

# Real-time Haptic Rendering and Haptic Telepresence Robotic System for the Visually Impaired

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## ABSTRACT

This paper presents a robotic system that provides telepresence to the visually impaired by combining real-time haptic rendering with multi-modal interaction. A virtual-proxy based haptic rendering process using a RGB-D sensor is developed and integrated into a unified framework for control and feedback for the telepresence robot. We discuss the challenging problem of presenting environmental perception to a user with visual impairments and our solution for multi-modal interaction. We also explain the experimental design and protocols, and results with human subjects with and without visual impairments. Discussion on the performance of our system and our future goals are presented toward the end.

**Index Terms:** I.2.9 [ARTIFICIAL INTELLIGENCE]: Robotics—Operator interfaces, Sensors; I.2.10 [ARTIFICIAL INTELLIGENCE]: Vision and Scene Understanding—3D/stereo scene analysis; H.5.2 [INFORMATION INTERFACES AND PRESENTATION]: User Interfaces—Haptic I/O; I.3.6 [COMPUTER GRAPHICS]: Methodology and Techniques—Interaction techniques;

## 1 INTRODUCTION

According to the World Health Organization [7], about 285 million people worldwide are classified as visually impaired, and among them, over 39 million individuals are diagnosed as legally blind. For those, the use of non-visual sensory modalities for environmental perception becomes more necessary, which typically requires direct contact with the environment. To increase the accessibility to the world, this research aims to provide a means for transferring perception of and granting interaction within a remote environment to a visually impaired user through a telepresence robotic system. In order to transform remote spatial information into a form for non-visual modality, we present a framework for utilizing a RGB-D based depth camera, a mobile robot, and a haptic interface for 3D haptic rendering [10] to accomplish the goal of haptic exploration of a remote environment.

In order to achieve this objective, a haptic linkage is created between an assistive robotic system and a human user by delivering haptic perceptions of a remote environment. More specifically, this research tackles the problem of providing individuals with visual impairments with a sense of the world based on the environmental data acquired by the perceptual system of a mobile robotic platform. To accomplish this, the user is provided with the ability to haptically explore the real-world environment through the “eyes” of a robotic system.

The details of this approach are described in the following sections. Section 2 displays previous and on-going research in the related fields of assistive robotics and haptics research. Section 3

presents the system architecture for our telepresence robotic system and algorithms for generating haptic and auditory feedback. Section 3.4 describes the control architecture for the system and user interaction, and Section 4 explains the protocol and experimental design. Section 5 then presents the results of our experiments and user studies with both visually-impaired and non-visually impaired users. Finally, Section 6 discusses the performance of our system and concludes with our aims for future studies.

## 2 RELATED WORK

There are several research efforts that incorporate robotic systems for assisting the visually impaired. Ulrich et al. developed a robotic cane [12] that can sense the environment and guide an individual to avoid obstacles. Another approach from Kulyukin et al. incorporated a mobile-robot based guidance approach with RFID devices to help people with visual impairments navigate in their living areas [3]. More recent challenges consist of a driving system for the visually impaired that utilizes a semi-autonomous car and a haptic vest [2], and a user study for robotic shopping using a robotic cart [1]. These efforts present pioneer studies in the field of assistive robotics in the sense that they intend to enable a person with a visual impairment to maintain independence in daily living. However, these systems require the user to be either in direct contact with a device or the environment to be fully preset with assistive technology, such as objects with RFID tags, and they do not fully transfer the details of remote environmental perception to the user.

For presenting environmental perception to a user with visual impairments, haptic modality can be an effective means for interaction. This perspective is derived from the fact that the most common assistive tools for people with visual impairments are canes and Braille-notes that stimulate tactile sensations. The ability of these tools to transfer tactile and textural sensations along with force feedback adds another dimension of interaction between the human and the system. With the recent advances in virtual proxy methods [4], haptic rendering of 3D objects has become more convenient and effective, especially with the use of GPU-based optimizations [13].

With the recent advancement in the methods of structured-light depth sensing with RGB-D sensors (or more widely known by the product name Kinect<sup>TM</sup>), point-cloud based depth data is found to be effective in enabling 3D haptic rendering both in real-time and in off-line applications [8], [9], [11]. In continuous work from our previous studies [5] and [6], we present our recent study of real-time 3D haptic rendering without *a priori* models and a multi-modal telepresence robotic application for individuals with visual impairments.

## 3 HAPTIC TELEPRESENCE ROBOTIC SYSTEM

We believe that a telepresence robotic system for the visually impaired should be able to 1) use its sensory devices to perceive the environment and generate a 3D perception model of the environment, 2) transfer the environmental perception to the human user in a non-visual way, and 3) transform human controls on the haptic interface to enable teleoperation and telepresence for the human

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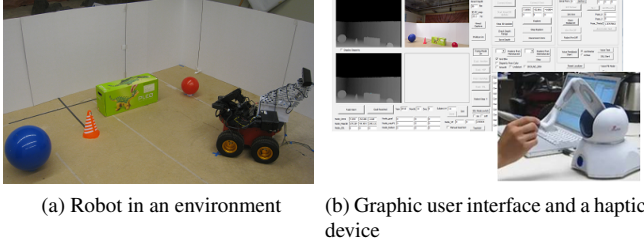


Figure 1: Telepresence robot for haptic exploration for the visually impaired.

user. In the following subsections, we present the overall structure of our system and describe the three-stage architecture—3D map building and virtual-proxy based haptic rendering algorithm, the transference of the 3D perception, and the control system with human input—of our telepresence robotic system.

### 3.1 System Configuration

The basic structure of our telepresence system consists of a mobile manipulator robotic system, a unified system controller, and a human interface with a haptic feedback channel. The mobile manipulator robotic system can include sensory devices such as a camera and a RGB-D sensor. The system controller controls the robotic system by the command of the human operator through the haptic interface, and also accesses the sensory devices to transfer the environmental perception of the robot to the human interface module.

With this configuration, the haptic interface in this framework functions in two ways: as a controller for the mobile manipulation system and as a generator for the environmental feedback to the human user. Likewise, the robotic system becomes a mediator between the user and environment working as an active agent that follows human operator’s intentions for movement and exploration as well as a remote agent that collects the environmental data for haptic 3D exploration of the remote environment.

The main hardware platform is composed of a robotic mobile base (Pioneer 3AT) equipped with the Kinect depth sensor. The robotic system is currently connected to a host PC through a wired connection, and a haptic interface (PHANTOM Omni) is also connected to the PC where a unified software is running that governs robotic control, perception, and haptic interaction.

### 3.2 3D-Map Building & Virtual-Proxy Algorithm

As presented in [6], we have developed a 3D haptic rendering algorithm based on 3D-point map generated by a stereo-vision process. We extend its application to incorporate the Kinect sensor. The Kinect sensor from Microsoft is an RGB-D sensor that provides depth and color images at a rate of approximately 30 frames per second (fps). To handle continuous depth and color image input-flow from the Kinect, a two-stage pipeline structure has been constructed.

#### 3.2.1 Real-time 3D Map Building from RGB-D sensor

In the first stage, the Kinect depth data frame is buffered, projected into a 3D coordinate system for each 640x480 data points, then transformed into a 3D point map (Figure 2). While converting a single frame depth data from the Kinect into the 3D map with 640x480x1024 points structure, the points behind the occupied points are marked as occupied as well, forming a 2.5D Map in essence. Once the 3D map is configured with occupied and unoccupied points, the haptic interaction point (HIP) corresponding to the user’s movement on the haptic interface can be projected into the

3D map, and the virtual proxy forces can be estimated (as to be explained in Section 3.2.2). This method differs from the approach of Rydén et al.([8]), which generates an averaged force vector on the HIP toward a certain region of neighbor point-cloud points. Our method tries to generate more precise force vectors given a complex shape and is used in correlation with the robotic mapping algorithms for generating more complete 3D maps during the robot’s movement. In addition, having an independent representation of 3D map can be useful in manipulating / augmenting the map to achieve extra objectives such as augmented reality or robotic mapping and planning.

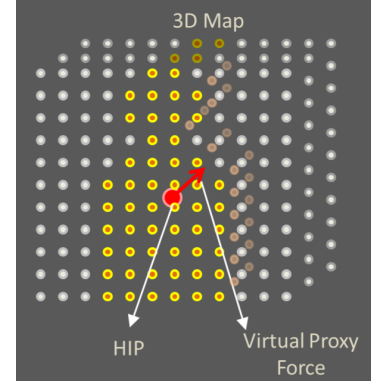
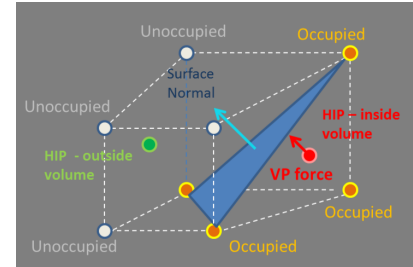
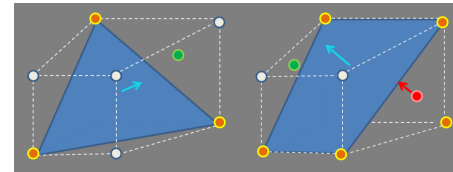


Figure 2: An example of a 3D point map of an object for the generation of virtual-proxy forces (For visual purposes, the points are arranged in a grid-style. Actual point cloud has shorter distances between points closer to the sensor and longer distances between points further away from the sensor, due to the optical characteristics of the sensor).



(a) Virtual-proxy force estimation within neighbor points.



(b) Example of the surface-normal estimation given neighboring 3D Points.

Figure 3: Illustration of a virtual-proxy force calculation within neighbor 3D points.

#### 3.2.2 3D Haptic Rendering from 3D Map

The second stage of the pipeline handles the haptic interaction, which constantly accesses the 3D map to calculate virtual proxy forces for the HIP (Figure 3a) while updating the 3D map if a new

map is built. To expedite the calculation for the virtual-proxy algorithm, neighbor-based surface normals are predefined in a look-up style as illustrated in Figure 3b.

To calculate the virtual-proxy force on the HIP given a 3D map, we first project the position of HIP in the 3D map which will be on or in-between any points in the map. To determine if the HIP is in any object volume, we check the occupancy of neighbor points. If the neighbors are partially occupied, meaning the HIP is near the surface of the object volume, we project the HIP on to the estimated surface within the neighbors and determine if the HIP is inside or outside of the volume. If it is inside the volume, then the projected vector can be used to generate the virtual-proxy force for the HIP's position. Also, if the neighbors are fully occupied, then the HIP is fully penetrating the volume, and we find the closest surface point by searching outward (where a directed search can be used given a previous virtual-proxy force vector). To increase the accuracy, we project a temporary HIP point near the closest surface point and use the surface virtual-proxy estimation to find the exact virtual-proxy position corresponding to the actual HIP that is fully penetrating the volume and generate the virtual-proxy force. This process is illustrated in Figure 4.

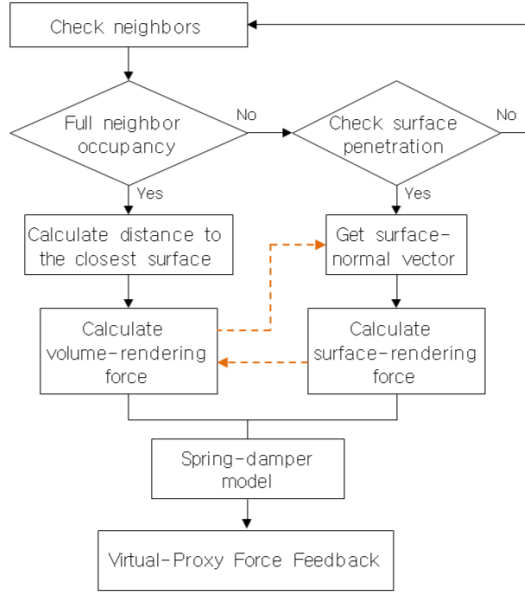


Figure 4: Haptic virtual-proxy algorithm for calculating interaction force with a 3D map.

For rendering the virtual-proxy force for the haptic interface, a typical spring-damper model is used as in Equation 1. Given the position vector of the proxy  $\vec{P}_{proxy}$ , a position vector of the probe  $\vec{P}_{probe}$ , and the velocities of the proxy and the probe,  $\vec{v}_{proxy}$  and  $\vec{v}_{probe}$ , a virtual-proxy force-feedback  $\vec{f}_{feedback}$  is composed of a penetration depth term (with a spring constant  $k$ ) and a damping term (with a damping constant  $b$ ) as shown in Equation 1. The penetration depth term is a static value representing the positional relationship between  $\vec{P}_{probe}$  and  $\vec{P}_{proxy}$  that conveys object shape and stiffness to the user. The damping term is a dynamic value that corresponds to the movement of the probe. The corresponding proxy position represents the surface friction or texture of the object.

$$\vec{f}_{feedback} = k \left( \vec{P}_{proxy} - \vec{P}_{probe} \right) + b \left( \vec{v}_{proxy} - \vec{v}_{probe} \right) \quad (1)$$

The advantages of this approach are direct haptic rendering from RGB-D perception and the capability for handling dynamic environments. These benefits encourage real-time interaction and exploration in the remote environment through the robotic platform, enabling the telepresence of the user. This real-time interaction allows the system to have the capacity for extra sensory modalities, i.e., encapsulating sound information of the remote environment, generating auditory feedback, or representing dynamic movements in real-time. Figure 5 shows a simple haptic interaction that mimics the experiments from the work of Rydén et al.([8]).

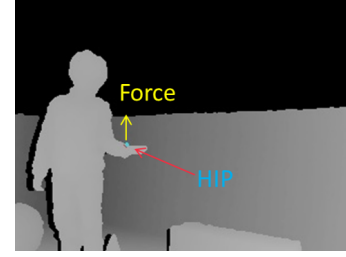


Figure 5: Haptic interaction with a human in the scene.

### 3.3 Teleperception: Transfer of the 3D Perception

With the algorithm described in Section 3.2, a typical user can feel the 3D environment with a haptic interface (and typically with a graphical user interface) and use this perception to control the system. However, we need to provide additional information to enable teleperception for a visually-impaired user besides haptic perception. Multiple feedback channels are necessary to fully gain knowledge of both the 3D perception of a remote environment and the status of the movement of a remote robotic agent. In this study we chose to utilize the haptic channel to convey the 3D environmental perception and add an auditory feedback channel to provide information on robot status and the visual details of a remote environment.

Auditory feedback contains two types of information. The first one is the verbal description of the status of our robotic system, which consist of “forward”, “left”, “right”, “backward”, and “stop”. These are reported only once when the status changes, i.e. it does not keep reporting “forward” while moving forward. The second type consists of verbal descriptions of color and distance information associated with an object the HIP is in contact with. The corresponding color image for each depth frame is buffered at the same time of the 3D map generation, and the color data of the scene is utilized if the HIP is determined to be inside volume of any objects in the 3D map. The list of colors recognized by our system is shown in Table 1. The distance information from the telepresence robot to the HIP on the surface of the 3D environment is extracted from the depth frame of the sensor, and is verbally translated using a text-to-speech(TTS) technology to the user to provide the user with the scale of the environment and the movement of the system.

One of the common misconception a typical person has about the visual impairment is that a visually-impaired person has no idea of colors. There can be different level of visual impairments, including low sight, partially blind, and full blind. In addition, the progress of visual impairment can be different from person to person: a person can have it from birth, or it can be degenerative. Accordingly, many people with visual impairments have knowledge of colors, and even if a person has been blind whole life, she/he gains knowledge of colors through education and interaction throughout one's life. From our experience with individuals with visual impairments through other projects, we have learned that people with visual impairments sense the environment through multi-modal senses, with

equal or more detailed accuracy compared to people with no visual impairments.

Table 1: List of color names differentiable with verbal feedback.

#	Color	#	Color	#	Color
1	Aqua	23	Indian red	45	Peach
2	Aquamarine	24	Ivory	46	Pink
3	Black	25	Khaki	47	Plum
4	Blue	26	Lawn green	48	Purple
5	Blue violet	27	Light blue	49	Red
6	Bronze	28	Light coral	50	Royal blue
7	Brown	29	Light gold	51	Scarlet
8	Cadet blue	30	Light sky blue	52	Sea green
9	Chartreuse	31	Lime green	53	Sienna
10	Chocolate	32	Magenta	54	Sky blue
11	Coral	33	Maroon	55	Slate blue
12	Dark green	34	Medium purple	56	Spring green
13	Dark grey	35	Midnight blue	57	Steel blue
14	Deep pink	36	Navy blue	58	Summer sky
15	Deep sky blue	37	Neon blue	59	Turquoise
16	Dodger blue	38	Olive	60	Violet
17	Forest green	39	Olive green	61	Violet red
18	Gold	40	Orange	62	White
19	Green	41	Orange red	63	Wood
20	Green yellow	42	Orchid	64	Yellow
21	Grey	43	Pale green	65	Yellow green
22	Hot pink	44	Pale violet		

### 3.4 Teleoperation: Control System with Human Input

Given that the user is provided with perception of the remote environment and the system, the system must now cope with the control input from the human and operate the robotic system properly. To integrate all functionalities into a unified system, we constructed a finite state machine (FSM) to run seamlessly while interacting with the human user (Figure 6). The FSM is composed of three states required for the experimental design of this study, which are *idle*, *static haptic-exploration*, and *dynamic haptic-exploration*. Each state has sub-states defined for the robotic platform and the haptic interface. The whole FSM configuration is governed by two scenarios designed to encapsulate the experimental setups used to validate the system’s functionality. These two scenarios are described as below:

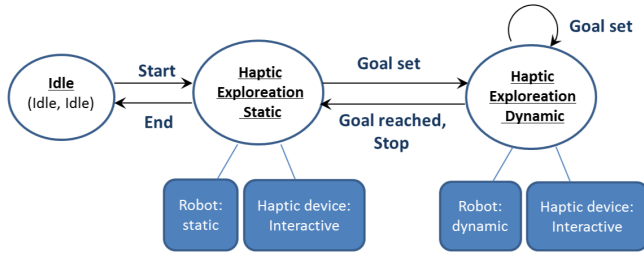


Figure 6: Finite-state machine (FSM) for our haptic telepresence system. Only three states are enabled for this study, but more states that connect multiple functionalities of the robotic system and the user interface are hidden.

- Scenario 1: Manual control with multi-modal tele-perception. The human subject holds the haptic interface to feel and ex-

plore the spatial distribution of the remote scene, and uses the traditional keyboard control of “up arrow” for making the robot move forward, “down arrow” for making the robot move backward, “left arrow” for making the robot turn left, “right arrow” for making the robot turn right, and “Ctrl key” for making the robot stop.

- Scenario 2: Semi-autonomous navigation with multi-modal tele-perception and haptic command, or “Clickable world”. The human subject explores the scene with the haptic interface, and upon finding a target, the subject can click on the position in the haptic space to command the robot to approach that position autonomously. Since the robot navigates on the user’s command, this is “semi-autonomous navigation.”

## 4 EXPERIMENTAL DESIGN

We designed our experiments to evaluate the performance of our system with the two operational scenarios. Specific purposes of this experimental design is to figure out whether our system can be used so a visually impaired user can gain 3D perception of a remote environment, find and distinguish objects in the scene, and intuitively learn to control the system for interacting within the environment through the telepresence system. The experiments are performed given a 3-DoF haptic interface for haptic exploration and an auditory feedback channel from the system, but with two distinctive control mechanisms of key control for manual navigation and haptic “clicking” of the semi-autonomous navigation. The spatial setup of a remote scene for the experiments is as shown in Figure 7, which includes a colorful box, a scarlet cone, two balls (blue and red), and a chair (not seen from the initial position) surrounded by walls forming a 5m x 6m indoor environment.

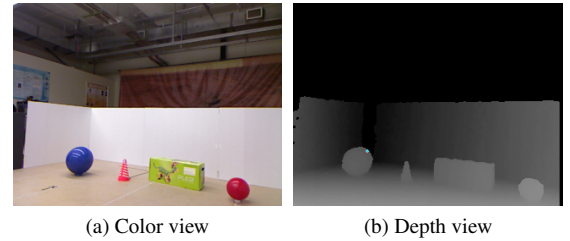


Figure 7: Experimental setup.

The procedures for our experiments are as follows:

- Subject consent or verbal assent are acquired prior to the experiment.
- Explanation on the robotic platform, computer system, and the haptic interface is given.
- Tutorial on the haptic exploration is provided. The subject can feel the 3D space with a box (different size and color from the box in Figure 7a) and a scarlet cone in it.
- Tutorial on the control methods—key control and haptic clicking—are given to the subject.
- Explanation on the auditory feedback—color, distance, and the status of the robot—are provided to the subject.
- The subject is asked to accomplish the task of finding a specific-colored ball (either blue or red ball, whose positions are subject to switch) by controlling the robot. The sighted subjects are blindfolded at this time to provide equal conditions for the experiment.

During the experiments, measurements are made on the following criteria: 1) Time for task completion, 2) the trajectory of the robot, and 3) the trajectory of and force feedback generated for subject's HIP position. The objective of each experiment is to accomplish a goal, which is to "find a blue/red ball and approach it," and it is only revealed to the human subject after they are given sufficient time for explanations and tutorials on the system.

## 5 RESULTS

A total of 12 human subjects (two female and ten male subjects) participated in our experiments. Among them, nine were sighted, one was legally blind (partially sighted), and two were fully blind. The age group was between 15 and 40. Each subject was given a tutorial of the system (about 5-10 minutes) and then given two tasks: 1) find and approach a blue ball in Scenario 1, and 2) find and approach a red ball in Scenario 2. The experimental results and analysis are discussed below.

### 5.1 User Performance for Task Completion

The success rate and task-completion time are measured for each subject during each scenario. The success rate is measured to inspect how well users interpret haptic and auditory information to make the right decision, and the task completion time is measured to compare the efficiency of operation between the two scenarios. The average time taken for Scenario 1 was 58.2 sec, and 32.38 sec for Scenario 2. The median values and the variations are depicted as a bar plot in Figure 8.

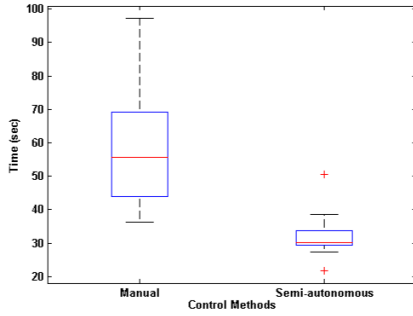


Figure 8: Task time comparison between manual and semi-autonomous control.

The success rates for Scenario 1 and 2 for all participants were 50% and 75% respectively. However, the group of subjects with no visual impairments marked the rates of 55.6% and 66.7%, and the group of subjects with visual impairments marked 33.3% and 100% respectively as shown in Table 2.

Table 2: Success rates of human subjects.

	Total human subjects	Sighted subjects (blind-folded)	Visually-impaired subjects
Scenario 1	50%	55.6%	33.3%
Scenario 2	75%	66.7%	100%

Based on these results of task time and success rates, we can see that the control method does affect the performance of the system with the user, and semi-autonomous control is more effective in achieving the goal. One thing we can note from Table 2 is that the visually-impaired subjects show a larger drop in the success rate

with manual control, which implies that the system is not providing enough support for the user in teleoperating the system. More closer look on the log files revealed that the subjects with visual impairments found it hard to determine where the robot was headed while the robot is turning to the left or right and missed the right timing to stop the robot. To resolve this issue, the robot's turning angle will be limited for each user's command for turning.

### 5.2 Trajectory of the Robot

Exemplary results of the user study from Scenario 1 and Scenario 2 are shown in Figure 9a and Figure 9b. Sequential images taken from the viewpoint of the robot during the task (at the events of user command) are also illustrated in Figures 10 and 11. We setup the experimental site in such a way that the robot can perceive all four objects at the beginning, so there will be no possibility of biasing the scenario. The results show that the semi-autonomous mode achieves the path toward the goal with shorter distance and smoother trajectory. More importantly, combined with the results of success rates, these results implies that if given the right control modality, our system can assist the user in remotely perceiving the environment and making a correct decision to achieve a goal in an efficient manner.

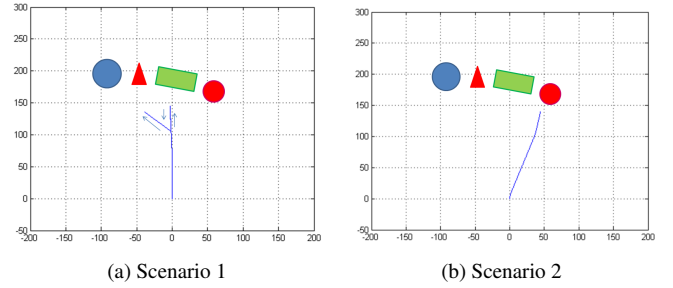


Figure 9: Resulting trajectories of Scenario 1 & 2.

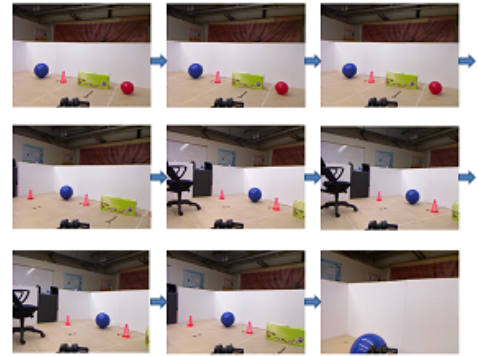


Figure 10: Sequential images on each control commands during Scenario 1.



Figure 11: Sequential images on each control commands during Scenario 2.



### 5.3 Learned Knowledge on Haptic Exploration

After observing the subjects' manner of exploring the remote environment with our haptic interface, we realized that the users showed a mixed set of behaviors for exploration: "touching" (contour following) of partial surfaces and "poking" on random positions, as roughly depicted in Figure 12. Since the user is perceiving a large 3D environment and not just feeling a single object, these behaviors seemed natural. However, subjects with previous experience with haptic devices and novice users on haptic interface showed different behaviors in haptically exploring, as shown in Figure 13. We plan to investigate these behaviors more in our future studies.

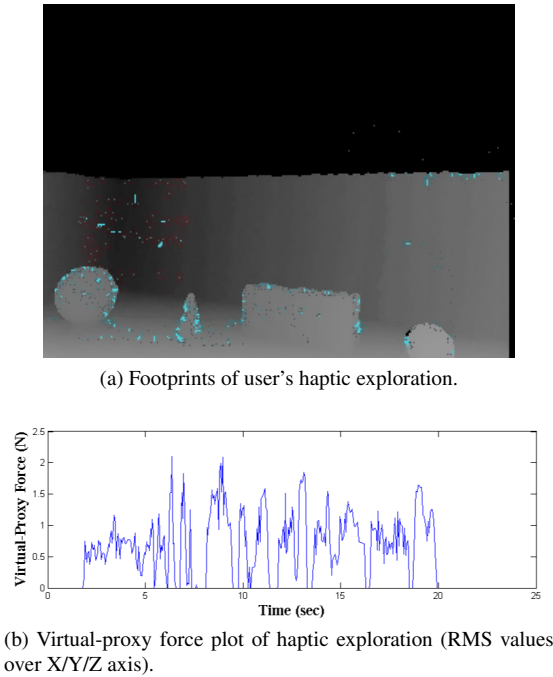


Figure 12: Visualization of haptic exploration for 20sec.

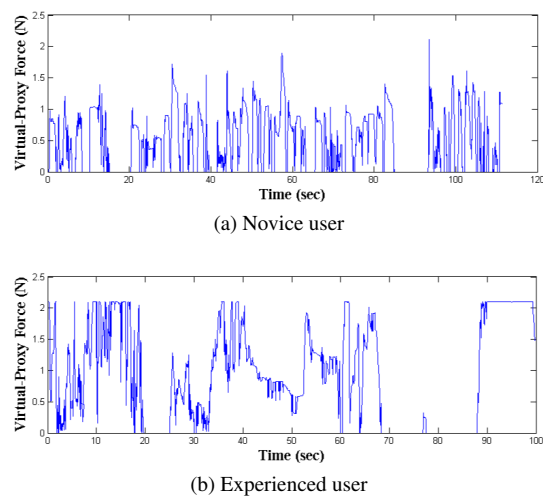


Figure 13: Haptic force plots for novice and experienced users for 120sec.

### 6 CONCLUSION

We have presented in this paper our approach for real-time haptic rendering using a RGB-D sensor and its application with a haptic telepresence robotic system for use by individuals with visual impairments. Although the tasks performed in this work can be regarded as preliminary in the full spectrum of teleoperation and telepresence, we have made a challenging effort to bring multi-disciplinary aspects of haptics, robotics, and assistive technology into a unified framework of assistive robotics and haptic telepresence. Considering that telepresence technology is also in an evolving stage, we aim to make further contributions in the field of haptic exploration and telepresence based on our gained knowledge in this study. For future work, we plan to make necessary upgrades to our system to increase accuracy of feedback and support for more intuitive user control, as well as more in-depth human-factor analysis with more subjects.

### ACKNOWLEDGEMENTS

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