HEAT TRANSFER TO A FLUID FLOWING TURBULENTLY IN A SMOOTH PIPE WITH WALLS AT CONSTANT TEMPERATURE

A THESIS

Presented to

the Faculty of the Graduate Division

By

Robert James Hefner

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Chemical Engineering

Georgia Institute of Technology

May 1958

"In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institution shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

7A
12T

HEAT TRANSFER TO A FLUID FLOWING TURBULENTLY

IN A SMOOTH PIPE WITH WALLS AT

CONSTANT TEMPERATURE

Approved:

____

____

____

____

Date Approved by Chairman: May 26, 1958

ACKNOWLEDGEMENTS

The author wishes to express his appreciation to those who gave invaluable assistance in the course of this investigation: to Dr. J. M. DallaValle and Dr. W. B. Harrison, who gave indispensable advice and guidance; and to Duane L. Franklet, who assisted in every phase of the project.

The author is also grateful to the Shell Oil Company for making this study possible through the award of the Shell Fellowship in Chemical Engineering during the academic years of 1956-57 and 1957-58.

TABLE OF CONTENTS

NOMENCLATURE

C_p	Specific heat at constant pressure	Btu/lb.$_m$ $^\circ$F
D	Diameter of pipe	ft.
f	Friction factor, $8g_c\tau_w/\rho u^2$	
g_c	Conversion factor	ft. lb.$_m$/hr.^{2}lb.$_f$
h	Coefficient of heat-transfer between fluid and surface	Btu/hr. ft.^{2}
k	Thermal conductivity of fluid	Btu/hr. ft.^{2} $^\circ$F/ft.
Nu	Nusselt number, hD/k	
p	Pressure	lb.$_f$/ft.^{2}
Pe	Peclet number, Re$\cdot$Pr	
Pr	Prandtl number, ν/α	
q	Heat-transfer rate	Btu/hr.
Re	Reynolds number, Du/ν	
r	Radial distance from pipe center	ft.
R	Radius of pipe	ft.
R^+	Radius of pipe in dimensionless form, $\frac{1}{2}$ Re $\sqrt{f/8}$	
t	Temperature of fluid at any point	$^\circ$F
u	Axial velocity component	ft/hr.
u^+	Dimensionless velocity, $\bar{u}/\sqrt{g_c\tau_w/\rho}$	
v	Radial velocity component	ft./hr.
w	Radial distance from pipe wall divided by radius of pipe	

x	Distance in axial direction	ft.
y	Radial distance from pipe wall, R-r	ft.
y^+	Radial distance from pipe wall in dimensionless form, $\sqrt{g_c \tau_w / \rho}\ y/\nu$	
z	Radial distance from pipe wall divided by radius of pipe	

GREEK LETTERS

α	Thermal diffusivity of fluid	$ft.^2/hr.$
ε_H	Eddy diffusivity of heat-transfer	$ft.^2/hr.$
ε_M	Eddy diffusivity of momentum-transfer	$ft.^2/hr.$
θ	Temperature at pipe wall minus temperature at any point, $t_w - \bar{t}$	^oF
ν	Kinematic viscosity of fluid	$ft.^2/hr.$
ρ	Density of fluid	$lb._m/ft.^3$
σ	Ratio of eddy diffusivities of heat and momentum,	
τ	Shear stress at any point	$lb._f/ft.^2$

SUBSCRIPTS

c	Quantity measured at pipe center
m	Bulk mean
w	Quantity measured at pipe wall

SUPERSCRIPTS

'	Fluctuating component
–	Temporal mean

LIST OF TABLES

LIST OF ILLUSTRATIONS

SUMMARY

Techniques utilizing analogy methods for the calculation of heat-transfer coefficients for the turbulent flow of fluids in circular pipes have been presented by several investigators. Most of these presentations have been for the case of a constant heat flux along the pipe wall. Seban and Shimazaki (Transactions of the American Society of Mechanical Engineers, 73, 803-809, 1951) used analogy methods to investigate the heat-transfer coefficients of fluids with low Prandtl numbers and found that the wall temperature profile is significant in such fluids.

In the present investigation a study of the heat-transfer coefficients of fluids flowing turbulently in a smooth pipe with walls at constant temperature has been made employing analogy methods in a manner essentially the same as that presented by Seban and Shimazaki with the exception that the method presented in this study is based on different assumptions as to the nature of the buffer region and thermal diffusivity.

The present investigation presents an analytical solution of the momentum equation and the energy equation for the system by employing an analogy between the eddy diffusivities of momentum and heat. The universal dimensionless velocity distribution equations of von Karman (Transactions of the American Society of Mechanical Engineers, 61, 705-710, 1939) have been used in this development. The character of the equations developed is such that the temperature distribution across

the pipe can be solved by iterative procedures. The heat-transfer coefficient for the system can then be computed directly by employing this temperature distribution.

Values of the heat-transfer coefficient in the form of Nusselt number have been computed on a Remington Rand 1101 digital computer for values of Prandtl number ranging from 0.01 to 10.0 and values of Reynolds number from 5,000 to 10,000,000. Values of the Nusselt number for Peclet numbers less than 150 have been corrected for axial conduction by employing a semi-empirical correlation postulated by Trefethen (Transactions of the American Society of Mechanical Engineers, 78, 1207-1212, 1956).

The values calculated in this investigation have been compared to existing experimental data and to values computed by Franklet (Ph.D. Thesis, Georgia Institute of Technology, 1958) for a system under conditions of constant heat flux along the pipe wall (linear wall temperature profile). The results indicate that fluids with Prandtl numbers less than unity exhibit substantially different heat-transfer coefficients depending upon the wall temperature profile.

The heat-transfer coefficients predicted in this investigation compare favorably with experimental data for Prandtl numbers less than unity. For fluids with Prandtl numbers greater than unity, the comparison is not as good. The discrepancy between the predicted and experimental values of the heat-transfer coefficient for fluids with large Prandtl numbers can probably be attributed to the velocity distribution used in the development presented in this study. If a more accurate equation for predicting the velocity distribution near the pipe wall were employed, the predicted and experimental values would probably be in somewhat better agreement.

CHAPTER I

INTRODUCTION

The use of analogy methods for the calculation of heat-transfer coefficients for the turbulent flow of fluids in circular pipes has been developed and presented by several investigators. The more important of these have been reviewed by Summerfield (1) and Jacob (2). Additional references not contained in the above reviews are listed in the bibliography. Most of these presentations, however, have been for the case of a constant heat flux along the pipe wall.

The effect of a constant wall temperature as contrasted to constant heat flux was first investigated by Reichardt (3) who found that fluids with Prandtl number greater than 0.70 exhibited only small variations of heat-transfer coefficients due to changes in the wall temperature profile.

Seban and Shimazaki (4) used the analogy methods based upon a model postulated by Martinelli (5) to investigate the heat-transfer coefficient of fluids with low Prandtl number and found that the wall-temperature profile is significant in such fluids.

This investigation employs essentially the same development as that presented by Seban and Shimazaki, with the exception that the development here presented is based upon a model postulated by Franklet (6) and this writer. This study presents an investigation of heat-transfer coefficients obtained under conditions of constant wall temperature with a comparison to those obtained under conditions of constant heat flux. The present investigation is restricted to flow in smooth circular pipes.

The heat-transfer coefficients are obtained analytically from the following differential equations of heat and momentum by means of an iterative method.

$$\frac{g_c}{\rho}\frac{\partial \bar{p}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\nu\frac{\partial \bar{u}}{\partial r}\right) - \frac{1}{r}\frac{\partial}{\partial r}\left(\overline{ru'v'}\right) \tag{1}$$

$$\bar{u}\frac{\partial \bar{t}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\alpha\frac{\partial \bar{t}}{\partial r}\right) - \frac{1}{r}\frac{\partial}{\partial r}\left(\overline{rv't'}\right) \tag{2}$$

The solutions of these equations are based upon considering three distinct regions of flow within the pipe as follows:

(a) Laminar sub-layer, where the eddy diffusivities are considered to be negligible compared to the molecular diffusivities.

(b) Buffer layer, where the molecular and eddy diffusivities are considered to be of the same order of magnitude.

(c) Turbulent core, where the molecular diffusivities are considered negligible compared to the eddy diffusivities.

Von Karman (7) postulated the following dimensionless universal velocity distribution for smooth circular pipes based on the data of Nikuradse (8).

$$u^+ = y^+ \qquad 0 \leq y^+ \leq 5 \tag{3a}$$

$$u^+ = -3.05 + 5.0 \ln y^+ \qquad 5 \leq y^+ \leq 30 \tag{3b}$$

$$u^+ = 5.0 + 2.5 \ln y^+ \qquad 30 \leq y^+ \tag{3c}$$

These equations are used in this investigation to represent the velocity in Equation 2. It should be noted here that Seban and Shimazaki, in their development, chose the limits of the laminar, buffer and turbulent regions to coincide with the limits of Equations 3a, 3b and 3c respectively. In this investigation, however, the buffer layer is considered to extend into the pipe to a y^+ value equal to seventy.

Equation 2 above assumes no axial transfer of heat. The validity of this assumption is dependent upon the physical properties of the fluid and the velocity within the pipe. For liquids and gases Deissler (9) has shown that a velocity large enough to produce turbulence is sufficient to render the axial conduction negligible compared to the bulk transport of heat in the axial direction. The thermal conductivity of liquid metals, however, is large enough to cause considerable axial conduction at velocities well above that required to produce turbulence. Trefethen (10) has postulated a semi-empirical correction as a function of the Peclet modulus to correct the heat-transfer coefficient for axial conduction. This correction has been applied to the results of Equations 1 and 2.

The mathematical development of the equations necessary for the calculation of the temperature distribution and heat-transfer coefficient are presented in Chapter II.

Discussion of the parameters necessary for the computation of heat-transfer coefficients by analogy methods is presented in Chapter III. There is also presented in Chapter III a comparison of the heat-transfer coefficients for liquid metals computed in this investigation with those computed for the case of constant heat flux as calculated by Martinelli (5) and Franklet (6), and with existing experimental data. There is also presented a brief discussion of the extension of the equations developed in this investigation to include fluids with Prandtl numbers greater than 0.70.

CHAPTER II

MATHEMATICAL DEVELOPMENT

For a fluid flowing in turbulent motion within a cylindrical pipe sufficiently far removed from the entrance to insure absence of end effects, the momentum and energy equations reduce to

$$\frac{g_c}{\rho}\ \frac{\partial \overline{p}}{\partial x} = \frac{1}{r}\ \frac{\partial}{\partial r}\left(r\nu\ \frac{\partial \overline{u}}{\partial r}\right) - \frac{1}{r}\ \frac{\partial}{\partial r}\left(\overline{ru'v'}\right) \tag{4}$$

$$\overline{u}\ \frac{\partial \overline{t}}{\partial x} = \frac{1}{r}\ \frac{\partial}{\partial r}\left(r\alpha\ \frac{\partial \overline{t}}{\partial r}\right) - \frac{1}{r}\ \frac{\partial}{\partial r}\left(\overline{rv't'}\right) \tag{5}$$

The system satisfying these equations is subject to the following restrictions:

(a) Fluid properties are independent of temperature.

(b) Fluid is incompressible.

(c) Steady state conditions exist.

(d) Mean velocity of fluid is in the x-direction, the x-axis coinciding with the center-line of the pipe.

(e) System is symmetrical about the x-axis.

(f) Axial diffusion is negligible with respect to bulk transport of energy in the x-direction.

If the mean values $\overline{u'v'}$ and $\overline{v't'}$ are expressed as $-\varepsilon_M \frac{\partial \overline{u}}{\partial r}$ and $-\varepsilon_H \frac{\partial \overline{t}}{\partial r}$ respectively, Equations 4 and 5 reduce to

$$\frac{g_c}{\rho}\frac{\partial\bar{p}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\nu\frac{\partial\bar{u}}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\varepsilon_M\frac{\partial\bar{u}}{\partial r}\right) \tag{6}$$

$$\bar{u}\frac{\partial\bar{t}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\alpha\frac{\partial\bar{t}}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\varepsilon_H\frac{\partial\bar{t}}{\partial r}\right) \tag{7}$$

The momentum equation for the system, Equation 6, can be rearranged and integrated once with respect to radius to yield

$$\frac{g_c r}{2\rho}\frac{\partial\bar{p}}{\partial x} = -(\nu + \varepsilon_M)\frac{\partial\bar{u}}{\partial r} \tag{8}$$

A force balance on an annulus of fluid with radius r becomes

$$\frac{1}{\rho}\frac{\partial\bar{p}}{\partial x} = -\frac{2}{R}\frac{\tau_w}{\rho} \tag{9}$$

and

$$\frac{\tau_w}{R} = \frac{\tau}{r} \tag{10}$$

Equation 10 can be written as

$$\frac{\tau}{\rho} = \frac{\tau_w}{\rho}(1 - y/R) \tag{11}$$

Substitution of Equation 9 into Equation 8 yields

$$\frac{g_c\tau}{\rho} = -(\nu + \varepsilon_M)\frac{\partial\bar{u}}{\partial r} = (\nu + \varepsilon_M)\frac{\partial\bar{u}}{\partial y} \tag{12}$$

Then on substitution of Equation 11 into Equation 12 and rearranging, there is obtained

$$\varepsilon_M = \frac{g_c \tau_w}{\rho} \frac{(1 - y/R)}{\partial \bar{u}/\partial y} - \nu \qquad (13)$$

or in terms of the dimensionless parameters u^+ and y^+

$$\varepsilon_M = \nu\left[\frac{(1 - y^+/R^+)}{\partial u^+/\partial y^+} - 1\right] \qquad (14)$$

where: $u^+ = \bar{u}/\sqrt{g_c \tau_w/\rho}$

$y^+ = \sqrt{g_c \tau_w/\rho}\, y/\nu$

Defining the ratio of the eddy diffusivities of heat and momentum by the following function

$$\sigma = \frac{\varepsilon_H}{\varepsilon_M} \qquad (15)$$

Equation 14 can then be expressed as

$$\frac{\varepsilon_H + \alpha}{\sigma\nu} = \left[\frac{(1 - y^+/R^+)}{\partial u^+/\partial y^+} + \frac{1}{\sigma Pr} - 1\right] \qquad (16)$$

In terms of $\theta = t_w - \bar{t}$ and the dimensionless parameters u^+ and y^+, the energy equation for the system, Equation 7, can be written in the form

$$R^2 \sqrt{g_c \tau_w/\rho} \quad u^+ \quad \frac{\partial \theta}{\partial x} =$$

$$\frac{1}{(1 - y^+/R^+)} \frac{\partial}{\partial(y^+/R^+)} [(1 - y^+/R^+)(\alpha + \varepsilon_H) \quad \frac{\partial\theta}{\partial(y^+/R^+)}] \tag{17}$$

Now for the system under consideration with a uniform wall temperature, the following lemma is developed in Appendix I

$$\frac{\partial\theta}{\partial x} = \frac{\theta}{\theta_m} \frac{\partial\theta_m}{\partial x} \tag{18}$$

Substituting this equality and rearranging, Equation 17 can be integrated once with respect to y^+/R^+ since $\frac{\partial\theta_m}{\partial x}$ is independent of radius to yield

$$R^2 \sqrt{g_c \tau_w/\rho} \ \frac{\partial\theta_m}{\partial x} \int_1^w \frac{\theta}{\theta_m} u^+ \ (1 - y^+/R^+) d(y^+/R^+)$$

$$= \int_1^w d\left[(1 - y^+/R^+)(\alpha + \varepsilon_H) \frac{d\theta}{d(y^+/R^+)}\right] \tag{19}$$

where $1 \geqq w \geqq 0$

For simplicity of notation let

$$F(w) \equiv \int_1^w \frac{\theta}{\theta_c} u^+ \ (1 - y^+/R^+) d(y^+/R^+) \tag{20}$$

Performing the indicated integration on the right hand side and rearranging, Equation 19 can be integrated a second time with respect to the radius ratio to become

$$R^2 \sqrt{g_c \tau_w/\rho} \ \frac{\theta_c}{\theta_m} \frac{\partial\theta_m}{\partial x} \ \frac{1}{\sigma\nu} \int_1^z \frac{F(w)}{(1 - w) \ \frac{\alpha + \varepsilon_H}{\sigma\nu}} dw = \theta - \theta_c \tag{21}$$

$$\text{where} \quad 1 \geqq z \geqq 0$$

Substituting Equation 16 into Equation 21 yields

$$R^2\sqrt{g_c \tau_w/\rho} \;\; \frac{\theta_c}{\theta_m} \; \frac{\partial\theta_m}{\partial x} \;\; \frac{1}{\sigma\nu} \; G(z) = \theta - \theta_c \tag{22}$$

where, for simplicity of notation

$$G(z) \equiv \int_1^z \frac{F(w)dw}{\left[(1 - w)du^+/dy^+ + \frac{1}{\sigma Pr} - 1\right](1 - w)} \tag{23}$$

$$\text{where} \quad 1 \geqq z \geqq 0$$

Also Equation 21 when integrated over the entire region from $z = 1$ to $z = 0$ becomes

$$R^2\sqrt{g_c \tau_w/\rho} \;\; \frac{\theta_c}{\theta_m} \; \frac{\partial\theta_m}{\partial x} \;\; \frac{1}{\sigma\nu} \; G(0) = -\theta_c \tag{24}$$

Thus dividing Equation 22 by Equation 24 an expression for the generalized temperature distribution can be obtained in the form

$$\frac{\theta}{\theta_c} = 1 - \frac{\theta - \theta_c}{-\theta_c} = 1 - \frac{G(z)}{G(0)} \tag{25}$$

As stated in the introduction the von Karman universal velocity distribution equations can be used in the evaluation of Equations 20 and 23. Introducing these equations into Equations 20 and 23 and considering the three regions of flow, the following are obtained.

Turbulent Core.—

$$u^+ = 5.0 + 2.5 \ln y^+ \qquad R^+ \geqq y^+ \geqq 70 \tag{26}$$

$$F_1(w) =$$

$$\int_1^w [5.0 + 2.5 \ln R^+(y^+/R^+)] \frac{\theta}{\theta_c} (1 - y^+/R^+) d(y^+/R^+) \tag{27}$$

where $1 \geqq w \geqq 70/R^+$

$$G(z) = \int_1^z \frac{F_1(w)\, dw}{\frac{R^+}{2.5} w(1 - w)} \tag{28}$$

where $1 \geqq z \geqq 70/R^+$

Buffer Region.—

$$\begin{aligned} u^+ &= 5.0 + 2.5 \ln y^+, \qquad 70 \geqq y^+ \geqq 30 \\ u^+ &= -3.05 + 5.0 \ln y^+ \qquad 30 \geqq y^+ \geqq 5 \end{aligned} \tag{29}$$

$$F_2(w) =$$

$$\int_{70/R^+}^w [5.0 + 2.5 \ln R^+(y^+/R^+)] \frac{\theta}{\theta_c} (1 - y^+/R^+) d(y^+/R^+) \tag{30}$$

where $70/R^+ \geqq w \geqq 30/R^+$

$$F_3(w) =$$

$$\int_{30/R^+}^{w} [-3.05 + 5.0 \ln R^+(y^+/R^+)] \frac{\theta}{\theta_c}(1 - y^+/R^+)d(y^+/R^+) \tag{31}$$

where $30/R^+ \geqq w \geqq 5/R^+$

$$G(z) = \int_{70/R^+}^{z} \frac{[F_1(w) + F_2(w) + F_3(w)]\, dw}{[(\frac{1}{\sigma Pr} - 1) + \frac{R+}{5.0} w(1 - w)]\, (1 - w)} \tag{32}$$

where $70/R^+ \geqq z \geqq 5/R^+$

<u>Laminar Sub-layer</u>.—

$$u^+ = y^+ \qquad 5 \geqq y^+ \geqq 0 \tag{33}$$

$$F_4(w) = \int_{5/R^+}^{w} R^+ \frac{\theta}{\theta_c} (y^+/R^+)(1 - y^+/R^+)d(y^+/R^+) \tag{34}$$

where $5/R^+ \geqq w \geqq 0$

$$G(z) = \int_{5/R^+}^{w} \frac{[F_1(w) + F_2(w) + F_3(w) + F_4(w)]\, dw}{\frac{1}{\sigma\, Pr}(1 - w)} \tag{35}$$

where $5/R^+ \geqq z \geqq 0$

Equations 28, 32 and 35 can thus be used in Equation 25 to evaluate the radial temperature distribution. It can be seen, however, that the solution of Equation 25 requires a knowledge of the very temperature distribution being sought. Therefore an iterative procedure must be used to converge on the desired temperature distribution.

An equation for the heat flow in a round circular pipe can be expressed in terms of θ as

$$\pi D h \, \theta_m = -\frac{\pi D^2}{4} C_p \rho \bar{u} \; \frac{\partial \theta_m}{\partial x} \tag{36}$$

which can be written in the form

$$Nu = \frac{hD}{\alpha C_p \rho} = -\frac{D^2}{4} \; \frac{1}{\theta_m} \; \frac{\partial \theta_m}{\partial x} \; \frac{\bar{u}}{\alpha} \tag{37}$$

Now Equation 24 can be rearranged to the form

$$\frac{D^2}{4} \; \frac{1}{\theta_m} \; \frac{\partial \theta_m}{\partial x} \; \frac{g_c \tau_w}{\rho} = -\frac{\sigma \nu}{G(0)} \tag{38}$$

From the definition of the friction factor it follows that

$$\sqrt{g_c \tau_w / \rho} = \bar{u} \sqrt{f/8} \tag{39}$$

Substituting Equation 39 into Equation 38 and combining the resulting equation with Equation 37 yields

$$Nu = -\frac{D^2}{4} \; \frac{1}{\alpha \theta_m} \; \frac{\partial \theta_m}{\partial x} \, \bar{u} \tag{40}$$

or

$$Nu = \frac{\sigma \, Pr}{\sqrt{f/8}} \; \frac{1}{G(0)} \tag{41}$$

The results herein obtained were calculated by utilizing a Remington Rand 1101 digital computer according to the foregoing iterative procedure. The integrations involved were accomplished on the computer utilizing the trapezoidal rule by using thirty intervals in the turbulent core, thirty in the buffer region and five in the laminar sub-layer. A linear temperature distribution was assumed as a first approximation with iterations continued until two successive values of the Nusselt number differed by less than 0.005. In general six iterations were required for sufficient convergence of the temperature distribution.

As a test of computational accuracy several values were recomputed utilizing twice the normal number of intervals for integration purposes and twice the normal number of iterations. In all cases tested the computed values differed by less than 0.2 per cent.

CHAPTER III

DISCUSSION OF RESULTS

Evaluation of heat-transfer coefficients by the solution of the energy equation for the system under consideration as defined in Chapter II is contingent upon a knowledge of the velocity distribution, thermal conductivity and eddy diffusivity of heat-transfer of the fluid. The accuracy of results obtained from an investigation of this type is obviously directly dependent upon the precision with which these three quantities are known. Also in using analogy methods the accuracy of results depends upon the eddy diffusivity of momentum-transfer as well as the ratio of the eddy diffusivities.

Some of the problems involved in the evaluation of these quantities are discussed in the following paragraphs.

Velocity Distribution.--As stated in the introduction, the von Karman universal velocity distribution equations have been used in this investigation. Ross (11), Ruth and Yang (12), and Rothfus, *et al.*, (13) have criticized the distribution predicted by these equations, especially in the region adjacent to the pipe wall. The results of this investigation indicate, however, that for liquid metals which exhibit low Prandtl numbers the region near the wall contributes very little to the overall transfer of energy within the system. This is not the case for ordinary liquids and gases for which the region near the wall contributes a major portion to the overall effect.

Van Driest (14) and Deissler (15) have postulated velocity distribution equations in an attempt to better describe the velocity adjacent to the pipe wall. Franklet (6) has calculated heat-transfer coefficients for the case of constant heat flux using both the universal velocity distribution and a modified form of the van Driest equation. His results indicate that either distribution gives favorable results for liquid metals but that the van Driest equation yields considerably better results for ordinary fluids. This comparison has not been made in this investigation due to the extreme complexity of computations involved. It can be assumed, however, that if a more accurate velocity distribution were used in the region near the pipe wall that the results of this investigation would be considerably improved for gases and ordinary liquids.

Eddy Diffusivity.—The eddy diffusivity of heat-transfer, ε_H, required for the solution of the energy equation is obtained in analogy methods from the diffusivity of momentum-transfer, ε_M, by employing the relation $\sigma \equiv \varepsilon_H/\varepsilon_M$. Thus the accuracy of ε_H is dependent upon the evaluation of ε_M and σ. The evaluation of ε_M can be accomplished directly from the momentum equation for the system. It should be noted, however, that numerical computation of ε_M from the momentum equation is dependent upon the evaluation of the velocity gradient within the pipe. Thus empirical equations which describe the velocity distribution with great accuracy may produce large errors when differentiated to evaluate the velocity gradient.

Probably the point of greatest doubt in the evaluation of the energy equation by analogy methods is the numerical value of the ratio of the two eddy diffusivities, σ. Reichardt (16) postulates that σ should be between one and two for all fluids. Jenkins (17) postulates a semi-empirical relationship which predicts that σ should be less than one for all fluids with Prandtl numbers less than one. The latter is not borne out, however, by extensive studies of air conducted by Sage, et al., (18), (19) and (20).

Isakoff and Drew (21) and Brown, Armstead and Short (22) have measured velocity and temperature distributions for mercury and have calculated the ratio of the eddy diffusivities from their results. Isakoff and Drew report values ranging up to approximately 1.7. Their values are questionable, however, due to the fact that the fluid temperature did not extrapolate well to the wall temperature. Brown, et al., report values of σ ranging up to approximately 0.95. Unfortunately, the spacing of their data does not satisfactorily facilitate a numerical analysis of the temperature distribution such that somewhat higher values can be obtained from their data if either graphical or numerical methods are employed. Jenkins' correlation predicts values ranging up to approximately 0.4 for mercury. Sleicher (23) adjusted these predicted values to correspond to his measured ratio for air. The result of this adjustment predicts somewhat higher values of the ratio.

As a result of the uncertainty of the numerical value of the ratio of the eddy diffusivities, a value of one has been used as a sort of mean in this investigation. The actual value is probably a function of the Reynolds number and Prandtl number as well as the position within the pipe. Figure 6 shows a comparison of the predicted Nusselt numbers using values of the ratio of the eddy diffusivities equal to 1.0 and 0.8 for a Prandtl number equal to 0.01 in order to illustrate the effect of this ratio on the prediction of the heat-transfer coefficient.

Thermal Conductivity.--The model employed in this investigation assumes the thermal conductivity and the eddy diffusivity of heat-transfer to be additive properties with the magnitude of the thermal conductivity becoming insignificant in the turbulent core. If the thermal conductivity is assumed to be constant across the pipe, numerical evaluation of the energy equation does not bear out the assumption that the thermal conductivity becomes insignificant in the turbulent region for liquid metals. This contradiction might be explained in the following way: Possibly the thermal conductivity and eddy diffusivity are so inter-related that their effect is not truly additive. That is, it could be true that even though the thermal conductivity is in itself constant throughout the pipe, its effectiveness is diminished by the influence of turbulence.

Discussion of Results.--Figure 7 shows the effect of Prandtl number on the temperature distribution for a constant Reynolds number. It can be seen that as Prandtl number increases the temperature at any point within the pipe increases relative to the center line temperature. As a result the mean temperature across the pipe increases with increasing Prandtl number. For fluids of large Prandtl number, the change in temperature between the wall and the fluid is localized in the laminar and buffer regions immediately adjacent to the pipe wall, and the temperature is almost constant across the entire turbulent core. Thus the magnitude of the mean temperature is only insignificantly affected by variation in the temperature of the wall.

Figure 8 shows the effect of Reynolds number on the temperature distribution for a constant Prandtl number. Here again the temperature at any point increases relative to the center line temperature for increasing Reynolds number. Thus for sufficiently large Reynolds number the mean temperature is essentially independent of variations in the wall temperature and becomes approximately equal to the center line temperature regardless of the Prandtl number.

The difference in temperature distributions indicated by the analysis of Franklet and by the present investigation is shown in Figure 9. It is seen that for the same wall to center line temperature difference the heat-transfer rate is substantially less for the case of constant wall temperature. This does not exactly reflect the difference in the heat-transfer coefficients, since the mean temperature magnitudes are also different for the two cases.

Table I lists the values for the heat-transfer coefficient expressed as Nusselt number as calculated in this investigation for constant wall temperature. It also lists the values for a system under conditions of constant heat flux for the same flow conditions as computed by Franklet as well as a comparison of these values expressed as a ratio of the Nusselt numbers for the two cases. From this table it can be seen that as Prandtl number increases the ratio between the Nusselt numbers for the two cases tends to unity for any particular Reynolds number. It can also be seen that the ratio tends to unity with increasing Reynolds number at any particular Prandtl number for Reynolds numbers less than 1,000,000. For values of Reynolds number above 1,000,000, however, the difference in Nusselt numbers for the two cases tends to become greater for a particular value of Prandtl number. The comparison of the two systems is also shown graphically in Figure 4.

In order to consider qualitatively the difference between the heat-transfer coefficients for the two systems, Equation 37 can be used to express the ratio of the systems under the same conditions of flow as

$$\frac{(Nu)_t}{(Nu)_q} = \frac{(t_m - t_w)_q (\partial t_m/\partial x)_t}{(t_m - t_w)_t (\partial t_m/\partial x)_q} \tag{42}$$

where the subscripts t and q refer to a system under conditions of constant wall temperature and constant heat flux respectively. For the case of constant heat flux, the term $\partial t_m/\partial x$ is constant for a particular system, whereas for the case of constant wall temperature $\partial t_m/\partial x$ is a function of Reynolds number and decreases with increasing Reynolds number. As stated above, for sufficiently large values of

Reynolds number, the mean temperature is essentially equal to the center line temperature. Therefore if the two cases are considered to have the same center line temperature, the heat-transfer coefficients become a function of the axial mean temperature gradient. Thus the ratio of the heat-transfer coefficients should decrease for sufficiently large Reynolds numbers. For low Prandtl numbers, however, at somewhat lower values of Reynolds number, where the wall temperature affects the mean temperature appreciably, both the magnitude of the mean temperature gradient and the difference between the mean and wall temperatures become significant and it has been found that in this region the ratio of the heat-transfer coefficients increase toward unity. The ratios of the heat-transfer coefficients computed in this investigation are shown graphically in Figure 5.

The values of the Nusselt number for Peclet numbers less than 150 used throughout this investigation have been corrected for axial conduction by employing a semi-empirical correlation postulated by Trefethen (10). Table II lists the uncorrected and corrected values for comparison.

Figure 1 shows a plot of experimental data obtained from the literature. The majority of these data were obtained under conditions of constant heat flux or conditions between constant flux and constant wall temperature. These values have been used for comparison of the Nusselt number values computed in this investigation, however, for lack of experimental data obtained under conditions of constant wall temperature.

Figure 2 reveals the predicted values of Nusselt number for Prandtl number values of 0.01 and 0.02 in comparison to the constant heat flux solution as computed by Martinelli (5) and Franklet and with the experimental data shown in Figure 1. The predicted values of Martinelli are plotted as originally reported along with the values corrected for axial conduction as applied in this investigation. Figure 3 shows the same comparison for Prandtl number values of 0.7, 1.0 and 10.0 as compared to those predicted by Rannie (24) and Franklet.

Conclusions.—The case of heat transfer to a fluid flowing turbulently in a smooth pipe with walls at constant temperature has been considered, and heat-transfer coefficients for fluids of Prandtl number less than 1.0 have been shown to be significantly different from those for the case of constant heat rate. Thus is illustrated the importance of describing the surface temperature in the direction of flow for fluids of low Prandtl number even sufficiently far from the entrance such that thermal entrance effects can be neglected.

The equations developed in this investigation appear to be satisfactory for predicting values of the heat-transfer coefficient of fluids with Prandtl number less than unity. They would probably predict satisfactory values of the heat-transfer coefficient for fluids of large Prandtl number if a better equation for the prediction of the velocity distribution of the fluid adjacent to the wall were employed. In the case of fluids of large Prandtl number, however, the heat-transfer coefficient for the case of constant wall temperature is not sufficiently different from that of the case of constant heat flux to warrant the use of the more complex iterative scheme required in the solution of the equations developed in this investigation.

APPENDIX I

AXIAL TEMPERATURE DISTRIBUTION

In order to integrate the energy equation for the system defined in Chapter II, it is desirable to make use of the following lemma.

$$\frac{\partial}{\partial x}\left[\frac{t_w - t}{t_w - t_m}\right] = 0 \tag{43}$$

This lemma can be verified for the system with a uniform wall temperature in the following manner. Consideration of a heat balance on a differential element of length dx yields

$$\frac{C_p \rho \bar{u} D}{4h} \frac{\partial t_m}{\partial x} = t_w - t_m \tag{44}$$

The mean radial temperature is defined by

$$t_m = \frac{\int_0^R turdr}{\int_0^R urdr} \tag{45}$$

which when substituted into Equation 43 yields

$$\frac{C_p \rho \bar{u} D}{4h} \frac{\partial}{\partial x} \frac{\int_0^R turdr}{\int_0^R urdr} = t_w - \frac{\int_0^R turdr}{\int_0^R urdr} \tag{46}$$

Now u and r are independent of x and R is a constant, thus Equation 43 can be written as

$$\frac{C_p \rho \bar{u} D}{4h} \frac{\int_0^R \frac{\partial t}{\partial x} urdr}{\int_0^R urdr} = t_w - \frac{\int_0^R turdr}{\int_0^R urdr} \tag{47}$$

which when cleared of fractions becomes on differentiation

$$\frac{C_p \rho \bar{u} D}{4h} \frac{\partial t}{\partial x} = t_w - t \tag{48}$$

Combining Equations 44 and 48, there is obtained

$$(t_w - t) \frac{\partial t_m}{\partial x} - (t_w - t_m) \frac{\partial t}{\partial x} = 0 \tag{49}$$

Now since the wall temperature is constant

$$\frac{\partial (t_w - t)}{\partial x} = - \frac{\partial t}{\partial x} \tag{50}$$

and

$$\frac{\partial (t_w - t_m)}{\partial x} = - \frac{\partial t_m}{\partial x} \tag{51}$$

Thus Equation 49 can be written as

$$(t_w - t) \frac{\partial (t_w - t_m)}{\partial x} -- (t_w - t_m) \frac{\partial (t_w - t_m)}{\partial x} \tag{52}$$

By multiplying Equation 52 by $(t_w - t_m)^2/(t_w - t_m)^2$, it can be reduced to

$$(t_w - t_m)^2 \frac{\partial}{\partial x} \frac{t_w - t}{t_w - t_m} = 0 \tag{53}$$

Now since in general $t_w - t_m$ is not equal to zero it follows that

$$\frac{\partial}{\partial x} \left[\frac{t_w - t}{t_w - t_m} \right] = 0 \tag{54}$$

For the case of uniform wall temperature here considered Equation 54 reduces to

$$\frac{\partial t}{\partial x} = \frac{t_w - t}{t_w - t_m} \frac{\partial t_m}{\partial x} \tag{55}$$

or in terms of $\theta = t_w - t$

$$\frac{\partial \theta}{\partial x} = \frac{\theta}{\theta_m} \frac{\partial \theta_m}{\partial x} \tag{56}$$

APPENDIX II

TABLES AND FIGURES

Table I

Calculated Values of Nusselt Number for Constant Wall Temperature and Constant Heat Flux

Pr	Re	Pe	Nu t_w const.	Nu* q_w const.	$\frac{(Nu)_t}{(Nu)_q}$
0.01	5,000	50	1.39	2.53	0.55
	10,000	100	1.89	3.21	0.57
	30,000	300	3.11	4.43	0.71
	100,000	1,000	6.19	7.72	0.80
	300,000	3,000	11.17	15.0	0.78
	1,000,000	10,000	20.4	34.0	0.60
	3,000,000	30,000	24.2	76.3	0.32
	10,000,000	100,000	25.1	193	0.13
0.02	5,000	100	2.76	4.75	0.58
	10,000	200	3.41	5.46	0.62
	30,000	600	5.76	7.79	0.74
	100,000	2,000	11.68	14.2	0.82
	300,000	6,000	22.4	28.2	0.80
	1,000,000	20,000	39.7	65.0	0.61
	3,000,000	60,000	47.8	147	0.32
	10,000,000	200,000	49.9	374	0.14
0.70	10,000	7,000	29.9	32.6	0.92
	100,000	70,000	170	179	0.95
	300,000	210,000	396	425	0.93
	1,000,000	700,000	863	1130	0.76
1.0	10,000	10,000	36.6	38.3	0.96
	100,000	100,000	210	220	0.96
	1,000,000	1,000,000	1072	1430	0.75
10.0	10,000	100,000	85.5	88.1	0.97
	100,000	1,000,000	599	612	0.98
	1,000,000	10,000,000	4060	4610	0.88

*Nu for constant heat flux computed by Franklet (6).

Table II

Correction for Axial Conduction Applied to Nusselt Number

Pr	Re	Pe	Nu uncorrected	Nu corrected
0.01	5,000	50	1.79	1.39
	10,000	100	1.97	1.82
0.02	5,000	100	3.10	2.76

Table III

Cross-Sectional Temperature Distributions

$\frac{y}{R}$	Pr = 0.01	Pr = 0.02	Pr = 0.7	Pr = 1.0	Pr = 10.0
1.000	1.000	1.000	1.000	1.000	1.000
0.935	0.981	0.982	0.992	0.993	0.998
0.871	0.954	0.956	0.982	0.984	0.996
0.806	0.925	0.930	0.970	0.974	0.993
0.741	0.896	0.901	0.958	0.964	0.990
0.676	0.864	0.871	0.945	0.952	0.987
0.612	0.830	0.839	0.931	0.940	0.984
0.574	0.793	0.804	0.915	0.926	0.980
0.482	0.753	0.766	0.898	0.911	0.976
0.418	0.708	0.723	0.879	0.894	0.971
0.353	0.658	0.676	0.856	0.875	0.966
0.288	0.601	0.621	0.830	0.852	0.959
0.224	0.532	0.555	0.798	0.824	0.951
0.160	0.445	0.471	0.756	0.787	0.940
0.094	0.319	0.350	0.694	0.732	0.924
0.029	0.051	0.091	0.555	0.608	0.888
0.028	0.047	0.085	0.548	0.602	0.886
0.026	0.045	0.081	0.542	0.596	0.884
0.024	0.042	0.077	0.535	0.590	0.882
0.023	0.039	0.072	0.527	0.583	0.880
0.021	0.037	0.068	0.519	0.576	0.877
0.019	0.034	0.063	0.510	0.567	0.875
0.018	0.031	0.058	0.500	0.559	0.872
0.016	0.028	0.054	0.490	0.549	0.869
0.014	0.026	0.049	0.478	0.538	0.866
0.013	0.023	0.044	0.465	0.526	0.862
0.011	0.019	0.037	0.433	0.496	0.850
0.008	0.015	0.030	0.389	0.453	0.834
0.006	0.012	0.023	0.335	0.398	0.812
0.004	0.0078	0.015	0.261	0.321	0.774
0.002	0.0039	0.0076	0.147	0.188	0.568
0.0016	0.0031	0.0061	0.118	0.150	0.454
0.0012	0.0024	0.0046	0.088	0.112	0.340
0.0008	0.0016	0.0030	0.059	0.075	0.227
0.0004	0.0008	0.0015	0.030	0.038	0.113
0	0	0	0	0	0

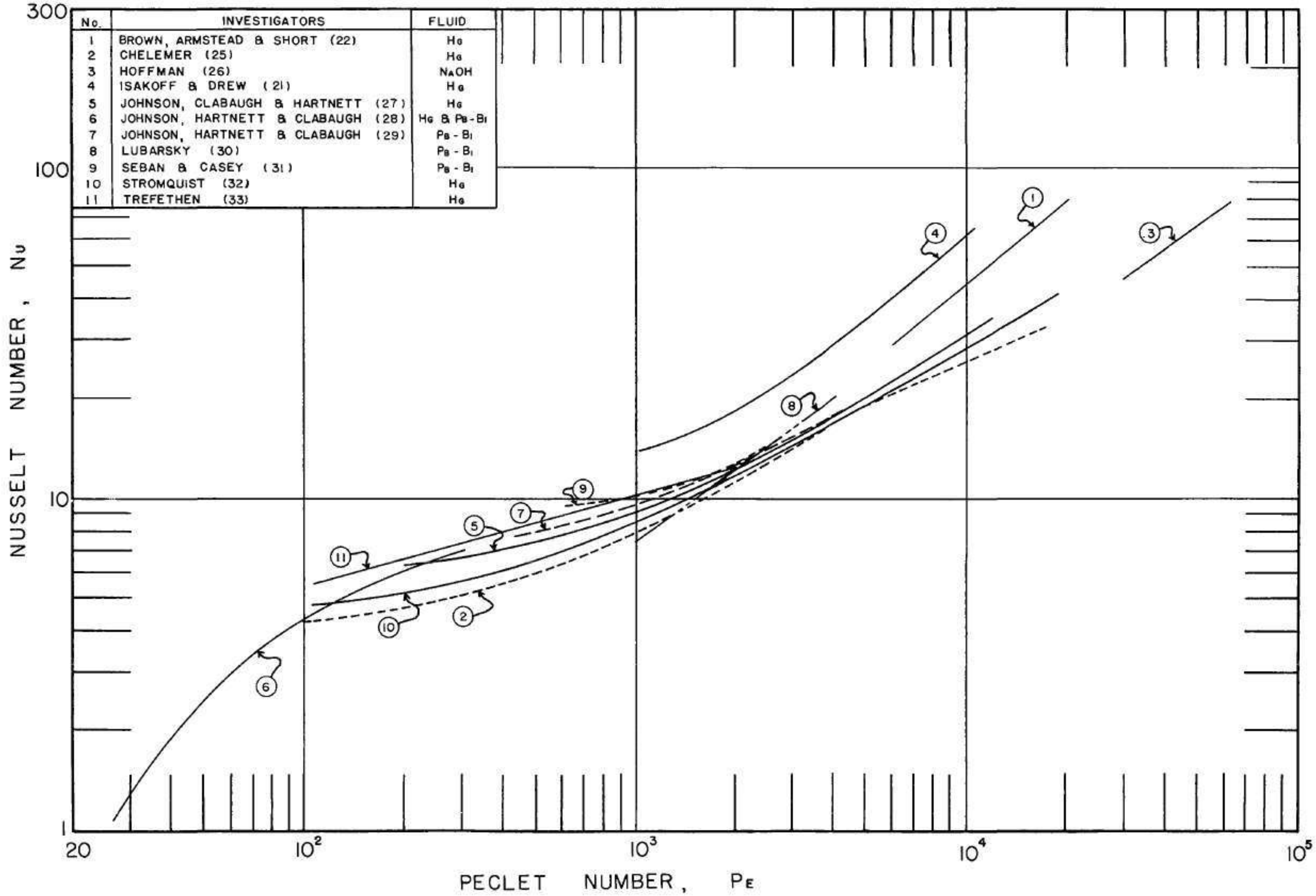

FIGURE 1. EXPERIMENTAL DATA

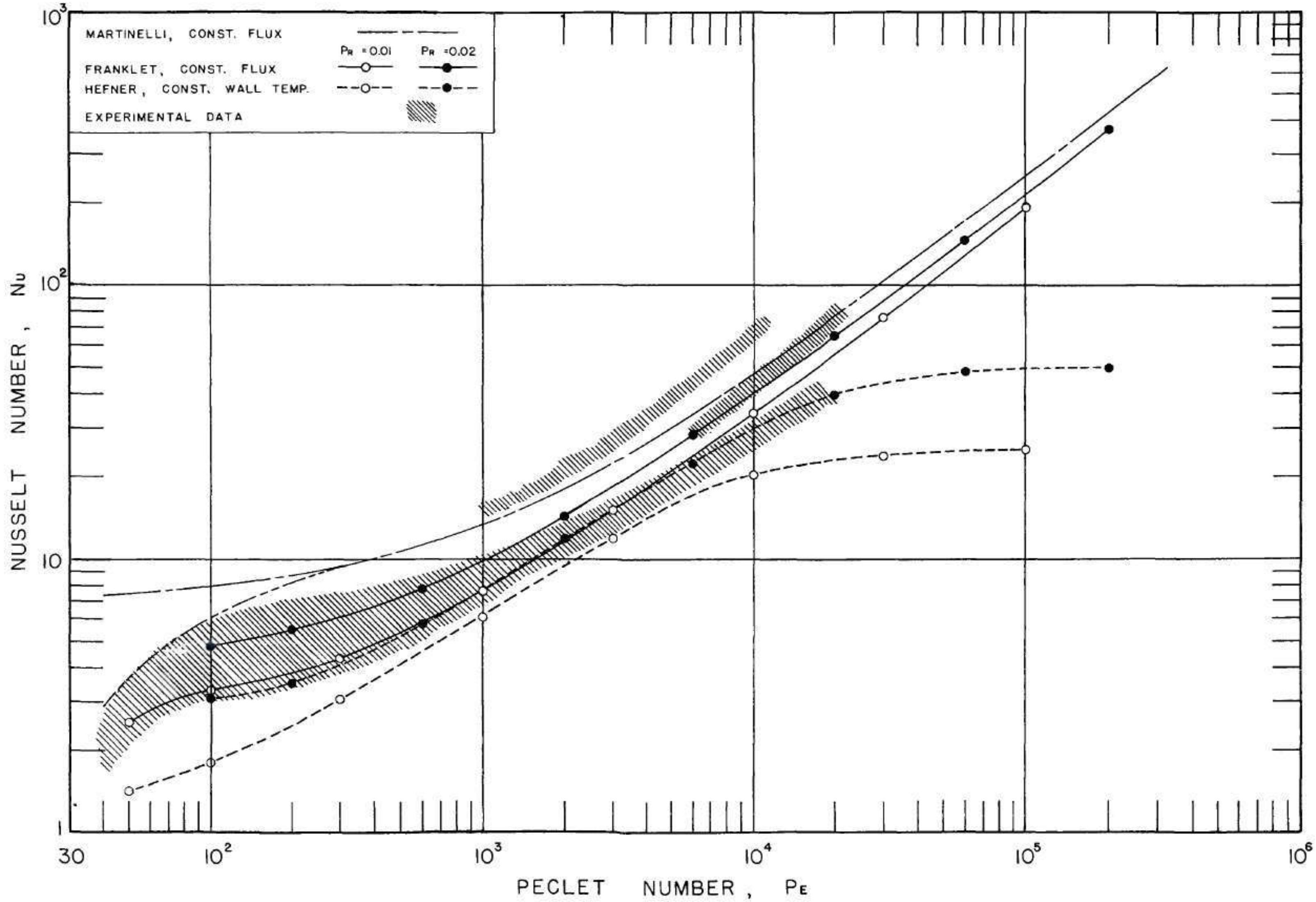

FIGURE 2. NUSSELT NUMBER AS A FUNCTION OF PECLET NUMBER FOR LOW PRANDTL NUMBERS

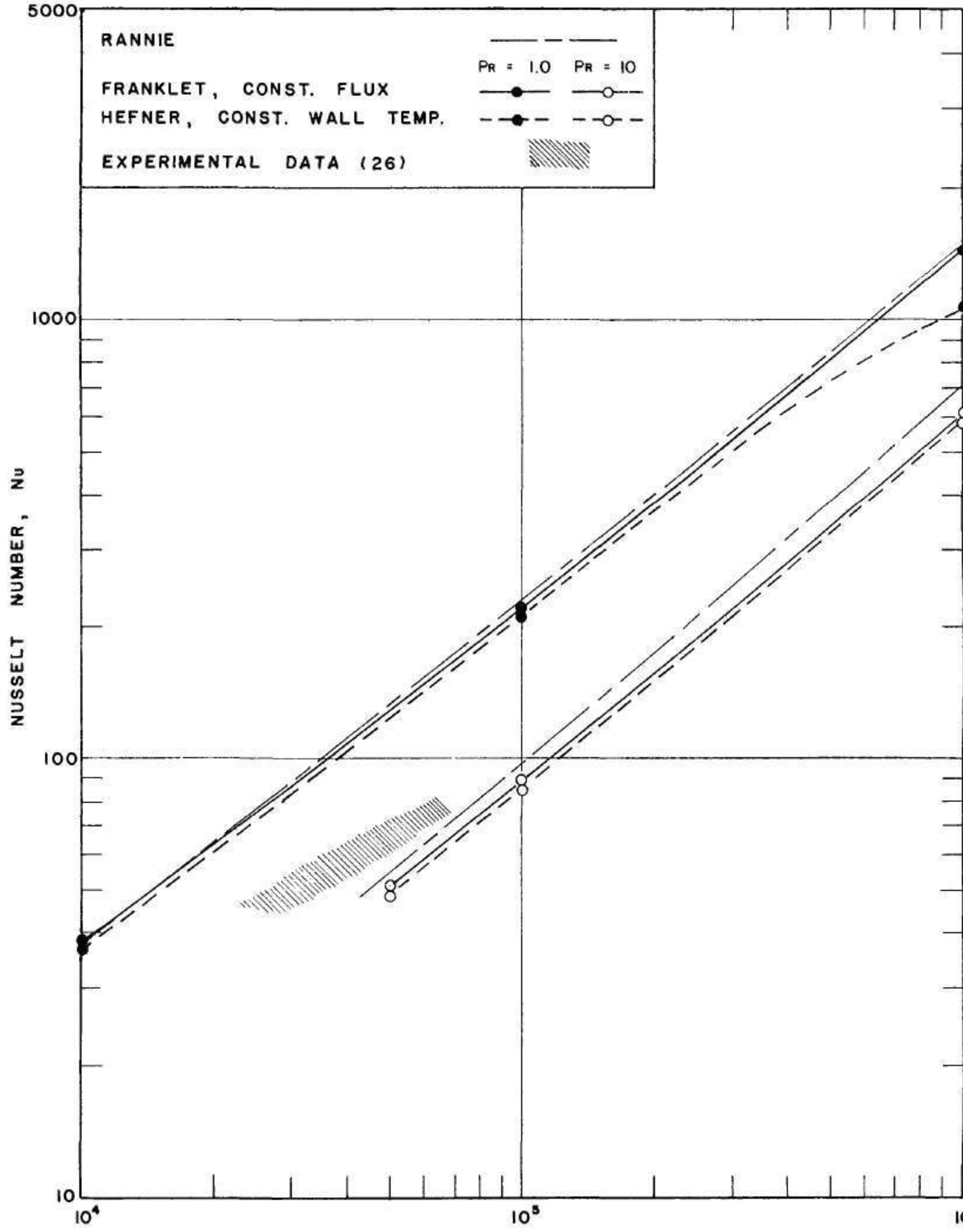

FIGURE 3. NUSSELT NUMBER AS A FUNCTION OF PECLET NUMBER FOR LARGE PRANDTL NUMBERS

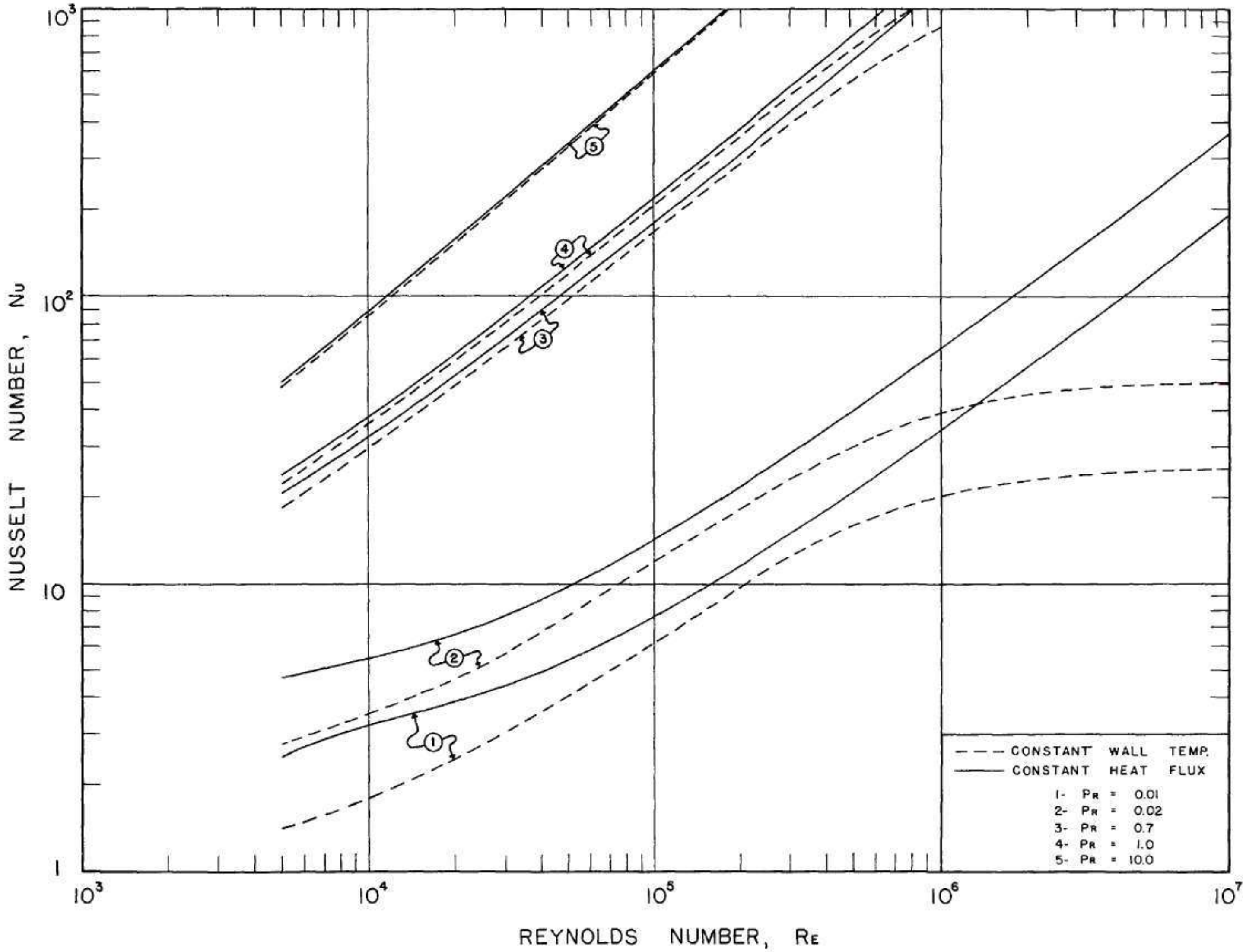

FIGURE 4. NUSSELT NUMBER AS A FUNCTION OF REYNOLDS NUMBER AND PRANDTL NUMBER

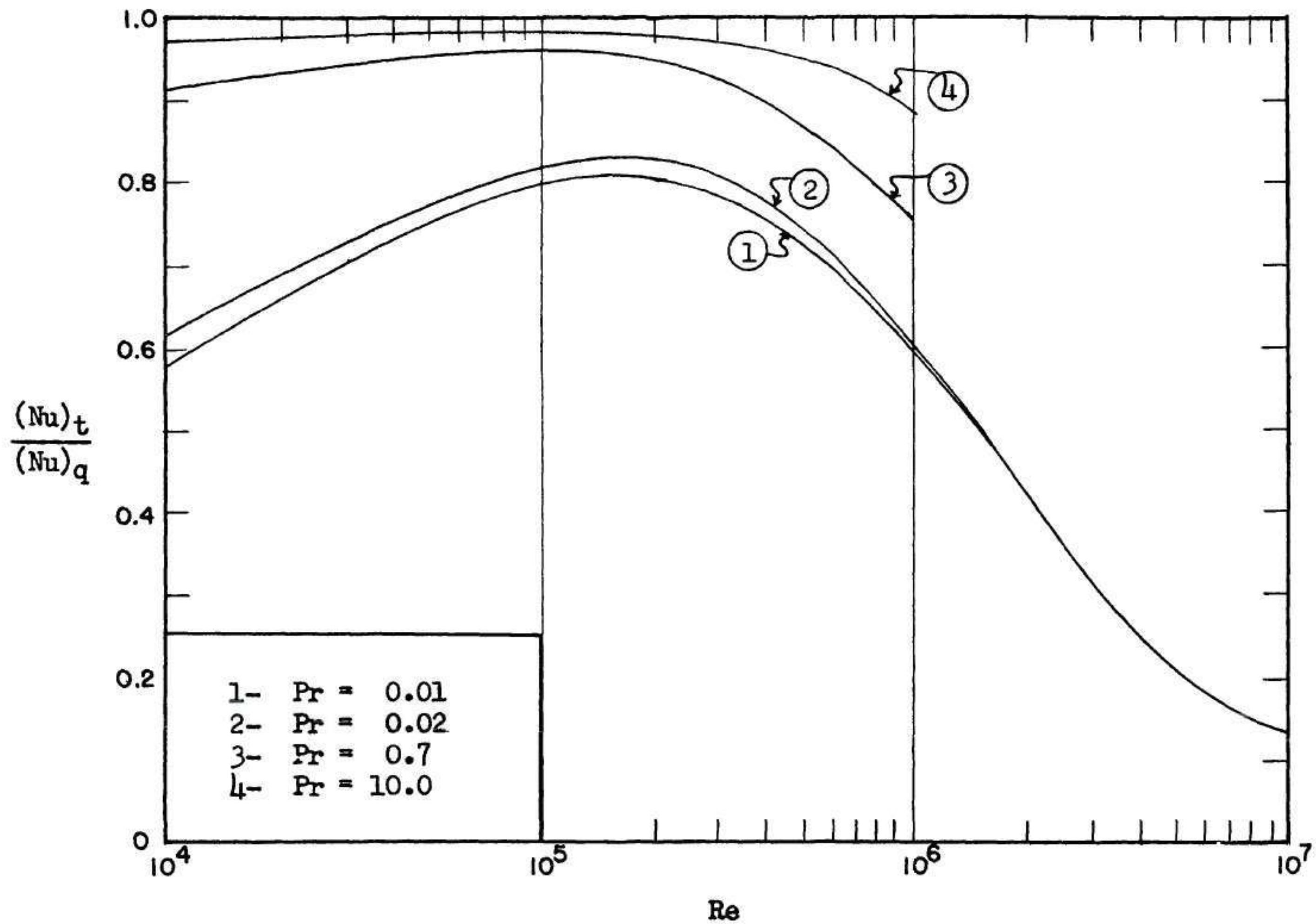

Figure 5. Ratio of Nusselt Numbers for Constant Wall Temperature to Constant Heat Rate as a Function of Reynolds Number and Prandtl Number

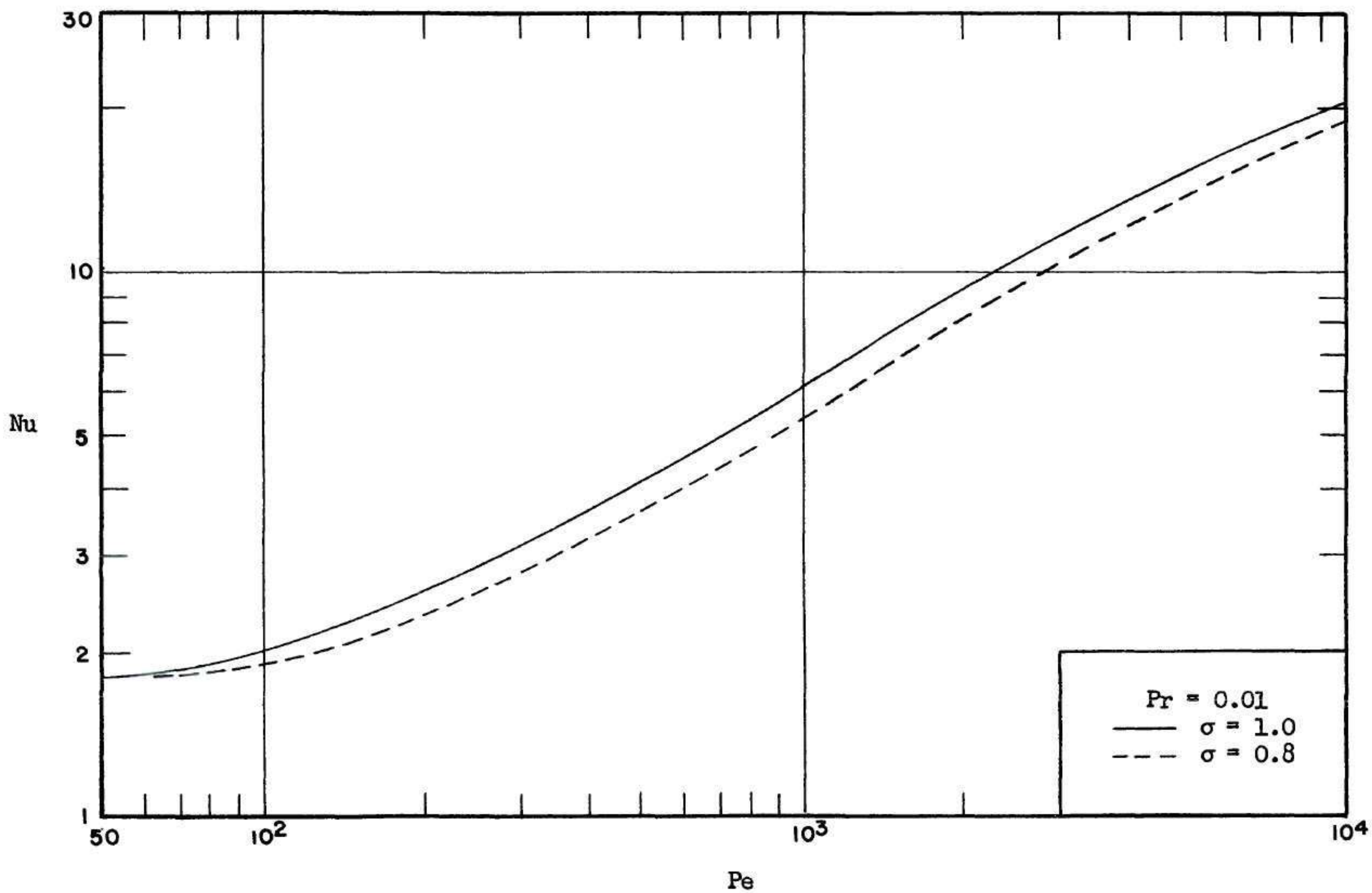

Figure 6. Nusselt Number as a Function of Peclet Number and the Ratio of Diffusivities of Heat and Momentum

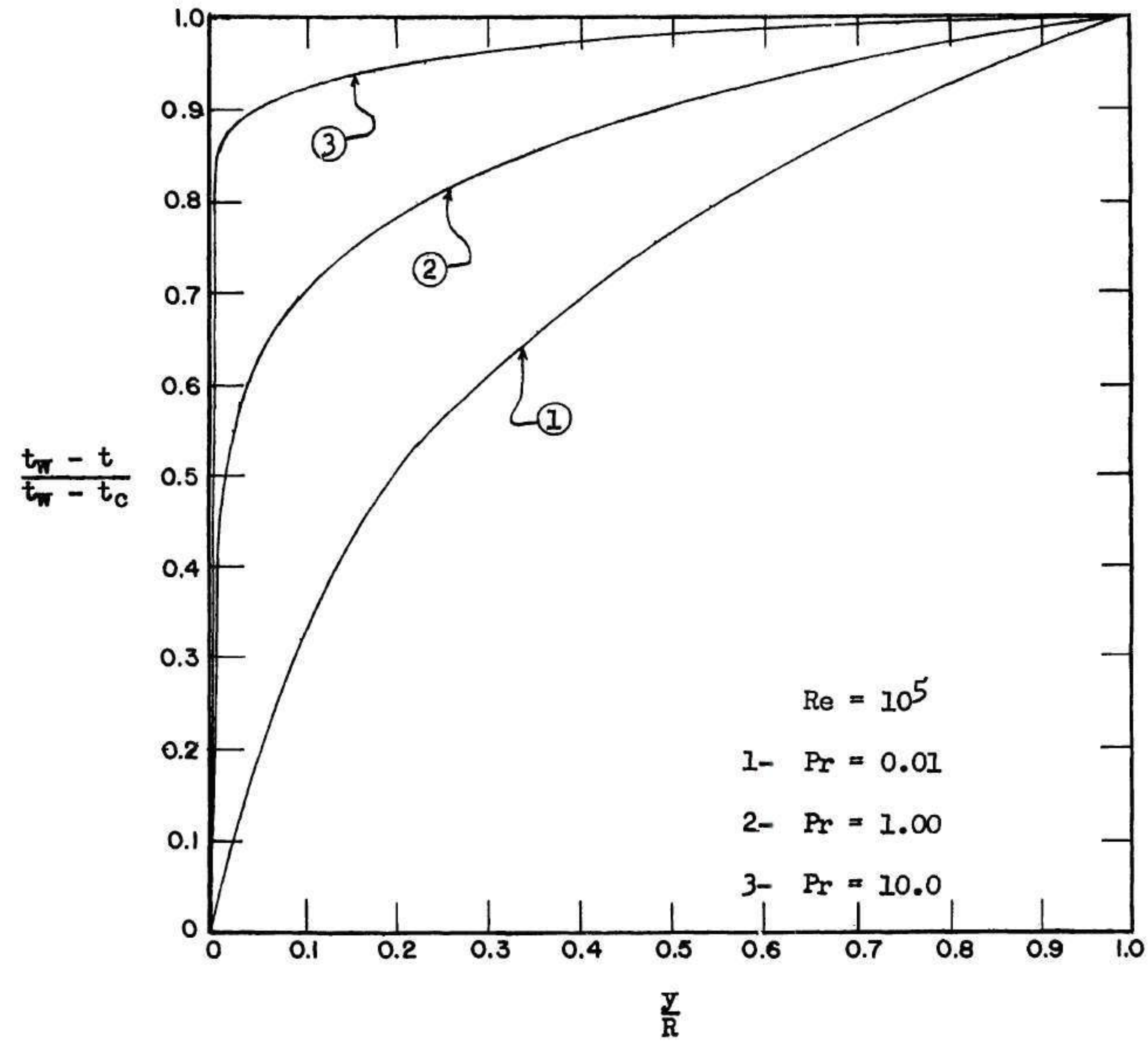

Figure 7. Temperature Distribution as a Function of Prandtl Number

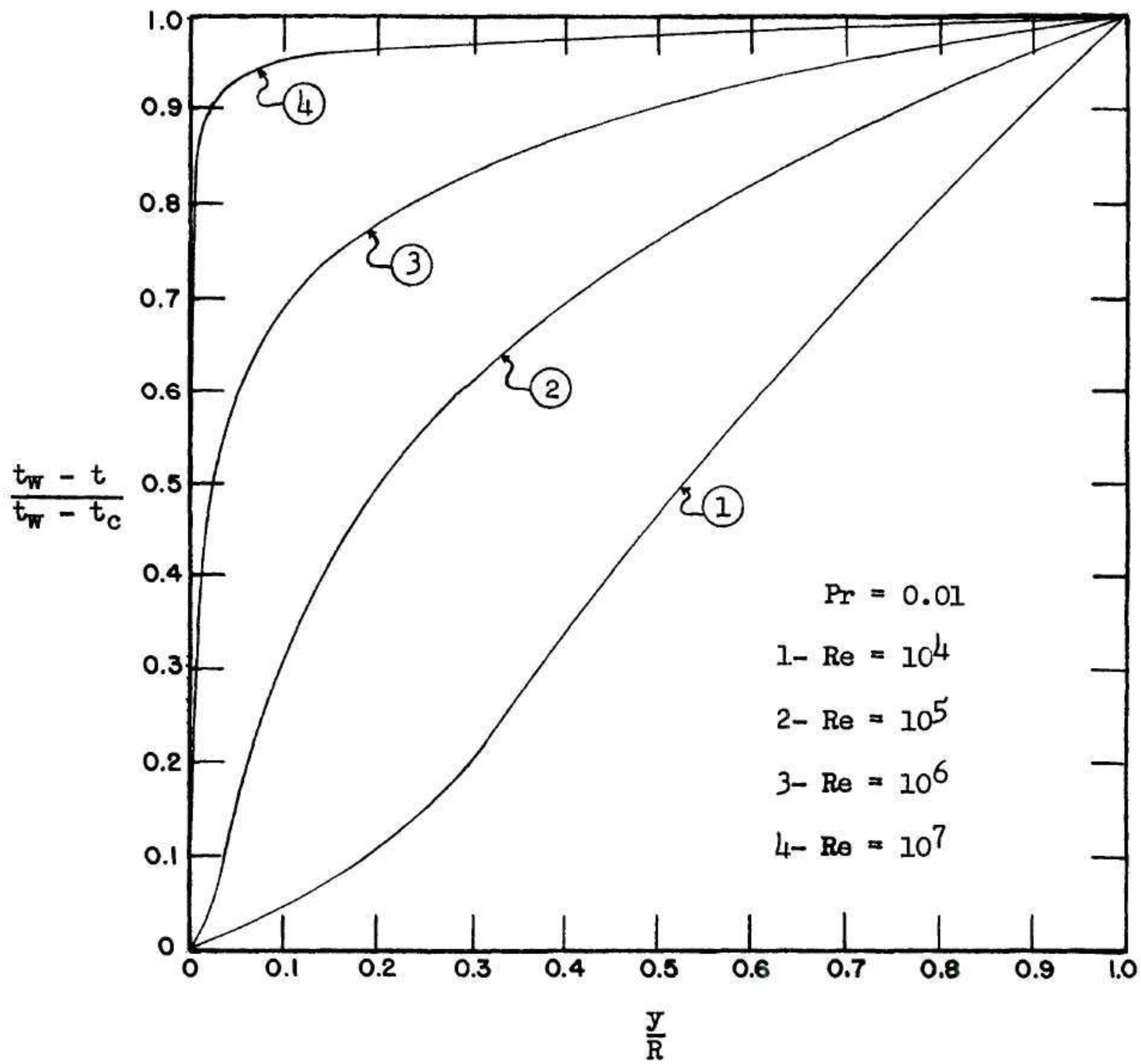

Figure 8. Temperature Distribution as a Function of Reynolds Number

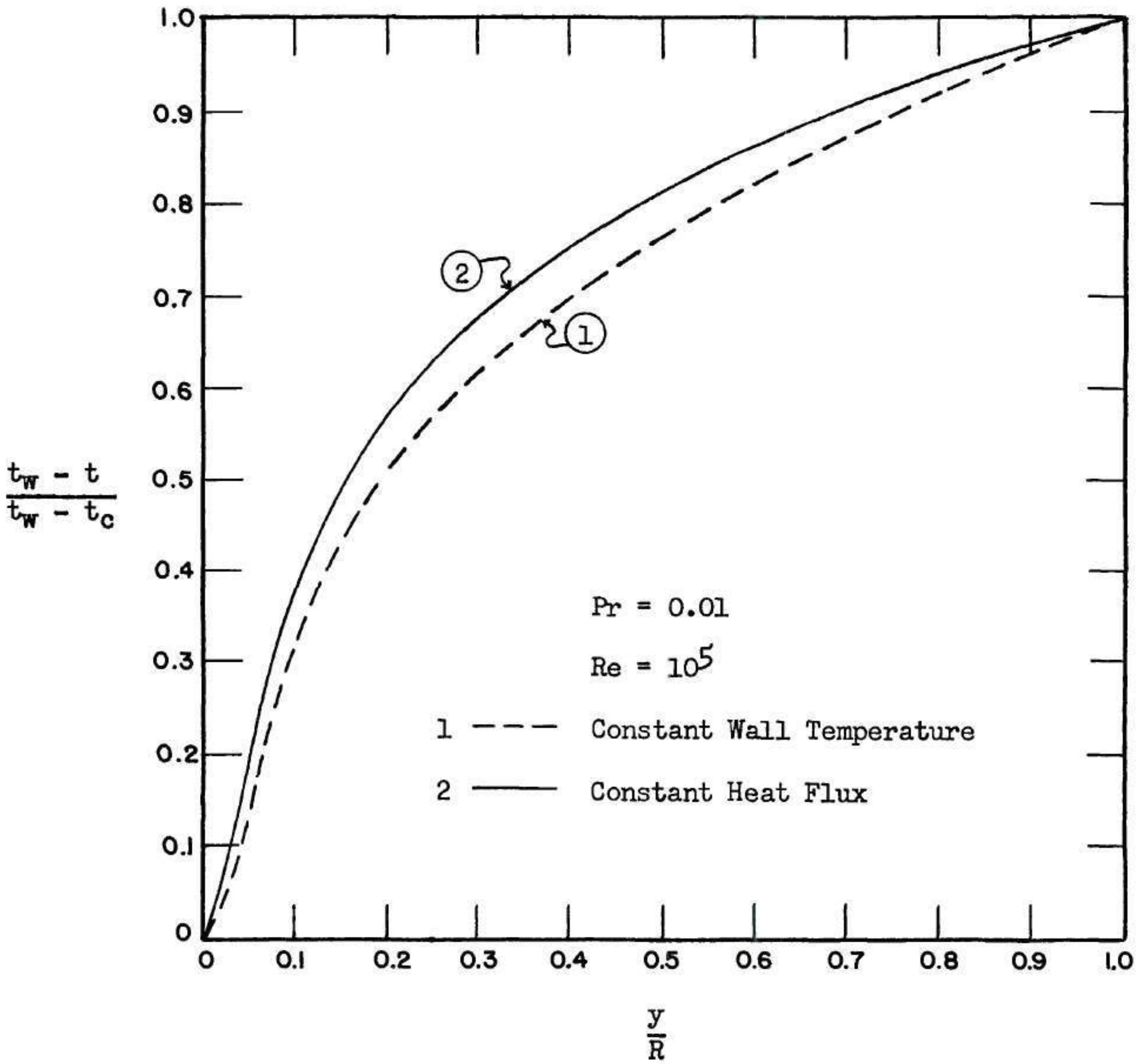

Figure 9. Temperature Distribution of Constant Wall Temperature and Constant Heat Flux

BIBLIOGRAPHY

1. Summerfield, M., "Recent Developments in Convective Heat Transfer with Special Reference to High-Temperature Combustion Chambers," Heat Transfer Symposium, University of Michigan, 156-169, (1953).

2. Jakob, M., Heat Transfer, Vol. I, John Wiley and Sons, Inc., New York, 500-521, (1956).

3. Reichardt, H., "Heat Transfer Through Turbulent Friction Layers," National Advisory Committee for Aeronautics, Technical Note 1047, (1940).

4. Seban, R. A., and Shimazaki, T. T., "Heat Transfer to a Fluid Flowing Turbulently in a Smooth Pipe with Walls at Constant Temperature," Transactions of the American Society of Mechanical Engineers 73, 803-808, (1951).

5. Martinelli, R. C., "Heat Transfer to Molten Metals," Transactions of the American Society of Mechanical Engineers 69, 947-959, (1947).

6. Franklet, D. L., Heat Transfer to Liquid Metals, Ph.D. Thesis, Georgia Institute of Technology (1958).

7. von Karman, T., "The Analogy between Fluid Friction and Heat Transfer," Transactions of the American Society of Mechanical Engineers 61, 705-710, (1939).

8. Nikuradse, J., "Gesetzmässigkeiten der Turbulenten Strömung in Glatten Rohren," Vereinigung Deutschen Ingenieurn, Verlag G.M.B.H., (1932).

9. Deissler, R. G., "Analysis of Turbulent Heat Transfer, Mass Transfer and Friction in Smooth Tubes at High Prandtl and Schmidt Numbers," National Advisory Committee for Aeronautics, Tech Note 3145, (1954).

10. Trefethen, L., "Measurement of Mean Fluid Temperatures," Transactions of the American Society of Mechanical Engineers 78, 1207-1212, (1956).

11. Ross, D., "Turbulent Flow in Smooth Pipe, A Reanalysis of Nikuradse's Experiments," The Third Midwestern Conference of Fluid Mechanics, University of Minnesota, (1953).

12. Ruth, B. F., and Yang, H. H., "An Empirical Correlation of Velocity Distribution of Turbulent Fluid Flow," Journal of the American Institute of Chemical Engineers 3, 117-120, (1957).

13. Rothfus, R. R., Archer, D. H., and Sikchi, K. G., "Distribution of Eddy Viscosity and Mixing Length in Smooth Tubes," American Institute of Chemical Engineers Journal 4, 27-32, (1957).

14. van Driest, E. R., "On Turbulent Flow Near a Wall," Heat Transfer and Fluid Mechanics Institute, Paper No. XII, University of California, (1955).

15. Deissler, R. G., and Eian, C. S., "Analytical and Experimental Investigation of Fully Developed Turbulent Flow in Air in a Smooth Tube with Heat Transfer with Variable Fluid Properties," National Advisory Committee for Aeronautics, Tech. Note 2629, (1952).

16. Reichardt, H., "The Principles of Turbulent Heat Transfer," National Advisory Committee for Aeronautics, Technical Note 1408, (1957).

17. Jenkins, R., "Variation of the Eddy Conductivity with Prandtl Modulus and Its Use in Prediction of Turbulent Heat Transfer Coefficients," 1951 Heat Transfer and Fluid Mechanics Institute, Stanford University, (1951).

18. Schlinger, W. G., Berry, V. J., Mason, J. L., and Sage, B. H., "Temperature Gradients in Turbulent Gas Streams," Industrial and Engineering Chemistry 45, 662-666, (1953).

19. Page, F. Jr., Schlinger, W. G., Breaux, D. K., and Sage, B. H., "Point Values of Eddy Conductivity and Viscosity in Uniform Flow between Parallel Plates," Industrial and Engineering Chemistry 44, 428-430, (1952).

20. Corcoran, W. H., and Sage, B. H., "Role of Eddy Conductivity in Thermal Transport," Journal of the American Institute of Chemical Engineers 2, 251-258, (1956).

21. Isakoff, S. E., and Drew, T. B., "Heat and Momentum Transfer in Turbulent Flow of Mercury," Proceedings of the General Discussion on Heat Transfer, pp. 405-409, 479-480, Institution of Mechanical Engineers, London, and American Society of Mechanical Engineers, New York; also Isakoff, S. E., Heat and Momentum Transfer in Turbulent Flow of Mercury, Ph.D. Thesis, Columbia University (1952).

22. Brown, H. E., Armstead, B. H., and Short, B. E., "Temperature and Velocity Distribution and Transfer of Heat in a Liquid Metal," Transactions of the American Society of Mechanical Engineers 79, 279-285, (1957).

23. Sleicher, C. A., Heat Transfer in a Pipe with Turbulent Flow and Arbitrary Wall-Temperature, Ph.D. Thesis, University of Michigan, (1956).

24. Rannie, W. D., Heat Transfer in Turbulent Shear Flow, Ph.D. Thesis, California Institute of Technology, (1951).

25. Chelemer, Harold, "Effect of Gas Entrainment on the Heat Transfer Characteristics of Mercury under Turbulent Flow Conditions," Ph.D. Thesis, University of Tennessee, (1955).

26. Hoffman, H. W., "Turbulent Forced Convection Heat Transfer in Circular Tubes Containing Molten Sodium Hydroxide," Oak Ridge National Laboratory 1370, (1952).

27. Johnson, H. A., Clabaugh, W. J., and Hartnett, J. P., "Heat Transfer to Mercury in Turbulent Pipe Flow," Report of U.S. Atomic Energy Commission, Contract AT-11-1-Gen 10, Project 5, Phase II, (1953).

28. Johnson, H. A., Hartnett, J. P. and Clabaugh, W. J., "Heat Transfer to Molten Lead-Bismuth Eutectic in Turbulent Pipe Flow," Final Report for U.S. Atomic Energy Commission Research, Contract No. AT-(40-1)-1061, Part 2(1951).

29. Johnson, H. A., Hartnett, J. P., and Clabaugh, W. J., "Heat Transfer to Lead-Bismuth and Mercury in Laminar and Transi- Pipe Flow," Report for U.S. Atomic Energy Commission, Contract No. AT-11-1-Gen 10, Project 5, Phase II, (1953).

30. Lubarsky, B., and Kaufman, S. J., "Review of Experimental Investigations of Liquid-Metal Heat Transfer," National Advisory Committee for Aeronautics, Technical Note 3336, (1955).

31. Seban, R. A., and Casey, D. E., "Heat Transfer to Lead-Bismuth in Turbulent Flow in an Annulus," Transactions of the American Society of Mechanical Engineers 79, 1514-1518, (1957).

32. Stromquist, W. K., Effect of Wetting on Heat Transfer Characteristics of Liquid Metals, Ph.D. Thesis, University of Tennessee, (1953).

33. Trefethen, L. M., Heat Transfer Properties of Liquid Metals, NP-1788 Technical Information Service, U.S. Atomic Energy Commission, Oak Ridge, (1950).