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DEVELOPMENT OF A LINEAR RECORDING METER

A THESIS

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by

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DEVELOPMENT OF A LINEAR RECORDING METER

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ACKNOWLEDGEMENTS

I wish to express my sincerest thanks to Professor M. A. Honnell for his suggestion of the problem and also for his valuable aid and guidance in its prosecution.

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LIST OF SYMBOLS

θ_i Input signal in volts

θ_o Output signal of potentiometer in volts

θ Error signal in volts

K Output torque per unit error voltage

F Friction torque per unit output speed

J Output moment of inertia

t Time in seconds

u Amplification factor of vacuum tube

e Input voltage to respective grid

e_{gl} Effective grid voltage of respective grid

r_p Dynamic plate resistance of vacuum tube

θ_T Transient error voltage

$\theta_{c.d.}$ Transient error voltage (Critically damped)

θ_{To} Transient error voltage (Oscillations)

E_c Bias developed across R_2-C_1

Z_m Mutual Impedance

Z_p Primary Impedance

Z_s Secondary Impedance

R_s Resistance of Secondary

R_p Resistance of Primary

a Transformer turns Ratio

LIST OF FIGURES

DEVELOPMENT OF A LINEAR RECORDING METER

INTRODUCTION

Since the beginning of World War II, automatic recording and controlling processes have been widely used.[1] However, of all the instruments developed to date, the servomechanism type has become most popular. Although, the use of the word servomechanism in the engineering field is rather recent, the field of automatic control, which it covers, is not new.

The purpose of this paper is to describe and analyze a graphic linear recording meter. The instrument, aside from its linear recording characteristic, has the following features:

1. High input
2. Error less than 5%
3. Records 1 cycle per minute d.c. signal changes (0 to 10 volts) on polar coordinate paper

Emphasis is placed on the relationship of the meter to a servomechanism. With slight modifications in design, the meter can be readily used as an automatic control device. This will be more evident when it is compared with a servomechanism.

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[1] "Electronic Recording Instruments," Electrical Engineering, January, 1946, pp. 36-44.

In the construction of the instrument, the following devices were necessary:

1. A suitable detector
2. A circuit to be responsive to polarity changes in the signal
3. A means of obtaining linearity in the recording process
4. A recording mechanism

The recording instrument of this thesis utilized a rotating table (polar table) for convenience in construction and overall test.

The general discussion of the next section will show the means by which the above devices were obtained.

GENERAL DISCUSSION OF THE LINEAR RECORDING METER

"Servomechanisms can be considered as a class of automatic regulators whose purpose is to keep a regulated quantity matched to a reference quantity."[2] To show that the linear recording meter of this thesis can be thought of as a servomechanism, the elements of a recording instrument are briefly presented.

In general, a recording or control process can be represented by Figure 1.[3] The primary element detects the variable to be measured, and its output is injected into the measuring circuit. The measuring circuit, in most cases, consists of some form of a bridge followed by several stages of amplifiers. The amplified output signal of the bridge may operate directly a recording mechanism. Part of the output of the measuring circuit actuates a controller. The controller in turn compares its input with a known reference, or it may operate the recording mechanism as well. The recording control process can be made more accurate by including in the controller a final control element, which returns to the primary element an intelligence to correct for any error in the position of the recording mechanism.

Figure 2, which is the block diagram of the linear

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[2] S. W. Herwald, "Forms and Principles of Servomechanisms," Westinghouse Engineer, September, 1946.

[3] D. M. Nielson, "Electric Measuring Instruments," Electrical Engineering, February, 1946, pp. 66-74.

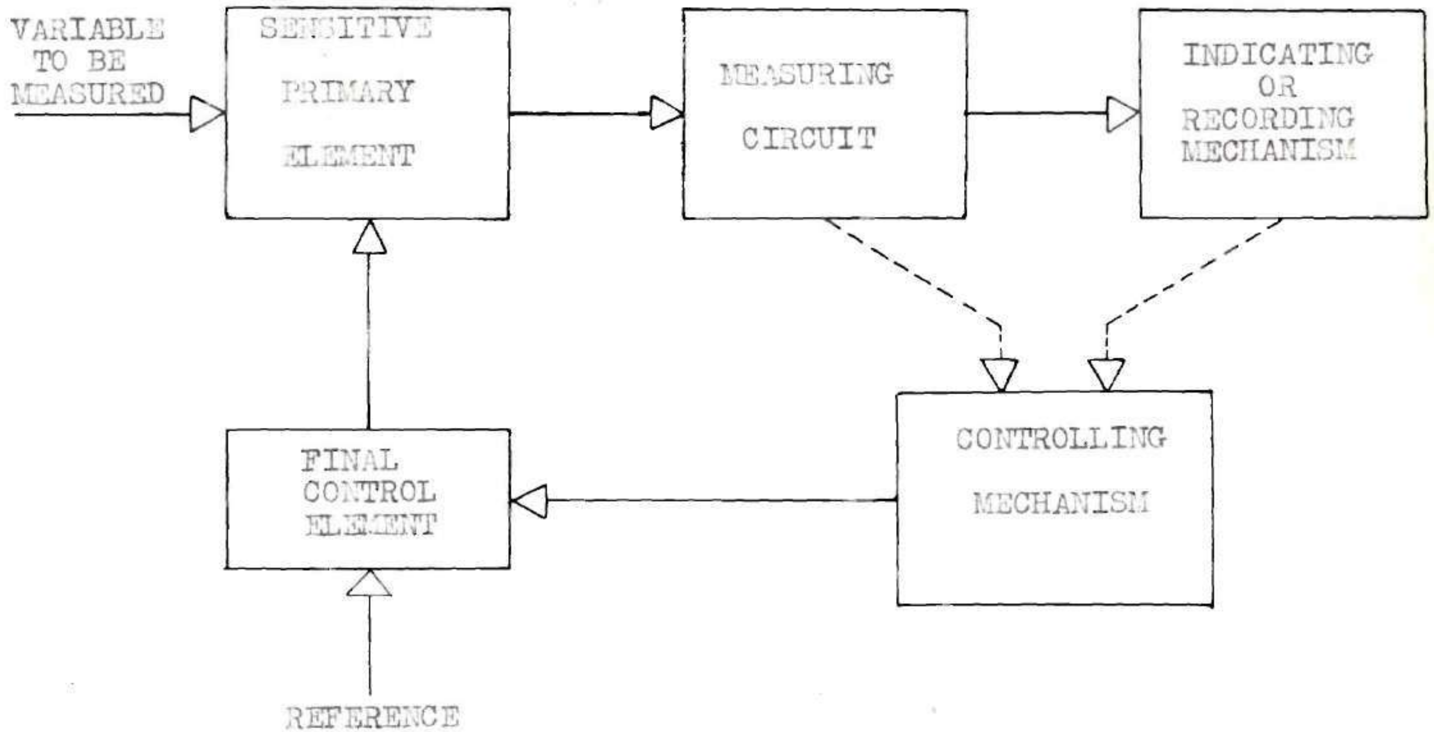

FIGURE 1 - General Block Diagram of a Recording Instrument

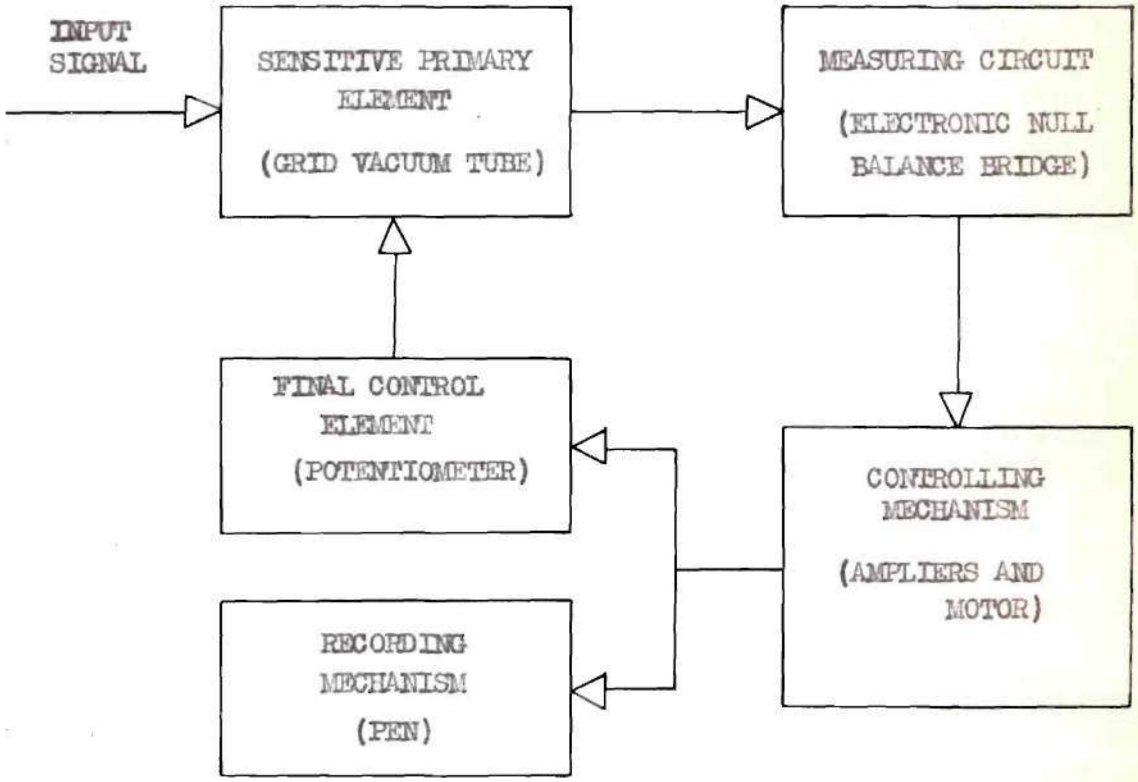

FIGURE 2 - Linear Recording Meter and its Relationship to the General Recording Instrument

recording meter developed by the writer, shows its relationship to the general type as indicated by Figure 1. The only difference in the two diagrams is that in Figure 2 the measuring circuit does not control the motion of the recording mechanism directly.

Figure 3 represents a general block diagram of a servomechanism of the control type.[4] The relationship to the recording meter is indicated below the diagram.

The linear recording meter of this paper can be likened to the servomechanism represented in Figure 3. A general discussion of the operation of the meter will point out the similarities.

Figure 4 is a complete schematic of the linear recording meter referred to in the following discussion.

Assume for the moment that the input terminals A and B are connected together and that the arm of potentiometer P_2 coincides with point B. This will represent zero input signal voltage to the grid. It is easily seen that there will be a particular setting of the potentiometer P_1 that will produce no a.c. voltage difference between points X and Y.

By considering the resistors R_1 as two arms of a Wheatstone bridge and the sections of the tube (6SN7GT) as the other arms, there will be a null developed between points X and Y when P_1 is varied. This assumes a constant a.c. plate

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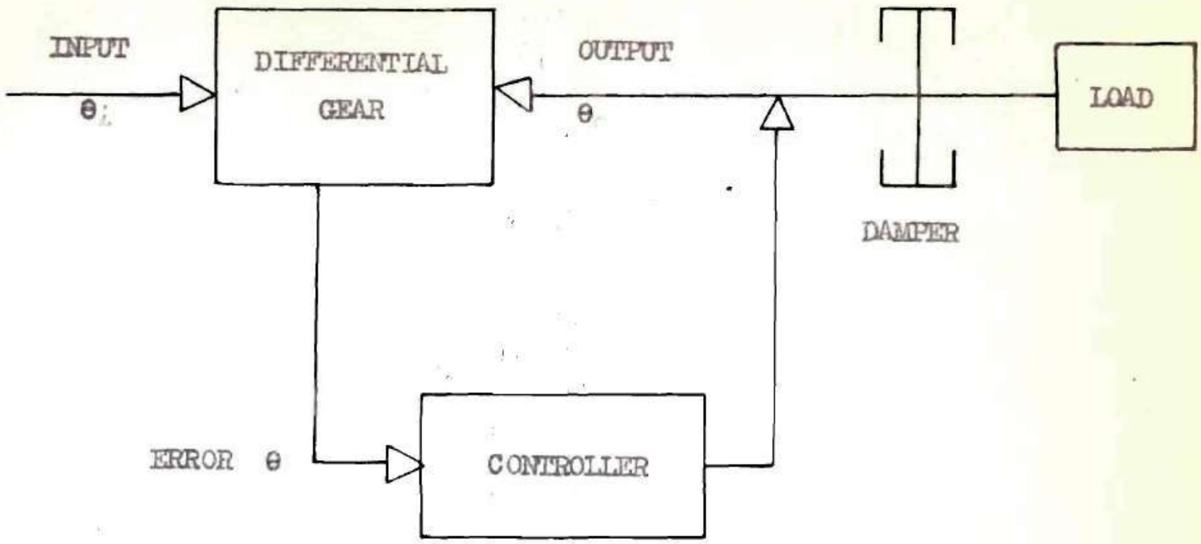
[4] H. Lauer, R. Lesnich, L. E. Matson, Servomechanism Fundamentals. New York: McGraw-Hill Book Company, 1947. p.64.



Input θ_i	∿	Input Grid Voltage (A-B) Fig. 4
Differential Gear	"	Electronic Null Balance Bridge
Damper	"	Friction Of System
Output θ_o	"	Output Of Potentiometer P_2 Fig. 4
Error θ	"	Output Of Bridge
Controller	"	Amplifiers - Thyratrons - Motor
Load	"	Inertial Mass

FIGURE 3 - Relationship of Simple Servomechanism to Linear Recording Meter

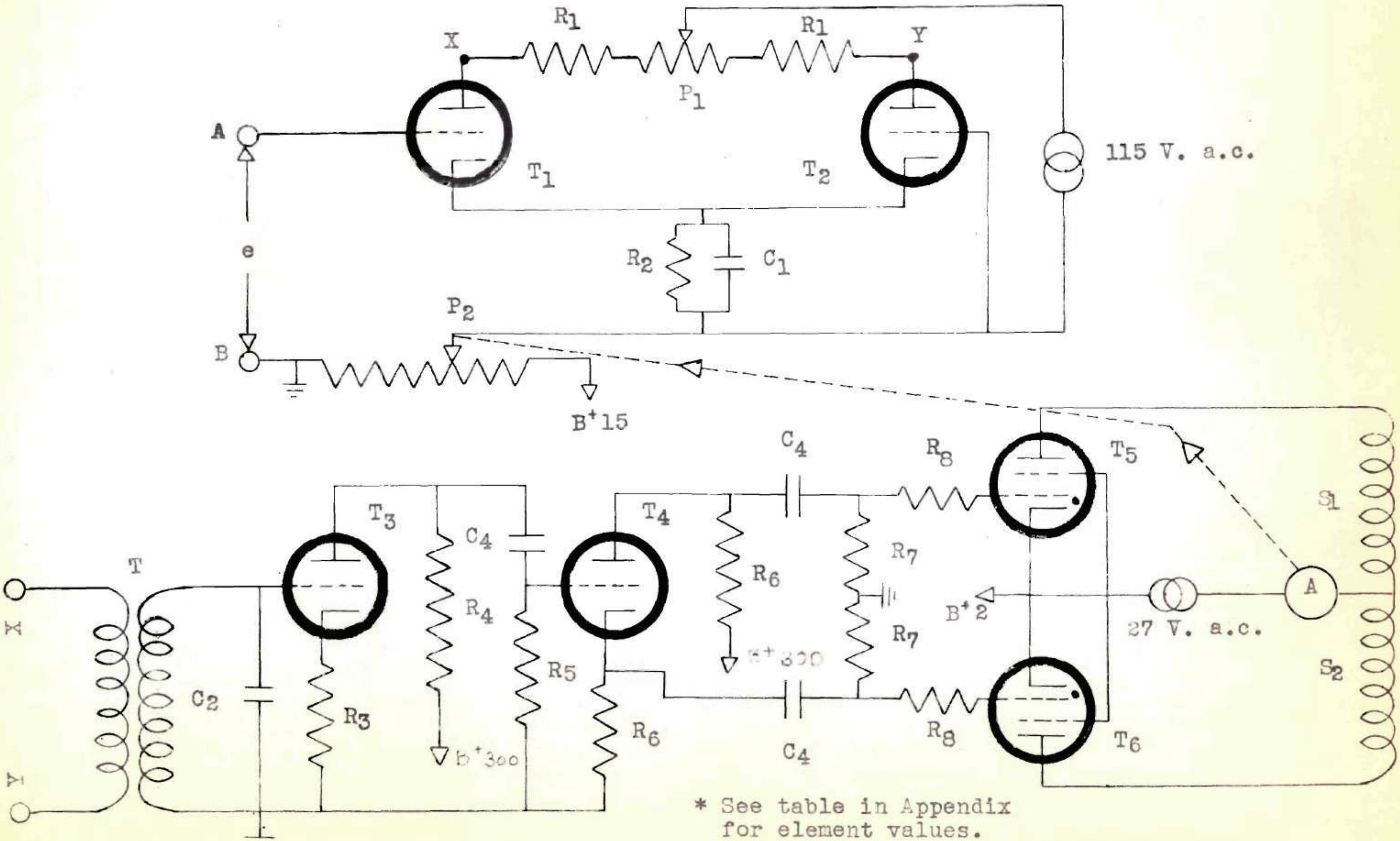


FIGURE 4 - SCHEMATIC OF LINEAR RECORDING METER

voltage.

As P_1 is varied about the null point, the a.c. voltage between X and Y is reversed in polarity. This reversal can be observed by the use of a cathode ray oscilloscope across the bridge output.

Assume that with the bridge balanced and that a d.c. voltage e is introduced between the terminals A and B. Immediately, the bridge becomes unbalanced due to the fact that the arm of the bridge, which contains vacuum tube T_1, has developed an equivalent voltage equal to μe in its plate circuit. When the polarity of the net input grid voltage is changed, the unbalanced voltage between points X and Y reverses in polarity also. This reversal in polarity is of the same character as previously discussed when P_1 was varied about the null point. Therefore, any change in magnitude or polarity of the d.c. input voltage will cause a proportional change in the output voltage of the bridge. This assumes linear operation of the tube.

At this point in the discussion, we can identify two important features of the recording meter. The grid of vacuum tube T_1 or the input terminals A and B are representative of the sensitive primary element whereas the Wheatstone bridge is seen to be the measuring circuit.

The output terminals X and Y, of the bridge, are connected to an isolating transformer (T). The transformer also furnishes an increase in the voltage output of the bridge.

This increase in voltage is not a necessary function as it feeds to a stage of voltage amplification.

The condenser (C_2), paralleling the transformer secondary, resonates the secondary to the 60 cycle a.c. frequency. Since the primary contains a half wave voltage, the secondary will contain a great deal of harmonics. Therefore, the transformer and condenser C_2 can be looked upon as a filter.

The single stage of voltage amplification is sufficient. The amplifier T_3 is a voltage amplifier utilizing current feedback to reduce the distortion and insures better stability.

At this stage, it should be pointed out, that odd symmetry in the wave form is important, otherwise the two thyratrons will not fire symmetrically. It is noted that the wave form has not been stipulated, but it must contain odd symmetry.

The a.c. output of the single stage amplifier is coupled to the grid of a degenerative amplifier phase inverter. The coupling condenser (C_3) and grid resistor (R_5) also serve to shift the fundamental a.c. component of the bridge voltage so that the voltages which are placed on the grids of the thyratrons, that follow later, are in phase with their respective plate voltages.

The phase inverter from its appearance seems to be an ordinary amplifier.[5] A closer examination reveals that the cathode and plate resistors (R_6) are of the same magnitude. Therefore, the a.c. voltage drops across the resistors are equal in magnitude and of opposite polarity. Consequently, two a.c. voltages of opposite polarity are obtained. Each voltage is coupled through a capacitor (C_4) and a resistor (R_7), as shown in Figure 4, to the corresponding grid of the thyratrons. The coupling capacitors (C_4) are used to eliminate any d.c. voltages to be impressed on the grids of the thyratrons and the resistors (R_7) to provide a d.c. path for the bias.

The thyratrons are used to control the direction of rotation of the reversible d-c motor. They are connected in the usual push-pull fashion with the load of each tube consisting of a series winding of the motor as indicated in Figure 4. The two tubes have initially a small d.c. bias to prevent firing of the tubes when no signal is applied to the grids. The plate supply of the thyratrons is sinusoidal and its value is determined by the rating of the d-c motor.

When the plate of one tube is positive at the same instant that its grid is positive, it will fire and cause the motor to operate. Similarly, as the plate and grid voltage of the other thyratron are simultaneously positive, it

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[5] Walther Richter, _Fundamentals of Industrial Electronics_. New York: McGraw-Hill Book Company, 1947, p. 285.

will pass current in the other series winding of the motor to rotate it in the reverse direction. To obtain the optimum operation of the motor, it is desirable to have the a.c. signal grid voltage in phase with the plate voltage. As previously discussed, C_3 and R_5 are utilized as a phase shifter for this purpose.

For the moment, let us return to the measuring circuit. It was pointed out that when a potential is introduced into terminals A and B, the polarity of point X with respect to Y depends upon the polarity of the grid voltage. To rebalance the bridge or reduce the voltage between X and Y to zero, several methods can be employed. The potentiometer P_1 can be varied in a direction to rebalance the bridge. A voltage of the same magnitude but opposite polarity can be introduced in series with the applied signal so that the resulting grid voltage input is zero. Of the methods outlined, the introduction of a voltage of opposite polarity will give the linear response desired in the meter. This voltage is obtained by means of a voltage divider P_2. The arm of the voltage divider is connected to the cathode and one end to the terminal B as shown in Figure 4.

Therefore, to rebalance the bridge, the arm of the voltage divider is moved in the direction to reduce the effective input voltage to the grid circuit to zero.

The motor shaft, Figures 5 and 6, is connected through reduction gears to a precisely ground threaded shaft. This

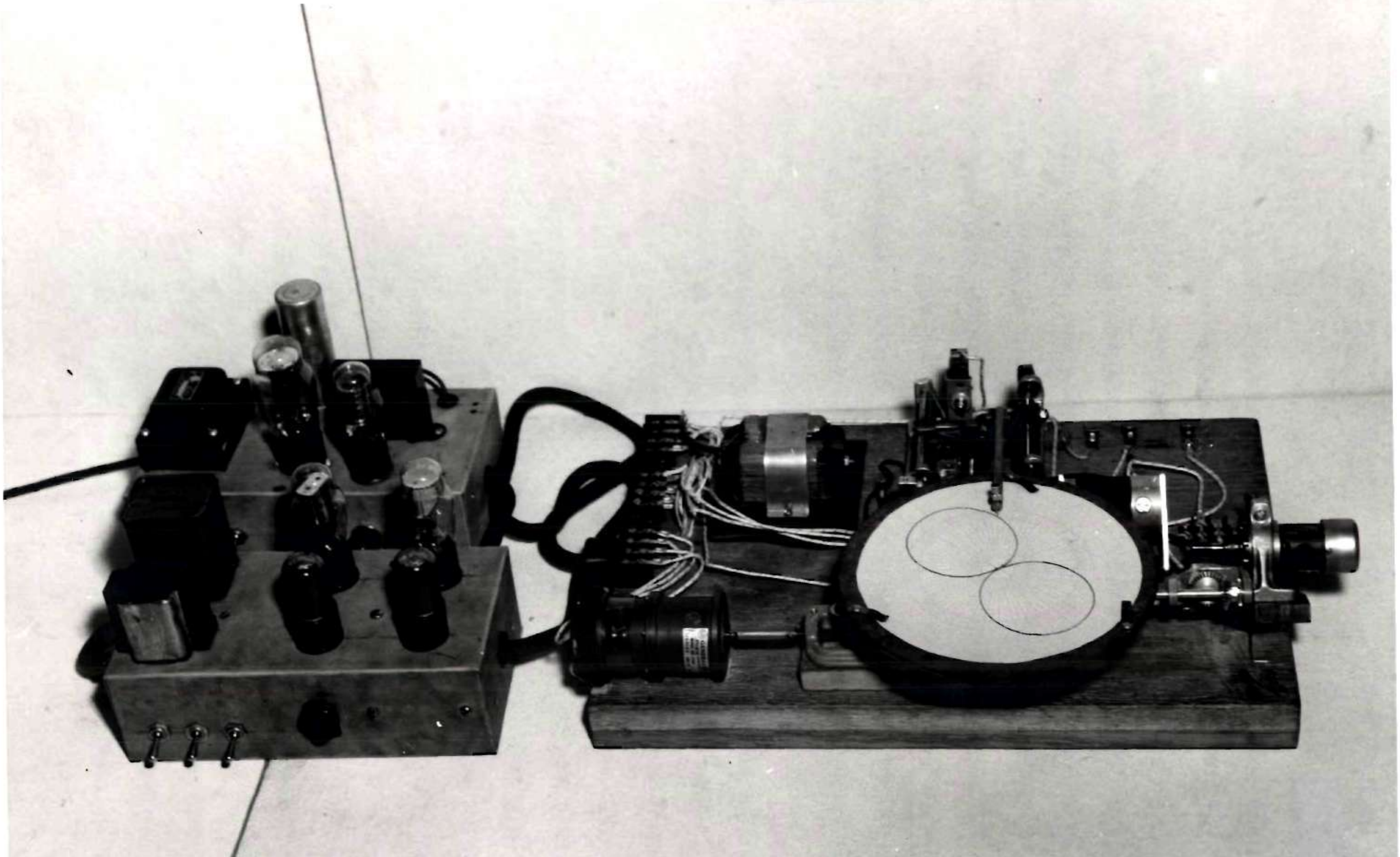

FIGURE 5 - Photograph of Linear Recording Meter

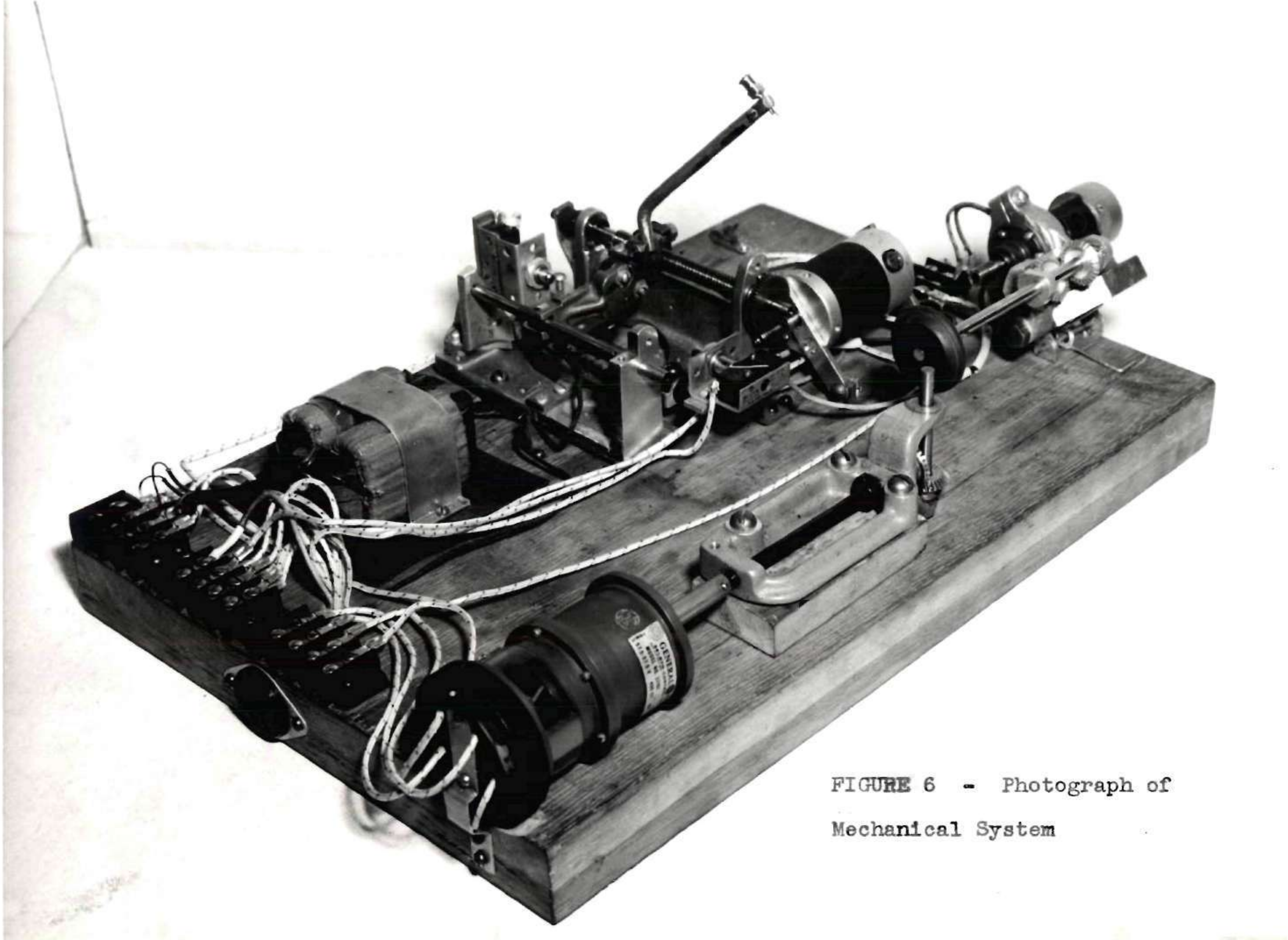

FIGURE 6 - Photograph of Mechanical System

shaft has on its assembly the arm of the voltage divider and the recording pen. As the screw, which has one degree of freedom, rotates, the assembly containing the contact arm and pen will move correspondingly. Since the voltage divider has uniform resistance, the voltage drop per unit section will be proportional to the displacement. Therefore, if a unit positive d-c voltage is applied to the input A-B, the motor will rotate in the direction dictated until the voltage divider introduces one volt of opposite polarity back into the input. The resulting polarity of the input will determine which thyratron will fire to cause the proper direction of rotation of the d-c motor.

In the previous example, let us assume the input signal is reduced to a half volt. Immediately, the resulting voltage on the grid is negative ($\frac{1}{2}$ volt) and consequently the motor is caused to rotate in the direction to reduce the voltage divider input to a negative half volt. Meanwhile, the pen, which is also attached to the same assembly as the contact arm, moves linearly. This is easily seen, as the contact arm moves proportional to the signal input voltage.

At this point, we can now identify the remaining parts of a recording meter. The amplifier, phase inverter, thyratrons, and motor represent the controller. The recording pen is the recording mechanism and last, the voltage divider is identified as the final control element.

From the servomechanism viewpoint, it is now seen how Figure 3 is justified. The signal input to the grid is represented by θ_i, whereas θ_o is obtained from the voltage divider. The difference in these two voltages will cause a corresponding change in the output of the bridge which can be considered as the error voltage θ. The load on the controller is the inertia of the associated mass and the frictional forces.

Since this section dealt with the system in general, the following sections will present an electrical and mechanical analysis.

ELECTRICAL ANALYSIS

In order to have a better understanding of the following analysis, each unit will be analyzed separately and then the results will be correlated into one unit.

Four sections of the meter will be considered in the following order:

1. Measuring section (Bridge)
2. Amplifier
3. Phase inverter
4. Thyratron circuit

Rather than consider particular values, general symbols are used throughout in order to generalize the analysis.

1. Measuring Section (Bridge)

The measuring section is reproduced in Figure 7(a) including the resonated output transformer since it represents a load on the tubes of the bridge circuit.

Although this circuit employs an alternating current plate supply voltage, the equivalent plate circuit theorem is used in the analysis.[6] Therefore, it is obvious that μ and r_p represent average values.

In Figure 7(b) is represented the equivalent circuit of the bridge section in Figure 7(a). It is assumed that the plate resistance of each tube is identical and that P_1 is set

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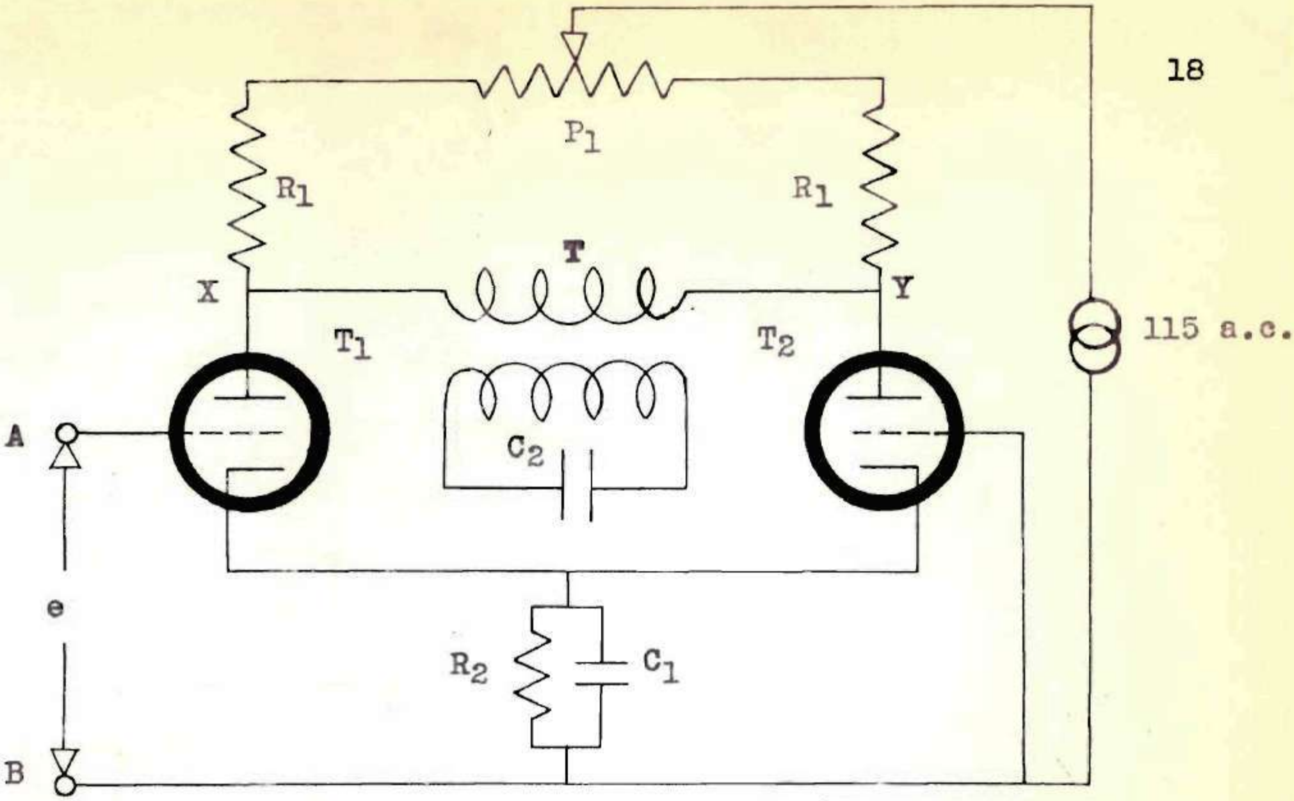
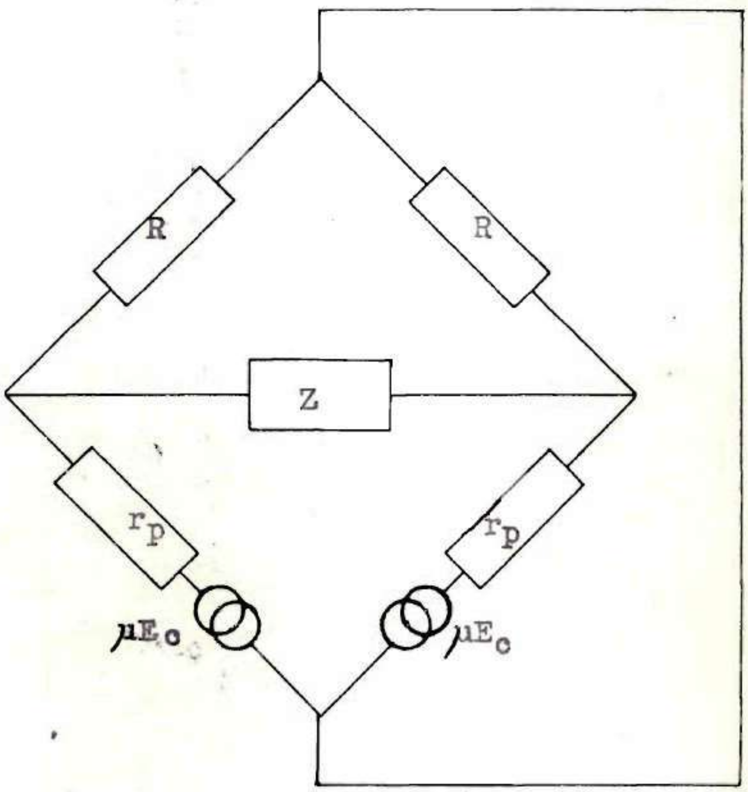
[6] Jacob Millman and Samuel Seely, Electronics. New York: McGraw-Hill Book Company, 1945, p. 518.



(a) Actual Circuit



(b) Equivalent Circuit

FIGURE 7 - Measuring Circuit (Bridge)

at its midpoint so that the combination of R_1 and $\frac{1}{2}$ of P_1 will represent R.

When no signal is present between terminals A and B, it is evident from the equivalent circuit that no voltage will be developed across Z when e is zero. However, as soon as a signal e is introduced, an equivalent voltage μe is introduced in the arm containing tube T_1. By the superposition theorem, the circuit of Figure 8(a) is derived. Consequently, any voltage developed across Z is evidently due to the unbalanced voltage resulting from the input signal.

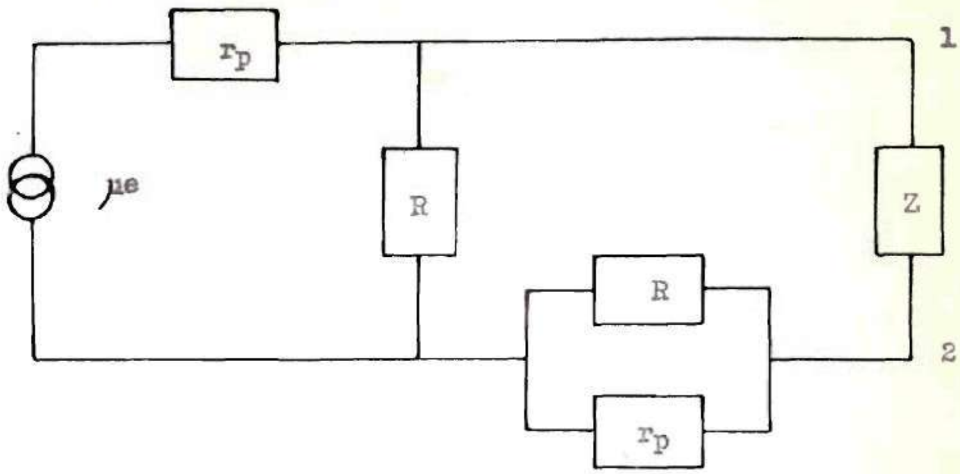
Applying Thevenin's theorem, Figure 8(b) is obtained. The voltage e_{eq} is the open-circuited voltage across terminals land 2. R_{eq} is the impedance looking into the terminals 1 and 2 with all generated e.m.f.'s short-circuited.

$$e_{eq} = \frac{\mu E R}{R + r_p} \tag{1}$$

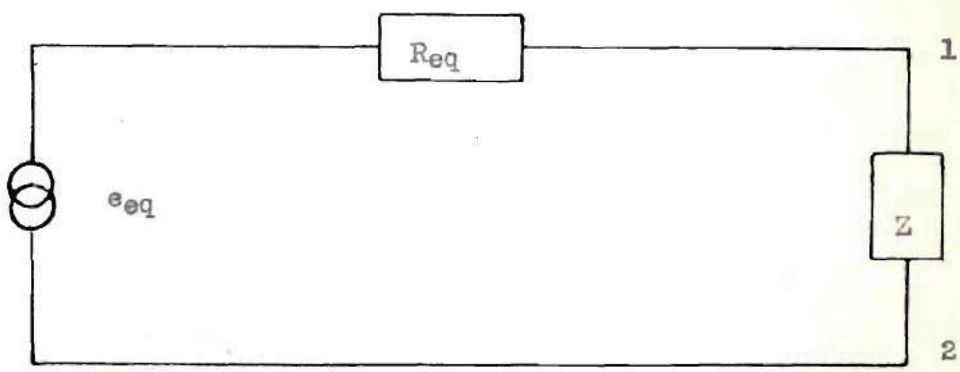
$$R_{eq} = \frac{R r_p}{R + r_p} + \frac{R r_p}{R + r_p} = \frac{2 R r_p}{R + r_p} \tag{2}$$

The value of the voltage E developed across Z is now easily obtained by applying the potentiometer rule to Figure 8(b).

$$E = \frac{e_{eq} Z}{R_{eq} + Z} = \frac{\mu e R Z}{2 R r_p + R Z + r_p Z} \tag{3a}$$



(a)



(b)

FIGURE 8 - Measuring Circuit Analysis

Rearranging equation 3a in the form of equation 3b, it is seen that the input voltage to the transformer is amplified.

$$E = \frac{\mu e}{\frac{2r_p}{Z} + 1 + \frac{r_p}{R}} \tag{3b}$$

If Z and R are both made large in comparison to r_p the input voltage to the transformer is approximately μe.

Since Z represents the input impedance looking into the transformer terminals X and Y, the usual method of transformer analysis is applied. Considering the core losses of the transformer to be negligible, since the frequency of operation is low, and for all practical purposes, the coefficient of coupling to be unity, the general expression for the impedance Z is obtained.[7]

$$Z = R_p + \frac{Z_p \frac{Z_2}{a^2}}{Z_p + \frac{Z_2}{a^2}} \tag{4}$$

The condenser (C_2) is resonated with the transformer secondary at 60 cycles.

2. Amplifier

The grid voltage e_1, determined in the Appendix, is the input voltage to the amplifier T_3 which can be analyzed using the equivalent plate circuit theorem. It is to be

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[7] See Appendix

noted that this amplifier contains negative current feedback.[8]

Figure 9(a) represents the actual circuit of the amplifier. Since R_5 is much greater than R_4, the effective output voltage is determined with simplicity by the use of Figure 9(b). The effective grid voltage is easily seen to be $e_1 - i_p R_3$.

$$i_p = \frac{\mu e_{g1}}{R_3 + R_4 + r_p} \qquad (5)$$

but

$$e_{g1} = e_1 - i_p R_3$$

therefore

$$i_p = \frac{\mu e_1}{R_3(\mu+1) + R_4 + r_p}. \qquad (6)$$

From the diagram, the grid voltage (e_2) of the next section is as follows:

$$e_2 = i_p R_4 = \frac{\mu e_1 R_4}{R_3(\mu+1) + R_4 + r_p} \qquad (7)$$

3. <u>Phase Inverter</u>

The analysis of the phase inverter T_4 is identical to the previous analysis. Figure 10(a), as far as calculation are concerned, is reduced to Figure 10(b). Therefore, replacing R_3 and R_4 by R_6 and e_1 by e_2, we obtain the grid voltage e_3.

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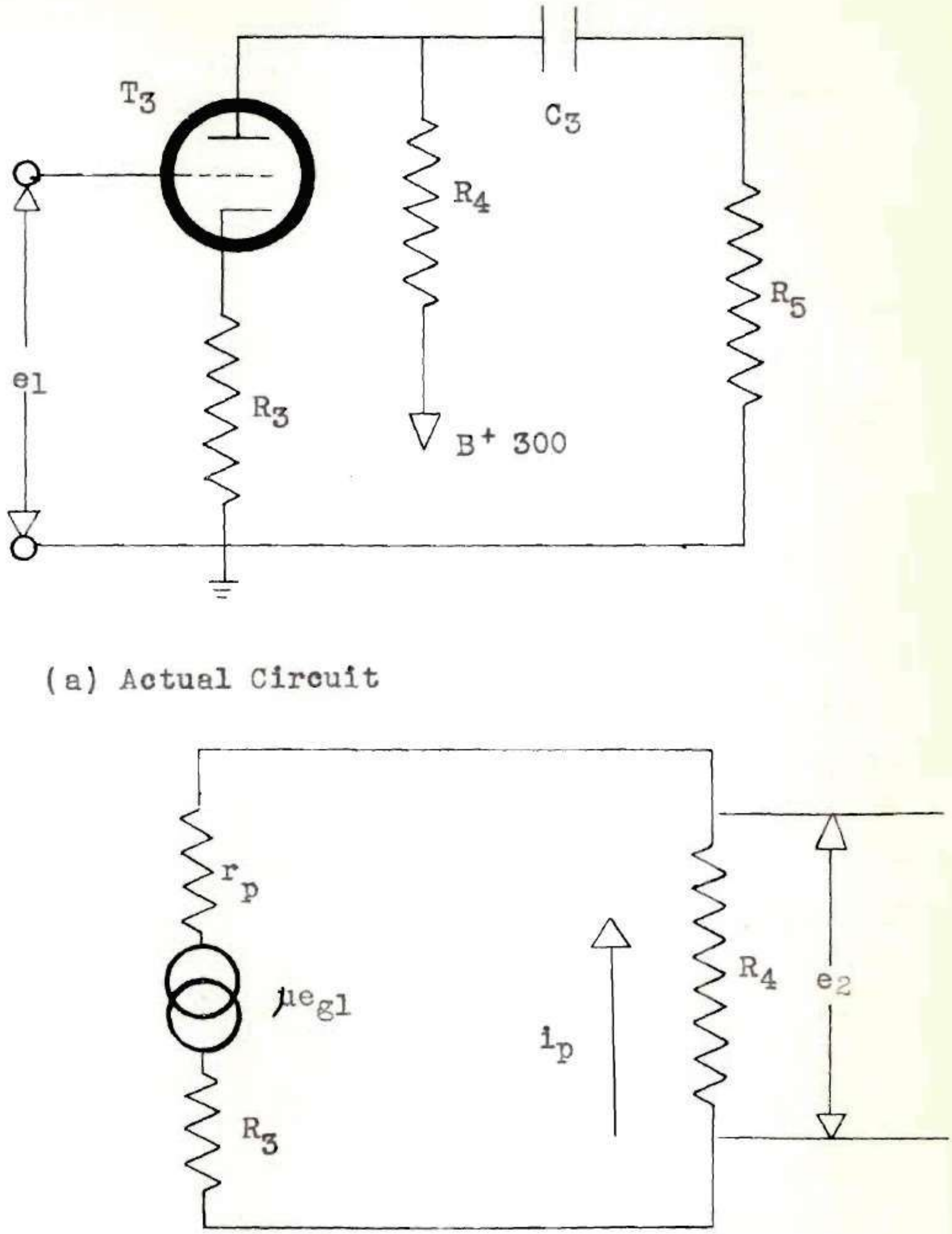
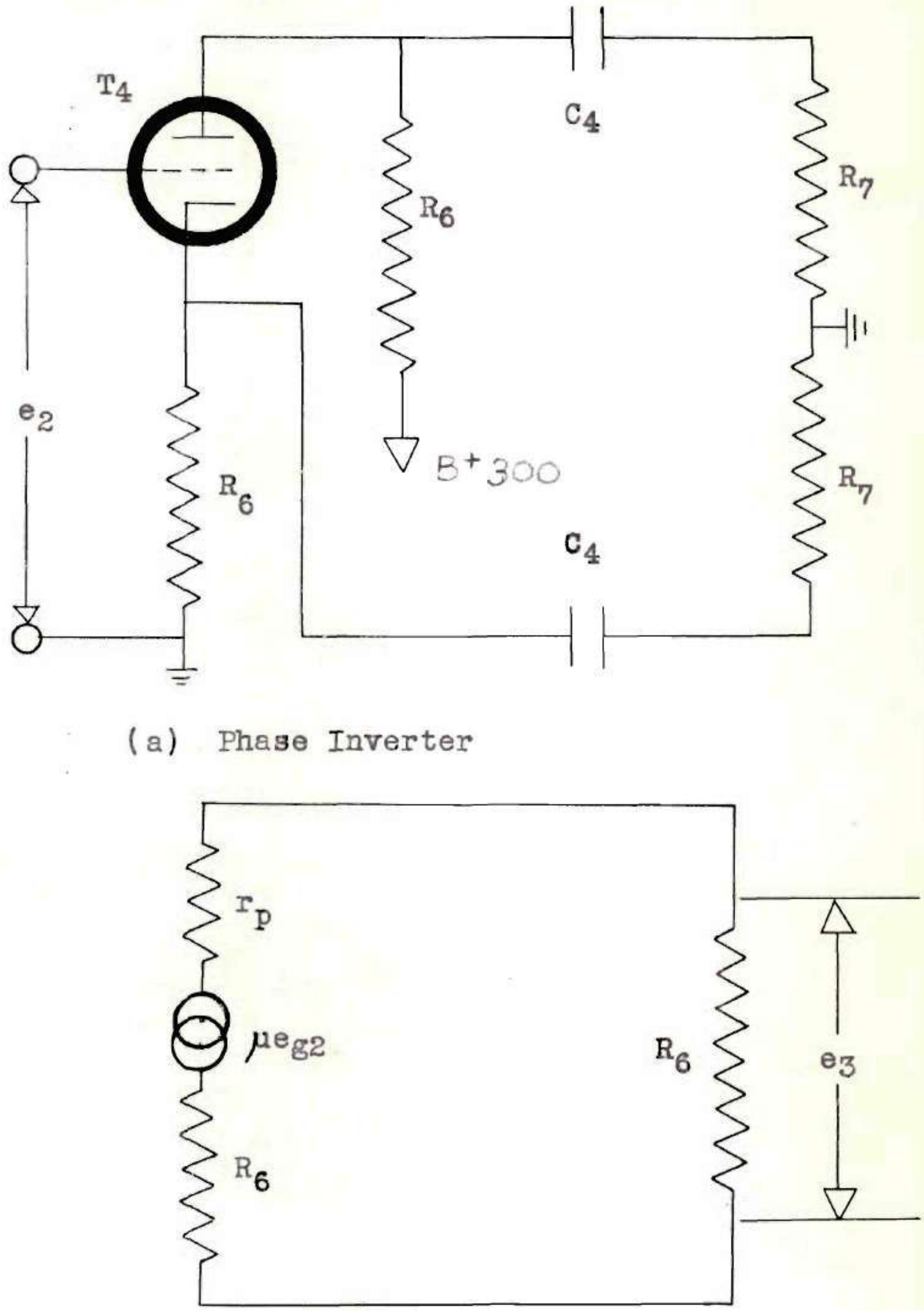
[8] Millman and Seely, Op. Cit., p. 612.



(b) Approximate Equivalent Circuit

FIGURE 9 - Voltage Amplifier



(a) Phase Inverter

(b) Approximate Equivalent Circuit

FIGURE 10 - Degenerative Amplifier-Phase Inverter

$$e_3 = \frac{\mu e_2 R_6}{R_6(\mu+2)+r_p} \tag{8}$$

Substituting for the values of e_1 and e_2 the following equation is obtained for the output voltage:

$$e_3 = \frac{\mu R_6}{R_6(\mu+2)+r_p} \cdot \frac{\mu R_4}{R_3(\mu+1)+R_4+r_p} \cdot \frac{a E Z_p Z_{c2}}{(R_p+Z_p)(R_5+Z_{c2})+a^2 R_p Z_p} \tag{9}$$

or

$$e_3 = A_1 \cdot A_2 \cdot A_3 \cdot e$$

where A_1 represents the amplification of the bridge, A_2 the amplification of the amplifier, and A_3 the amplification of the inverter.

4. Thyratron Circuit

The thyratron circuit is not analyzed as it represents an off and on type of switch. The requirements for operation is that the grid voltage of the thyratron should be great enough to override the d-c bias present.

Although a change of 0.1 volts in the input operates the motor, the actual voltage on the grid of the thyratron when it fires is the d-c bias plus 0.1 volts. The point at which the thyratron fires will depend upon the d-c bias on the grid.

When the thyratron fires, it will supply power to the motor which in turn drives the arm of the potentiometer. The potentiometer feeds a voltage into the input to counterbalance the signal voltage e. The time required to rebalance the bridge depends primarily upon two factors: the speed of the motor and the signal voltage change. The greater the signal voltage change, the farther the pen has to travel to rebalance the bridge.

MECHANICAL ANALYSIS

The electrical analysis just presented does not bring out the effects of the reversible d-c motor and its associated parts. If the electrical circuit is designed properly, there will be no danger from electrical oscillations. However, when moving parts are concerned, there is danger of mechanical oscillations or the possibilities of the system being overdamped.

The purpose of this section is to show the relationship of the operation of the recording meter with respect to its mechanical properties.

Figure 3 is redrawn with slight modification in Figure 11 to facilitate the analysis of the mechanical system.[9]

Applying Newton's laws to the motion of a system, the accelerating forces can be equated to the retarding forces. The only accelerating force ($K\theta$) as observed from the diagram is the motion of the reversible d-c motor as dictated by the error voltage θ. Thus the remaining forces acting within the system are evidently the retarding forces. These are seen to be due to the inertia ($J\ d^2\theta_o/dt^2$) and viscous friction ($F\ d\theta_o/dt$).

The forces acting in the system are described by the following equation:

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[9] Lauer, Lesnick, Matson, Loc. Cit.

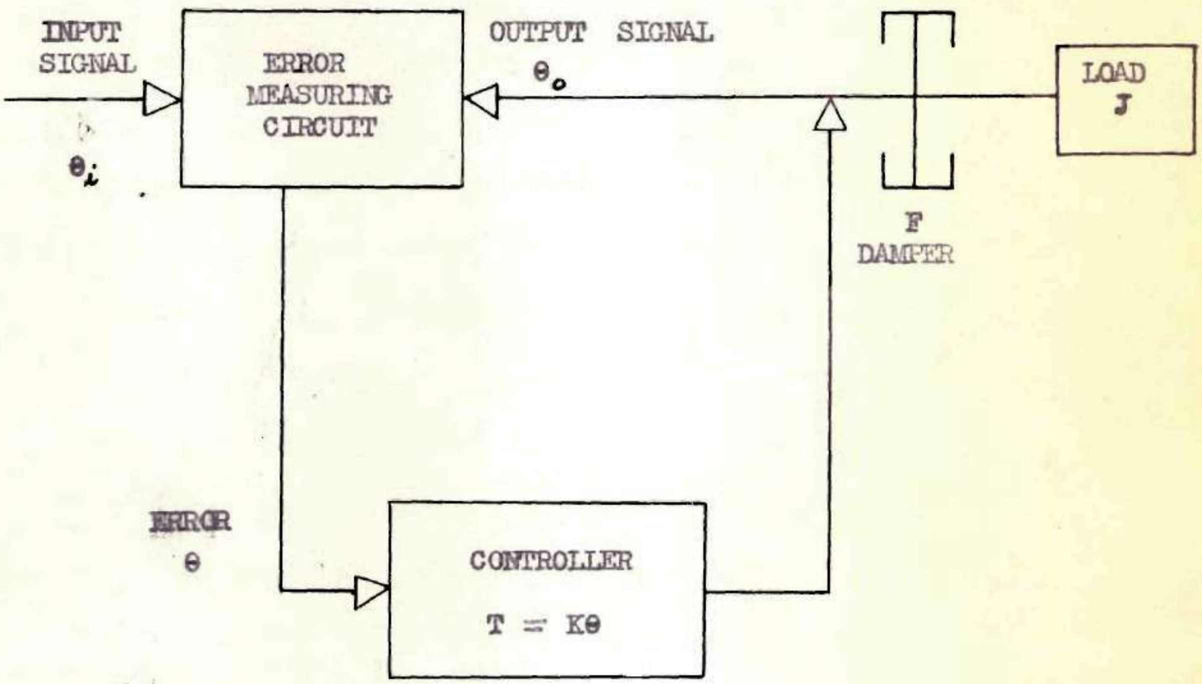

FIGURE 11 - Servomechanism Analogy of
Linear Recording Meter

$$J \frac{d^2\theta_o}{dt^2} + F\frac{d\theta_o}{dt} = K\theta \tag{10}$$

The error voltage θ is the difference between the input and the output voltages.

$$\theta = \theta_i - \theta_o$$
$$\text{or} \quad \theta_o = \theta_i - \theta \tag{11}$$

Substituting the value of θ_o into equation 10, an equation relating the error voltage to the signal input voltage is obtained.

$$J\frac{d^2\theta}{dt^2} + F\frac{d\theta}{dt} + K\theta = J\frac{d^2\theta_i}{dt^2} + F\frac{d\theta_i}{dt} \tag{12}$$

To obtain any meaning from this differential equation, the relation of θ_i as a function of t must be known. It appears from the equation, that the input signal must be a known factor. However, as it will be shown later, this will not be a necessity.

From the differential equation 12 it is seen that regardless of the input signal function, the general transient equation will be the same. It is the steady state error that will differ with the function of the input voltages.

Since it is the transient response that develops oscillations, its equation is derived. To obtain the transient solution, the differential equation is set equal to zero.[10]

$$J\frac{d^2\theta}{dt^2} + F\frac{d\theta}{dt} + K\theta = 0 \tag{13}$$

The solution of equation 13 can be obtained in many ways. However, the classical method is used here.[11]

$$\theta_T = Ae^{\left\{-\frac{F}{2J}+\sqrt{\frac{F^2}{4J^2}-\frac{K}{J}}\right\}t} + Be^{\left\{-\frac{F}{2J}-\sqrt{\frac{F^2}{4J^2}-\frac{K}{J}}\right\}t} \tag{14}$$

Examination of equation 14 indicates that the radical of the exponent is the controlling factor. The values for F, K, and J will determine whether the system is damped, critically damped, or oscillatory.

If $F^2/4J^2$ is less than K/J, than the value under the radical becomes negative and oscillations begin. This fact is evident with a mathematical substitution. For convenience, let $jb = j\sqrt{K/J-F^2/4J^2}$ and $a = F/2J$ where $j = \sqrt{-1}$. Therefore, equation 14 can be written as follows:

$$\theta_T = e^{-at}\left(Ae^{jbt} + Be^{-jbt}\right) \tag{15}$$

Substituting for e^{jbt} and e^{-jbt} their equals

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[10] Louis A. Pipes, Applied Mathematics for Engineers and Physicists. New York: McGraw-Hill Book Company, 1947, p. 109.

[11] Ibid, pp. 111-112.

$$e^{jbt} = \cos(bt) + j\sin(bt)$$
$$e^{-jbt} = \cos(bt) - j\sin(bt)$$

and collecting terms, the following is obtained:

$$\theta_{T_o} = e^{-at}\{D\cos(bt) + C\sin(bt)\} \tag{16}$$

where D = A + B and C = j(A-B).

The preceding equation is immediately recognized as a damped oscillatory wave. The constants C and D are determined from the conditions of the system. This requires that the steady state solution be known.

The critically damped case occurs when b is identically equal to zero. Applying the classical solution to differential equations with multiple roots, equation 17 is obtained.[12]

$$\theta_{T_{cd}} = e^{-at}(C_1 + C_2 t) \tag{17}$$

where C_1 and C_2 are arbitrary constants.

Equation 17 represents the ideal conditions under which this system would operate best. However, in general practice, this is never obtained.

Since in most systems, the controller constant (K) and mass (M) are difficult to control, the frictional factor (F)

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[12] Ibid., p. 112.

would be the remaining factor that could be varied in removing the oscillations. This was found to be effective in eliminating hunting of the recording pen.

EXPERIMENTAL RESULTS AND DISCUSSION OF MECHANICAL CONSTRUCTION

The mechanical design and construction of the meter is as important as the electrical design. Figure 6 shows a closer view of the mechanical details. Micro switches are placed beside each end of the threaded shaft to prevent the recording pen assembly from jamming at the ends. Thus, a signal voltage greater than the design limit will automatically stop the motor from rotating when the pen reaches 1/8" from the end of its pen travel.

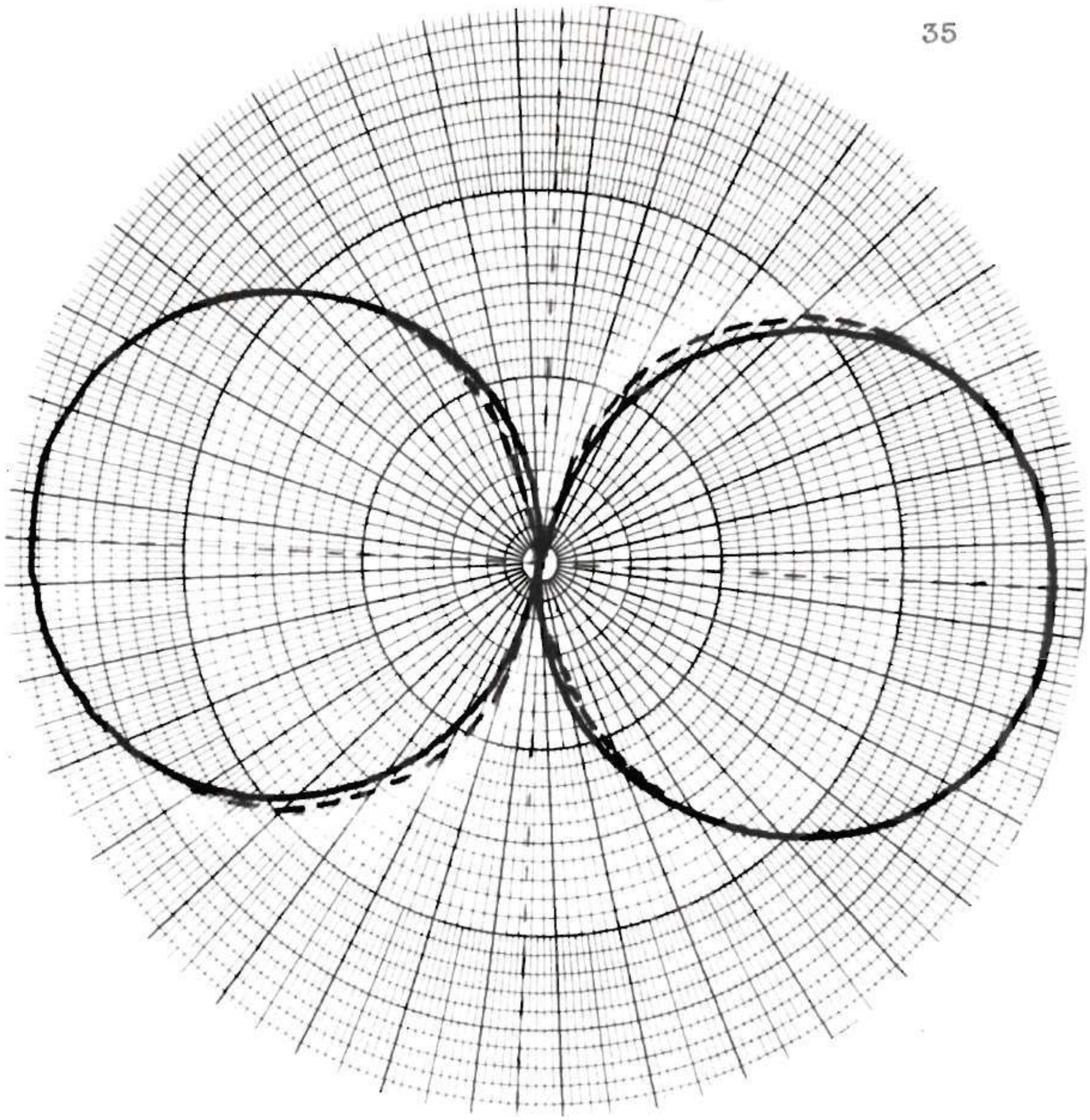
The potentiometer is located directly under the threaded shaft and the potentiometer arm. Its arm, which is a silver plated brush, extends downward from the pen assembly to the potentiometer.

The rectified output of a selsyn transformer was used to test the performance of the recording meter. Figures 5 and 6 show the selsyn mechanically coupled to the rotating disc. The disc, in turn, is rotated very slowly by a d-c motor. Therefore, as the recording disc (polar table) is rotated, the output of the selsyn will change correspondingly with respect to the position of the disc.

The output of the selsyn is rectified and used as the input signal voltage to the meter. The pattern obtained, with a signal of this nature is a figure eight. It is easily seen that as the selsyn is rotated, the envelope of the output voltage is sinusoidal.

In Figure 12 is shown the comparison between the actual plotted curve (dotted) and that obtained with the recording meter (solid curve). The error involved is less than five percent. The small error involved about the zero point can be due to several causes which are unbalance in the supply voltages to the system and the rapid change in the signal voltage in changing from zero to a miximum. The voltage change from maximum to zero is not as rapid as it is from zero to maximum. This was evidenced by means of a vacuum tube voltmeter. A cathode ray oscilloscope can be placed across the output of the bridge to show the error voltage change.

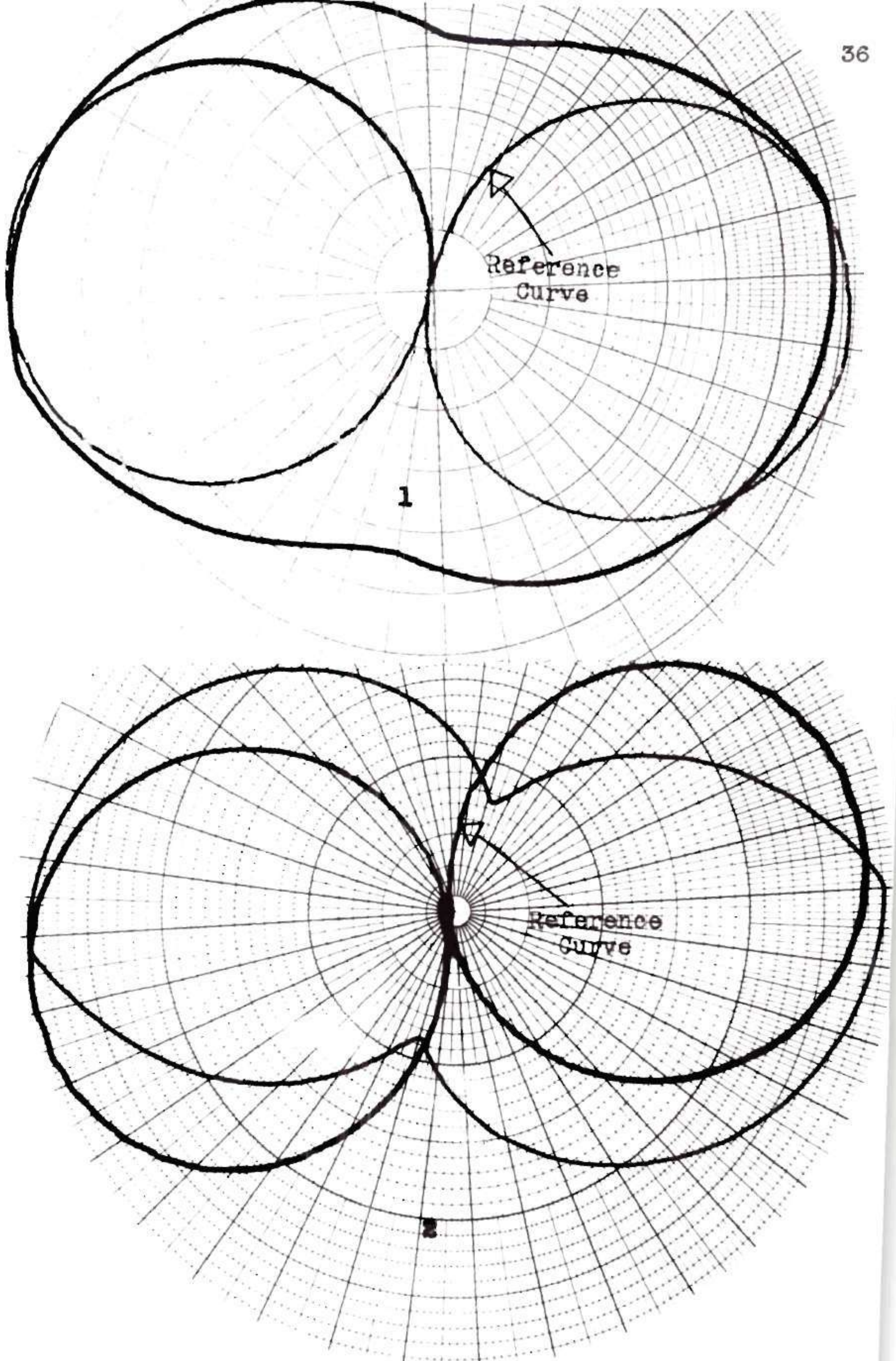
Figure 13 illustrates the effect of increasing the speed of the polar table. The reversible motor is not capable of following the rapid change in the input signal as shown by the error introduced. Since there is a time lag in the movement of the pen, that is it requires a definite time for the pen to rebalance the bridge, it is obviously seen that for a great change in the input signal with a small displacement of the disc, it will cause large errors if the disc is rotated fast. Therefore, a definite relationship exists between the rate of change of the recording disc and the input voltage. Consequently, a different speed of the disc seems necessary with each type of input signal. However, this can be avoided by adjusting the speed of the disc to



Maximum Voltage - 9.0 Volts D.C.

Dotted Curve - Actual Curve
Solid " - Plotted " by Meter
Table Speed - 1 RPM

FIGURE 12 - Graphic Plot of Recording Meter



1- Speed 2.0 RPM

2- " 1.5 "

FIGURE 13 - Effect of Rotational Speed on Graphic Plots

record a voltage change from zero to maximum scale reading with approximately 3° turn of the disc. In this manner, it will be certain that all important parts of the curve will be recorded. This points out the necessity of adjusting the angular velocity of the disc with respect to the rate of change of input signal voltage.

SUMMARY AND CONCLUSIONS

Examination of the graphic results indicates that a linear recording method has been obtained. The theory, as outlined in the discussion, is, therefore, well substantiated.

Although a polar table was used in the test, the instrument can also be adapted to record on cartesian coordinate paper. The necessary change would be to replace the polar table with a standard chart driving mechanism.

One immediate application of this recording meter is to plot a polar diagram of model antennas utilizing the rectified output of a crystal detector. Another application of the instrument is to employ it in connection with an antenna pattern calculator whose output is a modulated 60 cycle sine wave.

It appears a great many uses will be found for the recording meter.

The design of the meter could be further improved by the use of

a) Low voltage, low grid current, high current carrying capacity triode gas tubes

b) Continuous current rating reversible d-c motor.

It is believed a d.c. feedback path into the input could be arranged to act as a anti-hunt circuit for the d-c motor.

BIBLIOGRAPHY

"Electronic Recording Instruments," Electrical Engineering, January, 1947. pp. 36-44.

Harrison, T. R., W. P. Wills, and F. W. Side, "A Self Balancing Potentiometer," Electronic Industries, May, 1943. p. 68.

Herwald, S. W., "Forms and Principles of Servomechanism's," Westinghouse Engineer, September, 1946.

Keinath, George, "The Keinath Recorder," Instruments, Vol. 19, April, 1946. pp. 200-210.

Lauer, H., R. Lesnick, and L. E. Matson, Servomechanism Fundamentals. New York: McGraw-Hill Book Company, 1947. 277 pp.

Mikelson, W., and H. W. Bousman, "Rapid Recording A.C. Bridge," AIEE Transactions, Vol. 59, 1940. pp. 628-631.

Millman, Jacob, and Samuel Seeley, Electronics. New York: McGraw-Hill Book Company, 1945. 721 pp.

Nielson, D. M., "Electric Measuring Instruments," Electrical Engineering, February, 1946. pp. 66-74.

Pipes, Louis A., Applied Mathematics for Engineers and Physicists. New York: McGraw-Hill Book Compahy. 1947. 618 pp.

Richter, Walther, Fundamentals of Industrial Electronics. New York: McGraw-Hill Book Company, 1947. 277 pp.

APPENDIX

Determination of the Impedance Z and the Voltage e_1

To determine the input impedance of the transformer (T) several assumptions will be made. Core losses and leakages will be neglected and unity coupling will be assumed.

Applying Kirchhoff's laws to Figure 14a we obtain the following equations:

$$E = I_p(Z_p + R_p) + I_s Z_m \tag{A1}$$

$$0 = I_s Z_2 + I_s Z_s + I_p Z_m \tag{A2}$$

Solving for I_s from A2

$$I_s = -I_p \frac{Z_m}{Z_2 + Z_s} \tag{A3}$$

and substituting this value of I_s and A1 and solving for I_p we obtain

$$I_p = \frac{E(Z_2 + Z_s)}{(R_p + Z_p)(Z_2 + Z_s) - Z_m^2} \tag{A4}$$

Therefore, the impedance looking into the transformer terminals X and Y is the ratio of the input voltage to input current.

$$Z = \frac{E}{I_p} = Z_p + R_1 - \frac{Z_m^2}{Z_2 + Z_s} \tag{A5}$$

Since unity coupling is assumed, $Z_m = aZ_p$, where $a = Z_s/Z_p$. Replacing Z_m and Z_s into equation A5 and collecting terms

$$Z = R_p + \frac{Z_p \frac{Z_2}{a^2}}{Z_p + \frac{Z_2}{a^2}} \tag{A6}$$

An equivalent circuit for equation A6 is represented by Figure 14b. The impedance Z_2 is the secondary load impedance including R_s.

The determination of e_1, which represents the voltage drop across C_2, requires the equation of the secondary current I_s. Substituting equation A4 into equation A3 we obtain I_s.

$$I_s = \frac{EZ_m}{Z_m^2 - (R_p + Z_p)(Z_2 + Z_s)} \tag{A7a}$$

Replacing Z_m by its equal aZ_p

$$I_s = -\frac{aEZ_p}{(R_p + Z_p)Z_2 + a^2 R_p Z_p} \tag{A7b}$$

Considering Figure 14c, e_1 is seen to be $I_s Z_{c2}$.

$$e_1 = I_s Z_{c2} = -\frac{aEZ_p Z_{c2}}{(R_p + Z_p)Z_2 + a^2 R_p Z_p} \tag{A8a}$$

Since Z_2 was defined as the secondary load, it can be

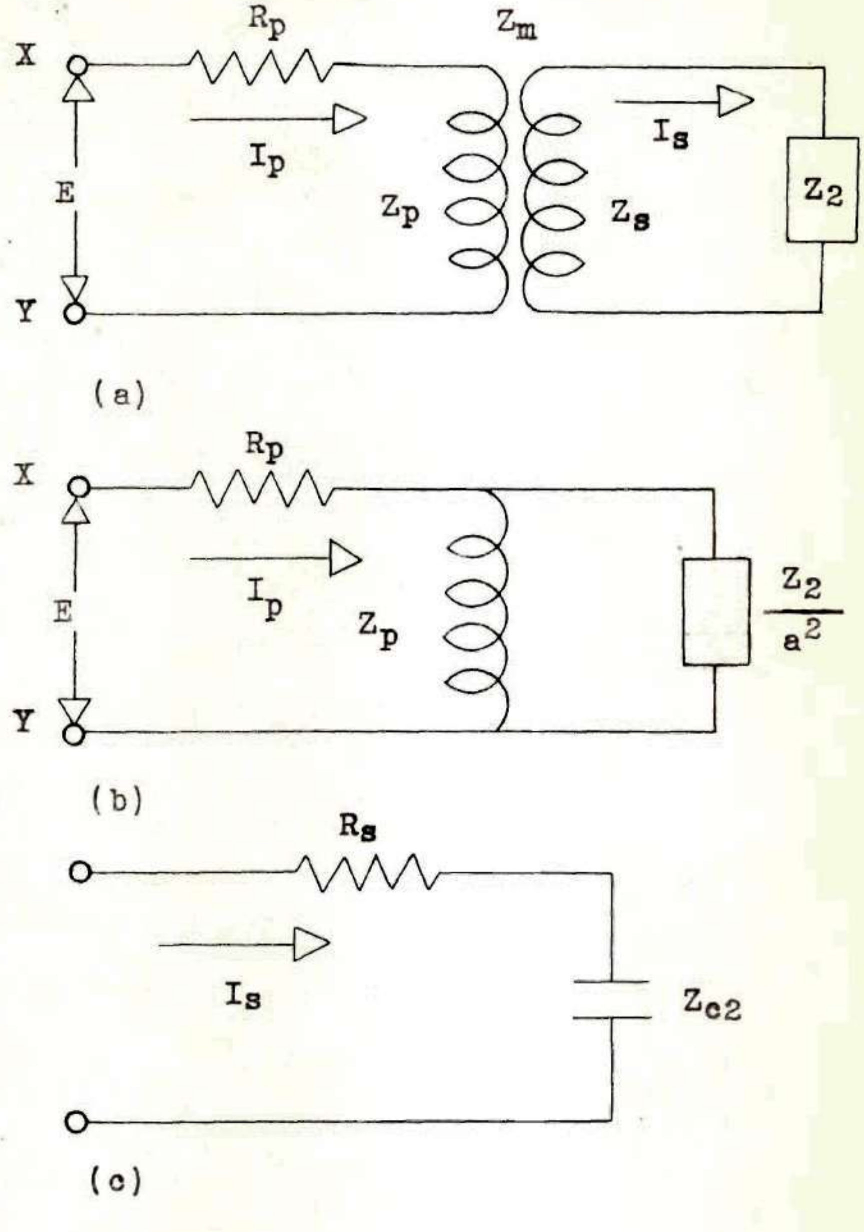


FIGURE 14 - Transformer Analysis

substituted into equation A8a, where $Z_2 = R_s + Z_{c2}$.

$$e_1 = \frac{a E Z_p Z_{c2}}{(R_p + Z_p)(R_s + Z_{c2}) + a^2 R_p Z_p} \qquad \text{(A8b)}$$

The negative sign is not included, since it indicates that e_1 is 180° out of phase with respect to the input voltage E.

Values of Components[13]

C_1	50 uf	R_1	25,000 Ohms
C_2	.04 "	R_2	1,000 "
C_3	.02 "	R_3	2,000 "
C_4	.5 "	R_4	50,000 "
		R_5	100,000 "
P_1	25,000 Ohms	R_6	50,000 "
P_2	110 Ohms	R_7	50,000 "
		R_8	1 Megohm

S_1 Series Field (Forward)

S_2 Series Field (Reverse)

A Armature of d-c motor

T_1 and T_2 are half sections of a 6SN7GT vacuum tube.

T_3 and T_4 " " " " " " " " " "

T_5 and T_6 " 2050 gas tubes.

Reversible d-c motor is rated at 27 volts

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[13] Refer to Figure 4.