Simple Newsvendor Heuristics for Inventory Distribution Systems

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Simple Newsvendor Heuristics for Inventory Distribution Systems

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SUMMARY

To date, closed form optimal solutions for stocking levels in arborescent multiechelon inventory systems have not been obtained. These problems exhibit the joint difficulties of requiring an allocation policy as well as a stocking policy, and the multidimensional nature of their state space makes dynamic programming formulations impractical. In this dissertation, we introduce procedures that approximate multiechelon networks with sets of single installation problems. We first use this technique to solve for base-stock levels in a distribution network with asymmetric retailers. Second, we use this technique to analyze delayed differentiation production processes and provide guidance as to when the strategy is most warranted. Third, we modify the technique to account for inventory that exhibits perishability and solve for stocking policies for distribution systems when the inventory has a fixed shelf life.

INTRODUCTION

Multiechelon distribution systems are pervasive in practice. They occur whenever multiple internal or external customers exist for the producer of a good. Examples are abundant, from Taco Bell's centralized production of foodstuffs sold through a distributed network of franchises to the internal consumption of computer components at Dell Computers. Poor supply chain management practices lead to significantly suboptimal tactical and strategic behavior, potentially costing firms millions of dollars (Singhal and Hendricks, 2003). In this dissertation, we introduce heuristic techniques for the management of distribution systems.

We begin by introducing a core technique, referred to as the Newsvendor Bounds Technique. The method begins by bounding from above and below the costs and stocking levels of a multiechelon supply chain. This reduces the distribution system to a set of serial chain problems. We approximate base-stock levels in these serial chains by single installation policies. The method thus transforms a very difficult problem into a number of readily solvable problems, providing a simple and surprisingly accurate solution.

We first apply the Newsvendor Bounds Technique to two-echelon distribution networks. This problem is the most frequently studied of those we consider in this dissertation. Notable recent works such as Graves (1996) and Cachon (2001) have progressed understanding of distribution networks, but have significant limitations. Our approach relaxes key assumptions of prior works, as well as providing both a simpler solution method and a reduction in the average error compared to other heuristic techniques. Using the simple structure of our solution technique, we also offer an analytical parametric study of the optimal cost and stocking behavior of distribution networks. We show that, holding all else equal, asymmetry in backordering costs or demand rate decreases the costs of operating a distribution system compared to a symmetric system. We argue this occurs due to a virtual pooling effect, causing the distribution system to behave similarly to a two-echelon serial system.

Next we turn our attention to the difficulties faced by firms offering wide product varieties. Various supply chain strategies have been explored to provide these products at minimum cost. We focus on one of these strategies, delayed differentiation, which enables the firm to maintain large stocks of work-in-process to fulfill demand for multiple products. Delayed differentiation allows firms to exploit risk pooling and aggregate forecasting to reduce the uncertainty inherent in offering product variety. Lee and Tang (1997) provide a seminal work in this area, but utilize a decoupling assumption in which the installations in the supply chain are assumed to operate independently. By removing this assumption and recognizing the interdependency of installations, we show that, under certain conditions, their analysis may result in supply chain costs that are significantly higher than optimal.

We also reveal a previously hidden factor in determining the correct delayed differentiation strategy. The pattern of holding costs assessed for the various stages of work in process, which we refer to as the holding cost profile, plays a role in the determination of the least-cost strategy. Prior work has established the importance of the absolute holding cost in this decision; in this dissertation, we show that the correct strategy incorporates not only these absolute levels but also the structure of the holding cost profile. Capturing large jumps in inventory holding costs is considerably more valuable than would be predicted based on previous literature. Thus we provide a qualitative test for identifying valuable differentiation opportunities in the production process.

Finally, we consider the effects of inventory perishability in distribution networks. This adds an additional level of complexity to the nonperishable distribution network problem. Although both economically and socially significant, it has gathered scant research attention. We first find serial chains and distribution systems behave quite differently, with the terminal locations of a serial chain behaving as if the inventory was nonperishable. Distribution systems, however, must account for an opportunity cost that arises in perishable contexts but is absent in nonperishable problems. This opportunity cost occurs due to the possibility of a unit of inventory expiring at an installation rather than satisfying demand elsewhere in the system. We develop well performing heuristic stocking policies for both serial and distribution systems. This work further suggests that the opportunity cost inhibits the exploitation of risk pooling opportunities. As lifetimes increase, the likelihood of outdating decreases, causing these opportunity costs disappear. Hence strategies to extend lifetime are particularly warranted when risk pooling opportunities are available.

The remainder of this dissertation is organized as follows. A review of the literature is presented in Chapter 2. The development of our core technique is presented in Chapter 3. Our models and preliminary results for our studies in distribution systems, delayed differentiation, and perishable inventory distribution systems are presented in Chapters 4, 5, and 6, respectively.

CHAPTER 2
LITERATURE REVIEW

2.1 Distribution System Inventory Theory

Two main challenges exist in determining optimal supply chain strategies for distribution systems: determining the stocking policies for each installation and the allocation policy of inventory to downstream stages when demand exceeds supply at an upstream stage. Prior work on these elements of the problem is discussed in §2.1.1 and §2.1.2 below.

2.1.1. Allocation Policies

Clark and Scarf (1960) inspired a long stream of literature on the domain of production and distribution networks with their analysis of serial systems, finding for systems with a single retailer, echelon inventory stocking policies are optimal. They further suggest arborescent systems may be approximated by a serial system under a balance relaxation of the traditional dynamic program formulation. This relaxation allows the warehouse to reallocate inventory by imposing negative inventory shipments on downstream installations. The balance relaxation is utilized frequently in this literature (e.g. see Eppen and Schrage (1981), Federgruen and Zipkin (1984a, b), Federgruen (1993), Verrijdt and de Kok (1996), Garg and Tang (1997), and van der Heijden et al. (1997)) although in practice such a policy may not be always feasible. Eppen and Schrage (1981) and Erkip et al (1990) provide simulation results suggesting that for high service level systems, such an allocation policy is feasible most of the time. Unfortunately, the balance relaxation may be inappropriate when downstream installations are substantially asymmetric in inventory cost profiles and lead times (e.g. see Clark and Scarf (1960), Federgruen and Zipkin (1984a), McGavin et al. (1993), Kumar and Jacobson (1998), and Axsater et al. (2002)). Additionally, such an assumption is unrealistic in practice, as it implies the existence of costless and instantaneous transshipment opportunities. While the balance relaxation serves to make the problem tractable, it is also severely limiting in addressing complex and realistic distribution systems; hence we avoid the use of such a relaxation in this work.

A number of allocation policies that do not rely on the balance relaxation have already been introduced in the literature. Graves (1996) utilizes a virtual assignment rule, where echelon inventory is devoted to a given retailer as demand occurs. This is essentially the opposite of the rebalancing assumption, in that rather than assigning inventory at the end of the supply chain, the assignment occurs before the inventory enters the system. These concepts suggest lower and upper bounds on system stock, respectively.

Erkip (1984), Jackson and Muckstadt (1989), McGavin et al. (1993) and Axsater et al. (2002) consider policies where retailers order from the warehouse two times during the warehouse's order cycle. These models show that splitting an arriving order at the warehouse into two quantities, one of which is shipped immediately and the other at some period before the next arrival of inventory at the warehouse, captures most of the risk pooling benefits. In periodic review systems such as are considered here, the warehouse may ship inventory in every period.

By using a random allocation policy, Cachon (2001) develops exact results for the retailer and warehouse costs, although such a policy does not consider the relative need for inventory at the retailers and is thus sub-optimal. Myopic allocation policies, used in our heuristic and by Federgruen and Zipkin (1984b) and Axsater et al. (2002), allocate inventory in an attempt to minimize the expected costs at the retailers in the period the inventory arrives (e.g. after the warehouse to retailer shipment lead-time). Federgruen and Zipkin (1984b) show that, for identical retailers, a myopic policy is approximately optimal when orders may be placed every period. Jackson and Muckstadt (1989) and Jackson (1988) use a similar allocation rule, denoted the "runout allocation rule", where the allocation is determined by solving an optimization problem over the horizon until the next arrival of inventory at the warehouse stage. The allocation rule used in this work is most similar to that of McGavin, Schwarz, and Ward (1993), who assume identical retailers and allocate stock so as to maximize the minimum retailer inventory position. We allow for non-identical downstream stages and instead minimize the maximum deviation between each installation's echelon inventory-transit position and its echelon base-stock level.

2.1.2 Stocking Policies

The traditional approach used in determining stocking levels for a distribution system is to formulate the problem as a stochastic dynamic program and apply relaxations or restrictions to the system to allow for tractability. Federgruen (1993) notes for most deterministic demand systems, efficient algorithms for determining optimal strategies do not exist, a situation exacerbated in models with stochastic demand. One particularly challenging issue, for example, is in the absence of the balancing relaxations, the optimal stock at an installation may be less than the actual stock on hand, given the state of other installations (e.g. consider two retailers, one with ample safety stock and the other carrying backorders). The resulting large solution space for the optimal policy is accompanied by considerable computational burden. Hence researchers tend to approximate the system to create a policy and then compare that policy via numerical solutions or simulation to either known bounds or the "best found system" (e.g. McGavin et al., 1993).

Because the literature in this area typically utilizes two-echelon models with a single warehouse and multiple retailers, we begin with a survey of models of this type. One approach is to treat the warehouse as a cross-dock which may not hold inventory. Here, Eppen and Schrage (1981) determine the average inventory and backorder levels assuming identical retailers and independent demands. Erkip et al. (1990) extend this model to allow for correlated demands and Garg and Tang (1997) extend it to an arbitrary number of echelons and retailers (assuming arborescence holds). Unlike these works, we allow the inventory to be held at upstream locations. This allows a centralized decision maker to better protect the retailers from uncertainty in demand over the lead-time from the exogenous supplier to the warehouse, in addition to exploiting potential holding cost savings at the warehouse.

When inventory is allowed to be held at the warehouse, Federgruen and Zipkin (1984a) show that one may approximate a two-echelon distribution system with identical retailers by relaxing their DP formulation, initially allowing rebalancing to determine the shipment quantity to a collapsed retailer. This provides a lower bound, and we adopt a similar approach in an arbitrary arborescent multiechelon system below. They also show their policy is essentially the same as a decentralized system where each installation follows its own critical number policy. Chen and Zheng (1994) provide lower bounds for the total inventory related costs after noting optimal policies for such a system are unknown. Earlier, Jackson (1988) provided an extension of the Eppen and Schrage (1981) model to allow the warehouse to hold inventory. Like Eppen and Schrage, the warehouse orders every *t* periods, but by allowing inventory to be held at the warehouse, Jackson captures the "depot effect" of allocating inventory to retailers late in the warehouse's order cycle, creating more balanced inventory positions (and hence service levels). As we do in this work, Jackson follows a base stock policy in each period that the warehouse holds sufficient inventory, which provides for a finer degree of control than the single mid-cycle allocations of Erkip (1984), Jonsson and Silver (1987 a, b), and Jackson and Muckstadt (1989). Jackson defines a cost function over a single warehouse order horizon of *t* periods, and sets retailer order up to levels based on an approximate problem. Our approach is similar to Jackson in that our stocking policy is a function of a sum of newsvendor cost functions. However, Jackson's approximate cost function is a nested optimization problem, where the internal newsvendor problems depend only on the decision of quantity to hold at the warehouse in the beginning of the warehouse order cycle. Thus while Jackson's' approximate cost function is minimized by a search over a single variable, we do not require recursive solutions.

Axsater, et al. (2002) consider a two-echelon multiple retailer distribution system with no ordering costs but where the orders occur in batches, and the warehouse orders in multiples of a batch size (a system-batch). They propose heuristics to avoid the computationally impractical solution of the stochastic dynamic programming problem. Their virtual assignment heuristic determines stocking levels and is related to Graves (1996) work. Axsater et al. (2002) decompose the system into multiple independent distributor-retailer systems. Future reallocation at the warehouse stage is permitted, but the warehouse orders as if it must fill each retailer's order separately. They argue this creates an upper bound on stocking levels and costs. We apply the same argument, and additionally provide a lower bound reminiscent of the collapsed system of Federgruen and Zipkin (1984b).

Thus far we have discussed a number of works where the solution technique has been to relax or constrain the problem to establish tractability. Cachon (2001) also considers a periodic review system with batch ordering, but provides exact results that may be obtained through a recursive process. In contrast to the works cited above, Cachon utilizes a random allocation policy. This allows an exact expression to be developed for each retailer's lead-time distribution, which in turn allows exact results for average inventory and backorder levels given a stocking policy. Cachon uses a bounded iterative search to determine stocking levels, and finds other simple and commonly used heuristics fail to reliably perform well. We confirm these findings while introducing a simple closed form heuristic that does perform well. We show as the allocation policy becomes more sophisticated, shifting from a random to a myopic allocation policy, the results of our approach outperform these other methods. This suggests, like the use of the rebalancing assumption, the use of random allocation policies improves tractability at the cost of decreased performance. Furthermore, no performance testing exists for such an allocation policy on more complex systems.

Although many authors argue their approach may be extended beyond two-echelon distribution systems, (e.g. Graves (1996), Cachon (2001)), the literature is sparse with analytical, numeric, or simulation results for generic arborescent topologies. Notable exceptions are provided by Federgruen and Zipkin (1984b), Garg and Tang (1997), and van der Heijden et al. (1997). Of these, only van der Heijden et al. allow inventory to be held at the non-retail stages, and they minimize inventory under a fill rate constraint. Our approach may be trivially extended to any number of arborescent echelons with non-identical retailers, and inventory may be held at every installation in the network.

2.2 Delayed Differentiation

Delayed differentiation, first introduced by Alderson (1950), refers to the redesign of production processes to delay the stage where a universal set of product components is modified to their final configurations. This differentiation delay typically allows for greater service levels at decreased inventory costs, as firms exploit risk-pooling effects, and these effects grow stronger the further down the supply chain that differentiation takes place (Lee, 1996). However, the redesign of these processes or the over-design of the universal components may increase manufacturing or component costs (Fisher et al., 1999). Lee and Tang (1997) provide an explicit analysis of these tradeoffs.

Increasing product variety has lead to considerable literature proposing solutions to the corresponding complexities, such as part commonalities, process sequencing, delayed differentiation, and lead-time reduction (see Chapters 15, 16, and 18 in Tayur et al. (1999) for a representative sample). Delayed differentiation exploits the variance reduction through the risk pooling effect, reducing the required safety stock to meet a given service level, as in Lee et al. (1993), Lee (1996), and Lee and Tang (1997). Garg and Tang (1997) add a second differentiation opportunity to the firm, resulting in greater benefits of late differentiation.

Historically, the analysis of delayed differentiation of multiple product lines typically assumes installations along the supply chain may be treated independently, an assumption referred to as the decoupling assumption (e.g. Lee and Tang, 1997, Ma, Wang, and Liu, 2002). The decoupling assumption results in single stage inventory policies. These assumptions are also made regularly in the related literature on component commonalities, such as Hsu and Wang (2003). However, the decoupling assumption is clearly suboptimal. By assuming the installations in a supply chain are independent, the assumption neglects significant interaction effects (e.g. Graves, 1996, and Zipkin, 2000). Hence in this work, we avoid the use of a decoupling assumption.

Baker et al. (1986) consider service levels in a two-product, two-level system, but assume zero lead-times between installations. They show a decrease in total inventory by using a common component. Gerchak, Magazine, and Gamble (1988) and Eynan and Rosenblatt (1996) show that under general demand distributions and correlations, the optimal levels of common component inventories are non-intuitive. Furthermore, Eynan and Rosenblatt (1996) and Su, Chang, and Ferguson (2005) show that it may be optimal to not utilize commonality when the costs of a common assembly are substantially more expensive than the specific assemblies it replaces. Although these works suggest that the optimal inventory policy of stages of a product line poses an interesting, non-intuitive problem, they also limit their analysis to two-echelon systems. In this work, our approach determines inventory base-stock levels for a generic m-echelon network, with any number of differentiation opportunities and asymmetric cost profiles.

2.3 Perishable Inventory Theory

We begin our review of the existent literature in perishable inventory theory with single installation models under periodic, stochastic demand. The seminal works are Van Zyl (1964), Nahmias (1975a) and Fries (1975). Van Zyl (1964) derived the optimal stocking policy for inventory with lifetimes of two periods. Nahmias (1975a) and Fries (1975) independently consider the expected one period cost of an installation controlling a single product with a lifetime of *r*. They both assume inventory arrives fresh from the supplier. Holding and shortage costs apply as well as outdating costs for units of inventory held at the end of the $r+1^{st}$ period after they were ordered. Important findings of Nahmias (1975a) are the existence of a bounded ordering region and once the system enters the region, it never subsequently leaves. The generation of an optimal ordering policy however, is complicated by the need to retain a multi-dimensional state variable (the quantity of inventory held of ages 1, …, *r*). Nahmias (1982) notes the computation of optimal policies for large *r* is prohibitively complex due to this complication.

To avoid these complexities, several approximate methods to solve the single stage problem have been introduced. Nahmias (1975b) considers three policies which only utilize information on the total quantity of system inventory rather than the quantities of inventory at each possible remaining lifetime. Of these, a critical number policy was found to be both highly accurate and simplest to implement. Nahmias (1976) approximates the problem with a myopic critical number policy through the use of an upper bound on the expected quantity of outdating. His solution for stocking policies is a modified newsvendor policy that we utilize in this work. Cohen (1976) developed an optimal critical number policy for two period lifetimes, through the construction of the stationary distribution of inventory. Nandakumar and Morton (1993) create myopic upper and lower bounds and take a ratio of the tightest ones to select the order quantity. Tekin, Gurler, and Berk (2001) show in a continuous review system, incorporating the age of inventory into the ordering policy improves performance. Cooper (2001) provides additional bounds on the outdate quantity, and provides numerical evidence that the critical number policies are nearly as good as the optimal policies. In this work, we also utilize fixed critical number policies.

Our work differs from the above primarily by expanding the analysis to multiechelon systems. The consideration of extended supply chains raises a number of complexities. The state vector now requires a second dimension to account for the position of each unit of inventory in the supply chain. Additionally, the above literature assumes all incoming inventory is fresh. In multiechelon problems where upstream stages follow their own ordering and allocation policies, downstream installations receive products with a distribution of inventory ages.

Due in part to these difficulties, works considering multiechelon inventory theory for perishable products are fairly sparse. Ferguson and Ketzenberg (2005) and Ketzenberg and Ferguson (2005) explicitly consider the effects of uncertain remaining lifetime of inventory upon receipt at the downstream stage of a two-echelon supply chain, and the value information sharing imparts to the system. Goh, Greensberg, and Matsuo (1993) consider a two-stage system when supply as well as demand is stochastic, and inventory may fill two separate types of age segregated demand. Fujiwara et al. (1997) also analyze a two-echelon serial system where the upstream stage holds a product that is decomposed into multiple subproducts. Their model allows for emergency expedition of orders in the event of stockout and for the lifetime of the product to vary by the installation at which it is held, but is restricted to i.i.d. demands for each product and a constant ratio of subproducts produced from a unit of the master product.

Contributions considering traditional distribution systems are less frequent still. Prastacos (1981) extends the work of Yen (1965) in considering the optimal myopic allocation policies of perishable inventory in distribution networks. Prastacos shows both stockouts and outages are minimized when inventory is allocated to equalize the probability of demand exceeding inventory for each age at each location. Leiberman (1958) and Pierskalla and Roach (1972) show with constant product utility, issuing the oldest inventory first (FIFO) is optimal. Prastacos's (1981) allocation, as well as ours, utilizes constant product utility and thus FIFO policies. Unlike our work, Prastacos does not develop stocking policies, and assumes random supply. To the best of our knowledge, this work is the first to develop stocking policies for traditional multiechelon distribution systems with perishable inventory.

CHAPTER 3
NEWSVENDOR HEURISTICS FOR ARBORESCENT MULTIECHELON SUPPLY CHAINS

3.1 Introduction

In this chapter, we develop a well performing heuristic used for the stocking, ordering, and issuing of inventory in an arborescent supply chain with multiple nonidentical retailers. The core procedure is capable of the analysis of any arbitrary supply chain assuming there are no ordering costs, unmet demand is fully backlogged, holding and stockout costs are linear, and the chain retains arborescence. In this chapter, we present a method by which these supply chains may be reduced to a set of single installation newsvendor problems. In later chapters we will show the application of this technique to a variety of supply chain problems.

Consider the problem of determining optimal stocking levels in a multi-echelon distribution network consisting of m echelons and n non-identical terminal locations. Inventory stocking levels are chosen and controlled by a central decision maker and inventory is monitored on a periodic basis. Optimal solutions of this problem are problematic because of the allocation policy at the branched locations. Both Clark and Scarf (1960) and Federgruen and Zipkin (1984b) propose heuristic solutions for this problem based on a stochastic dynamic programming formulation. The disadvantage of such a formulation lies in the very large state space needed for its solution, thus several simpler heuristics have since been proposed (e.g. Jackson (1988), McGavin et al (1993), Graves (1996), and Axsater et. al (2002)). All of these heuristics face the trade-off of performance and complexity and no rigorous comparison of them exists.

For serial supply chains, Shang and Song (2003) provide a series of single period newsvendor problems, the solution to which bound the optimal stocking levels as determined by Clark and Scarf. Newsvendor bounds have a number of valuable qualities: they are considerably less computationally intensive, allow for ready parametric analysis, and facilitate the development of intuition. In this chapter, we extend the newsvendor bounds technique to distribution systems.

Traditional depictions of two-echelon, single warehouse systems focus on minimizing the total supply chain costs by determining inventory stocking levels for each installation and applying an allocation policy for the warehouse to utilize when it cannot fill all retailer demands. Because of the large dimensionality of the resulting dynamic program, a common approach is to approximate the system and conduct a recursive search over stocking levels. Our newsvendor heuristic avoids such a search, requiring only the solution of a set of simple closed form functions to set base stock levels. We bound the costs and base-stock levels of the arborescent system by a single serial system on the low side and a set of n decomposed serial chains on the high side. We solve for the base-stock levels of the resulting serial systems using the Shang and Song (2003) heuristic, and take the average of the resulting system wide base stock levels as our heuristic for the original arborescent system.

3.2 Arborescent System Model

Consider a multi-echelon supply chain with a single supplier of an abundantly available commodity. Label the terminal stages of the system as 1_α where $\alpha = 1, \ldots, n$, (we omit the index α when only one installation exists within an echelon) and the furthest upstream stage as m, to denote an m-echelon, n-retailer system. Label intermediate stages beginning with the installation just upstream of a retailer as 2_α and so forth. The simplest example of such a system has a single distribution point and n retailers, and is depicted in Figure 3.1.

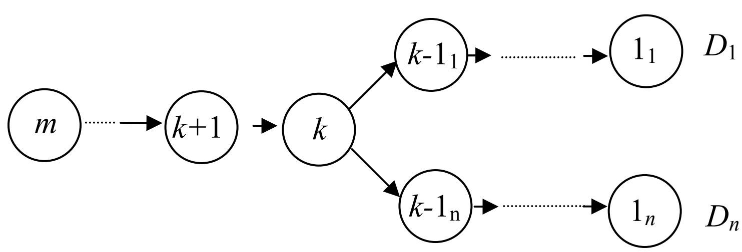


Figure 3.1: Model of Supply Chain Network

Let D_α^t denote the stochastic demand over t unit length periods at retailer 1_α, and f_α^t and F_α^t its probability and cumulative distribution functions (we omit the superscripts when $t = 1$). We assume demand to be stationary and independent across retailers and time, with known but not necessarily identical distributions across retailers. In each period, the following sequence of events occurs: previously shipped replenishments arrive at each installation, demand occurs at each retailer, excess demand is fully backordered, replenishment orders are placed, costs are assessed, and replenishment orders are shipped. Inventory is reviewed every period and a centralized decision maker places replenishment orders based on knowledge of the entire supply chain's inventory positions.

We assume per unit local inventory holding costs ($H_{i,\alpha}$) and backordering costs (b_α) are linear, and ordering costs throughout the system are zero, resulting in the optimality of base-stock policies at each installation. Before costs are assessed in each period, the following variables are measured:

$s_{i,\alpha}^{x,y}$ = the base stock level for installation i_α in topology x, as determined by method y, where $x \in (d, c, a)$ and $y \in (*, v)$

d, c = superscripts denoting decomposed and collapsed systems, respectively

a = superscript denoting the distribution system

$*$ = superscript denoting an optimal solution

v = superscript denoting the Newsvendor Heuristic

$h_{i,\alpha}$ = echelon holding cost at installation i_α, $h_{i,\alpha} = H_{i,\alpha} - H_{i+1,\alpha}$

B_α = number of backorders at installation 1_α.

$I_{i,\alpha}^{'}$ = on-hand inventory at installation i_α.

$T_{i,\alpha}$ = inventory in transit to stage i_α.

$I_{i,\alpha}$ = echelon inventory at installation i_α, $I_{i,\alpha} = I_{i,\alpha}^{'} + \sum_{j=1}^{i-1}\left(T_{j,\alpha} + I_{j,\alpha}^{'}\right)$ for $i = 1, \ldots, k\text{-}1$,

and $I_i = I_i^{'} + \sum_{j=k}^{i-1}\left(T_j + I_j^{'}\right) + \sum_{\alpha=1}^{n}\sum_{j=1}^{k-1}\left(T_{j,\alpha} + I_{j,\alpha}^{'}\right)$ for $i = k, \ldots, m$.

$IP_{i,\alpha}$ = echelon inventory-transit position at installation i_α, $IP_{i,\alpha} = I_{i,\alpha} - B_\alpha + T_{i,\alpha}$

$IO_{i,\alpha}$ = inventory orders outstanding for installation i_α, $IO_{i,\alpha} = s_{i,\alpha} - I_{i,\alpha}$

The total system cost in a period is the sum $\sum_{j=k}^{m} h_j I_j + \sum_{\alpha=1}^{m}\sum_{j=1}^{k-1} h_{j,\alpha} I_{j,\alpha} + \sum_{\alpha=1}^{m} b_{i,\alpha} B_{i,\alpha}$.

Replenishments for an installation arrive $L_{i,\alpha}$ periods after being shipped. In any period in which the warehouse has sufficient inventory on hand to fill all downstream demands, our allocation policy is to ship all on hand inventory while minimizing IO_i. Thus, the allocation policy allocates scarce inventory to installations on the basis of their relative need.

3.3 Branched Multi-Echelon Newsvendor Heuristic

In this section, we present a heuristic for determining echelon base-stock levels for a multiechelon distribution network. We construct two serial supply chain systems whose costs bound the optimal costs and echelon base-stock levels of distribution system from above and below. Our illustrative network, depicted in the center of Figure 3.1, faces demand processes $D_1, D_2, \ldots, D_n$ at the terminal ends of the chain segments.

To determine the upper bound, we restrict all installations at and upstream of the distribution point to designate and maintain retailer specific inventories. That is, the centralized decision maker specifies which retailer each unit of inventory will eventually be shipped to as that unit of inventory is ordered from the exogenous supplier. In spirit, this is similar to the virtual assignment approach of Graves (1996), who notes that because it may be desirable to un-commit stock, this assignment rule will not perform as well as a dynamic allocation policy. The restriction decomposes the distribution network into a set of *n* independent serial systems, one system for each retailer, as depicted on the right of Figure 3.2. We refer to these serial chains as 'decomposed'.

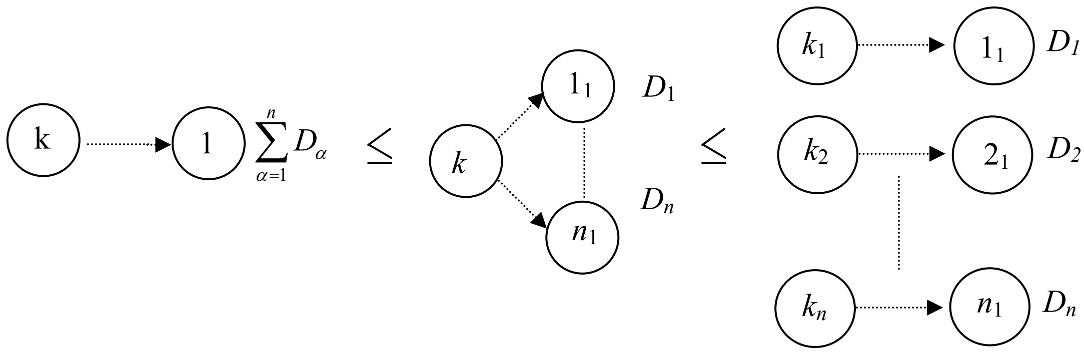


Figure 3.2: Constructed Serial Chains

To describe our Newsvendor Heuristic, we need the following notation.

μ_α = mean demand rate at installation i_α, $\mu_\alpha = \mathrm{E}[D_\alpha]$

$(x)^-$ = max{0, -x}

$\boldsymbol{s}^x$ = a vector of optimal echelon base-stock levels for all installations in topology x, where $x \in (d, c, a)$

$C^x\left(\boldsymbol{s}^x\right)$ = the expected per period cost of the topology x under echelon base-stock vector $\boldsymbol{s}^x$, where $x \in (d, c, a)$

The optimal base-stock policy of each decomposed serial chain may be determined as follows. Let $C_{0,\alpha} = b_\alpha + H_{1,\alpha}(x)^-$ and $s_{0,\alpha}{}^* = \infty$. For i = 1, 2, ..., m, solve the recursive optimization equations

$$C^s_{i,\alpha}(y) = \mathrm{E}\left[h_{i,\alpha}\left(y - D^i{}_\alpha\right) + C^s_{i-1,\alpha}\left(\min\left\{s^{d,*}_{i-1,\alpha}, y - D^i{}_\alpha\right\}\right)\right] \tag{3.1}$$

where $s^{d,*}_{i,\alpha} = \arg\min\left\{C^s{}_{i\alpha}\left(y\right)\right\}$. (3.2)

The optimal expected cost for each of the decomposed systems is $C^d_{m,\alpha}\left(s^{d,*}_{m,\alpha}\right)$, and the expected overall cost of the total system of decomposed chains is simply the sum

$$\sum_{\alpha=1}^{n} C^d_{m,\alpha}\left(s^{d,*}_{m,\alpha}\right) = C^d\left(\boldsymbol{s}^d\right). \tag{3.3}$$

Because this sum is obtained by applying a constraint to the warehouse, it is an upper bound for the optimal cost of the distribution network. Additionally, removing the decomposition constraint allows for risk pooling, suggesting that if backordering costs are sufficiently high to induce installations to carry positive safety stock, the sum

$$\sum_{\alpha=1}^{n} s_{i,\alpha}^{d,*} = s_i^{d,*} \tag{3.4}$$

is an upper bound for the optimal echelon base stock level at the warehouse. A similar argument is made by Gallego et al. (2003).

Having constructed an upper bound for the cost of the distribution network, we next construct a single serial system that serves as a lower bound. Here, our approach is similar to Federgruen and Zipkin (1984a), who assume that instantaneous and costless transshipments within an echelon are allowable. The result of this assumption is an artificial distinction between installations in an echelon. The stages downstream of a distribution point may thus be collectively treated as a single virtual installation, as shown on the left of Figure 3.2. We refer to this system as 'collapsed'.

As with the decomposed system, this serial system is solved by the above optimization equations. Let $\boldsymbol{s}^c$ and $C^c(\boldsymbol{s}^c)$ represent the optimal echelon base-stock policy and expected system wide cost of the collapsed system, respectively. By introducing inventory commitment constraints on the downstream stages of the collapsed system, we achieve the original distribution network. For identical retailers, $C^c(\mathbf{s}^c)$ is a lower bound for $C^a(\mathbf{s}^a)$ because the distribution network is the result of adding constraints on the collapsed network.

Additionally, the retailer echelon base-stock level under the collapsed system suggests lower bounds for the echelon base-stock levels for the distribution network. To see this, consider by combining the retail stages from the distribution network, we gain the opportunity to exploit risk pooling. Assuming the chain carries nonnegative safety stocks, the pooling potentially reduces inventory in this installation and the optimal echelon base-stock level of the warehouse. The decomposition and collapsed system results combine to give

$$C^c\left(s^c\right) \le C^a\left(s^a\right) \le \sum_{j=1}^{n} C_{m,\alpha}^{d}\left(s_{m,\alpha}^{d,*}\right). \tag{3.5}$$

and suggests that

$$\mathbf{s}_i^c \le \mathbf{s}_i^a \le \mathbf{s}_i^d . \tag{3.6}$$

We use these serial systems to approximate the optimal echelon base-stock levels for the distribution network. For identical installations in an echelon, our approach is to utilize the Shang and Song (2003) heuristic for each of the n+1 constructed chains. Using an illustrative two-echelon, two-retailer system, for the collapsed serial chain system, the stocking level at the warehouse is

$$s_2^{c,v} = \frac{F_2^{-1}\left(\dfrac{b}{b + h_2 + h_1}\right) + F_2^{-1}\left(\dfrac{b}{b + h_2}\right)}{2}. \tag{3.7}$$

For the decomposed serial chain system, the stocking levels at echelon i are, for our illustrative system,

$$s_{2,1}^{d,v} = \frac{F_{2,1}^{-1}\left(\dfrac{b_1}{b_1 + h_{1,1} + h_2}\right) + F_{2,1}^{-1}\left(\dfrac{b_1}{b_1 + h_2}\right)}{2} . \tag{3.8}$$

and

$$s_{2,2}^{d,v} = \frac{F_{2,2}^{-1}\left(\dfrac{b_2}{b_2 + h_{1,2} + h_2}\right) + F_{2,2}^{-1}\left(\dfrac{b_2}{b_2 + h_2}\right)}{2}. \tag{3.9}$$

The sum of these stock levels, $s_2^{d,v} = s_{2,1}^{d,v} + s_{2,2}^{d,v}$ represents an approximation for an upper bound of the echelon base-stock policy of the distribution system.

When backorder costs or holding costs differ between retailers, we must adjust the collapsed system equation (3.7). To do so, we use the mean demand weighted average backorder and holding costs for the distribution stage. Thus, for a two-retailer system, the terms in equation (3.7) are

$$b = \frac{\mu_1 b_1 + \mu_2 b_2}{\mu_1 + \mu_2} \text{ and } h_1 = \frac{\mu_1 h_{1,1} + \mu_2 h_{1,2}}{\mu_1 + \mu_2} \tag{3.10,3.11}$$

In this case, our argument that the total inventory costs of the distribution system are bounded from below by that of the collapsed system may not hold. However, in our numerical experiments, we find no instances where the collapsed system costs exceed that of the distribution system. Thus we present the results for asymmetric retailers under the numerical conjecture that the bound holds.

The Newsvendor Heuristic for the stocking level at the warehouse is a simple average of the stocking levels from the constructed systems:

$$s_2^{a,v} = \frac{s_2^{c,v} + s_2^{d,v}}{2}. \tag{3.12}$$

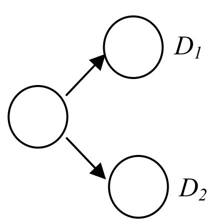
CHAPTER 4

EVALUATION OF TWO-ECHELON DISTRIBUTION SYSTEMS

In this chapter, we use the Newsvendor Heuristic developed in Chapter 3 to analyze two-echelon distribution networks. We begin by showing the accuracy of the heuristic approach, and then develop managerial insight into the behavior of distribution systems. Due to the unavailability of practical analytical solution methods, we test our heuristic through an extensive and rigorous simulation experiment and compare its performance against other common heuristics.

4.1 Simulation Methodology

A majority of previous papers on distribution system stocking policies use simulation to test the accuracy of dynamic programming relaxations because closed form cost equations do not exist for most common allocation policies. Thus, we also use simulation to test the performance of our approach against prior work and commonly used practitioner heuristics. We consider examples for both symmetric and asymmetric cost structures for two-echelon network topologies with either two or four terminal retail stages (we discuss in Chapter 5 a related study where we present the performance of the heuristic for three-echelon networks).



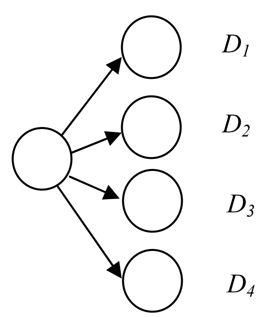


Figure 4.1: Network Topologies

Our simulation methodology is an unequal variance, two-stage screening-subset selection procedure presented in Nelson et al. (2001). We first create a set of base-stock level candidates. For distributions with finite support, these candidates are obtained by enumerating over the entire range of potential lead-time demands at each installation. For distributions without finite support, candidates cover a range of the expected minimizing base-stock level, +/- at least 5 inventory units for each installation. For the parameter settings in these examples, this range covers at least 50% of the cumulative distribution of the lead-time demand at each installation, centered on the cost minimizing stocking level as suggested by the Newsvendor Heuristic.

For each stocking level, we initially conduct a steady state simulation of our model and allocation policy for 50,000 periods. We batch periods into groups of 10 to reduce deviations from normality and correlations between single period costs. Based on the lead-times used in our study, we omit the first 10 periods to eliminate initialization effects. The remaining data points are used in the initial screening phase.

Stocking level vectors that survive the initial screening are candidates for our final solution, and are subjected to a second round of simulation experiments where we retain our batch mean sizes and generate a sufficient number of data points to eliminate all but one of the systems. After this experiment, the set of stocking levels that has the lowest per period cost is selected. This procedure ensures a confidence level of at least 1-γ that the selected system performs within a quantity δ of the best found system cost. Hence we refer to the selected system as a δ–best system. For our purposes, we consider $\gamma = 5\%$ and $\delta = 0.2\%$ of the average per period system cost of the best system found in the first stage.

The simulation model was verified by using the same approach to simulate a serial chain, whereupon the results are identical to those found by Shang and Song (2003). In the next section, we compare the performance of our NH to other simple and widely used heuristics.

4.2 Problem Design and Results

4.2.1 Symmetric Two-Echelon Networks

Our first experimental design considers two network topologies, with either two or four symmetric retailers. We test the heuristics using a full factorial design over a range of holding cost, backorder cost, and lead-time parameters. We consider (h_2, $h_1 = h_{1,1} = h_{1,2}$) = {(1,1), (1,2), (2,1)}, (L_2, $L_1 = L_{1,1} = L_{1,2}$) = {(1,1), (1,2), (2,1)}, and $b_1 = b_2 =$

$\{5,10,20\}$. We hold the total periodic system demand constant at 20 units per period, distributed according to a Poisson distribution. This demand is split among the retailers, resulting in $\mu_\alpha = 10$ for the 2-retailer network and $\mu_\alpha = 5$ for the 4-retailer network. These parameter values are similar to those used by Jackson (1988), Cachon (2001), Axsater et al. (2002) and Shang and Song (2003), and are summarized in Tables A4.1 and A4.2 in Appendix 1 for the two-retailer and four-retailer networks, respectively.

4.2.1.1 Random Allocation Policies

We compare the results of the Newsvendor Heuristic (NH) to those of Cachon (2001), whose results are optimal when a random allocation policy is used. These results are presented in Table A4.3 in Appendix 1 and are summarized in Table 4.1. Based on this test, we make the following three observations.

Table 4.1: Random Allocation Summary

% Error Under Random Allocation		
	Two-Retailer	Four-Retailer
Exact	0.00%	0.00%
Bounds	2.68%	3.38%

Observation 4.1: *A small but significant error exists from using the NH under a random allocation setting. The error grows as the number of retailers increase but the heuristic reacts to parametric changes in a similar manner as the exact procedure.*

Observation 4.2: *The exact strategy holds more inventory at the distribution point than does the NH.*

We argue below that this is a result of poor management of inventory at the distribution point.

Observation 4.3: *As backorder costs increase, the total system stock held by the NH falls relative to the exact analysis.*

For backorder rates of 5, the exact analysis tends to hold less inventory than the NH. For backorder rates of 10, there is no clear trend, but for backorder rates of 20, the NH carries less total inventory than the exact analysis. We discuss this further in the next section.

4.2.1.2 Myopic Allocation Policies

In this section we compare the systems generated by the NH to the δ-best system found via the simulation procedure described in §4.1. We also investigate the performance of three other alternative heuristics. First, we use the results of Cachon's (2001) exact analysis under random allocation as a heuristic under our proposed allocation policy. Since Graves (1996) finds that holding no safety stock at the upstream stage is frequently a good (and simple) heuristic, we also consider this approach (termed the zero safety stock policy in the results below). Finally, we investigate the performance of setting a fixed service rate at the warehouse stage, as is frequently encountered in practice. We choose a 99% fill rate because in practice, managers frequently require high fill rates from the warehouse (Lee and Tang, 1997). The results of these experiments are presented in Tables A4.4 and A4.5 in Appendix 1 for the two-retailer and four-retailer networks, respectively. From these results, we make the following observations.

Observation 4.4: *The NH performs best of all the tested heuristics. It is followed by the Cachon exact analysis and zero safety stock heuristics, while the fixed high fill-rate heuristic performs poorly in all problems*.

A summary of these results is presented in the symmetric columns in Table 4.2.

Table 4.2: Myopic Allocation Summary

% Error Under Myopic Allocation				
	Two-Retailer		Four-Retailer	
Heuristic	Symmetric	Asymmetric	Symmetric	Asymmetric
Newsvendor	0.40%	0.85%	0.48%	0.89%
Cachon	1.75%	NA	2.24%	NA
99% Fill Rate	22.06%	24.59%	21.21%	24.66%
Zero Safety Stock	2.21%	1.96%	2.95%	3.96%

Observation 4.5: *The additional upstream inventory held by Cachon's exact analysis causes it to under perform the NH when non-random allocation is allowed.*

By allocating inventory randomly, the exact analysis increases the variance of the demand placed upon the warehouse by the terminal stages, increasing the required inventory at the warehouse. In contrast, allocating inventory myopically is more efficient. A myopic allocation reduces the penalty induced by preventing the retailer from redistributing inventory (in a random allocation), allowing inventory to be placed further downstream, since the lower inventory at the distribution point results in less frequent stock outs. This effect strengthens as the backorder costs increase.

Observation 4.6: *All else held constant, increasing the number of retailers increases the total system cost. Additionally, increasing the holding costs, lead-times, or backorder costs also increases the total system cost.*

These effects are congruent with prior work and intuition. Increasing the number of retailers reduces risk-pooling savings, while increasing other parameters increases costs directly. We address these effects further in section 4.4.

4.2.2 Asymmetric Two-Echelon Networks

We now consider networks where the terminal stages are asymmetric or non-identical. We consider a full factorial design over $h_{1,\alpha} = \{1,2\}$ and $b_\alpha = \{5,10,20\}$ for both two and four-retailer chains, while holding $L_2 = L_1 = L_{1,1} = L_{1,2} = 1$ and the system demand as described in section 4.2.1. The parameters for each problem investigated are presented in Tables A1 and A2. We compare the performance of the NH, Zero Safety Stock, and 99% Fill Rate heuristics to that of the δ-best system. The results are presented in Tables A4.6 and A4.7 in Appendix 1 and are summarized in the asymmetric columns in Table 4.2.

Observation 4.7: *Observations 4.4 and 4.6 hold in the asymmetric case. Additionally, asymmetric networks introduce slightly more error in the NH performance.*

This increase is present in the other tested heuristics as well, and may be due in part to a larger number of candidate policies. The NH returns an average error of 0.87%, while the holding costs between retail locations vary by 100% and the backorder costs between locations vary by 400%. We believe this range covers most realistic distribution systems.

4.3 Heuristic Robustness Tests

Having established that the NH performs well over a broad range of cost parameters, we next examine its robustness. Our primary goal in this section is to determine where the performance of the NH degrades, thus the range for the tested parameter values may exceed those ever found in practice. We begin by examining the performance of the NH across other demand distributions than Poisson. We investigate three other demand distributions: discrete uniform (5,15), negative binomial (with $\mu = 10$ and $\sigma^2 = 16.54$), and a constructed bimodal distribution whose pmf is depicted in Figure 4.2. The variance of the negative binomial distribution was selected to match that of the constructed distribution, while the mean demand of each distribution matches those of the Poisson distribution from the previous tests.

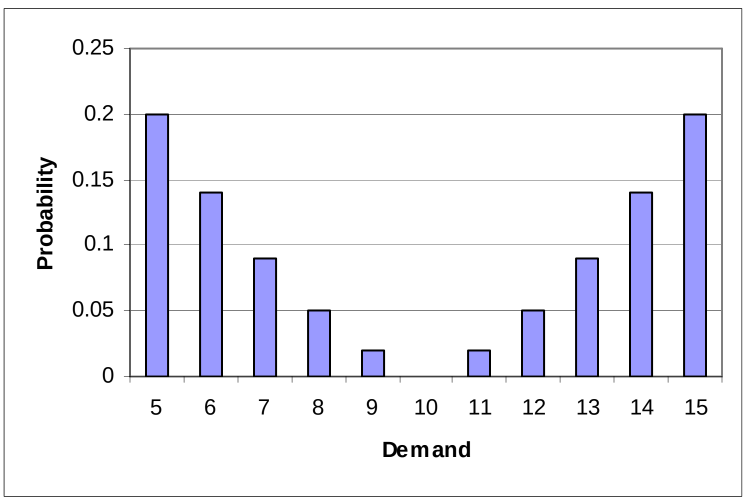


Figure 4.2: pmf of the Constructed Bimodal Distribution

We test the NH on these distributions over a broader range of parameters. We consider a full factorial design of $n = \{2,10\}$, $b_1 = b_2 = \{1,10,50\}$, $(h_2, h_{1,1} = h_{1,2}) = \{(1,1), (10,1), (1,10)\}$, and $(L_2, L_{1,1} = L_{1,2}) = \{(1,1), (1,3), (3,1)\}$, and continue to use the simulation methodology presented in §4.1. A summary of the results of these tests is presented in Table 4.3 (full results are available in Tables A4.8-A4.10), where we report the number of test cases where the cost exceeded the δ-best policy by the range given in the left most column.

Table 4.3: Robustness Tests Across Demand Distributions and Number of Retailers

	Number of Retailers					
	2			10		
Range of Error	Discrete Uniform	Negative Binomial	Constructed Bimodal	Discrete Uniform	Negative Binomial	Constructed Bimodal
0%	13	12	11	1	2	0
<1%	7	10	12	11	11	11
1% to 5%	5	5	4	4	4	6
>5%	2	0	0	11	10	10

Observation 4.8: *The accuracy of the NH does not significantly depend on the type of demand distribution.*

The error the NH incurs is approximately the same across each demand distribution investigated. Additionally, the errors generated by the two retailer tests are approximately equal to those under the Poisson tests in §4.2.1.2.

Observation 4.9: *Increasing the number of retailers significantly decreases the performance of the NH.*

As the number of retailers increase from two to ten, the performance of the NH drops precipitously. This trend is common across demand distributions but is not common across cost parameters. A further investigation into the trend cited in Observation 4.9 reveals that the NH's performance is heavily dependent on the relative holding cost patterns, as depicted in Table 4.4. This leads to our next observation.

Table 4.4: Performance of the NH with 10 Retailers

	Holding Cost (h_2, h_1)		
Range of Error	(1,1)	(10,1)	(1,10)
0%	2	1	0
<1%	12	21	0
1% to 5%	7	5	2
>5%	6	0	25

Observation 4.10: *The NH fails when there are many retailers and the holding cost increases dramatically between the warehouse and the retail stages.*

Interestingly, the warehouse echelon base-stock level under these conditions is approximately equal to that of the δ-best system. The majority of the error arises instead from the allocation of inventory within the system. In these cases, the NH carries too little inventory at the retail stages, overcompensating for the exceptionally high holding costs. Thus the NH is useful in setting the total inventory stock, but should not be used to determine the base-stock levels at the retailers.

We believe these scenarios, where holding costs at the retailers exceed that at the warehouse by orders of magnitude are rare in practice. For instance, in the electronics industry in the United States, warehouse and retail space rents are approximately \$4/ft^2 and \$7.1/ft^2, respectively (http://www.bizstats.com). On average, the firms generate \$355/ft^2 in sales on 13.8 inventory turns a year, generating \$58/ft^2 in gross profit. If the cost of capital is 10%, the capital cost component of local holding costs at the retail and warehouse locations are \$9.25/ft^2 and \$6.15/ft^2 for the average US electronics seller respectively. Taking backordering costs as solely the forfeited margin of lost sales, and scaling such that $h_1 = 1$, these correspond to cost parameter settings of $h_1 = 1$, $h_2 = 0.5$, and $b = 9.52$. These values easily fall within the parameter value range where the NH performs well.

We next consider the effects of asymmetry in the retail parameter values. There were no significant differences in the accuracy of our results between the four different demand distributions so we utilize the discrete uniform distribution in the following tests. Because the discrete uniform distribution has finite support, we are able to fully enumerate over all possible lead-time demands. We consider two sets of problems, each with two retailers, where the demand or lead-time to one retailer varies, respectively. The first set of examples is a full factorial design over the parameter values $(b_1, b_2) = \{(1,1), (1,50), (50,1)\}$, $h_2 = 1$, $L_2 = L_{1,1} = L_{1,2} = 1$, $(h_{1,1}, h_{1,2}) = \{(1,1), (1,10), (10,1)\}$, $D_1 \sim U(5,15)$, and $D_2 \sim \{U(15,25), U(35,45), U(55,65), U(75,85), U(95,105)\}$. This test captures asymmetries in retail demand; the results are summarized in Table 4.5 (full data are presented in Table A4.11).

Table 4.5: NH Performance over Demand and Cost Asymmetry

	Demand Ratio (μ_2/μ_1)				
Range of Error	2	4	6	8	10
0%	1	0	1	0	1
<1%	2	3	5	5	4
1% to 5%	6	6	2	3	3
>5%	0	0	1	1	1

The second set of test problems is a full factorial design over the parameter values $(b_1, b_2) = \{(1,1), (1,50), (50,1)\}$, $h_2 = 1$, $L_2 = L_{1,1} = 1$, $L_{1,2} = \{2,3,4,5\}$, $(h_{1,1}, h_{1,2}) = \{(1,1), (1,10), (10,1)\}$, $D_1 \sim U(5,15)$, and $D_2 \sim U(5,15)$. This test captures asymmetries in the lead-times between the warehouse and the two retailers. The results of this test are summarized in Table 4.6 (full data are presented in Table A4.12). Our final observation summarizes our asymmetric results.

Table 4.6: NH Performance over Lead-time and Cost Asymmetry

	Lead-time Ratio ($L_{2,1}/L_{1,1}$)			
Range of Error	2	3	4	5
0%	0	0	0	3
<1%	1	3	4	1
1% to 5%	8	5	4	4
>5%	0	0	1	1

Observation 4.11: *The NH is robust to asymmetry in both the lead-times and demand rates.*

Tables 4.5 and 4.6 show the NH is robust across widely varying cost, lead-time, and demand rates. Although the NH typically fails to identify the δ-best policy, it performs within 5% of the δ-best policy costs in 76 of the 81 test cases. That this performance occurs under a wide disparity in retailer parameter values gives further support to the robustness of the NH.

4.4 Cost Functions and Analysis of Parameter Value Effects

As noted by Shang and Song (2003), the simple newsvendor bounds presented above enable the analysis of the effects of the system parameters much more readily than previous solution methods. Although these bounds and cost functions are general, assuming normally distributed demand allows us to obtain some analytical results. Hence, for Propositions 4.1 and 4.2 below, we assume demand at each retailer is normally distributed.

Recall that our method of bounding the distribution system is through the construction of a set of serial systems. The analysis of the resulting cost functions has a number of parallels to the serial supply chain system studied by Shang and Song (2003). Under symmetric profiles, increasing either the backordering cost or the lead-time increases both total system costs and echelon stocking levels. Increasing the warehouse echelon holding cost rate increases system costs and stocking levels at the retailers, while increasing the echelon holding cost rate at the retailers while decreasing the echelon base stock levels at the warehouse. Thus the parametric results for symmetric distribution systems are identical to those of a serial chain. The analysis becomes slightly more complex when considering asymmetric problems. Here, a change in a given retailer's parameter value does not affect the base-stock levels of the other retailers. Proposition 4.1 describes the impact on the retailer whose parameter value is modified along with the impact on the warehouse.

Proposition 4.1. *For* $\alpha = 1, 2, \ldots n$, *and* $\alpha \neq j$

(a) *as* b_j *increases,* $C^a(\mathbf{s}^a)$ *increases,* s_2 *and* $s_{1,j}$ *increase, while* s_α *remains unchanged.*

(b) *as* h_j *increases,* $C^a(\mathbf{s}^a)$ *is non-decreasing,* s_2 *and* $s_{1,j}$ *decrease, and* s_α *remains unchanged.*

(c) *as* L_j *increases,* $C^a(\mathbf{s}^a)$ *increases,* s_2 *and* $s_{1,j}$ *increase, while* s_α *remains unchanged.*

Thus, increasing the backorder cost or lead-time at one retailer increases the total system costs and increases the echelon stocking levels of both the warehouse and that retailer. However, the stocking levels of the other retailers are independent of the effects of the change in the parameter value. An increase in the echelon holding cost at the retailer decreases the echelon stocking levels, while the total system costs are non-decreasing (and likely increasing).

Standard risk pooling arguments yield the intuition that, keeping the system demand constant, increasing the number of retailers increases both system stocking levels and total system costs. We formalize this intuition in Proposition 4.2.

Proposition 4.2. *For* $\alpha = 1, 2, \ldots n$, *and* $\beta = 1, 2, \ldots n, n+1$, *and assuming safety stocks are positive,*

$C^a(\mathbf{s}^a)$ *and* s_2 *are non-decreasing, and* $s_\beta < s_\alpha$.

Proposition 4.2 states that while increasing the number of retailers in a distribution network while keeping the total system demand constant reduces the inventory held at each retailer, it also likely increases the total amount of system stock and total system costs. These effects arise due to the limited ability of the centralized decision maker to exploit risk-pooling opportunities.

Finally, we examine the effects of increasing asymmetry in the retailer parameter values. We begin by addressing asymmetry in backorder costs. Consider an initially symmetric system, and increase b_1 while decreasing b_2 by Δ, such that $b_1 = b\ (1+\Delta)$ and $b_2 = b\ (1-\Delta)$. Because there is no closed form for the inverse of the normal cdf, we condition Propositions 4.3 and 4.4 assuming uniform lead-time demand distributions. Our numerical tests verify the results hold for normal distributions as well, although we note that certain pathological distributions exist for which the results will not hold.

Proposition 4.3. *For* $\alpha = 3, ..., n$, $b_1 = b\ (1+\Delta)$ *and* $b_2 = b\ (1-\Delta)$, s_2 *and* $C^a(s^{a^*})$ *are non-increasing with* Δ.

Proposition 4.3 states that increasing asymmetry in backordering costs does not increase and likely decreases stocking levels and system costs, as the decomposed echelon stock levels and system costs are non-increasing while the collapsed values remain unchanged. This seemingly counter intuitive result arises due to the tendency of the system to behave as a serial chain as asymmetry increases. Taken to an extreme, the retailer with the high backordering cost captures the majority of the inventory related costs. Thus, this high-backorder cost retailer dominates the system, which begins to resemble a serial chain consisting solely of the high-backorder cost retailer. Recall we expect the collapsed serial chain to serve as an approximate lower bound for the distribution system. In effect, we find symmetric sub-chains may be thought of as 'worst case' scenarios for total system costs. Proposition 4.3 is illustrated by a small set of numerical test problems, as presented in Figure 4.3. We compare the difference in $C^a(s^a)$ for two-retailer networks with $b = 10$, $\Delta = \{0,0.5\}$, normally distributed demands with $\mu = 20$ and $\sigma^2 = 20$, and three holding cost cases where $(h_2, h_1) = \{(1,1), (1,2), (2,1)\}$.

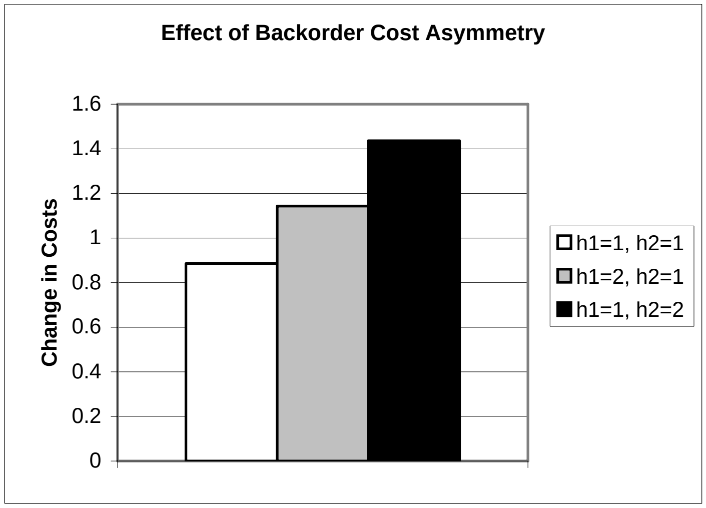


Figure 4.3: Effect of Backorder Cost Asymmetry

In a similar manner, we examine the effect of asymmetry in the holding costs in Proposition 4.4. To do so, we modify the retailer's echelon holding cost such that $h_{1,1} = h_1(1+\Delta)$ and $h_{1,2} = h_1(1-\Delta)$.

Proposition 4.4. *For $\alpha = 1, \ldots, n$, $h_{1,1} = h_1(1+\Delta)$ and $h_{1,2} = h_1(1-\Delta)$, $s_{1,2}$ increases while $s_{1,1}$ decreases with increasing Δ. Also, s_2 is non-increasing and $C^a(s^a)$ is non-decreasing in Δ.*

Proposition 4.4 shows that the system echelon stocking level is non-increasing but total system inventory costs are non-decreasing in asymmetry of the retailer holding costs. This is illustrated by a set of numerical test problems, as presented in Figure 4.4. We compare the difference in $C^a(s^a)$ for two-retailer networks with $h_1 = 1$, $\Delta = \{0, 0.5\}$ demand distributed normally with $\mu = 20$ and $\sigma^2 = 20$ and three backorder cases where $b = \{5,10,15\}$.

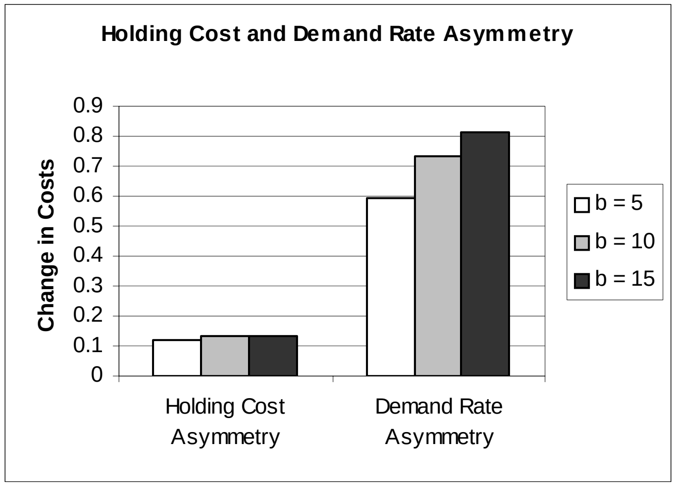


Figure 4.4: Effects of Holding Cost and Demand Rate Asymmetry

Finally, in Proposition 4.5 we present the effect of demand asymmetry on the stocking levels and supply chain costs. The critical fractile computations of our newsvendor approach are independent of the demand distribution, hence we return to considering normal distributions for this result.

Proposition 4.5. *For* $\alpha = 3, \ldots, n$, $\mu_1 = (1+\Delta)\mu_\alpha$, *and* $\mu_2 = (1-\Delta)\mu_\alpha$, *both* s_2 *and* $C^a(s^a)$ *are non-increasing with* Δ.

Proposition 4.5 states that increasing asymmetry in demand rates does not increase and often decreases both echelon stocking levels and system costs. By a similar argument as for Proposition 4.3, the results of Proposition 4.5 arise due to the tendency of the resulting network to more closely resemble a serial chain. Although the increase in asymmetry decreases the risk pooling savings at the warehouse, it also introduces a virtual pooling effect in the retail stages of the network. A numerical depiction of Proposition 4.5 is illustrated in Figure 4.4 where we compare the difference in $C^a(s^a)$ for two-retailer networks with $\mu_i = 10$ and $\Delta = \{0, 0.5\}$.

4.5 Concluding Remarks

In this chapter, we presented a simple heuristic for two-echelon distribution system with n non-identical retailers. The Newsvendor Heuristic requires only the computation of $4(n+1)$ newsvendor problems, but performs well over a wide range of parameters, resulting in an average cost that is 0.44% and 0.87% greater than the cost of the best found stocking policies for symmetric and asymmetric cost parameters, respectively, outperforming all other commonly used heuristics. The heuristic is robust over multiple demand distributions and widely varying cost parameters. For asymmetric (non identical) systems, the heuristic is shown to perform well with backordering costs ranging from 50% to 2,500% of the local holding costs, and mean demand and lead-times varying by up to 500%. Although the heuristic does break down when holding costs increase by 1,000% between the warehouse and retailers, even here the NH provides useful insights, accurately predicting the amount of total system stock. The simplicity of our heuristic also facilitates insights on parametric analysis that are difficult or impossible to obtain based on the competing heuristics. For example, we show that the supply chain's inventory and costs increase in the number of retailers, but decrease as backordering costs and demands at the retailers become asymmetric. These results simplify the teaching of supply chain distribution system concepts in the classroom, may be used as the foundation of a decision support tool, and provide practical insights for managers.

CHAPTER 5
EVALUATION OF DELAYED DIFFERENTIATION PRODUCTION

5.1 Introduction

In many industries and product lines, the growing diversity of customer demands is causing firms to dramatically increase product variety. For instance, in 2003 alone, 26,893 new food and household products were introduced, including 115 deodorants, 187 breakfast cereals, and 303 women's fragrances (Bianco, 2004). Such an increase poses significant challenges to firms as product proliferation is typically accompanied by increasingly inaccurate forecasts, higher inventories, and more frequent stockouts (Lee, 1996). The delay of differentiation between multiple product lines, through use of common components or modularity, has been examined as one solution to this problem. Delayed differentiation, first introduced by Alderson (1950), refers to the redesign of production processes to delay the stage where a universal set of product components is modified to their final distinct configurations. A delay in differentiation typically allows for greater service levels at decreased inventory costs, as firms exploit better information and risk-pooling effects. Thus, benefits of delayed differentiation tend to increase the further down the supply chain that differentiation takes place (Lee, 1996).

Unfortunately, implementing delayed differentiation is not free. There are often significant costs involved in redesigning (and in many cases, over-designing) the product (Fisher et. al., 1999). In addition, configuring the product further down the supply chain (for instance, at the warehouse or retailer stage) is rarely as cost efficient as at the primary manufacturing facility. Both of these costs (design and assembly) tend to increase the further down the supply chain that differentiation takes place. Since both the benefits and the costs of delayed differentiation increase the further down the supply chain, the question of where in the supply chain to optimally differentiate arises. In this chapter, we explicitly model this tradeoff to determine the optimal point of differentiation. While this problem has been previously modeled in the literature (Lee and Tang, 1997), we relax one of their major assumptions and find some instances where our solutions are significantly different.

As in Lee and Tang (1997), we assume the production process may be modeled by a series of discrete processing stages or installations. We refer to the last stage that the generic product exists as the Last Common Operation (LCO), and seek to determine which stage should be selected as the LCO to minimize total cost. For example, a comparison of the total cost of the two networks shown in Figure 5.1 will assist a firm in deciding if it is better to differentiate its product either one or two stages from the last stage.

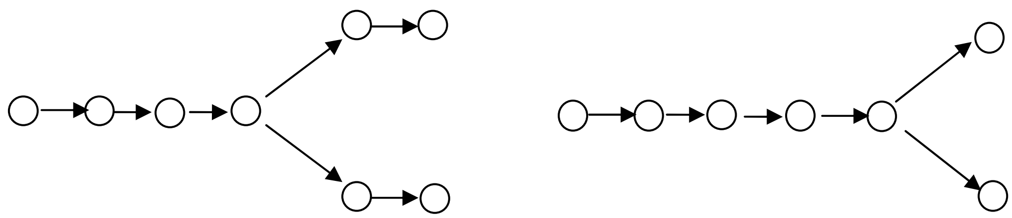


Figure 5.1: Delayed Differentiation Product Networks

When Lee and Tang (1997) choose the amount of product to stock at each stage, they assume the stages may be treated independently; i.e. there is no connection between the service level of one stage with the stocking level of the stage preceding it. This assumption is referred to as the decoupling heuristic (DH) because it allows each stage to be analyzed in isolation. While the DH is appropriate when every stage is required to maintain a high service level, the literature on multi-echelon inventory systems shows it is rarely optimal to maintain high service levels in the stages of the system that do not directly serve the final customer (see for example Chapter 8 in Zipkin, 2000). We relax the DH and provide guidance on the conditions where its use can lead to far from optimal decisions.

Because the analysis of arboreal multi-echelon supply chains without the DH becomes prohibitively complex, we approximate the echelon stocking levels for each installation with our Newsvendor Heuristic (NH). The heuristic is tested using these stocking levels as parameters in a series of supply chain simulation experiments and we compare the resulting supply chain costs to the costs obtained using the best stocking levels found from a full enumeration search. After verifying the heuristic performs well under common conditions, we compare the results against the results generated using the DH.

We show that the NH performs much better than the DH and the optimal point of differentiation shifts towards the end of the supply chain as backorder costs increase and towards the beginning of the supply chain as echelon holding costs increase. These results are in agreement with the results of Lee and Tang (1997), however, we also find the DH may *over or underestimate* the value of delaying differentiation. Specifically, the DH often overestimates the value of delaying differentiation, except for cases when the echelon holding costs are high in the initial stages of production and the backordering costs are also high. In these cases, the DH results in the firm failing to carry sufficient inventory in the intermediate stages, and correspondingly underestimates the risk pooling benefits of delaying differentiation. Both the over and underestimation are often significant and may lead a firm to select suboptimal supply chain structures.

Finally, by not assuming the stages are decoupled, we discover a non-intuitive and previously hidden insight that the *shape* of the holding cost profile (how much the holding cost increases from one stage to the next) significantly affects the choice of where in the process product differentiation occurs. We find the presence of sharp rises in local holding cost between stages is associated with increased cost savings due to the reduction in effective backordering costs at the downstream stages. Thus, capturing holding cost 'spikes' under a common component form is more valuable than previously believed, and may serve as justification for a more extensive use of delayed differentiation strategies.

The remainder of this chapter is organized as follows. Our model is defined in §5.2 and a simulation study is presented and discussed in §5.3. We state and explain our experimental results in §5.4 and discuss the value of delayed differentiation in §5.5. Section 5.6 presents holding cost profiles as a key qualitative driver of the value of delaying differentiation. We present a numerical example showing how previous work may lead to incorrect strategies in §5.7, and conclude with managerial implications of the study in §5.8. Appendix 1 contains additional numerical results and Appendix 2 provides proofs to our propositions.

5.2 Delayed Differentiation Model

We begin the presentation of our model of delayed differentiation by introducing the following terminology:

k = the stage of last common operations (LCO), our decision variable

S_k = the annuitized cost per period for an investment in the ability to perform a common operation at stage k

$P_{i,\alpha}$ = the processing cost of product α at stage i

$U_{i,\alpha}$ = the transport cost of product α at stage i

$I_{i,\alpha}^{k}$ = the echelon inventory of product α at stage i, given that the last common operation is stage k

For exposition, we will consider a process with two final products. Our objective is to select the point of differentiation that minimizes the total expected cost per period. Cost is comprised of annuitized investment costs that allow a product to remain generic until the LCO (stage k), processing costs, inventory holding and stockout costs, and transportation costs between the installations. Let the sum of these costs be $Z(k)$, so the firm's problem is to find the stage $\bar{k}$ where

$$\begin{aligned}\bar{k} = \underset{1\le k\le N-1}{\arg\min}\, Z(k) = &\sum_{j=1}^{k} S_k + \left[\sum_{j=k}^{m} P_j\left(\mu_1+\mu_2\right) + \sum_{j=1}^{k-1}\left(P_{j,1}*\mu_1\right) + \sum_{j=1}^{k-1}\left(P_{j,2}*\mu_2\right)\right] \\ &+\mathrm{E}\left[\sum_{j=k}^{m}\left(H_j * I_j^k\right) + \sum_{j=1}^{k}\left(H_{j,1}*I_{j,1}^k\right) + \sum_{j=1}^{k}\left(H_{j,2}*I_{j,2}^k\right)\right] \\ &+\left[\sum_{j=k}^{m} U_j\left(\mu_1+\mu_2\right) + \sum_{j=1}^{k-1}\left(U_{j,1}*\mu_1\right) + \sum_{j=1}^{k-1}\left(U_{j,2}*\mu_2\right)\right] + b_1\mathrm{E}\left[D_1 - I_{1,1}^k\right]^+ + b_2\mathrm{E}\left[D_2 - I_{1,2}^k\right]^+\end{aligned} \tag{5.1}$$

The first term in (5.1) is the total investment costs, the terms in the first bracket are the total processing costs, the second bracket are the total holding costs, the third bracket are the transportation costs, and the fifth and sixth terms are the backorder costs. As in Lee and Tang (1997), the optimal LCO is determined by comparing the objective function $Z(k)$ for stages k = 1 through m-1. Unlike their model however, which uses a decoupling argument to determine an inventory stocking level (or safety stock factor) at each stage in the chain, we determine echelon stocking levels. An optimal policy for this type of system has not yet been solved, although Federgruen and Zipkin (1984a) provide bounds and Zipkin (2000) provides approximations. The existing approximations are complex and difficult to compute, thus we solve the problem through the use of the Newsvendor Heuristic. We test the performance of the NH via simulation as described below in §5.3.

5.3 Simulation Methodology and Problem Design

The NH is extensively tested for a two-echelon distribution network in Chapter 4. To demonstrate its usefulness in a delayed differentiation analysis, however, we need to test it on systems of at least three echelons. Because closed form solutions are as of yet unavailable, we do so using simulation. We consider two candidate supply chains of three echelons with LCOs of 2 and 3, respectively, as shown below in Figures 5.2 and 5.3. Comparing these topologies captures the critical elements of a delayed differentiation process.

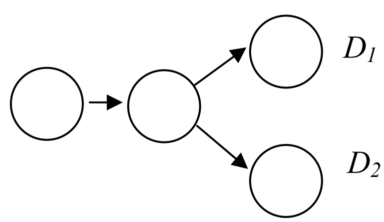


Figure 5.2: Three Echelon System with k=2

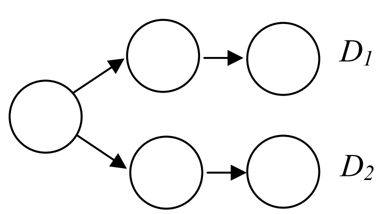


Figure 5.3: Three Echelon System with k=3

The simulations are conducted as follows; for a single, steady state replication, random demands are generated for 100,000 periods. In each period, demand is satisfied or backordered, orders are placed and filled, and linear holding and backordering costs are assessed. These costs are aggregated into 1000 batch means of 100 periods each. The first batch mean is removed to eliminate initialization effects. The removal of the first 100 periods of data is overly conservative because, under base-stock policies, the state spaces of the investigated supply chains are independent after $L_3 + L_2 + L_1 + 1$ periods. The average costs and standard deviations for the remaining 99 batch means (periods 1001 through 100,000) are reported. We utilize common random numbers across systems during demand generation, for computational simplicity and to potentially exploit variance reduction. Demand for both products is assumed Poisson with a mean rate of 10 units per period.

We compare the NH results to those generated by the DH used in Lee and Tang's (1997) procedure and to the set of echelon base-stock levels that results in the lowest average system cost found via the simulation. The DH stocking levels are determined by calculating the optimal local fill rate at the retail stages and applying this rate to each upstream installation. The lowest average system costs are found by conducting a full enumeration across the expected minimizing local base-stock level $\pm \frac{1}{2}\mu$. If lowest cost set of stock levels are potentially constrained by these limits, the study is widened so that the set contains no elements at the limits. We conduct simple difference of means tests between the NH and DH to establish statistically significant results. We achieve significant results in 46 of 48 experiments, while the remaining 2 experiments fail to achieve a significant difference at the 5% level using a two-tailed t-test.

5.4 Experimental Results

5.4.1 Symmetric Costs

The first series of experiments assumes the echelon holding costs and backordering costs for each installation at each stage are identical. We further assume that the processing and transportation costs are constant regardless of k (e.g. $P_j = P_{j,1} = P_{j,2}$), and $\sum_{n=1}^{k} S_k = 0$ for all k. Under these assumptions, differences in the solution to Equation (5.1) arise solely from inventory related effects.

We begin establishing the accuracy of the two heuristics by varying the backorder costs across a 4,000% range while keeping the echelon holding costs constant. For these examples, the echelon holding costs of each of three levels of installations is set to 1 (i.e. the local installation holding costs are 1, 2, and 3, respectively). Simulations were conducted as described in §5.2 and the results are depicted in Appendix 1 in Tables A5.1 and A5.2.

Observation 5.1: *The NH policies result in lower costs than the DH policies.*

For the non-delayed case, the NH generates inventory policies that exceed the cost of the best found policy by an average of 0.2%, compared to 1.7% for the DH. For the delayed case, the errors of the NH policies and DH policies are 0.7% and 2.0%, respectively.

Observation 5.2: *The NH performs progressively better at higher backordering costs. The DH also performs better as the backordering costs increase, but reaches a point where it begins to perform worse.*

As the backordering costs increase, the consistently high service levels at each installation required by the DH are justified. Thus it is unsurprising that here, the DH policies perform well. However, these policies are still outperformed by the NH. In the non-delayed case, the NH finds the best policy in over half of the experiments with the highest backordering costs.

Observation 5.3: *The largest discrepancies between the NH and the DH occur in the low backordering cost range.*

We focus on this area in the remainder of this chapter, believing the greatest contribution may be achieved in this range. Intuitively, it is in this region that the optimal upstream fill rates are significantly smaller than at the retail stage. Thus, in the low backordering cost range, the DH is particularly inappropriate. We specifically consider backordering to holding cost ratios of approximately 7:1 at the retailer, noting that this ratio describes a wide variety of products. For instance, by taking the backordering cost as the lost revenue of a sale of a product with a 50% profit margin, this ratio applies as long as the holding costs exceed 7% of the value of the product.

5.4.2 Asymmetric Costs

The second series of experiments addresses asymmetric costs where the backordering cost and/or the holding costs at the terminal stage are allowed to differ. We continue to assume the processing and transportation costs are constant regardless of the topology given by the selection of k, and that $\sum_{n=1}^{k} S_k = 0$ for all k, again resulting in an investigation into solely the role of inventory effects on Equation (5.1).

We test two levels of holding cost at each echelon, $h = \{1,2\}$ and three backordering costs, $b=\{5,10,20\}$. The NH and DH stocking levels are generated as before. The findings of these experiments are summarized in Appendix 1 in Tables A5.3 and A5.4.

Observation 5.4: *The NH is robust to asymmetric costs in the production network beyond the last common operation.*

For the non-delayed and delayed production networks, the NH produces an average error of 0.6% and 1.2%, respectively. In contrast, use of the DH leads to errors of 1.7% and 1.8%, respectively. We note that when both holding and backordering costs differ between the chains, the NH performs worse than the DH in the delayed network. The allocation policy utilized in the NH is suboptimal for asymmetric retailers but the errors induced by this allocation policy are small relative to the DH results. These results should also be viewed in the context of the range of diversity in holding costs (100%) and backordering costs (400%) between installations. Products originating from a set of common components are unlikely to experience this degree of cost parameter asymmetry. We leave an investigation for a slight correction to the heuristic for future work. For both the DH and NH, the errors are greater in the asymmetric cost experiments than in the symmetric cost experiments.

5.5 The Value of Delaying Differentiation

Having established the NH performs well in three-echelon topologies, we expand the experiment to compare the NH results to the DH results as the echelon holding costs and backordering costs vary. These results are summarized in Table 5.1.

Table 5.1: Comparative Costs

Echelon Holding Costs			Backorder Cost	Non-delayed Chain Costs		Delayed Chain Costs	
h_3	h_2	h_1	b	NH	DH	NH	DH
1	1	1	2.5	8110	8603	7970	8385
1	1	1	5	8928	9226	8733	8940
1	1	1	7.5	9426	9637	9183	9302
1	1	1	10	9814	9980	9544	9603
1	1	1	20	10676	10819	10331	10380
1	1	2	5	9341	9460	9132	9175
1	1	2	10	10411	10484	10136	10088
1	1	2	20	11592	11644	11155*	11146*
1	2	1	5	11473	12382	11180	11859
1	2	1	10	12597	13232	12236	12578
1	2	1	20	13773	14176	13286	13458
1	2	2	5	11798	12208	11507	11741
1	2	2	10	13159	13420	12667	12861
1	2	2	20	14561	14706	14076*	14027*
2	1	1	5	13428	14473	13201	14156
2	1	1	10	14621	15324	14308	14947
2	1	1	20	15852	16268	15399	15827
2	1	1	50	17461	17509	16880	17090
2	1	2	5	13840	14300	13613	13973
2	1	2	10	15175	15550	14842	15158
2	1	2	20	16643	16835	16210	16324
2	2	1	5	15934	17326	15663	16595
2	2	1	10	17317	18331	16919	17926
2	2	1	20	18845	19439	18322	18904
2	2	2	5	16221	17206	15940	16770
2	2	2	10	17856	18452	17467	17880
2	2	2	20	19574	19959	19087	19206

* Denotes an insignificant difference of means on a two-tailed t-test at the 5% level

Observation 5.5: *The experiments show that delaying differentiation is consistently valuable.*

This observation is expected since we assume the redesign cost to be zero.

Observation 5.6: *The NH outperforms the results of the DH.*

In 45 of the 48 cases the NH outperforms the DH at the 5% significance level. In 2 of the remaining 3 cases, the difference of means is statistically insignificant at the 5% level after 200,000 periods. In only one case did the DH significantly outperform the NH. Thus, when determining stocking levels for a multi-level production system, our results indicate NH stocking levels result in lower inventory costs for a given service level than DH stocking levels.

In designing the manufacturing process, a firm seeks to answer the question of whether the savings in inventory costs from delaying differentiation are worth the additional costs from processing, transportation, or redesign. To examine the effects of the DH on this decision, we revisit the data presented in Table A5.3 and compare the differences in the expected inventory costs between the non-delayed and delayed chains. This data is presented in Table 5.2

Table 5.2: Value of Delaying Differentiation

Echelon Holding Costs			Backorder Cost	Delay Value		
h_3	h_2	h_1	b	NH	DH	DH Overestimation
1	1	1	2.5	140.65	218.3	35.6%
1	1	1	5	194.56	286.28	32.0%
1	1	1	7.5	242.65	334.8	27.5%
1	1	1	10	270.13	376.8	28.3%
1	1	1	20	344.6	438.1	21.3%
1	1	2	5	209.02	284.44	26.5%
1	1	2	10	274.8	395.6	30.5%
1	1	2	20	437.8	498.1	12.1%
1	2	1	5	293.7	523.1	43.9%
1	2	1	10	360.7	654.7	44.9%
1	2	1	20	487	718.6	32.2%
1	2	2	5	291.4	467.9	37.7%
1	2	2	10	491.1	559.1	12.2%
1	2	2	20	485.2	678.5	28.5%
2	1	1	5	226.6	317.5	28.6%
2	1	1	10	312.6	376.9	17.1%
2	1	1	20	453.2	440.9	-2.8%
2	1	1	50	581	418.8	-38.7%
2	1	2	5	227	326.9	30.6%
2	1	2	10	333.8	391.6	14.8%
2	1	2	20	433.5	510.9	15.1%
2	2	1	5	271	730.3	62.9%
2	2	1	10	398.5	405.2	1.7%
2	2	1	20	523.3	534.7	2.1%
2	2	2	5	281.3	436.6	35.6%
2	2	2	10	389.1	572.6	32.0%
2	2	2	20	486.5	753.6	35.4%

Observation 5.7: *In the majority of cases studied, the DH overestimates the potential cost savings of delaying differentiation.*

In 93% of the cases investigated, the cost savings generated by delaying differentiation under the DH exceeds that generated by delaying differentiation under the NH, and this discrepancy ranged as high as 62.9%. The overestimation of value is greatest when a significant increase in holding cost occurs at the potential LCO stage. Here, the DH overestimates the cost savings by decreasing the inventory held at the LCO by amounts greater than the NH. Because the DH relies on the decoupling assumption, it carries excessive inventory at upstream stages. This excessive inventory is reduced by greater amounts upon realization of the pooling effect than the more appropriately set inventory levels under the NH. Thus, the DH often leads to an overestimation of the benefits of delayed differentiation.

Observation 5.8: *Use of the DH may also significantly underestimate the potential cost savings of delaying differentiation.*

The underestimation of value is greatest when the local holding costs are high at the beginning of the process and backordering costs are also high. This may occur when there is considerable value in the raw materials compared to the value added during the production process, or when dealing with materials that require expensive or dangerous handling such as molten metal. Here, the DH fails to capitalize fully on the benefits of holding product in intermediate stages. Because a majority of the holding costs are applied regardless of the position of the inventory, the effective cost of shifting base-stock levels downstream towards the intermediate installations decreases. This increases inventory at the potential LCO stage under the NH, affording a greater savings upon merging the chains and exploiting the risk pooling effect. By ignoring the effects of the delayed differentiation decision on installations upstream of the LCO echelon, the DH also fails to appreciate potential cost savings and may underestimate the value of delaying differentiation.

5.6 Holding Cost Profile Insights

The use of the NH allows the development of a non-intuitive and previously obscured result in the behavior of the value of delayed differentiation as a function of the local holding costs. To demonstrate, consider the data in Table 5.2 where $H_1 = 4$ and $b = 20$. Three local holding cost profiles meet this criteria; P1: $\{H_3 = 2, H_2 = 3, H_1 = 4\}$, P2: $\{H_3 = 1, H_2 = 3, H_1 = 4\}$, and P_3: $\{H_3 = 1, H_2 = 2, H_1 = 4\}$. These data are plotted below in Figure 5.4, where the plotted areas represent the local holding cost of a unit of product as it progresses through the production process, and the values in parenthesis are the associated values of delaying differentiation. Under the DH, the value of delaying differentiation under holding cost profiles P1 and P2 are equal, and larger than under P3, due to the inventory savings at the second echelon. Using the NH, however, clearly shows the value under profile P2 > P1 > P3.

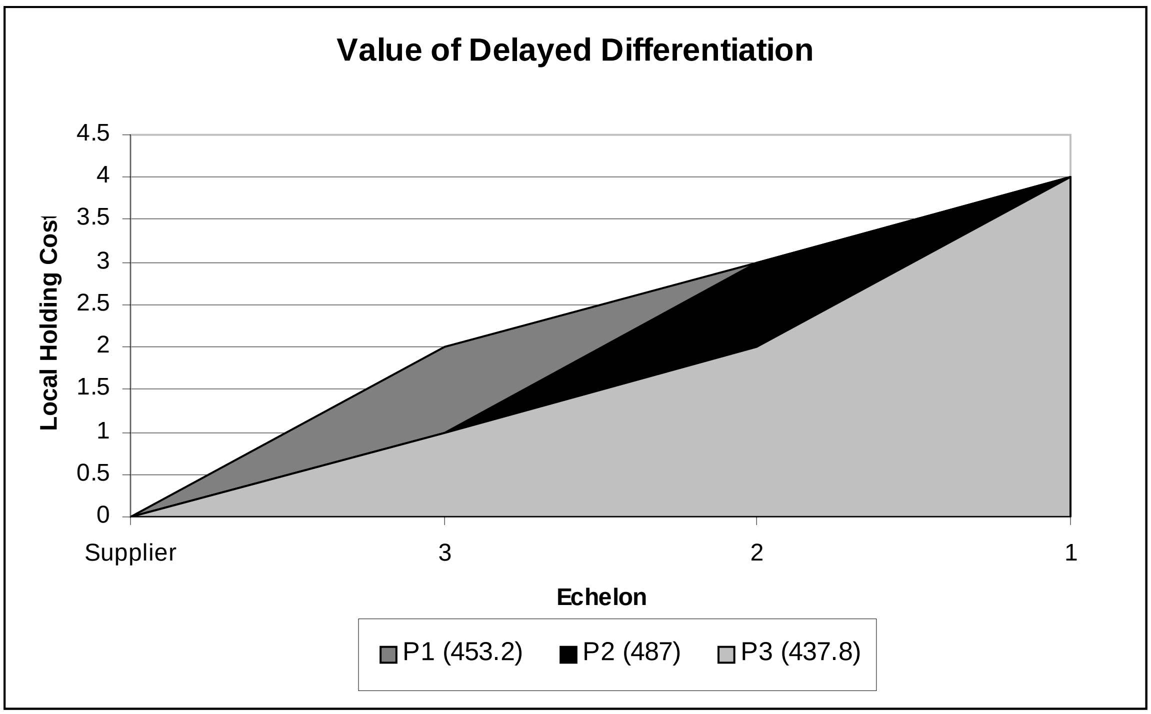


Figure 5.4: Values of Delaying Differentiation for Three Holding Cost Profiles

The example above indicates the value of delaying differentiation is related to the shape of the holding cost profile in addition to the absolute level of local holding cost. When comparing P1 and P2, we see that under P2, the holding costs increase at the potential point of delayed differentiation. We refer to this as a holding cost 'spike'; that is, a region in the production process where the slope of the holding cost profile is large. We find that it is more valuable than previously expected to capture this spike, a finding that is consistent with all of our numerical examples. This value may even exceed that obtained from the absolute holding cost savings. Note that in the above example, the increase in savings due to reduction of inventory when holding costs are 2 rather than 3 (e.g. P1 and P3) is 15.4 cost units. However, the additional benefit obtained from including a holding cost spike in delayed differentiation (e.g. P1 and P2) is an additional 33.8 cost units beyond that predicted from solely the absolute holding cost related savings. We formalize this finding in Propositions 5.1 and 5.2, whose proofs are presented in Appendix 2. The propositions require the following assumptions:

Assumption 5.1: The investment, processing, and transportation costs are independent of the selection of *k*. That is, $\sum_{n=1}^{k} S_k = 0$, $P_j = P_{j,1} = P_{j,2}$, and $T_j = T_{j,1} = T_{j,2}$. Assumption 1 limits our consideration to the effects of inventory costs only. Even if the conditions of the assumption never occur in practice, the analysis allows us to isolate the role of these costs from those of the investment, processing, and transportation costs, which are additive in the objective function and may be treated independently from the inventory considerations.

Assumption 5.2: Consider two network topologies, each with the same number of echelons, where the first consists of serial chains and the second consists of a distribution center at the top echelon, as depicted in Figures 5.5 and 5.6, respectively. We assume the addition of an upstream echelon that converts the networks into Figures 5.7 and 5.8, respectively, affects the costs of operating each system identically. That is, the difference in expected periodic costs between the topologies represented by Figures 5.5 and 5.6 is equal to the difference in expected periodic costs between the topologies represented by Figures 5.7 and 5.8.

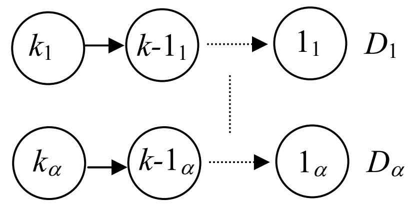


Figure 5.5: Decoupled Topology

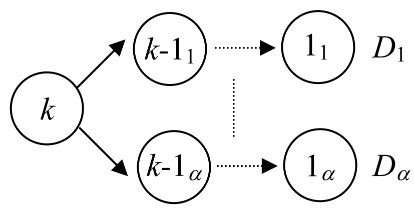


Figure 5.6: Distribution Topology

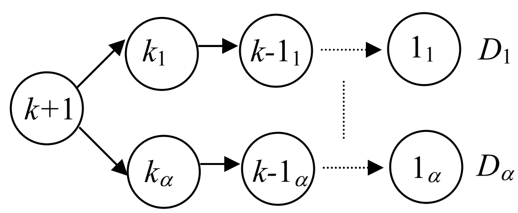


Figure 5.7: Extended Decoupled Topology

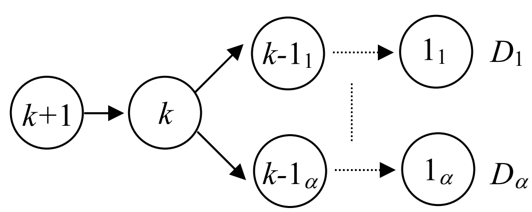


Figure 5.8: Extended Distribution Topology

Assumption 5.2 is weaker than a decoupling assumption, as we need not assume the upstream echelon behaves as an infinite supplier for both topologies but rather it affects the downstream system in identical ways. Under a base-stock policy, the demand process observed at echelon k+1 is identical under both topologies and the additional echelon holding costs and implied backordering costs are constant across topologies. We can now present our two propositions.

Proposition 5.1: *Let $h'_{k-1} = h_{k-1} + \Delta$ and $h'_k = h_k - \Delta$. Then, under Assumptions 5.1 and 5.2, and holding all other parameters constant, the value of delaying differentiation is non-increasing in Δ. This effect becomes more pronounced as the number of final product forms increase.*

Proposition 5.2: *Let $h'_k = h_k + \Delta$ and $h'_{k+1} = h_{k+1} - \Delta$. Then, under Assumptions 5.1 and 5.2, and holding all other parameters constant, the value of delaying differentiation is non-decreasing in Δ. This effect becomes more pronounced as the number of final product forms increase.*

Proposition 5.1 states when larger holding costs are applied early in the process, the value of delayed differentiation is greater than when the holding costs are applied later in the process. This result is largely intuitive and arises from the decrease in inventory at the second stage associated with the pooling effect. Proposition 5.2 considers changes in the local holding costs that apply before the stage of differentiation and states the value of delayed differentiation increases as holding costs shift downstream towards the point of differentiation.

This non-intuitive result arises from the decrease in the effective backordering cost for carrying insufficient inventory in the LCO echelon. The lower upstream holding cost allows for greater inventory to be held at the installation just upstream of the LCO, insulating the LCO from stockouts in a manner that has previously been unobserved. By failing to capture the role of the entire supply chain when determining the value of delayed differentiation, the DH misses these important cost savings. In this instance, the holding cost profile serves as a qualitative indicator for the presence of potential savings via delayed differentiation, and the presence of ‘spikes’ in the holding cost from one stage to the next gives rise to the most favorable conditions for the use of delayed differentiation strategies.

An example of Proposition 5.2 may be found in the products of Experimental Craftworks (http://www.experimentalcraftworks.com), a handmade jewelry boutique. Experimental Craftworks designs and produces faceted gemstone and woven seed glass chandelier earrings. The gemstone products are simple in design and require little labor (approximately one half-hour) in assembly, but have relatively expensive raw materials, costing on average \$35 per pair. The woven seed glass earrings, however, have inexpensive raw materials, costing only \$5 per pair, but require substantial assembly time and expertise. Thus, although the raw materials costs differ, at the earring drop (completed subassembly) state, their values are approximately equal. Both products are also candidates for delayed differentiation strategies, as customers are sensitive to the type of earring backing, such as French hook, lever back, clip-on, or post. These backings are finalized in a separate production stage. Proposition 5.2 states the woven seed crystal earrings are more suitable for delayed differentiation. In essence, the lower raw material value of the seed crystals allows larger quantities to be held in raw inventory, reducing the effective backordering rate of stockouts.

5.7 Numerical Example Demonstrating How the DH May Lead To More Costly Supply Chain Configurations

To see the possible implications of the error induced by the DH, consider the case of a manufacturer who produces a product in three separate steps. Initially the product utilizes a generic component that is differentiated upon completion of the first stage. For ease of exposition, suppose the processing, shipping, and holding costs of the intermediate stage product are identical to that of a possible generic component at the second stage (i.e. the case of delayed differentiation). In other words, let $H_2 = H_{2,1} = H_{2,2}$, $T_2=T_{2,1}=T_{2,2}$ and $P_2 = P_{2,1} = P_{2,2}$.

Suppose the manufacturer faces the following inventory and backorder costs: $H_3 = 1$, $H_2 = 3$, $H_{1,1} = H_{1,2} = 4$, and $b_1 = b_2 = 5$, corresponding to the parameters presented in the 9th row of example problems in Table 5.1. Suppose further that the firm faces an average demand of $\mu_1 = \mu_2 = 10$ units per period for each product, and the per-period annuitized cost of redesigning the process to allow for use of the generic component at the second stage is $S_2 = 5$. This is a set of parameters where we expect the DH to overestimate the value of delaying differentiation. From Table 5.2, we see that under the NH, the firm is only willing to pay up to 2.94 cost units per period to enable delayed differentiation, and will thus opt to not implement the strategy. However, under the DH, the firm will pay up to 5.23 cost units, and will delay the differentiation of the products. In this case, solving the problem using the DH compares an expected cost of

$$Z(3)_{\text{Decoupling}} = 123.82+20*[P_3+T_3]+10*[T_{2,1}+T_{2,2}+T_{1,1}+T_{1,2}]+10*[P_{2,1}+P_{2,2}+P_{1,1}+P_{1,2}]$$

to

$$Z(2)_{\text{Decoupling}} = 118.59+5+20*[P_3+T_3+P_2+T_2]+10*[T_{1,1}+T_{1,2}] + 10*[P_{1,1}+P_{1,2}] .$$

By assumption, the processing and transportation costs are equivalent in the two cases and thus are ignored, leaving $Z(3)_{\text{Decoupling}} = 123.82$ and $Z(2)_{\text{Decoupling}} = 123.59$. Because $Z(2)_{\text{Decoupling}} < Z(3)_{\text{Decoupling}}$, the firm delays differentiation, believing the savings in inventory costs exceed the cost of redesigning the product for delayed differentiation. By a similar analysis, if the firm utilizes the NH, it opts to *not* delay differentiation because $Z(3)_{\text{Bounds}} < Z(2)_{\text{Bounds}}$.

To see the importance of this difference, consider the difference between the total supply chain costs of the two strategies; that is, $Z(3)_{\text{Bounds}}$ - $Z(2)_{\text{Decoupling}}$. Again, the processing and transportation costs are equal and thus cancel. Therefore the difference in expected costs between the two options is 8.86 per period. This represents 7.7% of the total expected inventory related costs.

5.8 Concluding Remarks

When firms face increasing operational costs driven by product proliferation, they often turn to delayed differentiation as a potential cure. To properly assess the benefits of delayed differentiation, firms need to balance the savings from inventory risk-pooling with the costs of process and design modifications. In this chapter, we make three major contributions: I) we provide guidance for when the decoupling assumption used by Lee and Tang (1997) may mislead a firm attempting to determine the optimal point to differentiate its products, II) we verify the Newsvendor Heuristic is robust in three echelon topologies, and most importantly, III) we show the shape of the holding cost profile impacts the optimal point in the production process to delay differentiation.

We show in most cases, the benefits of delaying differentiation are smaller than those predicted by Lee and Tang (1997) due to their decoupling assumption, especially when the echelon holding costs at the last common operation are relatively large. This situation occurs when the majority of the value added processing occurs at the potential point of differentiation, because the inventory becomes relatively expensive at this point. Because the high value add stages also, sometimes incorrectly, appear to benefit the most from delaying differentiation, the decoupling assumption may lead to significant errors in supply chain design.

The decoupling assumption also underestimates the risk pooling savings when the echelon holding costs at stages upstream from the differentiating stage are high relative to the holding costs at other echelons and the backordering costs are high. This is due to a failure to exploit the holding cost structure in the intermediate installations, resulting in lower inventory levels, and consequently, smaller cost savings from risk pooling. This effect is exacerbated when inventories are large due to significant backordering costs.

Finally, we discover the non-intuitive and previously hidden insight that the shape of the holding cost profile significantly affects the optimal point in the process to delay differentiation. We show that the presence of sharp rises in local holding cost is associated with increased cost savings due to the reduction in effective backordering costs at the downstream stages. In other words, capturing holding cost spikes through the use of a common component is more valuable than previously believed, and may serve as an additional justification for using delayed differentiation strategies.

CHAPTER 6
EVALUATION OF MULTIECHELON SUPPLY CHAINS WITH PERISHABLE PRODUCTS

6.1 Introduction

The control of perishable products is increasingly important in supply chain management. In the past few decades, the need to properly manage such products has increased in many industries. For instance, in the technology and fashion industries, continually decreasing product lifecycles have increased the need for agile manufacturing practices. Shrinking margins place ever increasing stress on managing the $200 billion perishable product sales in the US grocery industry, which loses up to 15% of product due to spoilage. Numerous instances of perishable products exist in practice, such as photographic films, pharmaceuticals, blood, biotechnology products, foodstuffs, radioactive materials, electronic wafer fabs, and many chemicals. Fashion and technology goods may also be viewed as deteriorating or perishable products over sufficiently long time horizons. Recognizing the economic and social importance of perishable products, researchers have conducted substantial work in perishable inventory theory. However, with the exception of a handful of situationally specific papers, the multiechelon perishable inventory problem has yet to be addressed.

For illustration, consider the following motivating example. A firm consisting of a two-echelon serial supply chain (a distribution and retail stage) sells a perishable product from the retail stage. The firm purchases a product from an upstream supplier for $20 per unit, and unsold product perishes 3 periods after arriving. The firm faces uncertain demand, distributed according to a negative binomial distribution with a mean of 10 units per period and a coefficient of variation of 0.632. If demand exceeds the on hand inventory at the retail stage, then the firm incurs a backordering cost of $20 per unit. Holding costs at the two stages are $0.5 and $1 per period. The firm seeks inventory stocking levels that minimize their expected long-term operating costs per period.

The current state of the literature fails to give guidance under this scenario. One approach a firm may select is to treat the inventory as nonperishable. Using the technique for determining optimal base-stock levels in a serial supply chain described in Federgruen and Zipkin (1984), the firm will stock 12 units at the warehouse and 23 units at the retailer. Alternately, the firm may choose to use Nahmias' (1979) single location heuristic for each stage. For the retail stage, this procedure is straightforward, while for the warehouse, we utilize effective backorder costs analogously to Shang and Song (2003). Using this heuristic, the firm stocks 10 units at the warehouse but only 12 units at the retailer. Either of these ad-hoc policies creates significant errors. For this scenario, the base-stock levels that result in the minimal total costs are 4 units at the warehouse and 24 at the retailer. The increase in costs associated with these ad-hoc policies, for lifetimes of 2, 3, and 5 periods, are presented in Figure 6.1.

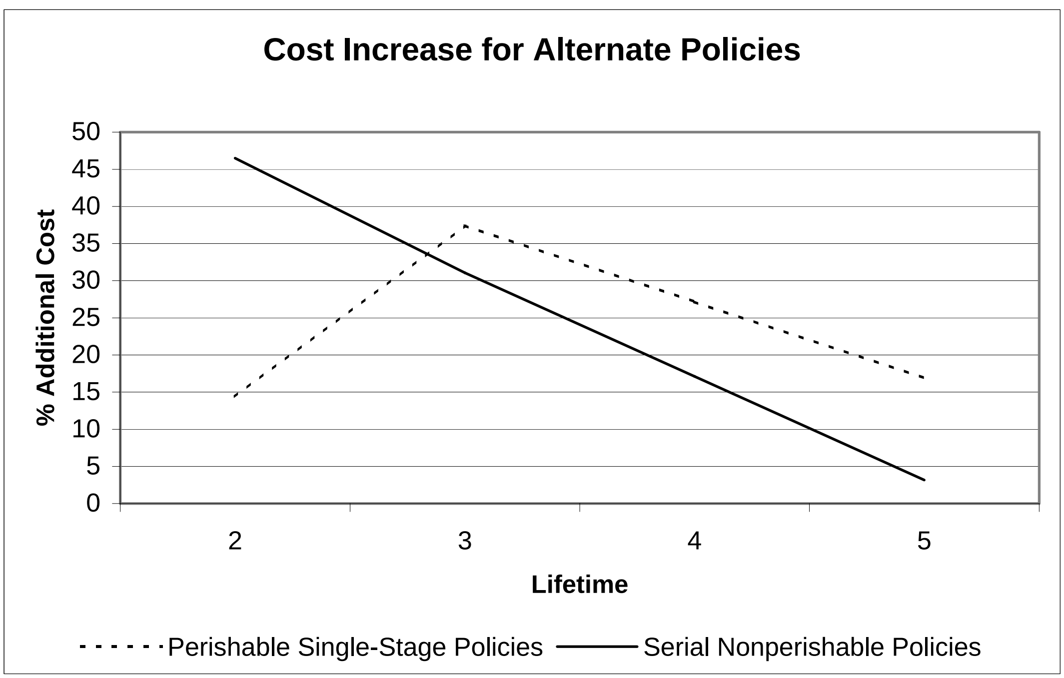


Figure 6.1: Additional Costs From Using Existing Policies

Motivated by the large cost penalties of the alternate policies, we provide managers with a simple and robust heuristic for solving problems such as the one described above.

Specifically, our heuristic solves for the base-stock levels for a two-level, serial or supply distribution system when the product is perishable with fixed lifetimes.

To do so, we link the calculation of stocking policies for supply chains of nonperishable products with the perishable inventory theory. We construct computationally simple and independent single-stage heuristic stocking policies for each supply chain installation, and test the performance of our heuristics via simulation. Our approach yields costs that exceed the best found policy over a wide parameter set by 2.18% for serial systems and 2.99% for distribution systems. The simple structure of our heuristic allows us to develop insights into the proper management of perishable inventory in a two-level supply chain. For instance, we show for serial supply chains, downstream installations behave as if they held nonperishable inventory, but in distribution systems, the allocation of inventory to retail installations must account for the additional opportunity costs of not allowing the expiring inventory to be re-allocated to the retailer who recently experienced the largest total demand. Thus, the inventory management of perishable products for a serial system is quite different than for a distribution system.

We argue the main driver of the differences in managing serial versus distribution systems is, in the latter case, a significant opportunity cost is assessed for inventory expiring at one retailer rather than being used to satisfy demand at a second retailer. For expensive products with short lifetimes, this opportunity cost is sufficiently large to prevent exploitation of the common risk-pooling effect at the warehouse stage. Thus, extending the product's lifetime is most valuable when there are fewer downstream customers.

The remainder of the chapter is organized as follows. We begin by discussing our model in §6.2. We develop our inventory control policies in §6.3. Due to the intractable nature of the problem, we test these policies via simulation. These procedures are presented in §6.4 and their results in §6.5. We provide a summary of our findings in §6.6.

6.2 Perishable Inventory Model

As in Chapter 4, we consider a two-echelon supply chain with n retail sites, labeled with index $\alpha = 1, 2, \ldots, n$,and a single warehouse denoted by W. Inventory is fresh when it arrives to the warehouse, but at the end of r periods after arrival, the inventory must be disposed of at a cost of p per unit. Before the age of the inventory exceeds r periods, it maintains a constant utility to the customer over its lifetime, i.e., the customer values a two-day old unit the same as a three-day old unit. Let D_i^t denote the stochastic demand over t periods at retailer i (we omit the superscript when t = 1), with respective probability and cumulative distribution functions f_i^t and F_i^t. We assume the demand process for each retailer is stationary over time, with the demand processes being independent across retailers. In each period, the following sequence of events occurs: previously shipped replenishments arrive at each level, demand occurs at each retailer, excess demand is fully backordered, inventory is aged (and disposed of if appropriate), replenishment orders are placed, costs are assessed, and replenishment orders are shipped. The inventory positions are reviewed every period and a centralized decision maker places replenishment orders based on knowledge of the entire supply chain's inventory positions.

We assume linear per unit local inventory holding costs (H_i) and backordering costs (b_i), and zero ordering costs throughout the system. Therefore we utilize echelon base-stock policies at each installation with reorder points s_i. Before costs are assessed in each period, the following variables are measured:

B_α = number of backorders at installation α.

O = inventory disposed of in the current period over all installations.

J_α = on-hand inventory at installation α.

T_α = inventory in transit to installation α.

I_α = echelon inventory at installation α, $I_\alpha = J_\alpha$ for α = 1, …, n.

I_W $= J_W + \sum_{j=1}^{n}\left(T_j + I_\alpha\right)$

IP_α = echelon inventory-transit position at installation α, = $I_\alpha - B_\alpha + T_\alpha$

IO_α = inventory orders outstanding for installation α, = s_α - IP_α

The objective is to minimize the expected steady state single-period total costs,

$$\min \mathrm{E}\left[h_W I_W + \sum_{j=1}^{n}\left(b_\alpha B_\alpha + h_\alpha I_\alpha\right) + pO \right] \tag{6.1}$$

Replenishments for each level in the supply chain arrive L_α periods after being shipped. Product is shipped from the warehouse in a first in, first out (FIFO) order, minimizing the outstanding orders (IO_α) in successive allotments of increasing remaining life. By minimizing the outstanding orders, the allocation policy allocates scarce inventory to installations on the basis of their relative need. In other words, the FIFO policy ships inventory that is more likely to expire to retailers who are more likely to use the inventory to satisfy demand before that unit's lifetime is exceeded. We do not claim that this allocation policy is optimal, although we note such an allocation minimizes expected backorders and stockouts (Prastacos, 1981). The policy's nonperishable analog has also been used previously by McGavin, Schwarz, and Ward (1993) for identical retailers and above in Chapter 4 for non-identical retailers.

6.3 Echelon Base-Stock Heuristic for Perishable Distribution Networks

In this section, we present a heuristic for determining echelon base-stock levels for a two-echelon distribution network. Based on the approach introduced in Chapter 3 for nonperishable inventory systems, we begin by constructing two serial supply chain systems whose mean costs bound the mean cost of the distribution system from above and below. We then use a "power approximation" regression model to identify robust echelon base-stock levels for these serial chains. Unfortunately, using serial chain stocking policies as approximations for the stocking levels in a distribution networks are not as accurate as we desire. Thus, we introduce a second adjustment for determining the retailer base-stock levels. From the power regressions and the retailer stocking level adjustment, our heuristic provides near-optimal echelon base-stock levels for the distribution system. We present the details of these steps in §6.3.1 through §6.3.4.

6.3.1 Constructed Serial Systems

As in Chapter 3, we construct two serial systems that provide cost and stocking level bounds for our distribution system. Here we present a brief review of our procedure, directing the reader to Chapter 3 for a more thorough discussion. We use these bounds in a heuristic to find the stocking levels at each stage and location in our distribution system.

To determine the upper bound, we remove risk pooling opportunities from the warehouse by requiring the decision maker to devote inventory to retailers as it is ordered from the supplier. This approach is due to Graves (1996), who notes because it may be desirable to un-commit stock, this assignment rule will not perform as well as a dynamic allocation policy. The restriction decomposes the distribution network into a set of n independent serial systems, one system for each retailer, as depicted on the right of Figure 3.2. We introduce the labels W_α to denote a warehouse installation that exclusively serves retailer i. We refer to these serial chains as "decomposed".

The construction of a lower bound on the mean system costs is based on an approach used by Federgruen and Zipkin (1984), who assume instantaneous and costless transshipment opportunities within an echelon. Under this assumption, the distinctions between installations within an echelon are artificial and the retailers may be treated collectively as a single virtual installation that fills all system demands, as shown on the left of Figure 3.2. We refer to this system as "collapsed". We use a combination of the stocking levels of these two serial systems as an approximation for the stocking levels of the distribution system.

It still remains, however, to determine stocking levels for the serial supply chain relaxations when inventory is perishable. In our pilot numerical studies, we found a systematic bias induced by traditional serial chain stocking policies developed for nonperishable products as an approximation for the stocking levels of a serial chain carrying perishable products. To adjust for this bias, we use a modification of the power approximation method developed by Ehrhardt (1979). Our method is described below.

6.3.2 Power Approximation for Perishable Inventory in Serial Systems

We formulate a regression model to approximate the echelon base-stock levels for the serial chains. We initially create two regressors to utilize in our model. First, we treat the supply chain as a single installation that receives fresh inventory from its upstream supplier. Nahmias (1976) shows that a good heuristic for stocking levels in this problem is the solution to the nonlinear equation

$$\frac{\partial \mathrm{G}_\alpha(s)}{\partial s} + pF_\alpha^{\,\mathrm{r}}\left(s\right) - pF_\alpha^{\,\mathrm{r+1}}\left(s\right) = 0 \tag{6.2}$$

$$\text{where } G_\alpha\left(s\right) = \sum_{x=0}^{s} h\left(s-x\right) f_\alpha\left(x\right) + \sum_{x=s+1}^{\infty} b_\alpha\left(x-s\right) f_\alpha\left(x\right). \tag{6.3}$$

For nonperishable products, keeping no safety stock at the upstream stage of a two-echelon system is frequently a good and simple heuristic (Graves, 1996). This observation suggests that the inventory arriving to the downstream stage of a two-echelon network has a remaining lifetime of approximately $r - L_\alpha$. Hence we calculate Nahmias' heuristic twice, first for the warehouse base-stock level $\left(s_{W_\alpha}^N\right)$ and then for the retailer base-stock level $\left(s_\alpha^N\right)$ with the reduced remaining lifetime.

Next we treat the inventory in the serial chains as nonperishable. In this case, optimal solutions are given by Clark and Scarf (1960) for a finite horizon and Federgruen and Zipkin (1984) for an infinite horizon. Approximately optimal solutions may be calculated via the simple newsvendor heuristics of Shang and Song (2003). These heuristics have the attractive property that they can be expressed in closed-form. Thus, we use the Shang and Song approximation for our second regressor by setting

$$s_\alpha^S = F_\alpha^{-1}\left(\frac{b_\alpha + h_{W_\alpha}}{b_\alpha + h_{W_\alpha} + h_\alpha}\right) \tag{6.4}$$

$$s_{W_\alpha}^S = \frac{1}{2}\left(F_{W_\alpha}^{-1}\left(\frac{b_\alpha}{b_\alpha + h_{W_\alpha} + h_\alpha}\right) + F_{W_\alpha}^{-1}\left(\frac{b_\alpha}{b_\alpha + h_{W_\alpha}}\right)\right) \tag{6.5}$$

for the retail and warehouse installations, respectively.

An initial pilot study also suggested that the demand variance (σ^2), product lifetime (r), backordering cost (b), outdating cost (o), and coefficient of variation (σ/μ) may also be significant. These regressors were entered into the two regression models below

$$s_\alpha^R = V_1^1\left(s_\alpha^N\right)^{V_1^2}\left(s_\alpha^S\right)^{V_1^3}\left(\sigma^2\right)^{V_1^4}\left(r\right)^{V_1^5}\left(b_\alpha\right)^{V_1^6}\left(p\right)^{V_1^7}\left(\sigma_\alpha/\mu_\alpha\right)^{V_1^8} \tag{6.6}$$

$$s_{W_\alpha}^R = V_2^1\left(s_{W_\alpha}^N\right)^{V_2^2}\left(s_{W_\alpha}^S\right)^{V_2^3}\left(\sigma_\alpha^2\right)^{V_2^4}\left(r\right)^{V_2^5}\left(b_\alpha\right)^{V_2^6}\left(p\right)^{V_2^7}\left(\sigma_\alpha/\mu_\alpha\right)^{V_2^8} \tag{6.7}$$

where V_i^x are variables to be fitted. We make the above expressions linear by taking the logarithms of (6.6) and (6.7), and use least-squares regression to fit the models over a grid of 144 values for s_α^R and $s_{W_\alpha}^R$. These values were determined by the simulation model described in Section 5, and were taken over the following parameter values: demand following the Poisson and negative binomial distributions with mean $\mu_\alpha \in \{10, 20\}$ and variance $\sigma_\alpha^2 \in \{\mu_\alpha, 2\mu_\alpha, 4\mu_\alpha\}$, $r \in \{2,3\}$, $b_\alpha \in \{5, 10, 20\}$, and $p \in \{0.5, 5, 10, 20\}$. The Poisson and negative binomial distributions were selected because of their versatility in representing a spectrum of low to high coefficients of variation. Our cost parameters are similar to those used in Nahmias (1976), and Nandakumar and Morton (1993) for perishable systems, and Cachon (2001), Axsater et. al (2002), Shang and Song (2003) and Lystad and Ferguson (2006) for nonperishable systems. The models (6.6) and (6.7) were refined by discarding insignificant factors (p -values less than 0.01) and were refitted, yielding the approximations

$$s_\alpha^R = 1.402\left(s_\alpha^S\right)^{.8306}\left(\sigma_\alpha^2\right)^{.0551}\left(b_\alpha\right)^{.0153} \tag{6.8}$$

$$s_{W_\alpha}^R = 1.707\left(s_{W_\alpha}^N\right)^{1.0399}\left(s_{W_\alpha}^S\right)^{-.2702}\left(\sigma_\alpha^2\right)^{.0964}\left(b_\alpha\right)^{.0399}\left(r\right)^{-.0935} \tag{6.9}$$

The proportions of explained variation for these regressions are 0.988 and 0.997, respectively (although we note that because the regressors are not independent, these values solely represent the fit of the resulting equation). These models were tested against a second grid of 162 values for s_α^R and $s_{W_\alpha}^R$, over the parameter values: demand following the Poisson and negative binomial distributions with mean $\mu_\alpha \in \{5, 15, 30\}$ and variance $\sigma_\alpha^2 \in \{\mu_\alpha, 2\mu_\alpha, 3\mu_\alpha\}$, $r \in \{2,3\}$, $b_\alpha \in \{2, 8, 32\}$, and $p \in \{2, 8, 32\}$. The average error in the resulting serial system costs is 3.43% (full data are presented in Tables A6.1 and A6.2 in Appendix 1).

Investigation of Equation (6.8) gives rise to an insight:

Observation 6.1: *The retailer base-stock levels of a serial chain may be calculated independently of any consideration of perishability.*

Our intuitive explanation is as follows. When the central decision maker sets the stocking level at the warehouse, s/he in effect determines the total system stock and thus the likelihood that the demand over the lifetime of the product is less than the quantity introduced to the system. Thus the full responsibility of outdating is realized at the warehouse. With the total system stock decision made, the retailer, unable to affect the outdating process, operates as if it held nonperishable inventory.

6.3.3 Retailer Base-Stock Approximations for a Distribution System

Observation 6.1 does not apply to distribution systems. In the absence of a rebalancing relaxation (where inventory is collected from the retailers and redistributed each period), carrying an additional unit of inventory at one retailer may cause that unit to expire rather than being used to satisfy demand at an alternate retailer. This opportunity cost is not present in serial systems. Thus we consider an adjustment to our serial system policy for setting the retailer base-stock levels in a distribution system. To develop this adjustment, we relax our problem by making the following simplifying assumptions:

1) All inventory units at the retailer are as fresh as is possible (e.g. with $r-1$ periods of life remaining).
2) The warehouse carries sufficient stock to ensure that all orders are filled in the following period.

These assumptions allow an approximation for the cost of allocating a unit of inventory to a retail stage. This cost is comprised of three elements: additional expected holding costs, a reduction in potential backordering costs, and an increase in expected outdating costs. The first element is simply the probability that demand in one period is less than the amount of inventory held at the retailer times the echelon holding cost at that retailer, $h_\alpha F_\alpha\left(s_\alpha\right)$. By moving a unit of inventory from the warehouse to the retailer, savings in expected backorder costs are achieved. To capture this, our second element is equal to the probability a stockout occurs times the backorder cost rate, $b_\alpha\left(1-F_\alpha\left(s_\alpha\right)\right)$. The previous two terms are analogous to the costs in the traditional Newsvendor problem. To capture the impact of perishable products, we add the increase in the cost associated with perishability. The probability that the unit expires at the retailer is $F_\alpha^{r-L_i}\left(s_\alpha\right)$. However, the unit perishes regardless of whether it was at the retailer or at the warehouse if the total cumulative demand at each other location is less than the inventory held at that location. In this case, we do not penalize the allocation decision. Thus the final cost element in our adjustment is the outdate cost times the joint probability that a unit expires at the retailer and at least one other retailer has sufficient demand so that the unit could have avoided perishing. This element is $pF_\alpha^{r-L_\alpha}\left(s_\alpha\right)\left(1-\prod_{\beta\neq\alpha}F_\beta^{r-L_\beta}\left(s_\beta\right)\right)$

The combined cost of allocating a unit of inventory to a retail stage is

$$h_\alpha F_\alpha\left(s_\alpha\right)-b_\alpha\left(1-F_\alpha\left(s_\alpha\right)\right)+pF_\alpha^{r-L_i}\left(s_\alpha\right)\left(1-\prod_{\beta\neq\alpha}F_\beta^{r-L_\beta}\left(s_\beta\right)\right) \tag{6.10}$$

We select as our stocking level adjustment, the inventory quantity that minimizes this cost.

$$s_\alpha^A=\arg\min\left(h_\alpha F_\alpha\left(s_\alpha\right)-b_\alpha\left(1-F_\alpha\left(s_\alpha\right)\right)+pF_\alpha^{r-L_i}\left(s_\alpha\right)\left(1-\prod_{\beta\neq\alpha}F_\beta^{r-L_\beta}\left(s_\beta\right)\right)\right) \tag{6.11}$$

6.3.4 Heuristic Policy

Our heuristic calculates the echelon base-stock levels as simple averages of the preceding calculations. For the warehouse, we follow the technique in Chapter 3 and average the warehouse echelon base-stock level under the collapsed serial chain $\left(s_W^R\right)$ with the sum of the warehouse echelon base-stock levels over the decomposed serial chains $\left(s_{W_\alpha}^R\right)$. That is,

$$s_W^H=\frac{1}{2}\left(s_W^R+\sum_{\alpha=1}^{n}s_{W_\alpha}^R\right). \tag{6.12}$$

For the retail stages, we take a weighted sum of the retailer base stock level under the decomposed serial chains ($s_{W_i}^R$) and the stocking level found by our approximation (s_i^A). The weights were calculated by regression, fitting the model

$$s_\alpha^H=V^9 s_\alpha^R+V^{10}s_\alpha^A \tag{6.13}$$

with data from experiments taken over the following parameter values: demand following the Poisson and negative binomial distributions with mean $\mu\in\{10, 20\}$ and variance $\sigma^2\in\left\{\mu,2\mu,4\mu\right\}$, $r\in\{2,3\}$, $b\in\{5, 10, 20\}$, and $p\in\{0.5, 5, 10, 20\}$. The resulting model,

$$s_\alpha^H=0.281s_\alpha^R+0.782s_\alpha^A \tag{6.14}$$

was tested against a second grid of values for s_i^H over the parameter values: demand following the Poisson and negative binomial distributions with mean $\mu\in\{5, 15\}$ and variance $\sigma^2\in\left\{\mu,2\mu,3\mu\right\}$, $r\in\{2,3\}$, $b\in\{2, 8, 32\}$, and $p\in\{2, 8, 32\}$. The average error in the resulting distribution system costs is 3.51% (the full results from this experiment is presented in Table A6.3 in Appendix 1).

Since the optimal solution to our problem is unknown, we test our heuristic against the best stocking policies found using simulation. Our methodology is similar to the methodology used in Chapters 4 and 5, and is described in the next section.

6.4 Simulation Methodology

Our simulation methodology is based on an unequal variance, two-stage screening-subset selection procedure presented in Nelson et al. (2001). We first create a set of base-stock level candidates covering a range of the expected minimizing base-stock levels (as predicted by our heuristic), ± at least 5 inventory units for each installation. For the parameter settings in these examples, this range covers at least 50% of the cumulative distribution of the lead-time demand at each installation.

For each stocking level, we initially make a long simulation run of our model over 20,020 periods. We use the method of batch means (Law and Kelton, 2000) by batching periods into groups of 20 to reduce deviations from normality and correlations between single period costs. We omit the first batch (20 periods) to mitigate initialization effects. The remaining data points are used in the initial screening phase.

Potential sets of stocking levels that survive the initial screening are subjected to a second round of simulation experiments, where we retain our batch sizes and generate a sufficient number of data points to eliminate all but one of the systems. After this experiment, the set of stocking levels that has the lowest per period cost is selected. This procedure ensures a confidence level of at least $1-\gamma$ that the selected system performs within a quantity δ of the best system's cost. Hence we refer to the selected system as a δ–best system. We set γ = 5% and δ = 0.2% of the average per period system cost of the best system found in the first stage.

6.5 Problem Design and Experimental Results

6.5.1 Symmetric Two-Echelon Networks

Our first experimental design considers two network topologies, with either two or four symmetric retailers. We test the heuristics using a full factorial design over a range of backorder costs, outdate costs, lifetimes, and demand variances. We assume the total system demand is distributed according to either a Poisson or negative binomial distribution with a mean of 20 units per period across topologies, and that the demand process at each retailer is i.i.d. Our remaining parameters are $b_i \in \{5,10,20\}$, $p \in \{5,10,20\}$, $r \in \{2,3\}$, and for negative binomial demand, $\sigma_i^2 \in \{2\mu, 4\mu\}$. The demand, backorder, and holding cost parameter values are similar to those used in nonperishable works by Jackson (1988), Cachon (2001), Axsater et al. (2002), Shang and Song (2003), and Lystad and Ferguson (2006). The demand, holding cost, outdate cost, and backorder cost parameter values are similar to those used by Nahmias (1976) and Nandakumar and Morton (1993) for single-stage perishable systems. In the latter works, the authors note that a perishable system quickly resembles a nonperishable system as lifetimes exceed two periods. We also observe this convergence at noted in the following observation.

Observation 6.2: *In both the serial supply chain and distribution network cases, increasing lifetimes cause a system to quickly converge to its nonperishable analog.*

This trend is apparent when comparing the inventory stocking levels in Tables A6.1 and A6.2 in Appendix 1. In Table A6.1, the best found stocking levels decrease quickly with outdate cost (while keeping other parameters constant). In the analogous results in Table A6.2, the stocking levels decrease at a significantly slower rate, suggesting the costs imposed by expired inventory are somewhat small. To illustrate the effects of increasing product lifetimes on total system costs, we plot the average total cost per period as lifetimes increase for systems with $b = 20$, $p = 20$, $h_W = h_\alpha = 0.5$ and $r \in \{2, 3, 5\}$. Serial systems with $\mu = 10$, $\sigma^2 \in \{10, 20, 40\}$ are presented in Figure 6.2. Two-retailer distribution systems with $\mu_i = 10$, $\sigma_i^2 \in \{10, 20, 40\}$ are presented in Figure 6.3.

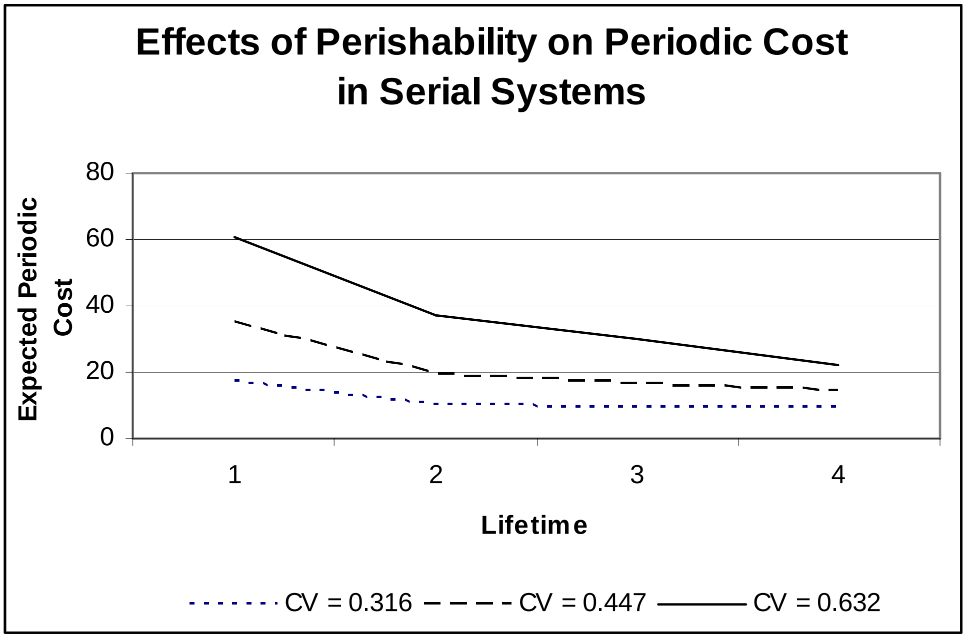


Figure 6.2: Effects of Perishability for Serial Systems

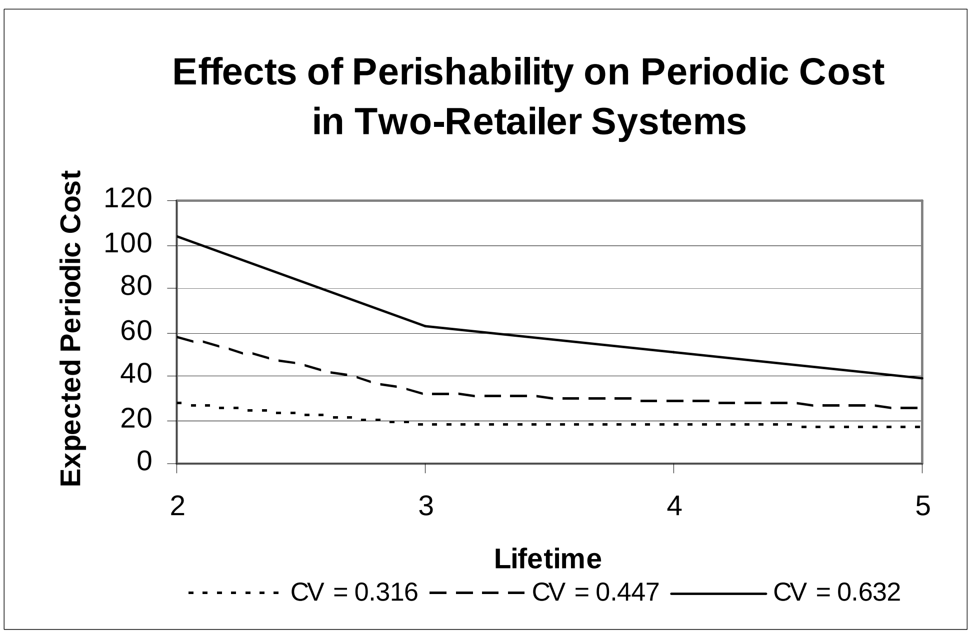


Figure 6.3: Effects of Perishability for Two-retailer Systems

The decrease in costs associated with increasing lifetimes occurs due to a decreasing frequency of outdating. The expected periodic cost converges to that of a nonperishable system as the lifetimes increase, but significant additional costs associated with perishability exist when either lifetimes are short or demand variance is high. Because the majority of perishable associated costs are eliminated by the third period of lifetime, we set our lifetimes to either 2 or 3 periods. These assignments are equivalent to the ones in Nahmias (1976) and Nandakumar and Morton (1993), adjusted to account for the lifetime lost in transit to the retail stages. The results of these tests are summarized in Table 6.1 and Table 6.2, for the two and four retailer systems, respectively, with the entire set of results displayed in Tables A6.4 and A6.5 in Appendix 1.

Table 6.1: Two-retailer Summary

% Error for Two-Retailer Systems		
	Lifetime	
p	2	3
0.5	0.94	1.09
5	0.54	0.92
10	0.93	1.59
20	1.94	1.07

Table 6.2: Four-retailer Summary

% Error for Four-Retailer Systems		
	Lifetime	
p	2	3
0.5	5.34	9.02
5	3.66	4.11
10	4.42	3.44
20	3.28	1.16

The results above lead to the following observation:

Observation 6.3: *The heuristic performs well, with small error rates. These errors are increasing as the number of retailers increases. The most significant errors correspond to low outdating costs*.

While the heuristic's error can exceed an average of 5%, it does so only for low outdating costs. Since the local holding cost rate at the retailers in these tests is set to one unit while the outdating cost is only 0.5 units, the firm prefers to dispose of expiring inventory rather than carry it an additional period. In practice, such scenarios are rare. Neglecting these cases, the heuristic's error is approximately 2.26%. The increase in the error when the number of retailers increases is partly due to discretization effects, where rounding errors imposed to ensure integer valued base-stock levels become more problematic because of the smaller inventory quantities. However, these errors are only slightly greater than those arising from leading heuristics for nonperishable distribution systems (see for comparison Chapters 4 and 5).

Observation 6.4: *All else held constant, increasing the outdate cost, backorder cost, or demand variance also increases the total system cost. Further, decreasing the lifetime tends to increase costs, except when the outdating cost is less than the local holding cost.*

This observation is in line with intuition and previous work in both nonperishable multiechelon and perishable single location systems. Increasing the variance of demand increases both the expected number of backorders and expected number of outdates. Increasing the costs of either of these increases mean system costs directly.

Observation 6.5: *As lifetimes increase, the mean system cost drops faster for two-retailer systems than for four-retailer systems.*

Previous work in nonperishable multi-echelon systems suggests that increasing the number of retailers inhibits the exploitation of risk-pooling benefits, leading to increased costs. This effect is apparent when lifetimes are long and our stocking levels approach those of a nonperishable system. When lifetimes are short, however, less inventory is held at the retailers because of the opportunity cost effect. The greater drop in system costs associated with the two-retailer systems results from the ability to exploit risk-pooling opportunities at the retailers.

Observation 6.6: *As lifetimes decrease, the system-wide inventory savings associated with risk-pooling for systems with fewer, large-volume retailers decreases.*

The increase in opportunity cost effects begins to dominate the risk pooling advantages the two-retailer network enjoys compared to the four-retailer network. The downstream risk-pooling savings can no longer be captured in light of the increased opportunity costs, thus the incentives for developing transshipment opportunities increase as lifetimes decrease. However, it should be mentioned that in practice transshipment opportunities may consume valuable lead-time. As noted above, unless lifetimes are very short, the system may be treated as if the inventory were nonperishable. Thus, transshipments that route inventory back through a distribution point, such as the traditional rebalancing relaxation in nonperishable work (e.g., see Clark and Scarf, 1960;

Federgruen and Zipkin, 1984; and Axsater et al., 2002), are particularly inappropriate in a perishable context. Rather, it is the ability to directly *satisfy customer demand* from multiple sources that becomes increasingly attractive rather than the ability to *rebalance inventories*.

6.5.2 Asymmetric Backorder Rates

In this section, we consider a warehouse that serves the perishable product to retailers with differing backorder rates. We continue to utilize Poisson and negative binomial demands with μ_α = 10. We set $b_1 = 5$, $b_2 \in \{10, 20\}$, $p \in \{0.5, 5, 10, 20\}$, $r \in \{2, 3\}$, and for negative binomial demand distributions, $\sigma_\alpha^2 \in \{2\mu, 4\mu\}$. These assignments generate a total of 48 cases. The results of the tests are summarized in Tables 6.3 and 6.4 for problems with lifetimes of 2 or 3, respectively. The complete set of results is given in Tables A6.6 and A6.7 in Appendix 1.

Table 6.3: Asymmetric Backorder Cost Summary for Two-period Lifetimes

% Errors for Two Period Lifetimes		
	b_2	
σ^2	10	20
10	0.77	1.15
20	0.95	0.64
40	1.31	2.35

Table 6.4: Asymmetric Backorder Cost Summary for Three-period Lifetimes

% Errors for Three Period Lifetimes		
	b_2	
σ^2	10	20
10	1.34	0.56
20	0.35	0.73
40	1.14	1.04

Observation 6.7: *The heuristic is robust to asymmetry in backordering costs. total system costs.*

The errors of the heuristic under asymmetric backorder cost profiles are similar to those of under symmetric backorder cost profiles. Thus the heuristic is robust to asymmetry in backordering costs of even up to 400%, atypical for a vast majority of practical examples.

Observation 8: *Although system-wide echelon inventories for asymmetric backorder cases are approximately the same as for symmetric backorder cases, greater inventory is held at the warehouse in the asymmetric cases. Also, backorder asymmetry decreases total system costs.*

Chapter 4 shows that for the management of nonperishable products, backorder cost asymmetry decreases echelon stocking levels and inventory costs. Here we find similar results for perishable products. The system controller may exploit virtual pooling effects as inventories increase at the retailer with high backorder cost. However, these virtual pooling savings are inhibited by the danger of inventory expiring at the high backorder retailer; hence the decision maker holds a portion of inventory at the warehouse rather than allocating it to the retail stages. This limits the cost savings that may be achieved through the risk-pooling effects.

6.5.3 Asymmetric Demand Rates

We next consider a warehouse that serves the perishable product to retailers with varying mean demand rates. As before, we assume Poisson or negative binomial demands, with means $\mu_1 = 5$ and $\mu_2 = 15$. The remaining parameters are $b_\alpha \in \{5,10,20\}$, $p \in \{0.5,5,10,20\}$, $r \in \{2,3\}$, and for negative binomial demands, $\sigma_\alpha^2 = 2\mu$, for a total of 48 problems. The results of these tests are summarized in Tables 6.5 and 6.6 for problems with lifetimes of 2 or 3, respectively. The complete set of results is contained in Tables A6.8 and A6.9 in Appendix 1.

Table 6.5: Asymmetric Demand Summary for Two-period Lifetimes

% Error for Two-period Lifetimes			
	b		
σ^2	5	10	20
15	0.76	0.96	1.61
30	1.00	0.56	0.18

Table 6.6: Asymmetric Demand Summary for Three-period Lifetimes

% Error for Three-period Lifetimes			
	b		
σ^2	5	10	20
15	2.31	2.61	4.53
30	3.31	1.79	1.94

Observation 9: *The heuristic is robust to asymmetry in demand.*

The errors in the heuristic under asymmetry in demand are approximately the same as under symmetric demand, and continue to be on par with those found for heuristics for nonperishable inventory control.

Observation 10: *There are no significant differences in total system costs between the symmetric and asymmetric demand cases. Slightly greater inventory is held in the asymmetric demand problems.*

Above we show that in nonperishable cases, demand rate asymmetry decreases echelon stocking levels and inventory costs. In this case, we find a slight increase in the warehouse echelon base-stock level with demand asymmetry. The presence of the opportunity cost effect prevents the exploitation of virtual risk-pooling as one retailer captures most of the system demand.

6.6 Importance of Perishable Inventory Policies

In the previous sections, we have presented heuristics to set inventory base-stock levels in supply chains when inventory is perishable. We have also argued that beyond very short lifetimes, the systems behave essentially as if they held nonperishable products. A natural question is to ask when a firm needs to consider the perishable aspect of its inventory and utilize the more complex inventory control policies.

To see the importance of inventory control policies that account for perishability, we compare our heuristic to the nonperishable newsvendor heuristics of Shang and Song (2003) and Lystad and Ferguson (2006). We consider scenarios with $b = 20$, $p = 20$, $h_W = h_\alpha = 0.5$, and $r \in \{2, 3, 5\}$. The results for serial systems with $\mu = 10$ and $\sigma^2 \in \{10, 20, 40\}$ are presented in Figure 6.4. The results for two-retailer distribution systems with $\mu_\alpha = 10$ and $\sigma_\alpha^2 \in \{10, 20, 40\}$ are presented in Figure 6.5.

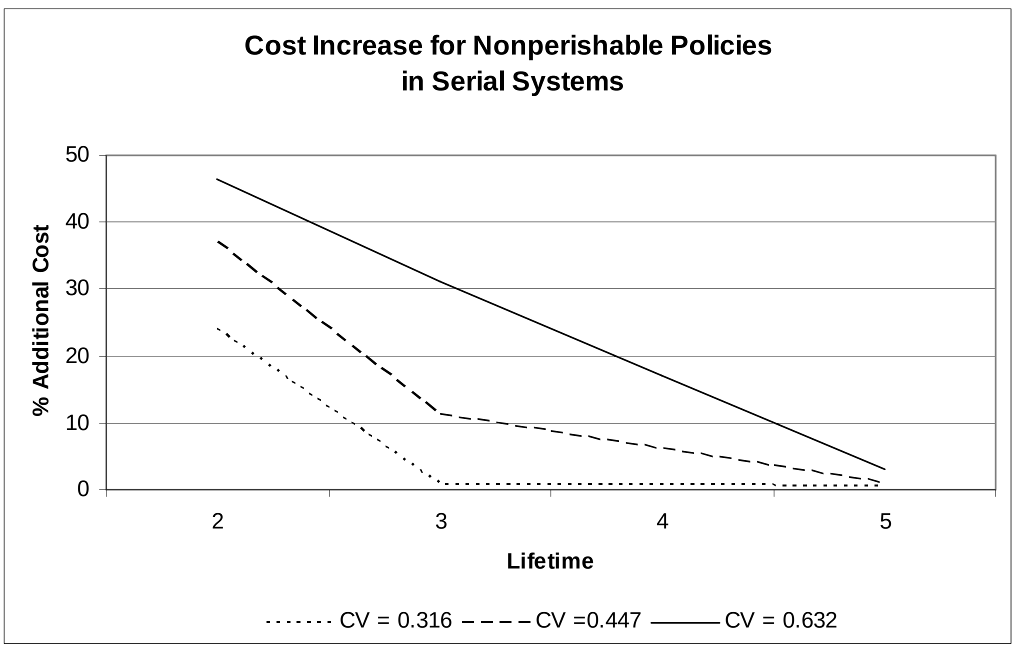


Figure 6.4: Nonperishable Heuristics in Perishable Serial Systems

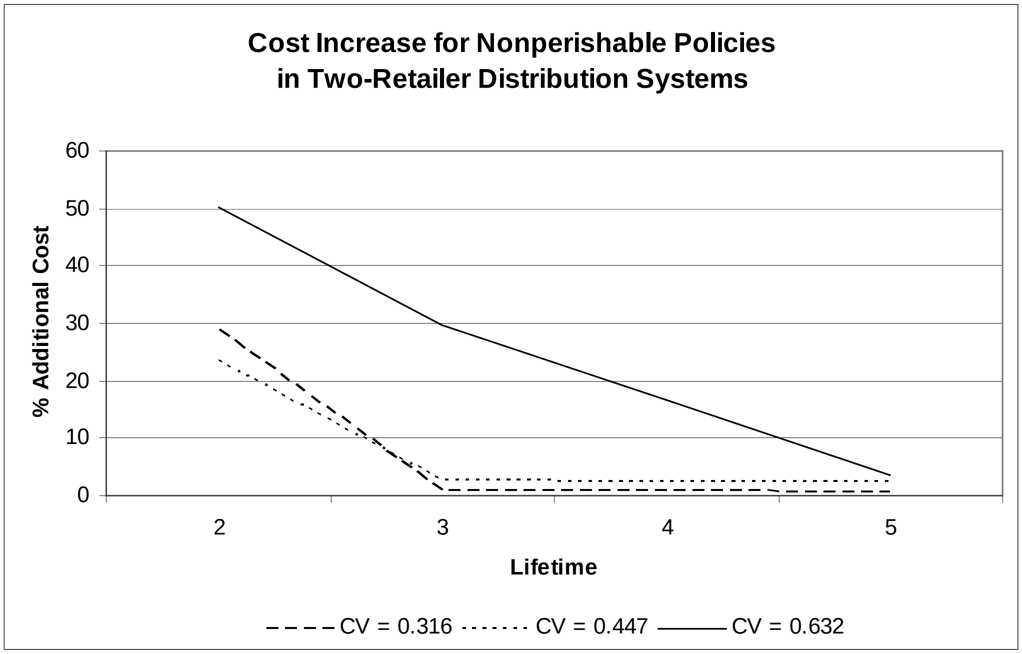


Figure 6.5: Nonperishable Heuristics in Perishable Two-Retailer Distribution Systems

Figures 6.4 and 6.5 reinforce our claim that supply chains resemble nonperishable systems as the likelihood of outdating decreases. They additionally show that when outdating is nonnegligible, the use of nonperishable inventory control policies yields significantly higher costs. Failure to account for the perishable aspect of inventory thus may lead managers to make costly mistakes. We found that the use of nonperishable policies increases costs by about 23% for the shortest lifetimes and lowest demand variances; this impact decreases with lifetime but increases with demand variance.

6.7 Concluding Remarks

Managers faced with perishable inventory may make costly mistakes when relying on nonperishable policies and intuition. In this paper, we presented a simple heuristic for two-echelon supply chains with fixed-life perishable inventory. We considered both serial chains and distribution networks with *n* non-identical retailers, and showed that the qualitative behaviors of the two types of topologies are distinct. Our heuristic treats installations within the supply chain independently, allowing for simple solutions that may be applied using spreadsheet applications.

Relative to the best found (via simulation) base-stock levels, our heuristic yielded a 2.18% and 2.99% average errors for the mean total cost per period in serial and distribution systems, respectively. Under serial systems, the retail stage of a two-echelon network behaves as if it holds nonperishable inventory because the warehouse determines the expected number of outdates per period by setting the echelon base-stock levels. For the parameter settings under consideration, the heuristic echelon base-stock levels were close to their nonperishable inventory analogs. As the lifetime increases, the likelihood of outdating drops quickly, since the probability of stock exceeding demand over the lifetime of the inventory unit becomes quite small.

For distribution systems, however, the allocation of inventory within the network may increase the probability of outdating; a unit of inventory held in stock at one retailer may expire while a "younger" unit is used to satisfy demand at another retailer. We referred to this as an opportunity cost of carrying inventory at the retail stages, and found this opportunity cost has important implications for the control of distribution networks; in particular, it inhibited the exploitation of risk-pooling benefits. As inventory lifetime increases, the opportunity cost effect diminishes, creating greater savings for systems with fewer retailers. Compared to the nonperishable inventory case, increasing the number of retailers while keeping the total system demand constant has a smaller effect on the stocking levels and the total amount of inventory in the system. This suggests investments made to extend lifetimes are more valuable when significant risk-pooling opportunities exist but cannot be exploited due to the perishable nature of the products. Alternately, strategies allowing customer demands to be filled from multiple retail sites remove the opportunity cost effect entirely, in addition to enabling the traditional cost savings.

APPENDIX 1

NUMERIC EXPERIMENTAL DATA AND RESULTS

Table A4.1: Two-Echelon Two-Retailer Problem Parameter Settings

Problem	h_W	h_1	h_2	b_1	b_2	L_W	L_1
1	1	1	1	5	5	1	1
2	1	1	1	10	10	1	1
3	1	1	1	20	20	1	1
4	1	2	2	5	5	1	1
5	1	2	2	10	10	1	1
6	1	2	2	20	20	1	1
7	2	1	1	5	5	1	1
8	2	1	1	10	10	1	1
9	2	1	1	20	20	1	1
10	1	1	1	5	5	1	2
11	1	1	1	10	10	1	2
12	1	1	1	20	20	1	2
13	1	2	2	5	5	1	2
14	1	2	2	10	10	1	2
15	1	2	2	20	20	1	2
16	2	1	1	5	5	1	2
17	2	1	1	10	10	1	2
18	2	1	1	20	20	1	2
19	1	1	1	5	5	2	1
20	1	1	1	10	10	2	1
21	1	1	1	20	20	2	1
22	1	2	2	5	5	2	1
23	1	2	2	10	10	2	1
24	1	2	2	20	20	2	1
25	2	1	1	5	5	2	1
26	2	1	1	10	10	2	1
27	2	1	1	20	20	2	1
28	1	1	1	5	10	1	1
29	1	1	1	5	20	1	1
30	1	1	1	10	20	1	1
31	1	1	2	5	5	1	1
32	1	1	2	10	10	1	1
33	1	1	2	20	20	1	1
34	1	1	2	5	10	1	1
35	1	1	2	5	20	1	1
36	1	1	2	10	20	1	1
37	1	2	1	5	10	1	1
38	1	2	1	5	20	1	1
39	1	2	1	10	20	1	1
40	2	1	1	5	10	1	1
41	2	1	1	5	20	1	1
42	2	1	1	10	20	1	1
43	2	1	2	5	5	1	1
44	2	1	2	10	10	1	1
45	2	1	2	20	20	1	1
46	2	1	2	5	10	1	1
47	2	1	2	5	20	1	1
48	2	1	2	10	20	1	1

Table A4.2: Two-Echelon, Four-Retailer Network Problem Parameter Settings

Problem	h_2	$h_{1,1}$	$h_{1,2}$	$h_{1,3}$	$h_{1,4}$	b_1	b_2	b_3	b_4	L_2	L_1
49	1	1	1	1	1	5	5	5	5	1	1
50	1	1	1	1	1	10	10	10	10	1	1
51	1	1	1	1	1	20	20	20	20	1	1
52	1	2	2	2	2	5	5	5	5	1	1
53	1	2	2	2	2	10	10	10	10	1	1
54	1	2	2	2	2	20	20	20	20	1	1
55	2	1	1	1	1	5	5	5	5	1	1
56	2	1	1	1	1	10	10	10	10	1	1
57	2	1	1	1	1	20	20	20	20	1	1
58	1	1	1	1	1	5	5	5	5	1	2
59	1	1	1	1	1	10	10	10	10	1	2
60	1	1	1	1	1	20	20	20	20	1	2
61	1	2	2	2	2	5	5	5	5	1	2
62	1	2	2	2	2	10	10	10	10	1	2
63	1	2	2	2	2	20	20	20	20	1	2
64	2	1	1	1	1	5	5	5	5	1	2
65	2	1	1	1	1	10	10	10	10	1	2
66	2	1	1	1	1	20	20	20	20	1	2
67	1	1	1	1	1	5	5	5	5	2	1
68	1	1	1	1	1	10	10	10	10	2	1
69	1	1	1	1	1	20	20	20	20	2	1
70	1	2	2	2	2	5	5	5	5	2	1
71	1	2	2	2	2	10	10	10	10	2	1
72	1	2	2	2	2	20	20	20	20	2	1
73	2	1	1	1	1	5	5	5	5	2	1
74	2	1	1	1	1	10	10	10	10	2	1
75	2	1	1	1	1	20	20	20	20	2	1
76	1	1	1	1	1	5	5	10	10	1	1
77	1	1	1	1	1	5	5	20	20	1	1
78	1	1	1	1	1	10	10	20	20	1	1
79	1	1	2	1	2	5	5	5	5	1	1
80	1	1	2	1	2	10	10	10	10	1	1
81	1	1	2	1	2	20	20	20	20	1	1
82	1	1	2	1	2	5	5	10	10	1	1
83	1	1	2	1	2	5	5	20	20	1	1
84	1	1	2	1	2	10	10	20	20	1	1
85	2	1	1	1	1	5	5	10	10	1	1
86	2	1	1	1	1	5	5	20	20	1	1
87	2	1	1	1	1	10	10	20	20	1	1
88	2	1	2	1	2	5	5	5	5	1	1
89	2	1	2	1	2	10	10	10	10	1	1
90	2	1	2	1	2	20	20	20	20	1	1
91	2	1	2	1	2	5	5	10	10	1	1
92	2	1	2	1	2	5	5	20	20	1	1
93	2	1	2	1	2	10	10	20	20	1	1

Table A4.3: Random Allocation Policy Results

Problem	Exact Results			Bounds Heuristic Results			
	s_2	s_1	Cost	s_2	s_1	Cost	% Error
1	41	22	48.15	39	23	48.40	0.53
2	43	24	55.29	39	25	56.44	2.08
3	44	26	62.28	40	26	64.57	3.68
4	42	21	54.22	41	22	54.76	0.99
5	43	23	64.32	42	23	64.47	0.23
6	44	25	74.58	43	24	75.52	1.26
7	38	22	77.35	35	24	79.12	2.29
8	41	23	88.56	36	25	90.67	2.38
9	42	25	99.71	38	26	102.27	2.56
10	40	33	72.42	37	35	73.34	1.27
11	42	35	80.75	38	36	81.90	1.42
12	43	37	88.89	38	38	91.70	3.16
13	41	31	79.95	40	33	80.87	1.15
14	42	34	91.80	40	35	92.72	1.01
15	44	36	103.67	41	36	104.78	1.07
16	38	32	122.56	34	35	124.40	1.50
17	40	34	135.45	34	37	137.96	1.85
18	42	36	148.42	35	38	152.71	2.89
19	62	22	49.91	61	23	49.91	0.01
20	64	24	57.38	60	25	58.33	1.65
21	66	26	64.69	62	26	66.15	2.25
22	62	21	56.03	62	22	56.48	0.80
23	64	23	66.50	64	23	66.50	0.00
24	66	25	77.04	65	24	77.84	1.04
25	58	22	79.85	56	24	81.08	1.54
26	60	24	91.86	58	25	92.62	0.82
27	63	25	103.76	59	26	105.61	1.79
49	41	11	57.35	39	12	58.08	1.27
50	43	12	67.60	39	13	68.46	1.27
51	44	14	77.35	39	14	80.19	3.67
52	42	10	65.74	42	11	66.90	1.76
53	43	12	80.28	42	12	80.31	0.04
54	45	13	94.32	43	13	94.68	0.38
55	38	11	89.05	32	13	93.33	4.81
56	40	12	104.10	36	13	106.35	2.16
57	42	13	119.62	36	14	124.19	3.82
58	39	17	83.73	37	18	84.94	1.44
59	42	18	95.32	33	20	99.02	3.89
60	43	20	106.86	34	21	112.41	5.20
61	40	16	94.19	39	17	96.40	2.35

Table A4.3, Continued							
62	43	17	110.60	40	18	111.80	1.09
63	44	19	127.73	41	19	128.53	0.63
64	37	16	136.55	29	19	140.77	3.09
65	40	17	154.55	30	20	158.54	2.58
66	41	19	172.19	30	21	179.29	4.12
67	62	11	58.94	61	12	59.52	0.99
68	63	13	69.47	61	13	69.64	0.24
69	65	14	79.40	62	14	80.25	1.07
70	63	10	67.45	63	11	68.44	1.48
71	64	12	82.15	64	12	82.15	0.00
72	66	13	96.55	65	13	96.69	0.14
73	57	11	91.15	53	13	94.71	3.90
74	61	12	106.99	58	13	108.17	1.10
75	63	13	123.20	58	14	125.13	1.56

Table A4.4: Myopic Allocation Policy Results for Two-Echelon,
Two-Retailer Symmetric Networks

Problem	δ-Best			Bounds			Cachon Heurisitc			99% Fill Rate			Zero Safety		
	s_2	s_1	Cost	s_2	s_1	%Error	S_2	s_1	%Error	s_2	s_1	%Error	s_2	s_1	%Error
1	17	14	38.53	19	13	0.00	21	12	1.21	31	13	23.44	20	13	0.50
2	20	14	43.47	19	15	0.54	23	14	2.49	31	15	23.12	20	15	1.32
3	18	16	47.82	20	16	1.63	24	16	6.33	31	16	20.14	20	16	1.63
4	19	12	42.88	21	12	0.96	22	11	1.48	31	12	20.23	20	12	0.31
5	21	13	49.99	22	13	0.62	23	13	1.42	31	13	14.93	20	13	0.45
6	22	14	57.40	23	14	0.28	24	15	3.35	31	14	10.95	20	14	1.69
7	14	14	63.66	15	14	0.18	18	12	0.65	31	14	39.90	20	14	7.61
8	16	15	71.22	16	15	0.12	21	13	1.73	31	15	33.03	20	15	4.66
9	19	15	78.75	18	16	0.64	22	15	2.90	31	16	27.71	20	16	2.73
10	14	26	64.11	17	25	0.29	20	23	0.79	31	25	18.09	20	25	2.49
11	18	26	70.70	18	26	0.00	22	25	1.19	31	26	13.63	20	26	0.60
12	18	28	76.18	18	28	1.27	23	27	2.79	31	28	14.15	20	28	1.83
13	17	24	70.60	20	23	0.46	21	21	1.06	31	23	13.77	20	23	0.46
14	17	26	80.13	20	25	0.24	22	24	0.30	31	25	11.97	20	25	0.24
15	19	27	88.09	21	26	1.63	24	26	3.22	31	26	10.35	20	26	1.59
16	11	26	110.56	14	25	0.22	18	22	0.93	31	25	24.27	20	25	5.62
17	9	29	120.20	14	27	0.28	20	24	1.23	31	27	22.93	20	27	5.50
18	15	28	130.40	15	28	0.00	22	26	1.82	31	28	18.22	20	28	2.71
19	40	13	40.64	41	13	0.27	42	12	1.32	55	13	26.97	40	13	0.23
20	40	15	46.05	40	15	0.00	44	14	1.09	55	15	24.97	40	15	0.34
21	41	16	51.35	42	16	-0.08	46	16	3.63	55	16	19.73	40	16	0.84
22	40	12	45.20	42	12	0.35	42	11	1.13	55	12	22.99	40	12	0.23
23	43	13	52.72	44	13	0.38	44	13	0.38	55	13	16.65	40	13	2.09
24	45	14	60.46	45	14	0.00	46	15	2.24	55	14	12.03	40	14	6.02
25	35	14	66.91	36	14	0.32	38	12	0.99	55	14	45.13	40	14	5.07
26	37	15	75.32	38	15	0.22	40	14	0.34	55	15	36.48	40	15	2.19

Table A4.5: Myopic Allocation Policy Results for Two-Echelon, Four-Retailer Symmetric Networks

Problem	δ-Best			Bounds Heuristic			Cachon Heuristic			99% Fill Rate Heuristic			Zero Safety Stock Heuristic		
	s_2	s_1	Cost	s_2	s_1	%Error	s_2	s_1	% Error	s_2	s_1	%Error	s_2	s_1	%Error
49	17	7	44.4	19	7	0.53	21	6	2.6	31	7	22	20	7	1.336
50	18	8	51.3	19	8	0.21	23	7	3.06	31	8	18.7	20	8	0.814
51	18	9	58.3	19	9	0.1	24	9	5.02	31	9	16.4	20	9	0.622
52	19	6	50.5	22	6	1.55	22	5	4.79	31	6	16.6	20	6	0.241
53	20	7	60.4	22	7	0.87	23	7	1.71	31	7	13.2	20	7	0
54	20	8	70.7	23	8	1	25	8	2.93	31	8	10.8	20	8	0
55	15	7	70.8	12	8	0.4	18	6	1.14	31	8	44	20	8	13.87
56	15	8	81	16	8	0.16	20	7	1.45	31	8	28.8	20	8	4.331
57	16	9	91.6	16	9	0	22	8	2.49	31	9	25.5	20	9	3.707
58	15	13	72.7	17	13	0.26	19	12	0.59	31	13	15.7	20	13	2.136
59	17	14	81.7	13	15	0.01	22	13	1.71	31	15	17.5	20	15	4.398
60	14	16	90.8	14	16	0	23	15	2.14	31	16	14.3	20	16	2.836
61	16	12	81.1	19	12	0.85	20	11	0.7	31	12	13.5	20	12	1.505
62	18	13	94.4	20	13	0.3	23	12	1.78	31	13	10.1	20	13	0.297
63	19	14	108	21	14	0.1	24	14	1.57	31	14	7.73	20	14	0
64	0	16	121	9	14	0.08	17	11	1.29	31	14	28.7	20	14	10.96
65	10	15	135	10	15	0	20	12	2.2	31	15	24.3	20	15	8.429
66	3	18	149	10	16	0.05	21	14	1.32	31	16	20.5	20	16	6.224
67	38	7	46.2	41	7	1.17	42	6	3.08	55	7	26	40	7	0.64
68	39	8	53.5	41	8	0.42	43	8	2.05	55	8	21.5	40	8	0.107
69	40	9	60.7	42	9	0.54	45	9	3.35	55	9	18.3	40	9	0
70	40	6	52.5	43	6	1.15	43	5	5.07	55	6	19.6	40	6	0
71	41	7	62.8	44	7	1	44	7	1	55	7	15.4	40	7	0.546
72	42	8	73.3	45	8	0.9	46	8	1.68	55	8	12.3	40	8	1.428
73	31	8	73.5	33	8	0.64	37	6	1.97	55	8	49.5	40	8	11.03
74	36	8	84.5	38	8	0.6	41	7	1.7	55	8	33.1	40	8	2.483
75	37	9	95.6	38	9	0.14	43	8	2.16	55	9	28.6	40	9	1.783

Table A4.6: Myopic Allocation Policy Results for Two-Echelon, Two-Retailer Asymmetric Networks

Problem	δ-Best System				Bound Heuristic				99% Fill Rate Heuristic				Zero Safety Stock Heuristic			
	s_2	$s_{1,1}$	$S_{1,2}$	Cost	s_2	$s_{1,1}$	$s_{1,2}$	% Error	s_2	$s_{1,1}$	$s_{1,2}$	% Error	s_2	$s_{1,1}$	$s_{1,2}$	% Error
28	19	13	14	40.97	20	13	15	1.31	31	13	15	23.33	20	13	15	1.31
29	19	13	16	43.42	21	13	16	1.26	31	13	16	20.85	20	13	16	0.45
30	19	15	16	45.93	20	15	16	0.92	31	15	16	20.75	20	15	16	0.64
31	19	13	12	40.66	20	13	12	0.39	31	13	12	21.84	20	13	12	0.37
32	20	14	13	46.73	21	15	13	0.91	31	15	13	18.77	20	15	13	0.47
33	22	15	14	52.88	22	16	14	0.72	31	16	14	14.33	20	16	14	0.79
34	20	13	13	44.29	21	13	13	0.46	31	13	13	18.55	20	13	13	0.00
35	19	13	15	47.97	22	13	14	1.02	31	13	14	16.19	20	13	14	0.50
36	21	14	14	50.41	21	15	14	0.65	31	15	14	16.34	20	15	14	1.15
37	20	12	14	43.18	20	12	15	1.19	31	12	15	21.61	20	12	15	0.80
38	19	12	16	45.65	21	12	16	0.97	31	12	16	19.51	20	12	16	0.53
39	21	13	15	49.21	21	13	16	0.61	31	13	16	16.60	20	13	16	0.69
40	15	14	15	67.39	16	14	15	0.27	31	14	15	36.33	20	14	15	5.87
41	17	13	16	71.27	18	14	16	1.51	31	14	16	32.96	20	14	16	4.92
42	16	15	16	74.98	17	15	16	0.35	31	15	16	30.21	20	15	16	3.26
43	17	13	12	65.47	17	14	12	0.59	31	14	12	34.76	20	14	12	4.62
44	18	14	13	74.04	18	15	13	0.41	31	15	13	28.18	20	15	13	2.17
45	18	16	15	82.93	19	16	15	0.79	31	16	15	25.21	20	16	15	1.79
46	16	13	14	70.35	18	14	13	1.15	31	14	13	30.79	20	14	13	3.33
47	17	13	15	75.33	18	14	15	1.33	31	14	15	30.06	20	14	15	3.47
48	18	14	15	79.09	18	15	15	0.52	31	15	15	27.57	20	15	15	2.49

Table A4.7: Myopic Allocation Policy Results for Two-Echelon, Four-Retailer Asymmetric Networks

Problem	δ-Best System						Bounds Heuristic						99% Fill Rate Heuristic						Zero Safety Stock Heuristic					
	s_2	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$	$s_{1,4}$	Cost	s_2	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$	$s_{1,4}$	% Error	s_2	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$	$s_{1,4}$	% Error	s_2	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$	$s_{1,4}$	% Error
76	18	7	7	8	8	47.8	19	7	7	8	8	0.5	31	7	7	8	8	20.4	20	7	7	8	8	1.2
77	18	7	7	9	9	51.2	20	7	7	9	9	1.0	31	7	7	9	9	19.1	20	7	7	9	9	1.0
78	18	8	8	9	9	54.7	19	8	8	9	9	0.3	31	8	8	9	9	17.6	20	8	8	9	9	0.8
79	19	7	6	7	6	47.4	21	7	6	7	6	1.3	31	7	6	7	6	19.1	20	7	6	7	6	0.5
80	19	8	7	8	7	55.8	20	8	7	8	7	0.4	31	8	7	8	7	15.9	20	8	7	8	7	0.4
81	19	9	8	9	8	64.5	21	9	8	9	8	0.7	31	9	8	9	8	13.3	20	9	8	9	8	0.3
82	18	7	6	8	7	51.6	21	7	6	8	7	1.2	31	7	6	8	7	17.4	20	7	6	8	7	0.5
83	19	7	6	9	8	55.8	21	7	6	9	8	1.2	31	7	6	9	8	16.1	20	7	6	9	8	0.2
84	19	8	7	9	8	60.1	21	8	7	9	8	1.0	31	8	7	9	8	14.6	20	8	7	9	8	0.4
85	15	7	7	8	8	75.9	14	8	8	8	8	0.4	31	8	8	8	8	36.0	20	8	8	8	8	8.5
86	15	7	7	9	9	81.0	16	8	8	9	9	2.0	31	8	8	9	9	33.9	20	8	8	9	9	8.2
87	15	8	8	9	9	86.2	16	8	8	9	9	0.3	31	8	8	9	9	27.4	20	8	8	9	9	4.2
88	15	7	7	7	6	73.2	14	8	7	8	7	0.9	31	8	7	8	7	40.2	20	8	7	8	7	11.2
89	15	8	8	8	7	84.9	16	8	8	8	8	1.0	31	8	8	8	8	30.3	20	8	8	8	8	6.2
90	16	9	9	9	8	97.1	16	9	9	9	9	0.9	31	9	9	9	9	26.8	20	9	9	9	9	5.5
91	16	7	6	8	7	79.0	15	8	7	8	8	1.2	31	8	7	8	8	35.0	20	8	7	8	8	8.5
92	16	7	6	9	8	85.1	15	8	7	9	9	0.9	31	8	7	9	9	32.6	20	8	7	9	9	7.8
93	17	8	7	9	8	91.0	16	8	8	9	9	0.9	31	8	8	9	9	28.4	20	8	8	9	9	5.8

Table A4.8: Constructed Bimodal Distribution Robustness Tests

Problem	h_W	h_i	b	L_W	L_i	n	s^*_W	s^*_i	s^a_W	s^a_i	Cost	% Error
bp1	1	1	1	1	1	2	10	13	11	14	30.1	0.26
bp2	1	1	10	1	1	2	23	15	23	15	41.6	0.00
bp3	1	1	50	1	1	2	28	15	28	15	44.3	0.00
bp4	1	10	1	1	1	2	18	6	23	5	35.4	4.34
bp5	1	10	10	1	1	2	21	12	23	12	81.2	0.20
bp6	1	10	50	1	1	2	27	15	25	15	91.5	3.07
bp7	10	1	1	1	1	2	10	8	0	13	216.7	0.02
bp8	10	1	10	1	1	2	10	15	10	15	285.4	0.00
bp9	10	1	50	1	1	2	20	15	21	15	351.2	0.16
bp10	1	1	1	3	1	2	50	14	50	14	32.4	0.00
bp11	1	1	10	3	1	2	66	15	66	15	47.4	0.00
bp12	1	1	50	3	1	2	77	15	77	15	54.4	0.00
bp13	1	10	1	3	1	2	58	6	61	5	38.0	1.68
bp14	1	10	10	3	1	2	64	14	65	12	87.2	4.49
bp15	1	10	50	3	1	2	77	15	72	15	99.5	0.61
bp16	10	1	1	3	1	2	32	15	32	15	222.6	0.00
bp17	10	1	10	3	1	2	50	15	50	15	306.8	0.00
bp18	10	1	50	3	1	2	63	15	64	15	397.9	0.18
bp19	1	1	1	1	3	2	10	34	10	34	74.9	0.00
bp20	1	1	10	1	3	2	14	41	14	41	97.1	0.00
bp21	1	1	50	1	3	2	20	44	19	44	108.4	0.45
bp22	1	10	1	1	3	2	12	25	23	24	86.7	3.57
bp23	1	10	10	1	3	2	26	31	27	31	158.0	0.27
bp24	1	10	50	1	3	2	26	36	30	36	227.1	0.35
bp25	10	1	1	1	3	2	10	25	0	31	627.0	0.09
bp26	10	1	10	1	3	2	10	34	0	40	726.8	0.02
bp27	10	1	50	1	3	2	6	44	6	44	836.0	0.00
bp28	1	1	1	1	1	10	68	12	49	14	140.9	0.01
bp29	1	1	10	1	1	10	102	15	96	15	171.8	2.18
bp30	1	1	50	1	1	10	118	15	113	15	180.3	1.25
bp31	1	10	1	1	1	10	55	11	118	5	169.0	14.58
bp32	1	10	10	1	1	10	101	15	103	12	276.9	33.26
bp33	1	10	50	1	1	10	114	15	100	15	260.6	17.23
bp34	10	1	1	1	1	10	68	8	8	13	1065.6	0.14
bp35	10	1	10	1	1	10	62	13	45	15	1396.8	0.04
bp36	10	1	50	1	1	10	97	15	89	15	1605.2	1.51
bp37	1	1	1	3	1	10	237	15	245	14	142.9	0.16
bp38	1	1	10	3	1	10	309	15	305	15	182.0	0.78
bp39	1	1	50	3	1	10	330	15	350	15	206.2	3.76
bp40	1	10	1	3	1	10	239	13	311	5	170.2	12.35
bp41	1	10	10	3	1	10	302	15	307	12	292.3	31.97

Table A4.8, Continued												
bp42	1	10	50	3	1	10	330	15	324	15	245.2	1.59
bp43	10	1	1	3	1	10	205	13	176	15	1079.1	0.31
bp44	10	1	10	3	1	10	251	15	245	15	1409.1	0.06
bp45	10	1	50	3	1	10	299	15	298	15	1675.0	0.07
bp46	1	1	1	1	3	10	68	33	45	34	362.4	0.04
bp47	1	1	10	1	3	10	60	43	45	41	462.9	5.13
bp48	1	1	50	1	3	10	91	45	60	44	547.6	17.51
bp49	1	10	1	1	3	10	65	29	121	24	403.5	6.87
bp50	1	10	10	1	3	10	58	40	117	31	633.9	23.50
bp51	1	10	50	1	3	10	86	44	114	36	845.1	44.80
bp52	10	1	1	1	3	10	50	24	6	32	3138.1	0.26
bp53	10	1	10	1	3	10	68	33	5	39	3600.7	0.01
bp54	10	1	50	1	3	10	66	40	8	44	4142.8	1.17

Table A4.9: Discrete Uniform Distribution Robustness Results

Problem	h_W	h_i	b	L_W	L_i	n	S^*_W	s^*_i	s^a_W	s^a_i	Cost	% Error
up1	1	1	1	1	1	2	11	13	15	12	29.1	1.48
up2	1	1	10	1	1	2	20	15	20	15	37.6	0.00
up3	1	1	50	1	1	2	25	15	25	15	41.6	0.00
up4	1	10	1	1	1	2	15	8	23	6	33.4	5.54
up5	1	10	10	1	1	2	25	10	26	10	70.2	0.71
up6	1	10	50	1	1	2	24	14	24	14	85.3	0.00
up7	10	1	1	1	1	2	11	9	0	15	213.3	0.01
up8	10	1	10	1	1	2	10	15	10	15	265.1	0.00
up9	10	1	50	1	1	2	18	15	18	15	318.9	0.00
up10	1	1	1	3	1	2	47	15	54	12	29.6	0.11
up11	1	1	10	3	1	2	63	15	63	15	42.0	0.00
up12	1	1	50	3	1	2	69	15	71	15	48.3	-0.03
up13	1	10	1	3	1	2	56	8	61	6	35.0	2.79
up14	1	10	10	3	1	2	66	12	68	10	74.8	6.89
up15	1	10	50	3	1	2	69	15	68	14	92.7	1.40
up16	10	1	1	3	1	2	35	15	35	15	217.8	0.00
up17	10	1	10	3	1	2	50	15	50	15	282.4	0.00
up18	10	1	50	3	1	2	59	15	61	15	354.6	0.13
up19	1	1	1	1	3	2	13	32	14	32	71.5	0.10
up20	1	1	10	1	3	2	17	38	17	38	88.6	0.00
up21	1	1	50	1	3	2	18	42	19	41	98.9	0.52
up22	1	10	1	1	3	2	14	27	23	25	81.1	3.62
up23	1	10	10	1	3	2	26	30	28	30	139.0	1.14
up24	1	10	50	1	3	2	25	36	24	36	179.7	-0.02
up25	10	1	1	1	3	2	13	25	9	36	621.4	0.05
up26	10	1	10	1	3	2	13	33	2	39	697.3	-0.07
up27	10	1	50	1	3	2	9	41	9	41	782.8	0.00
up28	1	1	1	1	1	10	76	12	69	12	130.3	0.50
up29	1	1	10	1	1	10	93	15	81	15	167.8	5.74
up30	1	1	50	1	1	10	108	15	101	15	175.5	3.86
up31	1	10	1	1	1	10	70	11	117	6	160.0	17.20
up32	1	10	10	1	1	10	85	15	114	10	279.6	45.12
up33	1	10	50	1	1	10	106	15	97	14	278.3	31.53
up34	10	1	1	1	1	10	62	9	6	15	1054.7	0.00
up35	10	1	10	1	1	10	57	14	45	15	1288.5	0.10
up36	10	1	50	1	1	10	85	15	75	15	1526.3	2.47
up37	1	1	1	3	1	10	269	13	266	12	132.9	0.97
up38	1	1	10	3	1	10	296	15	294	15	166.9	0.28
up39	1	1	50	3	1	10	318	15	324	15	183.1	0.66
up40	1	10	1	3	1	10	247	13	308	6	157.4	12.85
up41	1	10	10	3	1	10	287	15	323	10	292.3	44.88

Table A4.9, Continued												
up42	1	10	50	3	1	10	312	15	316	14	260.8	15.36
up43	10	1	1	3	1	10	220	13	187	15	1066.6	0.47
up44	10	1	10	3	1	10	269	13	245	15	1306.8	0.10
up45	10	1	50	3	1	10	288	15	285	15	1542.8	0.10
up46	1	1	1	1	3	10	68	33	66	32	348.1	0.23
up47	1	1	10	1	3	10	50	42	64	38	426.3	4.64
up48	1	1	50	1	3	10	66	44	64	41	501.8	14.14
up49	1	10	1	1	3	10	50	32	118	25	383.6	6.67
up50	1	10	10	1	3	10	60	39	123	30	589.4	27.03
up51	1	10	50	1	3	10	53	44	96	36	665.2	25.47
up52	10	1	1	1	3	10	58	26	7	33	3102.5	0.12
up53	10	1	10	1	3	10	68	33	5	39	3458.4	0.03
up54	10	1	50	1	3	10	56	40	25	41	3914.1	1.67

Table A4.10: Negative Binomial Distribution Robustness Results

Problem	h_W	h_i	b	L_W	L_i	n	s^*_W	s^*_i	s^a_W	s^a_i	Cost	% Error
nb1	1	1	1	1	1	2	11	12	15	11	29.0	0.28
nb2	1	1	10	1	1	2	19	16	19	16	45.2	0.00
nb3	1	1	50	1	1	2	20	20	22	19	57.5	0.93
nb4	1	10	1	1	1	2	13	8	21	6	34.5	3.01
nb5	1	10	10	1	1	2	24	10	25	10	78.1	0.16
nb6	1	10	50	1	1	2	27	14	27	14	123.9	0.00
nb7	10	1	1	1	1	2	9	9	1	13	212.2	-0.06
nb8	10	1	10	1	1	2	8	15	5	17	276.1	0.06
nb9	10	1	50	1	1	2	9	20	9	20	357.7	0.00
nb10	1	1	1	3	1	2	34	21	53	11	31.7	0.08
nb11	1	1	10	3	1	2	63	16	63	16	51.2	0.00
nb12	1	1	50	3	1	2	59	22	68	19	65.1	0.48
nb13	1	10	1	3	1	2	53	8	57	6	37.1	1.13
nb14	1	10	10	3	1	2	65	12	67	10	85.1	2.90
nb15	1	10	50	3	1	2	72	16	72	14	132.9	2.70
nb16	10	1	1	3	1	2	19	22	29	16	217.7	-0.06
nb17	10	1	10	3	1	2	44	17	44	17	299.8	0.00
nb18	10	1	50	3	1	2	46	23	52	20	402.0	0.01
nb19	1	1	1	1	3	2	8	32	11	32	73.1	-0.11
nb20	1	1	10	1	3	2	8	43	17	39	97.6	0.23
nb21	1	1	50	1	3	2	15	46	16	45	115.4	0.59
nb22	1	10	1	1	3	2	11	25	23	23	84.2	3.69
nb23	1	10	10	1	3	2	28	29	29	29	159.3	0.44
nb24	1	10	50	1	3	2	28	36	28	36	229.7	0.00
nb25	10	1	1	1	3	2	0	13	3	37	613.7	-0.07
nb26	10	1	10	1	3	2	10	33	4	41	710.7	-0.08
nb27	10	1	50	1	3	2	11	41	0	46	831.2	-0.01
nb28	1	1	1	1	1	10	34	15	79	11	133.1	-0.15
nb29	1	1	10	1	1	10	67	18	86	16	189.3	0.48
nb30	1	1	50	1	1	10	68	21	73	19	256.0	9.92
nb31	1	10	1	1	1	10	39	13	108	6	147.2	4.18
nb32	1	10	10	1	1	10	74	16	123	10	272.6	18.40
nb33	1	10	50	1	1	10	86	18	110	14	367.1	18.50
nb34	10	1	1	1	1	10	33	11	8	13	1048.7	0.05
nb35	10	1	10	1	1	10	42	15	25	17	1319.0	0.05
nb36	10	1	50	1	1	10	35	20	39	20	1668.6	0.05
nb37	1	1	1	3	1	10	198	18	275	11	137.2	0.29
nb38	1	1	10	3	1	10	223	22	295	16	198.6	0.98
nb39	1	1	50	3	1	10	263	22	310	19	245.3	2.22
nb40	1	10	1	3	1	10	160	21	301	6	153.4	5.89
nb41	1	10	10	3	1	10	248	18	327	10	319.3	31.44

Table A4.10, Continued												
nb42	1	10	50	3	1	10	259	21	334	14	427.9	32.53
nb43	10	1	1	3	1	10	141	19	166	16	1057.3	0.09
nb44	10	1	10	3	1	10	205	18	225	17	1351.8	0.14
nb45	10	1	50	3	1	10	214	22	248	20	1739.1	1.03
nb46	1	1	1	1	3	10	73	30	65	32	353.1	-0.20
nb47	1	1	10	1	3	10	50	42	65	39	455.1	2.35
nb48	1	1	50	1	3	10	69	46	50	45	554.3	9.24
nb49	1	10	1	1	3	10	46	30	131	23	407.8	10.58
nb50	1	10	10	1	3	10	67	38	137	29	682.1	31.99
nb51	1	10	50	1	3	10	75	44	114	36	832.5	32.33
nb52	10	1	1	1	3	10	76	21	6	32	3072.6	0.24
nb53	10	1	10	1	3	10	33	35	5	39	3514.6	0.04
nb54	10	1	50	1	3	10	67	39	8	44	4089.0	0.36

Table A4.11: Asymmetric Demand Robustness Results

Problem	h_1	h_2	b_1	b_2	μ_2	s^*_W	s^*_1	s^*_2	s^a_W	s^a_1	s^a_2	Cost	% Error
ad1	1	1	1	1	20	22	13	22	22	12	22	38.7	0.08
ad2	1	1	50	1	20	35	15	18	32	15	22	51.8	3.82
ad3	1	1	1	50	20	33	9	25	33	12	25	50.5	3.25
ad4	1	1	1	1	40	42	13	42	42	12	42	58.7	0.05
ad5	1	1	50	1	40	55	15	38	51	15	42	73.3	4.80
ad6	1	1	1	50	40	53	9	45	54	12	45	70.8	2.76
ad7	1	1	1	1	60	62	13	62	62	12	62	78.7	0.04
ad8	1	1	50	1	60	75	15	58	70	15	62	94.8	5.42
ad9	1	1	1	50	60	74	8	65	74	12	65	90.8	1.93
ad10	1	1	1	1	80	80	14	83	82	12	82	98.7	0.04
ad11	1	1	50	1	80	95	15	78	89	15	82	117.1	6.59
ad12	1	1	1	50	80	94	8	85	94	12	85	110.8	1.58
ad13	1	1	1	1	100	100	14	103	102	12	102	118.7	-0.03
ad14	1	1	50	1	100	114	15	97	109	15	102	137.1	5.50
ad15	1	1	1	50	100	114	8	105	114	12	105	130.8	1.33
ad16	10	1	1	1	20	27	7	20	26	6	22	41.7	1.87
ad17	1	10	1	1	20	24	10	17	25	12	16	41.3	0.22
ad18	10	1	1	1	40	47	7	39	46	6	42	61.7	1.14
ad19	1	10	1	1	40	48	10	36	45	12	36	61.3	0.76
ad20	10	1	1	1	60	67	7	60	67	6	62	81.7	0.98
ad21	1	10	1	1	60	66	11	57	65	12	56	81.3	0.59
ad22	10	1	1	1	80	85	8	81	87	6	82	101.7	0.64
ad23	1	10	1	1	80	88	10	76	85	12	76	101.3	0.46
ad24	10	1	1	1	100	107	7	100	107	6	102	121.7	0.66
ad25	1	10	1	1	100	106	11	97	105	12	96	121.3	0.39
ad26	10	1	50	1	20	35	14	18	31	14	22	91.5	2.54
ad27	1	10	50	1	20	35	15	16	35	15	16	51.4	0.00
ad28	10	1	50	1	40	55	14	38	50	14	42	112.5	3.07
ad29	1	10	50	1	40	54	15	36	53	15	36	70.8	0.30
ad30	10	1	50	1	60	72	15	57	70	14	62	132.5	0.66
ad31	1	10	50	1	60	73	15	56	73	15	56	91.5	0.00
ad32	10	1	50	1	80	95	14	78	89	14	82	154.5	3.56
ad33	1	10	50	1	80	95	15	75	92	15	76	112.1	0.71
ad34	10	1	50	1	100	115	14	98	109	14	102	174.6	3.19
ad35	1	10	50	1	100	115	15	95	112	15	96	132.1	0.60
ad36	10	1	1	50	20	34	6	25	37	6	25	52.3	3.53
ad37	1	10	1	50	20	33	10	24	31	12	24	89.4	1.60
ad38	10	1	1	50	40	54	6	45	58	6	45	73.1	3.76
ad39	1	10	1	50	40	54	9	44	52	12	44	109.2	1.02
ad40	10	1	1	50	60	74	6	65	78	6	65	93.2	2.97
ad41	1	10	1	50	60	73	10	64	72	12	64	129.2	0.95
ad42	10	1	1	50	80	94	6	85	98	6	85	113.2	2.43
ad43	1	10	1	50	80	94	9	84	92	12	84	149.2	0.75
ad44	10	1	1	50	100	114	6	105	118	6	105	133.2	2.06
ad45	1	10	1	50	100	113	10	104	112	12	104	169.2	0.73

Table A4.12: Asymmetric Lead-time Robustness Results

Problem	h_W	h_1	h_2	b_1	b_2	L_2	s^*_W	s^*_1	s^*_2	s^a_W	s^a_1	s^a_2	Cost	% Error
al1	1	1	1	1	1	2	11	13	21	15	12	22	40.4	1.19
al2	1	1	1	50	1	2	24	15	17	21	15	22	54.4	4.70
al3	1	1	1	1	50	2	18	10	29	20	12	29	55.7	1.68
al4	1	1	1	1	1	3	11	13	30	14	12	32	51.2	0.33
al5	1	1	1	50	1	3	24	15	26	21	15	32	65.5	5.27
al6	1	1	1	1	50	3	14	12	42	19	12	41	70.0	0.30
al7	1	1	1	1	1	4	14	11	38	13	12	43	62.3	0.56
al8	1	1	1	50	1	4	23	15	35	21	15	43	77.4	5.18
al9	1	1	1	1	50	4	17	10	52	18	12	53	83.4	0.02
al10	1	10	1	1	1	2	16	7	19	18	6	22	43.2	2.49
al11	1	1	10	1	1	2	17	10	14	19	12	15	44.6	2.83
al12	1	10	1	1	1	3	16	7	28	18	6	32	54.4	2.01
al13	1	1	10	1	1	3	12	13	24	18	12	25	56.5	2.11
al14	1	10	1	1	1	4	16	7	37	17	6	43	65.9	2.43
al15	1	1	10	1	1	4	12	13	33	19	12	34	68.7	2.97
al16	1	10	1	50	1	2	24	14	16	21	14	22	92.5	2.31
al17	1	1	10	50	1	2	23	15	13	26	15	15	55.1	1.49
al18	1	10	1	50	1	3	23	14	26	21	14	32	103.6	2.40
al19	1	1	10	50	1	3	23	15	21	26	15	25	68.1	2.54
al20	1	10	1	50	1	4	24	14	35	20	14	43	116.6	3.83
al21	1	1	10	50	1	4	23	15	31	27	15	34	80.4	3.46
al22	1	10	1	1	50	2	21	6	28	24	6	29	58.1	2.52
al23	1	1	10	1	50	2	23	8	24	22	12	25	111.7	0.51
al24	1	10	1	1	50	3	19	6	40	23	6	41	72.4	1.33
al25	1	1	10	1	50	3	21	9	35	22	12	36	137.7	0.56
al26	1	10	1	1	50	4	16	6	52	22	6	53	86.9	0.57
al27	1	1	10	1	50	4	19	10	46	23	12	46	162.2	0.10
al28	1	1	1	1	1	5	10	13	49	13	12	53	73.2	0.51
al29	1	1	1	50	1	5	23	15	45	20	15	53	89.7	5.30
al30	1	1	1	1	50	5	15	11	64	17	12	64	96.7	-0.24
al31	1	10	1	1	1	5	16	7	46	17	6	53	76.7	1.95
al32	1	1	10	1	1	5	13	11	41	20	12	43	80.7	1.84
al33		10	1	50	1	5	23	14	44	20	14	53	127.5	3.49
al34	1	1	10	50	1	5	23	15	39	27	15	43	91.6	2.63
al35	1	10	1	1	50	5	21	6	64	21	6	64	98.7	0.00
al36	1	1	10	1	50	5	21	9	56	21	12	57	184.0	-0.16

Table A5.1: Stocking Level and Costs for Non-Delayed Chain (k=3)

Non-Delayed (k=3) Cost Results					
Best Found Base Stock Policy					
b	s_3	s_2	s_1	Cost	% Error
2.5	15	9	13	8109.7	N/A
5	18	9	14	8935.5	N/A
7.5	17	11	14	9452.7	N/A
10	18	10	15	9822.9	N/A
20	19	11	16	10710.9	N/A
30	19	12	16	11210.8	N/A
50	20	12	17	11824.1	N/A
100	19	13	18	12599.2	N/A
Newsvendor Bounds Heuristic Policy					
b	s_3	s_2	s_1	Cost	% Error
2.5	18	9	13	8169.69	0.7
5	18	10	14	8985.39	0.6
7.5	18	11	14	9475.62	0.2
10	18	11	15	9864.56	0.4
20	19	11	16	10710.9	0.0
30	19	12	16	11210.8	0.0
50	20	12	17	11824.1	0.0
100	19	13	18	12599.2	0.0
Decoupling Heuristic Policy					
b	s_3	s_2	s_1	Cost	% Error
2.5	19	11	13	8588.5	5.9
5	19	11	14	9204.5	3.0
7.5	18	12	14	9606.4	1.6
10	17	12	15	9944.1	1.2
20	18	12	16	10758.6	0.4
30	17	13	16	11234.9	0.2
50	17	13	17	11850.4	0.2
100	17	13	18	12681.5	0.7

Table A5.2: Stocking Level and Costs for Delayed Chain (k=2)

Delayed (k=2) Cost Results					
Best Found Base Stock Policy					
b	s_3	s_2	s_1	Cost	% Error
2.5	17	16	13	7882.2	N/A
5	18	17	14	8632.6	N/A
7.5	19	19	14	9100.5	N/A
10	18	19	15	9444.2	N/A
20	19	20	16	10243.7	N/A
30	20	21	16	10694.3	N/A
50	21	21	17	11256.3	N/A
100	22	21	18	11952.2	N/A
Newsvendor Bounds Heuristic Policy					
b	s_3	s_2	s_1	Cost	% Error
2.5	18	18	13	7954.43	0.9
5	19	19	14	8715.86	1.0
7.5	20	20	14	9160.29	0.7
10	21	19	15	9519.34	0.8
20	21	20	16	10287.7	0.4
30	21	22	16	10748.9	0.5
50	22	22	17	11329.2	0.6
100	23	22	18	12017.9	0.5
Decoupling Heuristic Policy					
b	s_3	s_2	s_1	Cost	% Error
2.5	21	20	13	8376.9	6.3
5	22	19	14	8926.1	3.4
7.5	22	20	14	9282.6	2.0
10	22	19	15	9579.1	1.4
20	23	19	16	10338.5	0.9
30	23	20	16	10754.2	0.6
50	23	20	17	11289.7	0.3
100	24	19	18	12071.7	1.0

Table A5.3: Asymmetric Results for Non-Delayed Differentiation Network

Asymmetric Costs, Nondelayed (k=3) Results												
Echelon Holding Costs				Backordering Costs		Best Found Base Stock Policy						
h_3	h_2	$h_{1,1}$	$h_{1,2}$	b_1	b_2	s_3	$s_{2,1}$	$s_{2,2}$	$s_{1.1}$	$s_{1.2}$	Cost	%Error
1	1	1	2	10	10	18	11	12	14	13	10131	N/A
1	2	1	2	10	10	19	9	9	15	14	12891	N/A
2	1	1	2	10	10	15	10	12	15	13	14969	N/A
2	2	1	2	10	10	17	9	9	15	14	17625	N/A
1	1	1	1	5	10	18	10	10	13	15	9388	N/A
1	1	1	1	5	20	18	10	11	13	16	9839	N/A
1	1	1	1	10	20	18	10	11	13	16	10386	N/A
1	1	1	2	5	5	16	10	11	14	12	9134	N/A
1	1	1	2	5	10	18	10	12	13	13	9696	N/A
1	1	1	2	5	20	18	10	12	13	15	10274	N/A
1	1	1	2	10	5	18	11	10	14	12	9572	N/A
1	1	1	2	10	20	18	11	12	14	15	10709	N/A
1	1	1	2	20	5	19	12	10	15	12	10008	N/A
1	1	1	2	20	10	19	12	11	15	13	10566	N/A
1	1	1	2	20	20	18	12	12	15	15	11146	N/A
Echelon Holding Costs				Backordering Costs		Newsvendor Bounds Heuristic Policy						
h_3	h_2	$h_{1,1}$	$h_{1,2}$	b_1	b_2	s_3	$s_{2,1}$	$s_{2,2}$	$s_{1.1}$	$s_{1.2}$	Cost	%Error
1	1	1	2	10	10	18	11	12	15	13	10150	0.2
1	2	1	2	10	10	21	9	10	15	14	12988	0.8
2	1	1	2	10	10	15	11	12	15	14	15016	0.3
2	2	1	2	10	10	17	10	10	15	14	17703	0.4
1	1	1	1	5	10	18	10	11	14	15	9499	1.2
1	1	1	1	5	20	20	10	11	14	16	9930	0.9
1	1	1	1	10	20	19	11	11	15	16	10423	0.4
1	1	1	2	5	5	17	10	12	14	12	9174	0.4
1	1	1	2	5	10	19	10	12	14	13	9757	0.6
1	1	1	2	5	20	19	10	12	14	15	10332	0.6
1	1	1	2	10	5	18	11	12	15	12	9650	0.8
1	1	1	2	10	20	19	11	12	15	15	10764	0.5
1	1	1	2	20	5	19	11	12	16	12	10103	1.0
1	1	1	2	20	10	20	11	12	16	13	10618	0.5
1	1	1	2	20	20	19	11	12	16	15	11157	0.1

Table A5.3, Continued												
Echelon Holding Costs				Backordering Costs		Decoupling Heuristic Policy						
h_3	h_2	$h_{1,1}$	$h_{1,2}$	b_1	b_2	s_3	$s_{2,1}$	$s_{2,2}$	$s_{1.1}$	$s_{1.2}$	Cost	%Error
1	1	1	2	10	10	18	12	12	15	13	10226	0.9
1	2	1	2	10	10	18	12	11	15	14	13256	2.8
2	1	1	2	10	10	18	12	11	15	14	15345	2.5
2	2	1	2	10	10	18	12	11	15	14	18367	4.2
1	1	1	1	5	10	18	11	12	14	15	9575	2.0
1	1	1	1	5	20	19	11	12	14	16	10017	1.8
1	1	1	1	10	20	18	12	12	15	16	10475	0.9
1	1	1	2	5	5	19	11	11	14	12	9288	1.7
1	1	1	2	5	10	19	11	12	14	13	9840	1.5
1	1	1	2	5	20	19	11	11	14	15	10406	1.3
1	1	1	2	10	5	19	12	11	15	12	9730	1.7
1	1	1	2	10	20	18	12	11	15	15	10829	1.1
1	1	1	2	20	5	19	12	11	16	12	10080	0.7
1	1	1	2	20	10	18	12	12	16	13	10593	0.2
1	1	1	2	20	20	18	12	11	16	15	11195	0.4

Table A5.4: Asymmetric Results for Delayed Differentiation Network

Asymmetric Costs, Delayed (k=2) Results											
Echelon Holding Costs				Backordering Costs		Best Found Base Stock Policy					
h_3	h_2	$h_{1,1}$	$h_{1,2}$	b_1	b_2	s_3	s_2	$s_{1,1}$	$s_{1,2}$	Cost	% Error
1	1	1	2	10	10	19	21	14	13	9740.4	N/A
1	2	1	2	10	10	20	17	15	14	12373.4	N/A
2	1	1	2	10	10	17	19	15	14	14481.6	N/A
2	2	1	2	10	10	18	18	14	14	17033.9	N/A
1	1	1	1	5	10	18	19	13	15	9040.7	N/A
1	1	1	1	5	20	19	18	13	17	9456.5	N/A
1	1	1	1	10	20	19	20	14	16	9845.9	N/A
1	1	1	2	5	5	18	20	13	12	8826.0	N/A
1	1	1	2	5	10	19	19	13	14	9343.1	N/A
1	1	1	2	5	20	19	21	12	15	9893.7	N/A
1	1	1	2	10	5	19	20	14	12	9229.5	N/A
1	1	1	2	10	20	20	20	14	15	10266.8	N/A
1	1	1	2	20	5	19	20	16	12	9627.4	N/A
1	1	1	2	20	10	20	21	15	13	10137.6	N/A
1	1	1	2	20	20	20	21	15	15	10667.8	N/A
Echelon Holding Costs				Backordering Costs		Newsvendor Bounds Heuristic Policy					
h_3	h_2	$h_{1,1}$	$h_{1,2}$	b_1	b_2	s_3	s_2	$s_{1,1}$	$s_{1,2}$	Cost	% Error
1	1	1	2	10	10	20	21	15	13	9826.5	0.9
1	2	1	2	10	10	22	18	15	14	12520.6	1.2
2	1	1	2	10	10	18	20	15	14	14581.9	0.7
2	2	1	2	10	10	19	18	15	14	17150.7	0.7
1	1	1	1	5	10	20	19	14	15	9126.9	1.0
1	1	1	1	5	20	21	20	14	16	9623.2	1.8
1	1	1	1	10	20	21	20	15	16	9943.3	1.0
1	1	1	2	5	5	19	20	14	12	8905.7	0.9
1	1	1	2	5	10	20	21	14	13	9510.1	1.8
1	1	1	2	5	20	20	21	14	15	10040.3	1.5
1	1	1	2	10	5	20	21	15	12	9354.3	1.4
1	1	1	2	10	20	22	20	15	15	10392.8	1.2
1	1	1	2	20	5	20	22	16	12	9775.0	1.5
1	1	1	2	20	10	22	21	16	13	10265.6	1.3
1	1	1	2	20	20	21	21	16	15	10739.9	0.7

Table A5.4, Continued											
Echelon Holding Costs				Backordering Costs		Decoupling Heuristic Policy					
h_3	h_2	$h_{1,1}$	$h_{1,2}$	b_1	b_2	s_3	s_2	$s_{1,1}$	$s_{1,2}$	Cost	% Error
1	1	1	2	10	10	22	20	15	13	9869.2	1.3
1	2	1	2	10	10	22	19	15	14	12650.0	2.2
2	1	1	2	10	10	22	19	15	14	14946.1	3.2
2	2	1	2	10	10	22	19	15	14	17748.7	4.2
1	1	1	1	5	10	22	19	14	15	9254.2	2.4
1	1	1	1	5	20	22	20	14	16	9695.7	2.5
1	1	1	1	10	20	23	19	15	16	9995.7	1.5
1	1	1	2	5	5	21	20	14	12	9017.9	2.2
1	1	1	2	5	10	21	20	14	13	9498.5	1.7
1	1	1	2	5	20	22	19	14	15	10018.5	1.3
1	1	1	2	10	5	19	20	15	12	9247.8	0.2
1	1	1	2	10	20	22	19	15	15	10343.3	0.7
1	1	1	2	20	5	22	20	16	12	9739.8	1.2
1	1	1	2	20	10	22	20	16	13	10204.3	0.7
1	1	1	2	20	20	22	19	16	15	10695.5	0.3

Table A6.1: Serial System to Create Regressions

r	μ	σ^2	b	p	s_W^*	s_1^*	C^*
2	10	10	5	0.5	10	15	6.31
2	10	10	5	5	10	14	7.41
2	10	10	5	10	8	15	8.30
2	10	10	5	20	8	14	9.60
2	10	10	10	0.5	12	16	7.44
2	10	10	10	5	10	16	9.36
2	10	10	10	10	10	15	10.87
2	10	10	10	20	8	16	13.05
2	10	10	20	0.5	12	17	8.39
2	10	10	20	5	10	17	11.42
2	10	10	20	10	9	17	13.60
2	10	10	20	20	8	17	17.08
2	10	20	5	0.5	13	16	8.85
2	10	20	5	5	8	17	12.23
2	10	20	5	10	7	16	14.49
2	10	20	5	20	3	19	17.62
2	10	20	10	0.5	14	18	10.51
2	10	20	10	5	9	18	15.98
2	10	20	10	10	8	18	19.81
2	10	20	10	20	4	20	25.20
2	10	20	20	0.5	15	20	12.07
2	10	20	20	5	10	20	19.84
2	10	20	20	10	8	20	25.73
2	10	20	20	20	6	20	34.68
2	10	40	5	0.5	12	18	11.54
2	10	40	5	5	6	17	18.80
2	10	40	5	10	0	20	23.30
2	10	40	5	20	1	17	28.78
2	10	40	10	0.5	15	21	13.83
2	10	40	10	5	6	21	25.21
2	10	40	10	10	4	20	32.85
2	10	40	10	20	4	17	43.00
2	10	40	20	0.5	18	23	16.02
2	10	40	20	5	7	24	32.19
2	10	40	20	10	1	27	43.95
2	10	40	20	20	1	23	60.69
2	20	20	5	0.5	19	24	9.02
2	20	20	5	5	18	24	9.68
2	20	20	5	10	17	24	10.27
2	20	20	5	20	17	24	11.14
2	20	20	10	0.5	20	26	10.67
2	20	20	10	5	19	26	11.96

Table A6.1, Continued							
2	20	20	10	10	18	26	13.00
2	20	20	10	20	17	26	14.64
2	20	20	20	0.5	21	27	12.26
2	20	20	20	5	19	28	14.33
2	20	20	20	10	19	27	16.03
2	20	20	20	20	17	27	18.74
2	20	40	5	0.5	21	28	12.76
2	20	40	5	5	17	29	15.31
2	20	40	5	10	16	28	17.20
2	20	40	5	20	15	27	19.81
2	20	40	10	0.5	23	31	15.19
2	20	40	10	5	19	30	19.57
2	20	40	10	10	17	30	22.81
2	20	40	10	20	14	31	27.50
2	20	40	20	0.5	25	33	17.39
2	20	40	20	5	20	33	23.99
2	20	40	20	10	16	34	29.17
2	20	40	20	20	16	32	36.97
2	20	80	5	0.5	21	31	16.94
2	20	80	5	5	13	31	23.88
2	20	80	5	10	10	31	28.37
2	20	80	5	20	3	35	34.12
2	20	80	10	0.5	24	34	20.25
2	20	80	10	5	16	34	31.40
2	20	80	10	10	5	41	39.18
2	20	80	10	20	8	35	49.57
2	20	80	20	0.5	27	38	23.35
2	20	80	20	5	18	37	39.66
2	20	80	20	10	13	38	51.21
2	20	80	20	20	11	35	69.09
3	10	10	5	0.5	10	15	6.55
3	10	10	5	5	10	15	6.60
3	10	10	5	10	10	15	6.64
3	10	10	5	20	10	15	6.70
3	10	10	10	0.5	12	15	7.92
3	10	10	10	5	12	15	7.99
3	10	10	10	10	11	15	8.09
3	10	10	10	20	10	16	8.20
3	10	10	20	0.5	11	17	9.12
3	10	10	20	5	11	17	9.36
3	10	10	20	10	11	17	9.47
3	10	10	20	20	12	16	9.80
3	10	20	5	0.5	10	17	9.62
3	10	20	5	5	11	16	10.11

Table A6.1, Continued							
3	10	20	5	10	8	17	10.52
3	10	20	5	20	9	16	11.09
3	10	20	10	0.5	11	19	11.67
3	10	20	10	5	9	20	12.64
3	10	20	10	10	9	19	13.42
3	10	20	10	20	9	18	14.76
3	10	20	20	0.5	13	21	13.65
3	10	20	20	5	11	20	15.23
3	10	20	20	10	10	20	16.70
3	10	20	20	20	8	21	18.96
3	10	40	5	0.5	10	18	12.80
3	10	40	5	5	6	19	15.18
3	10	40	5	10	4	19	16.89
3	10	40	5	20	3	18	19.25
3	10	40	10	0.5	12	21	15.64
3	10	40	10	5	7	21	19.80
3	10	40	10	10	5	22	22.95
3	10	40	10	20	2	22	27.32
3	10	40	20	0.5	13	23	18.25
3	10	40	20	5	7	25	24.66
3	10	40	20	10	8	22	29.55
3	10	40	20	20	3	25	36.92
3	20	20	5	0.5	18	25	9.18
3	20	20	5	5	19	24	9.17
3	20	20	5	10	19	24	9.15
3	20	20	5	20	19	24	9.18
3	20	20	10	0.5	19	26	11.03
3	20	20	10	5	20	26	11.01
3	20	20	10	10	20	26	11.01
3	20	20	10	20	20	26	11.00
3	20	20	20	0.5	20	28	12.79
3	20	20	20	5	21	27	12.68
3	20	20	20	10	21	27	12.87
3	20	20	20	20	20	27	12.80
3	20	40	5	0.5	20	28	13.40
3	20	40	5	5	19	29	13.48
3	20	40	5	10	18	29	13.60
3	20	40	5	20	18	29	13.71
3	20	40	10	0.5	21	31	16.16
3	20	40	10	5	21	30	16.51
3	20	40	10	10	19	32	16.65
3	20	40	10	20	21	30	17.09
3	20	40	20	0.5	22	33	18.92
3	20	40	20	5	22	33	19.41

Table A6.1, Continued							
3	20	40	20	10	22	32	19.90
3	20	40	20	20	20	33	20.61
3	20	80	5	0.5	18	31	18.47
3	20	80	5	5	17	30	19.53
3	20	80	5	10	17	30	20.36
3	20	80	5	20	14	30	21.81
3	20	80	10	0.5	22	35	22.56
3	20	80	10	5	18	34	24.65
3	20	80	10	10	16	35	26.46
3	20	80	10	20	13	35	29.28
3	20	80	20	0.5	24	37	26.52
3	20	80	20	5	20	38	30.07
3	20	80	20	10	18	38	33.29
3	20	80	20	20	16	36	38.15

Table A6.2: Serial System Experimental Data Used to Test Regression Models

r	μ	σ^2	b	p	s_W^*	s_1^*	C^*	s_W^H	s_1^H	C^H
2	5	5	2	2	4	7	3.55	6	7	3.81
2	5	5	2	8	3	7	4.36	5	7	4.47
2	5	5	2	32	1	8	5.88	2	7	5.90
2	5	5	8	2	5	9	5.82	6	9	5.89
2	5	5	8	8	4	9	8.35	5	9	8.37
2	5	5	8	32	2	9	13.43	2	9	13.43
2	5	5	32	2	7	10	8.01	6	11	8.01
2	5	5	32	8	5	10	13.28	5	11	13.59
2	5	5	32	32	2	11	25.81	3	11	26.41
2	5	10	2	2	4	8	5.22	4	9	5.37
2	5	10	2	8	1	8	6.87	2	9	7.23
2	5	10	2	32	0	7	9.31	0	7	9.31
2	5	10	8	2	6	10	9.05	6	12	9.16
2	5	10	8	8	1	12	14.24	2	12	14.32
2	5	10	8	32	0	10	23.56	0	11	23.50
2	5	10	32	2	8	13	13.17	7	15	13.45
2	5	10	32	8	3	14	24.08	4	15	24.30
2	5	10	32	32	0	14	48.54	0	14	49.35
2	5	15	2	2	3	6	5.90	2	8	5.92
2	5	15	2	8	0	7	7.97	0	8	8.30
2	5	15	2	32	0	4	10.63	0	4	10.63
2	5	15	8	2	5	10	10.79	5	11	10.86
2	5	15	8	8	1	10	17.68	1	11	17.89
2	5	15	8	32	0	7	29.21	0	7	29.21
2	5	15	32	2	7	13	16.34	7	15	16.42
2	5	15	32	8	3	13	31.30	3	15	32.25
2	5	15	32	32	0	12	63.90	0	12	63.90
2	15	15	2	2	14	19	5.84	12	19	6.49
2	15	15	2	8	13	19	6.10	12	19	6.53
2	15	15	2	32	12	19	6.78	11	19	7.34
2	15	15	8	2	16	21	9.26	16	21	9.42
2	15	15	8	8	15	21	10.39	14	21	10.81
2	15	15	8	32	12	22	13.27	12	21	13.89
2	15	15	32	2	17	24	12.44	17	24	12.52
2	15	15	32	8	16	24	15.47	15	24	15.58
2	15	15	32	32	14	23	23.17	13	24	23.31
2	15	30	2	2	13	20	8.52	13	20	8.52
2	15	30	2	8	11	20	9.68	12	20	9.74
2	15	30	2	32	9	20	11.86	9	20	11.90
2	15	30	8	2	17	24	14.07	15	24	14.24
2	15	30	8	8	11	26	18.28	12	24	18.55
2	15	30	8	32	9	24	26.65	9	24	26.65

Table A6.2, Continued										
2	15	30	32	2	18	29	19.66	18	29	19.74
2	15	30	32	8	15	28	29.29	13	29	29.96
2	15	30	32	32	9	29	51.38	9	29	52.71
2	15	45	2	2	11	21	10.36	11	21	10.36
2	15	45	2	8	5	23	12.54	8	21	12.60
2	15	45	2	32	1	23	16.25	4	21	16.26
2	15	45	8	2	15	25	17.66	14	26	17.65
2	15	45	8	8	12	24	25.05	10	26	24.97
2	15	45	8	32	7	23	38.99	4	26	39.11
2	15	45	32	2	18	31	25.51	17	31	25.72
2	15	45	32	8	12	31	41.25	12	31	41.50
2	15	45	32	32	8	29	78.35	6	31	79.12
2	30	30	2	2	27	33	8.18	25	32	9.15
2	30	30	2	8	27	33	8.23	25	32	9.15
2	30	30	2	32	26	33	8.43	25	32	9.75
2	30	30	8	2	30	37	12.72	28	36	13.95
2	30	30	8	8	29	37	13.12	28	36	14.08
2	30	30	8	32	28	37	14.37	27	36	15.52
2	30	30	32	2	31	41	16.92	32	39	17.32
2	30	30	32	8	31	40	18.36	31	39	18.69
2	30	30	32	32	29	40	22.17	30	39	22.57
2	30	60	2	2	28	37	11.77	26	35	13.02
2	30	60	2	8	26	38	12.22	25	35	13.69
2	30	60	2	32	25	37	13.40	23	35	15.22
2	30	60	8	2	32	43	18.84	30	41	20.37
2	30	60	8	8	28	44	21.16	27	41	24.01
2	30	60	8	32	26	43	26.40	24	41	30.62
2	30	60	32	2	34	49	25.78	32	47	28.76
2	30	60	32	8	31	48	31.97	29	47	35.61
2	30	60	32	32	27	48	46.76	25	47	52.97
2	30	90	2	2	26	38	14.67	25	38	14.84
2	30	90	2	8	23	38	16.12	23	38	16.21
2	30	90	2	32	19	38	18.83	19	38	18.94
2	30	90	8	2	29	47	24.11	30	44	24.48
2	30	90	8	8	26	44	29.59	26	44	29.88
2	30	90	8	32	21	44	41.17	21	44	41.48
2	30	90	32	2	35	52	33.44	33	52	34.32
2	30	90	32	8	28	52	47.12	28	52	47.53
2	30	90	32	32	23	51	78.62	22	52	78.83
3	5	5	2	2	5	7	3.43	6	7	3.63
3	5	5	2	8	4	7	3.54	5	7	3.60
3	5	5	2	32	4	7	3.84	4	7	3.84
3	5	5	8	2	5	9	5.53	6	9	5.56
3	5	5	8	8	5	9	6.03	5	9	6.03

Table A6.2, Continued										
3	5	5	8	32	4	9	7.33	4	9	7.33
3	5	5	32	2	7	10	7.47	7	11	7.72
3	5	5	32	8	6	10	8.83	6	11	9.18
3	5	5	32	32	5	10	12.38	5	11	13.50
3	5	10	2	2	5	9	5.39	5	9	5.39
3	5	10	2	8	3	8	5.46	4	9	6.09
3	5	10	2	32	0	9	6.52	1	9	6.57
3	5	10	8	2	5	11	8.48	5	12	8.65
3	5	10	8	8	4	12	10.88	4	12	10.88
3	5	10	8	32	1	11	14.53	0	12	14.54
3	5	10	32	2	7	15	12.63	7	15	12.63
3	5	10	32	8	5	13	16.66	4	15	16.95
3	5	10	32	32	4	12	27.97	1	15	27.89
3	5	15	2	2	2	7	5.63	3	8	5.79
3	5	15	2	8	0	8	6.50	2	8	7.11
3	5	15	2	32	0	6	8.17	0	6	8.17
3	5	15	8	2	5	10	10.10	5	11	10.25
3	5	15	8	8	3	9	13.59	2	11	13.70
3	5	15	8	32	0	9	20.22	0	10	20.51
3	5	15	32	2	7	14	15.12	6	15	15.17
3	5	15	32	8	4	13	22.98	4	15	23.99
3	5	15	32	32	0	13	41.39	0	14	41.84
3	15	15	2	2	14	19	5.82	12	19	6.10
3	15	15	2	8	14	19	5.80	12	19	6.07
3	15	15	2	32	14	19	5.80	12	19	6.07
3	15	15	8	2	17	21	9.10	15	21	9.38
3	15	15	8	8	16	22	9.15	15	21	9.41
3	15	15	8	32	17	21	9.16	15	21	9.40
3	15	15	32	2	17	24	12.12	17	24	12.12
3	15	15	32	8	17	24	12.17	17	24	12.17
3	15	15	32	32	17	24	12.42	17	24	12.42
3	15	30	2	2	14	20	8.38	14	20	8.38
3	15	30	2	8	13	20	8.41	14	20	8.41
3	15	30	2	32	12	21	8.56	13	20	8.55
3	15	30	8	2	17	24	13.52	17	24	13.52
3	15	30	8	8	15	24	13.97	15	24	13.97
3	15	30	8	32	14	25	15.02	14	24	15.06
3	15	30	32	2	19	28	18.55	18	29	18.54
3	15	30	32	8	16	29	20.01	16	29	20.01
3	15	30	32	32	14	29	23.97	14	29	23.97
3	15	45	2	2	12	20	10.08	12	21	10.13
3	15	45	2	8	12	19	10.35	11	21	10.36
3	15	45	2	32	9	21	10.96	10	21	11.08
3	15	45	8	2	15	25	16.73	16	26	16.81

Table A6.2, Continued										
3	15	45	8	8	14	24	18.29	14	26	18.33
3	15	45	8	32	10	26	21.79	10	26	21.79
3	15	45	32	2	18	31	23.75	18	31	23.75
3	15	45	32	8	14	31	27.96	14	31	27.96
3	15	45	32	32	10	32	38.58	10	31	38.62
3	30	30	2	2	27	33	8.17	23	32	9.77
3	30	30	2	8	26	34	8.18	23	32	9.74
3	30	30	2	32	27	33	8.17	23	32	9.75
3	30	30	8	2	30	37	12.72	27	36	14.02
3	30	30	8	8	29	37	12.73	27	36	13.96
3	30	30	8	32	30	37	12.68	27	36	13.96
3	30	30	32	2	32	41	16.81	31	39	17.42
3	30	30	32	8	32	40	16.86	31	39	17.54
3	30	30	32	32	31	41	16.80	31	39	17.43
3	30	60	2	2	28	38	11.74	26	35	12.62
3	30	60	2	8	29	37	11.73	26	35	12.61
3	30	60	2	32	28	37	11.76	25	35	13.06
3	30	60	8	2	31	44	18.59	28	41	21.07
3	30	60	8	8	33	43	18.59	28	41	21.13
3	30	60	8	32	31	44	18.71	28	41	21.18
3	30	60	32	2	35	49	25.14	32	47	26.84
3	30	60	32	8	36	48	25.18	32	47	26.77
3	30	60	32	32	34	49	25.70	31	47	28.00
3	30	90	2	2	26	39	14.48	26	38	14.47
3	30	90	2	8	26	39	14.52	26	38	14.50
3	30	90	2	32	25	38	14.65	25	38	14.65
3	30	90	8	2	33	44	23.36	31	44	23.46
3	30	90	8	8	29	46	23.63	30	44	23.81
3	30	90	8	32	28	46	24.41	29	44	24.58
3	30	90	32	2	36	51	31.88	34	52	31.88
3	30	90	32	8	34	52	33.15	33	52	33.19
3	30	90	32	32	31	53	36.79	30	52	36.71

Table A6.3: Distribution System Experimental Data Used to Test Regression Model

r	μ	σ^2	b	p	s_W^*	s_1^*	C^*	s_W^H	s_1^H	Ca	ErrA
2	5	5	2	2	9	7	6.30	11	7	6.75	7.09%
2	5	5	2	8	9	6	7.48	11	6	8.21	9.77%
2	5	5	2	32	8	5	9.82	7	5	10.32	5.10%
2	5	5	8	2	11	8	10.50	12	8	10.48	-0.21%
2	5	5	8	8	9	8	14.25	10	8	14.48	1.59%
2	5	5	8	32	8	7	22.33	7	7	22.90	2.58%
2	5	5	32	2	12	10	14.60	12	10	14.60	0.00%
2	5	5	32	8	9	10	23.00	12	9	23.53	2.32%
2	5	5	32	32	8	9	42.75	8	9	42.75	0.00%
2	5	10	2	2	9	7	9.32	11	7	9.59	2.94%
2	5	10	2	8	7	6	11.95	9	6	12.18	1.89%
2	5	10	2	32	6	5	16.13	5	4	17.15	6.36%
2	5	10	8	2	12	10	16.51	16	9	17.55	6.33%
2	5	10	8	8	8	9	24.80	8	9	24.80	0.00%
2	5	10	8	32	7	7	40.59	7	7	40.59	0.00%
2	5	10	32	2	14	13	24.26	17	12	24.84	2.38%
2	5	10	32	8	11	11	42.50	14	11	44.34	4.33%
2	5	10	32	32	7	10	81.91	8	9	84.58	3.26%
2	5	15	2	2	7	6	10.62	3	8	10.80	1.67%
2	5	15	2	8	6	4	14.18	3	6	14.48	2.13%
2	5	15	2	32	4	3	18.83	0	4	19.84	5.39%
2	5	15	8	2	10	9	19.69	10	10	19.93	1.22%
2	5	15	8	8	7	7	31.49	5	9	32.03	1.71%
2	5	15	8	32	5	5	50.92	1	6	53.71	5.49%
2	5	15	32	2	13	13	30.17	15	13	30.24	0.22%
2	5	15	32	8	9	11	56.02	10	12	58.33	4.13%
2	5	15	32	32	7	8	110.09	3	10	113.43	3.03%
2	15	15	2	2	27	19	10.40	22	19	11.46	10.23%
2	15	15	2	8	28	18	10.60	24	18	11.43	7.84%
2	15	15	2	32	27	18	11.25	23	18	12.33	9.65%
2	15	15	8	2	30	21	16.50	25	22	17.74	7.48%
2	15	15	8	8	29	21	17.68	25	21	19.59	10.80%
2	15	15	8	32	28	20	20.94	23	21	23.32	11.35%
2	15	15	32	2	32	24	22.41	29	24	23.10	3.09%
2	15	15	32	8	29	24	26.05	26	24	27.53	5.67%
2	15	15	32	32	27	23	35.29	26	23	37.01	4.87%
2	15	30	2	2	26	20	15.12	26	19	15.16	0.28%
2	15	30	2	8	25	19	16.53	26	18	16.62	0.54%
2	15	30	2	32	25	17	19.69	24	17	19.77	0.44%

Table A6.3, Continued

2	15	30	8	2	30	24	25.09	29	23	25.71	2.47%
2	15	30	8	8	27	23	30.99	24	23	31.89	2.90%
2	15	30	8	32	26	21	43.18	24	21	43.07	-0.25%
2	15	30	32	2	34	28	35.51	33	27	35.70	0.52%
2	15	30	32	8	27	27	48.01	28	26	51.10	6.44%
2	15	30	32	32	28	24	80.06	26	24	84.63	5.70%
2	15	45	2	2	24	19	18.41	26	19	18.49	0.41%
2	15	45	2	8	23	17	21.48	24	18	21.71	1.03%
2	15	45	2	32	21	15	27.19	22	16	28.00	2.97%
2	15	45	8	2	27	25	31.59	27	25	31.59	0.00%
2	15	45	8	8	25	23	42.78	25	23	42.78	0.00%
2	15	45	8	32	23	19	64.57	21	21	65.52	1.48%
2	15	45	32	2	33	29	46.17	33	29	46.17	0.00%
2	15	45	32	8	27	27	70.07	29	27	71.12	1.49%
2	15	45	32	32	22	25	128.32	24	25	133.22	3.82%
3	5	5	2	2	9	7	6.11	10	7	6.24	2.24%
3	5	5	2	8	8	7	6.20	8	7	6.20	0.00%
3	5	5	2	32	8	7	6.57	6	7	6.93	5.41%
3	5	5	8	2	10	9	9.89	9	9	9.90	0.08%
3	5	5	8	8	9	9	10.54	7	9	11.14	5.68%
3	5	5	8	32	10	8	12.01	8	8	12.60	4.91%
3	5	5	32	2	12	10	13.52	13	10	13.66	1.02%
3	5	5	32	8	11	10	15.15	11	10	15.15	0.00%
3	5	5	32	32	9	10	19.64	11	9	19.95	1.57%
3	5	10	2	2	9	7	8.92	11	7	9.13	2.34%
3	5	10	2	8	8	7	9.45	12	6	10.02	5.94%
3	5	10	2	32	8	6	10.88	6	6	11.30	3.81%
3	5	10	8	2	11	10	15.37	11	10	15.37	0.00%
3	5	10	8	8	10	9	17.87	11	9	17.96	0.50%
3	5	10	8	32	9	8	24.07	4	9	26.72	11.02%
3	5	10	32	2	12	13	22.10	16	12	22.76	3.01%
3	5	10	32	8	11	12	28.90	11	12	28.90	0.00%
3	5	10	32	32	9	11	46.04	7	11	47.10	2.29%
3	5	15	2	2	8	6	10.11	4	8	10.34	2.24%
3	5	15	2	8	8	5	11.73	4	7	11.94	1.82%
3	5	15	2	32	6	3	14.62	1	5	15.24	4.29%
3	5	15	8	2	10	9	18.36	9	10	18.40	0.22%
3	5	15	8	8	4	10	24.18	4	10	24.18	0.00%
3	5	15	8	32	6	6	35.11	1	6	40.91	16.51%
3	5	15	32	2	13	13	27.49	13	13	27.49	0.00%
3	5	15	32	8	9	13	42.05	9	13	42.05	0.00%

Table A6.3, Continued											
3	5	15	32	32	9	9	72.53	3	11	72.65	0.16%
3	15	15	2	2	27	19	10.35	21	19	12.02	16.15%
3	15	15	2	8	27	19	10.35	21	19	11.94	15.35%
3	15	15	2	32	27	19	10.39	21	19	12.07	16.25%
3	15	15	8	2	31	21	16.41	24	22	18.41	12.23%
3	15	15	8	8	31	21	16.43	24	22	18.14	10.42%
3	15	15	8	32	31	21	16.31	24	22	18.14	11.24%
3	15	15	32	2	32	24	21.93	28	24	23.37	6.59%
3	15	15	32	8	32	24	21.78	28	24	23.00	5.59%
3	15	15	32	32	31	24	22.09	28	24	23.44	6.15%
3	15	30	2	2	26	20	14.89	27	19	14.93	0.24%
3	15	30	2	8	26	20	14.94	27	19	14.99	0.29%
3	15	30	2	32	26	20	15.04	25	19	15.29	1.63%
3	15	30	8	2	30	24	24.40	28	24	24.64	0.96%
3	15	30	8	8	29	24	24.54	26	24	25.24	2.87%
3	15	30	8	32	28	24	25.05	27	23	25.86	3.20%
3	15	30	32	2	33	28	33.35	33	27	33.73	1.12%
3	15	30	32	8	32	28	34.46	31	27	35.06	1.74%
3	15	30	32	32	31	27	38.36	28	27	39.56	3.14%
3	15	45	2	2	25	19	17.90	25	20	18.05	0.84%
3	15	45	2	8	24	19	18.08	26	19	18.20	0.67%
3	15	45	2	32	22	19	18.86	25	19	19.07	1.14%
3	15	45	8	2	28	25	29.85	30	25	30.02	0.56%
3	15	45	8	8	28	24	31.36	27	25	31.48	0.38%
3	15	45	8	32	26	23	35.91	25	24	36.20	0.79%
3	15	45	32	2	32	30	42.44	32	30	42.44	0.00%
3	15	45	32	8	28	30	47.31	28	30	47.31	0.00%
3	15	45	32	32	26	28	62.84	26	28	62.84	0.00%

Table A6.4: Two Symmetric Retailer Experimental Data

r	μ	σ^2	b	p	s_W^*	s_1^*	C^*	s_W^H	s_1^H
2	10	5	0.5	18	15	11.56	18	15	11.56
2	10	5	5	17	15	12.89	16	15	12.92
2	10	5	10	18	14	14.00	16	14	14.59
2	10	5	20	17	14	15.49	14	14	16.92
2	10	10	0.5	19	16	13.92	19	16	13.92
2	10	10	5	18	16	16.19	17	16	16.25
2	10	10	10	17	16	18.22	18	15	18.17
2	10	10	20	17	15	21.11	17	15	21.11
2	10	20	0.5	20	17	15.94	20	17	15.94
2	10	20	5	18	17	19.67	18	17	19.67
2	10	20	10	18	16	22.96	18	16	22.96
2	10	20	20	18	16	27.43	17	16	27.60
2	20	5	0.5	19	17	16.50	19	17	16.50
2	20	5	5	19	15	21.07	15	16	21.35
2	20	5	10	18	14	24.59	15	15	24.69
2	20	5	20	16	14	29.60	12	15	29.89
2	20	10	0.5	20	19	19.87	22	18	20.56
2	20	10	5	19	17	27.68	18	17	27.89
2	20	10	10	19	16	33.48	18	16	33.41
2	20	10	20	13	17	42.35	14	16	42.34
2	20	20	0.5	19	22	23.26	23	20	23.75
2	20	20	5	20	19	34.95	19	19	35.11
2	20	20	10	18	18	44.20	17	18	44.84
2	20	20	20	17	17	57.58	14	18	58.11
3	10	5	0.5	20	14	11.76	17	15	11.92
3	10	5	5	20	14	11.82	17	15	11.95
3	10	5	10	18	15	11.91	17	15	12.03
3	10	5	20	19	14	11.87	17	15	11.93
3	10	10	0.5	19	16	14.35	18	16	14.52
3	10	10	5	19	16	14.35	18	16	14.53
3	10	10	10	19	16	14.38	17	16	14.91
3	10	10	20	19	16	14.49	17	16	14.98
3	10	20	0.5	20	17	16.64	18	17	17.25
3	10	20	5	20	17	16.67	18	17	17.23
3	10	20	10	20	17	16.73	18	17	17.21
3	10	20	20	20	17	16.92	18	17	17.33
3	20	5	0.5	20	16	17.45	17	17	17.55
3	20	5	5	19	16	17.73	19	16	17.73
3	20	5	10	19	16	18.15	17	16	18.27
3	20	5	20	19	15	18.98	17	16	19.02
3	20	10	0.5	21	18	21.30	20	18	21.35
3	20	10	5	22	17	22.30	19	18	22.27
3	20	10	10	21	17	22.98	20	17	22.99
3	20	10	20	20	17	24.67	19	17	24.65
3	20	20	0.5	21	20	25.21	22	20	25.11
3	20	20	5	21	19	27.00	21	19	27.00
3	20	20	10	21	19	28.52	19	19	28.87
3	20	20	20	18	19	31.35	18	19	31.35

Table A6.5: Four Symmetric Retailer Experimental Data

r	μ	σ^2	b	p	s_W^*	s_1^*	C^*	s_W^H	s_1^H
2	5	5	0.5	15	8	11.14	16	8	11.13
2	5	5	5	17	7	12.83	14	7	13.26
2	5	5	10	19	6	14.15	14	7	14.23
2	5	5	20	17	6	15.98	12	7	16.29
2	5	10	0.5	20	8	13.47	21	9	14.72
2	5	10	5	19	7	16.16	14	8	16.33
2	5	10	10	18	7	18.27	14	8	18.43
2	5	10	20	15	7	21.40	11	8	21.72
2	5	20	0.5	19	9	16.09	20	10	17.10
2	5	20	5	19	8	20.52	16	9	20.78
2	5	20	10	17	8	24.06	13	9	24.45
2	5	20	20	18	7	28.89	14	8	29.39
2	10	5	0.5	18	8	15.54	19	9	16.09
2	10	5	5	19	6	19.87	16	8	20.97
2	10	5	10	16	6	22.59	8	8	23.49
2	10	5	20	13	6	26.40	9	7	27.08
2	10	10	0.5	20	9	18.92	20	11	20.45
2	10	10	5	19	7	25.79	16	9	27.15
2	10	10	10	20	6	30.31	13	9	33.10
2	10	10	20	12	7	36.70	8	8	37.24
2	10	20	0.5	22	10	22.95	22	12	24.08
2	10	20	5	21	8	33.97	14	11	35.82
2	10	20	10	20	7	41.52	15	10	45.77
2	10	20	20	16	7	51.31	9	10	56.67
3	5	5	0.5	18	7	11.45	19	8	12.29
3	5	5	5	18	7	11.54	16	8	11.62
3	5	5	10	18	7	11.65	13	8	11.67
3	5	5	20	17	7	11.81	13	8	11.85
3	5	10	0.5	18	8	13.92	18	9	14.79
3	5	10	5	17	8	14.13	15	9	14.28
3	5	10	10	17	8	14.33	12	9	14.47
3	5	10	20	16	8	14.69	12	9	14.88
3	5	20	0.5	18	9	16.94	17	10	17.79
3	5	20	5	17	9	17.35	15	10	17.87
3	5	20	10	20	8	17.71	16	9	17.67
3	5	20	20	20	8	18.38	16	9	18.45
3	10	5	0.5	16	8	16.35	17	10	18.72
3	10	5	5	18	7	17.07	13	9	17.58
3	10	5	10	17	7	17.68	10	9	18.42
3	10	5	20	16	7	18.82	11	8	19.02
3	10	10	0.5	18	9	20.20	19	11	23.01
3	10	10	5	19	8	21.49	18	10	23.68
3	10	10	10	17	8	22.74	15	10	25.15
3	10	10	20	15	8	24.77	11	9	25.16
3	10	20	0.5	19	10	24.73	19	12	26.48
3	10	20	5	20	9	27.34	13	12	29.21
3	10	20	10	18	9	29.54	14	11	30.99
3	10	20	20	14	9	33.51	9	11	34.30

Table A6.6: Asymmetric Backorder Cost Experiment Data

r	μ	σ^2	p	s_W^*	s_1^*	s_2^*	C^*	s_W^H	s_1^H	s_2^H	C^H
2	10	10	0.5	21	14	16	12.54	21	14	15	12.55
2	10	10	5	20	14	15	14.36	19	14	15	14.37
2	10	10	10	19	14	15	15.92	17	14	15	16.14
2	10	10	20	19	13	14	18.24	16	14	15	18.54
2	10	20	0.5	21	14	17	13.50	21	14	17	13.50
2	10	20	5	19	14	17	16.17	19	15	16	16.51
2	10	20	10	19	14	16	18.30	18	14	16	18.41
2	10	20	20	19	13	15	21.66	16	14	16	22.06
2	20	10	0.5	23	16	18	17.98	24	15	17	18.05
2	20	10	5	19	15	17	24.34	19	15	16	24.43
2	20	10	10	18	14	16	29.04	19	15	15	29.57
2	20	10	20	15	14	16	35.62	15	15	15	36.07
2	20	20	0.5	24	16	20	19.56	24	16	19	19.61
2	20	20	5	19	15	19	28.08	19	16	18	28.30
2	20	20	10	18	14	18	34.50	19	15	17	34.74
2	20	20	20	17	13	17	43.69	16	15	17	44.06
2	40	10	0.5	24	17	21	23.83	29	15	18	24.28
2	40	10	5	16	15	18	38.90	19	15	17	39.13
2	40	10	10	13	14	17	48.98	16	15	15	49.88
2	40	10	20	12	12	15	61.98	12	14	15	62.56
2	40	20	0.5	24	17	24	26.03	28	16	21	26.31
2	40	20	5	17	15	21	45.25	16	18	20	46.41
2	40	20	10	14	14	20	58.78	12	17	19	60.77
2	40	20	20	13	12	17	77.33	11	15	18	79.15
3	10	10	0.5	20	14	16	12.94	19	14	16	12.96
3	10	10	5	20	14	16	12.99	18	14	16	13.20
3	10	10	10	20	14	16	13.02	19	14	15	13.22
3	10	10	20	20	14	16	13.06	19	14	15	13.34
3	10	20	0.5	21	14	17	14.15	19	14	17	14.22
3	10	20	5	20	14	17	14.17	19	14	17	14.29
3	10	20	10	20	14	17	14.26	19	14	17	14.31
3	10	20	20	20	14	17	14.36	19	14	17	14.45
3	20	10	0.5	20	17	18	19.25	23	15	17	19.33
3	20	10	5	20	16	18	19.83	22	15	17	19.94
3	20	10	10	20	16	17	20.58	20	16	17	20.58
3	20	10	20	18	16	17	21.58	20	15	16	21.67
3	20	20	0.5	20	16	21	21.18	24	15	19	21.27
3	20	20	5	21	16	19	22.20	21	16	19	22.20
3	20	20	10	20	16	19	23.21	21	16	18	23.55
3	20	20	20	18	16	19	25.02	19	16	18	25.28
3	40	10	0.5	21	17	20	26.28	27	15	18	26.91
3	40	10	5	17	16	19	30.78	21	15	17	31.06
3	40	10	10	16	15	18	34.43	20	15	16	34.82
3	40	10	20	15	14	17	39.73	17	15	16	39.77
3	40	20	0.5	22	16	23	28.89	26	16	22	29.06
3	40	20	5	17	16	22	35.24	18	18	21	35.43
3	40	20	10	16	15	21	40.50	17	17	20	40.82
3	40	20	20	16	14	19	48.02	16	16	18	49.10

Table A6.7: Asymmetric Demand Experiment Data

r	σ_1^2	σ_2^2	b	p	s_W^*	s_1^*	s_2^*	C^*	s_W^H	s_1^H	s_2^H	C^H
2	5	15	5	0.5	20	8	21	11.28	21	8	20	11.26
2	5	15	5	5	20	7	20	12.77	18	7	22	12.95
2	5	15	5	10	18	7	21	13.97	17	7	22	14.09
2	5	15	5	20	19	6	20	15.70	17	6	21	15.84
2	5	15	10	0.5	21	9	22	13.49	22	9	22	13.57
2	5	15	10	5	20	8	22	16.07	18	8	24	16.44
2	5	15	10	10	18	8	23	18.25	17	8	24	18.42
2	5	15	10	20	15	8	25	21.19	15	8	25	21.19
2	5	15	20	0.5	23	10	23	15.50	23	10	23	15.50
2	5	15	20	5	20	9	24	19.65	19	9	26	20.28
2	5	15	20	10	20	8	23	23.01	17	9	26	23.21
2	5	15	20	20	19	8	23	27.94	15	9	26	28.60
2	10	30	5	0.5	22	9	23	16.03	25	8	21	16.40
2	10	30	5	5	17	8	23	20.79	20	7	22	20.92
2	10	30	5	10	17	7	22	24.08	15	7	24	24.37
2	10	30	5	20	16	6	21	28.65	17	6	21	28.61
2	10	30	10	0.5	24	11	25	19.31	25	10	24	19.33
2	10	30	10	5	19	9	25	27.06	19	9	25	27.06
2	10	30	10	10	18	8	24	33.00	16	9	26	33.36
2	10	30	10	20	17	7	23	41.17	12	9	27	41.61
2	10	30	20	0.5	26	12	27	22.37	27	11	26	22.46
2	10	30	20	5	19	11	28	34.21	19	11	28	34.21
2	10	30	20	10	15	11	29	43.28	15	11	29	43.28
2	10	30	20	20	15	9	27	56.64	14	10	28	56.81
3	5	15	5	0.5	19	8	21	11.59	20	8	19	11.90
3	5	15	5	5	19	8	21	11.70	19	8	20	11.88
3	5	15	5	10	19	8	21	11.81	17	8	21	12.23
3	5	15	5	20	21	7	20	11.94	18	7	21	12.11
3	5	15	10	0.5	21	9	22	14.05	20	10	21	14.83
3	5	15	10	5	21	8	22	14.25	19	9	22	14.54
3	5	15	10	10	21	8	22	14.38	17	9	24	14.70
3	5	15	10	20	21	8	22	14.68	18	8	24	14.77
3	5	15	20	0.5	21	10	23	16.36	21	10	23	16.45
3	5	15	20	5	22	9	23	16.70	18	10	25	16.96
3	5	15	20	10	22	9	23	17.03	14	10	29	18.00
3	5	15	20	20	20	9	24	17.63	15	9	29	19.46
3	10	30	5	0.5	20	9	23	16.98	25	9	20	17.75
3	10	30	5	5	20	8	23	17.66	22	7	23	18.13
3	10	30	5	10	19	8	23	18.21	20	7	24	18.63
3	10	30	5	20	19	7	23	19.15	20	6	24	19.87
3	10	30	10	0.5	21	10	25	20.82	26	10	23	21.27
3	10	30	10	5	20	10	25	22.23	20	9	27	22.37
3	10	30	10	10	20	9	25	23.32	18	9	28	23.78
3	10	30	10	20	20	8	25	25.16	17	8	28	25.76
3	10	30	20	0.5	22	12	28	24.50	26	11	26	24.60
3	10	30	20	5	21	11	28	26.90	19	11	30	27.26
3	10	30	20	10	21	10	27	28.97	15	11	33	29.91
3	10	30	20	20	19	10	28	32.13	16	10	32	33.03

APPENDIX 2
PROOFS OF ANALYTICAL PROPOSITIONS

Define the critical fractiles

$$\Theta_\alpha = \frac{b_\alpha + h_W}{b_\alpha + h_W + h_\alpha} \quad \Theta_{W,\alpha}^{l,d} = \frac{b_\alpha}{b_\alpha + h_W + h_\alpha} \qquad \Theta_W^{l,c} = \frac{b}{b + h_W + h_\alpha} \quad \Theta_W^{u,c} = \frac{b}{b + h_W}$$

Let

$z_\alpha = \Phi^{-1}\left(\Theta_\alpha\right)$, $z_{W,\alpha}^{l,d} = \Phi^{-1}\left(\Theta_{W,\alpha}^{l,d}\right)$, $z_{W,\alpha}^{u,d} = \Phi^{-1}\left(\Theta_{W,\alpha}^{u,d}\right)$, $z_W^{l,c} = \Phi^{-1}\left(\Theta_W^{l,c}\right)$ and $z_W^{u,c} = \Phi^{-1}\left(\Theta_W^{u,c}\right)$.

Let $\phi(\cdot)$ and $\Phi(\cdot)$ represent the standard normal pdf and cdf, respectively. Following the approach in Zipkin (2000) (see also Shang and Song, 2003),

$$s_\alpha = \mu_\alpha \tilde{L}_\alpha + z_\alpha \sqrt{\sigma_\alpha{}^2 \tilde{L}_\alpha} \tag{A1}$$

$$s_{W,\alpha}^{l,d} = \mu_\alpha \tilde{L}_W + z_{W,\alpha}^{l,d} \sqrt{\sigma_\alpha{}^2 \tilde{L}_{W,\alpha}} \tag{A2}$$

$$s_{W,\alpha}^{u,d} = \mu_\alpha \tilde{L}_W + z_{W,\alpha}^{u,d} \sqrt{\sigma_\alpha{}^2 \tilde{L}_{W,\alpha}} \tag{A3}$$

$$s_W^{l,c} = \mu \tilde{L}_W + z_W^{l,c} \sqrt{\sigma^2 \tilde{L}_W} \tag{A4}$$

$$s_W^{u,c} = \mu \tilde{L}_W + z_W^{u,c} \sqrt{\sigma^2 \tilde{L}_W} \tag{A5}$$

$$s_W^a = \mu \tilde{L}_W + \frac{(z_W^{u,c} + z_W^{l,c})}{4} \sqrt{\sigma^2 \tilde{L}_W} + \sum_{i=1}^{n} \frac{(z_{W,\alpha}^{u,d} + z_{W,\alpha}^{l,d})}{4} \sqrt{\sigma_\alpha{}^2 \tilde{L}_W} \tag{A6}$$

$$C_\alpha\left(s_\alpha\right) = \left(b_\alpha + h_\alpha + h_W\right)\phi\left(z_\alpha\right)\sqrt{\sigma_\alpha{}^2 \tilde{L}_\alpha} \tag{A7}$$

$$C_{W,\alpha}^{l,d}\left(s_{W,\alpha}^{u,d}\right) = \left(b_\alpha + h_W\right)\phi\left(z_{W,\alpha}^{u,d}\right)\sqrt{\sigma_\alpha{}^2 \tilde{L}_{W,\alpha}} + h_W \mu_\alpha \tilde{L}_{W,\alpha} \tag{A8}$$

$$C_W^{l,d} = \sum_{\alpha=1}^{n} C_{W,\alpha}^{l,d} \tag{A9}$$

$$C_{W,\alpha}^{u,d}\left(s_{W,\alpha}^{l,d}\right) = \left(b_\alpha + h_W + h_\alpha\right)\phi\left(z_{W,\alpha}^{l,d}\right)\sqrt{\sigma_\alpha{}^2 \tilde{L}_{W,\alpha}} + h_W \mu_\alpha \tilde{L}_{W,\alpha} \tag{A10}$$

$$C_W^{u,d} = \sum_{\alpha=1}^{n} C_{W,\alpha}^{u,d} \tag{A11}$$

$$C_W^{l,c}\left(s_W^{u,c}\right)=\left(b+h_W\right)\phi\left(z_W^{u,c}\right)\sqrt{\sigma^2\tilde{L}_W}+h_W\mu\tilde{L}_W \quad \text{(A12)}$$

$$C_W^{u,c}\left(s_W^{l,c}\right)=\left(b+h_W+h_\alpha\right)\phi\left(z_W^{l,c}\right)\sqrt{\sigma^2\tilde{L}_W}+h_W\mu\tilde{L}_W \quad \text{(A13)}$$

$$C_W^{l,d}\le C_W^d\le C_W^{u,d} \quad \text{(A14)}$$

$$C_W^{l,c}\le C_W^c\le C_W^{u,c} \quad \text{(A15)}$$

and from (3),

$$C^c\le C^a\left(s_W^a\right)\le C^d \quad \text{(A16)}$$

Proof of Proposition 4.1

Proposition 4.1 follows by inspection of equations A1 through A16.

(a) As b_j increases,

a. Θ_j, $\Theta_{W,j}^{l,d}$, $\Theta_{W,j}^{u,d}$, $\Theta_W^{l,c}$,and $\Theta_W^{u,c}$ increase, increasing A1 to A13.

b. Θ_α remains unchanged where $j\ne\alpha$

(b) As h_i increases,

a. Θ_j, $\Theta_{W,j}^{l,d}$, and $\Theta_W^{l,c}$, decrease, decreasing A1, A2, A4, and A6.

b. Examination of equations A7, A8, A10, A12, and A13 shows that for a fixed y, $C_i(y)$ increases with h_j due to the increase in the first coefficient in A7, A10, and A13. Meanwhile, A8 and A12 are independent of changes in h_j. Hence the collapsed system remains unchanged while the decomposed systems increase in costs.

c. Θ_j and hence A1 remains unchanged where $j\ne\alpha$

(c) As L_j increases,

a. equations A1 through A13 increase

b. equations A1 and A7 remain unchanged where $j\ne\alpha$

Proof of Proposition 4.2.

In this proposition, we hold the total system demand constant. Assuming demand is distributed normally with mean μ and variance σ^2, splitting among n identical terminal locations gives $\mu = \sum_{\alpha=1}^{n} \mu_\alpha$ and $\sigma^2 = \sum_{\alpha=1}^{n} \sigma_\alpha^2$, or $\mu_\alpha = \frac{\mu}{n}$ and $\sigma_\alpha^2 = \frac{\sigma^2}{n}$ while splitting the demand process across n+1 identical terminal locations gives $\mu_\beta = \frac{\mu}{n+1}$ and $\sigma_\beta^2 = \frac{\sigma^2}{n+1}$.

(a) We consider three cases, the retail stages, and the collapsed and decomposed serial systems.

a. For the retail stages, $\mu_\beta < \mu_\alpha$ and $\sigma_\alpha^2 < \sigma_\beta^2$, hence $s_\alpha > s_\beta$ from equations A1.

b. For the collapsed serial system, $\mu = \sum_{\alpha=1}^{n} \mu_\alpha$ and $\sigma^2 = \sum_{\alpha=1}^{n} \sigma_\alpha^2$ remain unchanged. Hence Equations A4 and A5 remain unchanged.

c. For the decomposed systems, consider $\sum_{\alpha=1}^{n} s_{W,\alpha}^{l,d} - \sum_{\beta=1}^{n+1} s_{W,\beta}^{l,d}$. From equations A2,

$$= \sum_{\alpha=1}^{n} \left(\mu_\alpha \tilde{L}_\alpha + z_\alpha^{l,d} \sqrt{\sigma_\alpha{}^2 \tilde{L}_\alpha} \right) - \sum_{\beta=1}^{n+1} \left(\mu_\beta \tilde{L}_\beta + z_\beta^{l,d} \sqrt{\sigma_\beta{}^2 \tilde{L}_\beta} \right)$$

$$= \sum_{\alpha=1}^{n} \left(\mu_\alpha \tilde{L}_\alpha + z_\alpha^{l,d} \sqrt{\sigma_\alpha{}^2 \tilde{L}_\alpha} \right) - \sum_{\beta=1}^{n+1} \left(\frac{n\mu_\alpha}{n+1} \tilde{L}_\alpha + z_\alpha^{l,d} \sqrt{\frac{n\sigma_\alpha{}^2}{n+1} \tilde{L}_\alpha} \right)$$

$$= \sum_{\alpha=1}^{n} \left(z_\alpha^{l,d} \sqrt{\sigma_\alpha{}^2 \tilde{L}_\alpha} \right) - \sum_{\beta=1}^{n+1} \left(z_\alpha^{l,d} \sqrt{\frac{n\sigma_\alpha{}^2}{n+1} \tilde{L}_\alpha} \right)$$

$$= z_\alpha^{l,d} \sqrt{\tilde{L}_\alpha} \left(n\sqrt{\sigma_\alpha^2} - (n+1)\sqrt{\frac{n\sigma_\alpha^2}{n+1}} \right)$$

$$= z_\alpha^{l,d} \sqrt{\tilde{L}_\alpha} \left(\sqrt{n^2\sigma_\alpha^2} - \sqrt{(n+1)n\sigma_\alpha^2} \right)$$

$$= -z_\alpha^{l,d} \sigma_\alpha \sqrt{n\tilde{L}_\alpha}$$

< 0

The above holds for $\sum_{\alpha=1}^{n} s_{W,\alpha}^{u,d} - \sum_{\beta=1}^{n+1} s_{W,\beta}^{u,d}$ as well.

Hence the echelon base stock level of the warehouse is non-decreasing in the number of retailers. To see the effects on the system costs, consider

d. For the retail installations, let K be a positive constant equal to $\left(b_\alpha + h_\alpha + h_W\right)\phi\left(z_\alpha\right)$. Then

$$\sum_{\alpha=1}^{n} C_\alpha - \sum_{\beta=1}^{n+1} C_\beta$$

$$= \sum_{\alpha=1}^{n}\left(K\sqrt{\sigma_\alpha{}^2 \tilde{L}_\alpha}\right) - \sum_{\beta=1}^{n+1}\left(K\sqrt{\sigma_\beta{}^2 \tilde{L}_\beta}\right)$$

$$= \sum_{\alpha=1}^{n}\left(K\sqrt{\sigma_\alpha{}^2 \tilde{L}_\alpha}\right) - \sum_{\beta=1}^{n+1}\left(K\sqrt{\frac{n\sigma_\alpha^2}{n+1}\tilde{L}_\alpha}\right)$$

$$= n\left(K\sqrt{\sigma_\alpha{}^2 \tilde{L}_\alpha}\right) - (n+1)\left(K\sqrt{\frac{n\sigma_\alpha^2}{n+1}\tilde{L}_\alpha}\right)$$

$$= K\left(n\sqrt{\sigma_\alpha{}^2 \tilde{L}_\alpha} - \sqrt{n(n+1)\sigma_\alpha^2 \tilde{L}_\alpha}\right)$$

$$= -K\sqrt{n\sigma_\alpha^2 \tilde{L}_\alpha}$$

$$< 0$$

e. For the collapsed system, $\mu = \sum_{\alpha=1}^{n} \mu_\alpha$ and $\sigma^2 = \sum_{\alpha=1}^{n} \sigma_\alpha^2$ remain unchanged.

Hence Equations A11 and A12 remain unchanged.

f. For the decomposed systems, let K be a positive constant equal to $\left(b\ + h_W\right)\phi\left(z_{W,\alpha}^{u,d}\right)$. Then

$$\sum_{\alpha=1}^{n} C_{W,\alpha}^{l,d} - \sum_{\beta=1}^{n+1} C_{W,\beta}^{l,d}$$

$$= \sum_{\alpha=1}^{n}\left(K\sqrt{\sigma_\alpha{}^2 \tilde{L}_\alpha} + h_W \mu_\alpha \tilde{L}_\alpha\right) - \sum_{\beta=1}^{n+1}\left(K\sqrt{\sigma_\beta{}^2 \tilde{L}_\beta} + h_W \mu_\beta \tilde{L}_\beta\right)$$

$$= \sum_{\alpha=1}^{n}\left(K\sqrt{\sigma_\alpha{}^2\tilde{L}_\alpha} + h_W\mu_\alpha\tilde{L}_\alpha\right) - \sum_{\beta=1}^{n+1}\left(K\sqrt{\frac{n\sigma_\alpha{}^2}{n+1}\tilde{L}_\alpha} + \frac{nh_W\mu_\alpha}{n+1}\tilde{L}_\alpha\right)$$

$$= \sum_{\alpha=1}^{n}\left(K\sqrt{\sigma_\alpha{}^2\tilde{L}_\alpha}\right) - \sum_{\beta=1}^{n+1}\left(K\sqrt{\frac{n\sigma_\alpha{}^2}{n+1}\tilde{L}_\alpha}\right)$$

$$= K\left(n\left(\sqrt{\sigma_\alpha{}^2\tilde{L}_\alpha}\right) - (n+1)\left(\sqrt{\frac{n\sigma_\alpha{}^2}{n+1}\tilde{L}_\alpha}\right)\right)$$

$$= -K\sqrt{n\sigma_\alpha{}^2\tilde{L}_\alpha}$$

$$< 0$$

The proof for $C_{W,\alpha}^{u,d}$ follows exactly as above if we instead let $K =$ $\left(b + h_W + h_\alpha\right)\phi\left(z_{W,\alpha}^{l,c}\right)$.

Hence the costs of both the retailer and warehouse echelons are nondecreasing in the number of retailers.

For Propositions 4.3 and 4.4, we assume the leadtime demand at retailer α is uniformly distributed. Specifically, we will consider Uniform(0,1) distributions. Let f(·) and F(·) represent the Uniform(0,1) pdf and cdf, respectively. The base stock levels become $F^{-1}(\Theta) = \Theta$.

Following the standard approach (e.g. see pp 205-209 in Zipkin (2000))

$$C_\alpha\left(s_\alpha\right) = \frac{1}{2}\left(1-\Theta_\alpha\right)\left(b_\alpha + h_W\right) \tag{A17}$$

$$C_{W,\alpha}^{l,d}\left(s_{W,\alpha}^{u,d}\right) = \frac{1}{2}\left(1-\Theta_{W,\alpha}^{u,d}\right)b_\alpha + h_W\mu_\alpha\tilde{L}_\alpha \tag{A18}$$

$$C_{W,\alpha}^{u,d}\left(s_{W,\alpha}^{l,d}\right) = \frac{1}{2}\left(1-\Theta_{W,\alpha}^{l,d}\right)b_\alpha + h_W\mu_\alpha\tilde{L}_\alpha \tag{A19}$$

$$C_W^{l,c}\left(s_W^{u,c}\right) = \frac{1}{2}\left(1-\Theta_W^{u,c}\right)b + h_W\mu\tilde{L}_\alpha \tag{A20}$$

$$C_W^{u,c}\left(s_W^{l,c}\right)=\frac{1}{2}\left(1-\Theta_W^{l,c}\right)b+h_W\mu\tilde{L}_\alpha \tag{A21}$$

Proof of Proposition 3.

Here we investigate the effects of backorder asymmetry. We consider two cases, the collapsed and decomposed systems. Let $b_1 = b_\alpha(1+\Delta)$ and $b_2 = b_\alpha(1-\Delta)$ where $\alpha \neq 1,2$.

(a) For the collapsed systems, $b=\frac{1}{\mu}\sum_{\alpha=1}^{n}\mu_\alpha b_\alpha$ remains unchanged. Hence the critical fractiles $\Theta_\alpha^{u,c}$ and $\Theta_\alpha^{l,c}$ remain unchanged and hence base stocking level equations A4 and A5, and equations A20 and A21 remain unchanged.

(b) For the decomposed systems, let $B = h_\alpha + h_W$, and note that $B > 0$. Consider

$$\begin{aligned}
&2s_{W,\alpha}^{l,d}-s_{W,1}^{l,d}-s_{W,2}^{l,d}\\
&=2F^{-1}\left(\Theta_{i,\alpha}^{d}\right)-F^{-1}\left(\Theta_{i,1}^{d}\right)-F^{-1}\left(\Theta_{i,2}^{d}\right)\\
&=2F^{-1}\left(\frac{b_\alpha}{b_\alpha+B}\right)-F^{-1}\left(\frac{b_1}{b_1+B}\right)-F^{-1}\left(\frac{b_2}{b_2+B}\right)\\
&=2F^{-1}\left(\frac{b}{b+B}\right)-F^{-1}\left(\frac{b+\Delta}{b+B+\Delta}\right)-F^{-1}\left(\frac{b-\Delta}{b+B-\Delta}\right)\\
&=2\frac{b}{b+B}-\frac{b+\Delta}{b+B+\Delta}-\frac{b-\Delta}{b+B-\Delta}\\
&=\frac{2(b)(b+B+\Delta)(b+B-\Delta)}{(b+B)(b+B+\Delta)(b+B-\Delta)}-\frac{(b+B)(b+\Delta)(b+B-\Delta)}{(b+B)(b+B+\Delta)(b+B-\Delta)}\\
&\qquad-\frac{(b+B)(b+B+\Delta)(b-\Delta)}{(b+B)(b+B+\Delta)(b+B-\Delta)}
\end{aligned}$$

After expansion and intermediate collection of like terms,

$$= \frac{2\left(b^3+2b^2B+bB^2-b\Delta^2\right)}{\left(b+B\right)\left(b+B+\Delta\right)\left(b+B-\Delta\right)} - \frac{b^3+2b^2B+bB\Delta+bB^2+B^2\Delta-b\Delta^2-B\Delta^2}{\left(b+B\right)\left(b+B+\Delta\right)\left(b+B-\Delta\right)}$$

$$- \frac{b^3+2b^2B+bB^2-b^2\Delta-bB\Delta-B^2\Delta-B\Delta^2}{\left(b+B\right)\left(b+B+\Delta\right)\left(b+B-\Delta\right)}$$

$$= \frac{2B\Delta^2}{\left(b+B\right)\left(b+B+\Delta\right)\left(b+B-\Delta\right)} > 0$$

The last expression is increasing in Δ.

The above also holds for $2s_{W,\alpha}^{u,d} - s_{W,1}^{u,d} - s_{W,2}^{u,d}$ if we define B $= h_W$.

Thus increasing asymmetry in backordering costs decreases equations A2 and A3.

To see the results for the effects on system costs, let $A = h_W + h_i$. First consider

$$2C_{W,\alpha}^{u,d}\left(s_{W,\alpha}^{l,d}\right) - C_{W,1}^{u,d}\left(s_{W,2}^{l,d}\right) - C_{W,1}^{u,d}\left(s_{W,2}^{l,d}\right)$$

$$= \left(1-\Theta_{W,\alpha}^{l,d}\right)\left(b_\alpha\right) - \frac{1}{2}\left(1-\Theta_{W,1}^{l,d}\right)\left(b_1\right) - \frac{1}{2}\left(1-\Theta_{W,2}^{l,d}\right)\left(b_2\right)$$
$$+2h_W\mu_\alpha\tilde{L}_\alpha - h_W\mu_1\tilde{L}_\alpha - h_W\mu_2\tilde{L}_\alpha$$

$$= \left(1-\Theta_{W,\alpha}^{l,d}\right)b_\alpha - \frac{1}{2}\left(1-\Theta_{W,1}^{l,d}\right)\left(b_\alpha+\Delta\right) - \frac{1}{2}\left(1-\Theta_{W,2}^{l,d}\right)\left(b_\alpha-\Delta\right)$$

$$= \left(1-\frac{b_\alpha}{b_\alpha+h_W+h_\alpha}\right)\left(b_i\right) - \frac{1}{2}\left(1-\frac{b_\alpha+\Delta}{b_\alpha+\Delta+h_W+h_\alpha}\right)\left(b_\alpha+\Delta\right) - \frac{1}{2}\left(1-\frac{b_\alpha-\Delta}{b_\alpha-\Delta+h_W+h_\alpha}\right)\left(b_\alpha-\Delta\right)$$

$$= b - \frac{b^2}{b+A} - \frac{\left(b+\Delta\right)}{2} + \frac{\left(b+\Delta\right)^2}{2\left(b+\Delta+A\right)} - \frac{\left(b-\Delta\right)}{2} + \frac{\left(b-\Delta\right)^2}{2\left(b-\Delta+A\right)}$$

$$= -\frac{b^2}{b+A} + \frac{\left(b+\Delta\right)^2}{2\left(b+\Delta+A\right)} + \frac{\left(b-\Delta\right)^2}{2\left(b-\Delta+A\right)}$$

$$= \frac{1}{2}\frac{2b^2\left(b+\Delta+A\right)\left(b-\Delta+A\right) - \left(b+A\right)\left(b+\Delta\right)^2\left(b-\Delta+A\right) - \left(b+A\right)\left(b-\Delta\right)^2\left(b+\Delta+A\right)}{\left(b+A\right)\left(b+\Delta+A\right)\left(b-\Delta+A\right)}$$

And after expansion and collection of terms,

$$= \frac{\Delta^2 A^2}{(b+A)(b+\Delta+A)(b-\Delta+A)} > 0$$

The last expression is increasing in Δ

Note that the above analysis also holds for $2C_{i,\alpha}^{l,d}\left(s_{i,\alpha}^{u,d}\right) - C_{i,1}^{l,d}\left(s_{i,2}^{u,d}\right) - C_{i,1}^{l,d}\left(s_{i,2}^{u,d}\right)$ if we define $A = h_W$. Thus asymmetry in backorder cost decreases equations A18 and A19. Combined with the above results, we find that both stocking levels and system costs are decreasing in backorder cost asymmetry.

Proof of Proposition 4.4.

We show the effects of holding cost asymmetry on stocking levels and total system costs. Beginning with the effects on stocking levels, we consider two cases, the collapsed and the decomposed systems.

(a) For the collapsed system, with $h_1 = h_\alpha(1+\Delta)$, and $h_2 = h_\alpha(1\text{-}\Delta)$, consider that the weighted holding cost $h = \frac{1}{\mu}\left(\sum_{\alpha\neq 1,2} h_\alpha + (h_1+\Delta) + (h_2-\Delta)\right)$ is independent of Δ. Hence $\Theta_W^{l,c}$ and $\Theta_W^{u,c}$, and thus the collapsed system stocking levels are independent of Δ.

(b) For the decomposed system, consider $2s_{W,\alpha}^{l,d} - s_{W,1}^{l,d} - s_{W,2}^{l,d}$ where $\alpha \neq 1,2$. Letting A = $h_W + h_\alpha$,

$$2s_{W,\alpha}^{l,d} - s_{W,1}^{l,d} - s_{W,2}^{l,d}$$

$$= 2\mu_\alpha \tilde{L}_W + 2z_{W,\alpha}^{l,d}\sqrt{\sigma_\alpha^2 \tilde{L}_{W,\alpha}} - \mu_1 \tilde{L}_W - z_{W,1}^{l,d}\sqrt{\sigma_\alpha^2 \tilde{L}_{W,\alpha}} - \mu_2 \tilde{L}_W - z_{W,2}^{l,d}\sqrt{\sigma_\alpha^2 \tilde{L}_{W,\alpha}}$$

$$= 2z_{W,\alpha}^{l,d}\sqrt{\sigma_\alpha^2 \tilde{L}_{W,i}} - z_{W,1}^{l,d}\sqrt{\sigma_\alpha^2 \tilde{L}_{W,\alpha}} - z_{W,2}^{l,d}\sqrt{\sigma_\alpha^2 \tilde{L}_{W,\alpha}}$$

$$= \sqrt{\sigma_\alpha^2 \tilde{L}_{W,\alpha}}\left(2z_{W,\alpha}^{l,d} - z_{W,1}^{l,d} - z_{W,2}^{l,d}\right)$$

$$=\sqrt{\sigma_\alpha^2 \tilde{L}_{W,\alpha}}\left(2\frac{b_\alpha}{b_\alpha+h_W+h_\alpha}-\frac{b_\alpha}{b_\alpha+h_W+h_\alpha+\Delta}-\frac{b_\alpha}{b_\alpha+h_W+h_\alpha-\Delta}\right)$$

$$=\sqrt{\sigma_\alpha^2 \tilde{L}_{W,\alpha}}\left(2\frac{b_\alpha}{b_\alpha+A}-\frac{b_\alpha}{b_\alpha+A+\Delta}-\frac{b_\alpha}{b_\alpha+A-\Delta}\right)$$

$$=\sqrt{\sigma_\alpha^2 \tilde{L}_{W,\alpha}}\left(\frac{2b_\alpha\left(b_\alpha+A+\Delta\right)\left(b_\alpha+A-\Delta\right)-b_\alpha\left(b_\alpha+A\right)\left(b_\alpha+A-\Delta\right)-b_\alpha\left(b_\alpha+A\right)b_\alpha+A+\Delta}{\left(b_\alpha+A\right)\left(b_\alpha+A+\Delta\right)\left(b_\alpha+A-\Delta\right)}\right)$$

After expansion and collection of terms,

$$=\frac{-2b_\alpha\Delta^2}{\left(b_\alpha+A\right)\left(b_\alpha+A+\Delta\right)\left(b_\alpha+A-\Delta\right)}<0$$

The last expression is decreasing in Δ

Note that the above also holds for $2s_{W,\alpha}^{u,d}-s_{W,1}^{u,d}-s_{W,2}^{u,d}$ if we let A = h_W. Thus the warehouse echelon base stock level is nondecreasing in holding cost asymmetry.

To see the effects of holding cost asymmetry on total system costs, consider

(c) For the collapsed systems, the weighted holding cost

$h=\frac{1}{\mu}\left(\sum_{\alpha\neq 1,2} h_\alpha+\left(h_1+\Delta\right)+\left(h_2-\Delta\right)\right)$ is independent of Δ. Hence equations A20 and A21 are independent of holding cost asymmetry.

(d) For the decomposed systems, let $A = h_W+h_\alpha$. First consider

$$2C_{W,\alpha}^{u,d}\left(s_{W,\alpha}^{l,d}\right)-C_{W,1}^{u,d}\left(s_{W,1}^{l,d}\right)-C_{W,2}^{u,d}\left(s_{W,2}^{l,d}\right)$$

$$=\left(1-\Theta_{W,\alpha}^{l,d}\right)b_\alpha+h_W\mu_\alpha\tilde{L}_\alpha-\frac{1}{2}\left(1-\Theta_{W,1}^{l,d}\right)b_\alpha-\frac{h_W\mu_\alpha\tilde{L}_\alpha}{2}-\frac{1}{2}\left(1-\Theta_{W,2}^{l,d}\right)b_\alpha-\frac{h_W\mu_\alpha\tilde{L}_\alpha}{2}$$

$$=\left(1-\Theta_{W,\alpha}^{l,d}\right)b_\alpha-\frac{1}{2}\left(1-\Theta_{W,1}^{l,d}\right)b_\alpha-\frac{1}{2}\left(1-\Theta_{W,2}^{l,d}\right)b_\alpha$$

$$=\left(1-\frac{b_\alpha}{b_\alpha+h_W+h_\alpha}\right)b_\alpha-\frac{1}{2}\left(1-\frac{b_\alpha}{b_\alpha+h_W+h_1}\right)b_\alpha-\frac{1}{2}\left(1-\frac{b_\alpha}{b_\alpha+h_W+h_2}\right)b_\alpha$$

$$= -\frac{{b_\alpha}^2}{b_\alpha + h_W + h_\alpha} + \frac{1}{2}\frac{{b_\alpha}^2}{b_\alpha + h_W + h_1} + \frac{1}{2}\frac{{b_\alpha}^2}{b_\alpha + h_W + h_2}$$

$$= -\frac{{b_\alpha}^2}{b_\alpha + h_W + h_\alpha} + \frac{{b_\alpha}^2}{b_\alpha + h_W + h_\alpha + \Delta} + \frac{1}{2}\frac{{b_\alpha}^2}{b_\alpha + h_W + h_\alpha - \Delta}$$

$$= -\frac{{b_\alpha}^2}{b_\alpha + A} + \frac{1}{2}\frac{{b_\alpha}^2}{b_\alpha + A + \Delta} + \frac{1}{2}\frac{{b_\alpha}^2}{b_\alpha + A - \Delta}$$

$$= \frac{-2{b_\alpha}^2\left(b_\alpha + A + \Delta\right)\left(b_\alpha + A - \Delta\right) + {b_\alpha}^2\left(b_\alpha + A\right)\left(b_\alpha + A - \Delta\right) + {b_\alpha}^2\left(b_\alpha + A\right)\left(b_\alpha + A + \Delta\right)}{2\left(b_\alpha + A\right)\left(b_\alpha + A + \Delta\right)\left(b_\alpha + A - \Delta\right)}$$

$$= \frac{{b_\alpha}^2\left(\left(b_\alpha + A\right)\left(b_\alpha + A - \Delta\right) + \left(b_\alpha + A\right)\left(b_\alpha + A + \Delta\right) - 2\left(b_\alpha + A + \Delta\right)\left(b_\alpha + A - \Delta\right)\right)}{2\left(b_\alpha + A\right)\left(b_\alpha + A + \Delta\right)\left(b_\alpha + A - \Delta\right)}$$

$$= \frac{{b_\alpha}^2\left(\left(b_\alpha + A\right)\left(b_\alpha + A - \Delta\right) + \left(b_\alpha + A\right)\left(b_\alpha + A + \Delta\right) - 2\left(b_\alpha + A + \Delta\right)\left(b_\alpha + A - \Delta\right)\right)}{2\left(b_\alpha + A\right)\left(b_\alpha + A + \Delta\right)\left(b_\alpha + A - \Delta\right)}$$

$$= \frac{{b_\alpha}^2\left(\left({b_\alpha}^2 - b_\alpha\Delta + Ab_\alpha + Ab_\alpha - A\Delta + A^2\right) + \left({b_\alpha}^2 + b_\alpha\Delta + Ab_\alpha + Ab_\alpha + A\Delta + A^2\right)\right)}{2\left(b_\alpha + A\right)\left(b_\alpha + \Delta + A\right)\left(b_\alpha + A - \Delta\right)}$$

$$- \frac{2{b_\alpha}^2\left({b_\alpha}^2 + Ab_\alpha - b_\alpha\Delta + Ab_\alpha + A^2 - A\Delta + b_\alpha\Delta + A\Delta - \Delta^2\right)}{2\left(b_\alpha + A\right)\left(b_\alpha + \Delta + A\right)\left(b_\alpha + A - \Delta\right)}$$

and collecting like terms gives

$$= \frac{{b_\alpha}^2\Delta^2}{\left(b_\alpha + A\right)\left(b_\alpha + \Delta + A\right)\left(b_\alpha + A - \Delta\right)}$$

> 0, and increasing in Δ.

Note that the above also holds for $2C_{W,\alpha}^{l,d}\left(s_{W,\alpha}^{u,d}\right) - C_{W,1}^{l,d}\left(s_{W,1}^{u,d}\right) - C_{W,2}^{l,d}\left(s_{W,2}^{u,d}\right)$ if we let $A = h_W$. Thus the total system costs are nondecreasing in holding cost asymmetry.

We next consider asymmetry in demand. Because demand does not appear in the critical fractiles, we revert to normal distributions.

Proof of Proposition 4.5.

We consider two cases, the collapsed and decomposed systems.

(a) For the collapsed system, note that $\mu = \sum_{\alpha=1}^{n} \mu_\alpha$ and $\sigma^2 = \sum_{\alpha=1}^{n} \sigma_\alpha^2$ are independent of Δ. Thus the collapsed stocking levels $s_W^{l,c} = \mu \tilde{L}_W + z_W^{l,c} \sqrt{\sigma^2 \tilde{L}_W}$ and $s_W^{u,c} = \mu \tilde{L}_W + z_W^{u,c} \sqrt{\sigma^2 \tilde{L}_W}$ are likewise independent of asymmetry in μ_α and σ_α^2. Also, the cost equations $C_W^{l,c}\left(s_W^{u,c}\right) = \left(b + h_W\right) \phi\left(z_W^{u,c}\right) \sqrt{\sigma^2 \tilde{L}_W} + h_W \mu \tilde{L}_\alpha$ and $C_W^{u,c}\left(s_W^{l,c}\right) = \left(b + h_W + h_\alpha\right) \phi\left(z_W^{l,c}\right) \sqrt{\sigma^2 \tilde{L}_W} + h_W \mu \tilde{L}_\alpha$ are likewise independent of asymmetry in μ_α and σ_α^2.

(b) For the decomposed system, first consider $2s_{W,\alpha}^{l,d} - s_{W,1}^{l,d} - s_{W,2}^{l,d}$

$$= 2\mu_\alpha \tilde{L}_W + 2z_{W,\alpha}^{l,d} \sqrt{\sigma_\alpha{}^2 \tilde{L}_{W,\alpha}} - \mu_1 \tilde{L}_W - z_{W,1}^{l,d} \sqrt{\sigma_1{}^2 \tilde{L}_{W,\alpha}} - \mu_2 \tilde{L}_W - z_{W,2}^{l,d} \sqrt{\sigma_2{}^2 \tilde{L}_{W,\alpha}}$$

$$= 2z_{W,\alpha}^{l,d} \sqrt{\sigma_\alpha{}^2 \tilde{L}_{W,\alpha}} - z_{W,1}^{l,d} \sqrt{\sigma_1{}^2 \tilde{L}_{W,\alpha}} - z_{W,2}^{l,d} \sqrt{\sigma_2{}^2 \tilde{L}_{W,\alpha}}$$

$$= z_{W,\alpha}^{l,d} \left(\sqrt{4\sigma_\alpha{}^2 \tilde{L}_{W,\alpha}} - \sqrt{\sigma_1{}^2 \tilde{L}_{W,\alpha}} - \sqrt{\sigma_2{}^2 \tilde{L}_{W,\alpha}} \right)$$

$$= z_{W,\alpha}^{l,d} \sqrt{\tilde{L}_{W,\alpha}} \left(\sqrt{4\sigma_\alpha{}^2} - \sqrt{\frac{\mu+\Delta}{\mu} \sigma_\alpha{}^2} - \sqrt{\frac{\mu-\Delta}{\mu} \sigma_\alpha{}^2} \right)$$

$$= z_{W,\alpha}^{l,d} \sqrt{\sigma_\alpha{}^2 \tilde{L}_{W,\alpha}} \left(\sqrt{4} - \sqrt{\frac{\mu+\Delta}{\mu}} - \sqrt{\frac{\mu-\Delta}{\mu}} \right)$$

$$\geq z_{W,\alpha}^{l,d} \sqrt{\sigma_\alpha{}^2 \tilde{L}_{W,\alpha}} \left(2 - \sqrt{\frac{\mu}{\mu}} - \sqrt{\frac{\Delta}{\mu}} - \sqrt{\frac{\mu}{\mu}} + \sqrt{\frac{\Delta}{\mu}} \right)$$

$$= z_{W,\alpha}^{l,d} \sqrt{\sigma_\alpha{}^2 \tilde{L}_{W,\alpha}} \left(2 - \sqrt{\frac{\mu}{\mu}} - \sqrt{\frac{\mu}{\mu}} \right) = 0$$

Note that the above follows for $2s^{u,d}_{W,\alpha} - s^{u,d}_{W,1} - s^{u,d}_{W,2}$ as well if we substitute $z^{u,d}_{W,\alpha}$ for $z^{l,d}_{W,\alpha}$. Thus the echelon base stock level at the warehouse is nonincreasing in demand rate asymmetry.

(c) Next, consider $2C^{u,d}_{W,\alpha}\left(s^{l,d}_{W,\alpha}\right) - C^{u,d}_{W,1}\left(s^{l,d}_{W,1}\right) - C^{u,d}_{W,2}\left(s^{l,d}_{W,2}\right)$

$$
\begin{aligned}
&= 2\left(b_\alpha + h_W + h_\alpha\right)\phi\left(z^{l,d}_{W,\alpha}\right)\sqrt{\sigma_i^{\,2}\tilde{L}_{W,\alpha}} + 2h_W\mu_\alpha\tilde{L}_{W,\alpha} - \left(b_\alpha + h_W + h_\alpha\right)\phi\left(z^{l,d}_{W,1}\right)\sqrt{\sigma_1^{\,2}\tilde{L}_{W,\alpha}} - h_W\mu_1\tilde{L}_{W,\alpha} \\
&\quad - \left(b_\alpha + h_W + h_\alpha\right)\phi\left(z^{l,d}_{W,2}\right)\sqrt{\sigma_2^{\,2}\tilde{L}_{W,\alpha}} - h_W\mu_2\tilde{L}_{W,\alpha} \\
&= \left(b_\alpha + h_W + h_\alpha\right)\left(2\phi\left(z^{l,d}_{W,\alpha}\right)\sqrt{\sigma_\alpha^{\,2}\tilde{L}_{W,\alpha}} - \phi\left(z^{l,d}_{W,1}\right)\sqrt{\sigma_1^{\,2}\tilde{L}_{W,\alpha}} - \phi\left(z^{l,d}_{W,2}\right)\sqrt{\sigma_2^{\,2}\tilde{L}_{W,\alpha}}\right) \\
&= \left(b_\alpha + h_W + h_\alpha\right)\phi\left(z^{l,d}_{W,\alpha}\right)\sqrt{\sigma_\alpha^{\,2}\tilde{L}_{W,\alpha}}\left(\sqrt{4} - \sqrt{\frac{\mu+\Delta}{\mu}} - \sqrt{\frac{\mu-\Delta}{\mu}}\right) \\
&\geq \left(b_\alpha + h_W + h_\alpha\right)\phi\left(z^{l,d}_{W,\alpha}\right)\sqrt{\sigma_\alpha^{\,2}\tilde{L}_{W,\alpha}}\left(2 - \sqrt{\frac{\mu}{\mu}} - \sqrt{\frac{\Delta}{\mu}} - \sqrt{\frac{\mu}{\mu}} + \sqrt{\frac{\Delta}{\mu}}\right) = 0
\end{aligned}
$$

Note that the above follows for $2C^{l,d}_{W,\alpha}\left(s^{u,d}_{W,\alpha}\right) - C^{l,d}_{W,1}\left(s^{u,d}_{W,1}\right) - C^{l,d}_{W,2}\left(s^{u,d}_{W,2}\right)$ as well if we substitute $z^{u,d}_{W,\alpha}$ for $z^{l,d}_{W,\alpha}$. Thus the total system inventory costs are nonincreasing in demand rate asymmetry.

Proof of Propositions 5.1 and 5.2

Propositions 5.1 and 5.2 investigate the difference in inventory costs associated with varying production network topologies. Under Assumption 5.2, we may analyze the network topologies below in Figures A.1 and A.2.

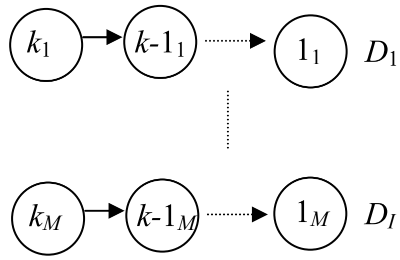


Figure A.1: Serial Topology

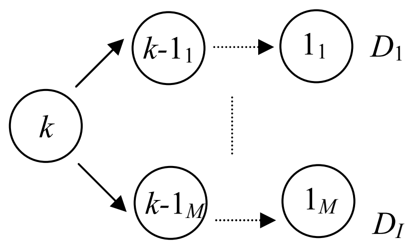


Figure A.2: Distribution Topology

Let the value of delaying differentiation,

$$V_d = Z(k+1)-Z(k) \tag{A21}$$

Proof of Proposition 1

By decomposing the network in Figure A2, a lower bound on V_d is obtained. Under the decomposition, the two networks are identical, thus

$$V_d \geq 0 \tag{A22}$$

By allowing instantaneous and costless transshipments between echelons in Figure A2, an upper bound on V_d is obtained. This transforms Figure A2 into a serial network. By reducing the serial chains to single stage problems (see Shang and Song (2003)), we have an upper bound for V_d which is in turn bounded by

$$C_k^{l,d}\left(s_k^{u,d}\right)-C_k^{l,c}\left(s_k^{u,c}\right) \tag{A23}$$

and

$$C_k^{u,d}\left(s_k^{l,d}\right)-C_k^{u,c}\left(s_k^{l,c}\right). \tag{A24}$$

Equation A23 is equal to

$$\begin{aligned}
&n\left(\left(b+\sum_{i=k}^{k+1}h_i\right)\phi\left(z_k^{u,c}\right)\sqrt{\sigma_\alpha{}^2\tilde{L}_k}+\sum_{j=1i}^{k-1}h_{i+1}\mu_\alpha\tilde{L}_i\right)-\left(b+\sum_{i=k}^{k+1}h_i\right)\phi\left(z_k^{u,c}\right)\sqrt{n\sigma_\alpha{}^2\tilde{L}_k}-\sum_{i=1}^{k-1}h_{i+1}n\mu_\alpha\tilde{L}_i \\
&=n\left(b+\sum_{i=k}^{k+1}h_i\right)\phi\left(z_k^{u,c}\right)\sqrt{\sigma_\alpha{}^2\tilde{L}_k}-\left(b+\sum_{i=k}^{k+1}h_i\right)\phi\left(z_k^{u,c}\right)\sqrt{n\sigma_\alpha{}^2\tilde{L}_k} \\
&=\left(b+\sum_{i=k}^{k+1}h_i\right)\phi\left(z_k^{u,c}\right)\sqrt{\left(n^2-n\right)\sigma_\alpha{}^2\tilde{L}_k}
\end{aligned} \tag{A25}$$

Let $h_{k-1}{}'=h_k+\Delta$ and $h_k{}'=h_k-\Delta$. Then Equation A25 becomes

$$\begin{aligned}
&\left(b+h_{k+1}+h_k{}'\right)\phi\left(\Phi^{-1}\left(\frac{b+h_k}{b+h_{k+1}+h_k{}'}\right)\right)\sqrt{(n^2-n)\sigma_\alpha{}^2\tilde{L}_k} \\
&=\left(b+h_{k+1}+h_k-\Delta\right)\phi\left(\Phi^{-1}\left(\frac{b+h_k}{b+h_{k+1}+h_k-\Delta}\right)\right)\sqrt{(n^2-n)\sigma_\alpha{}^2\tilde{L}_k} \\
&\le\left(b+h_{k+1}+h_k\right)\phi\left(\Phi^{-1}\left(\frac{b+h_{k+1}}{b+h_{k+1}+h_k}\right)\right)\sqrt{(n^2-n)\sigma_\alpha{}^2\tilde{L}_k}
\end{aligned} \tag{A26}$$

Hence Equation A23 is decreasing in Δ.

Likewise, Equation A24 is equal to

$$\left(b+\sum_{i\neq k,k-1}h_i+h_k{}'+h_{k-1}{}'\right)\phi\left(\Phi^{-1}\left(\frac{b+h_{k+1}}{b+\sum_{j\neq k,k-1}h_i+h_k{}'+h_{k-1}{}'}\right)\right)\sqrt{(n^2-n)\sigma_\alpha{}^2\tilde{L}_k}$$

$$=\left(b+\sum_{i\neq k,k-1}h_i+h_k+h_{k-1}\right)\phi\left(\Phi^{-1}\left(\frac{b+h_{k+1}}{b+\sum_{i\neq k,k-1}h_i+h_k+h_{k-1}}\right)\right)\sqrt{(n^2-n)\sigma_\alpha{}^2\tilde{L}_k} \qquad \text{(A27)}$$

Hence Equation A24 is independent of Δ.

Thus the lower bound of V_d is independent of Δ, while the upper bound is non-increasing (and potentially decreasing) in Δ. Hence shifting holding costs downstream from the point of differentiation may decrease the value of delaying differentiation. As *n* increases, the factor $\sqrt{(n^2-n)\sigma_\alpha{}^2\tilde{L}_k}$ also increases, hence the decrease in the value of delayed differentiation attributable to the holding cost shift is increasing in *n*.

Proof of Proposition 5.2

By a similar argument as in Proposition 1, we have

$$V_d \geq 0 \qquad \text{(A28)}$$

$$\begin{aligned}
&C_{k+1}^{l,d}\left(s_{k+1}^{u,d}\right)-C_{k+1}^{l,c}\left(s_{k+1}^{u,c}\right)\\
&=n\left(b+h_{k+1}+h_k\right)\phi\left(z_k^u\right)\sqrt{\sigma_\alpha{}^2\tilde{L}_k}+n\sum_{i=1}^{k-1}h_{i+1}\mu_\alpha\tilde{L}_i-\left(b+h_{k+1}+h_k\right)\phi\left(z_k^u\right)\sqrt{n\sigma_\alpha{}^2\tilde{L}_k}-\sum_{i=1}^{k-1}h_{i+1}n\mu_\alpha\tilde{L}_k\\
&=n\left(b+h_{k+1}+h_k\right)\phi\left(z_k^u\right)\sqrt{\sigma_\alpha{}^2\tilde{L}_k}-\left(b+h_{k+1}+h_k\right)\phi\left(z_k^u\right)\sqrt{n\sigma_\alpha{}^2\tilde{L}_k}\\
&=\left(b+h_{k+1}+h_k\right)\phi\left(z_k^u\right)\sqrt{\left(n^2-n\right)\sigma_\alpha{}^2\tilde{L}_k} \qquad \text{(A29)}
\end{aligned}$$

and

$$C_k^{u,d}\left(s_k^{l,d}\right)-C_k^{u,c}\left(s_k^{l,c}\right)$$

$$=n\left(b\ +\sum_{i=1}^{k+1}h_i\right)\phi\left(z_k^{l,d}\right)\sqrt{\sigma_\alpha{}^2\tilde{L}_k}+n\sum_{i=1}^{k-1}h_{i+1}\mu_\alpha\tilde{L}_i-\left(b+\sum_{i=1}^{k+1}h_i\right)\phi\left(z_k^{l,d}\right)\sqrt{n\sigma_\alpha{}^2\tilde{L}_k}-\sum_{i=1}^{k-1}h_{i+1}n\mu_\alpha\tilde{L}_i$$

$$=n\left(b+\sum_{i=1}^{k+1}h_i\right)\phi\left(z_k^{l,d}\right)\sqrt{\sigma_\alpha{}^2\tilde{L}_k}-\left(b+\sum_{i=1}^{k+1}h_i\right)\phi\left(z_k^{l,d}\right)\sqrt{n\sigma_\alpha{}^2\tilde{L}_k}$$

$$=\left(b+\sum_{i=1}^{k+1}h_i\right)\phi\left(z_k^{l,d}\right)\sqrt{(n^2-n)\sigma_\alpha{}^2\tilde{L}_k} \qquad \text{(A30)}$$

Let $h_2^{'}=h_2+\Delta$ and $h_3^{'}=h_3-\Delta$. Then (A29) becomes

$$\left(b+h_{k+1}{}^{'}+h_k{}^{'}\right)\phi\left(\Phi^{-1}\left(\frac{b+h^{'}{}_{k+1}}{b+h^{'}{}_{k+1}+h_k{}^{'}}\right)\right)\sqrt{\left(n^2-n\right)\sigma_\alpha{}^2\tilde{L}_k}$$

$$=\left(b+h_{k+1}+h_k\right)\phi\left(\Phi^{-1}\left(\frac{b+h_{k+1}-\Delta}{b+h_{k+1}+h_k}\right)\right)\sqrt{\left(n^2-n\right)\sigma_\alpha{}^2\tilde{L}_k}$$

$$\geq\left(b+h_{k+1}+h_k\right)\phi\left(\Phi^{-1}\left(\frac{b+h_{k+1}}{b+h_{k+1}+h_k}\right)\right)\sqrt{\left(n^2-n\right)\sigma_\alpha{}^2\tilde{L}_k} \qquad \text{(A31)}$$

Hence (A29) is increasing in Δ.

Likewise, (A30) becomes

$$\left(b+\sum_{i\neq k+1,k}h_i+h^{'}{}_{k+1}+h_k{}^{'}\right)\phi\left(\Phi^{-1}\left(\frac{b+h^{'}{}_{k+1}}{b+\sum_{i\neq k+1,k}h_i+h^{'}{}_{k+1}+h_k{}^{'}}\right)\right)\sqrt{(n^2-n)\sigma_\alpha{}^2\tilde{L}_k}$$

$$=\left(b+\sum_{i=1}^{k=1}h_i\right)\phi\left(\Phi^{-1}\left(\frac{b+h_{k+1}-\Delta}{b+\sum_{i=1}^{k=1}h_i}\right)\right)\sqrt{(n^2-n)\sigma_\alpha{}^2\tilde{L}_k}$$

$$\geq \left(b + \sum_{i=1}^{k=1} h_i \right) \phi \left(\Phi^{-1} \left(\frac{b + h_{k+1}}{b + \sum_{i=1}^{k=1} h_i} \right) \right) \sqrt{(n^2 - n)\sigma_\alpha{}^2 \tilde{L}_k} \tag{A32}$$

Hence (A30) is increasing in Δ.

Thus the upper bound of V_d is increasing in Δ, the shift of holding costs from the third echelon to the second, through the mechanism of effective backordering cost rate at the second echelon. The lower bound remains independent of Δ, hence the value of delaying differentiation is non-decreasing, and likely increasing in Δ. As *n* increases, the factor $\sqrt{(n^2 - n)\sigma_\alpha{}^2 \tilde{L}_k}$ also increases, hence the increase in the value of delayed differentiation attributable to the holding cost shift is increasing in *n*.

APPENDIX 3
SAMPLE SIMULATION CODES

This appendix presents two sample C++ programs written to generate data for the studies discussed above. The first program identifies a restricted subset of candidate base stock solutions. The second program takes this restricted subset and selects the best performing solution, as discussed in the Simulation Methodology sections of Chapters 4, 5, and 6. These programs specifically calculate data for symmetric, two-echelon, two-retailer systems under Poisson demand distributions; however, similar logic was followed for data generated throughout this dissertation.

```
// Program 1: Subset Restriction


#include <iostream>
#include <fstream>
#include <cstdlib>
#include <map>
#include <utility>
#include <string>
#include <time.h>
#include "mt19937ar.h"

using namespace std;

#define RAND_UNIFORM genrand_real1();
#define t_INCREMENT_TIMES 50010

double random_normal(double esp,double var)
{
      double u1=RAND_UNIFORM;
      double u2=RAND_UNIFORM;
      double z=sqrt(-2.0*log(u1))*cos(2*M_PI*u2);
      return esp+var*z;
}

unsigned long random_poisson(double lambda)
{
      double p=exp(-lambda);
      double g=p;
      double u=RAND_UNIFORM;
      unsigned long k=0;
      while (u>g)
      {
            p*=(lambda/(double)(++k));
            g+=p;
      }
```

```
      return k;
}

int main(int argc, char *argv[])
{

      float invPosM, invPosW, invPosR1, invPosR2;
      float aggOrdersW1, aggOrdersW2, aggOrdersM_W;
      float i1A, i1B, i2, i3, stockOutsA, stockOutsB;
      float shipToR1, shipToR2;
      float mean, var;
      multimap<int, float> ordersW1;
      multimap<int, float> ordersW2;
      multimap<int, float> ordersM_W;
      float max(float x, float y);
      int random_bimodal(float lambda);


      string fileName;

      int LEAD_W_R1;
      int LEAD_W_R2;
      int LEAD_M_W;
      int D1, D2;
      int lowBoundRA, lowBoundRB, lowBoundW;
      int highBoundRA, highBoundRB, highBoundW;
      int totOrders, dole;
      int supplier,i, j, k, k1, k2;

      float h1A, h1B;
      float h2;
      float h3;
      float bA, bB;
      float cost[5001];
      float lambda;
      float delta;
      int numkeep;
      int invPosWi[9341], invPosR1i[9341], invPosR2i[9341];
    float costi[9341], vari[9341],W[9341];
    float tval;
    int keep, point, numpoint;

// declarations of file pointer streams

      ifstream filePointerInLog;
      ofstream filepointerOutLog;
      ofstream filepointerOutDemands;

// Random Number Seed

    init_genrand(2);
    point = 0;

      cout << "Enter filename to write.\n";
      cin >> fileName;

 LEAD_M_W = 1;
```

```
LEAD_W_R1 = 1;

     h1B = h1A;


     cout << "Warehouse holding cost:\n";
     cin >> h2;
     h3 = 0;
     cout << "Retailer 1 holding cost:\n";
     cin >> h1A;
     cout << "Retailer 2 holding cost:\n";
     cin >> h1B;
     h1B = h1A;
     cout << "Retailer 1 Backorder cost:\n";
     cin >> bA;
     cout << "Retailer 2 Backorder cost:\n";
     cin >> bB;
     bB = bA;

   cout << "Supplier-DC Leadtime:\n";
   cin >> LEAD_M_W;
   cout << "DC-Retailer1 Leadtime:\n";
   cin >> LEAD_W_R1;
   cout << "DC-Retailer2 Leadtime:\n";
   cin >> LEAD_W_R2;

     cout << "Retailer 1 low stocking bound:\n";
     cin >> lowBoundRA;
     cout << "Retailer 1 high stocking bound:\n";
     cin >> highBoundRA;
     cout << "Retailer 2 low stocking bound:\n";
     cin >> lowBoundRB;
     cout << "Retailer 2 high stocking bound:\n";
     cin >> highBoundRB;


   cout << "Warehouse low stocking bound:\n";
   cin >> lowBoundW;
   cout << "Warehouse high stocking bound:\n";
   cin >> highBoundW;

   cout << "t-value: \n";
   cin >> tval;
   cout << "Indifference distance (Delta) : \n";
   cin >> delta;



     cout << "Lambda:\n";
     cin >> lambda; */


     supplier = 90000;
           for (int j = lowBoundW; j < highBoundW+1; ++j)
           {
                 for (int k1 = lowBoundRA; k1 < highBoundRA+1; ++k1)
                 {
```

```
            cout << j << "\n";
               float cost[5001];
                     for (int clear = 0; clear<5001; ++clear)
                     {cost[clear] = 0;}
                     invPosR1 = k1;
                     invPosR2 = k1;
                     invPosW = j;
                     invPosM = supplier;
                     totOrders = 0;
                     dole = 0;
                     int r = 0;
                     int modIndex = 0;
                     srand(r);
                     aggOrdersW1 = 0;
                     aggOrdersW2 = 0;
                     aggOrdersM_W = 0;
                     ordersW1.clear();
                     ordersW2.clear();
                     ordersM_W.clear();
                     i1A = 0;
                     i1B = 0;
                     i2 = 0;
                     i3 = 0;
                     shipToR1 = 0;
                     shipToR2 = 0;

                     for (int count = 0; count <5001; ++count)
                     {cost[count]=0;}

                     for (int t = 0; t < t_INCREMENT_TIMES; ++t)
                     {

// Receive any ordered inventory coming in today

                           multimap<int, float>::iterator
                                 orderSearch;
                           orderSearch = ordersW1.find(t);
                           if (orderSearch != ordersW1.end())
                           {
                           invPosR1 += (float) orderSearch->second;
                                 ordersW1.erase(orderSearch);
                           }
                           orderSearch = ordersW2.find(t);
                           if (orderSearch != ordersW2.end())
                           {
                           invPosR2 += (float) orderSearch->second;
                                 ordersW2.erase(orderSearch);
                           }

                           orderSearch = ordersM_W.find(t);
                           if (orderSearch != ordersM_W.end())
                           {
                           invPosW += (float) orderSearch->second;
                                 ordersM_W.erase(orderSearch);
                           }

                           invPosM = supplier;
```

```
// Calculate demand
                              D1 = random_poisson(lambda);
                              D2 = random_poisson(lambda);
                              invPosR1 -= D1;
                              invPosR2 -= D2;


// Calculate echelon inventories
                              i1A = 0;
                              stockOutsA = 0;
                              i1B = 0;
                              stockOutsB = 0;

                              if (invPosR1 > 0)
                                    i1A += invPosR1;
                              else
                                    stockOutsA -= invPosR1;
                              if (invPosR2 > 0)
                                    i1B += invPosR2;
                              else
                                    stockOutsB -= invPosR2;

                              i2 = i1A + i1B;
                              if (invPosW > 0)
                                    i2 += invPosW;

                  for (orderSearch = ordersW1.begin(); orderSearch
                               != ordersW1.end(); ++orderSearch)
                              {
                                    i2 += (float) orderSearch->second;
                              }
                  for (orderSearch = ordersW2.begin(); orderSearch
                               != ordersW2.end(); ++orderSearch)
                              {
                                    i2 += (float) orderSearch->second;
                              }

                              i3 = i2;
                              if (invPosM > 0)
                                    i3 += invPosM;
                  for (orderSearch = ordersM_W.begin(); orderSearch
                               != ordersM_W.end(); ++orderSearch)
                              {
                                    i3 += (float) orderSearch->second;
                              }

// Calculate cost
                  cost[modIndex] += (h1A * i1A) + (h1B * i1B)+(h2 *
                               i2) + (h3 * i3) + (bA * stockOutsA)+ (bB
                              * stockOutsB);

                               if ((t % 10) == 9)
                                    ++modIndex;

// Place orders
```

```
                    orderSearch = ordersW1.find(t+LEAD_W_R1);
                    if (orderSearch != ordersW1.end())
                    aggOrdersW1 += D1 - orderSearch->second;
                    else
                          aggOrdersW1 += D1;

                    orderSearch = ordersW2.find(t+LEAD_W_R2);
                    if (orderSearch != ordersW2.end())
                    aggOrdersW2 += D2 - orderSearch->second;
                    else
                          aggOrdersW2 += D2;

              aggOrdersM_W = j + k1 + k1 - i2 + stockOutsA +
                       stockOutsB;


// Ship Orders

        totOrders = (int) aggOrdersW1 + (int) aggOrdersW2;
        if (totOrders > invPosW) {totOrders = (int) invPosW;}

              for (int hold = 0; hold < totOrders; ++hold)
                    {dole = 1;
                    if (aggOrdersW2 >= aggOrdersW1) dole = 2;
              if (dole == 1) if (invPosW > 0) {shipToR1 += 1;
                          invPosW -= 1; aggOrdersW1 -= 1;}
              if (dole == 2) if (invPosW > 0) {shipToR2 += 1;
                          invPosW -= 1; aggOrdersW2 -= 1;}
              }

                  if (shipToR1 != 0)
                          ordersW1.insert(pair <int,
                                float>(t+LEAD_W_R1,
                          shipToR1));
                    if (shipToR2 != 0)
                          ordersW2.insert(pair <int,
                                float>(t+LEAD_W_R2,
                          shipToR2));

              if (aggOrdersM_W > invPosM)
                    {
                          ordersM_W.insert(pair<int,
                                float>(t+LEAD_M_W, invPosM));
                          aggOrdersM_W -= invPosM;
                          invPosM = 0;
                    }
                    else
                    {
                          ordersM_W.insert(pair<int,
                                float>(t+LEAD_M_W,
                          aggOrdersM_W));
                          invPosM -= aggOrdersM_W;
                          aggOrdersM_W = 0;
                    }
                    shipToR1 = 0;
                    shipToR2 = 0;
```

```
                        }   // Increment time


// Calculate mean cost

                        mean = 0;
                        var = 0;
                        int foo;
                        float bar;
                        for (foo = 1; foo < 5001; ++foo)
                        {
                              mean += cost[foo];
                        }
                        mean /= 5000;
                        cout << mean << "\n";

// Calculate standard deviation.


                        for (foo = 1; foo < 5001; ++foo)
                        {
                              bar = (mean - cost[foo]);
                        var += bar*bar;
                        }

                        var /= 4999;

                        invPosWi[point] = j;
                        invPosR1i[point] = k1;
                costi[point] = mean;
                cout << costi[point] << "  " << point << "\n";
                vari[point] = var;
                        point += 1;

         } // increment k1
            }   //Increment j


//  On to stage 2 Pruning


ofstream output(fileName.c_str());

numkeep = 0;
numpoint = int(highBoundW-lowBoundW)*int(highBoundRA-lowBoundRA)+1;

for (i=0; i<numpoint; ++i)
    {
        for (j=0; j<numpoint; ++j)
            {
             W[j]=sqrt(tval*(vari[i]+vari[j])/5000);
            }
        keep = 1;
        for (k=0;k<numpoint;++k)
            {
             if (i != k)
                {
```

```
                   if (-1*costi[i] < -1*costi[k]-max(0,W[k]-delta))
                       {
                       keep = 0;
                       }
                }
            }
        if (keep == 1)
        {   output << invPosWi[i] << "\n";
            output << invPosR1i[i] << "\n";
            output << costi[i] << "\n";
            output << vari[i] << "\n";
            numkeep += 1;
            output << numkeep << "\n";
        }
    }

      cout << numkeep;
      cout << "\n\nDone!\n\n";
    cout << mean / 10 << "\n" << var / 10 << "\n";
    system("PAUSE");
      return 0;
}

float max(float x, float y)
{  if (x >= y) {return x;}
   return y;
}
```

```
// Program 2:  Candidate Selection

#include <iostream>
#include <fstream>
#include <cstdlib>
#include <map>
#include <utility>
#include <string>
#include <time.h>
#include "mt19937ar.h"

using namespace std;

#define RAND_UNIFORM genrand_real1();

double random_normal(double esp,double var)
{
      double u1=RAND_UNIFORM;
      double u2=RAND_UNIFORM;
      double z=sqrt(-2.0*log(u1))*cos(2*M_PI*u2);
      return esp+var*z;
}

unsigned long random_poisson(double lambda)
{
      double p=exp(-lambda);
      double g=p;
      double u=RAND_UNIFORM;
      unsigned long k=0;
      while (u>g)
      {
            p*=(lambda/(double)(++k));
            g+=p;
      }
      return k;
}

int main(int argc, char *argv[])
{

      float invPosM, invPosW, invPosR1, invPosR2;
      float aggOrdersW1, aggOrdersW2, aggOrdersM_W;
      float i1A, i1B, i2, i3, stockOutsA, stockOutsB;
      float shipToR1, shipToR2;
      float mean, var;
      multimap<int, float> ordersW1;
      multimap<int, float> ordersW2;
      multimap<int, float> ordersM_W;
      float max(float x, float y);
      int t_INCREMENT_TIMES, numsys, tempWare, tempRet1;


      string fileName;
      string fileName2;

      int LEAD_W_R1;
      int LEAD_W_R2;
```

```
      int LEAD_M_W;
      int D1, D2;
      int lowBoundRA, lowBoundRB, lowBoundW;
      int highBoundRA, highBoundRB, highBoundW;
      int totOrders, dole;
      int supplier,i, j, k, k1, k2;
      int tMax;
      float check;

      float h1A, h1B;
      float h2;
      float h3;
      float bA, bB;
      float cost, tempCost, h;
      float lambda;
      float delta, deltac;
      int numkeep, cont;
      int invPosWi[100], invPosR1i[100];
    float costi[100], vari[100],W[100];
    float tval;
    int keep, point, numpoint;

// declarations of file pointer streams
      ifstream filePointerInLog;
      ofstream filepointerOutLog;
      ofstream filepointerOutDemands;

// Random Number Seed

    init_genrand(2);
    point = 0;

      cout << "Enter filename to write.\n";
      cin >> fileName;
      ofstream output(fileName.c_str());
      h3 = 0;

      do
      {
      for (int z = 0; z <101; ++z)
          {invPosWi[z] = 0;
          invPosR1i[z] = 0;
          costi[z] = 0;
          vari[z] = 1;}


        tempCost = 999999;
        cout << "Enter filename to read. \n";
        cin >> fileName2;
        ifstream input(fileName2.c_str());
          cout << "Warehouse holding cost:\n";
          cin >> h2;
          cout << "Retailer 1 holding cost:\n";
          cin >> h1A;
          h1B = h1A;
          cout << "Retailer 1 Backorder cost:\n";
          cin >> bA;
```

```
        bB = bA;
        cout << "Leadtime M_W:\n";
        cin >> LEAD_M_W;
        cout << "Leadtime W_R1\n";
        cin >> LEAD_W_R1;
        LEAD_W_R2=LEAD_W_R1;

      cout << "t-value: \n";
      cin >> tval;
      cout << "Indifference distance (Delta) : \n";
      cin >> delta;
      cout << "h-value: \n";
      cin >> h;
      cout << "Mean demand rate";
      cin >> lambda;
      check = 0;
      i = 0;

// Read input file data

    while(input)
    {input >> invPosWi[i];
    lowBoundW = invPosWi[i];
    highBoundW = lowBoundW;
    input >> invPosR1i[i];
    lowBoundRA = invPosR1i[i];
    highBoundRA = lowBoundRA;
    lowBoundRB = lowBoundRA;
    highBoundRB = lowBoundRB;
    input >> costi[i];
    input >> vari[i];
    deltac = delta * costi[i];



    t_INCREMENT_TIMES = (int)max(0,(h*h*vari[i]/deltac/deltac));

      supplier = 90000;
      cost = 0;
      j = lowBoundW;
      k1 = lowBoundRA;
      k2 = lowBoundRB;
      invPosR1 = k1;
      invPosR2 = k2;
      invPosW = j;
      invPosM = supplier;
      totOrders = 0;
      dole = 0;
      int r = 0;
      int modIndex = 0;
      srand(r);
      aggOrdersW1 = 0;
      aggOrdersW2 = 0;
      aggOrdersM_W = 0;
      ordersW1.clear();
      ordersW2.clear();
      ordersM_W.clear();
```

```
        i1A = 0;
        i1B = 0;
        i2 = 0;
        i3 = 0;
        for (int t = 0; t < t_INCREMENT_TIMES; ++t)
                          {

// Receive any ordered inventory coming in today

                          multimap<int, float>::iterator orderSearch;
                                orderSearch = ordersW1.find(t);
                          if (orderSearch != ordersW1.end())
                                {
                                invPosR1 += (float) orderSearch->second;
                                      ordersW1.erase(orderSearch);
                                }
                                orderSearch = ordersW2.find(t);
                          if (orderSearch != ordersW2.end())
                                {
                                invPosR2 += (float) orderSearch->second;
                                      ordersW2.erase(orderSearch);
                                }

                                orderSearch = ordersM_W.find(t);
                          if (orderSearch != ordersM_W.end())
                                {
                                invPosW += (float) orderSearch->second;
                                      ordersM_W.erase(orderSearch);
                                }
                                invPosM = supplier;

// Calculate demand
                                D1 = random_poisson(lambda)
                                D2 = random_poisson(lambda)
                                invPosR1 -= D1;
                                invPosR2 -= D2;


// Calculate echelon inventories
                                i1A = 0;
                                stockOutsA = 0;
                                i1B = 0;
                                stockOutsB = 0;

                                if (invPosR1 > 0)
                                      i1A += invPosR1;
                                else
                                      stockOutsA -= invPosR1;
                                if (invPosR2 > 0)
                                      i1B += invPosR2;
                                else
                                      stockOutsB -= invPosR2;

                                i2 = i1A + i1B;
                                if (invPosW > 0)
                                      i2 += invPosW;
```

```
                for (orderSearch = ordersW1.begin(); orderSearch
                                != ordersW1.end(); ++orderSearch)
                        {
                                i2 += (float) orderSearch->second;

                        }
                for (orderSearch = ordersW2.begin(); orderSearch
                                != ordersW2.end(); ++orderSearch)
                        {
                                i2 += (float) orderSearch->second;
                        }

                        i3 = i2;
                        if (invPosM > 0)
                                i3 += invPosM;
                for (orderSearch = ordersM_W.begin(); orderSearch
                                != ordersM_W.end(); ++orderSearch)
                        {
                                i3 += (float) orderSearch->second;
                        }

// Calculate cost
                cost += (h1A * i1A) + (h1B * i1B)+(h2 * i2) + (h3
                                * i3) + (bA * stockOutsA)+ (bB *
                                stockOutsB);

// Place orders

                        orderSearch = ordersW1.find(t+LEAD_W_R1);
                        if (orderSearch != ordersW1.end())
                        aggOrdersW1 += D1 - orderSearch->second;
                        else
                                aggOrdersW1 += D1;

                        orderSearch = ordersW2.find(t+LEAD_W_R2);
                        if (orderSearch != ordersW2.end())
                        aggOrdersW2 += D2 - orderSearch->second;
                        else
                                aggOrdersW2 += D2;

                   aggOrdersM_W = j + k1 + k1 - i2 + stockOutsA +
                                stockOutsB;


// Ship Orders

                totOrders = (int) aggOrdersW1 + (int) aggOrdersW2;
                if (totOrders > invPosW) {totOrders = (int) invPosW;}

                   for (int hold = 0; hold < totOrders; ++hold)
                        {dole = 1;
                        if (aggOrdersW2 >= aggOrdersW1) dole = 2;
                   if (dole == 1) if (invPosW > 0) {shipToR1 += 1;
                                invPosW -= 1; aggOrdersW1 -= 1;}
                   if (dole == 2) if (invPosW > 0) {shipToR2 += 1;
                                invPosW -= 1; aggOrdersW2 -= 1;}
                   }
```

```
                                if (shipToR1 != 0)
                                        ordersW1.insert(pair <int,
                                        float>(t+LEAD_W_R1, shipToR1));
                                if (shipToR2 != 0)
                                        ordersW2.insert(pair <int,
                                        float>(t+LEAD_W_R2, shipToR2));


                        if (aggOrdersM_W > invPosM)
                              {
                                        ordersM_W.insert(pair<int,
                                        float>(t+LEAD_M_W, invPosM));
                                        aggOrdersM_W -= invPosM;
                                        invPosM = 0;
                              }
                              else
                              {
                                        ordersM_W.insert(pair<int,
                                        float>(t+LEAD_M_W, aggOrdersM_W));
                                        invPosM -= aggOrdersM_W;
                                        aggOrdersM_W = 0;
                              }
                    shipToR1 = 0;
                    shipToR2 = 0;

                        }   // Increment time


            costi[i] = cost/t_INCREMENT_TIMES;



            cout << costi[i] << "  Done cost i  \n";
            numsys = i;
            input >> i;
            } // end While not end of file loop

// compare systems

            for (j = 0; j < numsys; ++j)
                {
                   if (tempCost > costi[j])
                      {
                           tempWare = invPosWi[j];
                           tempRet1 = invPosR1i[j];
                           tempCost = costi[j];
                      }
                }

            output << fileName2 << "," << tempWare << "," << tempRet1
                        << "," << tempCost << "\n";

            cout << "continue? (1 yes 0 no) : ";
            cin >> cont;
      }
      while (cont > 0) ;
```

```
      // End input file loop
        system("PAUSE");
        return 0;
}



float max(float x, float y)
{  if (x >= y) {return x;}
   return y;
}
```

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