GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION
SPONSORED PROJECT INITIATION

Date: 9/5/80

Project Title: Quantum Topology

Project No: G-41-610

Project Director: Dr. Finkelstein

Sponsor: National Science Foundation

Agreement Period: From July 1, 1980 Until ~~December 31, 1982~~ 12/31/84

Type Agreement: Grant No. PHY-8007921

Amount:

$31,450	G-41-610	(NSF Funds)
$27,680	G-41-340	(GIT Contribution)
$59,130	Total	

Reports Required: Annual Progress Report(s); Final Report

Sponsor Contact Person (s):

Technical Matters

NSF Program Official
J. Barry Cammarata
Theoretical Physics Program
Division of Physics
Directorate for Mathmatical
and Physical Sciences
National Science Foundation
Washington, D.C. 20550

(202) 357-7979

Contractual Matters
(thru OCA)

NSF Grants Official
Paulette L. Green
National Science Foundation
Washington, D.C. 20550

(202) 357-9630

Defense Priority Rating: N/A

Assigned to: Physics (School/~~Laboratory~~)

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CA-3 (3/76)

GEORGIA INSTITUTE OF TECHNOLOGY OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date 9/10/86

Project No. G-41-610 School/XXX Physics

Includes Subproject No.(s) N/A

Project Director(s) D. Finklestein GTRC /XXXX

Sponsor National Science Foundation, Washington, D.C

Title Quantum Topology

Effective Completion Date: 12/31/84 (Performance) (Reports)

Grant/Contract Closeout Actions Remaining:

All reports have been submitted to sponsor- (terminate per John Schonk).

- [ ] None
- [ ] Final Invoice or Final Fiscal Report
- [ ] Closing Documents
- [X] Final Report of Inventions — Questionnaire sent to P.I.
- [ ] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other

Continues Project No. Continued by Project No.

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FORM OCA 69.285

G41-610

NATIONAL SCIENCE FOUNDATION

GRANT RENEWAL PROPOSAL

June 1, 1982 to May 31, 1983

QUANTUM TOPOLOGY

David Finkelstein

School of Physics

Georgia Institute of Technology

Atlanta, Georgia 30332

January 5, 1982

INTRODUCTION

The main work of this period is the application of the quantum set theory formulated in the past year as a Clifford algebra over the integers. The principle application is to the construction of trial theories that in the continuum limit ("chronon" approaching 0) conform to the principles of 1. Special Relativity: Local Lorentz Invariance, 2. Quantum Theory: Hilbert space kinematics, and 3. Gauge Theory: Interactions express curvature. These models, by principles 1 and 2 and the previous work, must have a local real-tetradic structure (tensor products of four real factors) rather than the complex-binary structure (spinor space) studied previously. Turning to firmer ground for applications of the formalism, we attack the problem of a quantum theory of the topology of solids as a useful preliminary to a quantum theory of time space topology. It should now be possible to formulate and compute operators expressing for quantum solids curvature properties presently described by the Burger's vector in the classical limit. Concepts of field and source for quantum solid defects should be useful for the analogous entities in quantum time space.

1. Scientific Background

We continue to apply the tentative principles of atomism, asymmetry, and anomy formulated earlier. As a result of the developments of 1980-81, we adduce the following heuristic principles as well:

1.1 Discrete curvature

The idea that there might be a discrete quantum structure underlying the time space continuum too has long been entertained. The success of gauge theories in unifying weak, electromagnetic and strong interactions suggests that the underlying quantum theory be developed in a way that brings in the concept of gauge and curvature at an early stage. The best--perhaps only--example of a discrete physics with gauge and curvature is solid state physics, where the Burger's vector is the usual description of curvature. For comparison, the Burger's vector of time space for paths around the sun has length equal to the Schwarzschild radius of the sun, P. G. Bergmann has pointed out. Accordingly, we have developed a calculus for the description of discrete curvature in the absence of the usual time space framework. This is the "quantum set theory" formulated in the previous year of this program. It is gratifying that the most mathematically natural expression for this theory is a Clifford algebra (which we call S for "set") playing much the role of a Fock space, but generated by a single operation (called Br for "brace", the curly bracket of set theory), rather than by the continuous field of creation operators used in the usual Fock space of particle theory. In the process of formulating this theory, the long-vexing problem of the "quantum exponential" has been dissolved. The exponential Y**X or Hom(X,Y) is the usual way to make the algebra describing a field from the algebras X and Y describing the domain and range of the field functions. The problem is that for Hilbert space algebras the exponential is isomorphic to the product, which is physically inappropriate for a field theory. The resolution is that a Hilbert space algebra X is not a sufficient description for a set of many points, but only describes one "generic point" or "possibility", and tensor product structure must be added to describe a set of many actually-existing entities. This is done in S.

1.2 Real quantum logic

The conviction has strengthened in the previous period that time space quanta ("chronons") employ the real field of coefficients, not, say, the complex usual for quantum mechanics, or the quaternion field considered in some generalized quantum theories. I cite five theoretical heuristic indicators:

1. The simplest representation of the Lorentz group operates in L(2,C), the linear space of (binary) spinors, and so this space and its field are often used in work towards quantum time space. This representation, however, does not admit a Lorentz vector of operator components, and such a vector is needed to construct time space vectors. (It admits merely a Lorentz vector whose components are bilinear forms.) The simplest representation of such a vector of operators acts in the linear space L(4,R) of the Dirac-Majorana representation, which has real coefficients. (Thus L(2,C) is too small, L(4,C) is too big, L(4,R) is just right.)

2. It is hard to accept the usual quantum logic as basic, on the mathematical and intuitive grounds that it employs complex coefficients. One of the reasons for inventing set theory in the first place was to express the complex numbers in terms of more elementary notions. The quantum set theory S seems mathematically simpler than classical set theory only if it is formulated with real--indeed, integer--coefficients.

3. The i of the complex coefficients is a central ("superselection") operator: all observables commute with it. The only origin for central quantities we know (aside from baldly postulating them) is large numbers and

random phases. (For example, this is why macroscopic quantities and the results of measurements are central.) If i is central for such statistical reasons, it may not be central in the one-object case, the single "chronon" of the quantum time space network.

4. There is no actual loss of generality in working with integer matrices in the quantum theory (unlike the classical theory) of the Lorentz group. The 4x4 integer matrices define projective transformations of $L(4,R)$ that are dense in the projective representation of the Lorentz group.

5. The space of (quantum) binary sequences, formulated either in the language of "the space-time code" or S, has a "tangent space" generated by four finite operators

$$c', c'', d', d''$$

("create a 0", "create a 1", "delete a 0", "delete a 1", respectively). Over the real field, these generate an algebra whose group is already large enough to include the Lorentz group. (The Lorentz group is the subgroup leaving invariant not only the canonical commutation relations of the c's and d's but also a particular "i".)

1.3 Classical quantum effects

During the previous quarter a definite formulation has been found for the long-entertained idea that whenever physical determination processes interact with and change the systems they determine, logical structures resembling quantum logic (complementarity logic) appear. We pose the problem of determining the structure of an automaton by its responses to data inputs, in a context where the exact state of the automaton is not initially known, and the internal state of the automaton (and thus its future responses) may be modified by the input data. This problem is being analytically and experimentally studied by Shlomit Finkelstein (graduate student). The thesis is proposed that the (Galois) lattice of the relation between initial and final determination describes the logic of the intercomputer situation, as it does the logic of quantum mechanics. This form of the problem, however, is evidently a basic general form for many problems of artificial intelligence, and therefore support for this facet of the work is being sought elsewhere. In the present work, it will be supposed (as seems most likely) that the world has a quantum logic "all the way down".

2. Scientific Program

The following presentation employs some of the concepts developed in this program and presented as part of the scientific background for this proposal.

2.1 Quantum time space

It is necessary to go over the ground of the "space-time code" studies with the Clifford algebra S of quantum set theory as the basic language. One of the difficulties of the early work was the origin of the Clifford algebra of Dirac (the Clifford algebra of Minkowski space). We now have a formalism in which Clifford algebras arise quite naturally and with unambiguous meaning. It is likely that exact covariance under a group dense in the Lorentz group can be effective at the basic level, being represented by GL(4, Int) acting on the components of a quadruplet (4-component spinor) such as (c', c", d', d") of Section 1.2.

2.2 Quantum topology of solids

The remarkable successes of gauge theories make tenable the hypothesis that interactions express curvatures. We seek to formulate this hypothesis in a quantum topology.

There already exists a domain of physics where discrete topological structures give rise to curvature and its sources. That is solid-state physics. We take up the quantum formulation of the concepts of dislocation and Burger's vector already highly-developed for classical studies in a continuum limit, using our quantum set theory to formulate the necessary concepts of quantum path, quantum connection, and quantum curvature. This is not a study in the fine-structure of time space, but of the dynamics of small solid aggregates, and may use the usual Hamiltonian quantum mechanics. The unit cell of a crystal defines a transport of its edges to the corresponding edges of adjacent cells. This transport is integrable in the absence of defects. Thus defects appear as sources of curvature, a discrete analog of general relativity. The task is a quantum formulation of this relation between defects and curvature.

2.3 Quantum set theory

Ernesto Rodrigues (graduate student) is studying the theory of quantification of quantum set theory and the generalization to sets of infinite cardinality.

2.4 Nonholonomic quantum theory

A model of spinning particles previously studied by David Finkelstein leads to the problem of the quantization of systems with nonholonomic constraints, higher dimensional relativistic analogues of the rolling ball. David Luedtke (graduate student) is studying the canonical quantization of nonholonomic systems in this connection. His work has direct application to gauge fields, which often have nonholonomic constraints.

2.5 Black holes

The hydromagnetic (Rayleigh-Taylor-Krukal) stability of the accretion disk of a magnetic black hole is being studied by David Vogel (graduate student).

G-41-610

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

FINAL PROJECT REPORT

NSF FORM 98A

PLEASE READ INSTRUCTIONS ON REVERSE BEFORE COMPLETING

PART I--PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
Georgia Institute of Technology Atlanta GA 30032	Theoretical Physics	PHY800 7921
	4. Award Period From 80/7/1 To 82/12/31	5. Cumulative Award Amount $31 450

6. Project Title

QUANTUM TOPOLOGY

PART II--SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

The logical laws that work for large-scale mechanics have been revised for quantum mechanics since about 1936, but only the most primitive level of this new logic has been formulated till now. We have developed quantum logic further in order to use it to formulate models of particles and other systems below the atomic scale of sizes where present quantum logic works well. In particular we have made a simple quantum set theory to describe composite structures and relations between quanta. At the same time we have studied possibilities for still further revision of quantum logic at the smallest scale of time and size. A quantum logic using superposition with integer weights rather than complex numbers seems appropriate. Finally we have considered the possibility of a classical logic at the lowest level, by studying finite automata. We find quite simple classical automata appear like quantum systems to the outside observer, who is apt to cope with them by ascribing them wave-functions and superpositions.

PART III--TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGR Check (√)	Approx. Dat
a. Abstracts of Theses	x				
b. Publication Citations		x			
c. Data on Scientific Collaborators		x			
d. Information on Inventions	x				
e. Technical Description of Project and Results		x			
f. Other *(specify)*					

2. Principal Investigator/Project Director Name *(Typed)*	3.	4. Date
David Finkelstein		May 26,

1. Publications

1. D. Finkelstein, "Quantum Set Theory and Clifford Algebra", Intern. J. Theoret. Physics 1982 (in press)

2. D. Finkelstein, "Cosmological Choices", Synthese 50 (1982) 399.

3. D. Finkelstein, "A Search for Unity", in the Weizsaecker Festschrift volume to be published in 1982.

4. D. Finkelstein, "Coherence and Possibility", Kenyon Review 4 (1982) 95.

2. Scientific Collaborators

A workshop on Quantum Topology has been conducted since 1980. It is attended regularly by several graduate students and an outside visitor, and irregularly by other graduate students and faculty.

Graduate students collaborating on this research and taking part in the Quantum Topology workshop are:

David Luedtke
Ernesto Rodrigues
Shlomit Ritz Finkelstein

Collaborators in some of this work who have taken part in the workshop include

Gregory Chaitin, IBM
Prof. J. Williams, Okanaga U., B. C.