SOLUTION APPROACHES TO THE STOCHASTIC MULTI-ITEM INVENTORY SYSTEM UNDER A JOINT ORDER CONSTRAINT

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SOLUTION APPROACHES TO THE STOCHASTIC MULTI-ITEM

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ORDER CONSTRAINT

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Dedicated to the

Memory of My

Father

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SUMMARY

This thesis is concerned with the development of solution procedures to a particular class of stochastic multi-item inventory problems involving a joint order constraint, i.e., all items in the system must be ordered at the same time. A mathematical model based on the concept of a system reorder point is derived and compared with the more common periodic review model.

The determination of the optimal solutions for these models is accomplished with the utilization of a cyclic coordinate search algorithm.

In order to avoid the specification of backorder cost parameters, a service level approach is derived. The desired service level may be specified _a priori_ by management or selected via an interactive decision process which provides the inventory manager with relevant information concerning consequences of his service level selection.

These search algorithms are coded in Fortran and numerical examples are presented. Computer simulation solutions are also provided for comparative purposes.

CHAPTER I

INTRODUCTION

In many real world environments, the managing of inventories is accomplished under the influence of constraints imposed on the operating system either by management itself or by external factors that cannot be overriden. One common constraint facing inventory managers is a limit on capital available to be tied up in inventories. It is common to find management concerned with reducing inventories and increasing turnover rates which have as a result the decrease of capital invested in physical inventories.

Other relevant constraints include a limit on the warehouse space available for stocking of products and a limit on the number of orders that can be placed for a certain product in a given time horizon. In the management of a multi-item inventory system some or all of the above constraints may be present.

Description of the Problem

Very often one additional constraint may be very important in the operating system. This constraint involves the joint ordering of several or all the items in inventory at the same time. This case is commonly referred to as a joint order policy.

This constraint may be created by external factors as, for example, in the case of a warehouse which sends a delivery truck to the retailer only at some fixed time periods of the year and in consequence will be most likely to bring all or almost all different products that it sells to the retailer.

On the other hand, this constraint may be imposed on the system by the inventory manager himself who feels that he can save a part of the ordering costs by placing an order for more than one item at a time. This is often true when there is a single source of supply for several items and there is a high fixed cost incurred in each order placed.

Scope of the Research

This research is concentrated on this particular type of inventory policy, i.e., the joint order policy. For this purpose, three approaches are given. The first uses the well known R,T model for a multi-item inventory system. The second is based on a new "R,SR" model in which R represents as usual the base stock level of an item and SR is the system triggering level for the placement of an order, which will be called the "system reorder point". The third approach is based on a "joint marginal order point". Here a weighted probability of being out of stock is computed for the system. The decision maker specifies an acceptable probability of being out of stock, and an order is placed whenever the system reaches that limit.

Each of the first two approaches will be used within three different frameworks which can be described as follows:

A. The objective is to minimize the sum of the inventory costs (ordering cost, inventory holding cost, and backorder cost). It is assumed that all needed cost parameters can be either found in the accounting records or specified by management.

B. In the second formulation, the objective is to minimize the sum of the ordering and holding costs. The constraint is in the form of a minimum system service level that is specified by management. Here stockout cost parameters are not necessary, which is a real advantage since they are most often unquantifiable or at best difficult to quantify (whereas the other costs are more easily obtained).

C. Here the objective is the same as in formulation B above, that is, the minimization of the sum of ordering and holding costs. However, it is assumed that the system service level is not specified *a priori* at a single constraint value. Rather, it is obtained via a management decision process of comparing service levels and corresponding costs for the system when operating with each of these service levels. Here, a very useful tool is available in order to help the inventory manager make his decision. It is the method of interactive goal programming or interactive decision making, which, as the name suggests, uses the ideas of goal programming but puts them in an interactive format which is generalized to include nonlinear objective functions.

Survey of the Literature

The literature that deals with the multi-item inventory problem falls into one of the following two classes:

1. The system is treated as being composed of several independent items and individual order quantities and reorder points are derived in order to minimize the total operating cost of that system. In some cases constraints are imposed on the system in the form of maximum budget available for investment in inventory, floor space for storage, or maximum number of orders placed per year. These problems are solved by the Lagrange multiplier technique, as in [10], for example.

2. It is attempted to minimize total inventory costs by placing joint orders for several items at the same time. As mentioned in the introduction, this is often a very efficient policy to apply in an inventory system.

Balintfy [1] evaluates and compares classes of multi-item inventory problems, where joint order of several items saves a part of the set-up cost. He develops a new policy for reorder point-triggered multi-item systems which he calls the "random joint order policy". He defines a new inventory level called "can-order" point and the range determined by the difference between "can order" and reorder points replaces the triggering role of the reorder point. Then, whenever an order for a particular item must be issued, i.e., the stock of any item has dropped to the reorder level, the inventory levels of the rest of the items are checked, and all items which are in their reorder range are ordered jointly.

Danish [4] considers the case of reorder point calculations for a single item under conditions of uncertainty, given an accepted risk of stockout. He assumes that the frequency distributions of demand and lead time are given and then he calculates the marginal distribution of lead time demand in five different cases where various demand and lead time distributions are considered. Finally he studies the effect of truncation of the distributions of demand and lead time on the values of L, the reorder point, and α, the risk of stockout. He concludes that the risk of stockout decreases with an increase in the percentage of truncation and also that the truncation of distributions permits the setting of a lower reorder point associated with the given risk of stockout.

Freeman [7] considers the determination of an Rr inventory policy when the demand is distributed according to a Poisson distribution and the lead time is a random variable with any probability distribution. He derives formulas for the optimal order quantity and reorder point and points out the fact that there is no guarantee that among all possible policies the Rr policy is optimal but because of its ready applicability and simplicity it is highly desirable for commercial use.

Gavett [9] deals also with the determination of the reorder point under variable demand and lead time. He assumes that the probability of stockout is specified by management and that the reorder point for a particular item is based upon this specification. Charts are presented for the graphical determination of the reorder points.

Davis [5] considers a multi-item inventory system subject to restrictions on the system cost of obtaining and holding stock or on the system level of shortages. His decision variables are the reorder levels and reorder quantities for each item in the system.

The determination of the reorder point, given a desired customer service level, has been treated extensively in the literature. As an example, Estes [6] considered the case of inventory systems with a single item.

CHAPTER II

STOCHASTIC CONSIDERATIONS

Probabilistic Approach

This research is generally concerned with static probabilistic inventory models. The essential feature of a probabilistic model is that its measure of effectiveness--say, cost--depends not only on a set of decision variables $x_1, x_2, \ldots$ subject to the control of the decision maker but also on a set of uncontrollable variables $p_1, p_2, \ldots$ which are only known subject to probability distributions. Therefore, the following cost function can be written:

$$C = C(x_1, x_2, \ldots; \ p_1, p_2, \ldots)$$

Furthermore let $f(p_1, p_2, \ldots)$ be the joint probability distribution of the variables $p_1, p_2, \ldots$, so that

$$\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_1, p_2, \ldots) dp_1, dp_2, \ldots = 1$$

In order to define the notion of an optimal policy for probabilistic models, given the assumptions made above, the criterion for optimization most commonly used is the expected value of the measure of effectiveness, i.e.,

$$E(C) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} C(x_1,x_2,\ldots;\ p_1,p_2,\ldots) f(p_1,p_2,\ldots) dp_1 dp_2 \ldots$$

The particular policy $\hat{x}_1,\hat{x}_2,\ldots$ which minimizes $E(C)$ is called optimal. Note that $E(C)$ no longer depends on the uncertain variables. Although the expected value is almost exclusively used as the optimization criterion for probabilistic models, it does not represent the only possibility of defining a meaningful criterion. Restrictions which have to be fulfilled in a strict sense in deterministic models may have to be replaced by restrictions which must hold with a specified probability.

In principle, it is possible to generalize all deterministic models for the case of probabilistic variables. However, the demand rate usually is the variable of greatest interest. Furthermore, the great mathematical complexity added by the use of expected values limits the usefulness of more sophisticated probabilistic models. Therefore, the following chapters of this thesis will dwell upon relatively simple models with probabilistic demand and lead time. The static nature of the model is usually introduced by the assumption that demand and lead time come from independent, identical probability distributions. Demand materializes continuously in time in such a way that a linear approximation of the inventory curve is meaningful.

The final remark to be made is that in the deterministic case it is possible to determine reorder points in such a way that no shortages occur. In the probabilistic case, shortages can be prevented only with a specified probability α (confidence level).

Marginal Distribution Concepts

It is important to note that under the assumptions made in this thesis, the concept of a marginal distribution of lead time demand is particularly critical. That is, under the assumptions of stochastic lead time as well as stochastic demand per unit of time, the distribution of lead time demand is a marginal distribution.

Assume that demand per day is described by the distribution of Figure 1. Assume further that the lead time in days is described by the distribution of Figure 2. The lead time demand distribution would be described by one of the marginals of the joint probability distribution of the distributions in Figures 1 and 2.

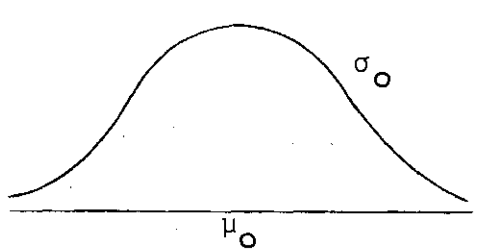

Figure 1

Probability Distribution of Demand

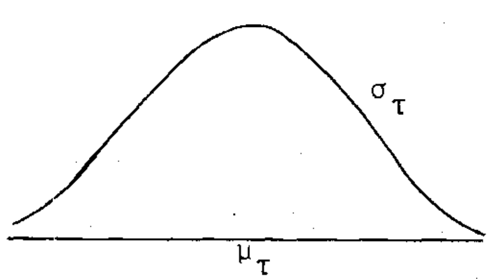

Figure 2

Probability Distribution of Lead Time

Let f(x,t) represent the joint probability distribution of demand, x, and lead time, t.

Let v(t) be the distribution of lead time. Then

$$h(x|t) = \frac{f(x,t)}{v(t)}$$

where h(x|t) is the conditional distribution of x given t. The marginal distribution of lead time demand, s, is

$$p(s) = \int_{-\infty}^{\infty} f(x,t)dt = \int_{0}^{\infty} f(x,t)dt$$

$$= \int_{0}^{\infty} h(x|t)v(t)dt \qquad \text{(II-1)}$$

With equation (II-1) it is possible to find the marginal distribution of lead time demand given the distributions of demand and lead time.

Now let μ_x be the expected value of demand, μ_t be the expected value of lead time, σ_x^2 be the variance of demand, and σ_t^2 be the variance of lead time. The expected value of lead time demand is:

$$E(s) = E(x.t)$$

Since the distributions of lead time and demand are independent, this yields:

$$E(s) = E(x) \cdot E(t) = \mu_x \cdot \mu_t = \mu_s \qquad \text{(II-2)}$$

The variance of lead time demand σ_s^2 is:

$$\sigma_s^2 = \int_{-\infty}^{\infty} (s - \mu_s)^2 \, p(s)ds =$$

$$= \int_0^{\infty} (s - \mu_x \cdot \mu_t)^2 \, p(s)ds$$

$$= \int_0^{\infty} [(s - \mu_x t) + (\mu_x t - \mu_x \cdot \mu_t)]^2 p(s)ds$$

From (II-1):

$$p(s) = \int_0^{\infty} h(s|t)v(t)dt$$

Then

$$\sigma_s^2 = \int_0^{\infty} [(s - \mu_x \cdot t) + (\mu_x t - \mu_x \cdot \mu_t)]^2 \, ds \int_0^{\infty} h(s|t)v(t)dt$$

$$= \int_0^{\infty} v(t) \int_0^{\infty} (s - \mu_x t)^2 \, h(s|t)dt \, ds$$

$$+ \int_0^{\infty} \mu_x^2 (t - \mu_t)^2 v(t)dt \int_0^{\infty} h(s|t)ds$$

$$+ 2 \int_0^{\infty} \mu_x (t - \mu_t) v(t) \int_0^{\infty} (s - \mu_x \cdot t) h(s|t) ds \cdot dt$$

But

$$\int_0^\infty (s - \mu_x t) h(s|t) ds = \int_0^\infty s.h(s|t) ds - \int_0^\infty \mu_x t \; h(s|t) ds$$

$$= t\mu_x - t\mu_x \int_0^\infty h(s|t) ds = 0$$

And

$$\int_0^\infty (s - \mu_x t)^2 \; h(s|t) ds = t \; \sigma_x^{\;2}$$

Then

$$\sigma_s^{\;2} = \int_0^\infty v(t) t \; \sigma_x^{\;2} \; dt + \mu_x^{\;2} \; \sigma_t^{\;2}$$

$$= \sigma_x^{\;2} \; \mu_t + \mu_x^{\;2} \; \sigma_t^{\;2} \qquad \text{(II-3)}$$

The mean and variance of the marginal distribution of lead time demand can be found with equations (II-2) and (II-3), given the means and variances of the distributions of demand and lead time.

As Danish [4] points out, however, it is not possible from this result to make any kind of statement about the shape of the marginal distribution of lead time demand. Although in practice the normal distribution is in general assumed, Danish shows that this is not always correct. From his analysis, the assumption of normality for the distribution of lead time demand is valid in the following cases:

A. Demand distribution: Poisson; lead time distribution: normal

B. Demand distribution: normal; lead time distribution: normal

However, it is incorrect to assume a normal distribution for the lead time demand in the following cases:

A. Demand distribution: Poisson; lead time distribution: exponential

B. Demand distribution: exponential; lead time distribution: normal

C. Demand distribution: normal; lead time distribution: exponential

There is an important result from probability theory called the "Central Limit Theorem" which is useful in this analysis. It can be stated as follows:

Let $X_1, X_2, X_3, \ldots$ be a sequence of independent distributed random variables where $E(X_i)$ exists. Let

$$E(X_i) = \mu_i \qquad \text{and} \qquad Var(X_i) = \sigma_i^{\,2}$$

Then $Y = X_1 + X_2 + X_3 + \ldots + X_n$ will be approximately normally distributed with mean $\mu = \mu_1 + \mu_2 + \mu_3 + \ldots + \mu_n$ and variance $\sigma^2 = \sigma_1^{\,2} + \sigma_2^{\,2} + \sigma_3^{\,3} + \ldots + \sigma_n^{\,2}$. As n increases, the approximation is better and the distribution of Y becomes closer to a normal curve.

From this theorem it can be concluded that if there is a large number of items in inventory and each item has an independent lead time demand distribution (which is almost always the case) then the total lead time demand will be approximately normally distributed with mean equal to the sum of the individual means and variance equal to the sum of the individual variances.

CHAPTER III

FORMULATION OF THE INVENTORY MODELS

The purpose in constructing a mathematical model of an inventory system is to utilize it as an aid in developing a suitable operating doctrine for the system. Usually one attempts to arrive at an operating doctrine that will make profits as large as possible or costs as small as possible. In other words, the criterion for selecting the operating doctrine is that of profit maximization or cost minimization. In some cases the mathematical model is so complicated that it is extremely difficult to work with it analytically. In such situations, one can use a computer simulation model to study various operating doctrines. In general, it is not possible to determine an optimal operating doctrine by use of simulation. About the best that can be done using simulation is to study a small number of operating doctrines and to select the one which seems to be the best.

As will be shown in the following chapters, the models derived for the inventory systems considered in this thesis are not suitable for analytical treatment. Therefore, use will be made of simulation as a first approach to the solution of the problem and, better yet, an efficient search algorithm will permit the determination of the best, or near-optimal solution of the inventory system.

The System Parameters

In order to proceed with the cost minimization it is necessary to identify and determine which are the relevant costs involved in the inventory system.

The first major class of costs is the procurement costs. It is customary to identify this procurement expense as ordering cost when outside suppliers are involved, and set-up cost, when the commodity is self-supplied.

These costs, in both cases, play the same role in the analytical formulation of the inventory problem. The ordering cost includes all those cost components which result from the processing of an order. It is often possible to determine the costs per order directly from cost accounting data. The self-supplier's procurement costs are called set-up costs. This name is, strictly speaking, only correct in the case of a company with a production line which makes a number of items on a job-order basis. Set-up cost then refers to the cost of changing over the production process to produce the ordered item.

The second class of costs are the inventory carrying costs. This includes a number of component costs, not all of which will necessarily be involved in the specific problems discussed here. Some of these component costs are the cost of the money tied up in inventory, the storage costs, the deterioration costs, and the insurance costs.

The third and final major class of costs is the shortage cost. There are two variants of this cost, depending on the reaction of the prospective customer to the out-of-stock situation. The first one, called backorder cost, occurs when the sale to the customer is not lost, there is only a delay in shipment of the item requested. The second one is called the cost of a lost sale. In this case the prospective customer cannot or does not want to wait for the item demanded and instead simply goes to a competitive supplier which has the item in stock.

Of the three classes of costs mentioned above, the most difficult to determine is undoubtedly the shortage cost. Although the ordering cost and the inventory carrying cost may be somewhat difficult to determine, the shortage cost is seldom quantifiable and, even then, it is mainly based on the subjective judgment of the inventory manager. As Hadley and Whitin [10] point out,

> . . . in any practical situation, it is very difficult to determine accurately the nature of the backorder cost. Backorder costs are inherently extremely difficult to measure since they can include such factors as loss of customers' goodwill. Other parts of the backorder cost can be somewhat easier to measure; however, these are usually a small part of the total backorder cost.

In this research the shortage costs will be considered in the backorder case only. The lost sales case will not be treated due to its less interesting mathematical formulation and its fewer number of practical applications.

Due to the difficulties that arise in the practical determination of the backorder cost, an alternative formulation of the inventory problem will be given considering the service level desired for the system.

The Inventory Models

The R,T Model

This section is concerned with the periodic review model for the multi-item inventory system. This operating doctrine requires that an order be placed for an item at each review time if there have been any demands at all for that item in the past review period (i.e., the time between two successive reviews). A sufficient quantity is ordered to bring the inventory position of item i (the amount on hand plus on order) up to a level R_i. In this system the quantity ordered for any item can vary from one review period to the next.

In this system the following assumptions are made:

1. The cost of making a review is independent of the variables R_i and T.

2. The unit cost of any item is constant, independent of the quantity ordered.

3. Backorders are incurred only in very small quantities. This implies that when an order arrives, it is almost always sufficient to meet any outstanding backorders.

4. It is assumed that orders are received in the same sequence in which they were placed, and furthermore, the lead times for different orders can be treated as independent random variables.

The following notation will be used:

R_i	base stock level for item i
T	time between reviews, or cycle length
n	number of different items
J	the cost of making a review
A	the cost of placing an order
I	the inventory carrying charge, equal for all items
C_i	the unit cost of item i
λ_i	mean rate of demand for item i
σ_i	standard deviation of demand rate for item i
$\bar{\tau}$	mean lead time
σ_τ	standard deviation of lead time distribution
μ_i	mean lead time demand for item i

The first part of the total cost equation to be computed is the average annual review and order cost. Since the time between reviews is T and an order is placed at each review time, this cost will be:

$$\frac{A + J}{T} = \frac{L}{T} \qquad \text{(III-1)}$$

where L is the combined cost of a review plus an order.

The average annual cost of holding inventory will be computed for one item and then the summation made over all items. In any period, the inventory position of item i immediately after the review and placement of an order is R_i. The net inventory of item i immediately after the arrival of a procurement is then R_i minus the demand during lead time for that item. Just prior to the arrival of the next order, the net inventory will be

$$R_i - (\text{lead time demand})_i - (\text{rate of demand})_i \cdot T$$

Taking expectations:

$$R_i - \mu_i - \lambda_i T$$

Then during one cycle the net inventory of item i will vary linearly between $R_i - \mu_i$ and $R_i - \mu_i - \lambda_i T$. The average number of units in inventory in any cycle is

$$\frac{1}{2}\left[(R_i - \mu_i) + (R_i - \mu_i - \lambda_i T)\right]$$

$$= R_i - \mu_i - \frac{\lambda_i T}{2}$$

which is also the expected number of units of item i in inventory during one year. Then the average annual cost of carrying inventory for all items will be:

$$\sum_{i=1}^{n} I\, C_i \left(R_i - \mu_i - \frac{\lambda_i T}{2}\right) \qquad \text{(III-2)}$$

Now the average number of backorders of item i incurred per year will be computed. First the case where the procurement lead time is a constant τ will be considered. In this model an order placed at time t will arrive in the system at time $t + \tau$, and the next procurement will arrive in the system at time $t + \tau + T$. After the order is placed at time t, the inventory position of item i is R_i. A backorder will occur between $t + \tau$ and $t + \tau + T$ if and only if the demand in the time period $\tau + T$ for item i exceeds R_i. Therefore the expected number of backorders of item i incurred per period is

$$\int_{R_i}^{\infty} (x_i - R_i) f_i(x_i | \tau + T) dx_i$$

where $f_i(x_i | \tau + T)$ is the probability distribution for the demand x_i of item i during the time interval of length $\tau + T$.

Now the procurement lead time τ is considered as a random variable with the probability distribution $g(\tau)$. Then if τ_1 and τ_2 are the lead times for the orders placed at times t and $t + T$, respectively, the expected number of backorders for item i incurred per period will be

$$\int_{o}^{\infty} \int_{o}^{\infty} \int_{R_i}^{\infty} (x_i - R_i) f_i(x_i | \tau_2 + T) g(\tau_2) g(\tau_1) dx_i \, d\tau_2 \, d\tau_1$$

But

$$\int_0^\infty g(\tau_1)d\tau_1 = 1$$

and

$$\int_0^\infty f_i(x_i|\tau_2 + T)g(\tau_2)d\tau_2$$

is the marginal distribution of demand for item i during $\tau_2 + T$, which will be represented by $h_i(x_i|T)$. Then the expected number of backorders for item i incurred per period will be

$$B_i(R_i,T) = \int_{R_i}^\infty (x_i - R_i)h_i(x_i|T)dx_i$$

and the expected number of backorders incurred per year will be

$$\frac{1}{T} B_i(R_i,T)$$

In order to simplify computations, it will be assumed that the distributions of demand for any item and the distribution of lead time are normal. Using the results of Chapter II, the marginal distribution of demand for item i during $\tau + T$ will be normally distributed with mean equal to

$$\mu_{h_i} = \lambda_i(\overline{\tau} + T)$$

and variance equal to

$$\sigma_{h_i}^{\ 2} = (\bar{\tau} + T)\sigma_i^{\ 2} + \lambda_i^{\ 2}\mathrm{Var}(\tau + T)$$

$$= (\bar{\tau} + T)\sigma_i^{\ 2} + \lambda_i^{\ 2}\ \sigma_\tau^{\ 2}$$

In the case of constant lead times, the variance of h_i reduces to

$$\sigma_{h_i}^{\ 2} = (\bar{\tau} + T)\sigma_i^{\ 2}$$

Now the expected number of backorders of item i incurred per year can be computed as follows:

$$\frac{1}{T} B_i(R_i, T) = \frac{1}{T}\int_{R_i}^{\infty} (x_i - R_i) h_i(x_i | T)\, dx_i$$

$$= \frac{1}{T}\int_{R_i}^{\infty} (x_i - R_i)\, \phi\left(\frac{x_i - \lambda_i(\bar{\tau} + T)}{\sqrt{(\bar{\tau} + T)\sigma_i^{\ 2} + \lambda_i^{\ 2}\sigma_\tau^{\ 2}}}\right) dx_i$$

$$= \frac{1}{T}\left[\sqrt{(\bar{\tau} + T)\sigma_i^{\ 2} + \lambda_i^{\ 2}\sigma_\tau^{\ 2}}\ \phi\left(\frac{R_i - \lambda_i(\bar{\tau} + T)}{\sqrt{(\bar{\tau} + T)\sigma_i^{\ 2} + \lambda_i^{\ 2}\sigma_\tau^{\ 2}}}\right)\right.$$

$$\left. + \left(\lambda_i(\bar{\tau} + T) - R_i\right)\Phi\left(\frac{R_i - \lambda_i(\bar{\tau} + T)}{\sqrt{(\bar{\tau} + T)\sigma_i^{\ 2} + \lambda_i^{\ 2}\sigma_\tau^{\ 2}}}\right)\right]$$

where $\phi(r) = \frac{1}{\sqrt{2\pi}}\, e^{-r^2/2}$ and $\Phi(r) = \int_r^{\infty} \phi(x)\, dx$. The total number of backorders incurred per year is simply

$$\frac{1}{T} \sum_{i=1}^{n} B_i(R_i,T) \tag{III-3}$$

The R,SR Model

The model to be derived in this section is based on the concept of a system reorder point. In order to utilize this model in an inventory system it is required that a transactions reporting system be established. That is, all transactions of interest be recorded as they occur, and the information be immediately made known to the decision maker. After each demand for any item, the on hand inventory levels of all items are recorded and added together to give a system on hand inventory. When this system level reaches the system reorder point, an order is placed for all items so that each item is brought up to its base stock level.

Since the rate of demand of each item is a random variable, the amount on hand at the time of reorder will also be a random variable. Therefore the quantities ordered for each item will vary from cycle to cycle.

In order to derive the cost components of this system, i.e., the ordering cost and holding cost, and the total number of backorders incurred per year, the following assumptions are made:

1. The cost of placing an order is independent of the amount ordered.

2. The unit cost of any item is constant independent

of the quantity ordered.

3. Backorders are incurred in very small quantities.

4. There is never more than one order outstanding.

In addition to the notation utilized in the derivation of the R,T model, the following will be used here:

N	expected number of cycles per year
SR	system reorder point
r_i	on hand inventory of item i when an order is placed
q_i	quantity ordered for item i

To find the yearly ordering cost of the system, it is necessary to compute the expected number of cycles or the expected number of orders placed in one year.

By definition:

$$\bar{r}_1 + \bar{r}_2 + \ldots + \bar{r}_n = SR \qquad \text{(III-4)}$$

Also be definition

$$q_i = R_i - r_i \quad , \quad i = 1,2,\ldots,n$$

And

$$E(q_i) = R_i - E(r_i)$$

$$\bar{q}_i = R_i - \bar{r}_i \qquad \text{(III-5)}$$

By definition of the joint order policy,

$$\frac{\lambda_1}{\bar{q}_1} = \frac{\lambda_2}{\bar{q}_2} = \ldots = \frac{\lambda_n}{\bar{q}_n}$$

And, using (III-5)

$$\frac{\lambda_1}{R_1 - \bar{r}_1} = \frac{\lambda_2}{R_2 - \bar{r}_2} = \ldots = \frac{\lambda_n}{R_n - \bar{r}_n} \qquad \text{(III-6)}$$

From (III-6)

$$\frac{\lambda_1}{R_1 - \bar{r}_1} = \frac{\lambda_i}{R_i - \bar{r}_i} \qquad i = 2,3,\ldots,n$$

Thus

$$\bar{r}_1 = R_1 + \frac{\lambda_1}{\lambda_i} (\bar{r}_i - R_i)$$

In the same way

$$\bar{r}_n = R_n + \frac{\lambda_n}{\lambda_i} (\bar{r}_i - R_i)$$

Then equation (III-4) yields

$$\begin{aligned}\bar{r}_i &= SR - \bar{r}_1 - \bar{r}_2 - \ldots - \bar{r}_{i-1} - \bar{r}_{i+1} - \ldots - \bar{r}_n \\ &= SR - R_1 - \frac{\lambda_1}{\lambda_i} (\bar{r}_i - R_i) - \ldots - R_{i-1} - \frac{\lambda_{i-1}}{\lambda_i} (\bar{r}_i - R_i) \\ &\quad - R_{i+1} - \frac{\lambda_{i+1}}{\lambda_i} (\bar{r}_i - R_i) - \ldots - R_n - \frac{\lambda_n}{\lambda_i} (\bar{r}_i - R_i)\end{aligned}$$

After adding and subtracting $R_i \lambda_i$ to the right-hand side of the above equation and rearranging its terms, the result is:

$$\bar{r}_i = \frac{R_i \sum_{j=1}^{n} \lambda_j - \lambda_i \sum_{j=1}^{n} R_j + SR\lambda_i}{\sum_{j=1}^{n} \lambda_j} \qquad \text{(III-7)}$$

With this equation it is possible to find the expected value of the on hand inventory of item i when an order is placed. The expected number of cycles per year will be

$$N = \frac{\sum_{j=1}^{n} \lambda_j}{\sum_{j=1}^{n} R_j - SR} \qquad \text{(III-8)}$$

Then the total yearly ordering cost is simply

$$\frac{A \sum_{j=1}^{n} \lambda_j}{\sum_{j=1}^{n} R_j - SR} \qquad \text{(III-9)}$$

The next cost component to be computed is the inventory holding cost. For any item, the inventory position immediately after the placement of an order is R_i. Its net inventory immediately after the arrival of a procurement is then $R_i - \mu_i$, where μ_i is the mean demand during lead time. Just prior to the arrival of the next order, the expected net inventory will be $\bar{r}_i - \mu_i$, where, as previously defined, $\bar{r}_i$ is the expected value of the on hand inventory of item i when an order is placed. Therefore the average on hand inventory level of item i in any cycle is

$$\frac{1}{2}\left[(R_i - \mu_i) + (\bar{r}_i - \mu_i)\right] = \frac{1}{2}(R_i - 2\mu_i + \bar{r}_i)$$

where $\bar{r}_i$ is given by (III-7). This is also the expected number of units of item i in inventory during one year. Then the average annual inventory holding cost for all items will be:

$$\sum_{i=1}^{n} \frac{I\,C_i}{2}(R_i - 2\mu_i + \bar{r}_i) \qquad \text{(III-10)}$$

It remains to evaluate the average annual number of backorders. The average number of backorders per year is simply the expected number of backorders incurred per cycle times the average number of cycles per year, N. Now the number of backorders $\eta_i(x_i,r_i)$ of item i incurred in a cycle will simply be the number of backorders on the books when a procurement arrives. If the lead time demand is x_i, the number of backorders will be

$$\eta_i(x_i,r_i) = \begin{cases} 0 & , \text{ if } x_i - r_i < 0 \\ x_i - r_i & , \text{ if } x_i - r_i \geq 0 \end{cases}$$

Thus the expected number of backorders of item i per period

$\bar{\eta}_i(r_i)$ is

$$\bar{\eta}_i(r_i) = \int_0^\infty \int_0^\infty \eta_i(x_i, r_i) h_i(x_i) g_i(r_i) dx_i dr_i$$

$$= \int_0^\infty \int_{r_i}^\infty (x_i - r_i) h_i(x_i) g_i(r_i) dx_i dr_i$$

$$= \int_0^\infty \int_{r_i}^\infty x_i h_i(x_i) g_i(r_i) dx_i dr_i$$

$$- \int_0^\infty \int_{r_i}^\infty r_i h_i(x_i) g_i(r_i) dx_i dr_i$$

$$= \int_0^\infty \left[\int_{r_i}^\infty x_i h_i(x_i) dx_i \right] g_i(r_i) dr_i$$

$$- \int_0^\infty \left[\int_{r_i}^\infty h_i(x_i) dx_i \right] r_i g_i(r_i) dr_i$$

where $h_i(x_i)$ is the marginal distribution of lead time demand of item i, given by

$$h_i(x_i) = \int_0^\infty f_i(x_i|\tau) g(\tau) d\tau$$

and $g_i(r_i)$ is the probability distribution of the on hand inventory level r_i. At this point it is convenient, in order to simplify the subsequent utilizations of this result, to assume that the distributions of demand for any item and the distribution of lead time are normal. Again using the results of Chapter II, it can be inferred that the marginal distribution of lead time demand for item i will be normally distributed with mean equal to

$$\mu_{h_i} = \lambda_i \bar{\tau}$$

and variance

$$\sigma_{h_i}^{\ 2} = \bar{\tau}\, \sigma_i^{\ 2} + \lambda_i^{\ 2}\, \sigma_\tau^{\ 2}$$

Then

$$\bar{\eta}_i(r_i) = \int_0^\infty \left[\sigma_i \phi\left(\frac{r_i - \mu_i}{\sigma_i}\right) + \mu_i \Phi\left(\frac{r_i - \mu_i}{\sigma_i}\right)\right] g_i(r_i)\,dr_i$$

$$- \int_0^\infty \Phi\left(\frac{r_i - \mu_i}{\sigma_i}\right) r_i g_i(r_i)\,dr_i$$

$$= \int_0^\infty \sigma_i \phi\left(\frac{r_i - \mu_i}{\sigma_i}\right) g_i(r_i)\,dr_i$$

$$+ \int_0^\infty \mu_i \Phi\left(\frac{r_i - \mu_i}{\sigma_i}\right) g_i(r_i)\,dr_i$$

$$- \int_0^\infty \Phi\left(\frac{r_i - \mu_i}{\sigma_i}\right) r_i g_i(r_i)\,dr_i$$

In order to proceed with the integrations involved in the above equation, the probability distribution of $r_i, g_i(r_i)$, should be determined. However, it turns out that this is not an easy task. Furthermore, the resulting integrals can only be computed by numerical methods. For these reasons, an approximate solution is necessary, substituting $\bar{r}_i$ for r_i. Thus

$$\bar{\eta}_i(\bar{r}_i) = \int_0^\infty \eta_i(x_i, \bar{r}_i) h_i(x_i) dx_i$$

$$= \int_{\bar{r}_i}^\infty (x_i - \bar{r}_i) h_i(x_i) dx_i$$

$$= \int_{\bar{r}_i}^\infty x_i h_i(x_i) dx_i - \bar{r}_i H_i(\bar{r}_i)$$

where $H_i(x_i)$ is the complementary cumulative distribution of $h_i(x_i)$, given by

$$H_i(\bar{r}_i) = \int_{r_i}^\infty h_i(x_i) dx_i$$

Therefore the average number of backorders of item i incurred per year will be

$$N\left[\int_{\bar{r}_i}^{\infty} x_i h_i(x_i)\,dx_i - \bar{r}_i H_i(\bar{r}_i)\right] = N\left[\int_{\bar{r}_i}^{\infty} x_i \frac{1}{\sigma_{h_i}} \phi\left(\frac{x_i - \mu_i}{\sigma_{h_i}}\right) dx_i - \bar{r}_i \Phi\left(\frac{\bar{r}_i - \mu_i}{\sigma_{h_i}}\right)\right]$$

$$= N\left[(\mu_i - \bar{r}_i)\Phi\left(\frac{\bar{r}_i - \mu_i}{\sigma_{h_i}}\right) + \sigma_{h_i}\phi\left(\frac{\bar{r}_i - \mu_i}{\sigma_{h_i}}\right)\right]$$

The total number of backorders incurred per year is then

$$N \sum_{i=1}^{n} \left[(\mu_i - \bar{r}_i)\Phi\left(\frac{\bar{r}_i - \mu_i}{\sigma_{h_i}}\right) + \sigma_{h_i}\phi\left(\frac{\bar{r}_i - \mu_i}{\sigma_{h_i}}\right)\right] = N \sum_{i=1}^{n} B_i(\bar{r}_i) \tag{III-11}$$

CHAPTER IV

CYCLIC COORDINATE METHOD: DESCRIPTION AND APPLICATIONS

The Search Procedure

As will be shown in the following sections of this chapter, the difficulties encountered in the attempt to minimize the total cost equations will be overcome by the utilization of a method described by Bazaraa [2]. It is a search procedure that he calls the "cyclic coordinate method". In his study he considers the following nonlinear programming problem P:

$$\begin{array}{lll} \text{minimize} & f(x) & \\ \text{subject to} & g_i(x) \leq 0 & i = 1,2,\ldots,m \\ & h_i(x) = 0 & i = 1,2,\ldots,k \end{array}$$

Bazaraa states that one of the popular schemes for solving the above problem is a penalty or barrier function approach, where the constrained problem is transformed into a single unconstrained problem, or a sequence of unconstrained problems. At least for the case of parametric penalty and barrier functions, however, the resulting unconstrained problem is very ill-conditioned such that, even the most efficient unconstrained minimizing techniques have a great deal of difficulty in handling the unconstrained penalty problem(s). It turns out that some of the most elementary procedures for unconstrained optimization, which are usually regarded in the literature as inefficient, can be successfully adopted to solve the penalty problem. (In fact, Bazaraa has developed a code based on the cyclic coordinate method that has been proved efficient for solving constrained optimization problems, by the aid of penalty functions.)

He uses the following notation:

d_i	i^{th} coordinate axis, d_i is a vector of zeros, except one at the i^{th} position
Δ	initial stepsize
Δ_{min}	minimum stepsize used
β	reduction rate of the stepsize
λ	initial penalty parameter
λ_{max}	maximum penalty parameter used
γ	penalty acceleration factor
ε	minimum displacement allowed per iteration
t	iteration counter
ℓ	switching factor beyond which the penalty parameter and/or the stepsize is changed
$\|x\|$	sum of the absolute values of the x components

Then the cyclic coordinate search procedure is described as follows:

Step 1.

Choose the starting vector x and appropriate values of the above parameters. Let $y_1 = x_1$, $i = 1$, and go to step 2.

Step 2.

Starting from y_i, minimize $p(\cdot,\lambda)$ along the direction d_i with the stepsize Δ. This leads to y_{i+1}. If $i \leq n$, replace i by i+1 and repeat step 2. Otherwise go to step 3.

Step 3.

Let $x_{t+1} = y_{n+1}$. If $|x_{t+1} - x_t| \geq \varepsilon$, let $i = 1$, $y_1 = x_{t+1}$, and repeat step 2. Now suppose that $|x_{t+1} - x_t| < \varepsilon$. If $\lambda \geq \lambda_{max}$ and $\Delta < \Delta_{min}$, then stop, with optimal x_{t+1}. Otherwise check whether $\lambda\Delta$ is smaller than ℓ or greater than or equal to ℓ. In the former case, λ is replaced by min $(\gamma\lambda, \lambda_{max})$, and in the latter case, λ is replaced by min $(2\lambda, \lambda_{max})$, and Δ is replaced by max $(\beta\Delta, \Delta_{min})$. Let $i = 1$, $y_1 = x_{t+1}$, and repeat step 2.

Bazaraa reports that nine standard test problems with varying degrees of difficulty were attempted and that the above algorithm was able to solve all the test problems. Furthermore, the computational time was comparable to, and in many cases was significantly less than, the computational time used by some of the successful penalty and nonpenalty based nonlinear programming codes.

In order to utilize the above algorithm to find the optimal solution of the total cost equations (IV-1) and

(IV-2), the following parameters were selected:

$\Delta = 1.0$ $\qquad$ $\lambda_{max} = 100\,000.0$

$\beta = 0.15$ $\qquad$ $\gamma = 10.0$

$\lambda = 1.0$ $\qquad$ $\varepsilon = 0.0004$

Each of the above parameters was chosen via a trial and error approach. In an actual application it would be important to make a prudent selection of these parameters.

Optimization of the Inventory Models

In the following sections the objective is to minimize the total yearly costs (the sum of the ordering cost, holding cost, and backorder cost) of the inventory system. As stated before, the cost of a backorder is assumed to be known for every item in the system.

The R,T Model

From equations (III-1), (III-2), and (III-3), the total cost function $K(R_i, T)$ can be written as

$$K(R_i, T) = \frac{L}{T} + \sum_{i=1}^{n} IC_i \left(R_i - \mu_i - \frac{\lambda_i T}{2}\right)$$

$$+ \sum_{i=1}^{n} \pi_i \frac{1}{T} B_i(R_i, T) \qquad \text{(IV-1)}$$

Then the optimum values of T and R_i will be solutions to

$$\frac{\partial K}{\partial T} = \frac{\partial K}{\partial R_i} = 0$$

$$\frac{\partial K}{\partial R_i} = IC_i + \frac{\pi_i}{T}\left(R_i h_i(R_i|T) - R_i h_i(R_i|T) - \int_{R_i}^{\infty} h_i(x_i|T)\,dx_i\right)$$

$$= IC_i - \frac{\pi_i}{T}\int_{R_i}^{\infty} h_i(x_i|T)\,dx_i$$

$$= IC_i - \frac{\pi_i}{T}\,\Phi\left(\frac{R_i - \lambda_i(\bar{\tau} + T)}{\sqrt{(\bar{\tau} + T)\sigma_i^2 + \lambda_i^2\sigma_\tau^2}}\right)$$

Let $R_i - \lambda_i(\bar{\tau} + T) = A$ and $\sqrt{(\bar{\tau} + T)\sigma_i^2 + \lambda_i^2\sigma_\tau^2} = B$

$$\frac{\partial K}{\partial T} = -\frac{L}{T^2} - \sum_{i=1}^{n}\frac{IC_i\lambda_i}{2}$$

$$+ \sum_{i=1}^{n}\frac{\pi_i}{T}\left\{\phi\frac{A}{B}\,\frac{1}{2B}\left[\frac{R_i^2}{\bar{\tau} + T} - \lambda_i^2(\bar{\tau} + T) + \sigma_i^2\right]\right.$$

$$+ \frac{\lambda_i(\bar{\tau} + T) - R_i}{2}\left[\frac{\lambda_i}{B} + \frac{R_i}{\lambda_i(\bar{\tau} + T)^{3/2}}\right]\phi\left(\frac{A}{B}\right)$$

$$\left. + \Phi\left(\frac{A}{B}\right)\lambda_i\right\} - \left[B\phi\left(\frac{A}{B}\right) - A\Phi\left(\frac{A}{B}\right)\right]\frac{\pi_i}{T^2}$$

However, the analytical difficulties encountered make it impossible to derive exact formulas for the optimum values of T and R_i, T^* and R_i^*, respectively. Therefore the search algorithm described above will be used to determine the "optimum" solution of equation (IV-1).

In order to minimize computer time utilization in the execution of this program, several runs were made to determine heuristically an appropriate starting x vector, i.e., one that is reasonably close to the optimal solution. The following starting vector was selected:

$$x_i = R_i = 1.5 \sqrt{\frac{2\lambda_i A}{nIC_i}} \quad i = 1,2,\ldots,n$$

$$x_{n+1} = T = \frac{182\ n}{I \sum_{i=1}^{n} C_i}$$

The R,SR Model

The same heuristic search procedure was adapted for the R,SR model. As before this model is based upon the selection of a system reorder point and individual base stock levels for each item.

The total cost function $K(R_i, SR)$ in this model is obtained from equations (III-9), (III-10) and (III-11) and is given by

$$K(R_i, SR) = \frac{A \sum_{i=1}^{n} \lambda_i}{\sum_{i=1}^{n} R_i - SR} + \sum_{i=1}^{n} \frac{IC_i}{2} (R_i - 2\mu_i + \overline{r}_i)$$

$$+ N \sum_{i=1}^{n} \pi_i B_i(\overline{r}_i) \qquad \text{(IV-2)}$$

The optimum values of R_i and SR will then be solutions to

$$\frac{\partial K}{\partial R_i} = \frac{\partial K}{\partial SR} = 0$$

$$\frac{\partial K}{\partial SR} = \frac{A \sum_{i=1}^{n} \lambda_i}{\left(\sum_{i=1}^{n} R_i - SR\right)^2} + \sum_{i=1}^{n} \frac{IC_i \ \lambda_i}{2 \sum_{j=1}^{n} \lambda_j}$$

$$+ \sum_{i=1}^{n} \pi_i \left[\int_{\bar{r}_i}^{\infty} x_i h_i(x_i)\,dx_i - \bar{r}_i H_i(\bar{r}_i)\right] \frac{\sum_{i=1}^{n} \lambda_i}{\left(\sum_{i=1}^{n} R_i - SR\right)^2}$$

$$+ \frac{\sum_{i=1}^{n} \lambda_i}{\sum_{i=1}^{n} R_i - SR} \left[- \sum_{i=1}^{n} \frac{\pi_i \ \lambda_i}{\sum_{i=1}^{n} \lambda_i} H_i(\bar{r}_i)\right]$$

$$\frac{\partial K}{\partial R_i} = \frac{- A \sum_{j=1}^{n} \lambda_j}{\left(\sum_{i=1}^{n} R_i - SR\right)^2} + IC_i - \sum_{i=1}^{n} \frac{IC_i}{2 \sum_{j=1}^{n} \lambda_j} \lambda_i$$

$$- \sum_{i=1}^{n} \pi_i \left[\int_{\bar{r}_i}^{\infty} x_i h_i(x_i)\,dx_i - \bar{r}_i H_i(\bar{r}_i)\right] \frac{\sum_{j=1}^{n} \lambda_j}{\left(\sum_{i=1}^{n} R_i - SR\right)^2}$$

$$+ \frac{\sum_{j=1}^{n} \lambda_j}{\sum_{i=1}^{n} R_i - SR} \left\{- \pi_i H_i(\bar{r}_i) + \sum_{i=1}^{n} \pi_i H_i(\bar{r}_i) \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i}\right\}$$

Again in this case it is not possible to solve analytically the resulting system of equations for the optimum values of R and SR. Thus the same approach will be utilized, i.e., the application of the search procedure described in the first section of this chapter.

After several runs, the starting x vector selected for this model was:

$$x_i = R_i = \sqrt{\frac{2\lambda_i A}{nIC_i}} + \lambda_i \frac{\sum_{i=1}^{n} \mu_i}{\sum_{i=1}^{n} \lambda_i} \quad i = 1,2,\ldots,n$$

$$x_{n+1} = SR = \sum_{i=1}^{n} \mu_i$$

A sample output of this program is shown in Figure 4. Figure 3 illustrates the sensitivity of the total inventory costs to changes in R_2 and SR, for the case of a 2-item inventory system.

The "optimal" solutions obtained with the utilization of the search algorithm were verified with an exhaustive trial and error program, which confirmed the resulting total costs.

The Service Level Approach

As discussed in Chapter III, it is evident that the task of determining backorder cost penalty parameters may

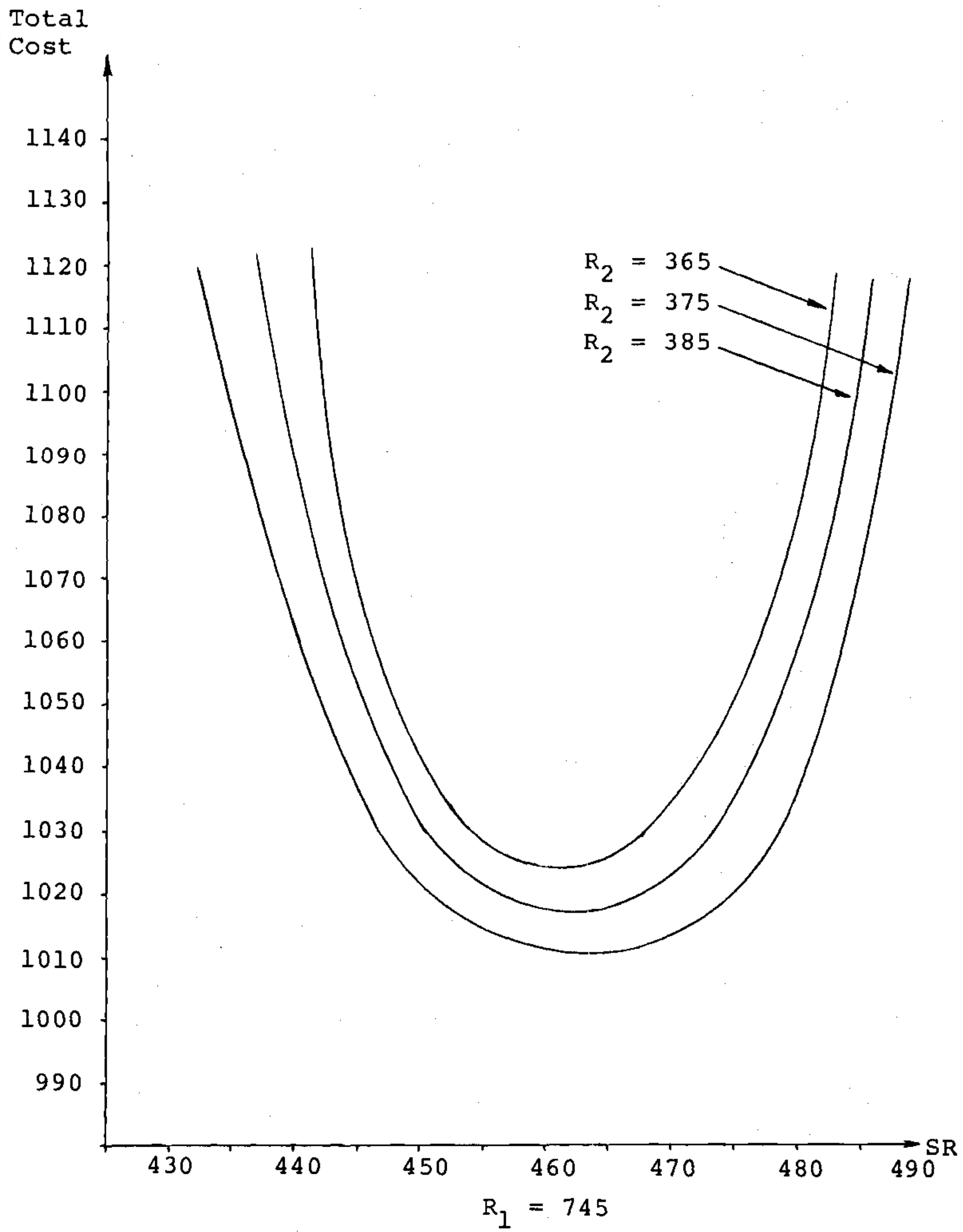

Figure 3. Graph of the Total Inventory Costs as a Function of the Decision Variables

```
ENTER THE NUMBER OF ITEMS (INTEGER) THEN RETURN.

2

ENTER THE FOLLOWING PARAMETERS:
1-MEAN RATE OF DEMAND, IN UNITS/YEAR
2-MEAN LEAD TIME DEMAND, IN UNITS
3-STANDARD DEVIATION OF LEAD TIME DEMAND, IN UNITS
4-THE COST OF AN ITEM, IN DOLLARS
5-THE COST OF A BACKORDER, IN DOLLARS
ALL ARE REAL SEPARATED BY COMMA
ENTER THE DATA FOR ONE ITEM PER LINE.

1000.0, 41.0, 4.0, 15.00, 5.00
2000.0, 82.0, 8.0, 30.00, 9.00

ENTER THE INVENTORY HOLDING RATE AND THE COST OF AN ORDER
BOTH REAL AND SEPARATED BY COMMA.

0.25, 20.00


FOR THIS   2 ITEM INVENTORY SYSTEM, THE EXPECTED YEARLY
COSTS COMPUTED BY THE SEARCH ALGORITHM ARE:

    ITEM          HOLDING COST          BACKORDER COST

       1             116.76                 5.95
       2             464.13                24.17

   TOTAL             580.89                30.12
THE ORDERING COST FOR THE SYSTEM IS       417.85
THE TOTAL COST FOR THE SYSTEM IS         1028.85

THE 'OPTIMAL' SOLUTION IS DESCRIBED BELOW:

THE SYSTEM REORDER POINT SHOULD BE  144 UNITS.
THE BASE STOCK LEVEL FOR ITEM    1 SHOULD BE    96 UNITS.
THE BASE STOCK LEVEL FOR ITEM    2 SHOULD BE   191 UNITS.


   ***** END OF PROGRAM *****
```

Figure 4. Application of the Search Algorithm to a 2-Item Inventory System

be difficult at best. To circumvent this problem, the concept of system service level may be substituted as an objective measure instead of backorder costs.

There are several different ways in which the concept of service level can be defined. In practice, it is usually defined as the average fraction of the time that demand is satisfied from stock, which is equivalent to 1 - (the expected number of backorders incurred per year divided by the average rate of demand). This, of course, can be expressed as a probability and is denoted by 1 - P (out) where P (out) is the probability of being out of stock. Then

$$\text{Service level} = 1 - P\ (\text{out}) = \frac{E\ (\text{demand satisfied from stock})}{E\ (\text{yearly demand})}$$

Often P (out) is used in place of service level since a 95 percent service level implies the item is out of stock 5 percent of the time.

In some instances the manager of the system will place limits on the service levels for certain items. He may wish to keep the probability of being out of stock, P (out), from exceeding some prescribed level α. Thus his objective is to keep $P\ (\text{out})_i$ very near the value α_i where α_i may be different from item to item. There are also other items in the system whose service levels are not critical to the operation of the enterprise. The manager, in that case, may wish to use as objectives the service levels of his critical items and the overall system service level. Thus, the problem can be reformulated as

minimize ordering cost + holding cost

subject to system service level $\geq \alpha$

service level for item i $\geq \beta_i$,

$i = 1,2,\ldots,n$

where α and β_i are to be specified by management at levels that will reflect directly or not the values assigned to backorders and the importance and criticality of the different items in the system.

The R,T Model

In this case, the formulation of the problem is

minimize

$$\frac{L}{T} + \sum_{i=1}^{n} IC_i \left(R_i - \mu_i - \frac{\lambda_i T}{2}\right) \qquad \text{(IV-3)}$$

subject to

$$\frac{\frac{1}{T} \sum_{i=1}^{n} B_i(R_i,T)}{\sum_{i=1}^{n} \lambda_i} \leq 1 - \alpha \qquad \text{(IV-4)}$$

and

$$\frac{\frac{1}{T} B_i(R_i,T)}{\lambda_i} \leq 1 - \beta_i \quad , \quad i = 1,2,\ldots,n \qquad \text{(IV-5)}$$

Now it is clear that if the costs in the objective function (IV-3) are to be minimized, the average fraction of time out of stock should be as large as possible. Hence the constraint (IV-4) will be active, i.e.,

$$\frac{1}{T} \sum_{i=1}^{n} B_i(R_i,T) = (1 - \alpha) \sum_{i=1}^{n} \lambda_i$$

Then from the theory of Lagrange multipliers, one should form the function

$$F(r_i,T,\theta) = \frac{L}{T} + \sum_{i=1}^{n} IC_i(R_i - \mu_i - \frac{\lambda_i T}{2})$$

$$+ \theta\left[\frac{1}{T} \sum_{i=1}^{n} B_i(R_i,T) - (1 - \alpha) \sum_{i=1}^{n} \lambda_i\right]$$

where θ is a Lagrange multiplier, and the optimal values of R_i,T and θ will be solutions to

$$\frac{\partial F}{\partial R_i} = \frac{\partial F}{\partial T} = \frac{\partial F}{\partial \theta} = 0 \quad ,$$

given that these values satisfy constraints (IV-5). If this is not the case then some of these constraints will be active and should be imposed upon the function $F(R_i,T,\theta)$, each one with a new Lagrange multiplier. The solution obtained is checked in constraints (IV-5) remaining and if some are violated, the above process is repeated. If not, the optimal solution has been reached. However, it turns out that the above procedure is not practical due to the tedious equations involved and the impossibility of working analytically with the resulting partial derivatives. However, the above problem can conveniently be solved by the search algorithm described in the first section of this chapter. In this case the objective function becomes

$$f(R_i,T) = \frac{L}{T} + \sum_{i=1}^{n} IC_i \; (R_i - \mu_i - \frac{\lambda_i T}{2})$$

and the constraints will be

$$g_1(R_i,T) = \frac{\frac{1}{T} \sum_{i=1}^{n} B_i(R_i,T)}{\sum_{i=1}^{n} \lambda_i} - 1 + \alpha$$

$$g_i(R_i,T) = \frac{\frac{1}{T} B_i(R_i,T)}{\lambda_i} - 1 + \beta_i \;\; , \; i = 2,3,\ldots,n+1$$

The resulting Fortran program is shown in Appendix E, and a sample output in Figure 5.

The R,SR Model

For this inventory model, the formulation of the problem becomes

minimize

$$\frac{A \sum_{i=1}^{n} \lambda_i}{\sum_{i=1}^{n} R_i - SR} + \sum_{i=1}^{n} \frac{IC_i}{2} (R_i + \bar{r}_i - 2\mu_i)$$

subject to

$$\frac{N \sum_{i=1}^{n} B_i(\bar{r}_i)}{\sum_{i=1}^{n} \lambda_i} \leq 1 - \alpha$$

and

$$\frac{NB_i(\bar{r}_i)}{\lambda_i} \leq 1 - \beta_i \quad , \quad i = 1,2,\ldots,n$$

The same reasoning utilized for the <R,T> model is valid here, i.e., the Lagrange multiplier technique could be attempted to solve the above problem, but the analytical treatment required becomes extremely difficult and an optimal solution, if any, is not possible to be determined by this method. Thus, again use will be made of the "cyclic coordinate method" to find the optimal solution of this system.

The objective function is

$$f(R_i, SR) = \frac{A \sum_{i=1}^{n} \lambda_i}{\sum_{i=1}^{n} R_i - SR} + \sum_{i=1}^{n} \frac{IC_i}{2} (R_i + \bar{r}_i - 2\mu_i)$$

and the constraints are

$$g_1(R_i, SR) = \frac{N \sum_{i=1}^{n} B_i(\overline{r}_i)}{\sum_{i=1}^{n} \lambda_i} - 1 + \alpha$$

and

$$g_i(R_i, SR) = \frac{NB_i(\overline{r}_i)}{\lambda_i} - 1 + \beta_i \quad , \quad i = 2,3,\ldots,n+1$$

Interactive Goal Programming

It should be noted that the specification of a desired service level *a priori* may also prove to be quite difficult for the inventory decision maker. As a practical alternative, it may prove helpful to devise a scheme whereby the decision maker can select a desirable service level after observing the effects of such a service level constraint on the other variables of the objective function. In such a scheme the service level constraint is obtained after a comparison between several feasible service levels and the resulting inventory costs for the system when operating with each of these service levels.

For this purpose, use will be made of the interactive goal programming algorithm proposed by Garrido and Deane [8]. The advantage of this procedure is that it overcomes some of the problems often encountered in application of Operations Research principles. By using multiple objectives (cost minimization and desired system service level) and an

```
ENTER THE NUMBER OF ITEMS (INTEGER) THEN RETURN.

2

IS EACH ITEM TO HAVE THE SAME MINIMUM SERVICE LEVEL?
(ENTER YES OR NO)

NO

ENTER THE FOLLOWING PARAMETERS:
1-MEAN RATE OF DEMAND,IN UNITS/YEAR
2-STANDARD DEVIATION OF DEMAND RATE, IN UNITS/YEAR
3-THE COST OF AN ITEM, IN DOLLARS
4-THE MINIMUM SERVICE LEVEL DESIRED FOR THE ITEM
ALL ARE REAL SEPARATED BY COMMA
ENTER THE DATA FOR ONE ITEM PER LINE.

1000.0, 100.0, 15.00, 0.55
2000.0, 200.0, 30.00, 0.65

ENTER THE INVENTORY HOLDING RATE AND THE COST OF
A REVIEW + AN ORDER. BOTH REAL AND SEPARATED BY COMMA.

0.25, 20.00

ENTER THE MEAN AND STANDARD DEVIATION OF LEAD TIME, IN DAYS,
BOTH REAL AND SEPARATED BY COMMA.

15.0, 2.0

ENTER THE DESIRED MINIMUM SYSTEM SERVICE LEVEL
A REAL NUMBER BETWEEN 0 AND 1.

0.94


FOR THIS   2 ITEM INVENTORY SYSTEM, THE EXPECTED YEARLY
COSTS COMPUTED BY THE SEARCH ALGORITHM ARE:

    ITEM          SERVICE LEVEL          HOLDING COST

      1                .96                  98.34
      2                .93                 347.06

   TOTAL               .94                 445.40
THE REVIEW + ORDER COST FOR THE SYSTEM IS  445.12
THE TOTAL COST FOR THE SYSTEM IS           890.52
```

Figure 5. Application of the Service Level Approach

```
THE 'OPTIMAL' SOLUTION IS DESCRIBED BELOW:

THE TIME BETWEEN REVIEWS FOR THIS SYSTEM SHOULD BE  16 DAYS.
THE BASE STOCK LEVEL FOR ITEM    1 SHOULD BE   89 UNITS.
THE BASE STOCK LEVEL FOR ITEM    2 SHOULD BE  173 UNITS.


DO YOU WANT TO CHANGE THE SYSTEM SERVICE LEVEL? (YES OR NO)

YES

DO YOU WANT TO CHANGE THE INDIVIDUAL SERVICE LEVELS?

NO


ENTER THE DESIRED MINIMUM SYSTEM SERVICE LEVEL
A REAL NUMBER BETWEEN Ø AND 1.

Ø.9Ø


FOR THIS   2 ITEM INVENTORY SYSTEM, THE EXPECTED YEARLY
COSTS COMPUTED BY THE SEARCH ALGORITHM ARE:

    ITEM          SERVICE LEVEL          HOLDING COST

       1                .97                 1ØØ.59
       2                .86                 248.69

   TOTAL                .9Ø                 349.28
THE REVIEW + ORDER COST FOR THE SYSTEM IS  486.67
THE TOTAL COST FOR THE SYSTEM IS           835.95

THE 'OPTIMAL' SOLUTION IS DESCRIBED BELOW:

THE TIME BETWEEN REVIEWS FOR THIS SYSTEM SHOULD BE  15 DAYS.
THE BASE STOCK LEVEL FOR ITEM    1 SHOULD BE   88 UNITS.
THE BASE STOCK LEVEL FOR ITEM    2 SHOULD BE  156 UNITS.


DO YOU WANT TO CHANGE THE SYSTEM SERVICE LEVEL? (YES OR NO)
NO
DO YOU WANT TO CHANGE THE INDIVIDUAL SERVICE LEVELS?
NO
```

Figure 5. Continued

interactive approach the languages of business and optimization can be brought closer together. Furthermore, as Garrido and Deane state,

> . . . extensive mathematical models exist to deal with numerous problem situations, yet in the business world they are seldom used. Reasons for this can be attributed to many factors but surely, the fact that businessmen do not fully trust the so-called optimum solutions arrived by "magical-mystical" means, must be at the top of the list. The operations research practitioner has heretofore been guilty of asking management for undeterminable cost parameter estimates and then showing the "optimal" solution back to the manager. Even when the solution procedures are reasonably well explained, and properly packaged and sold to management, most managers would rather trust their intuition.

The interactive procedure can be described as follows:

1. Obtain an initial value of α, α^o. This can be done by an arbitrary assignment or by asking the decision maker to give an initial estimate for this parameter.

2. Using the initial value of α (α^o) find the optimal values of the decision variables (R_i^* and T^* for the R,T model or R_i^* and SR^* for the R,SR model). This yields $C(R_i,T)^o$ or $C(R_i,SR)^o$, the initial value for the sum of ordering and holding costs.

3. Perturb (i.e., increase or decrease) the present value of α, say α^k, and obtain $C(R_i,T)^{k+1}$ or $C(R_i,SR)^{k+1}$ and α^{k+1}. If $C(R_i,T)^{k+1}$ or $C(R_i,SR)^{k+1}$ is the preferred resulting solution after all possible perturbations of α, terminate, otherwise increase k by one and repeat step 3.

The algorithm above was programmed in Fortran on a Univac 1108 computer in order to illustrate the tradeoffs which must be made by a decision maker when using this type of problem solving approach. It also serves to illustrate the interdependencies between the C(x)'s and α's. The program is shown in Appendix D. A computer printout of a numerical example is shown in Figure 6.

It is important to note that the problem considered in the first part of this chapter was a single objective problem. The objective was to minimize the total yearly variable cost of running an inventory system. Had any of the cost factors A, I, or π_i, not been available or obtainable, such a straight forward optimization procedure would not have been applicable. The approach given in this section would then be able to handle the problem and supply the manager with the answers he would be looking for.

```
ENTER THE NUMBER OF ITEMS (INTEGER) THEN RETURN.

2

IS EACH ITEM TO HAVE THE SAME MINIMUM SERVICE LEVEL?
(ENTER YES OR NO)

YES

ENTER THE COMMON MINIMUM SERVICE LEVEL FOR ANY AND ALL ITEMS
A REAL NUMBER BETWEEN Ø AND 1.

Ø.6Ø

ENTER THE FOLLOWING PARAMETERS:
1-MEAN RATE OF DEMAND,IN UNITS/YEAR
2-MEAN LEAD TIME DEMAND, IN UNITS
3-STANDARD DEVIATION OF LEAD TIME DEMAND, IN UNITS
4-THE COST OF AN ITEM, IN DOLLARS
ALL ARE REAL SEPARATED BY COMMA
ENTER THE DATA FOR ONE ITEM PER LINE.

1ØØØ.Ø, 41.Ø, 4.Ø, 15.Ø
2ØØØ.Ø, 82.Ø, 8.Ø, 3Ø.Ø

ENTER THE INVENTORY HOLDING RATE AND THE COST OF AN ORDER
BOTH REAL AND SEPARATED BY COMMA.

Ø.25, 2Ø.ØØ

ENTER THE DESIRED MINIMUM SYSTEM SERVICE LEVEL
A REAL NUMBER BETWEEN Ø AND 1.

Ø.94

                                   A           B

COST                           1Ø38.37     863.ØØ

SYSTEM SERVICE LEVEL            .99996      .94ØØØ

  ****** DO YOU PREFER A OR B? ******
  ** IF YOU WANT TO STOP TYPE STOP **

B
```

Figure 6. Application of the Goal Programming Approach

```
                              A          B

COST                        863.00     801.44

SYSTEM SERVICE LEVEL        .94000     .88000

   ****** DO YOU PREFER A OR B? ******
   ** IF YOU WANT TO STOP TYPE STOP **

A
                              A          B

COST                        913.04     863.00

SYSTEM SERVICE LEVEL        .97000     .94000

   ****** DO YOU PREFER A OR B? ******
   ** IF YOU WANT TO STOP TYPE STOP **

STOP

ENTER THE SYSTEM SERVICE LEVEL THAT YOU PREFER.

0.96

USING THE SERVICE LEVEL SELECTED ABOVE,
THE SYSTEM COSTS WILL BE:

    ITEM                SERVICE LEVEL   HOLDING COST

      1                     1.00            140.49
      2                      .94            448.48

   TOTAL                     .96            588.98
THE ORDERING COST FOR THE SYSTEM IS         300.28
THE TOTAL COST FOR THE SYSTEM IS            889.26

THE 'OPTIMAL' SOLUTION IS DESCRIBED BELOW:

THE SYSTEM REORDER POINT SHOULD BE  120 UNITS.
THE BASE STOCK LEVEL FOR ITEM   1 SHOULD BE  111 UNITS.
THE BASE STOCK LEVEL FOR ITEM   2 SHOULD BE  208 UNITS.

 ****** END OF PROGRAM ******
```

Figure 6. Continued

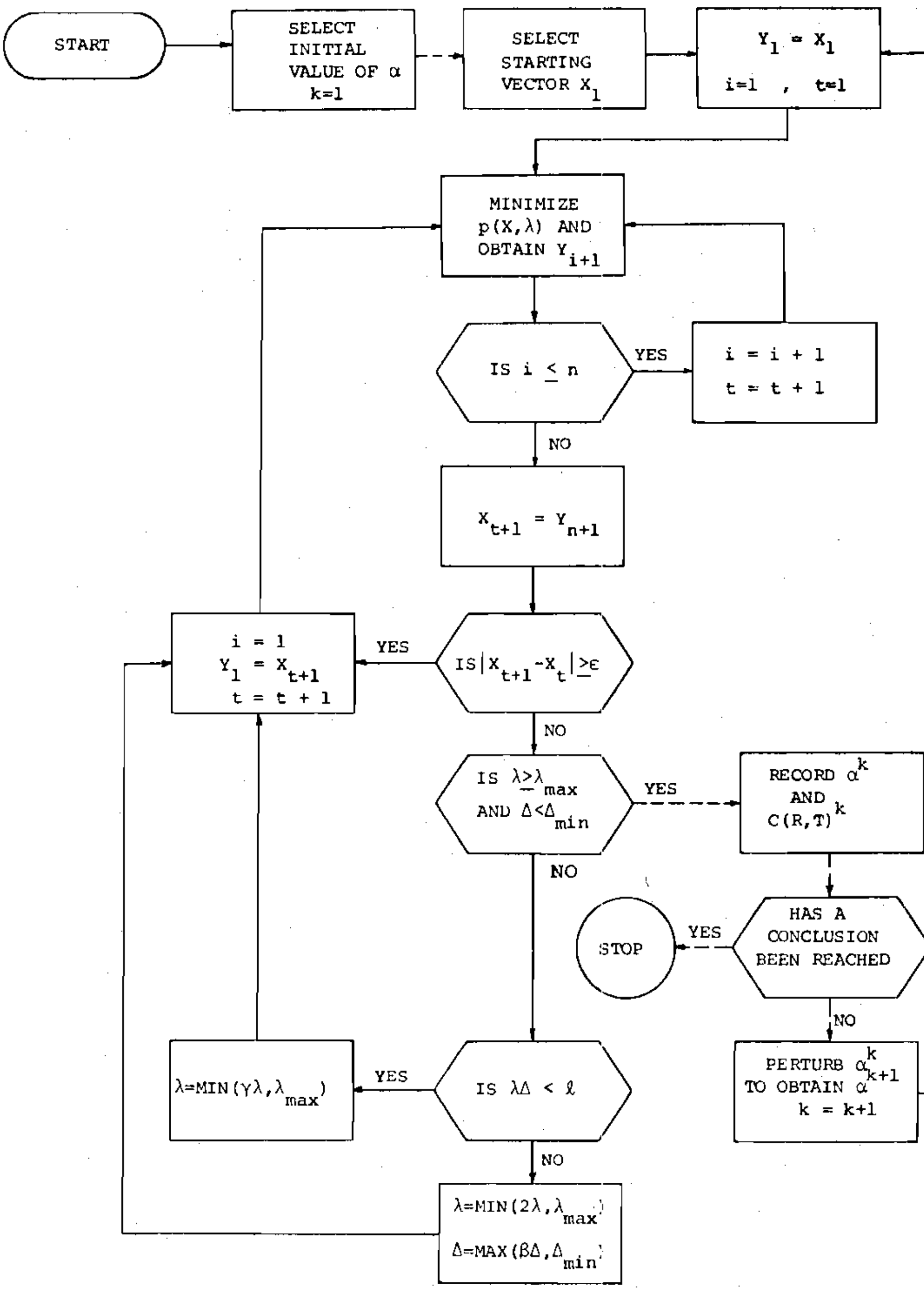

Figure 7. Flow Chart of the Search Procedure

CHAPTER V

SIMULATION EXPERIMENTS

This chapter provides a description of the simulation model used in this research along with an analysis of the results obtained. A computer simulation program written in GASP II was utilized, a programming language which inherently has the necessary housekeeping routines for conducting a simulation while the user writes subroutines pertinent to those events being simulated. This program models a multi-item inventory system which operates under a joint-order constraint with backorders permitted. It is capable of simulating demands, orders, receipts, and performing accounting functions and pertinent calculations to derive cost and statistical data for each item in the system.

Experimental Procedure

The procedure utilized to compare the results from the simulation program with the ones from the search algorithm can now be described.

A numerical example of a 2-item inventory system was selected as shown in Table 1. Chosen an arbitrary system service level, these values were used to run the simulation program of the "joint marginal order point" approach and the total yearly costs recorded. The same input values were utilized in the search algorithm programs of the <R,SR> and <R,T> models. The resulting total yearly costs were recorded and the optimal values of the decision variables (R_i and SR for the R, SR model and R_i and T for the R,T model) used as inputs in the corresponding simulation programs, whose resulting total costs were also recorded. This process was repeated for nine different system service levels, ranging from 0.9997 to 0.8706. The results are shown in Table 2.

Discussion of Results

From Table 2 it appears that the programs for the "system reorder point" model and the periodic review model give very consistent results, that is, as the system service level is decreased, the resulting policy costs also decrease.

Although the total yearly costs for the R,SR model are lower than for the other two models, it cannot be concluded that this is an optimal policy and should be utilized in real situations whenever possible. In fact, due to the analytical difficulties involved in the derivation of optimal solutions for these models, only a careful and exhaustive experimental design can yield results that would lead to meaningful conclusions.

It is important to note that the time between reviews, T, in the <R,T> model is, in general, one half of the cycle length in the <R,SR> model.

For this numerical example suppose there are two items in the system and their characteristics are given in the table below.

Table 1. Data Set for Numerical Example

	Item 1	Item 2
λ	1000	2000
σ	100	200
μ	41	82
C	15	30

Let I = 0.25 be used as the inventory holding rate (the same for all items) and A = 20.00 be the cost of an order. The lead time has a mean τ = 15 days and standard deviation σ_τ = 2 days.

Table 2. Results for Data Set of Table 1

System Service Level	<R,β> Model Simulation	<R,SR Model> System Reorder Pt.		<R,T Model> Periodic Review	
		Search Algorithm	Simulation	Search Algorithm	Simulation
0.9997	1095.00	1037.90	1062.30	1265.60	1226.90
0.9920	1008.10	936.60	952.90	1078.80	1081.90
0.9807	971.60	947.20	972.30	1009.60	1038.40
0.9717	943.80	910.30	910.30	971.80	925.40
0.9593	909.10	888.20	898.10	936.10	876.80
0.9403	877.60	863.20	906.60	890.60	839.10
0.9240	845.10	828.30	852.70	859.20	792.60
0.9060	812.80	830.70	863.50	851.90	814.60
0.8706	789.50	813.30	874.00	793.90	774.30

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Inventory stockage situations are fundamentally alike, each involving some aspects of cost, service, and usage. The objective in any given situation is to make that set of decisions which will minimize total costs and provide an acceptable--or economical--service level at the expected demand or usage rate. It is necessary that appropriate cost parameters, desired service levels, and forecasts of demand and replenishment characteristics be determined for each item in the system.

Here, for all practical purposes, the similarity ends. Each inventory problem will differ somewhat in the specific use of quantitative methods available for control and management. The level of sophistication required in employing decision models will depend on the unique characteristics of each situation. Thus, the models and methods discussed in this thesis should be considered primarily as a specific approach to the joint-order constrained inventory system and not as a solution to be utilized in every real world situation.

Conclusions

As a result of the research conducted in the preparation of this thesis, the following conclusions have been made:

1. With the sets of data used in this research, the <R,SR> model appeared to work effectively, yielding in most cases solutions with lower total costs than the solutions from the other models.

2. The search algorithm utilized in the optimization of the total cost equations has provided consistent solutions and has proved efficient, in the sense that computer time utilization has been negligible for most of the test runs made in this research.

3. With the utilization of the service level concept, the interactive goal programming approach can be utilized by management whenever a decision on the most suitable system service level has to be reached.

4. The simulation model utilized to compare the results from the search algorithm produced good results and is an efficient process of studying the effects of different inventory policies. However, an attempt to find the optimum solution to a total cost equation representing a specific inventory model by the utilization of the simulation program should seldom be made. Only in very simple cases should this method be utilized.

Recommendations

Although there seems to be good indications that suggest that the system reorder point model is operationally superior to the periodic review model, there is a need for a systematic experimental design study to confirm this hypothesis. Furthermore, the same type of comparison can be made with other models in the literature.

This research has demonstrated the applicability of the search algorithm to the optimization of multi-item inventory models. It remains to be evaluated whether it is the most efficient algorithm along the others currently known.

Within the computer programs themselves several improvements could be attempted. For example, in studying the outputs of several different sets of data, a more precise starting vector could be derived for the search algorithm.

Other types of probability distributions of demand and lead time should be considered and inventory models derived for each case. The resulting total cost equations could be solved via the utilization of the search algorithm described in this research.

The service level concept utilized in this study could be modified and other variations of this measure made and applied to the inventory models.

APPENDICES

APPENDIX A

TESTING OF RANDOM NUMBERS GENERATED IN SIMULATION PROGRAMS

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TESTING OF RANDOM NUMBERS GENERATED IN SIMULATION PROGRAMS

During the utilization of the simulation programs described, need was found of a testing of the random numbers generated by subroutines in the simulation programs. The statistical test utilized was the Chi-square test for goodness of fit to the uniform distribution. This test is based on the following equation

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E)^2}{E}$$

where n is the number of class intervals in which the unit interval is divided. O_i is the observed frequency of each class interval. E is the expected frequency, the same for all class intervals. The degrees of freedom for this test is one less the number of class intervals.

Other statistical tests could be used, for example the test for independence of successive numbers or serial correlation and the test for runs up and down. However, in this study only the first test was utilized, which gave good results.

A more detailed and complete study of random number testing was done by Stephens and White [16] and Herrmann [11].

APPENDIX B

NOMOGRAPHY APPLICATION

APPENDIX B

NOMOGRAPHY APPLICATION

The development of nomographs can sometimes simplify the practical utilization of mathematical equations.

Figure 8 shows an example of a nomograph representing the following equation:

$$N = \frac{\sum_{i=1}^{n} \lambda_i}{\sum_{i=1}^{n} R_i - SR}$$

It is used for the graphical determination of the number of inventory cycles per year, given the rates of demand, base stock levels, and system reorder point.

The scales selected for the construction of this nomograph were

$$\sum_{i=1}^{n} \lambda_i = 0 \text{ to } 10000 \text{ units/year}$$

$$\sum_{i=i}^{n} R_i = 0 \text{ to } 1000 \text{ units}$$

$$SR = 0 \text{ to } 750 \text{ units}$$

Let

$$N = \frac{\sum_{i=1}^{n} \lambda_i}{\sum_{i=1}^{n} R_i - SR} = \frac{a}{b - c}$$

Then

$$m_b = \frac{X_b}{b} = \frac{20}{1000} = 0.020$$

$$m_c = m_b = 0.020$$

$$m_a = \frac{X_a}{a} = \frac{20}{10000} = 0.0020$$

Thus

$$m_N = \frac{m_a}{m_b} K = \frac{0.0020}{0.020} \cdot 15 = 1.5$$

$$X_N = m_N \cdot N = 1.5\ N$$

To utilize this nomograph, the procedure is as follows:

1. Draw a straight line from the point selected on the $\sum_{i=1}^{n} R_i$ scale to the point selected on the SR scale.

2. From the point selected on the $\sum_{i=1}^{n} \lambda_i$ scale, draw a line parallel to the first one. This line will encounter the N scale on the desired solution point.

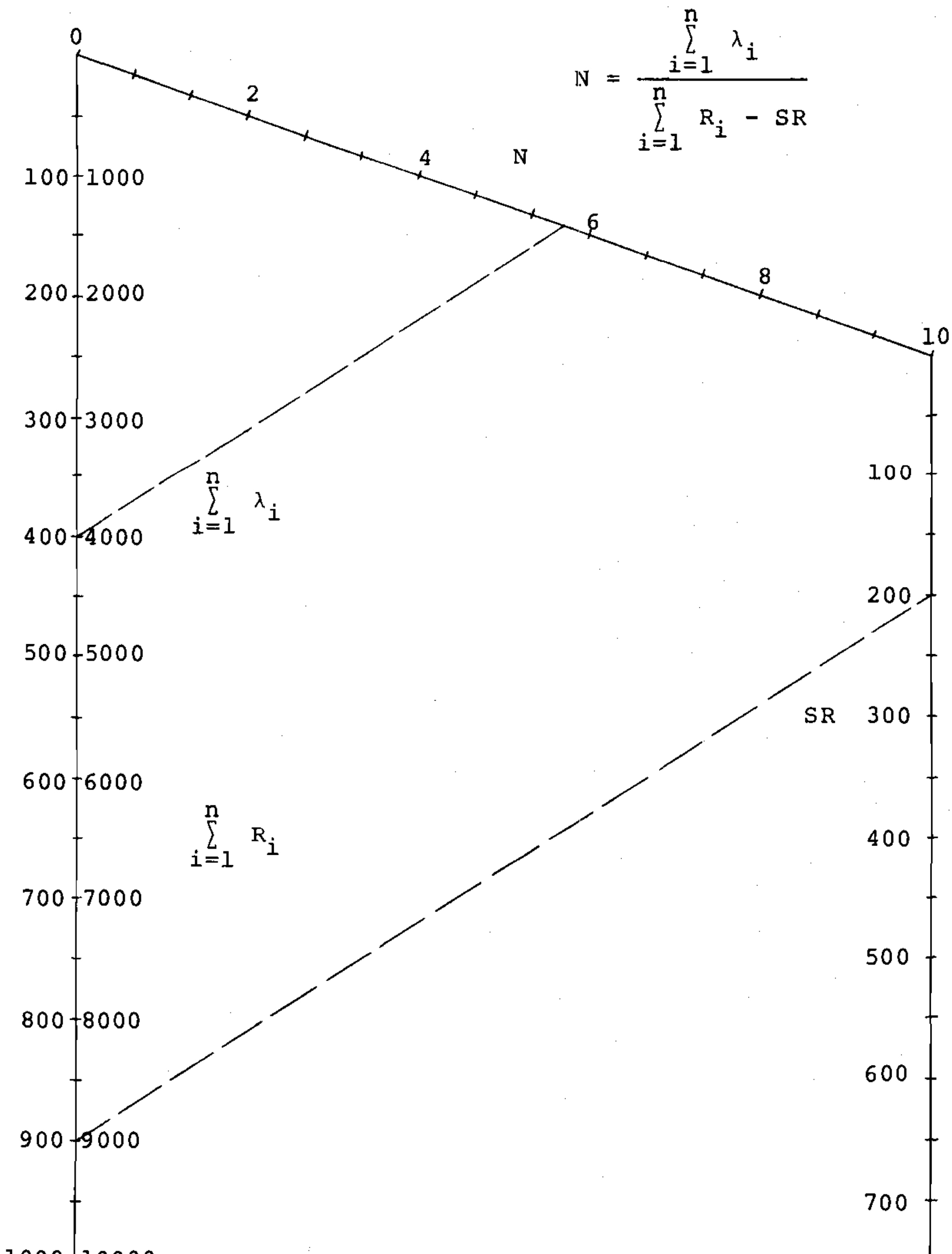

Figure 8. Nomograph

APPENDIX C

LISTING OF VARIABLES UTILIZED IN FORTRAN PROGRAMS

APPENDIX C

LISTING OF VARIABLES UTILIZED IN FORTRAN PROGRAMS

A	Cost of a review + an order, in dollars
ACC	Acceleration factor of penalty parameter
ALPHA	System service level
AMDA	Initial penalty parameter
AVLT	Mean of lead time distribution, in days
BETA(I)	Individual service level for item i
C(I)	Cost of item i, in dollars
CBACK(I)	Yearly cost of backorders of item i
CUMUH(I)	Cumulative standard normal distribution
DEL	Initial step size
DMIN	Displacement termination criterion
F	Total yearly costs of the inventory system
FMIN	Optimum value of the objective function
G(J)	Service level constraint for item j
G(N)	System service level constraint
HOLC	Holding cost for the system
HOLD(I)	Holding cost for item i
K	Number of equality constraints
M	Number of inequality constraints
N	Number of variables (number of items + one)
NBSL(I)	Base stock level for item i

NI	Number of items in the system
NSRP	System reorder point, in units
NTIME	Cycle length or time between reviews
ORDC	Ordering and reviewing cost for the system
ORDIN(I)	Ordinate value of the standard normal distribution
PI(I)	Cost of a backorder of item i
PMAX	Maximum size of penalty parameter
PRO	Protection period (lead time + one cycle)
RDEL	Reduction rate of step size
SERV	Resulting system service level
SERVI(I)	Resulting service level for item i
SIGMA(I)	Standard deviation of demand rate, in units/year
STDL	Standard deviation of lead time, in days
STVAL(I)	Standardized value of the base stock level
TBACK(I)	Total number of backorders of item i per year
XBACK(I)	Number of backorders of item i per cycle
XCYCLE	Number of cycles per year
XI	Inventory holding rate
XLAMB(I)	Mean rate of demand for item i, in units/year
XMU(I)	Mean lead time demand, in units
XSTAR(I)	Starting value of variable x(i)

APPENDIX D

FORTRAN PROGRAM FOR SYSTEM REORDER POINT MODEL: GOAL PROGRAMMING APPROACH

```
      DATA IYES/-YES-/NO/-NO-/IANA/-       -/
      DATA NA/-A-/NB/-B-/NSTOP/-STOP-/IANI/-       -/
      COMMON/COM/X(100),G(50),XSTAR(100),H(50)
      DIMENSION NBSL(100),SERVI(100)
      COMMON NI,  SLAMB,A,SUMR,SB, ,ALPHA(100)
      COMMON XI,      XLAMB(100),XMU(100),C(100),SIGMA(100)
      COMMON BETA(100)
      COMMON IP,IPP,XCYCLE
      COMMON XULT(100)
      COMMON    CUMUH(100),FI(100),XBACK(100),TBACK(100)
      COMMON    FMIN(100),ORDC(100),HOLC(100),SERV(100)
      COMMON    RBAR(100),HOLD(100),STVAL(100),ORDIN(100)
      IPP=1
      WRITE(6,5)
    5 FORMAT(//,- ENTER THE NUMBER OF ITEMS (INTEGER) THEN
     4RETURN.-)
      READ(5,10) NI
   10 FORMAT()
      N=NI+1
      WRITE(6,40)
   40 FORMAT(/,- IS EACH ITEM TO HAVE THE SAME MINIMUM SERVICE
     5LEVELU-,/,- (ENTER YES OR NO)-)
      READ(5,42)IANA
   42 FORMAT(A6)
      IF(IANA.EQ.NO)GO TO 48
      WRITE(6,44)
   44 FORMAT(/,- ENTER THE COMMON MINIMUM SERVICE LEVEL FOR
     5ANY AND ALL ITEMS-,/,- A REAL NUMBER BETWEEN 0 AND 1.-)
      READ(5,10) BETO
      DO 46 II=1,NI
      BETA(II)=BETO
   46 CONTINUE
      WRITE(6,700)
  700 FORMAT( /,- ENTER THE FOLLOWING PARAMETERS--,/,
     1- 1-MEAN RATE OF DEMAND,IN UNITS/YEAR-,/,
     2- 2-MEAN LEAD TIME DEMAND, IN UNITS-,/,
     3- 3-STANDARD DEVIATION OF LEAD TIME DEMAND, IN UNITS-,/,
     4- 4-THE COST OF AN ITEM, IN DOLLARS-,/,
     5- ALL ARE REAL SEPARATED BY COMMA-,/,
     6- ENTER THE DATA FOR ONE ITEM PER LINE.-,/)
      DO 705 I=1,NI
      READ(5,10 )XLAMB(I),XMU(I),SIGMA(I),C(I)
  705 CONTINUE
      GO TO 709
   48 WRITE(6,50)
   50 FORMAT( /,- ENTER THE FOLLOWING PARAMETERS--,/,
     1- 1-MEAN RATE OF DEMAND,IN UNITS/YEAR-,/,
     2- 2-MEAN LEAD TIME DEMAND, IN UNITS-,/,
     3- 3-STANDARD DEVIATION OF LEAD TIME DEMAND, IN UNITS-,/,
     4- 4-THE COST OF AN ITEM, IN DOLLARS-,/,
```

```
   7- 5-THE MINIMUM SERVICE LEVEL DESIRED FOR THE ITEM,
   6BETWEEN 0 AND 1.-,/,
   5- ALL ARE REAL SEPARATED BY COMMA-,/,
   6- ENTER THE DATA FOR ONE ITEM PER LINE.-,/)
    DO 52 I=1,NI
    READ(5,10 )XLAMB(I),XMU(I),SIGMA(I),C(I),BETA(I)
 52 CONTINUE
709 WRITE(6,710)
710 FORMAT( /,- ENTER THE INVENTORY HOLDING RATE AND THE
   7COST OF AN ORDER-,/,- BOTH REAL AND SEPARATED BY COMMA
   8.-)
    READ(5,10 )XI,A
    SLAMB=0.0
    DO 718 I=1,NI
    SLAMB=SLAMB+XLAMB(I)
718 CONTINUE
    STEP=0.10
723 WRITE(6,725)
725 FORMAT(/,- ENTER THE DESIRED MINIMUM SYSTEM SERVICE
   8LEVEL-,/,- A REAL NUMBER BETWEEN 0 AND 1.-)
    READ(5,10)GAMA
    HIGH=GAMA+STEP
    IF(HIGH.LE.1.0)GO TO 60
    HIGH=1.0
    STEP=1.0-GAMA
 60 XLOW=GAMA
    ALPHA(1)=HIGH
    ALPHA(2)=XLOW
    CALL MINI(ALPHA,X)
 65 WRITE(6,69)
 69 FORMAT(//,-    ****** DO YOU PREFER A OR B0 ******-,/,-
   9** IF YOU WANT TO STOP TYPE STOP **-)
    READ(5,42)IANI
    IF(IANI.EQ.NSTOP)GO TO 92
    IF(IANI.EQ.NA)GO TO 80
 70 ALPHA(1)=ALPHA(2)
    ALPHA(2)=ALPHA(1)-STEP
    CALL MINI(ALPHA,X)
    WRITE(6,69)
    READ(5,42)IANI
    IF(IANI.EQ.NSTOP)GO TO 92
    IF(IANI.EQ.NB)GO TO 70
    STEP=STEP/2.0
    ALPHA(2)=ALPHA(1)
    ALPHA(1)=ALPHA(2)+STEP
    CALL MINI(ALPHA,X)
    GO TO 65
 80 IF(ALPHA(1).GE.0.9999)GO TO 85
    ALPHA(2)=ALPHA(1)
    ALPHA(1)=ALPHA(2)+STEP
```

```
      IF(ALPHA(1).LT.1.0)GO TO 82
      STEP=STEP/2.0
      HIGH=1.0
      XLOW=HIGH-STEP
      ALPHA(1)=HIGH
      ALPHA(2)=XLOW
   82 CALL MINI(ALPHA,X)
      WRITE(6,69)
      READ(5,42)IANI
      IF(IANI.EQ.NSTOP)GO TO 92
      IF(IANI.EQ.NA)GO TO 80
      STEP=STEP/2.0
      ALPHA(1)=ALPHA(2)
      ALPHA(2)=ALPHA(1)-STEP
      CALL MINI(ALPHA,X)
      GO TO 65
   85 STEP=STEP/2.0
      HIGH=1.0
      XLOW=HIGH-STEP
      ALPHA(1)=HIGH
      ALPHA(2)=XLOW
      CALL MINI(ALPHA,X)
      WRITE(6,69)
      READ(5,42)IANI
      IF(IANI.EQ.NSTOP)GO TO 92
      IF(IANI.EQ.NA)GO TO 85
      IF(IANI.EQ.NB)GO TO 70
   92 WRITE(6,96)
   96 FORMAT(/,- ENTER THE SYSTEM SERVICE LEVEL THAT YOU
     1PREFER.-)
      READ(5,10)ALPHA(1)
      IPP=100
      CALL MINI(ALPHA,X)
      WRITE(6,93)
   93 FORMAT(/,- USING THE SERVICE LEVEL SELECTED ABOVE,
     2-,/,- THE SYSTEM COSTS WILL BE--,/)
      WRITE(6,95)
   95 FORMAT(T5,- ITEM-,T22,-SERVICE LEVEL-,T 38,-HOLDING
     4COST-,/)
      DO 228 I=1,NI
      SUMR=0.0
      DO 255 KL=1,NI
      SUMR=SUMR+X(KL)
  255 CONTINUE
      SERVI(I)=1.0-(TBACK(I)/XLAMB(I))
      WRITE(6,227)I,SERVI(I),HOLD(I)
  227 FORMAT(T5,I4,T20,F9.2,T38,F8.2)
  228 CONTINUE
      WRITE(6,230)SERV(IP),HOLC(IP)
  230 FORMAT(/,T5,-TOTAL-,T20,F9.2,T38,F8.2)
```

```
      WRITE(6,232)ORDC(IP)
  232 FORMAT(- THE ORDERING COST FOR THE SYSTEM IS-,F9.2)
      WRITE(6,234)FMIN(IP)
  234 FORMAT(- THE TOTAL COST FOR THE SYSTEM IS-,F12.2)
      WRITE(6,231)
  231 FORMAT(/,43H THE -OPTIMAL- SOLUTION IS DESCRIBED BELOW-)
      NSRP=XULT(N)
      WRITE(6,500)NSRP
  500 FORMAT(/,- THE SYSTEM REORDER POINT SHOULD BE-,I5,-
     6UNITS.- )
      DO 530 IJ=1,NI
      NBSL(IJ)=XULT(IJ)
      WRITE(6,510)IJ,NBSL(IJ)
  510 FORMAT(- THE BASE STOCK LEVEL FOR ITEM-,I4,- SHOULD BE
     7-,I5,- UNITS.-)
  530 CONTINUE
      WRITE(6,94)
   94 FORMAT(/,-  ****** END OF PROGRAM ******-,/)
      STOP
      END


      SUBROUTINE MINI(ALPHI,X)
      DOUBLE PRECISION   PFN,FEAS,ZSTAR,DEL,X,G(50),XSTAR
      DOUBLE PRECISION H(50)
      DIMENSION X(100)
      DIMENSION XSTAR(100)
      DIMENSION Q(100),R(100)
      COMMON NI,  SLAMB,A,SUMR,SB,  ALPHA(100)
      COMMON XI,     XLAMB(100),XMU(100),C(100),SIGMA(100)
      COMMON BETA(100)
      COMMON IP,IPP,XCYCLE
      COMMON XULT(100)
      COMMON    CUMUH(100),FI(100),XBACK(100),TBACK(100)
      COMMON    FMIN(100),ORDC(100),HOLC(100),SERV(100)
      COMMON    RBAR(100),HOLD(100),STVAL(100),ORDIN(100)
      IP=1
  728 N=NI+1
      M=N
      K=0
      RDEL=0.15
      PMAX=100000.0
      DMIN=0.0004
      AMDA=1.0
      DEL=1.0
      ACC=10.0
      SSMU=0.0
      DO 22 I=1,NI
      SSMU=SSMU+XMU(I)
   22 CONTINUE
```

```
      XSTAR(N)=(ALPHA(IP)**2.0)*SSMU
      DO 28 I=1,NI
      AIND=A/NI
      Q(I)=SQRT(2.0*XLAMB(I)*AIND/(XI*C(I)))
      R(I)=XSTAR(N)/SLAMB*XLAMB(I)
      XSTAR(I)=Q(I)+R(I)
   28 CONTINUE
      SW=0.5
      DO 30 I=1,N
      X(I)=XSTAR(I)
   30 CONTINUE
      ZSTAR=999999999.0
      DISP=0.
      FEAS=0.
      IEVAL=0
      XMOVE=1.
  100 DO 170 I=1,N
  110 X(I)=XSTAR(I)+XMOVE*DEL
      CALL FUNC(X,F,G,H)
      IEVAL=IEVAL+1
      IF(M.EQ.0)GO TO 125
      DO 120 J=1,M
      IF(G(J).LE.0.)GO TO 120
      P=G(J)
      FEAS=FEAS+P*P
  120 CONTINUE
  125 IF(K.EQ.0)GO TO 140
      DO 130 J=1,K
      P=ABS(H(J))
      FEAS=FEAS+P*P
  130 CONTINUE
  140 PFN=F+AMDA*FEAS
      IF(PFN.GE.ZSTAR)GO TO 150
      DISP=DISP+(ABS(XMOVE))*DEL
      XSTAR(I)=X(I)
      XMOVE=XMOVE+XMOVE
      ZSTAR=PFN
  145 FEAS=0.
      GO TO 110
  150 X(I)=XSTAR(I)
      IF(XMOVE.GT.1.1.OR.XMOVE.LT.0.9)GO TO 160
      XMOVE=-1.
      GO TO 145
  160 XMOVE=1.
      FEAS=0.
  170 CONTINUE
      FEAS=0.
      IF(DISP.LT.DMIN)GO TO 190
  180 DISP=0.
      XMOVE=1.
```

```
      GO TO 100
  190 IF(DEL*AMDA.LT.SW)GO TO 210
  200 DEL=RDEL*DEL
      IF(DEL.GT.0.3*DMIN)GO TO 205
      DEL=0.3*DMIN
      AMDA=2.0*AMDA
      IF(AMDA.GT.5.0*PMAX)GO TO 220
  205 GO TO 180
  210 IF(AMDA.GT.PMAX)GO TO 220
      AMDA=ACC*AMDA
      CALL FUNC(X,F,G,H)
      IF(M.EQ.0)GO TO 213
      DO 212 J=1,M
      IF(G(J).LE.0.)GO TO 212
      P=G(J)
      FEAS=FEAS+P*P
  212 CONTINUE
  213 IF(K.EQ.0)GO TO 215
      DO 214 J=1,K
      P=ABS(H(J))
      FEAS=FEAS+P*P
  214 CONTINUE
  215 ZSTAR=F+AMDA*FEAS
      FEAS=0.
      GO TO 200
  220 CALL FUNC(X,F,G,H)
      DO 222 I=1,N
      XULT(I)=X(I)
  222 CONTINUE
      FMIN(IP)=F
      ORDC(IP)=SLAMB*A/(SUMR-X(N))
      HOLC(IP)=FMIN(IP)-ORDC(IP)
      SERV(IP)=1.0-(SB/SLAMB)
      IF(IPP.GT.10)GO TO 69
      IP=IP+1
      IF(IP.EQ.2)GO TO 728
      WRITE(6,66)
   66 FORMAT(/,T28,-A-,T38,-B-)
      WRITE(6,67)FMIN(1),FMIN(2)
   67 FORMAT(/,- COST-,T23,F9.2,T33,F9.2)
      WRITE(6,68)SERV(1),SERV(2)
   68 FORMAT(/,- SYSTEM SERVICE LEVEL-,T25,F7.5,T35,F7.5)
   69 RETURN
      END


      SUBROUTINE FUNC(X,F,G,H)
      COMMON NI,  SLAMB,A,SUMR,SB,   ALPHA(100)
      COMMON XI,      XLAMB(100),XMU(100),C(100),SIGMA(100)
      COMMON BETA(100)
```

```
      COMMON IP,IPP,XCYCLE
      COMMON XULT(100)
      COMMON     CUMUH(100),FI(100),XBACK(100),TBACK(100)
      COMMON     FMIN(100),ORDC(100),HOLC(100),SERV(100)
      COMMON     RBAR(100),HOLD(100),STVAL(100),ORDIN(100)
      DOUBLE PRECISION X(100),G(50),H(50)
      N=NI+1
      SUMR=0.0
      DO 50 I=1,NI
      SUMR=SUMR+X(I)
   50 CONTINUE
      F1=SLAMB*A/(SUMR-X(N))
      DO 60 I=1,NI
      RBAR(I)=(X(I)*SLAMB-XLAMB(I)*SUMR+XLAMB(I)*X(N))/SLAMB
   60 CONTINUE
      SHOLD=0.0
      DO 80 I=1,NI
      HOLD(I)=XI*C(I)*(X(I)-2.0*XMU(I)+RBAR(I))/2.0
      SHOLD=SHOLD+HOLD(I)
   80 CONTINUE
      F2=SHOLD
      SUMB=0.0
      DO 90 I=1,NI
      STVAL(I)=(RBAR(I)-XMU(I))/SIGMA(I)
      ORDIN(I)=0.3989*(2.7183**(-(STVAL(I)**2.0)/2.0))
      CUMUH(I)=RNORM(STVAL(I))
      FI(I)=1.0-CUMUH(I)
      XBACK(I)=(XMU(I)-RBAR(I))*FI(I)+SIGMA(I)*ORDIN(I)
      XCYCLE=SLAMB/(SUMR-X(N))
      TBACK(I)=XBACK(I)*XCYCLE
      SUMB=SUMB+TBACK(I)
   90 CONTINUE
      SB=SUMB
      DO 95 J=1,NI
      G(J)=TBACK(J)-(1.0-BETA(J))*XLAMB(J)
   95 CONTINUE
      G(N)=SB-(1.0-ALPHA(IP))*SLAMB
      F=F1+F2
      RETURN
      END
```

APPENDIX E

FORTRAN PROGRAM FOR PERIODIC REVIEW MODEL:
SYSTEM SERVICE LEVEL APPROACH

```
      DATA IYES/-YES-/NO/-NO-/IANS/-      -/
      DATA IYES/-YES-/NO/-NO-/IANA/-      -/
      DATA IYES/-YES-/NO/-NO-/IANO/-      -/
      DATA IYES/-YES-/NO/-NO-/IANI/-      -/
      COMMON/COM/X(100),G(50),XSTAR(100),H(50)
      DOUBLE PRECISION   PFN,FEAS,ZSTAR,DEL,X,G,XSTAR,H
      DIMENSION NBSL(100),Q(100),SERVI(100)
      COMMON XI,XLAMB(100),C(100),SIGMA(100),BETA(100)
      COMMON NI,SLAMB,A,SB,N,ALPHA,AVLT
      COMMON CUMUH(100),FI(100),XBACK(100),TBACK(100)
      COMMON HOLD(100),STVAL(100),ORDIN(100)
      WRITE(6,5)
    5 FORMAT(//,- ENTER THE NUMBER OF ITEMS (INTEGER) THEN
     2RETURN.-)
      READ(5,10) NI
   10 FORMAT()
      WRITE(6,40)
   40 FORMAT(/,- IS EACH ITEM TO HAVE THE SAME MINIMUM
     3SERVICE LEVEL)-,/,- (ENTER YES OR NO)-)
      READ(5,42)IANA
   42 FORMAT(A6)
      IF(IANA.EQ.NO)GO TO 48
      WRITE(6,44)
   44 FORMAT(/,- ENTER THE COMMON MINIMUM SERVICE LEVEL FOR
     4ALL ITEMS.-,/,- A REAL NUMBER BETWEEN 0 AND 1.-)
      READ(5,10) BETO
      DO 46 II=1,NI
      BETA(II)=BETO
   46 CONTINUE
      WRITE(6,700)
  700 FORMAT( /,- ENTER THE FOLLOWING PARAMETERS--,/,
     1- 1-MEAN RATE OF DEMAND,IN UNITS/YEAR-,/,
     2- 2-STANDARD DEVIATION OF DEMAND RATE, IN UNITS/YEAR-,/,
     3- 3-THE COST OF AN ITEM, IN DOLLARS-,/,
     5- ALL ARE REAL SEPARATED BY COMMA-,/,
     6- ENTER THE DATA FOR ONE ITEM PER LINE.-,/)
      DO 705 I=1,NI
      READ(5,10)XLAMB(I),SIGMA(I),C(I)
  705 CONTINUE
      GO TO 709
   48 WRITE(6,50)
   50 FORMAT( /,- ENTER THE FOLLOWING PARAMETERS--,/,
     1- 1-MEAN RATE OF DEMAND,IN UNITS/YEAR-,/,
     2- 2-STANDARD DEVIATION OF DEMAND RATE, IN UNITS/YEAR-,/,
     4- 3-THE COST OF AN ITEM, IN DOLLARS-,/,
     7- 4-THE MINIMUM SERVICE LEVEL DESIRED FOR THE ITEM-,/,
     5- ALL ARE REAL SEPARATED BY COMMA-,/,
     6- ENTER THE DATA FOR ONE ITEM PER LINE.-,/)
      DO 52 I=1,NI
      READ(5,10)XLAMB(I),SIGMA(I),C(I),BETA(I)
```

```
 52 CONTINUE
709 WRITE(6,710)
710 FORMAT( /,- ENTER THE INVENTORY HOLDING RATE AND THE
   5COST OF-,/,- A REVIEW + AN ORDER. BOTH REAL AND
   6SEPARATED BY COMMA.-)
    READ(5,10 )XI,A
    WRITE(6,714)
714 FORMAT(/,- ENTER THE MEAN AND STANDARD DEVIATION OF
   7LEAD TIME, IN DAYS,-,/,- BOTH REAL AND SEPARATED
   8BY COMMA.-)
    READ(5,10)AVLT,STDL
    SLAMB=0.0
    DO 718 I=1,NI
    SLAMB=SLAMB+XLAMB(I)
718 CONTINUE
723 WRITE(6,725)
725 FORMAT(/,- ENTER THE DESIRED MINIMUM SYSTEM SERVICE
   9LEVEL-,/,- A REAL NUMBER BETWEEN 0 AND 1.-)
    READ(5,10 )ALPHA
728 N=NI+1
    M=N
    K=0
    RDEL=0.11
    PMAX=150000.0
    DMIN=0.0001
    AMDA=1.3
    DEL=1.4
    ACC=7.5
    SUMC=0.0
    DO 15 I=1,NI
 15 SUMC=SUMC+C(I)
    XSTAR(N)=15.0
    DO 28 I=1,NI
    AIND=A/NI
    Q(I)=SQRT(2.0*XLAMB(I)*AIND/(XI*C(I)))
    XSTAR(1)=100.0
    XSTAR(2)=200.0
 28 CONTINUE
    SW=0.5
    DO 30 I=1,N
    X(I)=XSTAR(I)
 30 CONTINUE
    ZSTAR=999999999.0
    DISP=0.
    FEAS=0.
    IEVAL=0
    XMOVE=1.
100 DO 170 I=1,N
110 X(I)=XSTAR(I)+XMOVE*DEL
    CALL FUNC(X,F,G,H)
```

```
      IEVAL=IEVAL+1
      IF(M.EQ.0)GO TO 125
      DO 120 J=1,M
      IF(G(J).LE.0.)GO TO 120
      P=G(J)
      FEAS=FEAS+P*P
  120 CONTINUE
  125 IF(K.EQ.0)GO TO 140
      DO 130 J=1,K
      P=ABS(H(J))
      FEAS=FEAS+P*P
  130 CONTINUE
  140 PFN=F+AMDA*FEAS
      IF(PFN.GE.ZSTAR)GO TO 150
      DISP=DISP+(ABS(XMOVE))*DEL
      XSTAR(I)=X(I)
      XMOVE=XMOVE+XMOVE
      ZSTAR=PFN
  145 FEAS=0.
      GO TO 110
  150 X(I)=XSTAR(I)
      IF(XMOVE.GT.1.1.OR.XMOVE.LT.0.9)GO TO 160
      XMOVE=-1.
      GO TO 145
  160 XMOVE=1.
      FEAS=0.
  170 CONTINUE
      FEAS=0.
      IF(DISP.LT.DMIN)GO TO 190
  180 DISP=0.
      XMOVE=1.
      GO TO 100
  190 IF(DEL*AMDA.LT.SW)GO TO 210
  200 DEL=RDEL*DEL
      IF(DEL.GT.0.3*DMIN)GO TO 205
      DEL=0.3*DMIN
      AMDA=2.0*AMDA
      IF(AMDA.GT.5.0*PMAX)GO TO 220
  205 GO TO 180
  210 IF(AMDA.GT.PMAX)GO TO 220
      AMDA=ACC*AMDA
      CALL FUNC(X,F,G,H)
      IF(M.EQ.0)GO TO 213
      DO 212 J=1,M
      IF(G(J).LE.0.)GO TO 212
      P=G(J)
      FEAS=FEAS+P*P
  212 CONTINUE
  213 IF(K.EQ.0)GO TO 215
      DO 214 J=1,K
```

```
      P=ABS(H(J))
      FEAS=FEAS+P*P
  214 CONTINUE
  215 ZSTAR=F+AMDA*FEAS
      FEAS=0.
      GO TO 200
  220 CALL FUNC(X,F,G,H)
      FMIN=F
      ORDC=365.0*A/X(N)
      HOLC=FMIN-ORDC
      SERV=1.0-(SB/SLAMB)
      WRITE(6,225)NI
  225 FORMAT(//,- FOR THIS-,I4,- ITEM INVENTORY SYSTEM, THE
     1EXPECTED YEARLY COSTS COMPUTED-,/,- BY THE SEARCH
     2ALGORITHM ARE--,//,T6,-ITEM-,T19,-SERVICE LEVEL-,T42
     3,-HOLDING COST-,/)
      DO 228 I=1,NI
      PRO=(AVLT+X(N))/365.0
      HOLD(I)=XI*C(I)*(X(I)-XLAMB(I)*AVLT/365.0-XLAMB(I)
     4*X(N)/730.0)
      STVAL(I)=(X(I)-XLAMB(I)*PRO)/(SIGMA(I)*PRO)
      ORDIN(I)=0.3989*(2.7183**(-(STVAL(I)**2.0)/2.0))
      CUMUH(I)=RNORM(STVAL(I))
      FI(I)=1.0-CUMUH(I)
      XBACK(I)=SIGMA(I)*PRO*ORDIN(I)+(XLAMB(I)*PRO-X(I))*FI(I)
      XCYCLE=365.0/X(N)
      TBACK(I)=XBACK(I)*XCYCLE
      SERVI(I)=1.0-(TBACK(I)/XLAMB(I))
      WRITE(6,227)I,SERVI(I),HOLD(I)
  227 FORMAT(T6,I4,T20,F9.2,T43,F8.2)
  228 CONTINUE
      WRITE(6,230)SERV,HOLC
  230 FORMAT(/,T5,-TOTAL-,T20,F9.2,T43,F8.2)
      WRITE(6,232)ORDC
  232 FORMAT(- THE REVIEW + ORDER COST FOR THE SYSTEM IS-
     5,F8.2)
      WRITE(6,234)FMIN
  234 FORMAT(- THE TOTAL COST FOR THE SYSTEM IS-,F17.2)
      WRITE(6,231)
  231 FORMAT(/,43H THE -OPTIMAL- SOLUTION IS DESCRIBED BELOW-)
      NTIME=X(N)
      WRITE(6,500)NTIME
  500 FORMAT(/,- THE TIME BETWEEN REVIEWS FOR THIS SYSTEM
     6SHOULD BE-,I4,- DAYS.-)
      DO 530 IJ=1,NI
      NBSL(IJ)=X(IJ)
      WRITE(6,510)IJ,NBSL(IJ)
  510 FORMAT(- THE BASE STOCK LEVEL FOR ITEM-,I4,- SHOULD BE
     6-,I5,- UNITS.-)
  530 CONTINUE
```

```
      WRITE(6,370)
  370 FORMAT(///,- DO YOU WANT TO CHANGE THE SYSTEM SERVICE
     7LEVELO (YES OR NO)-)
      READ(5,390)IANS
  390 FORMAT(A6)
      WRITE(6,375)
  375 FORMAT(/,- DO YOU WANT TO CHANGE THE INDIVIDUAL SERVICE
     8LEVELSO-)
      READ(5,390)IANO
      IF(IANS.EQ.IYES.AND.IANO.EQ.IYES)GO TO 605
      IF(IANS.EQ.IYES.AND.IANO.EQ.  NO)GO TO 665
      IF(IANS.EQ.  NO.AND.IANO.EQ.IYES)GO TO 620
      IF(IANS.EQ.  NO.AND.IANO.EQ.  NO)GO TO 670
  605 WRITE(6,610)
  610 FORMAT(/,- ENTER THE DESIRED MINIMUM SYSTEM SERVICE
     9LEVEL-,/,- A REAL NUMBER BETWEEN 0 AND 1.-)
      READ(5,10 )ALPHA
  620 WRITE(6,630)
  630 FORMAT(/,- IS EACH ITEM TO HAVE THE SAME MINIMUM SERVICE
     9LEVELO-,/,- (ENTER YES OR NO)-)
      READ(5,390)IANI
      IF(IANI.EQ.NO)GO TO 650
      WRITE(6,635)
  635 FORMAT(/,- ENTER THE COMMON MINIMUM SERVICE LEVEL FOR
     1ALL ITEMS.-,/,- A REAL NUMBER BETWEEN 0 AND 1.-)
      READ(5,10) BETO
      DO 640 J=1,NI
      BETA(J)=BETO
  640 CONTINUE
      GO TO 728
  650 WRITE(6,655)
  655 FORMAT(/,- ENTER THE INDIVIDUAL SERVICE LEVELS DESIRED,
     2REAL NUMBERS BETWEEN -,/,- 0 AND 1, ONE PER LINE.-)
      DO 660 J=1,NI
      READ(5,10)BETA(J)
  660 CONTINUE
      GO TO 728
  665 GO TO 723
  670 WRITE(6,440)
  440 FORMAT( //,-     ***** END OF PROGRAM ***** -,/)
      STOP
      END


      SUBROUTINE FUNC(X,F,G,H)
      COMMON XI,XLAMB(100),C(100),SIGMA(100),BETA(100)
      COMMON NI,SLAMB,A,SB,N,ALPHA,AVLT
      COMMON CUMUH(100),FI(100),XBACK(100),TBACK(100)
      COMMON HOLD(100),STVAL(100),ORDIN(100)
      DOUBLE PRECISION X(100),G(50),H(50)
```

```
      F1=365.0*A/X(N)
      SHOLD=0.0
      DO 80 I=1,NI
      HOLD(I)=XI*C(I)*(X(I)-XLAMB(I)*AVLT/365.0-XLAMB(I)*X(N)
     3/730.0)
      SHOLD=SHOLD+HOLD(I)
   80 CONTINUE
      F2=SHOLD
      SUMB=0.0
      DO 90 I=1,NI
      PRO=(AVLT+X(N))/365.0
      STVAL(I)=(X(I)-XLAMB(I)*PRO)/(SIGMA(I)*PRO)
      ORDIN(I)=0.3989*(2.7183**(-(STVAL(I)**2.0)/2.0))
      CUMUH(I)=RNORM(STVAL(I))
      FI(I)=1.0-CUMUH(I)
      XBACK(I)=SIGMA(I)*PRO*ORDIN(I)+(XLAMB(I)*PRO-X(I))*FI(I)
      XCYCLE=365.0/X(N)
      TBACK(I)=XBACK(I)*XCYCLE
   90 SUMB=SUMB+TBACK(I)
      SB=SUMB
      DO 95 J=1,NI
      G(J)=TBACK(J)-(1.0-BETA(J))*XLAMB(J)
   95 CONTINUE
      G(N)=SB-(1.0-ALPHA)*SLAMB
      F=F1+F2
      RETURN
      END
```

APPENDIX F

FORTRAN PROGRAM FOR SYSTEM REORDER POINT MODEL: UTILIZATION OF BACKORDER COST PARAMETERS

```
      COMMON/COM/X(100),G(50),XSTAR(100),H(50)
      DOUBLE PRECISION    PFN,FEAS,ZSTAR,DEL,X,G,XSTAR,H
      DIMENSION           NBSL(100),Q(100),R(100)
      COMMON XI,      XLAMB(100),XMU(100),C(100),SIGMA(100)
      COMMON PI(100),CBACK(100)
      COMMON NI,SLAMB,A,SUMR,N
      COMMON CUMUH(100),FI(100),XBACK(100),TBACK(100)
      COMMON RBAR(100),HOLD(100),STVAL(100),ORDIN(100)
      WRITE(6,5)
    5 FORMAT(//,- ENTER THE NUMBER OF ITEMS (INTEGER) THEN
     1RETURN.-)
      READ(5,10) NI
   10 FORMAT()
      WRITE(6,700)
  700 FORMAT( /,- ENTER THE FOLLOWING PARAMETERS--,/,
     1- 1-MEAN RATE OF DEMAND,IN UNITS/YEAR-,/,
     2- 2-MEAN LEAD TIME DEMAND, IN UNITS-,/,
     3- 3-STANDARD DEVIATION OF LEAD TIME DEMAND, IN UNITS-,/,
     4- 4-THE COST OF AN ITEM, IN DOLLARS-,/,
     7- 5-THE COST OF A BACKORDER, IN DOLLARS-,/,
     5- ALL ARE REAL SEPARATED BY COMMA-,/,
     6- ENTER THE DATA FOR ONE ITEM PER LINE.-,/)
      DO 705 I=1,NI
      READ(5,10 )XLAMB(I),XMU(I),SIGMA(I),C(I),PI(I)
  705 CONTINUE
      WRITE(6,710)
  710 FORMAT( /,- ENTER THE INVENTORY HOLDING RATE AND THE
     2COST OF AN ORDER-,/,- BOTH REAL AND SEPARATED BY
     3COMMA.-)
      READ(5,10 )XI,A
      SLAMB=0.0
      DO 718 I=1,NI
      SLAMB=SLAMB+XLAMB(I)
  718 CONTINUE
  728 N=NI+1
      M=N
      K=0
      RDEL=0.15
      PMAX=100000.0
      DMIN=0.0004
      AMDA=1.0
      DEL=1.0
      ACC=10.0
      SSMU=0.0
      DO 22 I=1,NI
      SSMU=SSMU+XMU(I)
   22 CONTINUE
      XSTAR(N)=SSMU
      DO 28 I=1,NI
      AIND=A/NI
```

```
      Q(I)=SQRT(2.0*XLAMB(I)*AIND/(XI*C(I)))
      R(I)=XSTAR(N)/SLAMB*XLAMB(I)
      XSTAR(I)=Q(I)+R(I)
   28 CONTINUE
      SW=0.5
      DO 30 I=1,N
      X(I)=XSTAR(I)
   30 CONTINUE
      ZSTAR=999999999.0
      DISP=0.
      FEAS=0.
      IEVAL=0
      XMOVE=1.
  100 DO 170 I=1,N
  110 X(I)=XSTAR(I)+XMOVE*DEL
      CALL FUNC(X,F,G,H)
      IEVAL=IEVAL+1
      IF(M.EQ.0)GO TO 125
      DO 120 J=1,M
      IF(G(J).LE.0.)GO TO 120
      P=G(J)
      FEAS=FEAS+P*P
  120 CONTINUE
  125 IF(K.EQ.0)GO TO 140
      DO 130 J=1,K
      P=ABS(H(J))
      FEAS=FEAS+P*P
  130 CONTINUE
  140 PFN=F+AMDA*FEAS
      IF(PFN.GE.ZSTAR)GO TO 150
      DISP=DISP+(ABS(XMOVE))*DEL
      XSTAR(I)=X(I)
      XMOVE=XMOVE+XMOVE
      ZSTAR=PFN
  145 FEAS=0.
      GO TO 110
  150 X(I)=XSTAR(I)
      IF(XMOVE.GT.1.1.OR.XMOVE.LT.0.9)GO TO 160
      XMOVE=-1.
      GO TO 145
  160 XMOVE=1.
      FEAS=0.
  170 CONTINUE
      FEAS=0.
      IF(DISP.LT.DMIN)GO TO 190
  180 DISP=0.
      XMOVE=1.
      GO TO 100
  190 IF(DEL*AMDA.LT.SW)GO TO 210
  200 DEL=RDEL*DEL
```

```
      IF(DEL.GT.0.3*DMIN)GO TO 205
      DEL=0.3*DMIN
      AMDA=2.0*AMDA
      IF(AMDA.GT.5.0*PMAX)GO TO 220
  205 GO TO 180
  210 IF(AMDA.GT.PMAX)GO TO 220
      AMDA=ACC*AMDA
      CALL FUNC(X,F,G,H)
      IF(M.EQ.0)GO TO 213
      DO 212 J=1,M
      IF(G(J).LE.0.)GO TO 212
      P=G(J)
      FEAS=FEAS+P*P
  212 CONTINUE
  213 IF(K.EQ.0)GO TO 215
      DO 214 J=1,K
      P=ABS(H(J))
      FEAS=FEAS+P*P
  214 CONTINUE
  215 ZSTAR=F+AMDA*FEAS
      FEAS=0.
      GO TO 200
  220 CALL FUNC(X,F,G,H)
      FMIN=F
      ORDC=SLAMB*A/(SUMR-X(N))
      WRITE(6,225)NI
  225 FORMAT(//,- FOR THIS-,I4,- ITEM INVENTORY SYSTEM, THE
     4EXPECTED YEARLY -,/,- COSTS COMPUTED BY THE SEARCH
     5ALGORITHM ARE-,//,T6,-ITEM-,T19,-HOLDING COST-,T38,
     6-BACKORDER COST-,/)
      BACK=0.0
      DO 228 I=1,NI
      SUMR=0.0
      DO 255 KL=1,NI
      SUMR=SUMR+X(KL)
  255 CONTINUE
      RBAR(I)=(X(I)*SLAMB-XLAMB(I)*SUMR+XLAMB(I)*X(N))/SLAMB
      HOLD(I)=XI*C(I)*(X(I)-2.0*XMU(I)+RBAR(I))/2.0
      STVAL(I)=(RBAR(I)-XMU(I))/SIGMA(I)
      ORDIN(I)=0.3989*(2.7183**(-(STVAL(I)**2.0)/2.0))
      CUMUH(I)=RNORM(STVAL(I))
      FI(I)=1.0-CUMUH(I)
      XBACK(I)=(XMU(I)-RBAR(I))*FI(I)+SIGMA(I)*ORDIN(I)
      XCYCLE=SLAMB/(SUMR-X(N))
      TBACK(I)=XBACK(I)*XCYCLE
      CBACK(I)=TBACK(I)*PI(I)
      BACK=BACK+CBACK(I)
      WRITE(6,227)I,HOLD(I),CBACK(I)
  227 FORMAT(T6,I4,T20,F9.2,T38,F8.2)
  228 CONTINUE
```

```
      HOLC=FMIN-ORDC-BACK
      WRITE(6,230)HOLC,BACK
  230 FORMAT(/,T5,-TOTAL-,T20,F9.2,T38,F8.2)
      WRITE(6,232)ORDC
  232 FORMAT(- THE ORDERING COST FOR THE SYSTEM IS-,F9.2)
      WRITE(6,234)FMIN
  234 FORMAT(- THE TOTAL COST FOR THE SYSTEM IS-,F12.2)
      WRITE(6,231)
  231 FORMAT(/,43H THE -OPTIMAL- SOLUTION IS DESCRIBED BELOW-)
      NSRP=X(N)
      WRITE(6,500)NSRP
  500 FORMAT(/,- THE SYSTEM REORDER POINT SHOULD BE-,I5,-
     7UNITS.- )
      DO 530 IJ=1,NI
      NBSL(IJ)=X(IJ)
      WRITE(6,510)IJ,NBSL(IJ)
  510 FORMAT(- THE BASE STOCK LEVEL FOR ITEM-,I4,- SHOULD BE-
     8,I5,- UNITS.- )
  530 CONTINUE
      WRITE(6,440)
  440 FORMAT( //,-     ***** END OF PROGRAM ***** -,/)
      STOP
      END


      SUBROUTINE FUNC(X,F,G,H)
      COMMON XI,     XLAMB(100),XMU(100),C(100),SIGMA(100)
      COMMON PI(100),CBACK(100)
      COMMON NI,SLAMB,A,SUMR,N
      COMMON CUMUH(100),FI(100),XBACK(100),TBACK(100)
      COMMON RBAR(100),HOLD(100),STVAL(100),ORDIN(100)
      DOUBLE PRECISION X(100),G(50),H(50)
      N=NI+1
      SUMR=0.0
      DO 50 I=1,NI
      SUMR=SUMR+X(I)
   50 CONTINUE
      F1=SLAMB*A/(SUMR-X(N))
      SHOLD=0.0
      DO 80 I=1,NI
      RBAR(I)=(X(I)*SLAMB-XLAMB(I)*SUMR+XLAMB(I)*X(N))/SLAMB
      HOLD(I)=XI*C(I)*(X(I)-2.0*XMU(I)+RBAR(I))/2.0
      SHOLD=SHOLD+HOLD(I)
   80 CONTINUE
      F2=SHOLD
      SUMB=0.0
      DO 90 I=1,NI
      STVAL(I)=(RBAR(I)-XMU(I))/SIGMA(I)
      ORDIN(I)=0.3989*(2.7183**(-(STVAL(I)**2.0)/2.0))
```

```
      CUMUH(I)=RNORM(STVAL(I))
      FI(I)=1.0-CUMUH(I)
      XBACK(I)=(XMU(I)-RBAR(I))*FI(I)+SIGMA(I)*ORDIN(I)
      XCYCLE=SLAMB/(SUMR-X(N))
      TBACK(I)=XBACK(I)*XCYCLE
      CBACK(I)=TBACK(I)*PI(I)
      SUMB=SUMB+CBACK(I)
   90 CONTINUE
      F3=SUMB
      DO 95 J=1,N
      G(J)=1.0-X(J)
   95 CONTINUE
      F=F1+F2+F3
      RETURN
      END
```

APPENDIX G

EXAMPLE OF A 6-ITEM INVENTORY SYSTEM

Table 3. Example of a 6-Item Inventory System

	Input Data				Output Values		
	λ	σ	C	α	R	T	Hold.Cost
1	100	10	10.00	0.87	7	13	5.39
2	350	35	40.00	0.66	22	13	21.65
3	789	10	5.00	0.55	56	13	14.22
4	1230	100	40.00	0.73	86	13	160.59
5	456	34	4.00	0.61	34	13	8.49
6	1990	160	12.00	0.76	161	13	137.84

Table 4. Effect of Service Level on Yearly Costs for System of Table 3

System Service Level	System Yearly Costs		
	Ordering	Holding	Total
0.950	1476.10	411.25	1887.35
0.910	1476.10	348.18	1824.28
0.860	1476.10	292.38	1768.48
0.810	1476.10	270.75	1746.86
0.750	1476.10	224.76	1700.86

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