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OPTIMUM OPERATING CONDITIONS  
OF A MULTI-GRID FREQUENCY  
CONVERTER

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## PREFACE: MEANING OF SYMBOLS USED

- $I_m$ .....Bessel's Function of 1st kind, order  $m$ , and  
imaginary argument.
- $G_m$ .....Signal electrode to plate transconductance.
- $G_c$ .....Conversion transconductance.
- $E_{c1}$ .....Bias of first electrode from cathode.
- $E_{c3}$ .....Bias of third electrode from cathode.
- $e_s$ .....Total signal electrode voltage.
- $e_o$ .....Total oscillator electrode voltage.
- $\omega_o$ .....Angular frequency of the oscillator electrode voltage.
- $\omega_s$ .....Angular frequency of signal electrode voltage.
- $\omega_{IF}$ .....Angular intermediate frequency.
- $i_p$ .....Alternating component of plate current.
- $i_{\omega_{IF}}$ .....Alternating component at  $\omega_{IF}$  of plate current.
- $E_s$ .....Amplitude of alternating component of signal voltage.
- $E_o$ .....Amplitude of alternating component of oscillator  
voltage.
- $R_L$ .....Plate load resistance.
- $k$ .....Boltzmann's Constant.
- $T_c$ .....Cathode temperature in degrees Kelvin.
- $Y_o$ .....Input admittance in mho.
- $\Delta f$ .....Frequency band width in cycles per second.
- $a_n, b_n, C_n$ .....Empirical coefficients of plate family.
- $N$ .....Noise Ratio.



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# OPTIMUM OPERATING CONDITIONS OF A MULT-GRID FREQUENCY CONVERTER

## INTRODUCTION

An important unit in any superheterodyne receiver is the converter or mixer. The superheterodyne receiver is characterized by the fact that most of its amplification or gain is secured from a fixed amplifier tuned to some relatively low radio frequency. In order that such a receiver can be operated over a frequency band, it is necessary that the incoming signal of any frequency within the band be modified to give a constant output frequency, which is termed the intermediate frequency, and is the frequency to which the fixed radio frequency (or intermediate frequency) amplifiers are tuned.

This signal frequency conversion to a constant intermediate frequency is obtained by feeding both the incoming signal and a locally generated oscillator voltage to a non-linear impedance. Then the intermediate frequency is related to the signal and oscillator frequencies by the relationship

$$f_{i.f.} = Mf_o \pm Nf_s$$

where M & N are any integers, usually unity.



The circuit in which this frequency conversion is performed is termed the frequency converter if the local oscillator is incorporated into the same stage, and is called a mixer if it utilizes a separate local oscillator. It is the purpose of this thesis to study the multi-grid frequency converter or mixer, since the same general considerations apply to both.

The essential feature of any frequency converter is that it must be a non-linear impedance. For a number of years non-linear inductances have been available (as any iron core inductor is non-linear), and lately non-linear dielectric materials have been developed; but both these have seen but very limited use as converters. The great majority of frequency converters make use of non-linear resistance in order to obtain the necessary impedance characteristic.

Of the group of non-linear resistances, some of the more common types are germanium and silicon crystalline structures and most common of all are vacuum tubes operated as non-linear resistors. It is our purpose to study a further sub-category of the latter. That is, we shall confine our attention in this thesis to a study of frequency converters of the type which use multi-grid vacuum tubes as non-linear resistors.

## OBJECT

In recent years many works have appeared in the literature upon frequency converters. Most of these have been quite general in nature. With this in mind this thesis was undertaken in an attempt to correlate and unify existing works upon converters and to offer recommendations as to their optimum operation to the engineer, who must use them in the field.

## METHOD OF APPROACH

In view of the broad scope of the problem of frequency converters, it was necessary to limit this work to a special group of converters, the multi-grid vacuum tube converters, and generalize the results obtained to include other groups. Due to the predominant usage of the multi-grid converter using separate electrodes for the applied voltages, it was decided to further confine the attention of this thesis to the method of operation in which the locally generated oscillator voltage is applied to an inner grid, and the incoming signal voltage to an outer grid.

In multi-grid frequency converter or mixer tubes, often the local oscillator is incorporated into the same tube envelope as the mixer section. Hence, in such devices one must, for a complete understanding of the circuit, also study the characteristics of the local oscillator.



The types of oscillators suitable for use with frequency converters are treated in the literature. This thesis is therefore to place emphasis upon the mechanism of frequency conversion.

The method of investigation consists first, of the theoretical development of the method of operation and second, of the experimental check on the developed theory.

#### GENERAL BASIS OF OPERATION

Basically, frequency converters can be catalogued<sup>1</sup> into one of two different groups.

Group one is composed of those converters whose operation is such that both local oscillator and signal voltages are impressed upon the same electrode--thus, obtaining non-linear operation by the movement of the operating point along a fixed characteristic. These converters are said to be of the sliding Q point type.

Group two of frequency converters are those which utilize separate electrodes for the local oscillator and signal voltages. The non-linearity of operation of this group of converters arises from the wide variation in

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1. H. Stockman, "Frequency Converter Terminology", Electronics, Vol. 16, p. 144, November 1946.

oscillator electrode voltage, which causes a continuous non-linear shifting of the operating point of the signal electrode. Thus, this second group of converters are called the shifting Q point group. It is to this latter group that the multi-grid converters belong.

The operation of the multi-grid converter is such that a locally generated oscillator voltage is utilized to vary the signal grid-to-plate transconductance at the oscillator frequency. Thus, the operation of this type of converter is much the same as the suppressor grid modulator. The distinction between the modulator and converter is in the magnitude and frequency of the signal voltage relative to the oscillator voltage.

The oscillator electrode of the multi-grid converter is commonly operated in class B or C, and the signal voltage is used to modulate the resulting plate current pulses. The usual spacing in the spectrum of the frequencies of the local oscillator and signal voltages is such that by the use of a tuned load, one is able to select the desired frequency component of the resultant output.



## METHODS OF ANALYSIS

Any general analysis of the multi-grid converter must include some assumptions as to the nature of the plate current in class C or B operation.

If a theoretical treatment of such class B or C converters is to be based upon the use of continuous analytical expressions, it must be assumed that at all times there is a very small but finite minimum plate current. Thus the plate current can be expressed as a function of the electrode potentials--this function being continuous and continuous in all derivatives.

Writing the plate current as

$$i_p = f(e_{g1}, e_{g2}, e_{g3}, e_{g4}, \text{-----} e_p) \quad (2)$$

the function may then be expanded in an absolutely convergent series of the most desirable and informative form.

There are several types of series into which the plate current may be expanded. These series developments<sup>2</sup> are listed below with some comments as to application.

(a). "One alternating voltage applied, the volt-ampere characteristic varied by other". This is called the method of variational conductance, because

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2. H. Stockman, "Calculation of the Output from Non-linear Mixers", Journal of Applied Physics, pp. 110-120, February, 1946.

it approaches the problem from the variation of the transconductance between the signal electrode and the plate at the oscillator frequency. The disadvantage of this method is that the actual obtaining of results must involve either graphical methods or dynamic measurements.

(b). "Empirical equation written as an exponential series of 'n' variables developed into a Fourier series with Bessel function coefficients". This method utilizes an exponential series with empirical coefficients to describe the static plate family. Such a series possesses the advantage that a complete analysis of converter operation may be made with knowledge of only the static plate family. This series can be only approximate because the coefficients are not unique and consequently must be empirical. Other than the stated approximations, the main disadvantage is the considerable labor involved in the coefficient evaluation.

(c). " $i = f(e)$  written as a Taylor's series of 'n' variables where 'n' is the number of electrode potentials." A Taylor series is not directly applicable as a general method of convertor analysis, because the coefficients of a Taylor series must be evaluated from the partial derivatives of plate current. In class C operation the operating point (about which the partial derivatives are evaluated) is in the region of zero



plate current; therefore, the partials are not defined. Of course, the method could be applied to converters operated in class A.

Of the above methods of analysis, the second and third are the most valuable for a general converter analysis. Therefore, this thesis will develop these two methods, drawing some results from each and making recommendations as to applications to specific converters.

Of the two analyses, the "variational conductance" possesses the advantage of having a simpler developmental theory and end result, but the "empirical exponential equation" gives a much more complete analysis. The accuracies of the two methods are approximately the same.

#### A. METHOD OF VARIATIONAL CONDUCTANCE<sup>3</sup>

Frequency converters of the type in question are operated with a small signal voltage and large applied oscillator voltage. Therefore, the grid to which the signal voltage is applied (the signal grid) is operated over a linear region. Under these conditions, the transconductance between the signal electrode and the plate may be considered as a function of the oscillator electrode voltage only, and the plate current

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3. E. W. Herold, "Operation of Frequency Converters and Mixers", Proc. I. R. E., Vol.30, No.2, p.84, February, 1942.

resulting from the two applied voltages is written as

$$i_p = f(e_{gs}, g_{ms}) = f(e_{gs}, h(e_{go})) \quad (3)$$

$$\text{where } g_{ms} = h(e_{go}) \quad (4)$$

where the signal electrode to plate transconductance may be written as a Fourier series.

$$g_{ms} = a_0 + a_1 \cos w_0 t + a_2 \cos 2w_0 t + \dots a_n \cos nw_0 t \quad (5)$$

and the oscillator electrode voltage is equal to  $E_0 \cos w_0 t$ . Thus, because the plate current bears a linear relationship to the signal voltage, the plate current may be expressed by the equation

$$i_p = g_{ms} e_s \sin w_s t = a_0 e_s \sin w_s t + e_s \sum_{n=1}^{\infty} a_n \sin w_s t \cos nw_0 t \quad (6)$$

$$i_p = a_0 e_s \sin w_s t + \frac{e_s}{2} \sum_{n=1}^{\infty} a_n \sin(w_s + nw_0)t + \frac{e_s}{2} \sum_{n=1}^{\infty} a_n \sin(w_s - nw_0)t \quad (7)$$

where the signal electrode voltage equals  $e_s \sin w_s t$ .

Note that the assumption of a linear relationship of plate current to signal voltages neglects all effects of cross modulation products (all interaction terms).

Conversion transconductance is defined as the ratio of the amplitude of the plate current component at the desired intermediate frequency to the amplitude of the applied signal voltage. Thus conversion



transconductance, usually designated by the symbol  $g_c$ , may be written

$$g_{c_n} = \frac{i_{w_s} \frac{1}{e_s} n w_o}{2} = \frac{a_n}{2} \quad (8)$$

where  $a_n$  = Fourier coefficient of cosine series expansion

$$a_n = \frac{1}{\pi} \int_0^{2\pi} g_m \cos n w_o t \, d(w_o t) \quad (9)$$

Therefore,

$$g_{c_m} = \frac{1}{2\pi} \int_0^{2\pi} g_m \cos n w_o t \, d(w_o t) \quad (10)$$

Thus, this method of analysis requires a Fourier analysis of the transconductance between the signal electrode and the plate curve as a function of time. This can be obtained most easily by graphically projecting the oscillator voltage upon the static transconductance versus oscillator electrode potential curve and applying a harmonic analysis to the resultant  $g_m = f(t)$  curve. In a later paragraph, it is shown that the resultant  $i_p = f(t)$  curve from the harmonic analysis will be valuable in the problem of maximizing the converter output.<sup>4</sup>

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4. See Curves #3 and #4, Appendix V.

## B. METHOD OF EMPIRICAL EXPONENTIAL EQUATION

Once the previously stated approximations are realized, the plate family of the converter tube may be represented as an "n" dimensional surface on which plate current is a function of the "n" electrode potentials. For the case of "n" tube electrodes, the plate current may thus be expressed by the equation

$$i_p = f(e_{g1}, e_{g2}, e_{g3}, e_{g4}, \dots e_n). \quad (11)$$

In the practical case of a multi-grid converter tube, the plate current need be studied only as a function of the variable oscillator and signal electrode voltages, the other electrode potentials being held fixed. Thus

$$i_p = f(e_{go}, e_{gs}) \quad (12)$$

where the voltages  $E_o \sin w_o t$ , and  $E_s \sin w_s t$  are applied to the oscillator and signal grids respectively, in addition to the fixed bias potentials which are applied to various electrodes.

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5. M.J.O. Strutt, "On Conversion Detectors", Proc. I. R. E., Vol. 22, p. 981, August, 1934.

6. M.J.O. Strutt, Moderne Mehrgitter Elektronenrohren, Edward Brothers Inc., Ann Arbor, Michigan, 1943.



The above explicit function of plate current may be expanded into a series of the form

$$i_p = \sum C_n \epsilon^{a_n e_o + b_n e_s} \quad (13)$$

where  $C_n$ ,  $a_n$ , and  $b_n$  are empirical coefficients determined for the particular tube and circuit used. If in the above series, expressions for  $e_o$  and  $e_s$  are substituted, the equation may be further expanded. Substituting

$$e_o = E_{c1} + E_o \sin w_o t \quad (14)$$

and 
$$e_s = E_{c2} + E_s \sin w_s t \quad (15)$$

the equation becomes

$$i_p = \sum C_n \epsilon^{a_n (E_{c1} + E_o \sin w_o t) + b_n (E_{c2} + E_s \sin w_s t)} \quad (16)$$

This equation can now be expanded into an expression for the complete plate current in terms of all its frequency components.<sup>7</sup> Any desired frequency component may be selected from this array, and the conversion transconductance for this frequency computed.

Now proceeding to expand the plate current expression use is made of the following equation.<sup>8, 9, 10</sup>

7. For complete development, see Appendix I.

8. Strutt, op. cit., "On Conversion Detectors".

9. G. N. Watson, Theory of Bessel Functions, p. 14 par. 21, eq. 1, University Press, Cambridge, England, 1944.

10. H. C. Barlett, "Calculation of Modulation Products", Philosophical Mag., Vol. 16, pp. 845-47, (1933).

$$\begin{aligned}
e^{-jz \cos \theta} &= J_0(z) + 2 \sum_{m=1}^{\infty} J_{2m}(z) \cos 2m\theta \\
&+ \frac{2}{j} \sum_{m=0}^{\infty} J_{2m+1}(z) \sin(2m+1)\theta \quad (17)
\end{aligned}$$

from which, if  $z = ja_n$  and  $\theta = w_0 t$  we get

$$\begin{aligned}
e^{a_n E_0 \sin w_0 t} &= I_0(ja_n E_0) + 2 \sum_{m=1}^{\infty} I_{2m}(ja_n E_0) \cos 2m w_0 t \\
&+ \frac{2}{j} \sum_{m=0}^{\infty} I_{2m+1}(ja_n E_0) \sin(2m+1)w_0 t \quad (18)
\end{aligned}$$

which when substituted into the plate current expression gives the expansion

$$\begin{aligned}
i_p = \sum C_n e^{a_n E_{c1}} &\left\{ I_0(ja_n E_0) + 2 \sum_{m=1}^{\infty} I_{2m}(ja_n E_0) \cos 2m w_0 t \right. \\
&+ \left. \frac{2}{j} \sum_{m=0}^{\infty} I_{2m+1}(ja_n E_0) \sin(2m+1)w_0 t \right\} e^{b_n E_{c2}} \\
&\left\{ I_0(jb_n E_s) + 2 \sum_{m=1}^{\infty} I_{2m}(jb_n E_s) \cos 2m w_s t \right. \\
&+ \left. \frac{2}{j} \sum_{m=0}^{\infty} I_{2m+1}(jb_n E_s) \sin(2m+1)w_s t \right\} \quad (19)
\end{aligned}$$

where  $I_m(jx)$  is a complex Bessel function of the first kind,  $m^{\text{th}}$  order, and imaginary argument, as given by the defining series

$$I_0(jx) = 1 + \frac{x^2}{2^2} + \frac{x^4}{2^4(2!)^2} + \frac{x^6}{2^6(3!)^2} + \dots + \frac{x^{2k}}{2^{2k}(k!)^2} \quad (20)$$



$$I_m(jx) = \sum_{k=0}^{\infty} (-1)^k (j)^{m+2k} \frac{x^{2k+m}}{2^{2k+m} k! (k+m)!} \quad (21)$$

From this expression the desired frequency component may be obtained

$$i_{w_o - w_s} = \sum C_n 2 I_1(j a_n E_o) I_1(j b_n E_s) e^{a_n E_{c1} + b_n E_{c2}} \quad (22)$$

Thus, it is obvious that the conversion transconductance is given by

$$g_c = \frac{i_{w_o - w_s}}{E_s} = \frac{2}{E_s} \sum C_n e^{a_n E_{g1} + b_n E_{g2}} I_1(j a_n E_o) I_1(j b_n E_s) \quad (23)$$

## DEDUCTIONS FROM ANALYSIS AS TO OPTIMUM OPERATING CONDITIONS

### A. The Problem of Maximizing the Converter Output

In the multi-grid type of converter the plate resistance of the mixer section is very high. Thus, the conversion transconductance may be used as a figure of merit of the relative gain through the mixer. That is, the plate current at the desired heterodyne frequency may be expressed as a product of the conversion transconductance and the signal voltage.

That is 
$$i_{w_o - w_s} = g_c e_{g_s} \quad (24)$$

The problem of maximizing the converter gain can hence be treated by a study of the effects upon the conversion transconductance of the various electrode bias potentials and of the two radio frequency voltages.

Under normal operating conditions, a multi-grid tube is operated at a plate voltage above the knee of its characteristic curve; in such cases, plate current is essentially independent of plate voltage. This fact is also true of the multi-grid frequency converters. Thus, conversion transconductance is independent of the plate voltage over the normal operating range.



A typical multi-grid converter has five grids in addition to the usual plate and cathode electrodes, although in some cases not all the grids are present. These five grids are normally for inner grid oscillator injection, the order taken from cathode to plate, the oscillator grid, the oscillator anode grid, the control grid, the screen grid, and the suppressor grid.

The effects of each of these grids on the conversion transconductance are considered below.

As in amplifiers, the suppressor grid is normally operated at cathode potential, and hence need not be further considered in the present problem.

The bias applied to both the oscillator anode and the screen grid is usually determined primarily from tube maximum power ratings, and stability of frequency in the case of the oscillator anode grid. Thus, while assuming operation of oscillator anode and screen grids under recommended conditions, the conversion transconductance can be maximized without regard to either of these two grids.

The problem of maximizing the converter output is thus reduced to a study of plate current at the intermediate frequency as a function of the biases and driving voltages of the oscillator and of the signal grids.

As the frequency converter is most commonly used in the reception of modulated waves, it is obvious that it must be operated so as to result in minimum distortion. That is, the signal grid must be operated over a linear region of its characteristic.

Thus, conversion transconductance is independent of signal grid voltage over the linear region of the signal grid transfer characteristic, so that the output voltage varies linearly with the signal voltage as stated by the equation below.

$$\text{Output voltage} = g_c R_L e_{g_s} \text{ if } r_p \text{ is much greater than } R_L \quad (25)$$

As was previously stated, the oscillator driving voltage varies the signal grid to plate transconductance at the oscillator frequency  $W_o$ . If the signal grid bias is adjusted to the value corresponding to a maximum transconductance between the signal and plate, this same value of bias will give a maximum conversion transconductance. By definition, the transconductance between the signal grid and the plate ( $g_{m_{36}}$ ) is the partial derivative of plate current with respect to the electrode voltage; hence,  $g_{m_{36}}$  can be maximized by adjusting the the signal grid bias to such a value that it gives a maximum rate of change of plate current for a fixed increment of signal grid voltage. In practice this of course could be carried out as a direct-current test.



The plate current is again written as an empirical function of signal and oscillator voltages.

$$i_p = \sum C_n e^{a_n(E_{c1}) + b_n(E_{c3})} \quad (26)$$

and 
$$g_{m36} = \frac{\partial i_p}{\partial E_{c3}} = \sum C_n b_n e^{a_n(E_{c1}) + b_n(E_{c3})} \quad (27)$$

$$\frac{\partial g_{m36}}{\partial E_{g3}} = \frac{\partial^2 i_p}{\partial E_{c3}^2} = \sum C_n b_n^2 e^{a_n(E_{c1}) + b_n(E_{c3})} \quad (28)$$

Thus, for a maximum  $g_c$ ,  $E_{c1}$  should be given by the solution of the equation below.

$$\sum C_n b_n^2 e^{a_n(E_{c1}) + b_n(E_{c3})} = 0 \quad (29)$$

This, of course, is the mathematical way of expressing the statement that a maximum conversion transconductance can be obtained when the bias of the signal grid is such that the operating point is at the region of maximum slope of the characteristic families.

There remains now only to consider the effects of oscillator driving voltage and bias of grid one on conversion transconductance. Later it will be shown that for maximum conversion transconductance the oscillator grid must be driven considerably positive, which results in grid current. The voltage drop produced by the flow of this current through the grid return is commonly used as grid leak bias for the oscillator grid. Therefore, oscillator driving voltage and grid bias are interdependent.

The primary purpose of this thesis is to investigate the mixer section. Therefore, the operation of the oscillator must be relegated to a secondary position. Numerous works have appeared in the literature covering oscillators of types suitable for application in a frequency converter. It must be remembered that any consideration of mixer performance as a function of its oscillator grid potentials will be of necessity limited by considerations of the oscillator itself. Some of the considerations are available oscillator output, oscillator voltage, oscillator frequency stability, as well as the allowable maximum ratings of the mixer proper.

Previously it was mentioned that the oscillator driving voltage and oscillator grid bias are interdependent, when grid leak bias is used with a fixed grid leak resistor. This arises from the fact that the bias depends on grid current, which in turn is a function of the driving voltage. That is  $I_{g1} = f(e_{g1})$  and  $E_{g1} = I_{g1} R_{g1}$  (30)

The integral form of the equation<sup>11</sup> expressing conversion transconductance contains information pertinent to our problem.

$$g_{cn} = \frac{1}{2\pi} \int_0^{2\pi} g_{m36} \cos n\omega_0 t \, d(\omega_0 t) \quad (31)$$

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11. E. W. Herold, op. cit., p. 86.



Investigating this relationship carefully, it is seen that over the negative half cycle of oscillator voltage the transconductance between the signal grid and the plate must be zero, if the conversion transconductance is to be a maximum. This restriction means that at least over  $180^\circ$  of the oscillator cycle plate current should be zero. Thus, if a converter is to have maximum conversion transconductance, it must be operated in either class B or C operation. It is also seen from the above equation that the peak positive swing of the grid should be to the maximum allowable value consistent with tube ratings.

The data of curve #1 shows that a grid of a multi-electrode structure has a transconductance which falls off as it is carried into the positive region.<sup>12</sup> It is obvious that as the oscillator grid positive swing is increased, diminishing results are obtained. This fact must be considered along with oscillator limitations and tube ratings in determining the amount of positive oscillator grid swing desirable.

To maximize conversion transconductance with respect to the oscillator grid voltage, the oscillator grid is driven positive the maximum allowable amount and a grid leak bias resistor is then selected so as to give class B or C operation. The exact value of bias desirable is dependent on the available oscillator driving voltage.

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12. For typical curves, see Appendix IV.

In determining the load on the oscillator, it should be noted that the oscillator grid dynamic resistance ( $r_g$ ) must be considered when the grid is operated in the positive region.

The conversion transconductance has been considered so it now remains to treat conversion gain itself. The conversion gain of a mixer stage depends on its load impedance.

$$\text{Conversion Gain} = \frac{g_c R_L}{r_p + R_L} \quad (32)$$

#### B. Other Problems in Optimum Operation of Converters

In addition to available gain, other considerations must be investigated in the use of frequency converters. Those which will be considered in this thesis are random noise, harmonic whistles, image interference, stability, frequency limitations, and current consumption.

Random noise is a major limitation in the use of multi-grid frequency converters, especially in the receiver, where there is a constant demand for a lower minimum receivable signal level.

In another work,<sup>13</sup> it has been shown that the noise ratio centered at  $W_{IF} = W_o - W_s$  measured in the output of a multi-grid converter is given by the equation

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13. M. J. O. Strutt, "Noise in Frequency Mixers", Proc. I. R. E., vol. 34, December, 1946, p. 942.



$$N = 4k T_c Y_o \frac{\Delta f}{g_c^2 E_s^2} \quad (33)$$

where

$k$  = Boltzman's universal gas constant =  $1.38 \times 10^{-23}$

$T_c$  = Cathode temperature

$Y_o$  = Input admittance of converter

$\Delta f$  = Frequency Band Width

$G_c$  = Conversion Transconductance

$E_s$  = Signal Voltage

$N$  = Noise Ratio--the ratio of the noise output power  
to the available signal output power

As shown in the above equation, the operating conditions for minimum random noise are those of maximum conversion transconductance. If some reduction in conversion gain can be tolerated, the noise of a multi-grid converter may be reduced to about that of a triode amplifier by the use of feedback.<sup>14</sup>

The next important consideration in the use of frequency converters is their inherent tendency to produce harmonic whistles, image, and cross modulation interference.<sup>15</sup>

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14. Ibid.

15. M. J. O. Strutt, "On Conversion Detectors", Proc. I. R. E., vol. 22, August, 1934, p. 981.

The oscillator grid of a frequency converter is operated in a non-linear fashion, giving rise to oscillator frequency harmonics. These harmonics may react with undesired incoming signals and produce interfering whistles or beat notes at the intermediate frequency. One means of reducing such interference is by reducing the oscillator harmonic content which requires a reduction in the oscillator grid driving voltage ( a condition incompatible with the condition of maximum conversion transconductance). Such interfering whistles may be also reduced by the use of tuned stages ahead of the converter, thus discriminating against undesired signals. Such tuned circuits preceding the converter in a receiver are called preselector stages.

The use of preselector stages will also reduce image interference. Image response arises from the fact that a converter will respond to a signal differing in frequency from the oscillator by either plus or minus the intermediate frequency. The use of a preselector will hence discriminate against the unwanted signal and result in a reduced image interference.

Cross modulation effects arise when the signal grid is operated in a non-linear fashion. Such operation results in distortion of the modulation envelope. Cross modulation may be reduced to a minimum by the simple expedient of using only small signal voltages (of the order of one volt or less) so as to maintain signal grid linearity. This will,



of course, necessitate the use of added gain in the intermediate frequency amplifiers of a receiver, if a given voltage input to the detector is to be maintained.

Of the remaining considerations, only one is of primary importance in the use of multi-grid converters. This is a frequency pulling effect between the oscillator and signal grids due to the formation of a virtual cathode (space charge) in this region. This results in frequency instability and generally poor operation in the higher frequency bands. The pulling effect may be greatly reduced by using a neutralizing circuit from signal to oscillator grids.<sup>16</sup>

Other considerations on frequency converters primarily apply to the oscillator rather than the mixer, and hence, will not be treated in this thesis.

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16. E. W. Herold, "Operation of Frequency Converters and Mixers", Proc. I. R. E., vol. 30, no. 2, February, 1942, pp. 84-103.

## METHODS OF MEASUREMENT

Inasmuch as the frequency converters of the type being studied in this thesis are usually operated in a non-linear fashion, any attempt to obtain an explicit theoretical answer must in general involve an infinite series, the coefficients of which must be determined experimentally. With the above facts in mind and with heavy weight upon the problem of maximizing the conversion transconductance, this thesis seeks from direct measurements to obtain information which first allows the computation of the value of conversion conductance and secondarily to obtain as much information on the operation of the converter as possible. With this data in hand, it should then be possible to compare the results of analytical deductions with those deductions arising from direct experiment.

In order to carry out the above object, measurements were made on a typical frequency converter, the 6SA7. The converter was operated in a separately excited (that is as a mixer) manner.

Due to the availability of equipment, the experimental work was carried out at audio frequencies. The results, however, are independent of frequency until the frequency reaches the region of 100 megacycles.



## EXPERIMENTAL MEASUREMENTS PERFORMED

In order to enable the actual conversion conductance of a converter to be calculated, certain experimental information must be obtained. In the use of the empirical exponential series method of describing converter operation, the coefficients  $a_n$ ,  $b_n$ , and  $C_n$  of the equation must be determined empirically from a knowledge of the complete plate characteristic family.

The first measurements made were to determine the complete static plate family to enable the above computation.

In order to make use of the variational conductance method of converter analysis, it is necessary to have some means of evaluating the coefficients in the equation  $G_m = f(t)$ . The easiest method of doing this is by obtaining a curve of the transconductance between the signal grid and the plate, as a function of oscillator grid voltage, and by graphically constructing a curve  $G_m = f(t)$  with the known oscillator voltage  $e_o = f(t)$ . Applying to this  $G_m = f(t)$  curve a harmonic analysis, the Fourier coefficients may be determined, and  $G_m = f(t)$  written as a definite Fourier series.

The curve of  $G_m$  as a function of oscillator grid voltage can be obtained from either dynamic measurement of the  $\frac{\partial i_p}{\partial E_{g3}}$  as a function of  $E_{g1}$  bias, where  $E_{g3}$  is the

signal grid voltage and  $E_{g1}$  is the oscillator grid bias of the actual tube under study, or from the static plate family by the use of increments of  $i_p$ . For the remainder of this thesis, numerical subscripts refer to various electrodes of the 6SA7 tube. That is,  $E_{g1}$  and  $E_{g5}$  refer to the grid voltages of grids #1 to #5. The curve  $G_{m3p}$  versus  $C_{g1}$  could be determined from the static curve family by the use of small increments. In this thesis, use was made of the former method in view of obtaining a possibly greater accuracy. The dynamic measurements were performed by application of a small known signal of frequency  $W_s$  to grid #3 and measuring the voltage of frequency  $W_s$  developed in the plate circuit across a known resistor as the grid #1 bias was varied.

To determine the proper oscillator driving voltage, it was necessary to secure a curve of average grid current through a grid leak as a function of the oscillator grid voltage. In obtaining this curve, an oscillator voltage of suitable frequency was applied to the #1 grid through an R C network. The grid leak was selected so as to obtain the correct bias, and the condenser was large enough to have a negligible reactance at the frequency used. In this case  $R_{gL}$  as used was equal to 19,600 ohms.



It must be mentioned that in this method the internal impedance of the oscillator must be sufficiently low as to cause a negligible distortion of the oscillator voltage when the grid is swung positive.

The measurements described above give sufficient data for the indicated calculations. After these calculations were performed, they were checked against directly measured values of conversion transconductance. The conversion transconductance was measured in terms of the voltage of the desired frequency developed across a known non-reactive load with applied oscillator voltage  $E_o \sin W_o t$  and applied signal voltage  $E_s \sin W_s t$ . In order to accurately measure the output voltage at various frequencies, with available equipment, a harmonic wave analyzer was used in the audio frequency band. The use of a harmonic wave analyzer allowed measurement of the voltage of all frequency components present in the output of the converter, thus providing a means of measuring the conversion transconductance at various sum and difference frequencies.

Two other sets of measurements were made in order to check the predicted variation of conversion transconductance first, with the signal grid bias, and second, with oscillator driving voltage.

The variation of conversion transconductance with grid #3 bias was determined by reading the voltage, of difference frequency, developed across the load as the grid #3 bias was varied.

In a similar manner the variation of conversion transconductance with oscillator voltage was determined by measuring the output voltage at the desired difference frequency, as the oscillator driving voltage was varied.



## ACCURACY OF MEASUREMENTS

The accuracy of the various methods of obtaining values for conversion transconductance were obtained from a knowledge of the various experimental errors.

In the method of variational conductance for determination of the conversion transconductance, the expected accuracy was limited by the possible errors in output and input voltage readings and the accuracy to which the load resistance was known. In this thesis, the voltmeter used had a maximum error of  $\pm 2\%$  at full scale, with a possible  $\pm 3\%$  average error. The load resistor was known to an accuracy of  $\pm 1\%$  as measured on a commercial bridge. The errors arising in the graphical construction were about  $\pm 2\%$  and the harmonic analysis errors were about  $\pm 2\%$ . These errors gave a possible error of  $\pm 11\%$  in this method of analysis.

In use of the empirical exponential equation analysis, the computed value suffers errors due principally to the accuracy of the empirical plate family coefficients and due to meter errors in the static plate family characteristics. In this thesis no attempt was made to obtain extreme accuracy in the empirical coefficients, because the primary objective was to justify and explain the method of analysis. The errors of the two methods of analysis were approximately the same. In the case

of the empirical method in this thesis the coefficients were determined with an average error of  $\pm 10\%$ , while the plate family suffered meter errors of  $\pm 2\%$  per reading or a possible total error of  $\pm 6\%$ . Thus the empirical method of analysis on the 6SA7 converter presented in this thesis contains a possible error of  $\pm 16\%$ .

As was previously stated, the direct measurements of conversion transconductance were made in the audio frequency band. The frequencies used were the oscillator frequency, equal to five kilocycles, and the signal frequency, equal to two kilocycles. The actual measurements made of conversion transconductance were subject to an average error of  $\pm 6\%$  in the harmonic wave analyzer, in addition to the input voltage error of  $\pm 2\%$ , and resistance load error of  $\pm 1\%$ ; thus, the actual measured conversion transconductance had a possible error of  $\pm 9\%$ .

The accuracies of the determinations of variations of conversion transconductance with  $E_{g3}$  bias and  $e_{g1}$  driving voltage were respectively  $\pm 11\%$  and  $\pm 12\%$ .



## RESULTS OF MEASUREMENTS

The calculated values of conversion transconductance check with the measured value well within the possible error.<sup>17</sup> In the case of the variational conductance method, the computed value of conversion transconductance was 418 micromhos  $\pm 11\%$ . The empirical equation method yields 358 micromhos  $\pm 16\%$ . The measured value of 362 micromhos  $\pm 9\%$  is in excellent agreement with the latter and within the range of the former calculations.

The point at which a maximum conversion transconductance occurred was computed and found to be at a bias  $E_{g3}$  of -3.2 volts  $\pm 16\%$ , and the measured value was found to be -3.7 volts  $\pm 12\%$ .

The results of these measurements and computations substantiate the correctness of the methods of analysis as presented herein.

A significant and convenient fact is that a maximum conversion transconductance occurred when the grid #3 bias is such as to give a maximum slope of the static transfer characteristic.<sup>18</sup> The data also showed that the conversion transconductance increased at a

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17. See Appendix IIc.

18. See Appendices III & IV.

decreasing rate as #1 oscillator driving voltage was increased; that is, it approached a region of saturation.<sup>19</sup> Also it should be noted that the amplitudes of the cross modulation components with the applied signal voltage are negligible.<sup>20</sup>

In the case of the 6SA7 converter tube, operated in the manner described in this thesis, the operation may be described by either of the following equations.<sup>21</sup>

$$\text{When } e_{g1} = 14.14 \cos W_0 t \quad e_{g3} = E_{g3m} \sin W_s t$$

$$R_{gL} = 19,600 \text{ ohms}$$

$$i_p = 10^{-6} \times (495 + 836 \cos W_0 t + 490 \cos 2W_0 t + 96.2 \cos 3W_0 t + 44.2 \cos 4W_0 t + 40.2 \cos 5W_0 t + 62.4 \cos 6W_0 t) E_{g3m} \sin W_s t \quad (34)$$

or

$$i_p = 11.0 \times 10^{-3} e^{0.174 e_{g1} + 0.144 e_{g3}} - 1.72 \times 10^{-3} e^{0.120 e_{g1} + 0.80 e_{g3}} \quad (35)$$

where  $e_{g1} = E_{g1} + E_{g1m} \sin W_0 t$ ,  $e_{g3} = E_{g3} + E_{g3m} \sin W_s t$ .

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19. Ibid.

20. See Appendix III.

21. See Appendix II.



## RECOMMENDATIONS AS TO OPERATION

In the use of multi-grid frequency converters, the plate voltage should be above the knee of its characteristic curve. That is, in the region where all currents are independent of the plate voltage. The screen voltage should be at about the potential recommended for the converter tube used.

In order to obtain maximum conversion transconductance, the signal grid bias should be adjusted to the point which will give a maximum  $G_{m36}$ . The oscillator driving voltage should be adjusted to a maximum magnitude consistent with tube ratings. This driving voltage should produce an oscillator bias such that the oscillator grid operates in either class B or C.

If automatic volume control bias is to be used in a receiver incorporating a frequency converter, it is desirable that the converter be operated at a fixed bias, the AVC bias being applied to the IF and RF amplifiers.

In order to reduce the distortion introduced in the converter and yet have sufficient selectivity ahead of the converter to minimize image and harmonic interference, a tuned RF amplifier should be used ahead of the converter input. The output of the RF amplifier should be small enough to operate the converter over a linear region of the signal grid under maximum input signal strength. This will insure a minimum of cross modulation effects.

The converter should be operated into a tuned resonant load at the intermediate frequency, so as to obtain a maximum gain throughout the converter stage. This load should still have a sufficient band width to pass the modulation envelope without frequency discrimination. In some cases a relatively complex network is required to meet these objectives.



## SUMMARY

The operation of a frequency converter may be completely described from a knowledge of the complete static characteristics of the converter tube. The output may be calculated by either of the two methods presented in this thesis. Accuracies obtainable by the two methods are comparable; the method of an empirical equation possesses the advantage of giving a more complete final result. The method of an empirical equation has a serious disadvantage in the amount of work necessary to obtain the empirical coefficients.

The conditions for optimum operation of a multi-grid frequency converter follow. A value of signal grid bias which gives the maximum slope of the static transfer characteristic should be employed. The class of operation should be B or C. The oscillator grid should be driven positive the maximum amount allowed by the tube ratings. The signal voltage should be small enough to allow operation over the linear portion of the signal grid transfer characteristic. Other tube electrode potentials should be as recommended by the tube manufacturer.

The above recommendations in general apply to all converters which use an inner grid for the oscillator and an outer grid for the signal. With slight modifications the recommendations also apply to other vacuum-tube converters.

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## APPENDIX I

## Derivations

Empirical Exponential Method of Converter Analysis 22

Let the static plate current be represented by the equation

$$i = \sum C_n e^{a_n e_o + b_n e_s} \quad (36)$$

where  $C_n$ ,  $a_n$ , and  $b_n$  are empirically evaluated coefficients.  $e_o$  = the oscillator grid voltage, and  $e_s$  = the signal grid voltage. If one substitutes the following

$$e_o = E_1 + E_o \sin W_o t \quad e_s = E_3 + E_s \sin W_s t$$

in the equation (36), he obtains

$$i = \sum C_n e^{a_n(E_1 + E_o \sin W_o t) + b_n(E_3 + E_s \sin W_s t)} \quad (37)$$

which may be written as

$$i = \sum C_n e^{a_n E_1 + b_n E_3} e^{a_n E_o \sin W_o t} e^{b_n E_s \sin W_s t} \quad (38)$$

A useful relationship, equation (47), may be derived from

$$e^{\frac{1}{2}Z(t - \frac{1}{t})} = \sum_{m=-\infty}^{\infty} t^m J_m(Z) \quad (39)$$

$$\text{and } J_{-m}(Z) = (-1)^m J_m(Z) \text{ where "m" is any integer} \quad (40)$$

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22. G. N. Watson, Theory of Bessel Functions, p.14, par. 2.1, eq. 1, University Press, Cambridge, England.



thus

$$e^{\frac{Z}{2}(t - \frac{1}{t})} = J_0(Z) + \sum_{m=1}^{\infty} \left\{ t^m + (-1)^m t^{-m} \right\} J_m(Z) \quad (41)$$

substituting  $t = -e^{i\theta}$  in the above

$$e^{-\frac{Z}{2}(e^{i\theta} - e^{-i\theta})} = J_0(Z) + \sum_{m=1}^{\infty} \left\{ e^{im\theta} + (-1)^m e^{-im\theta} \right\} (-1)^m J_m(Z) \quad (42)$$

$$\text{if "m" is even } e^{2mi\theta} + e^{-2mi\theta} = 2\cos 2m\theta \quad (43)$$

and if "m" is odd

$$e^{(2m+1)i\theta} - e^{-(2m+1)i\theta} = 2i[\sin(2m+1)\theta] \quad (44)$$

thus

$$e^{-iZ\sin\theta} = J_0(Z) + 2 \sum_{m=1}^{\infty} J_{2m}(Z) \cos 2m\theta - 2i \sum_{m=0}^{\infty} J_{2m+1}(Z) \sin(2m+1)\theta \quad (45)$$

$$\text{letting } Z = iA_n E_0, \theta = W_0 t, \text{ and } I_m(ix) = (i)^{-m} J_m(ix) \quad (46)$$

$$\begin{aligned} e^{-i(iA_n E_0)\sin W_0 t} &= I_0(A_n E_0) + 2 \sum_{m=1}^{\infty} I_{2m}(A_n E_0) \cos 2mW_0 t \\ &\quad + 2 \sum_{m=0}^{\infty} I_{2m+1}(A_n E_0) \sin(2m+1)W_0 t. \end{aligned} \quad (47)$$

Therefore equation 38 may be written as below by using equation 47.

$$\begin{aligned} i = \sum C_n e^{A_n E_1 + b_n E_3} &\left[ I_0(A_n E_0) + 2 \sum_{m=1}^{\infty} I_{2m}(A_n E_0) \cos 2mW_0 t \right. \\ &\quad \left. + 2 \sum_{m=0}^{\infty} I_{2m+1}(A_n E_0) \sin(2m+1)W_0 t \right] \end{aligned}$$

$$\left[ I_0(b_n E_s) + 2 \sum_{m=1}^{\infty} I_{2m}(b_n E_s) \cos 2m W_s t + \frac{2}{j} \sum_{m=0}^{\infty} I_{2m+1}(b_n E_s) \sin(2m+1) W_s t \right] \quad (48)$$

or

$$\begin{aligned} i = \sum C_n e^{a_n E_1 + b_n E_s} & \left\{ I_0(a_n E_0) I_0(b_n E_s) \right. \\ & + 2 I_0(b_n E_s) \sum_{m=1}^{\infty} I_{2m}(a_n E_0) \cos 2m W_0 t \\ & \quad + \frac{2}{j} I_0(b_n E_s) \sum_{m=0}^{\infty} I_{2m+1}(a_n E_0) \sin(2m+1) W_0 t \\ & + 2 I_0(a_n E_0) \sum_{m=1}^{\infty} I_{2m}(b_n E_s) \cos 2m W_s t \\ & \quad + 4 \sum_{m=1}^{\infty} I_{2m}(b_n E_s) \cos 2m W_s t \sum_{m=1}^{\infty} I_{2m}(a_n E_0) \cos 2m W_0 t \\ & \quad + \frac{4}{j} \sum_{m=0}^{\infty} I_{2m+1}(a_n E_0) \sin(2m+1) W_0 t \sum_{m=1}^{\infty} I_{2m}(b_n E_s) \cos 2m W_s t \\ & \quad + \frac{2}{j} I_0(a_n E_0) \sum_{m=0}^{\infty} I_{2m+1}(b_n E_s) \sin(2m+1) W_s t \\ & \quad + \frac{4}{j} \sum_{m=1}^{\infty} I_{2m}(a_n E_0) \cos 2m W_0 t \sum_{m=0}^{\infty} I_{2m+1}(b_n E_s) \sin(2m+1) W_s t \\ & \quad \left. - 4 \sum_{m=0}^{\infty} I_{2m+1}(a_n E_0) \sin(2m+1) W_0 t \sum_{m=0}^{\infty} I_{2m+1}(b_n E_s) \sin(2m+1) W_s t \right\}. \end{aligned} \quad (49)$$



The sum and difference frequency components of the plate current must arise from the terms

$$\frac{4}{j} \sum_{m=0}^{\infty} I_{2m+1}(a_n E_o) \sin(2m+1)W_o t \sum_{m=1}^{\infty} I_{2m}(b_n E_s) \cos 2mW_s t \quad (50)$$

$$\text{or } \frac{4}{j} \sum_{m=1}^{\infty} I_{2m}(a_n E_o) \cos 2mW_o t \sum_{m=0}^{\infty} I_{2m+1}(b_n E_s) \sin(2m+1)W_s t \quad (51)$$

$$\text{or } -4 \sum_{m=0}^{\infty} I_{2m+1}(a_n E_o) \sin(2m+1)W_o t \sum_{m=0}^{\infty} I_{2m+1}(b_n E_s) \sin(2m+1)W_s t. \quad (52)$$

This thesis is principally concerned with the sum and difference frequencies  $W_o \pm W_s$  as given by

$$i_{i.f.} = \sum C_n e^{a_n E_1 + b_n E_3} \left[ \frac{4}{j} I_1(b_n E_o) I_1(b_n E_s) \sin W_o t \sin W_s t \right] \quad (53)$$

$$\text{or } i_{i.f.} = 2 \sum C_n e^{a_n E_1 + b_n E_3} I_1(a_n E_o) I_1(b_n E_s) \cos(W_o \pm W_s t). \quad (54)$$

Equation 54 gives simply the desired result

$$G_c = \frac{i_{i.f.}}{E_s} = \frac{2}{E_s} \sum C_n e^{a_n E_1 + b_n E_3} I_1(a_n E_o) I_1(b_n E_s). \quad (55)$$

Note in the equation 55  $I_1$  is a Bessel's function as defined below, if "n" is an integer equal to or greater than zero.

$$I_m(jx) = \sum_{k=0}^{\infty} (-1)^k (j)^{n+2k} \frac{x^{n+2k}}{2^{n+2k} k! (n+k)!} \quad (56)$$

## APPENDIX II

## Calculations

## A. Empirical Coefficients of the Exponential Equation for a 6SA7 Tube.

The basic equation is

$$i_p = \sum C_n e^{a_n E_1 + b_n E_3}. \quad (57)$$

Since 
$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x}{n!}$$

the coefficients  $C_1$ ,  $C_2$ ,  $a_1$ ,  $a_2$ ,  $b_1$ , and  $b_2$  may be evaluated by choosing points of the characteristics, thus obtaining simultaneous equations from which the coefficients may be found.

At the point  $E_{g1} = 0$ ,  $E_{g3} = -4$ ,  $i_p = 6.40 \text{ m.a.}$ , the equation becomes

$$0.0064 = C_1 \left[ 1 - 4b_1 + 8b_1^2 - 10.67b_1^3 \right] + C_2 \left[ 1 - 4b_2 + 8b_2^2 - 10.67b_2^3 \right]. \quad (58)$$

At the point  $E_{g1} = 0$ ,  $E_{g3} = -10$ ,  $i_p = 1.90 \text{ m.a.}$ , the equation becomes

$$0.0019 = C_1 \left[ 1 - 10b_1 + 50b_1^2 + \frac{1000b_1^3}{6} \right] + C_2 \left[ 1 - 10b_2 + 50b_2^2 + \frac{1000b_2^3}{6} \right]. \quad (59)$$



A solution of the simultaneous equations 58 and 59 is the approximate equation

$$(1-10b_1+50b_1^2-167b_1^3)\frac{C_1}{1.90} = \frac{C_1}{6.40} (1-4b_1+8b_1^2-10.67b_1^3) \quad (60)$$

$$\text{or } b_1^3 - 0.291b_1^2 + 0.0539b_1 - 0.00430 = 0. \quad (61)$$

The solution of this equation yields  $b_1 \doteq 0.144$ .

At the point  $E_{g3} = -1$ ,  $E_{g1} = 0$ ,  $i_p = 9.10$  m.a., the equation becomes

$$0.00910 = C_1 [1 - b_1 + b_1^2 - b_1^3 + \dots] + C_2 [1 - b_2 + b_2^2 - b_2^3 + \dots] \quad (62)$$

At the point  $E_{g1} = 0$ ,  $E_{g3} = -4$ ,  $i_p = 6.40$  m.a., the equation becomes

$$0.0064 = C_1 [1 - 4b_1 + 8b_1^2 - 10.67b_1^3] + C_2 [1 - 4b_2 + 8b_2^2 - 10.67b_2^3] \quad (63)$$

A solution of the simultaneous equations 62 and 63 is the approximate equation

$$b_2^3 - 0.732b_2^2 + 0.3305b_2 - 0.02975 = 0. \quad (64)$$

The solution of this equation yields  $b_2 \doteq 0.8$ .

At the point  $E_{g1} = 0$ ,  $E_{g3} = 0$ ,  $i_p = 9.3$  m.a., the equation (57) becomes

$$0.0093 = C_1 + C_2. \quad (65)$$

At the point  $E_{g1} = 0$ ,  $E_{g3} = -4$ ,  $i_p = 6.4$  m. a., the equation (57) becomes

$$0.0064 = 0.551C_1 + 0.0409C_2. \quad (66)$$

The equations (65) and (66) when solved simultaneously, give

$$C_1 = 0.011, \quad C_2 = -0.00172.$$

At the point  $E_{g1} = -2$ ,  $E_{g3} = 0$ ,  $i_p = 6.83$  m. a., the equation becomes

$$0.00683 = C_1 [1 - 2a_1 + 2a_1^2 - 1.33a_1^3] + C_2 [1 - 2a_2 + 2a_2^2 - 1.33a_2^3]. \quad (67)$$

At the point  $E_{g3} = 0$ ,  $E_{g1} = -9$ ,  $i_p = 0.43$  m.a., the equation becomes

$$0.00043 = C_1 [1 - 9a_1 + 40.5a_1^2 - 121.5a_1^3] + C_2 [1 - 9a_2 + 40.5a_2^2 - 121.5a_2^3]. \quad (68)$$

The equations (67) and (68) when solved simultaneously, give

$$\frac{1}{6.83} [1 - 2a_1 + 2a_1^2 - 1.33a_1^3] = \frac{1}{0.43} [1 - 9a_1 + 40.5a_1^2 - 121.5a_1^3] \quad (69)$$

$$\text{or } a_1^3 - 0.332a_1^2 + 0.0730a_1 - 0.00770 = 0. \quad (70)$$

The solution of equation 70 yields  $a_1 = 0.174$ .



Since  $b_1=0.144$  and  $b_2=0.8$ , at the point  $E_{g3}=-2.45$ ,  $E_{g1}=4$ ,  $i_p=9.8$  m.a., the equation(57) becomes

$$0.00980 \doteq C_1 \left[ 1 + 4a_1 + 8a_1^2 + 10.67a_1^3 \right] e^{-0.352} \\ + C_2 \left[ 1 + 4a_2 + 8a_2^2 + 10.67a_2^3 \right] e^{-1.96}. \quad (71)$$

The simultaneous solution of equations (67) and (71) gives

$$\frac{1}{6.83} \left[ 1 - 2a_2 + 2a_2^2 - 1.333a_2^3 \right] \doteq \frac{0.702}{9.80} \left[ 1 + 4a_2 + 8a_2^2 + 10.67a_2^3 \right] \quad (72)$$

$$\text{or} \quad 6.54a_2^3 + 1.91a_2^2 + 3.955a_2 - 0.511 \doteq 0. \quad (73)$$

The solution of equation(73) yields  $a_2 \doteq 0.120$ .

Thus, the static plate family of the 6SA7 tube may be represented by the approximate equation

$$i_p = 0.011 e^{0.174E_{g1} + 0.144E_{g3} - 0.00172 e^{0.120E_{g1} + 0.8E_{g3}}}. \quad (74)$$

The method as used above in obtaining the coefficients  $a_n$ ,  $b_n$ , and  $C_n$  is a modification of the method of undetermined coefficients from the classical series theory.

Other methods applicable to the determination of these coefficients are graphical analyses,<sup>23</sup> and successive approximations. These latter methods were not used because of the greater effort involved in their use.

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23. J. Lipka, Graphical and Mechanical Computation, John Wiley & Sons, New York, N.Y., 1921, p. 156.

# B. Harmonic Analysis of $G_m = f(t)$ <sup>24</sup> and Calculation of $G_c$

The  $G_m = f(t)$  equation may be expanded into a Fourier series of cosine terms, if the time  $t = 0$  is chosen at the point where  $G_m$  is a maximum.

It is found that for a 6SA7 tube using  $e_{g1} = 14.14 \sin W_o t$ ,  $E_{c1} = -10.0$ ,  $E_{c3} = -2.45$ , the Fourier series of  $G_m$  is

$$G_m = f(t) \doteq 10^{-6} \left[ 495 + 836 \cos W_o t + 490 \cos 2W_o t + 96.2 \cos 3W_o t + 44.2 \cos 4W_o t + 40.2 \cos 5W_o t + 62.4 \cos 6W_o t \right] \quad (75)$$

When  $e_{g3}$  is small, the plate current is given by

$$i_p = G_m E_s \cos W_s t$$

$$\doteq E_s \times 10^{-6} \cos W_s t \left[ 495 + 836 \cos W_o t + 490 \cos 2W_o t + 96.2 \cos 3W_o t + 44.2 \cos 4W_o t + 40.2 \cos 5W_o t + 62.4 \cos 6W_o t \right] \quad (76)$$

thus

$$i_p = E_s \times 10^{-6} \left\{ 495 \cos W_s t + 418 \left[ \cos (W_o + W_s) t - \cos (W_o - W_s) t \right] + 245 \left[ \cos (2W_o + W_s) t - \cos (2W_o - W_s) t \right] + 48.1 \left[ \cos (3W_o + W_s) t - \cos (3W_o - W_s) t \right] + 22.1 \left[ \cos (4W_o + W_s) t - \cos (4W_o - W_s) t \right] + 20.1 \left[ \cos (5W_o + W_s) t - \cos (5W_o - W_s) t \right] + 32.1 \left[ \cos (6W_o + W_s) t - \cos (6W_o - W_s) t \right] \right\} \quad (77)$$

---

24. For values, see Table VII.



The preceding equations neglect intermodulation terms.

The equation(77) yields

$$G_c = \frac{i_{W_o-W_s}}{E_s} = 418 \text{ micromhos.} \quad (78)$$

C. Calculation of  $G_c$  from  $i_p = \sum C_n e^{a_n E_{g1} + b_n E_{g3}}$

It has been shown that

$$G_c = \frac{2}{E_s} \sum C_n e^{a_n E_{g1} + b_n E_{g3}} I_1(a_n E_o) I_1(b_n E_s). \quad (79)$$

For a particular 6SA7 tube operated with  $e_s = 1.414 \sin W_s t$ ,  $e_o = 14.14 \sin W_o t$ ,  $E_{c1} = -10.0$ , and  $E_{c3} = -3.0$ , the conversion transconductance is given by<sup>25</sup>

$$G_c = \frac{2}{1.414} \left[ .0110 e^{-0.174 \times 10.0 - 0.144 \times 3.00} I_1(0.174 \times 14.14) I_1(0.144 \times 1.414) - .00172 e^{-0.120 \times 10.0 - .8 \times 3.00} I_1(0.120 \times 14.14) I_1(0.8 \times 1.414) \right]. \quad (80)$$

Therefore

$$G_c = 358 \text{ micromhos} \pm 16\%.$$

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25. For tables of  $I_m(x)$ , see Watson, G. N., Theory of Bessel Functions, University Press, Cambridge, England, 1944.

or

Jahnke, E., Emde, F., Tables of Functions, John Wiley & Sons, New York, N. Y., 1938.



#### D. Value of Grid #3 Bias for a Maximum $G_c$ .

If  $G_c$  is to be a maximum,  $G_m$  must be a maximum at the operating point. Thus, the  $\frac{\partial i_p}{\partial E_{c3}}$  must be a maximum at the point of maximum  $G_c$ .

Therefore

$$\frac{G_{m36}}{E_{c3}} = \frac{\partial^2 i_p}{\partial E_{c3}^2} = \sum C_n a_n^2 \epsilon^{a_n(E_{c3} + e_s) + b_n(E_{c1} + e_o)} = 0. \quad (81)$$

When one solves the above equation for  $E_{c3}$ , one obtains the value of grid #3 bias for a maximum value of  $G_c$ .

When the coefficients  $C_1$ ,  $C_2$ ,  $a_1$ ,  $a_2$ ,  $b_1$ , and  $b_2$ , as found in Appendix II A, are substituted, the equation (81) is

$$11.0(0.144) \epsilon^{2 \cdot 0.174 E_{c1} + 0.144 E_{c3}} - 1.72(0.8) \epsilon^{2 \cdot 0.120 E_{c1} + 0.8 E_{c3}} = 0 \quad (82)$$

or

$$\epsilon^{0.656 E_{c3}} = \frac{11.0(0.144)^2}{1.72(0.64)} \epsilon^{0.054 E_{c1}}. \quad (83)$$

Therefore, the value of grid #3 bias for maximum  $G_c$  is given by the equation

$$E_{c3} = -\frac{1}{0.656} \ln(1.56 - 0.054 E_{c1}). \quad (84)$$

Thus, for a grid #1 bias of -10 volts, the maximum value of  $G_c$  obtainable by varying the grid #3 bias occurs when

$$E_{c3} = -3.2 \pm 16\% \text{ volts}. \quad (85)$$

# APPENDIX III

## Tables of Data

TABLE I: Static Plate Characteristics for a 6SA7 Tube  
 $E_p=250$ ,  $E_{g2}=105$ ,  $E_{g5}=0$

$E_{g1}$	$E_{g3}$	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13
<u><math>I_p</math> in M. A.</u>															
+4		14.0	12.6	11.2											
+3		13.3	11.4	10.9											
+2		12.0	10.4	9.5											
+1		10.9	9.39	8.6											
0		9.30	8.4	7.7	6.88	5.99	5.08	4.15	3.4	2.7	2.18	1.81	1.50	1.30	1.17
-1		7.90	7.2	6.5	5.78	5.13	4.35	3.61	3.0	2.39	1.91	1.60	1.30	1.13	1.00
-2		6.52	5.92	5.47	4.85	4.27	3.70	3.10	2.52	2.05	1.63	1.36	1.10	0.95	0.85
-3		5.20	4.80	4.35	3.90	3.49	3.03	2.53	2.11	1.72	1.38	1.12	0.91	0.78	0.70
-4		4.11	3.78	3.40	3.10	2.78	2.40	2.00	1.68	1.38	1.09	0.86	0.71	0.60	0.54
-5		3.00	2.70	2.49	2.25	2.01	1.70	1.45	1.20	0.98	0.78	0.63	0.51	0.44	0.39
-6		2.08	1.84	1.65	1.48	1.35	1.13	0.95	0.80	0.64	0.51	0.40	0.33	0.28	0.25
-7		1.32	1.10	1.05	0.91	0.80	0.68	0.55	0.45	0.37	0.29	0.24	0.19	0.17	0.14
-8		0.67	0.61	0.55	0.48	0.40	0.33	0.27	0.23	0.18	0.14	0.11	0.09	0.08	0.07
-9		0.34	0.28	0.25	0.20	0.18	0.14	0.11	0.09	0.08	0.06	0.05			
-10		0.13	0.10	0.08	0.07	0.06	0.05	0.04	0.03	0.03	0.02				



TABLE II: Grid #1 Rectification Data

$$E_{g2} = 105, \quad E_p = 250, \quad E_{g3} = -3, \quad E_{g5} = 0$$

$$R_{g1} = 19,600 \text{ ohms}, \quad C_c = 0.5 \text{ microfarad}, \quad f = 5 \text{ k.c.}$$


---

$e_{g1}$ <u>Effective Volts</u>	$I_{g1}$ <u>Direct Current in M. A.</u>
0	0.017
1.0	0.057
2.0	0.110
3.0	0.163
4.0	0.217
5.0	0.269
6.0	0.311
7.0	0.368
8.0	0.411
9.0	0.461
9.8	0.500
11.0	0.558
12.0	0.611
13.0	0.667
14.4	0.732

---

TABLE III: 6SA7 Grid #3 Transconductance as  
a Function of Grid #1 Bias Voltage

$E_p = 250, E_{g2} = 105, E_{g3} = -3, E_{g5} = 0$   
-----

$E_{g1}$	$G_{m3p}$
<u>D. C. Volts</u>	<u>Micromhos</u>
4.5	1840
4.0	1718
3.5	1590
3.0	1500
2.5	1378
2.0	1275
1.5	1173
1.0	1110
0.5	1000
0.0	919
-0.5	834
-1.0	754
-1.5	671
-2.0	598
-2.5	530
-3.0	467
-4.0	306
-5.0	274
-6.0	190
-7.0	126
-8.0	76
-9.0	39
-10.0	16

-----



TABLE IV: Measured Output Voltages  
Across a Resistance Load

$R_L = 4910$  ohms,  $R_{g_L} = 19,600$  ohms,  $C_c = 0.5$  microfarad  
 $E_{g_3} = -3$  volts,  $e_{g_1} = 10.0$  volts eff.,  $I_{g_1} = 0.50$  m. a.  
 $E_{g_2} = 105$  volts,  $E_p = 250$  volts,  $f_o = 5$  k.c.,  $f_s = 2$  k.c.  
 $e_{g_3} = 1.0$  volts eff.

Effective Voltage Across $R_L$					
$e_{f_o-f_s}$	$e_{f_o}$	$e_{f_s}$	$e_{f_o+f_s}$	$e_{2f_o}$	$e_{2f_s}$
3kc.	5kc.	2kc.	7kc.	10kc.	4kc.
1.77	16.5	2.08	1.77	8.63	0.041
$e_{2f_o+f_s}$	$e_{2f_o-f_s}$	$e_{f_o-2f_s}$	$e_{f_o+2f_s}$	$e_{2f_o-2f_s}$	$e_{2f_o+2f_s}$
12kc.	8kc.	1kc.	9kc.	6kc.	14kc.
0.880	0.880	0.040	0.040	0.030	0.030

From the above voltage data, it is seen that intermodulation effects are small.

At the difference frequency of 3kc., the measured value of  $G_c$  is readily found from the known voltages.

$$G_{cf_o-f_s} = \frac{e_{f_o-f_s}}{R_L e_{g_3}} = \frac{1.77}{4910 \times 1.0} = 362 \text{ micromhos}$$

TABLE V: Conversion Transconductance as a  
Function of Grid #3 Bias Voltage

$e_{g3} = 1.0$  volts eff.,  $I_{g1} = 0.5$  m.a.,  $E_p = 250$  volts

$E_{g2} = 105$  volts,  $R_{gL} = 19,600$  ohms,  $f_o = 5$ kc.,  $f_s = 2$ kc.

---

$E_{g3}$ <u>D. C. Volts</u>	$G_c$ <u>Micromhos</u>
0.0	282
-0.5	304
-1.0	322
-1.5	337
-2.0	343
-3.0	357
-4.0	360
-5.0	347
-6.0	310
-7.0	259
-8.0	204
-9.0	151
-10.0	107
-11.0	81
-12.0	56
-13.0	43
-14.0	35
-15.0	29

---



TABLE VI: Conversion Transconductance as a  
Function of Grid #1 Driving Voltage

$E_p = 250$ ,  $E_{g2} = 105$ ,  $E_{g3} = -3.0$ ,  $e_{g3} = 1.0$  eff. volts

$R_{g1} = 19,600$  ohms,  $f_o = 5\text{kc.}$ ,  $f_s = 2\text{kc.}$

---

$I_g$ <u>Direct Current</u> <u>in M. A.</u>	$e_g$ <u>Eff. Volts</u>	$G_c$ <u>Micromhos</u>
0.013	0	0
0.053	1.0	101
0.080	1.5	149
0.100	2.0	175
0.151	3.0	233
0.202	4.0	267
0.252	5.0	286
0.301	6.0	300
0.351	7.0	308
0.398	8.0	324
0.440	9.0	335
0.500	10.0	349
0.547	11.0	361
0.600	12.0	375
0.641	13.0	388
0.703	14.0	400
0.759	15.0	412
0.820	16.0	426
0.870	17.0	435
0.921	18.0	443
0.971	19.0	451
1.02	20.0	460
1.07	21.0	464
1.25	25.0	482

---

TABLE VII: Harmonic Analysis of  $G_m = f(t)$  Curve

$E_{g3} = -3$  volts,  $e_{g1} = 10.0$  eff. volts,  $I_{g1} = 0.5$  m.a.

$E_p = 250$  volts,  $E_{g2} = 105$  volts,  $R_{gL} = 19,600$  ohms

t	$G_m$	$G_m \cos \theta$	$G_m \cos 2\theta$	$G_m \cos 3\theta$	$G_m \cos 4\theta$	$G_m \cos 5\theta$	$G_m \cos 6\theta$
0°	1840	1840	1840	1840	1840	1840	1840
5°	1785	1780	1759	1725	1679	1620	1548
10°	1748	1720	1642	1513	1340	1122	874
15°	1696	1638	1469	1199	848	439	0
20°	1605	1510	1230	803	278	-278	-803
25°	1498	1357	963	388	-260	-859	-1299
30°	1370	1187	785	0	-685	-1186	-1370
35°	1236	1011	423	-320	-946	-1230	-1070
40°	1000	766	174	-500	-940	-940	-500
45°	945	668	0	-669	-945	-669	0
50°	800	514	-139	-694	-752	-274	400
55°	650	372	-222	-629	-498	-57	564
60°	505	203	-203	-505	-303	303	605
65°	388	164	-250	-375	-67	328	336
70°	275	94	-211	-238	48	271	138
75°	180	47	-156	-127	90	174	0
80°	107	19	-101	-54	82	82	-53
85°	58	5	-57	-15	56	25	-50
90°	35	0	-34	0	35	0	-35
95°	4	0	-4	1	4	-1	-3
100°	0	0	0	0	0	0	0
105°	0	0	0	0	0	0	0
110°	0	0	0	0	0	0	0
115°	0	0	0	0	0	0	0
Total	17835	15046	8807	1730	796	723	1122
Av.	495	418	245	48.1	22.1	20.1	31.2

Thus, the equation  $G_m = f(t)$  is

$$G_m = 495 + 836 \cos W_0 t + 490 \cos 2W_0 t + 96.2 \cos 3W_0 t \\ + 44.2 \cos 4W_0 t + 40.2 \cos 5W_0 t + 62.4 \cos 6W_0 t$$

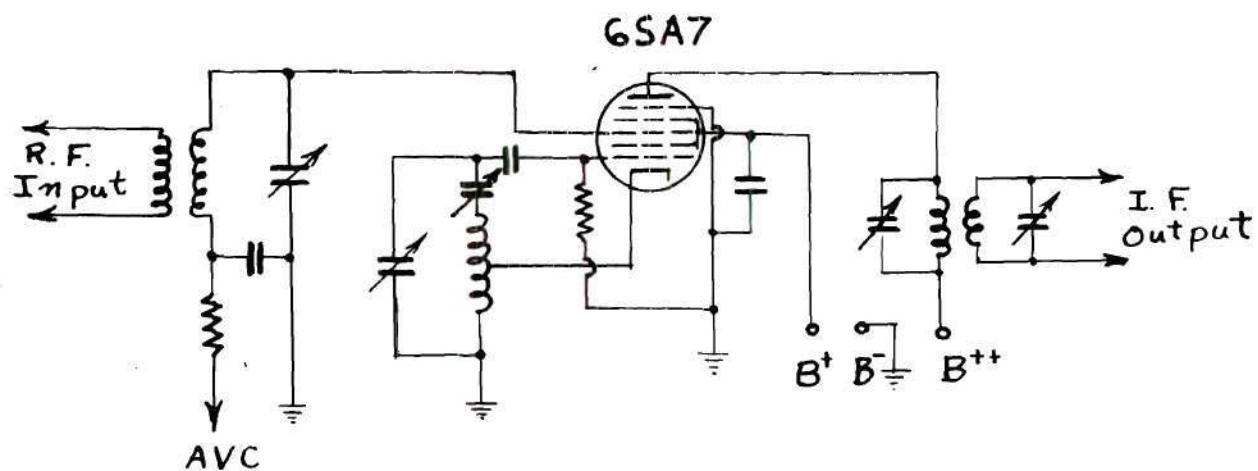
- where  $G_m$  is given in micromhos.



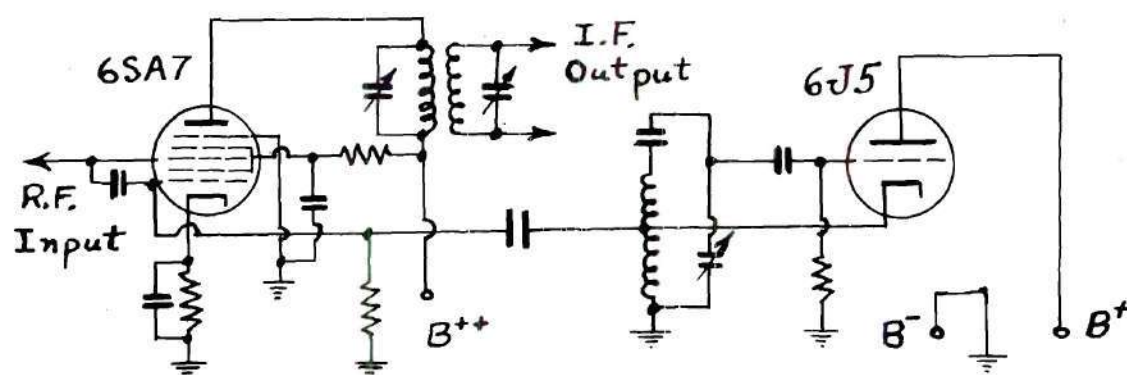
## APPENDIX IV

## Diagrams

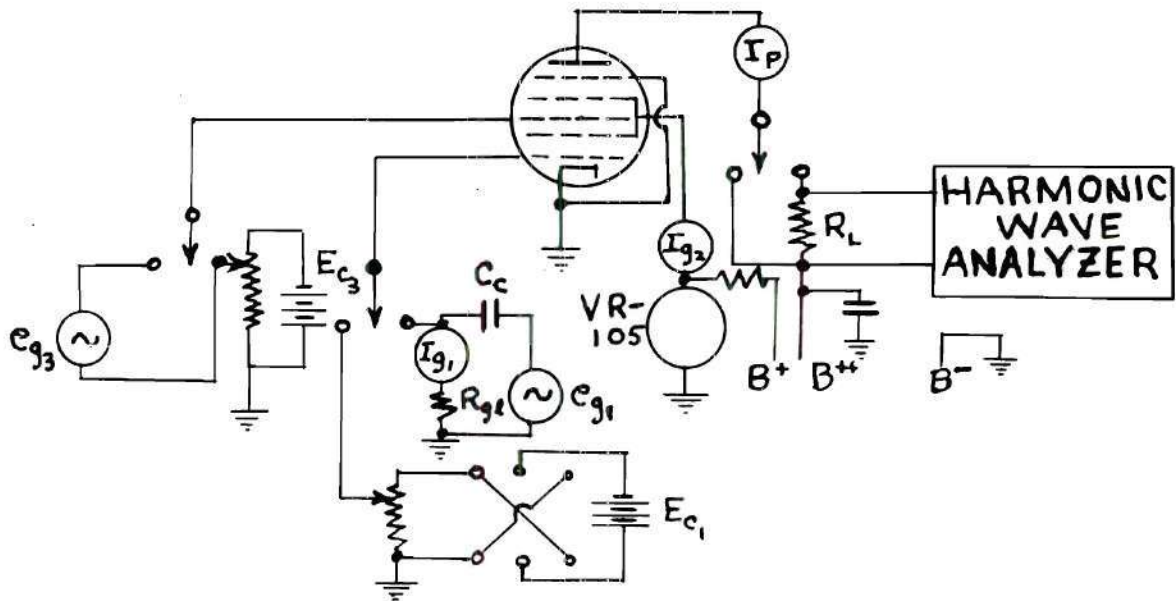
1. Typical 6SA7 Converter Circuits  
 (a) Self-Excited



(b) Separately Excited



## 2. Experimental Data Circuit



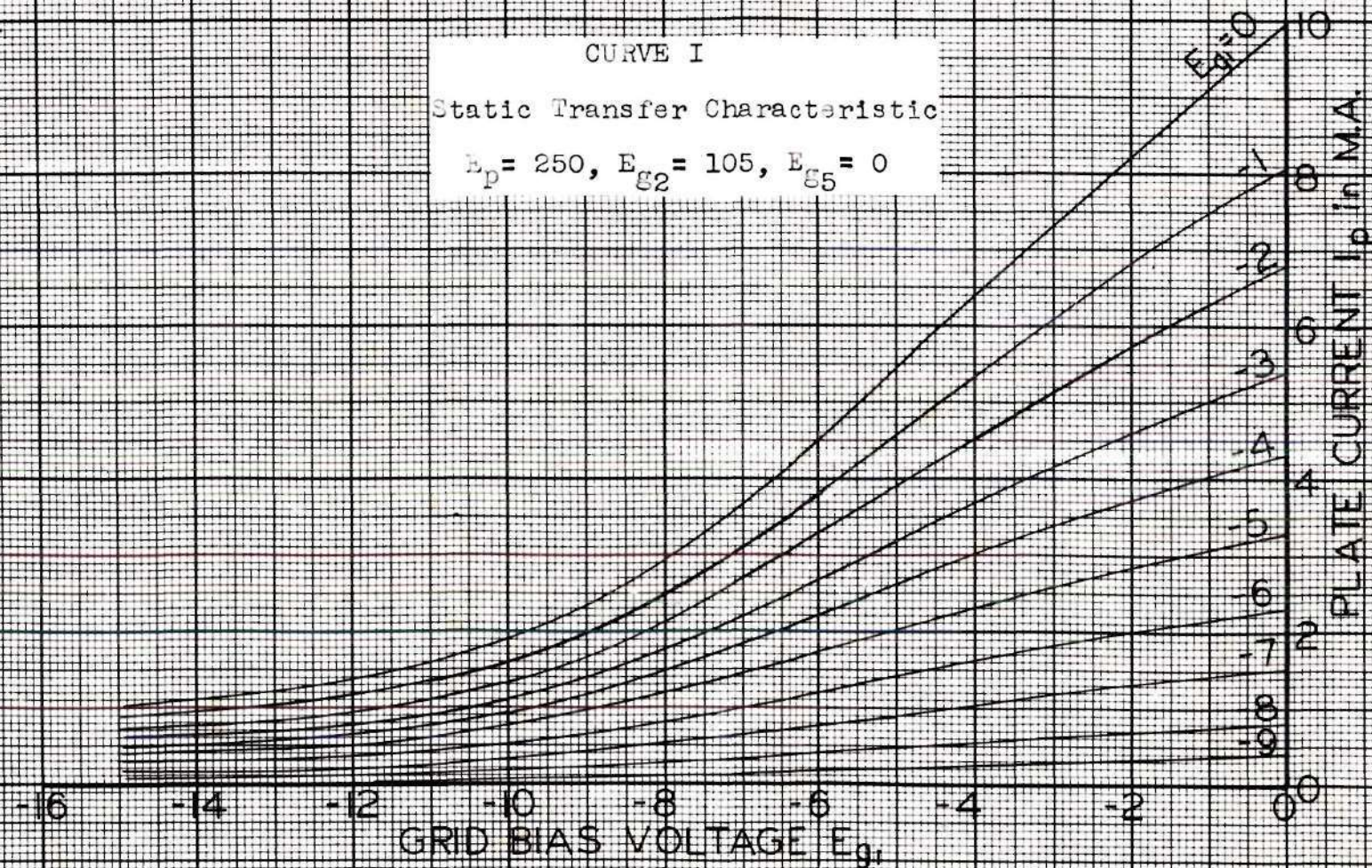
Note: In the above circuits  $B^+$  indicates the screen grid voltage, and  $B^{++}$  indicates the plate supply voltage.



## CURVE I

Static Transfer Characteristic

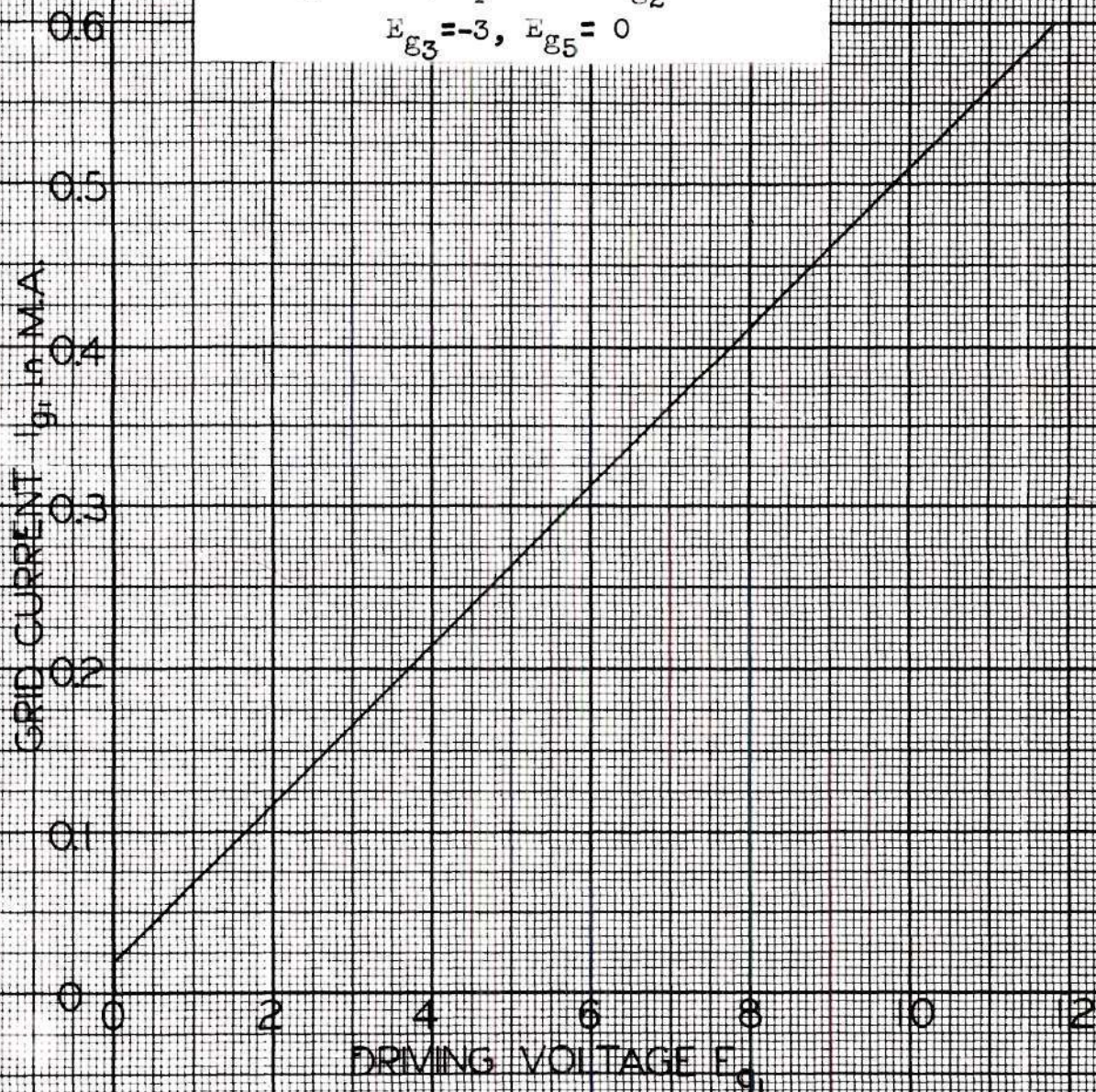
$$E_p = 250, E_{g2} = 105, E_{g5} = 0$$





## CURVE II

Grid #1 Rectification Characteristic

 $C_c = 0.5\mu\text{fd.}$ ,  $R_{g1} = 19,600\text{ ohms}$  $f_o = 5\text{kc.}$ ,  $E_p = 250$ ,  $E_{g2} = 105$  $E_{g3} = -3$ ,  $E_{g5} = 0$ 



## CURVE III

Signal Grid Transconductance  
vs. Grid #1 Bias

$E_p = 250$ ,  $E_{g2} = 105$ ,  $E_{g3} = -3$

$e_{g1} = 1.0$  eff.,  $E_{g5} = 0$

$G_{m3p}$  in MICROMHOS

GRID BIAS VOLTAGE  $E_{g1}$

-10 -8 -6 -4 -2 0 2 4

1800

1600

1400

1200

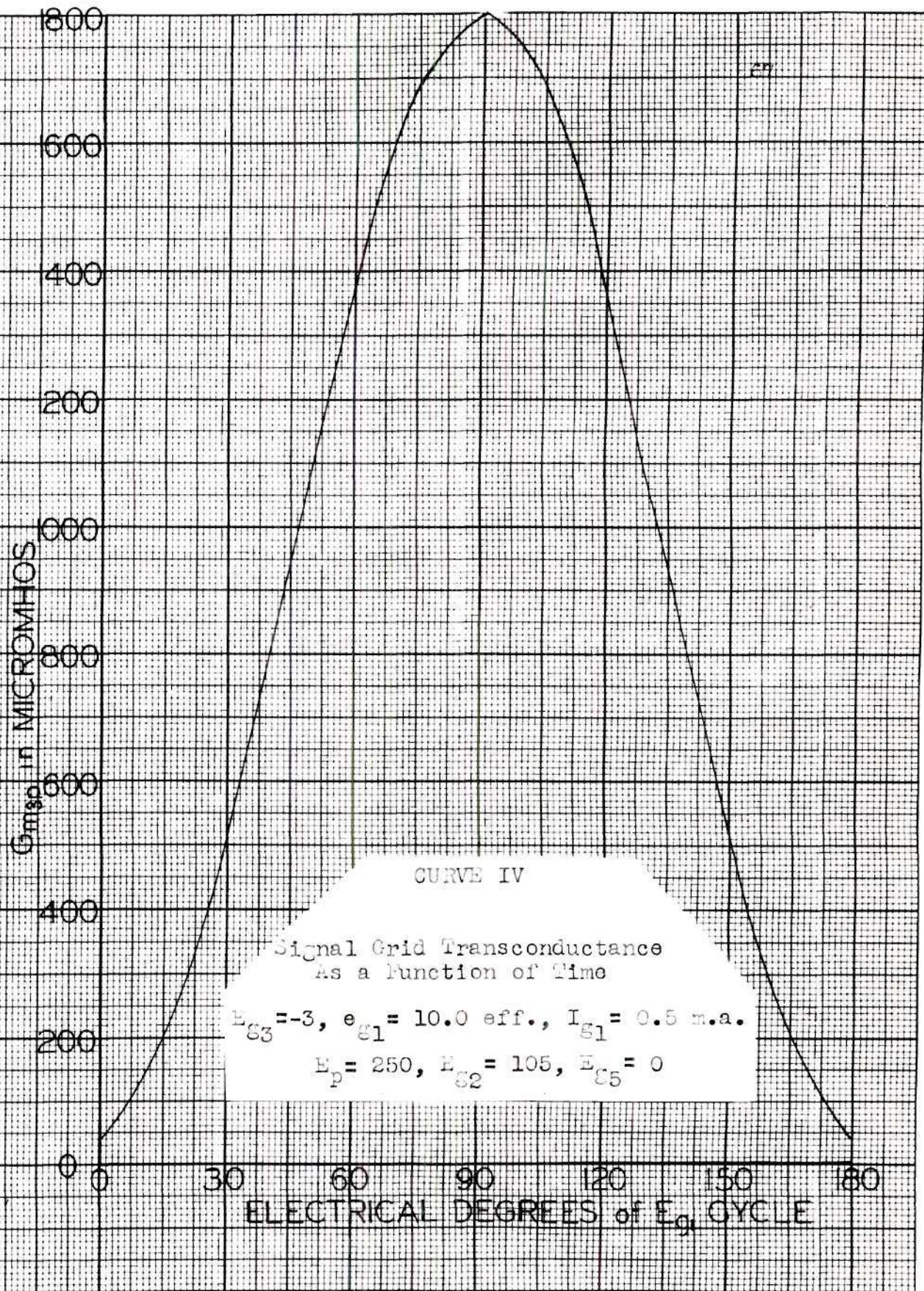
800

600

400

200







## CURVE V

Conversion Transconductance  
Vs. Grid #1 Current

$E_{g3} = -3$ ,  $E_p = 250$ ,  $E_{g2} = 105$ ,  $R_{g1} = 19,600$  ohms

$f_o = 5\text{kc.}$ ,  $f_s = 2\text{kc.}$ ,  $\theta_{g3} = 1.0$  eff.





CURVE VI

Conversion Transconductance  
Vs. Grid #3 Bias

$f_o = 5\text{kc.}, f_s = 2\text{kc.}, I_{g1} = 0.5 \text{ m.a.}$   
 $R_{g1} = 19,600 \text{ ohms}, e_{g1} = 10.0 \text{ eff.}$   
 $e_{g3} = 1.0 \text{ eff.}, E_p = 250, E_{g2} = 105$

