Optimal Customized Pricing in Competitive Settings

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In this paper, we study pricing situations where a firm provides a price quote in the presence of uncertainty in the competitive landscape and the preferences of the buyer. We review two possible customized-pricing bid-response models used in practice which can be developed from the historical information available to the firm based on previous bidding opportunities. We show how these models may be used to exploit the differences in the market segments to generate optimal price quotes given the characteristics of the current bid opportunity. We also show how the models may be adjusted depending on the amount of historical bid information available to the user. Finally, we test the two methods on two industry datasets to compare their performance and estimate the percent improvement in expected profits that may be possible from their use.

Key words: bid-response functions, customized pricing, price optimization, bid-pricing.

1. Introduction

While the majority of the previous literature in the price-optimization area focuses on the pricing of consumer goods or the optimal design of auctions, a large percentage of firms face pricing decisions in a business-to-business setting where a customer requests bids from a small set of competing firms and the firms vying for the customer's business respond with a single price quote for the product or service. When the total dollar amount of the customer does not justify a dedicated sales person on behalf of the firm responding to the bid, many firms use bid-response models to provide customized pricing recommendations on what price to offer for the business being bid upon. Customized-pricing bid-response models (CPBRMs) provide a probability of winning for every possible price response, allowing a firm to balance a decreasing margin with an increasing win probability needed in a price optimization model. Examples of firms using

CPBRMs include United Parcel Service (UPS) when responding to bids for from their small to medium size customers (Kniple, 2006), banks replying to interest rate requests for personal and business loans (Phillips, 2005a), and BlueLinx, the largest building products distributor in the U.S, responding to requests for products from construction companies (Dudziak, 2006). The financial impact from using CPBRMs can be significant. UPS reported an increase in profits of over $100 million per year by optimizing their price offerings using CPBRMs (Boyd et al. 2005).

In determining the winning bid probability, CPBRMs effectively determine the price segment the current bid falls in. Price segments are defined as sets of transactions, classified by customer, product, and transaction attributes, which exhibit similar relative pricing level and price sensitivity. Customer attributes may include customer location, size of the market the customer is in, type of business the customer is in, the way the customer uses the product, customer purchase frequency, customer size, and customer purchasing sophistication. Product attributes may include product type, lifecycle stage, and the degree of commoditization. Transaction attributes may include order size, other products on the order, channel, specific competitor, when the order is placed, and what the urgency is of the bidder. In addition, some models assume knowledge of the historical and current bid-price of competing firms participating in the bid.

A common characteristic of situations where firms employ CPBRMs is when the bidder with the lowest price does not always win the bid. Thus, markets are characterized by product differentiation where a given firm may command a positive price-premium over its competitors; dependent upon the particular customer offering the bid. Even assuming a firm collects enough historical data to perfectly derive its price premium for a given customer, there may still be some inherent amount of uncertainty in the bid winning probability due to the bid-requesting firm randomly allocating its business to different competitors to ensure a competitive market for future bids. Therefore, a firm will never be able to remove all uncertainty from the bid-price response process and must work with probabilistic models.

Another common characteristic of situations where firms have used CPBRMs is when the size of the bid opportunities is not large enough to justify a dedicated sales person for each bid opportunity. Thus, the most common alternatives to using CPBRMs is either to charge a fixed price to all customers or to have a sales agent respond to each separate bid opportunity with a customized price. Charging a fixed price leads to missed opportunities to price discriminate between different customer segments, a practice that has been well publicized for significantly increasing a firm's profit in many different industries. The other alternative, relying on a sales agent to respond to multiple bid opportunities, is also problematic. Theoretically, the sales agent should have knowledge of the market, based on a history of former bid-responses with the customer requesting the bid, allowing the sales agent to respond with a customized price that optimizes this inherent trade-off between decreasing margins, due to lowering the price, and increasing probabilities of winning the bid. In reality, sales agents often do not make good trade-offs in these situations, either because of lack of historical knowledge, the inability to process this historical knowledge into probability distributions, or mis-aligned incentives (Garrow et al., 2006). The judicious use of CPBRMs allows firms to capture historical bid information, process it, and present non-biased price recommendations to bidding opportunities. If there is additional information available regarding the bidding opportunity that can not be captured in the CPBRMs, the CPBRM's recommended price may serve as one of possibly many inputs to the person responsible for making the bid-response decision.

To summarize, CPBRMs apply to situations where a firm selling a non-commodity product must respond to frequent request for small to medium sized bids from a number of different customers where the bid-winning criteria is not always the lowest price. To use a CPBRM, a firm must have access to their historical bid history that includes, as a minimum, the price the firm bid at each opportunity and the corresponding bid result (win or loss). Other useful historical information used in developing CPBRMs is, for each historical bid opportunity, the customer, the length of the relationship with the customer, the size of the order, delivery date requirements, competitors' bids, and any other pertinent information useful for market segmentation. When CPBRMs are used as an input to a price optimization model, there is also the implicit assumption the actions of the competitors can be determined probabilistically and independently of the decision maker's action. If all competitors have similar analytic capabilities and jointly optimize against each other, competitive response modeling techniques such as game theory must be used.

In this paper, we evaluate two CPBRMs, namely the Logit and Power functions, which model the response of the buyer subject to the segmentation criteria described above. Based on numerical comparisons using two industry datasets, we provide observations on where each function is preferred. We find the Logit function is preferred when there is less historical bid data available or little data about each bid is available for determining customer segmentations. When detailed information is available about each former bidding opportunity such as the competitors' prices and the size of the order, the Power function outperforms the Logit on our test datasets. We demonstrate how to modify the functions to incorporate various degrees of segmentation data available to the firm. We then test both functions using the data available from the two industry datasets to analyze the relation between the nature of information available to the firm and the improvements generated by our approach. As expected, we find the percent improvement in expected profits increases in the amount of historical bid data available and in the amount of information recorded from each bid opportunity. Yet, even under the worst case conditions of little historical data, we continue to see significant improvements in expected profits over the un-optimized strategies the companies were following before using CPBRMs.

The rest of the paper is organized as follows. In §2 we review the academic literature and industry practices related to the modeling of bid-price responses. In §3 we present two CPBRMs that are used in practice and show how they can be modified to use under three different levels of availability of historical and competitive information. In §4 we present the results from applying the two CPBRMs to two industry datasets where we measure the percent improvement in expected profits under different information levels. In §5 we summarize our observations from the numerical comparison and conclude with some limitations and managerial implications of using CPBRM's.

2. Literature Review

In this section, we discuss the academic literature on bid-price response models and how CPBRMs are unique. We also discuss the motivation from industry practices related to such competitive pricing settings.

Several papers develop bid-price response models where price is the only attribute of the model. Friedman (1956) and Gates (1967) both develop models which use the historical bid information available. Morin and Clough (1969) build on their work by identifying key competitors and capturing temporal sensitivity to changes in strategy by giving recent data more importance. However, these models consider price as the sole criterion for winning a bid and only consider the objective of maximizing profits. Chapter 4 in Lilien et al. (1992) provides an overview of competition oriented pricing where the firm makes a trade-off between margin and probability of winning the bid. This is the same trade-off the firm makes in our models, the difference, however, occurs in the estimation of the winning probabilities. In their model, the lowest bid always wins so the probabilities are based on the number of competitors and each competitor's estimated bid-to-cost ratio. King and Mercer (1991) discuss estimation methods for determining the distributions for these ratios. The models we review are more general; they include non-price factors such as order size and continue to hold when factors other than just price are included in the buyer's decision. In addition, price optimization using CPBRMs can accommodate several strategic firm objectives such as increasing or maintaining market share.

Papaioannou and Cassaigne (2000) provide a detailed review of bid-price response models and develop a "*ServPrice*" model which, like CPBRMs, accommodates several firm objectives and accounts for both price and non-price attributes. However, their model relies only on sales or pricing agents to make tradeoffs and analyze historical information. On the other hand, CPBRMs help the firm obtain a non-biased input to the bid-response decision by processing the relevant historical information statistically. Lawrence (2003) develops a prescriptive model for quotes with high potential revenues, which predicts the outcome of a bid as a function of its attributes. His model requires more extensive bid history than a typical CPBRM and uses a machine-learning approach. It also only uses the outcomes of known transactions and doesn't exploit any additional information that is available to the firm. CPBRM's can exploit much more extensive information, if available, pertinent to a particular market segment. We also study the difference in improvements when CPBRMs are used with different levels of information knowledge.

This paper is most closely related to the work presented in Chapter 11 of Phillips (2005b) and the U.S. patent of Boyd et al. (2005), who discuss the use of CPBRMs in industry and develop models using a Logit function as a bid-response function. These models capture the inherent preference uncertainty and non-price factors which play a critical role in winning a bid. The contributions of our work over the methods suggested by Phillips and Boyd et al. are as follows: We extend the Logit bid-response function to include the competitor's price which helps to capture the competitive dynamics. We also present another CPBRM, the Power function (sometimes found in practice) which includes a parameter of the ratio of the bidder's price to the expected bid price of the bidder's competitor(s). We numerically test both models on two industry data sets representing best and worst case scenarios of where CPBRMs may be applied. Based on the performance of each model on the data-sets, we give observations on when each model may be preferred. To our knowledge, this is the first academic paper to present CPBRMs and test them on actual industry data.

3. Customized-Price Bid-Response Models

In this section we describe what CPBRMs are, present two CPBRMs used in practice, and discuss how they may be used in a price optimization model. CPBRMs calculate the probability of winning a bid opportunity for each possible price response given the market characteristics and competitive dynamics for a particular customer segment. The parameter values for these models are statistically estimated from historical bid information and include, at a minimum, the bidding firm's historical bids and the outcome from each bidding opportunity (win or loss). Intuitively, if the price quoted by a firm is very low compared to its competitor's price, the probability of winning the bid should be close to 100%. If it is very high by comparison, the probability of winning should be close to zero. This probability of "winning the bid" should monotonically decrease with an increase in price (or price ratio). Also, the slope of the response curve should be steeper for prices close to the competitor's price as compared to prices far higher or lower than the competitor's price. Hence, the bid response curve is generally S-shaped in nature. In a single competitor setting with no price premium enjoyed by either firm, pricing equal to the competitor's price should result in a 50% chance of winning the bid opportunity. In practice, however, one of the firms usually enjoys some price-premium over the other. Determining what this price-premium is for each customer segment is one of the uses of CPBRMs.

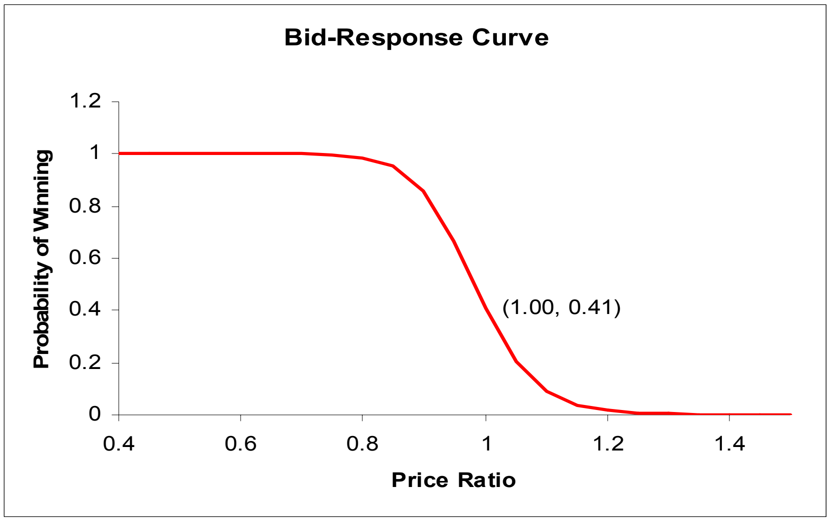

Figure 1

Figure 1 shows a CPBRM curve applied to one of our test case datasets. The price ratio on the x-axis is the ratio of the firm's price relative to its competitor's price. For the particular firm corresponding to this bid-response curve, a price equal to its competitors price (price ratio = 1) results in a probability of winning the bid of 40.91% (a price ratio = .98 equates to a 50% win probability for this firm). Thus, this firm has a negative price-premium and must price below its competitor's price for an equal opportunity of winning the business of the firm offering the bid. Before presenting the two CPBRM functions reviewed in this paper, we first introduce some notation.

Table 1: Notation

p_i	Unit price quoted by the firm for bid opportunity *i* (our decision variable)
$\rho(p_i)$	Bid-Response Function, i.e. probability of winning bid opportunity *i* given a price of p_i
i	Index for bid opportunities, *i* = 1, 2…
a_j, α_j	Parameters related to non-price factors for a segment *j*
b_j, γ_j	Parameters related to price factors for a segment *j*
j	Index for segments, *j* = 1, 2…
c_c	Parameter for the price quote of the competitor
$p_{c,i}$	Unit price quoted by the competitor(s) for bid opportunity *i*

c_q	Parameter for the order size
Q_i	Order size for bid opportunity *i*
$r(p_i)$	Price ratio $= p_i / p_{c,i}$
x_j	Indicator variable for segment *j* (binary)
$\varepsilon(p_i)$	Elasticity of the bid- response function
c_p	Marginal costs for the firm
W_i	Win/Loss indicator variable for a bid opportunity *i* (binary)

3.1 Two Common CPBRMs

In this section we describe the two CPBRMs (both commonly used in practice) compared in this paper and discuss how they can be adjusted to include segmentation and competitive pricing information. The inclusion of segmentation and competitive price parameters has been conjectured to significantly enhance the predictive power of a CPBRM. Bid-responses may differ based on customers, channels, or product attributes such as warranty or payment terms. We capture these possible aspects in our models through a single counting variable *j*, where *j* = 1, 2,… represents the number of distinct, discrete customer segments. Other factors such as the size of the order or the competitive price can often be modeled (depending on the CPBRM) on a continuous scale and may sometimes be treated separately.

Logit Bid-Response Function

Philips (2005b) & Boyd et al. (2005) both present the Logit function as their representation of a CPBRM. As discussed in Philips (2005b, pg. 289), for a dataset with *j* distinct segments, the general form for the Logit function is:

$$\rho(p_i) = \frac{1}{1 + e^{\sum_j a_j + \sum_j b_j p_i}}.$$

One of the main advantages of the Logit function is the ease of adding additional segmentation factors such as the size of the order, Q_i and the competitor's price quote, $p_{c,i}$. If the segmenting variables can be used as continuous variables, the model may be adjusted to include these segmentations by adding the parameters c_q to measure the effect of order quantity segmentation and c_c to measure the effect of the competitor's price segmentation. These parameters are multiplied by Q_i and $p_{c,i}$ respectively:

$$\rho(p_i \mid Q_i, p_{c,i}) = \frac{1}{1+e^{\sum_j a_j + \sum_j b_j p_i + c_q Q_i + c_c p_{c,i}}}.$$

Note that a relative price ratio may also be used in the Logit function by replacing $c_c p_{c,i}$ with $c_c(p_i / p_{c,i})$ in the equation above. In our performance test, we found little difference between these two representations so we present the simpler form with just the competitor's price for the remainder of this paper. Using the simplest form of the Logit function: $\rho(p) = \frac{1}{1+e^{a+bp}}$, the slope $\rho'(p)$ and elasticity $\varepsilon(p)$ of the Logit function is (Philips 2005b pg. 284):

$$\rho'(p) = -b\rho(p)(1-\rho(p)) \text{ and } \varepsilon(p) = bp(1-\rho(p)).$$

Power Bid-Response Function

An alternate CPBRM sometimes used in practice is the Power function, defined in its general form as: $\rho(p_i) = \frac{\alpha_j}{\alpha_j + r(p_i)^{\gamma_j}}$. The main advantage of the Power function is that, compared to the Logit function, competitive price dynamics are explicitly captured. The main disadvantage is that it is more cumbersome to adjust the model for non-price, continuous variable attributes. Segmentation parameters can be added to the Power function but only through a discrete characterization. Thus, a variable such as order size must be broken into discrete intervals and captured through the parameter γ_j, where the subscript j now represents the discrete intervals of the order size. Using the simplest form of the power model with no segmentation, $\rho(p) = \dfrac{\alpha}{\alpha + r(p)^{\gamma}}$, the slope and elasticity of the Power function is

$$\rho'(p) = -\frac{\gamma}{p}\rho(p)(1-\rho(p)) \text{ and } \varepsilon(p) = \gamma(1-\rho(p)).$$

For a CPBRM to be a strictly decreasing function in p, the price dependent parameters must be strictly greater than zero. More specifically, for the Power function: $\gamma > 0$. The parameter γ is a measure of the price sensitivity of the buyer with higher values implying greater price sensitivity. The effect of the parameter γ on the probability of winning is shown in Figure 2.

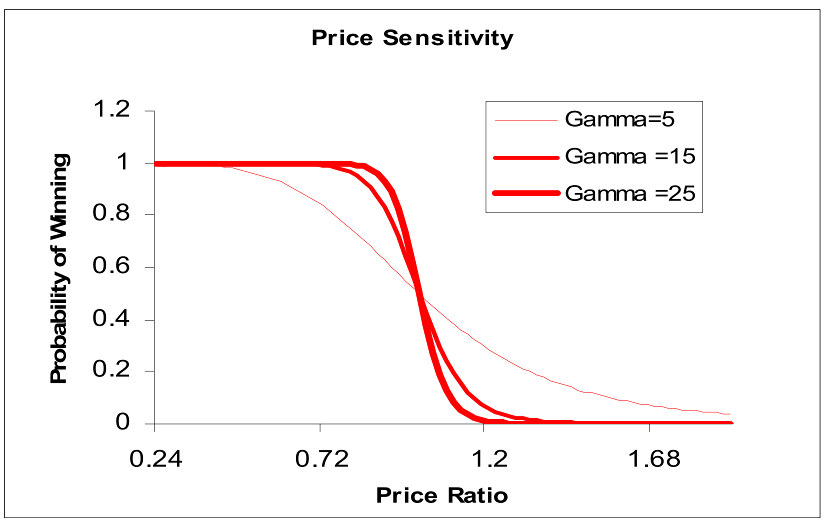

Figure 2

The parameter α is a measure of the price premium the firm enjoys, with a higher value implying a higher premium on the market. Thus, an increasing value of α allows the firm to charge a higher price for the same probability of winning. The effect of the parameter α on the probability of winning is shown in Figure 3.

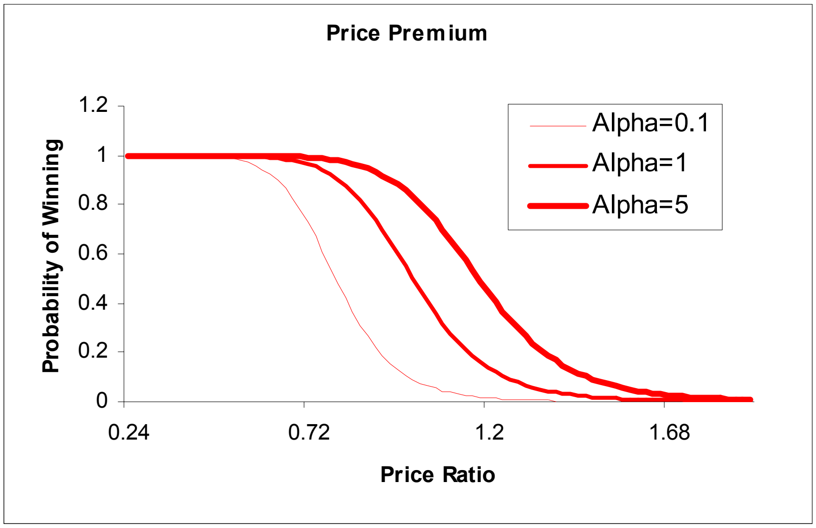

Figure 3

3.2 Estimation of Parameter Values

The parameter values of a CPBRM can be estimated statistically by fitting a curve to the available bid-history data based on minimizing the squared errors or using maximum-likelihood estimates. We briefly describe each method below, using the Power function as the CPBRM of reference. (Philips 2005b, pg. 285) describes how each method is applied to the Logit function.

Estimation by Minimizing the Squared Errors

The first method for estimating parameter values is by minimizing the squares of the error terms from the curve of the CPBRM to the actual wins and losses of each historical bid opportunity. The procedure is as follows. Start by assigning the indicator variables W_i =1 if the bid was won and W_i = 0 if the bid was lost for each historical bid opportunity. Remember that a CPBRM provides a probability of winning a bid for a given price. Thus, to determine the best estimates of the parameter values for a CPBRM, we want the bid-response to be as close to W_i as possible. This can be achieved by solving the unconstrained optimization problem

$$\underset{\alpha,\gamma}{Minimize} \quad \sum_i [\rho(p_i \mid \alpha,\gamma) - W_i]^2 .$$

Maximum-Likelihood Estimation

The second method for estimating the parameter values is to choose parameter values that mimic, as close as possible, the same pattern of wins and losses as the actual outcomes. Assuming all bids are independent, the probability of realizing the actual outcome for a particular bid i is $L_i(\alpha,\gamma) = \rho(p_i \mid \alpha,\gamma)W_i + [1-\rho(p_i \mid \alpha,\gamma)](1-W_i)$. Therefore, the parameters can be chosen in such a way that they maximize the probability of realizing the actual outcomes over all observations. This can be achieved by solving

$$\underset{\alpha,\gamma}{Maximize} \quad \prod_i [\rho(p_i \mid \alpha,\gamma)W_i + [1-\rho(p_i \mid \alpha,\gamma)](1-W_i)].$$

A drawback of the method above, however, is when the amount of historical bid data is small, the likelihood of even a good predictor is also very small. Working with the product of very small numbers often creates computational problems. To avoid this problem, we instead maximize the natural logarithms of the likelihood. Since the logarithm is also an increasing function of the probability, the parameter values that maximize the original problem will also maximize the natural logarithm expressed by

$$\underset{\alpha,\gamma}{Maximize} \quad \sum_i \ln[\rho(p_i \mid \alpha,\gamma)W_i + [1-\rho(p_i \mid \alpha,\gamma)](1-W_i)].$$

The number of bid attributes (segments) that can be accurately estimated depends on the amount of historical bid-information available. If extensive information is available, greater degrees of segmentation can be achieved without compromising the accuracy and robustness of the statistical estimation of the parameter values. After estimating the parameter values of a CPBRM using historical bid data, the CPBRM can now be used to determine the optimal bid-response price for an upcoming bid opportunity. This process is described in the next section.

3.3 Use of a CPBRM in Price Optimization

We now look at how CPBRM curves can be used in price optimization. For the following discussion, we use the objective of maximizing expected profits. However, other strategic or operational objectives can be easily accommodated such as increasing market shares or including constraints on capacity, inventory, price or margin. The price optimization problem for bid opportunity i is

$$\underset{p_i}{Maximize} \quad \pi(p_i) = \rho(p_i) \times (p_i - c_p) \times Q_i \,.$$

Note, the margin $(p_i - c_p)$ is strictly increasing in price (Figure 4) but the probability of winning the bid is strictly decreasing in price (Figure 5). Therefore, the expected profit is a unimodal function as shown in Figure 6.

Figure 4

Figure 5

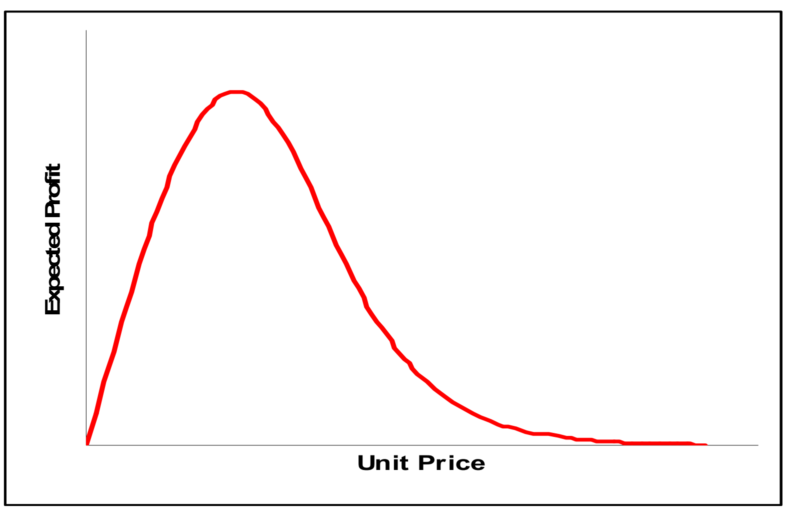

Figure 6

Determining the optimal price involves finding a global maxima for the expected profit which is unimodal in nature. The profit-maximizing price occurs where the elasticity of the expected profit function is equal to the inverse of the marginal contribution ratio,

$$\varepsilon(p) = \frac{p^*}{p^* - c_p}.$$

The derivation is available from the authors by request.

We have described two CPBRMs and explained how they can be used to find an optimal price response for a specific bid opportunity. In the next section we demonstrate how to apply the CPBRMs to historical bid data and test them on two industry datasets corresponding to two extremes of information available to the user.

4. Numerical Comparisons of CPBRMs on Industry Data

In this section, we compare the performance of the two CPBRMs described in the previous section using two bid-history industry datasets. The first dataset contains a single-competitor setting where extensive bid history is available including the competitor's price at each bid opportunity. This dataset represents a very favorable setting for applying CPBRMs. The second dataset contains a multi-competitor setting where very limited bid history is available, representing a more challenging environment to implement CPBRMs. Such disparate datasets allow us to contrast the improvements achieved from using CPBRMs at the extremes of their suitability and compare the effectiveness of the two competing models under different environments.

4.1 Test Case Scenarios

We test the two CPBRMs under a wide set of scenarios pertaining to: 1) the amount of historical bid data available, 2) the amount of knowledge of the competitors' price response to the current bid request, and 3) the amount of segmentation included in the models based on the size of the order in each bid opportunity. We capture sensitivity to the amount of historical bid data by testing the models on two industry datasets, one with a large amount of historical bid data (Dataset 1) and the other with a small amount (Dataset 2). We describe the two datasets below.

Bid History Datasets

Dataset 1: Single Competitor and Extensive Historical Data

The firm providing our first dataset manufactures and sells medical testing equipment to laboratories at hospitals, clinics, and universities across North America. One of their popular products is a gas chromatograph refill cartridge that has a list price of $11.85. The marginal cost associated with each unit is $6.00. The refill cartridges are ordered in batches ranging in size from 100 to over 1000. Orders for fewer than 200 units are handled through the company's website or through resellers with no associated discount from the list price. At the other extreme, the company receives about 100 orders per year for more than 1000 units. These large deals are negotiated by a national account manager, usually as part of a much larger sale. Orders for 200-1000 units are handled by regional sales staff that has considerable leeway with regard to discounting. We only look at this middle-size segment to apply the CPBRMs. The requested size of the order for each bid opportunity is also recorded, allowing us to test both segmented and unsegmented versions of the CPBRMs. Because of the specialized nature of the product, the firm has only one significant competitor and they are able to capture their competitor's price after each bid opportunity. Their bid history information is exhaustive, with approximately 2400 records of previous bid opportunities. A snapshot of this dataset is shown in Table 2.

Table 2: Bid History for a Medical Device Company

Bid Number	Win	Firm's Bid	Competitor's Bid	Order Size
1	Y	$8.44	$10.92	353
2	N	$11.88	$9.99	773
3	N	$11.29	$10.59	974
4	Y	$9.78	$11.52	857
5	Y	$9.28	$11.47	283

For this application, the probability of winning the bid at a price equal to the competitor's price (i.e. price ratio is one) is 51% (Figure 7). This percentage implies the firm doesn't enjoy any significant positive or negative price premium compared to its competitor.

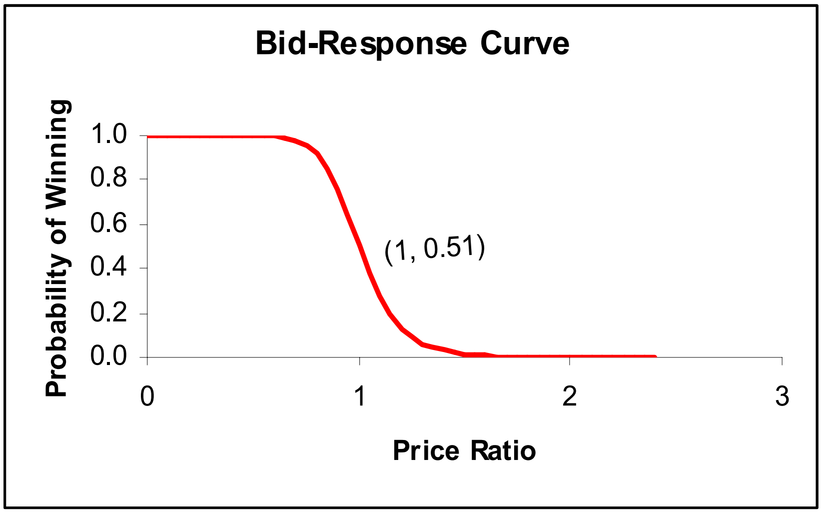

Figure 7: Bid-response Curve for Dataset 1

Dataset 2: Several Competitors and Limited Historical Data

Our second application is historical bid data from a federal agency (Foreign Food Assistance Program) accepting bids from food mills to provide bulk food commodity items such as wheat, soybeans, and vegetable oil. Unlike the dataset from the medical device company, the amount of historical bid data available to us was very limited (approximately 50 historical bid opportunities). For our test, we choose one of the food mills that participated in the majority of the bid opportunities.

Unlike in the first dataset, in this dataset there were multiple bids from competing food mills for each bid opportunity. Thus, we used an *average* of the competitors' bids to represent the competitive bid parameter in our CPBRMs. We chose an average of the competitors' bids rather than a minimum or the winning bid because this bidding situation is truly a case where the lowest bid does not always win. Delivering food supplies to foreign nations involves both purchasing the food from the mills and transporting the food to the nation in need. While the food mills can control the price they respond to the bid request, they have no control over the bids the transportation companies also respond with for delivering the food from the mill to the foreign country. Thus, there is an inherent degree of randomness in these bid opportunities. We assumed a marginal per-unit cost (cost of providing 1000 pounds) of $200.00 for our test mill. A snapshot of the data (after substituting in the average competitors' bid) is provided in Table 3.

Table 3: Bid History for Bulk Food Providers

Bid Number	**Win**	**Firm's Bid**	**Average of Competitors' Bids**	**Order Size (000) lbs**
1	N	$246.85	$238.34	110
2	N	$258.26	$258.49	720
3	Y	$228.55	$234.27	1060
4	N	$321.59	$313.93	1300
5	Y	$217.09	$222.20	2130

In both datasets, the competitors' prices were recorded for each bid opportunity. Thus, the data available is how the competitors priced on *past* bid opportunities. The Power function (and the Logit function with a competitors' price segmentation parameter) requires an input of the competitors' price for the *current* bid opportunity. In some cases, firms may have very limited knowledge of how their competitors will price in an upcoming bid opportunity while, in other cases, firms may have substantial knowledge. We describe how we capture these differing knowledge levels below.

Knowledge Level of Competitors' Pricing

We tested the two CPBRMs under three different levels of knowledge a firm may posses regarding its competitors' pricing, i.e. worst, medium, and best cases. Historical competitive bid-price information is often available in many B2B applications through either formal or informal channels, depending on the relationship the bidder shares with the buyer. UPS, for example, obtains competitors' bids in approximately 40% of the parcel shipping bid opportunities they participate in (Kniple, 2006). In some business-to-business scenarios, a firm may even be provided with the competitors' bids and asked to respond with a quote of their own (note that for reasons explained earlier, providing the lowest bid does not always guarantee a win in these situations). In many business-to-consumer markets such as loan and insurance quotes, information about the competitor's price may be available from a simple web-page search.

Worst Case: No Price Information Case: In this case, the firm has no historical price information on its competitors, nor does it have any information about how its competitors will price for the current bid opportunity. This scenario is rare in practice but, for our analysis, serves as a lower bound on the knowledge of competitors' pricing. With no competitor price information, the Logit function is the only CPBRM available, as the Power function requires an estimate of the competitor's price in the current period (via the price ratio).

Medium Case: Naïve Price Estimation Case: In this medium case of competitive pricing knowledge, the firm has no information about how its competitors will price in the current period except for the price history of its competitors on past bidding opportunities. Thus, the firm can estimate its competitors' prices for the current period through some type of forecasting or regression model. In our analysis we use a simple 10-period moving average to predict the competitors' prices in the current period. While a moving average of the historical prices is most likely not the best forecasting method for this application, it was chosen because it represents a technique that is common in practice. We experimented with moving averages of different numbers of periods but found the 10-period moving average resulted in the most accurate and least biased estimates for the future competitor's bid. Now that we have an estimate for the competitor's price, we can test both the Logit and Power functions as we now have an estimate for the price ratio required to use the Power function.

Best Case: Perfect Competitive Price Knowledge: In this best case of competitive pricing knowledge, the firm knows exactly what its competitors' bids will be in the current period. This can be considered an upper bound on the firm's forecasting capabilities. It also applies to cases where the buyer provides competitors' bids before requesting a bid from the firm or in applications where a firm can check its competitors' prices (possibly via their web pages) before responding with its own price quote.

The chart below summarizes which knowledge levels were tested for each CPBRM.

	Logit	Power
Worst Case	✔	✕
Medium Case	✔	✔
Best Case	✔	✔

The next section describes the procedure we used to test the two CPBRMs on the datasets and scenarios described above.

4.2 Procedure for Testing CPBRMs

The approach we used for both datasets is as follows:

1. We divided the datasets into two distinct sets; the first for estimation of the model parameter values and the second for performance evaluation (similar to a holdout sample commonly used in forecast method evaluations). We used the first 90% of the historical bid records as our *estimation data* and the remaining 10% as our *performance test data*. While the choice of 10% for the performance test may seem arbitrary, it is a common choice for holdout samples in forecast methods evaluations. Sensitivity test with different percentages of the historical data used for measuring performance were also performed. The changes in the parameter estimates and performance results on dataset 1 were insignificant when tested over a range of 10% - 20% for the performance test dataset. The same was not true for the second dataset however, due to the small sample of historical bids available. Thus, we decided to stay with the 10% sample to provide the best opportunity for obtaining good parameter value estimates.

2. Using the estimation data, we calculated the parameter values for both the Logit and Power functions using ordinary least squares and maximum likelihood estimators. We found little difference in the fit of the models between the two estimation methods so we present the values found using maximum likelihood estimators for the remainder of this procedure. The parameter values from the un-segmented analysis of Dataset 1 are:

	Logit Model			Power Model	
	a	B	c_c	α	γ
Worst Case	-8.272	0.825	NA	NA	NA
Medium Case	-0.299	1.0784	-1.05	1.03	10.55
Best Case	-0.299	1.0784	-1.05	1.03	10.55

and the parameter values from the un-segmented analysis of Dataset 2 are:

	Logit Model			Power Model	
	a	B	c_c	α	γ
Worst Case	-1.375	0.0051	NA	NA	NA
Medium Case	-0.293	1.0784	-1.0516	0.6924	20.665
Best Case	-0.293	1.0784	-1.0516	0.6924	20.665

Because the Power function requires an estimate of the competitors' price, it can not be used under the Worst information case. The reason the parameter values are the same for the Medium and Best information cases is because past bid opportunities are used for estimating the parameter values when the competitors' price is known with certainty (note the information cases pertain to knowledge of the competitors' price in the current period; the past prices are assumed to be known with certainty). In the next section, we describe how we also used the order size as a segmentation variable.

3. After estimating the parameter values for each model, we used the CPBRMs to optimize the bid-prices for all the bids in the performance test data subset. It is important to test the models on a different set of data than was used to build the models; else the fit of the models will be misleadingly high.

4. Finally, we computed the percent improvements over expected profits and over actual profits as explained in section 4.3. This provided us with two metrics of performance for each of our cases.

The incorporation of competitors' prices is only one possible input to CPBRMs (although for the Power function it is a required input). Another common input is the size of the order request. It is reasonable to assume the price sensitivity of customers only ordering a few units will differ significantly from customers ordering large quantities. Thus, we describe how we incorporated different levels of order size segmentation below.

Segmentation Based on Order Size

In Dataset 1, order quantities range between 200 and 1000. For segmentation based on the order size using the Logit function, we used the order size as a continuous variable and estimated a statistical constant dependent on the order size Q as follows:

$$\rho(p_i \mid Q, p_c) = \frac{1}{1+e^{a+bp_i+c_qQ+c_cp_c}}$$

For discrete segmentation, a separate parameter must be estimated for each segment but for a continuous variable, the estimation of only one parameter is required. Therefore, it is easier to use a continuous variable if the model allows it. For segmenting based on the order size using the Power model however, we had to use a discrete approach. We chose the eight segments below.

Order Size Between	200-299	300-399	400-499	500-599	600-699	700-799	800-899	900-999
Segment	2	3	4	5	6	7	8	9

To estimate the parameter value for each segment, we used a binary indicator variable x_j, j = 2, 3, ..9, which was assigned a value of one for the order size segment a specific bid fell under. This classification scheme is demonstrated in the table below:

Bid Number	Win	Firm's Bid	Competitor's Bid	Order Size	Indicator Variables for Order Size Segments							
					2	3	4	5	6	7	8	9
1	1	$8.44	$10.92	353	0	1	0	0	0	0	0	0
2	0	$11.88	$9.99	773	0	0	0	0	0	1	0	0

The bid response for the Power function was calculated by estimating a different value of γ_j (our estimates for α did not change so we held it constant) for each order size segment q as follows:

$$\rho(p_i \mid q) = \frac{\alpha}{\alpha + r(p_i)^{\gamma_j x_j}}.$$

Based on our maximum likelihood fits, the estimated parameter values for the segmented analysis of Dataset 1 are:

Knowledge of Comp Price									
	Logit function								
	a	b	c_c	c_q					
Worst Case	-8.436	0.828	NA	0.0003					
Medium Case	-0.498	1.0802	-1.05	0.0003					
Best Case	-0.498	1.0802	-1.05	0.0003					
	Power function								
	α	γ_i							
		2	3	4	5	6	7	8	9
Medium Case	1.02	9.46	9.10	9.45	7.51	7.35	15.33	19.26	15.35
Best Case	1.02	9.46	9.10	9.45	7.51	7.35	15.33	19.26	15.35

The amount of historical bid data available in Dataset 2 was insufficient to segment the data based on the order size. Therefore, we did not perform a segmented analysis on this dataset.

In summary, we compared the performance of the Logit and Power CPBRMs using two industry datasets, three levels of knowledge of the competitors' prices, two levels of segmentation on the order size, and using two performance measures. Thus, we had a total of 24 scenarios to base our observations. Table 4 summarizes the various scenarios.

Table 4: Summary of Test Scenarios

Historical Bid Data Available	**Knowledge of Competitors' Price**	**Segmentation on Order Size**	**Performance Measure**
Large (Dataset 1)	Worst Case	Segmented	Actual Profits
Small (Dataset 2)	Medium Case	Unsegmented	Expected Profits
	Best Case		

In the next section we describe how we measured the performance of the two CPBRMs.

4.3 Measures of Performance

To test the impact of using CPBRMs on the industry datasets, we used two performance metrics: percent improvement in profits over un-optimized actual profits and percent improvement in profits over un-optimized expected profits. To understand the difference between the two performance metrics, consider the following numerical example from Dataset 1:

Bid	Win	Order Size	Original Bid	Optimal Bid	Probability of Win at Original Bid	Probability of Win at Optimized Bid
1	Y	353	$ 8.44	$ 9.35	0.79	0.64

Applying the unsegmented, worst information case, Logit CPBRM with the parameter values obtained through the procedure outlined in section 4.2 we get: $\rho(p) = \dfrac{1}{1+e^{-8.272+.825p}}$.

Substituting in the original bid price of $8.44, we calculate the probability of winning for the un-optimized bid = $1/(1+e^{-8.271+0.825*\$8.44})$=0.79. Applying the optimization procedure described in section 3.3, we calculate the optimal bid price for this bid opportunity should have been $9.35. Substituting in this price results in a probability of winning for the optimized bid = $1/(1+e^{-8.271+0.825*\$9.35}) = 0.64$.

The actual profit from this bid opportunity is = (Original Unit Price- Marginal Cost)* Order Size* Win/Loss Indicator Variable = $(8.44-6)*353*1 = $ 861.32. If the original bid had resulted in a loss, the actual profit would be zero. The original bid expected profit = (Original Unit Price-Marginal Cost)* Order Size * Probability of Win at the Original Bid Price = $(8.44-6)*353*0.79 = $ 680.44. Note the expected profit is always smaller than the actual profit when the bid was won and is always larger when the bid was lost. The optimized bid expected profit = (Optimized Price- Marginal Cost)*Order Size* Probability of Win at the Optimized Bid Price = $ (9.35-6)*353*0.64= $ 756.83.

We now compare the percent improvement of the latter case over the first two:

- Percent Improvement in Optimized Bid Expected Profits over Un-Optimized Bid Actual Profits = ($ 756.83-$861.32)/$861.32 = -0.121 = -12.1 %
- Percent Improvement in Optimized Bid Expected Profits over Un-Optimized Bid Expected Profits = ($756.83-$680.44)/$680.44 = 0.112 = 11.20%.

The calculations above were performed for every bid opportunity in the performance test data subset and the average of each measure (over each bid opportunity in the performance test data subset) was used as the performance metrics presented in the next section.

4.4 Results from Numerical Comparisons

Performance on Dataset 1

For Dataset 1, the un-optimized bid actual and expected profits over the performance test subset of our dataset were exactly the same. Therefore, we only present one comparison of the percent improvements in expected profits for each scenario (segmented versus unsegmented, three levels of information, and two CPBRMs). Figure 8 presents the comparison of the models when the models were not adjusted for order size segmentation. For this comparison, the Logit function outperforms the Power function in the absence of segmentation.

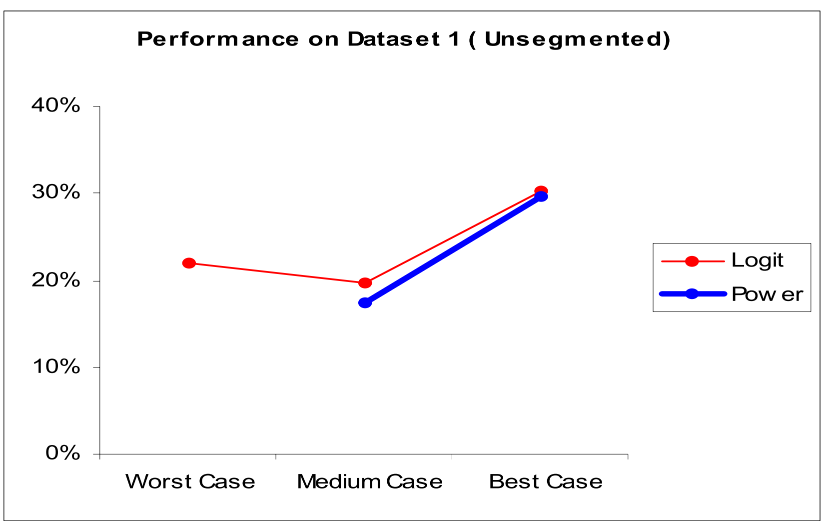

Figure 8: Percent Improvement in Expected Profits (Unsegmented)

Figure 9 presents the comparison with an adjustment for order size segmentation. In the presence of order size segmentation, the Power function outperforms the Logit function.

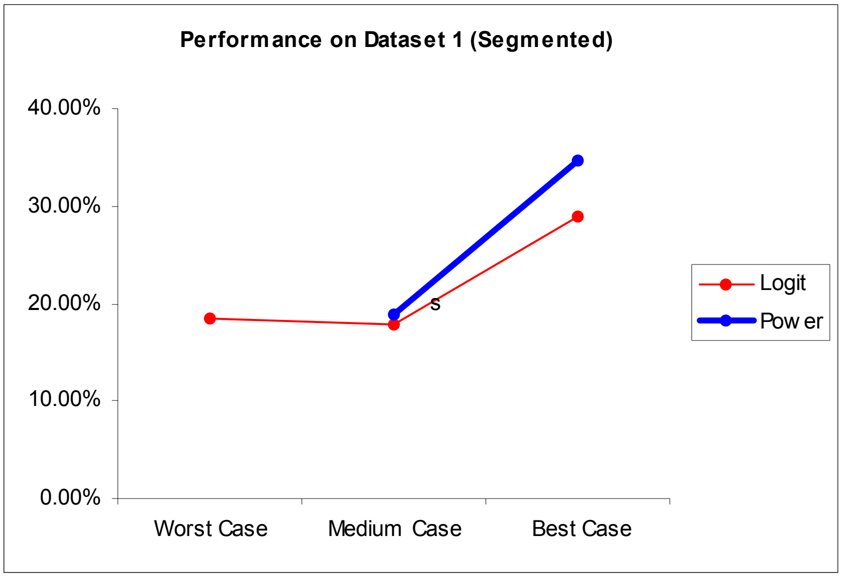

Figure 9: Percent Improvement in Expected Profits (Segmented)

In both the unsegmented and segmented comparisons, the ability to perfectly forecast information about the competitors' pricing results in the largest improvements. Having access to historical data on competitors' bids can sometimes make a company worse off however, as evidenced by the worst information case (no historical competitors' price data) outperforming the medium information case (10 period moving average of competitors' prices). A quick check of our 10-period moving average forecast which we used to provide the competitor's price estimate showed the forecast was unbiased but had a large variance in the forecast error. Thus, for this dataset, the past historical competitor's bids were poor indicators of how the competitor will bid in the future. The use of these poor estimates led to worse performance using the CPBRMs than if no estimate of the competitor's price was used at all.

Performance on Dataset 2

For this application, the probability of winning the business at a price equal to the average competitors' price (i.e. a price ratio of one) was 40.91% (Figure 1 provides the actual bid-response curve for the Power function). Due to the small number of historical bid records available to us in this dataset, we performed our analysis for the un-segmented scenario only. The percent improvements from using the optimized price from the CPBRMs on the performance test part of the datasets are shown in Figures 10 and 11 compared against actual and expected profits respectively. Similar to our comparisons in the first dataset, the case of perfect competitors' price information results in the largest improvements for both models and the Logit function outperforms the Power function in all information cases. Unlike in the previous dataset however, there were differences in actual and expected profits using the un-optimized price and the medium information case resulted in larger improvements than the worst information case.

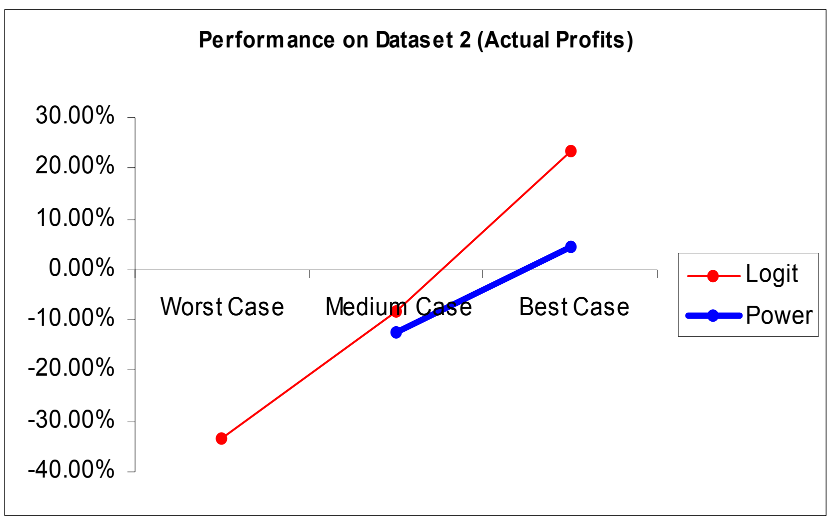

Figure 10: Percent Improvement in Actual Profits (Unsegmented)

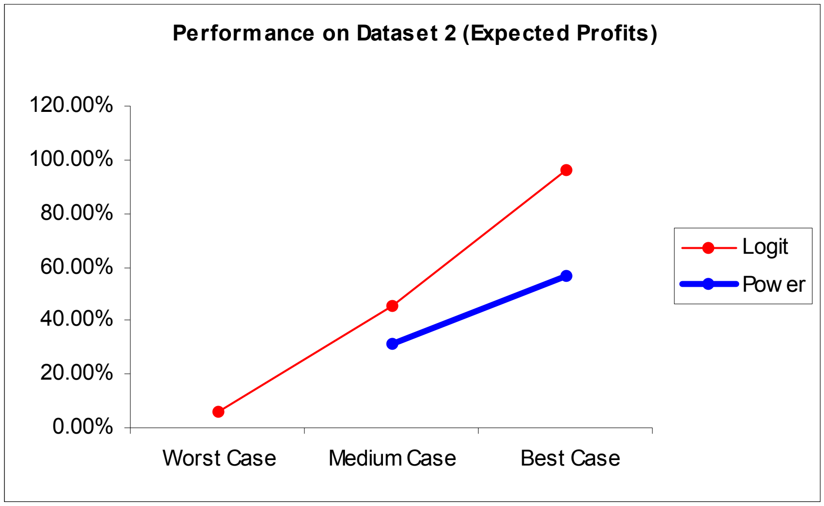

Figure 11: Percent Improvement in Expected Profits (Unsegmented)

Figure 10 shows the improvements when compared to the actual profits are negative for the two lower cases of information on the competitors' price. We suspect this is the result of insufficient data available for the performance test data as improvements over expected profits were positive for all three information cases as shown in Figure 11.

5. Observations from Numerical Comparisons and Conclusions

In this section, we summarize our observations based on our numerical comparisons and attempt to answer the question: "Given a particular set of conditions, which CPBRM should a firm use to optimize prices?" We then summarize our work and provide areas for future research. Our observations come with the following caveats; they are based purely on the performance of the models on our two available datasets and may not be generalizable to applications different than the ones tested. Thus, a firm should rigorously test the models using their own bid history data before drawing conclusions on the suitability of a particular model for their specific application. Our primary purpose is to describe a testing procedure for firms who wish to do so. Based on the performance on our two industry datasets, we provide three main observations:

Observation 1. ***If enough historical bid data is available to segment based on the order size, the Power function, adjusted for each discrete customer segment, outperforms the Logit function.***

This observation is evident in Figure 9 which compares the models developed with a large amount of historical bid data and segmented based on the order size. If the firm has no knowledge about the competitors' prices for the current bid opportunity however, the Power function can not be used.

Observation 2. ***If sufficient historical bid data is not available to a firm or the firm does not segment based on the order size, the Logit function outperforms the Power function.***

This observation is evident in Figures 8, 10, and 11. Figure 8 compares the models developed with a large amount of historical bid data but not segmented based on the order size. Figures 10 and 11 compare the models developed using the much smaller dataset where order size segmentation is not an option.

Observation 3. ***Incorporating historical competitor prices into a CPBRM does not ensure better performance.***

This observation is evident in Figures 8 and 9 which compare the models developed with a large amount of historical bid data. For both the unsegmented and segmented versions of the model, the worst case of competitor pricing information (estimated competitors' prices are not included in the model) outperforms the medium case (competitors' prices estimated through a 10 period moving average). Thus, firms who have access to historical competitors' bid prices but are not proficient in using this data to forecast future prices may be better off leaving this variable out of their CPBRMs.

A firm adopting CPBRMs for price optimization needs to be aware of their limitations. CPBRMs assume the bid opportunities are exogenous and are not affected by the bid responses suggested through the optimization model. In reality, a firm's pricing strategy may have a significant impact on customer retention, especially if the optimization model recommends consistently pricing higher than the competition for a particular customer class. Also, CPBRMs, and their corresponding optimization models, do not assume any response from the firm's competitors. Instead, they assume the actions of competitors can be determined probabilistically and independently of the decision maker's actions. In reality, competitors may react to a firm's new pricing strategy causing the historical bid opportunity data to be unrepresentative of future bid price responses. To help detect these possibilities, mechanisms should be put in place to monitor and evaluate the performance of the CPBRMs over time. If competitors change their bid-pricing behavior due to the implementation of a CPBRM, more involved models using concepts from game theory should be employed.

In summary, we present two CPBRMs used in practice and explain how they may be used to calculate optimal bid-response prices and discuss how they may be adjusted to accommodate segmentation based on the different levels of information available. We present a numerical analysis on two industry datasets and, based on these results, offer a set of recommendations about the type of CPBRM a firm should use depending on the availability of past information and the level of competitive knowledge available to a firm.

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