $f(\vec{x})$  is the objective function to be minimized through manipulation of the set of control variables  $\vec{x}$ . The functions  $\vec{g}(\vec{x})$  and  $\vec{h}(\vec{x})$  are vectors of inequality and equality constraints respectively, and  $\vec{x}_L$  and  $\vec{x}_U$  are vectors of upper and lower bounds on the design variables  $\vec{x}$ . When considering multiple objectives, the objective function becomes a vector, and this equation (2.41) is expressed as:

Minimize: 
$$\vec{f} = \{f_1(\vec{x}), f_2(\vec{x}), ..., f_m(\vec{x})\}$$
 (2.45)

Where m is the number of objectives. By placing different priorities on the individual objectives in equation (2.45), it is possible to obtain many solutions to the multi-objective problem. This range of compromise solutions is known as a Pareto set, curve or frontier [63]. The solutions along this curve are defined as non-dominated, meaning there is no other feasible solution that improves one or more objectives without worsening the others.

Design solutions are rarely evaluated on the basis of a single objective, but rather upon how well they balance multiple objectives often associate with cost, efficiency, environmental impact, robustness, and reliability. Therefore, in order to effectively pursue a balance between these multiple objectives many techniques have been proposed for generating Pareto sets of solutions. The simplest of these is a weighted sum approach, where the weighted sum of an objective function, Z, is expressed as a linear additive combination of the multiple objectives:

$$Z = \sum_{i=1}^{m} W_i f_i$$
 (2.46)

Where  $W_i$  is the weight for the *i*<sup>th</sup> objective,  $f_i$ , and *m* is the number of objectives. This approach is simple to implement and understand, and through variation of the weights it is possible to generate a family of Pareto solutions to the multi-objective problem given in equations (2.42)-(2.45). However, among other more foundational criticisms, if a single multi-objective solution is sought, it is difficult to determine *a priori* an appropriate set of weight coefficients that yield an appropriate compromise solution that is not dominated by a single or few objectives relative to the entire set.

In order to remedy this problem the compromise DSP implements objectives based upon goal programming. The focus of goal programming is to establish goals for each objective and attain each of them to the extent possible [15]. The corresponding mathematical formulation is as follows. For each objective, an achievement function,  $A_i(\vec{x})$ , represents the value of the objective as a function of the set of design variables,  $\vec{x}$ , and the goal value,  $G_i$ , is established for each objective. Deviation variables,  $d_i^+$  and  $d_i^-$ , represent the extent to which the achievement underachieves or overachieves its goal:

$$A_{i}(\vec{x}) + d_{i}^{-} - d_{i}^{+} = G_{i}$$
(2.47)

The overall objective function is therefore expressed as a function of the deviation variables as:

$$Z = \int_{i=1,\dots,m} \left( d_i^+, d_i^- \right)$$
(2.48)

The conceptual basis of the compromise DSP is to minimize the difference between what is desired,  $G_i$ , and what can be achieved,  $A_i(\vec{x})$ , represented by the deviation variables  $d_i^+$  and  $d_i^-$ . Therefore as expressed in equation (2.48) the objective function is exclusively a function of the deviation variables. The same weighted sum approach as taken in mathematical programming in equation (2.46) is used to aggregate the multiple goals into a composite Archimedean objective function:

$$Z = \sum_{i=1}^{m} \left( W_i^+ d_i^+ + W_i^- d_i^- \right)$$
(2.49)

Because it is impossible to simultaneously overachieve and underachieve a goal, restrictions are placed on the deviation variables to limit them to positive values and ensure than only deviation variable is positively valued at any point in the design space:

$$d_i^+ \ge 0, d_i^- \ge 0, d_i^+ \bullet d_i^- = 0 \tag{2.50}$$

Although not strictly part of goal programming, equality and inequality constraints are supported in the compromise DSP, the formation of which follows mathematical programming:

$$g_i(\vec{x}) \ge 0, i = 1, \dots, p$$
 (2.51)

$$h_i(\vec{x}) = 0, \, i = 1, \dots, q$$
 (2.52)

Where p and q are the number of inequality and equality constraints respectively. Bounds are also placed on the control variables, again following the formulation of mathematical programming:

$$x_{i,L} \le x_i \le x_{i,U}, i = 1, \dots, n$$
 (2.53)

Where *n* is the number of design variables, and  $x_{i,L}$  and  $x_{i,U}$  are the lower and upper bounds, respectively, for the *i*<sup>th</sup> design variable. With the mathematical foundations of the compromise DSP covered, their implementation in formulating a problem to be solved is described next.

### 2.3.2 Formulating the Compromise DSP

The objective function formulation and constraints borrowed from goal programming and mathematical programming respectively are unified into a single construct for problem definition and solution, the compromise DSP template, shown below in Table 2.2.

## Given

An alternative to be improved through modification Assumptions used to model the domain of interest The system parameters:

- *n* number of system variables
- *p* number of inequality constraints
- *q* number of equality constraints
- *m* number of system goals

#### Find

System Variables	$x_{i}$	$i=1,\ldots,n$
Deviation Variables	$d_i^+, d_i^-$	i = 1,, m

#### Satisfy

Inequality Constraints	$g_i(\vec{x}) \leq 0$	i = 1,, p
Equality Constraints	$h_i(\vec{x}) = 0$	i = 1,, q
Goals	$A_i(\vec{x}) - d_i^+ + d_i^- = G_i$	i = 1,, m
Bounds	$x_{i,L} \le x_i \le x_{i,U}$	<i>i</i> = 1,, <i>n</i>
	$d_i^+ \ge 0; d_i^- \ge 0; d_i^+ \bullet d_i^- = 0$	i = 1,, m

#### Minimize

Deviation Function: Archimedean formulation

$$Z = \sum_{i=1}^{m} W_i \left( d_i^+ + d_i^- \right) \qquad i = 1, ..., m$$

There are alternative approaches to formulating the objective function in the compromise DSP, such as the lexicographic approach [55], or using Utility Theory [91]. However, in this thesis the Archimedean formulation is utilized exclusively as it is the most applicable formulation, given the nature and associated information available of the problems encountered in the robust design of data center server cabinets.

The formulation and application of the constraint functions,  $g_i(\vec{x})$  and  $h_i(\vec{x})$  are straightforward. If the response  $g_i$  is greater than zero, or response  $h_i$  is not equal to zero, the solution is infeasible. The formulation of the goals such that the deviation from the target is minimized is more complex, and is described in the following section.

## 2.3.3 Mathematical Formulation of Goals in the Compromise DSP

There are three difference goal scenarios used in the compromise DSP, each resulting in a different mathematical formulation of the goal function [55]. The three different scenarios are: (I) the maximization of an objective, (II) the minimization of an objective, and (III) matching a target objective value. All formulations use the same nomenclature and variables as the goal programming approach, and equation (2.47). Each of these scenarios is discussed in turn below.

#### 2.3.3.1 Scenario I: Objective is to be maximized

To maximize the achievement,  $A_i(\vec{x})$ , a target  $G_i$  is selected that is greater or equal to the maximum expected value, such that the ratio in equation (2.54) is always true:

$$\frac{A_i(\vec{x})}{G_i} \le 1 \tag{2.54}$$

The deviation variables are added in to transform the problem to an equality formulation, and thus the deviation variables will vary between 0 and 1:

$$\frac{A_i(\vec{x})}{G_i} + d_i^- - d_i^+ = 1$$
(2.55)

In this formulation the deviation variable  $d_i^+$  will always be 0, and no overachievement is possible by definition. Therefore the goal becomes to minimize the underachievement  $d_i^-$ :

$$d_i^- = 1 - \frac{A_i(\vec{x})}{G_i}$$
(2.56)

## 2.3.3.2 Scenario II: Objective is to be minimized

To minimize the achievement,  $A_i(\vec{x})$ , a target  $G_i$  is selected that is lesser or equal to the minimum expected value, such that the ratio in equation (2.57) is always true:

$$\frac{G_i}{A_i(\vec{x})} \le 1 \tag{2.57}$$

The deviation variables are added in to transform the problem to an equality formulation, and thus the deviation variables will vary between 0 and 1:

$$\frac{G_i}{A_i(\vec{x})} + d_i^- - d_i^+ = 1$$
(2.58)

In this formulation the deviation variable  $d_i^-$  will always be 0, and no underachievement is possible by definition. Therefore the goal becomes to minimize the overachievement  $d_i^+$ :

$$d_i^+ = 1 - \frac{G_i}{A_i(\vec{x})}$$
(2.59)

If the target  $G_i$  is set to be zero, as estimate of the maximum value of the achievement  $A_i(\vec{x})$  is obtained,  $A_{i,\max}$ . The system goal is then formulated as:

$$\frac{A_i(\vec{x})}{A_{i,\max}} + d_i^- - d_i^+ = 0$$
(2.60)

In this formulation again the deviation variable  $d_i^-$  will always be 0, and no underachievement is possible by definition. Therefore the goal becomes to minimize the overachievement  $d_i^+$ :

$$d_i^+ = \frac{A_{i,\max}}{A_i(\vec{x})} \tag{2.61}$$

#### 2.3.3.3 Scenario III: Objective is to be matched

If it is desired that the achievement,  $A_i(\vec{x})$ , is to equal a target  $G_i$ , the approach depends upon the orientation of the target. If the target value  $G_i$  is approached from below by  $A_i(\vec{x})$  the problem is treated as a maximization problem, and equations (2.55)-(2.56) are applied. If the target value  $G_i$  is approached from above by  $A_i(\vec{x})$  the problem is treated as a minimization problem, and equations (2.58)-(2.59) are applied. If the target is zero, the problem is treated as a minimization problem and equations (2.60)-(2.61) are applied.

#### 2.3.4 Use of the Compromise DSP for the Application of Robust Design

The flexibility of the compromise DSP to represent all of these different goal formulations, and aggregate them into a single objective function is very useful for the

application of robust design, where multiple objectives of maximization, minimization, and target matching (particularly of response variability which is often set to zero) exist and must be solved concurrently. This capability to find solutions that are dominated, i.e. lie within the feasible design space but are not part of the Pareto frontier, and solutions that are dominant, i.e. lie on the Pareto frontier, through the setting of the targets associated with the goals is shown below in Figure 2.12



Figure 2.12 - Pareto solutions and goal targets in the Compromise DSP

In this figure two objectives are being balanced, represented by the x and y axes. The system constraints bound the feasible design space, limiting the attainment of both of these objectives. In scenario A the targets are set low, and hence both objectives are met, and the solution is found despite the fact that it is dominated by other solutions along the Pareto frontier. If the goals are set outside of the feasible design space, as is done in scenario B, the minimum deviation from the objectives is sought, yielding solution B. In traditional optimization dominated solution points such as A are considered undesirable, as points along the Pareto frontier are superior with no loss in either objective. However, in robust design, as demonstrated in Figure 2.10 and Figure 2.11, the desired solution point may not be part of this optimal Pareto set. An interior point, while initially unattractive, leads to greater flexibility in later design stages as constraints are tightened and objectives shifted, as there is still freedom to move within the feasible design space, whereas solutions on the Pareto frontier are already constrained to moving along the vertex, and may simply become infeasible solutions. This flexibility to generate families of solutions either on or off the Pareto frontier is possible without reformulating the problem simply by changing the goal targets and/or weighting values. This makes the compromise DSP an ideal construct for finding robust solutions to data center server configuration problems.

In order to determine a solution that simultaneously achieves the required performance goal as well as having low variability required for the implementation of robust design, two approaches are possible. The first is to set an achievable performance target, resulting in a feasible solution region within which the minimum of the second goal of variation minimization is obtained. The second is to set unobtainable goals for both performance and variability, and search the resulting Pareto frontier created by these two goals. Both approaches are viable, and while the second approach will yield a superior result, the first approach has greater flexibility to account for changing design specifications further along the design process. With these three core constructs described, a gap analysis is performed to find what augmentation or additional models or methods are required.

#### 2.4 Gap Analysis: What is needed

In this section the limitations of the three constructs described in Sections 0, 2.2, and 2.3 with respect to an approach for the robust configuration of data center server cabinets is completed. What further model development is needed in order to alleviate these shortcomings is then discussed.

The POD and PODc methods in themselves are only a highly efficient method of data representation. It still requires initial observations, and cannot improve over any shortcomings of these observations, the generation of which is computationally expensive. For efficient fluid flow modeling an augmentation to the method must be made in order to reconstruct the flow field based upon design variables which span a limited part of the domain, such as the server air inlet velocity, as the complete solution will not be available. This augmentation to create the POD based flow model is derived and validated in Section 0.

The POD is effective for the modeling of turbulent fluid flow, however the solution of a conjugate heat transfer problem is still under development. With the flow solution computed, the heat transfer solution is decoupled under the assumption of forced convection (valid for the flow regimes encountered in data center server cabinets). Therefore a convective diffusive energy equation solver to compute the temperature

profile is required. The derivation, analysis and validation of this model is presented in Section 3.3.

Robust design principles implemented though the compromise DSP formulation constitute an effective approach to robust design. However, the weighted sum formulation of the designer's preferences is sensitive to the bounds of the achievement function, and does not represent these preferences well in a complex non-linear system [58]. Therefore a full Pareto set of optimal to robust solutions is developed when applicable in order to obtain the full set of solutions to fully investigate the effectiveness of pursuing a robust solution, as implemented in Section 5.7 and 6.4.3. The development of these models is presented and validated in the following chapter.

## 2.5 Chapter Synopsis and Validation Summary

In this chapter the background, mathematical derivation, explanation, and application of the three core constructs used in the approach developed in this thesis was presented, consisting of the POD, robust design principles and application, and the compromise DSP. The quadrants of the validation square that have been addressed in this chapter are presented below. How the validation performed in this chapter falls within the complete validation roadmap can be determined from viewing Table 1.3.

## Theoretical Structural Validity

 Literature survey of previous applications of the POD was completed in Section 2.1.1.

- The mathematics of each construct is presented and explained thoroughly, with reference to their original derivation and formulation in Sections 2.1.2-2.1.6.
- The application of the POD to the RANS equations and theoretical considerations were made in Sections 2.1.7-2.1.8.

# Empirical Structural Validity

- Principal axis of a 2D random point scatter found by POD and direct analysis, showing identical results in Section 2.1.5.
- The reconstruction of a 2D flow field by projection of the POD modes onto the solution was presented, demonstrating the feasibility of the POD as a reduced order modeling tool in Section 2.1.8.

With the mathematical constructs discussed, and the requirements for model development completed, the core analysis models used in this thesis are derived.

# **CHAPTER 3**

# ANALYSIS MODEL DEVELOPMENT

In this chapter the core analysis models for the fluid flow and heat transfer simulations are developed and validated. The fluid flow model is based upon POD reconstructions using the flux matching procedure, and the heat transfer model is based upon a finite difference discretization of the energy equation, approximated using the power law. In Section 0 the flow model is developed, and validated in Section 3.2. In Section 3.3 the heat transfer model is developed and validated in Section 3.4. The chapter synopsis and validation summary is presented in Section 3.6.

How this chapter falls into the overall structure of the thesis and the validation square is presented in Figure 3.1. This chapter builds upon the constructs identified and described in Chapter 2 through the construction of the POD based flow model using the Flux Matching Procedure, and the finite difference heat transfer solver. This in turn addresses the empirical performance validity, where the appropriateness and capability of the individual aspects of the approach are tested to produce accurate and useful results.



Figure 3.1 - Thesis and validation roadmap: Chapter 3

#### **3.1** Flow Model Development

The fundamentals of the POD and PODc have been explained in Section 0, and its ability to successfully create an accurate model of a turbulent flow with greatly reduced DOF demonstrated. However, this reconstruction was only applied to an observation, a state for which the complete solution already existed. The challenge is to develop an approach in which an accurate flow field is reconstructed using parameters that are meaningful in an engineering design and analysis sense. In particular it is useful to be able to specify flow rates or integral fluxes across specific boundaries, in a similar manner to creating a CFD model, as these quantities are often most easily quantifiable and specified in engineering design practice. Based on this concept, two different approaches to find the weighting coefficients  $a_i$  are developed in Section 3.1.3. These approaches work with the bases created with either the POD or PODc routines.

The reconstruction of a field is only accurate within the range of observation used to create the bases. Extrapolation of the bases outside of this range has been investigated [83], and was found to only have acceptable accuracy within the bounds of the observations, following the statement "*you can't model what you haven't seen*". Fundamental arguments why reconstruction of the complete field based only on partial data is feasible are given in Section 3.2.1. The POD has been used primarily in this thesis for flow modeling, however it is also used and useful for modeling other phenomena such as the pressure or turbulence parameters. The approach taken in this thesis for the development of the flow models used is described in the following sections.

#### **3.1.1** Generating the Observations

The first step in creating the POD model is to generate the observations. In this thesis all observations are generated using the commercial CFD solver FLUENT v. 6.1.22 [25], although other sources of data such as PIV or detailed hot wire anemometry can be used. The CFD modeling and convergence criteria used are specific to each application and are discussed in turn. These observations are designed to span a range of input design parameters, such as an inlet or outlet flow velocity, or the parameters of a fan model. For a simple example with only a single parameter of inlet velocity, the observations would be created by varying the inlet velocity between the minimum  $V_{min}$  and the maximum  $V_{max}$  in increments  $\Delta V$ , creating the set:

$$V^{o} = \left\{ V_{\min} V_{\min} + \Delta V, V_{\min} + 2\Delta V, ..., V_{\max} \right\}$$
(3.1)

The number of observations dictated by the value of  $\Delta V$  is dependent upon the complexity of the flow and the range  $[V_{min}, V_{max}]$ . More information on observation density is available in Section 4.2.2, and [83]. For cases with multiple parameters a design of observations, akin to a design of experiments is required. In this thesis a simple ad-hoc factorial combination using symmetry is used, shown in Section 4.2.1. However this issue is far from trivial or being resolved [37], further discussion on an approach to design of observations for the flow regimes encountered in data centers is discussed in Section 7.2.2.

## 3.1.2 Performing the POD

The observation covariance matrix U is assembled using equation (2.5), where each column  $\vec{u}_i$  is a single observation. If the data is vector valued, such as the flow field,  $\vec{u}_i$  is a vertical concatenation of the u and v or u, v, and w data for two and threedimensional flows respectively. This use of extended state vectors is not unique [103], however its use here is justified as the flow field components are not independent, as their combination determines the velocity magnitude, a measure of the energy and dynamics of the flow. Furthermore, independent weighting of the individual velocity components enables the reconstruction of nonphysical solutions, which is undesirable and nullifies one of the key strengths of the POD given in Section 3.2.1. If the data is simply a scalar, such as the pressure or turbulent kinetic energy of the flow, no concentration is required. In either case, the POD or PODc routine is applied as described in Section 0.

For complex, detailed problems, particularly in three dimensions, both m and n can become very large, and thus the SVD algorithm requires a lot of memory to determine the solution. However, in practice it has been found that the memory limit of solving the CFD analysis required to generate the observations is similar to the limit of performing the SVD on the resultant data set. Therefore, if the computer used is able to solve the CFD analyses, as long as efficient memory management is employed in the algorithm, the POD modes of the data set can be found.

If the concept of principal component analysis and explanation given in Section 2.1.5 is unclear, it is also possible to simply think of the POD and PODc routines as a "black box". This model is depicted below in Figure 3.2. In this diagram a series of

observations is input to the model, the "handle" is turned applying variational mathematics, and the basis functions are output for reconstruction of the system.



Figure 3.2 - Black box POD model diagram

With the construction of the POD modes complete, an explanation of how they are used to reconstruct arbitrary solutions within the range of original observations is presented in the following section.

## 3.1.3 Reconstructing an Arbitrary Solution

With the POD modes computed a method is developed to enable the reconstruction of an arbitrary solution within the bounds of the original observations. As stated previously, the reconstructions are based on the input of the same control parameters as was used to create the observations, such as boundary flow conditions. Two approaches to this are described in this section. Both approaches have their individual strengths and are applied in this thesis; however, the flux matching procedure is the generally superior approach and is recommended for future application and development, as discussed further in the following sections.

#### 3.1.3.1 Coefficient Interpolation

This method represents the simplest approach for reconstruction, and has previously been applied to single parameter reconstructions [53, 54, 64, 65, 103], and is described and further augmented in this section. The POD mode weighting vector a in equation (2.2) used to reconstruct an observation u can be found by projecting each of the POD modes onto the observation in turn. This was applied in the examples in Sections 2.1.5 and 2.1.8 and is computed as:

$$a_i = \vec{u} \cdot \vec{\varphi}_i, \text{ for } i = 1, \dots, p \tag{3.2}$$

Where  $p \le m$  is the number of modes to be used in the reconstruction. This can be computed for all observations within the ensemble *U* as:

$$a_{i,j} = \vec{u}_j \cdot \vec{\varphi}_i$$
, for  $i = 1, ..., p$  and  $j = 1, ..., m$  (3.3)

This complete weighting matrix  $a \in \mathbb{R}^{p \times m}$ , in which each column is the weighting vector to reconstruct the corresponding observation from the ensemble *U*, can be more efficiently computed as:

$$a = \vec{\varphi}^+ \bullet \vec{U} \tag{3.4}$$

Where  $(\cdot)^+$  is the Moore-Penrose pseudo-inverse giving the least squares solution [104]. The resulting reconstructions are computed using equation (3.5).

$$\vec{u}_r = \vec{\varphi} \cdot a \tag{3.5}$$

Where subscript *r* signifies an approximate reconstructed solution. As  $a_i$  has been found for all observations, each of which represents the solution under a specified control parameter value or combination of values from the set  $V^o$ , reconstruction is possible through the interpolation of the weight coefficients  $a_i$  between observations corresponding to the parameter values desired for the reconstruction *V*. In other words, rather than directly interpolating between observations, interpolation is performed in the POD mode space using the weighting coefficients  $a_i$ .

For single parameter cases this interpolation can be done through linear or the slightly more accurate piecewise cubic spline interpolation between coefficients. For example, consider a simple flow with only one control parameter, the inlet velocity. Observations are created by varying this inlet velocity from 0.5 m/s to 1.5 m/s in 0.25 m/s increments, creating a set following equation (3.1):

$$V^{o} = \{0.5, 0.75, 1.0, 1.25, 1.5\}$$
(3.6)

Therefore, to reconstruct the flow field with an inlet velocity of 0.9 m/s the interpolation of  $a_2$  and  $a_3$  corresponding to the weights of the reconstruction of the 2<sup>nd</sup> and 3<sup>rd</sup> observation at the intermediate position of:

$$\frac{V - V_{lower}^o}{V_{upper}^o - V_{lower}^o} = \frac{0.9 - 0.75}{1.0 - 0.75} = 0.6$$
(3.7)

The resulting approximation is accurate ever for small changes in the inlet velocity parameter, despite the coarse incrementing of the inlet velocity parameter used to generate the set of observations. This accuracy using such a coarse observation parameter set is a key strength of the POD approach. This interpolation reconsutruction approach can be extended to multiple parameter reconstructions using multi-dimensional interpolation approaches, such as krieging or multivariate adaptive regression splines (MARS). The accuracy of these reconstructions is good, as investigated in Section 5.3.3, because the first few modes which capture the dominant system dynamics have weight coefficients that change smoothly, enabling accurate interpolation. The higher order modes however have weight coefficients that tend to vary significantly and oscillate, but as they do not contribute significantly to the reconstruction this is not important. The limitation of this approach, like any interpolation routine, is its dependency upon the density of the observations. This becomes a significant problem for problems with many control parameters, as the number of observations required for accurate interpolation will increase exponentially, and greatly exceed the number of observations required simply to create the POD mode bases. A further problem with this interpolation approach is that it unlike the flux matching procedure explained next, it cannot be used with the PODc decomposition as the bases change with the location of the desired reconstruction.

#### 3.1.3.2 Flux Matching

The concept of the flux matching procedure is to reconstruct a solution using the POD modes such that the sum of the weighted modes satisfy the boundary conditions. This is possible because the POD modes are themselves solutions to the governing equations (as discussed in Section 3.2.1), and is a derivative of the Galerkin tau method, developed by Gottlieb and Orszag [29]. This approach has also been utilized in the field of flow control [52, 54, 64, 65], where data from sensors at a few finite positions is used

to estimate the velocity field over the whole domain. The same concept is applied here, where the flow at a few specified locations such as the boundaries is reconstructed using the POD modes, yielding the complete flow solution. This is computed as follows.

In the analysis of complex flow, the exact velocity profile across a boundary is often unknown. However, from a design perspective the more important quantity is usually the integral solution, such as the mass or energy flux across the control surface  $\Gamma_i$ . This can be mathematically represented as a flux function:

$$F(\vec{u},\beta) = \int_{\Gamma} \rho \beta \vec{u} \cdot \hat{n} ds \tag{3.8}$$

Depending upon the transport phenomena being modeled, the parameter  $\beta$  can be changed to describe the flow of mass ( $\beta = 1$ ), momentum ( $\beta = \vec{u}$ ), energy ( $\beta = E$ ), or species concentration ( $\beta = c_i$ ). The case of mass flux is used for the reconstruction of the velocity field, and thus the application of equation (3.8) to a boundary  $\Gamma_i$  yields the mass flow rate  $\vec{m}$ . Note that F in equation (3.8) is not a mathematical function by rigorous definition, but it can be considered as a subroutine that outputs F for a velocity field input. To reconstruct an approximate solution the fluxes are expressed as a vector of goals  $G \in \mathbb{R}^q$ , for which a specific mass flux goal is desired through each of the set of corresponding control surfaces  $\Gamma = {\Gamma_1, \Gamma_2, ..., \Gamma_q}$ , where q is the number of boundaries to be matched. This flux function defines the desired reconstructed flow field  $\vec{u}_r$  such that  $G = F(\vec{u}_r)$ , and thus the desired mass flow rates across  $\Gamma$  are achieved. The solution procedure is then to find the set of weight coefficients that minimize the error on the set  $\Gamma$ :

$$\min\left(\left\|G'-\sum_{i=1}^{p}a_{i}F(\vec{\varphi}_{i})\right\|\right) \text{ where } G'=G-F\left(\vec{u}_{o}\right)$$
(3.9)

The corrected mass flux goal vector G' is required as the POD modes are mean centered, and thus the goals must be defined as deviations from the mean also. The modal summation is carried to  $p \le m$  modes because the optimal reconstruction may require less than the full spectrum of modes. This is true if the summation in equation (3.9) is not convergent, and thus is truncated at the point giving the lowest error. The weight coefficients  $a_i$  are found by assembling a coefficient matrix by operating equation (3.9) on the *q* surfaces of the *p* POD modes:

$$C = F(\vec{\phi}) \in \mathbb{R}^{m \times q} \tag{3.10}$$

Equation (3.11) can then be applied, where  $(\cdot)^+$  is the pseudo-inverse, as used in equation (3.4):

$$a = C^+ \bullet G' \tag{3.11}$$

An explicit example of the formulation of the goal vector G and coefficient matrix C using the flux function is given in Section 4.2.3.

The application of the flux function given in equation (3.8) is possible as the POD modes span the identical physical space as the observations; thus a data point from an observation at index *i* and the data point from a POD mode at index *i* represent the same

point in the physical system's Cartesian coordinate system. The advantage of this flux matching procedure is that only enough POD modes need to be generated in order to accurately represent the system dynamics, as no interpolative procedures are employed, as have been used in previous POD based reconstruction approaches [23, 53, 65, 103].

Integral fluxes are used as goals rather than specific point velocity values because the exact profile is unknown. However, because the POD modes satisfy the governing equations [83], discussed further in Section 3.2.1, their superposition will create a solution that most closely matches the desired goals, but constrained by the system physics, and thus the correct boundary profile for the flux specified is retained. This means that reconstructions of flows that could not occur using the CFD analysis will not be reconstructed, even if the goals set in *G* specify it. Thus an accurate boundary profile for the flux specified is retained in the reconstruction, despite using an integral formulation. The only disadvantage to this flux matching method is that a quantity of flux must be measurable through the boundaries, whereas the interpolation method can be applied to any arbitrary parameter.

## 3.1.4 General Parameter Transformation Approach

The flux matching procedure is shown to be useful for producing accurate reconstructions of many various flow solutions, including velocity and turbulence parameters. However, this computation of the flux function can be time consuming on large data sets. Furthermore, because the PODc basis changes with each computation, this flux function must also be computed with each solution.

Although these computational efficiency concerns are important, some phenomena cannot be computed using the flux function. For example, the heat flux can be computed based upon a temperature difference and the knowledge of the thermal conductivity. For turbulent boundary layers, the areas where heat fluxes are applied in simulations of data centers and data center server cabinets, this heat flux cannot be computed for the POD modes for non-conjugate problems. For conjugate simulations, the heat flux can be computed using adjacent nodal temperature values in the solid region, and the thermal conductivity of the solid material. However, in the fluid region, the temperature of the wall is unknown. The temperature of the adjacent fluid node is known, however, the heat flux is also unknown. This means the standard heat diffusion equation cannot be solved, as there are two unknowns and only one equation.

$$q = -k\delta x \frac{T_{wall} - T_{fluid}}{\delta y}$$
(3.12)

The solution to this problem is a general transformation from the observation space to the POD space. This transformation produces the same multi-dimensional rotation and scaling that is performed to create the POD modes. The transformation is computed as:

$$T = U^+ \bullet \Psi \tag{3.13}$$

Where  $(\cdot)^+$  is the pseudo-inverse, and  $\Psi$  the ensemble of POD modes, similar to the ensemble of observations U is computed as:

$$\Psi = \left\{ \vec{\varphi}_1, \vec{\varphi}_2, ..., \vec{\varphi}_m \right\}$$
(3.14)

Therefore the transformation T will take any data from the observation, and transform it to the equivalent value of the POD mode. For example, given a vector of velocity boundary conditions for a set of m observations:

$$V^{o} = \{V_{1}, V_{2}, ..., V_{m}\}$$
(3.15)

The values of the inlet velocities of the POD modes, used to create the coefficient matrix, as computed by the flux function,  $C = F(\vec{\phi})$ , can now be computed as:

$$C = T \bullet V^o \tag{3.16}$$

This approach avoids the use of the flux function, yet produces the same result. This allows the flux matching procedure to be applied using values used in the CFD simulations, including parameters that cannot be obtained directly from the POD modes, such as heat flux of non-conjugate systems.

The validity of this approach can be tested through computing the weighting coefficients to reconstruct an observation, a, using the same approach as the coefficient interpolation procedure by employing equation (3.4):

$$a = \vec{\varphi}^+ \bullet U \tag{3.4}$$

It can be shown that:

$$T \bullet a = I \tag{3.17}$$

Where  $I \in \mathbb{R}^{m \times m}$  is the identity matrix. This is because the transformation *T* takes a point from the observation space to the POD space, and the transformation *a* takes a point from the POD space to the observation space.

It should also be noted that for computation efficiency, the complete pseudo inverse of the observation and POD mode ensemble does not need to be taken. Rather, only a fairly over-determined system is required for accurate results, and thus the matrices used in equation (3.13) can be:

$$U \in \mathbb{R}^{m+d,m} \text{ and } \Psi \in \mathbb{R}^{m+d,m}$$
(3.18)

Where *m* is the number of observations, and  $d \ge 1$ ,  $d+m \le n$  is some extra number of elements in the matrix to create an over determined system. In practice, the pseudo-inverse algorithm is computationally efficient enough to compute instantly, even for very large problems. Lastly, because this approach was determined at the end of this thesis work, it has not been applied to any of the example problems, but has been validated and is suggested for future use.

#### **3.2** Flow Model Validation

As the POD method is foundational to the work performed in this thesis, this section serves to present the theoretical and empirical performance validity of the construct, as tied to the validation square presented in Table 1.3.

### 3.2.1 Theoretical Considerations

The POD based flow modeling approach has many strengths than are inherent to the POD method that are brought to light explicitly in this section. Some of these strengths are revealed when the basis, equation (2.1) is projected onto the RANS momentum continuity equation (3.19), part of the Galerkin projection method, shown below in equation (3.20).

$$\nabla \cdot \vec{u} = 0 \tag{3.19}$$

$$\nabla \cdot \vec{u} = \nabla \cdot \sum_{i=1}^{N} a_i \vec{\varphi}_i = \sum_{i=1}^{N} a_i \left( \nabla \cdot \vec{\varphi}_i \right) = 0$$
(3.20)

As the weight coefficients  $a_i$  are independent, equation (3.20) shows the POD modes are divergence free, and this each POD mode satisfies the constraint of incompressibility. Therefore the pressure field is not required for the reconstruction of solutions of the Navier-Stokes equations. Furthermore, the POD modes as solutions to the governing equations, and satisfy the boundary conditions of the problem  $\vec{\varphi} = 0$  where  $u(\partial \Omega) = 0$ . This property is very important, as it acts as a constraint that stops the reconstruction of unphysical solutions, as discussed above.

As the POD modes contain successively smaller and more detailed flow dynamics, at a certain point these details are obscured by numerical noise [21]. However, the location of the cut off point is made clear by the eigenvalue spectrum associated with the POD modes. A cut off can either be specified at the point where machine error becomes intolerable  $\lambda < O(10^{-12})$ , or simply by the minimum acceptable reconstruction

accuracy. For example, if  $\lambda < 0.01$  the associated mode only contributes less than 1% of the total system dynamics, and thus if only a coarse reconstruction is required, further modes can be truncated.

The flux matching procedure is shown in this thesis, as well as [77, 82, 83] to be an accurate approach for reconstruction. Direct comparison with the Galerkin projection approach for laminar flow by Rambo [82] has shown that the weighting coefficients found are similar, and often more accurate using the POD modes directly with the flux matching procedure for the reconstruction of laminar flows. Further validation of the linear nature, and thus smooth solution space of the POD mode weighted reconstructions is shown below in Figure 3.3.



Figure 3.3 - Reconstruction error for a range of weighting vectors using 3 POD modes
In this figure the axes represent the values of the weights for the first 3 POD modes of the cold aisle example given in Section 2.1.8. The iso-surfaces are generated and color coded for the error in velocity field reconstruction, computed using equations (3.21) and (3.22).

$$vel = \sqrt{u^2 + v^2} \tag{3.21}$$

$$e_2 = \frac{\left\| vel_r - vel_t \right\|}{\left\| vel_t \right\|} \tag{3.22}$$

The smooth convergent shape of the iso-surfaces is to be expected for the linear representation of the POD modes, and confirms that the pseudo-inverse fitting approach is valid, and there are no singularities or local minima to be concerned with.

#### 3.2.2 Empirical Performance Validity

An example of the applicability of the POD to reconstruct a flow field to within 5~10% error has been presented in Section 2.1.8. Therefore no further specific examples are presented to validate its use, as this work is the focus of Rambo. A complete investigation of the accuracy of the flow model described in this section is available in [83]. The accuracy of the flux based flow model is computed for the problems in Section 4.2.4 and 6.2.6. The accuracy of the interpolation based flow model is computed for the problem in Section 5.3.3. Further investigations of the validity of the POD based flow model are available in [82].

### 3.2.3 Flow Model Limitations

There are few limitations of the POD based flow model, considering the DOF reduction it achieves. These limitations are: (1) The POD requires sufficient observation density in order to extract the principal modes. The use of the bounding values of the parameters is insufficient; several intermediate observations are also required. (2) The apriori specification of a design of observations to efficiently generate POD modes is not available, nor is it a trivial task to determine. Currently the best approach is ad-hoc, this is discussed further in Section 7.2.2. (3) Currently, the POD cannot be used for observations with geometry as a variable. However, this capability is on the horizon with ongoing research [106]. (4) The POD and PODc yields at best less than a few percent reconstruction error using the flux matching procedure. This is very good, however reconstruction using partial information will never yield an exact reconstruction. (5) Lastly, the reconstruction is dependent upon the accuracy of the original observations, it can never be more accurate than the data put into the ensemble. This is important as the accuracy of turbulence models for the flow regimes encountered in data centers is questionable [79, 115]. The saying "you can't model what you haven't seen" is a good rule of thumb for the application of the POD based flow model.

#### **3.3 Heat Transfer Model Development**

In this section the development of a two-dimensional turbulent convective diffusive heat transfer model is presented. This yields a two-dimensional steady state elliptic system, which is solved using a finite difference approach. The development of this thermal model is important as the interest in a response from data center and server cabinet simulations is in the temperature, which the POD based flow model does not compute at this time. Therefore a robust, efficient and easily adaptable energy equation solver is required to determine the temperature field of the system. The system under consideration may also contain a degree of non-linearity, requiring iteration of the system to achieve convergence to steady parameter values, and hence the enegy equation solver must be able to handle this as well.

The fluid flow in all work in this thesis is assumed to be forced convection only and thus independent from the thermal system. This allows the energy equation to be solved independently for a prescribed flow field. Experimental work presented by [96] and [14] indicates that this is a valid assumption. The POD flow model presented in the preceding section is used for determining fluid flow as well as the k and epsilon fields, and how it is imported is described in each application in this thesis.

### 3.3.1 The CV Approach

The two dimensional conductive and convective heat transfer problem forms an elliptic partial differential equation, commonly referred to as as the energy equation, given below in equation (3.23). This formulation assumes no viscous heating, which is valid for the flow velocities encoutered in data centers and server cabinets.

$$\frac{d}{dt}(\rho T) + \frac{d}{dx}\left(\rho u T - \Gamma \frac{dT}{dx}\right) + \frac{d}{dy}\left(\rho v T - \Gamma \frac{dT}{dy}\right) = S$$
(3.23)

In this equation S is the volumetric heat generation (divided by  $c_p$ ), T is the temperature,  $\rho$  is the fluid density, u and v are the flow velocities in the x and y directions

respectively,  $\Gamma$  is the thermal diffusivity, given by equation (3.24) below, where k is thermal conductivity and  $c_p$  is the specific heat capacity at constant volume.

$$\Gamma = \frac{k}{c_p} \tag{3.24}$$

In this problem the transient term goes to 0 as it is a steady state problem. It is also convinient to lump the conductive and convective flux terms together. This makes the calculation of the numerical flux balance easier and is important for stability reasons to be explored later. This results in the following equation form given below in equations (3.25)-(3.27).

$$\frac{dJ_x}{dx} + \frac{dJ_y}{dy} = S \tag{3.25}$$

$$J_x = \rho u T - \Gamma \frac{dT}{dx}$$
(3.26)

$$J_{y} = \rho v T - \Gamma \frac{dT}{dy}$$
(3.27)

The basis of the numerical method is the conversion of the general differential equation 1 to an algebraic equation relating the temperature of the point under consideration, P, to the temperatures of the surrounding points N, E, S, W. This results in a flux balance into and out of the control volume as shown in Figure 3.4 below.



Figure 3.4 - Example control volume

These flux equations are given in equations (3.25)-(3.27). However, there are multiple ways to discretize them. There are explored in the next section, however, before this it is important to define the dimensionless numbers that are used in these discretization schemes and help to characterize the system.

#### 3.3.1.1 Diffusion Conductance

The diffusion conductance D is the quantity of heat flux from the diffusive term of the equation. The subscript  $D_e$  is the diffusion conductance across the control volume face e, as shown in Figure 3.4. The diffusion conductance  $D_e$  is given by equation (3.28) . This is a more appropriate approximation to the average thermal conductivity between nodes P and E using the harmonic mean. This is because the averaging technique better represents the conduction between two materials with very different thermal conductivities than a simple direct average. This becomes important when computing the heat flux between solid and fluid regions, as well as within turbulent fluid regions, as explained in Section 3.3.6.

$$D_e = A_e \frac{1}{\frac{\left(\delta x_e\right)_{e^-}}{\Gamma_P} + \frac{\left(\delta x_e\right)_{e^+}}{\Gamma_E}}$$
(3.28)

# 3.3.1.2 Mass Flow Rate

The mass flow rate F is the quantity of mass flowing across a boundary. The subscript  $F_e$  is the mass flow rate across the control volume face e. The mass flow rate  $F_e$  is obtained from equation (3.29) shown below.

$$F_e = (\rho u)_e A_e \tag{3.29}$$

#### 3.3.1.3 Peclet Number

The Peclet number is defined locally across each face of the control volume, yielding a maximum cell Peclet number. The general form of the Peclet number is shown in equation (3.30) and the Peclet number specific to the face e,  $Pe_{e}$ , in equation (3.31).

$$P = \frac{\rho u L}{\Gamma} \tag{3.30}$$

$$Pe_e = \frac{F_e}{D_e} \tag{3.31}$$

Equations (3.30) and (3.31) show that the Peclet number is in fact the ratio of conductive flux to diffusive flux. This is a critical parameter in the analysis of the possible discretization schemes explored next.

#### **3.3.2** Discretization Approaches

To discretize the equation in space, the coefficients to multiply the surrounding nodes by must be found. This general discretization is given in equation (3.32), where a represents the coefficients of the surrounding nodes and b is equal to the heat flux input to the element.

$$a_{P}T_{P} = a_{E}T_{E} + a_{W}T_{W} + a_{N}T_{N} + a_{S}T_{S} + b$$
(3.32)

The following equations give the general form of the coefficients for each side of the control volume. These coefficients are functions of the dimensionless parameters defined earlier, and a function of the Peclet number A(|Pe|). In these equations [[a,b]] represents the maximum of a and b.

$$a_{E} = D_{e}A(|Pe_{e}|) + [[-F_{e}, 0]]$$
(3.33)

$$a_{w} = D_{w}A(|Pe_{w}|) + [[F_{w}, 0]]$$
(3.34)

$$a_{N} = D_{n}A(|Pe_{n}|) + [-F_{n}, 0]$$
(3.35)

$$a_s = D_s A(|Pe_s|) + \llbracket F_s, 0 \rrbracket$$
(3.36)

$$a_P = a_E + a_W + a_N + a_S \tag{3.37}$$

These coefficients *a* are related to the fluxes into the element  $J_x$  and  $J_y$  given in equation (3.24) through equation (3.38), shown in general form as the flux through the control volume face between two adjacent elements. Here the capitalized letters represent opposite face coefficients from adjacent nodes.

$$J^* = BT_i - AT_{i+1} \tag{3.38}$$

Substituting one of equations (3.33)-(3.37) with an appropriate function A(|Pe|) and equation (3.38) yields the function for the total flux entering a specific control volume face. There are many approaches to approximating the function A(|Pe|), these are all dependent upon the Peclet number, *Pe*. This choice of approximation also determines the accuracy and the stability of the method, as investigated next.

# 3.3.3 Upwinding Schemes Approximation & Stability

In order to determine the most accurate and stable approximation of the flux function J several schemes for the function A(|Pe|) are tested. These are based upon the Taylor series expansion of equations (3.25)-(3.27), an approximation or exact solution to the differential equation between two nodes, or a combination of these. These schemes are presented below in equations (3.39)-(3.42) in Table 3.1.

Scheme	Formula for A( Pe )	
Central difference	1 - 0.5  Pe	(3.39)
Upwind	1	(3.40)
Power law	$\left[\!\left[0,\!\left(1\!-\!0.1\big Pe\big \right)^5\right]\!\right]$	(3.41)
Exponential (exact)	$\frac{ Pe }{\exp( Pe )-1}$	(3.42)

Table 3.1 - Discritization schemes for conductive and convective flux

The value of these functions A(|Pe|) versus the input Peclet Number is shown below in Figure 3.5.



Figure 3.5 - Flux function A(|Pe|) versus Peclet number

# 3.3.3.1 Central Difference

The central difference scheme is based upon a piece-wise linear profile for T between nodes. Viewing the plot above all schemes produce a physically realistic solution, except for the central difference scheme which produces values outside of the [0,1] range when the Peclet number is greater than 2. This means that the mesh would have to be fine enough to keep the grid Pe < 2 so as to keep the scheme stable, as Pe is defined on a local scale.

### 3.3.3.2 <u>Upwind</u>

The upwind scheme, or upwind-difference scheme, assumes that the value of T at the interface is equal to T at the upwind node. This assumption creates a more stable scheme, however its accuracy is questionable, as to be shown next.

# 3.3.3.3 Exponential

The exponential scheme is based upon the analytical solution to equation (3.26), which is the equivalent to substituting equation (3.42) into equations (3.32)-(3.37). This scheme therefore gives the exact solution regardless of the grid spacing, however the exponential function is computationally expensive to compute.

# 3.3.3.4 <u>Power law</u>

The power law scheme is an approximation to the exponential scheme that produces very accurate results. An added advantage beyond the increased computational speed is that if a fluid flow speed of 0 is encountered the equation does not fail, as the exponential scheme does.

## 3.3.4 Up-winding Schemes Accuracy

With the stability of the schemes established, their accuracies must be established. This is done through the application of a simple one-dimensional system given below in equations (3.43) and (3.44).

$$a_P T_P = a_E T_E + a_W T_W \tag{3.43}$$

$$T_P = \frac{a_E}{a_E + a_W} \tag{3.44}$$

In this system the diffusion D is taken to equal 1 thus T is a function of Pe only.  $T_W$  is taken as 0 while  $T_E$  is taken to be 1. This results in the following values of T for a range of Pe shown below in Figure 3.6.



Prediction of  $\phi_{\text{p}}$  by Various Schemes

Figure 3.6 - Prediction of T<sub>P</sub> by various schemes

The results of this plot indicate that the power law is an almost exact approximation to the exact solution generated by the exponential scheme, and also that the upwind scheme yields good results when Pe < 1, and the upwind scheme when Pe > 10. Therefore the selection of scheme depends upon the cell Peclet number as given by the fluid properties and the FLUENT generated flow field.

### 3.3.5 Cell Peclet Number Consideration

For this simulation and all further runs, the following fluid parameters are used for air at 300K, given by Incopera and DeWitt [39], displayed in Table 3.1. Because the bulk temperature of the fluid does not change very much, these constant properties are an accurate approximation.

Property	Value
Density, $\rho$	$1.1614 \text{ kg/m}^3$
Thermal conductivity, k	26.3e-3 W/m <sup>2</sup>
Specific heat capacity, $c_p$	1.007E3 J/KG-K

Table 3.2 - Thermal properties of air at room temperature

The velocities encountered in simulations of data center server cabinets range from near stagnant flows to up to 10m/s. Furthermore, because the energy equation is to be discretized on a similar scale and density mesh as the fluid flow, simply increasing the grid density is not an option, nor is it computationally efficient. Therefore the A(|Pe|)function selected is the power law, as given in equation (3.41). Substitution of this equation into equation (3.38) yields the final flux across the control volume interface *e* given by equation (3.45) below.

$$J_{e}A_{e} = F_{e}T_{P} + \left\{ D_{e}A(|Pe_{e}|) + [-F_{e}, 0] \right\} (T_{P} - T_{E})$$
(3.45)

Substitution of this same A(|Pe|) function into equations (3.32)-(3.37) yields the final discrimination, used in the finite difference schemes presented in Section 3.3.7.

# **3.3.6** Incorporating Turbulence Effects

The formulation of energy equation discretization in the preceding section assumes that the flow field, u and v, as well as the fluid thermal conductivity k is known. For laminar flow, the molecular value of the thermal conductivity of air is utilized, as given in Table 3.2. However, in non-laminar flow, the effects of the turbulence increase the effective thermal conductivity of the air. The turbulence model used in this thesis is the incompressible, steady, Reynolds-Averaged Navier Stokes (RANS) equations with an isotropic eddy viscosity closure, which is solved using the standard k- $\varepsilon$  model using the commercial CFD program FLUENT. This formulation in two-dimensions has 5 variables and hence 5 coupled partial differential equations, shown below.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho u_i) = 0 \tag{3.46}$$

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_i}{\partial x_i} \right) \right] + \frac{\partial}{\partial x_j} (-\rho \overline{u'_i u'_j}) \quad (3.47)$$

where, 
$$-\rho \overline{u'_{i}u'_{j}} = \mu_{t} \left( \frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{i}} \right) - \frac{2}{3} \left( \rho k + \mu_{t} \frac{\partial u_{i}}{\partial x_{i}} \right) \delta_{ij}$$
 (3.48)

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_i}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k + G_b - \rho \varepsilon - Y_M + S_k$$
(3.49)

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \frac{\partial}{\partial x_i}(\rho\varepsilon u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial\varepsilon}{\partial x_j} \right] + C_{1\varepsilon} \frac{\varepsilon}{k} (G_k + C_{3\varepsilon}G_b) - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} + S_{\varepsilon} \quad (3.50)$$

Equations (3.46)-(3.48) comprise of the continuity modeling of the flow, and equations (3.49) and (3.50) are the turbulent kinetic energy, k, and turbulent dissipation,  $\varepsilon$ , equations for closing the turbulence model. Further explanation of these equations

and how they are solved numerically is available in [25]. Through obtaining the values of k and  $\varepsilon$  using these equations, the turbulent viscosity of the fluid  $\mu_t$ , and the effective thermal conductivity  $k_{eff}$  can be computed as described in [17, 25] using equations (3.51) and (3.52) respectively, where the turbulence constant  $C_{\mu} = 0.09$  and the turbulent Prandtl number  $Pr_t = 0.85$  as used in FLUENT [25].

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \tag{3.51}$$

$$k_{eff} = k + \frac{c_p \mu_t}{Pr_t}$$
(3.52)

In the cabinet geometries investigated in this thesis,  $k_{eff}$  was found to increase several orders of magnitude in areas of high turbulence over the molecular value. This means including this thermal conductivity variability in the thermal modeling should increase its accuracy. In order to include this variable thermal conductivity in the energy equation solver derived in Section 3.3, equation (3.52) is substituted into equation (3.24), no further modifications are necessary.

#### 3.3.6.1 <u>Reconstructing the Turbulent Viscosity Field</u>

The problem becomes how to obtain the values of k and  $\varepsilon$  or  $\mu_t$  at every node in the model in order to compute  $k_{eff}$ , as the POD based flow model only solves for u and v, and the other variables from the RANS equations are de-coupled for the reconstruction. Rather than re-compute the RANS equations at great computational cost, the unknown variables are also reconstructed using the POD approach. This is done using either the flux matching procedure, as described in Section 3.1.3.2 and implemented to reconstruct *k* and  $\varepsilon$  in the study in Chapter 6, or the coefficient interpolation method, described in Section 3.1.3.1 and implemented to reconstruct  $\mu_t$  in Chapter 5. The accuracy of both of these methods is discussed in Section 3.2.2. The reconstruction of this effective thermal conductivity covers the majority of the effects of the turbulence, however there are further considerations near walls that demand special modeling, described in the following section.

#### 3.3.6.2 <u>Turbulent Wall Functions Approach</u>

Near a wall boundary, modeling turbulence becomes more complex because of the no-slip condition at the wall, where the flow transitions to laminar flow and the molecular properties of the fluid dominate. Also, near the wall area the velocity and other properties vary very rapidly at a short distance from the wall, creating a boundary layer with a sharper profile. Therefore, numerical modeling requires many grid cells close to the wall in order to maintain accuracy of the variation and physics. Another approach is to employ functions that are approximate expressions for the each variable in the near wall region. These functions essentially bridge the gap between the regular mesh and the wall. These approaches are shown graphically below in Figure 3.7. Further discussion of the theory, application and validation is available in [17, 75].







y

#### Figure 3.7 - Near-wall model vs. wall function approach to boundary layer turbuence modeling

In order to evaluate  $k_{eff}$  near the wall, the local k and  $\varepsilon$  values must be computed. This is done using the Shultz-Gronow log-law [75], an empirical relationship between local Reynolds number  $Re_x$  and the friction factor  $c_f$  given in equations (3.53) and (3.54) respectively.

$$\operatorname{Re}_{x} = \frac{\rho u_{\infty} y_{p}}{\mu}$$
(3.53)

$$c_f = 0.370(\log_{10} \operatorname{Re}_x)^{-2.584}$$
 (3.54)

In equation (3.53)  $u_{\infty}$  is the parallel flow free stream velocity, computed at one grid cell above the cell under consideration,  $y_p$  is the distance from the wall of the point under consideration, equal to one half of the  $\Delta y$  of a grid cell, the dynamic viscosity of the fluid  $\mu = 184.6e - 7$  N-s/m<sup>2</sup>, and  $\rho$  is the fluid density as given in Table 3.2. The wall Reynolds stress,  $\tau_w$ , can then be computed using equation (3.55).

$$\tau_w = \frac{\frac{V_2 \rho u_\infty^2}{c_f}}{c_f} \tag{3.55}$$

Next the friction velocity,  $u_t$ , is computed using equation (3.56).

$$u_t = \sqrt{\frac{\tau_w}{\rho}} \tag{3.56}$$

With the various fluid and turbulence properties computed, k and  $\varepsilon$  of the grid cell under consideration can be computed using equations (3.57) and (3.58) respectively.

$$k_p = \frac{u_t^2}{\sqrt{c_m}} \tag{3.57}$$

$$\varepsilon_p = \frac{u_l^3}{\kappa y_p} \tag{3.58}$$

In equation (3.58) the Von Kármán constant,  $\kappa = 0.42$  [25]. It should be noted that some of equations (3.51)-(3.58) are based on the FLUENT grid cell unit length scale  $y^*$  defined in equation (3.59), while others are based on the viscous length scale  $y^+$  defined in equation (3.60) shown below [25].

$$y^{*} = \frac{\rho C_{\mu}^{V_{4}} k_{p}^{V_{2}} y_{p}}{\mu}$$
(3.59)

$$y^{+} = \frac{\rho u_{\tau} y_{p}}{\mu} \tag{3.60}$$

In equilibrium boundary layers these values are equal, and this was tested and validated for the full range of velocities for which the wall function is valid [25]. Because this wall function approach determines the thermal conductivity of the upper bound of the cell, and the heat flux is applied from the bottom, more accurate results were obtained when an average of the molecular thermal conductivity and the effective turbulent thermal conductivity was taken as a blending function of the laminar sub layer and turbulent boundary layer. Finally, this wall function approach, while found to improve the model accuracy as tested in Section 5.3.3, is only a crude approximation. This is because all wall function correlations are only valid for flow parallel to the wall boundary, and not for flow with separation [17]. Hence the wall functions, employed in both the FLUENT CFD simulations and the energy equation solver described here are quite accurate. The blending function may be made more accurate through a relationship 134

with  $y^+$  and the thickness of the laminar sub layer, but for the purposes of this model the 0.5 approximation is adequate.

#### **3.3.7** Solution Approach Investigation

Three different solution methods are investigated for their use in this thesis work, and compared with regard to their speed. All methods are implicit, the solutions obtained through the inversion of the resulting *NxM* by *NxM* stiffness matrix. The methods used are direct inversion of the stiffness matrix through Gauss Elimination and the alternating direction line by line iterative method. The direct inversion method is applied to the full matrix as well as a "sparse" matrix, a bandwidth reduced matrix format used by MATLAB for comutational efficiency. The problem investigated is the same cold aisle simulation described in detail in Chapter 4, using the constant inlet temperature model.

#### 3.3.7.1 Stiffness Matrix Construction

The overall routine of constructing and solving the matrix system is performed by first assembling a stiffness matrix by a routine that scans through each line of the flow field matrix and fills in the computed coefficients from equations (3.32)-(3.37) into the appropriate elements. This was done through referencing another matrix, that contained the mesh information such as  $\Delta x$ ,  $\Delta y$ , thermal conductivity k, and heat generation q. This means that every node can have a different size and conductivity, and the algorithm will automatically fill in the correct coefficient in the stiffness matrix utilizing the power law flux approximation. This adds significant flexibility to the solver, which is important if it is to be used to solve problems with different geometries and boundary conditions. The boundary conditions can be adiabatic, outflow, or with an input flux either from the air inlet at a specified temperature, or from the heat generated from the surface of the racks. These are implemented through changing the appropriate coefficient of the flux between elements.

### 3.3.7.2 Alternating Line by Line Iteration

The alternating line by line solution works by considering only a single line and setting the flux input to elements from the lines above and below the line to constants using the most recently computed temperature values. This creates a tri-diagonal matrix system, which is solved for by the Thomas algorithm. This is then repeated for orthogonal lines, in this case alternating between lines of x and y. In order to alternate lines, two stiffness matrices are constructed, one for the normal geometry and second for the geometry rotated 90 degrees. This allows the same solution function to be applied, increasing efficiency. A transformation algorithm is then applied to transform the solution temperature vector into the order required for multiplication with the rotated stiffness matrix for the next iteration of the routine. This is repeated until the required convergence criterion is met.

### 3.3.7.3 Sparse Matrix Inversion

The Gauss Elimination of the full stiffness matrix is performed using the internal MATLAB routines for both the full and sparse matrix systems. The full matrix structure has a banded penta-diagonal structure, which cannot be solved as efficiently as a tridiagonal matrix. The sparse or bandwidth reduced bandwidth matrix is initialized using only the exact required number of elements that will be filled, thus reducing memory to the furthest extent possible, creating a very large increase in computation speed. Both of these matrix inversion algorithms are proprietary to the Mathworks, and are explained briefly in the MATLAB help files [107].

# 3.3.7.4 Comparison

The simulation was run for three different cases representing a low flow velocity case, a high side velocity case, and an asymmetric velocity case. These cases yield different temperature profiles and thoroughly test the convergence speed capabilities of the different methods. The exact observations simulated were Observations 1, 3 and 7 all with  $V_{in} = 0.5$ m/s shown in Table 4.1 in Section 4.2.1. The results are presented below in Table 3.3. The nomenclature used is "Full" for the full direct matrix inversion method, "Sparse" for the sparse matrix inversion method, and "Line" for the line by line method.

Computational time for each algorithm (seconds)				
Case	Full	Line	Sparse	
1	7.375	1.680	0.046	
2	7.390	0.938	0.047	
3	7.375	1.890	0.047	

Table 3.3 - Computation speed analysis of energy equation solution algorithms

Analysis of the results above show that the Line-by-Line solution is considerably faster than the direct matrix inversion. However, when the sparse matrix bandwidth reduction function is employed, the inversion becomes much faster because of the vast memory savings, and hence is implemented as the main solver in this thesis.

### 3.3.8 Handling Non-linearity

In order to deal with temperature dependent thermal properties or boundary conditions, such as used in the investigation in Chapter 4, a simple iteration wrapper is implemented using the existing thermal simulation at its core. An initial guess of the temperature dependent conditions is used, and the solution found. This result is then used as the guess for the next iteration, and the process continues until a specified minimum change in the dependent parameter is achieved. Figure 3.8 below shows the convergence behavior of the temperature dependent parameter, the inlet temperature of the boundary above the cold aisle, as described in detail in Sections 4.1.4.3 and 4.3.2.



Figure 3.8 - Convergence of the inlet temperature above the cold aisle with  $\eta = 0.2$ 

The algorithm converges in 7 iterations using convergence criterion of less than 0.01°C change in inlet temperature between iterations. As this is the only temperature

dependent problem investigated in this thesis, this result indicates that this simple iterative approach is adequate for handling non-linearity and thermal dependence during the determination of the system temperature profile.

#### 3.4 Heat Transfer Model Validation

In order to determine if the energy equation solver is accurate it is compared to an analytically computed conduction problem and a commercial CFD program's [22] solution to a convective problem. This is because there are no analytical solutions to a problem similar enough to the problems investigated in this thesis to test the energy equation solver against.

#### 3.4.1 Pure Conduction

To test the simulations accuracy of conduction the output is compared to an analytical solution to a square region with one side at higher temperature at steady state as follows in the partial differential given in equation (3.61).

$$T_{xx} + T_{yy} = 0 \tag{3.61}$$

The analytical series solution to this problem is given below in equation (3.62), where  $\lambda_n L = n\pi$ . This derivation of this solution can verified in [59].

$$T(x, y) = T_0 + 2(T_L - T_0) \sum_{n=1}^{\infty} \frac{1 - \cos(\lambda_n L)}{\lambda_n L \sinh(\lambda_n L)} \sinh(\lambda_n x) \sinh(\lambda_n y)$$
(3.62)

The resulting temperature distribution is plotted in Figure 3.9 below.



Figure 3.9 – Conductive test temperature distribution

The comparison of the numerical simulation with the analytical solution for lines of y and x are shown in Figure 3.10 and Figure 3.11 respectively. Viewing the figures it is shown that the results match perfectly, showing the simulation represents the conductive flux excellently.



Figure 3.10 - Comparison model for constant y



Figure 3.11 - Comparison model for constant x

#### 3.4.2 Conduction and Convection

The convective case selected was made similar to the system being simulated. Thus a square region measuring 2m by 2m with a vertical flow of 0.5m/s with an inlet temperature of 25 degrees C was subjected to a heat flux of 0.2W/m on the right and 0.4W/m on the left. These conditions were set up identically in both FEMLAB and the finite difference simulation. The resulting temperature profile is shown below in Figure 3.12.



Figure 3.12 - Convective test temperature profile

The comparison of the numerical simulation with the analytical solution for lines of y and x are shown in Figure 3.13 and Figure 3.14 respectively below.



Figure 3.13 - Comparison model for constant y



Figure 3.14 - Comparison model for constant x

The figures above show an almost perfect match, except for some slight deviation along the boundary. Some difference is to be expected from the comparison of different numerical methods employing different solution techniques, as FEMLAB uses the second order Finite Element method while the temperature solver presented in this thesis uses a first order power law based Finite Difference approach. However this is still a strong result that the simulation matches a commercial CFD package.

# 3.4.3 Thermal Model Limitations

The temperature solver model developed and presented in this section has some limitations that are discussed here. Firstly, although the model can accommodate variations in grid size, these changes must be uniform across the domain in order to maintain grid continuity. For example, if the  $\Delta x$  of the cells were to change, this change would have to be implemented for the entire column of grid cells. This means that local spatial grid density can change, but it may have consequences in creating highly skewed grid cells elsewhere in the grid. For this reason a uniform grid is employed for all cases solved with this model. Secondly, the model uses the power law approximation, which is both fast, accurate and stable. However, it is not as accurate as the second order upwind scheme employed by FLUENT's energy equation solver.

The largest problem however is the construction of the stiffness matrix during the solution process, described in Section 3.3.7.1. This process takes the bulk of the processing time, as measured using MATLAB's internal "profiler" function. Unfortunately there is no way to speed this up, as it is already as efficient as possible in its current state. The time required for matrix assembly increases exponentially with the

number of grid cells, as an N by M problem results in an  $N \times M$  by  $N \times M$  stiffness matrix.

To overcome these limitations a commercial energy equation solver can be used, such as utilized in Chapter 6, however the file I/O time required for large problems is still prohibitive. Ultimately, the development of a POD approach for the energy equation directly would be most useful. However, the development of this energy equation sovler has proven to be useful, as well as a vaulable leaning experience.

### 3.5 Formulating the Steps of an Approach

With the core constructs described in Chapter 2, and the analysis model formulation desicribed in this chapter, the steps of the approach for the robust design of data center server cabinets can be formulated. This is an approach, not a formal design method, as some steps are interchangeable, and can be solved using different models or algorithms, as have been employed in the three example applications in Chapters 4, 5, and 6. The steps are organized loosely into four main phases, with sub-steps in some phases as shown in Figure 3.15.



Figure 3.15 - Steps of the robust server cabinet design approach

The POD construct is integrated into the second phase of this approach, the compromise DSP and robust design constructs are integrated into the third phase. This approach is best explained through application, which is performed in the following three chapters. Most of the steps are applied in all three applications, with differing levels of complexity in the analysis models and some difference heat transfer models employed, however the same main four phases are always followed.

# 3.6 Chapter Synopsis and Validation Summary

In this chapter the core analysis models for the fluid flow and heat transfer simulations were developed and validated. The fluid flow model was based upon POD reconstructions using the flux matching procedure, and the heat transfer model was based upon a finite difference discretization of the energy equation, approximated using the power law. The quadrants of the validation square that have been addressed in this chapter are presented below. How the validation performed in this chapter falls within the complete validation roadmap can be determined from viewing Table 1.3.

# Theoretical Structural Validity

- Theoretical considerations of the POD basis projected onto the RANS governing equations are considered in Section 3.2.1
- The foundational accuracy and stability of the power law advective heat flux approximation is discussed in Sections 3.3.3-3.3.5.
- The use of log law functions for turbulent boundary layer approximations is discussed from literature and the FLUENT CFD software implementation in Section 3.3.6.2.
- Non-linearity in the energy equation solver is handled using a simple iterative method, a simple but effective approach to converge to a final solution in Section 3.3.8.

• The consideration of turbulence in the energy equation solver is tackled through the implementation of a local grid cell thermal conductivity  $k_{eff}$ , this does not change anything fundamental in the energy equation solver, shown in Section 3.3.6.1.

# Empirical Structural Validity

- The reconstruction error for three modes is explicitly computed using exhaustive search to determine the existence of local minima and the general shape of the error space in Section 3.2.2.
- The accuracy and stability of the power law advective heat flux approximation is considered against other commonly implemented approximations in Section 3.3.3.
- The  $y^+ \equiv y^*$  assumption is tested for all range of boundary layer flow Reynolds numbers in Section 3.3.6.2.
- The speed of convergence is tested for three different discretized energy equation solvers in Section 3.3.7.
- The conduction aspect of the heat transfer model is compared to a steady state analytical solution in Section 3.4.1.
- The combined conduction and advection aspect of the heat transfer model is compared to a simple FEMLAB FEA solution in Section 3.4.3.

With the core analysis models derived, discussed and validated, the first case study is presented: the flow configuration of a pair of server cabinets in the cold aisle of a data center with the objective of energy efficiency.

# **CHAPTER 4**

# **COLD AISLE STUDY**

In this chapter the first example problem is investigated, the energy efficient flow configuration of a cold aisle of a data center. This problem uses the simplest geometry, and is the first work to back up the hypothesis that flow and heat generation parameter based design is applicable to data center thermal management. In Section 4.1 the role of the study is presented, as well as the system geometry and boundary conditions. In Sections 4.2 and 4.3 the fluid flow and heat transfer solutions are investigated respectively. In Section 4.4 the evaluation of the temperature response is discussed, and in Section 4.5 the compromise DSP is developed and applied, and the results discussed. The chapter synopsis and validation summary is presented in Section 4.6.

How this chapter falls into the overall structure of the thesis and validation square is presented in Figure 4.1. This chapter builds upon the analysis models developed and the steps of the approach developed and presented in Chapter 3 through their application to the simplest example in this thesis. This in turn addresses the empirical performance validity of the approach, its capability to produce effective results, in this case, server cabinet configurations with greater thermal efficiency and operational stability. The role of this study as it pertains to the overall thesis motivation and validation approach is discussed in the following section.



Figure 4.1 - Thesis and validation roadmap: Chapter 4

### 4.1 Study Introduction

### 4.1.1 Motivation for this study

In this chapter an introduction to the first work undertaken integrating the POD based flow modeling approach with the compromise DSP and a direct energy equation solver is presented. The focus is to determine the feasibility of the approach, employing the test case used in Jeff Rambo's PhD proposal and [77]. Although the problem geometry is comparatively simple, this investigation is included for several reasons:

- Multi-dimensional POD flux matching The cold aisle open cabinet geometry investigated has 10 boundary fluxes to match. Because the vertical flow cabinet style investigated in Chapter 5 and Chapter 6 only have one boundary flux to match, this geometry is used to investigate the effectiveness of the POD based flow model when multiple boundary flow conditions need to be matched.
- Open cabinet geometry The open front horizontal flow cabinet geometry used in this investigation is commonly used in higher power data centers. Because of this use it is pertinent to investigate the impact of efficient cabinet configuration on this design of server cabinet as well as the enclosed vertical flow designs investigated more thoroughly in Chapter 5 and Chapter 6.
- Design space investigation Having the most simple geometry and least design parameters, the design space of this problem is easily visualized

and can be used to get a handle on the complexity, concavity, and linearity of the design space. This is useful as the latter example problems increase in dimensionality and complexity, making this task difficult.

In addition to the three points stated above, this problem also acts as an introductory application of the flow and heat transfer models described in Chapter 3. With the role of this investigation made clear, the problem derivation and geometry is described.

# 4.1.2 Problem Solution Process Organization

How this cold aisle study as presented in this thesis ties into the steps of the robust server cabinet design approach, as given in Section 3.5, is shown below in Figure 4.2. This figure in conjunction with the material presented in this chapter gives a good representation of what performing the cabinet design approach entails.
Sections



Figure 4.2 - Relationship between cabinet design appraoch and thesis chapter organization

# 4.1.3 Partitioning the Problem

The problem geometry investigated is the cooling of servers in a cold aisle of a data center. The center level cooling airflow scheme is presented in section 1.1.3. In this investigation the longitudinal similarity of the aisle is utilized to partition the problem into a two dimensional cross-section of two cabinets on opposite sides of the cold air supplying perforated tiles. The system boundaries are the perforated tiles through which

the cold air is supplied by the plenum on the bottom, the exhaust fans of the servers on each side, and the space above the cold aisle at the same level as the tops of the cabinets. This two-dimensional flow representation is only valid for cabinets away from the edges of the cold aisle. As shown by [12, 70, 78, 79, 99], cabinet airflows at the edges of the aisles contain re-circulation around the end of the aisle that is not captured in a two dimensional simulation. The section of the cold aisle investigated with respect to a typical data center layout is shown below in Figure 4.3, and a section system diagram is presented in the next section.



Figure 4.3 - Cold aisle section location within a data center

The cabinet models used are quite coarse, containing only 4 servers with no internal geometry. The heat load is modeled as a uniform heat flux from the bottom of the server. This model is similar to previous data center level analyses [12, 70, 78, 79, 99], and is used to show the effectiveness of the POD based flow modeling to a flow problem that has already been analyzed using traditional CFD analysis. Because the recirculation effects the hot exhaust air being drawn into the top of the cold aisle are not explicitly modeled, some assumptions are made to partition this cold aisle section from

the rest of the data center. These assumptions are described in detail in the following section.

# 4.1.4 System Geometry and Boundary Conditions

The problem geometry, fluid flow, and thermal system to be investigated is shown below in Figure 4.4. This geometry shows the two server cabinets with four racks each on both sides of a cold aisle. The cold aisle acts as the central flow, and acts as the plenum to distribute the air to the individual server racks. The boundary conditions for the model are specified to best simulate a cabinet's airflow.



Figure 4.4 - Cold aisle geometry, fluid flow, and system variables

# 4.1.4.1 Geometry

The complete system as shown in Figure 4.4 measures 2 m high by 3.1 m wide. Each server cabinet is 1 m wide by 2 m high, containting four 0.25 m high servers. This server height is very large, but as stated this coarse simple geometry has been selected for its similarity to the coarse data center level models completed in past work. The cold aisle width is 1.1 m, and it is also 2 m high. This boundary at the level of the top of the racks partitions the cold aisle model from the complex return airflow near the ceiling above the cabinets.

## 4.1.4.2 <u>Airflow Boundary Conditions</u>

The arrows in Figure 4.4 show the direction of airflow. The top set of double arrows indicate that the flow can either enter or exit, depending upon the mass continuity of the system. This means that if the mass sum of the exit flow is greater than that provided by the inlet flow, air is drawn in from the ambient air above the aisle, leading to re-circulation of hot exhaust air.

The inlet flow at the bottom of the system from the plenum through the perforated tiles is modeled as uniform and normal to the boundary. The top boundary of the cold aisle is modeled to have zero (gage) pressure, to enable the flow either in or out of the system depending upon continuity. A cubic pressure-velocity relationship or fan model, where pressure is specified as a function of velocity, is applied to simulate an induced draft fan at the exit boundary of each server. This cubic fan model is given in equation (4.1) below.

$$p(u) = 112.4 - 27.43u + 2.561u^2 - 0.1024u^3$$
(4.1)

This pressure-velocity relationship model is important, because it better represents the air flow distribution relationship between the plenum supply rate and the server fan draw rate. The fans heating effect upon the air is assumed to be negligable for all modeling work performed in this thesis. A simple uniform normal velocity boundary specification does not capture these complex interactions accurately. This is important as this supply rate to server draw rate relationship is investigated later.

## 4.1.4.3 Thermal Boundary Conditions

The grey lines in Figure 4.4 represent the chips on the surface of the PCB, however their effect of disrupting the flow was not modeled, and the heat generated is modeled as a uniform heat flux from the entire bottom surface of the server to the air above. All solid walls of the servers were modeled as adiabatic. The server outlets are specified as Neumann outflow boundaries, such that the temperature derivative, or heat flux normal to the boundary equals zero.

The plenum flow inlet temperature was specified as a dirichlet boundary condition, with the temperature equal to the specified value  $T_{in}$ . The top of the cold aisle is a variable boundary condition, modeled either as a dirichlet temperature inlet boundary specified at a value  $T_{\infty}$  if the flow is entering the domain, or an outflow boundary if the flow is leaving the domain.

Because the air leaving the domain though the servers (indicated in Figure 4.4 by the arrows labeled  $V_{out}$ ) is recycled to some degree through the top inlet, the double arrows labeled Continuity, this upper cold aisle inlet temperature  $T_{\infty}$  is dependent upon the temperature leaving the domain. These two variables can be coupled creating a nonlinear problem to solve for the steady state  $T_{\infty}$  value. This re-circulation of this exhausted air is investigated using both the specified and coupled  $T_{\infty}$  value.

# 4.1.5 System Variables

The system variables represent the flow velocities and heat generation rates within the server cabinets and cold aisle. These are classified as design variables, over which the designer has control, noise factors, parameters with inherent variation over which the designer does not have contorl over, constants, variables that are held constant, and response parameters, used to evaluate the performance of the system.

## 4.1.5.1 Design Variables

The design variables for this investigation are:

- *T<sub>in</sub>* The air inlet temperature from the under floor plenum that enters the cold aisle through the perforated tiles.
- *V<sub>in</sub>* The velocity the air enters the cold aisle from the plenum through the perforated tiles.
- V<sub>out,i</sub>, i = 1,...,8 The velocity of the air exiting the servers, provided by the server exhaust fans. There is one outlet velocity specified for each server.

All velocities are measured in meters per second, all temperatures in degrees Celsius, and power in Watts. The CRAC units that supply and control both the rate and temperature of the air to the plenum have advanced control algorithms that regulate temperature very well [49]. Furthermore, this inlet temperature has a direct one to one linear relationship with the response temperature (discussed in section 4.1.5.4). Therefore, the interest in this investigation is into the nonlinear variables, the inlet and outlet flow velocities, as such  $T_{in}$  is specified at a constant 25°C for this investigation. As there are eight servers,  $V_{out}$  must be specified for each of them. However, in this investigation all servers are specified a single  $V_{out}$  and hence it is treated as a single control variable for the reduction of computer computational time.

## 4.1.5.2 Noise Factors

The noise factors for this investigation are:

•  $T_{\infty}$  – The temperature of the inlet air drawn in from above the cold aisle.

 $T_{\infty}$  in this investigation is primarily assigned a constant value of 10°C above the inlet temperature  $T_{in}$ , equaling 35°C. This value is a rough average of the re-circulated air temperature from the full scale data center simulation work by Rambo [78]. During the investigation of the effects of the amount of heat re-circulated into the cold aisle  $T_{\infty}$  and the corresponding cold aisle to ambient boundary condition  $\partial \Omega$  is determined using equation (4.2) given below:

$$\partial \Omega_{cold \ aisle \ inlet} = \begin{cases} T_{\infty} = \frac{\sum_{i=1}^{8} \dot{m}_{out,i} \cdot T_{out,i}}{-\dot{m}_{aisle}} \cdot \eta, \text{if } \dot{m}_{aisle} \le 0\\ \frac{\partial T}{\partial y} = 0, \text{if } \dot{m}_{aisle} > 0 \end{cases}$$
(4.2)

Where  $\dot{m}_{out}$  is the mass flux out of a single server,  $T_{out}$  is the mean temperature of the exhaust air from a server,  $\dot{m}_{aisle}$  is the mass flux exiting the top of the cold aisle (and

hence is negative when air is entering when re-circulation occurs), and  $\eta$  is the recirculation coefficient, ranging from 0 to 1. If  $\dot{m} > 0$  in equation (4.2) the definition of  $T_{\infty}$  becomes redundant as the boundary becomes an outflow condition, and there is no air entering the domain through the boundary, and thus no definition of  $T_{\infty}$  is required.

# 4.1.5.3 Constants

The held constant parameters in this investigation are:

Q<sub>i</sub>, i = 1,...,8 – The power dissipated by each server. There is a specific power level for each server.

The heat flux from the boards is of prime importance in this investigation, although the designer does not have control over it. This is because in a physical data center operation the cooling system must be configured to meet the needs of the servers being used. This heat flux would change, depending upon the level of usage and the power of the processors in the servers. In a similar manner to  $V_{out}$ , the server exhaust fan velocity, the heat fluxes for all servers is uniform for this investigation. In this manner the cooling system flow rates alone must meet the heat dissipation needs of the system, without utilizing power re-distribution as using in the later applications in Chapter 5 and Chapter 6. This means heat flux as the only source of variation and thus the robustness of the system to changes in flux can be easily computed, as performed in Section 4.5.4.

# 4.1.5.4 <u>Response Parameters</u>

The response parameters for this investigation are:

*T<sub>i</sub>*, *i* = 1,...,8 – The maximum board surface temperature of each server.
 There is a specific temperature computed for each server.

The response used for computing the system constraints and objective values is the maximum board surface temperature of each server. These eight responses are treated as individual quantities for constraint handling purposes. The sum of the board temperatures yields a single metric of the cooling performance of the control variables, however in this investigation the minimization of this metric is not the focus, as described in the following section.

# 4.1.6 System Objective and Constraint Derivation

One of the current practices in data center thermal regulation employs variable fans in the server racks, combined with a variable speed drive in the CRAC unit to vary the airflow in the cabinets such that the maximum safe operating temperature is achieved. This practice translates to the following goals and constraints:

# System Design Objectives:

• Minimize airflow from CRAC units:

$$\min(V_{in}) \tag{4.3}$$

Minimize airflow of server fans:

$$\min(V_{out,i}), i = 1,...,8$$
 (4.4)

System Design Constraints:

Maintain all servers at operational temperature:

$$T_i \le T_c, i = 1, ..., 8$$
 (4.5)

The minimization of the inlet velocity goal is directly proportional to the volume of air supplied by the CRAC units, representing a significant portion of the cost of data center operation [48]. The minimization of the server fan velocity is a less important goal, as the operating cost of the server rack fan is much less than that of a CRAC unit. However, as investigated in Section 4.4, this goal ensures that the minimum server fan velocity that mates with the required inlet velocity for most efficient operation is utilized. This is important as lower fan speeds are both more energy efficient and less irritating to the data center operators. This is pertinent as the ultra high flow velocities required to cool state of the art high density servers such as an IBM blade server can become so loud that in a fully populated data center hearing protection is required.

Because the constraints commonly applied to computer systems is a maximum chip operating temperature  $T_c$ , the average server temperature is not considerd in this investigation, only the critical point. This maximum chip temperature constraint was set as  $T_c = 100$  °C, as measured from the maximum surface temperature of the heat generating surface in the server. The models used to determine this maximum chip temperature as a function of the air velocity and heat flux are described next.

## 4.1.7 System Systhesis Model

The control variables, noise factors, and problem constants are input into the cold aisle model, and the response of the server temperatures monitored creating a system model. These values are used to evaluate the goals and constraints in the compromise DSP, and the process iterated until convergence is achieved, shown in Figure 4.5.



Figure 4.5 - Cold aisle system model diagram

The derivation of this cold aisle flow model, yeilding the server temperatures, is described in the following section.

## 4.2 Determining Cold Aisle Airflow

#### 4.2.1 Generating the Observations

The first step in the POD is generating the observations. However, in this investigation the POD modes were created by Jeff Rambo for his PhD proposal work and [77] and applied directly. For completeness the details of the generation of the observations are described. The observations were created using the commercial CFD program FLUENT v. 6.1.22. The incompressible, steady, Reynolds-averaged Navier Stokes (RANS) equations with an isotropic eddy viscosity closure, were solved using the standard k-epsilon model. Second order up-winding was used to discretize the 164

convective fluxes, and the SIMPLEC procedure was used to couple the pressure and velocity fields. All fluid properties are assumed to remain constant. The boundary conditions employed are described in Section 4.1.4, and a model mesh measuring 63 nodes in x by 41 nodes in y was used, resulting in 2,583 nodes and 12,915 degrees of freedom. The standard FLUENT convergence criteria were used [25], and further iteration was shown to produce no change in the solution.

A series of observations spanning the range of server outlet velocities of 0.15, 0.3, 0.45, and 0.6 m/s, and inlet velocities of 0.5, 1, and 1.5 m/s are created. Adjacent server fan velocities are coupled together creating upper and lower pairs for each cabinet to reduce the number of combinations. The resulting server fan velocity combinations are tabulated below in Table 4.1. All 16 of these combinations are computed for each inlet velocity, resulting in a total of 48 obervations.

Obeservation	$V_{out,1,2}$	$V_{out,3,4}$	$V_{out,5,6}$	Vout, 7,8
1	0.15	0.15	0.15	0.15
2	0.3	0.3	0.3	0.3
3	0.45	0.45	0.45	0.45
4	0.6	0.6	0.6	0.6
5	0.15	0.15	0.3	0.3
6	0.15	0.15	0.45	0.45
7	0.15	0.15	0.6	0.6
8	0.3	0.3	0.45	0.45
9	0.3	0.3	0.6	0.6
10	0.45	0.45	0.6	0.6
11	0.3	0.15	0.3	0.15
12	0.45	0.15	0.45	0.15
13	0.45	0.3	0.3	0.45
14	0.6	0.3	0.3	0.6
15	0.6	0.45	0.45	0.6
16	0.6	0.15	0.15	0.6

Table 4.1 - Cold aisle observation server velocity parameter combinations

The velocity ranges selected are based upon the typical values encountered in data centers. The number of observations was reduced by employing symetry. It should be noted that this work represents the first application of the POD for RANS flow modeling, and as such the span and boundary condition combinations employed are not the most efficient for the problem investigated. The simplicity of the problem geometry and coarseness of the mesh enabled more observations than required be generated, future research into efficient arrays of observation generation are discussed in Section 7.2.2.

# 4.2.2 Generating the POD Modes

The FLUENT observations are exported as node centered ASCII files and imported in MATLAB for further analysis. The POD of the 48 observations described above yields 48 modes. These observations were not mean centered, as performed in the POD application in the following chapters. This results in a particularly large eigenvalue of the first mode, as it contains the mean flow physics of the problem, as shown in Figure 4.6 below.



Figure 4.6 - Eigenvalue spectrum of cold aisle POD modes [77]

The vector field of the cold aisle portion of the domain is shown in Figure 2.7 in Section 2.1.8. Although these modes are easily physically interpreted, the higher order modes with correspondingly smaller eigenvalues fields become random noise fields as the small flow details are obscured by the accuracy of the eigen decomposition algorithm and machine accuracy [21]. This is because of the number of observations used to generate the POD modes, and its ramifications of flow field reconstruction are investigated below in Section 4.2.4.

# 4.2.3 Reconstructing an Arbitrary Flow Field

To reconstruct the complete flow field using the POD modes, the flux matching procedure is used, as described in Section 3.1.3. The goals to be matched are the inlet velocity across the perforated tiles, the server fan velocities across all 8 servers, and the resulting inlet or outlet velocity at the top of the cold aisle in order to maintain the mass balance across the domain. These goals are ordered into a vector g shown below in equation (4.6), where the subscript g indicates that this is the desired value of the mass flux  $\dot{m}$ .

$$g = \begin{bmatrix} \dot{m}_{out,1,g} \\ \dot{m}_{out,2,g} \\ \vdots \\ \dot{m}_{out,8,g} \\ \dot{m}_{in,g} \\ \sum_{i=1}^{8} \dot{m}_{out,i,g} \end{bmatrix}$$
(4.6)

The coefficient matrix, C, is constructed of concatenated columns of each POD modes attainment towards the goal g, following the formulation given by equation (3.10), 167

computed using the flux function in equation (3.8). In this case, each element is the mass flux across a boundary,  $\dot{m}$ , of the individual POD mode. In equation (4.7) below, for a server mass flux  $\dot{m}_{out,i,j}$  subscript *i* refers to the server, and subscript *j* refers to the POD mode the mass flux is extracted from.  $\dot{m}_{in}$  refers to the mass flux entering the aisle through the perforated tiles from the under-floor plenum.

$$C = \begin{bmatrix} \dot{m}_{out,1,1} & \dot{m}_{out,1,2} & \cdots & \dot{m}_{out,1,p} \\ \dot{m}_{out,2,1} & \dot{m}_{out,2,2} & \cdots & \dot{m}_{out,2,p} \\ \vdots & \vdots & \ddots & \vdots \\ \dot{m}_{out,8,1} & \dot{m}_{out,8,2} & \cdots & \dot{m}_{out,8,p} \\ \dot{m}_{in,1} & \dot{m}_{in,2} & \cdots & \dot{m}_{in,p} \\ \dot{m}_{in,1} - \dot{m}_{aisle,1} & \dot{m}_{in,2} - \dot{m}_{aisle,2} & \cdots & \dot{m}_{in,p} - \dot{m}_{aisle,p} \end{bmatrix}$$
(4.7)

The coefficient vector, a, is computed using the pseudo-inverse lease squares approximation as described in section 3.1.3.2. The influence of the number of modes used in the reconstruction, p, is investigated in the following section.

# 4.2.4 Evaluation of the Flow Model

As discussed in Section 3.1.2, the POD modes are ordered by their associated eigenvalue, a measure of the level of system dynamics captured by that specific mode. Because the observation generation array includes many server fan flow rate combinations that cannot occur in this investigation when all server fans operate at the same speed, some of the dynamics picked up in the POD modes are redundant. As the flux matching procedure aims to reconstruct the complete flow field using only partial information defined at the boundaries, inclusion of these higher order complex modes may decrease the accuracy of the complete reconstruction, as the algorithm does not

penalize the weighting of higher order modes. Therefore it is pertinent to investigate the number of modes to be used in reconstruction for maximum accuracy. Because of the speed of the algorithm used for finding the modal weighting coefficients, accuracy is the only concern, as reconstruction computational speed with the number of modes used in this problem is negligible.



Figure 4.7 - Flow field reconstruction error vs. number of POD modes used

The error plot in Figure 4.7 shows the L<sub>2</sub> velocity error norm,  $e_2$ , of the reconstruction of observation 1 from Table 4.1 using a  $V_{in}$  of 0.5 m/s, computed using equations (4.8) and (4.9) below, where subscript r is the reconstruction and subscript t is correct complete solution, *vel* is the velocity magnitude, u the horizontal and v the vertical component of the flow velocity.

$$vel = \sqrt{u^2 + v^2}$$
(4.8)  
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$$e_2 = \frac{\left\| vel_r - vel_t \right\|}{\left\| vel_t \right\|} \tag{4.9}$$

For the reconstruction of observation 1, as well as most of the other observations tested, using three modes provided the most accurate reconstruction, with an  $e_2$  of less than 0.02. Although this number initially seems small, the flow being modeled in this investigation is fairly simple, and a very accurate reconstruction using the complete known field was computed using 5 modes as shown in Figure 2.8 in Section 2.1.8.

# 4.3 Heat Transfer Solution

## 4.3.1 Importing the Flow Field

The complete flow field as computed using the POD approximation is used to determine the advective heat flux across the boundary of each temperature nodes control volume, as described in Section 3.3. This requires knowledge of the flow across the center each face of the control volume, thus a staggered grid is employed, as shown in Figure 4.8 below.



Figure 4.8 - Staggered grid of flow and temperature nodes [67]

In this figure the shaded area is the control volume element of constant temperature, and the node at the center is the point at which the temperature is computed. The dashed lines represent the edges of the temperature cells. The intersection of these dashed lines are the solid lines are where the horivontal and vertical (u and v) components of the velocity are computed using the POD flow model. This results in the temperature matrix being one element smaller in each dimension with nodes offset by one half of a grid cell in both x and y directions.

## 4.3.2 Computation and Re-circulation Considerations

The stiffness matrix for solving the energy equation is assembled as described in Section 3.3, and is solved using the direct sparse matrix inversion approach. Because the inlet air temperature of the air above the cold aisle,  $T_{\infty}$ , is modeled as a constant 10°C rise, it is pertinent to quantify the amount of air re-circulation and thermal mixing occurring above the cold aisle that would lead to these conditions. Equation (4.2) is used to compute  $T_{\infty}$  for a range of  $\eta$  values from 0 to 0.5, shown below in Figure 4.9.



Figure 4.9 - Inlet temperature,  $T_\infty$  vs. re-circulation coefficient,  $\eta$ 

Analysis of Figure 4.9 indicates that the constant  $T_{\infty}$  of 10°C is achieved through around 7.5 % re-circulation of server exhaust air to the top of the cold aisle. This value of  $\eta$  is conservative minimum, as with higher  $V_{in}$ , there is less re-circulation and hence a lower  $T_{\infty}$ . This value of  $T_{\infty}$  also decreases with  $V_{out}$  as the domain mass balance means a higher  $V_{out}$  yields a higher  $\dot{m}_{out}$  and a proportionally higher  $\dot{m}_{aisle}$ , which means the numerator/denominator ratio is at a maximum when  $V_{out}$  is minimized. Thus the value of  $\eta$  found for the constant 10°C is the minimum possible within the range of parameters used in this investigation, indicating the data center modeled has a reasonable amount of re-circulation.

## 4.3.3 Analysis Model Evaluation

It should be noted that the energy equation solver used to determine the temperature in this investigation uses the assumption that the thermal conductivity of the air is constant. This assumption means the influence of the turbulence in creating a higher value of thermal conductivity is ignored, yielding a conservative result. However, because of the relatively low velocity of the flow, the effects of this assumption are not tremendous, as shown in Section 5.3. Because of the coarseness of the model being used, only the trends in the results are of interest at this point, and thus these results are not compared to the original FLUENT solutions for accuracy. Fundamentally, the use of this constant thermal conductivity assumption will not change the trends in temperature response, and thus this model is usable for the purposes set forth in this investigation.

## 4.4 Evaluating the Temperature Response

With the development of quickly computing fluid flow and temperature fields through the computational models described previously, a parametric investigation of the design space is possible. This will determine a coarse temperature response with respect to the control variables,  $V_{in}$  and  $V_{out}$ . These results can then be used for obtaining good starting points for use in the compromise DSP, as well as obtaining a feel for the shape of the design space. This is useful as the effects of the input variables are not linear or easily predictable. This is because of the nature of turbulent flow and the manifolding effects of the combination of  $V_{in}$  and  $V_{out}$  parameters. The response investigated is the maximum surface temperature of all of the servers, computed using equation (4.10).

$$T_{\max} = \max(T_i), i = 1, \dots, 8 \tag{4.10}$$

The maximum response surface plots are shown below in Figure 4.10 using a constant heat flux of 45W/m on all the servers.



Figure 4.10 - Temperature response vs. inlet & server fan velocities for constant heat flux

Inspecting the plot, there is a definitive valley shape in which the most thermally efficient combination of  $V_{in}$  and  $V_{out}$  parameters can be obtained. This indicates there should be a balance between the cold air pumped in through the plenum into the cold aisle as well as the amount or air being drawn in through the server racks themselves. To investigate what causes this temperature response, the temperature profiles of the four bounding cases are determined, as specified by Table 4.2 below.

Scenario	$V_{in}$	Vout
a	0.5	0.15
b	0.5	0.6
с	1.5	0.15
d	1.5	0.6

Table 4.2 - Bounding flow conditions for cold aisle

The temperature profiles of all four cases are plotted in Figure 4.11 - Figure 4.14 below.



Figure 4.11 - Temperature profile of Scenario a



Figure 4.12 - Temperature profile of Scenario b



Figure 4.13 - Temperature profile of Scenario c



#### Figure 4.14 - Temperature profile of Scenario d

Analysis of the Figures above shows that the critical server that is hottest changes depending upon the flow condition. If warm air is being recirculated from above the cold aisle, the ciritical server is the uppermost unit. However, the most conter-intuitive condition is case c. Here, too much cold air is being forced in, it is simply driven through the top of the aisle, as the server fans cannot provide the pressure required to draw the air into the servers, as shown in Figure 4.13. This results in the lower servers being the hottest. These figures help explain the tempeature response profile presented in Figure 4.10.

# 4.5 The Compromise DSP for Energy Efficient Cold Aisle Server Configuration

# 4.5.1 Constructing the compromise DSP

Following the mathematical formulation outlined in Section 2.3.2 and [55] the following compromise DSP for the most thermally efficient flow conditions for the cold aisle is developed using the control variables, goals, and constraints outlined in Sections 4.1.5 and 4.1.6 using equations (4.3) - (4.5). The complete formulation is shown below in Table 4.3, and each section discussed in turn.

# Table 4.3 - The compromise DSP for thermally efficient cold aisle flow distribution Given

- Response model of server temperature, server fan air velocity, and inlet air velocity as functions of  $x_1$ ,  $x_2 = V_{in}$ ,  $V_{out}$
- Constant value of server powers,  $Q_i \in [100, 170]$ ,  $i = 1, \dots, 8$
- Constant value of inlet temperature,  $T_{in} = 25 \text{ }^{\circ}\text{C}$
- Target for inlet air velocity,  $G_{vin} = 0.5 \text{ m/s}$
- Target for server fan air velocity,  $G_{vout} = 0.15 \text{ m/s}$
- Number of system variables, *n* = 4
- Number of inequality constraints, p = 1
- Number of equality constraints, q = 0
- Number of system goals, m = 3

## Find

The values of control factors:  $x_1$ , inlet air velocity,  $V_{in}$   $x_2$ , server fan air velocity,  $V_{out}$ The values of deviation variables  $d_i^+, d_i^-, i = 1,...,n$ 

## Satisfy

The constraint:

The individual server boards maximum temperature cannot exceed 100 °C

$$T_i \le 100, i = 1, \dots, 8$$
 (4.11)

## Table 4.3 - The compromise DSP for thermally efficient cold aisle flow distribution cont.

The goals: Minimize inlet air velocity

$$\frac{G_{vin}}{x_1} + d_1^- - d_1^+ = 1 \tag{4.12}$$

Minimize server fan air velocity

$$\frac{G_{vout}}{x_2} + d_2^- - d_2^+ = 1 \tag{4.13}$$

The bounds

$$0.5 \le x_1 \le 2 \,(\text{m/s}) \tag{4.14}$$

$$0.15 \le x_2 \le 0.6 \,(\text{m/s}) \tag{4.15}$$

$$d_i^+ \bullet d_i^- = 0$$
, with  $d_i^+, d_i^- \ge 0, i = 1, ..., m$  (4.16)

Minimize

The total objective function:

$$f = \sum_{i=1}^{m} W_i (d_i^+ + d_i^-), \text{ with } \sum_{i=1}^{m} W_i = 1, W_i \ge 0, i = 1, ..., m$$
(4.17)

## 4.5.1.1 Given (from Table 4.3)

Using the system model shown in Figure 4.5, the givens for the problem are derived. Most of the constants given have already been discussed in the derivation of the system variables, goals and constraints. The targets for the design variables are set at their lower bounds, given in equations (4.14) and (4.15), as defined by the minimum flow parameters of the POD based flow model. The range of server powers was found through a coarse parametric sweep starting from 100 W/m until the system constraints could no longer be met.

## 4.5.1.2 Find (from Table 4.3)

The design variables, and the associated deviation from the goal value associated with each design variables, as discussed in Section 2.3.2, are the parameters to be found in this investigation.

# 4.5.1.3 Satisfy (from Table 4.3)

The constraint given in equation (4.5) in Section 4.1.6 directly translates to the formulation used in the compromise DSP, given in Section 2.3.2, leading to the constraint equation (4.11). Each of the maximum surface temperatures of the servers must be below 100 °C or the solution is infeasible. The goals of this investigation are simple linear function minimizations, as given in equations (4.12) and (4.13). These equations are the transformation of equations (4.3) and (4.4) using the minimization goal formulation described in Section 2.3.3 with the goal values in the "Given" part of the compromise DSP. The bounds of the design variables are set as the upper and lower limits of the velocities that can be input into the POD based flow model, and the final constraint imposed on the system ensures that there is no simultaneous under and over achievement of any goal.

## 4.5.1.4 Minimize (from Table 4.3)

As the values of the deviation from both goals are normalized between 0 and 1 because of the mathematical formulation of the compromise DSP, the designer's preferences are represented through the selection of the weighting vector W. The weighting vector used is stated and explained in each application of the compromise

DSP, and its selection has significant effect on the resulting value of the total objective function *f* as well as the design variables  $\vec{x}$ .

### 4.5.2 Single Variable Formulation

This problem serves to test the empirical structural validity of the implementation of the compromise DSP. By using only one variable the comparison of the computed result to the parametric study plot to determine if the algorithm converged correctly to the most efficient point is possible. The parametric plot of maximum server temperature,  $T_{max}$  versus  $V_{in}$  for a flux input of 55 W/m and a constant  $V_{out}$  of 0.45 m/s is shown below in Figure 4.15. Because this is a single objective problem there is no need to define a weighting vector for goal prioritization.

Analysis of the figure shows the intersection of the plot with 100°C occurs around 0.85, which is very close to the compromise DSP computed value of 0.853. The execution of the compromise DSP is implemented using the "fmincon" function in MATLAB [108] to implement the SQP algorithm [27], using the same constant 55W/m heat flux and 0.45 m/s  $V_{out}$  values. Initial starting point for the algorithm was set at the lower bounds of both control variables, although other points were tested to converge to the same solution. The algorithm converged after 17 iterations, with one active constraint. In practice, the compromise DSP result has extraneous decimal places, the models used are not accurate to warrant this accuracy, however, the convergence tolerances can easily be changed, and the empirical performance validity of the algorithm has been shown.



Figure 4.15 - Parametric plot of  $T_{max}$  vs.  $V_{in}$  for constant heat flux Q and  $V_{out}$ 

# 4.5.3 Multi Variable Formulation

In this problem the compromise DSP is run again using both  $V_{in}$  and  $V_{out}$  control variables in a multivariable objective function and constraint function. The weighting of the two goals set forth in equations (4.12) and (4.13) are equal, as such W is defined as follows in equation (4.18).

$$W = \{0.5, 0.5\} \tag{4.18}$$

The resulting characterization curve of the inlet to server fan air velocity relationship for heat flux generations Q from 100 W/m to 170 W/m in 5 W/m steps is developed. Beyond 170W/m the system could not provide adequate cooling using the air 182

inlet temperature,  $T_{in}$ , provided. The resulting plot is shown below in Figure 4.16. The general trends are easily recognizable, however small fluctuations can be attributed to computational error.



Figure 4.16 - Characterization of the cold aisle, Vin vs. Vout for increasing heat loads

A plot such as Figure 4.16 for specific servers using a more detailed and experimentally validated model could be used to determine guidelines for CRAC unit operation and data center cabinet layout. Through measurement of the flow through the perforated tiles using a flow hood, the ideal CRAC unit flow rate and/or positioning of the perforated tiles could be determined through matching the plenum supply rate,  $V_{in}$ , with the required  $V_{out}$  of the servers being used, as specified by their manufacturer. This would enable the data center cold aisle cabinets to be positioned and configured more quickly and efficiently than if this flow distribution was completely unknown.

## 4.5.4 Sensitivity Analysis

A study of the effect of altering the noise variables is also conducted. The linearity of the thermal system means a simple slope computation is performed using a central difference approach. The slope of this relationship is important for determining the sensitivity of the response variables to changes in the noise variables, in this case the heat flux from each server, Q, and the inlet air temperature,  $T_{in}$ . The resulting sensitivity equations are presented below.

$$\Delta T_{\rm max} = \Delta T_{in} \tag{4.19}$$

$$\Delta T_{\rm max} = 0.72 \cdot \Delta Q \tag{4.20}$$

The slope of the maximum temperature with respect to inlet temperature, equation (4.19), is a one to one relationship, as is to be expected, as  $T_{in}$  is the baseline temperature of the entire simulation. The slope of the maximum temperature with respect to heat flux, equation (4.20), is computed as 0.72, which represents the increase in surface temperature in degrees C per unit increase in heat flux in W/m. This is the more important consideration, as the CRAC unit temperature is accurately controlled by on board controllers [49] and hence is unlikely to vary significantly. However, a spike in computing load would result in a corresponding spike in heat flux, and hence the maximum server temperature is going to respond accordingly. Because of the delay time involved ramping up the CRAC units output or lowering the inlet temperature, and the sensitivity of the chips to excessive temperatures, it would be wise to use a factor of safety in the operating conditions of the data center to accommodate for any sudden heat

flux spikes. A more efficient implementation is the use of robust design, as applied in Chapter 5 and Chapter 6.

## 4.5.5 Search Algorithm Convergence

The Karesh-Kuhn-Tucker optimality conditions dictate a set of specifications that should be met for a solution to be a minimum [10]. These specifications can be rewritten as part of the problem formulation, or applied afterwards to test the solution. The KKT conditions are an extension of Lagrangian function and method, and hence are difficult to apply to a problem with an objective function that is not a mathematical equation. Therefore the conditions are considered afterwards to test the solutions as guidelines for search algorithm convergence evaluation.

The first conditions are fairly straightforward to test and apply. In this problem there is only one inequality constraint, the server surface temperatures must be less than or equal to 100 °C, which is active for all solutions found, and shown for the single parameter case in Figure 4.15 and multi parameter case in Figure 4.17.

The next test is to find the direction of the objective function vector at the converged point and test if it is orthogonal to the active constraint surface. The gradient function was output from the fmincon function, and plotted on a contour plot similar to Figure 4.10, for the converged value of  $V_{in} = 0.85$ ,  $V_{out} = 0.3$ . Visual evaluation of the gradient vector against the constraint boundary, which is the 100 °C contour upon which the tail end of the vector lies, shows that it is normal to the constraint.



Figure 4.17 - Cold aisle objective function vector and constraint contour

The final test is to evaluate points surrounding the final converged solution and see if the value of the objective function is greater. Evaluation of the objective function at all surrounding points were found to have worse functional evaluations, or be infeasible, showing that the solution point is not a saddle point or point of inflection.

It is also known that the result is a global minimum of the solution space. This is through viewing the parametric surface plot in Figure 4.10, as there is a distinct valley shape within the feasible design space, without any local minima to trap the search routine. This was validated through the use of multiple starting points from different corners of the feasible design space. Although this is not a formal approach to testing for a global minima, the small size of the solution space and the speed of the POD based flow model make it straightforward to perform a parametric sweep as performed in Figure 4.10.

# 4.6 Chapter Synopsis and Validation Summary

In this chapter the first example problem is investigated. This problem uses the simplest geometry, and is the first work to back up the hypothesis that flow and heat generation parameter based design is applicable to data center thermal management. The quadrants of the validation square that have been addressed in this chapter are presented below. How the validation performed in this chapter falls within the complete validation roadmap can be determined from viewing Table 1.3.

# Empirical Structural Validity

- In this study the accuracy of the POD flow model is demonstrated, as referred to in the validation from Chapter 3.
- The convergence accuracy of the SQP minimization search algorithm was validated through graphical analysis and application of the Karesh-Kuhn-Tucker optimality conditions in Section 4.5.5.

# Empirical Performance Validity

• The cold aisle model geometry used, although only two dimensional, is still representative of the flow regime encountered in many data center layouts as shown in Section 4.1.4.1.

- The flow and heat transfer parameters used, as well as the goals used in the compromise DSP formulation are representative of physical data center server cabinet configuration problems as shown in Sections 4.1.4.2 and 4.1.4.3.
- The maximum heat dissipation is found to be a combined function of both the cold aisle and server rack flow rates, indicating valid design variables were chosen, shown in in Section 4.5.3.
- The resulting characterization plot of the cooling characteristics of the cold asile for increasing heat loads would be of great use to data center designers and operators if performed for specific server models and cabinets, indicating this type of approach has overall empirical performance validity, as shown in Section 4.5.3.

With the implementation and results of the cold aisle study presented, the second case study of a vertical flow 2U server cabinet is presented, constituting the primary investigation into the effectiveness of robust design.

# **CHAPTER 5**

# **2U SERVER CABINET INVESTIGATION**

In this chapter the second example problem is investigated, the energy efficient and thermally robust configuration of a vertical flow server cabinet is presented. This problem is two dimensional like the study in Chapter 4, however the geometry is more detailed and much more representative of a physical cabinet. In this study the feasibility and effectiveness of the application of robust design is investigated through variations in the amount of cooling air supplied, the heat load distribution within the cabinet, and the interchanging of the individual server fans. In Section 5.1 the study is introduced, the motivation for the work, and the problem geometry and boundary conditions. In Section 5.2 and 5.3 the cabinet airflow and heat transfer solutions are investigated respectively. The temperature response of the cabinet is investigated in Section 5.4, plotting out the rough design space. The compromise DSP for the thermally efficient and thermally robust configuration of the cabinet is derived and applied. This work is expanded in Section 5.6 to develop a family of solutions along a Pareto Frontier, representing all potential solutions from optimal to least varient, enabling the design to select their final operating conditions. In Section 5.8 an investigation into different server fan configurations is completed, utilizing a merit function to evaluate in the selection of a final configuration from 27 individual configurations found using the compromise DSP, creating a sequential compromise-selection decision. The chapter synopsis and validation summary is presented in Section 5.9.
How this chapter falls into the overall structure of the thesis and validation square is presented in Figure 5.1. This chapter builds upon the the steps of the approach developed and presented in Chapter 3 and their application in Chapter 4 through their application to a more complex and representative example. Furthermore, the sensitivity of the system and tradeoffs between the optimal and most invariant solutions are explicitly investigated. This in further addresses the empirical performance validity of the approach, its capability to produce effective results, in this case, server cabinet configurations with greater thermal efficiency and operational stability. The role of this study as it pertains to the overall thesis motivation and validation approach is discussed in the following section.



Figure 5.1 - Thesis and validation roadmap: Chapter 5

#### 5.1 Study Introduction

#### 5.1.1 Motivation for this study

The focus of this study is to determine the effectiveness of a robust design approach using flow and heat transfer design parameters upon an individual cabinet. The geometry employed is two dimensional, and is identical to the test case used in [83]. This was done in order to directly connect the two papers to show the applicability of the POD modeling construct to a design problem. Furthermore, this two dimensional problem is less computationally intensive to solve and easier to visualize and interpret. The core elements of this investigation are:

- Applicability of robust design As shown in the literature review in Section 1.2, little work has been done on the optimization of data center layouts or the configuration of the server cabinets housed within them, let alone the application of design for robustness. The inherent variability of a turbulent flow system like a data center and the associated high operational stability requirements make this first investigation of robust designs' effectiveness at generating solution insensitive configurations of prime interest.
- Pareto frontier development The use of linear weightings for the representation of preferences is simplistic for an application as complex as this. The development of a Pareto frontier spanning the entire solution set from optimal to least varient can enable the quick identification of the

operating conditions of a data center based on the amount of inherent variability and the designers preferences towards operational stability.

- Full flow parameter investigation The computational efficiency of solving the cabinet geometry used in this problem enables the investigation of the effectiveness of varying all the server flow parameters, as well as the heat fluxes, for a full analysis of the effectiveness of all potential server cabinet configuration techniques.
- Server cabinet geometry The rack mounted 2U server cabinet modeled is

   a commonly used design in data center processing equipment. The
   analysis of this design, although only two dimensional, shows the
   applicability of this approach to this server cabinet layout.

As the problem is only two dimensional, the emphasis in this investigation is on the applicability of the robust design approach, and the trends obtained in the results, not on the exact values of the results obtained or modeling of the cabinet. This specific modeling and validation work is presented in Chapter 6.

## 5.1.2 Problem Solution Process Organization

How this cold aisle study as presented in this thesis ties into the steps of the robust server cabinet design approach, as given in Section 3.5, is shown below.

Step: Sections:

(1) 5.1.1 - 5.1.5

- (2) 5.2-5.4
- (3) 5.1.5, 5.5.1
- (4) 5.5.2-5.7

This list in conjunction with the material presented in this chapter gives a good representation of what performing the cabinet design approach entails. This list provides the same information as Figure 4.2 in a more succinct format.

## 5.1.3 Partitioning the Problem

In the present study the server cabinets are considered partially isolated from the data center, interacting only through the supply of cool air from the raised floor plenum, and the exhausted hot air through the top of the cabinet. This allows the cabinet system to be decoupled from the overall data center system, shown below in Figure 5.2, linked only through the flow input and exhaust. Cabinet to cabinet interactions are not considered in this simplified treatment. In this manner the configuration of a complete data center, shown in Figure 5.2, can be broken down into individual server cabinet configuration sub-problems, shown in Figure 5.3.



Figure 5.2 - Layout of a single cabinet within a data center

The following design reconfiguration possibilities are considered. (1) Equipment of differing power density can be distributed within the cabinets for more efficient cooling. This can be implemented through physical relocation of the hardware, and/or by distributing the processing tasks to reduce the load on critical equipment [14, 68, 98]. (2) The volume of cooling air supplied to the cabinet can be increased. This can be accomplished via a CRAC unit output increase, however this will incur a penalty of greater operating costs and the associated environmental impact, making it a less attractive approach. Flow redistribution can also be accomplished by physically moving the cabinet to a position within the data center that receives a greater supply of air from the plenum. This can be measured using a tile flow meter, as described by [76, 87, 88] or as performed by Charles Fraley on the Georgia Tech experimental Data Center lab facility, compiled in Appendix A. A combination of these reconfiguration options is explored through the following problem geometry.

## 5.1.4 System Geometry and Boundary Conditions

A fully enclosed vertical cabinet containing ten individual rack mounted servers has been selected as the example system for investigation. A two dimensional model of a cross-section of a typical cabinet is constructed as described in this section. This two dimensional model is a representative although simple model of the system dynamics because of the orientation and symmetry of the servers. It is noted that the formulation described be easily extended to three dimensions at added computational cost, as performed in Chapter 6.

## 5.1.4.1 Geometry

The complete cabinet geometry is shown below in Figure 5.3. The cabinet dimensions are height H = 1.93 m and width W = 0.87 m, which are typical for this design of server cabinet, although geometric coincidence with any specific commercial cabinet is not intended.



Figure 5.3 - Cabinet configuration & variables

The 10 individual servers modeled are 2U dual processor units, the geometry of which is shown below in Figure 5.4, where  $L_s = 0.61$  m and  $H_s = 0.09$  m. This simple

model has two iso-flux blocks in the channel, representing the processors, which also act as flow obstructions.



Figure 5.4 - Server configuration & variables

The cabinet is divided into three sections: a, b and c, corresponding to the lower two, middle three, and upper five servers as shown in Figure 5.3. The subscripts in  $Q_{a,b,c}$ denote the heat generation of each processor in the respective cabinet section. This sectioning of the cabinet was performed in order to reduce the number of design variables in the illustrative example considered but is not a limitation of the approach. The exact sectioning is based on the temperature response of the individual servers, described in Section 5.4.

# 5.1.4.2 Airflow Boundary Conditions

Air enters the server cabinet enclosure from the bottom cutout,  $L_c = 0.39$  m at uniform velocity  $V_{in}$  normal to the boundary. This air is then distributed within the cabinet and drawn through the various servers, as shown by the flow arrows in Figure 5.3 and Figure 5.4. Although internal flow patterns can be complex, a mass balance exists under steady state conditions between the air entering the cabinet and leaving through the top exhaust vent. Therefore this boundary is modeled as an outlet vent with zero (gauge) pressure.

The shaded areas in Figure 5.3 represent blank server racks where no air can flow. This alternating blank and active server placement is often implemented to enhance the cooling and reduce the power density of the server cabinet. These blank servers, and all solid surfaces in the cabinet are modeled as no-slip boundaries.

The flow through each server is provided through a 140 CFM fan, modeled by the cubic pressure–velocity relationship given in equation (5.1), where pressure is measured in Pa and velocity in m/s.

$$p(u) = 112.4 - 27.43u + 2.561u^2 - 0.1024u^3$$
(5.1)

As with the cold aisle flow investigation, the use of a fan model over a velocity specification for the internal boundary is a better representation of the server fans that have a pressure drop to flow rate relationship.

### 5.1.4.3 Thermal Boundary Conditions

The inlet air supplied to the cabinet through the bottom inlet from the under floor plenum of the data center enters the domain at temperature  $T_{in}$ . Both iso-flux blocks in all 10 servers have a constant heat generation rate Q, modeled as a surface heat flux, which is dissipated through convective heat transfer to the air flowing through the server. All other surfaces are considered adiabatic. Note that these heated blocks are referred to as "chips" for this illustrative design problem, although the two dimensional nature of the simulation means the heated blocks are the same unit depth as the entire server. The simulated power dissipation using a surface heat flux load requires lower heat generation levels to maintain realistic chip temperatures as chip level thermal management is not being considered.

## 5.1.5 System Variables

The system variables represent the flow velocities and heat generation rates within the server cabinet. Again these variables are classified as design variables, over which the designer has control, noise factors, parameters with inherent variation the designer does not have contorl over, constants, variables that are held constant, and response parameters, used to evaluate the performance of the system.

# 5.1.5.1 Design Variables

The control parameters for this investigation are:

- $T_{in}$  The air inlet temperature from the under flow plenum that enters the cabinet through the bottom inlet.
- $V_{in}$  The velocity of the air entering the cabinet through the bottom inlet
- Q<sub>a,b,c</sub> The power dissipated by each iso-flux block in the servers in sections a, b, and c of the cabinet respectively.
- $p(u)_{a,b,c}$  The fan model applied to each section of the cabinet. This model represents the use of a low, moderate, or high flow rate server fan.

All velocities are measured in meters per second, all temperatures in degrees Celsius, and power in Watts. As was discussed in Section 4.1.5, the CRAC units control the flow rate and temperature of the air supplied to the cabinet. Because the inlet velocity is the non-linear and more interesting of the two variables, *Tin* is considered a constant 15  $^{\circ}$ C for this investigation. This is acceptable because the response to variations in this parameter is linear and uncoupled from the rest of the control factors. The server fan model *p*(*u*) is kept constant as defined in equation (5.1) for all cabinet sections a, b, and c in this portion of the investigation. The effects of changing the fan model are investigated in Section 5.8.

## 5.1.5.2 Noise Factors

Sources of noise in this system come from variation in the cabinet geometry due to manufacturing tolerances, which has a negligible effect on the temperature and flow fields and hence no effect on the system response. Hence accounting for this variation is a trivial problem and not considered in this investigation.

## 5.1.5.3 Constants

The held constant parameters in this investigation are:

•  $Q_{total}$  – The total amount of power dissipated in the cabinet, defined by equation (5.2).

The amount of power dissipated by the entire cabinet is of prime importance in this investigation. However, although the distribution of the power within the cabinet in being modeled as a design variable, the total amount of power the cabinet must dissipate is not flexible. This value  $Q_{total}$  is linked to the control parameters  $Q_{a,b,c}$  by equation (5.2) below.

$$Q_{total} = 4Q_a + 6Q_b + 10Q_c \tag{5.2}$$

This relationship is derived based on the number of isoflux blocks, representing the server processors, in each of the cabinet sections a, b, and c. This parameter is held constant for two reasons. (1) In physical data center operation, in line with the primary research question, a certain amount of power must be dissipated in a set space. This means that although reconfigurable, the total amount of power to be dissipated cannot be changed. (2) Through modeling this power as a constraint, as shown in the following section, a greater amount of analysis on cabinets of differing power levels can be performed, finding an ideal configuration for each level.

# 5.1.5.4 <u>Response Parameters</u>

The response parameters for this investigation are:

*T<sub>i</sub>*, *i* = 1,...,10 – The maximum chip surface temperature of either chip in each of the 10 servers.

The response used for computing the system constraints and objective values is the maximum chip surface temperature of each server. This is computed as the maximum surface tempeature of either of the two isoflux blocks. These 10 responses are treated as individual quantities for constraint handling purposes. The sum of the chip temperatures yields a single metric of the cooling performance of the control variables, as described in the following section.

# 5.1.6 System Objective and Constraint Derivation

In any design problem the first step is to define the objectives and specifications, forming the problem goals and constraints. In this problem, the cabinet is to be configured such that it operates effectively and efficiently with minimum performance variation while using the minimum cooling air flow rate. This yields the following design objectives and specifications:

## System Design Objectives:

Minimize flow rate of cooling air supplied to cabinet by the CRAC units

$$\min(V_{in}) \tag{5.3}$$

Minimize server chip temperatures

$$\min(T_i), i = 1, ..., 10$$
 (5.4)

Minimize sensitivity of configuration to changes in cabinet operating conditions

$$\min\left\{\frac{\partial T_i}{\partial V_{in}}, \frac{\partial T_i}{\partial Q_a}, \frac{\partial T_i}{\partial Q_b}, \frac{\partial T_i}{\partial Q_c}\right\}, i = 1, ..., 10$$
(5.5)

## System Design Constraints:

• All server chips must be operate at under 85°C

$$T_i \le T_c, i = 1, ..., 10$$
 (5.6)

Total cabinet power must equal the target value

$$Q_{total} = G_{power} \tag{5.7}$$

The minimization of the inlet velocity goal is directly proportional to the volume of air supplied by the CRAC units, representing a significant portion cost of data center operation, as discussed in the previous example. The minimization of the chip temperatures is simply to find the most thermally efficient cooling parameters, as there are many combinations of design parameters that can meet the constraints. Any reliability arguments that are under scrutiny for their validity are not considered in this investigation. The minimization of the sensitivity of the cabinet to changes in all of the design variables is important because of the emphasis on stability for data center operation. Therefore, the server configuration should minimize the potential impact of one server's thermal load on the rest of the system. Through the consideration of the minimization of the chip temperature variation with respect to all system parameters, the consequences of one server overheating or the variation in cooling air supply can be greatly reduced.

A constraint is placed on the maximum server chip temperatures, no individual server chip temperature is to exceed  $T_c = 85$  °C. This value is a rule of thumb, but

adequate for this investigaiton to demonstrate the effect of robsut constraint handling. The second constraint is the total cabinet power must equal the goal  $G_{power}$ . The reasoning for this is given in Section 4.1.5.3, being the center has a speficied ammount of computing power that must be distributed as efficiently as possible. It is implemented as an equality constraint to allow better searching for specific configurations to different total power cabients. The models used to determine the maximum chip temperature as a function of the air velocity and heat fluxes are described next.

#### 5.1.7 System Systhesis Model

The control variables, noise factors, and problem constants are input into the server cabinet model, and the response of the chip temperatures monitored. These values are used to evaluate the goals and constraints in the compromise DSP, and the process iterated until convergence is achieved. This is shown schematically below in Figure 5.5.



Figure 5.5 - Server Cabinet system model diagram

The derivation of this server cabinet model, yeilding the server temperatures, is described in the following section.

## 5.2 Determining Cabinet Airflow

## 5.2.1 Generating FLUENT Observations

Before the POD model can be performed and validated, the CFD analysis of the cabinet is required to generate a series of observations. Initial estimates based on hydraulic diameter of the cabinet inlet and a velocity of 0.2 m/s indicate the Reynolds number is  $\sim 5.0 \times 10^6$ . This results in turbulent conditions in the cabinet, thus the RANS equations are used to model the flow using the standard k-E model implemented in FLUENT. The effects of buoyancy are again neglected, decoupling the energy and momentum equations. The mesh was successively refined until convergence of less than 1% solution change, yielding 21,701 grid cells and a total of 108,505 DOF. The standard FLUENT convergence criterion of a residual less than 10<sup>-3</sup> for velocity and 10<sup>-6</sup> for the energy equation was used. When reduced by an order of magnitude further, the solution did not change within the range of convergence tolerance, demonstrating iteration convergence. The resulting flow profile is shown in Figure 5.6 for the case of  $V_{in} = 0.95$ m/s. The cabinet temperature profile was found to be nearly isothermal, except for the air directly surrounding the chips, and thus the contour plot is essentially isothermal at the cabinet scale.



Figure 5.6 - Cabinet velocity contour and vectors

The CFD generated observations for a sequence of inlet velocity conditions from 0 to 2 m/s in 0.25 m/s increments are used, yielding the set  $V^o$ :

$$V^{o} = \{0, 0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0\} \text{ m/s}$$
(5.8)

These observations parameters of horizontal and vertical velocity components, u and v, as well as the turbulent effective viscosity  $\mu_{eff}$  are used to create the POD modes, as described in the next section.

### 5.2.2 Generating the POD Modes

The FLUENT observations are exported as node centered ASCII files and imported in MATLAB for further analysis. The POD of the 9 observations described above yields 2 sets of 9 POD modes, one for the reconstruction of the velocity field, the other the effective viscosity field. These observations are first mean centered, as described in Section 0, and therefore the accuracy of the reconstruction using these modes is as good if not superior to the reconstruction used in the example in Chapter 4, as discussed in Section 5.2.4. Note that the normal POD approach was used over the PODc because the PODc method was under development at the time of this investigation.

The resulting eigenvalue spectrum, and first 3 POD modes from [83] are shown below in Figure 5.7 and Figure 5.8 respectively.



Figure 5.7 - Cabinet eigenvalue spectrum [83]



Figure 5.8 - Cabinet 1<sup>st</sup> 3 POD modes [83]

Note the shallower decay of the eigenvalue spectrum in Figure 5.7 over the spectrum in Figure 4.6 because of the mean centering, thus a better decomposition of the system dynamics is possible. The reconstruction approach using these POD modes is discussed next.

#### 5.2.3 Reconstructing an Arbitrary Field

The flux matching procedure described in Section 3.1.3 is applied using the velocity POD modes. The flow is matched across the cabinet air inlet boundary to a specified goal mass flux, associated with the desired value of  $V_{in}$ . This creates a simpler problem than the procedure in Section 4.2.3 as only one goal is required to be matched, and the continuity of the POD modes ensures the top boundary has an identical mass flux out of the domain. Therefore the goal vector g has only one value,  $\dot{m}_{in}$ , and the coefficient matrix C only one row, computed as:

$$C_i = F(\vec{\varphi}_i), i = 1, ..., 9$$
 (5.9)

Where *F* determines the mass flux  $\dot{m}$  across the inlet boundary of the cabinet, applied to each of the POD modes in equation (5.9). The values of the weighting coefficients *a* are determined using the pseudo-inverse approach given in equation (3.11).

The coefficient interpolation procedure is used to reconstruct the effective viscosity field. This approach is used over the flux matching as matching a viscosity "flux" does not make sense, as well as to evaluate the effectiveness of the approach for something other than a velocity field. The weight coefficients to reconstruct the original 9 observations are shown in Table 5.1 below.

	Observation								
Mode	$V^{o}_{l}$	$V_{2}^{o}$	$V^{o}_{3}$	$V^{o}_{4}$	$V_{5}^{o}$	$V_{6}^{o}$	$V_{7}^{o}$	$V_8^{o}$	$V^{o}_{9}$
1	-0.333	-0.023	0.030	-0.276	0.152	0.177	0.100	0.101	0.073
2	-0.438	-0.143	0.036	-0.790	0.248	0.301	0.317	0.164	0.305
3	-0.066	0.239	0.240	-0.135	0.048	-0.060	-0.185	0.154	-0.235
4	0.214	0.101	-0.017	-0.170	-0.080	-0.064	0.028	-0.068	0.055
5	-0.083	0.071	0.036	0.026	-0.063	-0.050	0.001	-0.033	0.096
6	0.019	-0.039	0.009	-0.006	0.015	-0.029	-0.061	0.040	0.052
7	-0.003	0.031	-0.033	-0.001	0.021	0.037	-0.042	-0.022	0.013
8	-0.005	0.016	-0.032	0.002	0.012	-0.023	0.010	0.022	-0.001
9	0.000	0.001	-0.007	0.000	-0.023	0.014	-0.003	0.017	0.000

Table 5.1 - Cabinet observation reconstruction weight coefficients

Piecewise cubic splines interpolation is used to evaluate the weight coefficients of each of the POD modes at  $V_{in}$  values in between the set  $V^o$ . The effectiveness of both of these approaches is discussed next.

#### 5.2.4 Evaluation of the Flow Model

The full analysis of the accuracy of the cabinet flow field using the POD is documented in [83]. A summary of that evaluation, as well as the evaluation of the turbulent viscosity field is presented below. Three test cases that were not part of the original observation set are generated, with the inlet flow conditions defined by:

$$V^t = \{0.33, 0.95, 1.65\}$$
(5.10)

The POD based flow model reconstructions, denoted by the subscript r, are then compared to these test cases. The error is quantified using both the  $L_{\infty}$  and  $L_2$  norms of the error field, measuring the maximum absolute point-wise error and the average relative error respectively. The velocity magnitude error,  $vel_e$ , is computed using equation (5.11), where subscript r denotes the reconstruction and subscript t represents the test case solution.

$$vel_e = \sqrt{(u_t - u_r)^2 + (v_t - v_r)^2}$$
 (5.11)

The relative  $L_2$  norm error for the velocity is then computed using equations (5.12) and (5.13):

$$vel_t = \sqrt{u_t^2 + v_t^2}$$
 (5.12)

$$\|vel_e\|_2 = \frac{\|vel_e\|_2}{\|vel_t\|_2}$$
 (5.13)

The relative  $L_2$  norm of the effective viscosity field is computed using equation (5.14):

$$\|\mu_e\|_2 = \frac{\|\mu_r - \mu_t\|_2}{\|\mu_e\|_2}$$
(5.14)

The  $L_{\infty}$  norms are computed operating on  $vel_e$  and  $\mu_r - \mu_t$  directly. Of the two error measures, the  $L_2$  norm is the most important, as it defined how useful the reconstruction is. The  $L_{\infty}$  is used to get a measure of how bad the reconstruction is at its worst point, however this may not be representative of the rest of the reconstruction, nor may this maximum error be in an area of the flow that affects the final temperature solution in a great way. The reconstruction errors using the  $L_2$  norm for all three test cases for the velocity field using all 9 POD modes is shown below in Table 5.2.

Case	$\ vel_e\ _2$
$V_1^t$	0.39
$V_2^t$	0.10
$V_3^t$	0.41

 Table 5.2 - Cabinet velocity reconstruction error

Note the more accurate reconstruction for the test case very close to the mean center of the observation ensemble. Further analysis on this reconstruction using all available POD modeling approaches are available in [83]. The  $L_{\infty}$  and  $L_2$  norm of the effective viscosity reconstruction using the interpolation approach is shown below in Table 5.3.

Case	Method	$\ vel_e\ _2$	$\ vel_e\ _{\infty}$
$V_1^t$	Obs	0.0093	0.049
$V_1^t$	POD	0.0093	0.052
$V_2^t$	Obs	0.0077	0.047
$V_2^t$	POD	0.0073	0.047
$V_3^t$	Obs	0.0089	0.033
$V_3^t$	POD	0.0071	0.032

Table 5.3 - Cabinet effective viscosity reconstruction error

For comparison, the direct linear interpolation of the observations was also used, and the results computed in the same manner. The two approaches are labeled in Table 5.3 as "Obs" for direct observation interpolation and "POD" for weight coefficient interpolation using the POD modes. Comparison of the results show that the POD approach is more accurate, even with an observation density as great as  $V^o$ , showing that even a simple reconstruction approach is effective due to the power of the POD. In summary, the reconstruction of both the velocity and effective viscosity is very sufficient for design purposes and use in the heat transfer model described next.

## 5.3 Heat Transfer Solution

#### 5.3.1 Importing the Flow Field

The complete flow field as computed using the POD approximation is used to determine the advective heat flux across the boundary of each temperature nodes control volume, as described in Section 3.3. This requires knowledge of the flow across the center each face of the control volume, as well as the grid cell's effective thermal conductivity. The cell's effective thermal conductivity is computed as described in Section 3.3.6, using equation (5.15), where  $\mu_t$  is imported from the POD reconstruction.

$$k_{eff} = k + \frac{c_p \mu_t}{Pr_t}$$
(5.15)

The effective thermal conductivity between two cells is computed using the harmonic mean approach, as given in equation (3.28). In order to bring the FLUENT solution, which is on an irregular mesh, to a regular grid, linear interpolation is performed. The grid size used is  $\Delta x = \Delta y = 0.01$  m, yielding a grid measuring M = 193 by N = 87 yielding 16,791 nodes. This interpolation to a coarser mesh, while still numerically stable and accurate, reduces the mesh refinement around the iso-flux blocks. The ramifications of this are discussed next.

## **5.3.2** Evaluation of Temperature Profile

The stiffness matrix for solving the energy equation is assembled as described in Section 3.3, and is solved using the direct sparse matrix inversion approach. The boundary conditions applied are as described in Section 5.1.4.3. The effective thermal conductivity near the iso-flux blocks is computed using the turbulent wall functions as described in Section 3.3.6, using the appropriate parallel flow velocity from the cell above, perpendicular to the wall, as an estimate of the free stream velocity. As with the CFD solution , the computed thermal profile is nearly isothermal except around the iso-flux blocks. The temperature profile of Server 1 is shown below in Figure 5.9 with  $V_{in} = 0.5$  m/s and Q = 20 W/m.



Figure 5.9 - Temperature profile of Server 1 with  $V_{in} = 0.5$  m/s and Q = 20 W/m

With the temperature profile computed the maximum chip temperatures are extracted as the maximum nodal temperatures of the iso-flux blocks' surfaces. The maximum of these values is output as the response for each server. The total computation time for the temperature solution is  $\sim 10$  seconds on a high end desktop PC.

#### 5.3.3 Analysis Model Evaluation and Validation

The temperature model was implemented for the cabinet geometry and a heat generation rate of 60 W/m per chip and compared to the FLUENT CFD simulation of the same conditions. In order to not introduce any further error into the solution, the exact CFD flow field was input into the temperature model instead of the POD approximation. The finite difference model gives slightly more diffusive results than the CFD simulation, as a result of the power law approximation over the more accurate second order upwind approximation, that when combined with the flow re-circulation effects yielded slightly higher chip temperatures in the upper cabinet region. However, the difference in maximum chip temperatures, the absolute  $L_{\infty}$  norm, is in good agreement between models as shown below in Table 5.4.

Server	Finite Difference ( $^{o}C$ )	FLUENT (°C)	Difference (%)
1	54.6	54.3	0.2
2	40.5	40.2	0.3
3	39.8	37.3	2.5
4	37.0	36.3	0.7
5	37.1	35.9	1.3
6	36.5	35.5	1.0
7	33.9	35.4	1.5
8	37.8	34.0	3.8
9	36.9	32.7	4.1
10	41.0	35.8	5.2

Table 5.4 - Finite difference temperature model vs. FLUENT CFD results

The average difference in chip temperature is less than 2 °C, which is accurate enough for this investigation. The core reason for the disagreement in server temperatures is not rooted in the thermal model, but rather the grid interpolation. This is because depending upon the location of the node interpolated to from the FLUENT flow mesh, in the areas of recirculation directly in the iso-flux blocks wake the flow pattern is distorted and thus the convective flux is incorrectly computed, resulting in the inaccuracies shown above. There is no way to remedy this problem without an unstructured grid temperature solver, and regardless, this small degree of inaccuracy is acceptable for the needs of this investigation.

In order to determine the effect of adding the effective thermal conductivity computed from the effective turbulent viscosity over a constant molecular thermal conductivity, the same case is run keeping k constant. This yields the following results, shown in Table 5.5 below.

Server	Finite Difference ( $^{o}C$ )	$FLUENT ( \ ^{o}C)$	Difference (%)
1	61.6	54.3	7.26
2	51.0	40.2	10.77
3	45.2	37.3	7.90
4	42.5	36.3	6.14
5	42.5	35.9	6.67
6	36.1	35.5	0.63
7	35.8	35.4	0.43
8	37.0	34.0	2.92
9	36.9	32.7	4.10
10	37.0	35.8	1.19

Table 5.5 - Finite difference temperature with constant k vs. FLUENT CFD results

These results are not as bad as to be expected, with an average difference of nearly 5 °C. This result is more conservative, as is to be expected. The reason for the relatively small loss of accuracy is that the biggest changes in the thermal conductivity of the air are away from the thin boundary layers of the chips, where the molecular value dominates the solution. It should be noted that this model achieves greater accuracy in the upper 3 servers because of the combination of error factors, not because of any inherent accuracy. These results indicate that the trends identified in Chapter 4 are representative, as the model used is accurate enough for its application there.

# 5.4 Evaluating the Temperature Response

In order to obtain an estimate of the design space and parameter response the fluid and thermal models were run with a constant heat generation of 60 W/m per chip through a range of inlet velocities from 0.2 to 1 m/s. The results are plotted below in Figure 5.10.



Figure 5.10 - Cabinet chip temperatures vs. inlet air velocity

The responses are grouped by the sections of the cabinet, a, b, and c. Analysis of the responses show how these groupings came to be, as the temperatures are similar and the response trends the same. As stated before, this reduction in the number of design variables is not a limitation of the approach, rather simply done to reduce the computational time.

## 5.5 The Compromise DSP for Thermally Efficient Cabinet Configuration

## 5.5.1 Constructing the Compromise DSP

Following the mathematical formulation outlined in Section 2.3.2 and [55] the following compromise DSP for the most thermally efficient flow conditions and power loading configuration for the server cabinet is developed using the control variables,

goals, and constraints outlined in Section 5.1.5 and 5.1.6 using equations (5.3)-(5.7). The complete formulation is shown below in Table 5.6, and each section discussed in turn, equation numbers are referenced from Table 5.6 in their derivations in the subsequent sections.

## Table 5.6 - The compromise DSP for thermally efficient cabinet configuration

#### Given

- Response model of Total Cabinet Power, Inlet Air Velocity, and
- Server Temperature as functions of  $x_1, x_2, x_3, x_4$ , =  $V_{in}, Q_a, Q_b, Q_c$
- $\Delta V_{in} = 0.1 \text{ m/s}$
- $\Delta Q_{a,b,c} = f(x_i) = -0.1x_i + 22 \text{ W/m}, i = 2,3,4$  (5.16)
- Collected vector of design variability bounds,  $var = \{\Delta V_{in}, \Delta Q_a, \Delta Q_b, \Delta Q_c\}$ (5.17)
- Target for total cabinet power,  $G_{power} = 1800-2400 \text{ W/m}$
- Target for inlet velocity,  $G_{vin} = 0.1 \text{ m/s}$
- Target for total chip temperature sum and their total maximum
- possible variation  $G_{temp} = 300 \text{ °C}, \delta T_{max} = 7657 \text{ °C}$
- number of system variables, n = 4
- number of inequality constraints, p = 1
- number of equality constraints, q = 1
- number of system goals, m = 3
- number of servers, s = 10

#### Find

The values of control factors:

- $x_1$ , Inlet velocity,  $V_{in}$
- $x_2$ , Chip power for Section a,  $Q_a$
- $x_3$ , Chip power for Section b,  $Q_b$
- $x_4$ , Chip power for Section c,  $Q_c$

The values of deviation variables  $d_i^+, d_i^-, i = 1, ..., n$ 

# Satisfy

The constraints:

The individual server chip temperatures cannot exceed 85 °C

$$T_j + \sum_{i=1}^n \left| \frac{\delta T_j}{\delta x_i} \right| \cdot var_j \le 85, j = 1, \dots, s$$
(5.18)

#### Table 5.6 - The compromise DSP for thermally efficient cabinet configuration cont.

The mean total cabinet power must equal value  $G_{power}$ 

$$4x_2 + 6x_3 + 10x_4 = G_{power} \tag{5.19}$$

The goals:

Minimize inlet air velocity

$$\frac{G_{vin}}{x_1} + d_1^- - d_1^+ = 1$$
(5.20)

Bring chip temperatures to target

$$\frac{G_{temp}}{\sum_{i=1}^{s} T_i} + d_2^- - d_2^+ = 1$$
(5.21)

Minimize variation of chip temperatures

$$\frac{\sum_{j=1}^{n}\sum_{i=1}^{s} \left(\frac{\delta T_{i}}{\delta x_{j}}\right)^{2} var_{j}^{2}}{\delta T_{max}} + d_{3}^{-} - d_{3}^{+} = 0$$
(5.22)

The bounds:

$$0.2 \le x_1 \le 1 \ (\text{m/s}) \tag{5.23}$$

$$20 \le x_i \le 200, i = 2,3,4 \,(W/m)$$
 (5.24)

$$d_i^+ \bullet d_i^- = 0$$
, with  $d_i^+, d_i^- \ge 0, i = 1, ..., m$  (5.25)

# Minimize

The total objective function:

$$f = \sum_{i=1}^{m} W_i (d_i^+ + d_i^-), \text{ with } \sum_{i=1}^{m} W_i = 1, W_i \ge 0, i = 1, ..., m$$
(5.26)

#### 5.5.1.1 Given (from Table 5.6)

Using the system model shown in Figure 5.5 and the computational models developed in Sections 5.2 and 5.3, a response model of the server cabinet is developed of the form  $y = f(\vec{x})$  where y is a system response as a function of the control variables  $\vec{x}$ . This model uses the POD based flow model with input  $x_1$ , the inlet air velocity. The flow field generated is passed to the finite difference heat transfer model with inputs  $x_2$ ,  $x_3$ ,  $x_4$ , the chip heat generation rates for each cabinet section.

The variation of the control variables is determined through literature review and experience. Manufacturers' or experimental statistical data can also be used if available for more accurate representation. For this investigation, a value of  $\Delta V_{in} = 0.1$  m/s corresponds to a ±5% velocity at the upper bound of 1 m/s. The variation of  $\Delta Q_{a,b,c}$  is given by equation (5.16) to determine the heat generation variation in the different cabinet sections.

$$\Delta Q_i = -0.1Q_i + 22, \ i = a, b, c \ (W/m)$$
(5.16)

Processors that are running continually will have a fairly constant heat generation rate. To reduce the workload and hence heat generation on a processor, its computational load is staggered creating a cyclic heat generation when the processor is computing or waiting, and this cyclic process increases the variation of the heat generation rate. Equation (5.16) represents this increased variation with a simple linear function. Note that this representation is a simple approximation, and ignores the thermal mass of the processors and cabinet system. With the interval bounds representing the maximum variation of each design variable defined, they are assembled into a vector *var*:

$$var = \left\{ \Delta V_{in}, \Delta Q_a, \Delta Q_b, \Delta Q_c \right\}$$
(5.17)

Target values for the responses are determined for the minimization goals by using the lower bound of the response; as such this goal cannot be exceeded. This is 15  $^{\circ}$ C for the chip temperatures and 0.2 m/s for the inlet velocity. The chip temperature goal,  $G_{temp}$  is computed using the sum of the minimum server chip temperatures and rounding down. For goals with a target of 0, such as the chip temperature variation goal, the maximum total chip temperature variation of the system is computed using equation (5.27).

$$\delta T_{\max} = \sum_{j=1}^{n} \sum_{i=1}^{s} \left( \max_{x_j} \left( \frac{\delta T_i}{\delta x_j} \right) \right)^2 var_j^2$$
(5.27)

Where  $\delta T_{\text{max}}$  from the Given section of the compromise DSP is found applying equation (5.27), where the maximum value is found using upper bound of  $x_{2,3,4}$  and the lower bound of  $x_1$ .

#### 5.5.1.2 Find (from Table 5.6)

The design variable values, and the associated deviation from the goal associated with each design variables, as discussed in Section 2.3.2, are the parameters to be found in this investigation.

## 5.5.1.3 <u>Satisfy (from Table 5.6)</u>

The constraints given in equations (5.6) and (5.7) are used to formulate the constraint equations (5.18) and (5.19) in the compromise DSP. Equation (5.19) simply implies that that the sum of the chip powers must equal the specified value, based on equation (5.2). Because this is an equality constraint, no variability is included as the average value alone must equal this desired power. The derivation of the chip temperature inequality constraint is more complex. As discussed in the robust design constraint formulation in Section 2.2.3.2, the constraints using a deterministic model are computed using equation (2.39), shown here as equation (5.28), where *p* is the number of inequality constraints.

$$g_j(\vec{x}) + \Delta g_j \le 0, j = 1, \dots, p$$
 (5.28)

Where the worst case variation, giving the variable value most likely to violate the constraint, is computed using equation (2.40), shown here as equation (5.29):

$$\Delta g_j = \sum_{i=1}^n \left| \frac{\partial g_j}{\partial x_i} \Delta x_i \right|, j = 1, \dots, p$$
(5.29)

Where *n* is the number of design variables. Equation (5.29) is applied directly to the server chip temperatures forming equation (5.18), where *n* is the number of design variables and *s* is the number of servers.

$$T_j + \sum_{i=1}^n \left| \frac{\delta T_j}{\delta x_i} \right| \cdot var_i \le 85, j = 1, \dots, s$$
(5.18)

Here the absolute value of the variation of the server temperature response is computed for each of the design variables and added together, yielding the maximum possible temperature. This is computed for all servers to ensure this constraint is met for the entire cabinet.

The goals given in equations (5.3)-(5.5) are used to formulate the goal equations (5.20)-(5.22) in the compromise DSP. Equation (5.20) follows the compromise DSP formulation of a minimization goal, where the target is the minimum possible inlet velocity. Equation (5.21) follows the same formulation, however the response is computed using the sum of the server chip temperatures, as the minimization of this summation is equivalent to the minimization of each server individually with equal emphasis on each. The derivation of equation (5.22) is more complex, and follows the derivation of a goal with a target of 0 in the compromise DSP, as described in Section 2.3.2. Following the robust design goal formulation description in Section 2.2.3.1, the variation of the system response is computed using the equation (2.37) shown here as equation (5.30):

$$\sigma_y^2 = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i}\right)^2 \Delta x_i^2$$
(5.30)

Where n is the number of number of design variables. The temperature variation is to be minimized for all servers, accounting for variation in all design variables. Therefore the summation of the variation of the response for each server is computed, and repeated for all design variables, resulting in the double summation:

$$\frac{\sum_{j=1}^{n}\sum_{i=1}^{s} \left(\frac{\delta T_{i}}{\delta x_{j}}\right)^{2} var_{j}^{2}}{\delta T_{max}} + d_{3}^{-} - d_{3}^{+} = 0$$
(5.22)

Where n is the number of design variables and s is the number of servers. Following the formulation of absolute minimization goals for the compromise DSP, this value is divided by the maximum possible variation, as computed in the Given section of the compromise DSP in Table 5.6.

The bounds on the control factors, given in equations (5.23)-(5.25), keep the problem from diverging during the search, as well as providing simple constraints. These bounds are established by evaluating sensible limits based on the computational models and system response. In this investigation these bounds for the flow variables are given by the limits of the input variables to the POD based flow model.

#### 5.5.1.4 Minimize (from Table 5.6)

The solution to the compromise DSP is the combination of control factors that minimize the total Archimedean deviation function, equation (5.26). The priority of the multiple goals is implemented though weighting each deviation variable. Because the deviation variables are bounded by 0 and 1, as set by the goal formulation process, the sum of the weights must equal 1 in order to keep the deviation function bounded between 0 and 1 also. Tweaking of these weights can be performed to change designer preferences of one goal over another, yielding different solutions. The investigation into the use of these weightings to determine the different between optimal and least variant solution is performed in Section 5.7.

### 5.5.2 Solving the Compromise DSP

The execution of the compromise DSP is implemented using the "fmincon" function in MATLAB to implement the SQP algorithm [27], part of the MATLAB optimization toolbox [108], as was used for the previous investigation. This approach is still valid despite the more complex solution space as the SQP algorithm has been shown to be reliable even for non-linear and non-convex solution spaces [27]. A user defined gradient function is supplied to the algorithm to improve the speed of convergence, based upon the computation of the Hessian matrix:

$$H = \begin{bmatrix} \frac{\partial^2 T}{\partial x_1^2} & \frac{\partial^2 T}{\partial x_1 \partial x_2} & \frac{\partial^2 T}{\partial x_1 \partial x_3} & \frac{\partial^2 T}{\partial x_1 \partial x_4} \\ * & \frac{\partial^2 T}{\partial x_2^2} & \frac{\partial^2 T}{\partial x_2 \partial x_3} & \frac{\partial^2 T}{\partial x_2 \partial x_4} \\ * & * & \frac{\partial^2 T}{\partial x_3^2} & \frac{\partial^2 T}{\partial x_3 \partial x_4} \\ * & * & * & \frac{\partial^2 T}{\partial x_4^2} \end{bmatrix}$$
(5.31)

Where the \* terms indicate symmetry in the matrix. The computation of a Hessian matrix is useful as in order to determine the gradient of a gradient based function (such as the variability formulas in Table 5.6), the second order derivative is required, which is computed for all variables in equation (5.31). This approach is efficient as the numerical computation of gradients requires the evaluation of many points to apply the central differencing method:

$$\frac{df}{dx} = \frac{f(x - \delta x) - f(x + \delta x)}{2\delta x} + O(\delta x)^2$$
(5.32)
Evaluation of points where the flow field changes required the reconstruction of the stiffness matrix for the heat transfer solution computation, a time consuming process. Efficiency is added to the temperature solving routine described in Section 3.3 by reusing the stiffness matrix if the flow solution does not change, and simply changing the matrix elements corresponding to the updated heat flux boundary conditions. This means that only the computation of the first row of the matrix *H* requires the computation of the cabinet air flow as they have partial derivatives with respect to  $x_1$ . This more efficient computation of the gradient, over the default direct numerical computation in the fmincon function improves the speed of convergence of the algorithm by a factor of 4.

Multiple starting points were used to ensure the solutions found were not local minima. Further evaluation of the solutions found were performed following the KKT convergence criterion, however as this was thoroughly investigated for the previous problem, the full analysis is not presented again.

# 5.6 Cabinet Configuration for Increasing Heat Loads

With the server cabinet design problem specified, it is solved in two different scenarios. Each scenario has different design objectives to highlight the flexibility of the robust design method to achieve the desired results. Before these cases can be run, a baseline evaluation is performed for comparison of the more efficient configurations.

# 5.6.1 Baseline Evaluation

For a baseline case the design variables  $x_2$ - $x_3$ , the server chip heat generation rates, are lumped into a single variable. The maximum cabinet total power was found to

be just over 1600 W/m with an inlet air velocity of 0.54 m/s, constrained by the 85 °C temperature constraint for server 1. Note that the maximum allowable cabinet power was found before the design variable  $V_{in}$  reached its upper bound, indicating that because of the flow distribution within the cabinet, simply supplying more cold air from the CRAC units is not an effective solution. The more effective configurations using a power distribution profile are evaluated next.

### 5.6.2 Increasing Thermal Efficiency

In this scenario, an existing data center facility receives a batch of new high power density servers to be integrated into the existing facility. This problem translates to how to place the high power servers in the cabinet, and what volume of cooling air to supply the cabinet with in order to meet the increase in total cabinet power requirements.

To investigate this problem, the total cabinet heat generation was incremented from 1800-2400 W/m, beyond with the problem constraints could not be met. This heat load range represents the lower bound where the minimum flow rate of cooling air is required, to the maximum total cabinet power that can be sustained. For each of these incremental heat loads the most energy efficient configuration is found that simultaneously minimizes the volume of cooling air, the chip temperatures, and the variation of the chip temperatures, as established by objective equations (5.20)-(5.22). The weighting of the goals was established as the following.

$$W = \{0.5, 0.25, 0.25\} \tag{5.33}$$

This weighting puts equal emphasis on the cooling energy conservation objective and server reliability objectives. The resulting values of inlet air velocity and chip power for each cabinet section for increasing total cabinet power levels are presented in Figure 5.11 and Figure 5.12 respectively.



Figure 5.11 - Inlet air velocity vs. total cabinet power

From Figure 5.11, the volume of cooling air required to maintain reliable server operation increases in an exponential fashion. This increase is to be expected, and from this curve a general estimate of cooling costs for various heat loads could be extrapolated based on CRAC unit operating costs for the facility. At the inlet velocity of 0.54 m/s as used in the most efficient baseline case, the cabinet is dissipating nearly 2250 W/m when using a more thermally efficient power distribution, as shown below in Figure 5.13. This shows that through utilizing the airflow distribution within the server cabinet, much more

power can be reliably dissipated using the same volume of cooling air over trying to using a uniform power distribution.



Figure 5.12 - Sectional power level vs. total power level

In Figure 5.12 it is evident that as the total power level increases, the server power distribution also must change, adapting to the new flow conditions and resulting temperature fields for maximum efficiency. Analysis of this plot in conjunction with Figure 5.11 demonstrates that with increased inlet air velocity the servers in Section a are supplied with an inadequate supply of air and thus have a reduced power load. The upper level servers have their loads increased as the higher flow velocity distributes the air higher up in the cabinet, allowing for more effective cooling.



Figure 5.13 - Sectional average chip temperatures

The effects of this flow and power redistribution on server chip temperatures are shown in Figure 5.13. In this plot the maximum chip temperature from each cabinet section is plotted for increasing total cabinet power, yielding the critical values. It is interesting to note the interaction of the increased airflow from Figure 5.11 on the server temperatures. In Section c, from 2100 W/m onwards, the mean server temperature decreases although the power levels continue to climb. This is because of reduced air recirculation in the upper cabinet region.



Figure 5.14 - Maximum chip temperature and bounds

In order to check that the optimization algorithm has correctly converged, the maximum temperature constraint is presented in Figure 5.14. In this figure the maximum chip temperature from all the servers is plotted versus total cabinet power level. It is evident that the maximum chip temperature constraint is never broken, as set by the worst case scenario constraint in equation (5.18). In this manner the temperature upper bound is continually at 85 °C, not the mean value. It is also evident in this figure how this mean temperature responds as the possible temperature range changes with increasing heat loads and inlet air velocity.

# 5.6.3 FLUENT Validation

To validate the solutions of the compromise DSP, several converged cases for a range of power levels were simulated in the CFD model. It was found that the CFD

results yielded similar chip temperatures, as shown in Table 5.7. These results give strong validation of the accuracy and capability of the reduced order models.

Total Cabinet	Mean Chip Temp.
Power (W/m)	Difference ( °C)
1600	3
2100	9
2400	3

Table 5.7 – Validation of Results using CFD analysis

It is expected that the lower power cases using a lower inlet velocity will be less accurate, as the POD based flow model is most accurate at reconstructing flow fields that are close to the ensemble average, in this case 1 m/s. Thus, the 2400 W/m case is the most accurate, and the 2100 W/m case less accurate. The accuracy of the 1600 W/m case is surprisingly accurate, as it is expected to be the worst reconstruction. The only modeling explanation for this would be more power is being dissipated by the lower servers, which the thermal model creates a more accurate solution for than the upper servers.

On a higher level of validation, the power distribution of the servers found to be most efficient yields an approximate hyperbolic tangent power profile. This profile has been demonstrated to be an efficient configuration by [81]. This result is encouraging, as the investigation was computed using a very high fidelity three dimensional CFD analysis of a cabinet with close to 2 million nodes. This indicates that the trends identified in this study are indeed accurate.

# 5.6.4 Discussion of Results

These initial results of the application of robust design to server rack cabinet configuration are promising, indicating that over 50% more power than the baseline maximum power level can be reliably maintained as shown in Figure 5.15, and validated through CFD analysis. Furthermore, these configurations are insensitive to changing environmental and operating conditions, adding integral reliability to the data center system. Further analysis and quantification of this level of robustness is investigated in the following section. Furthermore, the small degree of analysis error incurred through assumptions and approximate models is nullified through the robustness of the solutions obtained.



Figure 5.15 - Baseline uniform power distribution vs. efficient power distribution

Because a range of total cabinet powers have been investigated, as shown in Figure 5.12, this redistribution of heating load is beneficial to any power server cabinet. A plot such Figure 5.12 can be developed using more accurate models of specific server cabinets under different loading to create guidelines for data center operators to configure their equipment based on airflow measurements made in their facility. This is because the linearity of the energy equation means that the server does not have to dissipate exactly the maximum found in the plot, but simply be less than the specified value.

This work presented in this section takes a step towards addressing the lifecycle mismatch problem. The energy efficient cabinet configuration approach can enable extended data center infrastructure life cycles. This in turn reduces load on all of the ancillary systems and the inputs/outputs associated with them (disposal of old refrigeration units, coolant, etc.). In addition, through maximizing the thermal efficiency, energy is conserved, considerably reducing the energy costs and environmental impact of operating a data center. Because this configuration approach is applied without significant operational disruption, it is applicable within the current lifecycle of a data center without waiting many years for a complete center level redesign. An investigation of the sensitivity of the solutions to the designer's preferences, and the differences between an optimal and least variant configuration is presented next.

#### 5.7 **Pareto Frontier Development**

The linear weighting system used in the compromise DSP gives only a rough mathematical translation of the designer's emphasis upon the goals sought in its formulation. This is because the variables are all normalized between 0 and 1, and as the sum of W is also equal to one, and thus the assignment of the fractional weights translate to the proportional emphasis placed on each goal. However, the reciprocal function used in the formulation of a minimization goal, and the sometimes very large difference between the potential value used as a goal and what is actually achievable result in nonlinear functions, meaning this linear mapping may not lead to results that represent the designers preferences well at all. Thus the *a priori* selection of numerical values that accurately represent the designer's preferences for a complex, non-linear system such as the server cabinet example is very difficult. This is very important in the tradeoff between the goals of optimal energy efficiency (the goal of minimizing the supply air rate) and the least varient solution (the minimization of curvature of the temperature response). In order to investigate the differences in robust and optimal solutions, as well as how the cabinet configuration changes between these two goals, a Pareto frontier is developed between the optimal and least variant solution.

A Pareto frontier can be thought of as a trade-off curve, plotted with two design variables as the axes. This curve represents the boundary of feasible solutions, where no point on the curve is "better" than any other point with respect to the objectives, therefore no improvements can be made to any objective without worsening the others. This curve can be traced out through changing the weights in the Archimedean objective function in the compromise DSP.

In any physical data center, the operational stability requirements will be different, and be coupled to the amount of variability in the center, and thus the designers preferences for a more robust or optimal solution will change accordingly. The plotting of a full Pareto frontier will also identify any design variables that dominate the solution obtained. This approach is of plotting a Pareto curve between the optimal and least variant solution points is investigated in [58] for simple design problems, however the focus is upon the development of this frontier for problems where a linear weighting may not identify all points along the feasible design space. Because this was not a problem for this investigation, the development of the Pareto frontier using linear weighting is described next.

# 5.7.1 Robust Versus Optimal Cabinet Configuration

In this scenario the differences between optimal and robust cabinet configurations are investigated. To accomplish this, a Pareto frontier for a constant total cabinet power,  $Q_{total} = 2300$  W/m is constructed. This frontier shows the feasible limit of each design variable as the goal changes from a fully optimal to a fully robust solution. To generate this frontier the weighting of the inlet air velocity minimization goal and minimization of the variation of chip temperatures goal are varied from 0 to 1 and 1 to 0 respectively, while the minimization of chip temperatures goal is weighted with a 0, defining *W* as:

$$W(i) = \{1 - i, 0, i\}, i = 0, 0, 1, ..., 1$$
(5.34)

The resulting Pareto frontier is plotted in Figure 5.16 for the response and all variable combinations.



Figure 5.16 - Pareto frontiers with changing weighting

The limits of the feasible design space are shown in Figure 5.16 subplots (a-c). The variation in the response is shown in subplot (y). The leftmost point corresponds to the optimal solution parameters, the rightmost to the least variant solution parameters. The line connecting the two endpoints represents design parameters for a combination of both goals, where the minimum inlet air velocity is plotted against the maximum heat generation for each server section, shown in subplots (a-c) corresponding to the cabinet section a-c. Any region to the right of this curve is feasible, but only points on the frontier represent most efficient configurations.

This plot demonstrates the differences in design parameters that would occur if the data center were highly efficient and had little variability, lending itself to a more optimal solution, or a data center that was more loosely controlled or needed a high level of reliability, requiring a more robust solution. The point of a Pareto frontier is to investigate the requirements of obtaining a more robust solution. Viewing Figure 5.16, as the priority changes from optimal to robust, the point spacing increases slightly, and showing more cooling air flow is required for only a slightly more robust solution. Subplot (y) further shows that the chip temperatures to not decrease linearly either. This means that a point somewhere in the middle of the curve, before it flattens out, represents the best balance of minimization of cooling air flow rate and temperature variation minimization. The designer, accounting for the amount of variability in the system under consideration, specifies the location of this point, yielding the final design parameters.

The Pareto frontier is good for visualizing the variation of the design parameters with each other as the objectives change, however it is difficult to see how the parameters change absolutely with changing preferences. The Pareto frontier shows a clustering of points near the robust solution, indicating that the effect of changing the linear preferences is not having a linear effect on the control parameter values found. In order to clarify what is happening, Figure 5.17 is plotted below.



Figure 5.17 - Design variable values vs. optimal to robust solution preference

In this figure the responses of the cabinet section power as well as the inlet air velocities are plotted versus the value of i from equation (5.34), which defines the weighting vector W and thus the preferences for an optimal to robust solution. Here it is evident that as the inlet velocity increases, the power distribution changes in the same manner as found in the investigation above.

What is more important than analysis of the server chip temperatures is the amount of variability in the temperature response. In order to create a measure for this value for the entire cabinet the sum of the absolute value of the slope of the temperature response with respect to the design variables is computed:

$$S_{Vin} = \sum_{i=1}^{s} \left| \frac{\delta T_i}{\delta x_1} \right|$$
(5.35)

$$S_{Q} = \sum_{j=2}^{n} \sum_{i=1}^{s} \left| \frac{\delta T_{i}}{\delta x_{j}} \right|$$
(5.36)

Where *n* is the number of design variables and *s* is the number of servers. This is divided into two functions as the units of the slopes are different. Equation (5.35) is used to compute the slope of the temperature response with respect to  $V_{in}$ , and equation (5.36) the slope with respect to the sectional chip powers  $Q_{a,b,c}$ , assuming a worst case scenario. Plotting these responses as a function of the weighting value *W* as it is changed from optimal to robust yields the following plot:



Figure 5.18 - Cabinet chip temperature variability for optimal to robust design objectives

The implications of this change in variability, and the tradeoffs made in its achievement are discussed next.

### 5.7.2 Discussion of Server Cabinet Configuration Results

Viewing Figure 5.18, computing the rough average temperature variability per Watt increase in power generation for each server is possible by dividing S by 10. The more robust solution point reduces the potential variation in chip temperatures by an average of 7 °C per m/s change in  $V_{in}$  and 0.4 °C per W/m change in Q. This means using the fairly conservative bounds of variability used in this investigation, the average variability is reduced by close to 5 °C. This means that the temperature set point of the CRAC units could be set higher, yielding substantial cost savings. Although this  $5^{\circ}$ C may seem insignificant, it is important to remember that the CRAC units can accurately control the room temperature to a single degree, and the CRAC units are operational continuously, 24 hours a day, 7 days a week, 365 days a year. Most importantly, this increased operational stability is obtained not through changing the source of the variability, but only by re-configuring the cabinet. The cost of this increased stability is a redistribution of the power load, which has no negative connotations, and an increase in the output of the CRAC units to provide the server cabinet with an increase of 0.2 m/s flow rate of supply air. An added benefit of this is the reduction of chip temperatures by 3 °C. Therefore the final tradeoffs between a robust solution, optimal solution, or anywhere in between are known to the designer. The final decision will be based upon the amount of variability in the data center, and the cost of increasing the flow rate of the CRAC units versus the cost of lowering their temperature set points. Overall, this Pareto approach gives the designer a much greater ammount of information and freedom in configuring the data center cabinets for their desired goals over a single application of the weighted sum approach.

### 5.8 Server Fan Configuration Investigation

# 5.8.1 Overview

In order to supply high powered servers with an adequate supply of cold air to cool them, high power fans are used. In the highest power density cabinets, such as the IBM blade servers, 400 CFM fans are employed to move the huge volumes of air required to maintain operational temperatures. To this end, the effect of changing the fan model in the server cabinet model is investigated. Although a more powerful server that outputs more heat will usually come with a more powerful fan, it is also possible to retrofit a new fan or add additional fans to augment the airflow. The effectiveness of this fan augmentation or upgrading approach is investigated in this section, and as such the air inlet velocity is fixed at 0.5 m/s to avoid having any influence on the results.

### 5.8.2 Discrete Variable Challenges

Variable speed server fans could be employed, as are used in many high end cabinets and servers, and was modeled in the investigation in Chapter 4. However, in this investigation three discrete fan models are used. This is done to use a different approach, and to better model the augmentation or replacement of a server fan, as variable speed fans require controllers and are significantly more expensive. The challenges associated with discrete variables are gradient based optimization algorithms, such as SQP used in the other studies in this thesis, cannot be employed. The approach used to get around this problem is discussed later.

# 5.8.3 Fan Model Description

Three different fan models, are used in this investigation, corresponding to a 140, 320, and 600 CFM fan. The 140 CFM fan was used in the previous cabinet investigation, and the two new fans provide twice and three times as the flow rate of this base fan model. This is equivalent to adding another one or two base fans in parallel, and changes the cubic fan curve by the coefficient b the number of based fans in parallel:

$$p(u) = \sum_{i=0}^{3} b^{i} C_{i} u^{i}$$
(5.37)

For example, for two base fans in parallel, equation (5.37) expands to:

$$p(u) = C_0 + 2C_1u + 2^2C_2u^2 + 2^3C_3u^3$$
(5.38)

Therefore, the equations for all three fan models are given below in equations (5.39)-(5.41) respectively.

$$p_1(u) = 112.4 - 54.856u + 10.245u^2 - 0.8192u^3$$
(5.39)

$$p_2(u) = 112.4 - 27.428u + 2.5613u^2 - 0.1024u^3$$
(5.40)

$$p_3(u) = 112.4 - 13.714u + 0.6403u^2 - 0.0128u^3$$
(5.41)

The plots of these equations, yielding the fan curves, are shown below in Figure 5.19. Fan curves are usually empirically measured and the plotted by the manufacturer to characterize the fan's performance.



Figure 5.19 - Fan model pressure vs. flow velocity

It is common for CFD programs to emulate these pressure velocity relationships using a polynomial. The cubic polynomial used gives a good approximation of a high power fan empirical fan curve, as shown for the fans modeled in Section 6.1.3. Viewing the figure above it is possible to get a feel for how much more powerful the 600 CFM fan is over the original fans used in the simulation. The high flow rate fans are usually employed in the open server cabinet design investigated in the cold aisle simulation, and will be less effective in the enclosed server design. However, their effectiveness is still likely to be significant.

# 5.8.4 Explicit Enumeration Approach

The modeling of the air flow throughout the cabinet when changing the individual server fans using the POD and PODc based modeling approaches is investigated thoroughly in [83]. However, because of the sectioning of the cabinet employed in this investigation, a complete FLUENT fluid and thermal simulation can be run for all combinations, creating a  $3^3$  factorial which totals only 27 runs, making this explicit enumeration approach to the flow modeling feasible. This flow modeling approach is possible because the energy equation is linear, and thus when the flow field does not change, the chip temperatures vary linearly with the remaining design variables,  $Q_{a,b,c}$ , and thus a linear response model can be fit for each of the 27 discrete flow cases with near 100% accuracy. This means that the consideration of reduced accuracy through the use of less accurate computationally efficient models is not necessary. The  $3^3$  full factorial design of the combination of fans models for each server section is shown below in Table 5.8.

Fan Model Used					
Section a	Section b	Section c			
1	1	1			
2	1	1			
3	1	1			
1	2	1			
2	2	1			
3	2	1			
1	3	1			
2	3	1			
3	3	1			
1	1	2			
2	1	2			
3	1	2			
1	2	2			
2	2	2			
3	2	2			
1	3	2			
2	3	2			
3	3	2			
1	1	3			
2	1	3			
3	1	3			
1	2	3			
2	2	3			
3	2	3			
1	3	3			
2	3	3			
3	3	3			

Table 5.8 - Full factorial design of server sections with all three fan models

MATLAB's scripting functionality is at the center of this approach, as the fan models are entered into FLUENT, the simulation converged using the same criteria as the previous investigation. The flow and turbulence equations are then turned off, and a high and low heat flux boundary conditions are applied to all combinations of server sections a, b, and c, creating a  $3^2$  factorial design, shown below in Table 5.9.

Chip Powers (W/m)						
Section $a, Q_a$	Section b, $Q_b$	Section $c$ , $Q_c$				
100	100	100				
200	100	100				
100	200	100				
200	200	100				
100	100	200				
200	100	200				
100	200	200				
200	200	200				

Table 5.9 - Full factorial design of server sections with two chip power levels

The computation of these 8 cases is very fast as the energy equation alone solves very quickly in FLUENT. Thus with the maximum chip temperature responses computed for the above design, a linear regression model for each server is fit using the standard least squares fit:

$$T_{i} = \beta_{i,3}Q_{c} + \beta_{i,2}Q_{b} + \beta_{i,1}Q_{a} + \beta_{i,0}, \text{ for } i = 1,...,s$$
(5.42)

This translates into the matrix problem:

$$T_i = X\beta_i + \varepsilon_i, \text{ for } i = 1, \dots, s$$
(5.43)

Where  $T_i \in \mathbb{R}^{n \times 1}$  is the vector of temperature responses for a single server,  $\beta_i \in \mathbb{R}^{m \times 1}$  is the vector of coefficients of the linear model,  $X \in \mathbb{R}^{n \times m}$  is the matrix of control variable values, given in Table 5.9, and  $\varepsilon_i \in \mathbb{R}^{n \times 1}$  the vector of errors. The coefficient matrix  $\beta_i$  is computed as:

$$\beta_i = (X'X)^{-1} X'T_i, \text{ for } i = 1,...,s$$
  
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(5.44)

With the response model coefficients computed for all 10 servers, the model can be used in the compromise DSP to determine the cabinet power distribution. A further advantage to the analytical model is the derivatives are simply the first three coefficients in  $\beta_i$  for each design variable respectively. The compromise DSP solved for each flow case is described next.

# 5.8.5 The Compromise DSP for Thermally Efficient Cabinet Fan Configuration

Formulating the compromise DSP for this problem is very similar to the last formulation for the configuration of the cabinet. However, in this instance the only objective is to maximize the total cabinet power,  $Q_{total}$ , and the constraint is that all chip temperatures must be less than 85 °C. This single objective forms the three goals of maximizing the three chip power control variables, formulated using the compromise DSP goal maximization formulation given in Section 2.3.2. Note that as  $V_{in}$  is fixed, it is no longer a control variable. How the evaluation of the robustness is factored into this problem is discussed in the merit function definition, in the next section. The complete formulation is shown below in Table 5.10, and each section discussed in turn.

#### Table 5.10 - The compromise DSP for thermally efficient cabinet configuration

#### Given

- Individual response models of Total Cabinet Power, and Server Temperature as functions of  $x_1, x_2, x_3 = Q_a, Q_b, Q_c$
- $\Delta Q_{a,b,c} = f(x_i) = -0.04x_i + 5 \text{ W/m}, i = 1,2,3$
- $V_{in} = 0.5 \text{ m/s}$
- Collected vector of design variability bounds,  $var = \{\Delta Q_a, \Delta Q_b, \Delta Q_c\}$
- Target for cabinet section power,  $G_{power} = 2000 \text{ W/m}$
- number of system variables, n = 3
- number of inequality constraints, p = 1
- number of system goals, m = 3

 Table 5.10 - The compromise DSP for thermally efficient cabinet configuration cont.

## Find

The values of control factors:  $x_1$ , Chip power for Section a,  $Q_a$   $x_2$ , Chip power for Section b,  $Q_b$   $x_3$ , Chip power for Section c,  $Q_c$ The values of deviation variables  $d_i^+, d_i^-, i = 1, ..., n$ 

# Satisfy

The constraints:

The individual server chip temperatures cannot exceed 85  $^{\rm o}{\rm C}$ 

$$T_j + \sum_{i=1}^n \left| \frac{\delta T_j}{\delta x_i} \right| \cdot var_j \le 85, j = 1, \dots, s$$
(5.45)

The goals:

Maximize the total cabinet power dissipation

$$\frac{4x_2}{G_{power}} + d_1^- - d_1^+ = 1$$
(5.46)

$$\frac{6x_3}{G_{power}} + d_2^- - d_2^+ = 1 \tag{5.47}$$

$$\frac{10x_4}{G_{power}} + d_3^- - d_3^+ = 1$$
(5.48)

The bounds:

$$10 \le x_i \le 200, i = 1,2,3 \, (W/m)$$
 (5.49)

• 
$$d_i^+ \cdot d_i^- = 0$$
, with  $d_i^+, d_i^- \ge 0, i = 1, ..., m$  (5.50)

# Minimize

Minimize the total objective function:

$$f = \sum_{i=1}^{m} W_i(d_i^+ + d_i^-), \text{ with } \sum_{i=1}^{m} W_i = 1, W_i \ge 0, i = 1, ..., m$$
(5.51)

The above compromise DSP is solved to find the cabinet power profiles of all 27 fan configurations. The function used to evaluate and rank these 27 cases is described next.

## 5.8.5.1 <u>Given (from Table 5.10)</u>

The problem givens are identical to the problem presented in Table 5.6, with the removal of the air inlet velocity,  $V_{in}$ , as a control variable and instead fixed at a constant 0.5 m/s. The amount of variability of the chip powers,  $\Delta Q_{a,b,c}$ , is also reduced to the function:

$$\Delta Q_{a,b,c} = -0.04 x_{1,2,3} + 5 \quad (W/m) \tag{5.52}$$

This function means the variability will vary from a maximum of 5 W/m at the lower bound of the heat flux to 1 W/m at the upper bound. This change in the variability of the heat output of the chips is made because of the lower inlet velocity, and hence lower chip powers.

### 5.8.5.2 Find (from Table 5.10)

The design variable values, and the associated deviation from the goal associated with each design variables, as discussed in Section 2.3.2, are the parameters to be found in this investigation.

### 5.8.5.3 Subject To (from Table 5.10)

The same constraints as the problem presented in Table 5.6 are employed in the solution of this compromise DSP.

### 5.8.5.4 Objective (from Table 5.10)

The objective function of this compromise DSP is simply to maximize the heat output of all of the chips, subject to the system constraints.

# 5.8.6 Merit Function Definition

In order to compare the 27 different fan combination the following merit function is defined in equation (5.53).

$$Merit = \frac{\sum_{i=1}^{s} (Q_{chip,i} - Q_{fan,i})}{\sum_{i=1}^{s} (T_i - T_{in})}$$
(5.53)

Where *s* is the number of servers, equal to 10. This equation initially looks similar to the inverse of the thermal resistance of the cabinet, however in the numerator the power used to drive the fans is subtracted from the input heat flux, in order to penalize the configurations using the more powerful fans, that in turn require more power to operate. The values of  $Q_{fan}$  are given below in Table 5.11. In order to use this merit function, a unit depth is considered to change the units of chip power from W/m to W.

Table 5.11 - Fan model power consumption values

Fan Model	$Q_{fan}(W)$
1	16
2	32
3	48

These values are derived from the experimental server cabinet investigated in Chapter 6. The servers use four 150 CFM fans, rated to use 4 W of power each [62]. Extrapolation to the other three models yields the values in the table above.

This merit function represents the objectives of maximizing the cooling efficiency using the minimal amount of cooling energy. As  $V_{in}$  is fixed, the energy consumed by the CRAC units is constant for all configurations, and this does not enter into the merit function. The use of this merit function is important as the compromise DSP objective is simply to maximize the total cabinet power, and thus tradeoffs in the energy required to by the different fan configurations is not considered.

The robustness of the configurations is measured by the merit function indirectly through the consideration of the problem constraints. Because the only control variables considered in the application of the compromise DSP are the server powers, which are linear, the temperature variability of each server is constant for each fan configuration case. Therefore addition into the objective function is redundant. However, the change in variability is considered in the constraint function, and thus the less variability, the closer the control variable can be placed to the constraint, and the higher the chip heat output. Therefore the robustness of each solution is included in the consideration, but the addition of a separate function would be redundant as both the flux minimization and robustness function would not be linearly independent. The evaluation of the different configurations using this merit function is performed next.

### **5.8.7** Finding the Most Efficient Configurations

The results of the application of the merit function to all 27 configurations, rank ordered, and displayed in Table 5.12. This table shows the merit function value, the fan model used for each cabinet section, the total amount of power consumed by all the cabinet fans, the chip powers for each cabinet section, and the total cabinet power dissipation. The top 7 performing configurations are highlighted, as well as the baseline configuration.

Analysis of this table shows which fan configurations are most efficient. In general, the enhanced cooling capacity of the more powerful fans outweighs the higher power requirements, however the top merit rank is not the case with all of the most powerful fan. This shows the merit function does trade off between absolute cooling capability and power requirements. Also of interest is the baseline configuration, which ranks only slightly below the middle of the pack. Because the merit function is only an interpretation of the designer's preferences, and not an absolute metric, these rankings are subjective, and thus discretion should be used in the evaluation of all of the results presented in the table. Considering the robustness of the solutions the maximum and mean cabinet chip temperature variations are also tabulated above. Taking both values into account show the top 7 configurations also have among the lowest variations, showing the merit function does credit insensitive configurations. Therefore, these top configurations are not only energy efficient, but some of the least varient configurations available.

Fan model Total (		Total Cabinat	t Chip power for		Cabinet	Max	Mean			
Merit	for	cabi	inet	Fan Powar	cabinet section		Power	Temperature	Temperatur	
Value Section		n	- (W)		(W/m)		(W/m)	Variation	e Variation	
	а	b	c	(")	а	b	с		(°C)	(°C)
1.311	1	1	3	20	35.86	40.89	40.82	796.96	6.33	2.04
1.366	1	3	3	26	35.93	47.55	48.42	913.23	6.27	1.64
1.372	1	2	3	23	36.21	44.93	45.35	867.88	6.22	1.78
1.427	2	1	3	22	37.44	45.39	45.27	874.77	5.96	1.77
1.452	3	1	3	24	38.62	47.85	47.68	918.38	5.71	1.64
1.464	1	3	2	21	36.60	45.16	45.32	870.59	6.13	1.78
1.468	1	1	2	15	35.68	39.61	39.35	773.87	6.38	2.14
1.497	1	2	2	18	36.43	43.04	43.01	834.07	6.19	1.91
1.519	1	3	1	16	36.60	41.76	41.39	810.86	6.17	2.00
1.554	2	2	3	25	37.22	50.96	51.41	968.76	5.97	1.50
1.560	2	1	2	17	37.41	43.63	43.22	843.66	5.97	1.88
1.569	3	1	2	19	38.73	45.74	45.24	881.75	5.70	1.76
1.580	1	2	1	13	36.13	40.22	39.82	783.99	6.27	2.10
1.593	1	1	1	10	35.13	37.66	37.30	739.39	6.50	2.30
1.605	2	3	3	28	36.28	54.90	55.85	1033.00	6.18	1.35
1.615	2	3	1	18	39.02	45.87	45.23	883.61	5.65	1.76
1.619	3	2	3	27	37.93	54.41	54.79	1026.00	5.82	1.37
1.623	3	3	1	20	40.61	47.99	47.19	922.27	5.35	1.65
1.625	2	2	2	20	38.42	47.86	47.68	917.64	5.75	1.64
1.629	2	3	2	23	38.42	50.82	50.82	966.78	5.74	1.51
1.641	3	1	1	14	38.07	42.66	41.90	827.21	5.86	1.95
1.651	2	1	1	12	36.87	40.92	40.28	795.80	6.11	2.06
1.662	2	2	1	15	38.22	44.05	43.40	851.16	5.81	1.86
1.663	3	2	1	17	39.74	46.00	45.21	887.07	5.51	1.75
1.665	3	2	2	22	39.68	50.71	50.41	967.04	5.50	1.52
1.671	3	3	3	30	35.79	58.51	59.37	1087.90	6.24	1.22
1.678	3	3	2	25	39.52	53.95	53.78	1019.60	5.52	1.39

Table 5.12 - Results of server cabinet fan configurations

# 5.8.8 Discussion of Server Fan Configuration Results

Analysis of the top 7 configurations shows a common trend of increasing the airflow to the bottom servers in the cabinet. This makes sense, as these servers have already been identified as prone to overheating from lack of airflow. All of these configurations place higher power fans in the lower portion of the cabinet, with little or moderate flow enhancement in the rest of the cabinet. In fact, simply upgrading the bottom server fans to the fan model 2, a configuration requiring the lowest fan power apart from the baseline case, results in the 6<sup>th</sup> highest ranked configuration. What is more interesting is the resulting cabinet power distribution for those cases where only the bottom section of the cabinet has the fans upgraded. These are shown in Figure 5.20.



Figure 5.20 - Cabient power distribution with upgraded fans in Section a

With increased flow to the bottom servers there is increased power distribution to the lower cabinet, but even more power can be dissipated in the rest of the cabinet as well. This is an unexpected result that simply shows that the overall flow throughout the cabinet is improved through better flow through the lower two servers.

Disregarding the merit function temporarily and ranking the configurations purely based on total cabinet power dissipation, plotted against total cabinet fan power creates the distribution shown below in Figure 5.21.



Figure 5.21 - Total cabinet power vs. total cabinet fan power

The general tend shown in this plot is that with increased server airflow comes increased power dissipation. However, the coincidence of data points on the x axis with different power dissipations shown how some configurations using the same fan powers

are more thermally efficient than others. Looking at the highest point, it is possible to dissipate an addition 350 W/m of power through increasing the server fans flow rates. The trend in increased fan power and flow rates with total cabinet power dissipation is fairly linear, and starts to tail off as the fans get very powerful. This means there is a point of diminishing returns where simply increasing the volume of air flowing through the server is no longer effective. The tradeoff on a data center level between the server fan power and CRAC unit power will depend upon the number of servers being fed from a single CRAC unit, and hence is not performed here. However, in general, upgrading the fans in the lower portion of the server cabinet to a higher flow rate than the rest of the cabinet, regardless of the server flow rates, appears to be beneficial for this type of server cabinet design.

### 5.9 Chapter Synopsis and Validation Summary

In this chapter the second example problem is investigated, the energy efficient and thermally robust configuration of a vertical flow server cabinet was presented. In this study the feasibility and effectiveness of the application of robust design was investigated through variations in the amount of cooling air supplied, the heat load distribution within the cabinet, and the interchanging of the individual server fans. The quadrants of the validation square that have been addressed in this chapter are presented below. How the validation performed in this chapter falls within the complete validation roadmap can be determined from viewing Table 1.3.

# Empirical Structural Validity

- In this study the accuracy of the turbulent heat transfer model is demonstrated through comparison with FLUENT simulations in Section 5.3.3.
- In this study the accuracy of the combined POD flow model and heat transfer model is demonstrated through comparison with FLUENT simulations in Section 5.2.4 and 5.6.3.

# Empirical Performance Validity

- The 2U cabinet model geometry used, although two dimensional, is representative of a very popular and commonly implemented cabinet architecture, as shown in Section 5.1.4.1.
- The flow and heat transfer parameters used, as well as the goals used in the compromise DSP formulation are representative of physical data center server cabinet configuration problems, as shown in Sections 5.1.4.2, 5.1.4.3, and 5.1.5.
- The maximum heat dissipation is found to be a function of both the supply rate of cooling air, cabinet server power dissipation profile, and server fan models used, indicating valid design variables were chosen, as shown in Sections 5.6.2, 5.8.7, and 5.8.8.

• The effect of changing the weighting for a more robust or optimal solution was found to have significant effects on the amount of chip temperature variation with minimal tradeoffs in energy efficiency, indicating the validity of using the minimization of temperature variation goal, as shown in Section 5.7.

With the implementation and results of the 2U server cabinet presented, the third and most complex case study of the vertical flow experimental mock blade server cabinet is presented. This investigation constitutes the core validation work, through comparison of the 3D POD flow model and solution with the FLUNET CFD results and experimental measurements.

# **CHAPTER 6**

# **BLADE SERVER CABINET INVESTIGATION AND VALIDATION**

This core purpose of the analysis presented in this chapter is to strengthen the validity of this approach. The applicability of all three constructs to a very complex three dimensional cabinet model is demonstrated, and similar results and conclusions to the study in Chapter 5 are found. Furthermore, the experimental cabinet is introduced, and its role in this investigation as a validation tool is also presented. In Section 0 the study is introduced, the motivation for the work, and the problem geometry and boundary conditions. The cabinet airflow is determined, using the complimentary POD in Section 6.2. The heat transfer solution computed using FLUENT is discussed in Section 6.3. The compromise DSP for the thermally efficient and thermally robust blade cabinet configuration is derived and solved in Section 6.4, including the development of a full Pareto Frontier for all three objectives. The experimental mock blade cabinet is described in Section 6.5, including the data acquisition system and process, and the development and comparison with the CFD simulation. The chapter synopsis and validation summary is presented in Section 6.6.

How this chapter falls into the overall structure of the thesis and validation square is presented in Figure 6.1. This chapter builds upon the the steps of the approach developed and presented in Chapter 3 and thier application in Chapters 4 and 5 through their application to a highly complex and representative example. Furthermore, the sensitivity of the system and tradeoffs between the optimal and most invariant solutions are explicitly investigated. Finally, the performance of the CFD models, upon which all other analysis models are built, is empirically validated. This in further addresses the empirical performance validity of the approach, its capability to produce effective results in a very strong and thorough manner. The role of this study as it pertains to the overall thesis motivation and validation approach is discussed in the following section.


Figure 6.1 - Thesis and validation roadmap: Chapter 6

## 6.1 Study Introduction

## 6.1.1 Motivation for this study

This chapter provides the application of the POD approach and robust design to the most detailed server cabinet simulation performed for this thesis. Furthermore, the model developed is based upon a physical cabinet mock-up, enabling the comparison of the results with experimentally gathered results, providing validation of the models and approach. In summary, the example problem presented in this chapter provides:

- Application of the POD to a complex 3D model The complexity of fluid flow modeling increases significantly when moving from two to three dimensions. The analysis of the 3D server cabinet system is the largest, most complex, 3D flow simulation using the POD approach to date. Its effectiveness for this type of modeling is demonstrated in this chapter.
- Analyze application of robust design to a physical cabinet In the previous investigations the variability of the design variables was derived from manufacturers data and logical analysis. Because the system modeled in this chapter is physical, statistical data can be obtained to measure there variances, and hence the true effectiveness of robust design on an actual server cabinet can be determined.
- Validation of FLUENT and POD modeling Comparison of the server chip temperatures with measured experimental results enable the identifications of the RANS based CFD model's capability to accurately

simulate the system. Furthermore, this grounding in physical measurement enables the comparison of the CFD simulations, experimental results, and the POD reconstructions.

The emphasis in this study is upon the development of a complex, 3D server cabinet system, obtaining the experimental results, and validation of the models. The applicability of robust design is also investigated, with respect to the same objectives as the study in Chapter 5. However the focus in this chapter is upon the model and method, as the results of this design problem are already thoroughly investigated in Chapter 5. This is essentially a similar investigation, with a much more complex and accurate model, using experimentally determined variation values and validation.

#### 6.1.2 **Problem Solution Process Organization**

How this cold aisle study as presented in this thesis ties into the steps of the robust server cabinet design approach, as given in Section 3.5, is shown below.

Step: Sections:

- (5) 6.1.1 6.1.5
- (6) 6.2-6.4
- (7) 6.1.5, 6.5.1
- (8) 6.5.2-6.7

This list in conjunction with the material presented in this chapter gives a good representation of what performing the cabinet design approach entails. This list provides the same information as Figure 4.2 in a more succinct format.

#### 6.1.3 System Geometry and Boundary Conditions

The system studied in this investigation is the experimental mock blade server cabinet, originally fabricated by Ben Hodgkinson. This experiment is used as a test bed in this thesis for the validation of the CFD modeling approach, and the POD reduced order model based upon the CFD generated observations. This server cabinet is cooled using vertically oriented air flow distributed to seven servers, each server containing 10 blade units. Alternating server spaces are filled with blank racks to block the airflow.

#### 6.1.3.1 Geometry

The cabinet schematic is shown below in Figure 6.2. The complete cabinet measures 0.6 m wide by 0.8 m deep by 2 m tall shown in the x, y, and z coordinates respectively. The flow inlet, shown as the red outline in Figure 6.2, measures 0.355 m by 0.325 m. The airflow entering from this inlet comes from the under floor plenum of the data center, as the cabinet has feet to stand on top of a removed floor tile and thus has unobstructed flow access to the CRAC supply air through the plenum. The lowest server rack is 20 mm above this inlet vent. The flow through the cabinet is provided by a 550 CFM exhaust fan on top of the cabinet, shown by the blue outline in Figure 6.2. The fan measures 0.3 m in diameter, with a 92 mm inner shroud. For modeling purposes, this is implemented as a rectangular boundary measuring 0.23 m by 0.28 m, which yields the same effective area as the physical fan.



Figure 6.2 - Experimental mock blade server cabinet schematic

Flow motivation through the server racks is provided by four 20 CFM fans, each measuring 80 mm by 80 mm. These are shown by the green outlines in Figure 6.2. The servers are numbered one through sever, with the lowest server being Server 1. The details of the server rack and blade geometries are shown below in Figure 6.3.



Figure 6.3 - Experimental bock blade server rack schematic

Each server rack measures 0.44 m wide by 0.72 m deep by 0.132 m tall, shown by the x, y, and z coordinates respectively. The green outlines again represent the server rack fans. The blade servers are represented by the channels formed using large pieces of FR4 board with a foil heater in the center on one side simulating the chip as dividers. The chip measures 32 by 32 mm, and is shown by the red outlines in Figure 6.3. The FR4 board is an epoxy-copper laminate, measuring 360 mm long by 132 mm by 1.6 mm thick, shown in the y, z, and x coordinates respectively in Figure 6.3. The FR4 contains only one copper layer on its surface, measuring 1 oz (0.0355 mm) thick. The foil heater is placed on this copper surface, facing in the positive x direction in Figure 6.3. Thus the ten mock blade server units are numbered starting as labeled, from the second channel, proceeding in the positive x direction.

## 6.1.3.2 Airflow Boundary Conditions

The inlet vent boundary, as shown in Figure 6.2, is modeled as a velocity inlet boundary condition. Air enters at a velocity  $V_{in}$  normal to the boundary. In order to determine the value appropriate for comparison with the physical cabinet, a flow hood was used to measure the mass flow of air exiting the cabinet, and thus the appropriate velocity could be measured. The flow hood computes backpressure compensation, and thus accounts for any additional flow resistance added. The resulting airflow throughout the cabinet is similar to the 2D 2U server modeled in chapter 5, as shown below in Figure 6.4.



Figure 6.4 - Experimental cabinet airflow schematic

The inlet k and  $\varepsilon$  values are determined using the turbulent intensity and hydraulic diameter computation in FLUENT. This approach makes is easier to estimate

realistic turbulence parameters than arbitrary selection of k and  $\varepsilon$  values. The hydraulic diameter is computed using equation (6.1).

$$D_h = \frac{4A_c}{P} \tag{6.1}$$

Where  $A_c$  is the cross-sectional area of the inlet cutout, and P is its perimeter. A turbulence intensity value of 5% is employed, indicating moderate turbulent conditions, estimated from the Reynolds number computation.

The cabinet exhaust fan is modeled using a cubic pressure-velocity relationship given in equation (6.2), where pressure is measured in Pa and velocity in m/s.

$$P(u) = -0.0828u^3 + 1.8112u^2 - 16.738u + 89.348$$
(6.2)

This relationship is determined from the manufacturer's data [62]. The comparison of the manufacturer's provided fan curve and the cubic interpolation, performed by standard regression techniques, is shown below in Figure 6.5





Figure 6.5 - Exhaust fan curve and polynomial approximation

The fit is quite good, demonstrated by the computed  $R^2$  value of 0.9907. This statistic is a measure of how well the curve fits the data, in this case indicating that 99% of the variation in the pressure is accounted for by the cubic approximation. More information of the computation of the  $R^2$  value is available in [32]. The same cubic interpolation of the manufacturer's data is applied to model the server rack fans [62], resulting in the relationship given in equation (6.3), where pressure is measured in Pa and velocity in m/s.

$$p(u) = -0.0027u^3 + 0.0836u^2 - 2.2709u + 25.604$$
(6.3)

The comparison of the manufacturer's provided fan curve and the cubic interpolation is shown below in Figure 6.6.

Server Fan Performance



Figure 6.6 - Server fan curve and polynomial interpolation

Again the fit is quite good, demonstrated by the computed  $R^2$  value of 0.9886. The accuracy of these fan models is important as they provide the pressure to velocity relationship that determines the airflow patterns and distribution within the cabinet. If these models are not accurate, the resulting temperature and flow profiles will also be different from the physical cabinet. All walls and solid regions in the cabinet, including the blank server sections, are modeled as non-slip boundary conditions.

## 6.1.3.3 Thermal Boundary Conditions

The inlet air supplied to the cabinet through the bottom inlet from the under floor plenum of the data center enters the domain at temperature  $T_{in}$ . All simulated chips have a heat generation rate Q, modeled as a surface heat flux, which is dissipated through convective heat transfer to the air flowing through the blade server. All other surfaces are considered adiabatic.

Note that this simulated power dissipation using a surface heat flux load requires lower heat generation levels to maintain realistic chip temperatures as chip level thermal management is not being considered. However, the heat spreading effect through the FR4 requires special consideration, as is modeled using an anisotropic thermal conductivity and the shell conduction model in FLUENT. The anisotropic thermal conductivity of the FR4 board is computed across the plane and into the plane using equations (6.4) and (6.5) respectively.

$$k_{\parallel} = k_{cu} \left( \frac{\Delta Z_{cu}}{\Delta Z_{cu} + \Delta Z_{epoxy}} \right) + k_{epoxy} \left( \frac{\Delta Z_{epoxy}}{\Delta Z_{cu} + \Delta Z_{epoxy}} \right)$$
(6.4)

$$k_{\perp} = \frac{1}{\left(\frac{1}{k_{cu}}\left(\frac{\Delta Z_{cu}}{\Delta Z_{cu} + \Delta Z_{epoxy}}\right) + \frac{1}{k_{epoxy}}\left(\frac{\Delta Z_{epoxy}}{\Delta Z_{cu} + \Delta Z_{epoxy}}\right)\right)}$$
(6.5)

Where  $k_{cu} = 400$  W/mK,  $k_{epoxy} = 0.4$  W/mK,  $\Delta Z_{cu} = 0.0355$  mm, corresponding to 1 oz, measured with a micrometer, and  $\Delta Z_{epoxy} = 1.6$  mm, measured with calipers. This yields values of through and cross thermal conductivities of  $k_{\perp} = 0.2044$  W/mK and  $k_{\parallel} =$ 9.0737 W/mK. Thus the through plane conduction is assumed to be negligible, as the thermal resistance is several order of magnitudes smaller than the convective flux, as determined further in the investigation.

## 6.1.4 System Variables

The system variables represent the flow velocities and heat generation rates within the server cabinet. Again these variables are classified as design variables, over which the designer has control, noise factors, parameters with inherent variation the designer does not have control over, constants, variables that are held constant, and response parameters, used to evaluate the performance of the system. These variables are very similar to those used in the investigation in Chapter 5.

## 6.1.4.1 Design variables

The control parameters for this investigation are:

- $T_{in}$  The air inlet temperature from the under flow plenum that enters the cabinet through the bottom inlet.
- $V_{in}$  The velocity of the air entering the cabinet through the bottom inlet
- $Q_{i}$ , i = 1, ..., 7 The power dissipated by the chips of each blade module in each server rack of the cabinet.

All velocities are measured in meters per second, all temperatures in degrees Celsius, and power in Watts. In this investigation, the CRAC units control the flow rate and temperature of the air supplied to the cabinet, however the flow rate is augmented by the exhaust fan pulling air through the cabinet. Because the inlet velocity is the non-linear and more interesting of the two variables,  $T_{in}$  is considered a constant 26.85 °C for this investigation. This is acceptable because the response to variations in this parameter

is linear and uncoupled from the rest of the control factors. The rack server fan model p(u) is kept constant as defined in equation (6.3) throughout this investigation.

## 6.1.4.2 Noise factors

Sources of noise in this system come from variation in the cabinet geometry due to manufacturing tolerances, which has a negligible effect on the temperature and flow fields and hence no effect on the system response. Hence accounting for this variation is a trivial problem and not considered in this investigation.

#### 6.1.4.3 Constants

The parameters held constant in this investigation are:

•  $Q_{total}$  – The total amount of power dissipated in the cabinet, defined by equation (6.6).

The amount of power dissipated by the entire cabinet is of prime importance in this investigation. However, although the distribution of the power within the cabinet in being modeled as a design variable, the total amount of power the cabinet must dissipate is not flexible. This value  $Q_{total}$  is linked to the control parameters  $Q_i$  by equation (6.6) below.

$$Q_{total} = \sum_{i=1}^{7} 10 \cdot Q_i \tag{6.6}$$

This relationship is derived based on the number of foil heaters, representing the server processors in each blade unit, in each of the cabinet rack sections. The reasoning for this parameter being held constant is the same as given in Section 5.1.5.3.

## 6.1.4.4 <u>Response parameters</u>

The response parameters for this investigation are:

•  $T_i$ , i = 1,...,7 – The maximum chip surface temperature of any blade module processor in each of the 7 servers.

The response used for computing the system constraints and objective values is the maximum chip surface temperature of each server. This is computed as the maximum surface tempeature of any of the 10 blade module processors, as shown in equation (6.7).

$$T_i = \max_j (T_{i,j}), i = 1,...,s, j = 1,...,b$$
 (6.7)

Where *s* the number of server racks equals 7, and *b* the bumber of blade modules equals 10. These 7 responses are treated as individual quantities for constraint handling purposes. The sum of the chip temperatures yields a single metric of the cooling performance of the control variables, as described in the following section.

## 6.1.5 System Goals and Constraints

In any design problem the first step is to define the objectives and specifications, forming the problem goals and constraints. In this problem, the cabinet is to be configured such that it operates effectively and efficiently with minimum performance variation while using the minimum cooling air flow rate, in an identical manner to the investigation in Chapter 5. This yields the following design objectives and specifications:

# System Design Goals:

Minimize flow rate of cooling air supplied to cabinet by the CRAC units

$$\min(V_{in}) \tag{6.8}$$

Minimize server chip temperatures

$$\min(T_i), i = 1, ..., 7$$
 (6.9)

Minimize sensitivity of configuration to changes in cabinet operating conditions

$$\min\left\{\frac{\partial T_i}{\partial V_{in}}, \frac{\partial T_i}{\partial Q_1}, \frac{\partial T_i}{\partial Q_2}, \dots, \frac{\partial T_i}{\partial Q_7}\right\}, i = 1, \dots, 7$$
(6.10)

System Design Constraints:

• All server chips must be under 85°C

$$T_i \le T_c, i = 1, ..., 7$$
 (6.11)

• Total cabinet power must equal the target value

$$Q_{total} = G_{power} \tag{6.12}$$

The reasoning for these goals and constraints is identical to those given in Section 5.1.6.

## 6.1.6 System Systhesis Model

The control variables, noise factors, and problem constants are input into the server cabinet model, and the response of the chip tempeatures monitored. These values are used to evaluate the goals and constraints in the compromise DSP, and the process iterated until convergence is achieved. This is shown schematically below in Figure 6.7.



Figure 6.7 - Server Cabinet system model diagram

The derivation of this server cabinet model, yeilding the server temperatures, is described in the following section.

## 6.2 Determining Cabinet Airflow

## 6.2.1 Generating FLUENT Observations

Again before the POD model can be performed and validated, the CFD analysis of the cabinet is required to generate a series of observations. This model is also used to compare the chip temperature responses with the experimental mock cabinet, discussed in Section 6.5.7. Initial estimates of the Reynolds number in the blade modules, computed below, indicate the flow is turbulent, and hence the flow is modeled using the standard k- $\epsilon$  model implemented in FLUENT. The effects of buoyancy are again neglected, decoupling the energy and momentum equations. The final mesh contained 626,143 nodes, and 526,062 cells, requiring around 12 hours to converge from an initialized state, computed using a top of the line desktop workstation (a 4 GHz Pentium 4 processor with 2 GB of RAM).

The CFD generated observations for a sequence of inlet velocity conditions from 0.25 to 2 m/s in 0.25 m/s increments are used, yielding the set  $V^{\circ}$ :

$$V^{o} = \{0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0\} \text{ m/s}$$
(6.13)

These observations parameters of the velocity components, u, v, and w, as well as the turbulent kinetic energy, k, and turbulent dissipation,  $\varepsilon$  are used to create the POD modes, as described in Section 6.2.4.

## 6.2.2 Mesh Generation

The quality of the mesh is of utmost importance in a large, detailed, complex simulation such as this. The mesh needs to be refined in areas of sharp gradients, yet coarse enough to keep the problem feasible with the finite computational resources available. Although adaptive procedures are employed, a high quality initial mesh is also needed as too many adaptations can create convergence errors. Furthermore, this adaptive procedure employs a hanging node procedure of subdividing cells, which does not work with shell conduction zones. As shell conduction is employed to simulate the FR4 boards, the mesh of the blade modules must be made correctly initially without any need for further adaptation. Thus the mesh is built up in sections, starting with a blade module, moving to a server rack section, and then assembling the complete cabinet. This process is described below.

The computation of the Reynolds number for the possible flow rates encountered within a channel formed by the blade module is computed using equation (6.14):

$$Re_{x} = \frac{\rho u_{\infty} x}{\mu} \tag{6.14}$$

Where x is the position along the channel, the fluid viscosity,  $\mu = 1.789e-5$ , the free stream velocity  $u_{\infty} \in [0.25, 2]$  m/s based on the velocity input set  $V^{\circ}$ , and the fluid density  $\rho = 1.225$  kg/m<sup>3</sup> [39].

$$\delta = 0.37 x R e_x^{-\frac{1}{5}} \tag{6.15}$$

Using equation (6.14) the minimum Reynolds number was computed as 4.93e4, indicating the turbulent flow modeling approach is valid. Equation (6.15) applied to the range of  $u_{\infty}$  values indicate a boundary layer from 1.9 mm to 15.3 mm thick. The width of each channel created by the FR4 to simulate a blade module is 40 mm. Based on these boundary layer estimates, an initial mesh was constructed for the blade modules and refined until the solution no longer changed. This mesh is shown below in Figure 6.8.



Figure 6.8 - Blade module meshes

With the convergence of the blade module mesh, the complete server rack mesh is created as shown in Figure 6.9.



Figure 6.9 - Server rack mesh

The server geometry is slightly simplified, all geometry is based on a unit length scale, in the case of this model, the width of the blade module. This means the 1.6 mm width of the FR4 boards is modeled as an infinitely thin conduction plane. This simplification enables the creation of an almost perfectly orthogonal mesh, resulting in very high element quality, with all elements having an equi-axial skew less than 0.01. A perfectly orthogonal mesh could not be created without changing the placement of the chips, which would result in a significant compromise in model accuracy. The effects of these modeling simplifications are explored in Section 6.5.7 through comparison with the experimental data.

## 6.2.3 Mesh and Iteration Convergence

With the completion of a good first mesh, the solution was converged for the maximum inlet velocity of 2 m/s. Initially the power law is implemented as the discretization scheme for all transport phenomena to quickly converge to the solution area, then the under-relaxation factors are reduced and the second order upwind approach is applied to converge to the final solution. The SIMPLEC procedure is used to couple the pressure and velocity fields, with the pressure field discretized using the PRESTO! scheme. The final under-relaxation factors used are given in Table 6.1.

Transport Phenomena	Under-relaxation Factor
Pressure	0.3
Momentum	0.3
Turbulent Kinetic Energy, k	0.3
Turbulent Dissipation, $\varepsilon$	0.5
Turbulent Viscosity, $\mu_t$	0.5
Energy	0.7

Table 6.1 - Under-relaxation factors used in final simulation

The adaptive mesh procedure is employed because the traditional approach of decreasing the grid size by a factor of 2 will result in 8 times the number of nodes for this problem, hence making the model unsolvable. This approach has been widely implemented in large complex CFD simulations, such as [81]. The mesh adaptation is performed on the velocity curvature, identifying the regions of sharpest gradients and increasing the resolution using the hanging-node approach. The maximum system values of velocity, temperature, and turbulence parameters are recorded to check the convergence of the mesh. Each adaptation is performed to around 5% of the total number

of grid cells, as a larger value than this can cause errors. The results are shown below in Table 6.2.

Step	Grid Cells	V <sub>max</sub>	% Change	T <sub>max</sub>	% Change	k <sub>max</sub>	%Change	E <sub>max</sub>	% Change
0	455252	7.535	-	301.98	-	5.527	-	901.2	-
1	480676	8.2049	8.89%	302.08	0.03%	5.492	-0.63%	902.418	0.14%
2	503531	8.1887	-0.20%	302.085	0.00%	5.493	0.02%	900.9723	-0.16%
3	528549	8.1811	-0.09%	302.0875	0.00%	5.4896	-0.06%	903.65	0.30%

Table 6.2 - Mesh adaptation of full 3D cabinet

As the changes in values between steps 2 and 3 are less than a single percent, the mesh is considered converged after 2 adaptations, showing the quality of the original mesh.

With the mesh convergence complete, the final mesh was converged again using iteration converge parameters an order of magnitude lower than the default values of 1e-3 for continuity, velocity, and turbulence parameters, and 1e-6 for energy. The solution was found not to change, indicating the default values adequate for iteration convergence. Finally, the continuity of mass and energy are tested throughout the domain, and the quantities entering and leaving the system were measured. All values are found to be within the tolerances of convergence used.

#### 6.2.4 Generating the POD Modes

The FLUENT observations are exported as cell centered ASCII files and imported into MATLAB for further analysis. The cell centering is required as the FLUENT interpolation file is required in order to import the POD reconstructed flow solution back into FLUENT to solve for the temperature field. The PODc of the 8 observations yields 3 sets of 9 POD modes, one set for the velocity, k, and  $\varepsilon$  fields. The velocity observation ensemble  $\vec{U}$  is the concatenation of the u, v, and w velocities, as the flow is three dimensional. The PODc procedure is described in Section 0. Note that the case of  $V_{in} =$ 0 m/s is excluded from the observation set  $V^o$  as very low velocity flow conditions are not of interest, and thus its inclusion is deemed unnecessary. The resulting eigenvalue spectrum is shown below in Figure 6.10.



Figure 6.10 - Eigenvalue spectrum of U for 3D cabinet model

Note the sharp decay in the eigenvalue spectrum shown in Figure 6.10, similar to the decay in Figure 5.7 for the 2D cabinet model. This indicates that increasing the dimensionality of the flow from two to three dimensions does not have significant implications regarding the system's decomposition into the POD basis.

## 6.2.5 Reconstructing an Arbitrary Flow Field

The flux matching procedure described in Section 3.1.3 is applied using the velocity, k, and  $\varepsilon$  POD modes. The flow is matched across the cabinet air inlet boundary to the corresponding specified goal flux, associated with the desired value of  $V_{in}$ . In order to determine the k and  $\varepsilon$  goals to match at the inlet, the flux function  $F(\vec{\varphi}_i)$  is applied to the observations, and the resulting relationships between  $V_{in}$  and the turbulence parameters are established, shown below in Figure 6.11 and Figure 6.12.



Figure 6.11 - Inlet velocity vs. inlet turbulent dissipation value



Figure 6.12 - Inlet velocity vs inlet turbulent kinetic energy value

The smooth relationships exist because of the boundary conditions applied to the inlet, which is proportional to the inlet velocity. Interpolation of these plots with the desired  $V_{in}$  value is used to determine the goal k and  $\varepsilon$  values. As in the 2D cabinet problem the goal vector g has only one value, and the coefficient matrix C only one row,

$$C_i = F(\vec{\varphi}_i), i = 1, ..., 9$$
 (6.16)

Where *F* determines the flux across the inlet boundary of the cabinet, applied to each of the POD modes. Three coefficient matrices are determined,  $C_{Vin}$ ,  $C_k$ , and  $C_{\varepsilon}$  for the computation of the flux matching procedure for the velocity, turbulent kinetic energy, and turbulent dissipation fields respectively. This application of the flux matching procedure over the coefficient interpolation procedure is simpler to implement, and made possible because of the smooth relationship between  $V_{in}$  and k and  $\varepsilon$ .

#### 6.2.6 Evaluation of the Flow Model

The accuracy of the PODc reconstruction is evaluated through the reconstruction of the 8 observations using the flux matching procedure to match the inlet velocities associated with each observation. The relative L2 norm for the velocity field is computed using equations (5.11)-(5.13), and the results are plotted below in Figure 6.13.



Figure 6.13 - 3D Cabinet PODc velocity reconstruction error

The accuracy of the PODc reconstructions is impressive, with less than 6.5% average error, computed using the  $L_2$  norm ratio, across the entire velocity range, and less than 5% error in reconstructions above 1 m/s. This slight bias incurred in the accuracy of

the reconstructions is inherent to the way the POD modes are computed, as the higher energy observations are preferentially detected by the decomposition. However, the complimentary subspace approach used in the PODc goes a long way to improve upon this, as the initial reconstruction error using the normal POD was over 30% in the lower velocity region. The reconstruction of the *k* and  $\varepsilon$  fields are found to follow a similar curve, but with slightly greater error of an  $L_2$  norm under 10% and 15% respectively. Because these values are less important in the reconstruction of the temperature solution, and the regions of high gradients are far away from the thermal areas of interest, the PODc reconstruction approach performs very well for this problem, computing all solutions in under 5 seconds.

## 6.3 Heat Transfer Solution

#### 6.3.1 Importing the Flow and Turbulence Field

The complete velocity, k and  $\varepsilon$  fields as computed by the PODc are imported back into FLUENT as a cell centered interpolation file. FLUENT is then used to solve the energy equation only, and the other parameters are kept constant. This allows convergence in only a few fast iterations, taking around 10 seconds. The most time consuming part of this approach is writing and reading the ASCII interpolation files, which takes around 120 seconds. However, the efficiency of the FLUNET solver makes this a superior approach to solving this problem directly in MATLAB with a custom energy equation solver. Furthermore, if even greater accuracy is required, the imported PODc solution can be used as an initial guess for the complete solver, allowing convergence of the complete CFD solution in a much shorter time than without such a good initialization.

# 6.3.2 Evaluation of the Heat Transfer Model

As the same energy equation solver used to create the observations is used to create the reconstructed solutions, the heat transfer model does not need to be investigated for accuracy. The only error induced in the reconstructions is because of the PODc approach, and hence only the complete reconstructions are considered for accuracy validation in Section 6.4.4. The only issue with the use of the FLUENT energy equation solver is a slight increase in the number of iterations required to converge because of the slight errors on the PODc reconstructions used in the solution.

As the measured velocity of the experimental cabinet is above 1 m/s, and the accuracy of the PODc is greatest from 1-2 m/s, the inlet velocity range is truncated from 1 m/s to 2 m/s. The resulting values of maximum server rack chips temperatures with a heat generation of 2W per chip, computed using equation (6.7), are shown below in Figure 6.14.



Figure 6.14 - Server chip temperature responses

The server temperature response is non-linear, with different servers responses forming convex or concave profiles. This requires some special considerations when solving for the minimum objective function in the compromise DSP, as discussed in the next section.

## 6.4 The Compromise DSP for Thermally Efficient Blade Server Configuration

## 6.4.1 Constructing the Compromise DSP

Following the mathematical formulation outlined in Section 2.3.2 and [55] the following compromise DSP for the most thermally efficient flow conditions and power loading configuration for the server cabinet is developed using the control variables, goals, and constraints outlined in Section 6.1.4 and 6.1.5 using equations (6.8)-(6.12). The complete formulation is shown below in Table 6.3, and each section discussed in 290

turn, equation numbers are referenced from Table 6.3 in their derivations in the subsequent sections.

Table 6.3 - The compromise DSP for thermally efficient blade cabinet configuration

#### Given

- Response model of Total Cabinet Power, Inlet Air Velocity, and
- Server Temperature as functions of  $x_1, x_2, ..., x_8$ , =  $V_{in}, Q_1, ..., Q_7$
- $\Delta V_{in} = 0.188 \text{ m/s}$  (6.17)
- $\Delta Q_i = 0.05 \text{ W}, i = 1, ..., 7$  (6.18)
- Collected vector of design variability bounds,  $var = \{\Delta V_{in}, Q_1, ..., \Delta Q_7\}$ (6.19)
- Target for total cabinet power,  $G_{power} = 600-655 \text{ W}$
- Target for inlet velocity,  $G_{vin} = 1 \text{ m/s}$
- Target for total chip temperature sum and their total maximum possible variation  $G_{temp} = 200 \text{ }^{\circ}\text{C}, \delta T_{max} = 320 \text{ }^{\circ}\text{C}$  (6.20)
- number of system variables, n = 8
- number of inequality constraints, p = 1
- number of equality constraints, q = 1
- number of system goals, m = 3
- number of servers, s = 7
- number of blade modules, b = 10

## Find

The values of control factors:  $x_i$ , Inlet velocity,  $V_{in}$   $x_{i+1}$ , Chip power for server rack i,  $Q_i$ , i = 1,...,7The values of deviation variables  $d_i^+, d_i^-, i = 1,...,n$ 

## Satisfy

The constraints:

The individual server chip temperatures cannot exceed 85 °C

$$T_j + \sum_{i=1}^n \left| \frac{\delta T_j}{\delta x_i} \right| \cdot var_j \le 85, j = 1, \dots, s$$
(6.21)

The mean total cabinet power must equal value  $G_{power}$ 

$$\sum_{i=1}^{s} 10 \cdot Q_i = G_{power} \tag{6.22}$$

# Table 6.3 - The compromise DSP for thermally efficient blade cabinet configuration cont. The goals:

Minimize inlet air velocity

$$\frac{G_{vin}}{x_1} + d_1^- - d_1^+ = 1$$
(6.23)

Bring chip temperatures to target

$$\frac{G_{temp}}{\sum_{i=1}^{s} T_i} + d_2^- - d_2^+ = 1$$
(6.24)

Minimize variation of chip temperatures

$$\frac{\sum_{j=1}^{n}\sum_{i=1}^{s} \left(\frac{\delta T_{i}}{\delta x_{j}}\right)^{2} var_{j}^{2}}{\delta T_{max}} + d_{3}^{-} - d_{3}^{+} = 0$$
(6.25)

The bounds:

$$1 \le x_1 \le 2$$
 (m/s) (6.26)

$$1 \le x_i \le 20, i = 2,...,8$$
 (W) (6.27)

• 
$$d_i^+ \cdot d_i^- = 0$$
, with  $d_i^+, d_i^- \ge 0, i = 1, ..., m$  (6.28)

#### Minimize

The total objective function:

$$f = \sum_{i=1}^{m} W_i(d_i^+ + d_i^-), \text{ with } \sum_{i=1}^{m} W_i = 1, W_i \ge 0, i = 1, ..., m$$
(6.29)

Because of the similarity of this compromise DSP and the compromise DSP solved in Chapter 5, only the different sections and derivations are discussed below. The complete reasoning and derivations of Table 6.3 can be found in Section 5.5.1.

## 6.4.1.1 <u>Given (from Table 6.3)</u>

Using the system model shown in Figure 6.7 and the computational models developed in Sections 6.2 and 6.3, a response model of the server cabinet is developed of the form  $y = f(\vec{x})$  where y is a system response as a function of the control variables  $\vec{x}$ . This model uses the POD based flow model with input  $x_1$ , the inlet air velocity. The flow field generated is passed to FLUENT as the heat transfer model solver inputs  $x_2,...,x_8$ , the chip heat generation rates for each cabinet rack section.

The variation of the control variables in this problem is determined through experimental measurement. The inlet velocity variation as given in equation (6.17) is computed using three standard deviations of the inlet velocity measurements, shown in equation (6.30).

$$\Delta V_{in} = 3\sigma_{Vin} \tag{6.30}$$

The use of three times the standard deviation value of 0.188 results in an upper bound of 99.74% probability that the inlet velocity deviation will not exceed this value, based on the assumption of a normal distribution given in equation (6.31).

$$P(\mu - c\sigma \le X \le \mu + c\sigma) \ge \Phi(c) - \Phi(-c), \text{ where } c = 3$$
(6.31)

Where  $\mu$  is the mean value,  $\sigma$  is the standard deviation, X is the random variable, in this case the inlet velocity, and  $\Phi$  is the Standard Normal Distribution:

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-x^{2}/2} dy, \text{ for } x \in [-\infty, \infty]$$
(6.32)

This test of a normal distribution fit to the measured cabinet flow rate data is shown below in Figure 6.15.



Figure 6.15 - Normal probability plot of measured cabinet flow data

The fit in Figure 6.15 is acceptable, indicating the distribution is normal. Even assuming a worst case scenario, the use of equation (6.30) to determine the bounds results in a lower bound of 88.89% probability the inlet velocity will not exceed this value, given any distribution of the velocity deviation, computed using the Chebychev Inequality [32]:

$$P(\mu - c\sigma \le X \le \mu + c\sigma) \ge 1 - \frac{1}{c^2}$$
, for  $c = 3$  (6.33)

Thus the value of  $\Delta V_{in}$  given in equation (6.17). The value of  $\Delta Q_i$  given in equation (6.18) is based on the manufacturers data of the power supply [5], and the

recordings of the change in thermal resistance of the foil heaters during the experimental measurements.

Again, with the interval bounds representing the maximum variation of each design variable defined, they are assembled into a vector *var*:

$$var = \left\{ \Delta V_{in}, Q_1, \dots, \Delta Q_7 \right\}$$
(6.19)

Target values for the responses are determined for the minimization goals by using the lower bound of the response; as such this goal cannot be exceeded. The maximum chip temperature deviation,  $\delta T_{\text{max}}$ , given in equation (6.20) is computed using equation (6.34).

$$\delta T_{\max} = \sum_{j=1}^{n} \sum_{i=1}^{s} \left( \max_{x_j} \left( \frac{\delta T_i}{\delta x_j} \right) \right)^2 var_j^2$$
(6.34)

Where the maximum values of the deviation in the response from each control variable is used, finding the upper bound in variability.

## 6.4.1.2 <u>Find (from Table 6.3)</u>

The design variable values, and the associated deviation from the goal associated with each design variables, as discussed in Section 6.1.4, are the parameters to be found in this investigation.

## 6.4.1.3 Satisfy (from Table 6.3)

The constraints and goals given in equations (6.21)-(6.28) are identical in formulation with the compromise DSP given in Table 5.6, and thus are not repeated here.

## 6.4.1.4 Minimize (from Table 6.3)

The solution to the compromise DSP is the combination of control factors that minimize the total Archimedean deviation function, equation (6.29). The priority of the multiple goals is implemented though weighting each deviation variable. Because the deviation variables are bounded by 0 and 1, as set by the goal formulation process, the sum of the weights must equal 1 in order to keep the deviation function bounded between 0 and 1 also. Tweaking of these weights can be performed to change designer preferences of one goal over another, yielding different solutions. The investigation into the use of these weightings to determine the different between optimal and robust solution is performed in Section 6.4.3.

# 6.4.2 Solving the cDSP and Finding the Maximum Cabinet Power Dissipation

The nonlinearity of the system response and existence of local minima means additional considerations must be made in solving this cDSP. For this problem, Monte-Carlo techniques are integrated with the previously employed SQP method implemented in finding the minimum of the total objective function. 15 random starting points are used, as well as 2 points at the upper and lower limit of  $x_1$ , and the lower limits of  $x_{2-8}$ . If the solution is not found after 500 iterations to a tolerance of 1e-8 for both objective and constraint convergence, a total deviation value of 1 is assigned, as in equation (6.35).

$$f_{i} = \begin{cases} \sum_{i=1}^{m} W_{i}(d_{i}^{+} + d_{i}^{-}), \text{ if } g(\vec{x}) \le 0 \text{ and } h(\vec{x}) = 0\\ 1, \text{ if } g(\vec{x}) > 0 \text{ or } h(\vec{x}) \ne 0 \end{cases}$$
(6.35)

Therefore, the final value of the total deviation function is given by:

$$Z = \min(f_i) \tag{6.36}$$

In order to determine the maximum reliable power dissipation the cabinet can dissipate, the compromise DSP given in Table 6.3 is solved recursively until the constraints cannot be met, solving the problem:

$$\min f(\vec{x}, \vec{W}), \text{ s.t. } f(\vec{x}) < 1$$
 (6.37)

For this scenario, the weighting vector W is defined to provide approximately equal weighting between the energy conservation and reliability goals:

$$\tilde{W} = \{0.5, 0.25, 0.25\}$$
 (6.38)

The search is conducted using interval bisection, and a final maximum cabinet power,  $Q_{total} = 654.6$  W is found. The search is not completed to more than a single decimal point as fractions of a Watt are insignificant in this problem. The resulting cabinet configuration uses an inlet velocity of 1.209 m/s, and server power distribution as given below in Table 6.4.

Table 6.4 - Power levels of maximum cabinet power dissipation

Chip Power (W)								
$Q_1$	Q2	Q3	Q4	Q5	Q6	Q7		
10.62	8.39	9.42	9.23	9.06	8.69	9.58		
The power distribution is rather discontinuous because of the airflow distribution within the cabinet, as the maximum server temperature is found to be in the middle blades for the lower servers, and the edge blades for the upper servers. This airflow distribution is studied in more detail in Section 6.5.9. It is also interesting that this maximum dissipation occurs at a lower inlet velocity than the maximum. This again is a result of the complex interaction of airflow with the position of the blade modules. This result cannot be generalized to blade cabinet architecture in general, however, it does serve to show the importance of characterization of data center server cabinets for efficient cooling. Without this knowledge, it is likely this cabinet would be operating outside its most efficient parameters, as the trend in data center equipment that is overheating is to simply supply more cooling air to the cabinet as a whole, not the specific areas in need of cooling. This flow can compound the cooling difficulties in other sections of the cabinet, as found in this mock blade cabinet architecture.

## 6.4.3 Complete Pareto Analysis

In the previous 2D cabinet investigation different cabinet power distributions and the required inlet air velocity are found for increasing power loads in the cabinet, as well as variation of the fan models in order to maximize the potential cabinet power dissipation. This problem has been solved in the previous section under the ideal conditions to find the maximum possible cabinet heat dissipation. This analysis is not completed again, as it would be very similar results but with a different system model.

Instead, in this section a full Pareto Analysis is performed. Using a total cabinet power dissipation of 650W, a full investigation of how the inlet velocity and cabinet

power distribution change with variation of the designer's preferences towards all three goals of inlet velocity minimization, chip temperature minimization, and chip temperature variation minimization. Consequently, this analysis also determines how the power distribution within the cabinet changes for realistic values of inlet velocity. This approach is more appropriate than simply sweeping through all possible inlet velocities and determining the ideal power distribution as it is rooted in the designer's preferences.



Figure 6.16 - Triangular design space of designers preferences

In order to effectively visualize and display the tradeoffs between the three goals, a triangular design space is constructed, as shown in Figure 6.16 (a). In this plot, the three corners represent the full weighting of a specific goal. The lines perpendicular to the edges represent a linear decline from a weight of 1 for that goal, to a weight of 0 at the opposite edge. Therefore, the key points of interest are the intersections of all three lines in the center, where the weighting of each goal is 1/3, and the intersection of the perpendicular lines with the triangle edges, where the weighting of the two goals along that edge are each 1/2. This weighting scheme is similar to the shape functions used in Finite Element Analysis, this weighting structure is represented graphically in three dimensions in Figure 6.16 (b) for the weighting of the  $V_{in}$  goal. Here the function is 299

highest at the corresponding vertex, and decreases linearly to 0 towards the opposite edge.

The use of this triangular structure performs a similar function to the Pareto frontier developed in 5.7; however it allows the visualization of the tradeoffs between three goals simultaneously. The cDSP is solved for all possible tradeoffs within this design preference triangle. The evaluation points are computed using a full factorial design, the results of which are sorted for the sum of the rows being equal to 1. An example of these evaluation points using 4 levels of preferences for each variable, and thus the resulting value of the weighting vector W is shown below in Table 6.5.

Evaluation Point	$W_1$	$W_2$	$W_3$
1	1	0	0
2	0.75	0.25	0
3	0.5	0.5	0
4	0.25	0.75	0
5	0	1	0
6	0.75	0	0.25
7	0.5	0.25	0.25
8	0.25	0.5	0.25
9	0	0.75	0.25
10	0.5	0	0.5
11	0.25	0.25	0.5
12	0	0.5	0.5
13	0.25	0	0.75
14	0	0.25	0.75
15	0	0	1

Table 6.5 - Preference weighting evaluation points for 4 level investigation

In the previous investigation, only the tradeoffs between the minimization of the inlet velocity and temperature variation goals were investigated. Those goals best represent the optimal and least varient solutions respectively, however the influence of

the minimization of the chip temperature goal is also of interest and its effects on the solution obtained investigated for completeness. The value of inlet velocity with respect to the weighting of all three goals, as described above, is shown below in Figure 6.17.



Figure 6.17 - Inlet velocitiy vs. designer preferences

In this figure it is evident that the dominant tradeoff is between the minimization of the inlet velocity and the temperature variation goals, representing the optimal and least varient solutions respectively. This creates the response shape that transitions across the center of the triangle. The tradeoffs between chip temperature and inlet velocity, and chip temperature and temperature variability do not have an effect on the inlet velocity.

The results of these tradeoffs along the edges of the triangle is shown more clearly for the resulting cabinet power distributions in Figure 6.18, Figure 6.19, and Figure 6.20.



Figure 6.18 - Cabinet power distribution for inlet velocity to temperature variation minimization

goals



Figure 6.19 - Cabinet power distribution for inlet velocity to temperature minimization goals



Figure 6.20 - Cabinet power distribution for temperature to temperature variation minimization goals

Again, the effects of weighting the minimization of chip temperature goal are uninteresting. This is to be expected, as with full weighting of this goal, the values of the chip power generations will simply be their lower bound. It is interesting to note how this goal is dominated by the other two. In practice, this goal is only implemented in the cDSP to find the solution among multiple feasible solutions that has the lower temperature, as it is the more energy efficient solution, and hence is the lowest priority goal.

Viewing Figure 6.18 specifically, it is interesting to see how the cabinet power distribution changes in response to the increased inlet velocity and preferences for a more stable solution over an optimal one. Lastly, it is possible to see how closely the designers goals can be achieved by plotting the value of the total objective function, f, along the

tradeoff between the inlet velocity and temperature variation minimization goals. This is shown below in Figure 6.21.



Figure 6.21 - Total objective function value vs. inlet velocity to temperature minimization goal

This shows that the minimization of the inlet velocity goal can be more closely matched than the temperature variation minimization goal. This plot is purely of academic interest to the designer, as it shows how the placement of their targets can potentially affect the outcome of the tradeoff between multiple goals in the cDSP. However, it does not have significant implications for the configuration of data center server cabinets. The physical and practical implications of this tradeoff are discussed in the previous investigation in Section 5.7.2.

The effectiveness of a fully robust versus an optimal solution in the reduction of the variation of the temperature of the chips is computed in a similar manner as the previous chapter. This change in weighting corresponds to the line between the minimization of inlet velocity and temperature variation goals on the triangle in Figure 6.16. In order to create a measure for this value for the entire cabinet the sum of the absolute value of the slope of the temperature response with respect to the design variables is computed:

$$S_{Vin} = \sum_{i=1}^{s} \left| \frac{\delta T_i}{\delta x_1} \right|$$
(6.39)

$$S_{Q} = \sum_{j=2}^{n} \sum_{i=1}^{s} \left| \frac{\delta T_{i}}{\delta x_{j}} \right|$$
(6.40)

Where n is the number of design variables and s is the number of servers. This is divided into two functions as the units of the slopes are different, as in the previous investigation. Plotting these responses as a function of the weighting value W as it is changed from optimal to robust yields the following plot:



Figure 6.22 -Reduction in temperature variability from optimal to robust solutions

The total amount of variability in this cabinet is less than in the previous investigation, primarily because more conservative bounds of the control variable variation are used for the power dissipation. The maximum computed temperature variation is 7.5 °C for the optimal solution, and 4.7 °C for the least varient solution, leading to a reduction of nearly 5 °C. This is still significant, and the power dissipation of 650 W is very close to the maximum potential cabinet dissipation, this these results are not exaggerated by an under constrained system operating point. Using the larger variation bounds of the chip heat flux of 20 W, the reduction in temperature variation goes from 15.1 °C to 6.2 °C, a 9 °C difference. These results indicate that through redistribution of the power load in the cabinet, the temperature variation of the chips can be significantly reduced, even at close to the maximum potential power dissipation of the cabinet.

### 6.4.4 CFD Validation

In order to validate the solutions found using the combination of the PODc flow model, and FLUENT temperature solver, the converged most efficient case as given in Section 6.4.2 and

Table 6.4. The approximate model, FLUENT, and temperature differences are given below in Table 6.6.

<i>Temperature (°C)</i>											
Server	Approximate Model	FLUENT	Difference								
1	84.7	85.3	-0.7								
2	82.5	81.7	1.0								
3	84.5	85.2	-0.8								
4	84.4	85.6	-1.4								
5	84.4	86.3	-2.3								
6	81.9	81.5	0.5								
7	83.1	81.5	1.9								

Table 6.6 - Comparison of FLUENT CFD and approximate model cabinet server temperatures

The temperatures are not exactly equal to the constrained values because of the robust handling of the constraints. The differences found are very small, which is to be expected given the accuracy of the PODc flow model, as the same temperature solver is used for both models. However, even with this small amount of inaccuracy, some of the server temperatures are found to be above the constraint value in the CFD solution. This indicates that use of a more conservative estimate of the power variation, to cover this inaccuracy, is in order if this were a physical data center cabinet. This is because very small values of  $\Delta Q$  are used in this investigation. With the rooting of the accuracy of this approximate model in the CFD simulations complete, the accuracy of the CFD

simulations are investigated, to tie the entire approach to physical results in order to demonstrate its accuracy and feasibility.

## 6.5 The Experimental Cabinet

## 6.5.1 Experimental Characterization

The experimental mock blade server cabinet, as originally constructed by Ben Hodgkinson, required significant work in order to be used to effectively acquire results that would be compared with the CFD simulations. The major challenges involved rewiring the system for more reliable and accurate temperature measurement, the routing of all wires to minimize flow interference, set-up and use of a new data acquisition system, and complete wiring of an interchangeable blade module system to enable the quick acquisition of temperature data from the complete cabinet. The overall cabinet wiring is described in Section 6.5.3.

# 6.5.2 Cabinet Description

The physical geometry and dimensions of the experimental mock blade server cabinet are described in Section 6.1.3.1. A chassis from a standard 42 U cabinet design with moveable rails for rack mounted servers forms the primary cabinet structure. The four sides of the cabinet have been replaced with Plexiglas sheets for flow visualization. Photographs of the experimental cabinet are shown from the front, front right, and back in Figure 6.23 (a), (b), and (c) respectively.



Figure 6.23 - Experimental mock blade server cabinet photos

The cabinet is set up with seven 3 U blade server enclosures; the remaining six enclosures are blank units. The server units are constructed of 0.5" thick Lexan sheets. The blade server enclosures contain grooves for the FR4 boards to slide into from the front, and four Papst 8830N fans at the back of the enclosure provide the flow. The top of the server units are left open, and the top surface provided by the bottom of the unit above it. Therefore, a single Lexan top cover is provided for the uppermost server. These servers are screwed into the rail mounts at the front and back of the cabinet, as can be seen in Figure 6.23. This design allows varied configurations of the server units, however in this thesis only the staggered blank-mock server configuration is considered.

The primary flow through the cabinet is provided by a Caravel CLE2T2 AC tube axial fan mounted above the top panel of the cabinet. This modified placement is more

representative of the fan placement used in industrial data centers, as well as providing a superior flow exit path.

The chips are simulated by Minco 10  $\Omega$  foil heaters, which are adhesive backed for easy and secure attachment to the copper side of the FR4 boards, ensuring a good thermal conductance path. The thermal resistance of the adhesive layer is assumed to be negligible for all experimental and simulation work. An Agilent 6644A 200 W DC power supply is used to power the heaters. Because of the requirements of simultaneously powering and acquiring data from 10 blades per server, for all seven servers is prohibitive, only one server rack can be heated at a time. This created a challenge in the DAQ and power system wiring such that the powered server rack could be reconfigured with the minimum time interval for efficient data collection.

### 6.5.3 Data Acquisition System and Wiring

In this section the design of the data acquisition system (DAQ) and modifications made to the cabinet to accommodate the system are described. The temperature measurements are made using a National Instruments Field Point thermocouple modules (NI FP-TC-120) modules connected to an Ethernet base module for computer connectivity (NI FP-1601). Each thermocouple module has eight inputs for direct measurement of temperature from standard thermocouple types. With signal conditioning, double-insulated isolation, input noise filtering, and a high-accuracy deltasigma 16-bit analog-to-digital converter, the module delivers reliable, accurate temperature or millivolt measurements. An onboard microcontroller compensates and linearizes thermocouple readings to the NIST-90 standard, using an advanced linearization routine for maximum accuracy, and automatic scaling to engineering units. Each module comes with a NIST-traceable calibration certificate ensuring accurate, reliable measurements. The modularity of the system easily enables as many thermocouple modules to be added to the base unit as required.

The high quality, insulated, Omega Engineering type T copper-constantan thermocouples are used for all temperature measurements. The thermocouple wire used is 36 gauge throughout the cabinet. This fine gauge wire is routed through the cabinet along paths that minimize the flow interference, where terminal strips are used to connect to thicker, braided 18 gauge wire with superior cladding for greater noise resistance. The terminal strips allow for easy connection and replacement of the fine thermocouple wire, which is fragile but used because of its low flow interference properties. An example of this wire routing along an FR4 board is shown below in Figure 6.24.



Figure 6.24 - Wire routing on FR4 board for mock blade module

The thermocouple tips are epoxied to the surface of the foil heaters on the FR4, as can be seen in Figure 6.24. The thermocouple wire and fine gauge heater wire is attached flush to the surface of the FR4 with high temperature low profile Kapton tape. The thermocouples used to monitor the air flow temperatures are placed mid flow in the middle of the server racks at blade module channels 2 and 9, and at the middle of the outer two server fans. This use of two thermocouples per server enables an accurate measurement in spite of any potential temperature gradients or thermocouple failures. Thermocouples are also placed in the center of the inlet flow vent, and the top exhaust fan. Therefore, a total of 50 thermocouples are used, placed throughout the cabinet as shown in Figure 6.25.



Figure 6.25 - Thermocouple point temperature measurement locations

In order to facilitate the easy switching of the heated server section, the thermocouple and heater power wires are bundled and routed out the front of the server,

along the bottom edge so as to minimize the flow interference. Detachable clips are used to route the wiring in this fashion as the powered FR4 boards are moved up and down in the cabinet. As all wires come out the front, the powered server is moved by unclipping the wires, sliding out the FR4 boards, switching the boards with the server to be heated, and clipping the wires in their new position.

# 6.5.4 Power Supply Calculations and Data Acquisition Measurements

The power supply has a maximum current throughput of 5 amps, and thus attempting to connect all 10 heaters in parallel results in a total resistance of  $1\Omega$  results in an over current situation. Therefore the heaters are wired in a combination of series and parallel, as shown in Figure 6.26, where the dashed rectangle represents the boundaries of the cabinet.



Figure 6.26 - Cabinet circuit diagram

The total resistance of the foil heaters within the cabinet is computed as:

$$R_{cabinet} = \frac{1}{\frac{1}{\sum_{i=1}^{5} R_i} + \frac{1}{\sum_{i=6}^{10} R_i}} = 25.3\Omega$$
(6.41)

Therefore this arrangement provides enough resistance of  $25\Omega$  per side that adequate power can be dissipated, yet the FR4 boards can still be removed easily as each side of the cabinet is supplied independently. The power dissipated through the foil heaters is computed using equation (6.42), where *i* is the current computed using equation **Error! Reference source not found.** from the precision resistor, and *V* is the voltage drop across the cabinet.

$$P = iV_p \tag{6.42}$$

This calculation ensures that changes in the resistance of the heaters do not affect the measured value of the power dissipated by the heaters.

The powers at which the temperature is measured are 2, 4, and 6 W which correspond to the nominal settings on the power supply given in Table 6.7.

Power (W)	Supply Current (A)	Supply Voltage (V)
2	0.894	22.094
4	1.265	31.314
6	1.549	38.479

 Table 6.7 - Nominal power supply settings

The values given above are used as the initial settings, as the values of  $V_c$  and  $V_p$  change, the supply voltage is adjusted to maintain a constant accurate power to the

heaters. This monitoring and control is possible through a simple feedback loop implemented in LabVIEW between the monitored power and the output power supply settings.

The temperature readings from all thermocouples are monitored simultaneously at a frequency of 2Hz using a custom LabVIEW program. The low response frequency of the system dictates that this sample rate is adequate, and no aliasing or other potential measurement inaccuracies are occurring. The maximum sample rate of the DAQ system is 200kHz, and thus the signal multiplexing time is not an issue either, again as the system response is very slow. In order to ensure steady state conditions are reached, a running linear regression of the most recent 20 sample data points is taken. Once this value reaches and stays below a value of 10e-3 °C/second, 120 data points are acquired, representing a minute of continuous acquisition. This steady state monitoring approach is far more accurate and consistent than eyeballing a graph, as it is not dependent upon the scaling of the graph which throws off slope estimation, and accounts for small fluctuations and noise in the temperature readings. Further real time analysis of the data is available in the LabVIEW program through graphs of the temperature profile of the blade modules, and cabinet enclosure, along with the temperature-time history of the chips and other data.

# 6.5.5 Experimental Results

The chip temperature response of the cabinet is measured and the response described according to the chip's position within the cabinet. The server and blade position dictate the position of the chip on the blade module within the cabinet. The servers are numbered in ascending order, starting from the bottom, and the blades are also numbered in ascending order, starting from the leftmost unit, looking at the cabinet from the front, as labeled in Figure 6.3. The average chip temperatures from all three power levels are shown, grouped by server position, in Figure 6.27-Figure 6.29.



Figure 6.27 - Blade chip temperatures with power generation of 2W



Figure 6.28 - Blade chip temperatures with power generation of 4W



Figure 6.29 - Blade chip temperatures with power generation of 6W

The resulting temperature profile of the blade modules within the server is almost identical in the three plots, as to be expected with a linear thermal system. The shape of the curve is decidedly off-center, this is to be expected as the chips are on one side of the FR4, which has a highly anisotropic thermal conductivity, and hence the profile is not expected to be symmetrical. Experimental data for server position 1 could not be obtained because of the configuration of the cabinet, the blade modules could not fit in the bottom server because the cabinet chassis blocked the lower half of the server. The constant low temperature of blade module 7 is likely due to the bonding of the thermocouple to the surface of the heater; the tip of the thermocouple is likely embedded in the epoxy, and not in direct contact with the surface of the heater. However, this problem cannot be fixed, as the results would likely be less accurate if another thermocouple were placed off-center of the foil heater, as the temperature profile of the FR4 changes rapidly, as is shown later in this chapter. The data depicted in Figure 6.27-Figure 6.29 is compiled in Table 6.8-

Table 6.10 below.

	Blade										
Server	1	2	3	4	5	6	7	8	9	10	
2	28.6	25.6	23.4	23.9	23.5	24.0	22.1	24.4	27.3	31.6	
3	30.6	26.7	23.2	23.0	23.1	23.9	22.9	26.1	28.6	32.8	
4	30.4	26.7	22.7	22.8	22.5	22.7	22.6	27.2	30.1	34.8	
5	30.2	28.1	24.2	23.6	22.9	23.0	23.1	27.0	30.2	35.2	
6	28.6	27.0	23.5	24.3	24.2	24.0	23.4	26.4	30.2	34.4	
7	30.1	26.8	25.0	25.2	24.8	25.3	25.6	26.9	29.8	32.4	

 Table 6.8 - Chip temperatures at 2W

		Blade										
Server	1	2	3	4	5	6	7	8	9	10		
2	56.8	51.1	46.8	47.6	47.0	48.0	44.2	48.9	54.9	62.4		
3	59.8	52.9	45.9	45.6	45.7	47.3	45.3	51.4	56.3	63.8		
4	60.7	53.8	45.1	45.3	44.7	45.1	45.0	54.4	60.8	69.1		
5	58.8	55.3	47.7	46.2	44.8	45.1	45.3	53.1	59.3	68.6		
6	55.2	52.6	46.0	47.4	47.5	46.9	45.7	51.6	59.0	66.8		
7	58.6	52.1	48.5	49.0	48.2	49.2	50.0	52.4	58.4	63.6		

Table 6.9 - Chip temperatures at 4W

Table 6.10 - Chip temperatures at 6W

	Blade									
Server	1	2	3	4	5	6	7	8	9	10
2	84.0	76.1	69.9	70.6	70.1	71.6	66.1	73.0	80.8	89.4
3	87.2	78.1	68.3	67.9	67.7	70.3	67.4	76.4	83.4	93.0
4	88.5	79.1	67.1	67.2	66.3	67.1	66.9	80.5	89.7	99.5
5	86.4	81.7	70.7	68.4	66.3	66.9	67.1	78.8	87.4	99.3
6	80.6	77.2	67.7	69.9	69.4	69.3	67.3	75.7	85.6	95.5
7	86.6	77.6	72.2	72.8	71.8	73.4	74.4	78.1	87.3	94.5

Plots of the chip temperature with the blade position held constant using the server position as the x variable are difficult to decipher. Thus, the statistical technique, the Analysis of Variance (ANOVA) is used to determine if the position of the blade module within the server is statistically significant. The Randomized Block Design ANOVA approach is used, an extension of paired sampling approach [32], as there are two factors, the blade position, and the server position. A randomized block design consists of a set of blocks, which contain one data sample from each of the treatment factor levels under consideration. In this application, the blocks are the server positions,

and the treatment factors are the blade positions. Both the statistical significance of the blocked factor and the factor under consideration are computed, thus the process does not need to be repeated. This approach enables the computation of the effect of the significance of the blade position, and the server position, independent of each other, and acts to ensure the variation due the server position is not missed because of the greater variation due to the blade position. Further information on this approach is available [32].

Creating a null hypothesis for statistical testing, it is that temperature does not vary as a function of horizontal position. The results of testing this hypothesis are presented in an ANOVA table, as shown in Table 6.11, computed using the statistical software MINITAB. The computation of this ANOVA table is completed using the following. Note that the notation used below is specific to this ANOVA of the sever cabinet experiment, and not the general nomenclature often used.

In the first column the sources of variance are defined, in this case being the blade position, the server position, the error, variance which is not accounted for by either of the other two factors, and total, the sum of these three sources of variation. The second column gives the degrees of freedom of each of these sources defined as:

$$DOF_{blade} = b - 1 \tag{6.43}$$

$$DOF_{server} = s - 1 \tag{6.44}$$

$$DOF_{error} = (s-1)(b-1)$$
 (6.45)

$$DOF_{total} = sb - 1 \tag{6.46}$$

Where b is the number of blade positions, equal to 10, and s is the number of server positions, equal to six. The third column contains the sum of squares of each of the sources, computed as:

$$SS_{blade} = \sum_{i=1}^{s} b(T_{i.} - \overline{T}_{..})^2$$
, where  $\overline{T}_{..} = \frac{1}{sb} \sum_{i=1}^{s} \sum_{j=1}^{b} T_{i,j}$  (6.47)

$$SS_{server} = \sum_{j=1}^{b} s(\overline{T}_{.j} - \overline{T}_{.})^2$$
, where  $\overline{T}_{.j} = \hat{\mu}_j = \frac{1}{s} \sum_{i=1}^{s} T_{i,j}$  (6.48)

$$SS_{error} = \sum_{i=1}^{s} \sum_{j=1}^{b} (T_{ij} - \overline{T}_{i.} - \overline{T}_{.j} + \overline{T}_{..})^2, \text{ where } \overline{T}_{i.} = \hat{\mu}_i = \frac{1}{b} \sum_{j=1}^{b} T_{i,j}$$
(6.49)

$$SS_{total} = \sum_{i=1}^{s} \sum_{j=1}^{b} (T_{ij} - \overline{T}_{..})^2$$
(6.50)

Equation (6.47) measures the variability between the blade positions, equation (6.48) the variability among the server positions, equation (6.49) the differences between the measured data and the statistically estimated cell means  $\hat{\mu}_{ij}$ . Equation (6.50) measures the total variability in the data set, such that  $SS_{total} = SS_{blade} + SS_{server} + SS_{error}$ .

The fourth column is the mean square errors, which is computed by dividing the sum of the squares by the degrees of freedom for each source:

$$MS_{blade} = \frac{SS_{blade}}{b-1} \tag{6.51}$$

$$MS_{server} = \frac{SS_{server}}{s-1} \tag{6.52}$$

$$MS_{error} = \frac{SS_{error}}{(s-1)(b-1)}$$

$$321$$
(6.53)

The fifth column contains the F statistic for the two known sources of variation, computed as the ratio of the mean squares value over the mean squares error.

$$F_{blade} = \frac{MS_{blade}}{MSE} \tag{6.54}$$

$$F_{server} = \frac{MS_{server}}{MS_{error}}$$
(6.55)

The last column contains the p-value. The p-value for testing the null hypothesis that the treatment factor level means, in this case the server positions, are all equal is computed using an F-test:

$$p - value = P(T \ge F_{blade}) \tag{6.56}$$

Where the random variable, in this case the chip temperatures T, is distributed as the F distribution:

$$T \sim F_{b-1,(s-1)(b-1)}$$
 (6.57)

The *p*-value for measuring the plausibility of the blocks being indistinguishable from each other is also measured using an F-test:

$$p - value = P(T \ge F_{server}) \tag{6.58}$$

Where *T* is distrusted as the F distribution:

$$T \sim F_{s-1,(s-1)(b-1)}$$
 (6.59)

The results of this analysis are shown below in Table 6.11.

Table 6.11 - Randomized block design ANOVA tables for the cabinet at all power levels

2W chip power case:

#### Two-way ANOVA: Temperature versus Blade, Server

Source DF SS MSF Ρ Blade 9 659.550 73.2834 94.83 0.000 5 15.229 3.0457 Server 3.94 0.005 34.776 0.7728 Error 45 Total 59 709.555 S = 0.8791 R-Sq = 95.10% R-Sq(adj) = 93.57%

4W chip power case:

#### Two-way ANOVA: Temperature versus Blade, Server

Source DF SS MS F P Blade 9 2492.24 276.915 87.65 0.000 Server 5 24.61 4.922 1.56 0.191 Error 45 142.18 3.159 Total 59 2659.02 S = 1.777 R-Sq = 94.65% R-Sq(adj) = 92.99% 6W chip power case:

#### Two-way ANOVA: Temperature versus Blade, Server

Source DF SS MS F P Blade 9 4785.29 531.699 87.21 0.000 Server 5 71.00 14.199 2.33 0.058 Error 45 274.36 6.097 Total 59 5130.65 S = 2.469 R-Sq = 94.65% R-Sq(adj) = 92.99%

The *p*-value of 0 for all three power levels for the blade position is to be expected, there is no statistical probability that the effect of blade position has no effect on the chip temperature. The *p*-value of the server, measuring the effectiveness of the block, is very low for the 2W and 6W cases. These values indicate there is a 95% probability that the server position has an effect on the temperature response. The higher *p*-value for the 4 W chip power case in Table 6.11 is unexpected, but simply indicative of a greater amount of noise in the data for this run. Overall, the value is low enough for general guidelines on

rejection of the null hypothesis for this kind of analysis. Analysis of the scatter plot residuals appear to be random and the normal scores plot indicates the residual errors are normally distributed within reason. Therefore it is safe to conclude that the server position does have influence on the temperature response for the experimental cabinet.

An analysis of the standard deviation of the data is performed in order to determine what factors, if any, have an effect on the variability of the chip temperatures. These data are important not only in the scope of experimental accuracy, but also from a robust design standpoint, to determine better approximations of the variability of the control variables and noise factors. The mean standard deviation of the chip temperatures is 0.12 °C, which is very small, with a maximum of 0.47 °C, which is still within the margin of experimental measurement error, as discussed in the next section. However, it is still pertinent to investigate any trends in the variability of the standard deviation with cabinet position or the power dissipated by the cabinet.

The standard deviations are computed and tabulated in two manners; using the mean of the standard deviation across the other factors not considered, or the maximum value of the standard deviation of the other factors. For example, in

Table 6.12 below, the standard deviation of the temperature with respect to blade position is displayed. The mean value, in column 2, is the average of the standard deviation of blade 1 for all servers, and all power levels. The max value, in column 3, is the maximum standard deviation found in blade 1 in any server, for any power used.

Blade	Mean	Max
1	0.17	0.33
2	0.12	0.35
3	0.08	0.23
4	0.07	0.16
5	0.10	0.26
6	0.08	0.22
7	0.10	0.25
8	0.10	0.19
9	0.17	0.38
10	0.23	0.47

 Table 6.12 - Standard deviation variation with blade position

The trend in Table 6.12 is that not only do the middle blade modules have lower operating temperatures, but also are more stable, having a smaller standard deviation value. In fact, blade module 10, the hottest and furtherst right module, has the highest standard deviation in the entire cabinet.

Server	Mean	Max
2	0.12	0.16
3	0.13	0.19
4	0.12	0.14
5	0.14	0.20
6	0.15	0.25
7	0.11	0.15

 Table 6.13 - Standard deviation variation with server position

The standard deviation does not appreciably vary with server position, as shown in Table 6.13. There is a slight trend of increasing variability in the upper cabinet, however the topmost server has the lowest variability. No strong conclusions can be drawn from these data.

Power	Mean	Max
2W	0.10	0.13
4W	0.12	0.15
6W	0.17	0.25

Table 6.14 - Standard deviation variance with power level

The standard deviation increases with the amount of power dissipated by the chips, as shown in Table 6.14. This indicates that the use of a function, linear or otherwise, is appropriate for modeling the variance inherent in the chip power generation for constant source heaters. Unfortunately, while useful for determining the robust design of a mock server cabinet such as this one, this result cannot be broadly applied to estimate any predictions of the temperature variation of real operational processors in operational server cabinets.

### 6.5.6 Validation and Error Analysis

It is important to consider the accuracy of any experimentally gathered data when using it for analysis and comparison with computer simulations, as is performed in this thesis. The standard deviation of the data shown in this chapter has been completed, however there are many other further factors that can affect the accuracy of the results obtained.

The accuracy of the thermocouples is tested using a bath containing mixture of crushed ice and water. The thermocouples used are all from the same spool of high quality Omega thermocouple wire, and thus only a few probes are tested. The temperature of the ice bath is measured at the same location using the thermocouple system as well a precision mercury thermometer, certified accurate to 0.2 °C. The difference between the two temperature readings did not exceed 0.1 °C, indicating that both the thermocouples, and cold junction compensation and thermocouple calibration built into the DAQ system is accurate. Further considerations of this accuracy are investigated by moving the thermocouple connection around the terminal connection on the DAQ module. Although the modules use large copper blocks to create an isothermal connection for the cold junction compensation, a temperature gradient, either from the unit's power supply or other sources can create error in the reading. No appreciable temperature difference is found between either end or module of the DAQ system. No further thermocouple calibration is required as the build in system is found to be adequate for the small temperature range used in this investigation.

Another estimate of the variance in the cabinet is performed using linear regression. The linear regression of each chip temperature for all three power levels is performed, and the  $R^2$  value computed. The average  $R^2$  value is 0.998, indicating that 99.8% of the variability of the temperature variability in the cabinet is attributed to the changes in the power dissipated by the chips. This value is not the most precise estimate, as only three points are used in the analysis, however it is still valuable to get an idea of how much external factors influence the chip temperatures.

The last source of error to consider is in the power supplied to the chips. Although the power is measured accurately using the combination of the measured current and voltage, there are still sources of error. The voltage drop across each individual chip is unknown. This is because it is impractical and difficult to wire a voltage tap across the resistive portion of the heater within the interchangeable cabinet wiring system. This means that the voltage drop across the entire system must be used. Furthermore, there is potential for a slight difference in resistance between the heaters on the left and right sides of the server rack. However, because the resistances of the heaters are measured to be within  $0.05\Omega$ , the difference is very unlikely to cause any difference outside the range of the standard deviation of the temperature measurements.

Finally, the repeatability of the data is checked. Sample data was acquired on several different days, and normalized against inlet temperature in the same manner as the data presented here. With the removal of the primary external noise factor, the temperatures are found to be within a degree, with similar standard deviations. These differences are likely caused by differences in relative humidity, as although regulated in operational data centers, this condition is not regulated in the CEETHERM experimental data center laboratory. However, this difference is still very small, and hence the results are repeatable. A complete data set is presented in Appendix B.

#### 6.5.7 Comparison to CFD Model

In order to determine if the turbulent CFD simulations employed in this thesis are adequate modeling tools for use in this robust design approach, a CFD analysis of the experimental mock blade cabinet is performed, and the simulated chip temperatures compared with the experimental data. In order to generate the flow conditions needed to replicate the flow within the experimental mock cabinet, a flow hood is used to measure the flow out the top of the cabinet. Using the mass conservation property of the cabinet, the inlet velocity is then measured as a mean of 1.63 m/s with a standard deviation of 0.0627 m/s. This standard deviation is applied in the robust compromise DSP formulation in Section 6.4. This inlet velocity boundary condition use is more accurate than using an inlet vent boundary condition as it accounts for flow provided by the CRAC units, as well as converging significantly faster than a pressure inlet boundary condition.

The simulation is performed with all chips generating 2 W, 4 W, and 6 W of power. The servers are heated individually to create identical heating conditions as the experimental cabinet. The percent temperature differences of all the chips is computed as the simulated value minus the experimental value over the experimental value, shown in Table 6.15-6.17.

	Blade										
Server	1	2	3	4	5	6	7	8	9	10	
2	2.8	2.0	-0.6	1.6	-4.3	-0.3	-4.2	1.2	1.8	1.6	
3	0.7	1.4	-4.6	-1.9	-0.7	2.7	-2.5	2.5	-1.6	-1.1	
4	-0.4	0.2	-4.3	-3.2	-3.7	0.4	-3.7	4.9	0.1	-0.3	
5	-1.0	4.0	-2.5	-1.4	-3.1	-0.3	-3.2	4.0	-0.3	-2.3	
6	-4.4	-0.3	-4.6	-1.0	-0.6	1.5	-3.5	1.3	1.7	-4.0	
7	7.2	2.1	-2.1	-0.7	-0.9	3.8	4.3	5.8	9.5	-1.9	

 Table 6.15 - Simulation error percentage for 2W power

Table 6.16 - Simulation error percentage for 4W power

	Blade										
Server	1	2	3	4	5	6	7	8	9	10	
2	1.0	1.2	-0.3	1.1	-2.1	-0.3	-2.6	0.6	0.7	2.6	
3	0.2	0.7	-2.4	-1.5	-1.0	1.4	-1.8	1.2	-1.3	1.0	
4	-0.3	-0.8	-2.1	-1.8	-2.2	-0.3	-2.3	2.9	0.0	0.0	
5	-1.0	1.9	-1.2	-0.6	-1.9	0.3	-2.0	2.3	-0.4	-0.4	
6	-2.8	-0.2	-1.9	-0.2	-0.7	0.8	-2.0	1.1	0.8	-0.8	
7	4.3	0.7	-1.2	-0.8	-0.7	2.4	2.9	3.3	5.4	0.1	

Table 6.17 - Simulation error percentage for 6W power

	Blade											
Server	1	2	3	4	5	6	7	8	9	10		
2	2.0	1.7	0.3	2.5	3.0	0.1	2.0	1.1	1.1	9.3		
3	1.1	1.9	1.7	0.4	1.7	3.9	0.1	2.9	1.6	8.5		
4	2.0	0.3	4.7	0.6	0.8	2.0	1.4	3.5	3.4	3.6		
5	1.1	3.5	0.2	1.4	0.6	2.3	0.0	3.7	1.4	4.8		
6	1.6	1.7	2.8	1.9	2.5	3.7	0.3	2.9	2.1	5.4		
7	6.0	2.7	0.3	1.5	1.3	4.5	4.7	5.4	6.7	5.1		

The average percent differences of the 2 W, 4 W, and 6 W cases are 2.4%, 1.4%, and 1.1% respectively, and the maximum percent differences 9.5%, 5.4%, and 5.2% respectively. Overall the results match closely, with only the rightmost blades in the upper servers having a greater difference than 5%. However, all these values are within 10% of the simulated values. This is demonstrated in Figure 6.30, which shows the normalized temperature response, temperature divided by power input, for the CFD model and the three experimental powers used, along with error bars at 10%.



Figure 6.30 - Normalized cabinet temperature response an error bars

This figure quickly shows where the areas of greatest difference are. These areas are also of concern as they have the highest variability, both in the experimental readings, and the robustness of the temperature response with variations in the power levels (as the two are related). Thus these outermost blade modules are by far the most at risk of overheating or having other thermal related problems. The top server is also the least accurate, likely because of the complexity of the fan swirl and turbulence effects that are not modeled, and are difficult to implement accurately in the CFD simulation.

The CFD model is likely slightly biased towards conservative temperatures, therefore as the heat flux increases, so does the radiation heat transfer, which is not

modeled in the CFD simulation. This is likely why the CFD model goes from being under predictive to over predictive, as when the chip temperatures approach 100 °C the heat transfer by radiation may no longer be negligible, even with the forced convective conditions. The CFD model is also more accurate for the higher heat flux conditions, which are close to the conditions found in the cDSP. Therefore, it is likely that these converged solutions are accurate, as the reduced order model was found to be quite accurate in Section 6.4.4. Overall, the results of the CFD analysis quite closely predict the experimental results, including all trends, using the standard k- $\varepsilon$  model and boundary conditions. This provides strong validation that the use of the robust cabinet design approach, based on CFD generated results, is capable of producing designs that are effective in a physical data center.

## 6.5.8 Natural Convection and Radiation Considerations

The CFD simulation of the cabinet in this chapter ignores the heat transfer effects of natural convection and radiation. Natural convection was found to be insignificant through experimental work [14]. However, this assumption can be tested through a simple comparison of the Reynolds and Grashof dimensionless numbers [39], given in equations (6.60) and (6.61) respectively.

$$Re_L^2 = \left(\frac{u_0 L}{\nu}\right)^2 \tag{6.60}$$

$$Gr_{L} = \frac{g\beta(T_{s} - T_{\infty})L^{3}}{v^{2}}$$
 (6.61)

In these equations  $u_o$  is the free stream velocity, L the characteristic length, v the fluid dynamic viscosity, g gravitational acceleration,  $T_s$  the heated surface temperature, and  $T_{\infty}$  the ambient temperature.  $\beta$  is computed as 1/T, where T is the average of  $T_s$  and  $T_{\infty}$  in degrees Kelvin. The ratio of these numbers is displayed in Figure 6.31 for the lowest range of velocities found in the blade channels in the cabinet simulation.



Figure 6.31 - Grashof number to Reynolds number ratio

Provided this ratio is significantly less than 1, the assumption that natural convection is negligible holds true. This calculation was made using the maximum possible temperature difference of 100 °C, using the characteristic length of the chip, and the mean maximum temperature of the entire FR4, using the entire vertical length of the FR4 board, yielding the upper limit of this ratio. As this worst case scenario still yields numbers slightly less when natural convection needs to be included, the forced
convection assumption is valid. The natural convective velocity is computed as 0.3 m/s using equation (6.62), indicating at the lowest channel flow velocities natural convection may become significant if channel velocity falls any lower.

$$u_{conv} = \sqrt{g\beta\Delta TL} \tag{6.62}$$

Radiation scales with absolute temperature to the fourth, and thus becomes significant in cooling of electronics at around the same point natural convection becomes significant and the temperatures become high. The radiative heat flux from the heater is computed using equation (6.63), where  $\sigma = 5.67e-8 \text{ W/m}^2\text{K}^4$ .

$$q = A\varepsilon\sigma(T_s^4 - T_\infty^4) \tag{6.63}$$

This formulation assumes a small emissive body in an enclosed cavity, yielding the upper bound of the effect of radiative heat transfer. The difficulty is finding an appropriate value of the emissivity,  $\varepsilon$ . This value can vary significantly for metals depending upon their level of polish and surface finish. Therefore, values of 1, 0.4, and 0.05, representing the upper, median, and lowest possible values [39] are used in the computation of Figure 6.32.



Figure 6.32 - Radiative heat flux with increasing sureface temperature

In this figure the heat dissipated is computed for an increasing chip surface temperature. Using the emissivity value of 0.4, corresponding to stably oxidized copper, and a surface temperature of 100 °C, 0.25 W of power are dissipated. This corresponds to 4.2% of the 6 W dissipated by the forced convection in the CFD simulation. This additional heat dissipation, which is not accounted for by the CFD simulation, corresponds well with the increase in temperature modeling over approximation from less than 5% to just under 10%. Therefore it is likely that if higher chip temperatures are to be modeled in server cabinets, some handling of radiative heat transfer as well as natural convection is desirable for very accurate results.

# 6.5.9 Cabinet Flow Analysis

As the cabinet CFD model has been shown to be accurate, analysis of the CFD generated flow patterns is possible to gain insight into the causes of the temperature distribution found. The maximum server temperature distribution at this inlet velocity is quite uniform, as shown in Figure 6.14, the Velocity of 1.63 m/s is just before the temperature responses almost all converge. With this in mind, the flow distribution throughout the cabined is analyzed.



Figure 6.33 - Cabinet cross-section planes in y

In Figure 6.34 below, the flow velocity is plotted for three different cross-sections of the server. The cross-section through blade 1, blade 3, and blade 5, as shown in Figure 6.33 above, representing Figure 6.34 (a), (b), and (c) respectively.

Velocity Magnitude (m/s) 0.5 1 1.5 2 2.5 3 3.5 4 4.5



Figure 6.34 - Cabinet flow cross-section through (a) blade 1 (b) blade 3 (c) blade 5

In Figure 6.34 above, the flow velocity is plotted for three different cross-sections of the server. The cross-section through blade 1, blade 3, and blade 5 going from the left to right images respectively. The transition of the flow velocity through the cabinet is displayed clearly, both with respect to blade position and server position. The lowest server receives a fair amount of air, however the distribution of the airflow is greatest on

the blades at the edges, and the middle blades receive less cooling air. This creates a low maximum temperature, despite the odd flow conditions, as shown in Figure 6.14 and discussed further below. The middle five servers all have similar flow conditions, with a slight decrease in the flow velocity magnitude as the server position moves higher, and appropriately higher server temperature again shown in Figure 6.14. The highest server has quite uniform flow velocity throughout the blade modules. This means that despite its lower flow velocity, the critical blade positions 1 and 10 receive enough flow to reduce the critical maximum server temperature to the same range as the rest of the servers. It is likely that this mal flow distribution causes the temperature response differences between the servers in Figure 6.14.

In order to obtain a better concept of the flow distribution within the servers, Figure 6.35 above shows the flow velocity magnitude at the mid plane of all the servers. The figure is divided in order to cleanly show all the servers. The flow patters visible are the same as those described above, however more detail of the distribution within the blade channels is visible. Of particular note is blade position of the maximum flow velocity with respect to the server vertical position. It moves from the outermost blades in server 1, to blades 3 and 7 in server 2, then to the center blade in the rest of the cabinet. This gradual transition is not visible in Figure 6.34, and helps explain the temperature responses found for server 2, as the middle blade heaters are hotter than those directly beside them towards the sides of the server. Also visible again is the more uniform flow distribution in the uppermost server.



Figure 6.35 - Server velocity profile for servers (a) 1, 3, 5, 7 (b) 2, 4, 6

The three different flow conditions in the servers within the cabinet are now analyzed in detail. The following three figures show the temperature profile of the 10 FR4 boards and foil heaters, enabling visualization of the resulting chip temperature profile across the server, and the heat spreading into the FR4. Stream traces of the flow at the midplane of the servers are also plotted, giving detailed visualization of the airflow patters through the blade modules and server. Because the flow in servers 2 through 6 is very similar, the results of only one of these servers are presented.



Figure 6.36 - Flow streamtraces and FR4 temperature distribution of server 1

In Figure 6.36 the recirculation of the air around the middle blades is evident through the streamtraces. This results in the higher chip temperatures of the middle blades, also visible in the figure above. This recirculation is likely because of the position of the fans interacting with the high velocity of the air blowing by the entrance

of the server. The resulting chip temperatures are low, but have high variability because of the unstable flow pattern. This is accounted for in the robust configuration approach applied.



Figure 6.37 - Flow streamtraces and FR4 temperature distribution of server 5

In Figure 6.37 the flow conditions have settled in the middle blades, as the streamtraces are straight and uniform. The edge blades however, have significant recirculation and some vertical momentum made particularly visible in this plot. This results in the chip temperature distribution between blades also visible in the figure.



Figure 6.38 - Flow streamtraces and FR4 temperature distribution of server 7

In Figure 6.38 the flow is almost entirely smooth and uniform, as shown by the streamtraces. Unfortunately, the streamtraces to not show flow velocity magnitude well, and hence it is slightly difficult to see why the chip temperature of blade 10 is higher than the remaining blades. However, it is evident from the chip and FR4 temperatures that this uniform flow results in much more uniform temperature conditions also. Overall, the analysis of the complex flow patterns with data center server cabinets, as demonstrated in the figures above, can be used to gain insight into why local areas of cabinet are having thermal problems, and thus approaches to their resolution can also be derived from this information. It is important to remember that detailed analysis of a CFD model alone for a complex problem such as this is risky without validation of the model first. Without validation, preferably through experimental comparison, only the trends in the model should be considered.

# 6.5.10 Improving the CFD Simulation

There are many factors that can potentially be tweaked to improve the accuracy of this CFD simulation to match the experimental results more closely. The most influential of these factors is the thermal conductivity measurements of the FR4 boards. Although these are simulated using an anisotropic shell conduction model, heat conduction through the boards between blade modules is not considered. This assumption is adequate, as the difference in thermal resistance between the air side and conduction through the FR4 is nearly two orders of magnitude, however changing this assumption combined with very accurate measurements of the thermal conductivity could increase the accuracy of the simulation.

The second most influential factor is the turbulence conditions in the cabinet. Although the hydraulic diameter – turbulence intensity approach is a good general approximation, the use of hot wire or PIV data to back out the plenum and cabinet entry turbulence parameters could also increase the accuracy of the simulation. Some geometric simplifications and assumptions are made in this CFD model for the sake of convergence speed, mesh quality, and number of elements. However, with the new high power computer cluster in the data center lab, a very large, very accurate model can be constructed, which could enable very accurate models to be constructed.

Lastly, PIV data can be collected of the cabinet for a complete flow profile, and thus an idea of the accuracy of the model on a local as well as a system scale can be determined. This could be used to identify failures in the CFD foundational flow models and equations for regions of turbulent flow, and obtain a much better concept of where the CFD model breaks down, rather than the blanket "less than 10% difference" results obtained here. This should be tested at multiple inlet velocities to find any Reynolds number dependence of these local modeling inaccuracies.

Testing of different velocities for the cabinet temperature profile was attempted using various CRAC unit combinations. However, it was found for all cases, the small change in temperature from the increased flow was made statistically insignificant by the drastic increase in the temperature standard deviation, making the results obtained useless.

## 6.6 Chapter Synopsis and Validation Summary

In this chapter the applicability of all three constructs to a very complex three dimensional cabinet model was demonstrated, and the models used grounded through comparison with experimental data. In this study the feasibility and effectiveness of the application of robust design was investigated through variations in the amount of cooling air supplied, the heat load distribution within the cabinet. The quadrants of the validation square that have been addressed in this chapter are presented below. How the validation performed in this chapter falls within the complete validation roadmap can be determined from viewing Table 1.3.

## Empirical Structural Validity

• The PODc reduced order flow model was shown to be accurate and effective even for large, complex three dimensional flow simulations, this

problem is the largest and most complex application of the PODc flow model to date, as shown in Section 6.2.6.

# Empirical Performance Validity

- The blade cabinet model geometry used is representative of the new blade style cabinet architecture, which is commonly used for the highest performance computing cluster applications, shown in Section 6.1.3.1.
- The flow and heat transfer parameters used, as well as the goals used in the compromise DSP formulation are representative of physical data center server cabinet configuration problems, as shown in Sections 6.1.3.2, 6.1.3.3, and 6.1.4.
- The maximum heat dissipation is found to be a function of both the supply rate of cooling air, the server rack position, and blade module position, indicating the validity of consideration of these design variables, as shown in Section 6.4.2.
- The effect of changing the weighting for a more robust or optimal solution was found to have significant effects on the amount of chip temperature variation with minimal tradeoffs in energy efficiency, indicating the validity of using the minimization of temperature variation goal, shown in Section 6.4.3.
- Through comparison with accurate experimental data the CFD models upon which all work in this thesis is based was found to be within 10% of

the measured temperatures for all chips, and most within 5%, given in Section 6.4.4.

# Theoretical Performance Validity

- Although the geometry considered in this chapter is simplified in order to match the experimental set up, the accuracy of the results and verified, and the trends indicate that considerable reduction in temperature variation and increases in thermal efficiency can be made using this approach, shown in Sections 6.5.6 and 6.5.7.
- The consideration of all three case studies presented in this thesis, combined with the extensive validation of every aspect of the work, indicates a degree of theoretical performance validity for the configuration of data center server cabinets, shown in Chapters 4, 5, and 6.
- A greater level of theoretical performance validity cannot be claimed from the work presented in this thesis, further work and development is required, as discussed in Chapter 7.

With the implementation and results of the mock blade server cabinet presented, a critical review of the approach is performed. After this, augmentations to the approach are discussed, however empirical performance validity is not addressed in the next chapter, and hence these ideas and approaches need further rigorous validation.

# **CHAPTER 7**

# **CRITICAL REVIEW AND EXTENSIONS OF APPROACH**

In review, the principal goal for this thesis is to:

Establish an approach for the design of data center server cabinets for efficient cooling, accounting for the inherent variability in both internal and external operating conditions, and enabling effective tradeoff between the goals of energy efficiency and reliability, with the potential for broader multi-scale thermal-fluid simulation based design applications.

The motivation for the development of this approach is given in Chapter 1. In this chapter a summary and critical review of the work undertaken is presented. From this review, directions for future work and validation are presented to the end of giving direction to developing this approach into a formal method for broader application to multi-scale thermal-fluid simulation based design. In Section 0 a critical review of the work and approach is performed. In Section 7.2 a discussion of future augmentations to the approach developed is discussed, to the end of achieving a greater potential for broader application. In Section 7.3 a discussion of future directions for superior data center design, considering the integration of all length scales and aspects is presented. The boarder applications and potential impacts of the work presented in this thesis are discussed in Section 7.4.3. Finally, the concluding statements are made in Section 7.5.

How this chapter falls into the overall structure of the thesis and validation square is presented in Figure 7.1. This chapter works to extend the approach developed in Chapter 3 and its validation through application in Chapters 4-6. This chapter primarily addresses the theorectical performance validity of the approach, demonstrated through the general applicability of the core constructs used to build it, and its effectiveness at solving the example problems presented in this thesis.



Figure 7.1 - Thesis and validation roadmap: Chapter 7

# 7.1 Critical Review of the Work

# 7.1.1 Overall Effectiveness of the Approach

The primary challenges in the development of the approach, as identified in Section 1.3, are:

Requirement	Completed	Sections
Systematic approach	V	2.3, 3.5, 4.5, 5.5, 5.8, 6.4
Reduced order modeling	V	2.1, 3.1, 3.2, 3.3, 3.4, 4.2, 4.3, 5.2, 5.3, 6.2, 6.3,
Multi-scale cabinet level analysis	$\mathbf{\nabla}$	1.2, 4.1, 5.1, 6.1
Variability consideration		1.2, 2.2, 4.1, 4.5, 5.1, 5.5, 6.1, 6.4
Multi-objective tradeoffs	$\overline{\mathbf{A}}$	2.3, 3.5, 4.5, 5.5, 5.7, 5.8, 6.4
Experimental validation	V	6.1, 6.2, 6.5

Table 7.1 - Requirements review checklist

There requirements are based upon the challenges regarding data center design that have not been addressed well or at all by the existing work on data centers. Though addressing each and all of these requirements, the primary and subsidiary research questions, presented in Section 1.6, have been answered, proving the hypotheses put forth to be valid. This linking of the research challenges, questions, hypotheses, and tasks was given in Table 1.2. The validation summary of all work undertaken throughout this thesis is presented in Section 7.1.4. How effectively the approach presented in this thesis tackles the individual challenges listed above is addressed for each challenge in turn:

#### 7.1.1.1 Systematic Approach

The primary driver behind the need for an effective, efficient, systematic approach for server cabinet configuration is the life cycle mismatch, identified and explained in Section 1.3.1. The integration of the POD based reduced order flow model and robust design principles in the compromise DSP provide a mathematically rigorous, systematic approach for the thermally efficient configuration of data centers and data center server cabinets. This approach is implemented for three different test cases in Chapters 4, 5, and 6 to create a total of four compromise DSP templates that are solved. The word formulation of the compromise DSP works well in this application, as any designer can look at the template and all pertinent information is conveyed quickly and efficiently. These templates can also be coupled, as was performed to find the maximum cabinet power dissipation in Section 6.4.2, or for multiple cabinets to find the configuration of many cabinets within a center concurrently. Therefore this challenge has been met thoroughly.

#### 7.1.1.2 <u>Reduced Order Modeling</u>

The need for reduced order modeling because of the complexity of the CFD simulations and high resulting computational expense is presented in Section 1.3.2. The POD is introduced as an efficient reduced order modeling construct in Section 0, and the development of this construct into a flow model and further augmentations in Section 0. Through the models application, validation, and evaluation in Chapters 4, 5, and 6 it is shown as a very effective and efficient reduced order flow modeling approach. The validity of the use of CFD generated solutions is shown in Chapter 6, which is required as

the reduced order models are all based upon the POD of an ensemble of CFD generated observations. One area not thoroughly addressed is an approach for the generation of observations. This is not a trivial problem, as identified in [37], however an approach leading to the minimum or close to minimum number of observations required to accurately represent a system would be very beneficial. The foundations for such an approach are presented in Section 7.2.2; however thorough validation is not presented. Overall, the POD based flow modeling approach is shown to be very effective, meeting this challenge thoroughly.

## 7.1.1.3 Multi-Scale Cabinet Level Analysis

The need for consideration of the cabinet at a greater resolution than a black box model is provided by Rambo [78, 84], and discussed further in Section 1.3.3. The parsing and coupling of the cabinet system to the greater data center level system is discussed for each example investigation in turn in Chapters 4, 5, and 6. Analysis of the results of Chapters 4 and 5 show the importance of consideration of the airflow distribution vertically within the cabinet, and the resulting vertical temperature profile of the server cabinet chips. The results presented in Chapter 6 show that not only is this vertical distribution important for Blade servers, but also the horizontal placement of the Blade modules within a server rack. Without this multi-scale approach, the details of the cabinet airflow conditions, which dictate the chip temperatures, are obscured. An approach for the effective, efficient estimation of the airflow rates provided to the cabinets at the data center level is provided in Appendix C. Overall, through the investigation and validation of the server cabinet configurations performed in this thesis this requirement has been met sufficiently.

#### 7.1.1.4 Variability Consideration

The importance of the implementation of robust design in order to effectively deal with the inherent variability in data centers is provided in Section 1.3.4. The implementation of robust design principles through the simultaneous goals of bringing the mean performance to target and minimizing the variability of the response is handled through the multi-objective formulation of the compromise DSP. The effectiveness of this approach for reducing the variability in the server chip temperatures is performed in Chapter 5 and Chapter 6, reducing the potential temperature variability by up to 15% or more, depending upon the final operating point selected from the Patero set of solutions. Therefore the requirement of achieving thermal efficiency while simultaneously accounting for the inherent variability in data center server cabinet systems is met thoroughly.

## 7.1.1.5 <u>Multi-Objective Tradeoffs</u>

The need for mathematically rigorous, effective handling of multiple objectives is discussed in Section 1.3.5. Of particular interest for this application of robust design is the tradeoff between ultimate thermal efficiency and the amount of variation in the server chip temperatures. In the example applications in Chapter 5 and Chapter 6 a Pareto Frontier is established between the optimal and the least varient solution points. This curve is independent of the designer's preferences, and hence a final operating point can be found based upon the operating characteristics of the data center and the designer's preferences. However, in this more general form, the full data set is represented, adding a greater degree of flexibility and augmented handling of the multiple objectives associated with data center design.

#### 7.1.1.6 Experimental Validation

The lack of experimental validation of temperature simulations in previous literature and the need for strong experimental validation is presented in Section 1.3.6. The experimental mock blade server cabinet, and the associated set up, data acquisition, analysis, and validation is presented in Section 6.5 in Chapter 6. The results at multiple cabinet power levels are compared with a detailed CFD simulation, and the results agree within 5% for most responses, and within 10% for the worst handful of points. This good agreement in temperatures, and very strong capturing of the thermal trends throughout the cabinet provide strong validation, and indicate that CFD analysis using the standard  $k - \varepsilon$  model is valid for the simulation of data center server cabinets.

# 7.1.2 Limitation of Models

In this section the limitations of the models are discussed, both for the fluid flow and heat transfer simulations. Although these models have been strongly validated for all of their applications in this thesis, there are a few small remaining issues and considerations regarding their implementation for modeling of other complex thermalfluid systems.

#### 7.1.2.1 Flow Model

The POD flow model has very few limitations, if any, within its realm of application within this thesis. One limitation is its accuracy is entirely dependent upon the accuracy of the observations the decomposition operates on. Therefore, if the CFD generated observations are inaccurate or make invalid assumptions, the POD approximations will also be incorrect. The CFD solutions are validated against experimental data for the mock blade server model, however, validations against multiple inlet velocities was not possible with the current experimental set-up. With direct control of the variable CRAC flow output and some ducting to the cabinet, accurate analysis of the temperature profile at a range of inlet velocities may be possible. Also, although not a direct limitation of the reduced order modeling approach, the issue of determining the number and parameters of the observations *a priori* is still unresolved, however guidelines with respect to the types of flows dealt with in this thesis are given in Section 7.2.2.

The POD approach as it stands at this time does not work for problems with parametric variations in geometry. This type of problem has been investigated by [106] for a flow through a diffuser with a variable angle. The data was acquired using Particle Imaging Velocimitry (PIV), and focuses upon the low dimensional reconstruction of a given flow field for a set velocity and diffuser angle. Although the representation with only a few modes is found to be accurate, although the ensemble consisted of over 800 modes, there was some spatial leakage resulting from the variation in geometry. This problem may require additional considerations in the flux matching procedure if variations in geometry are considered and as accurate results as possible are desired.

### 7.1.2.2 Heat Transfer Model

The energy equation solution approach using the power law approximation has been shown to be a valid and accurate approximation [67], and hence is commonly implemented in commercial fluid flow and heat transfer solver packages. The key limitation in the model developed in this thesis is the constraint of requiring a uniform grid. The interpolation of the irregular meshes used in the CFD and POD solutions can cause errors or even singularities in the stiffness matrix inversion and hence lead to inaccurate results. However, this limitation is not found to be of large concern for the range of model applications in this thesis. The heat transfer model also deals with the turbulent convection transport adequately through the combination of the POD based turbulent viscosity field reconstruction and the empirical wall function computations, without resorting to a full coupled  $k-\varepsilon$  solution. However, the selection of the reference point used for the free stream velocity can strongly influence the effective heat transfer coefficient obtained, and must be chosen with care. The energy equation solvers in FLUENT are both accurate and robust for many types of convective heat transfer problems. However, the primary limitation of using this solver is the importing of the ASCII data, which can take up to 180 seconds for a large model, such as the blade server cabinet model. With the recent development of the accurate conjugate PODc approach, a separate energy equation solver is no longer required, ultimately making these limitations obsolete.

# 7.1.3 Limitations of Case Studies

The three case studies presented in this thesis cover all major cabinet designs and flow configurations. The most prominent limitation is the simplicity of the cold aisle study model, however, a more detailed model is not required to show the utility of the approach developed, the primary goal of this thesis. A detailed three dimensional model of a horizontal flow cabinet is not investigated either. However, comparing the similarity of the results between the 2D and 3D enclosed vertical flow cabinets, it is highly unlikely that increasing the complexity of the model to three dimensions would bring any major development, or reveal any new flaws in the approach.

The only major limitation in the case studies presented is the lack of detailed center level server configuration with multiple interactions. The center level considerations are trivial for the servers modeled in Chapter 5 and Chapter 6 because there is no inter cabinet hot exhaust recirculation in this style of cabinet. The inlet flow conditions to the server placement in the data center can be predicted using a flow hood and the Design of Experiments methodology presented in Appendix C. The recirculation considerations in the cold aisle modeling in Chapter 4 show that the approach is still valid under any range of hot exhaust air re-circulation, and hence application to a complete cold aisle model. Furthermore, the detail of the blade server CFD model is greater than many of the full center CFD simulations performed in existing literature, which indicates generating the POD models for a full data center system does not present any significant challenges either.

#### 7.1.4 Validation Summary

In this thesis the validation square, as proposed by Pederson and coauthors [73], has been followed as a strategy for validation. The concept of this method is to build confidence in the proposed approach through following the four quadrants of the square, ultimately indicating its applicability to a broad range of engineering design problems. In this thesis, the validation of the first three quadrants of the validation square; theoretical structural validity, empirical structural validity, and empirical performance validity are thoroughly investigated. The concept of the fourth quadrant, the theoretical performance validity, is presented in this chapter; however no rigorous validation is presented. Further explanation of the four individual quadrants and how they are investigated is presented in Section 1.7.

The complete breakdown of which sections of each chapter address the specific quadrants of the validation square is presented in Table 1.3. Details upon what exactly was performed in each chapter, with references to the exact sub-section are presented in Sections 2.5, 3.6, 4.6, 5.9, and 6.6. At the end of this thesis, having the details covered throughout the individual chapters and summarized at the end, the big picture is again bought into focus.

Figure 7.2 is shown to again demonstrate how the validation square is applied to each chapter of this thesis, and thus how the details fit into the big picture again, with the objective of demonstrating theoretical performance validity through building confidence in the approach.



Figure 7.2 - Thesis and validation roadmap: thesis validation organization

The theoretical structural validity is presented in Chapters 2, and 3, regarding the constructs and modeling approaches respectively. The theoretical structural validity is considered in this thesis through searching and referencing literature related to each of the constructs employed in the design approach, and the underlying assumptions of the constructs, as well as the modeling approaches employed, and their overall applicability to the problem under consideration. This applies specifically to the POD, robust design, and compromise DSP constructs, as well as the fluid and thermal modeling equation systems. In this manner the theoretical structural validity has been demonstrated.

Empirical structural validity is established primarily in this thesis in Chapters 3 and 4, where the validity of the analysis models as well as the search algorithms are investigated. The empirical structural validity is investigated through the consideration of the example problems chosen for illustrating and verifying the performance of the individual components of the design approach. These components include physics modeling accuracy, reduced order modeling accuracy, and searching algorithm accuracy and convergence. This applies specifically to the the POD fluc matching reconstructions, the thermal modeling capability, and the convergence of the search algorithm. In this manner the empirical structural validity has been demonstrated.

The empirical performance validity is determined through the application of the approach to representative example problems to evaluate the outcome in terms of its usefulness in Chapters 4, 5, and 6. In the case of data center server cabinet design, the core metrics are increased energy efficiency and reduction of variability, which are considered for all three cabinet architectures investigated, and all four cabinet

configuration problems solved. In Chapter 6 particularly strong empirical performance validity is considered through experimental validation of the foundational CFD modeling method, its accuracy propagated through to the final solutions obtained using the method. The overall usefulness of the approach as it pertains to physical data center operation could be extended through finding the operating point on the Pareto Frontier using data center operational costs. This information could be obtained using the data center flow measurement technique described in Appendix C, and the cost of operation of the CEETHERM data center laboratory at Georgia Institute of Technology. This validation is shown specifically by the three example problems of a cold aisle, 2U server, and blade server cabinet architectures of increasing fidelity and complexity. In this manner the empirical performance validity has been demonstrated.

The theoretical performance validity of the approach developed in this thesis pertains to the last part of the primary objective: potential applicability to a boarder range of thermal-fluid simulation based design problems. Although the data center server cabinet problems are representative of many thermal-fluid problems encountered in engineering design, the "leap of faith" cannot be made from this work alone. It has been shown the POD based modeling approach also works well for laminar flow problems [82], extending its domain of applicability significantly. The range of previous literature on POD based modeling presented in Section 1.2 give a great number of examples where POD based modeling is applicable; however, they have not been directly tackled by the approach given in this thesis. Furthermore, validation of the proposed approaches for observation generation, discussed in the next section, is required for the approach developed to become a more formal design method. Overall, the work presented,

literature reviewed, and through validation of the first three quadrants of the validation Square yield a very strong foundation upon which a little further work and validation is required in order to complete the "leap of faith" in order to achieve theoretical performance validity.

## 7.2 Future Work

In this section a discussion of future developments of the approach developed in this thesis are presented, consisting of a more through multi-scale modeling approach, a formal observations generation approach, and more advanced robust design implementations.

#### 7.2.1 Multi-Scale Modeling

In the examples investigated in this thesis the multi-scale approach is undertaken implicitly, through parsing the problem and decoupling the plenum system from the server cabinets, linked through the flow inlet boundary condition. However, there is no limitation for integrating the POD flow model with a zonal modeling approach. In this manner, the fluxes between POD models are matched, for example the inlet and exhaust flow from a server are matched to the higher level cabinet model, which has its flow boundary conditions matched to a data center model. In practice, this becomes like stitching together different models of many length scales. The advantage is it enables the concurrent solution of each length scale at a very high fidelity, much greater than could be accomplished with a single CFD model. For example, a cabinet model of 600,000 nodes could be linked to 20 individual high fidelity server models of 200,000 nodes each,

and the cabinet model is linked to a 1.2 million node data center model, containing 14 cabinets. This model would contain 65.6 million nodes, or 6.56e8 DOF. However, the POD model would contain only 308p DOF, where *p* is the number of POD modes used in the reconstruction, assuming each server has its own flow rate. This number is still a trivial problem for the pseudo-inverse algorithm used in the flux matching procedure.

The challenge in this approach is the accurate modeling of the boundary conditions across length scales. Different approaches to this problem include the high fidelity approach used by Rambo [78, 84], where the POD of the different zones from a single CFD solution would be performed. Another option would be to replicate the range of inlet flow boundary profiles and vary them as the inlet boundary condition to a separate CFD model, and POD that model only, in a "smear" trans-length scale approach. Different combinations of high and low-fidelity boundary modeling approaches may be required depending upon the independence and coupling of the boundary under consideration. In either case, the flux matching may also need to include the momentum balance parallel to the boundary in order to accurately reconstruct the boundary flow profile. However, there are no theoretical or fundamental technical limitations to this approach, and its empirical performance validity is being investigated by Qihong Ni from the METTL lab group.

Further integration of the POD flow modeling approach with liquid cooling, thermoelectric, or other cooling approaches is also not fundamentally difficult. This is because the flux matching procedure works in a similar manner to a CFD model, and hence the flux through a specific area of the model, corresponding to a thermoelectric block or heat exchanger, can have a corresponding heat flux input in the POD flow model. This results in a coupling of the POD flow model and the equations that simulate the performance of the heat exchanger or thermoelectric, which can be solved simultaneously for the complete solution. The empirical performance validity of this approach is also being investigated by Qihong Ni.

## 7.2.2 Formal Observation Generation Approach

As discussed in [37], the problem of determining the appropriate number and parameter combinations of observations *a priori* is still unresolved. This is an important consideration, as the smallest observation ensemble that produces a POD basis that accurately represents the system dynamics represents a substantial savings in computational expense. This is because the generation of the CFD observations is by far the most computationally expensive part of the approach developed in this thesis. Although this is a very complex and likely system dependent problem, an approach for efficient observation generation for the types of flows encountered in this thesis is presented in this Section.

It has been shown by Rambo [77] that the use of a two level factorial design for generation of the POD basis results in a poor representation, and that at least 3-5 levels are required for decent representation of the cabinet model investigated. Using this information, it is likely that a central composite Design of Experiments (DOE) approach utilizing a high alpha value, resulting in axial points outside of the range of parameters may be effective for initial parameter combinations for observation generation. This is

because these designs have up to 5 levels per factor, and thus should enable the decomposition to extract an accurate principal basis.

For problems with large numbers of parameters, such as the cabinet investigated by Rambo with 10 independent flow rates to match [77], this DOE approach will likely produce too large of an ensemble, as this problem only required 20 observations for accurate representation. It has been a rule of thumb for the application of the flux matching POD model that m + 1 POD modes are required for the accurate representation, where m is the number of independent flow parameters. This rule of thumb is not rigorously validated, but has held true for all applications of the approach thus far. Thus, if perfect knowledge were attainable, only this many observations would be required. However, without this knowledge, an approach for judging the quality of the POD basis is required.

The "quality" of the POD basis, in the sense of the amount of system dynamics captured by the basis, is measured by the eigenvalue spectrum. The decay of the eigenvalue spectrum shows how many dominant flow structures or patterns have been extracted. A very steep eigenvalue spectrum indicates that all the dominant flow physics have been captured by a single mode, and thus usually corresponds to variation in only one flow parameter. Thus, the POD eigenvalue spectrum would hopefully have the sum of first m + 1 normalized eigenvalues correspond to the desired system representation, such as 95% or more of the total system variability, as given by equation (7.1):

$$\sum_{i=1}^{m+1} \lambda_i \ge 0.95 \tag{7.1}$$

This eignevalue spectrum will also converge as further observations are added to the ensemble, thus as observations are added, and the eigenvalues change less than a specified small convergence criterion, the observation ensemble may be considered complete.

Using this eigenvalue spectrum convergence as a foundation, and adaptive approach for the formulation of the POD observation ensemble can be created. This approach is similar to the ideas behind augmenting designs using D-Optimal DOE [61]. Given an initial set of observations, the weighting coefficients for the observation reconstructions using the POD modes is performed, using equation (3.4) from the coefficient interpolation procedure described in Section 3.1.3.1. Analysis of the curvature of the coefficients, their partial derivatives with respect to the control variables, will show the areas where the system dynamics are changing rapidly. These control parameter values from these regions should be used to generate additional observations, in order to better capture the changes in flow dynamics in this region. This approach is similar the adaptive meshing procedures used in Finite Element Analysis (FEA). Lastly, the quality of the POD basis can also be tested by reconstruction of the observations using the flux matching procedure, however this approach is slightly biased, and ideally should be tested against different test cases, not from the observation ensemble. Again, there are not fundamental limitations of this approach, but its empirical validity has not been rigorously shown.

# 7.2.3 Application of More Advanced Robust Design Implementations

In this thesis, the application of the designer's preferences to obtain a final operating point for the server cabinets is deliberately avoided. Instead, a Pareto frontier of all possible solutions ranging from least varient to optimal is presented. The final operating point is determined by a knowledgeable data center operator, from a combination of their preferences and the economics of the data center and CRAC operation. These requirements and preferences change with each data center, thus in order to keep the approach general, this last step is not performed. However, the designer's preferences and data center cost functions can be encapsulated in a utility function, which can then be applied to the Pareto frontier and the final operating point determined. More advanced implementations of robust design, such as consideration of higher order representations of the system variation [60], and the use of Utility Theory [91] when sufficient data is available, can be used to augment the fundamental approach developed and utilized in this thesis.

## 7.2.4 Variability Versus Uncertainty

In this thesis the focus has been upon dealing with the variability in data center server cabinets from a variety of physical sources as the latter stages of design are under investigation, the system is well characterized. This variability can be quantified, physically measured, and a distribution plotted, and dominates over any uncertainty present in the measurements as shown in Sections 6.5.3, 6.5.4, and 6.5.6.

However, there is some error and uncertainty in the analysis models used. The error in the POD and CFD simulations is non-deterministic, it is always the same amount. This is why Type III robust design is not used. This error is dealt with implicitly, the maximum bounds of the error are found, and hence the potential variability in the design is ensured to encompass this potential difference in solution. Thus the Type II robust design application do account for this, albeit not very cleanly.

Different CFD turbulence models and assumptions lead to slightly different results, as investigated by Rambo and Joshi [79]. With different models available varying in fidelity, assumptions, and accuracy, as well as experimental data and thorough characterization of the POD based models error; this system variability and model uncertainty can be separated and dealt with explicitly. This cleaner separation of variability and uncertainty is more flexible, as different models can be used with greater knowledge of the implications of using models that are less accurate and hence the uncertainty in the design that results from their use can be quantified and dealt with effectively.

## 7.2.5 Integration into Existing Approach Steps

The strength of the approach developed, and the steps laid out in Section 3.5, is the flexibility and modularity, enabeling the changing and augmentation of the approach. In this section how the future developments discussed prior can be integrated into the approach. The four main developments discussed are:

- Observation Set Generation A method for efficiently determining the parameters for CFD observation set generation.
- Multi-Scale Modeling Augmentation of the Flux Matching Procedure to link models of multiple length scales and mesh densities.
- Integrated Modeling Integration of the Flux Matching Procedure flow models with flow network modeling, thermoelectric flux calculations, and/or other thermal models.
- Advanced Robust Design Integration of higher order and more advanced robust design computations and applications into the compromise DSP.


Figure 7.3 - Integrating future developments into the approach

How these four augmentations fit into the steps of the appraoch is shown in Figure 7.3. However, this potential augmentation is not limited to only these specific points, but more general model and approach integration. Of particular interest is the integration of analysis models specific to different parts of the data center. For example, the POD and flux matching procedure works well for air cooled server modeling, but the fluxes from these models can be used to integrate flow network modeling for the liquid cooling system and CRAC units, and the loads on the chiller and the global energy balances can be determined. This integration of models and constructs is essentially what was performed to create the method in the first place, all tied together into the compromise DSP template.

### 7.3 Looking Ahead: Data Center Design

In this section a discussion of the future of data center design, and suggested direction based upon the research performed in this thesis and its ultimate limitations is presented.

#### 7.3.1 Limitations of Air-Cooling Approach

There are fundamental limits to air cooling and the amount of heat that can be extracted using air as the working medium. This is because of the low thermal capacity of air, and thus the high flow rates that are required in order to dissipate large heat fluxes, such as the latest generation 30 kW IBM blade server [90]. The flow rates required to dissipate these large heat fluxes makes the data center environments incredibly unpleasant to human occupants, because of the large flow rates, thermal gradients, and

almost deafening noise levels. The fundamental theoretical limits to many different approaches to electronics thermal management, including air and single phase liquid cooling, is discussed in [31].

Faced with these fundamental limitations, several approaches for the integration of liquid cooling, specifically using water as the working fluid, have been proposed [34, 41, 90, 117]. These approaches all focus on the use of an air-water heat exchanger integrated into an enclosed cabinet to provide a greater amount of heat dissipation. However, although shortening the thermal path, these designs still employ air as the working medium for heat extraction from the chips. This is primarily because of the reluctance of data center designers and operators to run chilled water lines through their multi-million dollar computer systems, however, with the trends in data processing equipment indicating 60 kW cabinets are on the horizon, air cooling will be insufficient. It should also be noted that these systems use, on average, one quarter of the required supply of chilled water of a CRAC unit, and thus really only offer the advantage of reduced footprint, not greater efficiency. Ultimately, as indicated by Gurram, et. al. [31], direct liquid cooling of the server processors will be required for the heat loads anticipated. This will integrate the developments in processor cooling technology developed by other METTL researchers [4] with data center server design. This possibility is discussed further in the following section.

#### 7.3.2 Communication Between All Aspects of Data Center Design

Regardless of the heat flux and total power to be dissipated in a data center, greater communication between all levels and aspects of data center design, from the

servers and processor packaging to the overall center layout designs and guidelines need to be implemented at all stages of design. The multi-scale design approach is considered in this thesis, however, this approach focuses upon configuration and detailed design stages. This multi-scale design consideration and integration needs to occur as part of the original design processes. Examples of this disjointed design approach include individual air conditioning units on cabinets that exhaust their hot air directly into other servers and cabinets, or a new dense blade packaging design with vertical flow orientation that requires a flow rate that simply cannot be provided by most under floor plenums. However, new initiatives, such as the CEETHERM at Georgia Institute of Technology and University of Maryland, as well as research groups such as those lead by Dr. Schmidt at IBM, are working towards this goal, with more industry partners, representing all aspects of data centers from the air handling units to cabinet enclosures and the servers housed within them joining to contribute to complete multi-scale design consideration.

#### 7.4 Contributions

In this section a discussion of the research contributions is presented, classified into three sections: the specific and validated data center server cabinet design work, future extensions and augmentations of the approach with respect to data center design and analysis, and higher level possibilities and applications of the approach. The focus of this work has not been on finding the most efficient server cabinet, although the approach can be used to answer this question, but rather the more general challenge of the robust design of complex thermal-fluid systems. With this in mind, the contributions are as follows.

### 7.4.1 Cabinet Design

As shown and discussed in Section 7.1 the approach developed in this thesis addresses the six requirements set forth in this thesis, and hence also answers and validates the research questions and hypotheses. Rather than readdress these points, the greater value of the approach is discussed in the context of server cabinet design.

#### 7.4.1.1 Cabinet Configuration

The approach developed in this thesis is driven by the application to data center server cabinet configuration design. This is a fairly short term deisgn problem, in which new equipment is integrated in 2-3 year intervals. This approach enables data center designers and operators to determine the best cabinet configurations for their given heat loads, increasing the thermal efficiency, reducing costs, and increasing the life cycle of their current data center facility.

#### 7.4.1.2 Cabinet Heat Dissipation Prediction

The greatest strength of this approach is its value as a predictive tool, enabling the comparison of the potential heat dissipation of different cabinet architectures. This can be used as employed in this thesis, for the short term objective of housing higher powered equipment. However, it can also be used longer term, to determine the design potential of different architectures in various data center environments. Through simulation experimental mockup fabrication costs and time can be avoided.

#### 7.4.1.3 POD and PODc Validation

The POD and PODc are elegant mathematical constructs; however, they have yet to be applied to a real engineering problem. Specifically, the POD of a large, complex, RANS CFD simulation had yet to be tested. Through the POD and PODc's application and validation in this thesis, these constructs as well as the flux matching procedure have been shown to be very effective reduced order modeling approaches.

#### 7.4.1.4 General POD Parameter Transform

The computation of the fluxes from the individual POD modes in the flux matching procedure can be complex as node numbers from CFD simulations are ordered for bandwidth reduction, not for easy interpretation of the results. Furthermore, some fluxes, such as the heat flux to a turbulent fluid from a surface, cannot be computed. The general parameter transform is simple, quick, and transforms any parameters from the observation space to the POD space, thus avoids the difficult or infeasible task of determining each POD modes contribution to the flux goals specified.

## 7.4.1.5 Pareto Frontier Mapping

Pareto Frontiers are not a new concept, however mapping the frontier between the robust and most invariant solution points to determine the best solution with respect to the designer's objectives is new. Although Mourelatos and co authors [58] has also looked at this type of Pareto mapping, their focus was different, and the final considerations required by the designer were not discussed. This explicit mapping of the tradeoffs between optimality and minium variance enable the designer to identify the

operating point that best satisfies their needs rather than blindly relying upon a preference weighting scheme.

## 7.4.2 Further Data Center Design Work

The approach developed and applied in this thesis can easily be augmented and extended in application, as discussed in Section 7.2. The two main extensions of this approach as it pertains to data center design are discussed in this section.

#### 7.4.2.1 Data Center Design and Analysis

The approach can easily be extended to the design of a complete data center. The flow distribution through the plenum and network of perforated tiles is complex to model, but can be completed with reasonable accuracy [115]. Alternately, physical measurements can be used to create a simple meta-model, as described in Appendix B. Using this flow input, the flow throughout the data center can be modeled either using black box servers or multi-scale flux matching models. The block box approach results in a model with less DOF than the full scale 3D simulation used in this thesis, and hence there are no fundamental reasons why the POD modeling approach would not work.

## 7.4.2.2 Transient Modeling and Considerations

Transient scenarios have not been considered in this thesis as the steady state operation is of dominant interest in long running data center environments. However, the consideration of transient scenarios, such as CRAC unit failure have been considered in previous data center CFD analyses [71]. In transient computations the thermal mass of the cabinets, chips, and other equipment must be considered. This is not fundamentally difficult to model, however additional information regarding the physical properties of the data center equipment is required. The POD modeling approach can be coupled with an explicit solver to march forward in time, ensuring the cell Courant number meets the CFL stability condition. This will enable the simulation exploration of the concepts and control schemes suggested for dynamic control and global distribution of computing load of data centers [14, 69, 98].

## 7.4.2.3 Global and Complete System Simulation and Design

External auxiliary systems to the data center, such as the chiller load, are also not considered in this analysis. However, accurate system level modes for this type of industrial equipment exist had have been extensively used by the HVAC field. Integration of global heat fluxes and flow rates through the data center, computed using POD based models can be integrated with models of chiller operations and the associated plumbing system in a system of ordinary differential equations which can easily be solved using traditional numerical methods. This includes the consideration of water cooled cabinets, such as [34], because additional load is placed on the chiller directly, with some or no load on the CRAC units and circulated air within the center.

## 7.4.3 Broader Applications and Impacts

The final statement in the objective set forth in this thesis reads: "with the potential for broader multi-scale thermal-fluid simulation based design applications." The idea behind this statement is that although the primary motivation for the work in this thesis is the thermally efficient design of data center server cabinets, the approach

developed can also be applied to any complex and possibly multi-scale thermal-fluid design problem. This is important because turbulent fluid systems have proven problematic for the application of robust design because of the computational expense of the CFD models needed, and the many numerical derivative computations required to implement robust design.

The thermal-fluid problems presented in this thesis, while all server cabinets, actually space a decent range of geometry complexity and scale. From the previous POD applications research presented in Section 2.1.1, it is shown the POD reconstructions work accurately for a large range of geometry and flow regimes. Thus, the flux matching procedure, the core enabling construct used in this thesis, is also applicable to these problems. The flux matching procedure is augmented by the development of the general parameter transform approach, extending the range of applicability and computational efficiency of the flux matching procedure. Furthermore, there are no fundamental issues with the consideration of multi-scale modeling using this flux matching approach. This approach is finally integrated through the compromise DSP, providing all relevant information in an easily interpretable format. Therefore, overall this approach serves as the foundation of an effective design approach for one of the most complex and challenging areas of simulation based design.

The three core constructs used in this approach, the POD, robust design, and the compromise DSP have all been successfully applied to many domains, as discussed in Sections 2.1, 2.2, and 2.3 respectively. Hence there is no fundamental reason this

proposed approach cannot be extended to the more general domain of the robust design of thermal-fluid systems with equally successful results.

# 7.5 Closing Comments

In these closing remarks, the overarching goal of creating an approach for the thermal efficient and robust design of data center server cabinets is addressed. Through careful development, application to several representative example problems and thorough validation the approach development is solid. However, taking a step back and looking at the big picture, did the approach achieve the goal set forth at the beginning of the thesis?

- The initial results of the application of this robust design approach to server rack cabinet configuration are promising, the key results are:
- 50% more power than a uniform distribution can be reliably dissipated while maintaining equal emphasis on energy efficiency and stability.
- 15 °C reduction on the average potential variability of the processors can be achieved thorough emphasis design robustness.
- Any solution between the optimal and least varient can be selected from the family of solutions along the Pareto frontier generated by the compromise DSP.

• The small degree of analysis error incurred through assumptions and approximate models is nullified through the robustness of the solutions obtained, verified through CFD analysis and experimental measurement.

This approach applied and presented in this thesis takes a step towards addressing the challenge of reliable data center thermal management. The energy efficient cabinet configuration approach can be used to increase the thermal efficiency, considerably reducing the energy costs and environmental impact of operating a data center, while simultaneously increase the operational stability of the center also, reducing the cost associated with downtime and backup system maintenance.

The objective set forth in this thesis is the development of an approach to enable the robust design of data center server cabinets, with potential boarder application to any complex turbulent thermal-fluid system. The approach presented is founded upon the integration of three constructs; the POD, robust design principles, and the compromise DSP, to solve the challenges of flow complexity, system variability, and multiple objective tradeoffs, as described in Chapters 2 and 3. The viability of the approach is demonstrated through the application to the data center server cabinets, in the example problems investigated in Chapter 4, 5, and 6. The results obtained show that the approach enables the computation of superior solutions, both in ultimate power dissipation and reduction in variability, over a uniform power distribution, described in Chapter 5. Current research augmentations of the POD modeling method include the solution of multi-scale and conjugate heat transfer problems, as well as a formal approach for observation generation, to extend the domain of applicability of the approach presented, as discussed in Chapter 7. In summary, the principal objective set forth at the beginning of this thesis has been thoroughly completed.

## 7.6 Reflection

In this thesis the focus has been upon the development of an approach for the robust design of complex thermal fluid systems using simulation based design. The fundamental applicability of simulation based design is not questioned. In this section the applicability of the approach developed in this thesis is questioned and discussed.

The approach developed is reliant upon accurate CFD models, and their integration into the compromise DSP. Practically, this means integration of complex FEA analyses with a programming tool, such as MATLAB, C++, JAVA, or some other capable mathematical programming language. This requires significant ability of the user of this approach; they must be competent at CFD modeling and simulation, as well as programming and model integration. In its current form, although conceptually solid, much work remains to be done in order to bring simulation based design to a broader market.

The expense of the equipment housed in data centers and investment required makes the hiring of an expert consultant to model and integrate the simulations required in this approach feasible. However, ideally this approach should be applicable by anyone with an undergraduate engineering or technical degree. As it stands, this approach does not really meet this goal. Primarily what are needed are accurate, detailed models of the servers and cabinets. If these models were provided by the OEMs it would make large strides towards simplifying data center simulation work. Advanced automatic mesh generation programs like Simmetrix [2] that work with industry standard CAD programs and models such as Pro-Engineer go along way towards making simulation based design feasible on a broader scale, as accurate meshing is the most challenging task in FEA. Therefore, because every commercial product has these detailed CAD models created during their design (including servers and cabinets), the provision of these CAD models would facilitate easy simulation. Attempts to integrate models in a user friendly GUI environment such as Phoenix Integration's Model Center that also integrate easy DOE and optimization routines using these models help to address the second challenge, integration of the models and finding a solution. Finally, the POD modeling can be integrated into CFD analysis programs, such as FLUENT or FEMLAB as open source user defined functions.

As this work stands, it provides only an approach, something other knowledgeable and experienced engineers working with simulation and design can employ. Without the tools for the mass market to easily use simulation design, along with some checks and balances to limit its misuse, it will remain a good idea with limited practical applicability. The configuration of data centers as it stands today is best left to guidelines like ASHRAE using a flow hood and temperature probe. However, simulation based design approaches such as this can be used to explore the potential design space, leading to guidelines published for the general public implementation. Thus, this approach has its true value in this application at the current time.

# **APPENDIX A**

The work presented in this appendix serves to show what is required in order to use the approach developed in this thesis to configure an entire data center, not just a single server cabinet, as discussed in Section 7.4.2. In order to determine the airflow for the CRAC units through the raised floor plenum to the server cabinets a model of the airflow distribution is required. This can be estimated from CFD analysis, however with the multitude of blockages and obstructions the accurate modeling can become very complex [115]. Therefore physical measurements leading to a meta-model of the perforated tile flow rates can be developed. An example of perforated tile flow rates measured at the CEETHERM data center lab facility at Georgia Institute of Technology is presented in this appendix. The development of a model of the flow distribution at the room level using these measurement procedures is presented in Appendix B.

The layout of the perforated tiles creating the cold aisles in the CEETHERM data center lab is shown below in Figure A.1. The variables L1, L2, W1 and W2 are measurements of the position of the aisles with respect to the origin in the upper left corner. The gird unit of measure is a floor tile, measuring 2' x 2', the highlighted regions are the cold aisles measuring 2 by 8 tiles, and the rectangles on the outer regions are the CRAC units, the two downflow units are labeled.



Figure A.1 - Schematic of CEETHERM data center floor plan and cold aisle locations

The data was aquired by Charles Fraley, a local teacher from Dunwoody High School working as part of a technology and education fellowship. The measurements were made using a flow meter, shown below in Figure A.2. Seven measurements were made per tile to obtain an accurate mean and standard deviation estimate. The flow hood corrects for backpressure across the unit yielding accurate measurements.



Figure A.2 - Flow hood used to measure floor tile flow rates

The flow measurements were obtained with the CRAC units at 100% output, ensuring the maximum flow rate. The resulting values of mean flow rate and standard deviation for the tiles are plotted below. These results are important for comparison with the variations used in the cabinet investigations as the inlet flow rate variable is dominant in all cases. The resulting flow rates presented in the figures below are for  $W_1 = W_2 = 3$ ,  $L_1 = 5$  and  $L_2 = 12$ . The different bars give the plus/minus one standard deviation in measured flow rate, thus the middle stacked bar is the mean. This graphically shows the amount of variability through the perforated tiles, and thus why a robust treatment is appropriate.



Figure A.3 - Perforated tile flow rates for aisle 1



Figure A.4 - Perforated tile flow rates for aisle 2

Although no formal analysis of the data in the above figures is performed, the difference in both mean flow rate and variability within the tiles in a single cold aisle is considerable. This result backs us the need for an efficient, flexible approach for cabinet configuration for different data center air flow conditions.

# **APPENDIX B**

The work presented in this appendix serves to show what is required in order to use the approach developed in this thesis to configure an entire data center, not just a single server cabinet, as discussed in Section 7.4.2. In order to determine the airflow for the CRAC units through the raised floor plenum to the server cabinets a model of the airflow distribution is required. This can be estimated from CFD analysis, however with the multitude of blockages and obstructions the accurate modeling can become very complex [115]. Therefore physical measurements leading to a meta-model of the perforated tile flow rates can be developed. Sample flow rates for single specific cold aisle locations and the measurement technique is presented in Appendix A. In this section the approach for constructing a meta-model from these measurements is presented.

There are five variables under consideration in the CEETHERM data center lab, the positions of each cold aisle, represented by the variables  $L_1$ ,  $L_2$ ,  $W_1$ , and  $W_2$  in Figure B.1, and the flow configuration of the CRAC units, being both or one of the two down flow units on at a time. The measurement process is very time consuming, at about 1 hour per tile configuration. Thus Design of Experiments (DOE) is employed to reduce the number of runs required to obtain an accurate meta-model.



Figure B.1 - Schematic of CEETHERM data center floor plan and cold aisle locations

The DOE suggested is a Central Composite Design (CCD) [61], as it yields a quadratic response surface model which give an estimate of the variables interactions as well as a second order estimate of the main effects. In cases such as the data center aisle configuration for the CEETHERM facility, a square design region is not available, and thus a D-optimal design approach is preferred [61]. In the case of the CEETHERM lab, there is a constraint acting on  $L_1$  and  $L_2$ :

$$L_1 - L_2 > 3$$
 (B.1)

This results in a triangular arrangement of feasible test points. The D-optimal criterion [61] is shown below in equation B.2.

$$\frac{(b-\beta)'X'X(b-\beta)}{pMSE} \le F(1-\alpha; p, n-p)$$
(B.2)

This is the equivalent of the minimization of equation B.3, as shown in [61].

$$D = \left| (X'X)^{-1} \right| \tag{B.3}$$

The objective of a D-optimal design is to find the experimental points that will yield the smallest volume confidence interval for a given model, number of experiments to run, and a set of feasible experimental points. The algorithms used to solve these problems are out of the scope of this thesis, however the popular approach the coordinate exchange algorithm is part of the MATLAB statistical toolbox [109]. Using a quadratic response model, and 12 experiments, and the geometry of the CEETHERM data center lab the following D-optimal design for a response of the cold aisle perforated tile flow rates is obtained. The entire set of feasible points is obtained using a full factorial design of all cold aisle locations. The resulting points for  $L_1$  and  $L_2$  along with all feasible points are displayed below in Figure B.2.



Figure B.2 - D-optimal design of experiments for  $L_1$  and  $L_2$  for CEETHERM data center lab floor tile flow measurements

The complete results are tabulated below in Table B.1.

Run	Ll	<i>L2</i>	W1 = W2
1	4	11	5
2	4	16	0
3	9	16	0
4	9	16	5
5	4	16	5
6	7	14	0
7	4	11	0
8	4	14	0
9	6	13	3
10	6	16	5
11	7	16	2
12	4	14	3

Table B.1 - D-optimal experimental points, complete results

The strengths of the D-optimal approach for data center floor tile flow rate metamodeling is in the discrete nature of the tile locations. This means that a starting set of feasible test points is easily obtained, as arbitrary discretization of a continuous variable is not performed. This approach for flow measurement is presented as it is related to the thesis work; however, it is not directly part of the approach. This documentation serves to enable other METTL researchers to obtain accurate models of the CEETHERM facilities tile flow rates and further investigate the effectiveness of this meta-modeling approach.

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