

Improving Uniform Ultimate Bounded Response of Neuroadaptive Control Approaches Using Command Governors

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In this paper, we develop a command governor-based architecture in order to improve the response of neuroadaptive control approaches. Specifically, a command governor is a linear dynamical system that modifies a given desired command to improve transient and steady-state performance of uncertain dynamical systems. It is shown that as the command governor gain is increased, the neuroadaptive system converges to the linear reference system. Simulation results are used to validate the effectiveness of the proposed framework.

I. Introduction

All models of real world phenomena are approximations, and the design of successful control systems must take this fact into account. Furthermore, systems can be subject to disturbances and other uncertainties such as unpredictable adverse conditions. Adaptive controllers are designed to handle uncertain terms in the model by actively changing the controller law to achieve a desired result. Model reference adaptive controllers accomplish this effect by propagating a reference system with the desired dynamics. The output from the reference system is compared to the output of the plant, and the error is used to drive the adaptation. The class of errors which can be handled by the model reference adaptive controller is partially dependent on the approximation function used. Multilayer neural networks have attractive properties for this role in adaptive control; they are universal function approximators, meaning they can approximate any continuous function to any degree of accuracy given enough hidden layer neurons.

It is desired that the adaptive controller quickly approximate the error. In theory, fast approximation can be achieved by using high adaptation gain. However, high gain controllers can excite high frequency unmodeled dynamics and cause instability in the system.¹ A novel command governor architecture was constructed in Ref. 2 to address the problem of obtaining predictable transient response with adaptive controllers for uncertain dynamical systems without requiring high-gain learning rates. Specifically, the command governor is a linear dynamical system which adjusts the trajectories of a given command in order to follow an ideal reference system (capturing a desired closed-loop system behavior) in transient-time. That is, by choosing the design parameter of the command governor, the controlled uncertain dynamical system approximates a Hurwitz linear time-invariant dynamical system with \mathcal{L}_∞ input-output signals. This allows a low-gain adaptive element to slowly adapt to the modeling error. Application of this architecture to autonomous helicopter control is developed in Ref. 3.

In this paper, the command governor architecture is used to improve the transient response of a neuroadaptive controller. The command governor improves transient response during the adaptation phase, which

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allows a lower adaptation gain to be used while achieving the same performance. Simulation results are presented to validate the proposed system.

The notation used in this paper is fairly standard. Specifically, \mathbb{R} denotes the set of real numbers, \mathbb{R}^n denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n \times m}$ denotes the set of $n \times m$ real matrices, \mathbb{R}_+ (resp., $\overline{\mathbb{R}}_+$) denotes the set of positive (resp., nonnegative-definite) real numbers, $\mathbb{R}_+^{n \times n}$ (resp., $\overline{\mathbb{R}}_+^{n \times n}$) denotes the set of $n \times n$ positive-definite (resp., nonnegative-definite) real matrices, $\mathbb{S}^{n \times n}$ denotes the set of $n \times n$ symmetric real matrices, $(\cdot)^T$ denotes transpose, and $(\cdot)^{-1}$ denotes inverse. In addition, we write $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix A , $\det(A)$ for the determinant of the Hermitian matrix A , $\text{tr}(\cdot)$ for the trace operator, A^L for the left inverse $(A^T A)^+ A^T$ of $A \in \mathbb{R}^{n \times m}$, P_A for the projection matrix AA^L of $A \in \mathbb{R}^{n \times m}$, $\|\cdot\|_2$ for the Euclidian norm, $\|\cdot\|_\infty$ for the infinity norm, and $\|\cdot\|_F$ for the Frobenius matrix norm.

II. Neuroadaptive Control

We begin by presenting a standard model reference neuroadaptive control problem. Specifically, consider the nonlinear uncertain dynamical system given by

$$\dot{x}(t) = Ax(t) + B[u(t) + \delta(x(t))], \quad x(0) = x_0, \quad t \in \overline{\mathbb{R}}_+, \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector available for feedback, $u(t) \in \mathbb{R}^m$ is the control input, $\delta: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is an *uncertainty*, $A \in \mathbb{R}^{n \times n}$ is a known system matrix, and $B \in \mathbb{R}^{n \times m}$ is a known control input matrix such that $\det(B^T B) \neq 0$ and the pair (A, B) is controllable.

Assumption 1. The uncertainty in (1) is parameterized as

$$\delta(x) = W^T \sigma(V^T x) + \epsilon(x), \quad |\epsilon(x)| < \bar{\epsilon}, \quad x \in S \subset \mathbb{R}^n, \quad (2)$$

where $W \in \mathbb{R}^{s \times m}$ and $V \in \mathbb{R}^{n \times s}$ are *unknown* weight matrices and $\sigma: \mathbb{R}^s \rightarrow \mathbb{R}^s$ is a known function of the form $\sigma(z) = [\sigma_1(z_1), \sigma_2(z_2), \dots, \sigma_s(z_s)]^T$. The set S is a compact, simply connected set. The term $\epsilon(x)$ defines the approximation error of the neural network to the true uncertainty. Let us further define the following matrix

$$Z = \begin{bmatrix} W & 0 \\ 0 & V \end{bmatrix} \quad (3)$$

Assumption 2. The matrices W and V have known upper bounds

$$\|W\|_F \leq \bar{W}, \quad \|V\|_F \leq \bar{V} \quad (4)$$

and therefore

$$\|Z\|_F \leq \bar{Z} \quad (5)$$

Assumption 3. Desired trajectory x_r had a known upper bound

$$\|x_r\| \leq Q \quad (6)$$

Next, consider the ideal reference system capturing a desired closed-loop dynamical system performance given by

$$\dot{x}_r(t) = A_r x_r(t) + B_r c(t), \quad x_r(0) = x_{r0}, \quad t \in \overline{\mathbb{R}}_+, \quad (7)$$

where $x_r(t) \in \mathbb{R}^n$ is the reference state vector, $c(t) \in \mathbb{R}^m$ is a bounded command for tracking (or $c(t) = 0$ for stabilization), $A_r \in \mathbb{R}^{n \times n}$ is the Hurwitz reference system matrix, and $B_r \in \mathbb{R}^{n \times m}$ is the command input matrix. Also, there exist matrices $K_1 \in \mathbb{R}^{m \times n}$ and $K_2 \in \mathbb{R}^{m \times m}$ such that $A_r = A + BK_1$, $B_r = BK_2$, and $\det(K_2) \neq 0$ hold.

Consider the feedback law

$$u(t) = u_n(t) + u_a(t), \quad (8)$$

where $u_n(t)$ is the nominal feedback control law given by

$$u_n(t) = K_1 x(t) + K_2 c(t), \quad (9)$$

Using (8) and (9) in (1) subject to Assumption 1 gives

$$\dot{x}(t) = A_r x(t) + B_r c(t) + B[u_a(t) + W^T \sigma(V^T x) + \epsilon(x)]. \quad (10)$$

Next, let the adaptive feedback control law $u_a(t)$ be given by

$$u_a(t) = -\hat{W}^T \sigma(\hat{V}^T x(t) + v(t)), \quad (11)$$

where $\hat{W}(t) \in \mathbb{R}^{s \times m}$ and $\hat{V}(t) \in \mathbb{R}^{s \times n}$ are the estimates of W and V respectively, satisfying the weight update laws

$$\dot{\hat{W}}(t) = \Gamma_w^{-1} \left[\left(\sigma(\hat{V}^T x(t)) - \sigma'(\hat{V}^T x(t)) \hat{V}^T x(t) \right) e^T(t) P B + \kappa \|e\| \hat{W} \right], \quad \hat{W}(0) = \hat{W}_0, \quad t \in \overline{\mathbb{R}}_+, \quad (12)$$

$$\dot{\hat{V}}(t) = \Gamma_v^{-1} \left[x(t) e^T(t) P B \hat{W}^T \sigma'(\hat{V} x(t)) + \kappa \|e\| \hat{V} \right], \quad \hat{V}(0) = \hat{V}_0, \quad t \in \overline{\mathbb{R}}_+, \quad (13)$$

where $\Gamma_w \in \mathbb{R}_+^{s \times s} \cap \mathbb{S}^{s \times s}$ and $\Gamma_v \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$ are the learning rate matrices, $e(t) \triangleq x(t) - x_r(t)$ is the system error state vector, and $P \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$ is a solution of the Lyapunov equation

$$0 = A_r^T P + P A_r + R, \quad (14)$$

where $R \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$ can be viewed as an additional learning rate. Note that since A_r is Hurwitz, it follows from converse Lyapunov theory⁴ that there exists a unique P satisfying (14) for a given R . The term $v(t)$ is a robustifying term and is defined as

$$v(t) = -k_v \left(\|\hat{Z}\|_F + \bar{Z} \right) B^T P e(t) \quad (15)$$

Now, subtracting (10) from (7) gives system error dynamics

$$\dot{e}(t) = A_r e(t) + B [a_n + W^T \sigma(V^T x) + \epsilon(x)], \quad e(0) = e_0, \quad t \in \overline{\mathbb{R}}_+. \quad (16)$$

where $e_0 \triangleq x_0 - x_{r0}$. Defining $\widetilde{W} = W - \hat{W}$ and $\widetilde{V} = V - \hat{V}$ we note that

$$\dot{\widetilde{W}} = -\dot{\hat{W}}, \quad (17)$$

$$\dot{\widetilde{V}} = -\dot{\hat{V}}. \quad (18)$$

Taking (16) along with (11), (15), (17), and (18) yields a system of dynamic equations for the state and weight errors.

Proof of the uniform ultimate boundedness of this system subject to assumptions 1, 2, and 3 can be found in Ref. 5.,

III. Command Governor-based Neuroadaptive Control

The recently developed command governor architecture may be applied to a variety of adaptive and non-adaptive control frameworks. This section overviews the command governor architecture applied to the neuroadaptive control problem described in the previous section. Specifically, let the command $c(t)$ be given by

$$c(t) = c_d(t) + G\eta(t), \quad (19)$$

where $c_d(t) \in \mathbb{R}^m$ is a bounded external command for tracking (or $c_d(t) \equiv 0$ for stabilization) and $G\eta(t) \in \mathbb{R}^m$ is the command governor signal with $G \in \mathbb{R}^{m \times n}$ being the matrix defined by

$$G \triangleq K_2^{-1}B^L = K_2^{-1}(B^T B)^{-1}B^T, \quad (20)$$

and $\eta(t) \in \mathbb{R}^n$ being the command governor output generated by

$$\dot{\xi}(t) = -\lambda\xi(t) + \lambda e(t), \quad \xi(0) = 0, \quad t \in \overline{\mathbb{R}}_+, \quad (21)$$

$$\eta(t) = \lambda\xi(t) + (A_r - \lambda I_n)e(t), \quad (22)$$

where $\xi(t) \in \mathbb{R}^n$ is the command governor state vector and $\lambda \in \mathbb{R}_+$ is the command governor gain.

The addition of the command governor signal $G\eta(t)$ to the command for tracking $c_d(t)$ in (19) does not change the system error dynamics, and hence, the weight update law (12) for $\hat{W}(t)$ remains the same. In this case, however, (7) and (16) change to

$$\dot{x}_r(t) = A_r x_r(t) + B_r c_d(t) + P_B \eta(t), \quad (23)$$

$$\dot{x}(t) = A_r x(t) + B_r c_d(t) + P_B \eta(t) + B[u_a(t) + W^T \sigma(V^T x) + \epsilon(x)], \quad (24)$$

where $P_B = BB^L = B(B^T B)^{-1}B^T$. Even though this implies the modification of the reference system with the signal $P_B \eta(t)$, as we see later, by properly choosing the command governor gain λ it is possible to suppress the effect of $B[a_n + W^T \sigma(V^T x) + \epsilon(x)]$ in (24) through $P_B \eta(t)$.

For the following theorem, we assume that the choice of R in (16) satisfies $R = R_0 + \gamma \lambda I_n$, where $R_0 \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$ and $\gamma \in \mathbb{R}_+$ is an arbitrary constant that can be chosen to be sufficiently small. Therefore, this assumption is technical and does not place restrictions on the selection of R .

Theorem 1. Consider the nonlinear uncertain dynamical system given by (1) subject to Assumptions 1, 2, and 3, the reference system given by (7) with the command given by (19), the feedback control law

given by (8) along with (9), (11), (12) and (13), and the command governor given by (21) and (22). Then, the solution $(e(t), \widetilde{W}(t), \widetilde{V}(t), \xi(t))$ of the closed-loop dynamical system given by (16), (17), (18), and (21) is uniform ultimate bounded.

Proof. Consider the following Lyapunov candidate

$$L = e^T P e + \gamma \xi^T \xi + \text{tr} \widetilde{W}^T \Gamma_w \widetilde{W} + \text{tr} \widetilde{V}^T \Gamma_v \widetilde{V} \quad (25)$$

Taking the derivative along $(e(t), \widetilde{W}(t), \widetilde{V}(t), \xi(t))$ and inserting (16) and (21) yields

$$\dot{L} = -e^T R e - 2\gamma \xi^T (\lambda(\xi - e)) + 2\text{tr} \widetilde{W}^T \Gamma_w \dot{\widetilde{W}} + 2\text{tr} \widetilde{V}^T \Gamma_v \dot{\widetilde{V}} \quad (26)$$

$$= -e^T R_0 e - \gamma \lambda e^T e - 2\gamma \lambda \xi^T \xi + 2\gamma \lambda \xi^T e + 2\text{tr} \widetilde{W}^T \Gamma_w \dot{\widetilde{W}} + 2\text{tr} \widetilde{V}^T \Gamma_v \dot{\widetilde{V}} \quad (27)$$

$$= -e^T R_0 e - \gamma \lambda \|e - \xi\|^2 - \gamma \lambda \xi^T \xi + 2\text{tr} \widetilde{W}^T \Gamma_w \dot{\widetilde{W}} + 2\text{tr} \widetilde{V}^T \Gamma_v \dot{\widetilde{V}} \quad (28)$$

The command governor adds two terms to the Lyapunov function derivative, $-\gamma \lambda \|e - \xi\|^2$ and $-\gamma \lambda \xi^T \xi$, which are negative semidefinite. Therefore the proof of uniform ultimate boundedness is not affected by the addition of the command governor. ■

A. Improving the Transient Performance

Theorem 1 demonstrates that the addition of the command governor does not negatively affect the stability properties of the neuroadaptive system. On the contrary, it will be demonstrated in this section that the command governor affects the dynamics of the system such that, in the limit of the command governor gain, the original nonlinear system converges to the linear reference model.

Consider the dynamics of the command governor given in the frequency domain:

$$\mathcal{G}_{e \rightarrow \eta} = A_r e(s) - \frac{s}{\frac{1}{\lambda} s + 1} e(s) \quad (29)$$

The second term, $\frac{s}{\frac{1}{\lambda} s + 1} e(s)$, is a low-pass filter on the error dynamics derivative, and therefore the command governor output in Equation (22) can be rewritten

$$\eta(t) = A_r e(t) - \dot{e}_{lf}(t) \quad (30)$$

where $\dot{e}_{lf}(t)$ is the low frequency portion of the error dynamics derivative. Note that we define $\dot{e}_{hf}(t)$ to be the corresponding high frequency portion, and that $\dot{e}(s) = \dot{e}_{lf}(s) + \dot{e}_{hf}(s)$. In addition, note that

$$\eta(t) \rightarrow A_r e(t) - \dot{e}(t) \quad \text{as } \lambda \rightarrow \text{inf} \quad (31)$$

By rearranging the state dynamics the influence of the command governor becomes apparent. First note that the error dynamics in Equation (16) can be rewritten as

$$\dot{e}(t) - A_r e(t) = B (u_a(t) + W^T \sigma(V^T x) + \epsilon(x)) \quad (32)$$

Now, consider the state dynamics of the original system in Equation 24:

$$\dot{x}(t) = A_r x(t) + B_r c_d(t) + P_B \eta(t) + B[u_a(t) + W^T \sigma(V^T x) + \epsilon(x)] \quad (33)$$

$$\dot{x}(t) = A_r x(t) + B_r c_d(t) + P_B^T \eta(t) + B(B^T B)^{-1} B^T B[u_a(t) + W^T \sigma(V^T x) + \epsilon(x)] \quad (34)$$

$$\dot{x}(t) = A_r x(t) + B_r c_d(t) + P_B [\eta(t) + B(u_a(t) + W^T \sigma(V^T x) + \epsilon(x))] \quad (35)$$

Inserting (32) into (35) gives

$$\dot{x}(t) = A_r x(t) + B_r c_d(t) + P_B [\eta(t) + \dot{e}(t) - A_r e(t)] \quad (36)$$

Finally, inserting (30) into (37) results in

$$\dot{x}(t) = A_r x(t) + B_r c_d(t) + P_B [\dot{e}_{h_f}(t)] \quad (37)$$

As the command governor gain approaches infinity, $\dot{e}_{h_f}(t) \rightarrow 0$ and the system dynamics converge to the reference model.

At high gain, the feedback of the high frequency error dynamics in the command governor architecture can have the undesirable effect of amplifying the measurement noise present in the system. If this is the case, additional conditioning of the command governor signal can be performed as described in Reference 2.

IV. Simulation Results

To illustrate the behavior of the modified neuroadaptive controller, the proposed system was applied to a standard wing-rock aircraft model.⁶ Consider a nonlinear controlled wing rock aircraft dynamics model given by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [u(t) + \delta(t)], \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad t \in \bar{\mathbb{R}}_+ \quad (38)$$

where x_1 represents the roll angle in radians and x_2 represents the roll rate in radians per second. In (38), $\delta(x)$ represents an uncertainty of the form $\delta(x) = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 |x_1| x_2 + \alpha_4 |x_2| x_2 + \alpha_5 x_1^3$, where $\alpha_i, i = 1, \dots, 5$ are unknown parameters that are derived from the aircraft aerodynamic coefficients. For our numerical example, we set $\alpha_1 = 0.1414$, $\alpha_2 = 0.5504$, $\alpha_3 = -0.0624$, $\alpha_4 = 0.0095$, and $\alpha_5 = 0.0215$. We choose $K_1 = [-0.16, -0.57]$ and $K_2 = 0.16$ for the nominal controller design that yields to a reference system with a natural frequency of $\omega_a = 0.40$ rad/s and a damping ratio of $\zeta = 0.707$. For the standard neuroadaptive controller, $\Gamma_w^{-1} = 0.1I_3$, $\Gamma_v^{-1} = 0.1I_2$, $\bar{Z} = k_v = 0.1$ and $\kappa = .01$. Three hidden layer neurons were used to approximate the uncertainty. In the command governor case, the gain $\lambda = 50$.

Figures 1, 2 and 3 shows the results of a 120 second simulation where the desired response is a shaped $\pm 10^\circ$ input. Figure 1 shows that the baseline controller becomes oscillatory and does not converge to the command within the 120 second period. Likewise the actuator commands are oscillatory also. The results of increasing the adaptation gain by a factor of 100 are shown in Figure 2. Better tracking is observed, but at the expense of highly active actuator commands which would be unrealistic. Figure 3 show the response with the addition of the command governor at the original adaptation gain. Note the behavior of command governor-affected input c , which is the command driving the neuroadaptive controller. The attitude tracks the desired command well throughout and the actuator commands are realistic.

V. Conclusion

This paper presented a novel method to address the transient response and uniform ultimate boundedness of neuroadaptive controllers through the application of the command governor. It was shown that the command governor architecture causes the dynamics of the original neuroadaptive system to approximate the linear reference model. Simulation results illustrated the effectiveness of the approach using an aircraft wing-rock model.

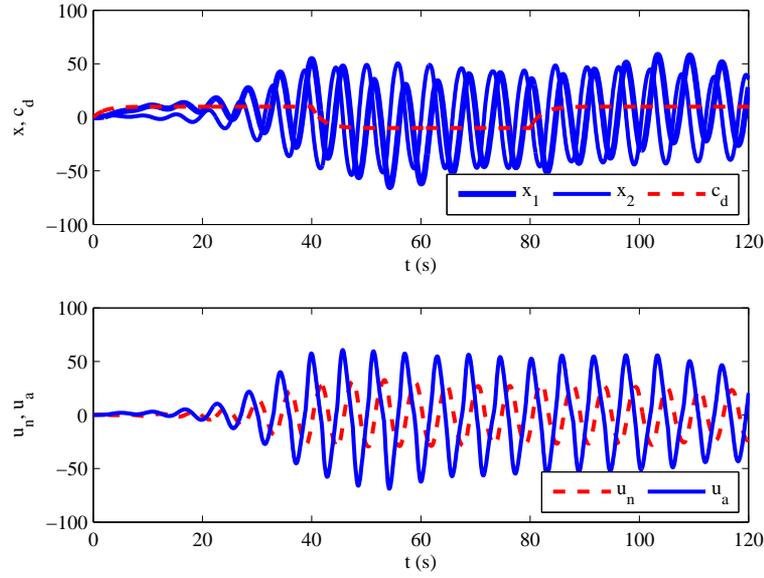


Figure 1. Wing-rock model response for standard neuroadaptive control for a 10 degree input with state feedback. Adaptation gains are set to $\Gamma_w^{-1} = 0.1I_3$, $\Gamma_v^{-1} = 0.1I_2$.

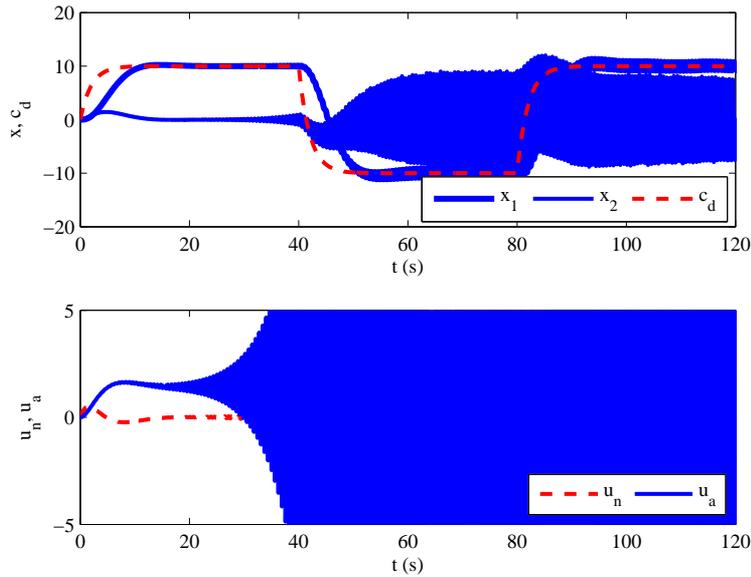


Figure 2. Wing-rock model response for standard neuroadaptive control for a 10 degree input with state feedback. Adaptation gains are set to $\Gamma_w^{-1} = 10I_3$, $\Gamma_v^{-1} = 10I_2$.

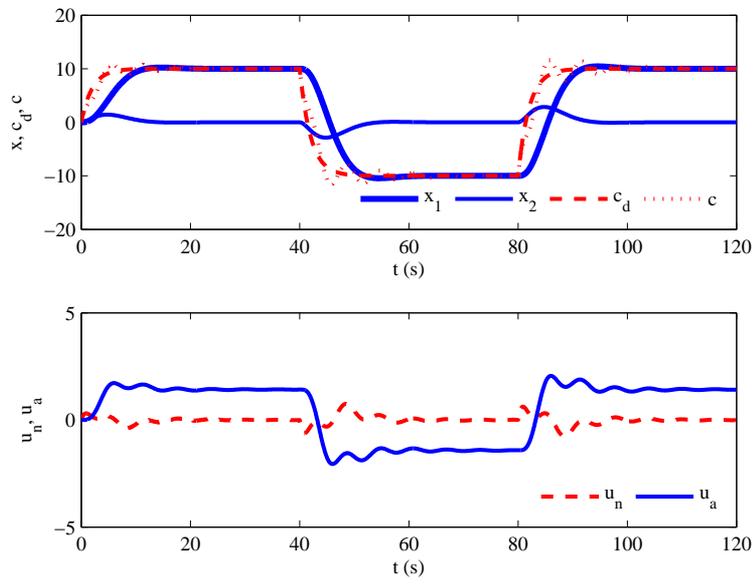


Figure 3. Wing-rock model response for command governor-based neuroadaptive control for a 10 degree input with state feedback with $\lambda = 50$. Adaptation gains are set to $\Gamma_w^{-1} = 0.1I_3$, $\Gamma_v^{-1} = 0.1I_2$

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