

ONR FINAL TECHNICAL REPORT

"Nonlinear Dynamics of Josephson Junction Parametric Amplifier Arrays"

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I. Summary of Work Accomplished

A. Introduction

The basic goal of the project was to investigate the behavior of Josephson junction arrays using theoretical tools from dynamical systems theory. Emphasis was placed on the stability of coherent operation of arrays, and their sensitivity near the onset of instabilities. Good progress has been made in these areas; all of this work has appeared (or is scheduled to appear) in the published literature: for a listing, see references 1-7 in Section III. This primary work is summarized in Section IB.

In addition, we were able to make progress on a number of other problems, which share mathematical features with the Josephson junction system. This was possible because the approach of dynamical systems theory extracts features which are not sensitive to the details of the physical system; consequently, these behaviors are typical of a broader class of systems. The results on these other topics also have been published in the technical literature: for a listing, see references 8-18 in Section III. This additional work is summarized in Section IC.

B. Josephson Junction Arrays.

For practical applications, it is most desirable to operate arrays in the *in-phase state*, so that each oscillator has precisely the same waveform, and all oscillators are in perfect synchrony. Consequently, the first order of business is to determine the conditions under which stable in-phase operation is possible. The stability regions for a variety of circuit configurations has been mapped out, first via numerical simulations[1] and then via approximate analytic calculations[2,3]. In fact, we discovered that for certain circuit designs, the dynamics possessed a fundamental symmetry that prohibits stable in-phase orbits[4,5]. We conclude immediately that such designs should be avoided for any practical

application. (For example, this applies to small capacitance "point contact" junctions shunted by a purely resistive load.)

The next step taken was to investigate the response of the in-phase state to random fluctuations. For small noise strengths (corresponding to very low temperatures), the effects show up in two fundamentally different ways: (i) as broadening the sharp spectral lines, and (ii) as additional broadband "skirts" of Lorentzian character[6]. As the stability boundary is approached, these effects become increasingly strong -- that is, the quality of the output is degraded. How great is the degradation depends only on the symmetry properties of the nearing instability. In particular, if the instability corresponds to a symmetry breaking, we find a remarkable robustness possessed by the array: though each element suffers large-scale fluctuations, the total voltage oscillations across the array remain small! On the other hand, if the instability is symmetry preserving, the fluctuations become increasingly strong as the bifurcation is approached[7].

For use as parametric amplifiers, we also considered the array response to periodic perturbations (representing the input signal). Here, we found that substantial amplification is expected only in the case of symmetry preserving instabilities. In fact, the amplification of this extrinsic signal is expected to be greater than the degradation (mentioned above) due to intrinsic fluctuations as a function of increasing array size. Specifically, for an array consisting of N elements, the extrinsic signal enjoys a power gain proportional to N^2 , while the intrinsic noise shows a gain in power proportional to N .

Finally, we discovered the possibility of a new kind of noise sensitivity that can affect an array of N junctions, which is absent for the single junction problem. This phenomenon, called attractor crowding[8,9], is the result of global dynamical considerations (in contrast with the local considerations described above). In particular, if the stable in-phase orbit coexists with a particular type of orbit called the splay-phase orbit (as is often the case in the Josephson circuits), then noise becomes increasingly effective at destroying the coherence of the array. Direct simulations of the circuit equations have verified this picture[9]. This problem can be avoided by operating the Josephson junction array in a parameter regime where no stable splay-phase orbits exist.

C. Additional Work

Globally Coupled Arrays. The primary spin-offs of the research on Josephson junction arrays rest on the structure of these equations, in particular the property of global coupling. This property allow certain general progress, which can also be applied to certain laser systems[10,11], and also to other systems[12]. The laser system we studied was a diode pumped Nd:YAG laser with an intracavity frequency-doubling crystal. We were able to predict the conditions under which stable steady-state behavior was expected[10], and also the appearance of splay-state orbits[11]. Both sets of predictions were directly verified by experiments carried out by R. Roy's group at Georgia Tech. In another project, we looked at a room temperature electrical circuit consisting of an array of p-n junctions, which has structural similarities with the Josephson junction array circuit. We tested whether certain combinatoric ideas might be useful in describing the degree of coherence exhibited by the array. In fact, the results were surprisingly encouraging[12], and we conclude that such a statistical approach might be valuable in handling cases where the array elements are not nearly identical.

Stochastic Resonance. This concerns the behavior of bistable dynamical systems which are driven by a strong noise source and a weak periodic modulation, and the possibility of improved signal-to-noise ratios as the input noise is increased. Previously, in collaboration with R. Roy, we demonstrated this effect in a laser experiment. The experiment set off a number of theoretical and experimental efforts, including the publication of our own detailed theory (which formed the second part of B. McNamara's Ph.D. dissertation)[13].

Self-Organized Criticality. This refers to a phenomenon observed in model spatially extended dynamical systems which (like the Josephson junction arrays) have large numbers of metastable states, where the statistical steady state is characterized both spatially and temporally by power law fluctuations. Considerable interest has been generated, since this offers a paradigm in which

the balance between steady external stress and internal transport results in complex spatial and temporal behavior, rather than a simple time-independent equilibrium. We established new examples of cellular automata which exhibit SOC[14], including the first case of a completely deterministic system[15]. We have also provided some of the first analytic results on the subject, and deduced that the "critical state" has the structure of a neutrally stable attractor in the appropriate phase space.

Classical Analog of Squeezed Fluctuations. Experiments by M. Bocko (U. Rochester) previously showed that a classical analog of the squeezed fluctuations observed in quantum optics can occur in a driven p-n junction near the onset of simple bifurcations. We developed a theory[16] showing that such squeezing is a generic property of a broad class of nonlinear systems which includes the one studied by Bocko. The theory suggests a limit to the maximum amount of squeezing, in contrast to (presumably more fundamental) quantum optical descriptions.

References

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- [2] P. Hadley, M.R. Beasley and K. Wiesenfeld, Physical Review B, volume 38, pp. 8712-8719 (1988).
- [3] S. Nichols, "Dynamical Stability in Josephson Junction Arrays", Masters Thesis, Georgia Institute of Technology (1990), unpublished.
- [4] K.Y. Tsang, R.E. Mirollo, S.H. Strogatz, and K. Wiesenfeld, Physica D, volume 48, pp. 102-112 (1991).
- [5] K.Y. Tsang, S.H. Strogatz, and K. Wiesenfeld, Physical Review Letters, volume 66, pp. 1097-1100 (1991).
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- [7]. "Amplification by Globally Coupled Nonlinear Arrays: Coherence and Symmetry", K. Wiesenfeld, accepted for publication in Physical Review A (to appear, 1991).

- [8] K. Wiesenfeld and P. Hadley, Physical Review Letters, volume 62, pp. 1135-1138 (1989).
- [9] K.Y. Tsang and K. Wiesenfeld, Applied Physics Letters, volume 56, pp. 495-496 (1990).
- [10] G.E. James, E. Harrell II, C. Bracikowski, K. Wiesenfeld, and R. Roy, Optics Letters, volume 15, pp. 1141-1143 (1990).
- [11] K. Wiesenfeld, C. Bracikowski, G. James, and R. Roy, Physical Review Letters, volume 65, pp. 1749-1752 (1990).
- [12] L. Fabiny and K. Wiesenfeld, Physical Review A, volume 43, pp. 2640-2648 (1991).
- [13] B. McNamara and K. Wiesenfeld, Physical Review A, volume 39, pp. 4854-4869 (1989).
- [14] B. McNamara and K. Wiesenfeld, Physical Review A, volume 41, pp. 1867-1873 (1990).
- [15] K. Wiesenfeld, J. Theiler, and B. McNamara, Physical Review Letters, volume 65, pp. 949-952 (1990).
- [16] K. Wiesenfeld and J.S. McCarley, Physical Review A, volume 42, pp. 755-760 (1990).

II. Personnel

During the period of the contract, several people who participated in the research described received support or partial support. Bruce McNamara is presently an Assistant Professor at Reed College in Portland, Oregon. Kwok Yeung Tsang is now a scientist at the Naval Research Laboratory in Washington, D.C. Each of these two individuals had filled the postdoctoral position funded by the grant. The grant has also been used at times to support two graduate students: Li-Shi Luo is scheduled to receive his Ph.D. by the end of the calendar year 1991; Steven Nichols completed his Masters thesis in the summer of 1990, and is presently working toward his Ph.D. in our group.

III. Index of Publications

The following is a list of all publications of the Principal Investigator during the period covered by the contract.

1. "Attractor Crowding in Oscillator Arrays", K. Wiesenfeld and P. Hadley, Physical Review Letters, volume 62, pp. 1135-1138 (March, 1989).
2. "Noise Driven Fluctuations of Josephson Junction Series Arrays", P. Hadley, M.R. Beasley, and K. Wiesenfeld, I.E.E.E. Transactions on Magnetics, volume 25, pp. 1088-1091 (March, 1989).
3. "Attractor Crowding in Josephson Junction Arrays", K.Y. Tsang and K. Wiesenfeld, Applied Physics Letters, volume 56, pp. 495-496 (January, 1990).
4. "Dynamics of a Globally Coupled Oscillator Array", K.Y. Tsang, R.E. Mirollo, S.H. Strogatz, and K. Wiesenfeld, Physica D, volume 48, pp. 102-112 (1991).
5. "Reversibility and Noise Sensitivity of Josephson Arrays", K.Y. Tsang, S.H. Strogatz, and K. Wiesenfeld, Physical Review Letters, volume 66, pp. 1097-1100 (February, 1991).
6. "On the Comparison Between Josephson Junction Array Variations", K.Y. Tsang and K. Wiesenfeld, accepted for publication in Journal of Applied Physics(to appear, 1991).
7. "Amplification by Globally Coupled Nonlinear Arrays: Coherence and Symmetry", K. Wiesenfeld, accepted for publication in Physical Review A (to appear, 1991).
8. "Theory of Stochastic Resonance", B. McNamara and K. Wiesenfeld, Physical Review A, volume 39, pp. 4854-4869 (May, 1989).
9. "A Physicist's Sandbox", K. Wiesenfeld, C. Tang, and P. Bak, Journal of Statistical Physics, volume 54, pp. 1441-1458 (1989).

10. "Period doubling bifurcations: what good are they?", K. Wiesenfeld, in Noise in Nonlinear Systems: Theory, Experiment, and Simulation, F. Moss and P.V.E. McClintock, editors (Cambridge University Press, Cambridge, 1989).
11. "Comment on 'Relaxation at the Angle of Repose'", P. Bak, C. Tang, and K. Wiesenfeld, Physical Review Letters, volume 62, pg. 110 (January, 1989).
12. "Self Organized Criticality in Vector Avalanche Automata", B. McNamara and K. Wiesenfeld, Physical Review A, volume 41, pp. 1867-1873 (February, 1990.)
13. "Suppressed Fluctuations and Incipient Instabilities", K. Wiesenfeld and J.S. McCarley, Physical Review A, volume 42, pp. 755-760 (July, 1990).
14. "Self Organized Criticality in a Deterministic Automaton, K. Wiesenfeld, J. Theiler, and B. McNamara, Physical Review Letters, volume 65, pp. 949-952 (August, 1990).
15. "Elimination of Chaos in an Intracavity-Doubled Nd:YAG Laser", G.E. James, E. Harrell II, C. Bracikowski, K. Wiesenfeld, and R. Roy, Optics Letters, volume 15, pp. 1141-1143 (October 1990).
16. "Observation of Antiphase States in a Multimode Laser", K. Wiesenfeld, C. Bracikowski, G. James, and R. Roy, Physical Review Letters, volume 65, pp. 1749-1752 (October, 1990).
17. "Unfolding a Chaotic Bifurcation", O. Rossler, H.B. Stewart, and K. Wiesenfeld, Proceedings of the Royal Society of London A, volume 431, pp. 371-383 (1990).
18. "Clustering Behavior in Globally Coupled Oscillators", L. Fabiny and K. Wiesenfeld, Physical Review A, volume 43, pp. 2640-2648 (March, 1991).