A HYBRID EXTERNAL MULTIDIMENSIONAL UNFOLDING APPROACH FOR THE ANALYSIS OF SELF-SIMILARITY EMOTION JUDGMENTS

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SUMMARY

A hybrid model approach to Carroll's hierarchy of preference models (1972) is presented to 1) provide a more parsimonious fit for preference judgments, 2) minimize the number of anti-ideal points that typically arise from External Multidimensional Unfolding (EMDU) models, and 3) guarantee that all model terms are statistically significant. The term "hybrid model" refers to situations in which the optimal regression model within Carroll's hierarchy has terms that are not all statistically significant, and consequently, such terms are eliminated. This elimination of terms from Carroll's original models leads to hybrid models in which alternative representations of preference may operate across stimulus dimensions. This is in stark contrast to Carroll's original models which assume that preference operates identically across all dimensions. This methodology was grounded in the idea that there may be a few interpretable anti-ideal points in an EMDU solution, but they should account for a statistically significant amount of variation in the preference responses. The new approach was applied to self-similarity judgments in the context of facial affect. Specifically, photos depicting facial emotions were scaled using multidimensional scaling of pairwise similarity judgments among photos, and then 1,564 subjects were located jointly in that same emotion space using single-photo, self-similarity judgments. When the optimal model was selected for each subject, more than 95% of these models were hybrids rather than traditional models. Additionally, this new approach reduced the number of anti-ideal points by approximately 25% by allowing these points to become vectors in the group space. The results of this research illustrate that a hybrid approach to EMDU is an intuitive extension

of Carroll's hierarchy that can provide more parsimonious fit, reduce the number of anti-deal points, and represent preferences across stimulus dimensions in a less "all-or-none" fashion.

CHAPTER 1

INTRODUCTION

External multidimensional unfolding (EMDU) is a method used to jointly represent participants and stimuli in the same multidimensional space. This is achieved by first generating a stimulus space using multidimensional scaling (MDS) techniques and then analyzing preference judgments about the stimuli to place participants jointly onto the same space. MDS techniques allow researchers to reveal a hypothetical spatial "structure" based on the perceived similarity among stimuli. Individual preference judgments are then modeled using the locations of stimuli from the resulting MDS space.

The most common EMDU technique involves the implementation of Carroll's hierarchy of preference models (1972): the general Euclidean model, the weighted Euclidean model, the simple Euclidean model, and the vector model. Each of the four models in this hierarchy utilizes a different regression equation for calculating subject parameter estimates. Four different joint spaces are possible depending on which regression model is used. An optimal model is selected for an individual's preferences based on the fit of the model relative to the other three (assessed traditionally using a series of F-tests). Three of Carroll's models are Euclidean "unfolding" models. In these models, an individual's preference is assumed to form an ideal position in the multidimensional space (otherwise known as an ideal point), and the degree to which a given stimulus is preferred is a function of its proximity to that ideal point in the space. Carroll's fourth model is a cumulative, vector model in which an individual's preference is expected to take the shape of a vector moving through the space. An individual is expected to prefer stimuli that are located farther along on the vector within the stimulus space.

The most highly parameterized model in Carroll's (1972) hierarchy is the general Euclidean model. The general Euclidean model is a nonlinear distance model which allows for preference ratings to possess nonmonotonic properties. This model estimates a subject's ideal point coordinate on each dimension utilizing the following formula:

$$\delta_{ij} = \sum_k w_{ik}^2 \left(x_{jk} - x_{ik}\right)^2 + \sum_{(k,k')} w_{ki}\, w_{k'i}\, r_{ikk'} \left(x_{jk} - x_{ik}\right)\left(x_{jk'} - x_{ik'}\right) + c_i \qquad (1)$$

where:

k and k' are dimension indices with $k \neq k'$

j is a stimulus index

i is a subject index

δ_{ij} is a measure of preference for subject i for stimulus j

w_{ik}^2 is a weight parameter for subject i on dimension k

x_{ik} is an ideal point coordinate for subject i on dimension k

x_{jk} is a stimulus coordinate for stimulus j on dimension k

$r_{ikk'}$ is a subject interaction parameter between dimensions k and k'

c_i is an additive constant for subject i

Fitting the general Euclidean model via OLS regression yields estimates of these parameters along with a measure of fit, the multiple correlation coefficient R_G. This model allows for an individual to orthogonally rotate and then weight dimensions in the group space to create their own personal space. The remaining three models are successively constrained versions of Equation 1, and thus, they form a nested hierarchy of models. For example, if the subject interaction parameters ($r_{ikk'}$) are set to zero, the model becomes identical to the weighted Euclidean model:

$$\delta_{ij} = \sum_k w_{ik}^2 \left(x_{jk} - x_{ik}\right)^2 + c_i \tag{2}$$

In this case, subjects may weight dimensions in the group space, but they may no longer orthogonally rotate the space in an individualized manner. If, in addition to constraining the subject interaction parameters, the subject weights (w^2_{ik}) are constrained to be identical across K dimensions, then the model becomes what Carroll referred to as the simple Euclidean model.

$$\delta_{ij} = \sum_k w_i^2 (x_{jk} - x_{ik})^2 + c_i \tag{3}$$

Consequently, subjects may no longer weight each dimension differently in the group space, and all dimensions are presumed to be equally salient to a given subject. In order to obtain the linear vector model, the simple Euclidean model must be further deconstructed. By squaring the parenthetical expression and rearranging the terms, an alternative formula for the simple Euclidean model is obtained:

$$\delta_{ij} = w_i^2 \sum_k x_{jk}^2 + \sum_k (-2w_i^2 x_{ik} x_{jk}) + \sum_k w_i^2 x_{ik}^2 + c_i \tag{4}$$

By defining c_i^* as

$$c_i^* = \sum_k w_i^2 x_{ik}^2 + c_i \tag{5}$$

and defining b_{ik} as

$$b_{ik} = -2w_i^2 x_{ik} \tag{6}$$

We may reparameterize the simple Euclidean model as follows:

$$\delta_{ij} = w_i^2 \sum_k x_{jk}^2 + \sum_k b_{ik} x_{jk} + c_i^* \tag{7}$$

By setting the remaining subject weight to zero (w_i^2) we obtain the linear vector model:

$$\delta_{ij} = \sum_k b_{ik} x_{jk} + c_i \tag{8}$$

In this model, subject preferences are mapped as vectors projecting through the multidimensional space.

Over the past five decades, several researchers have scaled emotion stimuli using MDS and these efforts suggest that the emotion space is a multidimensional concept (Roberts & Wedell, 1994; Russell, 1980). Russell's emotion circumplex (1980) proposed a two-dimensional emotion space that consisted of a pleasure-displeasure dimension and a level of arousal dimension. Other researchers have suggested a similar representation derived from non-MDS techniques (Heller, 1990; Schlosberg, 1952; Shaver, Schwartz, Kirson, & O'Connor, 1987). It appears that, despite a lack of common labels for the dominant dimensions in these studies, there is a common thread in that valence and activation both appear to account for a notable amount of variance in perceptions of emotion stimuli. Later studies on the scaling of emotion have suggested the existence of a third dimension. Russell and Bullock (1985) added on to Russell's (1980) original circumplex and suggested a third "assertiveness" or "boldness" dimension. Roberts and Wedell (1994) have suggested a three-dimensional space with "potency" as the third dimension which, among other things, distinguishes between anger and fear. They argued that this dimension was necessary because these two basic emotions would otherwise occupy similar positions on the emotion circumplex since they reflect approximately equal levels of both valence (negative) and activation (high).

While there is substantial evidence for a multidimensional emotion space based on previous research using MDS techniques, jointly placing an individual's perceived self on a multidimensional map of emotion has not previously been done. The possibility of a joint space using facial emotion stimuli may have applications in a number of applied research areas within the domain of psychology, such as providing an alternative method of assessing the emotional state of individuals who are unable to complete tasks requiring written or oral directions or responses (e.g., individuals who are illiterate or have been diagnosed with an Autism Spectrum Disorder). In this case, individuals may rate the degree to which emotion stimuli are similar to their own feelings at a given point in time. Such ratings might be binary ("this is like me") or graded in nature. Ratings of the similarity between a subject's own emotional state and a given emotion stimulus (i.e., self-similarity) are expected to function like traditional preference ratings in the EMDU domain. More specifically, the subject's emotional state is expected to form an ideal position in the emotion space, and the degree to which a given stimulus judged as similar to that ideal point should be related to its proximity to that ideal point in the space.

This author's earlier pilot research in which EMDU techniques were used to map self-similarity judgments onto a scaled multidimensional space of emotion faces resulted in large numbers of both ideal points and "anti-ideal" points for individuals who fit an unfolding model. Carroll (1972) describes anti-ideal points as being coordinates that indicate the location of an individual's least preferred stimuli on the associated dimension(s). These anti-ideal points occur when the weight parameters in the unfolding models (Equations 1, 2 or 3) are negative. Carroll (1972) believes that anti-ideal points are as interpretable as ideal points. Specifically, the individual prefers (or in this case, feels more similar to) stimuli that are more and more distant from the anti-ideal point in any direction on the corresponding dimension. While some may argue that the negative weights in the models are cause of concern, Carroll argues that it is not necessary to constrain these weights to be positive (Davison, 1983; Srinivasan & Shocker, 1973). This opinion is also supported in Green and Carmone (1970) who contend that there are real world situations that can be represented with anti-ideal points.

Carroll's acceptance of anti-ideal points has been the subject of debate and criticism among quantitative psychologists. For example, Davison (1983) argues that anti-ideal points should not be included in EMDU models. He states that the existence of negative weights leads to "unrealistic results," and he developed a quadratic programming approach that allows the option of constraining dimensional weights to be nonnegative (Davison, 1976). Other opponents of Carroll's incorporation of anti-ideal points in preference models include Srinivasan and Shocker (1973). Their answer to the anti-ideal point argument was to develop a model that allows the existence of multiple ideal points (rather than a single point) from which preferences emanate (Srinivasan & Shocker, 1973). Another proposed solution to anti-ideal points is to force subjects with anti-ideal points into cumulative models to remove the notion of ideal points altogether (Schiffman, Reynolds, & Young 1981).

Unfortunately, there has been no resolution to the argument for or against the inclusion of anti-ideal points in EMDU models. Many textbooks that reference EMDU typically mention the argument without proposing guidelines for researchers (Borg & Groenen, 2005; Cox & Cox, 2001). Therefore, the decision is left up to the researcher as to whether or not to allow anti-ideal points in his or her preference models. Although the appropriateness of anti-ideal points in such models is debatable, there is a substantial amount of agreement that the presence of anti-ideal points makes the interpretation of the joint space much more complicated (Davison, 1983; Green & Carmone, 1970; Srinivasan

& Shocker, 1973). Therefore, it seems wise to avoid them to the extent that they are not truly required to adequately represent preference or self-similarity data.

In the case of emotion measurement, it is tenable that self-similarity judgments may take the form of either anti-ideal points or ideal points in a multidimensional space. Anti-ideal points may exist on one dimension of the space, but possibly not for a second dimension (resulting in a function that takes the shape of a saddle point). One example of a potential instance of an anti-ideal point in an emotion space would be a scenario in which a participant reports feeling both happy and calm. In Russell's circumplex model, happy is a positive, active emotion, whereas calm is a positive, inactive emotion. Consequently the stimuli are not in the same quadrants of the emotion space, and an individual who reports feeling both happy and calm might be represented by a point between these two emotions such that the weight is positive for valence (i.e., an ideal point coordinate) and negative for activation (i.e., an anti-ideal point coordinate). It has been suggested that emotions appear to lack definitive borders that differentiate one particular emotion from another (Russell & Fehr, 1994). Additionally, most individuals tend to report feeling more than one positive emotion at one time (Watson & Clark, 1992). It is likely that in these instances anti-ideal points between positive stimuli are interpretable. A different example arises when an individual reports being both angry and afraid. Roberts & Wedell (1994) found that these two emotions were similar with respect to both valence (negative) and activation (positive), but were on opposite ends of a third dimension which they identified as potency. Thus, an individual who experienced both anger and fear within a short span of time might be represented by an anti-ideal point located between these emotions on the third dimension. Nonetheless, there are also many cases where interpretation of anti-ideal points is difficult. Indeed, such interpretational difficulties have led some researchers to dismiss anti-ideal points as relevant psychological phenomena and prevent them from occurring (Davison, 1976; Srinivasan and Shocker, 1973).

In addition to anti-ideal points, one may encounter other results from Carroll's hierarchy of preference models that are difficult to interpret. One such condition is when extreme ideal points are encountered. Coombs (1950) suggested that if estimated ideal points (or anti-ideal points) are far outside of the range of stimuli, then corresponding preferences could be fit equally well with a vector model as compared to an unfolding model. Following this logic, if an individual's location falls far outside of the stimulus range for one dimension but is located in a more moderate position with respect to a second dimension, then a vector model from Carroll's hierarchy (1972) may hold for the first dimension while an unfolding model might be appropriate for the other dimension. This would result in what may be best described as an "ideal line" where an individual's preferences are best fit by a vector model on one dimension, but a weighted Euclidean model on a second dimension. Such a model would no longer conform to Carroll's hierarchy of preference models, because his modeling scheme requires preferences to operate the same way across all dimensions. In other words, if a vector model is used to model preferences on one dimension, then a vector model must be implemented across all dimensions. The scenario described above would not fit well within Carroll's framework, but would instead be a type of "hybrid" model in which preference follows different models in Carroll's hierarchy (i.e., vector, simple Euclidean, etc.) for different dimensions of the space. These hybrid solutions may be more appropriate for modeling the multitude of ways individual's experience and perceive emotion. As an example of this hybrid model, suppose a subject was asked to indicate his or her ideal emotional state. It is easy to imagine an ideal emotion that is extremely positive (more is better), but is moderate with respect to activation (there is an ideal amount that is neither too high nor low). More generally, one can think of preferences in other contexts which might function in this manner. For example, when choosing among ideal jobs, preference may follow a vector model with respect to compensation (more is better), but may follow some type of unfolding model with respect to other dimensions (e.g., there is an ideal amount of intellectual challenge, administrative activities, managerial activities, etc.). In short, it is easy to envision variants of Carroll's modeling hierarchy which include hybrids formed by mixing his original four models across dimensions in a meaningful way.

In the context of Carroll's method, the term "hybrid model" will be used to refer to situations in which the optimal regression model within his hierarchy has terms that are not all statistically significant. When these non-significant terms are weights from the general or weighted Euclidean models, or interaction effects from the former model, then the functional process underlying preference can change across dimensions when such terms are removed. The implications of the statistical significance of parameters incorporated in Carroll's hierarchy of models have not been addressed previously in the literature; however, the interpretation of these models could drastically change if non-significant terms are eliminated from his basic models. Again, the examples pertaining to ideal emotional states and preferences for ideal jobs illustrate the types of interpretational changes that may result from eliminating model terms that are not statistically significant.

One might argue that hybrid models are not necessary because parameters in any of Carroll's models that are not statistically significant will function as though they were zero. This is not true for at least two reasons. First, when the standard error of a parameter is high, then the parameter value may be noticeably different from zero even though it is not statistically significant. Moreover, it is the value of the parameter that is subsequently used to construct the joint space with no regard to the corresponding standard error. The second reason relates to the notion of anti-ideal points. Such points arise when weights in either the simple, weighted or general Euclidean model are negative. If a weight is not statistically significant, then it is not statistically different from zero, but may fluctuate around the value of zero randomly according to the form of its sampling distribution. Thus, it may randomly assume positive or, more importantly, negative values. A negative weight that is not statistically discernable from zero must be interpreted as an anti-ideal point if it is left in the model. This leads to a very different interpretation of the very same point than if the weight had assumed a positive value that was not statistically significant from zero. There is a <u>qualitative</u> difference in the type of conclusion that is reached, yet from a statistical perspective, neither conclusion is justified because the weight may be equal to zero in the population. In instances of a statistically non-significant interaction term, a given subject's preference/self-similarity space may appear rotated from the group space orientation in cases where there is no true rotation required. This leads to yet another interpretation of the solution that may erroneously represent an individual's preferences when the ideal points generated from this solution are mapped onto a group space. To summarize this logic, if a statistically non-significant weight parameter is eliminated from one of Carroll's (Euclidean) models, then 1) a subject's preference (or self-similarity) judgments can be represented by a vector running in the direction of the associated dimension(s) and 2) this statistically limits the emergence of an anti-ideal point that is due simply to chance. With respect to the second point, the hybrid model will mitigate the emergence of anti-ideal points statistically as opposed to an estimation method that would not allow them to occur at all like that proposed by Davison (1976). In addition to the elimination of statistically non-significant weight parameters, the removal of statistically non-significant interaction parameters from Carroll's general Euclidean model will avoid unjustified individual rotations of the preference/self-similarity space.

In this author's previous pilot research, a three-dimensional MDS solution of emotion using NimStim faces was constructed. A total of 835 subject responses were fit to this solution using EMDU. Each subject was fit to one of Carroll's four models by comparing Akaike's information criterion (Akaike, 1973) across each of the models. From this analysis, several statistically non-significant parameters were discovered. These included many negative weights corresponding to anti-ideal points, and thus, the resulting joint space of individuals and emotion stimuli was not easily interpretable. This research addresses the anti-ideal points found in emotion data using Carroll's unfolding models and utilizes a different method to limit the emergence of anti-ideal points and statistically nonsignificant model parameters. This is accomplished using stepwise regression techniques to obtain hybrid models from Carroll's hierarchy in which all model parameters are statistically significant.

CHAPTER 2

METHOD

2.1 Participants

A total of 2001 undergraduate students participated in this study. Participants consisted of currently enrolled students attending the Georgia Institute of Technology who were 18 years old or older. Participation in the Psychology subject pool is required by all introductory psychology courses offered in the School. These data were collected in an ongoing, multi-year study conducted in the Psychometric Research and Development Laboratory.

2.2 Materials

The NimStim set of facial expressions developed by Tottenham, Tanaka, Leon, McCarry, and Nurse (2009) features images of actors from different ethnicities modelling a range of emotions from eight different categories (happy, sad, disgusted, fearful, angry, surprised, neutral, and calm). All photographs of the models were taken under identical conditions and each emotion was modelled using open- and close- mouthed variations of each expression. Three faces from each emotion category were selected for this study. Each of these sets features at least one European American model and at least one minority model (African American, Asian American, or Latin American). If two minority models were chosen, the ethnicities of the models had to be different. Each set of emotions included at least one male and at least one female model and no models were repeated to avoid the same model appearing twice during the pairwise comparison portion of the study. Faces depicting closed mouths were excluded for the happy, angry, and fearful emotion categories as Tottenham et al. (2009) found those emotion faces had higher validity with open mouths. In contrast, sad faces had higher validity with closed mouths, therefore sad emotion faces with open mouths were excluded.

2.3 Procedure

Data were collected across multiple sessions of up to five participants at a time using separate computers with a divider between each workstation. After the completion of the consent form, the researcher read instructions detailing the four phases of the study (paired comparisons, attribute ratings acquisition, single stimulus ratings, and the demographic survey). The paired comparisons phase of the study and the single stimulus response phase of the study were presented in a random order; however, the attribute ratings phase of the study consistently followed the paired comparisons phase to ensure that it did not prime subjects to discriminate among stimuli along any predetermined dimensions.

2.3.1 Paired Comparisons

Participants were presented with a pair of faces. The task for the participants was to rate how similar the emotions depicted in the two faces were to each other. Participants rated the similarity of the stimuli using a 9-point scale (where 1 represented very dissimilar and 9 represented very similar stimuli). Stimuli were assigned “photo numbers” which were displayed in pairs according to a previously established Ross ordering (Ross, 1934). The assignment of stimuli to photo numbers was randomized independently for each participant. Participants made a total of 276 paired comparisons during this portion of the study. At the end of these trials, 10 comparisons were repeated so that test-retest consistency could be examined for each subject.

2.3.2 Attribute Ratings

Attribute ratings were acquired by asking participants to rate the degree to which each stimulus exhibits a given characteristic (happy, sad, angry, afraid, surprised, disgusted, calm, submissive, dominant, bored, or energetic). Participants rated a given face on a 9-point scale (where 1 represents the lowest degree and 9 represent the highest degree of a given attribute). Participants made a total of 264 attribute ratings during this portion of the experiment (11 attributes x 24 stimuli) and each attribute was randomly selected and presented with all 24 stimuli in succession. The order of the 24 stimuli was determined randomly for each subject.

2.3.3 Self-Similarity Ratings

The next portion of the study asked participants to recall the emotional state they were in the previous evening. Participants were then presented with single stimuli and asked to rate how similar the emotion depicted in the face was to the emotional state they were experiencing the previous evening. Similarity judgments were made using a 6-point scale (where 1 represented an emotion that was very dissimilar to how they felt and 6 represented an emotion that was very similar to how they felt). This part of the study consisted of 24 unique trials and presented each of the NimStim faces in a randomized order. After these trials, five comparisons were repeated for the purposes of examining test-retest reliability for each subject.

2.3.4 Demographics

In the final phase of the study, participants answered demographic questions regarding gender, age, ethnicity, and social aspects of their lives such as their type of schooling, family life, and number of friends. These questions were always presented after all other types of data collection were completed.

2.4 Design

2.4.1 Multidimensional Scaling

An analysis of the pairwise similarity data collected from this study was first obtained in SPSS using the PROXSCAL procedure (Busing, Commandeur, & Heiser, 1997). Responses used in this analysis were limited to those that passed a test-retest reliability check ($r > 0.6$) where participant responses were correlated across the 10 repeated stimulus pairs. As multidimensional scaling uses dissimilarity judgments to approximate distances between stimuli, the similarity data were transformed into dissimilarities by the program before the analysis was performed (Kruskal & Wish, 1978).

This analysis provided the group-level emotion space coordinates for the stimuli. A Procrustes rotation (Schonemann & Carroll, 1970) of the initial solution was completed to better align the resulting MDS axes to those that are traditionally presented (i.e., valence and activation; Russell, 1980). Following this, 11 multiple linear regressions were performed using the mean response of the attribute ratings for each stimulus as the dependent variable and the MDS coordinates of each stimulus as the independent variables. The regression weights from each equation were normed to produce direction cosines for each attribute. These cosines provided the coordinates of vectors that illustrate which direction in the space best represents each attribute.

2.4.2 External Multidimensional Unfolding

After examining the test-retest reliability of the responses to the self-similarity portion of the study (by correlating responses to five repeated trials), the data from participants with reliable ($r > 0.6$) responses were analyzed using Carroll's four proposed models as well as stepwise regression variants of those models. Specifically, each of Carroll's models served as the starting point for the stepwise regression procedure. The relative fit of all starting and resulting models was compared. In some cases, the resulting model was not one of Carroll's four models because some of the terms were missing. These cases will be referred to as hybrid models. These models were fit independently for each subject and implemented using SAS software. SAS gives the user the option to choose between forward selection regression, backward elimination regression, and stepwise regression. Of these three options, backward elimination regression was implemented for the purposes of this study. Backward elimination regression begins with every predictor specified in the model. The predictor with the least contribution to the model (i.e., the predictor with the largest p-value) is removed during each successive step and this process continues until no further predictors are non-significant. In this case, after a predictor is removed from the regression model, it will not be included in the following steps. The backwards elimination method is preferred to other techniques for exploratory analyses (Kleinbaum et al., 2008). Backwards elimination typically results in more saturated models, and in comparison to forwards selection or stepwise techniques, there is less risk of making a Type II error by excluding predictors involved in suppressor effects (Field, A., Miles, J., & Field, Z., 2012). Suppressor effects refer to instances where more error variance in the model is accounted for by a parameter that does not have a significant relationship to the criterion variable, but to another parameter in the model (Field et al., 2012). It was theorized that the backwards elimination procedure would provide solutions that tend to favor Carroll's unfolding Euclidean models over his vector model because these are more highly parameterized models, and the backwards elimination model begins with the most parameterized model and eliminates terms in succession. Therefore, the backwards elimination procedure does not inadvertently preclude an unfolding model simply due to the methodological steps involved in this regression approach. Given that an unfolding mechanism is presumed to underlie at least some self-similarity judgments, this strategy seemed prudent.

2.4.2.1 Fit Indices

Carroll (1972) suggested that F-tests be used for comparing the fit of his four proposed models; however, these F-tests are only plausible when the models are nested. Therefore, it is no longer appropriate to use these techniques when using stepwise regression techniques resulting in hybrid models. Other measures of model fit that were examined were Akaike's information criterion (Akaike, 1973), Schwarz's Bayesian information criterion (Schwarz, 1978), the consistent Akaike's information criterion (Bozdogan, 1987) (commonly referred to as AIC, BIC, and CAIC, respectively), and the adjusted R-squared index, which has been used in previous research to compare preference models (Cooper & Nakanishi, 1983). These measures each include a penalty function that is based on the number of parameters in the model. The particular form of the penalty function varies across indices with adjusted R-squared penalizing the least and CAIC penalizing the most. The penalty for the general Euclidean model would be larger than the penalty for the weighted Euclidean model because the general model includes $K*(K-1)/2$ more parameters, where K is the number of dimensions. Similarly, the penalty for the weighted Euclidean model would be larger than that for the simple Euclidean model which, in turn, would be larger than that for the vector model. As might be expected, these indices did not generally favor the same model due to the number of parameters having an impact on how the indices were calculated. Additionally, there was no discernable way to interpret whether or not there were substantial differences between models using AIC, CAIC, and adjusted R-squared, which makes it more difficult to assess relationships present in the data. As it is hypothesized that there may be both solutions with up to nine parameters as well as solutions with zero parameters, it was necessary to compare the measures of fit to ascertain which of the indices allowed for both of these types of solutions. After a comparison of the different fit indices that were explored here, the BIC index was chosen as the most favorable index to use for the present analysis as this index did not tend to heavily favor either unfolding or vector models while still minimizing the total number of negative weights in a chosen model. Following the selection of an optimal backwards elimination regression model, individuals' self-reported emotions were mapped onto a group preference stimulus space using their resulting model.

2.4.2.2 Ideal Point Calculation

The individual-level coordinates were computed for each dimension. When the BER approach is used, preferences can be modeled as either cumulative or unfolding on any given dimension. Thus, it was necessary to generalize Carroll's original solutions for ideal points and vectors to this more general case.

2.4.2.2.1 Carroll's original method

A traditional MDS analysis results in a (J x K) matrix X that gives the locations for the jth stimulus on K dimensions (i.e., the coordinates of stimuli in the multidimensional space). For the general Euclidean model, it is assumed that both the known matrix of stimulus coordinates and the unknown (K x 1) matrix of ideal point coordinates for the ith subject (denoted as matrix Y) are rotated in the K dimensional space by an orthogonal (K x K) transformation matrix (denoted matrix T). These new axes are then stretched or shrunk in scale by a diagonal (K x K) weight matrix W_i^*. The squared distance between person i and stimulus j is computed from the resulting transformed values:

$$d_{ij}^2 = X_j T_i W_i^* T_i' X_j' - 2Y_i T_i W_i^*{}_i T_i' X_j' + Y_i T_i W_i^* T_i' Y_i' \qquad (9)$$

By defining R_i* as

$$R_i^* = T_i W_i^* T_i' \qquad (10)$$

and allowing c_i^* to represent the last term in Equation 9, Carroll reparameterizes the squared distance as:

$$d_{ij}^2 = X_j R_i^* X_j' - 2Y_i R_i^* X_j' + c_i^* \qquad (11)$$

Let δ_{ij} be the self-similarity judgment (or preference judgment as the case may be) from person i to stimulus j after reverse scoring the judgment so that higher numbers reflect less similarity (or less preference). In each of his Euclidean models, Carroll assumes that δ_{ij} is a linear function of the associated squared distance:

$$\delta_{ij} = a_i d_{ij}^2 + b_i + e_{ij} \qquad (12)$$

where a_i and b_i are arbitrary constants and e_{ij} is an error term. It is possible to solve for the *K* ideal point coordinates for the *i*th individual by substituting in Equation 11 and simplifying so that:

$$\delta_{ij} \approx X_j R_i X_j' + B_i X_j' + c_i \tag{13}$$

where:

$$R_i = a_i R_i^* = T_i a_i W_i^* T_i' = T_i W_i T_i' \quad , \tag{14}$$

$$B_i = -2Y_i R_i \quad , \tag{15}$$

and

$$c_i = a_i c_i^* + b_i + e_{ij} \tag{16}$$

Equation 13 is a re-parameterized version of the general Euclidean model previously defined in Equation 1. It illustrates the quadratic relationship that is assumed to operate between preference ratings and MDS stimulus coordinate values. The values of R_i and B_i can be estimated using a simple multiple regression technique. As an example, δ_{ij} can be regressed on the *K* coordinate locations for stimulus *j*, the *K* squared coordinate locations, and the *K*(*K*-1)/2 cross-products of coordinate locations as follows:

$$\delta_{ij} = b_0 + \sum_{k=1}^{K} b_{1k} x_{jk} + \sum_{k=1}^{K} b_{2k} x_{jk}^2 + \sum_{k=1}^{K} \sum_{k'>k}^{K} b_{3kk'} x_{jk} x_{jk'} + e_{ij} \tag{17}$$

The regression weights from Equation 17 can be used to complete matrices R_i and B_i. Specifically, the *K* values of b_{1k} can be used to form the rows of column vector B_i, and the *K* values of b_{2k} can be placed on the main diagonal of the square, symmetric matrix $R_{i..}$, The off-diagonal elements of R_i are then set to one-half of the corresponding $b_{3kk'}$ estimates. Once R_i and B_i have been set to fixed values, it is then possible to solve for the ideal point coordinates of each subject using the following equation:

$$Y_i = -\tfrac{1}{2} R_i^{-1} B_i . \tag{18}$$

In Carroll's (1972) paper, he mentions that the weighted Euclidean, simple Euclidean, and vector models are nested within the general Euclidean formula and can easily be obtained by following the different assumptions previously discussed for each model. For example, in the case of the weighted Euclidean model, the orthogonal transformation matrix T_i is constrained to be an identity matrix for each individual. Consequently, the R_i matrix is simply the W_i matrix mentioned in Equation 14. The simple Euclidean model is a special case of the weighted Euclidean model where each dimension is weighted the same amount. For this model, the weights on the diagonal matrix W_i are identical across dimensions, and, again, matrix W_i is equal to matrix R_i. However, in the solution for the vector model, Carroll explains that any quadratic terms included in the regression equations for the unfolding models are effectively set to zero, and he also removes the -1/2 term included Equation 18. Operationally, this amounts to setting W_i, and hence R_i, equal to a null matrix. Carroll then presents a linear regression equation to solve for vector coordinates:

$$\delta_{ij} = b_0 + \sum_{k=1}^{K} b_{1k} x_{jk} + e_{ij} \tag{19}$$

Clearly, a solution for individual vector coordinates cannot be derived from Equation 18 without some redefinition of terms. However, one can easily solve for <u>either</u> individual ideal point or vector coordinates using a slight modification of Carroll's original formula:

$$Y_i = Q_i \tilde{R}_i^{-1} B_i = Q_i [T_i (W_i + H_i) T_i']^{-1} B_i \,, \tag{20}$$

where:

Q_i is a KxK diagonal matrix with elements equal to $(-1/2)^U$ on the main diagonal,

U is a dummy variable that is equal to 0 in the case of a vector model and 1 when a solution for a

Euclidean model is required, and

H_i is set equal to an identity matrix in the case of a vector model, otherwise it is set to a null matrix.

In the case of a vector model, W_i is null matrix and H_i is an identity matrix, as is $T_i(W_i + H_i)T_i'$. Additionally, Q_i is equal to an identity matrix. If a solution to one of Carroll's Euclidean models is desired, then W_i contains the appropriate weights, H_i is a null matrix, and Q_i has elements equal to -1/2 on the main diagonal. When this alternative equation is used along with Carroll's original definitions for T_i and W_i, one can easily derive individual coordinates for either a Euclidean or vector model with a single equation.

2.4.2.2.2 Extending Carroll's method to a hybrid approach

Recall that Carroll's original technique required that one of his four preference models must operate across <u>all</u> K dimensions from the MDS solution. In contrast, the hybrid approach described in this thesis explicitly allows for preference models to differ across dimensions. Solving for individual parameters in Y_i that vary in nature (i.e., represent a vector coordinate or a Euclidean coordinate) across dimensions is easily accomplished using Equation 20 along with more general definitions of Q_i and H_i. Specifically, let:

U_k be a dummy variable that is equal to 0 if the coordinate on dimension k refers to a vector model coordinate and 1 if it refers to a Euclidean model coordinate,

$$Q_i = \begin{bmatrix} \left(\frac{-1}{2}\right)^{U_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \left(\frac{-1}{2}\right)^{U_k} \end{bmatrix}, \text{ and} \tag{21}$$

$$H_i = \begin{bmatrix} 1 - U_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1 - U_k \end{bmatrix}. \tag{22}$$

Note that the preceding definitions are equal to those following Equation 20 in the case of Carroll's four original models. However, the nature of W_i is more complicated. For a vector model dimension, the corresponding diagonal element in W_i is equal to zero, whereas it is nonzero for a Euclidean model dimension. In the case of a hybrid model, matrix R_i is built from regression coefficients and any missing coefficients are treated as zero. In order to add H_i to W_i, it is necessary to decompose R_i into eigenvectors and eigenvalues so that:

$$R_i = E\Lambda E' = T_i W_i T_i' \tag{23}$$

where:

E is a KxK matrix of eigenvectors

Λ is a KxK diagonal matrix of eigenvalues

Following this decomposition, matrix H_i can be added to W_i as in Equation 20 and the individual coordinates for Y_i can be calculated. By creating one equation that subsumes the four traditional models as well as any combination of hybrid models, the computational demands for calculating ideal point coordinates are streamlined and more efficient.

CHAPTER 3

RESULTS

3.1 Multidimensional Scaling

3.1.1 Analysis of Pairwise Similarity Judgments

Out of 2001 subject responses, 1357 reliable subject responses met the criterion ($r > 0.6$) to be used in this analysis. The initial analysis of pairwise data resulted in the normalized raw stress values for configurations of up to six dimensions. The plot of the normalized raw stress index from each dimension suggests a point of diminishing returns around the third dimension (see Figure 1). This suggests that a three-dimensional solution optimally minimizes normalized raw stress and adding more dimensions would not substantially decrease the raw stress value. The normalized raw stress value for the third dimension was equal to 0.00779 (see Table 1). There is no standard cut off for normalized raw stress; however, it is notable that minimized Stress-I values of less than .1 are traditionally acceptable. The Stress-I value, though not minimized by PROXCAL, dropped below .1 to 0.08828. It is interesting to note that there appears to be a local maximum on the sixth dimension, as the normalized raw stress value <u>increases</u> by approximately 0.0004. A secondary PROXCAL analysis was run using a Torgerson starting configuration. This configuration eliminated the presence of the local maximum yielded from the Simplex configuration and also suggested that a three-dimensional solution was appropriate for these data.

Table 1

MDS Stress Values for Six Dimensions

Number of Dimensions	Normalized Raw Stress	Stress-I
1	0.23198	0.48164
2	0.04982	0.22322
3	0.00779	0.08828
4	0.00370	0.06081
5	0.00141	0.03750
6	0.00187	0.04327

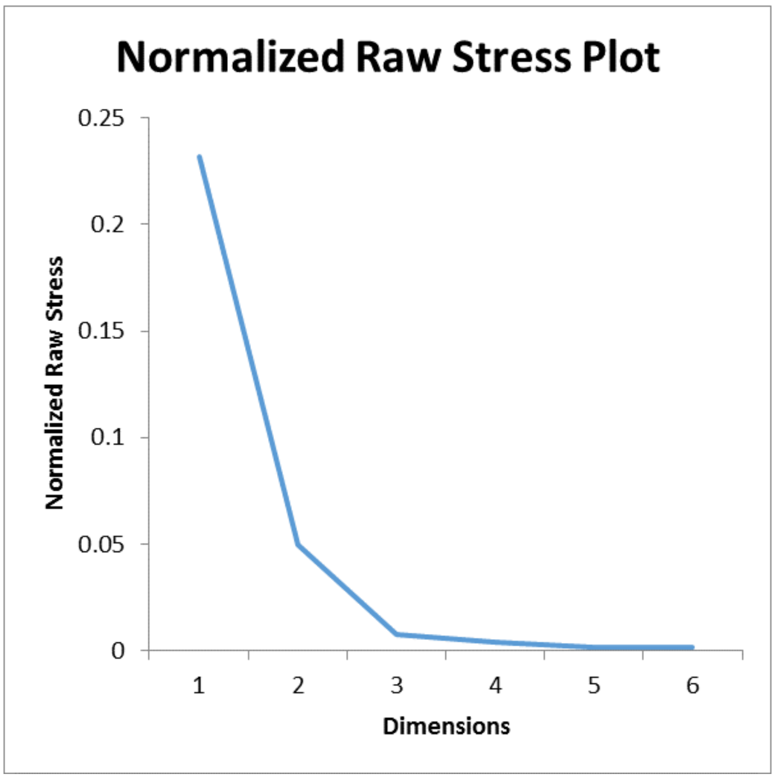

Figure 1. Normalized raw stress measures corresponding to the dimensionality of MDS configurations

Another analysis using PROXCAL was completed to extract only a three-dimensional solution using a Simplex starting configuration. This analysis yielded a normalized raw stress value of 0.00621 and a Stress-I value of 0.07883. These values differ from the initial analysis because PROXCAL begins estimating coordinates for the configuration with the highest dimensionality and then uses these estimates as initial values for configurations with a successively smaller number of dimensions. Therefore, the two analyses had differing starting values. In addition to the stress values, the plot of the transformed proximities by the distances yielded a linear pattern with small residuals, suggesting that this three-dimensional solution was a good fit for the data (see Figure 2).

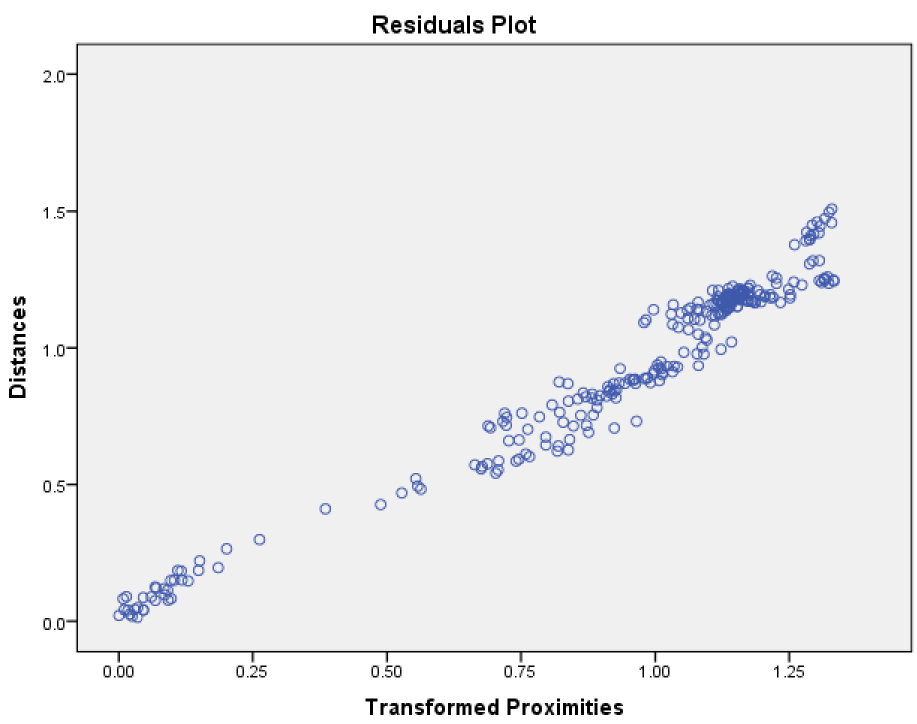

Figure 2. Transformed proximities vs. estimated distances

This analysis provided the final common space coordinates for the three-dimensional emotion space. To further validate the configuration produced from Simplex starting values, a three-dimensional solution was extracted using Torgerson starting values. The coordinates yielded from this configuration were nearly identical to the configuration extracted using Simplex starting values. Therefore, the coordinates yielded from the Simplex configuration were used for this study. A Procrustes rotation (Schonemann & Carroll, 1970) of the first two dimensions of the solution was completed to better align the resulting MDS axes to those that are traditionally presented (i.e., valence, activation; Russell, 1980). This increased the interpretability of the configuration. The final rotated coordinates are listed in Table 2. Because a three dimensional map of the clusters of face stimuli appears convoluted and difficult to read (see Figure 3), three two-dimensional maps have been provided in Figures 4, 6, and 8 for ease of interpretation. These two-dimensional maps ignore one of the three coordinates in the three-dimensional solution.

Following this, a total of eleven multiple linear regressions were performed using the three-dimensional MDS coordinates of each stimulus as the independent variables and the mean response of the attribute ratings for each stimulus as the dependent variable. The regression weights from each equation were normed to produce direction cosines for each attribute (see Table 3). As one proceeds in this direction through the space, stimuli along that direction will reflect more of the given attribute. These direction cosines were plotted as vectors overlaid on the MDS stimulus space in Figures 5, 7, and 9.

Table 2

Final Coordinates for the MDS Solution

Face	Final Coordinates Dimension 1	2	3
Angry1	-0.327	0.061	-0.707
Angry2	-0.312	0.061	-0.718
Angry3	-0.298	0.054	-0.738
Calm1	0.449	-0.442	0.034
Calm2	0.419	-0.464	0.022
Calm3	0.376	-0.524	0.060
Disgusted1	-0.606	0.041	-0.193
Disgusted2	-0.590	0.128	-0.072
Disgusted3	-0.519	0.243	-0.200
Afraid1	-0.346	0.360	0.326
Afraid2	-0.308	0.448	0.314
Afraid3	-0.233	0.449	0.358
Happy1	0.799	0.204	-0.163
Happy2	0.796	0.165	-0.175
Happy3	0.784	0.165	-0.135
Neutral1	0.191	-0.577	0.096
Neutral2	0.309	-0.560	0.055
Neutral3	0.306	-0.550	0.066
Sad1	-0.436	-0.360	0.364
Sad2	-0.510	-0.374	0.317
Sad3	-0.416	-0.388	0.388
Surprised1	0.212	0.615	0.224
Surprised2	0.125	0.615	0.246
Surprised3	0.133	0.630	0.234

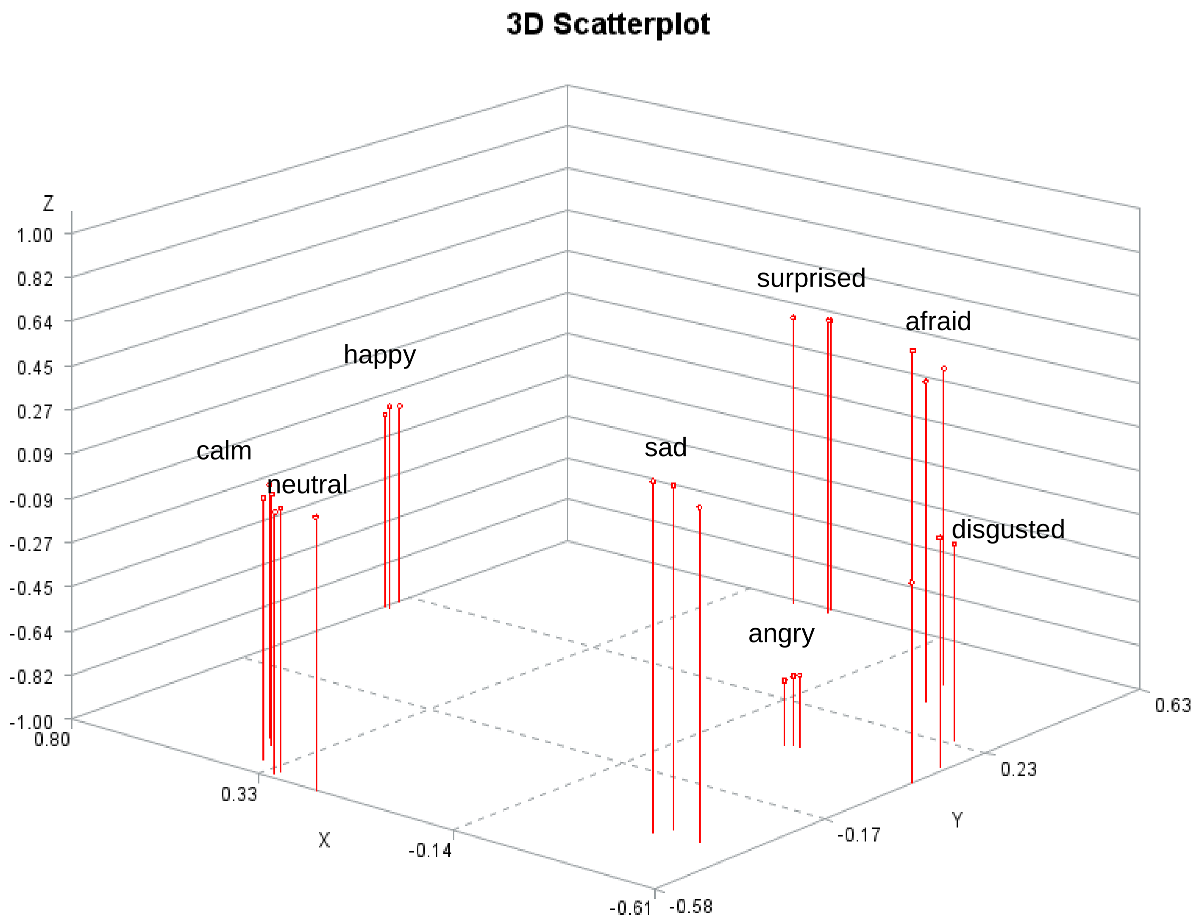

Figure 3. Three-dimensional MDS configuration with stimulus clusters labeled

Table 3
Attribute Vectors from the MDS Solution

Attribute	Vector Length	R^2	X-coordinate	Y-coordinate	Z-coordinate
Happy	0.955	0.872	0.978	0.208	-0.017
Sad	0.963	0.855	-0.564	-0.657	0.500
Afraid	0.824	0.716	-0.424	0.616	0.664
Surprised	0.925	0.851	0.025	0.914	0.406
Angry	0.946	0.885	-0.484	-0.107	-0.869
Disgusted	0.729	0.557	-0.954	0.235	-0.186
Calm	0.901	0.894	0.606	-0.781	0.155
Submissive	0.960	0.922	0.001	-0.580	0.815
Dominant	0.942	0.896	0.257	-0.185	-0.949
Energetic	0.985	0.882	0.565	0.758	-0.325
Bored	0.857	0.774	0.260	-0.941	0.218

3.1.2 Interpretation of MDS Solution

The plot of dimensions 1 and 2 (Figure 4) reveals a configuration with dimensions of positive-negative affect and degree of arousal. This plot strongly resembles Russell's emotion circumplex (1980). Russell's emotion circumplex (1980) displays all emotions in the shape of a circle. More positive emotions (e.g., happy and calm) are located farther on the positive end of the X-axis and negative emotions (e.g., disgusted, angry, sad, etc.) are found farther on the negative end of the X-axis. In addition, more aroused emotions (e.g., surprise) follow the positive end of the Y-axis and less aroused emotions (e.g., bored) follow the negative end of this axis. In this plot, angry was found to be closer to the center of this circle (Figure 4). However, this is due to the fact that the solution is multidimensional, and the cluster of angry stimuli is better represented as protruding outward in a sphere-like shape due to the third dimension.

Figure 5 shows the eleven attribute vectors mapped onto the plot of dimensions 1 and 2. The vectors for emotions with positive and negative valence very clearly point in different directions on the horizontal axis. It is also of note that attributes that would be described as "fast" (i.e., energetic and surprised) point in one direction on the vertical axis, while "slow" attributes point in the opposite direction. The vectors for the attributes angry, submissive, and dominant are relatively short in this configuration, implying that the first two dimensions of the MDS solution do not account for these attributes as well as the others.

The dimensions of positive-negative affect and submissiveness are portrayed in the plot of dimensions 1 and 3 (Figure 6). Once again, we see dimension 1 teasing apart valence. Most of the clusters of emotion faces are gathered towards the center of the axis on dimension 3 with the exception of angry, which is on the extreme end of the axis. The attribute vectors for dominant and submissive help define the axes on the third dimension. Although they do not perfectly align on the axis, these attribute vectors do appear to be what is defining this dimension (Figure 7). Moreover, it is clear that, among other things, this dimension distinguishes between anger and fear.

The plot of dimensions 2 and 3 reveal the dimensions of degree of arousal and submissiveness (Figure 8). Once again, dimension 3 distinguishes anger and fear. Dominant and submissive attribute vectors are still on opposing sides of the axis, and in this configuration angry and afraid are also on opposing sides of this third dimension (Figure 9). On the horizontal axis, which represents degree of arousal (i.e., activation), it can be seen that angry and afraid stimuli differ in their levels of activation. Afraid appears to be a more active emotion relative to anger. Interestingly, it can be noted that the dominant and submissive vectors which also define the third dimension differ in levels of activation. Dominant and angry vectors are in similar locations on the activation

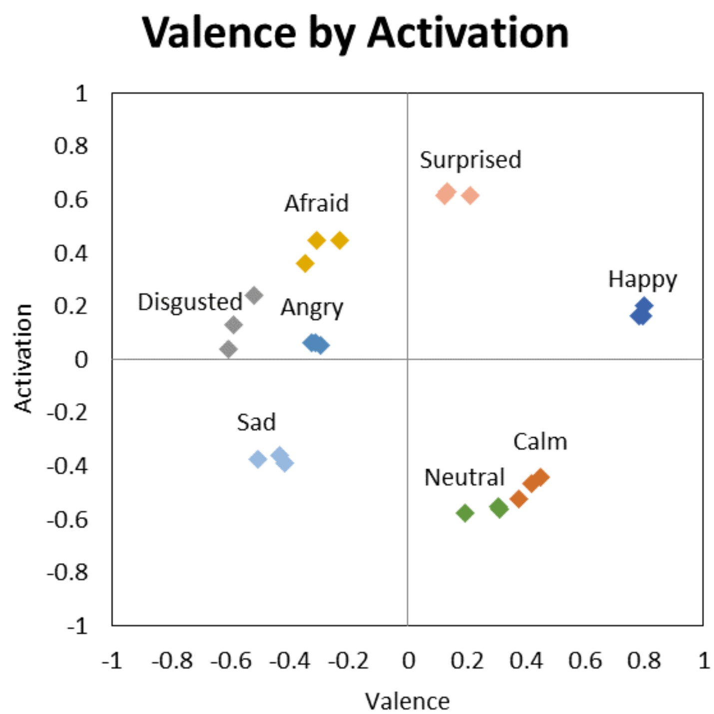

Figure 4. Plot of stimulus coordinates for dimension 1 (valence) and dimension 2 (activation) of the three-dimensional configuration with stimulus clusters labeled

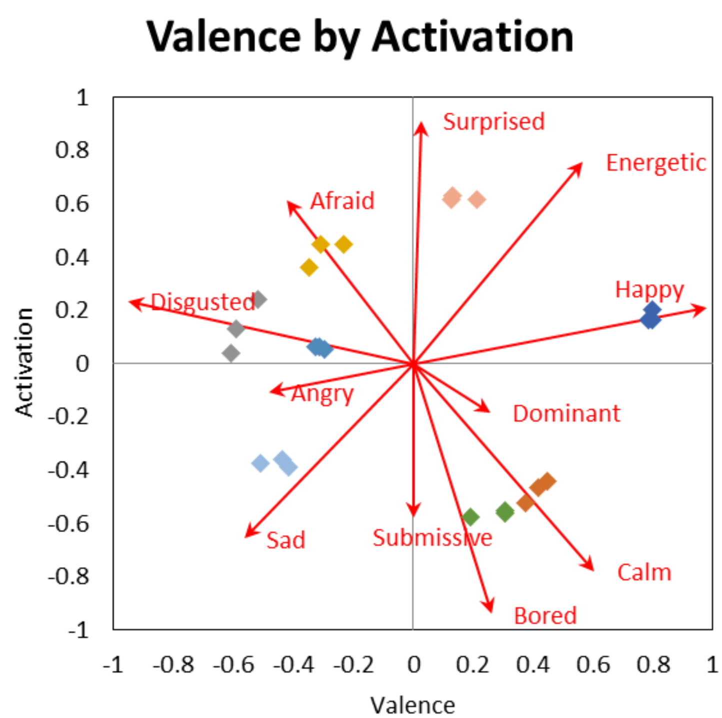

Figure 5. Plot of stimulus coordinates for dimension 1 (valence) and dimension 2 (activation) of the three-dimensional configuration with labeled attribute vectors

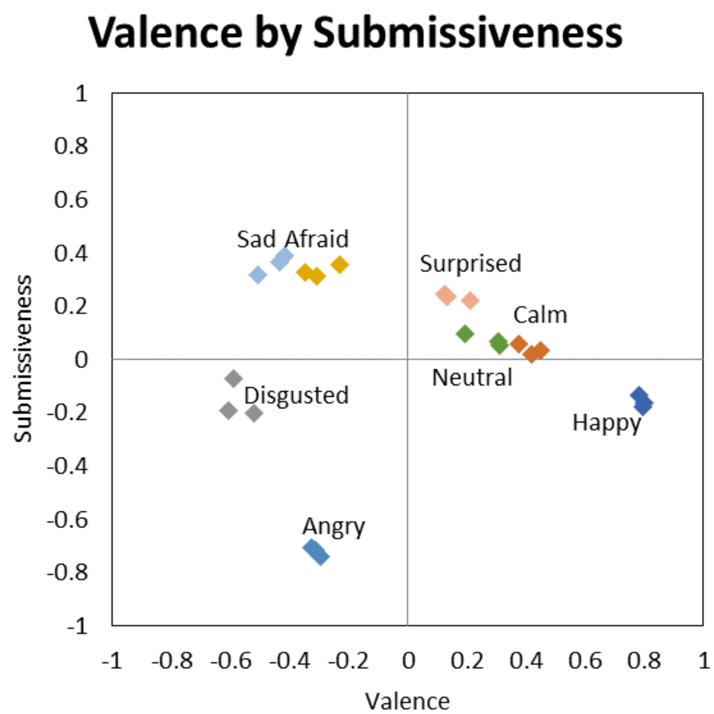

Figure 6. Plot of stimulus coordinates for dimension 1 (valence) and dimension 3 (submissiveness) of the three-dimensional configuration with stimulus clusters labeled

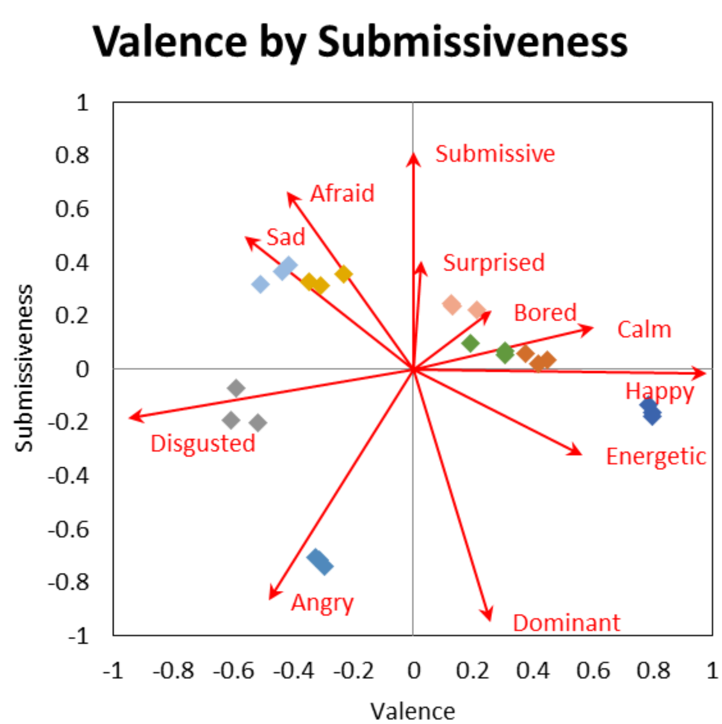

Figure 7. Plot of stimulus coordinates for dimension 1 (valence) and dimension 3 (submissiveness) of the three-dimensional configuration with labeled attribute vectors

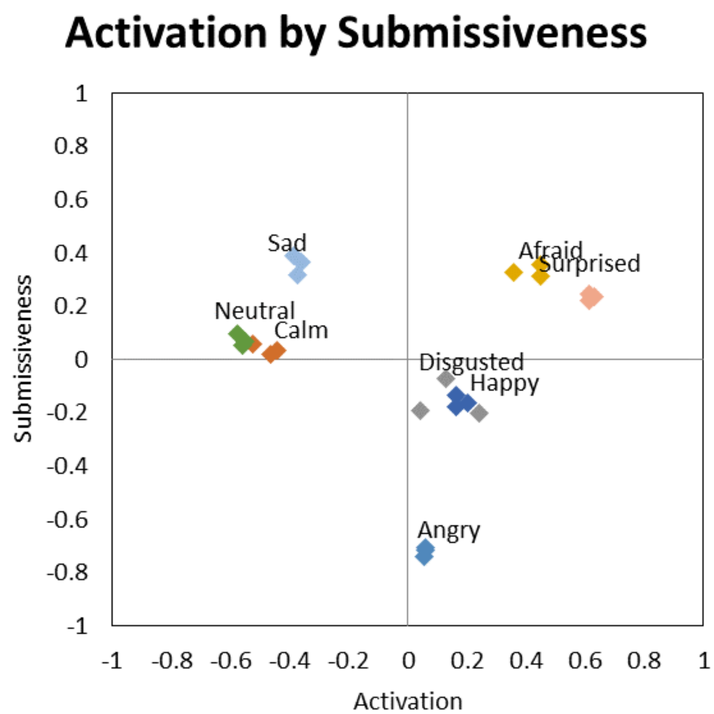

Figure 8. Plot of stimulus coordinates for dimension 2 (activation) and dimension 3 (submissiveness) of the three-dimensional configuration with stimulus clusters labeled

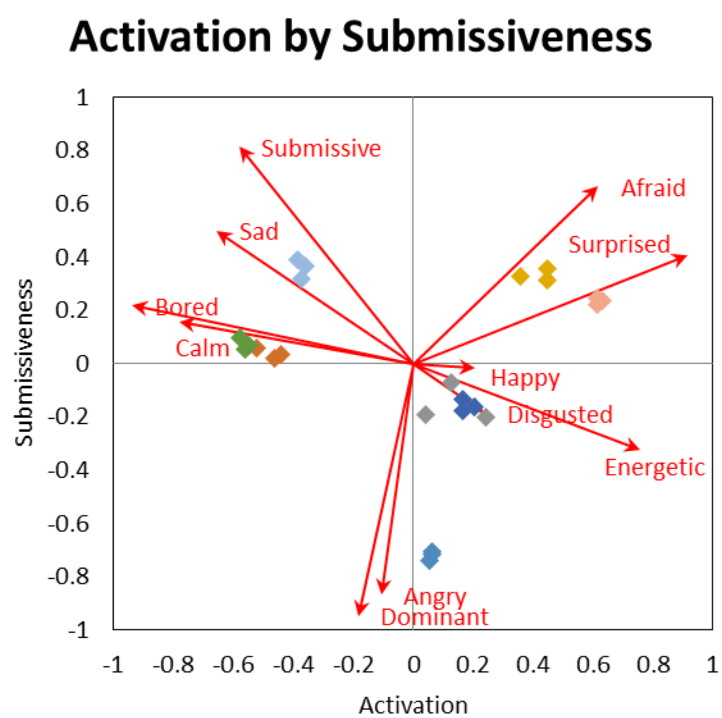

Figure 9. Plot of stimulus coordinates for dimension 2 (activation) and dimension 3 (submissiveness) of the three-dimensional configuration with labeled attribute vectors

dimension, whereas submissive and afraid vectors are on two different sides of the dimension. Whereas afraid appears to be a more active emotion, the submissive vector seems to characterize less active emotions. These four vectors, along with the surprised and sad attribute vectors, are primarily defining the vertical axes, whereas the remaining attribute vectors are more concentrated along the horizontal axis.

3.2 External Multidimensional Unfolding

A total of 1564 participants provided reliable ($r > 0.6$) responses to the self-similarity portion of the study. These responses were analyzed using both Carroll's four proposed models as well as backwards elimination (stepwise) regression variants of those models. Cases in which the resulting model was not one of Carroll's four models (as some of the terms were missing) will be referred to as hybrid models.

3.2.1 Non-Significant Parameters

The number of non-significant parameters removed using the backwards elimination regression (BER) approach was compared to the number of non-significant parameters retained in the traditional model approach. These results are presented in Table 4. By removing parameters that do not account for a significant amount of variance, it is possible to see previously non-significant parameters become significant due to less multicollinearity and larger error degrees of freedom within a resulting model. For example, when all subjects were fit with a traditional general Euclidean model, there was a total of 1109 models in which all three interaction terms were not significant. By implementing the BER approach, this number dramatically dropped to 421 non-significant interaction terms. The BER approach often produced more significant

Table 4

Frequency of Significant Parameters for Traditional EMDU and BER EMDU Approaches

Model Comparison	All Non-significant	Some Significant	All Significant
Traditional General Euclidean Model			
Location Parameters	643	862	59
Weight Parameters	1287	192	85
Interaction Parameters	1109	428	27
Traditional Weighted Euclidean Model			
Location Parameters	117	1265	182
Weight Parameters	678	540	346
Traditional Simple Euclidean Model			
Location Parameters	55	1226	283
Weight Parameter	911	n/a	653
Traditional Vector Model			
Location Parameters	60	1381	123
BER General Euclidean Model			
Location Parameters	104	1162	298
Weight Parameters	362	934	268
Interaction Parameters	421	1024	119
BER Weighted Euclidean Model			
Location Parameters	60	1249	255
Weight Parameters	328	808	428
BER Simple Euclidean Model			
Location Parameters	50	1217	297
Weight Parameters	875	n/a	689
BER Vector Model			
Location Parameters	63	1378	123

parameters by allowing non-significant ones to drop out of the model, and this occurred more often with highly parameterized models from Carroll's hierarchy.

3.2.2 Anti-Ideal Points

A large proportion of anti-ideal points were discovered in earlier research on this self-similarity data. Additionally, several of these anti-ideal points had non-significant weight parameters, which unnecessarily complicated the interpretation of the model coordinates. By ensuring that all parameters were significant and allowing for a hybrid model solution, the number of anti-ideal points greatly decreased. The results presented in Table 5 show that the overall number of both ideal points and anti-ideal points decreased across all three unfolding models as a result of the BER approach, and this resulted in a larger number of vector models on a given dimension.

3.2.3 Model Selection

An optimal BER model was selected for each individual by identifying the model with the minimum BIC value. In the case of ties, the most parsimonious model was selected as the optimal model. For example, if an individual had the same BIC value for the simple Euclidean starting model and the vector starting model, the vector model would be selected over the simple Euclidean model. It was discovered that approximately 7% of the individuals included in this analysis yielded results that suggest the self-similarity between a given emotion stimulus and their reported emotion is equally well represented by a BER approach to any of Carroll's four models. These BER models all resulted in identical vector model solutions, and consequently, the BIC values were identical. Identical solutions across all four starting models is ideal from a consistency standpoint, though this can only occur when the resulting solution is a vector model.

Table 5

Comparison of the Frequency of Ideal Points and Anti-Ideal Points for Traditional EMDU and BER EMDU Approaches

			Traditional		BER	
Model	Dimension	Weight	Frequency	Percent	Frequency	Percent
General	Valence	Ideal Point	840	53.71%	391	25.00%
		Anti-Ideal Point	724	46.29%	418	26.73%
		Missing	n/a	n/a	755	48.27%
	Activation	Ideal Point	879	56.20%	322	20.59%
		Anti-Ideal Point	685	43.80%	243	15.54%
		Missing	n/a	n/a	322	63.87%
	Submissiveness	Ideal Point	888	56.78%	525	17.39%
		Anti-Ideal Point	676	43.22%	272	49.04%
		Missing	n/a	n/a	767	33.57%
Weighted	Valence	Ideal Point	844	53.96%	432	27.62%
		Anti-Ideal Point	720	46.04%	442	28.26%
		Missing	n/a	n/a	690	44.12%
	Activation	Ideal Point	985	62.98%	516	32.99%
		Anti-Ideal Point	579	37.02%	269	17.20%
		Missing	n/a	n/a	779	49.81%
	Submissiveness	Ideal Point	939	60.04%	527	33.70%
		Anti-Ideal Point	625	39.96%	277	17.71%
		Missing	n/a	n/a	760	48.59%
Simple	Sum	Ideal Point	769	49.17%	380	24.30%
		Anti-Ideal Point	795	50.83%	309	19.76%
		Missing	n/a	n/a	875	55.95%

Optimal models were crosstabulated in Table 6 with respect to the starting BER model and the resulting model. Models with interaction terms were referred to as general Euclidean models or as hybrid general Euclidean models depending on whether all of Carroll's original terms were present in the resulting equation. Any models that did not have interaction terms but had unique dimensional weight terms were referred to as either weighted Euclidean models or as hybrid weighted Euclidean models. Models that retained the summed dimensional weight term were referred to as simple Euclidean models or hybrid simple Euclidean models, and any other models which only retained location parameters were referred to as vector Euclidean models or hybrid vector Euclidean models. In most cases, the chosen BER model for an individual determined its resulting model classification. For example, many individuals whose self-similarity judgments were classified as fitting best with the BER general Euclidean starting model yielded solutions that included at least one interaction parameter. However, nine of the 856 individuals fit best with this BER model did not yield solutions including interaction parameters; although these starting models may have been the general Euclidean model, the resulting models were classified as hybrid weighted Euclidean models.

The results presented in Table 6 also further illustrate the importance of implementing a hybrid EMDU approach. Out of a sample of 1564, only 59 sets of self-similarity judgments resulted in a traditional EMDU model solution. Every other resulting model was a hybrid variation that cannot be obtained using traditional techniques. However, it is of note that 15 out of 205 individuals had empty solutions where the only remaining parameter was the intercept. While these resulting models are not explicitly beyond the definition of what qualifies a hybrid vector model, these

Table 6
Starting BER Model against Resulting Model Classification

Starting BER Model	Model Classification: General Euclidean		Weighted Euclidean		Simple Euclidean		Vector		Empty Model	Total
	Traditional	Hybrid	Traditional	Hybrid	Traditional	Hybrid	Traditional	Hybrid		
General Euclidean	6	841	0	9	0	0	0	0	0	856
Weighted Euclidean	0	0	12	393	0	0	0	0	0	405
Simple Euclidean	0	0	0	0	24	74	0	0	0	98
Vector	0	0	0	0	0	0	17	173	15	205
Total	6	841	12	402	24	74	17	173	15	1564

individuals' preferences were located at the origin for all three dimensions as no one dimension was salient for that individual. It is tenable these 15 individuals reported self-similarity judgments that cannot be represented using EMDU techniques or that they made a preponderance of irrelevant judgments that passed the consistency check. To conclude, a substantially high number of starting BER models resulted in hybrid models as opposed to traditional EMDU models. Further, the resulting models obtained using this BER approach were typically hybrid variants of the starting BER model as opposed to variations of a nested model. These results not only demonstrate the value of a hybrid model approach, but also the importance of an individual-based model selection process.

3.2.4 Assessment of BER/EMDU Solution

Following the selection of an optimal BER model for each individual, the resulting optimal models were used to place respondents on a joint map of persons and stimuli. Coordinates for placement were derived with Equation 20. Figures 10-15 show the resulting map two dimensions at a time using all possible pairs of axes (in order to avoid portraying a 3-dimensional configuration on a 2-D substrate). Figures 10-12 show a cropped view of the solution that eliminates individuals with coordinate values substantially beyond the stimulus range of the MDS solution. Figures 13-15 portray all coordinates, including those cropped from the previous three plots. These plots depict whether or not a certain point corresponds to an ideal point, an anti-ideal point, or the head of a vector along a given dimension. As there are a total of nine different types of coordinates for each two-dimensional plot, coordinates have been assigned a particular shape and color to distinguish them from one another. The shape of a coordinate represents the type of point along the vertical dimension and the color of a coordinate

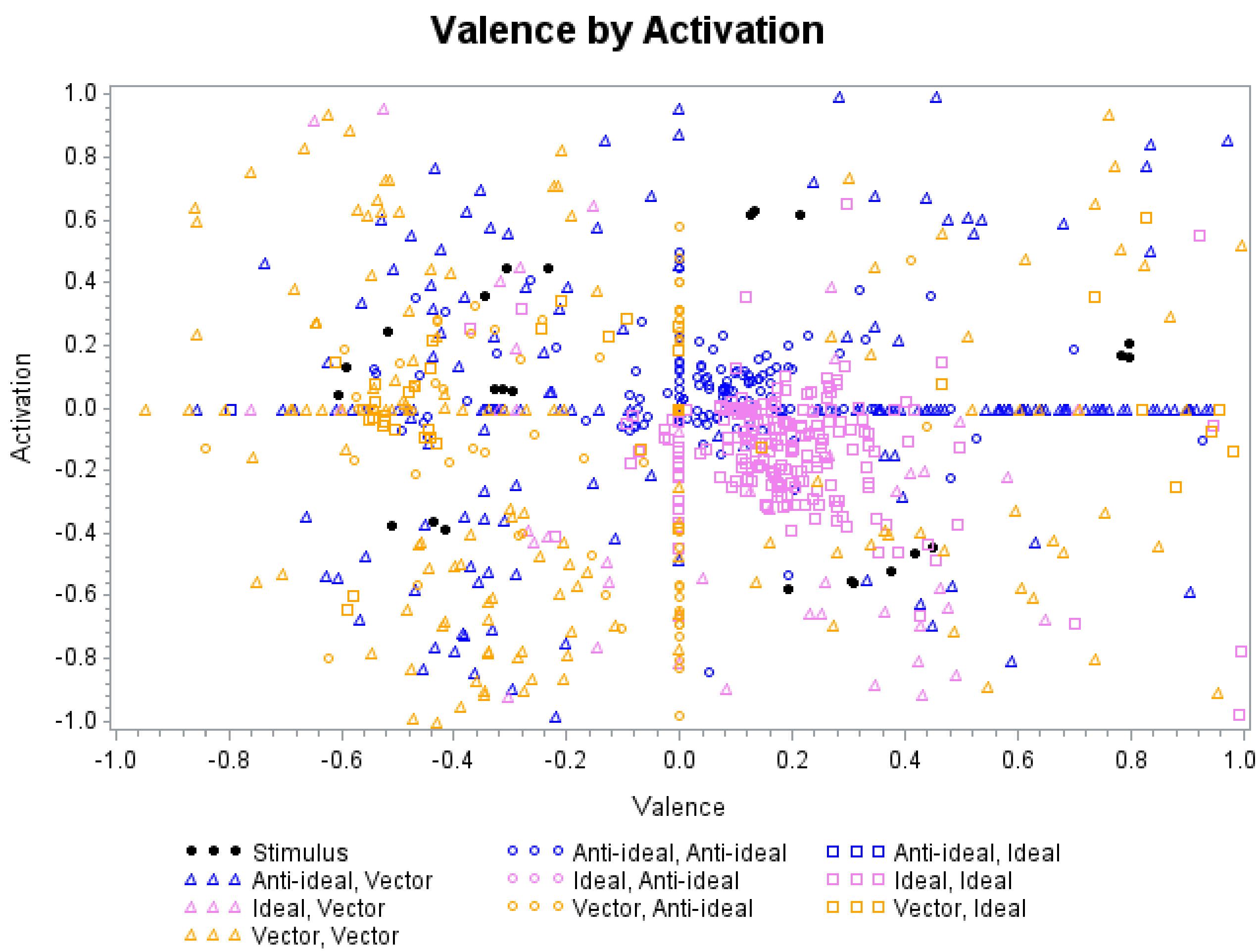

Figure 10. Plot of stimulus coordinates for dimension 1 (valence) and dimension 2 (activation) of the three-dimensional configuration with jointly mapped individual preferences (Note: the coordinate color in the legend corresponds to the valence dimension. The coordinate shape corresponds to the activation dimension)

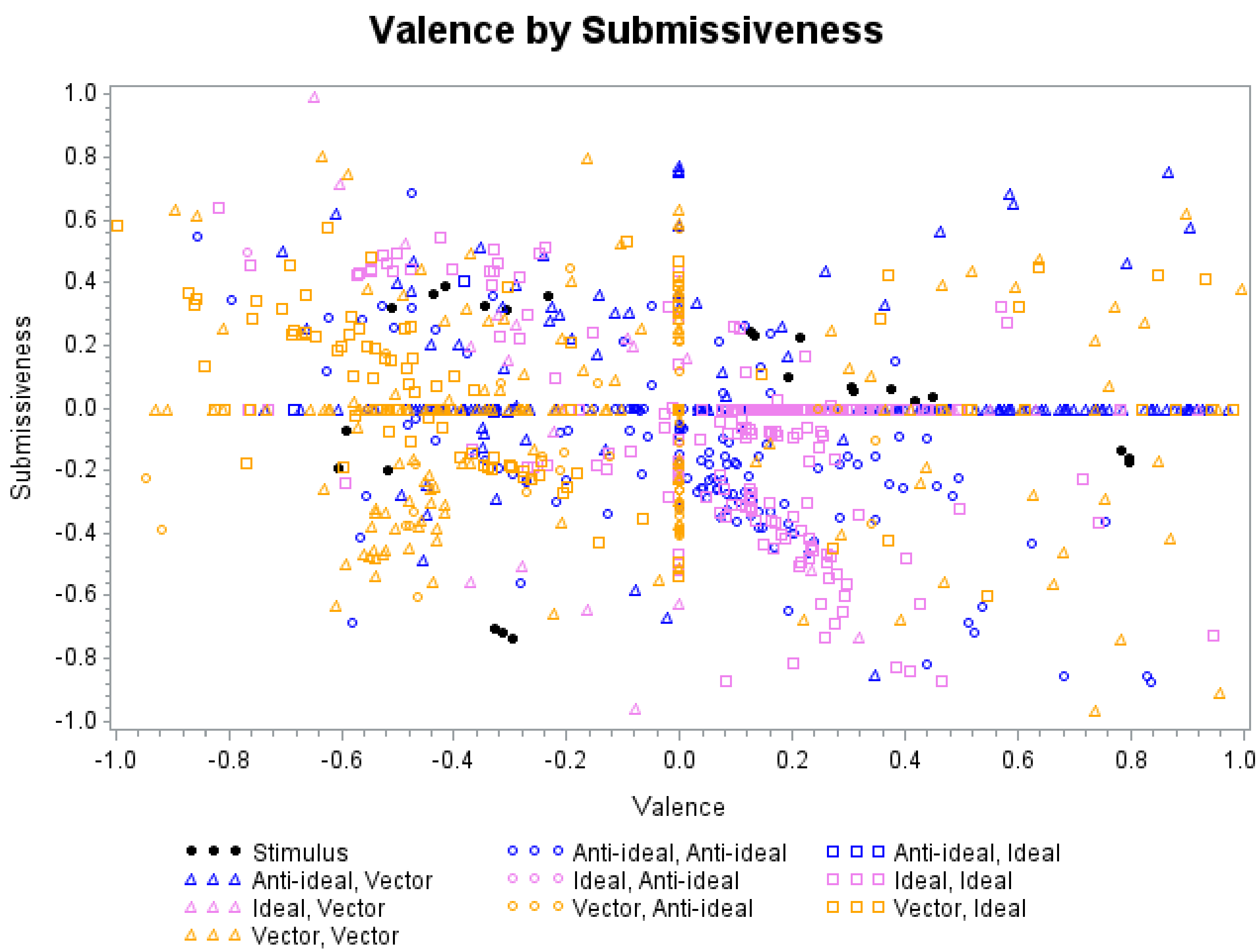

Figure 11. Plot of stimulus coordinates for dimension 1 (valence) and dimension 3 (submissiveness) of the three-dimensional configuration with jointly mapped individual preferences (Note: the coordinate color in the legend corresponds to the valence dimension. The coordinate shape corresponds to the submissiveness dimension)

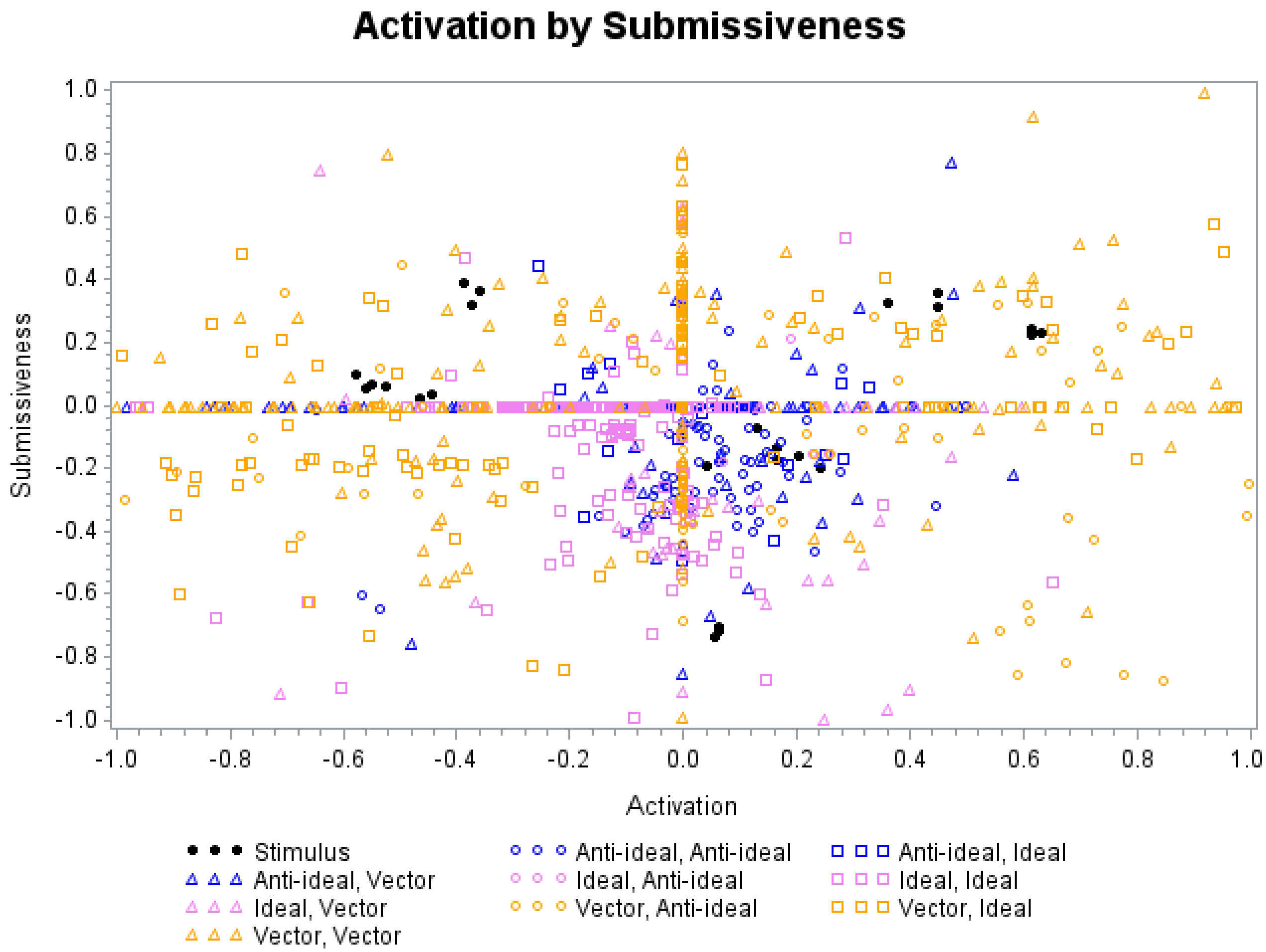

Figure 12. Plot of stimulus coordinates for dimension 2 (activation) and dimension 3 (submissiveness) of the three-dimensional configuration with jointly mapped individual preferences (Note: the coordinate color in the legend corresponds to the activation dimension. The coordinate shape corresponds to the submissiveness dimension)

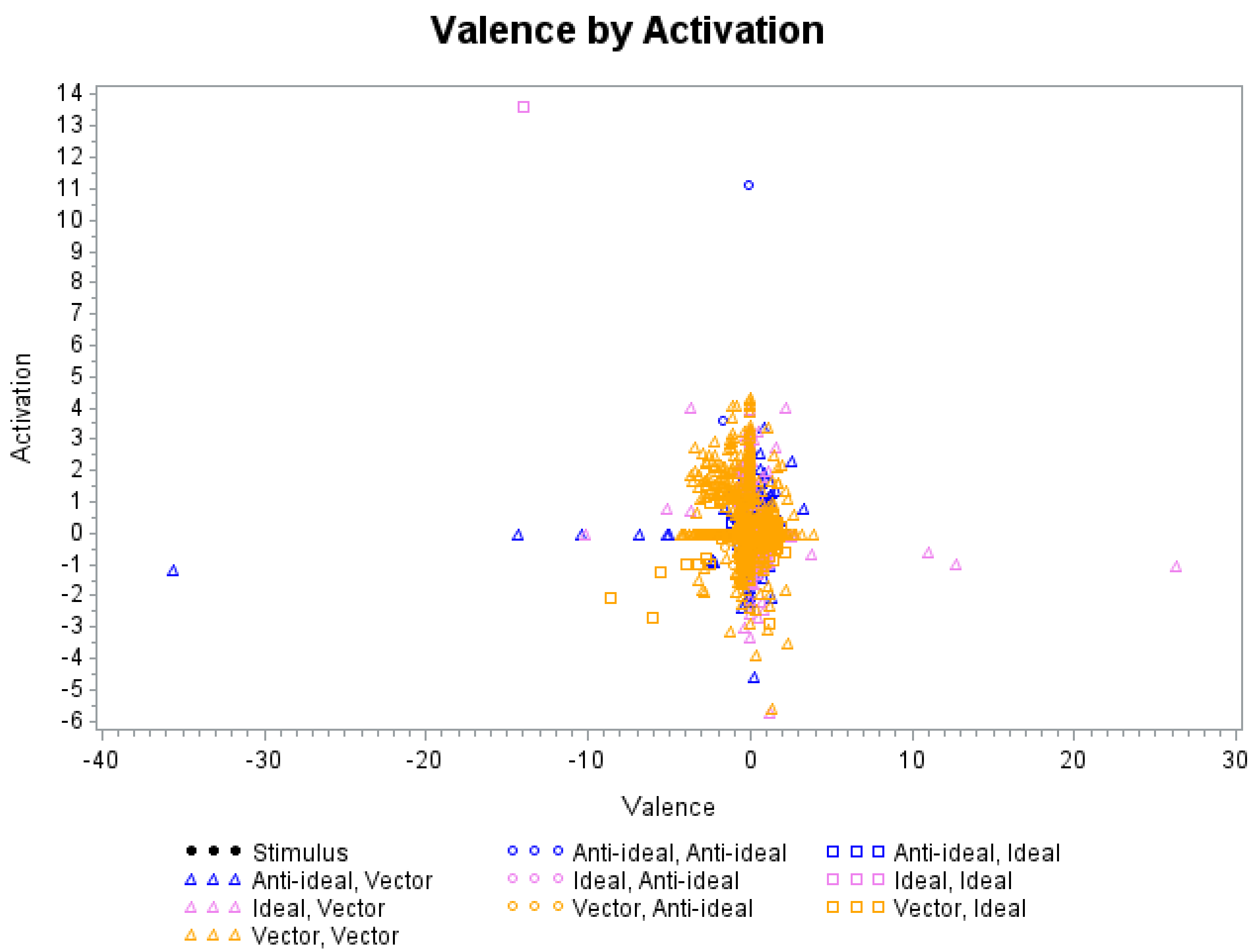

Figure 13. Plot of all calculated individual preferences for dimension 1 (valence) and dimension 2 (activation) (Note: the coordinate color in the legend corresponds to the valence dimension. The coordinate shape corresponds to the activation dimension)

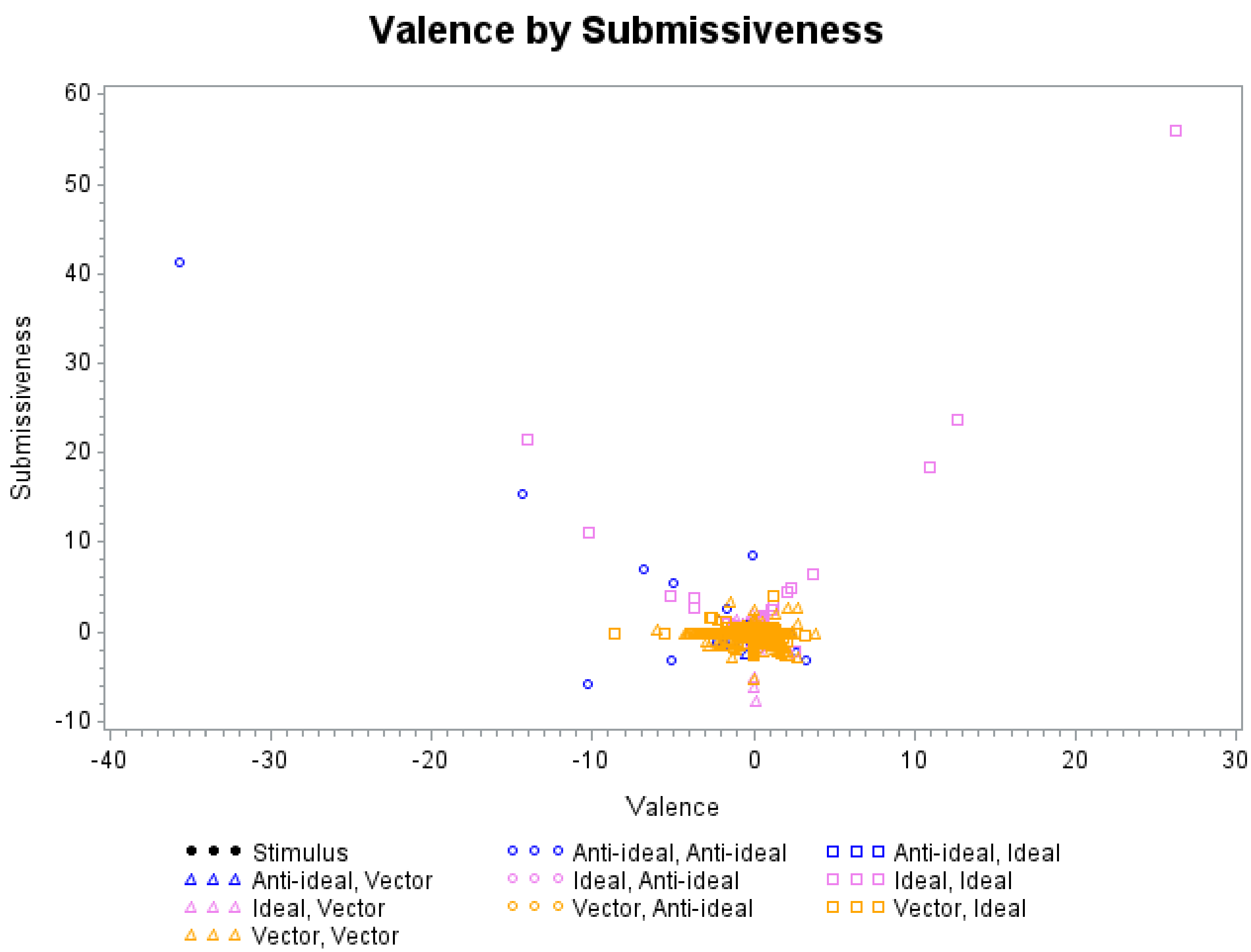

Figure 14. Plot of all calculated individual preferences for dimension 1 (valence) and dimension 3 (submissiveness) (Note: the coordinate color in the legend corresponds to the valence dimension. The coordinate shape corresponds to the submissiveness dimension)

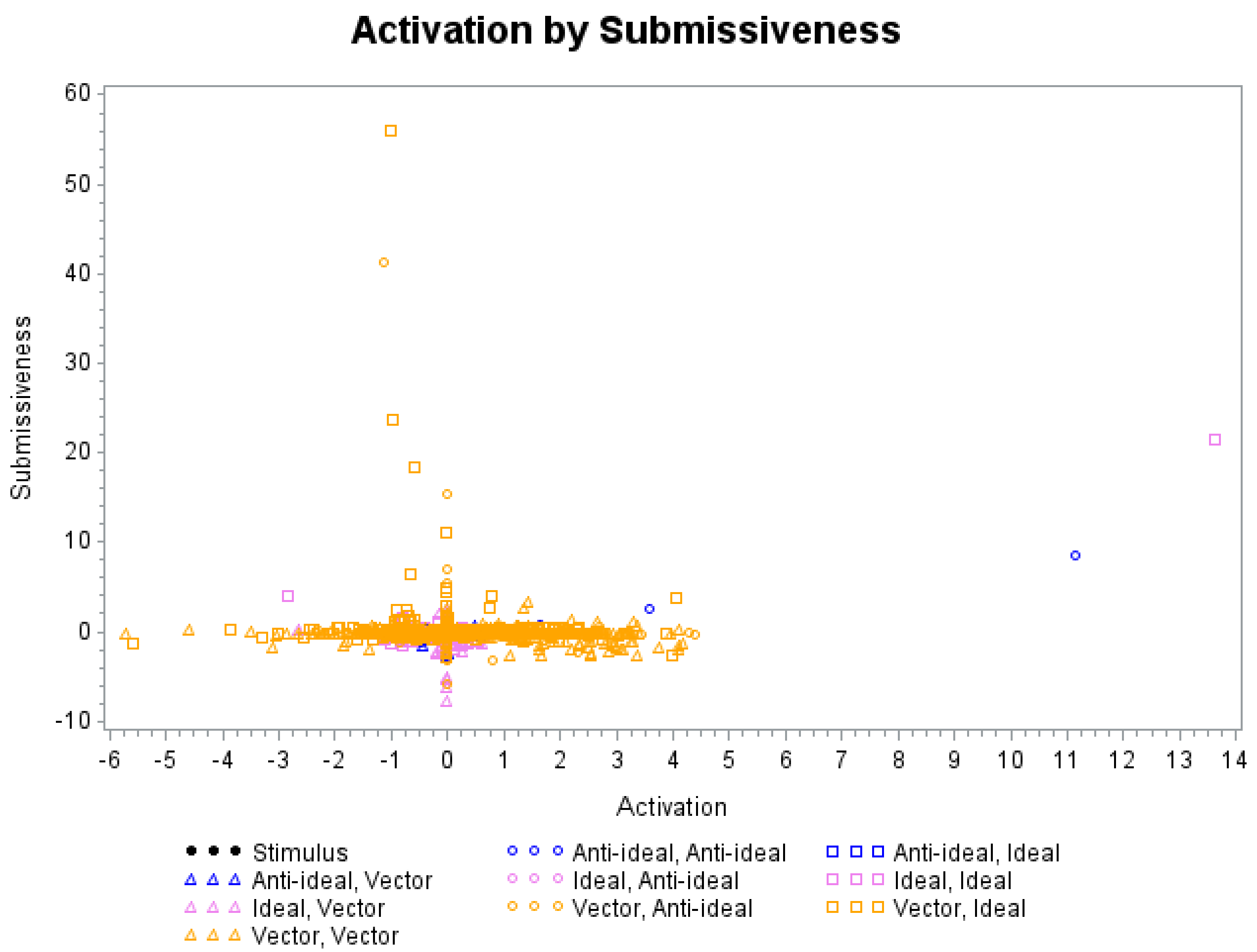

Figure 15. Plot of all calculated individual preferences for dimension 1 (valence) and dimension 3 (submissiveness) (Note: the coordinate color in the legend corresponds to the activation dimension. The coordinate shape corresponds to the submissiveness dimension)

represents the type of point along the horizontal dimension for a given plot. Specifically, an ideal point, an anti-ideal point, or a vector coordinate on the vertical dimension is represented by a square, a circle or a triangle, respectively. Correspondingly, an ideal point, an anti-ideal point, or a vector coordinate on the horizontal dimension is portrayed by a symbol color of violet, blue or orange. For example, a coordinate that is an anti-ideal point on the horizontal dimension and an ideal point on the vertical dimension is represented by a blue square. A coordinate that is an anti-ideal point on the horizontal dimension and the head of a vector on the vertical dimension is represented by a blue triangle.

The different coordinate combinations presented in the plots each uniquely represent an individual's reported emotion, and more importantly, how they are expected to respond to stimuli throughout the joint space. A point consisting of ideal point coordinates on both dimensions, for example, may be visually depicted as a two-dimensional parabolic shape with a maximum at the point represented by the two coordinates. This point best represents an individual's reported emotion. The self-similarity of an individual to a point in the emotion space decreases in any direction away from this point. In contrast, a point consisting of two anti-ideal point coordinates is represented by a two-dimensional parabolic shape with a minimum, where emotion self-similarity increases in any direction away from the identified point. If a point consists of an ideal point coordinate on one dimension and an anti-ideal point coordinate on the other, then the self-similarity surface on the two dimensions takes the shape of a saddle point, where self-similarity contains a maximum on one dimension, but a minimum on the other. Vector/vector coordinate combinations represent the magnitude of an individual's emotion self-similarity in a given direction in the emotion space. Self-similarity is represented by a vector emanating from the origin of the space in the direction of a given point. The further one travels along the vector the more accurately one represents an individual's reported emotion.

One potential limitation of this hybrid model approach is the difficulty of interpreting the final two coordinate combinations: ideal/vector and anti-ideal/vector points. In instances where there is an anti-ideal point coordinate or an ideal point coordinate on one dimension and a vector coordinate on another, it becomes cumbersome to illustrate these coordinates and to interpret these points. Ideal/vector points are best described visually as an "ideal line" projecting through the space from one of the coordinate axes. An "ideal line" is represented by a parabolic shape with a maximum traveling through the space in the direction of the vector coordinate. The paraboloid tilts upwards in the direction of the vector such that preference increases as one travels along a given dimension. For example, the individual represented by a violet triangle point at approximately (0.30, -0.55) in Figure 10 (Valence by Activation) has an ideal coordinate along the valence dimension. This individual may have been reporting an emotion with moderately positive valence. Along the activation dimension, the individual's reported emotion self-similarity has been modeled with a vector. This implies that this individual's reported emotion is better described the further one travels down a vector towards less active emotions. Therefore, it may be concluded that this individual was reporting an emotion similar to neutral or calm. The vector that extends to this point on the second dimension would emanate from approximately 0.30 on the valence axis, as opposed to the origin, which is seen in traditional EMDU, as self-similarity on the valence axis is defined by an ideal point coordinate. It should be emphasized that this example does not include the third dimension. To truly interpret an individual's emotion in a three-dimensional solution, his or her location on the third dimension would need to be ascertained and interpreted along with the other two dimensions.

Similarly, anti-ideal/vector points are best described visually as an "anti-ideal line" projecting through the space. An "anti-ideal line" is represented by a parabolic shape with a minimum (with either end of the shape best representing an individual's highest self-similarity on that given dimension) traveling through the space towards the vector coordinate. For example, the individual with an orange circle point on the Valence by Activation plot (Figure 10) around (-0.4, 0.05) has a vector along the valence dimension and an anti-ideal point along the activation dimension. Given only these two dimensions, one may interpret that this individual was reporting a negative emotion (e.g.., angry, disgust, sad) and that this individual's reported emotion is best represented by any place in the stimulus range away from emotions that are neutral along the activation dimension. From these coordinates, it may be ascertained that this individual was reporting an emotion similar to either afraid or sad. Though interpretations are more difficult in these situations, a BER model approach still offers the possibility of a more parsimonious representation of an individual's reported emotion.

In Figures 13-15, there are a large number of points that fall beyond the range of emotion stimuli. It has been proposed that ideal points falling outside the range of stimuli are better modeled using a vector model (Coombs, 1950). With this in mind, it is advantageous for this hybrid model approach to minimize the total number of undesirable ideal points and anti-ideal points falling outside of the stimulus range. As can be seen across the figures, many of the coordinates outside the range of emotion stimuli are vector coordinates (represented by orange triangles). Specifically, the number of outlying ideal point or anti-ideal point coordinates on any dimension is minimal, with 105 coordinates falling out of the range of stimuli for the valence dimension (approximately -0.7 to 0.8), 42 falling out of the range of stimuli for the activation dimension (approximately -0.6 to 0.7), and 122 falling out of the range of stimuli for the submissiveness dimension (approximately -0.8 to 0.4). This is a clear improvement over the number of ideal point or anti-ideal point coordinates falling outside of the stimulus range when using Carroll's traditional EMDU approach, where 232 coordinates were out of the stimulus range for the valence dimension, 165 were out of the stimulus range for the activation dimension, and 213 were out of the stimulus range for the submissiveness dimension. Although the current methods did not perfectly eliminate the appearance of these extreme anti-ideal points or ideal points, this improvement over traditional methods suggests that the hybrid BER approach is a superior alternative to minimizing these undesirable anti-ideal and ideal points in the extreme regions of the configuration.

Figures 10-12 show several points that fall on the coordinate axes. If a point falls on an axis, then an individual's emotion self-similarity falls at that point if both coordinates are ideal point coordinates, but if both coordinates are vector coordinates, an individual's preference falls along that dimension's axis and the opposing dimension is not salient for that individual. In vector/ideal or vector/anti-ideal cases where the vector coordinate falls at zero on a given dimension it is not possible to draw a vector to illustrate an individual's reported emotion, as the vector both starts and stops at that point. While an individual's emotion self-similarity can be represented with an ideal point on one dimension, preference neither increases nor decreases regardless of the direction traveled along the other dimension. In instances where there are vector coordinates on both dimensions and one vector coordinate falls along one of the coordinate axes, a vector may be drawn from the origin to this point such that emotion self-similarity increases the farther one travels along the axis. In both cases it may be concluded that one dimension is not salient for an individual.

3.2.4.1 Trends in the BER Solution

This BER/EMDU approach resulted in trends in the types of coordinates that appear across the two-dimensional plots. For the Valence by Activation plot (Figure 10), one can see the prevalence of a large cluster of violet square points (corresponding to ideal point coordinates for both dimensions) in the fourth quadrant of the plot. This may suggest that it is easier for individuals to relate to a more positive, though less active emotion stimulus (i.e., bored or calm) than it is the other emotion stimuli in the plot. The anti-ideal points on both dimensions that linger in the center of the plot do not offer much useful information to the researcher as to what emotion an individual reported feeling, as one could theoretically go in any direction from these anti-ideal points. It is tenable that in a study with more stimuli featuring a larger range of emotions these individuals would rate their emotions as being similar to a particular group of stimuli. It is also notable that there are a large number of triangle-shaped coordinates around and outside of the range of emotion stimuli and relatively few other shapes in these areas. This is anticipated, as this implies that these points include vector coordinates for at least one dimension. Additionally, though there is a clear presence of blue triangle points (corresponding to an anti-ideal point coordinate on the valence dimension and a vector coordinate on the activation dimension) across the plot, it appears as though many of these points fall on the valence axis on the positive end of the valence dimension. The simplest interpretation of these points is that many individuals reported feeling “anything but” emotions with positive valence (such as surprised, happy, or calm) and different levels of activation in the stimuli were not salient to these individuals based on their self-similarity judgments.

The trends prevalent in the Valence by Activation plot do not hold for the Valence by Submissiveness plot (Figure 11). Here, there are different types of clusters that appear in the fourth quadrant of the plot. The fourth quadrant is filled with a large number of blue circle points (corresponding to anti-ideal point coordinates for both dimensions) and violet square points (corresponding to ideal point coordinates for both dimensions), with a limited number of vector coordinates inside the stimulus range in this quadrant. It is possible that a large number of the anti-ideal points prevalent on the valence axis in Figure 10 are teased apart by the submissiveness dimension in Figure 11, which may explain the prevalence of blue circle coordinates in the fourth quadrant of the plot. In this case, individuals with these anti-ideal points were frequently reporting negative, submissive emotions. Interestingly, the largest number of violet square (ideal/ideal) points appears to pass through the second and the fourth quadrant of the plot in a somewhat decreasing pattern from left to right. This may relate to the shape of the stimulus space in this plot which seems to follow a similar pattern. Additionally, similar to Figure 10, there appears to be a large number of vector coordinates around and outside the range of emotion stimuli; however, there are more instances in this plot where anti-ideal and ideal point coordinates fall outside the range of stimuli relative to Figure 10.

The Activation by Submissiveness plot (Figure 12) does not appear to show explicit clusters of points as clearly as the previous two plots; nonetheless, it should be noted that the largest amount of ideal-ideal points appears to fall to the left of the origin. This is, perhaps, the most interesting trend prevalent across the plots, as these ideal points are falling approximately in the center of the stimulus space (to the left of the origin in this plot) making them difficult to interpret. It is tenable that in a three-dimensional plot these coordinates would show more of a spread throughout the space. Also, the largest number of anti-ideal point coordinates fall in the fourth quadrant around the "disgusted" and "happy" emotion stimuli. Without knowing the valence of these individuals' reported emotions, the interpretation of these coordinates becomes difficult. A rotatable, three-dimensional plot of these stimuli would assist in the ease of interpretation of points in the space, particularly in these instances.

CHAPTER 4

DISCUSSION

This thesis has proposed a new method to conduct EMDU that 1) provides a more parsimonious fit for preference or self-similarity judgments, 2) minimizes the number of anti-ideal points that typically arise from EMDU models, and 3) guarantees that all model terms are statistically significant, which leads to stronger interpretations of the solutions. The foundations for the methodology of this study were grounded in the idea that there may be a few interpretable anti-ideal points in any EMDU solution for an emotion space, but they should account for a statistically significant amount of variation in the preference responses. The hybrid models that may emerge from the BER technique give researchers additional flexibility in mapping preferences by allowing alternative models in Carroll's hierarchy to represent each dimension. The flexibility proposed by this BER model approach can provide a more parsimonious fit of preference (or self-similarity) data relative to traditional models, and can reduce the number of extreme ideal points that may emerge when preferences are primarily cumulative. The BER model approach to Carroll's original models reduced the number of anti-ideal points by approximately 20-25% by allowing these points to become vectors in the group space.

This technique is not without its flaws, however. One, it is not possible to include the simple Euclidean model in the starting general Euclidean or weighted Euclidean models as the simple Euclidean model contains a summed weight parameter not included in the parameterizations of the other models. If this parameter was included in the general and weighted Euclidean models, this method would be capable of better representing preferences or self-similarity than the current technique. This is theoretically feasible with a re-parameterization of the general Euclidean and weighted Euclidean models. For example, for a three-dimensional solution it may be feasible to include the average of the X, Y, and Z weights as a predictor along with the Y weight and Z weight such that all three individual weights as well as the sum weight can be mathematically extracted from the resulting model through an accumulation of the regression terms. For example, $b_1*(X + Y + Z) + b_2*Y + b_3*Z = b_1*X + (b_1 + b_2)*Y + (b_1 + b_3)*Z$.

Although stepwise regression is useful for exploratory analyses, it is not preferred in quantitative research in which hypothesis tests are sought due to its lack of Type I error rate control. The future development of this hybrid model method will rely on the exploration of other techniques to achieve similar results. Though this analysis has demonstrated the benefits of utilizing a hybrid approach, the techniques employed should be refined as opportunities to apply new techniques to this method become available. For example, the general monotone model (GeMM) model developed by Dougherty and Thomas (2012) serves as an alternative to stepwise regression by utilizing a genetic algorithm to model data that are either monotonic or linear.

One area for future research is to investigate an internal multidimensional unfolding model that uses only the self-similarity responses to jointly map both stimuli and individuals. Such models have been difficult to use in the traditional MDU contexts because of solutions that pervasively degenerate. However, an MDU model can be formed in the context of item response theory, and the additional information provided by the inherent probability function may be enough to avoid degenerate solutions. An example of such an approach is the Multidimensional Generalized Graded Unfolding

Model (MGGUM) developed by Roberts and Shim (2010). Internal models such as the MGGUM would drastically reduce the experimental effort in that paired comparison data would not be required. Furthermore, the model parameters do not include subject-level weights, so the resulting solution would be free of anti-ideal points.

It may also be interesting to explore the emotion circumplex with other EMDU models that allow for the possibility of multiple ideal points. Srinivasan and Shocker (1973) suggested a multiple ideal point model in which weight parameters are restricted to be positive. They hypothesized that anti-ideal points may be the unintended result of the preference judgments that are determined by more than one ideal point for a given individual. The model can account for multiple ideal points by completely eliminating the possibility of anti-ideal points. Such a solution might be expected from some individuals when responding to emotional stimuli. For example, it is easy to think of an individual who is both happy and calm, although the positions associated with these two emotions are quite distinct on the circumplex.

One of the potential values of this study is that it will open up discussion on EMDU and ideal points and bring to light questions about the influence of statistically non-significant model parameters that have not previously been mentioned in the psychometric literature. Hybrid “unfolding” models allow researchers to model preferences in a more tailored fashion in which a single individual can respond to stimulus dimensions in a less consistent manner than the all or none models that Carroll (1972) proposed. Modeling preferences using stepwise methods that optimally represent an individual’s preferences in a multidimensional space is a straightforward means to achieve these ends.

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