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A SIMULATION APPROACH TO THE ANALYSIS AND DESIGN OF SEQUENTIAL PROCESS PRODUCTION SYSTEMS

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SUMMARY

The sequential process production system has not solved by itself the continuity of the flow of items through the system. Many production systems that are effective have been developed without being completely understood. A more complete understanding of production systems behavior and, in particular, of those systems involving a sequential process is necessary to increase their efficiency.

The criterion in optimizing a production system is the cost of production. The variation of cycle times and the breakdown in the series of operations, both decrease the output rate while the intermediate inventories have the purpose of improving the efficiency of the system. A balance between the value of the improvement achieved by the use of intermediate inventories and the cost of this improvement is the objective.

It is very difficult to analyze sequential process systems by formal mathematical means. The present status of the theory does not permit a general solution of intermediate inventories between operations in sequential processes. A simulation approach is presented in order to analyze and design storage facilities in the sequence of operations. A computer model, using the General Purpose Systems Simulator (GPSS-II), which simulates the behavior of the system is developed for the purpose of showing what the design of a sequential process should be.

The application of such techniques which are relatively new to industry can assist in the development of even more efficient production systems and such studies are essential for a complete understanding and

development of production control systems.

Numerical results for 2, 4, 6, 10, and 20 operations in series at various intermediate capacities are presented and analyzed. The simulation approach developed is not restricted to any particular form of probability distributions; however, for the particular production system simulated, the production time, setting time, and interval time between breakdowns for all operations have exponential distributions.

CHAPTER I

INTRODUCTION

Cost and effectiveness are important variables in the analysis and design of production systems. There are many studies related to this field; some are concerned with the applications for particular problems, and some deal with various mathematical techniques devoted to analytical research.

In present days it is very common to find industries where the arrangement of machinery, materials, and men is governed by the product to be made, i.e., there is a sequential process involved in which the manufacturing operations are subject to interdependence among the elements of the systems, and to conditions which may vary according to the design or according to probabilistic patterns. The analysis and design of such production systems are very critical tasks. A primary problem is the allocation of space and storage equipment at all intermediate phases of the manufacturing process to which the efficiency of the system, and, consequently, the cost of production is related. The number of operations and variation of cycle times of each operation are also important variables in production systems.

There exists a variety of techniques for designing balanced production lines; however, these techniques in general assume the operation times of the various operations as being fixed numbers. They also generally assume that no breakdowns will occur in the various operations.

When a sequence of operations constitutes a production system, the entire system becomes interdependent in the sense that any delay caused by the variation of cycle times or breakdowns at any operation can disrupt the entire system. This can basically happen in two ways: first, if the succeeding operation is not free when a prior operation finishes an item, and there is no space between the two work stations in which to dispose the finished material, then a delay will occur because the operation cannot undertake more work until the path is cleared; and, second, if an operation is free to process another item and there is no new item coming from the preceding operation available, a delay will also occur. The consequence in both cases is lost production time.

The delay which can occur because of these conditions will decrease the output rate. The flow of items through the system is measured by the rate of production. The occurrence of delays will interrupt the flow of items as well as affect the number of items produced by the entire system per unit of time.

There are two main reasons for which a delay can occur in a production line. The most important factor affecting the output rate is the cycle time of the various operations, which may be fixed or variable according to the properties of the particular operation. In fact, considerable variation in the cycle time from one item to the next may be inevitable. In general, the cycle time follows a probability distribution and the question arises how this variation should be taken into account in the design of the system.

Another factor of importance in determining delays, or, in other words, the efficiency of a sequential process, is the occurrence of breakdowns which are characteristic of each stage. The interval time between breakdowns for each operation and the setting time in the case of breakdowns are also characterized by probability distributions.

An interruption at any point of the system affects the rest of the operations. To avoid discontinuity, a continuous supply of materials must be assured for any operation. These considerations imply that provision should be made for a substantial intermediate inventory between operations. Its purpose is to reduce the impact of cycle time variations and breakdowns characteristic of each operation on the output rate.

The major problems in the analysis and design of a sequential process system are the number and location of the intermediate inventories between the various operations, and the size of these pulsating stores.

There are several theoretical attempts reported in the literature for analyzing and designing production systems in which intermediate storages are involved. A mathematical approach, more specifically a queueing theory approach, has been used by some authors. Basically, a queueing process is centered around a service system which has one or more service facilities. The elements of a queueing process are: the input source, queue, service facilities, and the service discipline. In fact, the problem of a sequential process with intermediate inventories can be looked upon as a queue system in which the server is the operating unit which removes an item from storage, and where the items in storage comprise the waiting line. This concept is illustrated in Figure 1.

The basic problem in the manufacturing system represented by a queueing process is the probability that the queue will exceed or underpass a specified length (corresponding to the intermediate inventory)

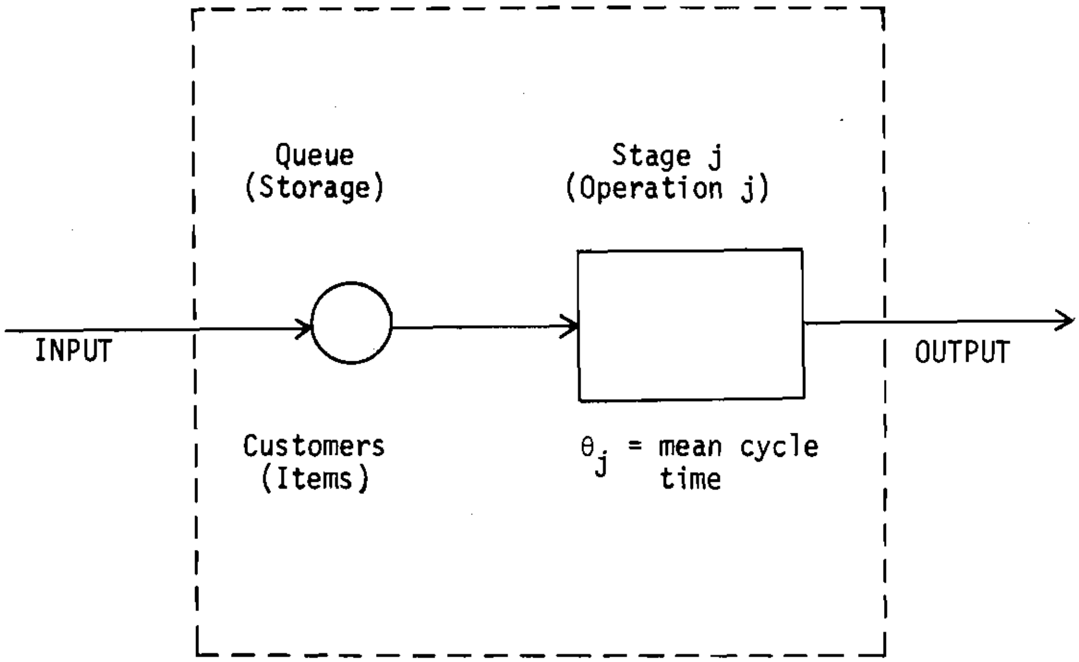


Figure 1. The Queueing Process.

and the output rate.

The queueing theory requires that the arrival rate must be exponentially distributed and the service rate constant or exponentially distributed. These constraints together with the requirements that the arrival rate of items into the line be independent of the service rate, that is the rate with which they are removed, are the most serious limitations in treating the production problem with queueing theory.

It must also be said that the variation of cycle times and setting times (caused by breakdowns) involve probabilistic events which make any formal mathematical formulation of the problem very difficult.

Another approach in evaluating the effects of these chance fluctuations is that of systems simulation. The description of the system by a simulation model allows to measure the effect of changing one or more variables in the system.

The objective of a systems simulation approach is not just to derive a solution from a mathematical model of the process, but rather to provide a means of observing the behavior of the elements of the system when conditions vary.

Because of the interdependence between the various operations in a sequential process system, a simulation study points out the way to make the components of a system work together in the best possible way. Mize and Cox[17] point out the following phases as adequate in describing a systems analysis study:

1. Formulating the problem.
2. Constructing the model to represent the system under study.
3. Deriving a solution from the model.

4. Testing the model and the solution derived from it.

5. Establishing controls over the solution.

6. Putting the solution to work: implementation.

For the purpose of this study the main emphasis shall be on the phases 1 through 3, since the main interest is to present an approach to the analysis and design of sequential process production systems rather than to develop a definite solution for a particular system. A systems simulation approach is developed and presented to analyze and design production systems which involve a sequence of operations. The various properties of the system are studied so that the substituted process can be designed to reproduce them. Because the number of samples to be observed is very large and the system involves several probability distributions which must be observed simultaneously, a computer model is developed and simulated for the purpose of deciding what the design of any sequential process should be. Rather than constraining the system variables to particular probability distributions or unrealistic assumptions, a simulator which can handle a general sequential process production system is designed.

Several special purpose computer languages have been developed which make simulation programming easier and more comprehensive. In particular, the General Purpose Systems Simulator (GPSS-II), developed by IBM Corporation is a part of the approach used. The reason for using the General Purpose Systems Simulator as the simulation language is due to its characteristics which are summarized below.

The General Purpose Systems Simulator is one of the most widely used simulation languages designed specificially for modeling queueing systems. The General Purpose Systems Simulator is a block-oriented language and the general correspondence between the elements of the production system under consideration and the terminology used in the computer language is as follows:

System type:	Sequential process production system with intermediate inventories.
Transaction:	Item to be produced.
Facility:	Operation (machine and operator).
Storage:	Intermediate inventory.
Logic switch:	Operation free or not, and storage space available or not.

To use the General Purpose Systems Simulator as the simulation language, the structure of a general sequential process production system with intermediate inventories is presented in the form of a block diagram with a fixed set of predetermined block types.

The criterion of effectiveness used is the output rate of the system. In the ideal case when the production line is perfectly balanced with constant cycle times and the system is not susceptible to breakdowns, then the output rate would be one item per cycle time, where the cycle time is the same for all operations. Since in real systems the cycle times are continuous variables which vary to some extent about a central value and as there are breakdowns, the production rate will be lower than in the hypothetical case of a perfectly balanced line and no breakdowns. As a consequence the output rate (production rate) measures the efficiency of the system with respect to the ideal hypothetical system. A coefficient of utilization (ρ) is defined in order to relate the mean output rate and the mean operations rate.

The cost of a production system has to be primarily considered when determining its proper design. First, providing too much efficiency (high utilization) by increasing the intermediate inventories would involve excessive cost. Second, not providing enough inventories would cause a low utilization rate of the equipment and a low output rate, and consequently an excessive cost of low utilization. Therefore, the ultimate goal is to achieve an economic balance between the cost of intermediate inventories and the cost of low utilization of the system. The underlying objective of the present study is to determine the level of intermediate inventories which minimizes the total of the average cost of utilization of the system and the cost of intermediate inventories. The conceptual solution is shown in Figure 2.

The objective of this investigation is to develop a systems simulation approach for analyzing and designing production lines with variable cycle times and subject to breakdowns. In particular, the effect of the number of operations and the amount of internal inventory capacity are formulated especially to represent a sequential process production system, however the results obtained also have considerable relevance for designing similar systems.

Many names have been given to material-in-process which is in either permanent or temporary storage within the system. In this study the material in storage between the various "operations" shall be called "intermediate inventory." In addition, the work "operation" shall be used to describe "work station." Also, the word "cycle time" shall be used to indicate the production time of any operation, and the word "breakdown" to indicate "stoppage or interruption."

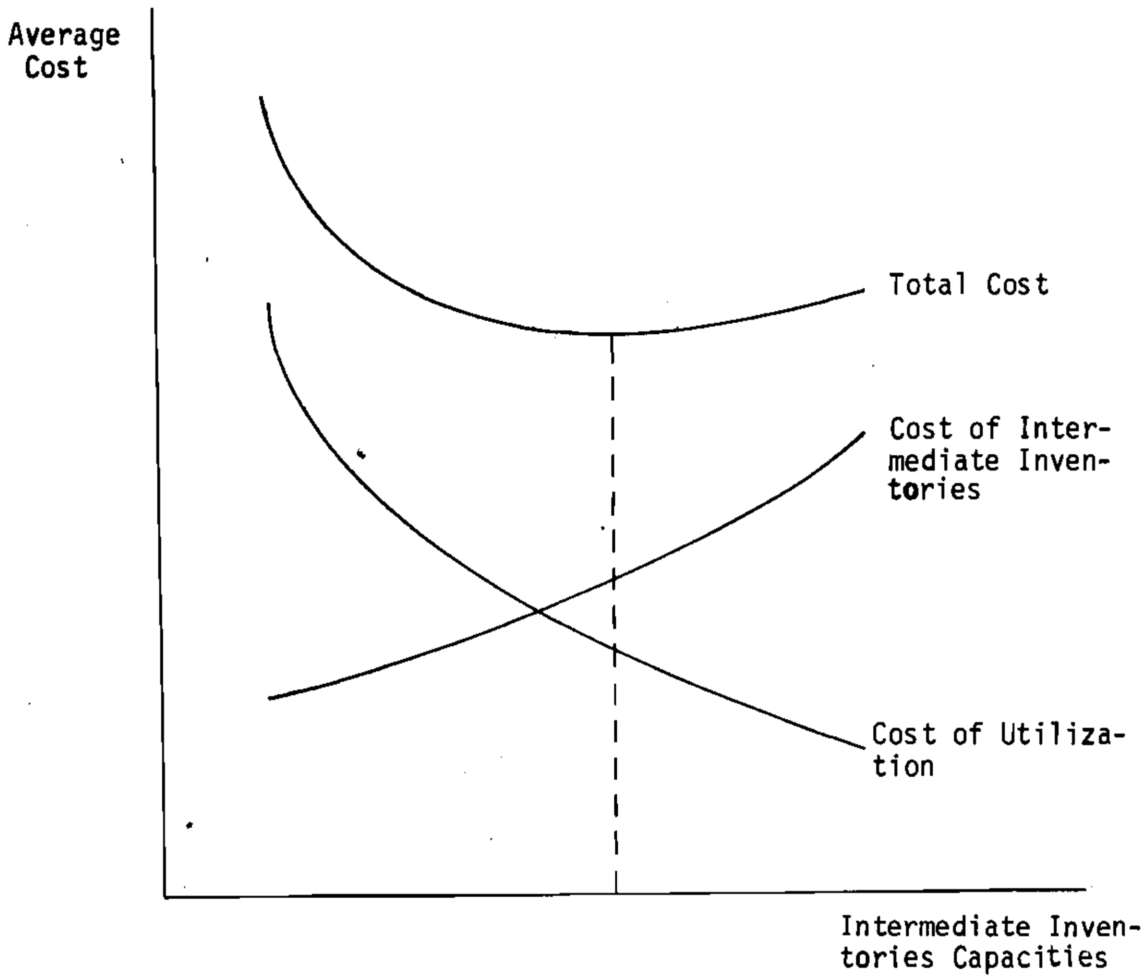


Figure 2. Conceptual Solution for the Production System Design Problem.

CHAPTER II

LITERATURE SURVEY

Considerable work has been done in the area of sequential process systems and in the evaluation of the effectiveness of internal inventories, especially using formal mathematical analysis. It is of pertinent importance to discuss only that research which has investigated the evaluation of production systems involving a sequential process and especially the effect of intermediate inventories on the efficiency of production systems subject to variable operation times and breakdowns.

Hunt[9] in a pioneering investigation applied a queueing theory approach to a problem involving a sequence of operations which must be performed on the units to be serviced. Hunt assumed Poisson arrivals and exponential service times. Breakdowns were not considered. Blocking is allowed and four cases are considered:

Case 1. Infinite queues are allowed in front of each service facility.

Case 2. No queues are allowed, with the exception that the first stage may have an infinite queue.

Case 3. Finite queues are allowed in front of each stage, with the exception that the first stage may have an infinite queue.

Case 4. No queues and no vacant facilities are allowed, with the exception that the first stage may have an infinite queue; the line moves all at once, as a unit.

Hunt uses the maximum possible utilization in the steady state (ratio of mean arrival rate to mean service time) and the average number of units in the system as an evaluation of the system.

In the first case no blocking can occur and the output equals the input in the steady state which requires that the arrival rate is less than or equal to the mean service rate. Hence, according to Hunt, the maximum possible utilization in this case is unity.

In the investigation of the second case an expression is derived for the maximum possible utilization for two and three operations in series with unequal service rates, as follows:

$$\rho = \frac{\lambda}{\mu} = \frac{\text{mean arrival rate}}{\text{mean service rate}}$$

If μ_i = mean service rate at the $i^{\underline{th}}$ station, then:

for two stages: $\rho_{max} = \dfrac{\mu_2(\mu_1 + \mu_2)}{\mu_1^2 + \mu_1\mu_2 + \mu_2^2}$

for three stages: $\rho_{max} = \dfrac{N}{D}$

where,

$$N = \mu_2\mu_3(\mu_2 + \mu_3)(\mu_1^4 + 2\mu_1^3\mu_2 + 3\mu_1^3\mu_3 + \mu_1^2\mu_2^2 + 4\mu_1^2\mu_2\mu_3 + $$
$$+ 3\mu_1^2\mu_3^2 + \mu_1^2\mu_2^2\mu_3 + 4\mu_1\mu_2\mu_3 + \mu_1\mu_3^3 + \mu_2^2 + \mu_3^2\mu_2)$$

$$D = \mu_1^5(\mu_2^2 + \mu_2\mu_3 + \mu_3^2)$$
$$+ \mu_1^4(2\mu_2^3 + 5\mu_2^2\mu_3 + 5\mu_2\mu_3^2 + 3\mu_3^2)$$
$$+ \mu_1^3(\mu_2^4 + 5\mu_2^3\mu_3 + 8\mu_2^2\mu_3^2 + 7\mu_2\mu_3^2 + 3\mu_3^4)$$
$$+ \mu_1^2(\mu_2^4\mu_3 + 5\mu_2^3\mu_3^2 + 8\mu_2^2\mu_3^2 + 5\mu_2\mu_3^4 + \mu_3^5)$$

$$+ \mu_1(\mu_2^4\mu_3^2 + 5\mu_2^3\mu_3^3 + 5\mu_2^2\mu_3^4 + \mu_2\mu_3^5)$$
$$+ \quad (\mu_2^4\mu_3^3 + 2\mu_2^3\mu_3^4 + \mu_2^2\mu_3^5)$$

The third case is more realistic and is the one treated in the present investigation. The expression for the maximum possible utilization in the case of different service rates and for two stations is given by

$$\max = \frac{\mu_2(\mu_1^{q+1} - \mu_2^{q+1})}{\mu_1^{q+2} - \mu_2^{q+2}}$$

where (q-1) is the length of the queue allowed in front of the second station.

Hunt points out that the fourth case can be treated in exactly the same manner as the previous cases. The problem has been solved completely for two stages, and a comparison is given with cases 1 and 2.

Hunt's investigation does not derive any general expression for n stations in series except in the first case and shows the difficulty of treating the problem by formal mathematical analysis.

Weber[26] developed a model to describe the effects of varying the amount and location of storage space between a group of operations arranged in series and subject to breakdowns. The model assumes that service and repair times are exponentially distributed and that the time between breakdowns follows a Poisson distribution. In spite of the fact that Weber develops a procedure for writing the specific expressions for any specified number of machines, no general expressions were derived when there are three or more machines. It is also worthwhile to note the following statement:

> The rapidly increasing complexity of the expressions as the size of the system increases suggests that economical application of the results may be limited to fairly simple systems.

Hutchinson[10] investigated in-process storage units on continuous production lines as a method for minimizing what he called "Balance and Delay Losses" and the associated production line costs. The evaluation of in-process storages is based on the measure of "Lost Time," defined as "Balance Loss" plus the expected value of "Delay Loss:"

$$LT = BL + DL$$

First a lost time measure is shown for production lines with no in-process storages and then the same measure is developed for production lines with intermediate storages by appropriate modifications to the measure.

Hutchinson assumes the cycle times of the various stages as constant and considers the "Balance Loss" as the "lost" time because the difference between a cycle time and the maximum cycle time. An expression for finding the balance loss is given. To find the delay loss, which occurs because of stoppages, a Monte Carlo procedure was developed. The final expression for the lost time is given and finally converted to an economic measure. The expression of the lost time is:

$$LT = \frac{E(M_w)}{C_{max}} \left(n\, C_{max} - \sum_{i=1}^{n} C_i \right) + E(DL)$$

where,

M_w = total working minutes per shift,

C_{max} = cycle time for the "bottle neck" operation in minutes per unit;

C_i = cycle time for the i^{th} operation in minutes per unit, and

DL = delay loss.

Hutchinson assumes that the occurrence of downtime on the production line is independent of the time interval between occurrence of downtime. Besides, it is assumed that the number of units in the in-process storage at the beginning of each shift is zero. The lost time measure is developed for the production line with intermediate storage for only two operations with one intermediate storage and the data used to determine the distribution for working interval, operation down and downtime interval are empirical.

The variation of cycle times, which are considered constant by Hutchinson, is in fact a cause of great losses. There are several limitations in the application of this procedure, mainly to the design of production lines.

Barten[2] constructed a simulator to allow the determination of the effect of the storage space between sequential operations on the production rate. For the cases in which there are 2, 4, 6, and 10 operations with intermediate inventories the effect on production rate is given. The effect of what Barten calls queue length (number of operations) on delay time for queues of 4, 6, and 10 operations is shown. The simulator flow chart for the case of two operations is presented. Barten does not consider breakdowns and assumes that cycle times are normally distributed. For the cases considered, a general formula for optimum storage capacity is developed based upon a general delay function inferred from the simulator runs.

Hillier and Boling[8] consider a sequential process system where each channel (operation) has an exponential or Erlang service time and a finite queue. The queue before the first stage is never empty, which means that there is never a shortage of material for the first operation. The number of customers in the system and the steady-state mean output rate are the measures for which a computationally procedure is described. Also for the case of exponential holding times, a procedure which is said to be exceptionally efficient by the authors is developed for approximating the mean output rate. The authors demonstrate that this procedure provides a good approximation for most cases and that it is computationally feasible for large problems, respected the assumptions made. Numerical results are also presented.

In spite of the fact that numerous problems dealing with production systems, especially those involving a sequential process, have been considered in the literature, none of the procedures permit a study of production systems involving a sequence of operations considering both the variation of cycle times and breakdowns for the general case of any number of operations and without constraining the system to particular probability distributions.

The simulation approach developed here makes no restrictions with regard to probability distributions and allows for the occurrence of breakdowns.

CHAPTER III

MODEL FORMULATION

The model used to represent a sequential process production system was selected to depict the case in which operations are subject to variation of cycle times and breakdowns. There is no attempt to develop a model yielding universal and general results but rather to point out an approach in analyzing and designing a sequential process system. The model is designed to indicate the relative effect of intermediate inventories between operations on the performance of the production system subject to variable cycle times and breakdowns occurring in each operation.

The series arrangement of operations as shown in Figure 3 and consists of a sequence of a number of operation facilities such that an item must go through one facility after another in a particular sequence before the final product is obtained.

The system under consideration may well be a sub-system of a larger system. For purposes of analysis it is sometimes convenient to subdivide large systems into subsystems.

In a production system involving a series arrangement of operations, the transfer of items from one operation to the next one may be done in one of the following two ways:

(1) There is no allowance for intermediate storages in which each operation is completely dependent on every other one.

(2) The item is disposed in a storage between the operations. In

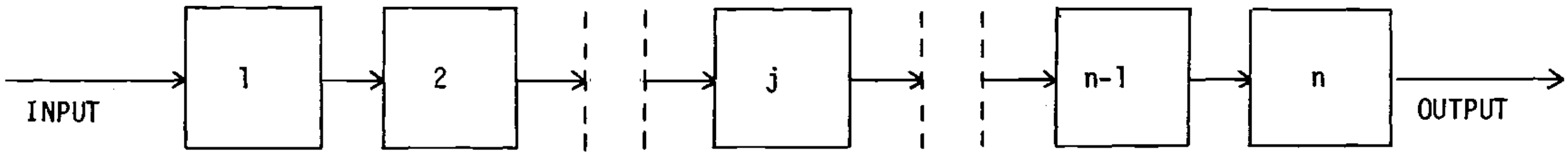


Figure 3. Series Arrangement of Operation Facilities.

fact, the first case may be considered as a particular case of the present one, in which the storage between operations has zero dimension. It can also be stated that the space provided for storages between operations ranges from zero to infinity. Infinite storage is the ideal from the point of view of time. However, it is impossible in reality and hence of little interest. In this case, every operation would operate independently of every other one. The output rate of such a system would be independent of cycle time variations and the total number of operations, and it would be determined by the cycle time of the last operation.

The case of finite storage capacity is more realistic. In general, production layouts allow at least some storage. There is, however, a question of how inventory investment should be balanced between these various intermediate inventories. The number of items in storage is a function of variables affecting the rate with which material is put into and taken out of the intermediate area. However, the rate with which material is put into the storage and taken out, is a function of many other variables representing an extremely complex system.

The upper limit of an intermediate inventory is any number c, dependent on physical and economical constraints. The model here developed considers all cases, i.e., all storage status from zero to c.

Figure 4 illustrates a sequential process with n operations and (n-1)) intermediate storages. The analysis and design of such a system requires the development of a model in order to point out how the time-based variables affect the problem and their interdependencies. The system is dynamic in nature and involves continuous changes.

The ideal system should provide a smooth output without interrup-

O_j - operation j

S_j - storage j

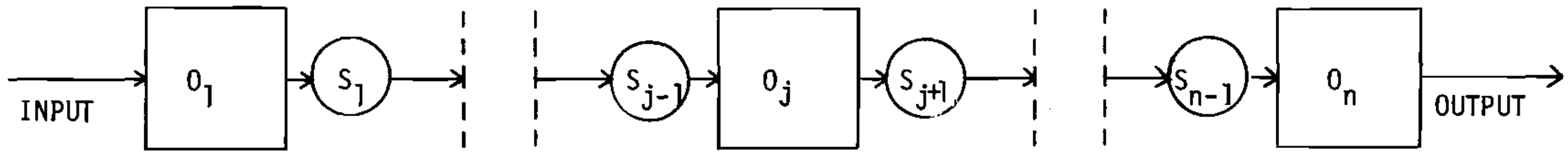


Figure 4. Sequential Process System with n Operations and (n-1) Intermediate Storages.

tions. It is important to note again that a discontinuity may happen in the flow of items through the system because of cycle time variations and breakdowns. The production efficiency and as well the production cost are dependent upon the rate of flow of items through the process. This rate of flow may be altered by using intermediate inventories, which, on the other hand, means an increase in the cost of facilities and in labor to provide additional items.

In the system shown in Figure 4, any operation O_j which has just completed an item(i-1) may face two situations:

(1) Storage S_j is full (or the storage capacity is zero) and the operation O_{j+1} is working on an item. In this case, there is no place to dispose the item (i-1) finished by operation O_j and a delay will occur because this operation cannot undertake more work until the path is cleared. This condition shall be called "blocking" and is one reason for delay.

(2) In the case that there is no restriction in storage S_j, and O_j is ready to begin to work on the next item i, it may happen that storage S_{j-1} is empty and so operation O_j must wait until operation O_{j-1} places an item into storage S_{j-1}. However, it may happen that operation O_{j-1} is waiting for an item to be produced by operation O_{j-2} because storage S_{j-2} is empty. This condition may continue up to operation O_1, which never has to wait because unlimited storage of raw material (input) is available.

The question turns out to be how long the operation O_j will be delayed. This delay will depend exactly on the previous and on the next storage status.

It may still happen that operation O_j, or any other operation, is down because of a breakdown. This fact, if it happens on operation O_j will also delay the production of an item until facility j is repaired. If the breakdown is occurring in another operation, the delay may be transferred to operation O_j some time later either because storage S_j will be full or storage S_{j-1} will be empty.

In the system described in this study there are n operations and (n-1) intermediate storages. It is assumed that:

(1) The space provided for intermediate inventories between operations is a finite one.

(2) The number of items available for the first operation O_1 is unlimited. Hence no runouts will happen in this initial supply. This assumption is consistent with the real production system.

(3) The space provided after the last operation O_n is such that it will never be full. Hence the last operation will never be blocked.

(4) There is no inspection in the line so that items cannot leave the system except at the terminal point. Thus only a final inspection is allowed.

(5) The production system is balanced.

A consequence of the above assumptions is that all sequential operations from O_2 to O_n could be delayed because of material shortages. In addition, all operations from O_1 to O_{n-1} could be delayed because the storage following the operation is full preventing the depositing of a completed item.

Measure of Effectiveness

Estimating the efficiency of a production system is necessary in planning a new system or in order to obtain an objective judgement of the capacity of the equipment available. The overall objective is to minimize the production cost by means of an economic balance between the cost of intermediate inventories and the cost of the utilization of the system. The purpose of this study is to show how to determine the level of intermediate inventories which minimizes the total average cost of maintaining intermediate inventories and the average cost of delays which cause a lower output rate. The coefficient of utilization (ρ) shall be defined as:

$\rho = \frac{\lambda}{\mu}$, where

λ = mean output rate, and

μ = mean operation rate.

The Mathematical Model

A mathematical model shall point out the behavior of the time-based variables and their relationships as well as their effect on the system. The following notation shall be followed:

$\theta_{j,i}$ = total cycle time taken by operation O_j to make item i; it includes the time to receive item i from storage S_{j-1}, position, work on, eject the item and transport it to storage S_j.

$R_{j,i}$ = the independent operation time of operation O_j on item i.

$W_{j,i}$ = the delay time spent by operation O_j waiting for item it to work upon.

$F_{j,i}$ = the delay time spent by operation O_j waiting for a space at storage S_j to deposit the finished item i.

$B_{j,i}$ = the delay time spent by operation O_j because of a breakdown during the time it was working on or waiting for item i. This time corresponds to the setting time, which is a chance variable characteristic of each stage.

The total cycle time of any operation O_j on item i shall be, thus:

$$\theta_{j,i} = R_{j,i} + W_{j,i} + F_{j,i} + B_{j,i}$$

where W, B, and F represent delays. It is possible to state that:

$$\theta_{j,i} = f(R_{j,i}, \text{delays}).$$

The analysis shows that $R_{j,i}$ is in fact independent of any other operation and is a characteristic of operation O_j itself. $W_{j,i}$ is the delay occurred because when operation O_j finished item (i-1) it had to wait for the completion of item i by operation O_{j-1}. This delay is dependent on all production rates and delays occurred in operations O_1 through O_{j-1}.

$B_{j,i}$ is also an independent variable, characteristic of each stage and is another factor of importance in determining the efficiency of a sequential process. Setting times are taken here to represent breakdowns, which follow a probability distribution.

The measure of effectiveness (ρ) requires the knowledge of the output rate. To compute the output rate it is necessary to know the total cycle time of the last operation for all items i, i.e., $\theta_{n,i}$ for $i=1,\ldots,m$.

It shall be assumed here that:

$$R_{j,i} + B_{j,i} + F_{j,i} = Y_{j,i}$$

where $Y_{j,i}$ is the sum of two independet variables and the delay time spent when operation O_j cannot undertake another item because it cannot deposit finished item i in storage S_j.

Hence,

$$\theta_{j,i} = Y_{j,i} + W_{j,i} \qquad \text{(I)}$$

where $W_{j,i}$ is a dependent variable.

The output rate of the operation O_{j-1}, i.e., the number of items delivered by operation O_{j-1} during the period of time t is determined by the independent rate R_{j-1}, B_{j-1} which is also an independent variable, by F_{j-1} and W_{j-1}.

The larger the value of j (i.e., the number of operations preceding operation O_j), the greater the probability of delays occurring at operation O_j. The larger the capacities of the intermediate inventories between operations the smaller the probability of occurring delays.

Consequently, the output rate of the system:

$$\lambda = \frac{m}{\sum_{i=1}^{m} \theta_{n,i}}$$

is a function of the following variables:

(1) Independent production rates of the various operations.

(2) Breakdowns occurring at the various operations.

(3) Number of operations.

(4) Capacities of the intermediate inventories in the system.

The mean output time can be computed as follows:

$$\frac{\sum_{i=1}^{m} \theta_{n,i}}{m} = \frac{\sum_{i=1}^{m} Y_{n,i}}{m} + \frac{\sum_{i=1}^{m} W_{n,i}}{m}$$

The value of $W_{n,i}$ can be said to be solely dependent upon the state of input storage S_{n-1} being empty or not. Thus consideration must be given to the factors that determine the state of any particular intermediate inventory in the system. The state of any intermediate inventory S_j is determined by the rate with which items are put into the storage by the oepration O_j and the rate with which they are removed by operation O_{j+1}.

Hence,

$$W_{n,i} = \theta_{n-1,i} - Y_{n,i-1}$$

where $W_{n,i}$ is the delay time spent by the last operation for an item i upon which to work. This expression is considered by Barten[2] with the difference that he does not include breakdowns in the value of Y.

However, $W_{n,i}$ may assume a real value only if the storage S_{n-1} preceding operation O_n is void at the time when operation O_n is ready to begin work on item i. Thus,

$$W_{n,i} = (\theta_{n-1,i} - Y_{n,i-1})\ p(V_{n-1}^{i}) \tag{1}$$

where $p(V_{n-1}^{i})$ is the state probability of an intermediate inventory, and should represent the probability that at the time when operation O_n is ready to begin to work on item i, the last storage S_{n-1} is void.

The relationship between the storage state probabilities and the independent production rates and setting times can be seen by developing the expression $p(V^i_{n-1})$:

$$p(V^i_{n-1}) = p(\theta_{n-1,i} > Y_{n,i-1})\ p(V^{i-1}_{n-1})$$

where

$$p(V^{i-1}_{n-1}) = p(\theta_{n-1,i-1} > Y_{n,i-2})\ p(V^{i-2}_{n-1})$$

Repeating the multiplication for all items i produced, it is obtained:

$$p(V^i_{n-1}) = \prod_{k=1}^{i-1} p(\theta_{n-1,i-k+1} > Y_{n,i-k})$$

Substituting:

$$\theta_{n-1,i} = Y_{n-1,i} + W_{n-1,i}$$

into equation (1) it is obtained:

$$W_{n,i} = \left[(Y_{n-1,i} + W_{n-1,i}) - Y_{n,i-1}\right]\ p(V^i_{n-1}) \qquad \text{(1a)}$$

where,

$$W_{n-1,i} = (\theta_{n-2,i} - Y_{n-1,i-1})\ p(V^i_{n-2})$$

substituting the above result into equation (1a), it is obtained:

$$W_{n,i} = \left[\ Y_{n-1,i} + (\theta_{n-2,i} - Y_{n-1,i-1})\ p(V^i_{n-2}) - Y_{n,i-1}\right]\ p(V^i_{n-1}) \qquad (2)$$

where,

$$\theta_{n-2,i} = Y_{n-2,i} + W_{n-2,i}$$

and substituting the above result into equation (2), it is obtained:

$$W_{n,i} = \left[Y_{n-1,i} + (Y_{n-2,i} + W_{n-2,i} - Y_{n-1,i-1})\ p(V^i_{n-2}) - Y_{n,i-1} \right]\ p(V^i_{n-1}) \qquad (2a)$$

Repeating this substitution for all n operations in the system:

$$W_{n,i} = (Y_{n-1,i} - Y_{n,i-1})\ p(V^i_{n-1}) + (Y_{n-2,i} - Y_{n-1,i-1})\ p(V^i_{n-2})\ p(V^i_{n-1}) + \ldots +$$

$$+ (Y_{1,i} - Y_{2,i-1})\ p(V^i_1) \ldots p(V^i_{n-1})$$

or

$$W_{n,i} = \sum_{j=1}^{n-1} (Y_{n-j,i} - Y_{n-j+1,i-1}) \prod_{k=1}^{j} p(V^i_{n-k}) \qquad (II)$$

where it is shown that the delay time spent by the last operation waiting for an item i is dependent on:

(1) The independent production rates.

(2) The independent setting times.

(3) The number of operations.

(4) The state of the intermediate capacities.

The expression (II) points out that the main problem is to determine the probabilistic terms. The present status of queueing theory does not permit the general solution for this problem.

The probability that an intermediate inventory with capacity c is full, is implicit in the value of Y because it includes the delay time spent due to the blocking effect.

All values $W_{n,i}$, for $i=1,\ldots,m$ should be computed in order to obtain the mean output rate λ.

CHAPTER IV

THE SIMULATION

The decision-maker determines the policies to guide the design of the system in order to transform a set of inputs into a set os outputs which have economic value.

Rather than trying to solve analytically the model developed in the previous chapter, a simulation approach shall be used to analyze and design the system. The simulation study can be applied not only to the design of a new system but also in the cases in which a change must be made in the structure of an existing system or an evaluation is needed of a system already working.

The important properties of the system shall be determined and analyzed so that it is possible to experiment with alternative policies. The behavior of the system shall be described according to the mathematical model developed previously, so that the attributes of the various components of the system and their relationships can be represented by a simulated model.

The simulation procedure will permit the consideration of changes in the state of the system through simulated time and under rules of operation. These rules may vary so that alternative policies can be considered without experimenting with the physical system or solving the mathematical model by pure mathematical tools. The procedure to be followed in analyzing and designing the production system shall follow the flow chart shown in Figure 5.

The number of samples to be observed and the amount of computations to be used in the simulation of a system which involves two or more operations and the number of trials necessary suggest that a computer model shall be used.

Analysis of System Properties

The expression:

$$\theta_{n,i} = Y_{n,i} + W_{n,i} \qquad \text{(I)}$$

shows that the mean output time and hence the output rate (λ) is dependent on the independent production times, setting times and delay times spent by the last operation in waiting for an item to work on. It is assumed that unlimited space is provided after the last operation so that no "blocking" will occur with relation to this operation.

Hence, in order to measure the output rate (λ), the simulator must:

(1) Determine the independent production time spent by the last operation on each item i, i=1,....,m.

(2) Determine the time spent by the last operation with setting times because of breakdowns.

(3) Determine the waiting times.

The waiting time is a dependent variable whose behavior is explained by equation (II):

$$W_{n,i} = \sum_{j=1}^{n-1} (Y_{n-j,i} - Y_{n-j+1,i-1}) \prod_{k=1}^{j} p(V^{i}_{n-k}) \qquad \text{(II)}$$

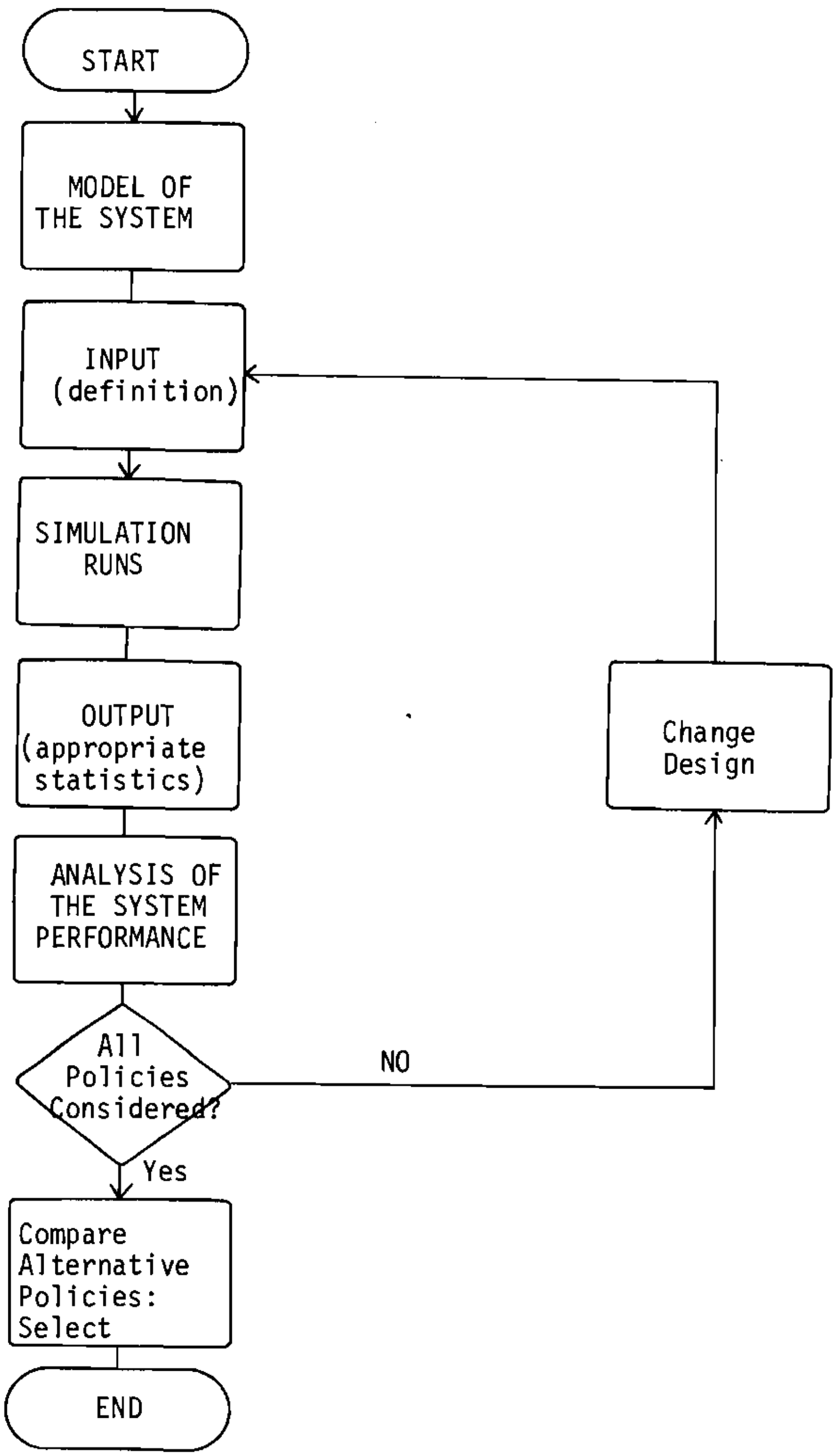


Figure 5. General Procedure of the Simulation Approach to the Analysis and Design of the Production System.

The waiting time of the last operation is dependent on the previous operations and the capacities of the intermediate inventories.

Thus, in order to compute $W_{n,i}$, the simulator must accomplish the following steps:

(1) Determine all previous production times and setting times of all operations, including the waiting time caused by blocking.

(2) Determine the state probabilities of the intermediate inventories, i.e., determine the current status of all intermediate inventories. This means the determination of the amount of items in each intermediate storage S_j after each item is produced by operation O_j or operation O_{j+1} begins to work on one.

(3) Compute the delay time for all operations O_j located between the last storage and the first non void storage with relation to item i. The delay $W_{n,i}$ on last operation is influenced by any delay on item i in each operation which is succeeding the first non void storage counting from the last storage at the time when the last operation is ready to work on item i. In order to determine $W_{n,i}$ it is necessary thus to determine the first non-void storage counting from the last one S_{n-1}.

All variables must have their status reproduced by the simulator. An analysis of the variables on which the system is dependent shows that:

(1) The number of operations for a specific design is constant.

(2) The capacity of intermediate inventories is a fixed variable for each design. It is the variable which will differentiate the different designs of the system in the present case. The different policies adopted by the analyst shall be concerned with the dimension of the internal storages.

(3) The production time of operation O_j on item i, i.e., $R_{j,i}$ is a random variable which varies both with j and i. The nature of this variation, however, has to be analyzed so that the simulator can reproduce its behavior. The simulator described here shall be able to handle any kind of probability distribution.

(4) The setting time which is characteristic of each stage is also a random variable. It is a factor of importance in the determination of the efficiency of the sequential process system. Setting times are taken here to represent breakdowns. The simulation model developed in this study makes also no restriction with respect to the kind of probability distribution followed either by the interval time between consecutive breakdowns or the setting times.

The consequence of variation in production times even if the mean value of the production times is equal ($R_{j,i}$ does not vary with j), is that there may be time spent by the various operations waiting for an item to work on. The existence of intermediate inventories is supposed to minimize the lost time because of delays of this kind. The increase in the storage capacities means a decrease in the probability that the state of any storage at any time is of emptiness.

It is apparent that a simulation of the system behavior requires the determination of the following probability distributions:

(1) Probability distribution of the production times for all operations.

(2) Probability distribution of the setting times for all operations.

(3) Probability distribution of the interval time between suc-

cessive breakdowns for all operations.

The Simulator

It has already been shown that there is an overlapping between queueing systems and sequential process production systems. A queueing process consists of the following basic elements: the input source, queue, service facilities, and service discipline. The main characteristic of a queueing system is that there are units (generally called customers) requiring service and that they must wait for service or the operations (service facilities) stand idle and wait for units (customers).

The input source of the production system under consideration is infinite and the arrival time distribution is governed by the first operation. A new unit enters the system instantaneously when the first operation comes free. The units stay in the system until finished ("patient" customers), because an inspection at the end of the production line is considered.

The queue refers to the units in a storage waiting for service. The queue is in this case finite because the capacities of the various storages are limited. When no intermediate inventories are allowed at all the permissible queue is of zero length.

The service facilities, commonly known as service channels, are disposed in a series arrangement. Hence a unit that comes in the system must go through one operation after another in a particular sequence before it is finished. The production time service has already been discussed and it has been shown to follow a probabilistic distribution.

The General Purpose Systems Simulator (GPSS-II) is a computer programming language designed specifically for modeling queueing systems. The system simulated has its structure described by the use of block diagrams in which a unique symbol system is used. The basic units, in the present case the items, have their movement through the system simulated. The General Purpose Systems Simulator uses a variable time incremental method by which computer running time is saved when the simulation is static for long period of clock time.

Figure 6 shows a general computer block diagram for the case of two operations and one intermediate storage in order show the behavior of the system. This block diagram assumes the availability of computer subroutines for generating stochastic variates having known probability distributions. The simulation for the case of two operations is very simple. However, the complexity of this flow diagram increases with the number of operations.

The steps followed by this flow diagram are the following:

(1) All required informations about the number of operations, capacities of intermediate storages and probability distributions are read.

(2) By comparing the accumulated times for operations O_1 and O_2, respectively T_1 and T_2, the lowest accumulated time is determined which will represent the event just occurred, i.e., if an item has been finished at O_1 or O_2 or both.

(3) For the operation which just finihsed an item, a new operation time and setting time (in the case breakdown occurs) are determined and added to the time T_j, which corresponds to the accumulated time for the given operation. It may occur that the accumulated time for both op-

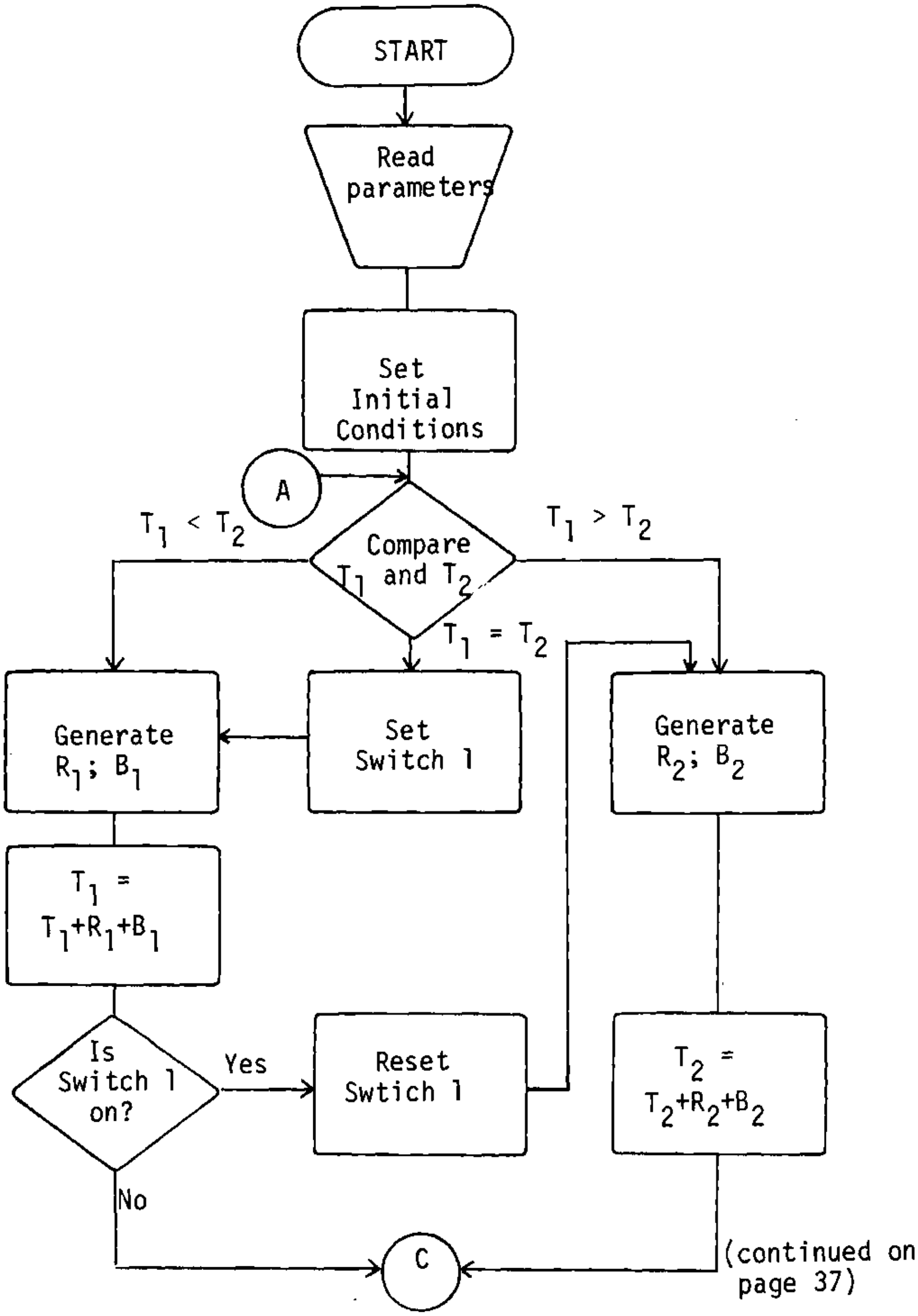
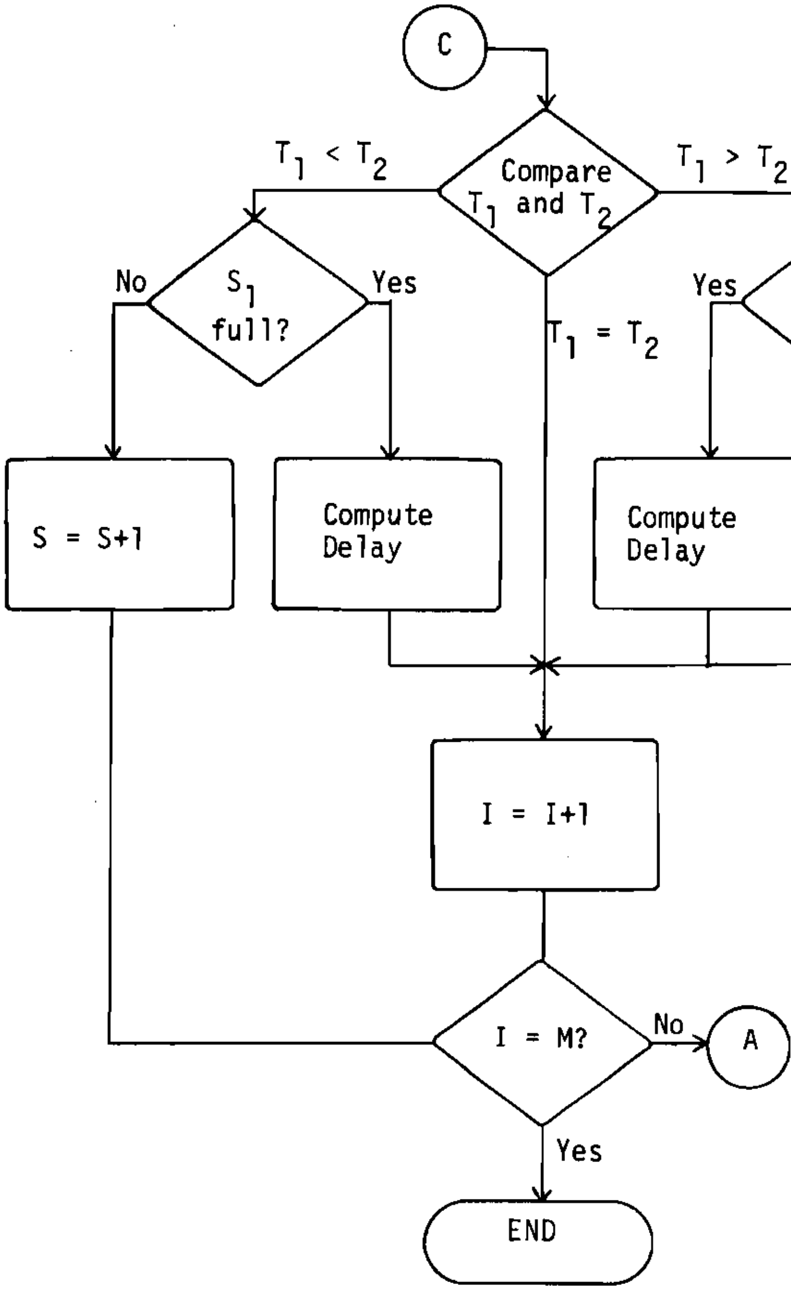


Figure 6. A Simulator Flow Chart for the Case of n=2 Operations. (Continued on next page.)



(continued on page 36)

Figure 6. A Simulator Flow Chart for the Case of n=2 Operations (concluded).

erations are equal because of delays or chance.

(4) By comparing the new values T_1 and T_2 the operation finishing first is determined (next event).

(5) In the case operation O_1 will finish first the status of the storage is examined to verify if it is full or not. If the storage is not full the operation can proceed with no delay and the status of the storage is adjusted. If the storage is full then a delay will occur and it is computed by taking the difference between the accumulated times, in which the storage state probabilities are implicit.

(6) In the case operation O_2 finishes first, the storage is examined. If it is not void, operation O_2 proceeds normally and the status of the storage is adjusted. If the storage is void a delay will occur, in which case it is computed.

The structure of the system for n operations employing the General Purpose Systems Simulator block types does not require any change for different values of n, except for the parameters values and functions coordinates.

The General Purpose Systems Simulator block diagram is presented in a logical way so that its clarity is not affected. A more sophisticated block diagram could be developed. However, that is not the purpose of this investigation.

It is possible to divide the simulator in three sections:

(1) The section consisting of the simulation of the operations and the flow of items through the sequence of operations, which is presented in Figure 7.

(2) The section representing the simulation of breakdowns in each

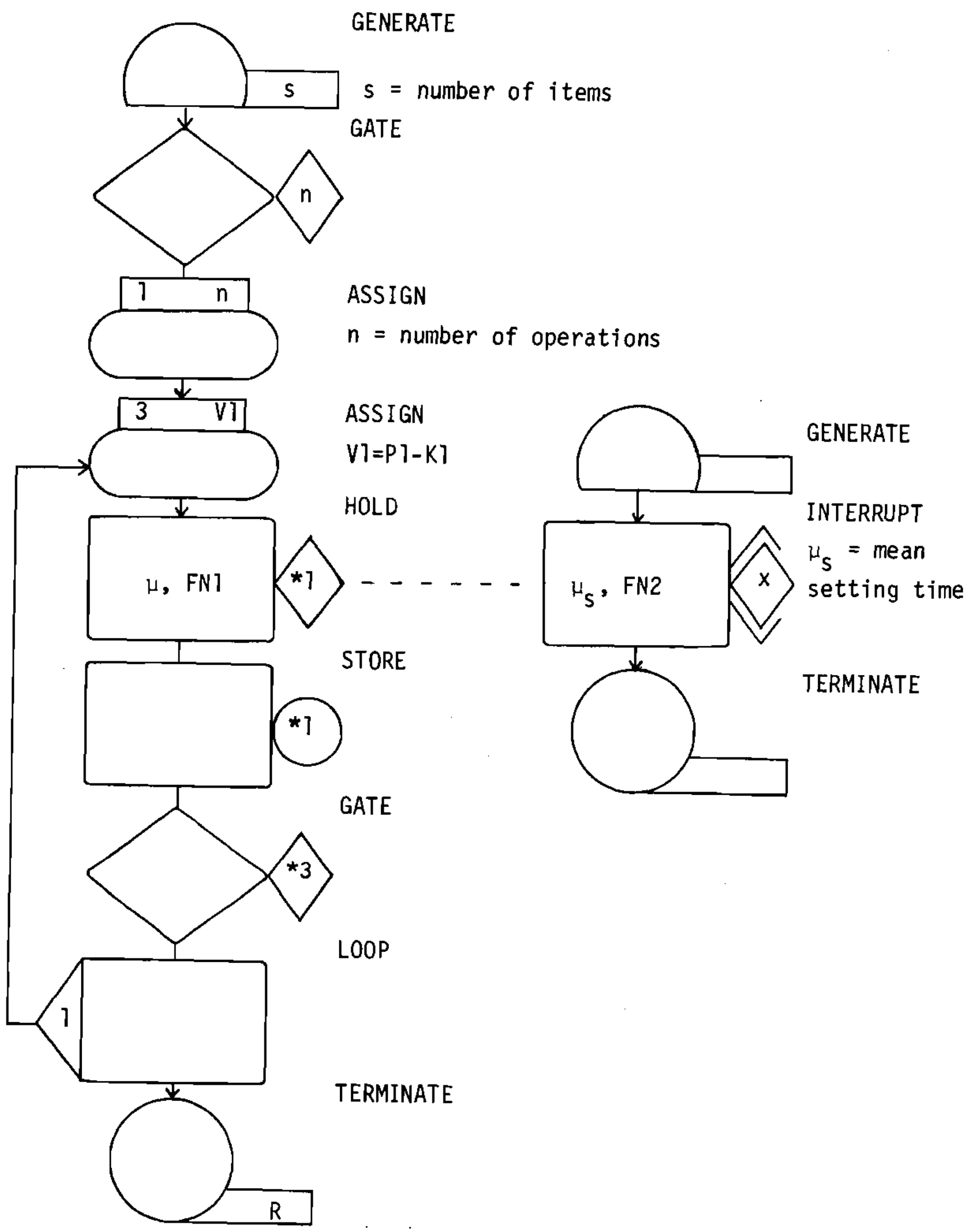


Figure 7. A GENERAL PURPOSE SYSTEMS SIMULATOR Flow Chart for a Simulation Model of a Sequential Process Production System of n Operations Subject to Breakdowns. (It is assumed that initial conditions are satisfied).

operation.

(3) The section representing the simulation of the initial conditions.

The service on an item (transaction) commences instantaneously when a unit arrives at an empty operation; and units are transferred from one operation to the next storage or operation after the completion of the service unless it is delayed because there is no space where to deposit the finished unit.

The simulation of the breakdowns is accomplished by the creation of transactions, which interrupt the normal operation. The time between the generation of consecutive transactions to represent the breakdowns follows a probability distribution. The time a transaction stays in an INTERRUPT block also follows a probability distribution.

The simulator used to obtain the solution of the sequential process production system with limited intermediate inventory operates in the following manner:

(1) The items generator is the common origin for all items to be processed through the n operations, except of those items which are placed at the intermediate inventories during an extra-time. A transaction (item) is nerated by a GENERATE block and enters the process instantaneously as the first operation O_1 becomes free.

(2) When the item enters the different operations, i.e., in a HOLD block, then a time for each of them is selected from the given probability distribution.

(3) These times are added to the current sum of all previous operation times, which is based in the variable time increment method.

(4) The event occurring first is determined, i.e., the operation which will finish an item first is determined.

(5) If the storage S_j, succeeding operation O_j which just finished to work on item i is full or if the storage S_{j-1} preceeding the operation is empty, then a delay will occur. This delay is computed by the program and the operation O_j will proceed as soon as an item is removed from storage S_j, in the case it was full or as soon as an item is put in the storage S_{j-1}, in the case it was empty.

(6) The status of each intermediate inventory is controlled through the STORAGE block by a counter keeping the storage contents.

(7) Every time an item is finished by the last operation O_n it is eliminated through the TERMINATE block and a record of the number of items finished is kept by the program.

(8) The simulation of breakdowns is accomplished by a generator of breakdowns (transactions). The transactions generated which represent the breakdowns enter an INTERRUPT block, interrupting the operation for a period of time also generated according to the probability distribution followed by the setting times.

(9) Before the production system starts to work all storages are filled up, which is also accomplished by the generation of transactions representing the extra-items.

(10) The process continues until the total time for which the system should work is over or until the finished items counter contains a quantity equal to the number of items to be produced (sample size).

An example of the computer program is presented in the Appendix. The simulation experiments were completed on an Univac-1108.

The mean cycle time of the last operation is computed by dividing the clock time by the number of finished items. Several runs were performed for a particular and hypothetical production system varying both the number of operations and the intermediate storage capacities. The analysis of the systems behavior is the subject of the following chapter.

CHAPTER V

ANALYSIS OF THE RESULTS

The operations downtime caused by the delays and breakdowns and on the other hand the maintenance of intermediate inventories between operations can be reduced to cost as a common denominator. The cost of a design improvement must be compared with the value of the improvement.

The system simulation allows an evaluation of the improvement whose cost must be compared with its value. In the production system under consideration the objective is to achieve an economic equilibrium between the cost of increasing intermediate inventories, changing the number of operations, diminishing the variation of cycle times or the frequency of breakdowns and the cost of the system utilization.

The simulation contributes with the information that is required for determining the behavior of the cost as a function of the coefficient of utilization.

There are many studies related to the behavior of the chance variables present in the system. The computer model developed is not restricted to particular probability distribution. However, in the system simulated for the purpose of this study it is assumed that the production times of all operations are exponentially distributed and have the same mean value (μ = 100 units of time). It has been found that the probability distribution followed by both, the interval time between breakdowns and setting times, is often of the exponential form. Hence these distributions are also considered as of the exponential form (mean value of setting times = 200 units of time; mean value of interval time between breakdowns = 100,000 units of time).

The experimental design is based on the gradual increase of the number of operations in the system; in this way 2, 4, 6, 10 and 20 operations in sequence were considered. For each case the capacity of intermediate inventories varies from zero to 15 units.

In this study the emphasis has been to present a possible approach to the analysis and design of sequential process production systems rather than a specific solution to such a problem. For the same reason no serious attempt was made in what Conway[4] calls strategic and tactical planning in simulation; the main emphasis is on the approach.

Analysis of the Effect of Intermediate Inventories

To illustrate the results obtained, the summary of the values of the coefficient of utilization (ρ) for the hypothetical system is presented in Table 1. Each value of ρ corresponds to one simulation run. It should be recalled that the simulated system has the following characteristics:

(1) Production time for all operations has exponential distribution.

(2) Setting time for all operations has exponential distribution.

(3) Interval time between breakdowns for all operations has exponential distribution.

Since graphs illustrate functional behavior more vividly than tables, the coefficient of utilization as a function of the intermediate

Table 1. Simulated Results for the Coefficient of Utilization (ρ).

c/n	2	4	6	10	20
0	0.67	0.50	0.46	0.42	0.39
1	0.76	0.62	0.58	0.54	0.53
2	0.81	0.69	0.64	0.62	0.61
3	0.85	0.74	0.71	0.67	0.66
4	0.86	0.77	0.76	0.72	0.71
6	0.88	0.82	0.81	0.79	0.77
15	0.95	0.92	0.91	0.90	0.89

inventories capacities is plotted in Figure 8 for the various number of operations.

The graphs show that the dependent function increases with the capacity of intermediate inventories and tends to the limit of one.

It is worthwhile to notice that the function is discrete; however, as a matter of convenience it can be considered continuous as shown in Figure 9.

It is difficult to justify a generalization from the results obtained from this simulation study. For the particular system, however the behavior of the coefficient of utilization (ρ) as function of the capacity of storages can be seen. The result of primary interest is that ρ increases with c. On the other hand, ρ will have its value remaining almost unchanged after a certain value of c, and approaching the limit of one as c increases.

It is also noticeable that the curves of ρ as a function of c are different for the various number of operations.

The role of the simulation in this problem is to furnish data as to the variation of the coefficient of utilization for different intermediate inventory capacities. Given these data and the cost of system idleness, and of the maintenance of storages, the total cost per unit of output can be determined for each assignment and the minimum found. There is much that can be done in this respect with a formula that shows the variation of ρ as a function of c. Considering:

C_1 = cost of utilization of the system per unit,

C_2 = cost of intermediate inventories per unit, and

C_T = total cost per unit,

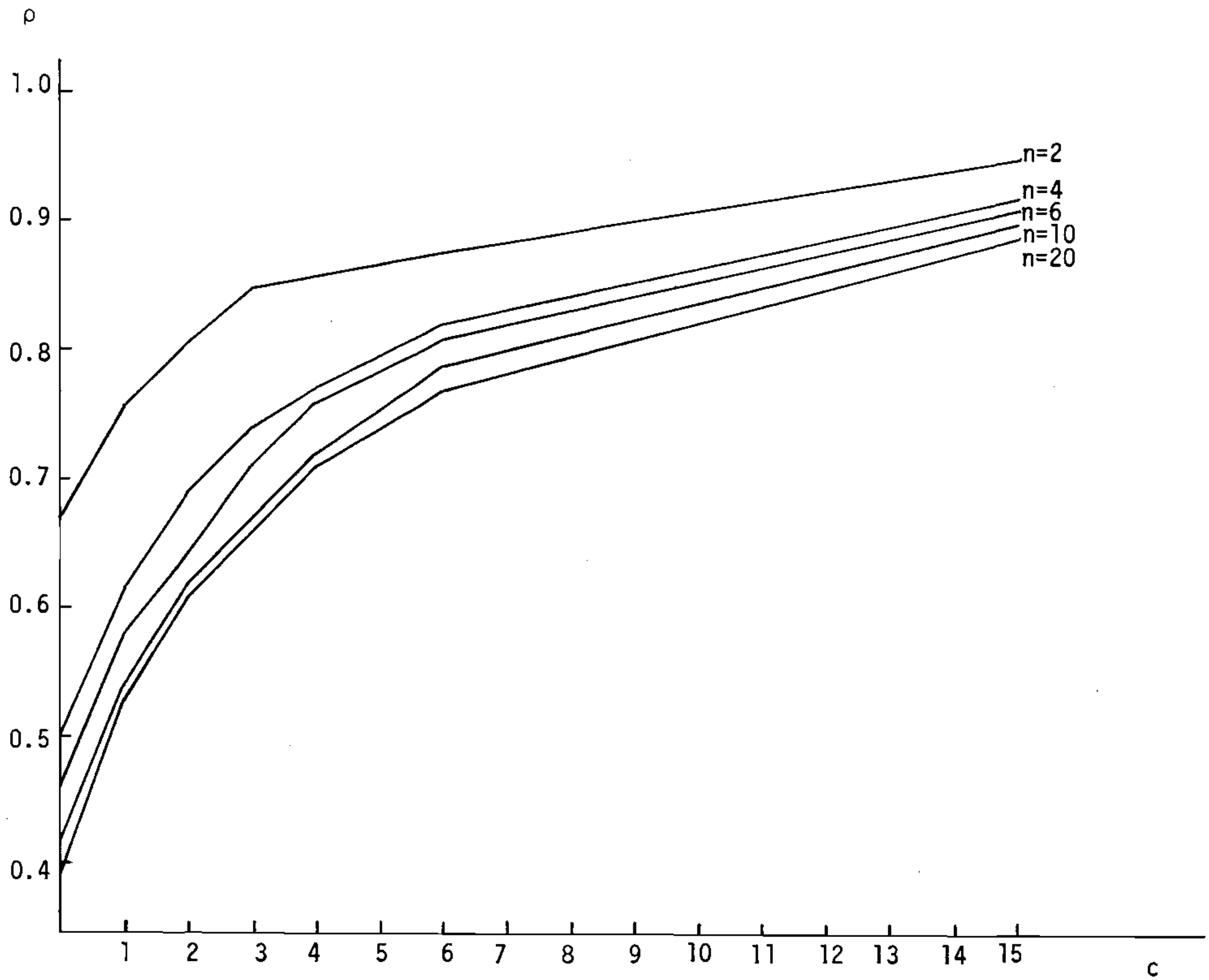


Figure 8. Coefficient of Utilization (ρ) for Various Number of Operations at Various Intermediate Inventory Capacities. (Discrete Function).

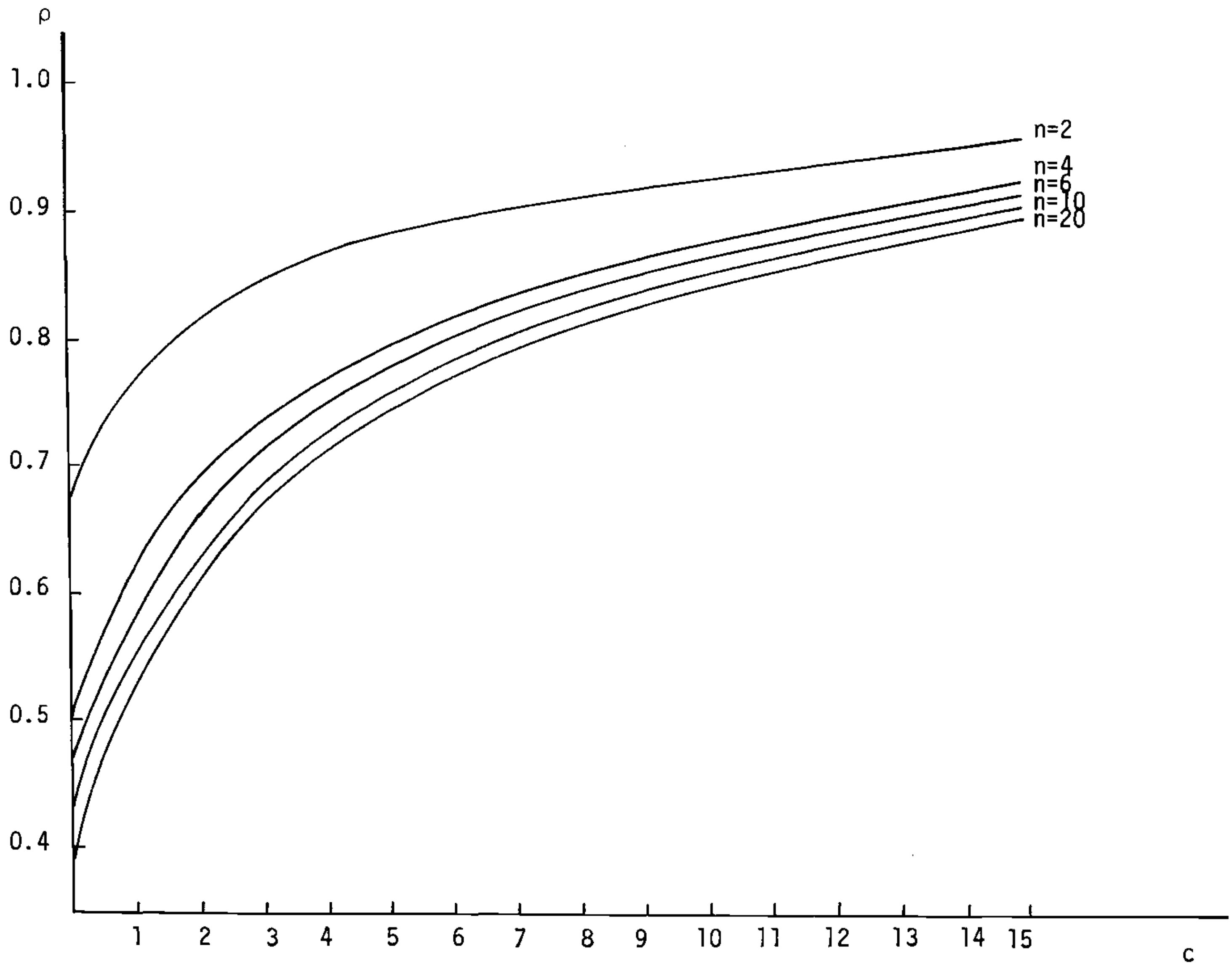


Figure 9. Coefficient of Utilization (ρ) for Various Number of Operations at Various Intermediate Inventory Capacities. (Continuous Function).

hence

$$C_T = C_1 + C_2$$

It can be noticed that C_1 is a function of ρ. However, ρ is a function of c. Hence both, C_1 and C_2, can be expressed in terms of c, allowing the determination of c for which C_T is minimum.

It should be observed that c can only assume integer values.

The principal difficulty is that it is often impossible to attach a specific dollar figure to the cost factors. This does not invalidate such a study because at least an approximate behavior pattern can be found and thus be very helpful to the analysis and design of the system.

The Number of Operations and the Coefficient of Utilization

The simulation was performed on sequences of 2, 4, 6, 10 and 20 operations. It can be seen that ρ decreases with the number of operations n if the capacity is held the same. This is to be concluded from Equation II. The delay on the last operation is a function of the state probabilities of the last storage, which turns to be a function of the rate with which items are put in and taken out. However, the operation O_{n-1} is dependent on storage S_{n-2} and so on. It can be seen that by increasing the number of operations the probability that the last storage is empty increases in frequency.

This difference decreases with the capacity of the intermediate storages because it diminishes the probability that a storage is void. In fact, as the storage capacity increases it can store a greater quantity so that any operation can use the items stored in the preceeding storage.

The capacity required to nullify the effect of the number of operations is in the neighborhood of 15 units and varies with the range of the operation times.

If the system were not balanced the effect of the intermediate storages would be different and probably the capacity required to produce the same effect would also be different.

The variation of ρ with the number of operations is shown in Figure 10.

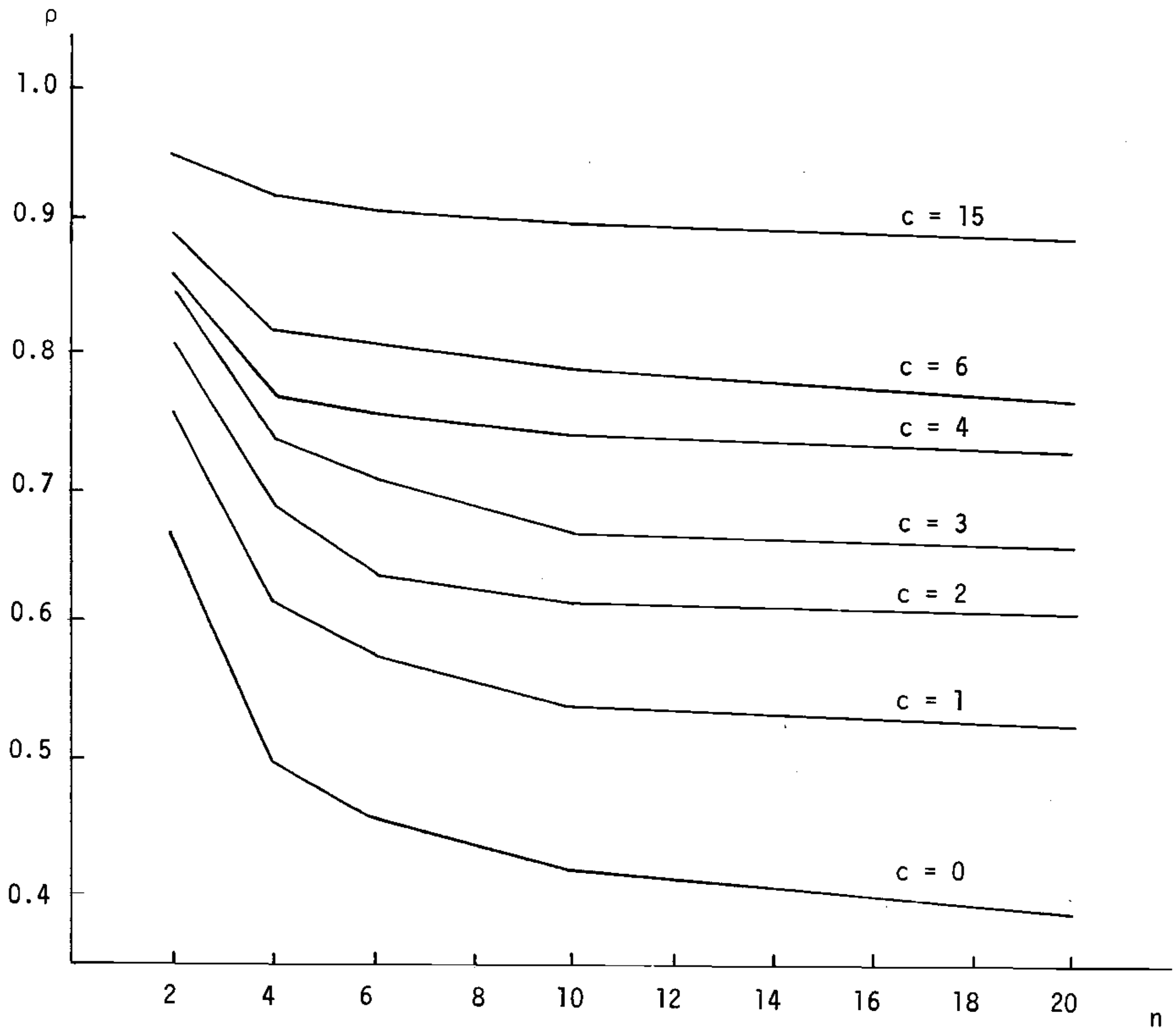


Figure 10. Coefficient of Utilization (ρ) for Various Number of Operations.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

It was the purpose of this study to develop a simulation approach to the analysis and design of sequential process production systems for the general case when any number of operations are arranged in series and subject to variation in the independent production times and to breakdowns. The simulation permits the measure of the coefficient of utilization in order to evaluate the performance of a particular system and the effect of intermediate inventories on this performance.

A mathematical model was formulated in order to analyze the behavior of the system. In addition, a computer model, using the General Purpose Systems Simulator (GPSS-II) was developed in order to simulate the system. An analysis of the results was presented to point out the effect of intermediate inventories and the number of operations on the system performance. A procedure for analyzing the effect of costs related to the intermediate inventories on the design of the system was developed.

The following conclusions can be derived from this study:

(1) The simulation approach presented is not restricted to system of machines in the strict sense; it may be applied to any system in which operations are arranged in series or a combination of series. There is no restriction with respect to the probability distribution of operation times, setting times or the probability distribution of the time

interval between breakdowns occurring in each operation. The breakdowns may be interpreted as any type of interruption and the setting times simply mean the times taken to restore operation.

(2) All the computer runs performed for the purpose of this study took less than 3 minutes.

(3) Features which limit the application of the method developed include the availability of an electronic computer suitable for the General Purpose Systems Simulator. Two other features which limit the usefulness of this approach are the knowledge of the various probability distributions and the costs involved. However, these are not serious limitations since the analyst may be more interested in the behavior of the system than in determining specific values.

(4) There are natural difficulties in this approach which are inherent to the technique of simulation as pointed out by Conway[4] and Ginsberg[6]. The principal problem in the analysis of results obtained from a simulation of the system is in estimating how precise the results are and how to make them more precise. The observations taken during a simulation are usually an autocorrelate, stochastic time series, which complicates the determination of the variability of results. There are several alternative methods in avoiding the autocorrelation and some authors consider a number of tactical questions that arise in the execution of simulated experiments on a digital computer.

Recommendations

(1) Additional research is needed for the development of universal expressions which may correlate the various factors affecting the co-

efficient of utilization.

(2) Additional simulation studies may result in the determination of general rules which may be applicable to all systems involving sequential processes.

(3) Further investigations with respect to the probability distributions involved in this kind of study should add considerably to the value of the approach.

(4) It is also recommended that an exact solution for the evaluation of the effect of intermediate inventories be developed because it would make possible a determination of the accuracy of the simulation approach.

APPENDIX

An example of the General Purpose Systems Simulator (GPSS-II) program for n=6 and c = 4.

```
        JOB
        FLOW
*
*
*
*                   SIMULATION OF A SEQUENTIAL PROCESS PRODUCTION SYSTEM
*
*
*                   NUMBER OF OPERATIONS     N=6
*                   CAPACITY OF INTERMEDIATE INVENTORIES    C=4
*
*
*
*       FUNCTION TO DETERMINE CYCLE TIMES, SETTING TIMES AND  TIME
*       INTERVAL BETWEEN BREAKDOWNS.      RANDOM MODE
*
 1      FUNCTION     RN1    C24
0.0     0     .1     .104   .2     .222   .3     .355   .4     .509   .5     .69
.6      .915  .7     1.2    .75    1.38   .8     1.6    .84    1.83   .88    2.12
.9      2.3   .92    2.52   .94    2.81   .95    2.99   .96    3.2    .97    3.5
.98     3.9   .99    4.6    .995   5.3    .998   6.2    .999   7.0    .9997 8.0
*
*       FUNCTIONS TO DETERMINE THE INITIAL CONDITIONS
*
 2      FUNCTION     N660   C2
0       1     5      6
*
 3      FUNCTION     N610   D5
3       2     7      3      11     4      15     5      19     6
*
*       FUNCTION TO DETERMINE THE PATH AFTER BLOCK 60
*
 4      FUNCTION     P1     D2
1       90    99999970
*
*
*
```

```
*
*      SIMULATION OF THE FLOW OF ITEMS THROUGH THE SYSTEM
*
 10    GENERATE     21     1000                   20             1      0
 20    GATE         NU6                           30
 30    ASSIGN       1      K6                     40
 40    ASSIGN       2      K100                   50
 50    ASSIGN       3      V1                     60
 60    HOLD         *1                    FN      4              *2     FN1
 70    STORE        *1                            80             0      0
 80    GATE         NU*3                          90
 90    LOOP         1                             40      100
 100   TABULATE     1                             110
 110   TERMINATE    R
*
*
*
*      SIMULATION OF BREAKDOWNS
*
 310   GENERATE     21                    BOTH    312     500    100000FN1
 312   COMPARE      N310   G      K1              315
 315   INTERRUPT    1                             500            200    FN1
*
 320   GENERATE     21                    BOTH    322     500    100000FN1
 322   COMPARE      N320   G      K1              325
 325   INTERRUPT    2                             500            200    FN1
*
 330   GENERATE     21                    BOTH    332     500    100000FN1
 332   COMPARE      N330   G      K1              335
 335   INTERRUPT    3                             500            200    FN1
*
 340   GENERATE     21                    BOTH    342     500    100000FN1
 342   COMPARE      N340   G      K1              345
 345   INTERRUPT    4                             500            200    FN1
*
 350   GENERATE     21                    BOTH    352     500    100000FN1
 352   COMPARE      N350   G      K1              355
```

```
 355   INTERRUPT   5                               500          200    FN1
*
 360   GENERATE    21                       BOTH   362    500   100000FN1
 362   COMPARE     N360   G     K1                 365
 365   INTERRUPT   6                               500          200    FN1
*
 500   TABULATE    2                               510
 510   TERMINATE
*
*
*      SIMULATION OF THE INITIAL CONDITIONS
*
 600   ORIGINATE          20                       610          1      0
 610   ASSIGN      1      FN3                      620
 620   ASSIGN      3      V1                       70
*
*
 650   ORIGINATE          6                        660          1      0
 660   ASSIGN      1      FN2                      670
 670   ASSIGN      4      V2                       680
 680   INTERRUPT   *1                              690          *4     0
 690   TERMINATE
*
*
*      TABLES
 1     TABLE       M1     0     50    100
 2     TABLE       M1     0     20    100
*
*
*      VARIABLES
 1     VARIABLE    P1-K1
 2     VARIABLE    K21-C1
*
*
*      CAPACITIES
 2     CAPACITY    4
 3     CAPACITY    4
```

```
 4    CAPACITY    4
 5    CAPACITY    4
 6    CAPACITY    4
*
*
*     CONTROL OF THE SAMPLE SIZE
      START       1000
      END
```

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