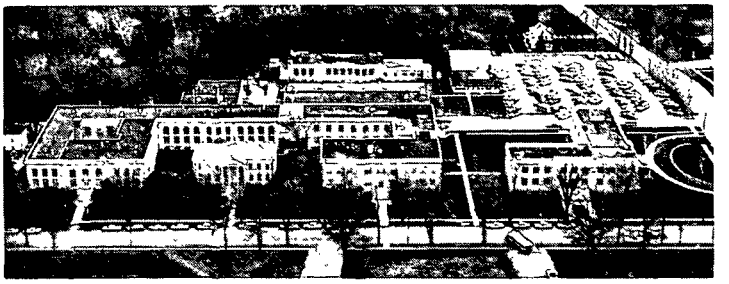

THE INSTITUTE OF PAPER CHEMISTRY, APPLETON, WISCONSIN

IPC TECHNICAL PAPER SERIES
NUMBER 54

ULTRASONIC PLATE WAVES IN PAPER

C. C. HABEGER, R. W. MANN, AND G. A. BAUM

MARCH, 1978

Ultrasonic Plate Waves in Paper

C. C. HABEGER, R. W. MANN, AND G. A. BAUM

INTRODUCTION

The ability of paper to resist failure when subjected to stresses is an important property in most all end-use applications. A variety of tests exist which are designed to measure "paper strength," but these are all destructive tests which must be performed off the paper machine. It would be desirable to develop a nondestructive test which could be used on the moving web to provide an estimate of paper strength. The investigation reported here is the first part of an effort to develop such instrumentation, based upon acoustical measurements.

The work described involves the development of a model which characterizes the propagation of acoustic waves in paper. The model predicts the relationships between sound velocity and measuring frequency, as functions of the elastic constants, thickness, and density of the paper. The model indicates that the low frequency range will be the most desirable in which to make measurements in paper. This is because the elastic parameters, certain of which correlate with strength properties, are most easily related to the measured velocities at low frequencies. The results also indicate that the anisotropy in the paper, especially in the thickness direction, plays an important role in the propagation of sound in paper. These observations provide a foundation for future efforts relating to the development of instrumentation for on-line strength measurements.

This paper is being submitted for publication in Ultrasonics.

Ultrasonic plate waves in paper

C. C. HABEGER; R. W. MANN; G. A. BAUM

ABSTRACT

A theoretical and experimental study of ultrasonic plate waves in machine-made paper is described. The paper is assumed to behave as a homogeneous orthotropic plate. The dispersion equation for orthotropic plate waves in principal directions is then developed analytically. It is shown that at low frequencies the orthotropic dispersion equation depends only on in-plane parameters. Using appropriate elastic constants, a computer is used to construct the dispersion curves and describe the internal motion for plate waves in paper. Finally, a portion of the dispersion curve is checked experimentally.

Introduction

An instrument capable of measuring paper strength on-machine would be useful in the paper industry for quality and process control. It is possible that this could be accomplished by measuring acoustical velocities in the moving web. Craver and Taylor[1] demonstrated experimentally that the velocity of the zeroth order symmetric mode at low frequencies correlated with paper tensile strength. The possibility of using piezoelectric transducers which contact the moving web and determining the ultrasonic velocities by transit time measurements has been investigated[2,3]. The extreme mechanical and electrical noise found in the mill, however, has prevented completion of a successful instrument. Luukkala et al[4] have described a noncontact method for measuring the phase velocity of plate waves in a dispersive region. This method shows promise in overcoming the

noise problems. An adaptation of the technique is used in this study and will be discussed later.

Machine-made paper exhibits a high degree of elastic anisotropy. The material displays the highest Young's modulus in the machine direction (MD). This is a result of a preferential alignment of fibers in this direction and also a consequence of the greater stresses which exist in the web in this direction during drying. By contrast Young's modulus in the cross-machine direction (CD) is about one-half as large. A good description of these phenomena has been presented by Van den Akker[5]. The difference between the MD and CD elastic moduli, however, is small compared to that observed between the in-plane and z-direction (ZD) moduli. Young's modulus in the thickness or z-direction is about 1/100th that in the MD. This can be demonstrated by comparing the velocity of a dilational bulk wave in the ZD to the low frequency velocity of a zeroth order symmetric plate wave.

Previous studies have ignored the effect of ZD properties on plate waves in paper. Rather, isotropic plate wave theory was employed[3,4] or it was assumed that paper could be treated as a planar material[6]. It will be shown here that the ZD elastic properties have a large influence on the plate wave dispersion curves and on the nature of the wave motion.

Theory

Before developing a theory for plate waves in machine-made paper, it will be beneficial to review qualitatively the nature of plate waves in an isotropic material[7,8]. The first step for the isotropic theory is to find the normal modes of oscillation in the bulk material. In an infinite, isotropic material there are two distinct types of simple harmonic motion.

These are called dilational waves and distortional waves. Dilational waves involve particle displacements in the direction of propagation and travel faster than distortional waves, whose particle motion is normal to the propagation direction. Both types of waves are nondispersive.

When a distortional or dilational wave is reflected from a free boundary, however, partial mode conversion occurs. A free boundary couples the two types of waves; so that (with the exception of distortional waves polarized in the plane of the plate) pure bulk modes cannot exist in a plate. The normal modes of oscillation in the plate can be expressed as a sum of two distortional waves and two dilational waves. The magnitudes and directions of these bulk waves are chosen so that the free boundary conditions are met at the plate surfaces. All four bulk waves have the same wavelength projected along the plate, but the transverse components of the wave vectors are positive for one distortional and one dilational wave, and negative for the other two. Since the distortional wave is slower, its direction of propagation is nearer to the plate normal. Along the plate, the plate waves are plane waves with a phase velocity, C, equal to the angular frequency divided by the component of the bulk wave vector along the plate. As the frequency changes, a corresponding change in the direction of the bulk waves is required in order to satisfy the boundary conditions. Figure 1 depicts the nature of this change in terms of dispersion curves which relate plate phase velocity to frequency. Plate waves can be described as symmetric or antisymmetric, depending on the symmetry about the midplane of the particle displacement along the plate. Symmetric waves are sometimes described as longitudinal modes and antisymmetric waves as flexural modes.

In this study the elastic properties of machine-made paper are assumed to obey orthotropic symmetry relations. The validity of this assumption has been experimentally established in the plane of the sheet[9], but not in the three-dimensional structure. An analysis of acoustic waves in orthotropic plates is more complicated than in the case of isotropic plates, because nine instead of two elastic constants are involved. However, the procedure outlined above can be used to find the orthotropic dispersion equation and describe plate wave motion. First the well-known relationships between orthotropic bulk wave velocities, displacement polarizations, and propagation directions will be derived.

For the orthotropic material, the stresses, τ_{ij}, can be expressed in terms of the strains, ε_{ij}, as follows.

$$
\begin{aligned}
\tau_{11} &= C_{11}\varepsilon_{11} + C_{12}\varepsilon_{22} + C_{13}\varepsilon_{33} \\
\tau_{22} &= C_{12}\varepsilon_{11} + C_{22}\varepsilon_{22} + C_{23}\varepsilon_{33} \\
\tau_{33} &= C_{13}\varepsilon_{11} + C_{23}\varepsilon_{22} + C_{33}\varepsilon_{33} \\
\tau_{23} &= 2C_{44}\varepsilon_{23} \\
\tau_{13} &= 2C_{55}\varepsilon_{13} \\
\tau_{12} &= 2C_{66}\varepsilon_{12}
\end{aligned}
\tag{1}
$$

The strain is defined in terms of U_i, the particle displacement from equilibrium, as

$$\varepsilon_{ij} = (U_{i,j} + U_{j,i})/2 , \qquad i,j = 1,2,3 \tag{2}$$

In paper, the MD, CD, and ZD are the three principal directions of symmetry, 1, 2, and 3, respectively. In the following analysis the discussion is

limited to the MD-ZD plane. All results, however, can be applied to the CD-ZD plane by changing the appropriate elastic constants.

The equations of motion for mechanical disturbances in an elastic medium are

$$\sum_{j=1}^{3} \tau_{ij,j} = \rho \ddot{U}_i, \qquad i = 1,2,3 \tag{3}$$

where ρ is the mass density of the medium. In order to construct plate waves in the principal directions, it is necessary to first find plane wave solutions to equation (3) whose direction of propagation and polarization are normal to one of the in-sheet principal directions. For bulk waves propagating the MD-ZD plane, U_2 and all derivatives with respect to the CD must be zero. Using equation (1) to express τ_{ij} in terms of U_i, equation (3) (for MD-ZD plane waves) becomes

$$\rho\ddot{U}_1 = C_{11}U_{1,11} + C_{13}U_{3,31} + C_{55}(U_{1,33} + U_{3,13}) \tag{4}$$

$$\rho\ddot{U}_3 = C_{33}U_{3,33} + C_{13}U_{1,13} + C_{55}(U_{1,13} + U_{3,11}) \tag{5}$$

Requiring that the solutions represent plane waves means that

$$U_1 = U_{10} \exp\,(i(k_x x + k_z z - \omega t)) \tag{6}$$

and

$$U_3 = U_{30} \exp\,(i(k_x x + k_z z - \omega t)) \tag{7}$$

So, equations (4) and (5) become

$$\rho U_{10}\omega^2 = C_{11}U_{10}k_x^{\,2} + (C_{55}+C_{13})\, U_{30}k_x k_z + C_{55}U_{10}k_z^{\,2} \tag{8}$$

and

$$\rho U_{30}\omega^2 = C_{55}U_{30}k_x^{\,2} + (C_{55}+C_{13})\, U_{10}k_x k_z + C_{33}U_{30}k_z^{\,2} \tag{9}$$

For a given frequency, ω, and wave vector in the MD, k_x, equations (8) and (9) can be used to find the allowed k_z's and corresponding values of U_{30}/U_{10}'s. Using equations (8) and (9) to eliminate U_{10} and U_{30}, it is found that k_z^2 must satisfy the following quadratic equation;

$$(C_{55}+C_{13})^2 k_x^2 k_z^2 = (\rho\omega^2 - C_{11}k_x^2 - C_{55}k_z^2)(\rho\omega^2 - C_{55}k_x^2 - C_{33}k_z^2) \tag{10}$$

Given a value of k_z, U_{30}/U_{10} can be determined from equation (11):

$$U_{30}/U_{10} = (\rho\omega^2 - C_{11}k_x^2 - C_{55}k_z^2)/(C_{55}+C_{13})k_x k_z \tag{11}$$

For $k_x^2 = 0$, equation (10) permits two possible values of k_z^2. One of the solutions is $k_z^2 = \rho\omega^2/C_{33}$. For this value of k_z^2, U_{30}/U_{10} is found from equation (11) to be infinity. This is a dilational wave traveling in the ZD with velocity $C_z = (C_{33}/\rho)^{1/2}$. The other solution is a distortional wave in the ZD with velocity $C_s = (C_{55}/\rho)^{1/2}$. If $k_z^2 = o$, equation (10) yields two other solutions. They are a distortional wave in the MD with velocity $C_x = (C_{11}/\rho)^{1/2}$ and a distortional wave in the MD with velocity C_s. Therefore, orthotropic bulk waves, traveling in a principal direction, are similar to isotropic bulk waves. That is, the disturbances for the two plane wave solutions are normal or parallel to the direction of travel.

For waves oriented at an angle to the principal axes ($k_x \neq 0$, $k_z \neq 0$) the polarization of the two solutions is neither normal nor parallel to travel. In this more general case, the two solutions of k_z^2 from equation (10) are represented by $k_z^2\pm$ in equation (12):

$$k_z^2\pm = k_x^2(- B \pm (B^2 - 4D)^{1/2})/2 \tag{12}$$

where $B = \rho[C_{33}(C_{11}/\rho - \omega^2/k_x^2) - C_{13}(2C_{55} + C_{13})/\rho - C_{55}\omega^2/k_x^2]/C_{33}C_{55}$, and $D = \rho^2(\omega^2/k_x^2 - C_{55}/\rho)(\omega^2/k_x^2 - C_{11}/\rho)/C_{33}C_{55}$. For a given frequency, ω, and component of the wave vector in the MD, k_x, equation (12) establishes the four possible values of k_z for orthotropic bulk waves. Equation (11) can then be used to find the polarization of the disturbance. These waves are nondispersive, but the velocity and the angle between the wave vector and the polarization depend on the direction of propagation.

Now that bulk waves have been studied, plate waves will be constructed from combinations of bulk waves. The MD plate modes must be plane waves along the machine direction. As shown above for any MD component of the wave vector there are four bulk plane waves. Since any combination of these four waves is a plane wave in the MD, sums of these bulk waves are sought which meet the free boundary conditions. It will turn out that a solution exists only for certain combinations of ω and k_x. Since the phase velocity, C, of a plate wave is ω/k_x, dispersion curves relating frequency to velocity can then be constructed from the permissible combinations of ω and k_x.

Before proceeding a few words on notation are in order. $k_z\pm$ are defined as the square roots of $k_z^2\pm$ having positive real parts. $\tan\phi\pm$ are defined as the values of U_{30}/U_{10} when $k_z\pm$ are inserted into equation (11). The $z = 0$ plane is at the center of the plate and the boundaries will be located at $z = \pm b$.

The possible plate wave solutions have the following form:

$$U_1 = \exp(i(k_x x-\omega t)) \, [M \exp(ik_{z+}z) + N \exp(-ik_{z+}z) + P \exp(ik_{z-}z) + Q \exp(-ik_{z-}z)] \tag{13}$$

$$U_3 = \exp(i(k_x x-\omega t))\ [\tan\phi+(M \exp(ik_{z+}z) - N \exp(-ik_{z+}z)$$

$$+ \tan\phi-(P \exp(ik_{z-}z) - Q \exp(-ik_{z-}z))] \tag{14}$$

The boundary conditions which must be met at $z = \pm b$ are

$$\tau_{33} = C_{33}U_{3,3} + C_{13}U_{1,1} = 0 \tag{15}$$

and

$$\tau_{31} = C_{55}U_{1,3} + C_{55}U_{3,1} = 0 \tag{16}$$

Substituting U_1 and U_3 from equations (13) and (14) into equations (15) and (16), imposes the four following conditions on M, N, P, and Q.

$$\begin{bmatrix} G_+\exp(ik_{z+}b) + G_+\exp(-ik_{z+}b) + G_-\exp(ik_{z-}b) + G_-\exp(-ik_{z-}b) \\ G_+\exp(-ik_{z+}b) + G_+\exp(ik_{z+}b) + G_-\exp(-ik_{z-}b) + G_-\exp(ik_{z-}b) \\ H_+\exp(ik_{z+}b) - H_+\exp(-ik_{z+}b) + H_-\exp(ik_{z-}b) - H_-\exp(-ik_{z-}b) \\ H_+\exp(-ik_{z+}b) - H_+\exp(ik_{z+}b) + H_-\exp(-ik_{z-}b) - H_-\exp(ik_{z-}b) \end{bmatrix} \begin{bmatrix} M \\ N \\ P \\ Q \end{bmatrix} = 0 \tag{17}$$

where

$$G\pm = C_{33}k_z\pm\tan\phi\pm + C_{13}k_x, \tag{18}$$

$$H\pm = k_z\pm + k_x\tan\phi\pm \tag{19}$$

There are nonzero solutions for U_i only if the determinant of the matrix in equation (17) is zero. This requirement reduces to:

$$\frac{\tan(k_{z+}b)}{\tan(k_{z-}b)} = \left[\frac{H_-G_+}{H_+G_-}\right]^{\pm 1} \tag{20}$$

Using equations (11) and (12) to eliminate the $\tan\phi$'s and k_z's, equation (20) becomes a relationship between ω and C that must be satisfied if a plate wave is to exist. This is the orthotropic plate

wave dispersion equation. The motion in a particular mode can be found by choosing values of ω and C that satisfy equation (20), finding M, N, P, and Q from equation (17), and substituting into equations (13) and (14) to find U_i as a function of x, z, and t. As in the isotropic case, the solutions can be classified by the symmetry of motion in the sheet direction. The plate waves corresponding to the +1 exponent in equation (20) are called symmetric modes. The antisymmetric modes are those solutions with a -1 exponent. For the symmetric modes M = N and P = Q, while antisymmetric modes have M = -N and P = -Q.

Low frequency approximations

The orthotropic dispersion equation is a complex transcendental equation with a large number of terms containing elastic constants. In general it can be studied only numerically with the aid of a computer. However, in the limit as $k_x b$ approaches zero, some rather surprising simplifications occur.

One interesting result is discovered when the zeroth order symmetric (S_o) mode is studied at low frequencies. This is done by taking the limit of the symmetric dispersion equation as $k_x b \to 0$. That is, the value of C that satisfies the following equation is found.

$$\lim_{k_x b \to 0} \frac{\tan(k_{z+} b)}{\tan(k_{z-} b)} = \frac{H_- G_+}{H_+ G_-} \tag{21}$$

From equation (12) $k_z^{2\pm} = f(C)_\pm^2 k_x^2$, where $f(C)_\pm^2 = [-B\pm(B^2-4D)^{1/2}]/2$. Expanding equation (21) to first order in $k_x b$ then gives

$$\frac{f(C)_+}{f(C)_-} = \frac{H_- G_+}{H_+ G_-} \tag{22}$$

The next step is to expand $f(C)_{\pm}$, $H_{\pm}$, and $G_{\pm}$ in terms of the C_{ij}'s and C. After considerable algebra, equation (22) simplifies to

$$C^2 = (C_{11} - C_{13}{}^2/C_{33})/\rho \tag{23}$$

Expressing the C_{ij}'s in terms of Young's moduli and Poisson's ratios, this takes a more familar form,

$$C^2 = E_1/\rho(1-\nu_{12}\nu_{21}) \tag{24}$$

Since equation (24) involves only in-sheet elastic parameters, the low frequency S_o velocity does not directly depend on ZD properties. This result becomes more reasonable when the equations of motion [equations (13) and (14)] are examined in the limit $k_x b \to 0$. It is easily shown for the symmetric case that $U_3(\max)/U_1(\max) \to 0$ as $k_x b \to 0$. Therefore, this mode does not involve ZD motion and its velocity should not depend on ZD properties. Notice that to first order this velocity is not a function of ω and the mode is nondispersive.

For an isotropic plate, equation (24) takes the well known form:

$$C^2 = E/\rho(1-\nu^2) \tag{25}$$

So if $E_1/\rho(1-\nu_{12}\nu_{21})$ in the orthotropic plate equals $E/\rho(1-\nu^2)$ in the isotropic plate, the low frequency S_o modes have equal velocities.

Paper is sometimes approximated as a planar material. This means that the ZD stresses are assumed to be zero and ZD motion is ignored. In this case it is easily shown that the velocity of the dilational mode in the MD is $[E_1/\rho(1-\nu_{12}\nu_{21})]^{1/2}$. This is identical to the plate wave velocity derived above. For the orthotropic S_o wave, τ_{11} is the only stress that

does not approach zero as $k_x b \to 0$. Therefore, in this limit the planar assumption is valid and is expected to provide the correct velocity. However, as the frequency increases ZD motion begins and the plate mode becomes dispersive.

Now the A_o mode at low frequency will be studied. Here $C \to 0$ as $\omega \to 0$, and to get additional information, $\tan(k_{z\pm}b)$ will be expanded one term further than above. In this case the dispersion equation becomes

$$\frac{f(C)_+[1 + k_x{}^2b^2f(C)_+^2/3]}{f(C)_-[1 + k_x{}^2b^2f(C)_-^2/3]} = \frac{H_+G_-}{H_-G_+} \tag{26}$$

A tedious algebraic manipulation, gives

$$C^2 = b^2k_x{}^2[(C_{11}-C_{13}{}^2/C_{33}) - \rho C^2(1+2C_{13}/C_{33}) - \rho^2C^4/C_{33}]/3\rho \tag{27}$$

Ignoring higher order terms ($\geq k_x{}^2C^2$), equation (27) becomes

$$C = bk_x[(C_{11}-C_{13}{}^2/C_{33})/3\rho]^{1/2} \tag{28}$$

Expressing equation (28) in terms of E's and ν's gives,

$$C = bk_x[E_1/3\rho(1-\nu_{12}\nu_{21})]^{1/2} \tag{29}$$

Again, only the in-plane elastic parameters are involved. The A_o mode has the same low frequency behavior as in the hypothetical isotropic plate. In summary, the low frequency orthotropic dispersion equation is identical to that in a properly chosen isotropic plate.

Dispersion curves

It is necessary to experimentally verify the dispersion curves predicted by equation (20). Before this can be done, the elastic constants in

equation (20) must be determined. For MD plate waves, this means C_{11}, C_{13}, C_{33}, and C_{55} are needed. The method used to establish these numbers is the subject of another paper[10]. Briefly the procedure is as follows. C_{55} is determined by measuring the velocity of the bulk distortional wave traveling in the CD. C_{33} is calculated from the velocity of the bulk dilational wave in the ZD. The velocity of the bulk dilational wave in the MD yields C_{11}. In order to make this measurement, a paper structure of thickness greater than the ultrasonic transducer width must be made. This is done by joining sheets together in a special way. Once C_{11} and C_{33} are known, C_{13} can be calculated from a measurement of the velocity of the zeroth order symmetric plate wave at low frequency. Since this mode involves no ZD motion, it is acceptable to use its velocity to set the bounds on a theory whose purpose is to predict the effects of ZD properties. Plate waves in the CD can be characterized by measuring C_{22}, C_{23}, C_{33}, and C_{44} in an identical way.

These values were measured for a 0.75 mm thick sheet of chipboard. A computer was used to obtain the solutions to equation (20). Figure 2 shows the results for plate waves in the MD. To illustrate the effect of ZD properties, these curves should be compared to the dispersion curves for the isotropic sheet depicted in Fig. 1. The elastic parameters used for the fictitious isotropic plate were chosen to best represent the MD characteristics of the chipboard sample. The same density and thickness were used for both calculations. The stiffness of the isotropic plate was set equal to C_{11} of the chipboard. The remaining isotropic elastic constant was specified by requiring that $E/(1-\nu^2) = E_1/(1-\nu_{21}\nu_{12})$. This causes the low frequency behavior of the two dispersion curves to be equal. Since ZD properties do not effect low frequency motion, this is a proper

choice. Figure 3 presents the dispersion curves for plate waves in the CD. The effect of changing the in-plane moduli is quite evident.

For the case of chipboard, with a low ZD modulus, a comparison of the isotropic and orthotropic dispersion curves reveals that for the **latter,**

(1) higher order modes begin to propagate at lower frequencies;

(2) the S_o becomes dispersive at a lower frequency;

(3) the A_o reaches a plateau at lower frequencies; and

(4) all modes asymptotically approach a smaller Rayleigh velocity at high frequencies.

[Figs. 1, 2, and 3 here]

Intuitively, the first three results are reasonable. Such phenomena would be expected to occur at frequencies where the wavelength of a ZD dilational wave is of the order of the plate thickness. This happens at lower frequencies because the low ZD modulus results in a lower ZD dilational wave velocity. At high frequencies the velocities of all modes approach the Rayleigh velocity. Here, all motion is concentrated at the surfaces, and the distinction between different modes is lost. The Rayleigh velocity increases rapidly with C_{55} (C_{44} for CD waves) which is small due to the low ZD moduli. Therefore, the fourth result also seems reasonable.

If the orthotropic plate thickness is increased, but the elastic moduli held constant, the various modes will also respond as noted in observations 1, 2, and 3 above, and conversely. The same comparisons between the isotropic and orthotropic cases, however, still apply.

Another interesting and unexpected result is the development of plateaus in the curves for higher order symmetric modes. These are nearly nondispersive regions at roughly the same velocity as the low frequency S_o mode. The isotropic theory predicts inflection points in the symmetric modes, but it does not allow for the broad flat areas shown in the orthotropic curves.

It is also possible to illustrate the nature of the particle motion associated with the respective plate waves. Once an acceptable combination of ω and C is chosen from a dispersive curve, the equations of motion can be solved as described earlier. Figure 4 illustrates the effect of a set of MD plate waves on a rectangular grid for some selected modes.

[Fig. 4 here]

Experiment

To test the predictions of the theory, plate wave velocities were measured on the chipboard sample. Since the low frequency S_o mode is nondispersive, it can be found by simply measuring the time delay of an ultrasonic pulse between two locations on a plate. However, this straight forward procedure is not applicable in general. To the authors' knowledge, the only successful measurements of dispersive waves in paper have been achieved using an air-paper resonance technique[4]. An adaptation of this procedure has been used here.

In the air-paper resonance technique, a sheet of paper is mounted between an ultrasonic transmitter and receiver. The two transducers are rigidly connected so that they always face each other, but they can be rotated with respect to the plane of the sheet. As the transducers rotate the

wavelength of the disturbance along the sheet changes. At an angle where the frequency and wavelength along the sheet correspond to those of a plate wave mode, optimum transfer of energy occurs. This results in a peak in the receiver signal. Therefore, the velocities of plate modes can be measured by recording peaks in the signal vs. angle curves at different frequencies. The technique used here differed from that described by Luukkala et al in one important aspect. Instead of using DC biased electrostatic transducers, permanently polarized dielectric films were used as the active medium in the transducers. These electric films were made by voltage cycling 0.125 mm F.E.P. Teflon films as described by Curtis[11].

The portion of the dispersion curves that can be measured using the air-paper resonance technique is limited by a number of factors. First, attenuation of ultrasonic waves in air limits the frequency range to below about 350 kHz. Also, since the wavelength along the sheet is always greater than in the direction of propagation, velocities below that of sound in air cannot be measured. This limits the measurements to velocities above 0.38 mm/μsec. One result of these limitations is that a sheet must be fairly thick ($\sim$ 0.4 mm) in order to excite plate waves by this method.

The results obtained on the chipboard sample using this technique are given in Fig. 5 and 6 for the MD and CD, respectively. Although the range of data is limited for the reasons cited above, the agreement with the theory is quite good. The A_o mode for the CD data does not appear because the velocity of this mode is apparently less than that of air. The low frequency S_o mode was not excited. This is believed to be due to the difficulty in using pressure waves to excite modes which do not involve large ZD displacements.

[Figs. 5 and 6 here]

The theoretical and experimental results presented above clearly demonstrate that plate waves in machine-made paper must be described using orthotropic theory rather than isotropic theory.

Acknowledgments

This work was supported by the pulp and paper industry through The Institute of Paper Chemistry.

Part of this report is taken from a dissertation submitted by R. W. Mann in partial fulfillment of the requirements for a Ph.D. degree at The Institute of Paper Chemistry, Appleton, Wisconsin.

References

1 Craver, J. K., Taylor, D. L., Tappi 48 (1965) 142.

2 Lu, M. T., Tappi 58 (1975) 80.

3 Papadakis, E. P., Tappi 56 (1973) 74.

4 Luukkala, M., Heikkila, P., Surakka, J., Ultrasonics 9 (1971) 201.

5 Van den Akker, J. A., Tappi 53 (1970) 388.

6 Taylor, D. L., Craver, J. K., Trans. of Cambridge Symp. Tech. Sect. of BPBMA, London, (1966) 852.

7 Viktorov, I. Rayleigh and Lamb Waves, Plenum Press, New York (1967).

8 Graff, K. Wave Motion in Elastic Solids, Ohio State University Press (1975).

9 Jones, A. R., Tappi 51 (1968) 203.

10 Mann, R. W., Baum, G. A., Habeger, C. C., submitted to Tappi.

11 Curtis, G., Ultrasonics 12 (1974) 148.

Charles C. Habeger, Jr., Ph.D.
Research Associate
Instrumentation Group
The Institute of Paper Chemistry
Appleton, Wisconsin 54911

Ronald W. Mann, M.S.
Student
The Institute of Paper Chemistry
Appleton, Wisconsin 54911

Gary A. Baum, Ph.D.
Group Leader
Instrumentation Group
The Institute of Paper Chemistry
Appleton, Wisconsin 54911

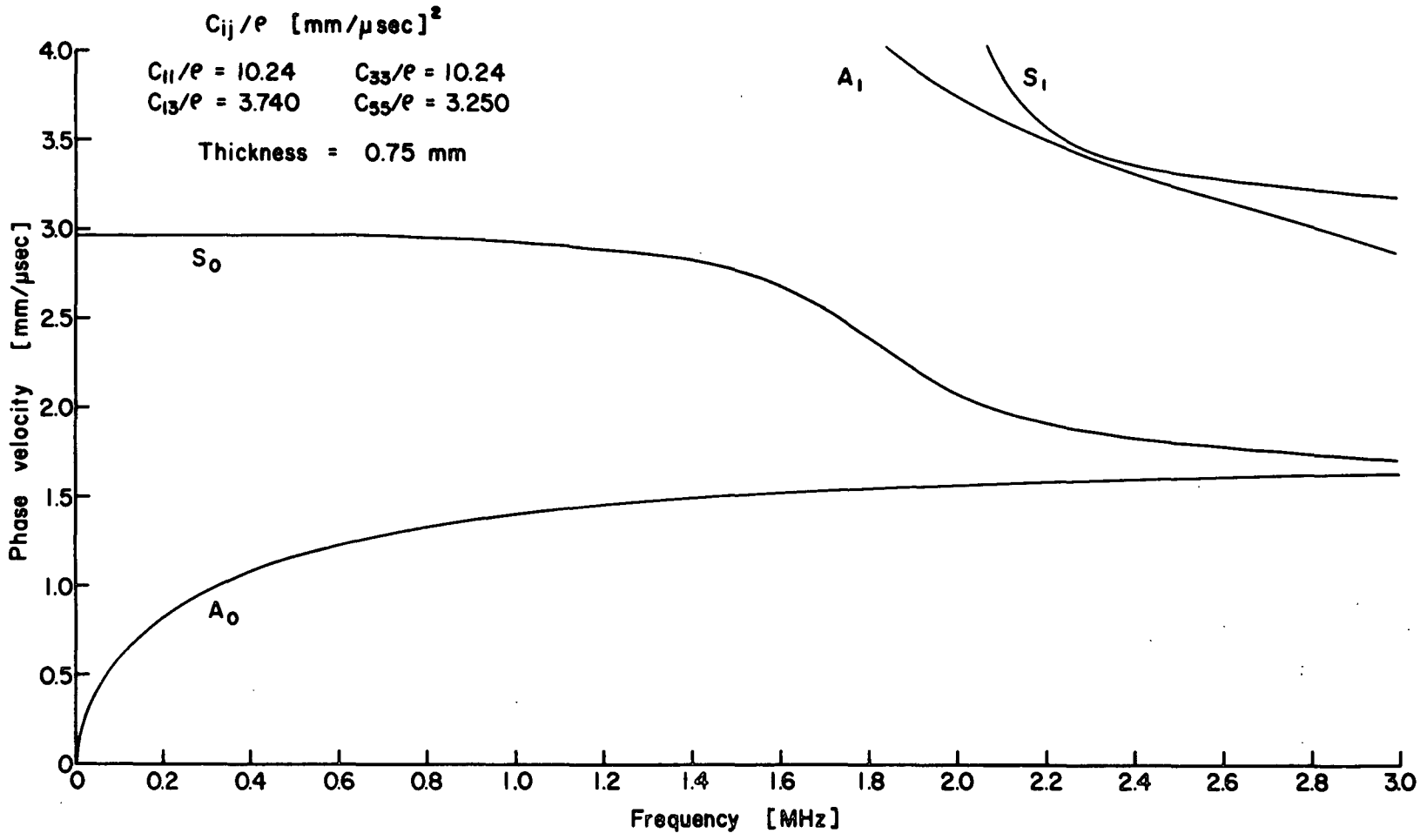

Fig. 1. Dispersion curves for plate waves in an isotropic medium. The low frequency behavior is chosen to be the same as for the orthotropic material in the MD in Fig. 2, as described in the text. The in-plane stiffness, the thickness, and the density are also the same as in Fig. 2.

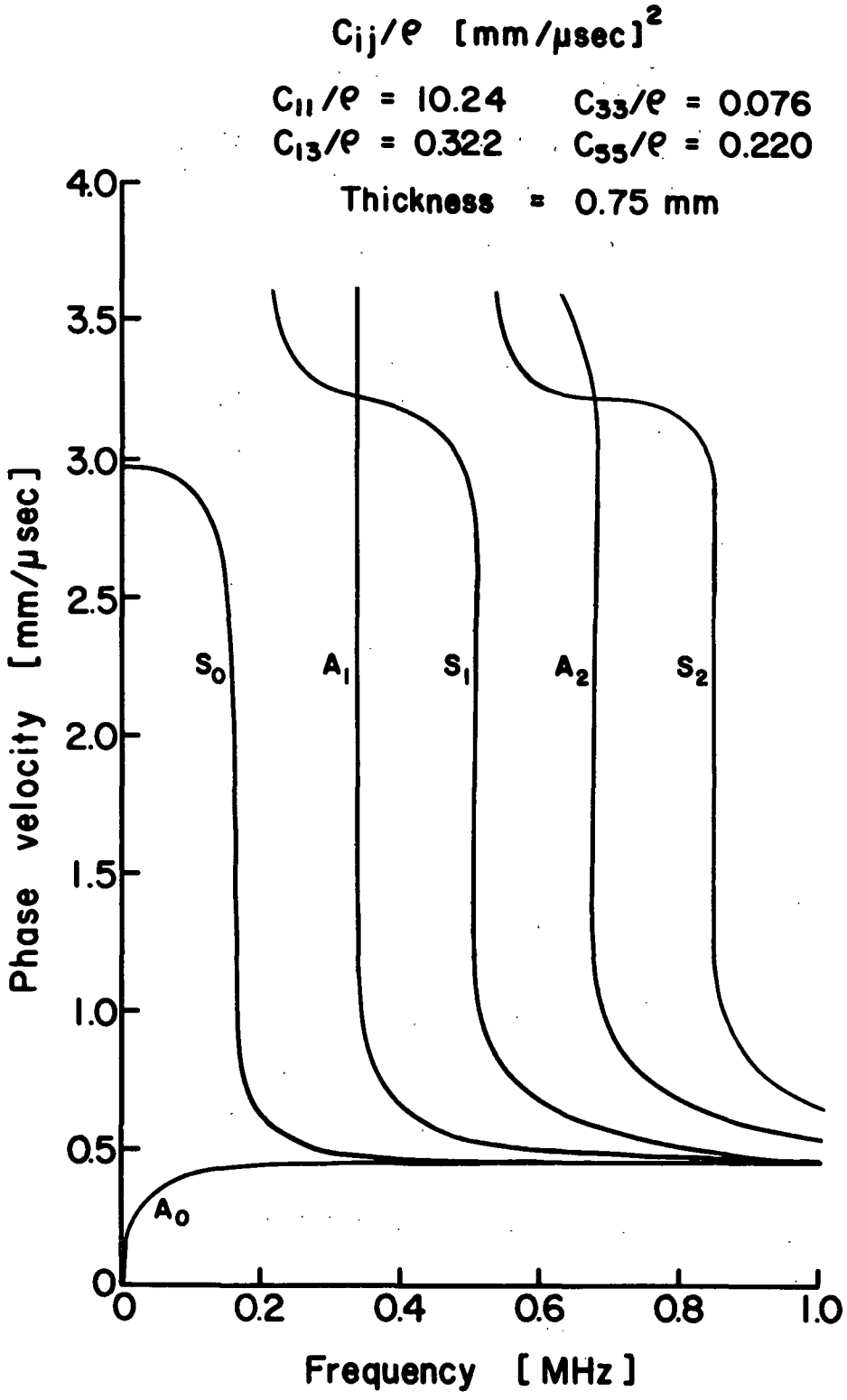

Fig. 2. MD dispersion curves for plate waves in an orthotropic medium. The elastic parameters are equal to those measured for an 0.75 mm chipboard sample.

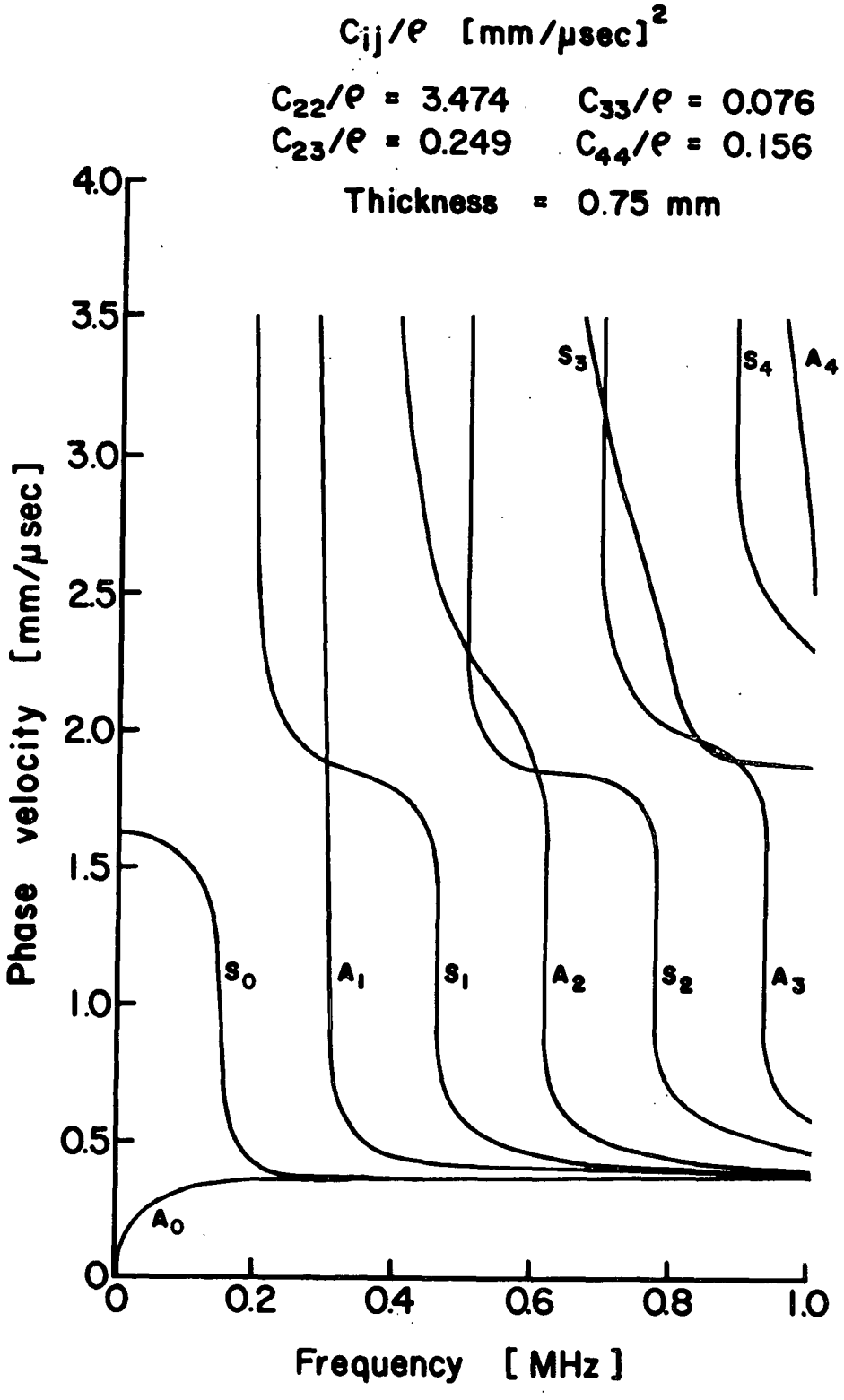

Fig. 3. CD dispersion curves for plate waves in an orthotropic medium. The elastic parameters are equal to those measured for an 0.75 mm chipboard sample.

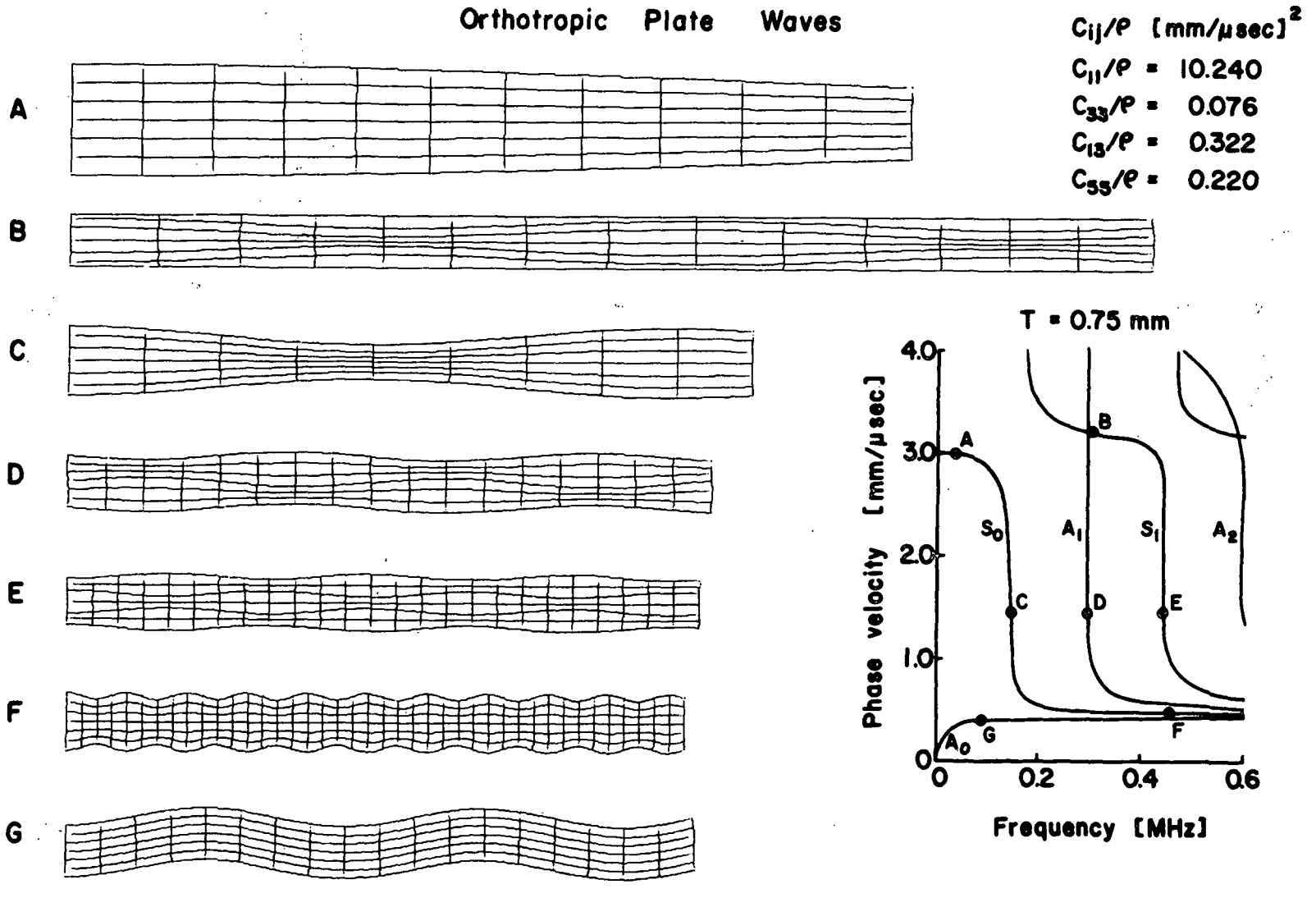

Fig. 4. Deformation of a rectangular grid caused by MD plate waves in an 0.75 mm chipboard sample.

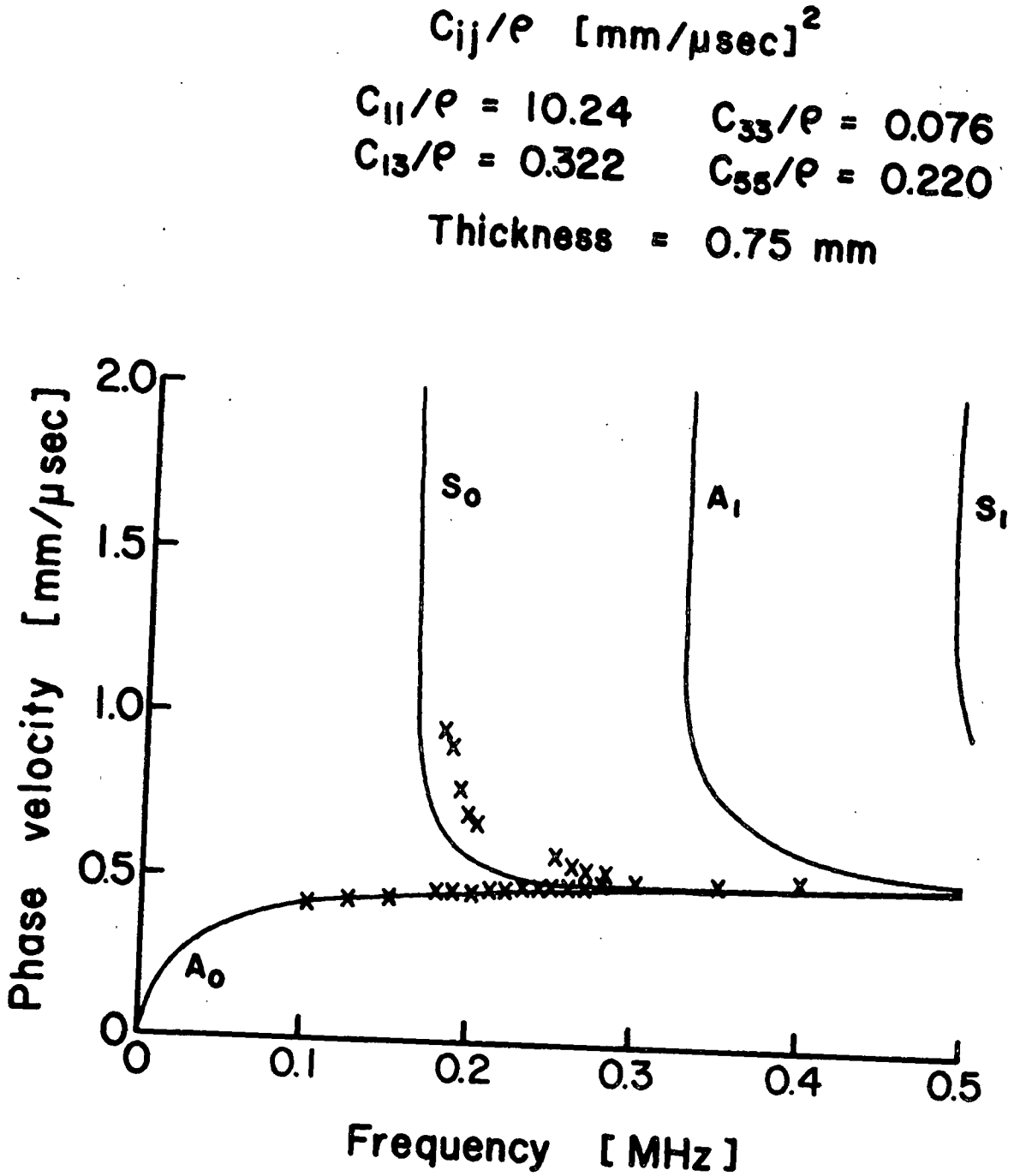

Fig. 5. Comparison of theoretical and experimental dispersion curves for an 0.75 mm chipboard sample in the MD.

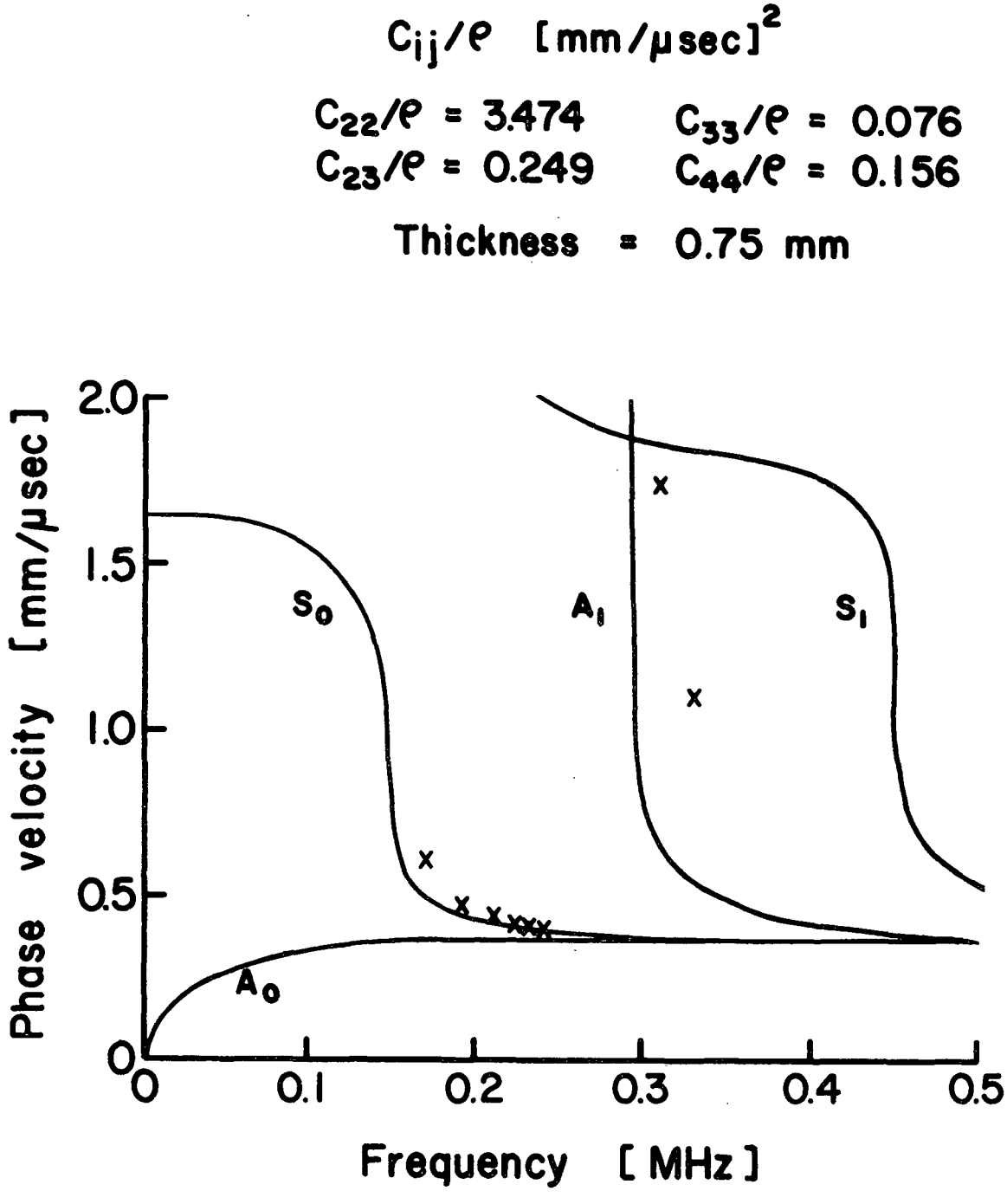

Fig. 6. Comparison of theoretical and experimental dispersion curves for an 0.75 mm chipboard sample in the CD.