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7/25/68

SYNTHESIS OF ADMITTANCE MATRICES USING

RC NETWORKS AND NONIDEAL AMPLIFIERS

A Thesis

Presented to

The Faculty of the Graduate Division

By

David Richard McElroy, Jr.

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

in the School of Electrical Engineering

Georgia Institute of Technology

September, 1969

SYNTHESIS OF ADMITTANCE MATRICES USING

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Approved:

Chairman

Date approved by Chairman: 12/11/69

ACKNOWLEDGMENTS

I wish to express my sincere appreciation to my thesis advisor, Dr. Kendall L. Su, for his guidance, encouragement, and assistance during the course of this investigation. I also wish to thank Dr. D. C. Fielder and Dr. D. C. Ray for their suggestions regarding the preparation of this thesis.

Special appreciation is given to the United States Department of Health, Education, and Welfare and the School of Electrical Engineering at the Georgia Institute of Technology for a fellowship and continued research support during the period when this investigation was conducted.

TABLE OF CONTENTS

Page

ACKNOWLEDGMENTS ii

LIST OF ILLUSTRATIONS v

SUMMARY vi

CHAPTER

I. INTRODUCTION 1

II. ADMITTANCE MATRIX SYNTHESIS USING VOLTAGE AMPLIFIERS 6

Analysis of the Network
Realization Procedure
An Example

III. ADMITTANCE MATRIX SYNTHESIS USING CURRENT AMPLIFIERS 31

Analysis of the Network
Realization Procedure
An Example

IV. SIMULTANEOUS REALIZATION OF TWO ADMITTANCES USING TWO VOLTAGE AMPLIFIERS 53

Case 1: Y_{11} and Y_{21}
Case 2: Y_{11} and Y_{22}
Case 3: Y_{12} and Y_{21}
Case 4: Y_{22} and Y_{12}
An Example

V. SIMULTANEOUS REALIZATION OF TWO ADMITTANCES USING TWO CURRENT AMPLIFIERS 85

Case 1: Y_{11} and Y_{12}
Case 2: Y_{11} and Y_{22}
Case 3: Y_{12} and Y_{21}
Case 4: Y_{22} and Y_{21}
An Example

TABLE OF CONTENTS (Continued)

CHAPTER — Page

VI. EXPERIMENTAL RESULTS 104

Realization of an Inductor
Measurement Procedures and Possible Errors

VII. CONCLUSIONS AND RECOMMENDATIONS 117

APPENDIX

I. SELECTION OF $\tilde{Y}_{11}$ 121

II. FACTORIZATION OF THE MATRIX [A] 123

BIBLIOGRAPHY 129

VITA 131

LIST OF ILLUSTRATIONS

Figure		Page
1.	Grounded Active N-Port RC Network Containing 2M Voltage Amplifiers	7
2.	Grounded Active N-Port RC Network Containing 2M Current Amplifiers	32
3.	Two-Port Network Driven by a Current Source	54
4.	Grounded Active Two-Port RC Network Containing Two Voltage Amplifiers	54
5.	Network Realizing Equation (237)	84
6.	Network Realizing the Current Transfer Function of Equation (216)	84
7.	Grounded Active Two-Port RC Network Containing Two Current Amplifiers	86
8.	Network Realizing Equation (293)	103
9.	Network Realizing the Driving-Point Admittances $Y_{11} = 1/s$ and $Y_{22} = 1/s + 2$	103
10.	Network Realizing Equation (307)	109
11.	Network Realizing the Driving-Point Admittance $Y = 1/s$	109
12.	Comparison of Experimental Data with Predicted Magnitude Variation for a One Henry Inductor	110
13.	Comparison of Experimental Data with Predicted Phase Variation for a One Henry Inductor	111
14.	Operational Amplifier Realization of a Voltage-Controlled Voltage Source	113

SUMMARY

This thesis involves the development of active synthesis procedures utilizing grounded networks containing only resistors, capacitors, and nonideal three-terminal amplifiers. Voltage-controlled voltage sources with finite gains and finite input conductances and current-controlled current sources with finite gains and finite output conductances are the active components used in the realization of N x N short-circuit admittance matrices and the simultaneous realization of two short-circuit admittance parameters. The conclusions of the study can be summarized in the following four theorems:

Theorem 1

To realize an arbitrary N x N short-circuit admittance matrix of real rational functions in the complex-frequency variable by a transformerless grounded active N-port RC network embedding three-terminal voltage amplifiers with negative finite constant gains greater than unity and finite input conductances, it is sufficient that the active network contains 2N amplifiers.

Theorem 2

To realize an arbitrary N x N short-circuit admittance matrix of real rational functions in the complex-frequency variable by a transformerless grounded active N-port RC network embedding three-terminal current amplifiers with negative finite constant gains greater than unity and finite output conductances, it is sufficient that the active network contains 2N amplifiers.

Theorem 3

To realize simultaneously any of the pairs of short-circuit admittance functions Y_{11} and Y_{21}, Y_{11} and Y_{22}, Y_{12} and Y_{21}, and Y_{22} and Y_{12}, where each admittance is a real rational function of the complex-frequency variable, by a grounded transformerless active two-port RC network, it is sufficient that the network contains two three-terminal voltage-controlled voltage sources with negative finite constant gains greater than unity and finite input conductances.

Theorem 4

To realize simultaneously any of the pairs of short-circuit admittance functions Y_{11} and Y_{12}, Y_{11} and Y_{22}, Y_{12} and Y_{21}, and Y_{22} and Y_{21}, where each admittance is a real rational function of the complex-frequency variable, by a grounded transformerless active two-port RC network, it is sufficient that the network contains two three-terminal current-controlled current sources with negative finite constant gains greater than unity and finite output conductances.

These theorems have been proved and illustrated with numerical examples, and an experimental verification of the procedure associated with Theorem 1 has been obtained.

To prove Theorem 1, the N x N admittance matrix of an N-port transformerless active RC network embedding 2M nonideal voltage amplifiers is expressed in terms of the amplifier gains and input conductances and the admittance parameters of an $(N+4M)$-port passive network. This expression with M = N is made to equal the prescribed admittance matrix through proper choices for the amplifier specifications and the admittances of the passive network. Also, the admittance parameters are identified so that the resulting matrix can be realized by a transformerless passive (5N + 1)-terminal RC network of two-terminal impedances with a common reference node and no internal nodes.

The proof of Theorem 2 is obtained by using a procedure similar to the one associated with Theorem 1. Theorems 3 and 4 are proved by developing four realization procedures for each theorem.

CHAPTER I

INTRODUCTION

The development of the transistor and more recently the rapid growth of integrated-circuit technology have aroused considerable interest in active network theory. These solid-state devices have provided the synthesist with small, light-weight, low-cost components for use in approximating the mathematical models which are utilized in rigorously developing a synthesis method. In the past decade, various active RC networks have been employed to obtain prescribed transfer and immittance functions. Consequences of these new synthesis procedures have been the elimination of the inductor and the realizations of more general functions than are possible with passive RLC circuits.

Many of the active synthesis methods and their related advantages and problems are discussed in a book by Su[1] and in an article by Mitra.[2] Each method employs one or more of the following types of active devices: controlled source, negative impedance converter, negative impedance inverter, negative resistance, and gyrator. In all of the procedures contained in these two references, the resulting active network is lumped, linear, finite, and time-invariant, but not passive and not necessarily reciprocal. Also, the models of the active components which have been employed usually are ideal and often must possess four terminals so that the resulting network may not be unbalanced. To broaden the knowledge about network synthesis, it would seem appropriate to develop active RC realization schemes utilizing nonideal three-terminal active components -- for example, voltage-controlled voltage sources with finite gains and finite input conductances or current-controlled current sources with finite gains and finite output conductances.

Sandberg[3] has shown that an arbitrary N x N matrix of real rational functions in the complex-frequency variable (a) can be realized as the short-circuit admittance matrix of a transformerless active RC N-port network containing N real-coefficient controlled sources, and (b) cannot, in general, be realized as the short-circuit admittance matrix of an active RC network containing less than N controlled sources. In addition, Sandberg[4] has extended his procedure by presenting a method for the synthesis of an arbitrary N x N short-circuit admittance matrix of real rational functions in the complex-frequency variable by an unbalanced transformerless active RC network requiring no more than the N controlled sources which are necessary for realizing N negative impedance converters. The passive RC network required in Sandberg's latter procedure can always be realized by a (3N + 1)-terminal network of two-terminal impedances with a common reference node and no internal nodes. However, as well as being ideal, the necessary N controlled sources must be either four-terminal devices or certain three-terminal degenerations which are formed so as to make them difficult to realize. As is the case in this investigation, Sandberg's work, especially his matrix factorization technique, has become an important part of several active synthesis procedures.

Hazony and Joseph[5] have considered the problem of synthesizing transfer matrices using grounded RC networks embedding ideal grounded unity-gain voltage-controlled voltage sources. Joseph and Hilberman[6,7] have gone somewhat further and have investigated the synthesis of immittance matrices by passive networks and ideal unity-gain voltage and current amplifiers. The procedures developed in these last three references will realize many, but not all, matrices of real rational functions in the complex-frequency variable. By eliminating the ground constraint on the unity-gain voltage amplifiers, Hilberman[8] has devised a method for realizing any rational transfer or admittance matrix with a common-ground active RC network.

Cox[9] has employed ideal operational amplifiers and RC networks to synthesize arbitrary short-circuit admittance matrices. For a prescribed N x N matrix, one of his methods yields a balanced RC network terminated in N operational amplifiers, and another a grounded RC network containing 2N amplifiers. Also, he developed procedures for the simultaneous synthesis of any pair of the admittance parameters associated with a two-port network by use of a balanced RC network and only one operational amplifier. Along with a method for realizing arbitrary short-circuit driving-point admittance functions, Sipress[10] presented techniques for the simultaneous synthesis of pairs of admittance parameters by an interconnection of grounded RC ladder networks and either a voltage-inversion or a current-inversion negative impedance converter. Rajasekaran and Rao[11] have also published methods for the realization of any driving-point admittance function and certain pairs of short-circuit admittance parameters of a two-port network. They employed grounded RC networks and an ideal grounded voltage-controlled differential-output voltage amplifier.

Various methods have been reported for the simulation of two-port transfer functions using controlled sources as the active elements. Kuh[12] utilized grounded RC networks and a practical transistor amplifier to realize most rational voltage transfer functions. Bobrow and Hakimi,[13] Hakim,[14] Cooper and Harbourt,[15] and Richards[16] all employed similar approaches to the synthesis of voltage transfer functions. In each method a grounded network consisting of RC immittances and controlled sources is assumed *a priori*, and procedures are given for finding the immittance values necessary to yield the prescribed transfer functions. Holt and Linggard[17] made liberal use of active elements in their procedure for the synthesis of all-pole transfer functions. They developed ladder networks with cascaded, low gain transistor amplifiers as the active devices. For the realization of transfer impedances and transfer ratios, Curtarelli[18] used two operational amplifiers with feedback as the active elements in an unbalanced RC network.

The aim of this research is the development of procedures for the realization of short-circuit admittance matrices by use of grounded networks with only resistors, capacitors, and nonideal three-terminal amplifiers as elements. Synthesis procedures which utilize, respectively, voltage-controlled voltage sources with finite gains and finite input conductances and current-controlled current sources with finite gains and finite output conductances will be presented. In addition to the matrix techniques, methods will be reported for simultaneously synthesizing any pair of the short-circuit admittance parameters of a two-port network through the use of a grounded RC network containing only two nonideal three-terminal amplifiers.

Some of the notations that will be utilized in this investigation are as follows:

1. A rectangular matrix will be denoted by $\bar{A}$, $[A]$, or $[A_{ij}]$ where A_{ij} denotes the element which appears in the i^{th} row and the j^{th} column of $[A]$.

2. A column matrix will be represented by $A]$.

3. The transpose of matrix $[A]$ will be indicated by $[A]^t$.

4. The inverse of matrix $[A]$ will be expressed as $[A]^{-1}$.

5. The adjoint of matrix $[A]$ will be denoted by adj $[A]$.

6. The determinant of a square matrix $[A]$ will be represented by det $[A]$.

7. Capital y's will be employed for the short-circuit admittance parameters of the active networks, and lower-case y's will be employed for the parameters of the passive RC networks.

8. The maximum degree of the polynomials which are elements of matrix $[A]$ will be indicated by deg $[A]$.

9. The degree of the polynomial q will be denoted by deg q.

Other notations will be presented as they are required.

CHAPTER II

REALIZATION OF ADMITTANCE MATRICES USING VOLTAGE AMPLIFIERS

In this chapter the realization of an N x N short-circuit admittance matrix by means of a transformerless grounded active RC network containing nonideal three-terminal voltage-controlled voltage sources will be considered. The amplifiers will have finite constant voltage gains and finite input conductances. A network of the type contemplated is shown in Figure 1.

In the development of the synthesis procedure, the selected type of network will be analyzed in order to specify its short-circuit admittance matrix in terms of the gains and input conductances of the voltage amplifiers and the admittance parameters of the passive section of the network. By comparing this derived matrix with the prescribed short-circuit admittance matrix, it will be possible to identify the amplifier specifications and the admittance parameters of the passive network so as to satisfy the equation and produce a matrix which is realizable as a transformerless grounded passive RC network.

Analysis of the Network

Consider the grounded transformerless active N-port RC network containing 2M nonideal voltage-controlled voltage sources as shown in Figure 1. Let the input voltage and current matrices of the active N-port network be represented by

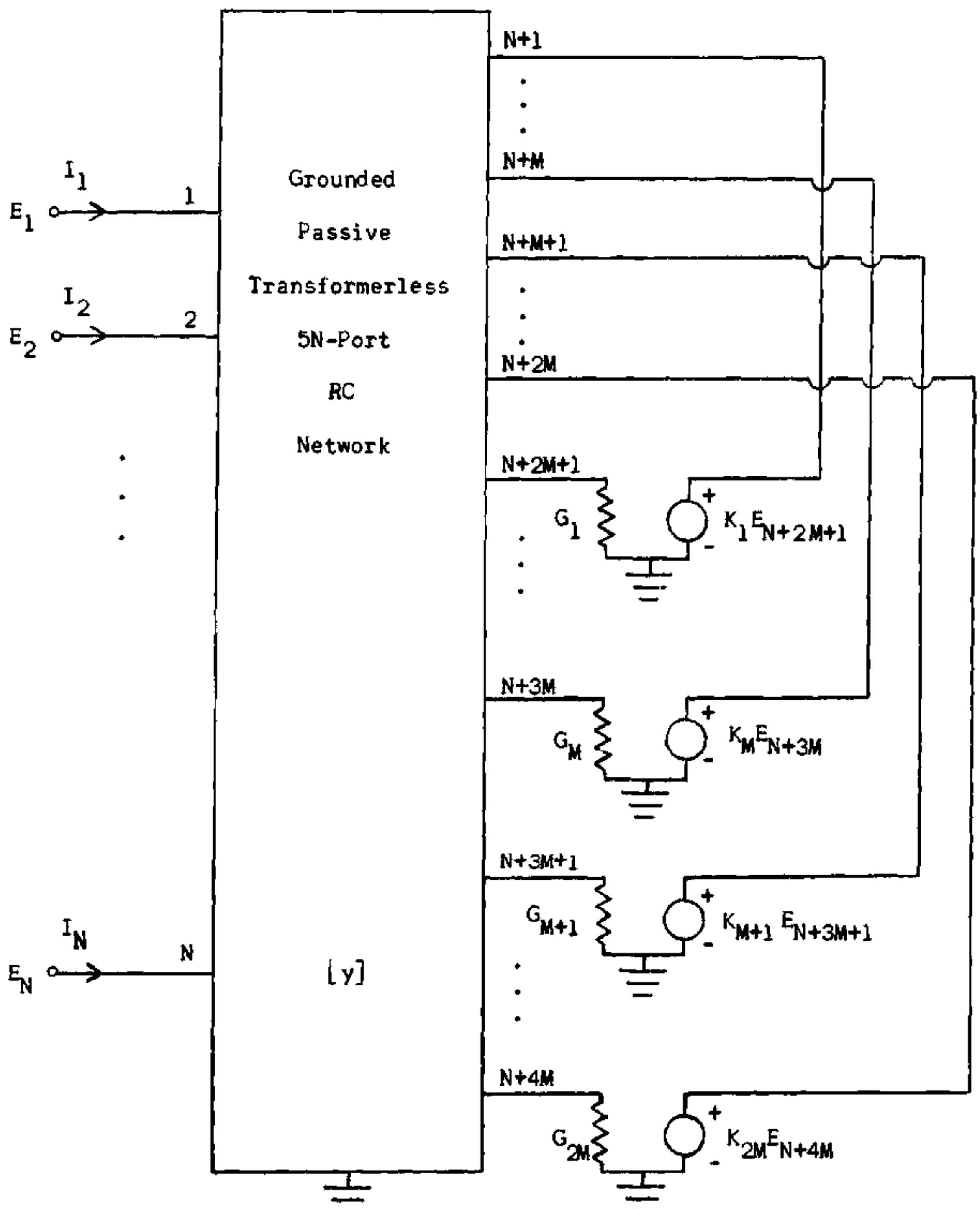


Figure 1. Grounded Active N-Port RC Network Containing 2M Voltage Amplifiers.

$$\bar{E} = \begin{bmatrix} E_1 \\ E_2 \\ \cdot \\ \cdot \\ \cdot \\ E_N \end{bmatrix} \qquad \bar{I} = \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ \cdot \\ I_N \end{bmatrix} \tag{1}$$

If the short-circuit admittance matrix of this active N-port is $\bar{Y}$, then the relationship among the voltage and current variables at the N accessible ports is

$$\bar{I} = \bar{Y}\,\bar{E} \tag{2}$$

Next, let the matrices of the voltages and currents at the outputs of the amplifiers be represented by

$$\begin{aligned} \bar{E}_a &= \begin{bmatrix} E_{N+1} \\ E_{N+2} \\ \cdot \\ \cdot \\ \cdot \\ E_{N+M} \end{bmatrix} \qquad \bar{I}_a = \begin{bmatrix} I_{N+1} \\ I_{N+2} \\ \cdot \\ \cdot \\ \cdot \\ I_{N+M} \end{bmatrix} \\ \bar{E}_b &= \begin{bmatrix} E_{N+M+1} \\ E_{N+M+2} \\ \cdot \\ \cdot \\ \cdot \\ E_{N+2M} \end{bmatrix} \qquad \bar{I}_b = \begin{bmatrix} I_{N+M+1} \\ I_{N+M+2} \\ \cdot \\ \cdot \\ \cdot \\ I_{N+2M} \end{bmatrix} \end{aligned} \tag{3}$$

Likewise, at the inputs to the amplifiers, let the voltage and current matrices be represented by

$$\bar{E}_c = \begin{bmatrix} E_{N+2M+1} \\ E_{N+2M+2} \\ \cdot \\ \cdot \\ \cdot \\ E_{N+3M} \end{bmatrix} \qquad \bar{I}_c = \begin{bmatrix} I_{N+2M+1} \\ I_{N+2M+2} \\ \cdot \\ \cdot \\ \cdot \\ I_{N+3M} \end{bmatrix}$$

$$\bar{E}_d = \begin{bmatrix} E_{N+3M+1} \\ E_{N+3M+2} \\ \cdot \\ \cdot \\ \cdot \\ E_{N+4M} \end{bmatrix} \qquad \bar{I}_d = \begin{bmatrix} I_{N+3M+1} \\ I_{N+3M+2} \\ \cdot \\ \cdot \\ \cdot \\ I_{N+4M} \end{bmatrix} \tag{4}$$

If $[y]$ represents the $(N+4M) \times (N+4M)$ short-circuit admittance matrix of the transformerless passive RC network in Figure 1 and if this matrix is partitioned after its first N and each succeeding group of M rows and columns, then $[y]$ is divided into 25 submatrices as

$$[y] = \begin{bmatrix} \bar{y}_{11} & \bar{y}_{12} & \bar{y}_{13} & \bar{y}_{14} & \bar{y}_{15} \\ \bar{y}_{21} & \bar{y}_{22} & \bar{y}_{23} & \bar{y}_{24} & \bar{y}_{25} \\ \bar{y}_{31} & \bar{y}_{32} & \bar{y}_{33} & \bar{y}_{34} & \bar{y}_{35} \\ \bar{y}_{41} & \bar{y}_{42} & \bar{y}_{43} & \bar{y}_{44} & \bar{y}_{45} \\ \bar{y}_{51} & \bar{y}_{52} & \bar{y}_{53} & \bar{y}_{54} & \bar{y}_{55} \end{bmatrix} \tag{5}$$

where

$$\begin{bmatrix} \bar{I} \\ \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \\ \bar{I}_d \end{bmatrix} = [y] \begin{bmatrix} \bar{E} \\ \bar{E}_a \\ \bar{E}_b \\ \bar{E}_c \\ \bar{E}_d \end{bmatrix} \qquad (6)$$

Since [y] describes a reciprocal network,

$$\bar{y}_{ij} = \bar{y}_{ji}^{t} \qquad (7)$$

for i = 1,2,3,4,5 and j = 1,2,3,4,5.

The constraints imposed on the network variables by the nonideal voltage-controlled voltage sources are

$$\begin{aligned} \bar{E}_a &= \bar{C}\,\bar{E}_c & \qquad \bar{I}_c &= -\,\bar{G}_c\,\bar{E}_c \\ \bar{E}_b &= \bar{D}\,\bar{E}_d & \qquad \bar{I}_d &= -\,\bar{G}_d\,\bar{E}_d \end{aligned} \qquad (8)$$

where $\bar{C}$, $\bar{D}$, $\bar{G}_c$, and $\bar{G}_d$ are M x M diagonal matrices with the amplifier gains as diagonal elements of $\bar{C}$ and $\bar{D}$ and the amplifier input conductances* as diagonal elements of $\bar{G}_c$ and $\bar{G}_d$. The i^{th} diagonal elements of $\bar{C}$ and $\bar{D}$ are the voltage gains of the amplifiers connected between ports N + 2M + i and N + i and ports N + 3M + i and N + M + i, respectively. Likewise, the i^{th} diagonal elements of $\bar{G}_c$ and $\bar{G}_d$ are the input conductances of the amplifiers connected between the same pairs of ports that are

*The resultant procedure can be easily modified to remain valid if the input admittance to each amplifier is a ratio of polynomials in the complex-frequency with only distinct negative-real zeros.

involved in $\bar{C}$ and $\bar{D}$, respectively.

If the amplifier constraints which are given in Equation (8) are introduced into Equation (6), it becomes

$$\begin{bmatrix} \bar{I} \\ \bar{I}_a \\ \bar{I}_b \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{y}_{11} & (\bar{y}_{12}\,\bar{C} + \bar{y}_{14}) & (\bar{y}_{13}\,\bar{D} + \bar{y}_{15}) \\ \bar{y}_{21} & (\bar{y}_{22}\,\bar{C} + \bar{y}_{24}) & (\bar{y}_{23}\,\bar{D} + \bar{y}_{25}) \\ \bar{y}_{31} & (\bar{y}_{32}\,\bar{C} + \bar{y}_{34}) & (\bar{y}_{33}\,\bar{D} + \bar{y}_{35}) \\ \bar{y}_{41} & (\bar{y}_{42}\,\bar{C} + \bar{y}_{44} + \bar{G}_c) & (\bar{y}_{43}\,\bar{D} + \bar{y}_{45}) \\ \bar{y}_{51} & (\bar{y}_{52}\,\bar{C} + \bar{y}_{54}) & (\bar{y}_{53}\,\bar{D} + \bar{y}_{55} + \bar{G}_d) \end{bmatrix} \begin{bmatrix} \bar{E} \\ \bar{E}_c \\ \bar{E}_d \end{bmatrix} \qquad (9)$$

Rows 4 and 5 of the above matrix equation yield the equations

$$(\bar{y}_{42}\,\bar{C} + \bar{y}_{44} + \bar{G}_c)\bar{E}_c + (\bar{y}_{43}\,\bar{D} + \bar{y}_{45})\,\bar{E}_d = -\bar{y}_{41}\,\bar{E} \qquad (10)$$

and

$$(\bar{y}_{52}\,\bar{C} + \bar{y}_{54})\,\bar{E}_c + (\bar{y}_{53}\,\bar{D} + \bar{y}_{55} + \bar{G}_d)\,\bar{E}_d = -\bar{y}_{51}\,\bar{E} \qquad (11)$$

Solving these two equations for $\bar{E}_c$ and $\bar{E}_d$ gives*

$$\begin{aligned} \bar{E}_c = [(\bar{y}_{43}\,\bar{D} + \bar{y}_{45})^{-1}\,(\bar{y}_{42}\,\bar{C} + \bar{y}_{44} + \bar{G}_c) \qquad (12) \\ -(\bar{y}_{53}\,\bar{D} + \bar{y}_{55} + \bar{G}_d)^{-1}\,(\bar{y}_{52}\,\bar{C} + \bar{y}_{54})]^{-1} \\ \cdot\,[(\bar{y}_{53}\,\bar{D} + \bar{y}_{55} + \bar{G}_d)^{-1}\,\bar{y}_{51} - (\bar{y}_{43}\,\bar{D} + \bar{y}_{45})^{-1}\,\bar{y}_{41}]\,\bar{E} \end{aligned}$$

and

---

*It is assumed that all necessary inverses exist.

$$\bar{E}_d = [(\bar{y}_{42}\bar{C} + \bar{y}_{44} + \bar{G}_c)^{-1} (\bar{y}_{43}\bar{D} + \bar{y}_{45})$$
$$- (\bar{y}_{52}\bar{C} + \bar{y}_{54})^{-1} (\bar{y}_{53}\bar{D} + \bar{y}_{55} + \bar{G}_d)]^{-1}$$
$$\cdot [(\bar{y}_{52}\bar{C} + \bar{y}_{54})^{-1} \bar{y}_{51} - (\bar{y}_{42}\bar{C} + \bar{y}_{44} + \bar{G}_c)^{-1} \bar{y}_{41}] \bar{E} \quad (13)$$

Substitution of Equations (12) and (13) into Row 1 of Equation (9) and comparison with Equation (2) yields

$$\bar{Y} = \bar{y}_{11} + \Big\{(\bar{y}_{12}\bar{C} + \bar{y}_{14})[(\bar{y}_{43}\bar{D} + \bar{y}_{45})^{-1} (\bar{y}_{42}\bar{C} + \bar{y}_{44} + \bar{G}_c)$$
$$- (\bar{y}_{53}\bar{D} + \bar{y}_{55} + \bar{G}_d)^{-1} (\bar{y}_{52}\bar{C} + \bar{y}_{54})]^{-1}$$
$$\cdot [(\bar{y}_{53}\bar{D} + \bar{y}_{55} + \bar{G}_d)^{-1} \bar{y}_{51} - (\bar{y}_{43}\bar{D} + \bar{y}_{45})^{-1} \bar{y}_{41}]\Big\}$$
$$+ \Big\{(\bar{y}_{13}\bar{D} + \bar{y}_{15})[(\bar{y}_{42}\bar{C} + \bar{y}_{44} + \bar{G}_c)^{-1} (\bar{y}_{43}\bar{D} + \bar{y}_{45})$$
$$-(\bar{y}_{52}\bar{C} + \bar{y}_{54})^{-1} (\bar{y}_{53}\bar{D} + \bar{y}_{55} + \bar{G}_d)]^{-1}$$
$$\cdot [(\bar{y}_{52}\bar{C} + \bar{y}_{54})^{-1} \bar{y}_{51} - (\bar{y}_{42}\bar{C} + \bar{y}_{44} + \bar{G}_c)^{-1} \bar{y}_{41}]\Big\} \quad (14)$$

If it is assumed that*

$$\bar{y}_{13} = \bar{y}_{15} = \bar{y}_{45} = \bar{y}_{53} = [0] \quad (15)$$

and

---

*There are, of course, other relationships which will simplify Equation (14). One such set is

$$\bar{y}_{13} = \bar{y}_{15} = \bar{y}_{45} = \bar{y}_{53} = [0]$$

and

$$\bar{y}_{43}\bar{D} = \bar{y}_{55} + \bar{G}_d$$

$$\bar{y}_{52}\bar{C} = \bar{y}_{55} + \bar{G}_d \text{ ,} \tag{16}$$

Equation (14) reduces to

$$\bar{Y} = \bar{y}_{11} - (\bar{y}_{14} + \bar{y}_{12}\bar{C})(\bar{y}_{44} + \bar{y}_{42}\bar{C} - \bar{y}_{43}\bar{D} + \bar{G}_c)^{-1}\, \bar{y}_{41} \tag{17}$$

To simplify the procedure, assume that the amplifier voltage-gain matrices $\bar{C}$ and $\bar{D}$ are given by

$$\bar{C} = -\, c\, \bar{U} \qquad\qquad \bar{D} = -\, d\, \bar{U} \tag{18}$$

where $\bar{U}$ is the M^{th} order identity matrix and c and d are positive real constants that will be determined later. Equations (16) and (17) may now be rewritten as

$$\bar{y}_{55} + \bar{G}_d = -\, c\, \bar{y}_{52} \tag{19}$$

and

$$\bar{Y} = \bar{y}_{11} - (\bar{y}_{14} - c\, \bar{y}_{12})(\bar{y}_{44} - c\, \bar{y}_{42} + d\, \bar{y}_{43} + \bar{G}_c)^{-1}\, \bar{y}_{41} \tag{20}$$

Equation (20) gives the short-circuit admittance matrix of the transformerless grounded active N-port RC network containing 2M nonideal voltage-controlled voltage sources. Of course, Equation (20) is valid only if the submatrices of $[y]$ fulfill the conditions of Equations (15), (18), and (19).

Realization Procedure

First, it will be assumed that M = N so that the resulting technique will require an active N-port RC network containing 2N nonideal voltage-controlled voltage sources. The assumptions in Equations (18) require N of these amplifiers to have a constant gain of -c and N a gain of -d. In order to prove that the network in Figure 1 with M = N is sufficient for the synthesis of a given N x N short-circuit admittance matrix, a realization procedure that is valid for any prescribed matrix will be presented.

Let the given N x N short-circuit admittance matrix be written as

$$\bar{Y} = \frac{[P]}{Q} \tag{21}$$

where $[P]$ is a polynomial matrix and Q is a polynomial in the complex frequency variable s. The function Q represents either the common denominator of all elements of $\bar{Y}$ if they are identical, or the least common multiple of all denominators if augmentation is necessary. Select an appropriate N x N matrix $\bar{y}_{11}$ which satisfies the conditions to be given below. Denote the chosen matrix by

$$\bar{y}_{11} = \frac{[p]}{q} = \frac{[p_{ij}]}{q} \tag{22}$$

where $[p]$ is a matrix of polynomials and q is the common denominator of all elements of $\bar{y}_{11}$. Subtracting of Equation (22) from Equation (21) yields

$$\bar{Y} - \bar{y}_{11} = \frac{[P]q - [p]Q}{Qq} = \frac{[A]}{Qq} \tag{23}$$

Now, the chosen short-circuit admittance matrix $\bar{y}_{11}$ should satisfy the following conditions:

(A) deg p_{ij} = deg $q = NL_o = T$ where $i = 1,2,\ldots,N$, $j = 1,2,\ldots,N$ and $L_o = \max$ [deg P_{ij}, deg Q];

(B) $\frac{p_{ii}}{q}$ and $\frac{p_{ij}}{q}$ $(i \neq j)$ are, respectively, positive and negative RC driving-point admittance functions with $p_{ij} = p_{ji}$ and $p_{ii}(0) \neq 0$;

(C) in the Foster expansion

$$\bar{y}_{11} = [B_o] + \sum_{\ell=1}^{T} [B_\ell] \frac{s}{s + \sigma_\ell}$$

where σ_ℓ are the zeros of q, the coefficient matrices $[B_o]$ and $[B_\ell]$ must satisfy the dominance* condition with an inequality;

(D) det $[A]$ contains NT distinct negative-real zeros;

(E) the polynomial matrix $[A]$, defined in Equation (23), can be written as the product $[A_1][A_2]$ of two N x N matrices of degrees, respectively, T and L_o; and

(F) the polynomial matrix $[A_2]$ has the property that det $[A_2]$, a polynomial of degree NL_o in s, has only distinct negative-real zeros which are different from those of q.

Conditions (A), (B), and (C) are easily satisfied by proper choices of the elements of $\bar{y}_{11}$. In Appendix I it is demonstrated that if Conditions (A) and (B) are fulfilled, then Condition (D) can be met by

---

*A symmetric matrix of real constants is said to be a dominant matrix if each of its main-diagonal elements is not less than the sum of the absolute values of all the other elements in the same row.[20]

sufficiently large choices for the p_{ii} elements of $\bar{y}_{11}$. Appendix II shows that Conditions (E) and (F) are fulfilled if Condition (D) is satisfied. Hence, for any specified $\bar{Y}$, a $\bar{y}_{11}$ can always be found such that the above six conditions are satisfied.

Equation (23) and Condition (E) yield the result

$$\bar{Y} - \bar{y}_{11} = \frac{[A_1][A_2]}{Qq} \tag{24}$$

and Equations (20) and (24) give

$$(\bar{y}_{14} - c\bar{y}_{12})(\bar{y}_{44} - c\bar{y}_{42} + d\bar{y}_{43} + \bar{G}_c)^{-1}\ \bar{y}_{41} = \frac{[A_1][A_2]}{Qq} \tag{25}$$

Solving the above equation for $\bar{y}_{44} - c\bar{y}_{42} + d\bar{y}_{43} + \bar{G}_c$ produces

$$\bar{y}_{44} - c\bar{y}_{42} + d\bar{y}_{43} + \bar{G}_c = -Qq\,\bar{y}_{14}^{t}[A_2]^{-1}[A_1]^{-1}(\bar{y}_{14} - c\bar{y}_{12}) \tag{26}$$

To begin the identification of the submatrices in $[y]$, let

$$\bar{y}_{14} - c\bar{y}_{12} = \alpha\,\frac{[A_1]}{q} \tag{27}$$

where α is a nonzero real constant to be specified later. By Condition (E), $[A_1]$ is of degree T so that $[A_1]/q$ is regular at infinity. Hence, if

$$q = L_1 \prod_{u=1}^{T} (s + \sigma_u) \tag{28}$$

where $0 < \sigma_1 < \sigma_2 < \ldots \sigma_T$ and L_1 is a real constant, then the Foster expansion of Equation (27) can be written as

$$\bar{y}_{14} - c\bar{y}_{12} = \alpha \sum_{u=0}^{T} \bar{F}_u \frac{s}{s + \sigma_u} \tag{29}$$

where the $\bar{F}_u$ are real coefficient matrices and $\sigma_o = 0$. The above equation can also be expressed as

$$\bar{y}_{14} - c\bar{y}_{12} = \alpha \sum_{u=0}^{T} \bar{G}_u \frac{s}{s + \sigma_u} - \alpha \sum_{u=0}^{T} \bar{H}_u \frac{s}{s + \sigma_u} \tag{30}$$

where the elements in $\bar{G}_u$ and $\bar{H}_u$ are real and nonnegative. With the identifications

$$\bar{y}_{12} = - \frac{\alpha}{c} \sum_{u=0}^{T} \bar{G}_u \frac{s}{s + \sigma_u} \tag{31}$$

and

$$\bar{y}_{14} = -\alpha \sum_{u=0}^{T} \bar{H}_u \frac{s}{s + \sigma_u} \tag{32}$$

Equation (30) is satisfied, and all of the elements of the submatrices $\bar{y}_{12}$ and $\bar{y}_{14}$ are negative RC driving-point admittance functions.

Let the matrix $\bar{y}_{14}$ be rewritten as

$$\bar{y}_{14} = \alpha \frac{[N_{14}]}{q_a} \tag{33}$$

where Equation (32) reveals that the zeros of q_a are contained in the set of zeros of q. Also, it is clear from the above two equations that

$$\deg [N_{14}] = \deg q_a = T_a \tag{34}$$

Upon substitution of Equations (27) and (33), Equation (26) becomes

$$\bar{Y}_{44} - c\bar{Y}_{42} + d\bar{Y}_{43} + \bar{G}_c = -\alpha^2 \frac{Q[N_{14}]^t(\mathrm{adj}[A_2])}{q_a \det [A_2]} \qquad (35)$$

From Condition (F), det $[A_2]$ contains only distinct negative-real zeros that are different from those of q.

Hence,

$$q_a \det [A_2] = L_2 \prod_{v=1}^{R} (s + \gamma_v) \qquad (36)$$

where $0 < \gamma_1 < \gamma_2 < \cdots < \gamma_R$, L_2 is a real constant, and R is the degree of the polynomial q_a det $[A_2]$.

By assuming that Equation (35) is regular at infinity, it can be written in its Foster expansion as

$$\bar{Y}_{44} - c\bar{Y}_{42} + d\bar{Y}_{43} + G_c = \sum_{v=0}^{R} \bar{V}_v \frac{s}{s + \gamma_v} \qquad (37)$$

where $\gamma_o = 0$ and the $\bar{V}_v$ are real coefficient matrices. In order to demonstrate that the above assumption is valid, it must be shown that for each element of the matrix equation the degree of the numerator is less than or equal to the degree of the denominator. This requires that

$$\deg Q + \deg [N_{14}]^t + \deg (\mathrm{adj}\, [A_2]) \leq \deg q_a + \deg (\det [A_2]) \quad (38)$$

Since the degree of a matrix and its transpose are identical,

$$\deg [N_{14}]^t = \deg q_a = T_a \qquad (39)$$

Also,

$$\deg\ (\mathrm{adj}\ [A_2]) = (N-1)\ L_o \tag{40}$$

and

$$\deg\ (\det\ [A_2]) = NL_o \tag{41}$$

Equation (37) then reduces to

$$\deg\ Q \leq L_o \tag{42}$$

Utilizing the definition of L_o from Condition (A), the above equation becomes

$$\deg\ Q \leq \max\ [\deg\ P_{ij},\ \deg\ Q] \tag{43}$$

which is always true. Hence, Equation (35) is regular at infinity, and its expansion as given in Equation (37) is valid. This latter equation can be rewritten as

$$\bar{y}_{44} - c\bar{y}_{42} + d\bar{y}_{43} + \bar{G}_c = \sum_{v=0}^{R} \bar{W}_v \frac{s}{s+\gamma_v} + \sum_{v=0}^{R} \bar{X}_v \frac{s}{s+\gamma_v} - \sum_{v=0}^{R} \bar{Z}_v \frac{s}{s+\gamma_v} \tag{44}$$

where the coefficient matrices $\bar{W}_v$, $\bar{X}_v$, and $\bar{Z}_v$ contain only nonnegative elements. In addition, $\bar{W}_v$ is a diagonal matrix whose diagonal elements are the positive diagonal elements of the coefficient matrix $\bar{V}_v$ in Equation (37), and $\bar{X}_v$ is composed of the remaining positive elements of

$\bar{V}_v$. Equation (44) can be augmented to yield

$$\bar{y}_{44} - c\bar{y}_{42} + d\bar{y}_{43} = \sum_{v=0}^{R} [\bar{W}_v + \bar{J}_v] \frac{s}{s + \gamma_v} + \sum_{v=0}^{R} \bar{X}_v \frac{s}{s + \gamma_v} - \sum_{v=0}^{R} [\bar{Z}_v + \bar{J}_v] \frac{s}{s + \gamma_v} - \bar{G}_c \tag{45}$$

where each $\bar{J}_v$ is a diagonal matrix whose diagonal elements are sufficiently large positive real numbers. A nonzero diagonal element in a $\bar{J}_v$ should be chosen to add additional terms to $\bar{y}_{44}$ for reasons that will be considered later. In the above equation, the diagonal matrix $\bar{G}_c$ is arbitrary, and it may be selected to contain nonnegative input conductance values which are readily realizable by the particular voltage amplifier design that is contemplated. Now, the following identifications can be made:

$$\bar{y}_{44} = \sum_{v=0}^{R} [\bar{W}_v + \bar{J}_v] \frac{s}{s + \gamma_v} \tag{46}$$

$$\bar{y}_{42} = - \frac{1}{c} \sum_{v=0}^{R} \bar{X}_v \frac{s}{s + \gamma_v} \tag{47}$$

and

$$\bar{y}_{43} = - \frac{1}{d} \sum_{v=0}^{R} [\bar{Z}_v + \bar{J}_v] \frac{s}{s + \gamma_v} - \frac{1}{d} \bar{G}_c \tag{48}$$

Note that each diagonal element of the submatrix $\bar{y}_{44}$ is a positive RC driving-point admittance function. The associated off-diagonal elements are zero, and hence, they are negative RC admittance functions. All of the elements of the submatrices $\bar{y}_{42}$ and $\bar{y}_{43}$ are negative RC driving-point admittance functions.

From Equation (19) with M = N, the relationship between the two nonzero submatrices which contain admittances from the last N rows of [y] may be augmented into the form

$$\bar{y}_{55} + c\bar{y}_{52} = -\bar{G}_d + \bar{L} - \bar{L} \tag{49}$$

where $\bar{G}_d$ and $\bar{L}$ are diagonal matrices with nonnegative diagonal elements that will be considered below. Let

$$\bar{y}_{55} = \frac{1}{(c - 1)} \bar{G}_d + \bar{L} \qquad (c > 1) \tag{50}$$

and

$$\bar{y}_{52} = \frac{-1}{(c - 1)} \bar{G}_d - \frac{1}{c} \bar{L} \qquad (c > 1) \tag{51}$$

As in the case of $\bar{G}_c$, $\bar{G}_d$ may be chosen for the convenience of the amplifier designs. If, after the selection of the diagonal elements of $\bar{G}_d$, det $\bar{G}_d$ is unequal to zero, then all of the elements of $\bar{L}$ should be taken to be zero. However, if one or more ideal voltage amplifiers are to be utilized, then det $\bar{G}_d$ will be equal to zero, and an arbitrary positive constant must be selected for each of the diagonal elements of $\bar{L}$. In either case the designations in Equations (50) and (51) are such that the inverses of the matrices $(\bar{y}_{55} + \bar{G}_d)$ and $\bar{y}_{52}$ are defined. The existence of these inverses is required for the validity of the results in the analysis section of this chapter. Note that $\bar{y}_{55}$ has been identified as a real matrix whose diagonal elements are positive RC driving-point admittance functions and whose off-diagonal elements are zero or negative RC admittances. Likewise, $\bar{y}_{52}$ contains only negative RC admittance functions.

Each of the submatrices appearing in Equations (19) and (20) have now been chosen so that these two equations with M = N may be satisfied for any prescribed admittance matrix $\bar{Y}$. To complete the realization procedure, it remains to be shown that the set of submatrices will form a short-circuit admittance matrix which is realizable by a transformerless grounded passive 5N-port RC network. It is known[19] that the necessary and sufficient conditions for the realization of [y] through the use of a (5N + 1)-terminal network of two-terminal admittances with a common reference node and no internal nodes are as follows:

(1) The diagonal elements are positive RC driving-point admittance functions;

(2) the off-diagonal elements are negative RC driving-point admittance functions; and

(3) all of the coefficient matrices in the Foster expansion of [y] are dominant.

Condition (B) and Equations (22), (46), and (50) reveal that the submatrices $\bar{y}_{11}$, $\bar{y}_{44}$, and $\bar{y}_{55}$, which are specified in the realization procedure, contain diagonal elements which satisfy Condition (1) and off-diagonal elements which satisfy Condition (2). Equations (15), (31), (32), (47), (48), and (51) show that the remainder of the specified submatrices meet the requirements of Condition (2). The unspecified submatrices $\bar{y}_{22}$, $\bar{y}_{33}$, and $\bar{y}_{23}$ may be chosen so that they satisfy

Conditions (1) and (2) as well as aid in the realization of [y]. Since Equations (31) and (32) show that the submatrices $\bar{y}_{12}$ and $\bar{y}_{14}$ contain a constant multiplier of α, the values of these parameters can be made as small as desired by choosing a small numerical value for α. Condition (C) requires the chosen $\bar{y}_{11}$ to fulfill the dominance condition with the inequality sign. Thus, the elements of the three nonzero submatrices which compose the first N rows of [y] can be made to satisfy the dominance condition by a sufficiently small value for α. Proper choices for the unspecified quantities $\bar{y}_{22}$ and $\bar{y}_{33}$ insure that rows (N + 1) through 2N and (2N + 1) through 3N meet the dominance requirements of Condition (3). If the matrix $\bar{L}$ is identically zero, the choices in Equations (15), (50), and (51) allow the last N rows of [y] to satisfy the dominance condition with the equality sign. In the instances when the diagonal elements of $\bar{L}$ must be nonzero, these last N rows fulfill the dominance requirement with an inequality. Note that Equation (50) restricts the voltage gain parameter c to the set of positive constants which are greater than unity. The presence of the inverse of the voltage gain factor d in the submatrix $\bar{y}_{43}$ allows this matrix to be reduced as the amplifier gains are increased. Thus, by taking d to be a constant greater than unity and by making proper choices for the augmentation coefficient matrices $\bar{J}_v$, rows (3N + 1) through 4N of [y] can be forced to fulfill Condition (3). In general, a nonzero element in a certain row of $\bar{J}_v$ should be selected when each element or sum of elements in the same row of $\bar{y}_{41}$, $\bar{y}_{42}$, and $\bar{y}_{43}$ possesses an absolute value which is larger than the corresponding element in $\bar{W}_v$ in Equation (46). By proper augmentation, large values for the amplifier gains are not necessary, and the elements in the passive network may be reduced. Furthermore, the realization procedure is successful if positive constants greater than unity are specified for c and d and if arbitrary nonnegative constants are assigned to $\bar{G}_c$ and $\bar{G}_d$ before beginning the synthesis of the desired matrix $\bar{Y}$.

Since the procedure has identified an admittance matrix $[y]$ which meets all of the requirements imposed on it, the passive RC network may be synthesized from this matrix. A method for realizing the 5N x 5N short-circuit admittance matrix $[y]$ by a transformerless passive (5N + 1)-terminal network composed of two-terminal impedances with a common reference node and no internal nodes is contained in reference (19). For a particular $\bar{Y}$, the 2N voltage amplifiers will be specified to have arbitrarily selected input impedances and certain negative gains. When these amplifiers are constructed and connected to the passive network as shown in Figure 1, the desired short-circuit admittance matrix $\bar{Y}$ is realized at the N remaining ports.

The realization technique which is contained in this chapter constitutes a proof of the following theorem:

Theorem 1

To realize an arbitrary N x N short-circuit admittance matrix of real rational functions in the complex-frequency variable by a transformerless grounded active N-port RC network embedding three-terminal voltage amplifiers with negative finite constant gains greater than unity and finite input conductances, it is sufficient that the active network contains 2N amplifiers.

An Example

As an example of the synthesis procedure employing voltage amplifiers, a 2-port active RC network containing 4 nonideal voltage amplifiers will be found to realize the short-circuit admittance matrix

$$\bar{Y} = \frac{1}{s+1}\begin{bmatrix} s+1 & s-3 \\ s+2 & s \end{bmatrix} = \frac{[P]}{Q} \qquad (52)$$

First, a 2 x 2 RC short-circuit admittance matrix will be selected for $\bar{y}_{11}$. Take

$$\bar{y}_{11} = \frac{1}{s+4}\begin{bmatrix} 40(s+3) & 0 \\ 0 & 40(s+2) \end{bmatrix} = \frac{[p]}{q} \qquad (53)$$

Note that to be conservatively sure that the resulting difference matrix $[A]$ can be factored into the desired form, a q of the second degree should be chosen. However, this particular requirement in Condition (A) is only sufficient, not necessary. Subtraction of $\bar{y}_{11}$ from $\bar{Y}$ yields

$$\bar{Y} - \bar{y}_{11} = \frac{\begin{bmatrix} -39s^2 - 155s - 116 & s^2 + s - 12 \\ s^2 + 6s + 8 & -39s^2 - 116s - 80 \end{bmatrix}}{(s+1)(s+4)} \qquad (54)$$

$$= \frac{[A]}{Qq}$$

By use of the matrix factorization technique in Appendix II,

$$[A] = \begin{bmatrix} -35.35s - 84.55 & 10.66s + 10.64 \\ -141.2s + 150.2 & -0.4063s - 0.8264 \end{bmatrix} \begin{bmatrix} 0.00341s + 0.006444 & 0.2732s + 0.5164 \\ -3.647s - 10.85 & s + 2.975 \end{bmatrix}$$

$$= [A_1][A_2] \tag{55}$$

From Equation (27)

$$\bar{y}_{14} - c\bar{y}_{12} = \alpha \frac{\begin{bmatrix} -35.35s - 84.55 & 10.66s + 10.64 \\ -141.2s + 150.2 & -0.4063s - 0.8264 \end{bmatrix}}{(s + 4)} \tag{56}$$

After taking a Foster expansion of the above equation, an allocation of terms as in Equations (31) and (32) yields

$$\bar{y}_{12} = -\frac{\alpha}{c} \begin{bmatrix} 0 & 2.66 \\ 37.55 & 0 \end{bmatrix} - \frac{\alpha}{c} \frac{s}{s+4} \begin{bmatrix} 0 & 8.00 \\ 0 & 0 \end{bmatrix} \tag{57}$$

and

$$\bar{y}_{14} = -\alpha \begin{bmatrix} 21.14 & 0 \\ 0 & 0.2066 \end{bmatrix} - \alpha \frac{s}{s+4} \begin{bmatrix} 14.21 & 0 \\ 178.8 & 0.1997 \end{bmatrix} \tag{58}$$

From Equations (35), (45), (46), (47), and (48),

$$\bar{y}_{44} = \alpha^2 \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} + \alpha^2 \frac{s}{s+1.80} \begin{bmatrix} 3.412 \times 10^2 + J_3 & 0 \\ 0 & J_4 \end{bmatrix}$$

$$+ \alpha^2 \frac{s}{s+2.975} \begin{bmatrix} J_5 & 0 \\ 0 & J_6 + 9.623 \times 10^{-4} \end{bmatrix} + \alpha^2 \frac{s}{s+4} \begin{bmatrix} J_7 & 0 \\ 0 & J_8 \end{bmatrix} \tag{59}$$

$$\bar{y}_{42} = -\frac{\alpha^2}{c}\begin{bmatrix}0 & 1.94\\ 0 & 0\end{bmatrix} - \frac{\alpha^2}{c}\frac{s}{s+4}\begin{bmatrix}0 & 9.575\\ 0 & 0\end{bmatrix} \tag{60}$$

$$\bar{y}_{43} = -\frac{\alpha^2}{d}\begin{bmatrix}1.118\times 10^{1} + \frac{d}{\alpha^2}G_1 + J_1 & 0\\ 0 & 2.699\times 10^{-4} + \frac{d}{\alpha^2}G_2 + J_2\end{bmatrix} \tag{61}$$

$$-\frac{\alpha^2}{d}\frac{s}{s+1.890}\begin{bmatrix}J_3 & 3.908\times 10^{-3}\\ 5.424\times 10^{-2} & J_4\end{bmatrix} - \frac{\alpha^2}{d}\frac{s}{s+2.975}$$

$$\begin{bmatrix}3.20\times 10^{2} + J_5 & 2.462\\ 0 & J_6\end{bmatrix} - \frac{\alpha^2}{d}\frac{s}{s+4}\begin{bmatrix}6.975\times 10^{2} + J_7 & 0\\ 1.18 & 2.271\times 10^{-3} + J_8\end{bmatrix}$$

where the nonnegative input conductances of the first two amplifiers form the matrix

$$\bar{G}_c = \begin{bmatrix}G_1 & 0\\ 0 & G_2\end{bmatrix} \tag{62}$$

Equations (15), (50), and (51) require that

$$\bar{y}_{13} = \bar{y}_{15} = \bar{y}_{45} = \bar{y}_{53} = [0] \tag{63}$$

$$\bar{y}_{55} = \frac{1}{(c-1)}\begin{bmatrix}G_3 & 0\\ 0 & G_4\end{bmatrix} \quad (c > 1) \tag{64}$$

and

$$\bar{y}_{52} = \frac{-1}{(c-1)} \begin{bmatrix} G_3 & 0 \\ 0 & G_4 \end{bmatrix} \quad (c > 1) \tag{65}$$

where the two amplifiers with a voltage gain of -d have positive input conductances that yield

$$\bar{G}_d = \begin{bmatrix} G_3 & 0 \\ 0 & G_4 \end{bmatrix} \tag{66}$$

In this example, all of the elements of $\tilde{L}$ have been taken to be zero.

The submatrices contained in Equation (20) have now been given values in Equations (53), (57), (58), (59), (60), and (61) such that Equation (20) with M = N is satisfied for the prescribed $\bar{Y}$. Equations (63), (64), and (65) specify values for the submatrices which must be constrained in order that Equation (20) is valid. The amplifier input conductances G_1, G_2, G_3, and G_4 can be chosen as discussed above, and the amplifier gain factors c and d may be selected to be any desired values greater than unity. The constant α can then be taken so that the first two rows of [y] satisfy the dominance condition. Because of the presence of c and d in the expressions for $\bar{y}_{42}$ and $\bar{y}_{43}$, the augmentation coefficient elements J_1 through J_8 can be selected so that $\bar{y}_{44}$ contains a pole with a larger positive residue than is contained in the sum of the absolute values of the residues for each pole of $\bar{y}_{14}$, $\bar{y}_{42}$, and $\bar{y}_{43}$. Thus, rows 7 and 8 of [y] are forced to be dominant. The values given in Equations (63), (64), and (65) insure that rows 9 and 10 satisfy the dominance condition with the equality sign. In selecting the remainder of the submatrices in Equation (14), the off-diagonal submatrix $\bar{y}_{23}$, which Equation (7) requires to equal $\bar{y}_{32}^{\,t}$, is arbitrary as long as each element is a negative RC driving-point admittance function. The on-diagonal submatrices $\bar{y}_{22}$ and $\bar{y}_{33}$ are to be chosen with diagonal elements which are positive RC driving-point admittance functions and off-diagonal elements which are negative RC admittance functions. The terms in $\bar{y}_{22}$ must contain the poles of $\bar{y}_{12}$, $\bar{y}_{23}$, $\bar{y}_{42}$, and $\bar{y}_{52}$ in their set of poles, and the elements of $\bar{y}_{33}$ must contain the poles of $\bar{y}_{23}$ and $\bar{y}_{43}$. The diagonal terms in $\bar{y}_{22}$ and $\bar{y}_{33}$ are to be taken large enough to satisfy the dominance condition, respectively, for rows 3 and 4 and for 5 and 6. A reduction in the number of elements required to realize $[y]$ may be obtained by selecting some of the J's and the diagonal elements of $\bar{y}_{22}$ and $\bar{y}_{33}$ such that rows 3 through 6 and some of the terms of rows 7 and 8 satisfy the dominance condition with equality.

Here, if $G_1 = G_2 = G_3 = G_4 = 0.10$ and $c = d = 2$, then any $\alpha < 0.1117$ will allow rows 1 and 2 to be dominant. Thus, take $\alpha = 0.10$ and then the following set of augmentation parameters are the minimum values which can be used to force rows 7 and 8 to satisfy the dominance condition:

$$J_1 = 4.559 \times 10^2 \qquad J_2 = 2.459 \times 10^1$$

$$J_3 = 0 \qquad J_4 = 5.424 \times 10^2$$

$$J_5 = 3.224 \times 10^2 \qquad J_6 = 0$$

$$J_7 = 4.287 \times 10^3 \qquad J_8 = 2.854 \times 10^2$$

Selection of appropriate values for $\bar{y}_{22}$, $\bar{y}_{33}$, and $\bar{y}_{23}$ will complete the specification of $[y]$.

The elements of the 10 x 10 short-circuit admittance matrix of the passive network have now been determined so that $[y]$ is realizable by a 10-port 11-terminal transformerless grounded passive RC network. After a Foster expansion of $[y]$, one of the procedures given in reference (19) may be used to synthesize the RC network. After the specified amplifiers are connected to ports 3 through 10, the prescribed short-circuit admittance matrix in Equation (52) is realized at ports 1 and 2.

CHAPTER III

REALIZATION OF ADMITTANCE MATRICES USING CURRENT AMPLIFIERS

This chapter is concerned with the synthesis of N x N short-circuit admittance matrices through the use of grounded RC networks and nonideal three-terminal current-controlled current sources. The following theorem will be verified:

Theorem 2

To realize an arbitrary N x N short-circuit admittance matrix of real rational functions in the complex-frequency variable by a transformerless grounded active N-port RC network embedding three-terminal current amplifiers with negative finite constant gains greater than unity and finite output conductances, it is sufficient that the active network contains 2N amplifiers.

To prove this theorem, a realization procedure which is valid for any N x N admittance matrix of real rational functions in the complex-frequency variable will be given. First, the chosen active network, which is shown in Figure 2, will be analyzed so that its short-circuit admittance matrix is expressed in terms of the gains and output conductances of the current sources and the admittance parameters of the passive portion of the network. This expression will be made to equal the prescribed matrix through proper choices for the amplifier gains and output conductances and the admittances of the passive

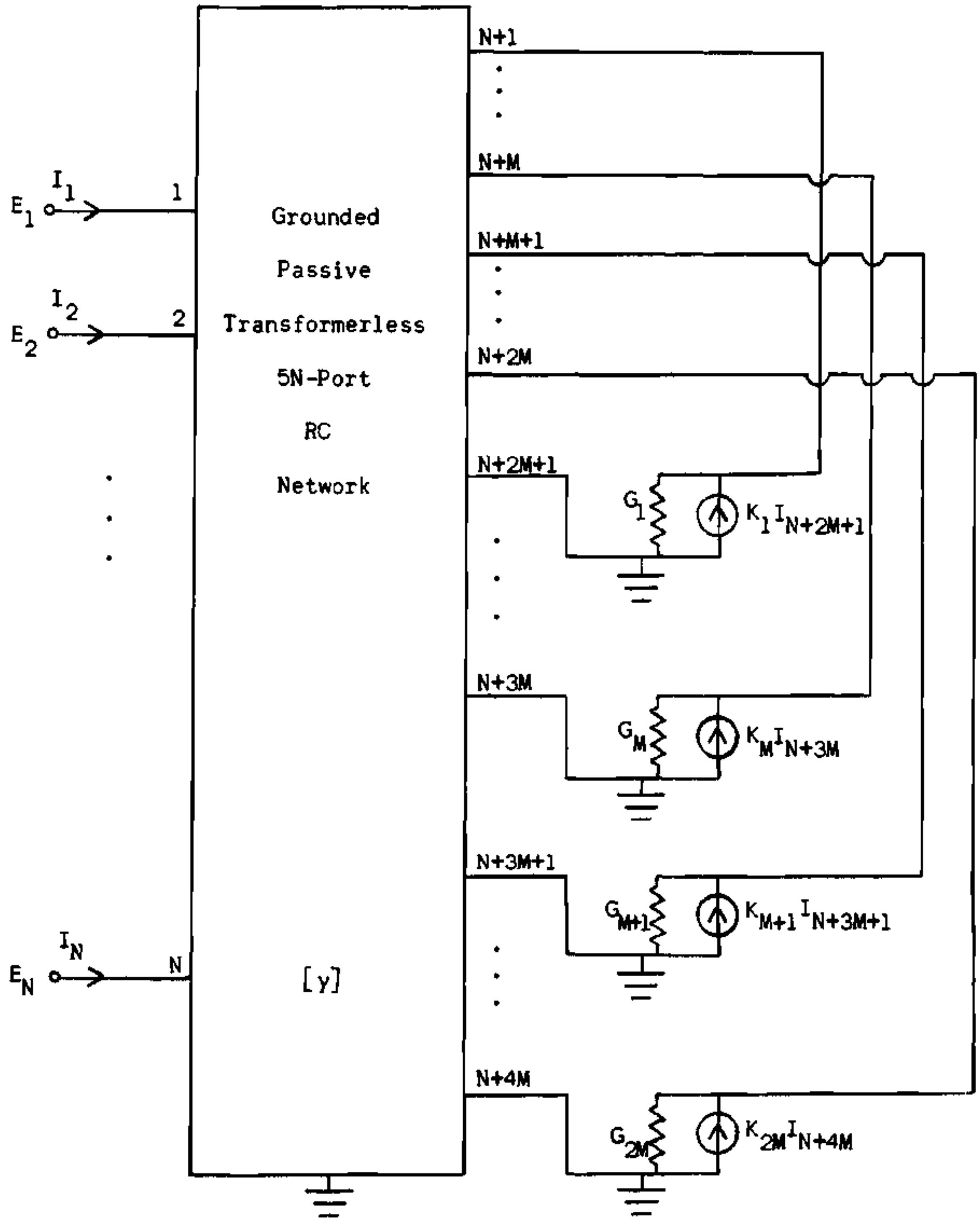


Figure 2. Grounded Active N-Port RC Network Containing 2M Current Amplifiers.

subnetwork. Finally, it will be shown that the passive portion of the network can always be realized as a transformerless, grounded RC network.

Analysis of the Network

By partitioning the short-circuit admittance matrix [y] of the passive RC subnetwork in Figure 2 after its $(N + kM)^{th}$ $(k = 0,1,2,3)$ rows and columns, the voltages and currents at the ports of this portion of the network are related by

$$\begin{bmatrix} \bar{I} \\ \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \\ \bar{I}_d \end{bmatrix} = \begin{bmatrix} \bar{y}_{11} & \bar{y}_{12} & \bar{y}_{13} & \bar{y}_{14} & \bar{y}_{15} \\ \bar{y}_{21} & \bar{y}_{22} & \bar{y}_{23} & \bar{y}_{24} & \bar{y}_{25} \\ \bar{y}_{31} & \bar{y}_{32} & \bar{y}_{33} & \bar{y}_{34} & \bar{y}_{35} \\ \bar{y}_{41} & \bar{y}_{42} & \bar{y}_{43} & \bar{y}_{44} & \bar{y}_{45} \\ \bar{y}_{51} & \bar{y}_{52} & \bar{y}_{53} & \bar{y}_{54} & \bar{y}_{55} \end{bmatrix} \begin{bmatrix} \bar{E} \\ \bar{E}_a \\ \bar{E}_b \\ \bar{E}_c \\ \bar{E}_d \end{bmatrix} \qquad (67)$$

where $\bar{I}$ and $\bar{E}$ are column matrices composed, respectively, of the currents and voltages at the first N ports and the remaining $\bar{I}$'s and $\bar{E}$'s are composed of the corresponding electrical quantities at each succeeding group of M ports. That is, $\bar{E}_a$ contains the voltages at ports $(N + 1)$ through $(N + M)$, and $\bar{E}_b$ contains the voltages at ports $(N + M + 1)$ through $(N + 2M)$. In Equation (67)

$$\bar{y}_{ij} = \bar{y}_{ji}^{\,t} \qquad (68)$$

for $i = 1,2,3,4,5$ and $j = 1,2,3,4,5$ because the network represented by [y] is reciprocal.

The nonideal current-controlled current sources force the following constraints upon the network parameters:

$$\bar{I}_a = -\bar{C}\,\bar{I}_c + \bar{G}_c\,\bar{E}_a \qquad \bar{E}_c = 0$$
$$\bar{I}_b = -\bar{D}\,\bar{I}_d + \bar{G}_d\,\bar{E}_b \qquad \bar{E}_d = 0 \tag{69}$$

where

$$\bar{C} = \operatorname{diag}[K_1,\ K_2, \ldots, K_M]$$
$$\bar{D} = \operatorname{diag}[K_{M+1}, K_{M+2}, \ldots, K_{2M}]$$
$$\bar{G}_c = \operatorname{diag}[G_1,\ G_2, \ldots, G_M] \tag{70}$$
$$\bar{G}_d = \operatorname{diag}[G_{M+1}, G_{M+2}, \ldots, G_{2M}]$$

As noted in the case of the voltage sources, the following synthesis procedure can be modified to remain valid if the elements of $\bar{G}_c$ and $\bar{G}_d$ are ratios of polynomials in the complex-frequency variable with only distinct negative-real zeros.

If Equation (69) is substituted into Equation (67), the result can be rearranged to yield

$$\begin{bmatrix} \bar{I} \\ -\bar{C}\bar{I}_c \\ -\bar{D}\bar{I}_d \\ \bar{I}_c \\ \bar{I}_d \end{bmatrix} = \begin{bmatrix} \bar{y}_{11} & \bar{y}_{12} & \bar{y}_{13} & \bar{y}_{14} & \bar{y}_{15} \\ \bar{y}_{21} & (\bar{y}_{22}-\bar{G}_c) & \bar{y}_{23} & \bar{y}_{24} & \bar{y}_{25} \\ \bar{y}_{31} & \bar{y}_{32} & (\bar{y}_{33}-\bar{G}_d) & \bar{y}_{34} & \bar{y}_{35} \\ \bar{y}_{41} & \bar{y}_{42} & \bar{y}_{43} & \bar{y}_{44} & \bar{y}_{45} \\ \bar{y}_{51} & \bar{y}_{52} & \bar{y}_{53} & \bar{y}_{54} & \bar{y}_{55} \end{bmatrix} \begin{bmatrix} \bar{E} \\ \bar{E}_a \\ \bar{E}_b \\ 0 \\ 0 \end{bmatrix} \tag{71}$$

The last four rows of this matrix equation produce the relationships

$$(\bar{y}_{22} + \bar{C}\,\bar{y}_{42} - \bar{G}_c)\bar{E}_a + (\bar{y}_{23} + \bar{C}\,\bar{y}_{43})\bar{E}_b = -(\bar{y}_{21} + \bar{C}\,\bar{y}_{41})\bar{E} \qquad (72)$$

and

$$(\bar{y}_{32} + \bar{D}\,\bar{y}_{52})\bar{E}_a + (\bar{y}_{33} + \bar{D}\,\bar{y}_{53} - \bar{G}_d)\bar{E}_b = -(\bar{y}_{31} + \bar{D}\,\bar{y}_{51})\bar{E} \qquad (73)$$

Solving Equations (72) and (73) for $\bar{E}_a$ and $\bar{E}_b$ supplies

$$\bar{E}_a = [(\bar{y}_{23} + \bar{C}\,\bar{y}_{43})^{-1}(\bar{y}_{22} + \bar{C}\,\bar{y}_{42} - \bar{G}_c) - (\bar{y}_{33} + \bar{D}\,\bar{y}_{53} - \bar{G}_d)^{-1}(\bar{y}_{32} + \bar{D}\,\bar{y}_{52})]^{-1}$$
$$\cdot\,[(\bar{y}_{33} + \bar{D}\,\bar{y}_{53} - \bar{G}_d)^{-1}(\bar{y}_{31} + \bar{D}\,\bar{y}_{51}) - (\bar{y}_{23} + \bar{C}\,\bar{y}_{43})^{-1}(\bar{y}_{21} + \bar{C}\,\bar{y}_{41})]\bar{E} \qquad (74)$$

and

$$\bar{E}_b = [(\bar{y}_{22} + \bar{C}\,\bar{y}_{42} - \bar{G}_c)^{-1}(\bar{y}_{23} + \bar{C}\,\bar{y}_{43}) - (\bar{y}_{32} + \bar{D}\,\bar{y}_{52})^{-1}(\bar{y}_{33} + \bar{D}\,\bar{y}_{53} - \bar{G}_d)]^{-1}$$
$$\cdot\,[(\bar{y}_{32} + \bar{D}\,\bar{y}_{52})^{-1}(\bar{y}_{31} + \bar{D}\,\bar{y}_{51}) - (\bar{y}_{22} + \bar{C}\,\bar{y}_{42} - \bar{G}_c)^{-1}(\bar{y}_{21} + \bar{C}\,\bar{y}_{41})]\bar{E} \qquad (75)$$

Since the voltages and currents at the N accessible ports of the active network are related by

$$\bar{I} = \bar{Y}\,\bar{E} \qquad (76)$$

the first row of Equation (71) and Equations (74) and (75) yield

$$\bar{Y} = \bar{y}_{11} + \left\{ \bar{y}_{12} [(\bar{y}_{23} + \bar{C}\,\bar{y}_{43})^{-1} (\bar{y}_{22} + \bar{C}\,\bar{y}_{42} - \bar{G}_c) - (\bar{y}_{33} + \bar{D}\,\bar{y}_{53} - \bar{G}_d)^{-1} \right. \tag{77}$$

$$\cdot\ (\bar{y}_{32} + \bar{D}\,\bar{y}_{52})]^{-1} [(\bar{y}_{33} + \bar{D}\,\bar{y}_{53} - \bar{G}_d)^{-1} (\bar{y}_{31} + \bar{D}\,\bar{y}_{51}) - (\bar{y}_{23} + \bar{C}\,\bar{y}_{43})^{-1}$$

$$\left. \cdot\ (\bar{y}_{21} + \bar{C}\,\bar{y}_{41})] \right\} + \left\{ \bar{y}_{13} [(\bar{y}_{22} + \bar{C}\,\bar{y}_{42} - \bar{G}_c)^{-1} (\bar{y}_{23} + \bar{C}\,\bar{y}_{43}) \right.$$

$$- (\bar{y}_{32} + \bar{C}\,\bar{y}_{52})^{-1} (\bar{y}_{33} + \bar{D}\,\bar{y}_{53} - \bar{G}_d)]^{-1} [(\bar{y}_{32} + \bar{D}\,\bar{y}_{52})^{-1} (\bar{y}_{31} + \bar{D}\,\bar{y}_{51})$$

$$\left. - (\bar{y}_{22} + \bar{C}\,\bar{y}_{42} - \bar{G}_c)^{-1} (\bar{y}_{21} + \bar{C}\,\bar{y}_{41})] \right\}$$

By assuming that

$$\bar{y}_{13} = \bar{y}_{23} = \bar{y}_{51} = \bar{y}_{53} = [0] \tag{78}$$

and

$$\bar{y}_{33} - \bar{G}_d = \bar{C}\,y_{43} \tag{79}$$

Equation (77) can be reduced to

$$\bar{Y} = \bar{Y}_{11} - \bar{y}_{12}(\bar{y}_{22} + \bar{C}\,\bar{y}_{42} - \bar{D}\,\bar{y}_{52} - \bar{G}_c)^{-1} (\bar{y}_{21} + \bar{C}\,y_{41}) \tag{80}$$

For an additional simplification, let the matrices which are composed of the current gains of the amplifiers be expressed as

$$\bar{C} = -\,c\bar{U} \qquad \bar{D} = -\,d\bar{U} \tag{81}$$

where $\bar{U}$ is the M^{th} order identity matrix and c and d are positive real constants which will be specified later. Equations (79) and (80) now become

$$\bar{y}_{33} - \bar{G}_d = -\,c\bar{y}_{43} \tag{82}$$

and

$$\bar{Y} = \bar{y}_{11} - \bar{y}_{12}(\bar{y}_{22} - c\bar{y}_{42} + d\bar{y}_{52} - \bar{G}_c)^{-1} (\bar{y}_{21} - c\bar{y}_{41}) \tag{83}$$

where the latter expression is the short-circuit admittance matrix of the transformerless grounded active N-port RC network containing 2M nonideal current-controlled current sources. For Equation (83) to be true, the submatrices of [y] must satisfy the demands of Equations (78), (81), and (82).

Realization Procedure

Using the results of the analysis of the selected type of network, it is now possible to develop the realization procedure. First, assume that M = N so that the resulting method will require an active N-port RC network containing 2N nonideal three-terminal current amplifiers. Equation (81) requires N of these amplifiers to have a constant current gain of -c and N a gain of -d. Let the prescribed N x N short-circuit admittance matrix be represented by

$$\bar{Y} = \frac{[P]}{Q} \tag{84}$$

where [P] and Q are, respectively, a matrix of polynomials and a polynomial in the complex-frequency variable, and Q is the least common multiple of all of the denominators of the elements of $\bar{Y}$. Select an appropriate N x N matrix $\bar{y}_{11}$ which satisfies the conditions to be presented below, and denote this matrix by

$$\bar{y}_{11} = \frac{[p]}{q} \tag{85}$$

where [p] is a matrix of polynomials and the polynomial q is the common denominator of all elements of $\bar{y}_{11}$. Subtraction of Equation (85) from Equation (84) produces

$$\bar{Y} - \bar{y}_{11} = \frac{[P]q - [p]Q}{Qq} = \frac{[A]}{Qq} \tag{86}$$

The chosen short-circuit admittance matrix $\bar{y}_{11}$ should fulfill the following conditions:

(A) deg p_{ij} = deg q = NL_o = T where
$i = 1,2,\ldots,N$, $j = 1,2,\ldots,N$ and
$L_o = \max$ [deg P_{ij}, deg Q];

(B) the diagonal and off-diagonal elements are, respectively, positive and negative RC driving-point admittance functions with $p_{ij} = p_{ji}$ and $p_{ii}(0) \neq 0$;

(C) in the Foster expansion

$$\bar{y}_{11} = [B_o] + \sum_{\ell=1}^{T} [B_\ell] \frac{s}{s + \sigma_\ell}$$

where σ_ℓ are the zeros of q, the coefficient matrices $[B_o]$ and $[B_\ell]$ must satisfy the dominance condition with an inequality;

(D) det [A] contains NT distinct negative-real zeros;

(E) the polynomial matrix [A], defined in Equation (86), can be written as the product $[A_4][A_3]$ of two N x N matrices of degrees, respectively, L_o and T; and

(F) the matric polynomial $[A_4]$ has the property that det $[A_4]$, a polynomial of degree NL_o, has only distinct negative-real zeros which are different from those of q.

Conditions (A), (B), and (C) can be met by proper selection of the elements of $\bar{y}_{11}$. In Appendix I it is shown that if Conditions (A) and (B) are satisfied, then Condition (D) can be fulfilled by sufficiently large choices for the diagonal elements of $\bar{y}_{11}$. Appendix II shows that if Condition (D) is satisfied, $[A]^t$ can be factored into

$$[A]^t = [A_1][A_2] \tag{87}$$

where the matrices $[A_1]$ and $[A_2]$ have the properties which Conditions (E) and (F) require, respectively, of $[A_3]$ and $[A_4]$. The transpose of Equation (87) is

$$[A] = [A_2]^t[A_1]^t \tag{88}$$

Since the requirements of Conditions (E) and (F) are such that $[A_1]^t$ and $[A_2]^t$ will fulfill them if $[A_1]$ and $[A_2]$ do, the desired factorization of $[A]$ can be obtained by letting

$$[A_3] = [A_1]^t \tag{89}$$

$$[A_4] = [A_2]^t \tag{90}$$

Hence, for any specified $\bar{Y}$, a $\bar{y}_{11}$ can always be chosen such that the above six conditions are satisfied.

Equations (83) and (86) and Condition (E) produce the relationship

$$\bar{y}_{12}(\bar{y}_{22} - c\bar{y}_{42} + d\bar{y}_{52} - \bar{G}_c)^{-1}\ (\bar{y}_{21} - c\bar{y}_{41}) = -\frac{[A_4][A_3]}{Qq} \tag{91}$$

Solving for $\bar{y}_{22} - c\bar{y}_{42} + d\bar{y}_{52} - \bar{G}_c$ yields

$$\bar{y}_{22} - c\bar{y}_{42} + d\bar{y}_{52} - \bar{G}_c = -Qq(\bar{y}_{21} - c\bar{y}_{41})[A_3]^{-1}[A_4]^{-1}\,\bar{y}_{12} \tag{92}$$

To initiate the selection of the submatrices in $[y]$, let

$$\bar{y}_{21} - c\bar{y}_{41} = \alpha\,\frac{[A_3]}{q} \tag{93}$$

where α is a nonzero real constant to be determined later. The expression $[A_3]/q$ is regular at infinity because Condition (E) requires the matrix $[A_3]$ to be of degree T. So, if

$$q = L_1 \prod_{u=1}^{T} (s + \sigma_u) \tag{94}$$

where the σ's are real distinct nonzero positive numbers and L_1 is a real constant, then the Foster expansion of Equation (93) can be expressed as

$$\bar{y}_{21} - c\bar{y}_{41} = \alpha \sum_{u=0}^{T} \bar{G}_u \frac{s}{s + \sigma_u} - \alpha \sum_{u=0}^{T} \bar{H}_u \frac{s}{s + \sigma_u} \tag{95}$$

where the elements in $\bar{G}_u$ and $\bar{H}_u$ are real and nonnegative and $\sigma_o = 0$. With the identifications

$$\bar{y}_{21} = -\alpha \sum_{u=0}^{T} \bar{H}_u \frac{s}{s + \sigma_u} \tag{96}$$

and

$$\bar{y}_{41} = -\frac{q}{c}\sum_{u=0}^{T} \bar{G}_u \frac{s}{s+\sigma_u} \tag{97}$$

Equation (95) is satisfied, and all of the elements of the submatrices $\bar{y}_{21}$ and $\bar{y}_{41}$ are negative RC driving-point admittance functions.

Let the matrix $\bar{y}_{21}$ be rewritten as

$$\bar{y}_{21} = \alpha \frac{[N_{21}]}{q_a} \tag{98}$$

where Equation (96) shows that the zeros of q_a are contained in the set of zeros of q. Also, Equations (96) and (98) reveal that

$$\deg [N_{21}] = \deg q_a = T_a \tag{99}$$

Substitution of Equations (93) and (98) into Equation (92) yields

$$\bar{y}_{22} - c\bar{y}_{42} + d\bar{y}_{52} - \bar{G}_c = -\frac{\alpha^2 Q(\operatorname{adj}[A_4])[N_{21}]^t}{q_a \det [A_4]} \tag{100}$$

Since Condition (F) requires the det $[A_4]$ to contain only distinct negative-real zeros which are different from those of q, the polynomial q_a det $[A_4]$ can be expressed as

$$q_a \det [A_4] = L_2 \prod_{v=1}^{R} (s + \gamma_v) \tag{101}$$

where $0 < \gamma_1 < \gamma_2 < \dots < \gamma_R$, L_2 is a real constant, and R is the degree of q_a det $[A_4]$.

Equation (100) is regular at infinity if the following inequality is valid:

$$\deg Q + \deg (\operatorname{adj} [A_4]) + \deg [N_{21}]^t \leq \deg q_a + \deg (\det [A_2]) \qquad (102)$$

But,

$$\deg [N_{21}]^t = \deg q_a = T_a \qquad (103)$$

because the degree of a matrix and its transpose are identical. Also,

$$\deg (\operatorname{adj} [A_4]) = (N - 1) L_o \qquad (104)$$

and

$$\deg (\det [A_4]) = N L_o \qquad (105)$$

so that Equation (102) reduces to

$$\deg Q \leq L_o \qquad (106)$$

By employing the definition of L_o from Condition (A), the above equation becomes

$$\deg Q \leq \max [\deg P_{ij}, \deg Q] \qquad (107)$$

which is always true. Consequently, Equation (100) is regular at infinity, and its Foster expansion can be written as

$$\bar{y}_{22} - c\bar{y}_{42} + d\bar{y}_{52} - \bar{G}_c = \sum_{v=0}^{R} \bar{V}_v \frac{s}{s + \gamma_v} \qquad (108)$$

where $\gamma_o = 0$ and the $\bar{V}_v$ are real coefficient matrices. This last equation can then be rewritten as

$$\bar{y}_{22} - c\bar{y}_{42} + d\bar{y}_{52} - \bar{G}_c = \sum_{v=0}^{R} \bar{W}_v \frac{s}{s+\gamma_v} + \sum_{v=0}^{R} \bar{X}_v \frac{s}{s+\gamma_v} - \sum_{v=0}^{R} \bar{Z}_v \frac{s}{s+\gamma_v} \tag{109}$$

where the coefficient matrices $\bar{W}_v$, $\bar{X}_v$, and $\bar{Z}_v$ contain only nonnegative elements. In addition, the nonzero elements of the matrices $\bar{W}_v$ and $\bar{X}_v$ are, respectively, the positive diagonal and positive off-diagonal elements of $\bar{V}_v$. Equation (109) can be augmented to produce

$$\bar{y}_{22} - c\bar{y}_{42} + d\bar{y}_{52} = \sum_{v=0}^{R} [\bar{W}_v + \bar{J}_v] \frac{s}{s+\gamma_v} + \sum_{v=0}^{R} \bar{X}_v \frac{s}{s+\gamma_v} - \sum_{v=0}^{R} [\bar{Z}_v + \bar{J}_v] \frac{s}{s+\gamma_v} + \bar{G}_c \tag{110}$$

where the $\bar{J}_v$ are diagonal matrices whose diagonal elements are sufficiently large positive real numbers which will be determined later. The diagonal matrix $\bar{G}_c$ is arbitrary, and it may be chosen to contain convenient finite nonnegative output conductance values. The following identifications will satisfy Equation (110):

$$\bar{y}_{22} = \sum_{v=0}^{R} [\bar{W}_v + \bar{J}_v] \frac{s}{s+\gamma_v} + \bar{G}_c \tag{111}$$

$$\bar{y}_{42} = -\frac{1}{c} \sum_{v=0}^{R} \bar{X}_v \frac{s}{s+\gamma_v} \tag{112}$$

and

$$\bar{y}_{52} = -\frac{1}{d}\sum_{v=0}^{R}[\bar{Z}_v + \bar{J}_v]\frac{s}{s+\gamma_v} \tag{113}$$

Note that each diagonal element of the submatrix $\bar{y}_{22}$ is a positive RC driving-point admittance function. The associated off-diagonal elements are negative RC admittance functions because they are identically zero. Also, all of the elements of $\bar{y}_{42}$ and $\bar{y}_{52}$ are negative RC driving-point admittance functions.

Equation (82) with M = N requires that the nonzero submatrices which are composed of elements from the last N rows of [y] must satisfy the constraint

$$\bar{y}_{33} + c\bar{y}_{43} = +\bar{G}_d \tag{114}$$

where the diagonal matrix $\bar{G}_d$ is arbitrary and may be chosen to contain convenient nonnegative conductance values. The above equation can be augmented into

$$\bar{y}_{33} - c\bar{y}_{43} = \bar{G}_d + \bar{L} - \bar{L} \tag{115}$$

where $\bar{L}$ is an N^{th} order diagonal matrix with arbitrary real positive nonzero constants as the diagonal elements.* Let

$$\bar{y}_{33} = \bar{G}_d + \bar{L} \tag{116}$$

$$\bar{y}_{43} = -\frac{1}{c}\bar{L} \quad (c \geq 1) \tag{117}$$

*The diagonal elements could be positive RC driving-point admittance functions, but this would require additional elements in the passive network.

Hence, $\bar{y}_{33}$ is a real matrix whose diagonal elements are positive RC driving-point admittance functions and whose off-diagonal elements are zero or negative RC admittances. Similarly, $\bar{y}_{43}$ contains only negative RC admittance functions. With the designations of Equations (116) and (117), the inverses of the matrices $(\bar{y}_{33} - \tilde{G}_d)$ and $c\bar{y}_{43}$ will exist. This existence is essential for the validity of the equations in the analysis section of this chapter.

All of the admittance submatrices which are contained in Equations (82) and (83) have now been selected so that the second of these two equations with $M = N$ may be made to equal any prescribed admittance matrix $\bar{Y}$. The synthesis procedure will be complete when it is demonstrated that this set of submatrices will form a short-circuit admittance matrix which is realizable by a transformerless grounded passive 5N-port RC network. The necessary and sufficient conditions for the realization of [y] through the use of such a network were presented as Conditions (1) through (3) in Chapter II.

An inspection of Condition (B) and Equations (85), (111), and (116) discloses that the submatrices $\bar{y}_{11}$, $\bar{y}_{22}$, and $\bar{y}_{33}$ fulfill the requirements of Conditions (1) and (2) because their diagonal and off-diagonal elements are, respectively, positive and negative RC driving-point admittance functions. As shown by Equations (78), (96), (97), (112), (113), and (117), the rest of the submatrices which are specified in the realization procedure satisfy Condition (2). The unspecified submatrices $\bar{y}_{44}$, $\bar{y}_{55}$, and $\bar{y}_{45}$ can be selected to meet the demands of Conditions (1) and (2). The submatrix $\bar{y}_{11}$ was chosen to satisfy the dominance condition with an inequality, and the submatrices $\bar{y}_{12}$ and $\bar{y}_{14}$

can be made arbitrarily small by a sufficiently small choice for their constant multipler α. Since $\bar{y}_{13}$ and $\bar{y}_{15}$ contain only zero elements, the admittance parameters in the first N rows of $[y]$ can be forced to fulfill the dominance requirement of Condition (3) by a sufficiently small value for α. The designations in Equations (116) and (117) for $\bar{y}_{33}$ and $\bar{y}_{43}$, which are the only submatrices with nonzero elements in rows $(2N+1)$ through 3N of $[y]$, permit these rows to fulfill the dominance condition. Note that Equation (117) requires the selection of the current gain parameter c to be any positive constant which is equal to or greater than unity. Due to the occurrence of the inverse of the current gain factor d as a constant multiplier in the submatrix $\bar{y}_{25}$, any desired value of d which is greater than unity allows a selection of augmentation coefficient matrices $\bar{J}_v$ so that rows $(N + 1)$ through 2N of $[y]$ are dominant. A nonzero diagonal element in a particular row of a $\bar{J}_v$ should be chosen when each element or sum of elements in the same row of $\bar{y}_{12}$, $\bar{y}_{24}$, and $\bar{y}_{25}$ has an absolute value which is larger than the element in the designated row of $\bar{W}_v$ in Equation (109). With proper augmentation, the synthesis of a prescribed matrix is possible even if before beginning the realization positive constants equal to or greater than unity and greater than unity are specified, respectively, for the gain factors c and d and if arbitrary nonnegative constants are assigned to $\bar{G}_c$ and $\bar{G}_d$. Also, certain augmentations can save elements in the passive network. The unspecified submatrices $\bar{y}_{44}$ and $\bar{y}_{55}$ can be chosen such that the last 2N rows of $[y]$ will meet the dominance requirements of Condition (3).

Since the method has produced an admittance matrix $[y]$ which fulfills all of the conditions required of it, the passive RC network may be realized from this matrix. A method for synthesizing the $5N \times 5N$ short-circuit admittance matrix $[y]$ by a transformerless passive $(5N+1)$-terminal network composed of two-terminal impedances with a common reference node and no internal nodes is contained in reference (19). When the specified current-controlled current sources are connected to the passive network as shown in Figure 2, the designated short-circuit admittance matrix $\bar{Y}$ is realized at the N remaining ports.

An Example

To exemplify the synthesis procedure of this chapter, a 2-port active RC network embedding 4 nonideal current-controlled current sources will be found to yield the short-circuit admittance matrix

$$\bar{Y} = \frac{1}{s+2}\begin{bmatrix} s+2 & s-1 \\ s+3 & s+1 \end{bmatrix} = \frac{[P]}{Q} \tag{118}$$

Due to its nonreciprocal nature, this matrix can not be realized by a passive network. For the two-by-two short-circuit admittance matrix $\bar{y}_{11}$, choose

$$\bar{y}_{11} = \frac{1}{s+3}\begin{bmatrix} 50(s+0.50) & 0 \\ 0 & 50(s+1) \end{bmatrix} = \frac{[p]}{q} \tag{119}$$

Since the requirement in Condition (A) concerning the degree T of the polynomial q is sufficient but not necessary, a lower value for T has been chosen. If the desired factorization of the difference matrix $[A]$

can not be obtained, the value of T must be increased. Subtraction of the above two equations gives

$$\bar{Y} - \bar{y}_{11} = \frac{\begin{bmatrix} -49s^2 - 120s - 44 & s^2 + 2s - 3 \\ s^2 + 6s + 9 & -49s^2 - 146s - 97 \end{bmatrix}}{(s+2)(s+3)} \tag{120}$$

$$= \frac{[A]}{Qq}$$

By use of the procedure in Appendix II, the matrix [A] can be factored as

$$[A] = \begin{bmatrix} 1.014s + 0.4632 & 0.0914s + 0.09026 \\ 0.1537s + 0.07021 & s + 0.9875 \end{bmatrix} \begin{bmatrix} -49.09s - 98.13 & 5.479s + 12.84 \\ 8.545s + 16.09 & -49.84s - 99.13 \end{bmatrix}$$

$$= [A_4][A_3] \tag{121}$$

From Equation (93),

$$\bar{y}_{21} - c\bar{y}_{41} = \alpha \frac{\begin{bmatrix} -49.09s - 98.13 & 5.479s + 12.84 \\ 8.545s + 16.09 & -49.84s - 99.13 \end{bmatrix}}{(s+3)} \tag{122}$$

After a Foster expansion of Equation (122), the identifications of Equations (96) and (97) produce

$$\bar{y}_{21} = -\alpha \begin{bmatrix} 32.71 & 0 \\ 0 & 33.04 \end{bmatrix} - \alpha \frac{s}{s+3} \begin{bmatrix} 16.38 & 0 \\ 0 & 16.80 \end{bmatrix} \tag{123}$$

and

$$\bar{y}_{41} = -\frac{\alpha}{c}\begin{bmatrix} 0 & 4.281 \\ 5.363 & 0 \end{bmatrix} - \frac{\alpha}{c}\,\frac{s}{s+3}\begin{bmatrix} 0 & 1.198 \\ 3.182 & 0 \end{bmatrix} \tag{124}$$

Equations (100), (110), (111), (112), and (113) give

$$\bar{y}_{22} = \alpha^2\begin{bmatrix} 143.2 + \frac{G_1}{\alpha^2} + J_1 & 0 \\ 0 & 67.86 + \frac{G_2}{\alpha^2} + J_2 \end{bmatrix} + \alpha^2\,\frac{s}{s+0.4568}\begin{bmatrix} J_3 & 0 \\ 0 & J_4 \end{bmatrix} \tag{125}$$

$$+ \alpha^2\,\frac{s}{s+0.9875}\begin{bmatrix} J_5 & 0 \\ 0 & J_6 \end{bmatrix} + \alpha^2\,\frac{s}{s+3}\begin{bmatrix} 6.441 + J_7 & 0 \\ 0 & 8.463 + J_8 \end{bmatrix}$$

$$\bar{y}_{42} = -\frac{\alpha^2}{c}\,\frac{s}{s+0.4568}\begin{bmatrix} 0 & 9.272 \\ 0 & 0 \end{bmatrix} - \frac{\alpha^2}{c}\,\frac{s}{s+0.9875}\begin{bmatrix} 0 & 0 \\ 3.888 & 0 \end{bmatrix} \tag{126}$$

and

$$\bar{y}_{52} = -\frac{\alpha^2}{d}\begin{bmatrix} J_1 & 13.22 \\ 10.18 & J_2 \end{bmatrix} - \frac{\alpha^2}{d}\,\frac{s}{s+0.4568}\begin{bmatrix} 100.6 + J_3 & 0 \\ 0 & J_4 \end{bmatrix} \tag{127}$$

$$- \frac{\alpha^2}{d}\,\frac{s}{s+0.9875}\begin{bmatrix} J_5 & 0 \\ 0 & 25.79 + J_6 \end{bmatrix} - \frac{\alpha^2}{d}\,\frac{s}{s+3}\begin{bmatrix} J_7 & 0.6036 \\ 1.251 & J_8 \end{bmatrix}$$

where the output conductances of the current sources of gain $-c$ form the matrix

$$\bar{G}_c = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} \tag{128}$$

By designating the output conductances of the remaining two amplifiers as G_3 and G_4, the matrix $\tilde{G}_d$ can be expressed as

$$\bar{G}_d = \begin{bmatrix} G_3 & 0 \\ 0 & G_4 \end{bmatrix} \tag{129}$$

Also, let the two-by-two diagonal matrix $\bar{L}$ be represented by

$$\bar{L} = \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix} \tag{130}$$

From Equations (78), (116), (117), (129), and (130),

$$\bar{y}_{13} = \bar{y}_{23} = \bar{y}_{51} = \bar{y}_{53} = [0] \tag{131}$$

$$\bar{y}_{33} = \begin{bmatrix} L_1 + G_3 & 0 \\ 0 & L_2 + G_4 \end{bmatrix} \tag{132}$$

and

$$\bar{y}_{43} = -\frac{1}{c} \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix} \tag{133}$$

To complete the numerical identification of the submatrices which are specified by the realization procedure, the matrix elements L_1 and L_2 must be taken to be finite positive constants, and the output conductances G_1, G_2, G_3, and G_4 should be chosen as any convenient finite nonnegative constants. Also, the current gain factors c and d should be selected to be positive constants which are, respectively, equal to or greater than unity and greater than unity. The value of the constant α can then be specified such that the first two rows of $[y]$ will satisfy the dominance condition. Next, the augmentation coefficient elements J_1 through J_8 can be chosen so that rows 3 and 4 of $[y]$ are dominant. From Equations (131), (132), and (133) it can be seen that rows 5 and 6 of $[y]$ are dominant with an inequality sign. The unspecified submatrices $\bar{y}_{44}$, $\bar{y}_{55}$, and $\bar{y}_{45}$ can always be taken to yield an admittance matrix $[y]$ which is realizable by the desired passive network. Each element of $\bar{y}_{45}$ and the off-diagonal elements of $\bar{y}_{44}$ and $\bar{y}_{55}$ are to be arbitrary negative RC driving-point admittance functions. The diagonal terms of $\bar{y}_{44}$ and $\bar{y}_{55}$ must be positive RC admittance functions with poles possessing large enough residues to satisfy the dominance condition for the last four rows of $[y]$. Some passive elements may be saved by choosing the J's and the diagonal elements of $\bar{y}_{44}$ and $\bar{y}_{55}$ so that certain elements of rows 5 and 6 and all of the elements of rows 7 through 10 fulfill the dominance condition with an equality sign.

In this example if $L_1 = L_2 = G_1 = G_2 = G_3 = G_4 = 1.0$ and $c = d = 5$, then any $\alpha < 0.2548$ permits rows 1 and 2 to be dominant. If α is chosen to be 0.10, the minimum values for the augmentation parameters are as follows:

$$J_1 = 108.2 \qquad J_2 = 205.7$$

$$J_3 = 27.47 \qquad J_4 = 0$$

$$J_5 = 0 \qquad J_6 = 7.420$$

$$J_7 = 196.8 \qquad J_8 = 199.7$$

The matrix [y] can be completed by selecting $\bar{y}_{44}$, $\bar{y}_{55}$, and $\bar{y}_{45}$ as discussed in the above paragraph.

One of the procedures contained in reference (19) may now be used to realize the 10 x 10 short-circuit admittance matrix [y] by a 10-port 11-terminal transformerless grounded passive RC network. When the designated current sources are connected to ports 3 through 10, the given short-circuit admittance matrix of Equation (118) is realized at ports 1 and 2.

CHAPTER IV

SIMULTANEOUS REALIZATION OF TWO ADMITTANCES USING TWO VOLTAGE AMPLIFIERS

The design criteria for a two-port network will sometimes prescribe only two of the network's four short-circuit admittance parameters. For example, the short-circuit current transfer function for a two-port network such as in Figure 3 is given by

$$\frac{I_2}{I_1} = \frac{Y_{21}}{Y_{11}} \tag{134}$$

In this instance many choices are possible for Y_{21} and Y_{11}, but the prescribed transfer function will establish their ratio. A specification of the transfer zeros of a nonreciprocal two-port network is another illustration of conditions being placed on only two admittance parameters.

The synthesis procedures of the two preceding chapters can be used to realize all four short-circuit admittance parameters of a two-port network by using four voltage or current amplifiers and a grounded transformerless 10-port RC network. While these methods will satisfy the design criteria when only two admittances are specified, they appear somewhat extravagant since the resulting network is forced to fulfill two unnecessary conditions. If only two amplifiers can be used, then a substantial savings in the number of elements is possible because two amplifiers and four of the ten ports with their related components are unnecessary.

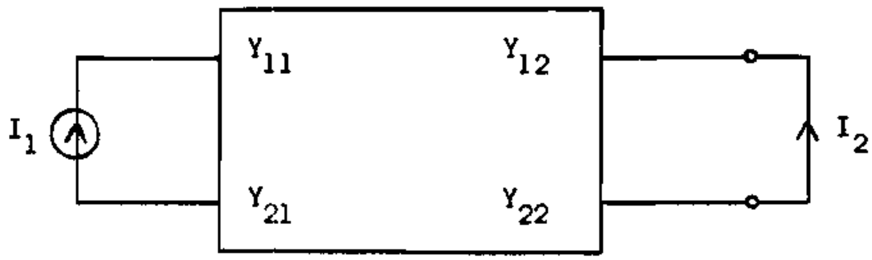


Figure 3. Two-Port Network Driven by a Current Source.

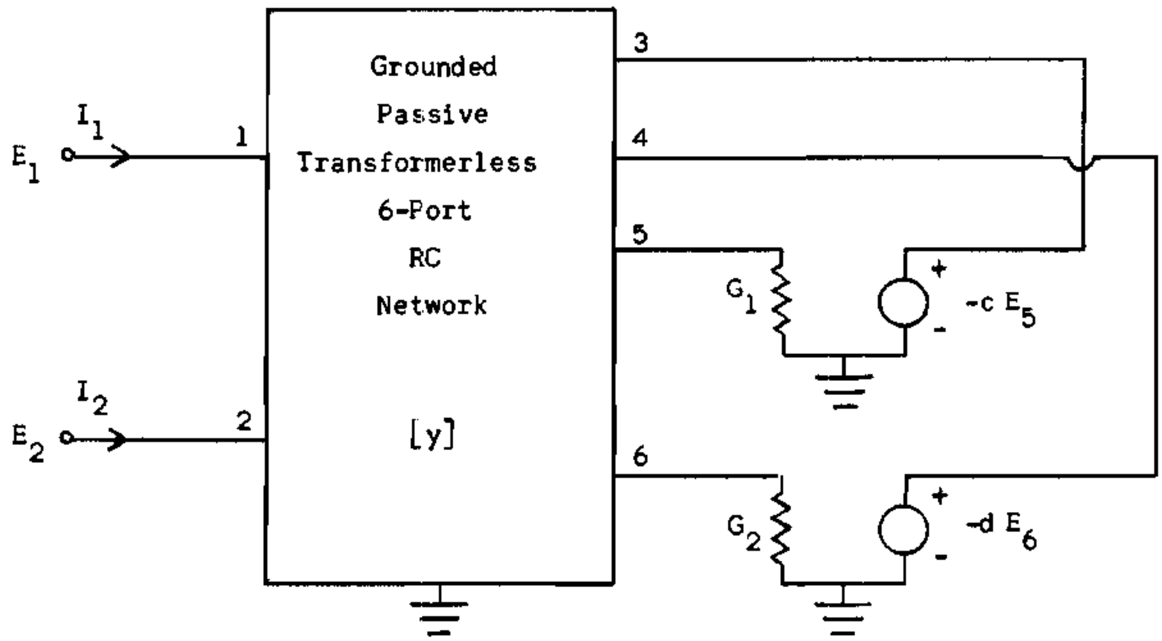


Figure 4. Grounded Active Two-Port RC Network
Containing Two Voltage Amplifiers.

Some techniques for the simultaneous realization of certain pairs of the four short-circuit admittance parameters of a two-port network will be presented in this chapter. The proposed circuit, which is shown in Figure 4, contains two nonideal voltage-controlled voltage sources and a grounded six-port passive RC subnetwork. Except for the need of demonstrating a scheme for each selected pair of admittances, the development of these realization procedures will follow the same general pattern as the previous ones. The short-circuit admittance matrix of the active two-port network will be written in terms of the admittance parameters of the passive network and the gains and input conductances of the voltage amplifiers. By comparing the two desired admittances with the corresponding matrix members, it will be possible to choose the amplifier specifications and a realizable set of admittance parameters for the passive grounded RC subnetwork.

If the active two-port network in Figure 4 has a short-circuit admittance matrix given by

$$\bar{Y} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \tag{135}$$

then the voltages and currents at the two accessible ports are related by

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \bar{Y} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \tag{136}$$

Next, let the 6 x 6 admittance matrix of the passive RC portion of the network be denoted by

$$[y] = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} \\ y_{21} & y_{22} & y_{23} & y_{24} & y_{25} & y_{26} \\ y_{31} & y_{32} & y_{33} & y_{34} & y_{35} & y_{36} \\ y_{41} & y_{42} & y_{43} & y_{44} & y_{45} & y_{46} \\ y_{51} & y_{52} & y_{53} & y_{54} & y_{55} & y_{56} \\ y_{61} & y_{62} & y_{63} & y_{64} & y_{65} & y_{66} \end{bmatrix} \tag{137}$$

where

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = [y] \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \end{bmatrix} \tag{138}$$

Notice that in Equation (137)

$$y_{ij} = y_{ji} \tag{139}$$

because the RC subnetwork is reciprocal.

Now, the network of Figure 4 is the same as the one in Figure 1 with N = 2 and M = 1. Thus, by partitioning after its second and each succeeding row and column, the matrix $[y]$ in Equation (137) can be put into the form of Equation (5). The analysis performed in Chapter II will then apply with N = 2 and M = 1. By substitution of the above partitioned submatrices of [y] into Equation (20), the short-circuit admittance matrix of the active two-port network of Figure 4 is given by

$$\bar{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{12} & y_{22} \end{bmatrix} - \frac{1}{y_{55} - cy_{35} + dy_{45} + G_1} \begin{bmatrix} y_{15}(y_{15} - cy_{13}) & y_{25}(y_{15} - cy_{13}) \\ y_{15}(y_{25} - cy_{23}) & y_{25}(y_{25} - cy_{23}) \end{bmatrix} \tag{140}$$

where Equation (18) requires the use of one amplifier with a voltage gain of -c and one with a gain of -d. For Equation (140) to be valid, Equations (15) and (19) impose the following constraints:

$$y_{14} = y_{24} = y_{16} = y_{26} = y_{46} = y_{56} = 0 \tag{141}$$

and

$$y_{66} + G_2 = -cy_{36} \tag{142}$$

When no more than two amplifiers are contained in the active network, the parameters y_{55} and $-cy_{35}$ must be allotted the same type residue terms. Hence, y_{35} is redundant, and in these cases passive elements may be saved by letting

$$y_{35} = 0 \tag{143}$$

When Equation (143) is inserted into Equation (140) and the result is compared with Equation (135), the four short-circuit admittance parameters of a two-port network containing two voltage amplifiers are

$$Y_{11} = y_{11} - \frac{y_{15}(y_{15} - cy_{13})}{y_{55} + dy_{45} + G_1} \tag{144}$$

$$Y_{12} = y_{12} - \frac{y_{25}(y_{15} - cy_{13})}{y_{55} + dy_{45} + G_1} \tag{145}$$

$$Y_{21} = y_{12} - \frac{y_{15}(y_{25} - cy_{23})}{y_{55} + dy_{45} + G_1} \tag{146}$$

$$Y_{22} = y_{22} - \frac{y_{25}(y_{25} - cy_{23})}{y_{55} + dy_{45} + G_1} \tag{147}$$

For the above equations to be true, the conditions imposed in Equations (141), (142), and (143) must be fulfilled.

The above results of the analysis of the proposed active two-port network will now be employed in the development of the realization techniques. A procedure will be presented for each of the four pairs of short-circuit admittance parameters which may be synthesized by a grounded transformerless passive six-port RC network terminated in two nonideal three-terminal voltage amplifiers.

Case 1: Y_{11} and Y_{21}

Let the two desired admittance functions be indicated by

$$Y_{11} = \frac{P_{11}}{Q} \tag{148}$$

and

$$Y_{21} = \frac{P_{21}}{Q} \tag{149}$$

where P_{11}, P_{21}, and Q are polynomials in the complex frequency variable and Q is the least common denominator of Y_{11} and Y_{21}. Select a positive RC driving-point admittance function

$$y_{11} = \alpha_1 \frac{p_{11}}{q} \tag{150}$$

and a negative RC driving-point admittance function

$$y_{12} = \frac{p_{12}}{q} \tag{151}$$

where

(A) deg q = t $\geq$ max (deg P_{11}, deg P_{21}, deg Q);

(B) y_{11} and y_{12} are regular at infinity;

(C) $p_{11}(0) \neq 0$; and

(D) α_1 is a real positive constant to be specified below.

Subtraction of Equation (150) from Equation (148) and Equation (151) from Equation (149) produces

$$Y_{11} - y_{11} = \frac{P_{11}q - \alpha_1 p_{11}Q}{Qq} = \frac{A_3}{Qq} \tag{152}$$

and

$$Y_{21} - y_{12} = \frac{P_{21}q - p_{12}Q}{Qq} = \frac{A_4}{Qq} \tag{153}$$

The above two equations reveal that the degrees of A_3 and A_4 must satisfy the following relationships:

$$\deg A_3 \leq \max [(\deg P_{11} + \deg q), (\deg p_{11} + \deg Q)] \leq 2t \qquad (154)$$

and

$$\deg A_4 \leq \max [(\deg P_{21} + \deg q), (\deg p_{12} + \deg Q)] \leq 2t \qquad (155)$$

The polynomial A_3 in the numerator of Equation (152) can be written as

$$A_3 = P_{11}q - \alpha_1 p_{11}Q = -\alpha_1 (p_{11}Q - \frac{P_{11}q}{\alpha_1}) \qquad (156)$$

where $P_{11}q/\alpha_1$ is a polynomial with all of its coefficients approaching zero as α_1 approaches infinity. Notice that, as α_1 approaches infinity, t of the zeros of A_3 approach the zeros of p_{11}. Hence, for a sufficiently large value of α_1, A_3 has at least t distinct negative-real zeros that are different from those of q.

The degree information in Equation (154) and the above discussion of the zeros of A_3 verify that the polynomial A_3 can be factored into

$$A_3 = A_1 A_2 \qquad (157)$$

where A_2 has t simple negative-real zeros that are not contained in q and A_1 is of degree less than or equal to t.

Using Equations (152), (153) and (157), Equations (144) and (146) can be written as

$$\frac{y_{15}(y_{15} - cy_{13})}{y_{55} + dy_{45} + G_1} = - \frac{A_1 A_2}{Qq} \qquad (158)$$

and

$$\frac{y_{15}(y_{25} - cy_{23})}{y_{55} + dy_{45} + G_1} = - \frac{A_4}{Qq} \qquad (159)$$

To begin the identification of the admittance parameters, let

$$y_{15} - cy_{13} = \alpha_2 \frac{A_1}{q} \qquad (160)$$

where α_2 is a constant to be specified later. Since deg $A_1 \leq t$, the expression A_1/q is regular at infinity. Thus, if

$$q = L_1 \prod_{u=1}^{t} (s + \sigma_u) \qquad (161)$$

where $0 < \sigma_1 < \sigma_2 < \ldots < \sigma_t$ and L_1 is a constant, then the Foster expansion of Equation (160) can be denoted by

$$y_{15} - cy_{13} = \alpha_2 \sum_{u=0}^{t} G_u \frac{s}{s + \sigma_u} - \alpha_2 \sum_{u=0}^{t} H_u \frac{s}{s + \sigma_u} \qquad (162)$$

where the coefficients G_u and H_u are real and nonnegative and $\sigma_o = 0$. Make the allocations

$$y_{13} = - \frac{\alpha_2}{c} \sum_{u=0}^{t} G_u \frac{s}{s + \sigma_u} \qquad (163)$$

and

$$y_{15} = -\alpha_2 \sum_{u=0}^{t} H_u \frac{s}{s+\sigma_u} \tag{164}$$

Note that y_{13} and y_{15} are defined such that they are negative RC driving-point admittance functions.

Let the admittance $\bar{y}_{15}$ be rewritten in the form

$$y_{15} = \alpha_2 \frac{N_{15}}{q_a} \tag{165}$$

where the polynomial q_a contains only negative-real zeros that are contained in the set of zeros of q. From the definition of y_{15} in Equation (164), it can be seen that this transfer admittance is regular at infinity. Hence,

$$\deg N_{15} = \deg q_a = t_a \leq t \tag{166}$$

After substitution of Equations (160) and (165) into Equation (158), the resulting equation can be rearranged to yield

$$y_{55} + dy_{45} + G_1 = -\frac{\alpha_2^{\,2} Q N_{15}}{q_a A_2} \tag{167}$$

Since A_2 contains only negative-real zeros different from those of q and q_a contains only negative-real zeros that are contained in q,

$$q_a A_2 = L_2 \prod_{v=1}^{m} (s + \gamma_v) \tag{168}$$

where $0 < \gamma_1 < \gamma_2 \cdots < \gamma_m$, L_2 is a constant, and m is the degree of the polynomial $q_a A_2$. Equations (157) and (166) and Condition (A) verify that the degree of the numerator of Equation (167) is less than or equal to the degree of its denominator. Thus, the latter equation is regular at infinity, and its Foster expansion can be written as

$$y_{55} + dy_{45} + G_1 = \sum_{v=0}^{m} W_v \frac{s}{s + \gamma_v} - \sum_{v=0}^{m} Z_v \frac{s}{s + \gamma_v} \tag{169}$$

where $\gamma_o = 0$ and the coefficients W_v and Z_v are real and nonnegative. Equation (169) may be augmented to yield

$$y_{55} + dy_{45} = \sum_{v=0}^{m} [W_v + J_v] \frac{s}{s + \gamma_v} - \sum_{v=0}^{m} [Z_v + J_v] \frac{s}{s + \gamma_v} - G_1 \tag{170}$$

where each J_v is a sufficiently large nonnegative number that will be chosen later. In the above equation, the input conductance G_1 can be any convenient nonnegative constant. The following identifications can now be made:

$$y_{55} = \sum_{v=0}^{m} [W_v + J_v] \frac{s}{s + \gamma_v} \tag{171}$$

and

$$y_{45} = -\frac{1}{d} \sum_{v=0}^{m} [Z_v + J_v] \frac{s}{s + \gamma_v} - \frac{1}{d} G_1 \tag{172}$$

With the above definitions, y_{55} and y_{45} are, respectively, positive and negative RC driving-point admittance functions.

Substitution of Equations (165) and (167) into Equation (159) yields

$$y_{25} - cy_{23} = \alpha_2 \frac{A_4}{qA_2} \tag{173}$$

In Equation (157), A_2 is chosen to have only distinct negative real zeros different from those of q. Hence,

$$qA_2 = L_3 \prod_{x=1}^{n} (s + \tau_x) \tag{174}$$

where the τ's are real distinct numbers which are greater than zero, L_3 is a constant, and n is the degree of the polynomial qA_2. An examination of Equations (155) and (157) and Condition (A) discloses that Equation (173) is regular at infinity. This equation may then be written in its Foster form as

$$y_{25} - cy_{23} = \alpha_2 \sum_{x=0}^{n} E_x \frac{s}{s + \tau_x} - \alpha_2 \sum_{x=0}^{n} F_x \frac{s}{s + \tau_x} \tag{175}$$

where the E_x and F_x are real and nonnegative and $\tau_o = 0$. The above equation can be satisfied with

$$y_{23} = - \frac{\alpha_2}{c} \sum_{x=0}^{n} E_x \frac{s}{s + \tau_x} \tag{176}$$

and

$$y_{25} = - \alpha_2 \sum_{x=0}^{n} F_x \frac{s}{s + \tau_x} \tag{177}$$

Hence, y_{23} and y_{25} both are negative RC driving-point admittance functions.

Equation (142) can be augmented into the form

$$y_{66} + cy_{36} = -G_2 + L - L \tag{178}$$

where G_2 and L are nonnegative constants which will be considered later. The nonzero admittances in the sixth row of [y] will satisfy the above equation if

$$y_{66} = \frac{1}{(c-1)} G_2 + L \quad (c > 1) \tag{179}$$

and

$$y_{36} = \frac{-1}{(c-1)} G_2 - \frac{1}{c} L \quad (c > 1) \tag{180}$$

The input conductance G_2 may be chosen to make the amplifier design easier. If a positive value is selected for G_2, then L can be chosen as zero. However, if G_2 is assumed to be zero, then a positive constant must be selected for L. In either instance the identifications in Equations (179) and (180) have been chosen so that the inverses of $(y_{66} + G_2)$ and y_{36} are defined. Also, y_{66} is a positive RC driving-point admittance function, and y_{36} is a negative one. Equations (141) and (143) require the transfer admittances y_{14}, y_{16}, y_{24}, y_{26}, y_{35}, y_{46}, and y_{56} to be zero.

Each of the admittance parameters of the passive network which appear in Equations (144) and (146) have now been selected so that these two equations are satisfied for any prescribed pair of admittances Y_{11}

and Y_{21}. The remaining parameters may be chosen to facilitate the realization of the admittance matrix [y]. A set of necessary and sufficient conditions for the synthesis of the short-circuit admittance matrix [y] by a transformerless grounded passive seven-terminal network composed of two-terminal impedances with a common reference node and no internal nodes are as follows:

(1) The diagonal terms are positive RC driving-point admittance functions;

(2) the off-diagonal terms are negative RC driving-point admittance functions; and

(3) the coefficient matrix for each pole in the Foster expansion of [y] are dominant.

The selection of y_{11} for Equation (150) and the specifications for y_{55} and y_{66} in Equations (171) and (179) reveal that Condition (1) is satisfied by these three diagonal terms. The unspecified parameters y_{22}, y_{33}, and y_{44} may be chosen to meet the restriction of Condition (1). As noted after each identification for a transfer admittance in the above procedure, the specified off-diagonal parameters of [y] are designated so that each one is a negative RC driving-point admittance function. In order to satisfy Condition (2) and to reduce the complexity of the network, the unspecified parameter y_{34} may be set equal to zero. The initial choices for y_{11} and y_{12} along with a sufficiently small value for the constant multiplier α_2, which is contained in y_{13} and y_{15}, may be used to insure that the first row of [y] meets the requirement of Condition (3). To save several elements, the unspecified diagonal elements y_{22}, y_{23}, and y_{44} may be designated such that rows 2, 3, and 4 meet the dominance condition with equality. Because of the presence of the inverse of the amplifier gain parameter d in the transfer admittance y_{45}, this off-diagonal term is reduced as d is increased. Thus, by taking d to be greater than unity and by then making proper choices for the augmentation residue terms J_v, the fourth row of [y] can be made to fulfill Condition (3). A nonzero J_v must be chosen for each residue or sum of residues in y_{15}, y_{25}, and y_{45} which is larger than the corresponding W_v in Equation (171). With the choices in Equations (179) and (180), the sixth row of [y] satisfies Condition (3) with the equality sign if the parameter L is zero. When L must be a positive constant, this row fulfills the dominance requirement with an inequality.

The short-circuit admittance matrix [y], which meets all of the required conditions, can now be used for the synthesis of the passive network. A realization procedure is contained in reference (19). When the two specified voltage amplifiers are connected from ports 5 and 6 to ports 3 and 4, respectively, the prescribed admittance functions are realized at ports 1 and 2.

Notice that the polynomial A_4 is not factored in this procedure. Thus, the admittance y_{12} may be chosen as zero. This identification results in an additional reduction in the necessary number of passive elements.

Case 2: Y_{11} and Y_{22}

Let the two prescribed admittance functions be denoted by

$$Y_{11} = \frac{P_{11}}{Q} \tag{181}$$

and

$$Y_{22} = \frac{P_{22}}{Q} \tag{182}$$

where Q is the least common denominator of Y_{11} and Y_{22}. Select two positive RC driving-point admittance functions

$$y_{11} = \alpha_1 \frac{p_{11}}{q} \tag{183}$$

and

$$y_{22} = \alpha_2 \frac{p_{22}}{q} \tag{184}$$

where

(A) α_1 and α_2 are real positive constants;

(B) $\deg q = t \geq \max (\deg P_{11}, \deg P_{22}, \deg Q)$;

(C) y_{11} and y_{22} are regular at infinity; and

(D) $p_{11}(0) \neq 0$ and $p_{22}(0) \neq 0$.

From Equations (181) through (184), form the differences

$$Y_{11} - y_{11} = \frac{P_{11}q - \alpha_1 p_{11} Q}{Qq} = \frac{A_3}{Qq} = \frac{A_1 A_2}{Qq} \tag{185}$$

and

$$Y_{22} - y_{22} = \frac{P_{22}q - \alpha_2 p_{22} Q}{Qq} = \frac{A_4}{Qq} = \frac{A_5 A_6}{Qq} \tag{186}$$

As in Case 1 of this chapter, α_1 and α_2 can be chosen large enough to force the remainder polynomials A_3 and A_4, respectively, to contain t negative-real zeros which are utilized in forming A_2 and A_6. Thus, the other factors A_1 and A_5 are each of degree less than or equal to t. Since t of the zeros of A_3 approach those of p_{11} as α_1 approaches infinity, it is possible to allot these zeros to A_2 and to select α_1 so that A_2/q is a positive RC driving-point admittance function. Likewise, by a proper allotment of the zeros of A_4 and a sufficiently large choice for α_2, A_6/q can be forced to be a positive RC admittance.

Substitution of Equations (185) and (186), respectively, into Equations (144) and (147) yields

$$\frac{y_{15}(y_{15} - cy_{13})}{y_{55} + dy_{45} + G_1} = -\frac{A_1A_2}{Qq} \tag{187}$$

and

$$\frac{y_{25}(y_{25} - cy_{23})}{y_{55} + dy_{45} + G_1} = -\frac{A_5A_6}{Qq} \tag{188}$$

Real positive constants α_3 and α_4, to be specified later, can be introduced into Equations (187) and (188), and these equations can be arranged so that it is possible to identify

$$y_{15} = -\alpha \frac{A_2}{q} \tag{189}$$

$$y_{15} - cy_{13} = \alpha_4 \frac{A_1}{q} \tag{190}$$

$$y_{25} = -\alpha_3 \frac{A_6}{q} \tag{191}$$

$$y_{25} - cy_{23} = \alpha_4 \frac{A_5}{q} \tag{192}$$

and

$$y_{55} + dy_{45} + G_1 = \alpha_3\alpha_4 \frac{Q}{q} \tag{193}$$

Equations (189) and (190) reveal that the admittance y_{13} is given by

$$y_{13} = -\frac{\alpha_3}{c} \cdot \frac{1}{q} \cdot [A_2 + \frac{\alpha_4}{\alpha_3} A_1] \tag{194}$$

Likewise, Equations (191) and (192) disclose that y_{23} is expressible as

$$y_{23} = -\frac{\alpha_3}{c} \cdot \frac{1}{q} \cdot [A_6 + \frac{\alpha_4}{\alpha_3} A_5] \tag{195}$$

Condition (B) shows that Equation (193) is regular at infinity. Hence, the Foster expansion of this equation can be written as

$$y_{55} + dy_{45} + G_1 = \sum_{v=0}^{t} W_v \frac{s}{s + \sigma_v} - \sum_{v=0}^{t} Z_v \frac{s}{s + \sigma_v} \tag{196}$$

where $0 = \sigma_o < \sigma_1 < \ldots < \sigma_m$ and the coefficients W_v and Z_v are real and nonnegative. Equation (196) can be augmented into

$$y_{55} + dy_{45} = \sum_{v=0}^{t} [W_v + J_v] \frac{s}{s + \sigma_v} - \sum_{v=0}^{t} [Z_v + J_v] \frac{s}{s + \sigma_v} - G_1 \tag{197}$$

where each J_v is a nonnegative real number which will be determined later. The value of the input conductance G_1 can be any convenient nonnegative constant. The following identifications will satisfy Equation (197):

$$y_{55} = \sum_{v=0}^{t} [W_v + J_v] \frac{s}{s + \sigma_v} \tag{198}$$

and

$$y_{45} = -\frac{1}{d} \sum_{v=0}^{t} [Z_v + J_v] \frac{s}{s + \sigma_v} - \frac{1}{d} G_1 \tag{199}$$

The requirements of Equation (141) and (143) are met by choosing

$$y_{14} = y_{16} = y_{24} = y_{26} = y_{35} = y_{46} = y_{56} = 0 \tag{200}$$

After selecting any nonnegative constant for the input conductance G_2, the constraints of Equation (142) can be fulfilled by designating y_{66} and y_{36} as given in Equations (179) and (180).

The above identifications for the admittance parameters have been chosen so that Equations (144) and (147) are satisfied for any choices for Y_{11} and Y_{22}. It remains to be shown that the resulting matrix $[y]$ can be realized by the desired grounded RC network. The necessary and sufficient conditions for this realization were given as Conditions (1) through (3) in Case 1 of this chapter. The selections of y_{11} and y_{22} for Equations (183) and (184) and the designations of y_{55} and y_{66} in Equations (179) and (198) are such that these diagonal elements of $[y]$ fulfill Condition (1). Since A_2/q and A_6/q are formed such that they are RC driving-point admittance functions, the definitions in Equations (189) and (191) reveal that y_{15} and y_{25} satisfy Condition (2). In Equations (194) and (195) the ratio of constants α_4/α_3 can be chosen small enough to force y_{13} and y_{23} to be negative RC driving-point admittance functions. Thus, these two admittances fulfill the requirement of Condition (2). Note that the ratio α_4/α_3 can be held fixed no matter what value is subsequently selected for α_3. Equations (180), (199), and (200) show that the remainder of the specified off-diagonal parameters of [y] satisfy Condition (2). The unspecified elements y_{33}, y_{44}, y_{12}, and y_{34} can be selected to meet the realizability conditions on [y]. Of course, the network will require less passive elements if y_{12} and y_{34} are taken to be zero and if the diagonal elements are chosen so that rows 3 and 4 are dominant with equality. The constant α_3 can be chosen sufficiently small to satisfy the dominance condition on rows 1 and 2 of [y]. As in Case 1, a positive constant greater than unity may be selected for the gain factor d, and then row 5 can be made dominant by proper choices for the augmentation coefficients J_v. Also, with c greater than unity the identifications in Equations (179), (180), and (200) are such that row 6 of [y] is dominant.

The matrix [y] can now be realized by the same procedure that was given in Case 1. After connecting the two specified voltage-controlled voltage sources from ports 5 and 6 to ports 3 and 4, respectively, the desired driving-point admittance functions are realized at ports (1) and (2).

Case 3: Y_{12} and Y_{21}

Represent the two desired admittance functions as

$$Y_{12} = \frac{P_{12}}{Q} \tag{201}$$

and

$$Y_{21} = \frac{P_{21}}{Q} \tag{202}$$

where Q is the least common denominator of Y_{12} and Y_{21}. Choose a negative RC driving-point admittance function

$$y_{12} = -\alpha \frac{p_{12}}{q} \tag{203}$$

where

(A) α_1 is a real positive constant;

(B) $\deg p_{12} = \deg q = t \geq \max [\deg P_{12}, \deg P_{21}, \deg Q]$; and

(C) $p_{12}(0) \neq 0$.

Subtraction of Equation (203) from Equations (201) and (202) yields

$$Y_{12} - y_{12} = \frac{P_{12}q + \alpha_1 p_{12}Q}{Qq} = \frac{A_3}{Qq} = \frac{A_1A_2}{Qq} \tag{204}$$

and

$$Y_{21} - y_{12} = \frac{P_{21}q + \alpha_1 p_{12}Q}{Qq} = \frac{A_4}{Qq} = \frac{A_5A_6}{Qq} \tag{205}$$

where, by a proof similar to the one given in Case 1 of this chapter, it can be shown that α_1 may be taken large enough to insure that the remainder polynomials A_3 and A_4 contain at least t negative-real zeros which are used to form A_2 and A_6. Hence, the factors A_1 and A_5 are each of degree less than or equal to t. Furthermore, by assigning to A_2 and A_6, respectively, the t zeros of A_3 and A_4 which approach those of p_{12} as α_1 approaches infinity and by choosing α_1 sufficiently large,

A_2/q and A_6/q can be forced to be RC driving-point admittance functions.

When Equations (204) and (205) are substituted, respectively, into Equations (145) and (146), the results produce

$$\frac{y_{25}(y_{15} - cy_{13})}{y_{55} + dy_{45} + G_1} = -\frac{A_1A_2}{Qq} \tag{206}$$

and

$$\frac{y_{15}(y_{25} - cy_{23})}{y_{55} + dy_{45} + G_1} = -\frac{A_5A_6}{Qq} \tag{207}$$

After inserting real positive constants α_2 and α_3, to be determined later, into Equations (206) and (207), the designation of the admittance parameters can be begun by letting

$$y_{15} = -\alpha_2 \frac{A_6}{q} \tag{208}$$

$$y_{25} = -\alpha_2 \frac{A_2}{q} \tag{209}$$

$$y_{15} - cy_{13} = \alpha_3 \frac{A_1}{q} \tag{210}$$

$$y_{25} - cy_{23} = \alpha_3 \frac{A_5}{q} \tag{211}$$

and

$$y_{55} + dy_{45} + G_1 = \alpha_2\alpha_3 \frac{Q}{q} \tag{212}$$

From Equations (208) through (211), the admittances y_{13} and y_{23} are given by

$$y_{13} = -\frac{\alpha_2}{c} \cdot \frac{1}{q} \cdot \left[A_6 + \frac{\alpha_3}{\alpha_2} A_1\right] \tag{213}$$

and

$$y_{23} = -\frac{\alpha_2}{c} \cdot \frac{1}{q} \cdot \left[A_2 + \frac{\alpha_3}{\alpha_2} A_5\right] \tag{214}$$

Since Condition (B) reveals that Equation (212) is regular at infinity, its Foster expansion can be expressed as shown in Equation (196) and augmented into the form of Equation (197). After selecting any convenient nonnegative constant for the input conductance G_1, the admittance parameters y_{55} and y_{45} can be identified as in Equations (198) and (199). Likewise, when a nonnegative constant is assumed for G_2, the restraints of Equation (142) are satisfied by taking y_{66} and y_{36} as in Equations (179) and (180). Finally, the requirements of Equations (141) and (143) are fulfilled by choosing

$$y_{14} = y_{16} = y_{24} = y_{26} = y_{35} = y_{46} = y_{56} = 0 \tag{215}$$

The three realizability conditions which the matrix [y] must satisfy were given in Case 1 of this chapter. Equations (179) and (198) indicate that the specified diagonal elements y_{55} and y_{66} fulfill the first of these conditions. The ratio of constants α_3/α_2 can be taken sufficiently small so that the admittances y_{13} and y_{23} satisfy Condition (2), and this ratio can be held invariant for any subsequent value which is chosen for α_2. An examination of Equations (180), (199), (203), (208), (209), and (215) reveals that the rest of the designated off-diagonal parameters are negative RC admittances. Since the functions y_{11}, y_{22}, y_{33}, y_{44}, and y_{34} do not appear in Equations (145) and (146), they may be selected such that rows (1) through (4) of [y] satisfy the realizability conditions. After any convenient value is taken for α_2, the number of passive elements needed to realize [y] can be reduced by choosing y_{34} to be zero and by selecting the unspecified driving-point admittances so that the first four rows fulfill the dominance requirement with equality. When positive constants greater than unity are selected for the gain factors c and d, row 6 is dominant, and the augmentation coefficients J_v can be chosen to make row 5 dominant.

After realizing the matrix [y] by the technique given by Weinberg[19] and connecting the designated voltage amplifiers as shown in Figure 4, the prescribed transfer admittance functions are obtained between ports (1) and (2).

Case 4: Y_{22} and Y_{12}

An inspection of Equations (147) and (145) discloses that if all of the subscripts 1 and 2 are changed to 2 and 1, respectively, these two equations are the same as Equations (144) and (146). Of course, it must be recalled that $y_{12} = y_{21}$. Hence, the procedure for the realization of Case 1 can be used for Y_{22} and Y_{12} if the specified changes in subscripts are made in all of the parameters used in the procedure.

In each of the realization procedures in this chapter, proper choices for the augmentation parameters J_v can be made such that row 5 of [y] is dominant without the necessity for large voltage gains. Some passive elements can be saved by choosing the J's to force the residues of some of the poles of the admittances in row 5 to satisfy the dominance condition with equality. Furthermore, the procedures are successful if positive constants greater than unity are designated for the gain factors c and d and if arbitrary nonnegative constants are selected for the input conductances G_1 and G_2 before starting the synthesis of the desired pair of admittances.

The realization procedure in this chapter constitute a proof of the following theorem:

Theorem 3

To realize simultaneously any of the pairs of short-circuit admittance functions Y_{11} and Y_{21}, Y_{11} and Y_{22}, Y_{12} and Y_{21}, and Y_{22} and Y_{12}, where each admittance is a real rational function of the complex-frequency variable, by a grounded transformerless active two-port RC network, it is sufficient that the network contains two three-terminal voltage-controlled voltage sources with negative finite constant gains greater than unity and finite input conductances.

An Example

To demonstrate the realization procedure of Case 1, a two-port network will be found to produce the short-circuit current transfer function

$$\frac{I_2}{I_1} = \frac{s}{s^2 + s + 1} \tag{216}$$

Since Equation (134) shows that this transfer function is given by

$$\frac{I_2}{I_1} = \frac{Y_{21}}{Y_{11}} \tag{217}$$

the two admittance functions Y_{21} and Y_{11} may be designated as

$$Y_{11} = 1 = \frac{s^2 + s + 1}{s^2 + s + 1} \tag{218}$$

and

$$Y_{21} = \frac{s}{s^2 + s + 1} \tag{219}$$

First, a positive and a negative RC short-circuit driving-point admittance function must be selected for y_{11} and y_{12}, respectively. Let

$$y_{11} = \frac{1}{2}\,\frac{(s+1)(s+3)}{(s+2)(s+4)} = \frac{p_{11}}{q} \tag{220}$$

and

$$y_{12} = 0 \tag{221}$$

Equations (152) and (153) yield

$$A_3 = \frac{1}{2}\,(s^2 + s + 1)(s^2 + 8s + 13) \tag{222}$$

and

$$A_4 = s(s + 2)(s + 4) \tag{223}$$

and Equation (157) indicates that

$$A_1 = \frac{1}{2}(s^2 + s + 1) \tag{224}$$

and

$$A_2 = (s^2 + 8s + 13) \tag{225}$$

From Equation (160),

$$y_{15} - cy_{13} = \alpha_2 \frac{0.50(s^2 + s + 1)}{(s + 2)(s + 4)} = \alpha_2 \frac{A_1}{q} \tag{226}$$

In this example let higher gain amplifiers be used by selecting

$$c = d = 100 \tag{227}$$

and

$$G_1 = G_2 = 0.01 \tag{228}$$

Because of the rather high value for c, the constant $(0.5563\ \alpha_2)$ was added to and subtracted from Equation (226) in order to keep the constant term in the expansion of y_{13} from becoming so small. Of course, this augmentation of Equation (226) is completely optional. After forming y_{13} and y_{15} as indicated in Equations (163) and (164), the dominance of row 1 was obtained by taking α_2 to be 0.3333. Hence,

$$y_{13} = -0.002063 - \frac{0.002708s}{s + 4} \tag{229}$$

and

$$y_{15} = -0.1854 - \frac{0.1250s}{s + 2} \tag{230}$$

Equations (167), (170), (171), and (172) give

$$y_{55} = J_o + \frac{(0.1250 + J_1)s}{s + 2} + \frac{J_2 s}{s + 2.268} + \frac{(0.1782 + J_3)}{s + 5.732} \tag{231}$$

and

$$y_{45} = -\frac{1}{100}(0.005243 + J_o) - \frac{1}{100}\frac{J_1 s}{s + 2} - \frac{1}{100}\frac{(0.2044 + J_2)s}{s + 2.268} - \frac{1}{100}\frac{J_3 s}{s + 5.732} \tag{232}$$

where the J's are the augmentation coefficients which must be selected so that row 5 of $[y]$ is dominant. Then, from Equations (173), (175), (176), and (177),

$$y_{23} = -\frac{0.0009622s}{s + 2.268} \tag{233}$$

and

$$y_{25} = -\frac{0.09622s}{s + 5.732} \tag{234}$$

Now that y_{25} is known, it can be determined that the following set of augmentation coefficients may be used:

$$J_0 = 0.1873 \qquad J_1 = 0$$

$$J_2 = 0.002065 \qquad J_3 = 0$$

With the above specification for G_2, the value of L can be taken as zero so that Equations (179) and (180) yield

$$y_{66} = 0.0001010 \tag{235}$$

and

$$y_{36} = -0.0001010 \tag{236}$$

Finally, to satisfy Equations (141) and (143), let the parameters y_{14}, y_{16}, y_{24}, y_{26}, y_{35}, y_{46}, and y_{56} be zero.

Since the parameters y_{22}, y_{33}, y_{44}, and y_{34} are not constrained by the procedure, y_{34} can be set equal to zero, and the three diagonal elements may be selected so that rows 2, 3, and 4 of $[y]$ fulfill the dominance condition with equality. The short-circuit admittance matrix $[y]$ of the passive network then becomes

$$[y] = \begin{bmatrix} 0.1875 & 0 & -0.002063 & 0 & -0.1854 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.002063 & 0 & 0.002164 & 0 & 0 & -0.0001010 \\ 0 & 0 & 0 & 0.001925 & -0.001925 & 0 \\ -0.1854 & 0 & 0 & -0.001925 & 0.1873 & 0 \\ 0 & 0 & -0.0001010 & 0 & 0 & 0.0001010 \end{bmatrix}$$

$$
+ \frac{s}{s+2}\begin{bmatrix} 0.1250 & 0 & 0 & 0 & -0.1250 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.1250 & 0 & 0 & 0 & 0.1250 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
+ \frac{s}{s+2.268}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0009622 & -0.0009622 & 0 & 0 & 0 \\ 0 & -0.0009622 & 0.0009622 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.002065 & -0.002065 & 0 \\ 0 & 0 & 0 & -0.002065 & 0.002065 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (237)
$$

$$
+ \frac{s}{s+4}\begin{bmatrix} 0.1875 & 0 & -0.002708 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.002708 & 0 & 0.002708 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$+\frac{s}{s+5.732}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.09622 & 0 & 0 & -0.09622 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.09622 & 0 & 0 & 0.1782 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The above admittance matrix can be realized by the grounded transformerless 7-terminal 6-port passive RC network which is shown in Figure 5. After the two designated voltage amplifiers are connected as shown in Figure 6, the desired output current is obtained at the short-circuited port 2. To obtain realistic values for the elements in Figure 5, any convenient magnitude and frequency scalings can be used.

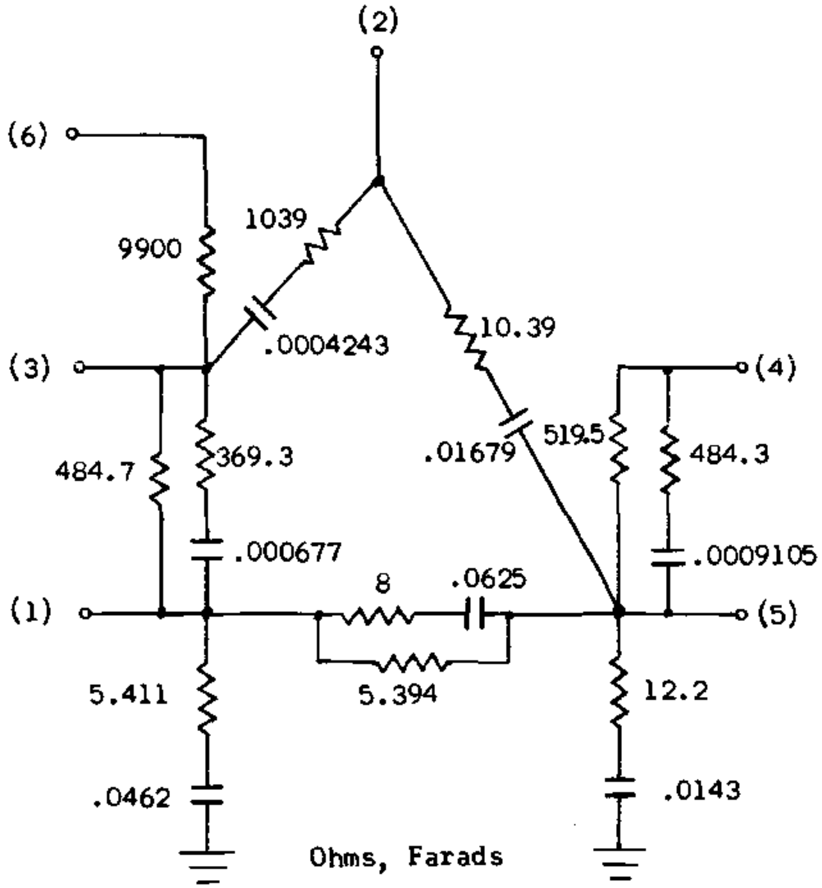


Figure 5. Network Realizing Equation (237).

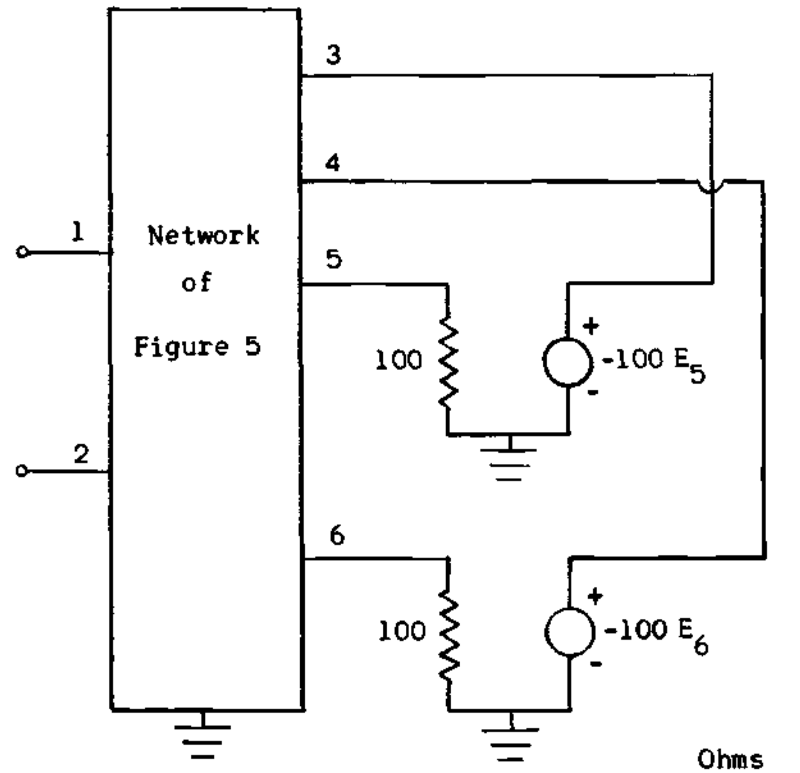


Figure 6. Network Realizing the Current Transfer Function of Equation (216).

CHAPTER V

SIMULTANEOUS REALIZATION OF TWO ADMITTANCES USING TWO CURRENT AMPLIFIERS

Through the use of current-controlled current sources as the active devices, the simultaneous realization of certain pairs of the short-circuit admittance parameters of a two-port network will be presented in this chapter. The desired circuit, which is shown in Figure 7, is a grounded six-port passive RC network terminated in two three-terminal nonideal current amplifiers. The following theorem will be proved:

Theorem 4

To realize simultaneously any of the pairs of short-circuit admittance functions Y_{11} and Y_{12}, Y_{11} and Y_{22}, Y_{12} and Y_{21}, and Y_{22} and Y_{21}, where each admittance is a real rational function of the complex-frequency variable, by a grounded transformerless active two-port RC network, it is sufficient that the network contains two three-terminal current-controlled current sources with negative finite constant gains greater than unity and finite output conductances.

The proof of this theorem can be effected by offering a synthesis procedure for each of the indicated pairs of admittances. As in the previous methods, the short-circuit admittance matrix of the active two-port network will be expressed as a function of the gains and output conductances of the current amplifiers and the admittance parameters

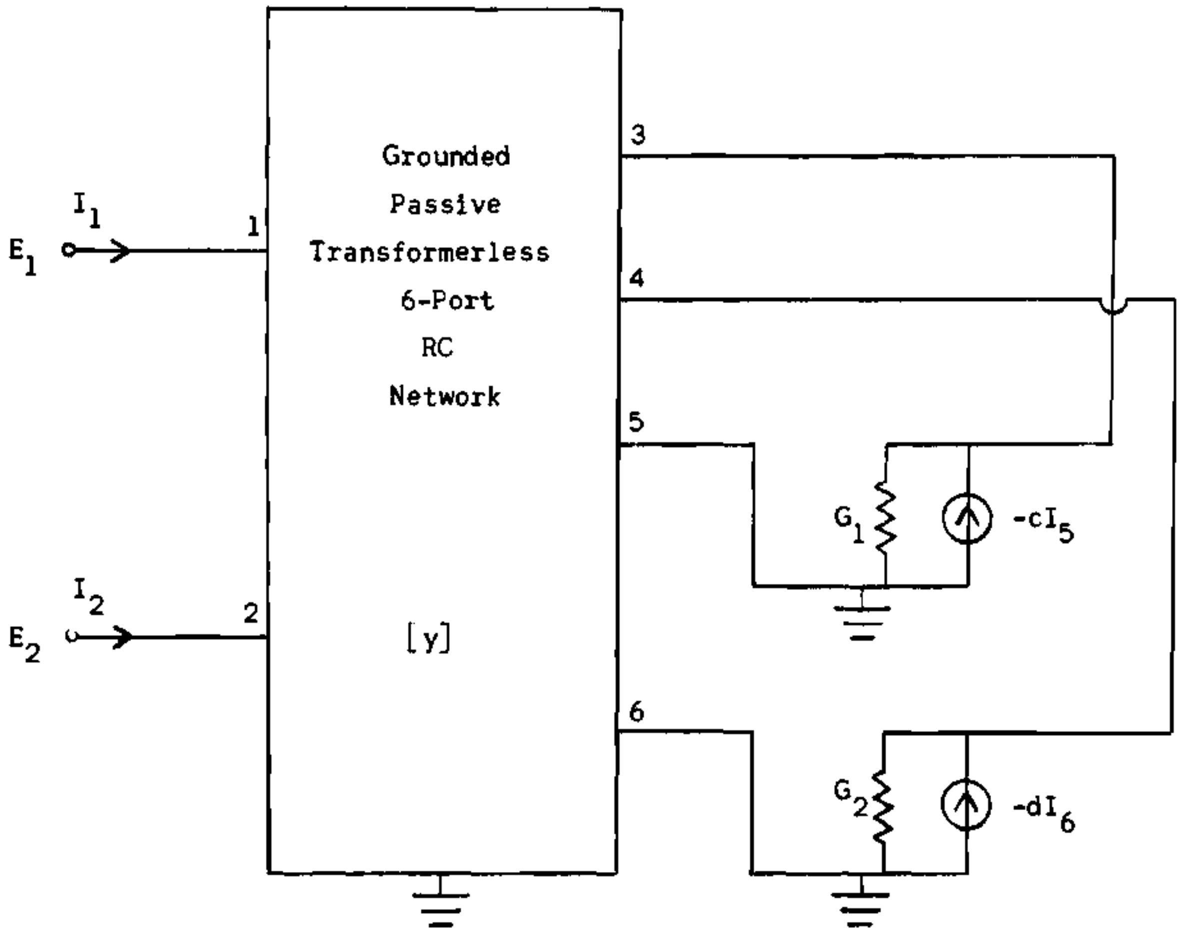


Figure 7. Grounded Active Two-Port RC Network Containing Two Current Amplifiers.

of the passive portion of the network. A comparison of the two prescribed admittance functions with the two corresponding matrix elements will then allow realizable selections for the amplifier specifications and the admittance parameters of the grounded passive RC subnetwork.

Let the 6 x 6 admittance matrix [y] of the passive section of the network in Figure 7 be expressed as in Equation (137) and constrained as in Equation (139). Notice that with N = 2 and M = 1 the network in Figure 2 is the same as the one in Figure 7. Hence, after partitioning of [y] into the form of Equation (67), Equation (83) reveals that the short-circuit admittance matrix of the active two-port network for this special case is given by

$$\overline{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{12} & y_{22} \end{bmatrix} - \frac{1}{y_{33} - cy_{35} + dy_{36} - G_c} \cdot \begin{bmatrix} y_{13}(y_{13} - cy_{15}) & y_{13}(y_{23} - cy_{25}) \\ y_{23}(y_{13} - cy_{15}) & y_{23}(y_{23} - cy_{25}) \end{bmatrix} \tag{238}$$

where

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \overline{Y} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \tag{239}$$

As indicated in Figure 7, the current gain of one of the amplifiers is -c and the other -d. For the above expression for $\overline{Y}$ to be valid, Equations (78) and (82) require that

$$y_{14} = y_{24} = y_{16} = y_{26} = y_{34} = y_{46} = 0 \qquad (240)$$

and

$$y_{44} - G_2 = -cy_{45} \qquad (241)$$

In the previous chapter it was noted that the presence of y_{35} is not required when only two amplifiers are used. Thus, passive elements may be saved by letting

$$y_{35} = 0 \qquad (242)$$

After denoting the admittance matrix $\bar{Y}$ by

$$\bar{Y} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \qquad (243)$$

the four short-circuit admittance parameters of a two-port network containing two current amplifiers can be written as

$$Y_{11} = y_{11} - \frac{y_{13}(y_{13} - cy_{15})}{y_{33} + dy_{36} - G_1} \qquad (244)$$

$$Y_{12} = y_{12} - \frac{y_{13}(y_{23} - cy_{25})}{y_{33} + dy_{36} - G_1} \qquad (245)$$

$$Y_{21} = y_{12} - \frac{y_{23}(y_{13} - cy_{15})}{y_{33} + dy_{36} - G_1} \qquad (246)$$

$$Y_{22} = y_{22} - \frac{y_{23}(y_{23} - cy_{25})}{y_{33} + dy_{36} - G_1} \qquad (247)$$

The above four equations are true as long as the requirements of Equations (240), (241), and (242) are met.

Using the results of the analysis of the desired network, a procedure will now be presented for each of the four pairs of short-circuit admittance parameters which can be realized by a grounded two-port network containing two current amplifiers.

Case 1: Y_{11} and Y_{12}

Let the two prescribed admittance functions be represented by

$$Y_{11} = \frac{P_{11}}{Q} \tag{248}$$

and

$$Y_{12} = \frac{P_{12}}{Q} \tag{249}$$

where P_{11}, P_{12}, and Q are polynomials in the complex-frequency variable and Q is the least common denominator of Y_{11} and Y_{12}. Specify a positive RC driving-point admittance function

$$y_{11} = a_1 \frac{p_{11}}{q} \tag{250}$$

and a negative RC driving-point admittance function

$$y_{12} = \frac{p_{12}}{q} \tag{251}$$

where deg p_{11} = deg p_{12} = deg q = t $\geq$ max (deg P_{11}, deg P_{12}, deg Q), $p_{11}(0) \neq 0$, and a_1 is a real positive constant. Subtractions of

Equation (250) from Equation (248) and Equation (251) from Equation (249) yield

$$Y_{11} - y_{11} = \frac{P_{11}q - \alpha_1 p_{11}Q}{Qq} = \frac{A_3}{Qq} = \frac{A_1A_2}{Qq} \tag{252}$$

and

$$Y_{12} - y_{12} = \frac{P_{12}q - p_{12}Q}{Qq} = \frac{A_4}{Qq} \tag{253}$$

As shown in Case 1 of Chapter IV, the constant α_1 can be selected sufficiently large to insure that the remainder polynomial A_3 contains at least t negative-real zeros which are utilized in forming the factor A_2. Thus, the other factor A_1 is of degree less than or equal to t. From before, the degree of A_4 is less than or equal to 2t.

Insertion of Equations (253) and (254), respectively, into Equations (244) and (245) produces

$$\frac{y_{13}(y_{13} - cy_{15})}{y_{33} + dy_{36} - G_1} = -\frac{A_1A_2}{Qq} \tag{254}$$

and

$$\frac{y_{13}(y_{23} - cy_{25})}{y_{33} + dy_{36} - G_1} = -\frac{A_4}{Qq} \tag{255}$$

To initiate the identification of the admittance parameters, let

$$y_{13} - cy_{15} = \alpha_2 \frac{A_1}{q} \tag{256}$$

where α_2 is a real positive constant. Since the degree of the polynomial A_1 is less than or equal to t, the function A_1/q is regular at infinity. Hence, the Foster expansion of Equation (256) can be expressed as

$$y_{13} - cy_{15} = \alpha_2 \sum_{u=0}^{t} G_u \frac{s}{s + \sigma_u} - \alpha_2 \sum_{u=0}^{t} H_u \frac{s}{s + \sigma_u} \tag{257}$$

where $\sigma_o = 0$, the σ_u $(u \neq 0)$ are distinct real positive constants, and the coefficients G_u and H_u are nonnegative. Identify

$$y_{13} = - \alpha_2 \sum_{u=0}^{t} H_u \frac{s}{s + \sigma_u} \tag{258}$$

and

$$y_{15} = - \frac{\alpha_2}{c} \sum_{u=0}^{t} G_u \frac{s}{s + \sigma_u} \tag{259}$$

Let the admittance $\bar{y}_{13}$ be rewritten as

$$y_{13} = \alpha_2 \frac{N_{13}}{q_a} \tag{260}$$

where the polynomial q_a has only negative-real zeros which are included in the set of zeros of q. The definition for y_{13} in Equation (258) reveals that

$$\deg N_{13} = \deg q_a = t_a \leq t \tag{261}$$

Substitution of Equations (256) and (260) into Equation (254) and rearrangement of the resulting equation produces

$$y_{33} + dy_{36} - G_1 = - \frac{\alpha_2^{\,2} Q N_{13}}{q_a A_2} \qquad (262)$$

The degree of the numerator of Equation (262) is less than or equal to $(t + t_a)$, and the degree of the denominator is equal to $(t + t_a)$. Thus, this equation is regular at infinity and its Foster expansion can be represented by

$$y_{33} + dy_{36} - G_1 = \sum_{v=0}^{m} W_v \frac{s}{s + \gamma_v} - \sum_{v=0}^{m} Z_v \frac{s}{s + \gamma_v} \qquad (263)$$

where the terms $(s + \gamma_v)$ with $v \neq 0$ are the distinct factors of $q_a A_2$, $\gamma_o = 0$, and the real coefficients W_v and Z_v are real and nonnegative. The above equation can be augmented and rearranged to produce

$$y_{33} + dy_{36} = \sum_{v=0}^{m} [W_v + J_v] \frac{s}{s + \gamma_v} - \sum_{v=0}^{m} [Z_v + J_v] \frac{s}{s + \gamma_v} + G_1 \qquad (264)$$

where each J_v is a nonnegative constant which will be determined later and the input conductance G_1 is any convenient nonnegative constant. Equation (264) is satisfied with the following identifications:

$$y_{33} = \sum_{v=0}^{m} [W_v + J_v] \frac{s}{s + \gamma_v} + G_1 \qquad (265)$$

and

$$y_{36} = -\frac{1}{d}\sum_{v=0}^{m}[Z_v + J_v]\frac{s}{s+\gamma_v} \tag{266}$$

Introduction of Equations (260) and (262) into Equation (255) gives

$$y_{23} - cy_{25} = \alpha_2 \frac{A_4}{qA_2} \tag{267}$$

Since the degree of A_4 is less than or equal to 2t, Equation (267) may be expanded as

$$y_{23} - cy_{25} = \alpha_2 \sum_{x=0}^{n} E_x \frac{s}{s+\tau_x} - \alpha_2 \sum_{x=0}^{n} F_x \frac{s}{s+\tau_x} \tag{268}$$

where $0 = \tau_o < \tau_1 < \tau_2 < \ldots < \tau_n$, n is the degree of the polynomial qA_2, and the E_x and F_x are real nonnegative constants. The restraints of the above equation can be met by letting

$$y_{23} = -\alpha_2 \sum_{x=0}^{n} F_x \frac{s}{s+\tau_x} \tag{269}$$

and

$$y_{25} = -\frac{\alpha_2}{c} \sum_{x=0}^{n} E_x \frac{s}{s+\tau_x} \tag{268}$$

Augmentation of Equation (241) produces

$$y_{44} + cy_{45} = G_2 + L - L \tag{267}$$

where the output conductance G_2 can be any convenient nonnegative constant and L is any positive constant. The above equation is satisfied if

$$y_{44} = G_2 + L \tag{268}$$

and

$$y_{45} = -\frac{1}{c} L \quad (c \geq 1) \tag{269}$$

These identifications are such that the inverses of $(y_{44} - G_2)$ and y_{45} are defined for any nonnegative value for the conductance G_2. To fulfill the requirements of Equations (240) and (242), choose

$$y_{14} = y_{24} = y_{16} = y_{26} = y_{34} = y_{35} = y_{46} = 0 \tag{270}$$

With the above designations for the admittance parameters, Equations (244) and (245) are fulfilled for any prescribed Y_{11} and Y_{12}. It must now be established that the resulting matrix [y] can be realized by the proposed grounded RC network. The necessary and sufficient conditions for this realization were given as Conditions (1) through (3) in Case 1 of Chapter IV. The choice for y_{11} and the identifications for y_{33} and y_{44} are such that these three diagonal elements fulfill Condition (1). Condition (2) is satisfied because the above designations for the off-diagonal elements of [y] are such that each parameter is a negative RC driving-point admittance function. The unspecified admittances y_{22}, y_{55}, y_{66}, and y_{56} can be selected to meet the three realizability conditions on [y]. Fewer passive components will be required if y_{56} is chosen as zero and if the diagonal elements are taken so that rows 2, 5, and 6 fulfill the dominance requirement with equality. The choices for y_{11}, y_{12}, and the constant multiplier α_2 can be used to make the first row of [y] dominant. After selecting convenient constants which are greater than or equal to unity and greater than unity, respectively, for the current-gain factors c and d, row 4 is dominant and row 3 can be made dominant by proper choices for the augmentation coefficients J_v.

Since the matrix [y] meets all of the required conditions, the realization procedure in reference (19) may be used for the synthesis of the passive network. After the two specified current amplifiers are connected from ports 5 and 6 to ports 3 and 4, respectively, the prescribed admittance functions are obtained at ports 1 and 2.

The admittance y_{12} may be chosen as zero because the remainder polynomial A_4 is not factored in this procedure. This identification produces an additional reduction in the required number of passive components.

Case 2: Y_{11} and Y_{22}

A procedure similar to the one in Case 2 of the previous chapter could be obtained for this pair of short-circuit driving-point admittance functions. However, a much simpler solution will be presented. An examination of Equations (144) and (147) discloses that by changing subscripts 3 and 6 to 5 and 4 and subscripts 4 and 5 to 6 and 3, respectively, Equations (144) and (244) are the same except for the sign of G_1. A corresponding result exists for Equations (147) and (247). Likewise, Equations (142) and (241) are the same except for the sign of G_2. In establishing these similarities it must be recalled that $y_{ij} = y_{ji}$. Due to these relationships, the method of Case 2 in Chapter IV may be easily modified for use in this case. After performing the subscript changes and reversing the signs of G_1 and G_2, the previous procedure should be followed except that the identifications in Equations (179), (180), (198), and (199) must be replaced by those in Equations (268), (269), (265), and (266), respectively. Of course, the parameters c and d must now be interpreted as current-gain coefficients, and G_1 and G_2 as output conductances.

Case 3: Y_{12} and Y_{21}

As in the above case, the synthesis procedure presented in Case 3 of Chapter IV can be employed in this case if the proper subscript replacements are made. If the subscripts of Equations (142), (145), and (146) are shuffled such that 5's are substituted for 3's, 3's for 5's, 4's for 6's and 6's for 4's and if the signs of G_1 and G_2 are changed, Equations (145) and (245) are the same, as are Equations (146) and (246) and Equations (142) and (241). Hence, by making the indicated changes in all of the subscripts and the signs of G_1 and G_2 in the method of the previous Case 3, that procedure can be used to realize the transfer admittances Y_{12} and Y_{21}. Notice that G_1 and G_2 are now output conductances, and c and d are current-gain coefficients.

Case 4: Y_{22} and Y_{21}

A close examination of Equations (247) and (246) reveals that if all of the subscripts 1 and 2 are changed to 2 and 1, respectively, these two equations are identical to Equations (244) and (245). However, it must be remembered that $y_{12} = y_{21}$. If the specified changes are made in the subscripts of the parameters in Case 1 of this chapter, that procedure can then be used to realize the admittances Y_{22} and Y_{21}.

Realization procedures have now been presented for the specified pairs of short-circuit admittance parameters, and the proof of Theorem 4 is finished.

An Example

To illustrate the synthesis procedure of Case 2, a two-port network possessing the driving-point admittances

$$Y_{11} = \frac{1}{s} \tag{271}$$

and

$$Y_{22} = \frac{1}{s + 2} \tag{272}$$

and containing two ideal current amplifiers with gains of -10 will be found. Since the proof of the technique in Case 2 of this chapter depended upon a modification of the method in Case 2 of Chapter IV, the equations mentioned in this example are the ones from Chapter IV with the proper subscript changes.

First, obtain a common denominator for Y_{11} and Y_{22} by taking

$$Q = s(s + 2) \tag{273}$$

so that

$$Y_{11} = \frac{s}{s(s + 2)} \tag{274}$$

and

$$Y_{22} = \frac{s}{s(s + 2)} \tag{275}$$

Then, choose two RC short-circuit driving-point admittance functions which satisfy Conditions (A) through (D). Let

$$y_{11} = \alpha_1 \frac{(s + 1)(s + 5)}{(s + 3)(s + 8)} \tag{276}$$

and

$$y_{22} = \alpha_2 \frac{(s + 1)(s + 5)}{(s + 3)(s + 8)} \tag{277}$$

From Equations (185) and (186)

$$A_3 = P_{11}q - \alpha_1 p_{11}Q = (s + 2)[s^2 + 11s + 24 - \alpha_1 s(s^2 + 6s + 5)] \tag{278}$$

and

$$A_4 = P_{22}q - \alpha_2 p_{22}Q = s[s^2 + 11s + 24 - \alpha_2(s^3 + 8s^2 + 17s + 10)] \tag{279}$$

By selecting $\alpha_1 = \alpha_2 = 1$, the polynomials A_3 and A_4 can be factored so that it is possible to identify

$$A_1 = -s(s + 2)(s - 2.275) \tag{280}$$

$$A_2 = (s + 2)(s + 5.275) \tag{281}$$

$$A_5 = -s(s - 1)$$

$$A_6 = (s + 2.586)(s + 5.414)$$

As required, A_2/q and A_6/q are RC driving-point admittance functions.

The above specifications for the current amplifiers require that $c = d = 10$ and $G_1 = G_2 = 0$. Hence, Equations (189), (194), (191), and (195) give

$$y_{13} = -\alpha_3 \frac{(s + 2)(s + 5.275)}{(s + 3)(s + 8)} \tag{284}$$

$$y_{15} = -\frac{\alpha_3}{10} \frac{1}{(s+3)(s+8)} \left[(s+2)(s+5.275) - \frac{\alpha_4}{\alpha_3}(s+2)(s-2.275) \right] \tag{285}$$

$$y_{23} = -\alpha_3 \frac{(s + 2.586)(s + 5.414)}{(s + 3)(s + 8)} \tag{286}$$

and

$$y_{25} = -\frac{\alpha_3}{10} \frac{1}{(s+3)(s+8)} \left[(s + 2.586)(s + 5.414) - \frac{\alpha_4}{\alpha_3} s(s-1) \right] \tag{287}$$

Equations (193), (197), (265), and (266) are satisfied if

$$y_{33} = \alpha_3\alpha_4 \left[J_o + \frac{J_1 s}{s + 3} + \frac{(1.20 + J_2)s}{s + 8} \right] \tag{288}$$

and

$$y_{36} = -\frac{\alpha_3\alpha_4}{10} \left[J_o + \frac{(0.20 + J_1)s}{s + 3} + \frac{J_2 s}{s+8} \right] \tag{289}$$

where the J's are the augmentation coefficients. The constraints of

Equations (268), (269), and (200) are fulfilled by letting

$$y_{44} = L \tag{290}$$

$$y_{45} = -\frac{1}{10} L \tag{291}$$

and

$$y_{14} = y_{16} = y_{24} = y_{26} = y_{35} = y_{46} = y_{34} = 0 \tag{292}$$

An examination of Equations (285) and (286) reveals that y_{15} and y_{25} are negative RC admittances if the constant α_4/α_3 is equal to or less than 0.1944. After taking $\alpha_4/\alpha_3 = 0.1944$, the first two rows of $[y]$ can be made dominant by selecting $\alpha_3 = 0.3246$. Row 3 may then be forced to fulfill the dominance condition by use of the following values for the augmentation coefficients:

$$J_0 = 18.02$$

$$J_1 = 3.867$$

$$J_2 = 12.03$$

The parameter L may be taken to be unity.

The unspecified transfer admittances y_{12} and y_{56} can be chosen as zero, and the diagonal elements y_{55} and y_{66} may be selected so that the last two rows of $[y]$ are dominant with equality. The short-circuit admittance matrix of the passive network may then be written as

$$[y] = \begin{bmatrix} 0.2083 & 0 & -0.1427 & 0 & -0.01547 & 0 \\ 0 & 0.2083 & -0.1849 & 0 & -0.01894 & 0 \\ -0.1427 & -0.1894 & 0.3690 & 0 & 0 & -0.03690 \\ 0 & 0 & 0 & 1.0 & -0.10 & 0 \\ -0.01547 & -0.01894 & 0 & -0.10 & 0.1344 & 0 \\ 0 & 0 & -0.03690 & 0 & 0 & 0.03690 \end{bmatrix} \tag{293}$$

$$+ \frac{s}{s+3} \begin{bmatrix} 0.2667 & 0 & -0.04924 & 0 & -0.007144 & 0 \\ 0 & 0.2667 & -0.02163 & 0 & -0.007209 & 0 \\ -0.04924 & -0.02163 & 0.07920 & 0 & 0 & -0.008329 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.007144 & -0.007209 & 0 & 0 & 0.01435 & 0 \\ 0 & 0 & -0.008329 & 0 & 0 & 0.008329 \end{bmatrix}$$

$$+ \frac{s}{s+8} \begin{bmatrix} 0.5250 & 0 & -0.1327 & 0 & -0.003545 & 0 \\ 0 & 0.5250 & -0.1136 & 0 & 0 & 0 \\ -0.1327 & -0.1136 & 0.2709 & 0 & 0 & -0.02464 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.003545 & 0 & 0 & 0 & 0.003545 & 0 \\ 0 & 0 & -0.02464 & 0 & 0 & 0.02464 \end{bmatrix}$$

The 6-port passive RC network shown in Figure 8 can be obtained from the above admittance matrix. When the specified current sources are connected as shown in Figure 9, the prescribed driving-point admittance functions are realized at ports 1 and 2.

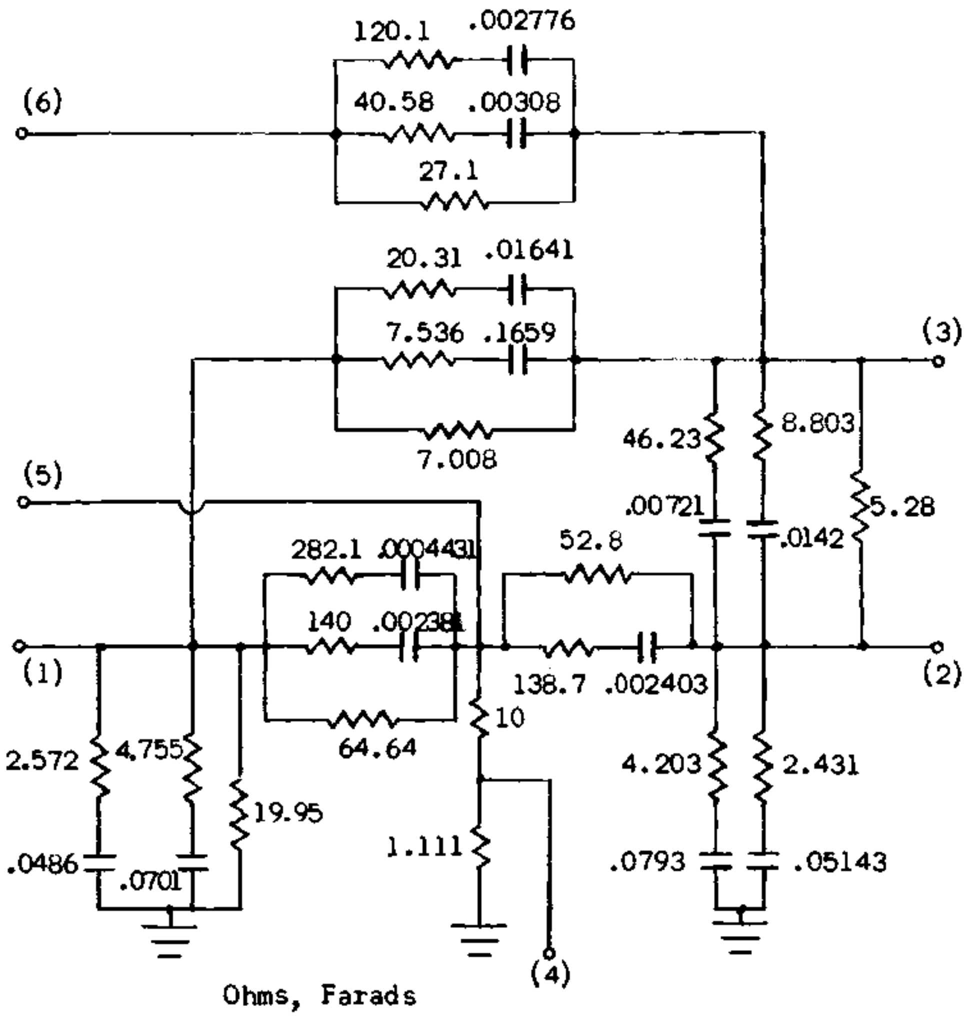


Figure 8. Network Realizing Equation (293).

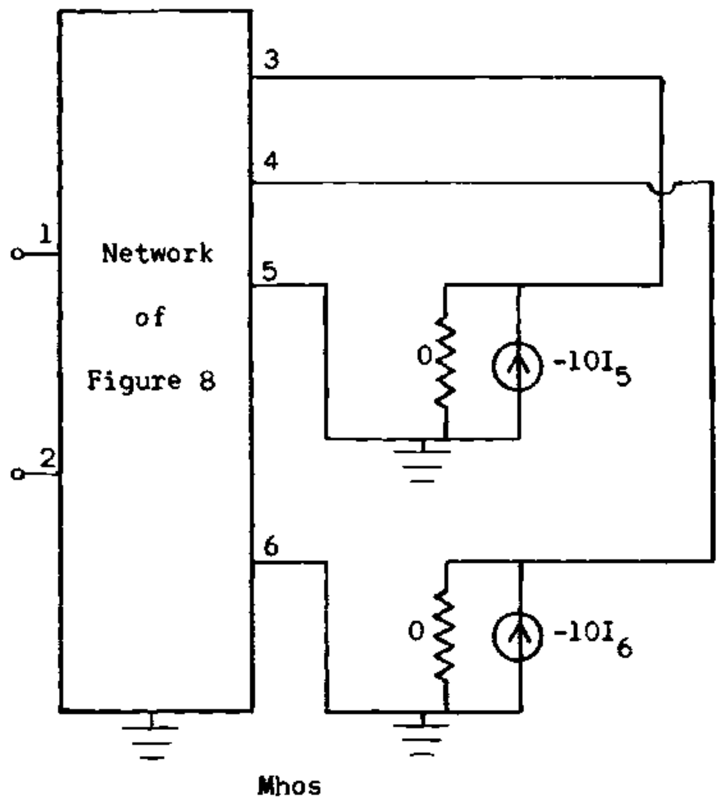


Figure 9. Network Realizing the Driving-Point Admittances $Y_{11} = 1/s$ and $Y_{22} = 1/s+2$.

CHAPTER VI

EXPERIMENTAL RESULTS

To establish that the realization procedures which have been developed in the preceding chapters can be implemented, one of the techniques was applied to a prescribed admittance function, and the resulting network was built and evaluated. A comparison was then made between the predicted and the actual behavior of the network.

Realization of an Inductor

Since the procedure in Chapter II is typical of the realization methods, a driving-point admittance function synthesized by this method was selected for testing. The desired function was chosen to be the admittance of a one-henry inductor. After the expression $Y = 1/s$ was realized, magnitude and frequency scalings were used to produce realistic values for the elements in the passive network. Of course, equal magnitude and frequency scaling factors had to be employed in order to retain the realization of the prescribed admittance.

In this example all of the matrices appearing in the equations in Chapter II are scalar quantities because N is unity. Since L_o is also unity, a first order function was a sufficient choice for the RC driving-point admittance y_{11}. Thus, y_{11} was chosen as

$$y_{11} = \frac{s + 1}{s + 2} = \frac{p}{q} \tag{294}$$

and Equation (23) gave

$$A = -s^2 + 2 \tag{295}$$

After factoring the function A, A_1 and A_2 were found to be

$$A_1 = -s + 1.414 \tag{296}$$

and

$$A_2 = s + 1.414 \tag{297}$$

From Equation (27)

$$y_{14} - cy_{12} = \alpha \frac{-s + 1.414}{s + 2} \tag{298}$$

so that the following identifications were possible:

$$y_{12} = -0.7071 \frac{\alpha}{c} \tag{299}$$

and

$$y_{14} = -\alpha \frac{1.707s}{s + 2} \tag{300}$$

Then, Equation (35) gave

$$y_{44} - cy_{42} + dy_{43} + G_c = \alpha^2 \frac{1.707s^2}{(s + 1.414)(s + 2)} \tag{301}$$

Augmentation and rearrangement of the above equation allowed the admittance parameters to be identified as

$$y_{44} = \alpha^2 \left[J_1 + \frac{J_2 s}{s + 1.414} + \frac{(J_3 + 5.828)s}{s + 2} \right] \tag{302}$$

$$y_{42} = 0$$

and

$$y_{43} = -\frac{\alpha^2}{d} \left[\left(J_1 + \frac{G_c}{\alpha^2}\right) + \frac{(J_2 + 4.121)s}{s + 1.414} + \frac{J_3 s}{s + 2} \right] \tag{303}$$

Equations (15), (50), and (51) were satisfied by choosing

$$y_{13} = y_{15} = y_{45} = y_{53} = 0 \tag{304}$$

$$y_{55} = \frac{1}{(c - 1)} G_d + L \tag{305}$$

and

$$y_{52} = \frac{-1}{(c - 1)} G_d - \frac{1}{c} L \tag{306}$$

The two voltage-controlled voltage sources were specified by taking $c = d = 100$ and $G_c = G_d = 0.01$. For dominance of row 1 of [y], the constant α was selected as 0.2929. Then, row 4 was forced to satisfy the dominance condition by use of the following values for the augmentation coefficients:

$$J_1 = 0.001177$$

$$J_2 = 0.04163$$

$$J_3 = 0$$

Since G_d was not zero, the parameter L was taken as zero.

After selecting y_{23} to be zero and choosing y_{22} and y_{33} so that rows 2 and 3 of [y] were dominant with equality, the above parameters were used to write

$$[y] = \begin{bmatrix} 0.50 & -0.002071 & 0 & 0 & 0 \\ -0.002071 & 0.002172 & 0 & 0 & -0.0001010 \\ 0 & 0 & 0.0001010 & -0.0001010 & 0 \\ 0 & 0 & -0.0001010 & 0.0001010 & 0 \\ 0 & -0.0001010 & 0 & 0 & 0.0001010 \end{bmatrix}$$

$$+ \frac{s}{s+1.414} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.003571 & -0.003571 & 0 \\ 0 & 0 & -0.003571 & 0.003571 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (307)$$

$$+ \frac{s}{s+2} \begin{bmatrix} 0.50 & 0 & 0 & -0.50 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -0.50 & 0 & 0 & 0.50 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The transformerless grounded 6-terminal 5-port passive RC network which is shown in Figure 10 was obtained from the short-circuit admittance matrix in Equation (307). Two amplifiers with voltage gains of -100 and input conductances of 0.01 mho were constructed and connected to the passive network as shown in Figure 11. Then, the desired admittance Y = 1/s was realized at port 1.

After magnitude and frequency scaling by a factor of 10^3, the network which was built had the resistance values in Figure 10 multiplied by 10^3 and the capacitance values by 10^{-6}. Also, the input resistances of the two controlled sources had the values of Figure 11 multiplied by 10^3. Comparisons of the predicted and measured impedance variations at port 1 are contained in Figures 12 and 13.

Measurement Procedures and Possible Errors

Since the intention of the experimental work was to exhibit a typical implementation of one of the realization procedures and not to produce a refined example, adequate but not elaborate construction and measurement methods were employed. Using discreet components, the networks were assembled on vector boards. Ceramic and carbon resistors of one and 10 per cent accuracy, respectively, and 15 per cent tolerance mylar capacitors were utilized. However, combinations that were within 5 per cent of the designed values were selected after the less accurate resistors and the capacitors were measured on an impedance bridge.

The synthesis procedure in Chapter II employed three-terminal voltage amplifiers which were assumed to have finite gains, finite input conductances, and zero output resistances. In the experimental

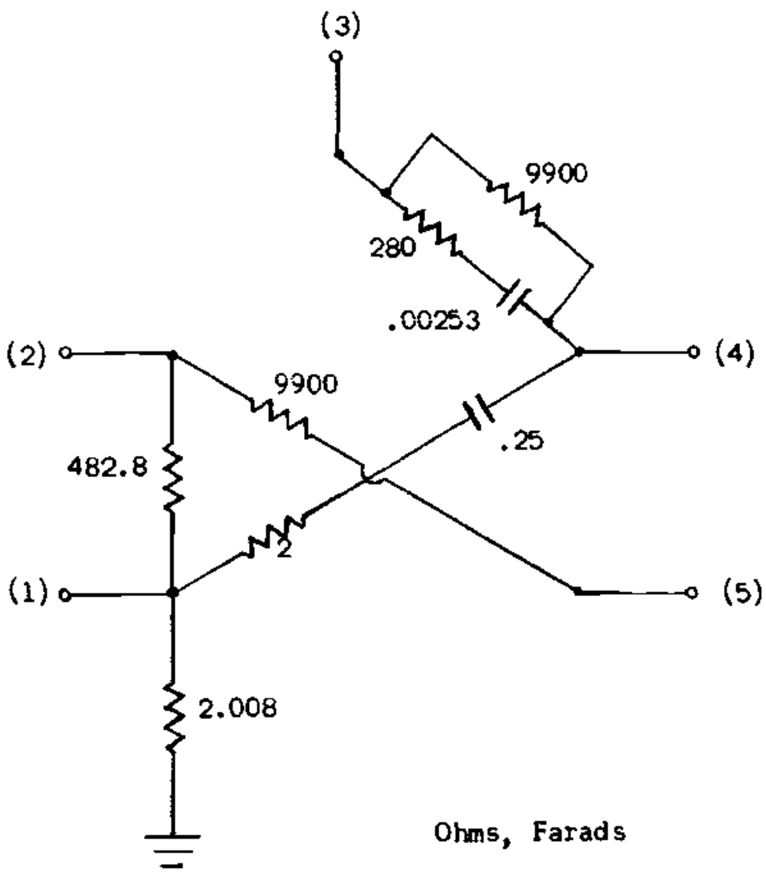


Figure 10. Network Realizing Equation (307).

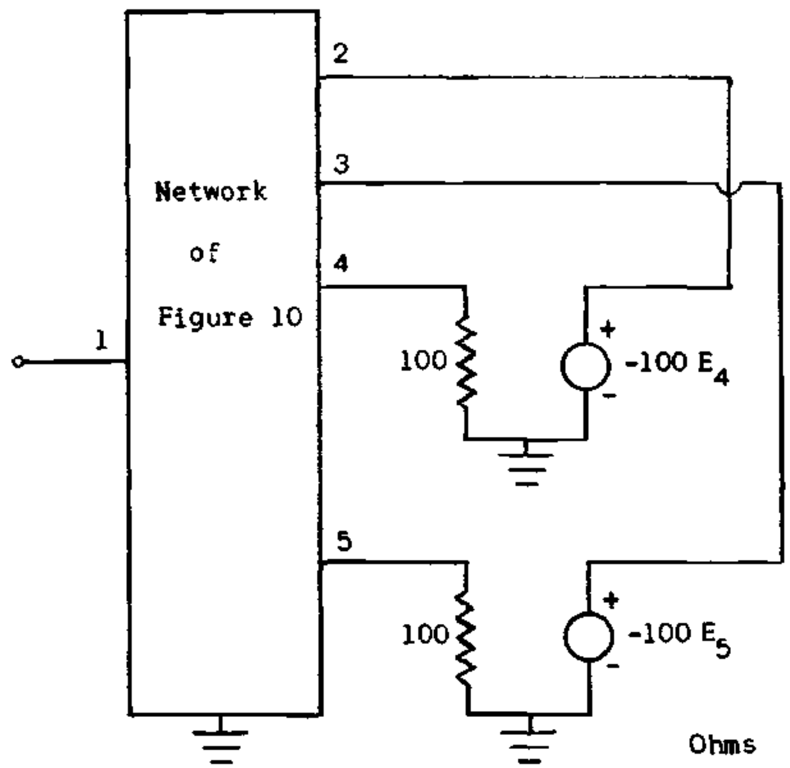


Figure 11. Network Realizing the Driving-Point Admittance Y = 1/s.

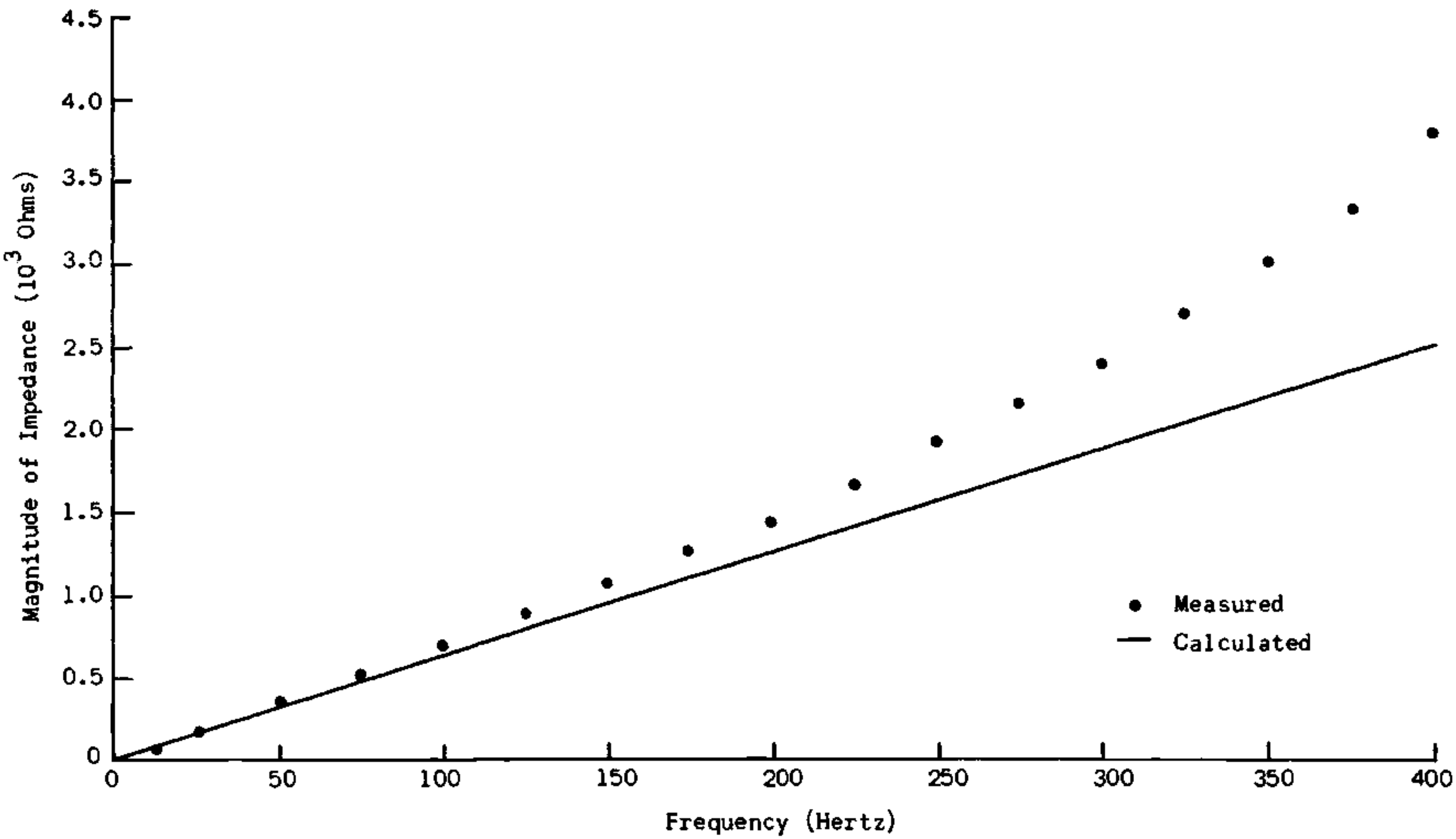


Figure 12. Comparison of Experimental Data with Predicted Magnitude Variation for a One Henry Inductor.

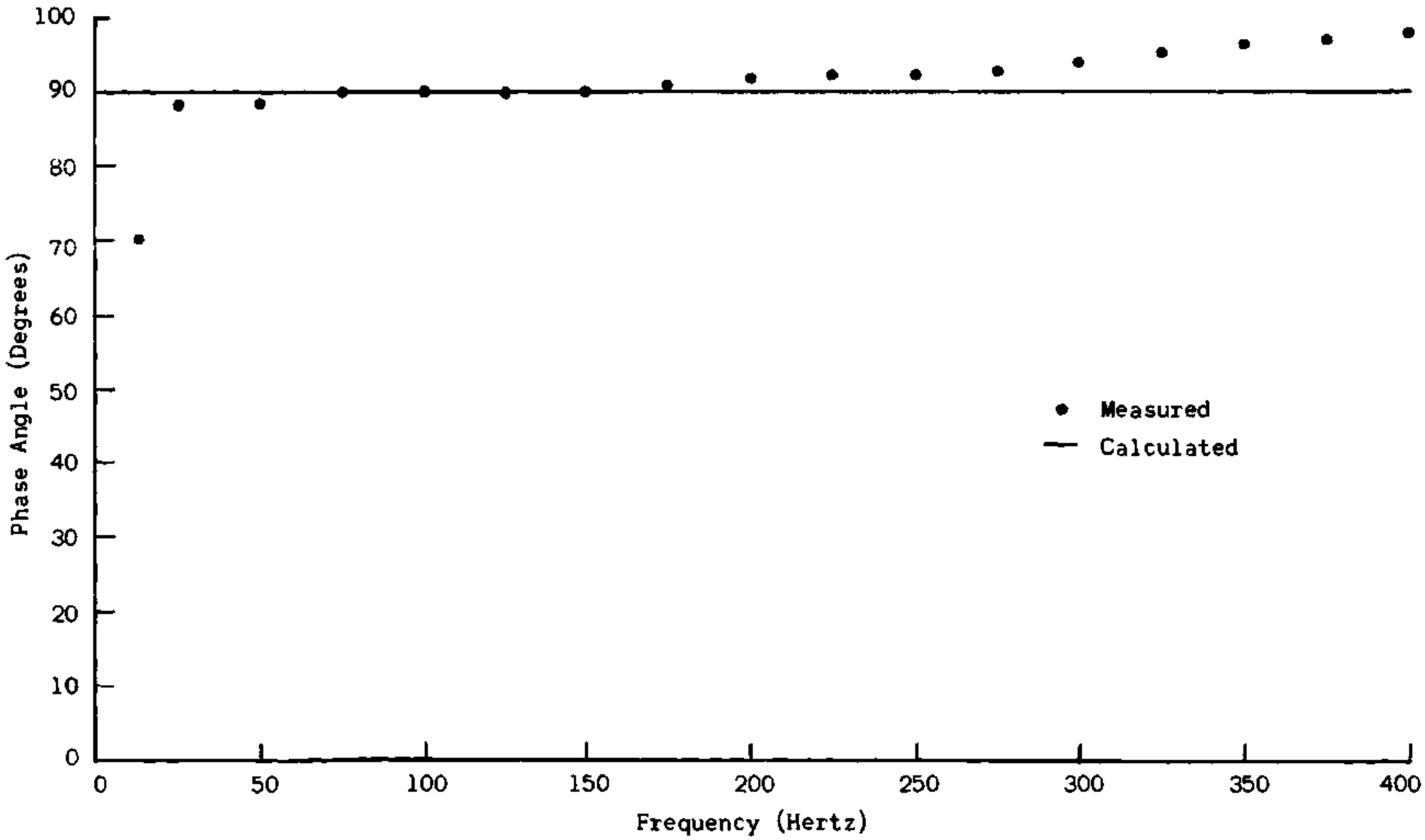
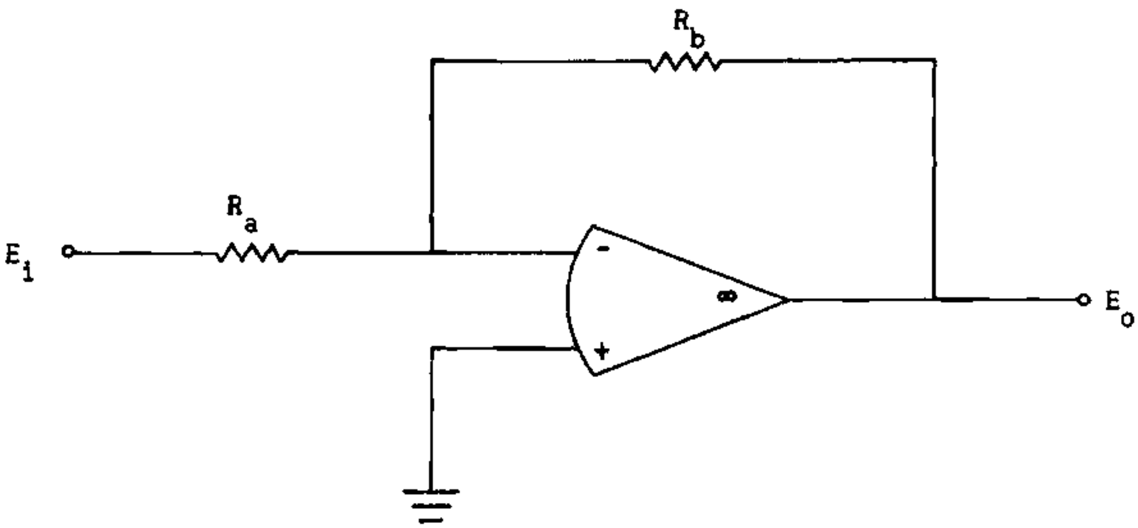


Figure 13. Comparison of Experimental Data with Predicted Phase Variation for a One Henry Inductor.

confirmation of this procedure, each of the amplifiers was approximated by using an operational amplifier and resistive feedback as is shown in Figure 14. The magnitude of the voltage gain was determined by the ratio R_b/R_a, and the input resistance was fixed by R_a. A Burr-Brown Model 1525 operational amplifier was utilized in the controlled source which was connected between ports 4 and 2 in Figure 11, and a Burr-Brown Model 1510 operational amplifier was used in the other controlled source. The D.C. open-loop gain, input resistance, and output resistance of the Model 1525 were approximately 106 db, 0.5 MΩ, and 5 KΩ, respectively, while the corresponding values for the Model 1510 were 90 db, 0.5 MΩ, and 0.1 KΩ. The designated voltage gains of -100 and the input resistances of 100 KΩ(after scaling) were achieved by using a 100 KΩ resistor for each R_a and a 10 MΩ resistor for each R_b. Using the information in reference (21), negligibly small output resistance values of 2.5Ω and 0.316Ω were calculated for the voltage amplifiers employing the Model 1525 and the Model 1510, respectively.

In order to measure the magnitude of the impedance of the network in Figure 11, a resistor and a Hewlett Packard Model 200 CD oscillator were connected in series with port 1. As the frequency was varied from about 10 to 400 hertz, a Keithley Model 103 differential input amplifier with a voltage gain of 40 db was employed to detect and amplify the voltage at the input of the original network and the differential voltage across the series resistor. The output voltage of the Keithley amplifier was measured with a Hewlett Packard Model 400 D vacuum tube voltmeter. The magnitude of the input impedance was then computed by dividing the voltage at the original port 1 by



$$\frac{E_o}{E_i} = -\frac{R_b}{R_a}$$

$$R_{in} = R_a$$

$$R_{out} \doteq 0$$

Figure 14. Operational Amplifier Realization of a Voltage-Controlled Voltage Source.

the voltage across the series resistor and multiplying the result by the value of the series resistor.

To determine the phase of the impedance of the constructed network, one Model 103 Keithley amplifier was connected to the original port 1, and another was connected across the series resistor. The amplifier outputs were attached to the vertical and horizontal input terminals of a Hewlett Packard Model 120A oscilloscope. By use of a Hewlett Packard Webb mask on the oscilloscope, the phase difference between the two voltages was measured as the frequency was varied.

Some problems developed during the testing of the network. The input voltage at port 1 had to be limited because the operational amplifiers saturated when the voltage at their outputs exceeded ±12 volts. However, a very low input voltage level was also unsatisfactory since the voltages at the inputs to the amplifiers became masked by noise, particularly 60 hertz. These problems were overcome by enclosing the network in an aluminum box, shielding the input and power supply leads, and keeping the oscillator output voltage as large as possible without saturating the amplifiers.

The network was found to be sensitive to changes in the amplifier gains and in the values of several of the resistances. The data recorded in Figures 12 and 13 was obtained when the gain of the voltage amplifier connected between ports 5 and 2 was adjusted until the phase of the impedance was approximately 90 degrees at 100 hertz. The magnitude of the impedance agreed well with its anticipated variation over the frequency range 25 to 200 hertz. However, at 275 hertz the error in the magnitude had increased to approximately 25 per cent. The measured phase of the impedance, which is shown in Figure 13, agreed much better with its predicted value. Over the frequency range 25 to 400 hertz, the error in the phase was less than 9 per cent.

The admittance parameters of the passive portion of the test network were calculated as functions of the complex-frequency by using the actual values of the resistors and capacitors in the network. Along with the designated values for c, d, G_1, and G_2, these calculated parameters were inserted into Equation (14), and the resulting expression was evaluated at several frequencies in the same range over which the measurements were taken. It was found that the calculated magnitude and phase values differed from the predicted ones by less than 5 per cent. Thus, part of the error between the measured and desired impedance values at the higher frequencies is due to the use of elements that differed from their designed values.

Though all of the reasons for the experimental error are difficult to determine, most of them are probably due to the following:

1. changes in the voltage gains as a function of frequency due to capacitive loading on the operational amplifiers[21];

2. noise which may have seriously affected the small voltages at the inputs to the amplifiers;

3. use of inaccurate components;

4. errors in measurements; and

5. parasitic effects, particularly stray capacitances, due to the network layout.

An extensive study of the causes of error was not undertaken since the only purpose of the experimental work was to verify one of the realization procedures. The data obtained accomplished this goal well enough to avoid further examination.

CHAPTER VII

RECOMMENDATIONS AND CONCLUSIONS

The advancement of integrated-circuit technology during the last decade has encouraged the development of active RC synthesis procedures. However, much of the work on these new methods is theoretical and is not suitable for the design of practical networks. One of the main problems in the application of the present active synthesis techniques is that many of the models of the active components have been so idealized that they can not be easily realized.

The three-terminal nonideal controlled sources which have been utilized in this study can be closely approximated by devices which are presently available. An operational amplifier with a resistive feedback arrangement as shown in Figure 14 can be used to produce each of the designated voltage amplifiers. In certain applications a single transistor is an adequate realization for one of the the specified current amplifiers, but other situations may require the use of an interconnection of transistors or operational amplifiers. The procedures developed in this thesis are valid regardless of the methods that are employed in realizing the necessary voltage-controlled voltage sources and current-controlled current sources. The experimental work indicates that these synthesis techniques can be implemented. Thus, they should be applicable to integrated-circuit methods.

The conclusion of this study can be summarized in the following four theorems:

Theorem 1

To realize an arbitrary N x N short-circuit admittance matrix of real rational functions in the complex-frequency variable by a transformerless grounded active N-port RC network embedding three-terminal voltage amplifiers with negative finite constant gains greater than unity and finite input conductances, it is sufficient that the active network contains 2N amplifiers.

Theorem 2

To realize an arbitrary N x N short-circuit admittance matrix of real rational functions in the complex-frequency variable by a transformerless grounded active N-port RC network embedding three-terminal current amplifiers with negative finite constant gains greater than unity and finite output conductances, it is sufficient that the active network contains 2N amplifiers.

Theorem 3

To realize simultaneously any of the pairs of short-circuit admittance functions Y_{11} and Y_{21}, Y_{11} and Y_{22}, Y_{12} and Y_{21}, and Y_{22} and Y_{12}, where each admittance is a real rational function of the complex-frequency variable, by a grounded transformerless active two-port RC network, it is sufficient that the network contains two three-terminal voltage-controlled voltage sources with negative finite constant gains greater than unity and finite input conductances.

Theorem 4

To realize simultaneously any of the pairs of short-circuit admittance functions Y_{11} and Y_{12}, Y_{11} and Y_{22}, Y_{12} and Y_{21}, and Y_{22}

and Y_{21}, where each admittance is a real rational function of the complex-frequency variable, by a grounded transformerless active two-port RC network, it is sufficient that the network contains two three-terminal current-controlled current sources with negative finite constant gains greater than unity and finite output conductances.

These theorems have been proved and demonstrated with examples, and an implementation of the procedure associated with Theorem 1 has been obtained.

This investigation has uncovered some additional issues which need further study. The development of techniques for minimizing the sensitivity of active RC networks to changes in the component values is a most difficult problem, but it must be solved before realization procedures employing controlled sources can be utilized to their full potential. Similarly, a search for an optimum value for the voltage and current gains of the amplifiers would be worthwhile.

Due to the effort required to perform the matrix synthesis methods, especially the matrix factorization step, a computerization of the design procedures would be helpful. With the aid of a computer, the designer would have a greater freedom of choice in the construction of a network because he could quickly obtain and analyze several networks which realize a particular prescribed function.

Integrated-circuit technology has progressed to the extent that it is no longer economically necessary to attempt to minimize the number of active components needed in a network. Consequently, an investigation should be made into the possibility of increasing the number of controlled sources in order to enhance the performance of the network or to reduce the number of passive elements.

In summary, this study has developed new procedures for the design of active RC networks. The models of the active components can be easily approximated with practical devices, and the resulting networks are grounded.

APPENDIX I

SELECTION OF $\bar{Y}_{11}$

The matrix factorization which is required by the synthesis procedures in Chapters II and III can always be performed if the determinant of the matrix $[A]$, which is to be factored, contains a specific number of negative-real zeros. Sandberg[4] has demonstrated a method which forces the determinant of $[A]$ to fulfill this requirement, and his proof will be given in this appendix.

In this investigation the matrix $[A]$ is obtained from the difference of two N x N matrices. Employ the notation

$$[A] = [P]q - [p]Q \tag{A-1}$$

where

(A) $\deg p_{ii} = \deg q = T$, and

(B) $\bar{y}_{11} = [p]/q$ is an arbitrarily selected short-circuit admittance matrix of a transformerless grounded passive RC network with $p_{ii}(0) \neq 0$.

Let y_{11} be represented by

$$\bar{y}_{11} = \frac{1}{q} \begin{bmatrix} \eta p'_{11} & p_{12} & \cdots & p_{1N} \\ p_{12} & \eta p'_{22} & \cdots & p_{2N} \\ \vdots & \vdots & & \vdots \\ p_{1N} & p_{2N} & \cdots & \eta p'_{NN} \end{bmatrix} \tag{A-2}$$

where η is a real positive constant. The polynomial det $[A]$ can be expressed as

$$\det [A] = \det ([P]q - [p]Q) = -(\eta)^N (Q^N \prod_{i=1}^{N} p'_{ii} + \frac{R}{\eta^N}) \qquad \text{(A-3)}$$

where all of the coefficients of the polynomial R/η^N approach zero as η approaches infinity. Notice that, as η approaches infinity, NT of the zeros of det $[A]$ approach the zeros of

$$\prod_{i=1}^{N} p'_{ii}$$

The choices for the zeros of this product are independent of the prescribed zeros of P_{ij} or Q. Hence, for a sufficiently large value of η, det $[A]$ has at least NT distinct negative-real zeros which are different from those of q.

APPENDIX II

FACTORIZATION OF THE MATRIX [A]

For the convenience of the reader, Sandberg's[3,4] procedure, which was later summarized by Su,[1] for the factorization of matric polynomials will be presented in this appendix. Let the N x N matrix which is to be factored be denoted by

$$[A] = [P]q - [p]Q \tag{A-4}$$

where

(A) $L_o = \max(\deg P_{ij}, \deg Q)$;

(B) $T = \deg p_{ij} = \deg q$; and

(C) [P] and Q are specified, but [p] and q may be selected.

First, assume that det [A] has m simple negative-real zeros and that these zeros are represented by the factors $(s + \beta_1)$, $(s + \beta_2)$,..., $(s + \beta_m)$. If it is possible to determine a nonsingular matrix of real constants

$$[X_i] = \begin{bmatrix} 1 & & & u_{1i} & & & 0 \\ & \ddots & & \vdots & & & \\ & & & u_{ii} & & & \\ & & & \vdots & \ddots & & \\ 0 & & & u_{Ni} & & & 1 \end{bmatrix} \tag{A-5}$$

such that the product $[A][X_i]$ contains one of the factors of det [A], $(s + \beta_i)$, in every element in the i^{th} column, then [A] can be expressed as

$$[A] = [A][X_i][X_i]^{-1} = [A_a] \begin{bmatrix} 1 & & & & & 0 \\ & \ddots & & & & \\ & & 1 & & & \\ & & & (s+\beta_i) & & \\ & & & & 1 & \\ & & & & & \ddots \\ 0 & & & & & 1 \end{bmatrix} [X_i]^{-1} \qquad \text{(A-6)}$$

The elements in $[A_a]$ are the same as those in $[A]$ except in the i^{th} column in which all polynomials are one degree lower.

Since $(s + \beta_i)$ is a factor of every element in the i^{th} column of $[A][X_i]$, N simultaneous equations in terms of the N u's can be obtained by evaluating each of these i^{th} column polynomials at $s = -\beta_i$ and setting the results equal to zero. Thus,

$$\begin{aligned} u_{1i}a_{11}(-\beta_i) + u_{2i}a_{12}(-\beta_i) + \cdots + u_{Ni}a_{1N}(-\beta_i) &= 0 \\ u_{1i}a_{21}(-\beta_i) + u_{2i}a_{22}(-\beta_i) + \cdots + u_{Ni}a_{2N}(-\beta_i) &= 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots & \\ u_{1i}a_{N1}(-\beta_i) + u_{2i}a_{N2}(-\beta_i) + \cdots + u_{Ni}a_{NN}(-\beta_i) &= 0 \end{aligned} \qquad \text{(A-7)}$$

Since

$$\det [A(-\beta_i)] = \det [a_{ij}(-\beta_i)] = 0 \qquad \text{(A-8)}$$

the linear equations in Equation (A-7) have a nontrivial solution. Hence, matrix $[X_i]$ can be determined once the u's are calculated.

To complete the demonstration of the existence of matrix $[X_i]$, it is only necessary to show that $[X_i]$ is nonsingular. This is true if

$$\det [X_i] \neq 0 \tag{A-9}$$

However, Equation (A-5) yields

$$\det [X_i] = u_{ii} \tag{A-10}$$

so that it must be shown that $u_{ii} \neq 0$. The Laplace expansion of [A] about its i^{th} column gives

$$\det [A] = \sum_{j=1}^{N} a_{ji}\Delta_{ji} \tag{A-11}$$

where Δ_{ji} is the cofactor of the j,i element of [A]. Let the greatest common factor of all the Δ_{ji} in Equation (A-11) be represented by b_i. It is then possible to write that

$$\det [A] = b_i \sum_{j=1}^{N} a_{ji}\Delta'_{ji} \tag{A-12}$$

where $\Delta'_{ji} = \frac{\Delta_{ji}}{b_i}$. If $(s + \beta_i)$ is a factor of $\sum_{j=1}^{N} a_{ji}\Delta'_{ji}$, it is not a factor of b_i, and hence, not a factor in all of the (N - 1)-rowed minors and cofactors that can be constructed by deleting the i^{th} column and one of the N rows from [A]. Consequently, all of these minors do not vanish at $s = -\beta_i$. If in Equation (A-7) u_{ii} is assumed to be zero, the solutions for the remaining u's are trivial.[20] However, it has already been determined that all of the u's are not zero. This suggests that $u_{ii} \neq 0$ and that $[X_i]$ is nonsingular.

Therefore, if det $[A]$ has at least one zero, β_i, which is different from those of b_i, a nonsingular matrix of constants $[X_i]$ can be determined such that each element in the i^{th} column of $[A][X_i]$ has a zero at $s = -\beta_i$. Equation (A-4) discloses that the masimum degree of the elements of $[A]$ is $r = L_o + T$ and that the degree of det $[A]$ is less than or equal to Nr. Since the degree of b_i can at most be $r(N-1)$, the factorization in Equation (A-6) is possible if

$$m > r(N - 1) \qquad \text{(A-13)}$$

From Equation (A-6) it can be determined that the determinant of $[A][X_i]$ contains all of the zeros of det $[A]$ because the determinant of the product of square matrices is equal to the product of the individual determinants. Thus, this factorization procedure can be applied to each column of $[A]$ as long as Equation (A-13) remains satisfied. A nonsingular real matrix $[F]$ may be specified such that

$$[A][F][F]^{-1} = [A_b] \begin{bmatrix} (s + \beta_1) & & 0 \\ & (s + \beta_2) & \\ 0 & & (s + \beta_N) \end{bmatrix} [F]^{-1} \qquad \text{(A-14)}$$

in which each element of $[A_b]$ is one degree lower than the corresponding element in $[A]$. Notice that all of the zeros of det $[A]$ are included in either det $[A_b]$ or the diagonal matrix in Equation (A-14).

If the determinant of the matrix $[A_2]$ has an adequate number of simple negative-real zeros, the factorization in Equation (A-14) can be repeated L_o times. After $(L_o - 1)$ repetitions, $N(L_o - 1)$ of the negative-real zeros of det [A] have been used, and $m - N(L_o - 1)$ of these zeros are left. At this point $(L_o - 1)$ linear factors have been removed from each element in [A]. Thus, the degree of the matrix which remains to be factored can be no more than $r - (L_o - 1)$, and the polynomial b in Equation (A-12) has a maximum of $(N-1)[r - (L_o - 1)]$ zeros. In order to perform the L_o^{th} repetition of the factorization technique, it is sufficient that

$$m - N(L_o - 1) > (N - 1)[r - (L_o - 1)] \qquad \text{(A-15)}$$

or

$$m > N(T + L_o) - T - 1 \qquad \text{(A-16)}$$

However, m can be made at least as large as NT by proper choices for the diagonal elements of [p]. One such selection is shown in Appendix I. Setting m = NT in Equation (A-16) gives

$$T > NL_o - 1 \qquad \text{(A-17)}$$

Notice that the relationship in Equation (A-17) is only a sufficient condition not a necessary one.

By use of the technique which has been presented, a prescribed matrix [A] can be factored such that

$$[A] = [A_1][A_2] \qquad \text{(A-18)}$$

Equation (A-14) reveals that the determinant of $[A_2]$ contains only simple negative-real zeros which are also contained in the determinant of $[A]$. All of the elements of $[A_2]$ are of degree L_0 while those of $[A_1]$ are of degree T.

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