MINIMUM WEIGHT DESIGN OF STIFFENED CIRCULAR CYLINDRICAL SHELLS SUBJECT TO UNIFORM HYDROSTATIC PRESSURE

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Mohan Aswani

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Approved:

G. J. Simitses, Chairman

C. V. Smith

L. W. Rehfield

Date approved by Chairman: 2/25/75

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NOTATIONS

A_r, A_{st}	Ring and stringer cross-sectional area, in^2
A_x, A_y	Ratio of flange thickness to web thickness for stringer and ring
B_x, B_y	Ratio of flange width to web depth of stringer and ring
C_x, C_y	Stringer and ring shape parameters
D	Flexural stiffness of skin, in-lb
D_{xx}, D_{yy}, D_{xy}	Orthotropic flexural and twisting stiffnesses, in-lb
D_{xxst}, D_{yyr}	Flexural stiffnesses of stringer and ring, in-lb
E, E_r, E_{st}	Young's modulus of elasticity of skin, ring, and stringer, psi
E_{xx}, E_{yy}	Orthotropic extensional stiffnesses, lb/in
E_{xxp}, E_{yyp}	Extensional stiffnesses of skin, lb/in
E_{xxst}, E_{yyr}	Extensional stiffnesses of stringer and ring, lb/in
G	Shear modulus of elasticity, psi
$(GJ)_{st}$, $(GJ)_r$	Contribution of stiffeners to torsional stiffness, in^2-lb
G_{xy}	Inplane skin shear stiffness, lb/in
I_{rc}, I_{stc}	Ring and stringer moment of inertia about their centroidal axes, in^4
L	Length of shell, in
M_{xx}, M_{yy}, M_{xy}	Moment resultants, in-lb/in
$\bar{N}$	Applied axial compression, lb/in
N_{xx}, N_{yy}, N_{xy}	Stress resultants, lb/in

R	Radius of shell, in
T	Applied torque, in-lb
W	Weight per unit length of shell, lb/in
$\bar{W}$	Non-dimensional weight parameter
W^*	Composite weight function, lb
$\bar{W}^*$	Non-dimensional composite weight function
Z	Curvature parameter
b_{fr}, b_{fst}	Flange width of ring and stringer, in
d_{wr}, d_{st}	Web depth of ring and stringer, in
e_r, e_{st}	Eccentricity of ring and stringer, in
$\bar{e}_x$, $\bar{e}_y$	Nondimensional eccentricities of stringer and ring
h	Skin thickness, in
$\bar{k}_{xx}$, $\bar{k}_{yy}$, $\bar{k}_s$	Buckling load parameters for axial compression, pressure and torsion
$\bar{k}_{yyp}$	Panel buckling load parameter
ℓ	Clear distance between two consecutive rings, in
ℓ_r, ℓ_{st}	Ring and stringer spacings, in
m, n	Number of longitudinal and circumferential waves for general instability
m_p, n_p	Number of longitudinal and circumferential waves for panel buckling
q	Hydrostatic pressure, (positive outward) psi
q^x, q^y, q^z	Applied load components in x, y, and z directions, psi
q_D^*	Nondimensional load parameter
t_{fst}, t_{fr}	Stringer and ring flange thickness, in
t_{st}, t_{wr}	Stringer and ring web thickness, in

u, v, w	Displacement components of a point on reference surface, in
x, y, z	Coordinate directions
α	Ratio $\bar{k}_{xx}/\bar{k}_{yy}$
$\bar{\alpha}_x, \bar{\alpha}_y$	Nondimensional radii of gyration of stringer and ring
γ	Shear strain at any point
γ_w	Density of immersion fluid lb/in^3
γ_{xy}	Shear strain of a point on reference surface
ϵ_x, ϵ_y	Normal strains at any point
$\epsilon_{xx}, \epsilon_{yy}$	Normal strains of a point on reference surface
$\kappa_{xx}, \kappa_{yy}, \kappa_{xy}$	Changes in curvature
λ	Lagrange multiplier
λ^*	Nondimensional Lagrange multiplier
$\bar{\lambda}_{xx}, \bar{\lambda}_{yy}$	Nondimensional extensional stiffnesses of stringer and ring
ν	Poisson ratio
$\rho_{sk}, \rho_r, \rho_{st}$	Weight density of skin, ring, and stringer, lb/in^3
$\bar{\rho}_{xx}, \bar{\rho}_{yy}$	Nondimensional flexural stiffnesses of stringer and ring
σ_y	Permissible yield stress, psi
$\sigma_{xxsk}, \sigma_{yysk}$	Prebuckling stresses in skin, psi
$\sigma_{xxst}, \sigma_{yyr}$	Prebuckling stresses in stringer and ring, psi
$\sigma_{xxsk_{cr}}, \sigma_{xxst_{cr}}$, $\sigma_{yyr_{cr}}$	Critical buckling stresses in skin, stringer, and ring, psi
Superscript "o"	Refers to membrane state
Superscript "1"	Refers to additional quantity necessary to bring the membrane state to the classical buckling state

GLOSSARY OF ABBREVIATIONS

GB	Gross buckling q_D/q_{cr}
IA	Inverted angle
PB	Panel buckling $q_D/q_{p_{cr}}$
PR	Rectangular ring
RS	Rectangular stringer
RR-RS	Rectangular ring and stringer
RB	Ring buckling $\sigma_{yyr}/\sigma_{yyr_{cr}}$
RY	Ring yielding σ_{yyr}/σ_y
SKB	Skin buckling $\sigma_{xxsk}/\sigma_{xxsk_{cr}}$
SKY	Skin yielding σ_s/σ_y
STB	Stringer buckling $\sigma_{xxsk}/\sigma_{xxsk_{cr}}$
STY	Stringer yielding σ_{xxsk}/σ_y
TR	Tee ring
TR-RS	Tee ring and rectangular stringer

SUMMARY

A methodology is developed by which minimum weight design of stiffened cylinders under hydrostatic pressure may be achieved. The precise statement of the problem is: Given a stiffened cylinder of specified material, radius, and length, find the size, shape, spacing of stiffeners, and the thickness of the skin, such that it can carry safely a hydrostatic pressure with minimum weight. The word 'safely carry' implies that none of the behavioral constraints are violated. These constraints include: general instability, panel instability, local instabilities of skin and stiffeners and the limitation on stress levels in various components of the cylinder.

The solution to the problem is accomplished in two stages. In the first stage, unconstrained minimization of the objective function (defining weight of the cylinder and including one active constraint as penalty function) is performed using a mathematical search technique. This yields a design space in which all the configurations satisfy the mode of failure that has been included in the objective function. In the second stage, this design space (represented by charts and tables) is used in arriving at final minimum weight configuration satisfying all the remaining constraints. A systematic procedure is given for accomplishing the design.

This methodology provides freedom to the designer to achieve and thus assess all equal weight designs. In addition, he knows what penalty in weight he pays, when moving arbitrarily in the design

space. By this approach simultaneous occurrence of failure modes can be avoided by paying least weight penalty. The availability of such information along with the study indicating the influence of type and shape of stiffeners on the weight of cylinder permits a designer to carry out trade-off studies and arrive at practical minimum weight design.

CHAPTER I

INTRODUCTION

Historical Review

During the last two decades, considerable progress has been made in structural analysis. With the aid of computers, structural problems with great degree of complexity can now be solved with relative ease. While these achievements are of great importance in assessing the behavior of structures, their full benefits will only be materialized when reflected in the improved designs of structures.

The aim of devising design procedures which satisfy all constraints of safety and performance and do it with least weight, or least cost is not a new one. The engineers have always strived for good designs by attempting investigations of several alternatives within the bounds of time and cost. However, only limited number of alternatives could be investigated in the absence of the aid of the present day computers. With this tremendous aid, the progress in achieving optimum solutions to the design problems has been outstanding.

Before attempting any type of solution to the problem of 'optimum' or 'the best' design of a particular structure, the first step is to decide the basis for which various designs can be compared. The basis of comparison, termed the criterion, defines the measure of value and accordingly enables one to choose between any two candidate design configurations. Often, it is difficult to establish and attain ideal criteria in practice. For example, the criterion of achieving minimum weight and minimum cost, in general, does not yield the same configuration. In such situations one looks for compromises between such requirements. Such a study or process of compromising between these requirements in the design criterion is known as the establishment of trade-offs. It is rather difficult to express analytically, in terms of the design variables the criterion, including for example, minimum weight and minimum cost. A possible diversion from this ideal situation will be to express the criterion analytically on one of these requirements and attempt a formulation which permits the designer to carry out limited trade-off studies.

Minimum weight is the primary consideration for design of aerospace vehicles and, more recently, of underwater structural systems, such as submarines and bathyscaphs. For these structures, the criterion of design is minimum weight. The function, expressed in terms of design variables, describing the weight is known as the merit function or the objective function.

For any manned underwater vehicle, the pressure hull is the most important component. It is essentially a stiffened cylindrical shell contributing one fourth to more than one half to the total weight of the vehicle. In order to carry the requisite pay load, while preserving adequate buoyancy, it is essential to design the hull for minimum weight. Increased operating depths have further necessitated such an investigation. The present investigation is an attempt to develop a methodology by which one can accomplish minimum weight design of pressure hulls.

Several attempts have been made in past for designing stiffened shells for minimum weight. A comprehensive survey related to optimization of aerospace structures was presented by Gerard [1]* in 1966. An excellent review on optimal structural design is given by Niordson and Pederson [2]. The most authoritative and complete surveys of optimum structural design in the context of mathematical programming procedures are those by Schmit [3-5].

The attempts made in the past for cylinders under various load conditions can broadly be classified into two categories. One approach is primarily based on the premise that minimum weight is accomplished if all modes of failure occur simultaneously. In this approach the design variables are established through parametric studies in conjunction with the mathematical equations that express the above premise. References [6-11] adopt such an approach. This conjecture, however, is disproved in some simple cases as shown by Spunt [12]. He shows that such a requirement puts severe restriction on the formulation and the solution of the problem. Furthermore, it prevents a designer from considering the families of alternative designs having equal weight but not satisfying the requirement of simultaneous mode occurrence. The recent studies by Thompson and Lewis [13] on optimal designs of thin walled compression members and by Thompson, Tulk and Walker [14] on stiffened plates have shown that a structural configuration which is designed for simultaneous

*Numbers in the square brackets designate references at the end of thesis.

occurrence of failure modes becomes more sensitive to geometric imperfections. These observations and the results of the second approach, which is discussed in the next paragraph, reject the formulation of the problem on the basis of simultaneous occurrence of failure modes.

The second approach is based on mathematical search technique applied for minimization of the objective function. The objective function defining the weight of the structure, contains all of the behavioral (limitations on stress levels) and geometric (limitations on dimensions of design variables) constraints as penalty functions. Such a composite objective function is expressed in terms of the design variables. By means of certain mathematical search techniques one finds the values of those design variables that correspond to minimum weight. References [15-21] adopt such an approach. The method of solution is, undoubtedly, in accord with the present day philosophy of achieving fully automated designs, but such an approach, in the opinion of the author, has certain limitations. The number of design variables associated with a cylinder stiffened with rectangular stiffeners is seven. Almost all the investigators who have used mathematical search techniques in seven dimensional space have reported great difficulties and algorithm failures. If one were to deal with other shapes of stiffeners, for example T-shape, the number of variables increases to 11, and the implementation of search techniques is further complicated. Some of the investigators [18] have attempted to fix certain design variables in the objective function. Such an assumption, however, does not indicate precisely how far from the real optimum solution one is. Even if these difficulties can be overcome, there still exist some questions regarding this approach. First, because of the complete automation the designer is virtually divorced from the design procedure and control over the design variables. This means that a designer can not introduce needed changes in the design variables with least weight penalty. Second, due to all the constraints included into the objective function, the resulting design represents only one feasible minimum weight configuration. Associated with this configuration, there may be two or more modes of failure that are active. There is no way of avoiding this simultaneous occurrence of two or more failure modes. Moreover, there may be many more feasible design configurations of equal or nearly equal weight which are not obtainable by this approach. The results of Pappas and Amba-Rao [22] and Jones and Hague [16] confirm such a doubt. These investigators have obtained several designs of nearly equal weight but with significantly different design variables. Simitses and Ungbhakorn [23] have explicitly shown that the minimum weight design is not unique in the case of stiffened cylinders subject to uniform axial compression. Third, because the formulation of the penalty function is dependent on which constraint is active, in many cases erroneous expressions have been used in the objective function. If, for example, skin wrinkling is the only active constraint. (see reference [16]) the expression for general instability is incorrect, because it is based on the assumption that the skin has not wrinkled.

Furthermore, the investigators in the past have considered only ring stiffened cylinders, (see reference [18]), for the minimum weight design of pressure hulls employing the equations which are mostly empirical. Rings, no doubt, are of most importance in resisting hydrostatic pressure, but in several situations the presence of light stringers can reduce the weight further. An approach that considers only ring stiffened geometry is therefore restrictive in nature. In addition, no attempt has been made in the past to study the influence of various shapes of stiffeners on the minimum weight. This aspect of study is important not only from the point of view of finding the best shape of stiffeners, but also in carrying out trade-off studies.

These observations obviously suggest that the approach to minimum weight design needs modification. The needed new methodology should provide freedom to the designer to achieve and thus assess all equal weight designs. In addition, he should know what penalty in weight he pays, when he moves arbitrarily in the design space. The availability of such information along with the study indicating the influence of type and shape of stiffeners on the weight is extremely desirable for obtaining practical minimum weight design and for carrying out trade-off studies.

Statement of the Problem

The methodology in the present investigation is based on the observation that for any given level of the specified parameters the design is governed by one or two failure modes. In very special cases three or more modes of failure may become active corresponding to the minimum weight configuration. In any case it is desirable, because of the findings of Thompson, Tulk and Walker to adjust the design variables so as to separate these failure modes. The development of a methodology which permits such a requirement to be satisfied is of tremendous importance. How this can be accomplished, by the present methodology, is discussed after giving precise statement of the problem.

The problem considered is: Given a stiffened cylinder of specified material, radius and length, find the size, shape, spacing of stiffeners, and thickness of the skin, such that it can carry safely a given hydrostatic pressure with minimum weight. The word 'safely carry' implies that none of the behavioral constraints are violated. These constraints include: general instability, panel instability, local instabilities of skin and stiffeners and the limitations on stress levels in various components of the cylinder. The constraints may also include certain geometric inequalities specifying limitations on dimensions of various design variables.

The design objective is minimum weight. The solution, therefore, requires minimization of the function defining the weight of the cylinder subject to the constraints defined above. In order to accomplish what is lacking in the earlier approaches, the objective function in the present case is formulated in different manner. The basic principle is similar to the one adopted by Ungbhakorn [24]. Instead of incorporating all the constraints as penalty functions along with the expression for the weight of the cylinder, only one active failure mode, expressed as an equality constraint is included as penalty function. By studying carefully the expressions of various modes of failure, the design variables are grouped so as to minimize the number of influential optimizing parameters. This point is explained in details in Chapter II.

The solution to the entire problem is accomplished in two stages. In the first stage, unconstrained minimization of the objective function (which includes one active constraint as penalty function) is performed using a mathematical search technique. This yields a design space in which all the configurations satisfy the mode of failure that has been included in the objective function. This design space is represented by means of design charts and design tables. These design charts and design tables give the values of optimizing parameters for each point in the design space. This is the first stage or phase I of the present methodology.

In the second stage or phase II, a designer moves in the design space in a systematic way, discussed in Chapter III, to arrive at a design which satisfies all the remaining failure modes and has minimum weight. One can look at the second stage as moving on the curve, that defines the active mode of failure, starting from a point that corresponds to the least weight to such a point where all the modes of failure are satisfied. It is obvious that the successful working of this approach depends on correctly identifying the active mode of failure. For the design of submarine pressure hulls, the two modes of failure, that are active corresponding to the minimum weight configuration, are general instability and skin yielding. If both modes of failure are active, one may formulate the problem on any one of these two modes. The weight of the final design configuration should work out the same in either case.

The minimization of the objective function is performed by the Nelder and Mead [25] search technique. There are several mathematical search techniques available in the literature, for example, (see reference [16]), which can possibly be used for the present problem. Since the aim of the present investigation is not to compare the relative merits of various mathematical search techniques, no such attempt is made here. This aspect of study is open to those interested in it.

CHAPTER II

MATHEMATICAL FORMULATION OF THE PROBLEM

The classical general instability parameter of thin stiffened cylindrical shell subject to a uniform hydrostatic pressure and axial compression (which is a known fraction of the hydrostatic pressure) with simply supported boundary conditions is given by (see Appendix A)

$$\bar{k}_{yy} = \frac{am^4 + bm^2 + c}{fm^2 + g} \tag{1}$$

where

$$a = \left[(1+\bar{\beta}^2)^2 + \bar{\lambda}_{xx} + \bar{\lambda}_{yy}\bar{\beta}^4 + \frac{2\bar{\beta}^2}{1-\nu}(\bar{\lambda}_{xx} + \bar{\lambda}_{yy} + \bar{\lambda}_{xx}\bar{\lambda}_{yy})\right]\left[(1+\bar{\beta}^2)^2\right.$$

$$\left. + \bar{\rho}_{xx} + \bar{\rho}_{yy}\bar{\beta}^4\right] + \frac{12Z^2}{\pi^4(1-\nu^2)}\left[\bar{e}_{st}^2\bar{\lambda}_{xx} + \frac{2}{1-\nu}\bar{e}_{st}^2\bar{\lambda}_{xx}(1-\nu+\bar{\lambda}_{yy})\bar{\beta}^2\right.$$

$$+ \left\{\bar{e}_{st}^2\bar{\lambda}_{xx}(1+\bar{\lambda}_{yy}) + 2\,\frac{1+\nu}{1-\nu}\,\bar{e}_{st}\bar{e}_r\bar{\lambda}_{xx}\bar{\lambda}_{yy} + \bar{e}_r^2\bar{\lambda}_{yy}(1+\bar{\lambda}_{xx})\right\}\bar{\beta}^4$$

$$\left. + \frac{2}{1-\nu}\bar{e}_r^2\bar{\lambda}_{yy}(1-\nu+\bar{\lambda}_{xx})\bar{\beta}^6 + \bar{e}_r^2\,\bar{\lambda}_{yy}\bar{\beta}^8\right]$$

$$b = \frac{12\,Z^2}{\pi^4(1-\nu^2)}\left[-2\nu\bar{e}_{st}\bar{\lambda}_{xx} + 2\left\{\bar{e}_{st}\bar{\lambda}_{xx}(1+\bar{\lambda}_{yy}) + \bar{e}_r\bar{\lambda}_{yy}(1+\bar{\lambda}_{xx})\right\}\bar{\beta}^2\right.$$

$$\left. - 2\nu\bar{e}_r\bar{\lambda}_{yy}\bar{\beta}^4\right]$$

$$c = \frac{12\ Z^2}{\pi^4(1-\nu^2)} [(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2]$$

$$f = \left(\frac{L}{\pi R}\right)^2 \left[(1+\bar{\beta}^2)(\bar{e}_{st}\bar{\lambda}_{xx}+\bar{e}_r\bar{\lambda}_{yy}\bar{\beta}^4) + \frac{2\bar{\beta}^2}{1-\nu}\ \bar{\lambda}_{xx}\bar{\lambda}_{yy}(\bar{e}_{st}+\bar{e}_r\bar{\beta}^2)\right]$$

$$+ \left(\frac{1}{2}-\alpha+\bar{\beta}^2\right)\left[(1+\bar{\beta}^2)^2 + \bar{\lambda}_{xx} + \bar{\lambda}_{yy}\bar{\beta}^4 + \frac{2\bar{\beta}^2}{1-\nu}\ (\bar{\lambda}_{xx}+\bar{\lambda}_{yy}+\bar{\lambda}_{xx}\bar{\lambda}_{yy})\right]$$

$$g = \left(\frac{L}{\pi R}\right)^2 \left[(1+\bar{\beta}^2)(\nu+\bar{\beta}^2)\right.$$

$$\left. + \frac{\bar{\beta}^2}{1-\nu}(2\bar{\lambda}_{xx}+\bar{\lambda}_{yy}+\nu\bar{\lambda}_{yy}+2\bar{\lambda}_{xx}\bar{\lambda}_{yy}) + \bar{\lambda}_{yy}\bar{\beta}^4\right]$$

$$\alpha = \frac{\bar{k}_{xx}}{\bar{k}_{yy}}\ ; \qquad \bar{k}_{xx} = \frac{\bar{N}L^2}{\pi^2 D}$$

$$\bar{k}_{yy} = \frac{qRL^2}{\pi^2\ D}\ ; \qquad \bar{\beta} = \frac{nL}{m\pi R} \tag{2}$$

and

$$\bar{\lambda}_{xx} = \frac{E_{st}A_{st}(1-\nu^2)}{E\ h\ \ell_{st}}\ ; \qquad \bar{\lambda}_{yy} = \frac{E_r A_r(1-\nu^2)}{E\ h\ \ell_r}$$

$$\bar{\rho}_{xx} = \frac{E_{st}I_{stc}}{D\ \ell_{st}}\ ; \qquad \bar{\rho}_{yy} = \frac{E_r I_{rc}}{D\ \ell_r}$$

$$\bar{e}_{st} = \frac{\pi^2 R}{L^2}\ e_{st}\ ; \qquad \bar{e}_r = \frac{\pi^2 R}{L^2}\ e_r \tag{3}$$

If the cylinder is to be designed for uniform hydrostatic pressure only, α is set equal to zero in the expression for general instability. The general instability critical load parameter $\bar{k}_{yycr}$ for a given cylinder and loading is obtained through minimization of Equation (1) with respect to integer values of m and n.

The prebuckling stresses in the skin, stringers, and rings are (see Appendix A)

$$\sigma_{xxsk} = -\frac{qR}{2h}\left[\frac{2\nu\bar{\lambda}_{xx}+\bar{\lambda}_{yy}(1+2\alpha)+(1-\nu^2)(1+2\alpha)}{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2}\right]$$

$$\sigma_{yysk} = -\frac{qR}{2h}\left[\frac{2\bar{\lambda}_{xx}+(1+2\alpha)\bar{\lambda}_{yy}+2(1-\nu^2)}{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2}\right]$$

$$\sigma_{xxst} = -\frac{qRE_{st}}{2hE}(1-\nu^2)\left[\frac{(1+\bar{\lambda}_{yy})(1+2\alpha)-2\nu}{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu^2}\right]$$

$$\sigma_{yyr} = -\frac{qRE_r}{2hE}(1-\nu^2)\left[\frac{2(1+\bar{\lambda}_{xx})-\nu(1+2\alpha)}{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})-\nu^2}\right] \qquad (4)$$

Formulation of the Objective Function

Assuming the eccentricities of the stiffeners to be small as compared to the radius of the shell, and ignoring the weight of the common material at the intersections of the stiffeners, the weight of the stiffened shell is

$$W_{ST} = 2\pi RLh\rho_{sk} + \rho_{st}\int_0^L\int_0^{2\pi R} (A_{st}/\ell_{st})dy\, dx + \rho_r\int_0^L\int_0^{2\pi R} (A_r/\ell_r)dy\, dx \quad (5)$$

Carrying out integrations in Equation (5) and using the nondimensional parameters $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ from Equation (3), the weight of the stiffened cylinder is given by

$$W_{ST} = 2\pi RLh\ \rho_{sk}\left[1 + \frac{1}{1-\nu^2}\left(\frac{E\rho_{st}}{E_{st}\rho_{sk}}\ \bar{\lambda}_{xx} + \frac{E\rho_r}{E_r\rho_{sk}}\ \bar{\lambda}_{yy}\right)\right] \quad (6)$$

Objective Function Based on Skin Yielding

The prebuckling stresses σ_{xxsk} and σ_{yysk} for the skin are obtained from Equation (6). The stress in the skin, σ_s, computed on the basis of von Mises-Henkey yield criterion is

$$\sigma_s = (\sigma_{xxsk}^2 + \sigma_{yysk}^2 - \sigma_{xxsk}\sigma_{yysk})^{\frac{1}{2}} \quad (7)$$

Let σ_y be the permissible yield stress for the material of the skin. The problem is stated as

$$\text{Minimize } W_{ST}$$

$$\text{such that } \sigma_s = \sigma_y \quad (8)$$

This constrained minimization problem is transformed to an unconstrained minimization problem, leading to the composite objective function

$$W^* = W_{ST} + \lambda|\sigma_s - \sigma_y| \quad (9)$$

where λ is a Lagrange multiplier.

Equation (9) can be put in the nondimensional form as

$$\bar{W}^* = \bar{W} + \lambda^* \left| PZ^2 - \sigma^* Z \right| \qquad (10)$$

where

$$\bar{W}^* = \frac{W^* Z}{2\pi R L^3 (1-\nu^2)^{\frac{1}{2}}}$$

$$\bar{W} = 1 + \frac{1}{(1-\nu^2)} \left[\frac{E\rho_{st}}{E_{st}\rho_{sk}} \bar{\lambda}_{xx} + \frac{E\rho_r}{E_r\rho_{sk}} \bar{\lambda}_{yy} \right]$$

$$\lambda^* = \frac{q\ R^2 \lambda}{2\pi L^5\ \rho_{sk}\ (1-\nu^2)} \qquad (11)$$

$$P = [(1-\nu+\nu^2)(4\bar{\lambda}_{xx}^2 + \bar{\lambda}_{yy}^2) - 2(1-4\nu+\nu^2)\bar{\lambda}_{xx}\bar{\lambda}_{yy}$$

$$- (1-\nu^2)(5\nu+1)(2\bar{\lambda}_{xx}+\bar{\lambda}_{yy})$$

$$+ 7(1-\nu^2)^2]^{\frac{1}{2}}/2[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2]$$

$$\sigma^* = \sigma_y \left(\frac{L}{R}\right)^2 \frac{(1-\nu^2)^{\frac{1}{2}}}{q}$$

In the functional form one can write

$$\bar{W}^* = \bar{W}^*(Z,\ \bar{\lambda}_{xx},\ \bar{\lambda}_{yy}) \qquad (12)$$

It can easily be verified that minimization of $\bar{W}^*$ on the basis of the skin yielding results in an unstiffened shell. Such a shell, undoubtedly, fails in general instability. Therefore, considering Z (or h) as an independent variable is meaningless in the present formulation. One can, however, approach the problem by considering $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ as independent variables. For a fixed value of Z (or h), one finds those values of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ which minimize $\bar{W}$. In this manner one generates sets of data that indicate the distribution of the material in the skin and the stiffeners such that the skin yielding constraint is satisfied. A systematic procedure, given in Chapter III, can then be followed to arrive at those values of the design variables which satisfy all the constraints and result in the minimum weight configuration. The values of λ^* in Equation (10) must be sufficiently large, Reference [26], so that the solution of the unconstrained problem approaches to that of the constrained problem.

Objective Function Based on General Instability

If general instability is the active failure mode, the objective function is formulated on the basis of this constraint. The problem is stated as

$$\text{Minimize } W_{ST}$$

$$\text{such that } q_{cr} = q_D \tag{13}$$

where q_{cr} is the general instability critical load, and q_D is the design load. This constrained minimization problem is transformed to an unconstrained minimization problem, leading to the objective function

$$W^* = W_{ST} + \lambda|q_{cr} - q_D| \tag{14}$$

where λ is a Lagrange multiplier.

Equation (14) can be put in the nondimensional form as

$$\bar{W}^{*} = \bar{W} + \lambda_G^{*}|\bar{k}^{*}_{yycr} - q_D^{*}| \qquad (15)$$

where

$$\lambda_G^{*} = \frac{\pi\, EL}{24R^4 \rho_{sk}}\, \lambda$$

$$q_D^{*} = \frac{12Z}{\pi^2 E}\left(\frac{L}{R}\right)^4 \frac{q_D}{(1-\nu^2)^{\frac{1}{2}}} \qquad (16)$$

$$\bar{k}^{*}_{yycr} = \frac{\bar{k}_{yycr}}{Z^2}$$

Thus $\bar{W}^{*}$ is a function of number of variables

$$\bar{W}^{*} = \bar{W}^{*}(Z, \bar{\lambda}_{xx}, \bar{\lambda}_{yy}, \bar{\rho}_{xx}, \bar{\rho}_{yy}, \bar{e}_{st}, \bar{e}_r, m^2, \bar{\beta}^2) \qquad (17)$$

It is observed, in this case, that $\bar{W}^{*}$ behaves like 1/Z, suggesting that there is no minimum with respect to finite Z. Equation (1) for the general instability load parameter $\bar{k}_{yy}$ indicates that the value of $\bar{k}_{yycr}$ increases with the increasing values of $\bar{\rho}_{xx}$, $\bar{\rho}_{yy}$, $\bar{e}_{st}$, and $\bar{e}_r$. But these parameters possess a certain upper limit, because any increase beyond that limit makes the local instabilities active. If these limits can somehow be found, a program can be set up for minimization of $\bar{W}^{*}$ with respect to $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ for fixed values of Z, $\bar{\rho}_{xx}$, $\bar{\rho}_{yy}$, $\bar{e}_{st}$, and $\bar{e}_r$.

Specifying limits on $\bar{\rho}_{xx}$, $\bar{\rho}_{yy}$, $\bar{e}_{st}$, and $\bar{e}_r$ as they appear in the expression for $\bar{k}_{yy}$ is rather difficult. This can, however, be accomplished by replacing these four parameters by four new parameters $\bar{\alpha}_x$, $\bar{\alpha}_y$, C_x, and C_y. The relations between the new and the old parameters are

$$\bar{\rho}_{xx} = \bar{\alpha}_x^{-2}\bar{\lambda}_{xx} \quad ; \qquad \bar{\rho}_{yy} = \bar{\alpha}_y^{-2}\bar{\lambda}_{yy}$$

$$\bar{e}_{st} = \frac{\pi^2(1-\nu^2)^{\frac{1}{2}}}{2Z}(1+C_x\bar{\alpha}_x) \; ; \quad \bar{e}_r = \frac{\pi^2(1-\nu^2)^{\frac{1}{2}}}{2Z}(1+C_y\bar{\alpha}_y) \tag{18}$$

The new parameters $\bar{\alpha}_x$ and $\bar{\alpha}_y$ denote the ratios of radii of gyration of stringers and rings to that of the skin per unit width respectively, and C_x and C_y are numbers specifying the shape of the stiffeners. These are called the stiffener shape parameters. By making such a substitution the $\bar{W}^*$ expression becomes

$$\bar{W}^* = \bar{W}^* \, [Z, \, \bar{\lambda}_{xx}, \, \bar{\lambda}_{yy}, \, m^2, \, \bar{\beta}^2, \, (\bar{\alpha}_x, \, \bar{\alpha}_y, \, C_x, \, C_y)] \tag{19}$$

Introduction of these four new parameters is helpful in two ways: (a) It is easier now to investigate various shapes of the stiffeners and (b) the range of these four new parameters can reasonably be fixed.

For a fixed value of Z (or h) and known load parameter q_D^*, values of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ are found in the space of $\bar{\alpha}_x$-$\bar{\alpha}_y$ such that $\bar{W}^*$ is minimum. The parameters C_x and C_y can be calculated for a particular shape of the stiffener. For some typical shapes the values of C_x and C_y are given in Appendix B.

In order to find the dimensions of the stiffeners, their spacings and the skin thickness, the results of first stage operation are used along with the inequalities describing various failure modes. The procedure to be followed for achieving the final design is discussed in Chapter III. Some typical design examples illustrating this procedure are given in Appendix C. A factor of safety of two is used against general instability, panel buckling, and local instabilities of skin and stiffeners. For yielding, a factor of safety of one is used.

CHAPTER III

METHOD OF SOLUTION

The solution to the present problem is accomplished, as stated earlier, in two stages. The design charts and tables are generated by performing the unconstrained minimization of the objective function by means of some mathematical search technique. This is the phase I of the present methodology. In the second phase, a design procedure (described in this Chapter) is followed systematically to arrive at the final minimum weight design configuration.

Phase I

Description of Mathematical Search Techniques

There are several mathematical search techniques available for optimization. A general distinction can be made between classical or indirect methods available to analytical solutions and mathematical programming and search methods which normally require a digital computer for finding a numerical solution to most realistic problems. The classical procedures are restricted to very few real world problems. In most cases one has to rely on the mathematical search techniques. In general, there are many techniques available that can be used for a particular problem. There is no single search technique that can uniquely be described as being the 'best' in all the situations, (see Reference [16]). In the present problem, the Nelder and Mean [25] mathematical search technique is used in the minimization of the objective function. This search technique proved to be quite effective. The general instability critical load parameter $\hat{k}_{yycr}$, needed for each iteration, in the general instability formulation, requires minimization with respect to m and n. This is accomplished by using either the golden section [27] or the modified sequential dichotomous [28] search technique.

Nelder and Mead Algorithm

This search technique is used for minimization of multivariable unconstrained nonlinear functions. The method is an extension of the simplex method given by Spendley, Hext, and Himsworth [29]. Both methods utilize a regular geometric figure called a simplex consisting of N+1 vertices, where N is the number of variables. The Nelder and Mead method accelerates the simplex method and makes it more general. This method adopts itself to the local landscape, using reflected, expanded, and contracted points to locate the minimum. The essential steps of the method are

1. Select a starting point.

2. Construct a starting simplex, refer [25] or Appendix E for appropriate expessions to obtain the remaining points of the simplex.

3. After forming the simplex, the objective function is evaluated at each point. The highest value of the objective function (the worst point) is replaced by a new point. Three operations used for this are reflection, contraction, and expansion. A point is first located by reflection of the worst point. The expression for this step is given in Appendix E, or Reference [25].

4. If the reflected point has the worst objective function value of the current points, a contracted point is located. The expression is given in Appendix E.

If the reflected point is better than the worst point but is not the best point, a contracted point is calculated from the reflected point. The objective function is now evaluated at the contracted point. If an improvement over the current points is achieved the process is restarted. If an improvement is not achieved, the points are moved one half the distance toward the best point. The process is then restarted.

5. If the reflected point calculated in step 3 is the best point, an expansion point is calculated. The program listing in Appendix E gives the expression for this operation.

If the expansion point is an improvement over the reflected point, the reflected point is replaced by the expansion point and the process is restarted. If the expansion point is not an improvement over the reflected point, the reflected point is retained and the process is restarted.

6. The procedure is terminated when the convergence criterion is satisfied or a specified number of iterations has been exceeded.

Method of Golden Section

This search technique is used for minimization of a nonlinear function of a single variable. The search commences with evaluation of the objective function at each end of the interval S, and at $d_1 = 2/(1+\sqrt{5})$ of the interval from both these bounding points. Refer

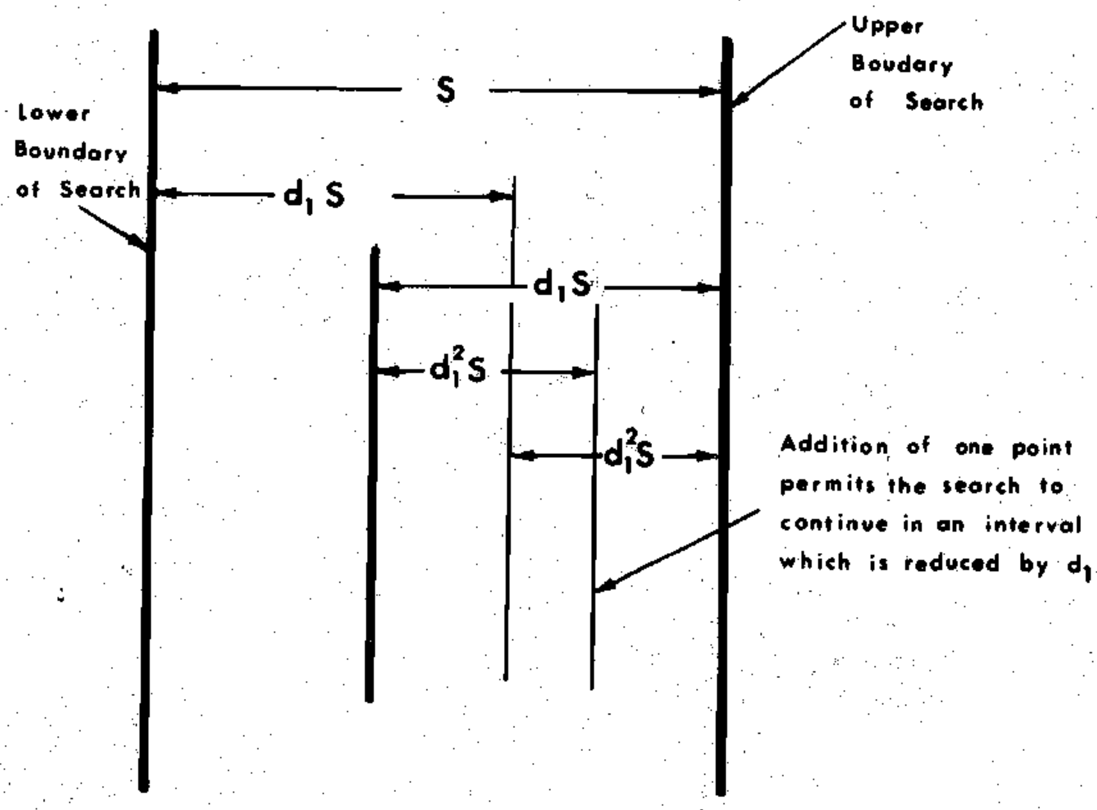

Figure 1. Golden Section Search Technique.

Figure 1.

On comparing the values of the objective function at these four points, the boundary point which is farthest from the lowest objective function value is discarded. The remaining three points are retained. The search now continues in the region which has been diminished in size by d_1. The internal point at which the objective function is known in the reduced interval is at a distance d_1 of the reduced interval from the remaining bounding point of the original interval. This is because $1-d_1 = d_1^2$. The search can, therefore, be continued in the reduced interval with only one additional evaluation of the objective function.

When the specified accuracy is achieved, the search is terminated. The method is based on the assumption of unimodality. It is, therefore, suggested that a set of different original intervals be attempted. The program is given in Appendix E.

Modified, Sequential, Dichotomous Search

This search technique is used for finding general instability critical load treating m and n as integer variables. The essential steps of the search technique are

1. Start from an arbitrary initial point m_1 and n_1. A one dimensional minimization is first performed with respect to m, using dichotomous search. The program in Appendix E gives the steps involved in this process. This search is continued until a minimum with respect to m is located.

2. For the fixed value of m located in step 1, the search procedure is repeated with respect to n. The search is terminated

when a set of search sequences fails to yield any change in the minimizing values of m and n.

Employing these search techniques the design charts and tables are generated for the two formulations of the objective function discussed in Chapter II. For the skin yielding formulation, values of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ are obtained for various values of Z (or h). The results are given in the form of tables and curves. These are discussed in Chapter IV.

For the general instability formulation, values of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ are obtained for fixed values of Z, C_x and C_y, in the space of $\bar{\alpha}_x - \bar{\alpha}_y$. The results are presented in the form of tables and charts as given in Chapter IV.

Phase II

Procedure for Design

In this phase the values of design variables are found by employing the design charts generated in phase I. The following quantities are known

1. The radius and length of the shell.

2. Applied hydrostatic pressure (or operating depth of submarine) and safety factors. A factor of safety of two is assumed against all the instability failure modes and a factor of safety of one is assumed against yielding.

3. The material of the skin and stiffeners and their properties.

4. The position of the stiffeners.

The design variables to be determined are the skin thickness, the sizes of the stiffeners and their spacings. The systematic approach to arrive at these design variables for the two formulations of the objective function is given as follows.

Design Procedure for General Instability Formulation

Ring Stiffened Shell

In general, the thickness corresponding to the minimum weight design is determined from a curve of Z (or h) versus W. In order to plot such a curve, designs are obtained for at least three different values of Z (or h). The following procedure is suggested for determining appropriate values of Z (or h).

The upper bound on the skin thickness of the shell is obtained from consideration of either skin yielding or buckling of an unstiffened shell. The optimum skin thickness is a fraction of the upper bound found from any of the above two considerations. The skin thickness obtained on the basis of skin yielding, given by $h_u = \frac{\sqrt{3}\ qR}{2\ \sigma_y}$, is much lower than the one obtained on the basis of buckling. It is, therefore, suggested to find the starting value of Z (or h) on this basis.

It is anticipated that the optimum thickness may be around $h_u/1.5$ $h_u/1.5$. As an initial guess take $Z = 1.2\ Z_u$; generate some data and design the shell according to steps 1 through 7. Let this weight be W_1. The procedure is repeated for $Z = 1.3\ Z_u$ and weight W_2 is obtained. If $W_1 < W_2$, use $Z = 1.1\ Z$. If $W_1 > W_2$, use $Z = 1.4\ Z_u$. If $W_1 \simeq W_2$, then the minimum weight configuration corresponds to a Z value between 1.2 Z_u and 1.3 Z_u. In this manner values of Z are selected for plotting the curve of Z versus W. The steps of designing a shell for a fixed Z are given as follows.

1. For a particular value of Z (or h) read values of $\bar{\lambda}_{yy}$ and $\bar{\alpha}_y$ corresponding to the minimum value of $\bar{W}$. Steps 2 through 9 are then followed such that the constraints defining failure modes are not violated. If any constraint is violated, move to a point of higher value of $\bar{W}$ and repeat the steps.

2. The ring spacing ℓ_r is determined from the criterion of panel buckling. This needs a few trials.

3. The ring dimensions are computed as follows

$$d_{wr} = \frac{h\,\bar{\alpha}_y(1+A_yB_y)}{\sqrt{(1+4A_yB_y)}} \;; \qquad t_{wr} = \frac{\bar{\lambda}_{yy}\ell_r\,h}{d_{wr}(1-\nu^2)(1+A_yB_y)}$$

$$t_{fr} = A_y t_{wr} \qquad ; \qquad b_{fr} = B_y d_{wr} \tag{20}$$

These expressions are written for T-rings. If rectangular rings are used, A_y and B_y in the above expressions are set equal to zero. For other shapes Table B1, in Appendix B, is used for calculating the dimensions of the ring.

4. The stresses in the ring and skin are calculated using Equations (A27), (A28) and (A31) given in Appendix A. These stresses are checked against permissible stress levels.

5. The critical ring stress is calculated from Equation (A21). This should be greater than the applied stress.

6. The ratios of actual load to failure load are calculated

to make sure that failure modes do not occur simultaneously. If this condition is not satisfied, either adjust ring spacing or proceed with another value of $\bar{W}$.

7. The weight of the shell is computed.

8. This procedure is repeated for at least three values of Z (or h), as suggested in the beginning, and $\bar{W}$ versus h is plotted. From this curve one finds the optimum value of h.

9. For this value of h (or Z) generate design data and follow the above steps. This step is needed if exact minimum weight is desired. The curve $\bar{W}$ versus h is relatively flat, indicating that there is a fairly large range of h that corresponds to almost same weight of the shell.

The steps given above are carried out conveniently through a computer program RSSH written for this purpose. A sample example is worked out in Appendix C.

Shell Stiffened with Rectangular Rings and Rectangular Stringers

The appropriate values of Z (or h) are determined as suggested under the ring stiffened shell design procedure.

1. From the design charts for this case, see for example, Figure 2, locate the minimum weight parameter $\bar{W}$ for each Z in the space of $\bar{\alpha}_x$-$\bar{\alpha}_y$. Corresponding to this value of $\bar{W}$, read values of $\bar{\alpha}_x$, $\bar{\alpha}_y$, $\bar{\lambda}_{xx}$, and $\bar{\lambda}_{yy}$. One then follows steps 2 through 12 such that the constraints are not violated. If any constraint is violated move to a point giving same value of $\bar{W}$, if there is any, or move to a point of higher value of $\bar{W}$. With few designs one could get clear indications as regards to the appropriate direction in which one

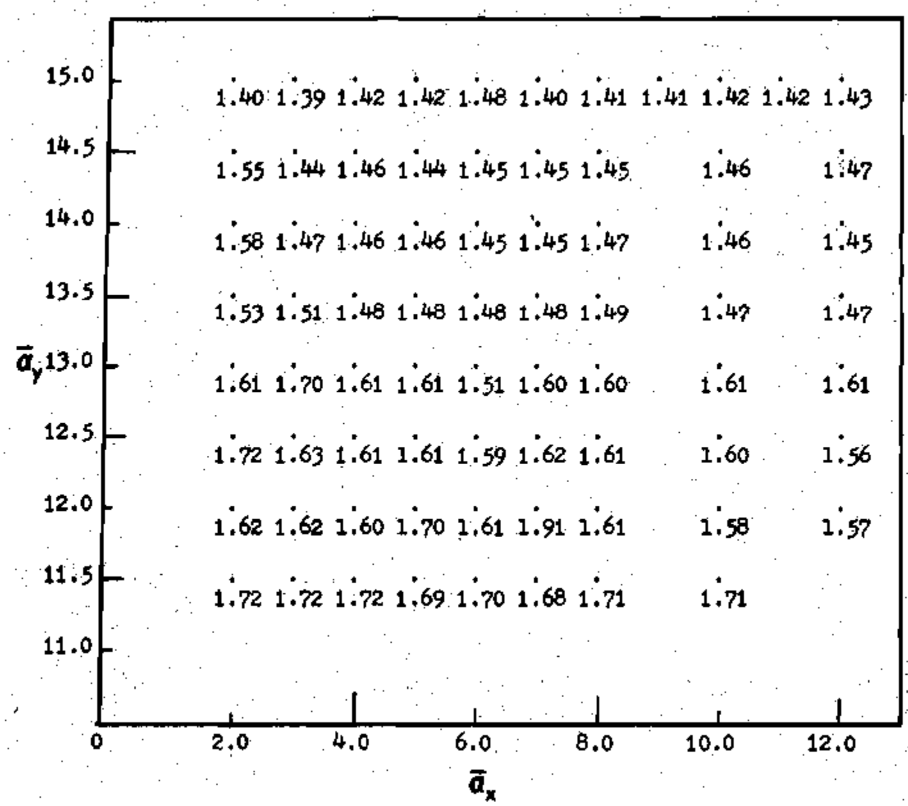

Figure 2. Design Chart for Internally TR-RS Stiffened Shell General Instability Formulation, High Strength Steel Operating Depth = 3000 feet, Z = 1200.

must move to get an acceptable design.

2. Through Equations (4) and (7) calculate stresses in the skin, stringers and rings. If these stresses are within the permissible levels specified, one proceeds to the next step.

3. The depths of stringers and rings are calculated from the following Equations.

$$d_{st} = h\,\bar{\alpha}_x \;; \qquad d_{wr} = h\,\bar{\alpha}_y \tag{21}$$

4. The ratios of stiffener thickness to the stiffener spacing are determined from the definitions of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ as

$$\frac{t_{st}}{\ell_{st}} = \frac{E\,\bar{\lambda}_{xx}\,h}{E_{st}d_{st}(1-\nu^2)} \;; \qquad \frac{t_{wr}}{\ell_r} = \frac{E\,\bar{\lambda}_{yy}\,h}{E_r\,d_{wr}(1-\nu^2)} \tag{22}$$

5. The ring spacing is determined from the requirement that the stress in the ring must be less than the critical stress or

$$\ell_r > \begin{cases} \sqrt{\dfrac{24(1-\nu^2)\sigma_{yyr}\,F_1}{\pi^2\,E_r}\,\dfrac{d_{wr}(1-\nu^2)}{\bar{\lambda}_{yy}\,h}\,(d_{wr}-d_{st})} \\ \text{or} \\ \sqrt{\dfrac{3(1-\nu^2)\sigma_{yyr}\,F_1}{\pi^2\,E_r}\,\dfrac{d_{wr}(1-\nu^2)}{\bar{\lambda}_{yy}\,h}}\,d_{st} \end{cases} \tag{23}$$

F_1 is factor of safety

The largest of the two values is taken as the required ring spacing. The spacing is selected such that the number of rings is an integer. For this ring spacing t_{wr} is calculated from Equation (22). Inequalities (23) are obtained for the case of rings deeper than the stringers. The portion of the ring equal in depth to the stringer is assumed as a plate simply supported on all four sides, whereas the portion of the ring projecting above the stringer is considered as free on one edge.

6. For the ring spacing determined in step 5, check for panel buckling.

7. The stringer spacing is calculated from the requirement that the stringer stress must be less than the critical stringer stress,

$$\ell_{st} > \sqrt{\frac{12(1-\nu^2)\sigma_{xxst}\, F_1}{\pi^2 E_{st}\left[\left(\frac{d_{st}}{\ell_r}\right)^2 + .425\right]}} \quad \frac{d_{st}^2\,(1-\nu^2)}{h\,\bar{\lambda}_{xx}} \tag{24}$$

Select ℓ_{st} such that the number of stringers is an integer and inequality (24) is satisfied. From Equation (22), one then calculates t_{st}.

8. For the values of ℓ_r and ℓ_{st} determined in steps 6 and 7, check against skin buckling using Equation (A18), Appendix A. If this constraint is satisfied proceed to the next step, otherwise, examine values of ℓ_r and ℓ_{st} and see if they can be adjusted, without violation of any other constraint, so as to satisfy the skin buckling failure mode. If such an adjustment is not possible, go back to

step 1.

9. Calculate ratios of actual load to the failure load. If there is simultaneous occurrence of two or more failure modes, adjust ℓ_r and ℓ_{st} to avoid this, or move to another point in the design space.

10. Compute the weight of the shell.

11. Repeat the above steps for a number of Z (or h) values and plot W versus h. For example, see Figure 5. At least three values of Z (or h) are needed for plotting the curve, see design procedure for ring stiffener shell. From the plot of W versus h one can locate the absolute minimum weight and corresponding value of Z (or h).

12. For this value of Z, design charts are generated and above steps repeated to finalize the design dimensions. This step is needed only if the exact minimum weight configuration is desired.

Shell Stiffened with T-Rings and Rectangular Stringers

The steps for the design in this case are similar to those given above. It may, however, be noted that the value of C_y is no longer equal to one and the expressions for $\bar{\alpha}_y$ and $\bar{\lambda}_{yy}$ are different than those given for the rectangular shapes. These expressions are given in Table B1, Appendix B.

The ring spacing in this case is calculated from the following inequality:

$$\ell_r > \begin{cases} \sqrt{\dfrac{3(1-\nu^2)\sigma_{yyr}\, F_1}{\pi^2\, E_r}\, \dfrac{(1 + A_y\, B_y)(1-\nu^2)d_{wr}}{\bar{\lambda}_{yy}\, h}}\, d_{st} \\ \\ \sqrt{\dfrac{3(1-\nu^2)\sigma_{yyr}\, F_1}{\pi^2\, E_r}\, \dfrac{(1 + A_y\, B_y)(1-\nu^2)d_{wr}}{\bar{\lambda}_{yy}\, h}}\, (d_{wr}-d_{st}) \end{cases} \tag{25}$$

In order to find the optimum value of C_y, a plot of W versus C_y is made. Such a plot, for example, is shown in Figure 7.

Various types of stiffener shapes can be examined by introducing proper values of C's from Table B1. The essential steps in the procedure for design remain the same.

Design Procedure for Skin Yielding Formulation
Ring Stiffened Shells

The steps to be followed for this case are similar to those given under general instability formulation. The difference lies in the fact that for each trial a check for general instability critical load is required. This is accomplished through a computer program, see Appendix E.

Shells Stiffened with Rectangular Rings and Rectangular Stringers

1. From the design charts or tables read the values of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ corresponding to a particular Z (a good starting guess is $Z = 1.1\ Z_u$). One then follows steps 2 through 7 such that the constraints are not violated.

2. This step is same as step 2 under corresponding case in general instability formulation.

3. In order to check against general instability failure mode, one must first determine values of $\bar{\alpha}_x$ and $\bar{\alpha}_y$. It is indicated by the present study that for minimum weight configuration rings are always deeper than the stringers. The ring spacing is obtainable from inequalities (23). In order that both expressions in that inequality yield the same value of ℓ_r, one easily finds that

$$d_{st} \simeq .74 \; d_{wr} \tag{26}$$

If the depth of the ring is limited by any geometric constraint, the starting value of d_{wr} is taken as equal to that limiting value. If no such limit is specified, d_{wr} is assumed as R/15.

4. With these values of d_{wr} and d_{st}, $\bar{\alpha}_x$ and $\bar{\alpha}_y$ are calculated from Equation (21).

5. For these values of $\bar{\alpha}_x$, $\bar{\alpha}_y$, $\bar{\lambda}_{xx}$, $\bar{\lambda}_{yy}$, and Z check the design against general instability. This is done through a computer program, see Appendix E. If $q_{cr} > q_D$, proceed to the next step. Otherwise, change values of $\bar{\alpha}_x$ and $\bar{\alpha}_y$. An increase in the value of $\bar{\alpha}_x$ and $\bar{\alpha}_y$ will increase value of q_{cr}. If the general instability constraint can not be satisfied, one must move to higher value of $\bar{W}$. A few trials are needed for this step.

6. The sizes and spacings of the stiffeners are found through the constraints of ring, stringer and skin buckling. The steps outlined under the general instability formulation are applicable in the present case also.

7. The ratios of actual load to failure load are calculated.

This is for making certain that there is no simultaneous occurrence of failure modes.

The procedure for shells stiffened with T-rings (or any other shape) and rectangular stringers is essentially the same except the changes pointed out under the general instability formulation. Some examples are worked out completely in Appendix C to illustrate the procedure for design.

CHAPTER IV

NUMERICAL RESULTS AND DISCUSSION

The following cases are considered during the course of this investigation to amply demonstrate the useful applicability of the present methodology.

Case 1. This case deals with the design studies employing skin yielding formulation for a shell of radius 198 inches and length 594 inches. The operating depths considered for this case are 1000 feet and 3000 feet. The material of construction for all the elements of shell is conventional steel with permissible yield stress of 60,000 psi. Both ring stiffened and ring-stringer stiffened shells are considered to arrive at minimum weight design.

Care 2. This case is similar to the case 1, except that the formulation of the objective function is based on general instability.

Case 3. For an operating depth of 3000 feet, the minimum weight designs are obtained for a shell of radius 198 inches and length 594 inches. The material of construction for all the elements of shell is high strength steel with permissible yield stress of 120,000 psi. Both ring stiffened and ring-stringer stiffened geometries are considered for this case also. The formulation of the objective function is based on general instability.

Case 4. This case is similar to case 3, except that instead of interior stiffeners, exterior stiffeners are considered here.

Case 5. Effect of varying L/R ratio on the minimum weight is studied in this case. A shell of radius 198 inches in considered. The L/R ratio is varied from one through five. The operating depth, type of steel and stiffening are same as in the Case 3.

Case 6. This case deals with the minimum weight design of a shell of radius 198 inches and length 594 inches when predominant hydrostatic pressure is combined with small uniform axial compression. The operating depth and type of steel used are same as in Case 3. The axial compression, in this case, is assumed to be .2qR, where q is the hydrostatic pressure and R is the radius of the shell.

The results of Cases 1 and 3, when the shell is stiffened with only rings, could be compared with the results of [18].

Tables 1 through 11 and Figures 3-28 give the results of the design studies listed above. Three sample design examples and corresponding design charts are given in Appendix C. The design charts for all the cases considered are given in a separate report [30].

For the objective function formulated on the basis of skin yielding, the results of Phase I, for the two operating depths considered, are given in Tables 1, 2, 4 and 5. The chart showing skin thickness versus W* is also plotted. Figure 3 and 8 are the design charts for ring stiffened shell with operating depth of 1000 feet and 3000 feet respectively.

Table 3 summarizes the results of the design studies for an operating depth of 1000 feet. The least weight is obtained, when the shell is stiffened with T-rings and rectangular stringers. The objective function for this case is formulated on the basis of

Table 1. Design Table for Shell Stiffened with Interior Ring Stiffeners. Skin Yielding Formulation.

Material of Construction - Conventional Steel.

Operating Depth	ν	σ_y	σ^*
1000 ft.	0.3000	60,000 psi	1147.78781
Z	h	$\bar{\lambda}_{yy}$	$\bar{W}$
1800.0	.94440	.45158	1.41305
1775.0	.95770	.42304	1.40291
1750.0	.97138	.39522	1.39326
1725.0	.98546	.36807	1.38405
1700.0	.99995	.34156	1.37528
1675.0	1.01488	.31565	1.36690
1650.0	1.03025	.29029	1.35891
1625.0	1.04610	.26547	1.35128
1600.0	1.06245	.24114	1.34399
1575.0	1.07931	.21729	1.33703
1550.0	1.09672	.19388	1.33038
1525.0	1.11470	.17089	1.32404
1500.0	1.13328	.14830	1.31797
1475.0	1.15249	.12609	1.31218
1450.0	1.17236	.10424	1.30665
1425.0	1.19293	.08272	1.30137
1400.0	1.21423	.06153	1.29633
1350.0	1.25920	.02004	1.28693

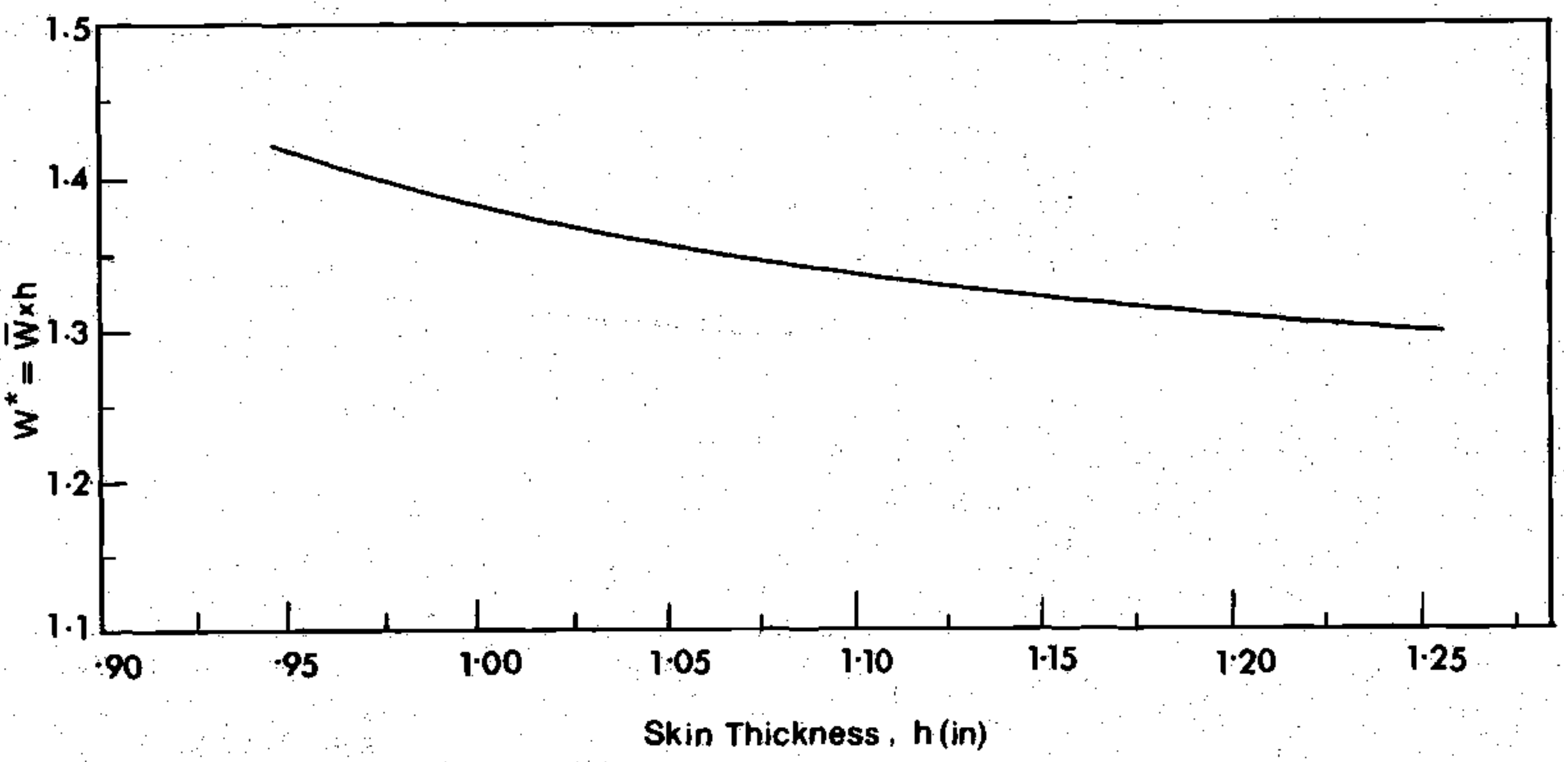

Figure 3. Design Chart for Internally Ring Stiffened Shell Skin Yielding Formulation, Conventional Steel Operating Depth = 1000 feet.

Table 2. Design Table. Shell Stiffened with Interior Ring-Stringer Stiffeners. Skin Yielding Formulation.

Material of Construction - Conventional Steel.

Operating Depth	ν	σ_y	σ^*
1000 ft.	0.300	60,000 psi	1147.78781

Z	h	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	$W^* = \bar{W} \times h$
1850.0	.91888	.00287	.50994	1.43669
1825.0	.93146	.00330	.47978	1.42593
1800.0	.94440	.00406	.45037	1.41600
1775.0	.95770	.00252	.42237	1.40485
1750.0	.97138	.00365	.39436	1.39623
1725.0	.98546	.00862	.36631	1.39147
1700.0	.99995	.00692	.34031	1.38088
1675.0	1.01488	.00883	.31426	1.37520
1650.0	1.03025	.00909	.28905	1.36778
1625.0	1.04610	.01464	.26376	1.36612
1600.0	1.06245	.01708	.23944	1.36194
1575.0	1.07931	.01328	.21615	1.35142
1550.0	1.09672	.02103	.19239	1.35391
1525.0	1.11470	.02567	.16940	1.35362
1500.0	1.13328	.02741	.14701	1.35049
1475.0	1.15249	.02594	.12511	1.34379
1450.0	1.17236	.01661	.10373	1.32740

yielding of skin. On comparing this result with corresponding case under general instability formulation, one notices that an improvement of about 4 percent in weight is realized.

Comparing the weight of ring stiffened shell to that of T-ring and rectangular stringer stiffened shell, it is found that the latter shows an improvement of about 13 percent in weight over the former. This is an appreciable saving in weight, indicating that one cannot ignore the importance of providing stringers without having a closer look at their contribution to the overall strength of the shell under hydrostatic pressure.

The results obtained by Pappas and Allentuch [18] are given in Table 3 along with the present results. The results indicate that, for ring stiffened shell, the minimum weight obtained in the present case is about 18.8 percent better than that of Pappas and Allentuch. One must, however, note that improved constraint equations are used in the present study. If, on the other hand, the comparison is made with respect to the best results obtained in the present case, the improvement in the weight is of the order of 34.3 percent.

The results of Pappas and Allentuch indicate that two or more failure modes occur simultaneously. This has been avoided, in the present results (see Table 3). It is possible, in most cases, to avoid simultaneous occurrence of failure modes, without any increase in the weight, just by adjusting various design variables. In some isolated cases, however, the failure mode interaction can be avoided only by increasing the weight of the shell. The present methodology enables one to achieve this with least weight penalty.

Table 3. Design Results.
Shell Stiffened with Interior Stiffeners.

Operating Depth = 1000 feet σ_y = 60,000 psi

		Skin Yielding Formulation		General Instability Formulation		Reference [18]
		TR	TR-RS	RR-RS	TR-RS	
W	lb/in	530.2	469.1	495.7	488.0	629.8
h	in	1.13328	1.06245	1.00000	1.00000	1.0631
d_{wr}	in	11.62590	12.88152	14.50000	11.34493	13.9570
t_{wr}	in	.64580	.33052	.42934	.28892	.7754
b_{fr}	in	3.48777	6.44076	....	5.67246	9.7698
t_{fr}	in	.64580	.33052	....	.28892	.7754
ℓ_r	in	23.76000	22.84615	16.50000	13.20000	22.8460
d_{st}	in	....	2.65612	10.00000	3.00000	....
t_{st}	in	....	.16669	.46056	.19000	....
ℓ_{st}	in	....	22.20428	73.14352	13.08884	....
GB		.91459	.94733	.96960	.96967	.3780
m , n		1 , 3	1 , 3	1 , 3	1 , 3	....
PB		.99228	.89584	.15920	.27763	1.0000
m_p , n_p		1 , 18	1 , 23	1 , 81	1 , 49	....
SKB		....	.92057	.84593	.35695	....
RB		.29038	.99607	.89475	.85093	1.0000
STB		....	.88892	.99269	.88626	....
SKY		1.00000	1.00000	.99365	.99769	.9940
RY		.80922	.93961	.91922	.92090	.6570
STY		....	.34646	.37655	.38247	....

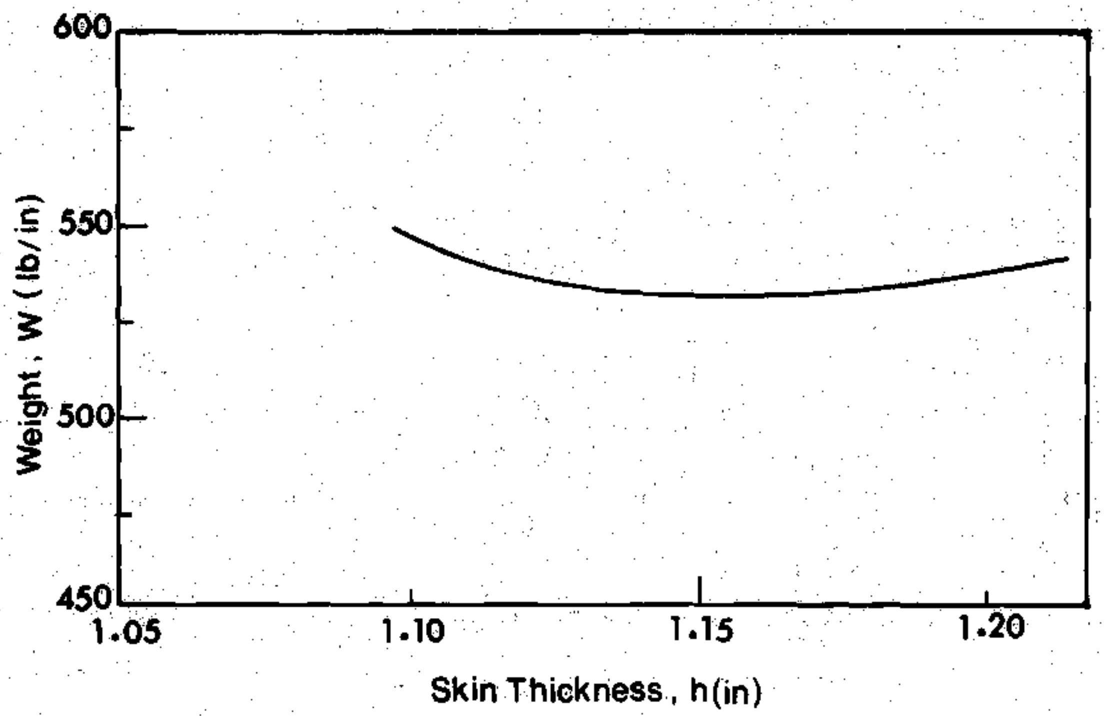

Figure 4. Determination of Optimum Skin Thickness. Internally TR Stiffened Shell, Operating Depth = 1000 feet. Skin Yielding Formulation, Conventional Steel.

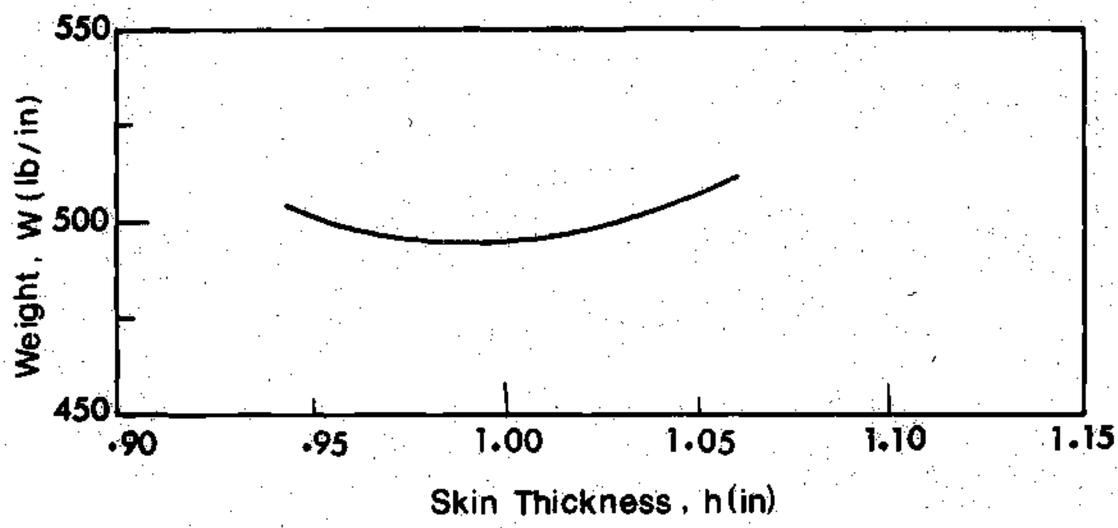

Figure 5. Determination of Optimum Skin Thickness. Internally RR-RS Stiffened Shell, Operating Depth = 1000 feet. General Instability Formulation, Conventional Steel.

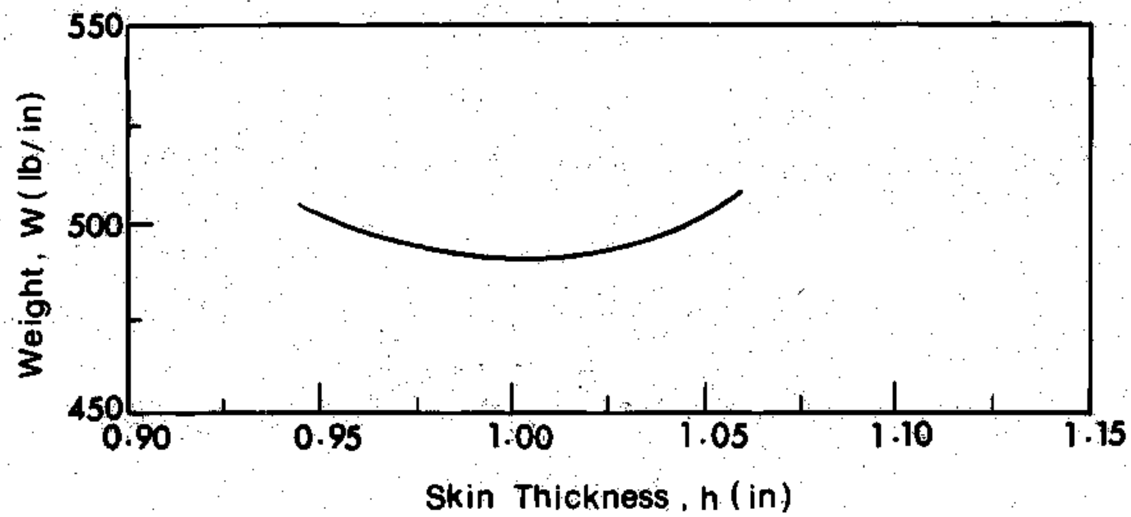

Figure 6. Determination of Optimum Skin Thickness. Internally TR-RS Stiffened Shell, Operating Depth = 1000 feet. General Instability Formulation, Conventional Steel.

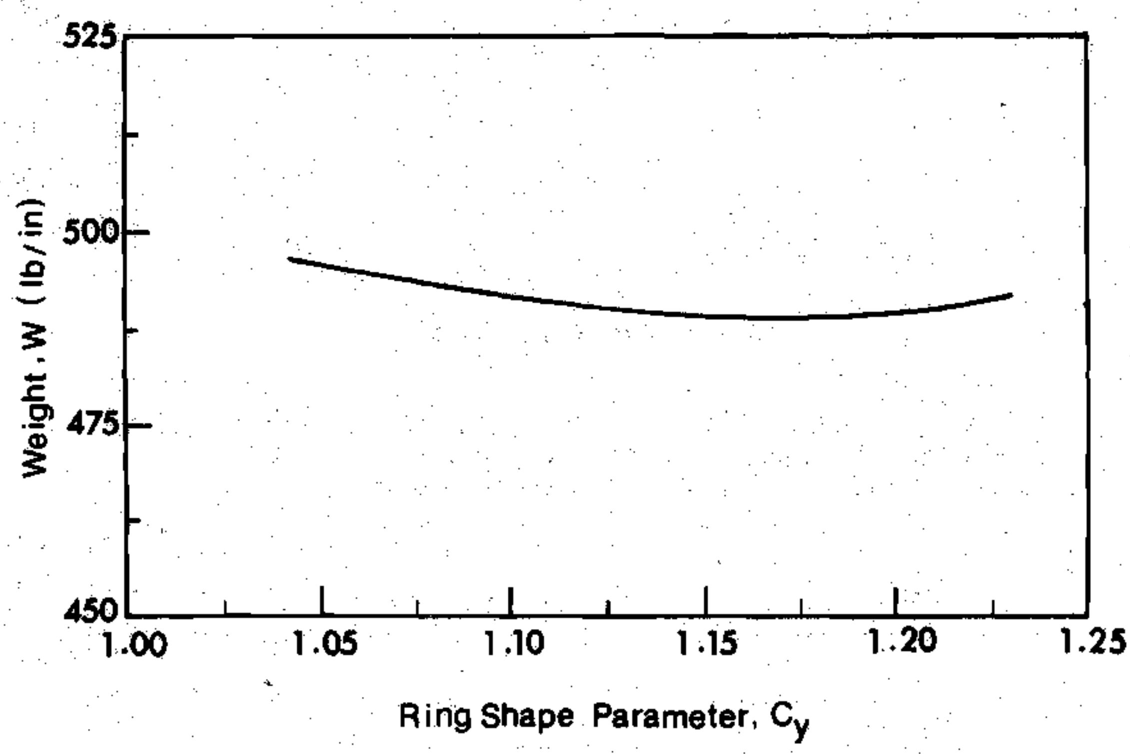

Figure 7. Determination of Optimum Ring Shape Parameter. Internally TR-RS Stiffened Shell, Operating Depth = 1000 feet. General Instability Formulation, Conventional Steel.

An observation of Table 3 indicates that the general instability coefficient is not equal to unity for the results obtained on the basis of general instability formulation. This is due to the fact that the parameter $\beta\left(=\frac{nL}{\pi R}\right)$ which minimizes $\bar{k}_{yy}$ does not necessarily yield integer value of n. In some cases, for example, minimizing value of β yield, $n = 3.2$. In such cases, $\bar{k}_{yy}$ is calculated for $n = 3$ and $n = 4$ and the least of these two values is taken as $\bar{k}_{yycr}$. This value is always slightly greater than the one obtained for $n = 3.2$. This is the reason for GB being less than unity.

The effect of the ring shape on the weight of the shell is studied by varying the value of parameter C_y. This study indicates that of all the shapes considered, T-rings prove to be most effective. Figure 7 shows the effect of varying parameters A_y and B_y (therefore C_y) on the weight of the shell. It helps in determining the optimum T-rings. It is obvious from the Figure 7 that the weight of the shell is not very sensitive to the variations of A_y and B_y (or C_y). This suggests that there is a large number of T-rings having different web depth to flange width ratio, and web thickness to flange thickness ratio, which give almost the same weight of the shell.

A similar study was conducted for stringers also. The results indicate that a rectangular shaped stringer is most effective. T-stringers give slightly higher weight. This phenomenon is understandable in the sense that the major contribution of the stringers is in strengthening the shell against panel buckling. Since local buckling of the stringers itself is not a critical failure mode, the shape of the stringer does not play a major role in reducing the

Table 4. Design Table. Shell Stiffened with Interior Ring Stiffeners. Skin Yielding Formulation

Material of Construction - Conventional Steel

Operating Depth	ν	σ_y	σ^*
3000 feet	0.300	60,000 psi	382.59594

Z	h	$\bar{\lambda}_{yy}$	$\bar{W} \times h$
875.0	1.94277	3.90202	10.27321
850.0	1.99991	2.56034	7.62677
825.0	2.06051	1.99755	6.58355
800.0	2.12490	1.64261	5.96049
775.0	2.19344	1.38463	5.53093
750.0	2.26656	1.18232	5.21139
725.0	2.34472	1.01585	4.96217
700.0	2.42846	.87424	4.76149
675.0	2.51840	.75077	4.59612
650.0	2.61526	.64104	4.45754
625.0	2.71987	.54203	4.33992
600.0	2.83320	.45158	4.23914
575.0	2.95630	.36807	4.15216
550.0	3.09076	.29029	4.07673
525.0	3.23794	.21729	4.01110
500.0	3.39984	.14830	3.95391
475.0	3.57878	.08272	3.90410

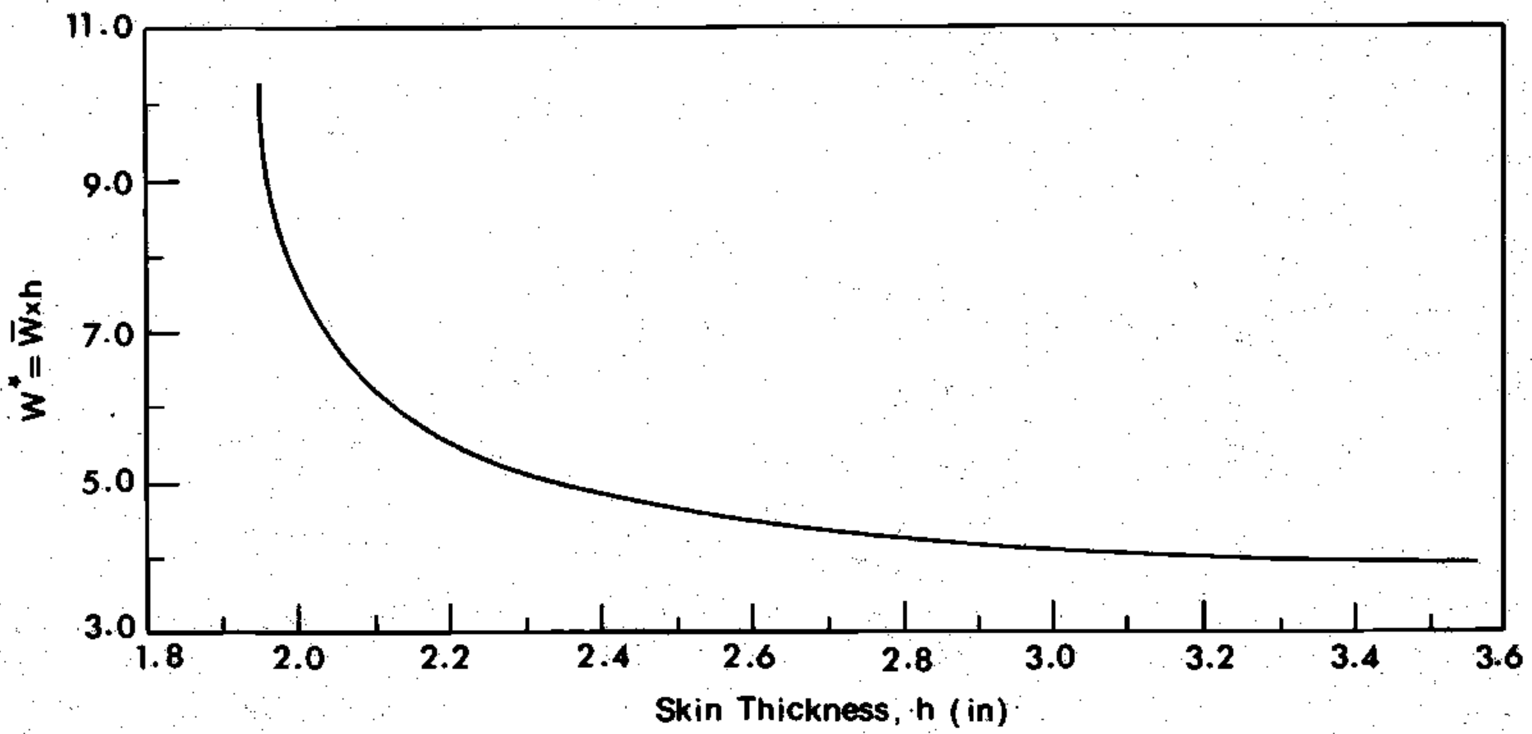

Figure 8. Design Chart for Internally Ring Stiffened Shell.
Skin Yielding Formulation, Conventional Steel.
Operating Depth = 3000 feet.

Table 5. Design Table. Shell Stiffened with Interior Ring-Stringer Stiffeners. Skin Yielding Formulation.

Material of Construction - Conventional Steel

Operating Depth	ν	σ_y		σ^*
3000 feet	0.300	60,000 psi		382.59594
Z	**h**	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	$W^* = \tilde{W} \times h$
500.0	3.39984	.02739	.14701	4.05138
525.0	3.23794	.01320	.21616	4.05403
550.0	3.09076	.00882	.28909	4.10255
575.0	2.95638	.00881	.36628	4.17494
600.0	2.83320	.00406	.45037	4.24801
625.0	2.71987	.00297	.54078	4.34507
650.0	2.61526	.01551	.63230	4.47701
675.0	2.51840	.00761	.74470	4.60036
700.0	2.42846	.03036	.84310	4.75941
725.0	2.34472	.06291	.93537	4.91687

weight of the shell. It is, therefore, decided to study only the rectangular stringers in the remaining cases.

The results of the design studies for an operating depth of 3000 feet, using conventional steel are given in Table 6. The governing critical mode of failure in this case is yielding of the skin. This mode of failure controls the design of the shell. Relatively large thickness of the skin is needed for preventing the stresses in the skin from exceeding the permissible level. As stated in Chapter III, for ring stiffened shells, the ring spacing is determined from the panel buckling criterion. However, in the present case, if the ring spacing is determined based on this failure mode, one finds that it results in very wide ring spacing. The widely spaced ring stiffened shell either failed in general instability or was of higher weight than the one with closely spaced rings. This suggests that panel buckling mode is not critical in the sense that one can disregard it temporarily. This means that the ring spacing obtained for the design that satisfies all the constraints except the panel buckling is much smaller than the one required by panel buckling constraint. In order to find the best ring spacing, a plot is made for the number of rings versus weight of the shell. Figure 9 shows such plots. The optimum number of rings (or ring spacing) is then found from such plots.

The results given in the Table 6 show that least weight is obtained for ring stiffened geometry under skin yielding formulation. Comparing with the results of Pappas and Allentuch, given in the same Table, one observes an improvement of about 5 percent in weight in the

Table 6. Design Results. Shell Stiffened with Interior Stiffeners.

Operating Depth = 3000 feet σ_y = 60,000 psi

		Skin Yielding Formulation		General Instability Formulation		Reference [18]
		TR	TR-RS	RR-RS	TR-RS	
W	lb/in	1387.2	1405.3	1414.2	1394.2	1456.0
h	in	3.39984	3.39984	3.23794	3.23794	3.20420
d_{wr}	in	12.68345	13.24957	14.89452	11.80754	17.93300
t_{wr}	in	.70464	.36478	.67036	.61154	.99630
b_{fr}	in	6.34173	6.62478	.....	4.72301	12.55300
t_{fr}	in	.70464	.36478	.....	.61154	.99630
ℓ_r	in	22.00000	13.20000	13.20000	13.20000	28.28600
d_{st}	in	.....	5.09976	11.00899	7.12346	.....
t_{st}	in	.....	.26263	.46216	.34398	.....
ℓ_{st}	in	.....	13.08884	54.06260	95.64923	.....
GB		.96942	.98113	.97920	.97910	.37400
m , n		1 , 3	1 , 3	1 , 3	1 , 3	.....
PB		.11571	.03112	.02614	.04160	.16700
m_p , n_p		1 , 9	1 , 30	1 , 45	1 , 22	.....
SKB		.....	.02980	.04856	.05585	.....
RB		.34835	.52999	.28312	.14185	.99800
STB		.....	.86934	.91623	.89856	.....
SKY		.99998	1.00000	1.00171	1.00096	1.00000
RY		.97075	.95856	.94783	.94392	.88400
STY		.....	.29883	.33873	.33924	.....

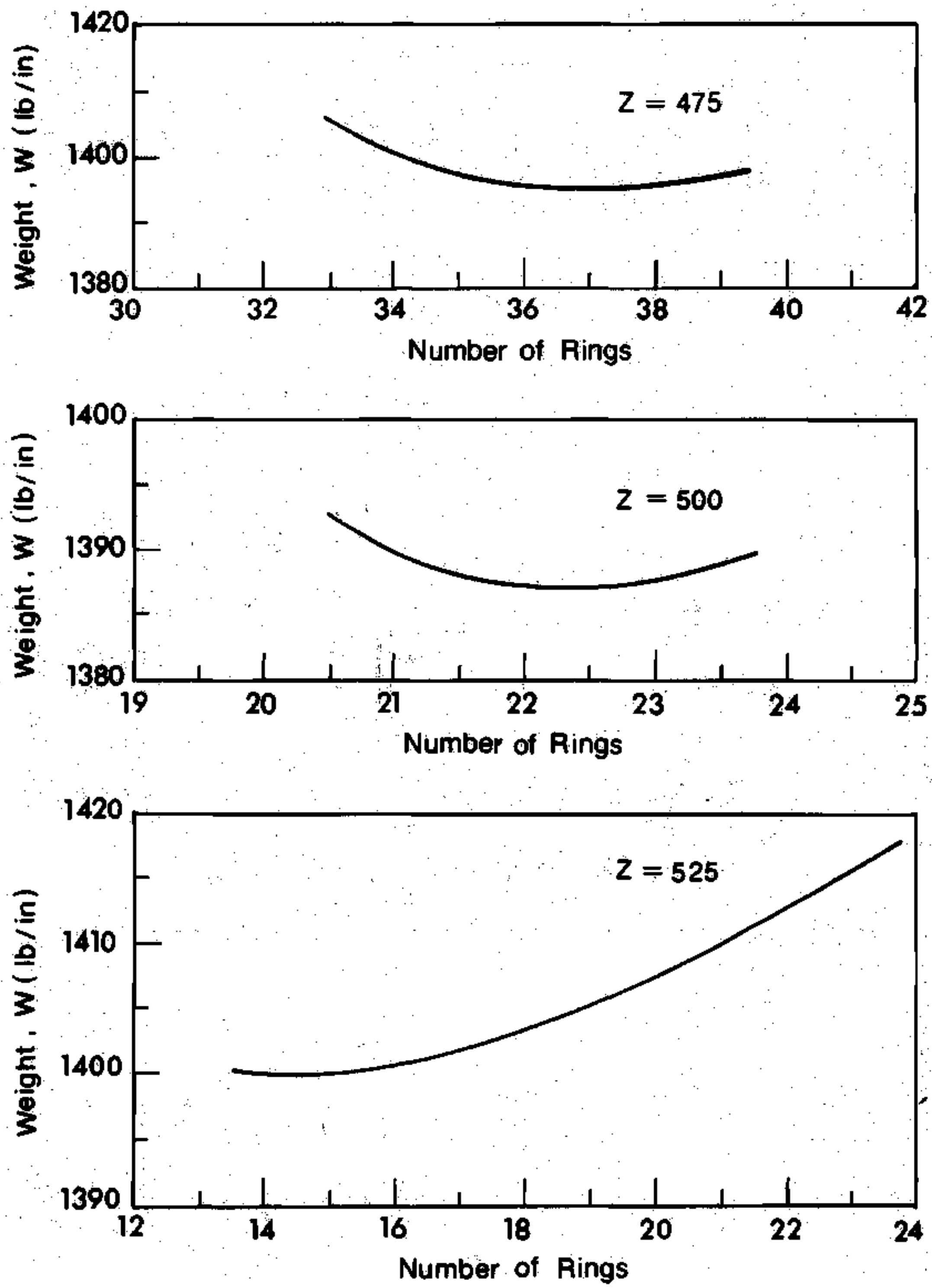

Figure 9. Determination of Optimum Ring Spacing. Internally TR Stiffened Shell, Operating Depth = 3000 feet. Skin Yielding Formulation, Conventional Steel.

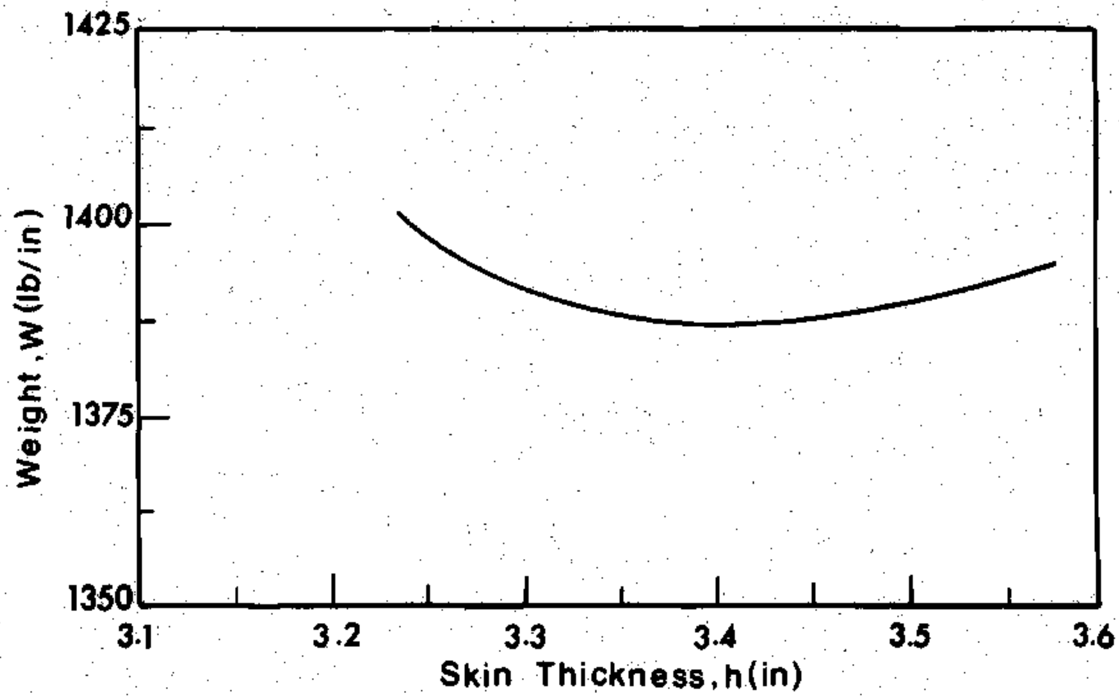

Figure 10. Determination of Optimum Skin Thickness. Internally TR Stiffened Shell, Operating Depth = 3000 feet. Skin Yielding Formulation. Conventional Steel.

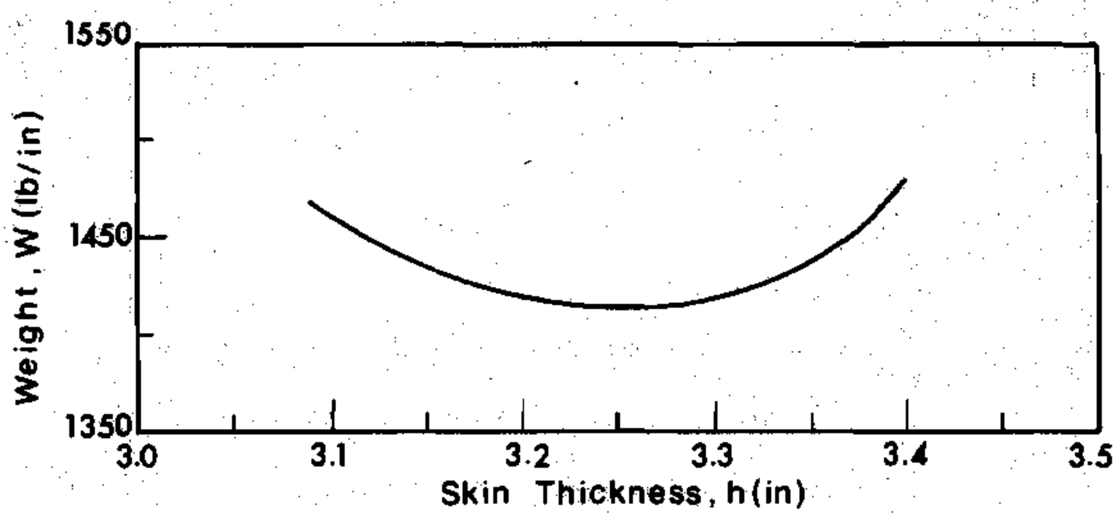

Figure 11. Determination of Optimum Skin Thickness. Internally RR-RS Stiffened Shell, Operating Depth = 3000 feet. General Instability Formulation, Conventional Steel.

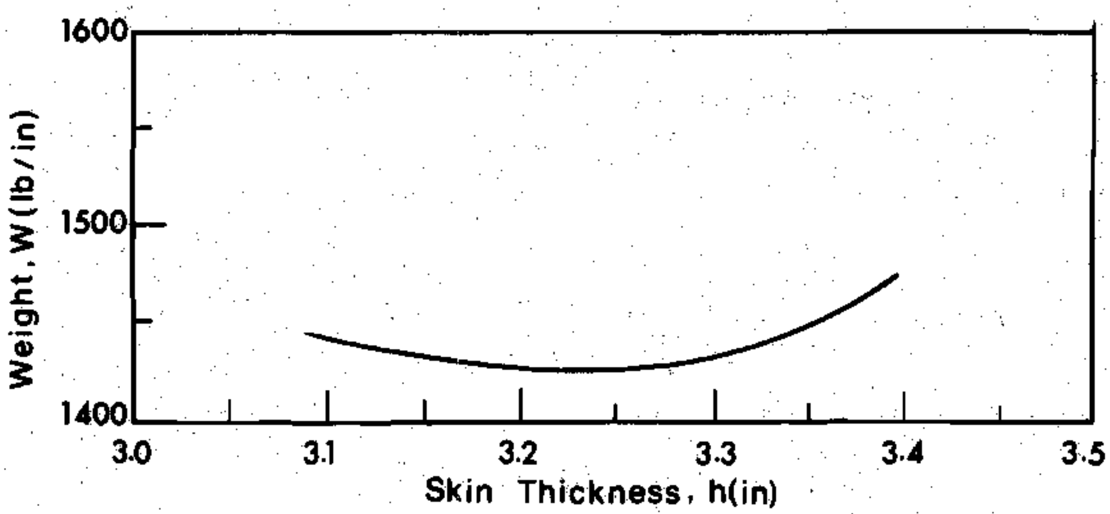

Figure 12. Determination of Optimum Skin Thickness. Internally TR-RS Stiffened Shell, Operating Depth = 3000 feet. General Instability Formulation, Conventional Steel.

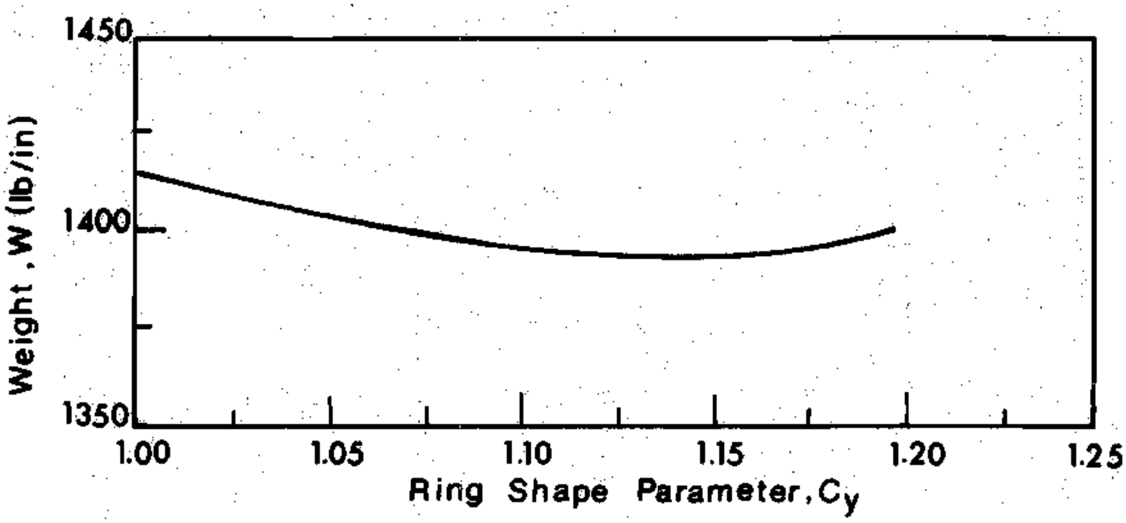

Figure 13. Determination of Optimum Ring Shape Parameter. Internally TR-RS Stiffened Shell. Operating Depth = 3000 feet. General Instability Formulation, Conventional Steel.

present case. It is also observed that the weight obtained in the case of ring-stringer stiffened shell is almost the same as in the case of ring stiffened geometry, the former being higher by about 1 percent. This indicates that the stringers are not effective in the present case. The reason is obvious, because neither general instability nor panel buckling is a critical mode of failure in the present case; therefore, stiffening by stringers does not show any improvement in the weight of the shell.

The design studies made under case 3 demonstrate the significance of using high strength steel. The results are given in the Table 7. The skin thickness obtained in this case is reduced to less than half the thickness required for the same depth when conventional steel is used. The governing critical mode of failure for this case is general instability. The skin yielding does not control the design.

Comparing the least weight obtained in the present case with that obtained for the same operating depth, refer Table 6, one observes that employing high strength steel enables reduction in the weight of 68.6 percent. This is significant weight saving, particularly so, for the submarine hulls, where adequate buoyancy is necessary for large depth operation. One must, however, take into account the cost and fabrication problems before assessing the advantages obtainable by the use of high strength steel.

Since panel buckling and the general instability are important modes of failure in controlling the design, stiffening in the longitudinal direction proves to be helpful in reducing the weight. It is observed during the course of present investigation that stiffening

Table 7. Design Results. Shells Stiffened with Interior Stiffeners.

Operating Depth = 3000 feet σ_y = 120,000 psi

		Formulation Based on General Instability			Reference [18]
		TR	RR-RS	TR-RS	
W	lb/in	821.7	848.3	772.7	979.8
h	in	1.78939	1.41660	1.41660	1.45600
d_{wr}	in	15.49656	17.70750	15.33515	18.30700
t_{wr}	in	.65296	.94687	.41284	1.01710
b_{fr}	in	7.74828	.....	7.66758	12.81500
t_{fr}	in	.65296	.....	.41284	1.01710
ℓ_r	in	24.75000	17.47058	13.20000	21.21400
d_{st}	in	.....	9.91620	8.49960	.....
t_{st}	in	.....	.76968	.57950	.....
ℓ_{st}	in	.....	62.17200	38.85720	.....
GB		.99999	1.00000	1.00000	.51800
m , n		1 , 3	1 , 3	1 , 3	1 , 3
PB		.89259	.21986	.13977	1.00000
m_p , n_p		1 , 14	1 , 69	1 , 85	.....
SKB		.....	.95752	.54165	.....
RB		.98629	.96160	.83695	1.00000
STB		.....	.86322	.93656	.....
SKY		.91628	.91808	.98447	.97700
RY		.79052	.80143	.89137	.63700
STY		.....	.43853	.41253	.....

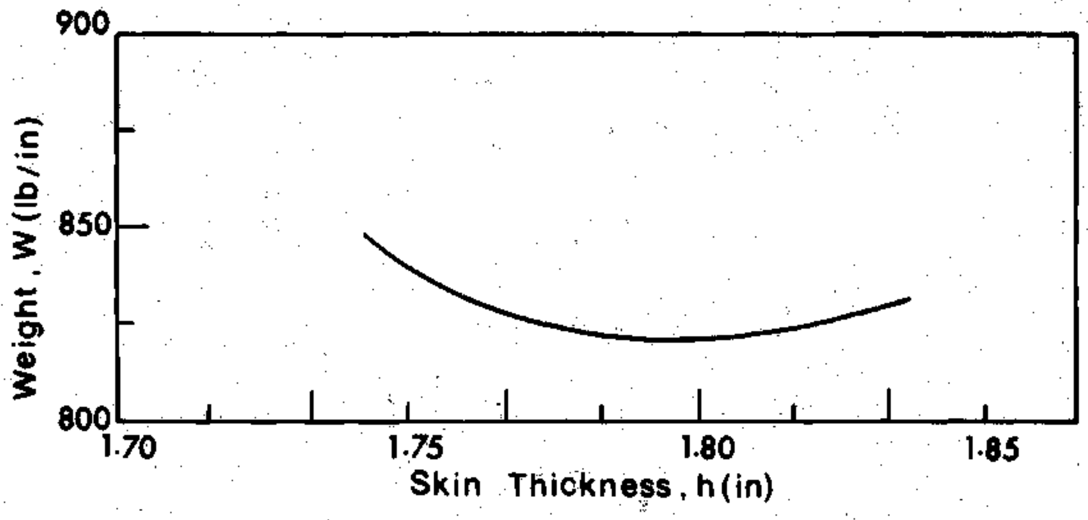

Figure 14. Determination of Optimum Skin Thickness. Internally TR Stiffened Shell, Operating Depth = 3000 feet. General Instability Formulation, High Strength Steel.

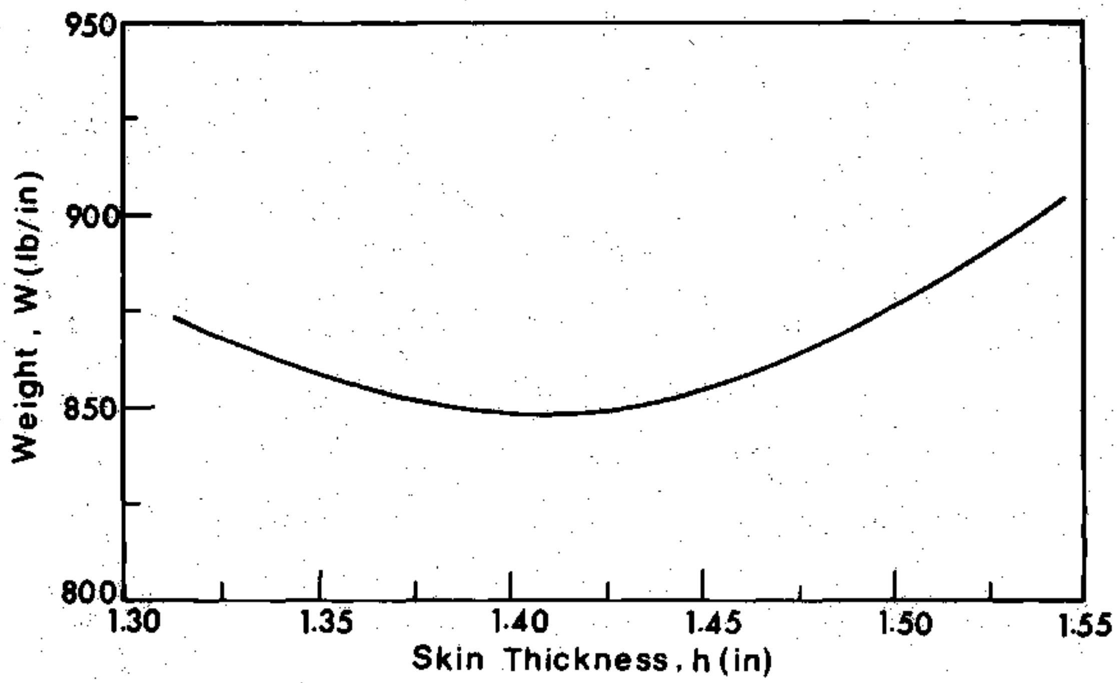

Figure 15. Determination of Optimum Skin Thickness. Internally RR-RS Stiffened Shell, Operating Depth = 3000 feet, General Instability Formulation, High Strength Steel.

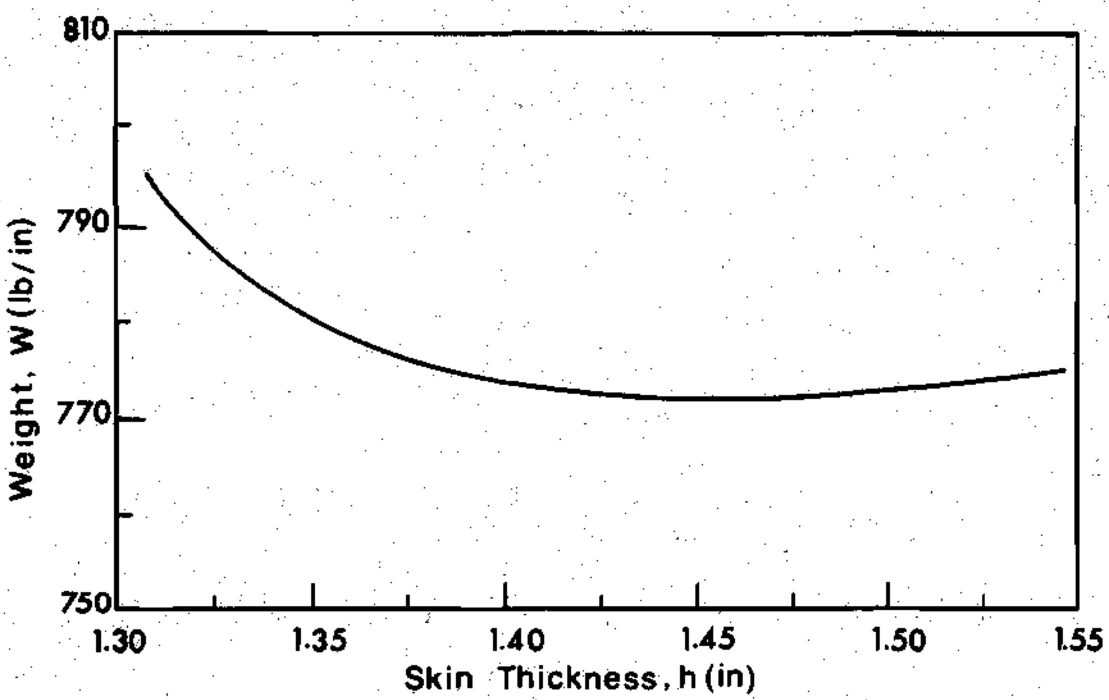

Figure 16. Determination of Optimum Skin Thickness. Internally TR-RS Stiffened Shell, Operating Depth = 3000 feet, General Instability Formulation, High Strength Steel.

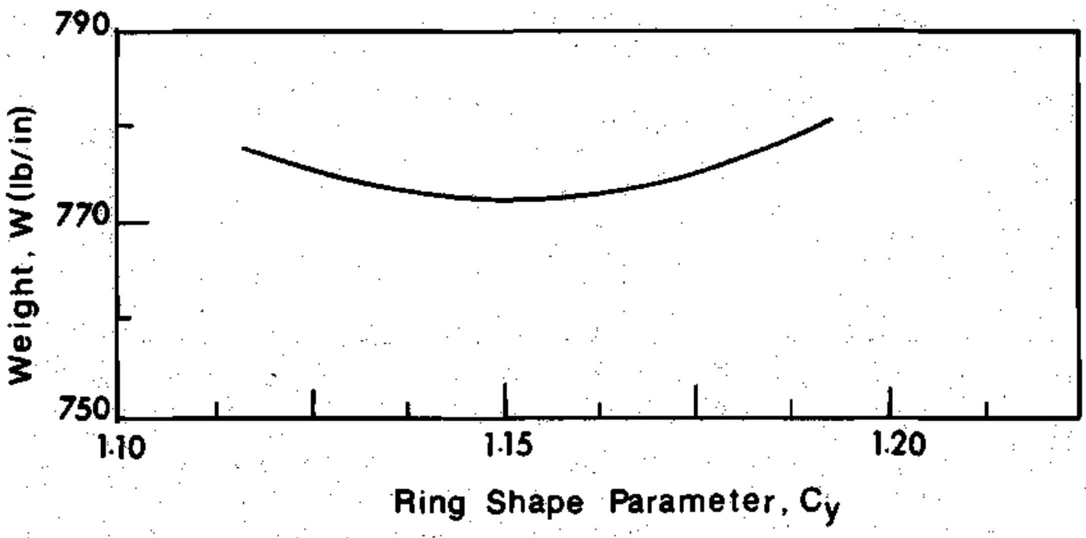

Figure 17. Determination of Optimum Ring Shape Parameter. Internallly TR-RS Stiffened Shell, Operating Depth = 3000 feet, General Instability Formulation, High Strength Steel.

with the stringers changes buckling mode from $m \neq 1$ to $m = 1$. This is also pointed out by Singer and Baruch [31] in connection with the effectiveness of the longitudinal stiffening for shells subject to hydrostatic pressure.

The results in the Table 7 indicate that the least weight is obtained when the shell is stiffened with T-rings and rectangular stringers. This weight shows an improvement of 7 percent over the weight of the shell when it is stiffened with rings only. Instead of T-rings if rectangular rings are used along with rectangular stringers, the weight obtained is 10 percent higher.

The results of Pappas and Allentuch are given in Table 7 for comparison purposes. On comparing their results with the present results, the ring stiffened geometry shows an improvement of 19.2 percent in weight in the present case. On the other hand the ring-stringer stiffened geometry shows an improvement in weight of 26.8 percent.

The results of using exterior stiffeners are given in the Table 8 and Figure 18. On comparing these results with the corresponding cases under Case 3 where interior stiffeners are used, it is noted that for ring stiffened shells interior stiffening yields 2 percent better weight, whereas T-ring and stringer stiffened shell shows an improvement of about 9 percent. The weight difference in the case of shells stiffened with rectangular rings and stringers is about 1 percent; the internal stiffening being better. Apart from the weight considerations, the location of stiffeners may depend on practical considerations also. If no such restriction is imposed, then the

Table 8. Design Results. Shell Stiffened with Exterior Stiffeners.

General Instability Formulation

Operating Depth = 3000 feet σ_y = 120,000 psi

		TR	RR-RS	TR-RS
W	lb/in	833.9	858.5	840.8
h	in	1.61897	1.41660	1.41660
d_{wr}	in	15.70319	19.83240	16.19392
t_{wr}	in	.67049	.72119	.38993
b_{fr}	in	7.85159	.....	8.09696
t_{fr}	in	.67049	.....	.38993
ℓ_r	in	21.21428	18.00000	13.20000
d_{st}	in	.....	14.16600	8.49960
t_{st}	in	.....	.83806	.55339
ℓ_{st}	in	.....	51.81000	19.12984
GB		1.00000	1.00000	1.00000
m , n		1 , 3	1 , 3	1 , 3
PB		.91936	.12981	.10462
m_p , n_p		1 , 15	1 , 95	1 ,102
SKB		.....	.96905	.44050
RB		.97826	.94766	.94523
STB		.....	.95861	.95485
SKY		.96284	.95425	.97749
RY		.80513	.86627	.89805
STY		.....	.39545	.38354

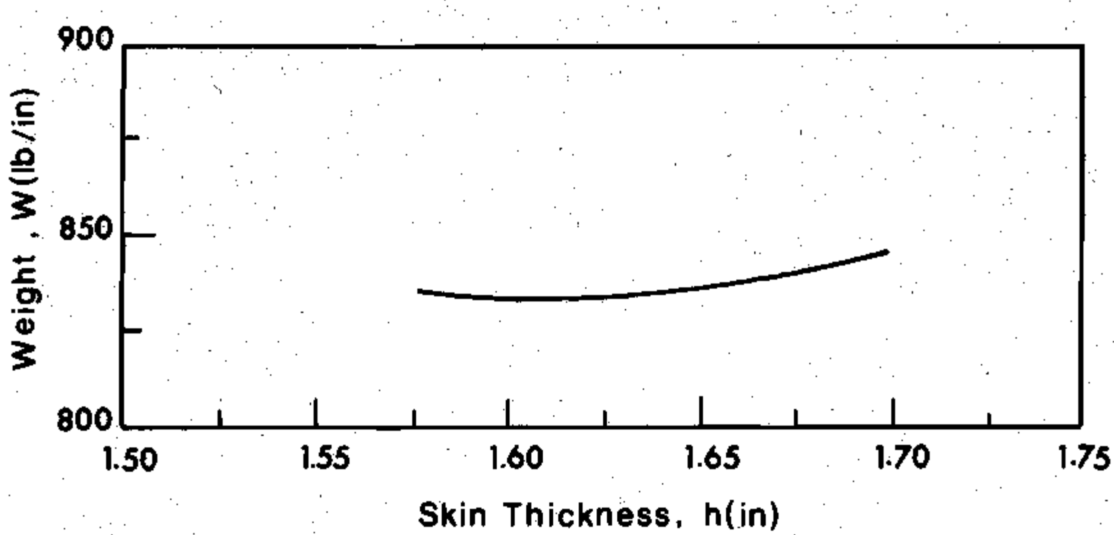

Figure 18. Determination of Optimum Skin Thickness. Externally TR-RS Stiffened Shell, Operating Depth = 3000 feet, General Instability Formulation, High Strength Steel.

internally stiffened shell, for the geometries considered in this study, is better.

The effect of varying L/R ratio, or in other words the effect of locating the heavy bulkheads on the weight of the shell has been studied in Case 5. The ratio is varied from one through five and the results are given in Tables 9 and 10 along with Figure 19-27. The results show that as the L/R ratio increases the weight per unit length also increases. However, it may be noted that this weight does not include the weight of the bulkheads. Therefore, for proper estimate of the weight, one must account for the weight of bulkheads, to arrive at the best L/R ratio which yields the least weight.
Certain functional requirement might override this phase of the study. That is, if L/R ratio is prespecified due to any practical considerations, it is not necessary to undertake this study. On the other hand if no such limitations are imposed, the results of this study help in arriving at the best ratio.

The results of this study indicate that for $L/R = 1$, the weight of the ring stiffened shell is 1.3 percent higher than that of the ring-stringer stiffened shell. This difference increases with the increasing L/R ratio. For $L/R = 5$, the difference is about eight percent. This indicates that stringers are more effective for higher L/R ratios. The results of Tables 9 and 10 indicate further, that the depth of the rings required for the minimum weight increases with increasing L/R ratio. This is another controlling factor in deciding for the most suitable L/R ratio.

The minimum weight design results for the case of the shell

Table 9. Design Results. Influence of Varying L/R Ratio on Minimum Weight. Shell Stiffened with Interior TR Stiffeners.

General Instability Formulation

Operating Depth = 3000 feet σ_y = 120,000 psi

L/R →		1	2	3	4	5
W	lb/in	739.5	784.8	821.7	885.5	930.9
h	in	1.88880	1.77769	1.78939	1.75193	1.74889
d_{wr}	in	10.30522	14.00970	15.49656	18.51003	19.68956
t_{wr}	in	.48927	.61032	.65296	.76315	.83851
b_{fr}	in	5.15261	7.00485	7.74828	9.25502	9.84478
t_{fr}	in	.48927	.61032	.65296	.76315	.83851
ℓ_r	in	28.28571	24.75000	24.75000	24.75000	24.75000
GB		.97105	.99983	.99999	.99989	1.00000
m , n		5 , 6	1 , 3	1 , 3	1 , 2	1 , 2
PB		.94774	.90915	.89259	.94710	.95171
m_p , n_p		1 , 14	1 , 14	1 , 14	1 , 14	1 , 15
RB		.87198	.96736	.98629	.94444	.83844
SKY		.94545	.93793	.91628	.90585	.89414
RY		.88734	.82881	.79052	.72475	.68647

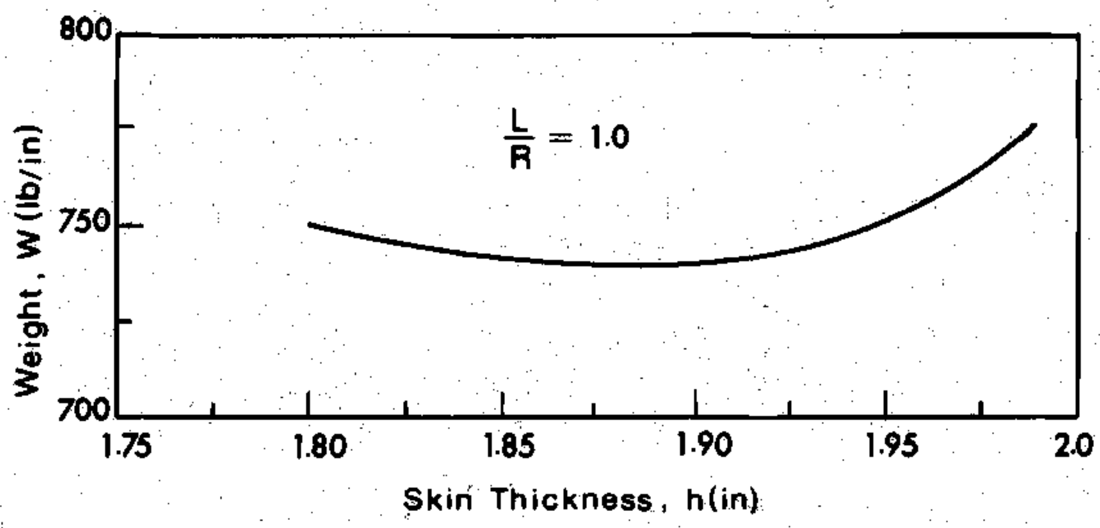

Figure 19. Determination of Optimum Skin Thickness. Internally TR Stiffened Shell, Operating Depth = 3000 feet, General Instability Formulation, High Strength Steel.

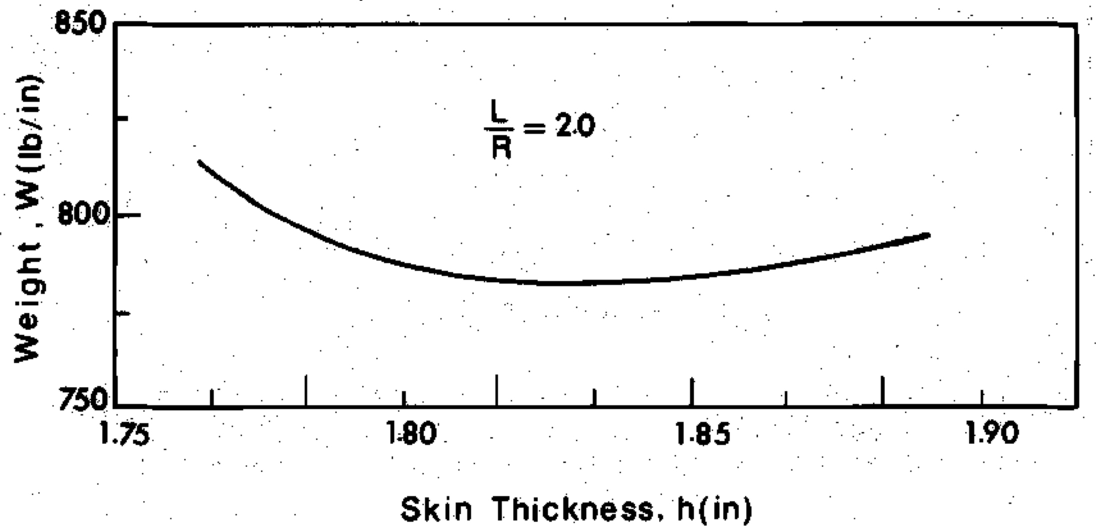

Figure 20. Determination of Optimum Skin Thickness. Internally TR Stiffened Shell, Operating Depth = 3000 feet, General Instability Formulation, High Strength Steel.

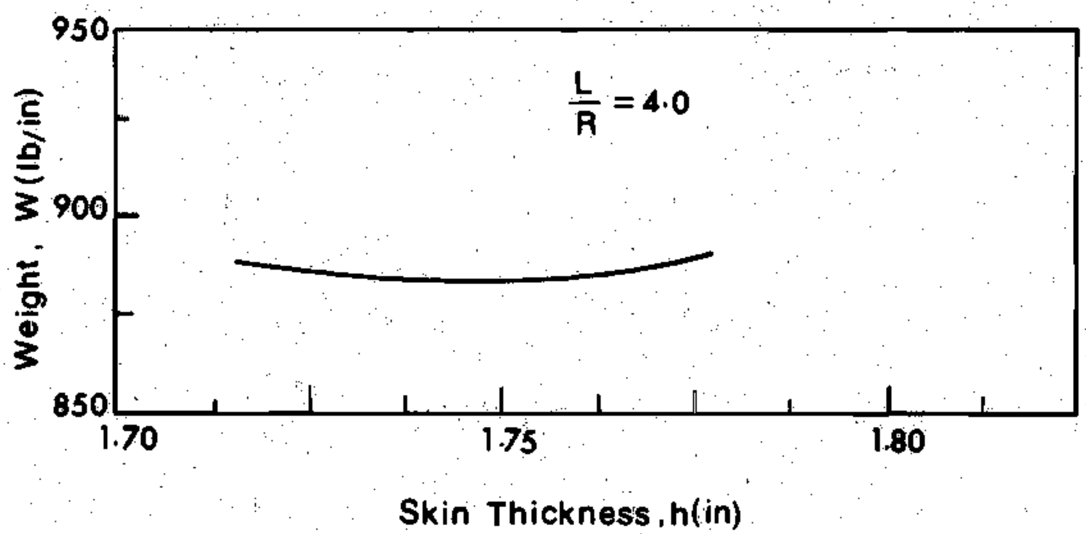

Figure 21. Determination of Optimum Skin Thickness Internally TR Stiffened Shell, Operating Depth = 3000 feet, General Instability Formulation, High Strength Steel.

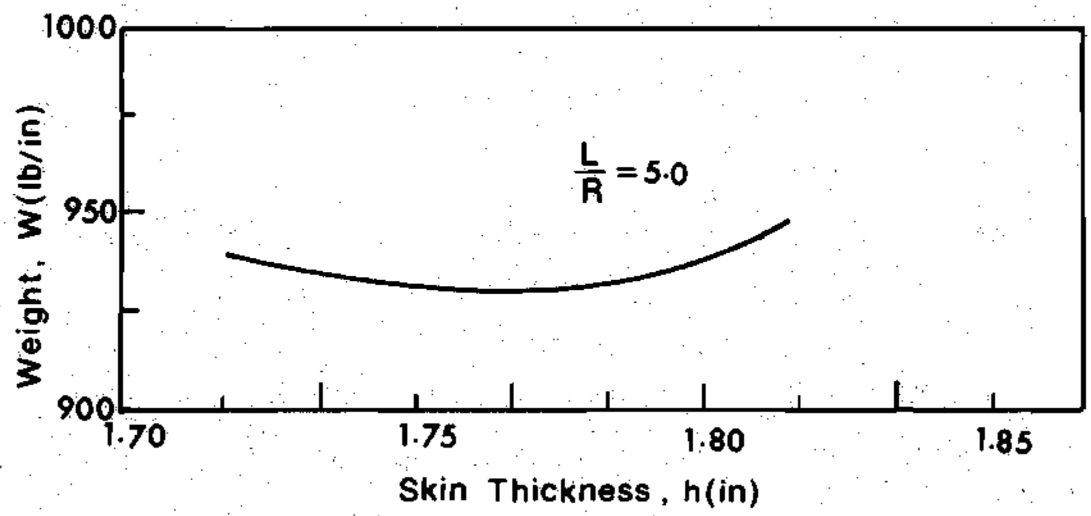

Figure 22. Determination of Optimum Skin Thickness Internally TR Stiffened Shell, Operating Depth = 3000 feet, General Instability Formulation, High Strength Steel.

Table 10. Design Results. Influence of Varying L/R Ratio on Minimum Weight. Shell Stiffened with Interior TR-RS.

General Instability Formulation

Operating Depth = 3000 feet σ_y = 120,000 psi

L/R →		1	2	3	4	5
W	lb/in	730.0	753.5	772.7	815.9	863.1
h	in	1.71707	1.51102	1.41660	1.4056	1.52321
d_{wr}	in	11.89621	15.04896	15.33515	19.47657	21.76579
t_{wr}	in	.34436	.37235	.41284	.53732	.53775
b_{fr}	in	5.94810	7.52434	7.66758	9.73828	10.88289
t_{fr}	in	.34436	.37235	.41284	.53732	.53775
ℓ_r	in	19.80000	15.84000	13.20000	16.50000	20.62500
d_{st}	in	6.86828	7.55510	8.49960	7.02800	9.13926
t_{st}	in	.50196	.53852	.57950	.58595	.69810
ℓ_{st}	in	34.54000	25.37632	38.85720	73.14352	40.11096
GB		1.00000	1.00000	1.00000	1.00000	1.00000
m , n		1 , 5	1 , 3	1 , 3	1 , 2	1 , 2
PB		.28197	.17782	.13977	.35405	.25759
m_p , n_p		1 , 41	1 , 68	1 , 85	1 , 49	1 , 57
SKB		.66709	.57615	.54615	.91155	.97132
RB		.82455	.85110	.83695	.95647	.98069
STB		.88979	.94909	.93656	.96900	.94344
SKY		.97634	.99741	.98447	.93240	.89679
RY		.93570	.93325	.89137	.80445	.80302
STY		.29245	.35474	.41253	.46098	.38601

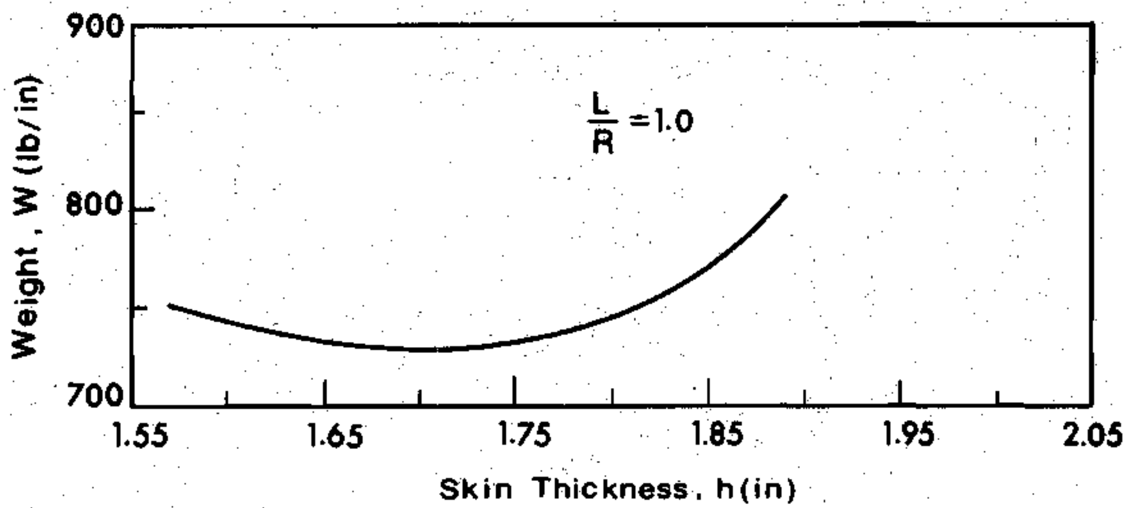

Figure 23. Determination of Optimum Skin Thickness. Internally TR-RS Stiffened Shell, Operating Depth = 3000 feet, General Instability Formulation, High Strength Steel.

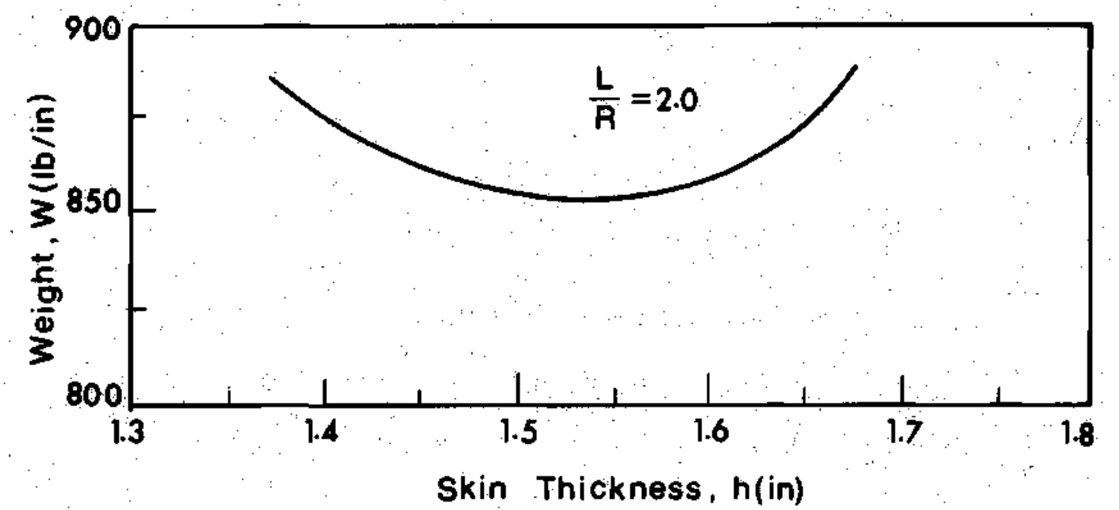

Figure 24. Determination of Optimum Skin Thickness. Internally TR-RS Stiffened Shell, Operating Depth = 3000 feet, General Instability Formulation, High Strength Steel.

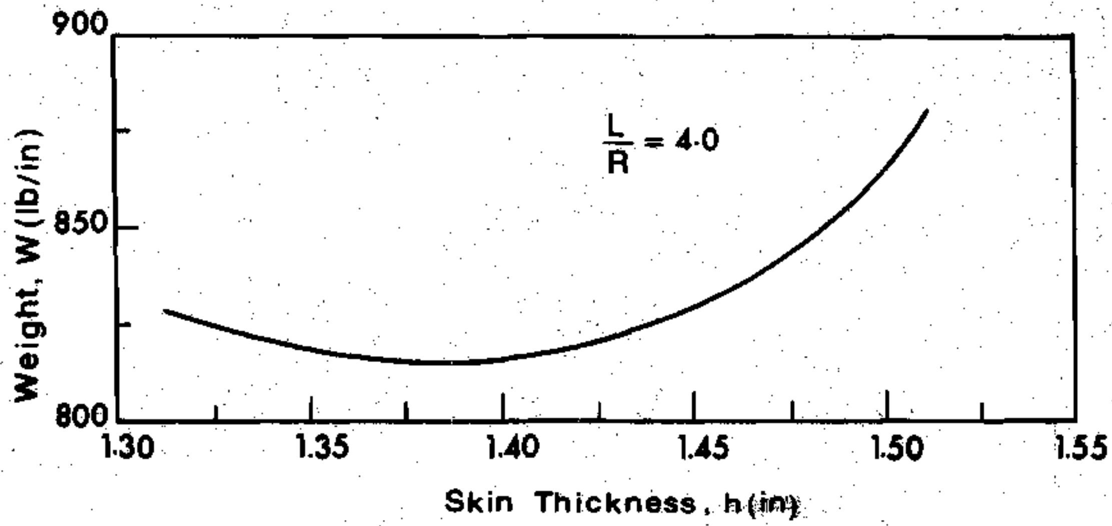

Figure 25. Determination of Optimum Skin Thickness. Internally TR-RS Stiffened Shell, Operating Depth = 3000 feet, General Instability Formulation, High Strength Steel.

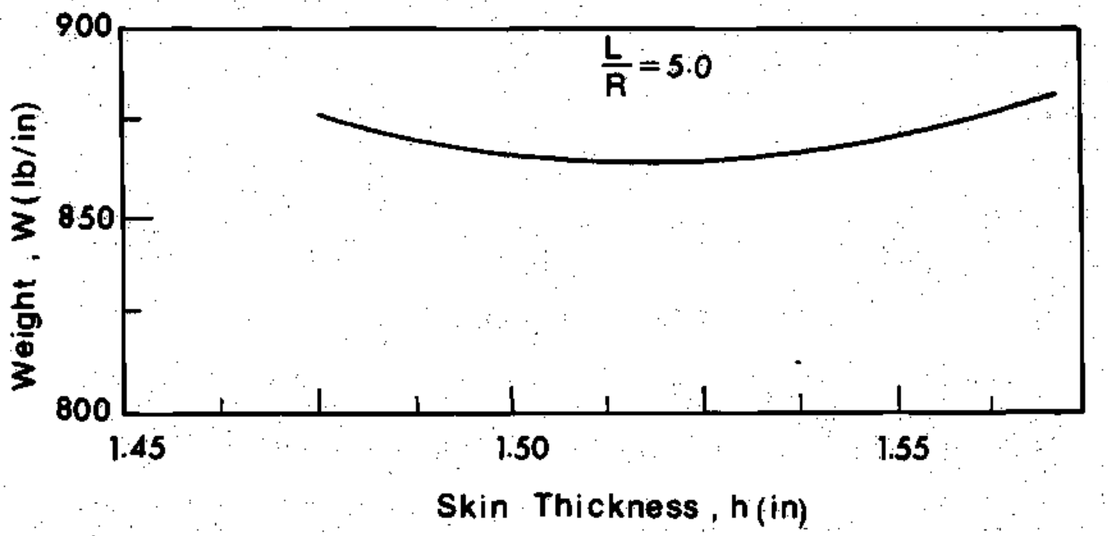

Figure 26. Determination of Optimum Skin Thickness. Internally TR-RS Stiffened Shell, Operating Depth = 3000 feet, General Instability Formulation, High Strength Steel.

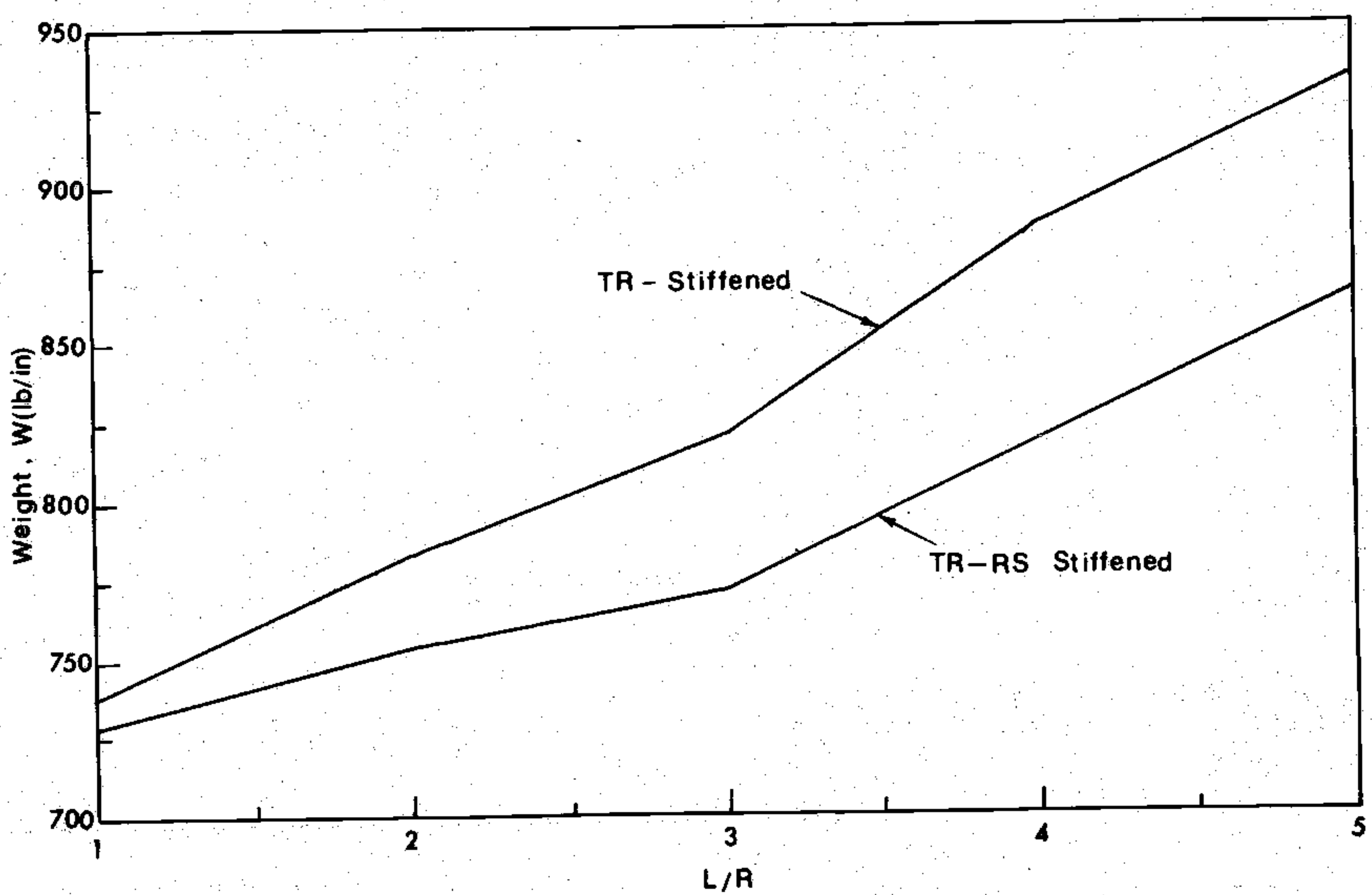

Figure 27. Influence of L/R on Minimum Weight Design. Internally TR, and TR-RS Stiffened Shell. Operating Depth = 3000 feet. General Instability Formulation, High Strength Steel.

subjected to the combined load (small axial compression combined with predominant hydrostatic pressure) are given in the Table 11 and Figure 28. The weight of the shell is higher than the corresponding case under hydrostatic pressure. The formulation is based on general instability. The skin thickness needed for this case is higher than the thickness needed for the shell with only hydrostatic pressure. The stringers are undoubtedly important for the present case. The design charts reveal that the weight of the shell is reduced significantly, when the stringers are of such proportion as to change the buckling mode from $m \neq 1$ to $m = 1$.

A general type of observation that is made during the present investigation refers to the determination of the optimum thickness of the skin or value of Z. The curves Z(or h) versus W in all the cases are relatively flat. This implies that there is sufficiently large range of skin thicknesses that give almost the same weight. This suggests that it is not necessary to determine very precisely the value of skin thickness which corresponds to minimum weight. The value of skin thickness in the neighbourhood of the minimum exhibited by the curve may be taken as optimum skin thickness.

The other observation is in connection with selecting the type of the formulation to be used for the objective function. The studies reveal that the selection primarily depends on the type of steel or material that is used for construction and the operating or design depth (or hydrostatic pressure, whichever is specified). For 1000 feet operating depth, it is observed that the two formulations (general instability and skin yielding), yield weights that differ by about

Table 11. Design Results for the Shell Subject to Combined Load. ($\bar{N} = .2qR$)
Shell Stiffened with Interior, TR-RS
General Instability Formulation.

Operating Depth 3000 feet		σ_y 120,000 psi
W	lb/in	832.0
h	in	1.69990
d_{wr}	in	16.92980
t_{wr}	in	.37581
b_{fr}	in	8.46490
t_{fr}	in	.37581
ℓ_r	in	18.00000
d_{st}	in	8.49950
t_{st}	in	.74893
ℓ_{st}	in	32.72210
GB		1.00000
m , n		1 , 3
PB		.15629
m_p , n_p		1 , 61
SKB		.68697
RB		.90759
STB		.94458
SKY		.97137
RY		.80101
STY		.53632

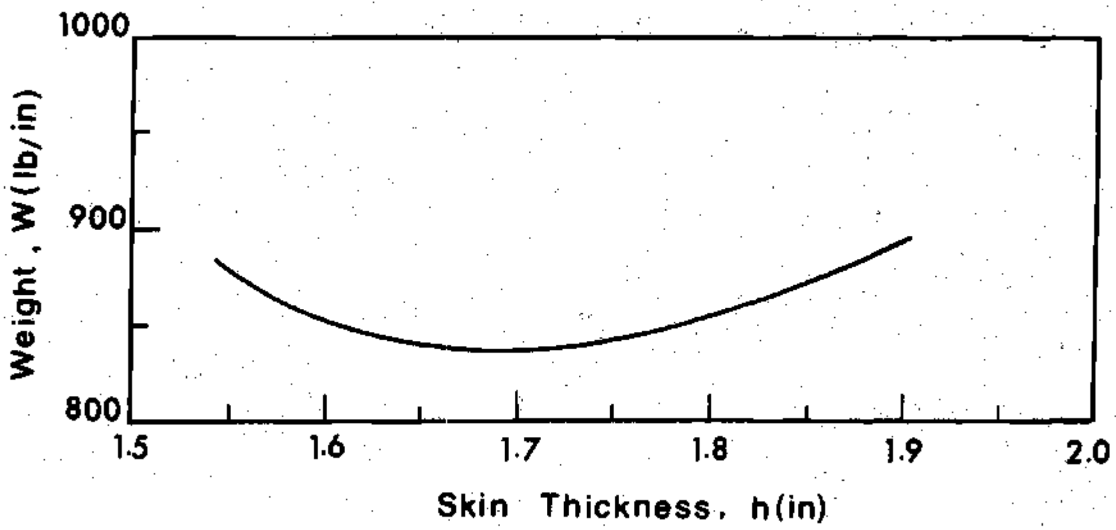

Figure 28. Determination of Optimum Skin Thickness. Axial Compression Combined with Hydrostatic Pressures. Internally TR-RS Stiffened Shell, High Strength Steel. General Instability Formulation. Operating Depth = 3000 feet, $\tilde{N}$ = .2qR.

four percent. If the shell is designed for higher operating depth, skin yielding takes over and the formulation must be based on skin yielding. This is true if conventional steel is used. One can, therefore, say that general instability formulation may be used for the operating depths of lower than 1000 feet and skin yielding formulation for higher depths.

If, on the other hand, high strength steel is used, general instability governs the design up to sufficiently high operating depths. From this study it is not possible to say what that upper limit is.

The plot of h is W and C_y versus W together with the choice of selecting from different design configurations corresponding to the same minimum weight provide important information to carry out trade-off studies. For example, if the minimum weight design configuration requires skin thickness which is difficult or expensive to manufacture, one can study and investigate easily the alternative neighbouring design configuration and make a compromise between the weight penalty and the cost.

CHAPTER V

CONCLUSIONS AND SUGGESTIONS

Conclusions

The main conclusions of the present investigation are

1. The objective function formulated on the basis of an active mode of failure (skin yielding or general instability in the present case) and accomplishing solution in two phases effectively leads to the minimum weight design. In addition, it enables a designer for carrying out important trade-off studies to arrive at practical minimum weight configurations.

2. On the basis of phase I, one can easily assess the need for stiffening in both directions for different shapes of stiffeners.

3. The present approach gives a designer full control over the design variables and it enables him to introduce needed changes or avoid interaction of failure modes by paying the least penalty in weight.

4. The minimum weight design is not unique. This implies that for a given level of the specified parameters the design variables can be adjusted so as to give several acceptable designs for the same weight.

5. The studies indicate that T-rings are most effective among all the shapes of stiffeners investigated. The ratios of flange width to web depth and flange thickness to web thickness does not appreciably affect the weight.

6. The curve for determining optimum skin thickness is relatively flat. Therefore, very precise determination of Z (or h) is not necessary for minimum weight design. This information is an important asset for the designer, as it enables him a wide choice in selecting skin thickness.

7. The use of high strength steel enables appreciable weight savings; but this aspect should be studied along with the cost and fabrication problems.

8. In certain cases stringers do help in saving weight. A designer must evaluate the cost of providing these stiffeners against the weight saving.

9. The weight of the shell increases with increasing L/R ratio. However, no account of the weight of heavy bulk heads is made in these computations. This study is more of qualitative than quantitative nature.

10. The interior stiffening proves to be better for the geometries considered in the present study.

Suggestions

The following suggestions are made for the future work

1. Minimum weight design of shells of shapes other than circular cylindrical.

2. Combined load case needs further study to investigate the entire pattern of interaction of these two loads viz. uniform hydrostatic pressure and axial compression.

3. Minimum weight design of stiffened cylindrical panels is

of significant importance.

4. Most important extension of the present work is to include cost factor.

APPENDICES

APPENDIX A

ANALYSIS OF STIFFENED CIRCULAR CYLINDRICAL SHELLS

In this appendix all the equations needed to analyze the stiffened cylinders under hydrostatic pressure and axial compression are presented. These include the general instability analysis of the cylinder as well as the buckling, stress and yield analyses of the skin and stiffeners.

The expression for the general instability failure mode of the stiffened cylinder is derived using Donnell's equations and smearing technique. An investigation for determining the accuracy obtainable from the Donnell's equations was carried out prior to undertaking the present work. The results of this investigation for uniform thin cylinders under lateral loading are given in Appendix D. It is indicated that the values obtainable by the Donnell's equations are within the acceptable engineering tolerances, especially in the practical range of R/h and L/R ratios. The comparison was made with the results obtained by Budiansky's equations.

The main assumptions for the stability analysis of the stiffened cylinders are

1. The shell is thin
2. The deflections are small
3. Rotations about normal are much smaller than the rotations about in-plane axes.
4. Normals to the reference surface before deformation remain

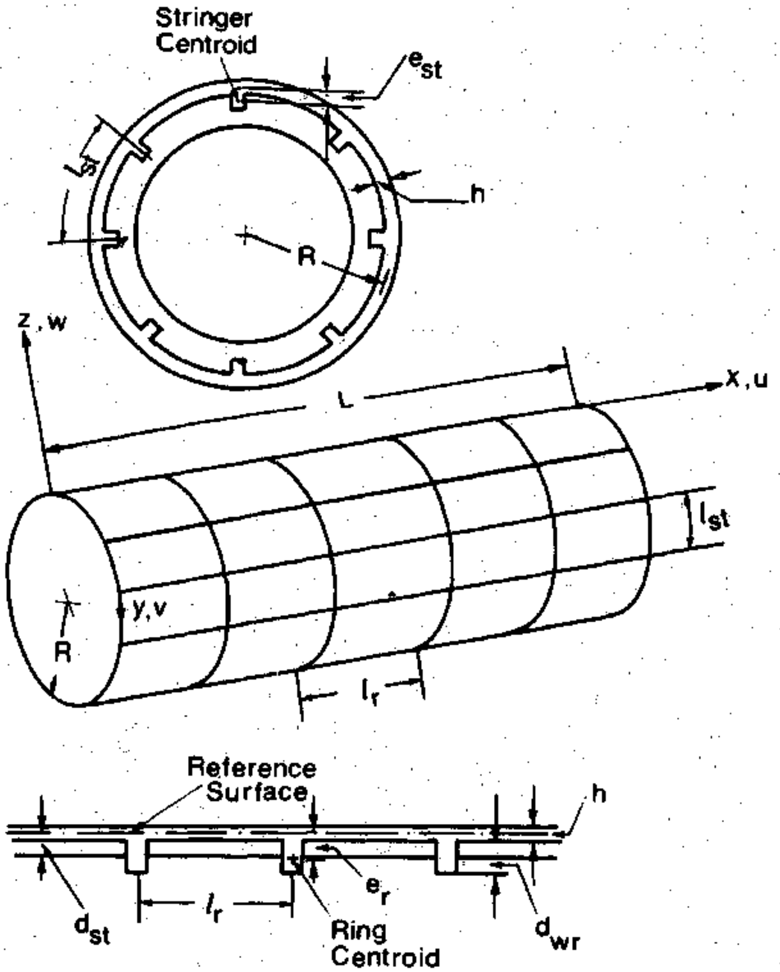

Figure A1. Geometry of Shell.

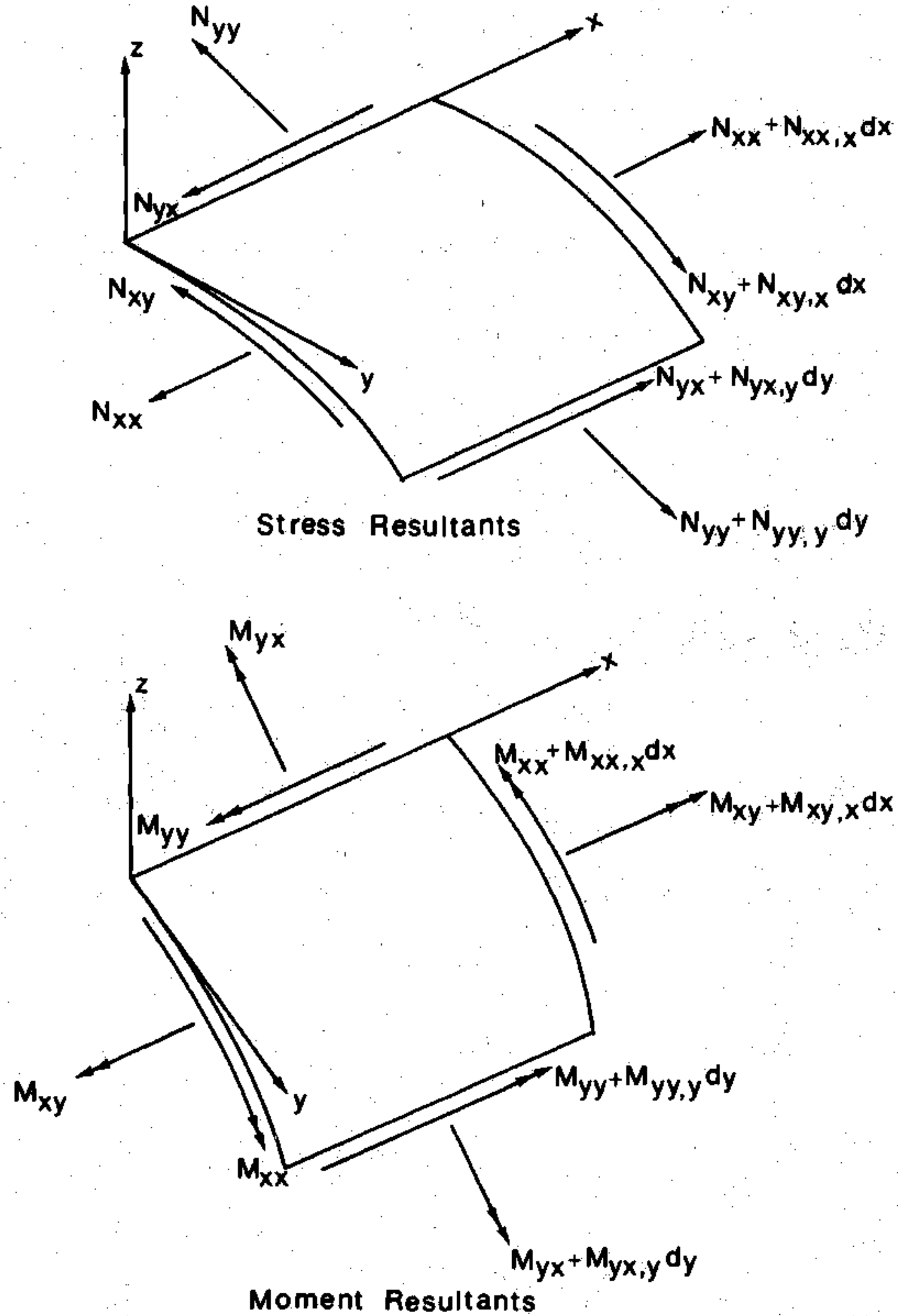

Figure A2. Sign Convention and Force Resultants.

normal to the reference surface after deformation, and they are inextensional.

5. The stiffeners are distributed or 'smeared' over the whole surface of the shell.

6. The stiffeners are along the directions of principal curvatures.

7. The connections of stiffeners to the skin are monolithic.

8. The stiffeners do not transmit shear. The shear is carried entirely by skin.

9. The stiffeners are torsionally weak.

Strain-Displacement Relations

The midsurface of the skin is taken as the reference surface. The coordinate system and sign convention are shown in Figure A1 and A2 respectively. Let u^1, v^1, and w^1 be the additional displacements in x, y and z directions respectively, required to bring the membrane state to the classical buckling state. The strain, curvature changes and rotations are given by

$$\epsilon_x = \epsilon_{xx} + z\kappa_{xx}$$

$$\epsilon_y = \epsilon_{yy} + z\kappa_{yy}$$

$$\gamma = \gamma_{xy} + 2z\kappa_{xy}$$

$$\epsilon_{xx} = \frac{\partial u^1}{\partial x}$$

$$\epsilon_{yy} = \frac{\partial v^1}{\partial x} + \frac{w^1}{R}$$

$$\gamma_{xy} = \frac{\partial u^1}{\partial y} + \frac{\partial v^1}{\partial x}$$

$$\kappa_{xx} = - \frac{\partial^2 w^1}{\partial x^2}$$

$$\kappa_{yy} = - \frac{\partial^2 w^1}{\partial y^2}$$

$$\kappa_{xy} = - \frac{\partial^2 w^1}{\partial x \partial y}$$

$$\varphi_x = - \frac{\partial w^1}{\partial x}$$

$$\varphi_y = - \frac{\partial w^1}{\partial y} \qquad \text{(A1)}$$

<u>Stress-Strain Relations</u>

The stress-strain relations, based on the assumption that skin is in biaxial state of stress, are:

$$\sigma_{xxsk} = \frac{E}{1-\nu^2} (\epsilon_{xx} + \nu \epsilon_{yy})$$

$$\sigma_{yysk} = \frac{E}{1-\nu^2} (\epsilon_{yy} + \nu \epsilon_{xx})$$

$$\tau_{xysk} = G \gamma_{xy} \qquad \text{(A2)}$$

The stiffeners are assumed to be in uniaxial state of stress, so that the stress-strain relations for longitudinal and circumferential stiffeners are:

$$\sigma_{xxst} = E_{st}\,\epsilon_{xx}$$

$$\sigma_{yyr} = E_r\,\epsilon_{yy} \tag{A2b}$$

Stress and Moment Resultants

The stress and moment resultants per unit length are obtained by performing integration of stresses over the thickness of the skin and then adding to these the corresponding stress and moment resultants per unit length in the stiffeners. Based on the assumption that the stiffeners are distributed over the whole surface, the stress and moment resultants per unit length in the stiffeners are obtained by dividing the resultant stress and moment by the stiffener spacing.

The stress resultants are:

$$N_{xx} = \int_{-h/2}^{h/2} \sigma_{xxsk}\,dz + \frac{1}{\ell_{st}} \int_{A_{st}} \sigma_{xxst}\,dA_{st}$$

$$N_{yy} = \int_{-h/2}^{h/2} \sigma_{yysk}\,dz + \frac{1}{\ell_r} \int_{A_r} \sigma_{yyr}\,dA_r$$

$$N_{xy} = \int_{-h/2}^{h/2} \tau_{xysk}\,dz$$

and moment resultants are

$$M_{xx} = \int_{-h/2}^{h/2} z\ \sigma_{xxsk}\ dz + \frac{1}{\ell_{st}} \int_{A_{st}} z\ \sigma_{xxst}\ dA_{st}$$

$$M_{yy} = \int_{-h/2}^{h/2} z\ \sigma_{yysk}\ dz + \frac{1}{\ell_r} \int_{A_r} z\ \sigma_{yyr}\ dA_r$$

$$M_{xy} = \int_{-h/2}^{h/2} z\ \tau_{xysk}\ dz + \frac{(GJ)_{st}}{\ell_{st}} \kappa_{xy}$$

$$M_{yx} = \int_{-h/2}^{h/2} z\ \tau_{xysk}\ dz + \frac{(GJ)_r}{\ell_r} \kappa_{yx} \tag{A3}$$

Substituting Equations (A1) and (A2) in Equation (A3) and performing appropriate integrations, one gets

$$N_{xx} = \frac{Eh}{(1-\nu^2)} (\epsilon_{xx} + \nu\ \epsilon_{yy}) + \frac{E_{st}\ A_{st}}{\ell_{st}} \epsilon_{xx} + \frac{E_{st}\ A_{st}}{\ell_{st}} e_{st}\ \kappa_{xx}$$

$$N_{yy} = \frac{Eh}{(1-\nu^2)} (\nu\ \epsilon_{xx} + \epsilon_{yy}) + \frac{E_r\ A_r}{\ell_r} \epsilon_{yy} + \frac{E_r\ A_r}{\ell_r} e_r\ \kappa_{yy}$$

$$N_{xy} = G\ \gamma_{xy}$$

and

$$M_{xx} = \frac{Eh^3}{12(1-\nu^2)} (\kappa_{xx} + \nu\ \kappa_{yy}) + \frac{E_{st}\ A_{st}}{\ell_{st}} e_{st}\ \epsilon_{xx}$$

$$+ \frac{E_{st}}{\ell_{st}} (I_{stc} + e_{st}^2 A_{st}) \varkappa_{xx}$$

$$M_{yy} = \frac{Eh^3}{12(1-\nu^2)} (\nu \varkappa_{xx} + \varkappa_{yy}) + \frac{E_r A_r}{\ell_r} e_r \epsilon_{yy}$$

$$+ \frac{E_r}{\ell_r} (I_{rc} + e_r^2 A_r) \varkappa_{yy}$$

$$M_{xy} = \frac{Eh^3}{12(1+\nu)} \varkappa_{xy} + \frac{(GJ)_{st}}{\ell_{st}} \varkappa_{xy}$$

$$M_{yx} = \frac{Eh^3}{12(1+\nu)} \varkappa_{yx} + \frac{(GJ)_r}{\ell_r} \varkappa_{yx} \qquad \text{(A4)}$$

since stiffeners are assumed to be torsionally weak

$$M_{xy} = M_{yx} = \frac{Eh^3}{12(1+\nu)} \varkappa_{xy}$$

A set of new parameters, described below, are introduced

$$E_{xxp} = E_{yyp} = Eh/(1-\nu^2)$$

$$E_{xxst} = E_{st} A_{st}/\ell_{st}$$

$$E_{yyr} = E_r A_r/\ell_r$$

$$G = Eh/2(1+\nu)$$

$$E_{xx} = E_{xxp} + E_{xxst}$$

$$E_{yy} = E_{yyp} + E_{yyr}$$

$$D_{xxp} = D_{yyp} = Eh^3/12(1-\nu^2)$$

$$D_{xxst} = E_{st}\ I_{stc}/\ell_{st}$$

$$D_{yyr} = E_r\ I_{rc}/\ell_r$$

$$D_{xy} = (1-\nu)\ D_{xxp}$$

$$D_{xx} = D_{xxp} + D_{xxst}$$

$$D_{yy} = D_{yyp} + D_{yyr} \tag{A5}$$

Substituting these new parameters in Equation (A4), the stress and moment resultants relations become

$$N_{xx} = E_{xx}\ \epsilon_{xx} + \nu\ E_{xxp}\ \epsilon_{yy} + e_{st}\ E_{xxst}\ \kappa_{xx}$$

$$N_{yy} = \nu\ E_{yyp}\ \epsilon_{xx} + E_{yy}\ \epsilon_{yy} + e_r\ E_{yyr}\ \kappa_{yy}$$

$$N_{xy} = G\ \gamma_{xy}$$

$$M_{xx} = (D_{xx} + e_{st}^2\ E_{xxst})\kappa_{xx} + \nu\ D_{xxp}\ \kappa_{yy} + e_{st}\ E_{xxst}\ \epsilon_{xx}$$

$$M_{yy} = \nu\ D_{yyp}\ \kappa_{xx} + (D_{yy} + e_r^2\ E_{yyr})\kappa_{yy} + e_r\ E_{yyr}\ \epsilon_{yy}$$

$$M_{xy} = D_{xy}\ \kappa_{xy} \tag{A6}$$

Buckling Analysis

The buckling equations, based on the Donnell's theory are

$$\frac{\partial N^1_{xx}}{\partial x} + \frac{\partial N^1_{xy}}{\partial y} = 0$$

$$\frac{\partial N^1_{xy}}{\partial x} + \frac{\partial N^1_{yy}}{\partial y} = 0$$

$$\frac{\partial^2 M^1_{xx}}{\partial x^2} + \frac{\partial^2 M^1_{yy}}{\partial y^2} + 2\,\frac{\partial^2 M^1_{xy}}{\partial x \partial y} + N^o_{xx}\,\frac{\partial^2 w^1}{\partial x^2} + N^o_{yy}\,\frac{\partial^2 w^1}{\partial y^2}$$

$$+ 2N^o_{xy}\,\frac{\partial^2 w^1}{\partial x \partial y} - \frac{N^1_{yy}}{R} = 0 \qquad \text{(A7)}$$

The buckling Equation (A7) can now be written in terms of displacements u^1, v^1, and w^1 by using stress-strain and strain-displacement relations. These equations are

$$\left(E_{xx}\,\frac{\partial^2}{\partial x^2} + G\,\frac{\partial^2}{\partial y^2}\right)u^1 + \left[(G_{xy} + \nu\,E_{yyp})\,\frac{\partial^2}{\partial x \partial y}\right]v^1$$

$$= \left[\left(q - \frac{\nu}{R}\,E_{yyp}\right)\frac{\partial}{\partial x} + e_{st}\,E_{xxst}\,\frac{\partial^3}{\partial x^3}\right]w^1$$

$$\left[(G_{xy} + \nu\,E_{xxp})\,\frac{\partial^2}{\partial x \partial y}\right]u^1 + \left(E_{yy}\,\frac{\partial^2}{\partial y^2} + G\,\frac{\partial^2}{\partial x^2}\right)v^1$$

$$= \left[\left(q - \frac{E_{yy}}{R}\right)\frac{\partial}{\partial y} + e_r\,E_{yyr}\,\frac{\partial^3}{\partial y^3}\right]w^1$$

$$\left(\frac{\nu}{R}\,E_{xxp}\,\frac{\partial}{\partial x} - e_{st}\,E_{xxst}\,\frac{\partial^3}{\partial x^3}\right)u^1 + \left(\frac{E_{yy}}{R}\,\frac{\partial}{\partial y} - e_r\,E_{yyr}\,\frac{\partial^3}{\partial y^3}\right)v^1$$

$$+ \left[\left(D_{xx} + e_{st}^2 E_{xxst}\right) \frac{\partial^4}{\partial x^4} \right.$$

$$+ 2\left(D_{xy} + \frac{\nu}{2} D_{xxp} + \frac{\nu}{2} D_{yyp}\right) \frac{\partial^4}{\partial x^2 \partial y^2}$$

$$\left. + \left(D_{yy} + e_r^2 E_{yyr}\right) \frac{\partial^4}{\partial y^4} + \frac{E_{yy}}{R^2} - 2 \frac{e_r}{R} E_{yyr} \frac{\partial^2}{\partial y^2} \right] w^1$$

$$= N_{xx}^o \frac{\partial^2 w^1}{\partial x^2} + N_{yy}^o \frac{\partial^2 w^1}{\partial y^2} + 2N_{xy}^o \frac{\partial^2 w^1}{\partial x \partial y} \qquad \text{(A8)}$$

These equations are for a stiffened cylinder subject to uniform axial compression, torsion and hydrostatic pressure. The pressure q is assumed to remain normal to the deflected midsurface during the buckling process. This is the true representation of the hydrostatic loading. The prebuckling stress resultants N_{xx}^o, N_{yy}^o and N_{xy}^o are given by

$$N_{xx}^o = qR/2 - \bar{N}$$

$$N_{yy}^o = qR$$

$$N_{xy}^o = T/2\pi R^2 \qquad \text{(A9)}$$

The following non-dimensional parameters, which help systematize the optimization procedure are introduced

$$\bar{\lambda}_{xx} = E_{xxst}/E_{xxp} = E_{st} A_{st} (1-\nu^2)/Eh \, \ell_{st}$$

$$\bar{\lambda}_{yy} = E_{yyr}/E_{yyp} = E_r\ A_r(1-\nu^2)/Eh\ \ell_r$$

$$\bar{\rho}_{xx} = D_{xxst}/D = 12\ E_{st}\ I_{stc}\ (1-\nu^2)/Eh^3\ \ell_{st}$$

$$\bar{\rho}_{yy} = D_{yyr}/D = 12\ E_r\ I_{rc}\ (1-\nu^2)/Eh^3\ \ell_r$$

$$\bar{e}_{st} = \pi^2 R\ e_{st}/L^2$$

$$\bar{e}_r = \pi^2 R\ e_r/L^2$$

$$Z = L^2 \sqrt{1-\nu^2}/Rh$$

$$\bar{k}_{xx} = \bar{N}L^2/\pi^2 D$$

$$\bar{k}_{yy} = qRL^2/\pi^2 D$$

$$\bar{k}_s = N^o_{xy}\ L^2/\pi^2 D \tag{A10}$$

It is possible to derive a single higher order Donnell-Batdorf type of equation by eliminating u^1 and v^1 in Equation (A8). This has been done in [32]. In terms of the parameters defined in Equation (A10), the buckling equation reduces to

$$\nabla_D w^1 + \nabla_E^{-1}\left[\frac{12Z^2}{1-\nu^2}\nabla_C w^1 - \left(\frac{L}{\pi R}\right)^2 \bar{k}_{yy}\ \nabla_P w^1\right]$$

$$= \left(\frac{L}{\pi}\right)^2 \left[\left(\frac{1}{2}\ \bar{k}_{yy} - \bar{k}_{xx}\right)\frac{\partial^2 w^1}{\partial x^2} + \bar{k}_{yy}\frac{\partial^2 w^1}{\partial y^2} + 2\bar{k}_s\frac{\partial^2 w^1}{\partial x \partial y}\right] \tag{A11}$$

where

$$\nabla_D = \left(\frac{L}{\pi}\right)^4 \left[(1 + \bar{\rho}_{xx}) \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + (1 + \bar{\rho}_{yy}) \frac{\partial^4}{\partial y^4}\right]$$

$$\nabla_E = \left(\frac{L}{\pi}\right)^4 \left[(1 + \bar{\lambda}_{xx}) \frac{\partial^4}{\partial x^4} + \frac{2}{1-\nu} \{1 + \bar{\lambda}_{xx})(1 + \bar{\lambda}_{yy}) - \nu\} \frac{\partial^4}{\partial x^2 \partial y^2} + (1 + \bar{\lambda}_{yy}) \frac{\partial^4}{\partial y^4}\right]$$

$$\nabla_P = \left(\frac{L}{\pi}\right)^6 \frac{1}{(1+\bar{\lambda}_{xx})^2} \left[\bar{e}_{st}\bar{\lambda}_{xx} \frac{\partial^6}{\partial x^6} + \left(1 + \frac{2\bar{\lambda}_{yy}}{1-\nu}\right)\bar{e}_{st}\bar{\lambda}_{xx} \frac{\partial^6}{\partial x^4 \partial y^2} + \left(1 + \frac{2\bar{\lambda}_{xx}}{1-\nu}\right)\bar{e}_r \bar{\lambda}_{yy} \frac{\partial^6}{\partial x^2 \partial y^4} + \bar{e}_r \bar{\lambda}_{yy} \frac{\partial^6}{\partial y^6} - \nu\left(\frac{\pi}{L}\right)^2 \frac{\partial^4}{\partial x^4} + \left(\frac{\pi}{L}\right)^2 \frac{1}{1-\nu} \{\nu(1+\nu) - (1 - \bar{\lambda}_{yy})(2\bar{\lambda}_{xx} + 1 + \nu)\}\frac{\partial^4}{\partial x^2 \partial y^2} - \left(\frac{\pi}{L}\right)^2 (1 + \bar{\lambda}_{yy}) \frac{\partial^4}{\partial y^4}\right] \tag{A12}$$

$$\nabla_C = \left(\frac{L}{\pi}\right)^8 \frac{1}{1+\bar{\lambda}_{xx}} \left[\bar{e}_{st}^2\bar{\lambda}_{xx}\frac{\partial^8}{\partial x^8} + 2\bar{e}_{st}^2\bar{\lambda}_{xx}\left(1 + \frac{\bar{\lambda}_{yy}}{1-\nu}\right) \frac{\partial^8}{\partial x^6 \partial y^2} + \left\{\bar{e}_{st}^2\bar{\lambda}_{xx}(1 + \bar{\lambda}_{yy}) + 2\bar{e}_{st}\,\bar{e}_r\bar{\lambda}_{xx}\bar{\lambda}_{yy}\left(\frac{1+\nu}{1-\nu}\right) + \bar{e}_r^2\bar{\lambda}_{yy}(1+\bar{\lambda}_{xx})\right\} \frac{\partial^8}{\partial x^4 \partial y^4}\right.$$

$$+ 2\bar{e}_r^{-2}\bar{\lambda}_y\left(1 + \frac{\bar{\lambda}_{xx}}{1-\nu}\right)\frac{\partial^8}{\partial x^2 \partial y^6} + \bar{e}_r^{-2}\bar{\lambda}_{yy}\frac{\partial^8}{\partial y^8}$$

$$+ 2\nu\left(\frac{\pi}{L}\right)^2 \bar{e}_{st}\bar{\lambda}_{xx}\frac{\partial^6}{\partial x^6}$$

$$- 2\left(\frac{\pi}{L}\right)^2 \{\bar{e}_{st}\bar{\lambda}_{xx} + \bar{e}_r\bar{\lambda}_{yy} + \bar{\lambda}_{xx}\bar{\lambda}_{yy}(\bar{e}_{st} + \bar{e}_r)\}\frac{\partial^6}{\partial x^4 \partial y^2}$$

$$+ 2\nu\left(\frac{\pi}{L}\right)^2 \bar{e}_r\bar{\lambda}_{yy}\frac{\partial^6}{\partial x^2 \partial y^4} + \left(\frac{\pi}{L}\right)^4 \{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2\}\frac{\partial^4}{\partial x^4}\Big]$$

where ∇^{-1} is an inverse differential operator such that $\nabla\nabla^{-1} = \nabla^{-1}\nabla = 1$.

Buckling Results for Cylinder Under Uniform Hydrostatic Pressure and Uniform Axial Compression

The buckling results are derived for general case of combined hydrostatic pressure and axial compression. The axial compression is a known fraction of hydrostatic pressure defined by a factor α. In the case of hydrostatic pressure alone, α is set equal to zero. With no torsion applied, the buckling Equation (A11) reduces to

$$\nabla_D w^1 + \frac{12Z^2}{1-\nu^2}\nabla_E\nabla_C w^1$$

$$- \left(\frac{L}{\pi}\right)^2 \bar{k}_{yy}\left[\frac{1}{R^2}\nabla_E^{-1}\nabla_P w^1 + \left(\frac{1}{2} - \alpha\right)\frac{\partial^2 w^1}{\partial x^2} + \frac{\partial^2 w^1}{\partial y^2}\right] = 0 \qquad \text{(A13)}$$

where

$$\alpha = \bar{k}_{xx}/\bar{k}_{yy}$$

The classical simply supported boundary conditions are

$$w^1(0,y) = w^1(L,y) = 0$$

$$v^1(0,y) = v^1(L,y) = 0$$

$$M^1_{xx}(0,y) = M^1_{xx}(L,y) = 0$$

$$N^1_{xx}(0,y) = N^1_{xx}(L,y) = 0 \tag{A14}$$

Constraint Equations

General Instability

The displacement function satisfying the boundary conditions (A14) is

$$w^1 = w_{mn} \sin \frac{m\pi x}{L} \sin \frac{ny}{R} \tag{A15}$$

The displacement function is substituted in the buckling Equation (13). Using the operators defined earlier, the expression for buckling load parameter is obtained. This expression when minimized with respect to integer values of m and n, representing the mode shape, yields the critical general instability load parameter $\bar{k}_{yycr}$.

Define $\beta = \frac{nL}{\pi R}$. The expression for buckling load parameter $\bar{k}_{yy}$ is then given by

$$-\bar{k}_{yy} = \Bigg[\Big\{(m^2+\beta^2) + \bar{\lambda}_{xx}m^4 + \bar{\lambda}_{yy}\beta^4$$

$$+ \frac{2m^2\beta^2}{1-\nu}(\bar{\lambda}_{xx} + \bar{\lambda}_{yy} + \bar{\lambda}_{xx}\bar{\lambda}_{yy})\Big\}\Big\{(m^2+\beta^2)^2 + \bar{\rho}_{xx}m^4 + \bar{\rho}_{yy}\beta^4\Big\}$$

$$+ \frac{12Z^2}{\pi^4(1-\nu^2)} \Big[\bar{e}_{st}^2\bar{\lambda}_{xx}m^8 + \frac{2}{1-\nu}\bar{e}_{st}^2\bar{\lambda}_{xx}(1-\nu+\bar{\lambda}_{yy})m^6\beta^2$$

$$+ \Big\{\bar{e}_{st}^2\bar{\lambda}_{xx}(1+\bar{\lambda}_{yy}) + 2\,\frac{1+\nu}{1-\nu}\,\bar{e}_{st}\bar{e}_r\bar{\lambda}_{xx}\bar{\lambda}_{yy}$$

$$+ \bar{e}_r^2\bar{\lambda}_{yy}(1+\bar{\lambda}_{xx})\Big\}m^4\beta^4 + \frac{2}{1-\nu}\,\bar{e}_r^2\bar{\lambda}_{yy}(1-\nu+\bar{\lambda}_{xx})m^2\beta^6$$

$$+ \bar{e}_r^2\bar{\lambda}_{yy}\,\beta^8 - 2\nu\bar{e}_{st}\bar{\lambda}_{yy}m^6$$

$$+ 2\{\bar{e}_{st}\bar{\lambda}_{xx}(1+\bar{\lambda}_{yy}) + \bar{e}_r\bar{\lambda}_{yy}(1+\bar{\lambda}_{xx})\}m^4\beta^2 - 2\nu\bar{e}_r\bar{\lambda}_{yy}m^2\beta^4$$

$$+ \{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy})$$

$$- \nu^2\}m^4\Big]\Bigg/\Big[\left(\frac{L}{\pi R}\right)^2 \Big\{(m^2+\beta^2)(\bar{e}_{st}\bar{\lambda}_{xx}m^4 + \bar{e}_r\bar{\lambda}_{yy}\beta^4 + \nu m^2 + \beta^2)$$

$$+ \frac{m^2\beta^2}{1-\nu}\,[2\bar{\lambda}_{xx} + \bar{\lambda}_{yy}(1+\nu) + 2\bar{\lambda}_{xx}\bar{\lambda}_{yy}(1+\bar{e}_{st}m^2 + \bar{e}_r\beta^2)]$$

$$+ \bar{\lambda}_{yy}\beta^4\Big\} + \Big\{(m^2+\beta^2)^2 + \bar{\lambda}_{xx}m^4 + \bar{\lambda}_{yy}\beta^4$$

$$+ \frac{2m^2\beta^2}{1-\nu}\,(\bar{\lambda}_{xx} + \bar{\lambda}_{yy} + \bar{\lambda}_{xx}\bar{\lambda}_{yy})\Big\}\left(\frac{m^2}{2} - \alpha m^2 + \beta^2\right)\Big] \qquad \text{(A16)}$$

Panel Buckling

This is the buckling mode in which stringers and skin between two adjacent rings participate. This is a special case of the general instability, so the expression for panel buckling can be derived by setting all ring parameters, Equation (A16), equal to zero. Thus

$$\bar{e}_r = 0 \;, \qquad \bar{\rho}_{yy} = 0$$

$$\bar{\lambda}_{yy} = 0 \;, \qquad L = \ell_r$$

The expression for panel buckling now becomes

$$- \bar{k}_{yyp} = \Big[\Big\{(m^2 + \beta^2)^2 + \bar{\lambda}_{xx} m^4 + \frac{2m^2\beta^2}{1-\nu}\bar{\lambda}_{xx}\Big\}\{(m^2 + \beta^2)^2 + \bar{\rho}_{xx} m^4\}$$

$$+ \frac{12Z^2}{\pi^4(1-\nu^2)}\{\bar{e}_{st}^2\bar{\lambda}_{xx} m^8 + 2\bar{e}_{st}^2\bar{\lambda}_{xx} m^6\beta^2 + \bar{e}_{st}^2\bar{\lambda}_{xx} m^4\beta^4$$

$$- 2\nu\bar{e}_{st}\bar{\lambda}_{xx} m^6 + 2\bar{e}_{st}\bar{\lambda}_{xx} m^4\beta^2 + (1 + \bar{\lambda}_{xx} - \nu^2)m^4\}\Big] \Big/ \Big[\Big(\frac{\ell_r}{\pi R}\Big)^2$$

$$\Big\{(m^2+\beta^2)(\bar{e}_{st}\bar{\lambda}_{xx} m^4 + \nu m^2 + \beta^2) + 2\,\frac{m^2\beta^2}{1-\nu}\bar{\lambda}_{xx}\Big\}$$

$$+ \Big\{(m^2+\beta^2)^2 + \bar{\lambda}_{xx} m^4 + \frac{2m^2\beta^2}{1-\nu}\bar{\lambda}_{xx}\Big\}\Big(\frac{m^2}{2} - \alpha m^2 + \beta^2\Big)\Big] \quad \text{(A17)}$$

Minimization of the expression (A17) with respect to integer values of m and n yields the critical load parameter for panel instability.

In Equations (A16) and (A17), $\bar{e}_r$ and $\bar{e}_{st}$ greater than zero correspond to the cylinder stiffened with exterior stiffeners; whereas, if these eccentricity parameters are less than zero they correspond to a cylinder stiffened with interior stiffeners.

Skin Buckling

For a shell which is stiffened with rings only, the skin buckling criterion and panel buckling criterion are identical. One can get the criterion from Equation (A17) for this case by setting ring and stringer parameters equal to zero and changing L to ℓ_r.

For the shell stiffened with rings and stringers, the skin is considered as a flat plate simply supported on four sides, and subject to biaxial compression. The buckling criterion is given by Timoshenko [33]. The critical stress is found from the following equation

$$\sigma_{xxsk}\, m^2 + \sigma_{yysk}\, n^2\left(\frac{\ell_r}{\ell_{st}}\right)^2 = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{h}{\ell_r}\right)^2\left[m^2 + \left(\frac{\ell_r}{\ell_{st}}\right)^2 n^2\right]^2 \quad \text{(A18)}$$

Stringer Buckling

In general the stringer is considered as a collection of flat plates of length ℓ_r. The appropriate boundary conditions and corresponding critical load depends on the relative depth of stringer and ring as well as the shape of the stringer. If the rings are deeper than the stringers, and the stringers are of rectangular cross-section, the boundary conditions for the stringer are: simply supported on three edges and free on one edge. If T, IA (inverted angle), Z, I shaped stringers are used, the web of the stringers is considered as simply supported on all the four edges, while the flange is considered as simply supported on three edges and free on the unloaded edge. The buckling criteria for various boundary conditions are taken from [33]. For rectangular and T shape stringers the buckling expressions are given as follows

(a) Rectangular Stringers

$$\sigma_{xxst_{cr}} = \frac{\pi^2 E_{st}}{12(1-\nu^2)} \left(\frac{t_{st}}{d_{st}}\right)^2 \left[\left(\frac{d_{st}}{\ell_r}\right)^2 + 0.425\right] \tag{A19}$$

(b) T-Stringers

$$\sigma_{xxst_{cr}} \text{ (Web)} = \frac{\pi^2 E_{st}}{3(1-\nu^2)} \left(\frac{t_{st}}{d_{st}}\right)^2$$

$$\sigma_{xxst_{cr}} \text{ (Flange)} = \frac{\pi^2 E_{st}}{12(1-\nu^2)} \left(\frac{2t_{fst}}{b_{fst}-t_{st}}\right)^2 \left[\left(\frac{b_{fst}-t_{st}}{2\ell_r}\right)^2 + 425\right] \tag{A20}$$

If the stringers are deeper than the rings, the portion of the stringer below the web of the ring is considered as a flat plate simply supported on four sides and length ℓ_r. The outstanding portion of the stringer is considered as simply supported on three edges and free on the fourth edge. The length of the plate in this case is L.

Ring Buckling

The ring is considered as an annular plate subjected to uniform compression along the circumference. For rectangular shape rings, the boundary conditions are assumed to be simply supported at one end and free at the other end. These are the boundary conditions for the case when the shell is stiffened with rings only. If the shell is stiffened with stringers and rings, the rings being deeper than the stringers, the boundary conditions for portion of the ring projecting above the stringers are simply supported at one edge and free at the other. For the portion of the ring which is equal to the stringer depth the boundary conditions are simply supported at both edges. If T, inverted angle (IA), I, channel, or Z shaped rings are used, the boundary conditions are simply supported at both edges. Furthermore, the depth of the rings is less than 1/10th of the radius of the shell, which means that the ratio of inner to outer radius of rings is of the order of 0.1, the annular plate can be approximated by a long narrow rectangular plate. This has been verified by Majumdar [34], and Yamaki [35]. Therefore, the buckling criterion for the ring is the same as for long narrow rectangular plate. A similar criterion has been used by Nickell and Crawford [6]. Under this assumption, the critical stress for rings is given by

$$\sigma_{yyr_{cr}} = K_R \frac{\pi^2 E_r}{12(1-\nu^2)} \left(\frac{t_{wr}}{d_{wr}}\right)^2 \tag{A21}$$

where $K_R = 4.0$, for rings with both ends simply supported and $K_R = \frac{1}{2}$ for the rings simply supported at one edge and free at the other.

Stresses in Skin and Stiffeners

It is assumed that membrane state exists prior to buckling. The stresses in the skin and the stiffeners are calculated based on this assumption. Under the membrane state displacement component u is assumed to be linear function of x only, where as displacement component v and w are independent of x and y. Denoting by superscript "o", the membrane state parameters are

$$\epsilon_x^o = \epsilon_{xx}^o = \frac{\partial u}{\partial x}$$

$$\epsilon_y^o = \epsilon_{yy}^o = \frac{w}{R}$$

$$\gamma^o = 0 \tag{A22}$$

The membrane state stress resultants are

$$N_{xx}^o = \frac{Eh}{1-\nu^2}(1 + \bar{\lambda}_{xx})\epsilon_{xx}^o + \frac{\nu Eh}{1-\nu^2}\epsilon_{yy}^o$$

$$N_{yy}^o = \frac{\nu Eh}{1-\nu^2}\epsilon_{xx}^o + \frac{Eh}{1-\nu^2}(1 + \bar{\lambda}_{yy})\epsilon_{yy}^o$$

$$N_{xy}^o = 0 \tag{A23}$$

For a circular cylindrical shell under uniform hydrostatic pressure and uniform axial compression

$$N_{xx}^o = -\frac{qR}{2} - \alpha qR$$

$$N_{yy}^o = -qR \tag{A24}$$

From equations (26) and (27), the prebuckling strains are

$$\epsilon_{xx}^o = -\frac{qR}{2}\frac{(1-\nu^2)}{Eh}\frac{[(1+\bar{\lambda}_{yy})(1+2\alpha) - 2\nu]}{[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2]}$$

$$\epsilon_{yy}^o = -\frac{qR}{2}\frac{(1-\nu^2)}{Eh}\frac{[2(1+\bar{\lambda}_{xx}) - \nu(1+2\alpha)]}{[(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2]} \tag{A25}$$

and the stresses in the skin, stringer and ring are

$$\sigma_{xxsk} = -\frac{qR}{2h}\left[\frac{2\nu\bar{\lambda}_{xx} + \bar{\lambda}_{yy}(1+2\alpha) + (1-\nu^2)(1+2\alpha)}{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2}\right]$$

$$\sigma_{yysk} = -\frac{qR}{2h}\left[\frac{2\bar{\lambda}_{xx} + \nu(1+2\alpha)\bar{\lambda}_{yy} + 2(1-\nu^2)}{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2}\right]$$

$$\sigma_{xxst} = -\frac{qR}{2h}\frac{E_{st}}{E}(1-\nu^2)\left[\frac{(1+\bar{\lambda}_{yy})(1+2\alpha) - 2\nu}{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2}\right]$$

$$\sigma_{yyr} = -\frac{qR}{2h}\frac{E_r}{E}(1-\nu^2)\left[\frac{2(1+\bar{\lambda}_{xx}) - \nu(1+2\alpha)}{(1+\bar{\lambda}_{xx})(1+\bar{\lambda}_{yy}) - \nu^2}\right] \tag{A26}$$

When the shell is stiffened by rings only, the stresses in the skin and ring are calculated by the analysis given by Salerno and Pulos [36]. The stresses calculated by this analysis are slightly higher than those calculated by membrane analysis. The stresses at the midsection between the two consecutive rings are

$$\sigma_{xxsk} = \sigma_{b\left(x=\frac{\ell}{2}\right)} - \frac{qR}{2h} \tag{A27}$$

σ_b being the bending stress.

$$\sigma_{yysk} = \sigma_y + \nu\sigma_b \tag{A28}$$

σ_y being the circumferential stress.

Further

$$\sigma_b = \pm \frac{qR^2}{2}\left(\frac{1-\frac{\nu}{2}}{1-\nu^2}\right) TJ \tag{A29}$$

$$\sigma_y = -\frac{qR}{h} + \left[\frac{qR}{h}\left(1-\frac{\nu}{2}\right)\right]\frac{TU}{H} \tag{A30}$$

The total load carried by the ring per unit length is given by

$$Q^* = \frac{qb - \frac{qR^2h^2W}{6(1-\nu^2)}\left[1 - \frac{\frac{\nu}{2}A_r}{A_r+bh}\right]}{\left[1 - \frac{R^2h^3W}{6(1-\nu^2)(A_r+bh)}\right]} \tag{A31}$$

Parameters W, J, U, H, and T are determined appropriately.

Expressions for W, J, U, H, and T

For hydrostatic pressure, the axial compression component $\bar{N}_x$ is given by

$$\bar{N}_x = -qR/2$$

Case I

$$\left(\frac{\bar{N}_x}{2D}\right)^2 < \frac{Eh}{DR^2}$$

For this case

$$W = \frac{-16ef\,(e^2+f^2)(\sinh^2 e + \sin^2 f)}{\ell^3(e \sin f \cos f + f \sinh e \cosh e)}$$

$$J = \frac{4(e^2+f^2)(e \cosh e \sin f - f \sinh e \cos f)}{\ell^2(e \sin f \cos f + f \sinh e \cosh e)}$$

$$U = -(f \sinh e \cos f + e \cosh e \sin f)$$

$$H = -(e \sin f \cos f + f \sinh e \cosh e)$$

$$T = \frac{A_r/(A_r+bh)}{\left[1 - \frac{R^2h^3W}{6(1-\nu^2)(A_r+bh)}\right]} \tag{A32}$$

where

$$e = \frac{c\ell}{2}$$

$$f = \frac{d\ell}{2}$$

$$c = \frac{\bar{\alpha}}{\ell}\left(1 - \frac{qR^3\alpha^2}{2Eh\ell^2}\right)^{\frac{1}{2}}$$

$$d = \frac{\bar{\alpha}}{\ell}\left(1 + \frac{qR^3\alpha^2}{2Eh\ell^2}\right)^{\frac{1}{2}}$$

$$\bar{\alpha} = \left[\frac{3(1-\nu^2)}{h^2R^2}\right]^{\frac{1}{4}}\ell \tag{A33}$$

<u>Case II</u>

$$\left(\frac{\bar{N}_x}{2D}\right)^2 = \frac{Eh}{DR^2}$$

For this case

$$W = -16\, g^3 \sin^2 g/(g + \sin g \cos g)\ell^3$$

$$J = 4g^2(\sin g - g\cos g)/(g + \sin g \cos g)\ell^2$$

$$U = g \sin g + \sin g$$

$$H = g + \sin g \cos g$$

$$T = \frac{A_r/(A_r + bh)}{\left[1 - \dfrac{R^2h^3W}{6(1-\nu^2)(A_r+bh)}\right]} \tag{A34}$$

where

$$g = \left(\frac{qR}{4D}\right)^{\frac{1}{2}} \frac{\ell}{2}$$

<u>Case III</u>

$$\left(\frac{\bar{N}_x}{2D}\right)^2 > \frac{Eh}{DR^2}$$

For this case

$$W = \frac{-16ef(f^2-e^2)(\sin^2 f - \sin^2 e)}{\ell^3(e \sin f \cos f + f \sin e \cos e)}$$

$$J = \frac{4(e^2-f^2)(f \sin e \cos f - e \cos e \sin f)}{\ell^2(f \sin e \cos e + e \sin f \cos f)}$$

$$U = f \sin e \cos f + e \cos e \sin f$$

$$H = e \sin f \cos f + f \sin e \cos e$$

$$T = \frac{A_r/(A_r+bh)}{\left[1 - \dfrac{R^2h^2W}{6(1-\nu^2)(A_r+bh)}\right]} \tag{A35}$$

where

$$e = \frac{c\ell}{2}$$

$$f = \frac{d\ell}{2}$$

$$c = \left[\frac{qR}{8D} - \frac{1}{2}\left(\frac{Eh}{DR^2}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

$$d = \left[\frac{qR}{8D} + \frac{1}{2}\left(\frac{Eh}{DR^2}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \qquad \text{(A36)}$$

In the above equations ℓ is the clear distance between the two adjacent rings.

APPENDIX B

PROPERTIES OF DIFFERENT SHAPES OF STIFFENERS

The derivation of $\bar{\alpha}$'s and shape parameters for various shapes of stiffener cross-section is given in this Appendix.

Rectangular Cross-Section

The radius of gyration of a rectangular cross-section is given by

$$\alpha_1 = \frac{d_w}{\sqrt{12}}$$

The radius of gyration of unit width of skin is

$$\alpha_2 = \frac{h}{\sqrt{12}}$$

Nondimensionalizing the radius of gyration of stiffener with respect to the radius of gyration of skin, one gets

$$\bar{\alpha} = \frac{d_w}{h}$$

The nondimensionalized flexural stiffness and the eccentricity parameters of the stiffeners are

$$\bar{\rho} = \frac{E_{stif}\, I_{stifc}}{\ell_{stif}\, D}$$

$$\bar{e} = \frac{\pi^2 Re}{L^2}$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

and

$$I_{stifc} = t_w d_w^3/12$$

The subscript 'stif' refers to the stiffener.

With simple algebraic operation, one can write

$$\bar{\rho} = \bar{\alpha}^{-2} \bar{\lambda}$$

$$\bar{e} = \frac{\pi^2(1-\nu^2)^{\frac{1}{2}}}{2Z} (1 + \bar{\alpha})$$

where

$$\bar{\lambda} = \frac{A_{stif} (1-\nu^2)}{\ell_{stif} h}$$

In similar way the relations for other shapes of the stiffener are obtained. Some of the shapes are shown in Figure B1. In deriving these expressions, assumption is made that the thickness of the web or flange is much smaller than the depth of the web of the stiffener. Thus, one has

$$\bar{\rho}_{xx} = \bar{\alpha}_x^{-2} \bar{\lambda}_{xx}$$

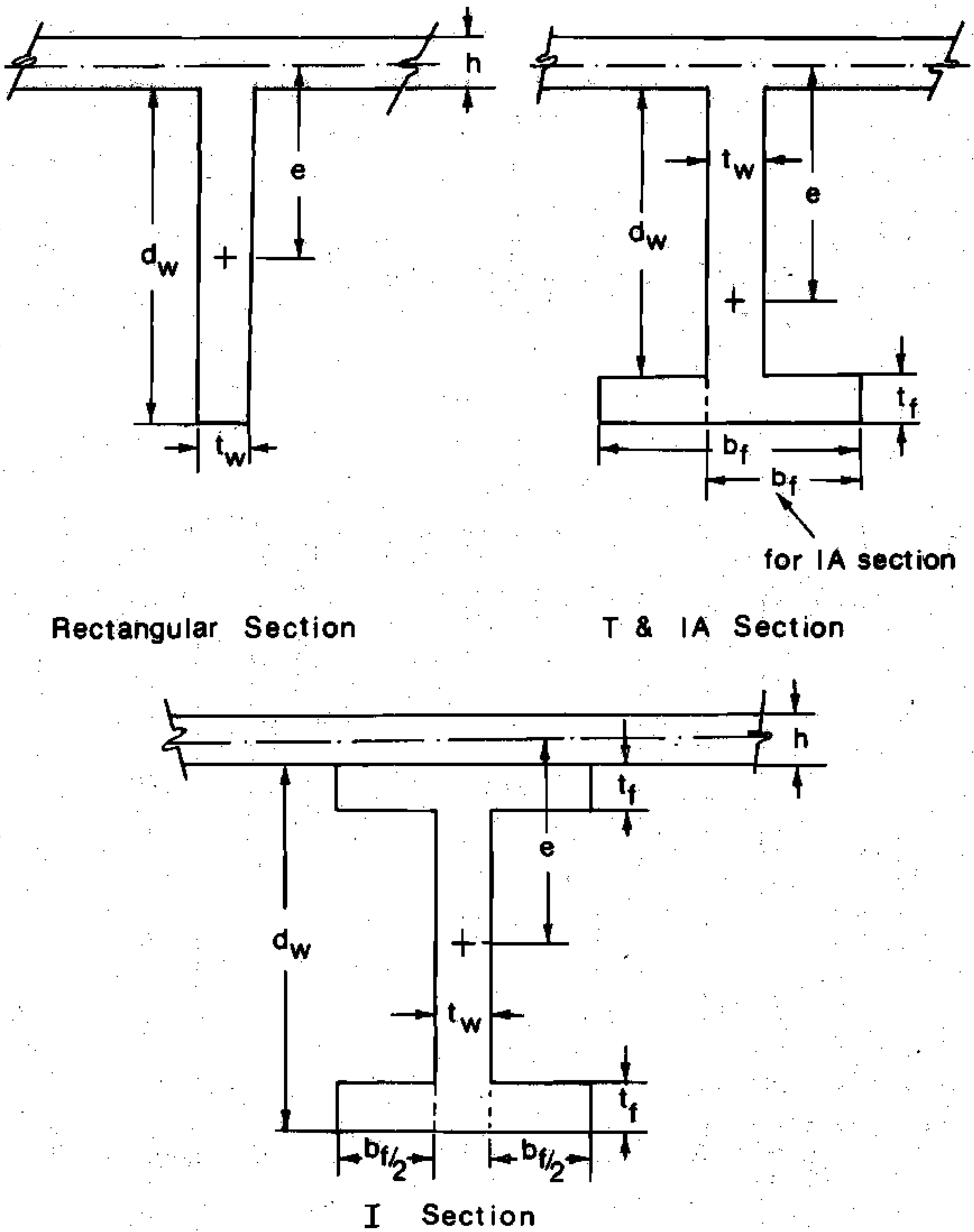

Figure B1. Properties of Various Shapes of Stiffeners.

$$\bar{\rho}_{yy} = \bar{\alpha}_y^2 \; \bar{\lambda}_{yy}$$

$$\bar{e}_x = \frac{\pi^2(1-\nu^2)^{\frac{1}{2}}}{2Z} (1 + C_x \; \bar{\alpha}_x)$$

$$\bar{e}_y = \frac{\pi^2(1-\nu^2)^{\frac{1}{2}}}{2Z} (1 + C_y \; \bar{\alpha}_y)$$

$$A_{stif} = t_w \; d_w \; k_1$$

Table A1 gives the values of different parameters for various shapes of the stiffeners.

Table B1. Properties of Various Shapes of Stiffeners.

Shape	Area	k_1	$\bar{\alpha}$	C
Rectangular	$t_w d_w$	1.0	d_w/h	1.0
T, IA	$t_w d_w(1+AB)$	1+AB	$\frac{\sqrt{1+4AB}}{1+AB} \frac{d_w}{h}$	$\frac{1+2AB}{\sqrt{1+4AB}}$
Channel, I, Z	$t_w d_w(1+2AB)$	(1+2AB)	$\sqrt{\frac{1+6AB}{1+AB}} \frac{d_w}{h}$	$\sqrt{\frac{1+2AB}{1+6AB}}$
Angle	$t_w d_w(1+AB)$	(1+AB)	$\frac{\sqrt{1+4AB}}{1+AB} \frac{d_w}{h}$	$\frac{1}{\sqrt{1+4AB}}$

APPENDIX C

SAMPLE DESIGN TABLES AND DESIGN EXAMPLES

In order to illustrate the procedure of design discussed under Phase II, three design examples are worked out in the Appendix. The design Tables corresponding to these design examples are also given here.

Example 1

For a shell stiffened with rectangular rings and rectangular stringers, the following data are known:

Operating Depth	= 3000 feet
Radius of the shell	= 198 inches
Length of the shell	= 594 inches
Permissible yield stress	= 120,000 psi
Poisson Ratio	= .300

Factor of safety

(a) For stress level limitations against yielding 1

(b) For all other failure modes 2

Modulus of elasticity $E = E_{st} = E_r = 30 \times 10^6$ psi

$\rho_r = \rho_{st} = \rho_{st} = .282$ lb./in.^{3}

Density of immersion fluid = .0374 lb/in.^{3}

From the design Table C1

$Z = 1200,\ C_x = C_y = 1.0$

$$h = \frac{L^2}{RZ}(1-\nu^2)^{\frac{1}{2}} = 1.41660 \text{ in.}$$

$\bar{\alpha}_x = 7.0$ $\bar{\lambda}_{xx} = .07886$

$\bar{\alpha}_y = 12.5$ $\bar{\lambda}_{yy} = .61651$

$m = 1$ $n = 3$

$\bar{W} = 1.75945$

Using Equation (4) the stresses in skin, ring, and stringer are calculated

$\sigma_{xxst} = 89{,}537.8$ psi , $\sigma_{yyr} = 96{,}172.8$ psi

$\sigma_{yysk} = 123{,}033$ psi , $\sigma_{xxsk} = 52{,}623.9$ psi

Using von Mises yield criterion, the stress in the skin is

$$\sigma_s = 110{,}170 \text{ psi}$$

The stresses in skin, stringer and ring being less than the permissible level, the constraints defining stress level limitations are satisfied.

The depths of stringer and ring are given by

$$d_{st} = \bar{\alpha}_x h = 9.91620 \text{ in.}$$

$$d_r = \bar{\alpha}_y h = 17.70750 \text{ in.}$$

From the ring buckling criterion, the thickness of the ring is given by

$$t_{wr} > \sqrt{\frac{24(1-\nu^2) \times 2\sigma_{yyr}}{\pi^2 E_r}}\,(d_r - d_{st})$$

Table C1. Design Table. Interior RR-RS Stiffened Shell.
General Instability Formulation.

Material of Construction - High Strength Steel

ν	C_x	C_y	Z	A_x	A_y	B_x	B_y	q_D^*
.300	1.0	1.0	1200	0	0	0	0	1.41384×10^{-5}

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{W}$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	m	n
2.0	14.0	2.07396	.11069	.87421	13	4
3.0	14.0	1.70229	.18203	.46368	1	3
4.0	14.0	1.62377	.09195	.47914	1	3
5.0	14.0	1.58382	.06845	.46533	1	3
6.0	14.0	1.57595	.06086	.46548	1	3
7.0	14.0	1.58925	.07344	.46546	1	3
8.0	14.0	1.61359	.09658	.46531	1	3
10.0	14.0	1.67914	.15864	.46531	1	3
12.0	14.0	1.61235	.09520	.46551	1	3
2.0	13.5	2.05919	.11496	.85693	13	4
3.0	13.5	1.72253	.15690	.50708	1	3
4.0	13.5	1.64456	.08168	.50825	1	3
5.0	13.5	1.63527	.07262	.50848	1	3
6.0	13.5	1.65531	.09170	.50842	1	3
7.0	13.5	1.61744	.05535	.50881	1	3
8.0	13.5	1.64406	.08075	.50869	1	3
10.0	13.5	1.68799	.12259	.50855	1	3
12.0	13.5	1.79240	.22224	.50803	1	3
2.0	13.0	2.07189	.11762	.86641	13	4
3.0	13.0	1.72480	.10707	.55754	1	3
4.0	13.0	1.72051	.10268	.55782	1	3

- continued -

Table C1. Design Table. (Continued)

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{W}$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	m	n
5.0	13.0	1.72253	.10444	.55799	1	3
6.0	13.0	1.71310	.09517	.55825	1	3
7.0	13.0	1.70972	.09178	.55840	1	3
8.0	13.0	1.67607	.04535	.57207	1	3
10.0	13.0	1.86975	.24535	.55769	1	3
12.0	13.0	1.73267	.11361	.55848	1	3
2.0	12.5	2.04181	.12200	.83501	12	4
3.0	12.5	1.80766	.12667	.61515	1	3
4.0	12.5	1.79363	.11267	.61563	1	3
5.0	12.5	1.77348	.09280	.61610	1	3
6.0	12.5	1.75268	.07240	.61647	1	3
7.0	12.5	1.75945	.07886	.61551	1	3
8.0	12.5	1.84643	.12715	.65037	4	4
10.0	12.5	1.80322	.12107	.61642	1	3
12.0	12.5	1.82070	.13800	.61631	1	3
2.0	12.0	2.06031	.12471	.84988	12	4
3.0	12.0	1.88168	.12725	.68304	1	3
4.0	12.0	1.84141	.08717	.68397	1	3
5.0	12.0	1.82312	.06896	.68440	1	3
6.0	12.0	1.79627	.04243	.68484	1	3
8.0	12.0	1.88739	.01302	.79545	12	4
12.0	12.0	1.95395	.19652	.68389	1	3

or

$$t_{wr} > .92850 \text{ in.}$$

Using the definition of $\bar{\lambda}_{yy}$, ℓ_r, the ring spacing is given by

$$\ell_r > \frac{d_{wr}\ t_{wr}(1-\nu^2)}{\bar{\lambda}_{yy}\ h}$$

$$\ell_r \quad 17.13140 \text{ in.}$$

Assuming 33 rings, the value of ℓ_r and corresponding t_{wr} are calculated as

$$\ell_r = 17.47058 \text{ in.}$$

$$t_{wr} = \quad .94687 \text{ in.}$$

The stringer spacing is calculated from Equation (24) as

$$\ell_{st} > \sqrt{\frac{12(1-\nu^2) \times \sigma_{xxst} \times F_1}{\pi^2 E_{st}\left[\left(\frac{d_{st}}{\ell_r}\right)^2 + .425\right]}}\ \frac{d_{st}^2(1-\nu^2)}{h\ \bar{\lambda}_{xx}}$$

or

$$\ell_{st} > 57.76287 \text{ in.}$$

Assuming 20 stringers, the spacing ℓ_{st} and corresponding t_{st} are calculated. These are

$$\ell_{st} = 62.17200 \text{ in.}$$

$$t_{st} = .76968 \text{ in.}$$

The critical stresses are now calculated for skin, stringer, and ring. These are

$$\sigma_{xxsk_{cr}} = 187{,}020 \text{ psi}$$

$$\sigma_{xxst_{cr}} = 121{,}994 \text{ psi}$$

$$\sigma_{yyr_{cr}} = 200{,}026 \text{ psi}$$

Using computer program for panel buckling check, the critical load obtained for the design variables given above is

$$q_{cr} = 12{,}247.9 \text{ psi}$$

and

$$m_p = 1, \quad n_p = 69$$

The ratios of actual load to the failure load are now calculated to ensure that interaction of failure modes does not occur

$$GB = \quad = 1.00000$$

$$PB = \quad = .21986$$

$$SKB = \quad = .95752$$

$$STB = \quad = .86322$$

RB = = .96160

SKY = = .91808

RY = = .80143

STY = = .43853

Finally the weight of the shell is calculated

$$W = 848.3 \text{ lb/in.}$$

Example 2

For shell stiffened with T-rings and rectangular stringers, the design Table C2 given in this Appendix is used. The known data are the same as in Example 1.

From the design Table C2, for Z = 1200, one has the following values:

$$C_x = 1.0$$

$$C_y = 1.155$$

$$A_y = 1.0$$

$$B_y = 0.5$$

$$\bar{\alpha}_x = 6.0$$

$$\bar{\alpha}_y = 12.5$$

$$\bar{\lambda}_{xx} = .08143$$

$$\bar{\lambda}_{yy} = .46216$$

$$m = 1$$

Table C2. Design Table. Interior TR-RS Stiffened Shell.
General Instability Formulation.
Material of Construction - High Strength Steel.

ν	C_x	C_y	Z	A_x	A_y	B_x	B_y	q_D^*
.300	1.0	1.155	1200	0	1.0	0	.5	1.41384×10^{-5}

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{W}$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	m	n
2.0	15.0	1.40189	.06792	.29929	1	3
3.0	15.0	1.39468	.06106	.29944	1	3
4.0	15.0	1.42192	.08653	.29932	1	3
5.0	15.0	1.42333	.08775	.29940	1	3
6.0	15.0	1.47740	.13828	.29918	1	3
7.0	15.0	1.40138	.06708	.29965	1	3
8.0	15.0	1.40781	.07304	.29967	1	3
9.0	15.0	1.41251	.07740	.29969	1	3
10.0	15.0	1.42015	.08450	.29969	1	3
11.0	15.0	1.41698	.08152	.29972	1	3
12.0	15.0	1.42846	.09223	.29969	1	3
2.0	14.5	1.55251	.04291	.46138	15	4
3.0	14.5	1.44101	.07958	.32369	1	3
4.0	14.5	1.45500	.09264	.32368	1	3
5.0	14.5	1.44380	.08201	.32387	1	3
6.0	14.5	1.45169	.08933	.32390	1	3
7.0	14.5	1.45044	.08808	.32398	1	3
8.0	14.5	1.44917	.04951	.36060	1	3
10.0	14.5	1.45811	.09513	.32409	1	3
12.0	14.5	1.47155	.10768	.32407	1	3
2.0	14.0	1.57600	.04368	.48214	15	4
3.0	14.0	1.46659	.07522	.35145	1	3

- continued -

Table C2. Design Table. (Continued)

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{W}$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	m	n
4.0	14.0	1.46078	.06962	.35161	1	3
5.0	14.0	1.45572	.06477	.35173	1	3
6.0	14.0	1.45368	.06294	.35181	1	3
7.0	14.0	1.44866	.05799	.35190	1	3
8.0	14.0	1.47446	.08221	.35182	1	3
10.0	14.0	1.46230	.07071	.35194	1	3
12.0	14.0	1.44672	.05604	.35202	1	3
2.0	13.5	1.52822	.06266	.42016	1	2
3.0	13.5	1.50783	.08162	.38306	1	3
4.0	13.5	1.47653	.05180	.38347	1	3
5.0	13.5	1.47716	.05232	.38353	1	3
6.0	13.5	1.48364	.05839	.38355	1	3
7.0	13.5	1.48735	.06184	.38358	1	3
8.0	13.5	1.46380	.03953	.38376	1	3
10.0	13.5	1.47499	.05006	.38375	1	3
12.0	13.5	1.47289	.04806	.38377	1	3
2.0	13.0	1.60890	.04540	.51066	15	4
3.0	13.0	1.69924	.07152	.56822	1	2
4.0	13.0	1.60974	.14082	.41903	1	3
5.0	13.0	1.60799	.13894	.41925	1	3
6.0	13.0	1.51096	.04650	.42013	1	3
7.0	13.0	1.60242	.13333	.41960	1	3
8.0	13.0	1.60306	.13383	.41970	1	3
10.0	13.0	1.61015	.14041	.41981	1	3
12.0	13.0	1.60853	.13884	.41984	1	3

- continued -

Table C2. Design Table. (Continued)

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{W}$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	m	n
2.0	12.5	1.72179	.04547	.61381	15	4
3.0	12.5	1.62547	.11236	.46137	1	3
4.0	12.5	1.61166	.09891	.46172	1	3
5.0	12.5	1.61089	.09800	.46189	1	3
6.0	12.5	1.59372	.08143	.46216	1	3
7.0	12.5	1.62059	.10702	.46207	1	3
8.0	12.5	1.60762	.09454	.46224	1	3
10.0	12.5	1.59940	.08659	.46238	1	3
12.0	12.5	1.55743	.04655	.46261	1	3
2.0	12.0	1.62400	.05922	.51140	1	3
3.0	12.0	1.62389	.05896	.51154	1	3
4.0	12.0	1.60206	.03772	.51192	1	3
5.0	12.0	1.70107	.03174	.60800	1	2
6.0	12.0	1.61065	.04587	.51197	1	3
7.0	12.0	1.91106	.33483	.50987	1	3
8.0	12.0	1.61334	.04835	.51205	1	3
10.0	12.0	1.58592	.02195	.51226	1	3
12.0	12.0	1.57103	.00765	.51235	1	3
2.0	11.5	1.71629	.08725	.56932	1	3
3.0	11.5	1.71908	.08974	.56951	1	3
4.0	11.5	1.71570	.08623	.56975	1	3
5.0	11.5	1.69033	.08040	.55204	1	3
6.0	11.5	1.69544	.07994	.55717	1	3
7.0	11.5	1.67799	.06706	.55346	1	3
8.0	11.5	1.71230	.08238	.57030	1	3
10.0	11.5	1.71249	.08246	.57040	1	3

$$n = 3$$

$$\bar{W} = 1.59372$$

Using Equation (4), the stresses in the skin, ring and stringer are calculated. These are

$$\sigma_{xxsk} = 89,663.9 \text{ psi}$$

$$\sigma_{yysk} = 133,864 \text{ psi}$$

$$\sigma_{yyr} = 106,965 \text{ psi}$$

$$\sigma_{xxst} = 49,504.7 \text{ psi}$$

von Mises yield criterion gives

$$\sigma_s = 118,137 \text{ psi}$$

All the stresses are within permissible limits.

The depths of the stringer and ring are given by

$$d_{st} = \bar{\alpha}_x \text{ h} = 8;49960 \text{ in.}$$

$$d_{wr} = \frac{\bar{\alpha}_y \times h \times (1+A_y B_y)}{\sqrt{(1+4A_y B_y)}} = 15.33515 \text{ in.}$$

$$b_{fr} = B_y \times d_{wr} = 7.66758$$

From ring buckling criterion, thickness of the ring is given by

$$t_{wr} > \sqrt{\frac{3 \times 2\sigma_{yyr} (1-\nu^2)}{\pi^2 E_r}}\, d_{st}$$

or

$$t_{wr} > .37721 \text{ in.}$$

Using definition of $\bar{\lambda}_{yy}$, ℓ_r is

$$\ell_r > 12.10316 \text{ in.}$$

Select ℓ_r as

$$\ell_r = 13.2 \text{ in.}$$

this gives

$$t_{wr} = .41284 \text{ in.}$$

and

$$t_{fr} = A_y \times t_{wr} = .41284 \text{ in.}$$

Thickness of stringer is found from stringer buckling criterion

$$t_{st} > \sqrt{\frac{12(1-\nu^2) \times 2\sigma_{xxst}}{\pi^2 E_{st} \left[\left(\frac{d_{st}}{\ell_r}\right)^2 + .425\right]}}\, d_{st}$$

or

$$t_{st} > .56077 \text{ in.}$$

this gives

$$\ell_{st} > 37.60039 \text{ in.}$$

Assuming

$$\ell_{st} = 38.8572 \text{ in.}$$

one gets

$$t_{st} = .5795 \text{ in.}$$

The critical stresses for skin, ring and stringer are now calculated, these are

$$\sigma_{xxsk_{cr}} = 331{,}076 \text{ psi}$$

$$\sigma_{xxst_{cr}} = 105{,}716 \text{ psi}$$

$$\sigma_{yyr_{cr}} = 255{,}604 \text{ psi}$$

The critical load for panel buckling is obtained as

$q_{cr} = 19{,}266$ psi, $m_p = 1$ and $n_p = 85$

Finally the ratios of actual load to the failure load are calculated to insure separation of these modes

$$GB = \frac{q_D}{q_{cr}} = 1.00000$$

$$PB = \frac{q_{D_{(PB)}}}{q_{cr}} = .13977$$

$$SKB = \frac{2\sigma_{xxsk}}{\sigma_{xxsk_{cr}}} = .54165$$

$$STB = \frac{2\sigma_{xxst}}{\sigma_{xxst_{cr}}} = .93656$$

$$RB = \frac{2\sigma_{yyr}}{\sigma_{yyr_{cr}}} = .83695$$

$$SKY = \frac{\sigma_{s}}{\sigma_{y}} = .98447$$

$$RY = \frac{\sigma_{yyr}}{\sigma_{y}} = .89137$$

$$STY = \frac{\sigma_{xxst}}{\sigma_{y}} = .41253$$

The weight of the shell is

$$W = 772.7 \text{ lb/in}$$

An alternative design giving the same weight as above is

$$\ell_{st} = 38.8572 \text{ in.}$$

$$t_{st} = .5795 \text{ in.}$$

$$\ell_{r} = 13.81395 \text{ in.}$$

$$t_{wr} = .43204 \text{ in.}$$

Example 3

This is the design example for ring stiffened shell. The operating depth is 3000 feet and high strength steel is used as material of construction. The design Table C3 is generated for this case. Program RSSH is used for finding the design variables. Before using above program, the ring spacing is first found from the criterion of panel buckling. Once the ring spacing is known, one proceeds with the design. The input data for the program RSSH are: Z, ELY(ring spacing), X(2)(value of $\bar{\lambda}_{yy}$), AY, BY, PBCR(panel buckling critical load) and ALY($\bar{\alpha}_y$). The results of the example are given on the next page.

Table C3. Design Table. Interior T-Ring Stiffened Shell.
General Instability Formulation.
Material of Construction - High Strength Steel.

ν	A_y	B_y	Z	C_y	q_D^*
.300	1.0	.5	1200.0	1.155	1.41384×10^{-6}

$\tilde{\alpha}_y$	$\tilde{W}$	$\bar{\lambda}_{yy}$	m	n
13.0	1.19311	.17573	1	3
12.8	1.19958	.18162	1	3
12.6	1.20639	.18782	1	3
12.4	1.21357	.19435	1	3
12.2	1.22116	.20125	1	3
12.0	1.22917	.20854	1	3
11.8	1.23765	.21626	1	3
11.6	1.24662	.22443	1	3
11.4	1.25615	.23309	1	3
11.2	1.26626	.24230	1	3
11.0	1.27701	.25208	1	3
10.8	1.28847	.26251	1	3
10.6	1.30069	.27363	1	3
10.4	1.31375	.28551	1	3
10.2	1.32773	.29823	1	3
10.0	1.34271	.31187	1	3

ENTER VALUES OF ZZ,ELY,BK,X(2), AY, BY, PBCR,ALY

950.,24.75,23.,.31187,1.,.5,3016.83,10.

DESIGN RESULTS

OPERATING DEPTH = 3000.

ZZ = 950.0 L = 594.0 R = 198.0

X(2) = .31187 , CY = 1.1547

WEIGHT PER INCH = 821.73

SKIN THICKNESS = 1.78939

DEPTH OF WEB = 15.49656

WEB THICKNESS = .65296

FLANGE WIDTH = 7.74828

FLANGE THICKNESS = .65296

RING SPACING = 24.75000

CKYR = 1212.20 M = 1 N = 3

QSTAR = 2692.82

QCR = 125604.04 SRY = 94862.00 PBCR = 3016.83

SX2 = -89348.22 SY2 = -122792.30 QQ = 7831.54

SKY = 109953.54

GB = .99999

PBC = .89259

RBC = .98629

RYC = .79052

SKYC = .91628

APPENDIX D

BUCKLING OF THIN CYLINDERS UNDER*

UNIFORM LATERAL LOADING

This Appendix presents a comparison of buckling loads for thin circular cylindrical shell based on different shell theories. This comparison includes three types of behavior of the lateral loading; 1) load normal to deflected surface (true pressure behavior); 2) load remaining constant-directional, and 3) load acting always toward initial center of curvature. The comparison covers the entire range of cylinder fineness ratios ($L/\pi R$) and the practical range of radius to thickness ratios. The primary conclusion of this work is that previous belief about the inaccuracy of the Donnell Equations for long cylinders is incorrect.

*This work is published in the form of a brief note in transaction of the ASME, Journal of Applied Mechanics, Vol. 41, No. 3, September 1974, pp. 827-829.

Nomenclature

D	Flexural Stiffness
E	Young's modulus of elasticity
H	Radius to thickness ratio
h	Thickness of shell
k_y	Applied load coefficient [$= qR^3/D$]
L	Length of shell
M_{xx}, M_{yy}, M_{xy}	Moment resultants
m	Number of longitudinal half waves
N_{xx}, N_{yy}, N_{xy}	Incremental stress resultants
n	Number of circumferential waves
q	Initial normal surface loading (positive outward)
q^x, q^y, q^z	Corrections to surface loading due to load behavior
R	Radius of shell
u, v, w	Incremental displacements
x, y	Lines of curvature coordinates
β	[$= L/\pi R$]
ϵ_x, ϵ_y, γ_{xy}	Incremental membrane strains
ν	Poisson's ratio
φ_x, φ_y, φ	Incremental rotations

Introduction

Donnell's equations defining small deformations of thin walled circular cylindrical shells have widely been used in the solution of problems of equilibrium and stability. From time to time doubt has been raised as to the accuracy of these equations. Hoff [1]* in 1955 compared and gave the range of basic parameters for which solutions to Donnell's and Flügge's equations are approximately equal. Dym [2] in 1973 compared buckling results obtainable from Donnell's equations with those obtained from Koiter-Budiansky [3-4] equations for cylinders in axial compression. The aim of the present work is to examine the accuracy obtainable from these equations for buckling of cylinders subjected to uniform lateral load.

As a basis of comparison, buckling loads obtained from Koiter and Budiansky's equations are used. Donnell's equations are much easier to solve than the Koiter and Budiansky's equations. They are, therefore, preferable in engineering applications if their accuracy is satisfactory. In order to have the complete picture, the comparison includes results based on Sanders [4-5] equations and the Von Mises [5] solution of Flügge's [6] equations. The Sanders equations are used with the assumption that the rotations about the normal are negligibly small.

The comparison is performed for large ranges of cylinder fineness ratios $[1/3 \leq L/\pi R \leq \infty]$ and radius to thickness ratios $[25 < R/h < 1000]$. In addition, the effect of load behavior during

*Numbers in square brackets designate references at the end of this Appendix.

the buckling process has been taken into account by studying the following three cases:

I. Load normal to the deflected surface (true pressure behavior)

II. Load remaining parallel to the original direction (a load model that has been used by many investigators for pressure buckling).

III. Load directed toward the original center of curvature.

The Equations of Koiter-Budiansky, Sanders, and Donnell

The Koiter-Budiansky buckling equations have been deduced from those given in the Appendix to Budiansky's paper. In terms of stress resultants, for a circular cylinder loaded by uniform pressure, q, which remains normal to the deflected surface, these equations are

$$N_{xx,x} + N_{xy,y} - \frac{1}{2R} M^{*}_{xy,y} + qR\left(\frac{1}{2}\gamma_{xy} - \varphi_{,y}\right)^{*} + q\varphi_x + q^x = 0$$

$$N_{xy,x} + N_{yy,y} + \frac{1}{R}\left(M_{yy,y} + M_{xy,x}\right)^{**} + \frac{1}{2R} M^{*}_{xy,x}$$

$$+ qR\left(\epsilon^{*}_{y,y} - \frac{\varphi^{**}_{y}}{R}\right) + q\varphi_y + q^y = 0$$

$$-\frac{N_{yy}}{R} + M_{xx,xx} + 2M_{xy,xy} + M_{yy,yy} - qR\left(\varphi_{y,y} + \frac{\epsilon^{*}_{y}}{R}\right)$$

$$+ q(\epsilon_x + \epsilon_y)^{*} + q^z = 0 \qquad \text{(D1)}$$

Here q^x, q^y, and q^z are corrections to surface loading, due to load behavior, being given by the following expressions for the three load cases

$$\text{I.} \qquad q^x = q^y = q^z = 0$$

$$\text{II.} \qquad q^x = qw,_x; \quad q^y = q\left(w,_y - \frac{\overset{**}{v}}{R}\right); \quad q^z = 0$$

$$\text{III.} \qquad q^x = qw,_x; \quad q^y = qw,_y; \quad q^z = 0 \tag{D2}$$

The relations between the stress and moment resultants on one hand and deformation components on the other are given below:

$$N_{xx} = \frac{Eh}{1-\nu^2}\left(u,_x + \nu w,_y + \nu\frac{w}{R}\right)$$

$$N_{yy} = \frac{Eh}{1-\nu^2}\left(v,_y + \frac{w}{R} + \nu u,_x\right)$$

$$N_{xy} = \frac{Eh}{2(1+\nu)}\left(u,_y + v,_x\right)$$

$$M_{xx} = \frac{Eh^3}{12(1-\nu^2)}\left(-w,_{xx} - \nu w,_{yy} + \nu\frac{\overset{**}{v},_y}{R}\right)$$

$$M_{xy} = \frac{Eh^3}{12(1+\nu)}\left[-w,_{yy} + \frac{\overset{**}{v},_x}{2R} + \left(\frac{v,_x}{4R} - \frac{u,_y}{4R}\right)^*\right]$$

$$M_{yy} = \frac{Eh^3}{12(1-\nu^2)}\left(w,_{yy} + \frac{\overset{**}{v},_y}{R} - \nu w,_{xx}\right) \tag{D3}$$

The corresponding boundary conditions are (at $x = 0$ and $x = L$)

$$u = 0 \quad \text{or} \quad N_{xx} = 0$$

$$v = 0 \quad \text{or} \quad N_{xy} + \frac{3}{2R} M_{xy} = 0$$

$$w = 0 \quad \text{or} \quad M_{xx,x} + 2M_{xy,y} = 0$$

$$w,_{x} = 0 \quad \text{or} \quad M_{xx} = 0 \tag{D4}$$

In the present investigation, the classical simply supported boundary conditions are used. These are

$$w(0,y) = w(L,y) = 0$$

$$M_{xx}(0,y) = M_{xx}(L,y) = 0$$

$$N_{xx}(0,y) = N_{xx}(L,y) = 0$$

$$v(0,y) = v(L,y) = 0 \tag{D5}$$

If terms marked with single asterisk are dropped, Equation (D1) through (D3) will give Sanders equations. In the same way if the terms marked with either single or double asterisk are dropped in these equations, one obtains Donnell's equations. The same convention will be followed throughout this paper.

The buckling Equation (D1) are expressed entirely in terms of displacements by employing Equation (D3). Using the convention discussed above, the equations in terms of displacements for all the three theories are given by Equation (D6). These equations take into account all the load cases also. The elements in the column matrices correspond to cases I, II and III, respectively.

$$u,_{xx} + \left(\frac{1-\nu}{2} + \frac{1-\nu^{*}}{96H^2} + \frac{k_y^{*}}{12H^2}\right)u,_{yy} + \left(\frac{1+\nu}{2} - \frac{1-\nu^{*}}{32H^2}\right)v,_{xy} + \frac{\nu}{R}\, w,_x$$

$$+ \frac{1-\nu}{24H^2}\, w,_{xyy}^{*} - \frac{k_y}{12H^2} \begin{pmatrix}1\\0\\0\end{pmatrix} = 0$$

$$\left(\frac{1+\nu}{2} - \frac{1-\nu^{*}}{32H^2}\right) u,_{xy} + \frac{1-\nu}{2}\left(1 + \frac{1^{**}}{12H^2} + \frac{5^{*}}{48H^2}\right)v,_{xx}$$

$$+ \left(1 + \frac{1^{**}}{12H^2} + \frac{k_y^{*}}{12H^2}\right)v,_{yy} - \frac{k_y}{12H^2 R}\, v \begin{pmatrix}0\\1\\0\end{pmatrix}^{**}$$

$$+ \frac{w,_y}{R} - \frac{R}{12H^2}\, w,_{yyy}^{**}$$

$$- \left(\frac{R^{**}}{12H^2} + \frac{1-\nu}{2}\frac{R^{*}}{12H^2}\right)w,_{xxy} - \frac{k_y}{12H^2}$$

$$- \frac{w,_y}{R} \left[\begin{pmatrix}1\\0\\0\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix}^{*} - \begin{pmatrix}1\\1\\1\end{pmatrix}^{**}\right] = 0$$

$$\left(\nu - \frac{k_y}{12H^2}\right)u,_x + \left(\frac{1-\nu}{24H^2}\right)R^2 u,_{xyy}^{*} + \left(1 + \frac{k_y^{**}}{12H^2}\right)v,_y - \frac{R^2}{12H^2}\, v,_{yyy}^{**}$$

$$- \frac{R^2}{12H^2}\left(1^{**} + \frac{1-\nu^{*}}{2}\right)v,_{xxy} + \frac{R^3}{12H^2}\nabla^4 w - \frac{Rk_y}{12H^2}\, w,_{yy}$$

$$+ \frac{w}{R} = 0 \qquad \text{(D6)}$$

Where

$$k_y = qR^3/D \quad ,$$

$$H = R/h$$

$$D = Eh^3/12(1-\nu^2) \tag{D7}$$

Solution and Results

The solution to Equation (D6) satisfying boundary conditions (D5) is given by

$$u = A_{mn} \cos \frac{m\pi x}{L} \cos \frac{ny}{R}$$

$$v = B_{mn} \sin \frac{m\pi x}{L} \sin \frac{ny}{R} \qquad n \geq 2$$

$$w = C_{mn} \sin \frac{m\pi x}{L} \cos \frac{ny}{R} \tag{D8}$$

Substitution of Equation (D8) into the differential Equation (D6) yields, with the usual arguments, the characteristic equation

$$\begin{vmatrix} a_{11} + \frac{k_y}{12H^2}\frac{n^{2*}}{m^2} & a_{12} & a_{13} - \frac{k_y}{12H^2\beta}\begin{pmatrix}1\\0\\0\end{pmatrix} \\ a_{12} & a_{22} + \frac{k_y n^{2*}}{12H^2} + \frac{k_y}{12H^2}\begin{pmatrix}0\\1\\0\end{pmatrix}^{**} & a_{23} - \frac{k_y n}{12H^2}\left[\begin{pmatrix}1\\0\\0\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix}^{*} - \begin{pmatrix}1\\1\\1\end{pmatrix}^{**}\right] \\ a_{13} + \frac{k_y^*}{12H^2\beta} & a_{23} + \frac{k_y n^*}{12H^2} & a_{33} + \frac{k_y n^2}{12H^2} \end{vmatrix} = 0 \quad \text{(D9)}$$

Where $\beta = L/\pi R$

$$a_{11} = \frac{1}{\beta^2} + \left(\frac{1-\nu}{2} + \frac{1-\nu}{96H^2}^*\right)\frac{n^2}{m^2}$$

$$a_{12} = \left(\frac{1+\nu}{2} - \frac{1-\nu}{32H^2}^*\right)\frac{n}{\beta}$$

$$a_{13} = \frac{\nu}{\beta} - \frac{1-\nu}{24H^2}\frac{n^2}{\beta}$$

$$a_{22} = \frac{1-\nu}{2}\left(1 + \frac{1}{12H^2}^{**} + \frac{5}{48H^2}^*\right)\frac{m^2}{\beta^2} + \left(1 + \frac{1}{12H^2}^*\right)n^2$$

$$a_{23} = n + \frac{n^3}{12H^2}^{**} + \frac{m^2n}{12H^2\beta^2}^{**} + \frac{1-\nu}{24H^2}\frac{m^2n}{\beta^2}^*$$

$$a_{33} = \frac{1}{12H^2}\left(\frac{m^2}{\beta^2} + n^2\right)^2 + 1 \tag{D10}$$

The characteristic equation is cubic for Koiter-Budiansky theory, it is linear for Donnell's theory and quadratic for Sanders theory except for the load case-II which yields linear equation. Buckling load parameter k_{ycr} is found through minimization with respect to integer values of m and n. The results are given in Tables D1 through D3. The plots, showing the effect of $L/\pi R$ and R/h on buckling load parameter k_{ycr} are given for Koiter-Budiansky's theory.

Infinitely Long Cylinder

When the length of the cylinder approaches infinity, the characteristic determinant reduces to

$$\begin{vmatrix} \bar{a}_{11} & 0 & 0 \\ 0 & \bar{a}_{22} + \bar{b}_{22} & \bar{a}_{23} - \bar{b}_{23} \\ 0 & \bar{a}_{23} + \frac{k_y n^2}{12H^2}^{*} & \bar{a}_{33} + \frac{k_y n^2}{12H^2} \end{vmatrix} \qquad \text{(D11)}$$

Where

$$\bar{a}_{11} = \left(\frac{1-\nu}{2} + \frac{1-\nu}{96H^2}^{*}\right)\frac{n^2}{m^2} \qquad \bar{a}_{23} = n + \frac{n^3}{12H^2}^{**}$$

$$\bar{a}_{22} = \left(1 + \frac{1}{12H^2}^{*}\right) n^2 \qquad \bar{a}_{33} = \frac{n^4}{12H^2} + 1$$

$$\bar{b}_{22} = \frac{k_y}{12H^2}^{**}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \frac{k_y n^2}{12H^2}^{*}$$

$$\bar{b}_{23} = \frac{k_y n}{12H^2}\left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}^{*} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}^{**}\right] \qquad \text{(D12)}$$

Table D1. Comparison of Critical Pressures for Load Case I (Load Remaining Normal to the Deflected Surface).

Table D2. Comparison of Critical Pressures for Load Case II (Load Remaining Parallel to Original Direction).

$\frac{L}{\pi R}$	$\frac{R}{h}$	Table D1: $-k_{y_{cr}}$ (m,n)				Table D2: $-k_{y_{cr}}$ (m,n)		
		BUDIANSKY-KOITER EQS.	SANDERS EQS. ($\varphi=0$)	DONNELL EQS.	FLUGGE EQS.	BUDIANSKY-KOITER EQS.	SANDERS EQS.	DONNELL EQS.
1/3		63.0808 (1,6)	63.2779 (1,6)	62.2779 (1,6)	56.8344	64.6432 (1,6)	64.8289 (1,6)	63.7369 (1,6)
1		18.5825 (1,4)	18.6763 (1,4)	16.8466 (1,3)	18.3576	19.7704 (1,4)	19.8677 (1,4)	18.5911 (1,3)
3		4.85714 (1,2)	4.90818 (1,2)	4.37979 (1,2)	4.81969	6.43292 (1,2)	6.50339 (1,2)	5.45461 (1,2)
4		3.64562 (1,2)	3.67395 (1,2)	3.62703 (1,2)	3.63389	4.84230 (1,2)	4.88133 (1,2)	4.52430 (1,2)
5		3.29146 (1,2)	3.30966 (1,2)	3.40090 (1,2)	3.28612	4.37790 (1,2)	4.40290 (1,2)	4.24540 (1,2)
6	25	3.15581 (1,2)	3.16848 (1,2)	3.31170 (1,2)	3.15280	4.20074 (1,2)	4.21800 (1,2)	4.13574 (1,2)
9		3.04256 (1,2)	3.04844 (1,2)	3.23388 (1,2)	3.04180	4.04399 (1,2)	4.06175 (1,2)	4.04066 (1,2)
12		3.01857 (1,2)	3.02203 (1,2)	3.21590 (1,2)	3.01832	4.02345 (1,2)	4.02778 (1,2)	4.01893 (1,2)
15		3.01016 (1,2)	3.01255 (1,2)	3.20924 (1,2)	3.01018	4.01289 (1,2)	4.01572 (1,2)	4.01094 (1,2)
18		3.00642 (1,2)	3.00811 (1,2)	3.20607 (1,2)	3.00648	4.00822 (1,2)	4.01011 (1,2)	4.00717 (1,2)
100		3.000 (1,2)	3.000 (1,2)	3.200 (1,2)	3.0000	4.000 (1,2)	4.000 (1,2)	4.000 (1,2)
1/3		70.5175 (1,6)	70.6990 (1,6)	69.2578 (1,6)	64.2228	72.2586 (1,6)	72.4332 (1,6)	70.9200 (1,6)
1		20.0807 (1,4)	20.1739 (1,4)	19.7320 (1,4)	19.8485	21.3630 (1,4)	21.4609 (1,4)	20.9160 (1,4)
3		6.44560 (1,2)	6.49959 (1,2)	5.32740 (1,2)	6.39328	8.53618 (1,2)	8.61205 (1,2)	6.63478 (1,2)
4		4.15841 (1,2)	4.18717 (1,2)	3.93360 (1,2)	4.14377	5.52306 (1,2)	5.56322 (1,2)	4.90672 (1,2)
5		3.50361 (1,2)	3.52174 (1,2)	3.40090 (1,2)	3.49730	4.65973 (1,2)	4.68504 (1,2)	4.40376 (1,2)
6	35	3.25877 (1,2)	3.27125 (1,2)	3.37319 (1,2)	3.25526	4.33747 (1,2)	4.35481 (1,2)	4.21254 (1,2)
9		3.06322 (1,2)	3.06884 (1,2)	3.24160 (1,2)	3.06219	4.08124 (1,2)	4.06175 (1,2)	4.04066 (1,2)
12		3.02521 (1,2)	3.02846 (1,2)	3.21977 (1,2)	3.02479	4.03202 (1,2)	4.02778 (1,2)	4.01893 (1,2)
15		3.01306 (1,2)	3.01518 (1,2)	3.21080 (1,2)	3.01283	4.01647 (1,2)	4.01572 (1,2)	4.01094 (1,2)
18		3.00796 (1,2)	3.00936 (1,2)	3.20684 (1,2)	3.00776	4.00822 (1,2)	4.01011 (1,2)	4.00717 (1,2)
100		3.0000 (1,2)	3.0000 (1,2)	3.2000 (1,2)	3.0000	4.000 (1,2)	4.000 (1,2)	4.000 (1,2)
1/3		81.2253 (1,7)	81.4036 (1,7)	80.4260 (1,7)	76.5172	82.7387 (1,7)	82.9154 (1,7)	81.8942 (1,7)
1		23.2624 (1,4)	23.3564 (1,4)	22.5340 (1,4)	23.0165	24.7470 (1,4)	24.8465 (1,4)	23.8860 (1,4)
3		8.71346 (1,3)	8.73450 (1,3)	7.34105 (1,2)	8.70401	9.79229 (1,3)	9.81609 (1,3)	9.14258 (1,2)
4		5.24750 (1,2)	5.27774 (1,2)	4.58502 (1,2)	5.22727	6.96932 (1,2)	7.01218 (1,2)	5.71928 (1,2)
5		3.95422 (1,2)	3.97247 (1,2)	3.52776 (1,2)	3.94605	5.25875 (1,2)	5.28465 (1,2)	4.74041 (1,2)
6	50	3.47728 (1,2)	3.48967 (1,2)	3.50387 (1,2)	3.47299	4.62809 (1,2)	4.64558 (1,2)	4.37573 (1,2)
9		3.10690 (1,2)	3.11219 (1,2)	3.27216 (1,2)	3.10553	4.13924 (1,2)	4.08892 (1,2)	4.05593 (1,2)
12		3.03909 (1,2)	3.04214 (1,2)	3.22793 (1,2)	3.03854	4.05034 (1,2)	4.03636 (1,2)	4.02376 (1,2)
15		3.01879 (1,2)	3.02075 (1,2)	3.21413 (1,2)	3.01847	4.02392 (1,2)	4.01922 (1,2)	4.01289 (1,2)
18		3.01086 (1,2)	3.01205 (1,2)	3.20849 (1,2)	3.01049	4.01354 (1,2)	4.01177 (1,2)	4.00813 (1,2)
100		3.0000 (1,2)	3.0000 (1,2)	3.2000 (1,2)	3.0000	4.000 (1,2)	4.000 (1,2)	4.000 (1,2)

Table D1. (Continued)

1/3	100	108.2980 (1,8)	108.4540 (1,8)	107.3240 (1,8)	104.617
1		32.6608 (1,5)	32.7237 (1,5)	32.1607 (1,5)	32.5114
3		10.2176 (1,3)	10.2393 (1,3)	9.91128(1,3)	10.2058
4		8.75986(1,3)	8.77140(1,3)	8.41691(1,2)	8.75541
5		6.60377(1,2)	6.62386(1,2)	5.38345(1,2)	6.58579
6		4.76085(1,2)	4.77390(1,2)	4.27270(1,2)	4.75375
9		3.36258(1,2)	3.36719(1,2)	3.42523(1,2)	3.36047
12		3.11991(1,2)	3.12235(1,2)	3.27610(1,2)	3.11942
15		3.05276(1,2)	3.05401(1,2)	3.23405(1,2)	3.05164
18		3.02692(1,2)	3.02740(1,2)	3.21803(1,2)	3.02649
100		3.0000 (1,2)	3.0000 (1,2)	3.2000 (1,2)	3.000
1/3	200	147.4830 (1,10)	147.6010 (1,10)	146.8130 (1,10)	145.089
1		46.0042 (1,6)	46.0491 (1,6)	45.5284 (1,6)	45.9022
3		16.2343 (1,3)	16.2578 (1,3)	14.7208 (1,3)	16.2129
4		10.6670 (1,3)	10.6938 (1,3)	10.2667 (1,3)	10.6766
5		9.13090(1,3)	9.13899(1,3)	9.01871(1,3)	9.12823
6		8.56219(1,3)	8.56709(1,3)	7.34714(1,2)	8.56120
9		4.38325(1,2)	4.38558(1,2)	4.03694(1,2)	4.38023
12		3.44510(1,2)	3.44548(1,2)	3.46877(1,2)	3.44295
15		3.18545(1,2)	3.18513(1,2)	3.31345(1,2)	3.18432
18		3.09366(1,2)	3.09109(1,2)	3.25589(1,2)	3.09052
100		3.0000 (1,2)	3.0000 (1,2)	3.2000 (1,2)	3.000
1/3	400	203.2650 (1,12)	203.2650 (1,12)	202.6430 (1,12)	201.584
1		64.3705 (1,7)	64.4039 (1,7)	63.8050 (1,7)	64.2952
3		20.6862 (1,4)	20.6942 (1,4)	20.0935 (1,4)	20.6762
4		16.8613 (1,4)	16.8657 (1,4)	16.4148 (1,3)	16.8589
5		12.2990 (1,3)	12.3050 (1,3)	11.9582 (1,3)	12.2915
6		10.0955 (1,3)	10.1005 (1,3)	9.78543(1,3)	10.0908
9		8.46744(1,2)	8.46824(1,2)	6.48378(1,2)	8.45927
12		4.75188(1,2)	4.73146(1,2)	4.24178(1,2)	4.73706
15		3.72294(1,2)	3.71222(1,2)	3.63218(1,2)	3.71506
18		3.35368(1,2)	3.33673(1,2)	3.40962(1,2)	3.34664
100		3.0000 (1,2)	3.0000 (1,2)	3.20000(1,2)	3.000
1/3	1000	313.523 (1,15)	313.569 (1,15)	312.8530 (1,15)	312.420
1		102.0960 (1,9)	102.0960 (1,9)	101.5460 (1,9)	101.983
3		33.0159 (1,5)	33.0000 (1,5)	32.3756 (1,5)	33.0023
4		26.0072 (1,4)	25.9950 (1,4)	24.7614 (1,4)	25.9913
5		19.5872 (1,4)	19.5433 (1,4)	19.0727 (1,4)	19.5422
6		17.2362 (1,4)	17.2028 (1,4)	16.9998 (1,4)	17.2100
9		10.5828 (1,3)	10.5247 (1,3)	10.1401 (1,3)	10.5494
12		8.82115(1,3)	8.79812(1,3)	8.75963(1,3)	8.81724
15		7.40[illegible](1,2)	7.39791(1,2)	5.83[illegible](1,2)	7.43017
18		5.21707(1,2)	5.20652(1,2)	4.4[illegible](1,2)	5.13949
100		3.0000 (1,2)	3.0000 (1,2)	3.2000 (1,2)	3.000

Table D2. (Continued)

1/3	100	109.871 (1,8)	110.028 (1,8)	108.860 (1,8)
1		33.9841 (1,5)	34.0499 (1,5)	33.4135 (1,5)
3		11.4700 (1,3)	11.5072 (1,3)	11.0033 (1,3)
4		9.84883(1,3)	9.86203(1,3)	9.69453(1,3)
5		8.78166(1,2)	8.81186(1,2)	6.72025(1,2)
6		6.33591(1,2)	6.35543(1,2)	5.33586(1,2)
9		4.47945(1,2)	4.48645(1,2)	4.27975(1,2)
12		4.15773(1,2)	4.16150(1,2)	4.09416(1,2)
15		4.06883(1,2)	4.07099(1,2)	4.04195(1,2)
18		4.03407(1,2)	4.03583(1,2)	4.02211(1,2)
100		4.000 (1,2)	4.000 (1,2)	4.000 (1,2)
1/3	200	148.888 (1,10)	149.007 (1,10)	148.198 (1,10)
1		47.2936 (1,6)	47.3399 (1,6)	46.7699 (1,6)
3		17.6832 (1,4)	17.6959 (1,4)	16.3429 (1,3)
4		12.0100 (1,3)	12.0235 (1,3)	11.4021 (1,3)
5		10.2678 (1,3)	10.2766 (1,3)	10.0178 (1,3)
6		9.62928(1,3)	9.63601(1,3)	9.17532(1,2)
9		5.83791(1,2)	5.84335(1,2)	5.04407(1,2)
12		4.59019(1,2)	4.59216(1,2)	4.33495(1,2)
15		4.24453(1,2)	4.24578(1,2)	4.14118(1,2)
18		4.11986(1,2)	4.12073(1,2)	4.06943(1,2)
100		4.000 (1,2)	4.000 (1,2)	4.000 (1,2)
1/3	400	204.628 (1,12)	204.719 (1,12)	203.994 (1,12)
1		65.6927 (1,7)	65.7271 (1,7)	65.0892 (1,7)
3		22.0579 (1,4)	22.1080 (1,4)	21.3390 (1,4)
4		17.9821 (1,4)	17.9865 (1,4)	17.7537 (1,4)
5		13.8294 (1,3)	13.8375 (1,3)	12.8285 (1,3)
6		11.3547 (1,3)	11.3593 (1,3)	10.8704 (1,3)
9		9.48360(1,2)	9.45811(1,2)	8.10134(1,2)
12		6.32005(1,2)	6.30613(1,2)	5.30097(1,2)
15		4.95645(1,2)	4.94838(1,2)	4.53954(1,2)
18		4.45386(1,2)	4.44820(1,2)	4.26157(1,2)
100		4.000 (1,2)	4.000 (1,2)	4.000 (1,2)
1/3	1000	314.916 (1,15)	314.932 (1,15)	314.207 (1,15)
1		103.342 (1,9)	103.311 (1,9)	102.789 (1,9)
3		34.4050 (1,5)	34.4660 (1,5)	33.6668 (1,5)
4		27.7359 (1,4)	27.7231 (1,4)	26.3049 (1,4)
5		20.8541 (1,4)	20.8439 (1,4)	20.4319 (1,4)
6		18.3556 (1,4)	18.3407 (1,4)	18.0611 (1,4)
9		11.8620 (1,3)	11.8309 (1,3)	11.2230 (1,3)
12		9.92311(1,3)	9.89723(1,3)	9.7324 (1,3)
15		9.35260(1,3)	9.33145(1,3)	7.29077(1,2)
18		6.8[illegible]41(1,2)	6.78040(1,2)	5.5[illegible]03(1,2)
100		4.000 (1,2)	4.000 (1,2)	4.000 (1,2)

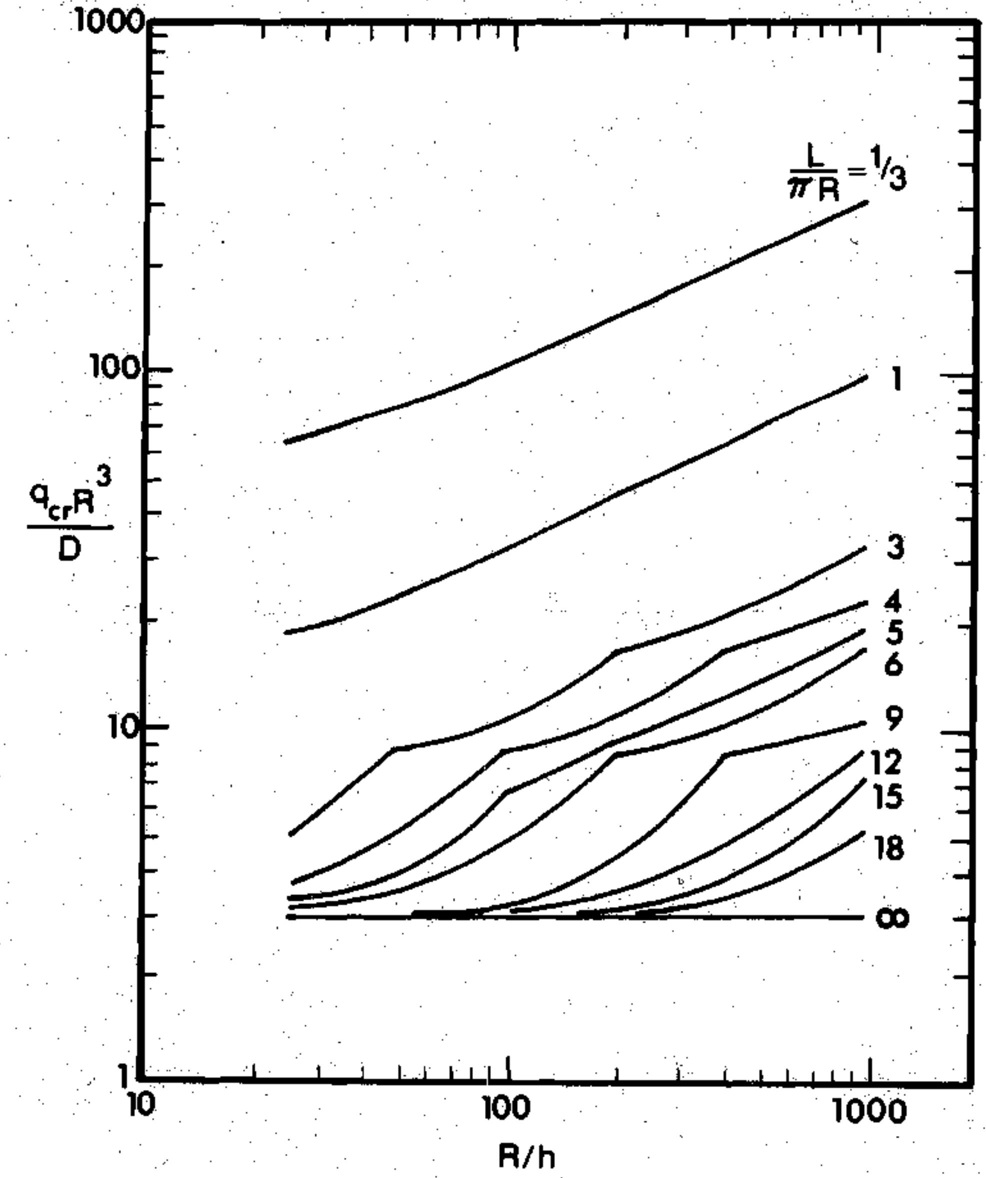

Figure D1. Effect of R/h and L/πR on Buckling Load Koiter-Budiansky Equations. Load Case I.

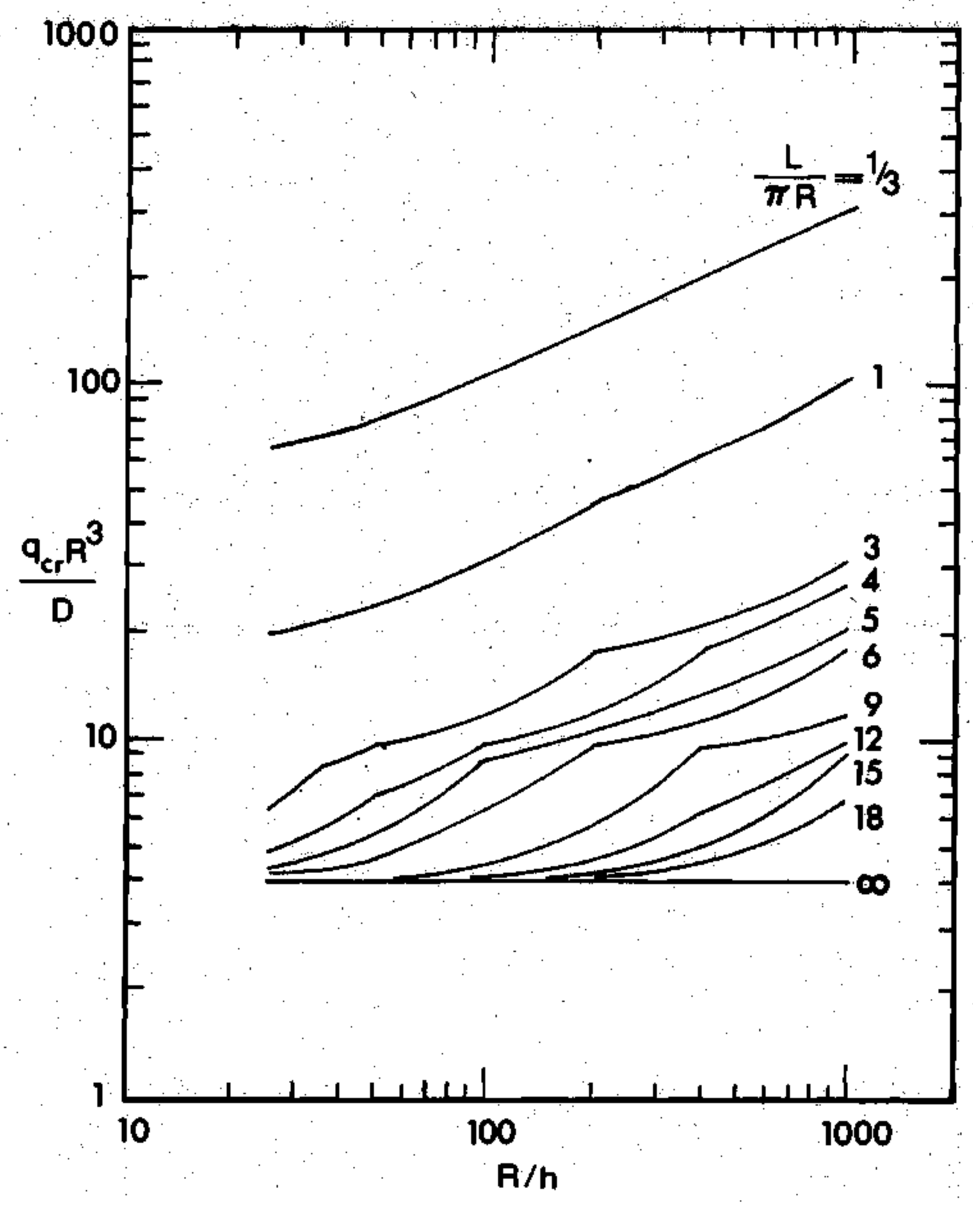

Figure D2. Effect of R/h and L/πR on Buckling Load Koiter-Budiansky Equations. Load Case II.

k_{ycr} is obtained through minimization with respect to integer values of m and n. For this particular case m = 1 and n = 2 lead to the critical load parameters. The values for the three load cases and the three theories are given in Table D4. An order analysis was performed to arrive at these values which are independent of R/H.

Discussion of Results

The characteristic Equation (D9) is solved numerically for all three shell theories (Koiter-Budiansky, Sanders, and Donnell) and for all three load cases through the UNIVAC 1108 High Speed Digital Computer. The results are presented in a tabular form in Tables D1 through D3 and graphically in Figures D2 through D5. In addition to the computed data the results of the Von Mises solution are presented in Table D1 for comparison purposes.

The comparison shows that for all the load cases and the entire range of the parameters considered ($L/\pi R$ and R-h) the results due to the Koiter-Budiansky and Sanders shell theories are virtually the same. The discrepancy is less than 1 percent. If the Donnell results are compared to those of the Koiter-Budiansky theory some discrepancies are observed. For load case I, it is seen from Table 1, that for each R/h value, the Donnell result is smaller than the Koiter-Budiansky result for small values of $L/\pi R$. Depending on the value of R/h as $L/\pi R$ increases a reversal takes place and the Donnell result is higher, with the discrepancy reaching a 6.7 percent at very large values of $L/\pi R$. For example, at R/h = 25 the reversal takes place somewhere between $L/\pi R$ equal four and five, for R/h = 35 the reversal takes place between $L/\pi R$ equal five and six, and in general the value

Table D3. Comparison of Critical Pressures for Load Case III.

(Load Directed Toward Original Center of Curvature)

$\frac{L}{\pi R}$	$\frac{R}{h}$	$-ky_{cr}$ (m,n) BUDIANSKY-KOITER EQS.	$-ky_{cr}$ (m,n) SANDERS EQS.	$\frac{R}{h}$	$-ky_{cr}$ (m,n) BUDIANSKY-KOITER EQS.	$-ky_{cr}$ (m,n) SANDERS EQS.
1/3	35	72.3192 (1,6)	72.4935 (1,6)	200	148.903 (1,10)	149.023 (1,10)
1		21.4614 (1,4)	21.5596 (1,4)		47.3328 (1,6)	47.3792 (1,6)
3		9.62484 (1,2)	9.68116 (1,2)		17.7625 (1,4)	17.7750 (1,4)
4		6.22179 (1,2)	6.26798 (1,2)		12.2014 (1,4)	12.2153 (1,3)
5		5.24687 (1,2)	5.27586 (1,2)		10.4321 (1,3)	10.4411 (1,3)
6		4.88276 (1,2)	4.90255 (1,2)		9.78245 (1,3)	9.78873 (1,3)
9		4.59280 (1,2)	4.60145 (1,2)		6.56944 (1,2)	6.57589 (1,2)
12		4.53688 (1,2)	4.54166 (1,2)		5.16476 (1,2)	5.16713 (1,2)
15		4.51913 (1,2)	4.52208 (1,2)		4.77557 (1,2)	4.77706 (1,2)
18		4.51165 (1,2)	4.51354 (1,2)		4.63516 (1,2)	4.63620 (1,2)
100		4.5000 (1,2)	4.5000 (1,2)		4.5000 (1,2)	4.5000 (1,2)
1/3	50	82.7761 (1,7)	82.9527 (1,7)	400	204.639 (1,12)	204.730 (1,12)
1		24.8608 (1,4)	24.9608 (1,4)		65.7290 (1,7)	65.7560 (1,7)
3		9.94901 (1,3)	9.97321 (1,3)		22.1543 (1,4)	22.1659 (1,4)
4		7.85087 (1,2)	7.90059 (1,2)		18.0626 (1,4)	18.0674 (1,4)
5		5.92125 (1,2)	5.95115 (1,2)		14.0496 (1,3)	14.0583 (1,3)
6		5.20982 (1,2)	5.22995 (1,2)		11.5353 (1,3)	11.5409 (1,3)
9		4.65799 (1,2)	4.66649 (1,2)		9.63450 (1,3)	9.63220 (1,3)
12		4.55741 (1,2)	4.68253 (1,2)		7.11114 (1,2)	7.09569 (1,2)
15		4.52744 (1,2)	4.53047 (1,2)		5.57656 (1,2)	5.56759 (1,2)
18		4.51562 (1,2)	4.51761 (1,2)		5.01094 (1,2)	5.00464 (1,2)
100		4.5000 (1,2)	4.5000 (1,2)		4.5000 (1,2)	4.5000 (1,2)
1/3	100	109.900 (1,8)	110.057 (1,8)	1000	314.880 (1,15)	314.938 (1,15)
1		34.0448 (1,5)	34.1104 (1,5)		103.353 (1,9)	103.327 (1,9)
3		11.6661 (1,3)	11.6914 (1,3)		34.4646 (1,5)	34.4488 (1,5)
4		10.0059 (1,3)	10.0193 (1,3)		27.8601 (1,4)	27.8472 (1,4)
5		9.52749 (1,3)	9.53651 (1,3)		20.9430 (1,4)	20.9371 (1,4)
6		7.13222 (1,2)	7.15462 (1,2)		18.4372 (1,3)	18.4302 (1,3)
9		5.04076 (1,2)	5.04888 (1,2)		12.0352 (1,3)	12.0270 (1,3)
12		4.67818 (1,2)	4.68253 (1,2)		10.0807 (1,3)	10.0544 (1,3)
15		4.57790 (1,2)	4.58040 (1,2)		9.50111 (1,3)	9.4608 (1,3)
18		4.53865 (1,2)	4.54067 (1,2)		7.70723 (1,2)	7.62858 (1,2)
100		4.5000 (1,2)	4.5000 (1,2)		4.5000 (1,2)	4.5000 (1,2)

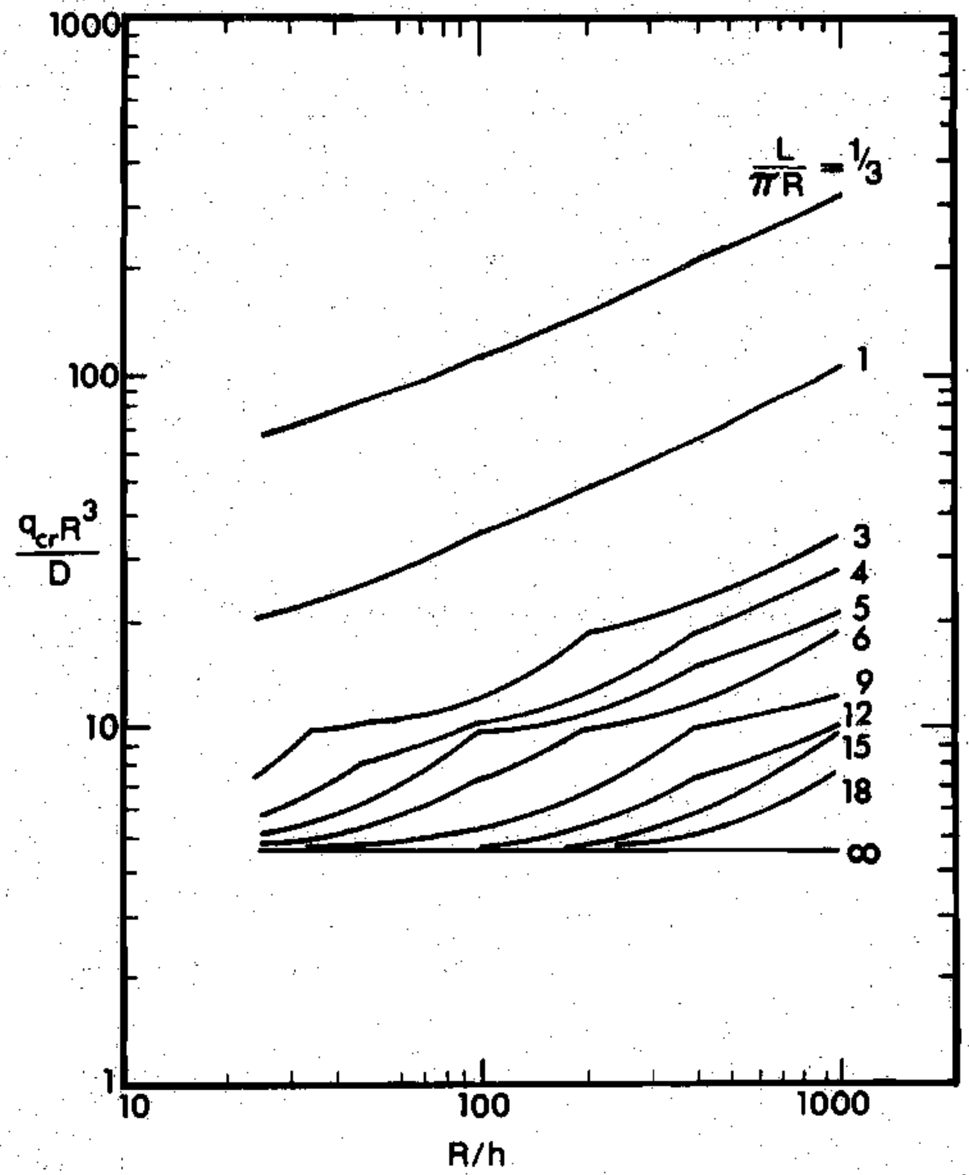

Figure D3. Effect of R/h and L/πR on Buckling Load Koiter-Budiansky Equations. Load Case III.

of $L/\pi R$ at which the reversal takes place increases with increasing R/h. In addition, it is seen that the discrepancy in the two results is appreciable only in a small range of $L/\pi R$ values for each R/h value. For example, at $R/h = 25$, the discrepancy is 12 percent at $L/\pi R = 3$ decreases with further increase in $L/\pi R$, and finally after the reversal takes place it reaches a maximum value of -6.7 percent as $L/\pi R \to \infty$. The value of $L/\pi R$ at which the discrepancy is the largest increases with increasing value of R/h. These critical loads are underlined in Table D1. The largest discrepancies occur at $L/\pi R$ values for which the circumferential mode changes to $n = 2$. But the discrepancy is not affected by the fact that $n = 2$ as seen from increasing values of $L/\pi R$. The maximum discrepancy computed is 24.3 percent at $L/\pi R = 9$ and $R/h = 400$. Finally, it is observed that for practical engineering uses of thin cylindrical shells, especially of the submarine hull type, for which $1 < L/\pi R < 4$, and $100 < R/h < 400$, the accuracy of the Donnell results is very good. It is also observed from Table D1 that the Von Mises solution which is based on Flügge's equations is extremely accurate (discrepancy less than one percent) except for short and relatively thick thin cylindrical shells ($R/h \leq 35$, $L/\pi R < 1$). For these geometries the discrepancy can be as large as 11 percent.

For load case II, the same conclusion and observations are made, based on the data presented in Table D2. There is only one exception, that there is no reversal taking place. The Donnell results are always smaller than the Koiter-Budiansky results and they become virtually identical for very long cylinders.

For load case III no attempt has been made to compare the Donnell results to those of the Koiter-Budiansky theory, because the Donnell equations do not differentiate between load case II and III. Because of this one might say that the Donnell results are in error for this load case.

The plots in Figures D1 through D3 show the effect of R/h and $L/\pi R$ on the critical pressure as obtained from the Koiter-Budiansky theory. It is observed from these plots (Figure D1) and Table D1 that the discrepancy between these results and the Donnell results is the largest when the curves exhibit sharp corners. The same is true for load case II (Figure D2 and Table D2).

Table D4. Comparison of Critical Pressures for Infinitely Long Cylinders.

	$-ky_{cr}$	
DONNELL EQUATIONS	SANDERS EQUATIONS	BUDIANSKY - KOITER EQUATIONS
3.2	3.0	3.0
4.0	4.0	4.0
	4.5	4.5

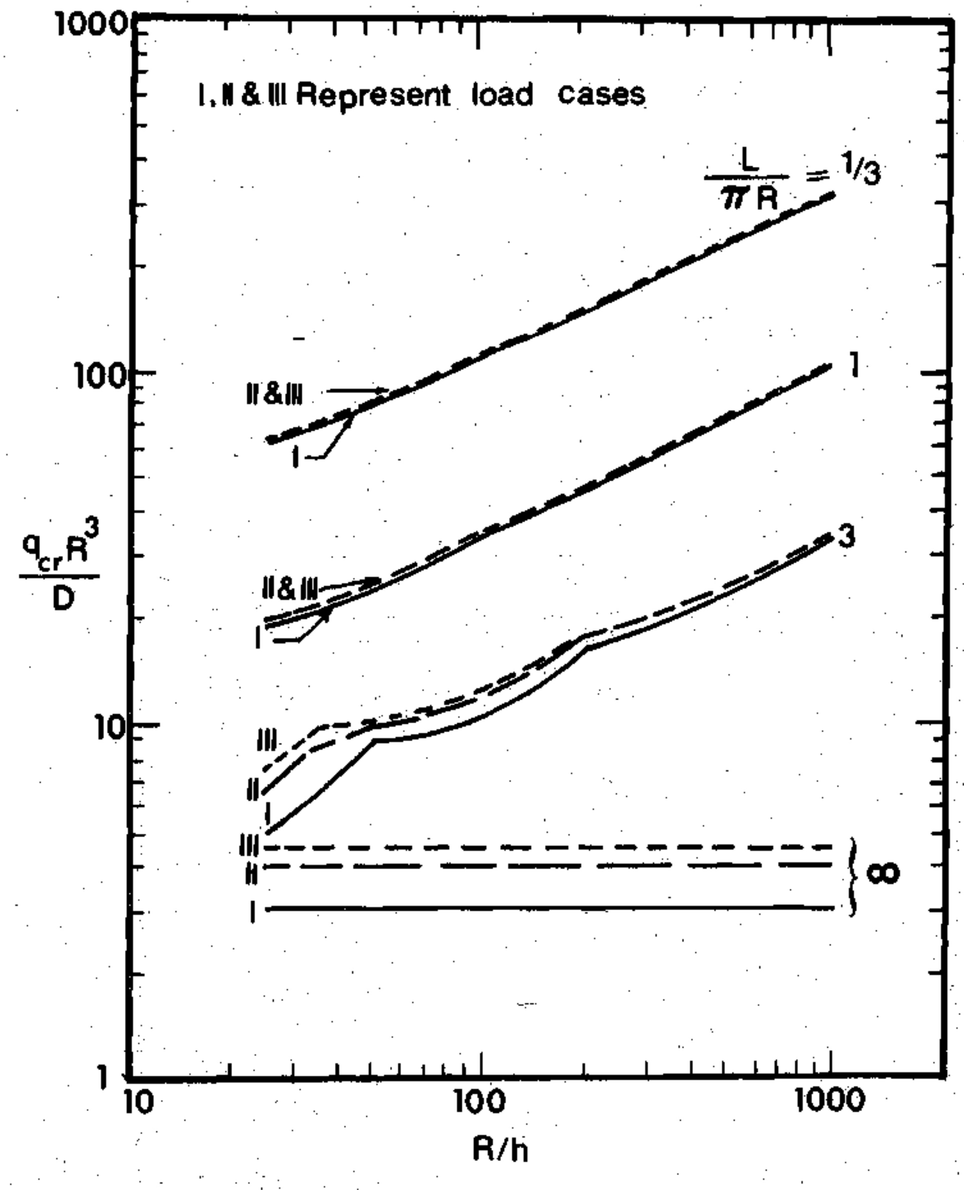

Figure D4. Effect of Load Behavior on Buckling Load Koiter-Budiansky Equations.

Finally, the plots in Figure D4 show the effect of load behavior on the critical pressures for the entire range of R/h and L/πR values. It is observed that for short to moderate length cylinders the difference among the results obtained is not appreciable. As the length increases the difference becomes more pronounced, especially for low R/h values, until for extremely long cylinders the difference reaches its maximum value and it is independent of R'h (see Table D4).

Conclusions

Among the most important conclusions of the present investigation one may list the following:

1. For each type of load behavior, the Donnell equations yield results which are within acceptable engineering tolerances, especially in the practical range of R/h and L/πR values.

2. Contrary to previous belief, the discrepancy is not associated with the number of circumferential waves. (The previous belief is that when n is very low (n=2) the discrepancy is the largest (of the order of 33 percent)). The authors contend that the fallacy of the above belief is attributed to the load behavior model rather than the Donnell equations.

3. Load case II may be used as a mathematical model for pressure buckling for short and moderate length cylinders. For extremely long cylinders this model is inaccurate and leads to overestimates of 33 percent (see Table D4).

References

1. Hoff, N. J., "The Accuracy of Donnell's Equations," Journal of Applied Mechanics, Vol. 22, No. 3, Trans. ASME, Vol. 77, September 1955, pp. 329-334.
2. Dym, C. L., "On the Buckling of Cylinders in Axial Compression," Journal of Applied Mechanics, Vol. 40, No. 2, Trans. ASME, June 1973, pp. 565-568.
3. Koiter, W. T., "General Equations of Elastic Stability for Thin Shells," Muster, D., ed., Proceedings, Symposium on the Theory of Shells, University of Houston, Houston, Texas, 1967.
4. Budiansky, B., "Notes on Nonlinear Shell Theory," Journal of Applied Mechanics, Vol. 35, No. 2, Trans. ASME, Vol. 90, Series E, June 1968, pp. 393-401.
5. Sanders, J. L., "Nonlinear Theories of Thin Shells," Quarterly of Applied Mathematics, Vol. 21, 1963, pp. 21-36.
6. Timoshenko, S. P., and Gere, J. M., Theory of Elastic Stability, McGraw-Hill Book Company, Inc.
7. Flügge, W., Stresses in Shell, Springer-Verlag, Berlin, 1962.

APPENDIX E

LISTING OF COMPUTER PROGRAMS

Following programs are written in this Appendix:

1. MAIN: Nelder and Mead algorithm employed for minimization of objective function formulated on the basis of general instability. The shell is ring-stringer stiffened.

2. MAINY: Nelder and Mead algorithm employed for minimization of objective function formulated on the basis of skin yielding. The shell is ring-stringer stiffened.

3. MAINR: Golden section search technique employed for minimization of objective function formulated on the basis of general instability. The shell is ring stiffened.

4. MAINP: Golden section search technique employed for checking panel buckling.

5. RSSH: The program is written for designing ring stiffened shell based on the results of Phase I.

6. SUBROUTINE START: This sets up the initial simplex for Program MAIN.

7. SUBROUTINE STARTY: This sets up initial simplex for program MAINY.

8. SUBROUTINE WSR: This defines the objective function for program MAIN.

9. SUBROUTINE WSRR: This defines the objective function for

program MAINR.

10. SUBROUTINE WSRY: This defines the objective function for program MAINY.

11. SUBROUTINE GENST: This program finds the general instability critical load parameter treating m and n as discrete variables.

12. SUBROUTINE CKYR: This gives the expression for $\bar{k}_{yy}$.

13. FUNCTION Q: This gives expression for $\bar{k}_{yy}$, treating m as continuous variable.

14. FUNCTION R: This gives expression for $\bar{k}_{yy}$, treating m as an integer.

The list of program variable names and corresponding mathematical notations are given as follows

Program Variable Name	Mathematical Notation
AFA	α
AL	L
ALX	$\bar{\alpha}_x$
ALY	$\bar{\alpha}_y$
AX	A_x
AY	A_y
BX	B_x
BY	B_y
BB	$\bar{\beta}$
BET	β
CX	C_x
CY	C_x

Program Variable Name	Mathematical Notation
DIFER	Standard deviation of $\bar{W}^*$
II	Number of iterations
KYCR	$\bar{k}_{yycr}$
M	m
N	n
PO	ν
QDS	q_D^*
QY	σ^*
RR or R	R
WBAR	$\bar{W}$
WSS or WSL	$\bar{W}^*$ minimum
WS(IN)	$\bar{W}^*$
X(1) or X1(KOUNT,1)	$\bar{\lambda}_{xx}$
X(2) or X1(KOUNT,2)	$\bar{\lambda}_{yy}$
X(3)	λ^*
ZZ or Z	Z

```
ASWANI-M-G*OPTIM.MAIN
C     MINIMIZATION OF WEIGHT OF STIFFENED CIRCULAR
C     CYLINDRICAL SHELL SUBJECT TO HYDROSTATIC
C     PRESSURE AND AXIAL COMPRESSION
C     NX IS NUMBER OF INDEPENDENT VARIABLES
C     STEP IS THE INITIAL STEP SIZE
C     X(I) IS THE ARRAY OF INITIAL GUESSES
C     X(1) IS NONDIMENSIONAL EXTENSIONAL
C     STIFFNESS PARAMETER FOR STRINGER
C     X(2) IS NONDIMENSIONAL EXTENSIONAL
C     STIFFNESS PARAMETER FOR RING
C     ZZ IS CURVATURE PARAMETER

C     M OR EM IS NUMBER OF LONGITUDINAL
C     HALF WAVES

C     QDS IS NONDIMENSIONAL DESIGN

C     LOAD PARAMETER
C     WS(IN) IS COMPOSITE WEIGHT
C     FUNCTION

C     WBAR IS WEIGHT PARAMETER
C     PO IS POISSON RATIO
C     AX IS STRINGER FLANGE TO WEB
C     THICKNESS RATIO
C     AY IS RING FLANGE TO WEB
C     THICKNESS RATIO
C     BX IS STRINGER FLANGE WIDTH
C     TO WEB DEPTH RATIO
C     BY IS RING FLANGE WIDTH TO

C     WEB DEPTH RATIO
C     CX IS STRINGER SHAPE PARAMETER

C     CY IS RING SHAPE PARAMETER

C     ALX IS NONDIMENSIONAL RADIUS OF
C     GYRATION OF STRINGER

C     ALY IS NONDIMENSIONAL RADIUS
C     OF GYRATION OF RING
C     RTO IS RATIO OF LENGTH TO
C     RADIUS OF SHELL
C     AFA IS RATIO KXX/KYY
C     N OR EN IS THE NUMBER OF
C     CIRCUMFERENTIAL WAVES
```

```
C       BB IS BETABAR IN KYY EXPRESSION
C       TN IS KYY CRITICAL
C       SET AFA ZERO FOR ONLY PRESSURE
        DIMENSION X1(10,10),X(10),WS(10)
        COMMON/S/X1,NX,STEP,K1,WS,IN
        COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZ
        COMMON/AAA/M,N
        COMMON/PPP/TN
        COMMON/SR/QDS
        COMMON/XX/AFA,RTO
        WRITE(6,1000)
   1000 FORMAT(//15X,'GENERAL INSTABILITY FORMULATION'//)
        READ(5,110)AX,AY,BX,BY,CX,CY,QDS,AFA,RTO,ZZ
        NX=2
        PO=.3
        X(3)=8.
        WRITE(6,1100)
   1100 FORMAT(/8X,'NU',3X,'CX',5X,'CY',9X,'ZZ',6X,'AX',4X,'AY
',4X,'BX',
       15X,'BY',5X,'QDS')
        WRITE(6,1200)PO,CX,CY,ZZ,AX,AY,BX,BY,QDS
   1200 FORMAT(6X,F5.3,F6.3,F7.3,3X,F8.2,4F7.4,E15.6//)
        WRITE(6,1300)
   1300 FORMAT(6X,'ALX',4X,'ALY',3X,'WBAR',9X,'KYCR',7X,'X(1)'
,5X,'X(2)',
       15X,'M',4X,'N',5X,'WPSTAR',4X,'DIFFER',5X,'II')
    100 READ(5,110,END=900)ALX,ALY
    110 FORMAT()
```

```
      C     START WITH ASSUMED VALUE OF X(1) AND X(2).
            X(1)=.1
            X(2)=.6
            STEP=.1
            ALFA=1.0
            BETA=0.5
            GAMA=2.0
            DIFER=0.
            XNX=NX
            IN=1
            CALL WSR
            K1=NX+1
            K2=NX+2
            K3=NX+3
            K4=NX+4
            CALL START
            DO 3 I=1,K1
            DO 4 J=1,NX
          4 X(J)=X1(I,J)
            IN=I
            CALL WSR
          3 CONTINUE
      C
         63 II=0
         28 II=II+1
            IF(II.LT.61) GO TO 60
            GO TO 888
      C
      C     SELECT LARGEST VALUE OF WS(I) IN SIMPLEX
         60 WSH=WS(1)
            INDEX=1
            DO 7 I=2,K1
            IF(WS(I).LE.WSH) GO TO 7
            WSH=WS(I)
            INDEX=I
          7 CONTINUE
      C     SELECT MINIMUM VALUE OF WS(I) IN SIMPLEX
            WSL=WS(1)
            KOUNT=1
            DO 8 I=2,K1
```

```
      IF(WSL.LE.WS(I)) GO TO 8
      WSL=WS(I)
      KOUNT=I
    8 CONTINUE
C     FIND CENTROID OF POINTS WITH I DIFFERENT THAN INDEX
      DO 9 J=1,NX
      WS2=0.
      DO 10 I=1,K1
   10 WS2=WS2+X1(I,J)
      X1(K2,J)=1./XNX*(WS2-X1(INDEX,J))
C     FIND REFLECTION OF HIGH POINT THROUGH CENTROID
      X1(K3,J)=(1.+ALFA)*X1(K2,J)-ALFA*X1(INDEX,J)
      IF(X1(K3,J).LT.0.) X1(K3,J)=0.
    9 X(J)=X1(K3,J)
      IN=K3
      CALL WSR
      IF(WS(K3).LT.WSL) GO TO 11
C     SELECT SECOND LARGEST VALUE IN SIMPLEX
      IF(INDEX.EQ.1) GO TO 38
      WSS=WS(1)
      GO TO 39
   38 WSS=WS(2)
   39 DO 12 I=1,K1
      IF((INDEX-I).EQ.0) GO TO 12
      IF(WS(I).LE.WSS) GO TO 12
      WSS=WS(I)
   12 CONTINUE
      IF(WS(K3).GT.WSS) GO TO 13
      GO TO 14
C     FORM EXPANSION OF NEW MINIMUM IF REFLECTION HAS PRODUC
ED ONE MINIMUM
```

```
      11 DO 15 J=1,NX
         X1(K4,J)=(1.-GAMA)*X1(K2,J)+GAMA*X1(K3,J)

         IF (X1(K4,J).LT.0.) X1(K4,J)=0.

      15 X(J)=X1(K4,J)
         IN=K4
         CALL WSR
         IF(WS(K4).LT.WSL) GO TO 16
         GO TO 14
      13 IF (WS(K3).GT.WSH) GO TO 17
         DO 18 J=1,NX
      18 X1(INDEX,J)=X1(K3,J)
      17 DO 19 J=1,NX
         X1(K4,J)=BETA*X1(INDEX,J)+(1.-BETA)*X1(K2,J)

         IF (X1(K4,J).LT.0.) X1(K4,J)=0.
      19 X(J)=X1(K4,J)

         IN=K4
         CALL WSR
         IF (WSH.GT.WS(K4)) GO TO 16

 C       REDUCE SIMPLEX BY HALF IF REFLECTION HAPPENS TO PRODUC
E A LARGER

 C       VALUE THAN THE MAXIMUM
         DO 20 J=1,NX
         DO 20 I=1,K1
      20 X1(I,J)=0.5*(X1(I,J)+X1(KOUNT,J))
         DO 29 I=1,K1
         DO 30 J=1,NX
      30 X(J)=X1(I,J)
         CALL WSR
         IN=I
      29 CONTINUE
```

```
      GO TO 26
   16 DO 21 J=1,NX
      X1(INDEX,J)=X1(K4,J)
   21 X(J)=X1(INDEX,J)
      IN=INDEX
      CALL WSR
      GO TO 26
   14 DO 22 J=1,NX
      X1(INDEX,J)=X1(K3,J)
   22 X(J)=X1(INDEX,J)
      IN=INDEX
      CALL WSR
   26 DO 23 J=1,NX
   23 X(J)=X1(K2,J)
      IN=K2
      CALL WSR
C     TO TERMINATE THE SEARCH DIFER MUST BE LESS THAN ERSILO
N
      DIFER=0.
      DO 24 I=1,K1
   24 DIFER=DIFER+(WS(I)/WS(K2)-1.)**2
      DIFER=SQRT(1./(XNX+1.0)*DIFER)
      IF(DIFER.GE..00001) GO TO 28
  888 WBAR=1.+(X1(KOUNT,1)+X1(KOUNT,2))/(1.-PO*PO)-
     1X1(KOUNT,1)*X1(KOUNT,2)/((1.-PO*PO)**2*ALY)
      WRITE(6,101)ALX,ALY,WBAR,TN,(X1(KOUNT,J),J=1,NX),M,N,W
SL,
     1DIFER,II
  101 FORMAT(1X,F8.1,F7.1,F10.5,F11.2,2F10.5,I5,I5,F10.5,E12
.5,I5)
      GO TO 100
  900 CONTINUE
      END
```

ASWANI-M-G*OPTIM.START

```
      SUBROUTINE START
C     THIS PROGRAM SETS UP THE INITIAL SIMPLEX
      DIMENSION X1(10,10),X(10),WS(10),A(10,10)
      COMMON/S/X1,NX,STEP,K1,WS,IN
      COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZ
      COMMON/XX/AFA,RTO
      VN=NX
      STEP1=STEP/(VN*SQRT(2.))*(SQRT(VN+1.)+VN-1.)
      STEP2=STEP/(VN*SQRT(2.))*(SQRT(VN+1.)-1.)
      DO 1 J=1,NX
    1 A(1,J)=0.
      DO 2 I=2,K1
      DO 2 J=1,NX
      A(I,J)=STEP2
      L=I-1
      A(I,L)=STEP1
    2 CONTINUE
      DO 3 I=1,K1
      DO 3 J=1,NX
    3 X1(I,J)=X(J)+A(I,J)
      RETURN
      END
```

```
ASWANI-M-G*OPTIM.WSR
      SUBROUTINE WSR
C     THIS SUBROUTINE DEFINESTHE OBJECTIVE
C     FUNCTION FORMULATED ON THE BASIS
C     OF GENERAL INSTABILITY
      DIMENSION X1(10,10),X(10),WS(10)
      COMMON/S/X1,NX,STEP,K1,WS,IN
      COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZ
C     COMMON/EE/BB,EM,CKYR
      COMMON/SR/QDS
      COMMON/XX/AFA,RTO
      COMMON/PPP/TN
      COMMON/AAA/M,N
      DO 10 J=1,NX
   10 IF(X(J).LT.0.) X(J)=0.
      CALL GENST(PCR)
      WS(IN)=1.0+(X(1)+X(2))/(1.-PO*PO)+10**X(3)*ABS(TN/(ZZ*
ZZ)-QDS*
     1ZZ)-X(1)*X(2)/((1.-PO*PO)**2*ALY)
      RETURN
      END
```

ASWANI-M-G*OPTIM.GENST

```
      SUBROUTINE GENST(TP)
C     THIS SUBROUTINE IS FOR FINDING KYYCR
C     TREATING M AND N AS INTEGERS
      COMMON/AAA/M,N
      COMMON/PPP/TN
      COMMON/YY/ROXX,ROYY,EX,EY
      IK=0
      IL=0
      IR=0
      IS=0
      NL=2
  102 N=NL
      IJ=1
      M=1
   40 NT=N
      MT=M
      NA=0
      MA=1
      NB=8
      MB=30
   17 N=N-1
      IF(N-NA) 42,41,41
   42 N=N+1
   41 CALL CKYR(TA,N,M,AL,IK)
      N=N+1
      CALL CKYR(TB,N,M,AL,IK)
      IF(TA-TB) 1,2,2
    1 IF(IR) 3,4,3
    2 N=N+1
      IF(N-NB) 46,46,45
   45 TN=TB
      N=N-1
      GO TO 7
   46 CALL CKYR(TC,N,M,AL,IK)
      IF(TB-TC) 10,10,11
    4 N=N-2
      IF(N) 43,44,44
   43 TN=TA
      N=N+1
      GO TO 7
   44 CALL CKYR(TD,N,M,AL,IK)
      IF(TA-TD) 5,5,6
```

```
    5 TN=TA
      N=N+1
      GO TO 7
    6 NB=N
      N=(NA+NB)/2
      GO TO 8
    3 IF(2-NB+NA) 6,6,9
    9 TN=TA
      N=N-1
      GO TO 7
   10 TN=TB
      N=N-1
      GO TO 7
   11 IF(IR) 15,13,15
   13 N=N+1
      IF(N-8) 48,48,47
   47 TN=TC
      N=N-1
      GO TO 7
   48 CALL CKYR(TO,N,M,AL,IK)
      IF(TC-TO) 14,14,15
   14 TN=TC
      N=N-1
      GO TO 7
   15 IF(2-NB+NA) 12,12,16
   16 TN=TC
      GO TO 7
   12 NA=N
      N=(NA+NB)/2
    8 IR=IR+1
      GO TO 17
    7 IF(IJ) 104,104,103
  103 T1=TN
      IJ=0
      NL=N
      N=3
      M=5
      IR=0
      GO TO 40
  104 M=M-1
      IF(M-MA) 49,50,50
```

```
   49 M=M+1
   50 CALL CKYR(TA,N,M,AL,IK)
      M=M+1
      CALL CKYR(TB,N,M,AL,IK)
      IF(TA-TB) 19,20,20
   19 IF(IS) 21,22,21
   20 M=M+1
      IF(M-MB) 51,51,52
   52 TN=TB
      M=M-1
      GO TO 25
   51 CALL CKYR(TC,N,M,AL,IK)
      IF(TB-TC) 28,28,29
   22 M=M-2
      IF(M) 53,53,54
   53 TN=TA
      M=M+1
      GO TO 25
   54 CALL CKYR(TD,N,M,AL,IK)
      IF(TA-TD) 23,23,211
   23 TN=TA
      M=M+1
      GO TO 25
  211 MB=M
      M=(MA+MB)/2
      GO TO 26
   21 IF(2-MB+MA) 211,211,27
   27 TN=TA
      M=M-1
      GO TO 25
   28 TN=TB
      M=M-1
      GO TO 25
   29 IF(IS) 33,31,33
   31 M=M+1
      IF(M-30) 55,55,56
   56 TN=TC
      M=M-1
      GO TO 25
   55 CALL CKYR(TD,N,M,AL,IK)
      IF(TC-TD) 32,32,33
   32 TN=TC
```

```
      M=M-1
      GO TO 25
   33 IF(2-MB+MA) 30,30,34
   34 TN=TC
      GO TO 25
   30 MA=M
      M=(MA+MB)/2
   26 IS=IS+1
      GO TO 7
   25 IF(N-NT) 40,36,40
   36 IF(M-MT) 40,37,40
   37 NB=N-3
      DO 60 I=1,2
      NB=NB+2
      IF(NB) 60,65,65
   65 IF(NB-8) 61,61,60
   61 MB=M-3
      DO 70 J=1,2
      MB=MB+2
      IF(MB) 70,70,64
   64 IF(MB-30) 62,62,70
   62 CALL CKYR(TA,NB,MB,AL,IK)
      IF(TA-TN) 63,63,70
   70 CONTINUE
   60 CONTINUE
      IF(N.EQ.0) GO TO 100
      IF(N-M) 101,100,100
  101 NI=N
      MI=M
      IF(T1.LT.TN) GO TO 105
      RETURN
  105 TN=T1
      N=NL
      M=1
      RETURN
   63 N=NB
      M=MB
      GO TO 40
  100 IL=IL+1
      IF(IL.GT.1) RETURN
      N=5
      M=30
      GO TO 40
      END
```

```
ASWANI-M-G*OPTIM.CKYR
          SUBROUTINE CKYR(U,N,M,AL,IR)
    C    THIS SUBROUTINE DEFINES THE PARAMETER KYYBAR
         DOUBLE PRECISION A1,A2,A3,A4,A,B,C,F,G
         DIMENSION X(10)
         COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZ
         COMMON/YY/ROXX,ROYY,EX,EY

          COMMON/XX/AFA,RTO
         IF(N.EQ.0) GO TO 2
         IF(IR.EQ.1) GO TO 1
         IR=1
         ROXX=ALX*ALX*X(1)
         ROYY=ALY*ALY*X(2)
         EX=-3.14*3.14*SQRT(1.-PO*PO)*(1.+CX*ALX)/(2.0*ZZ)
         EY=-3.14*3.14*SQRT(1.-PO*PO)*(1.+CY*ALY)/(2.0*ZZ)

       1 XN=N
         XM=M

         BB=XN*RTO/(3.14*XM)

         A1=(1.+BB*BB)**2+X(1)+X(2)*BB**4+2.*BB*BB*(X(1)+X(2)+X
(1)*X(2))/

        1(1.-PO)
         A2=(1.+BB*BB)**2+ROXX+ROYY*BB**4
         A3=12.*ZZ*ZZ/(3.14**4*(1.-PO*PO))
         A4=EX*EX*X(1)+2.0*EX*EX*X(1)*(1.-PO+X(2))*BB*BB/(1.-PO
)+(EX*EX*

        1X(1)*(1.+X(2))+2.0*(1.+PO)/(1.-PO)*EX*EY*X(1)*X(2)+EY*
EY*X(2)*(1.+

        2X(1)))*BB**4+2.0*EY*EY*X(2)/(1.-PO)*(1.-PO+X(1))*BB**6
+EY*EY*X(2)
```

```
     3*BB**8
      A=A1*A2+A3*A4
      A5=-2.*P0*EX*X(1)+2.*(EX*X(1)*(1.+X(2))+EY*X(2)*(1.+X(
1)))*BB*BB
     1-2.*P0*EY*X(2)*BB**4
      B=A3*A5
      C=A3*((1.+X(1))*(1.+X(2))-P0*P0)
      F=(RT0/3.14)**2*((1.+BB*BB)*(EX*X(1)+EY*X(2)*BB**4)+2.
*BB*BB/(1.-P0
     10)*(X(1)*X(2)*(EX+EY*BB*BB)))+(.5-AFA+BB*BB)*A1
      G=(RT0/3.14)**2*((1.+BB*BB)*(P0+BB*BB)+BB*BB/(1.-P0)*(
2.*X(1)+X(2)
     1+P0*X(2)+2.*X(1)*X(2))+X(2)*BB**4)
      U=(A*XM**4+B*XM**2+C)/(F*XM**2+G)
      RETURN
    2 U=1.E+30
      RETURN
      END
```

ASWANI-M-G*OPTIM.MAINR

```
C      THIS PROGRAM IS FOR MINIMIZATION OF
C      OBJECTIVE FUNCTION FORMULATED ON
C      THE BASIS OF GENERAL INSTABILITY
C      THE CYLINDER IS RING STIFFENED
C
       DIMENSION X(10),WS(10),X1(100),X2(100),X3(100),Y1(100)
,Y2(100),
      1DEL(100)
       COMMON/S/IN,WS
       COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZ
       COMMON/SR/QDS
       COMMON/PPP/TN
       COMMON/XX/AFA,RTO
       COMMON/AAA/M,N
       PO=.3
       ALX=0.
       X(1)=.0
       CX=1.
       AFA=-.0
       RTO=3.
       DATA X(3),AY,BY/8.,1.,.5/
       QDS=.141384E-5
       CY=(1.+2.*AY*BY)/SQRT(1.+4.*AY*BY)
       WRITE(6,90)
    90 FORMAT(6X,'ENTER VALUES OF ALY,ZZ'/)
       WRITE(6,1000)
```

```
      1000 FORMAT(//10X,'G.I. OPTIMIZATION FOR RING STIFFENED SHE
LL D-3000IR'

          1//)

           WRITE(6,1100)
      1100 FORMAT(/,8X,'NU',3X,'CY',9X,'ZZ',6X,'AY',4X,'BY',5X,'Q
DS')
           WRITE(6,1200)PO,CY,ZZ,AY,BY,QDS

      1200 FORMAT(6X,F5.3,F6.3,4X,F8.2,2F7.4,E15.6//)
           WRITE(6,1300)
      1300 FORMAT(6X,'ALY',3X,'WBAR',9X,'KYCR',7X,'X(2)',5X,'M',4
X,'N'
          1,4X,'WPSTAR')
       555 READ(5,111,END=999)ALY,ZZ
       111 FORMAT()
           DATA X1(1),X2(1),X3(1),F1,EPS/.001,.1,4.,0.381966011,.
000001/

           K=1

           L=0

        25 X(2)=X2(K)
           IN=1
           CALL WSRR
        11 X(2)=X3(K)
           IN=2
           CALL WSRR
           IF(WS(1)-WS(2)) 10,10,20
        20 X3(K)=X3(K)+0.2*X3(K)
           IF(X3(K).LT.15) GO TO 11
           L=L+1
           IF(L.LT.10) GO TO 20
           X1(1)=0.01
           X2(1)=.4
```

```
      X3(1)=12.0
      IF(L.LT.11) GO TO 25
   10 DEL(K)=X3(K)-X1(K)
   12 Y1(K)=X1(K)+F1*DEL(K)
      Y2(K)=X3(K)-F1*DEL(K)
      X(2)=Y1(K)
      IN=1
      CALL WSRR
      X(2)=Y2(K)
      IN=2
      CALL WSRR
      IF(WS(1)-WS(2)) 30,31,32
   30 DEL(K+1)=Y2(K)-X1(K)
      X1(K+1)=X1(K)
      X3(K+1)=Y2(K)
      K=K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 40
      GO TO 12
   31 DEL(K+1)=Y2(K)-X1(K)
      X1(K+1)=Y1(K)
      X3(K+1)=X3(K)
      K=K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 40
      GO TO 12
   32 DEL(K+1)=X3(K)-Y1(K)
      X1(K+1)=Y1(K)
      X3(K+1)=X3(K)
      K=K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 40
      GO TO 12
   40 X(2)=(X1(K)+X3(K))/2.
      IN=1
      CALL WSRR
      WSS=WS(1)
      WBAR=1.+X(2)/(1.-PO*PO)
      WRITE(6,101)ALY,WBAR,TN,X(2),M,N,WSS
  101 FORMAT(1X,F8.1,F10.5,F11.0,F10.5,I5,I5,F10.5)
      GO TO 555
  999 CONTINUE
      END
```

```
ASWANI-M-G*OPTIM.WSRR
                SUBROUTINE WSRR
      C         THIS SUBROUTINE DEFINES THE OBJECTIVE
      C         FUNCTION FOR RING STIFFENED SHELL
      C         BASED ON GENERAL INSTABILITY
                DIMENSION X(10),WS(10)
                COMMON/S/IN,WS
                COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZ
                COMMON/AAA/M,N
                COMMON/PPP/TN
                COMMON/SR/QDS
                CALL GENST(PCR)
                WS(IN)=1.+X(2)/(1.-PO*PO)+10**X(3)*ABS(TN/(ZZ*ZZ)-QDS*
ZZ)
                RETURN
                END
```

```
ASWANI-M-G*OPTIM.MAINY
C       THIS PROGRAM IS FOR MINIMIZATION
C       OF THE OBJECTIVE FUNCTION FORMULATED
C       ON THE BASISC OF SKIN YIELDING
C       OPTIMIZATION BASED ON SKIN YIELD
        DIMENSION X1(10,10),X(10),WS(10)
        COMMON/S/X1,NX,STEP,K1,WS,IN
        COMMON/SS/PO,X,ZZ,AL,R
        COMMON/SR/QY

        WRITE(6,1000)
  1000  FORMAT(//15X,'OPTIMIZATION FOR SHELL D-1000-I-T R-RS'/
/)

        NX=2
        STEP=.01

        PO=.3
        AL=594.
        R=198.
        YS=120000.
        DP=3000.
        GW=.0374
        Q=GW*DP*12.
        QY=YS*9.*SQRT(1.-PO*PO)/Q
        WRITE(6,1100)
  1100  FORMAT(8X,'NU',12X,'SIG.STAR')
        WRITE(6,1200)PO,QY
  1200  FORMAT(6X,F5.3,8X,F11.5)
        WRITE(6,1300)
  1300  FORMAT(8X,'H',10X,'X(1)',8X,'X(2)',8X,'WBAR',7X,'WSTAR
',8X,'DIFF
       1ER',5X,'II')

   100  READ(5, 110,END=900)ZZ
```

```
  110 FORMAT()
C     START WITH ASSUMED VALUE OF X(1) AND X(2).
      X(1)=.2
      X(2)=.8
      X(3)=8.
      ALFA=1.0
      BETA=0.5
      GAMA=2.0
      DIFER=0.
      XNX=NX
      IN=1
      CALL WSRY
      K1=NX+1
      K2=NX+2
      K3=NX+3
      K4=NX+4
      CALL STARTY
      DO 3 I=1,K1
      DO 4 J=1,NX
    4 X(J)=X1(I,J)
      IN=I
      CALL WSRY
    3 CONTINUE
C
   63 II=0
   28 II=II+1
      IF(II.LT.100) GO TO 60
      GO TO 888
C
C     SELECT LARGEST VALUE OF WS(I) IN SIMPLEX
   60 WSH=WS(1)
      INDEX=1
      DO 7 I=2,K1
      IF(WS(I).LE.WSH) GO TO 7
      WSH=WS(I)
      INDEX=I
    7 CONTINUE
C     SELECT MINIMUM VALUE OF WS(I) IN SIMPLEX
      WSL=WS(1)
      KOUNT=1
      DO 8 I=2,K1
      IF(WSL.LE.WS(I)) GO TO 8
```

```
      WSL=WS(I)
      KOUNT=I
    8 CONTINUE
C     FIND CENTROID OF POINTS WITH I DIFFERENT THAN INDEX
      DO 9 J=1,NX
      WS2=0.
      DO 10 I=1,K1
   10 WS2=WS2+X1(I,J)
      X1(K2,J)=1./XNX*(WS2-X1(INDEX,J))
C     FIND REFLECTION OF HIGH PO&NT TH-OUGH CENT-O&D
      X1(K3,J)=(1.+ALFA)*X1(K2,J)-ALFA*X1(INDEX,J)
      IF(X1(K3,J).LT.0.) X1(K3,J)=0.
    9 X(J)=X1(K3,J)
      IN=K3
      CALL WSRY
      IF(WS(K3).LT.WSL) GO TO 11
C     SELECT SECOND LARGEST VALUE IN S&MPLEX
      IF(INDEX.EQ.1) GO TO 38
      WSS=WS(1)
      GO TO 39
   38 WSS=WS(2)
   39 DO 12 I=1,K1
      IF((INDEX-I).EQ.0) GO TO 12
      IF(WS(I).LE.WSS) GO TO 12
      WSS=WS(I)
   12 CONTINUE
      IF(WS(K3).GT.WSS) GO TO 13
```

```
      GO TO 14
C     FORM EXPANSION OF NEW MINIMUM IF REFLECTION HAS PRODUC
ED ONE MINIMUM
   11 DO 15 J=1,NX
      X1(K4,J)=(1.-GAMA)*X1(K2,J)+GAMA*X1(K3,J)
      IF (X1(K4,J).LT.0.) X1(K4,J)=0.
   15 X(J)=X1(K4,J)
      IN=K4
      CALL WSRY
      IF(WS(K4).LT.WSL) GO TO 16
      GO TO 14
   13 IF (WS(K3).GT.WSH) GO TO 17
      DO 18 J=1,NX
   18 X1(INDEX,J)=X1(K3,J)
   17 DO 19 J=1,NX
      X1(K4,J)=BETA*X1(INDEX,J)+(1.-BETA)*X1(K2,J)
      IF (X1(K4,J).LT.0.) X1(K4,J)=0.
   19 X(J)=X1(K4,J)
      IN=K4
      CALL WSRY
      IF (WSH.GT.WS(K4)) GO TO 16
C     REDUCE SIMPLEX BY HALF IF REFLECTION HAPPENS TO PRODUC
E A LARGER
C     VALUE THAN THE MAXIMUM
      DO 20 J=1,NX
      DO 20 I=1,K1
   20 X1(I,J)=0.5*(X1(I,J)+X1(KOUNT,J))
      DO 29 I=1,K1
      DO 30 J=1,NX
   30 X(J)=X1(I,J)
      CALL WSRY
```

```
      IN=I
   29 CONTINUE
      GO TO 26
   16 DO 21 J=1,NX
      X1(INDEX,J)=X1(K4,J)
   21 X(J)=X1(INDEX,J)
      IN=INDEX
      CALL WSRY
      GO TO 26
   14 DO 22 J=1,NX
      X1(INDEX,J)=X1(K3,J)
   22 X(J)=X1(INDEX,J)
      IN=INDEX
      CALL WSRY
   26 DO 23 J=1,NX
   23 X(J)=X1(K2,J)
      IN=K2
      CALL WSRY
C     TO TERMINATE THE SEARCH DIFER MUST BE LESS THAN EPSILO
      DIFER=0.
      DO 24 I=1,K1
   24 DIFER=DIFER+(WS(I)/WS(K2)-1.)**2
      DIFER=SQRT(1./(XNX+1.0)*DIFER)
      IF (DIFER.GE.0.00001) GO TO 28
  888 WBARR=1.+(X1(KOUNT,1)+X1(KOUNT,2))/(1.-PO*PO)
      H=AL*AL*SQRT(1.-PO*PO)/(R*ZZ)
      WBAR=WBARR
      WRITE(6,101)H,(X1(KOUNT,J),J=1,NX),WBAR,WSL,DIFER,II
  101 FORMAT(3X,F10.5,2X,2F11.5,2X,F10.5,2X,F10.5,2X,E12.5,1
x,I5)
      GO TO 100
  900 CONTINUE
      END
```

```
ASWANI-M-G*OPTIM.STARTY
      SUBROUTINE STARTY
C     THIS SUBROUTINE SETS UP INITIAL
C     SIMPLEX FOR MAINY
      DIMENSION X1(10,10),X(10),WS(10),A(10,10)
      COMMON/S/X1,NX,STEP,K1,WS,IN
      COMMON/SS/PO,X,ZZ,AL,R
      VN=NX
      STEP1=STEP/(VN*SQRT(2.))*(SQRT(VN+1.)+VN-1.)

      STEP2=STEP/(VN*SQRT(2.))*(SQRT(VN+1.)-1.)
      DO 1 J=1,NX
    1 A(1,J)=0.
      DO 2 I=2,K1

      DO 2 J=1,NX

      A(I,J)=STEP2
      L=I-1
      A(I,L)=STEP1

    2 CONTINUE
      DO 3 I=1,K1
      DO 3 J=1,NX
    3 X1(I,J)=X(J)+A(I,J)
      RETURN
      END
```

```
ASWANI-M-G*OPTIM.WSRY
      SUBROUTINE WSRY
C     THIS SUBROUTINE DEFINES THE OBJECTIVE
C     FUNCTION FORMULATED ON THE BASIS
C     OF SKIN YIELDING
      DIMENSION X1(10,10),X(10),WS(10)
      COMMON/S/X1,NX,STEP,K1,WS,IN
      COMMON/SS/PO,X,ZZ,AL,R
      COMMON/SR/QY
      DO 10 J=1,NX
   10 IF(X(J).LT.0.) X(J)=0.
      AFA=.2

      A=(1.+X(1))*(1.+X(2))-PO*PO

      B=2.*PO*X(1)+(1.+2.*AFA)*(X(2)+1.-PO*PO)

      C=2.*(X(1)+1.-PO*PO)+PO*X(2)*(1.+2.*AFA)

      P=SQRT(B*B+C*C-B*C)/A
      WS(IN)=1.+(X(1)+X(2))/(1.-PO*PO)+10**X(3)*ABS(P/2.-QY/
ZZ)

      RETURN
      END
```

```
ASWANI-M-G*OPTIM.MAINP
      C     THIS PROGRAM IS FOR PANEL BUCKLING
      C     CHECK EMPLOYING GOLDEN SECTION
      C     SEARCH TECHNIQUE
      C
            DIMENSION X1(100),X2(100),X3(100),Y1(100),Y2(100),DEL(
100),X(10)
           1,M(5),GG(5),Z1(5)
            COMMON/SS/ALX,ALY,CX,CY,PO,X,Z
            COMMON/CC/A,B,C,F,G
            COMMON/DD/M,JJ
            COMMON/XX/AFA,RTO
            PO=.3
            CX=1.
            CY=1.
            ALY=0.
            X(2)=0.
        100 READ(5,140,END=999)Z,ALX,X(1),EL,RR,AL
        140 FORMAT()
            RTO=EL/RR
            DATA X1(1),X2(1),X3(1),F1,EPS/.00,4.00,5.00,0.38196601
1,0.01/
            ZZ=Z*EL*EL/(AL*AL)
            WRITE(6,1000)
       1000 FORMAT(7X,'ZZ',7X,'EL',7X,'X(1)',6X,'ALX',4X,'KYCR',6X
,'M',5X,'
           1BETA')
```

```
      K=1
      L=0
   11 IF(Q(X2(K))-Q(X3(K))) 10,10,20
   20 X3(K)=X3(K)+0.2*X3(K)
      IF(X3(K).LT.15) GO TO 11
      L=L+1
      IF(L.LT.10) GO TO 11
      X1(1)=0.00001
      X2(1)=0.8
      X3(1)=1.0
      IF(L.LT.11) GO TO 11
C     ATTEMPT A TRIAL VALUE FOR EM AS 1
      EM=1.0
      GO TO 8
   10 DEL(K)=X3(K)-X1(K)
   12 Y1(K)=X1(K)+F1*DEL(K)
      Y2(K)=X3(K)-F1*DEL(K)
      IF(Q(Y1(K))-Q(Y2(K))) 30,31,32
   30 DEL(K+1)=Y2(K)-X1(K)
      X1(K+1)=X1(K)
      X3(K+1)=Y2(K)
      K=K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 40
      GO TO 12
   31 DEL(K+1)=Y2(K)-X1(K)
      X1(K+1)=Y1(K)
      X3(K+1)=X3(K)
      K=K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 40
      GO TO 12
   32 DEL(K+1)=X3(K)-Y1(K)
      X1(K+1)=Y1(K)
      X3(K+1)=X3(K)
      K=K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 40
      GO TO 12
   40 BB=(X1(K)+X3(K))/2.
      QX=Q(BB)
      AM=-G/F+SQRT(G*G/(F*F)+C/A-B*G/(A*F))
      EM=SQRT(AM)
      BET=BB*EM
    8 JJ=1
      IF(EM-1.0) 41,41,42
   41 M(JJ)=1
```

```
      GO TO 49
   42 JJ=JJ+1
      M(JJ)=EM
      GO TO 49
   43 JJ=JJ+1
      M(JJ)=M(JJ-1)+1
      GO TO 49
   49 X1(1)=0.01
      X2(1)=4.5
      X3(1)=5.0
      K=1
      L=0
   71 IF (R(X2(K))-R(X3(K))) 72,72,73
   73 X3(K)=X3(K)+0.2*X3(K)
      IF(X3(K).LT.15.) GO TO 71
      L=L+1
      IF(L.LT.20) GO TO 71
      WRITE(6,101)
  101 FORMAT(15X,'BETA BAR LOST IN R')
      GO TO 898
   72 DEL(K)=X3(K)-X1(K)
   74 Y1(K)=X1(K)+F1*DEL(K)
      Y^
      Y2(K)=X3(K)-F1*DEL(K)
      IF(RP(Y1(K))-RP(Y2(K))) 75,76,77
   75 DEL(K+1)=Y2(K)-X1(K)
      X1(K+1)=X1(K)
      X3(K+1)=Y2(K)
      K=K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 78
      GO TO 74
   76 DEL(K+1)=Y2(K)-X1(K)
      X1(K+1)=Y1(K)
      X3(K+1)=X3(K)
      K=K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 78
      GO TO 74
   77 DEL(K+1)=X3(K)-Y1(K)
      X1(K+1)=Y1(K)
      X3(K+1)=X3(K)
```

```
      K=K+1
      IF(ABS((X3(K)-X1(K))/X3(K)).LT.EPS) GO TO 78
      GO TO 74
   78 Z1(JJ)=(X1(K)+X3(K))/2.
      GG(JJ)=RP(Z1(JJ))
      IF(JJ.EQ.1) GO TO 51
      IF(JJ.EQ.3) GO TO 44
      GO TO 43
   44 IF((GG(JJ)-GG(JJ-1))) 51,51,52
   51 CKYR=GG(JJ)
      BB=Z1(JJ)
      EM=M(JJ)
      GO TO 47
   52 CKYR=GG(JJ-1)
      BB=Z1(JJ-1)
      EM=M(JJ-1)
   47 CONTINUE
      BET=BB*EM
      M(JJ)=EM
      E=3.0E+7
      H=RTO*RTO*198.*SQRT(1.-PO*PO)/ZZ
      Q=3.14*3.14*E*H**3*CKYR/(EL*EL*198.*12.*(1.-PO*PO))
      EN=3.14*BET*198./EL
      WRITE(6,1001)ZZ,EL,X(1),ALX,CKYR,M(JJ),BET
 1001 FORMAT(5X,F8.2,1X,F6.2,F10.5,F8.1,F9.0,1X,I5,1X,F8.3)
      WRITE(6,222)
  222 FORMAT(5X,'EN',15X,'Q')
      WRITE(6,333)EN,Q
  333 FORMAT(3X,F7.1,5X,E14.7)
      QQ=2692.8
      PBCR=QQ/Q
      WRITE(6,909)PBCR
  909 FORMAT(6X,'PBCR=',F14.5////)
      GO TO 100
  999 CONTINUE
      END
```

```
ASWANI-M-G*OPTIM.Q
            FUNCTION Q(BB)
            DOUBLE PRECISION A1,A2,A3,A4,A5,A,B,C,F,G,T
            DIMENSION X(10)
            COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZ
            COMMON/CC/A,B,C,F,G
            COMMON/YY/AFA
            ROXX=ALX*ALX*X(1)
            ROYY=ALY*ALY*X(2)
            EX=-3.14*3.14*SQRT(1.-PO*PO)*(1.+CX*ALX)/(2.0*ZZ)
            EY=-3.14*3.14*SQRT(1.-PO*PO)*(1.+CY*ALY)/(2.0*ZZ)
            A1=(1.+BB*BB)**2+X(1)+X(2)*BB**4+2.*BB*BB*(X(1)+X(2)+X
(1)*X(2))/
           1(1.-PO)
            A2=(1.+BB*BB)**2+ROXX+ROYY*BB**4
            A3=12.*ZZ*ZZ/(3.14**4*(1.-PO*PO))
            A4=EX*EX*X(1)+2.0*EX*EX*X(1)*(1.-PO+X(2))*BB*BB/(1.-PO
)+(EX*EX*
           1X(1)*(1.+X(2))+2.0*(1.+PO)/(1.-PO)*EX*EY*X(1)*X(2)+EY*
EY*X(2)*(1.+
           2X(1)))*BB**4+2.0*EY*EY*X(2)/(1.-PO)*(1.-PO+X(1))*BB**6
+EY*EY*X(2)
           3*BB**8
            A=A1*A2+A3*A4
            A5=-2.*PO*EX*X(1)+2.*(EX*X(1)*(1.+X(2))+EY*X(2)*(1.+X(
1)))*BB*BB
           1-2.*PO*EY*X(2)*BB**4
            B=A3*A5
            C=A3*((1.+X(1))*(1.+X(2))-PO*PO)
```

```
      F=(3./3.14)**2*((1.+BB*BB)*(EX*X(1)+EY*X(2)*BB**4)+2.*
BB*BB/(1.-PO
     1)*(X(1)*X(2)*(EX+EY*BB*BB)))+(.5-AFA+BB*BB)*A1

      G=(3.0/3.14)**2*((1.+BB*BB)*(PO+BB*BB)+BB*BB/(1.-PO)*(
2.*X(1)+X(2)

     1+PO*X(2)+2.*X(1)*X(2))+X(2)*BB**4)
      T=-G/F+SQRT(G*G/(F*F)+C/A-B*G/(A*F))

      Q=(A*T*T+B*T+C)/(F*T+G)
      RETURN
      END
```

```
ASWANI-M-G*OPTIM.R
      FUNCTION R(BB)
C     R IS THE EXPRESSION OF KY WHEN M IS TREATED AS INTEGER
      DOUBLE PRECISION A1,A2,A3,A4,A5,A,B,C,F,G

      DIMENSION X(10),M(5)
      COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZ
      COMMON/DD/M,JJ
      COMMON/XX/RTO
      COMMON/YY/AFA
      ROXX=ALX*ALX*X(1)
      ROYY=ALY*ALY*X(2)
      EX=-3.14*3.14*SQRT(1.-PO*PO)*(1.+CX*ALX)/(2.0*ZZ)
      EY=-3.14*3.14*SQRT(1.-PO*PO)*(1.+CY*ALY)/(2.0*ZZ)
      A1=(1.+BB*BB)**2+X(1)+X(2)*BB**4+2.*BB*BB*(X(1)+X(2)+X
(1)*X(2))/
```

```
      1(1.-PO)
       A2=(1.+BB*BB)**2+ROXX+ROYY*BB**4
       A3=12.*ZZ*ZZ/(3.14**4*(1.-PO*PO))
       A4=EX*EX*X(1)+2.0*EX*EX*X(1)*(1.-PO+X(2))*BB*BB/(1.-PO
)+(EX*EX*
      1X(1)*(1.+X(2))+2.0*(1.+PO)/(1.-PO)*EX*EY*X(1)*X(2)+EY*
EY*X(2)*(1.+
      2X(1)))*BB**4+2.0*EY*EY*X(2)/(1.-PO)*(1.-PO+X(1))*BB**6
+EY*EY*X(2)
      3*BB**8
       A=A1*A2+A3*A4
       A5=-2.*PO*EX*X(1)+2.*(EX*X(1)*(1.+X(2))+EY*X(2)*(1.+X(
1)))*BB*BB
      1-2.*PO*EY*X(2)*BB**4
       B=A3*A5
       C=A3*((1.+X(1))*(1.+X(2))-PO*PO)
       F=(RTO/3.14)**2*((1.+BB*BB)*(EX*X(1)+EY*X(2)*BB**4)+2.
*BB*BB/(1.-PO
      1O)*(X(1)*X(2)*(EX+EY*BB*BB)))+(.5-AFA+BB*BB)*A1
       G=(RTO/3.14)**2*((1.+BB*BB)*(PO+BB*BB)+BB*BB/(1.-PO)*(
2.*X(1)+X(2)
      1+PO*X(2)+2.*X(1)*X(2))+X(2)*BB**4)
       R=(A*M(JJ)**4+B*M(JJ)*M(JJ)+C)/(F*M(JJ)*M(JJ)+G)
       RETURN
       END
```

ASWANI-M-G*OPTIM.RSSH

```
C     THIS PROGRAM IS FOR DESIGNING RING
C     STIFFENED SHELLS
C     SX2 IS STRESS IN SKIN SIGMAXX
C     SY2 IS STRESS IN SKIN SIGMAYY
C     CKYR IS KYY CRITICAL
C     QSTAR IS CRITICAL PRESSURE
C     SRY IS STRESS IN THE RING
C     QQ IS LOAD ON THE RING PER INCH
C     OF CIRCUMFERENCE
C     PBCR IS PANEL BUCKLING CRITICAL LOAD
C     QCR IS RING CRITICAL STRESS
C     SKY IS STRESS IN SKIN
C     GB IS GEN. INST. COEFFICIENT
C     PBC IS PANEL BUCKLING COEFFICIENT
C     RBC IS RING BUCKLING COEFFICIENT
C     RYC IS RING YIELDING COEFFICIENT
C     SKYC IS SKIN YIELDING COEFFICIENT
C     GW IS DENSITY OF IMMERSSION FLUID
C     DP IS OPERATING DEPTH
      DIMENSION X(10)
      COMMON/SS/ALX,ALY,CX,CY,PO,X,ZZ
      COMMON/PPP/TN
      COMMON/AAA/M,N
      COMMON/XX/AFA,RTO
      ALX=.0
      X(1)=.0
      CX=1.
    1 READ(5,3,END=4)ZZ,ELY,BK,X(2),AY,BY,PBCR,ALY,CY,R,AYS,
AFA,GW,DP,AL
```

```
    3 FORMAT()
      RTO=AL/R
      PO=.3
      GS=.282
      Q=12.*GW*DP
      E=.3E+8
      H=AL*AL*SQRT(1.-PO*PO)/(R*ZZ)
      DD=E*H**3/(12.*(1.-PO*PO))
      AA=Q*Q*R*R/(16.*DD*DD)
      BBB=E*H/(DD*R*R)
 1000 A=X(2)*ELY*H/(1.-PO*PO)
      DR=ALY*H*(1.+AY*BY)/SQRT(1.+4.*AY*BY)
      TR=A/(DR*(1.+AY*BY))
      TF=AY*TR
      WF=BY*DR
      AS=ELY*H
      TB=H*(1.+BK*A/(AS*BK+AS))
      QCR=4.*3.14*3.14*E*TR**3/(12.*(1.-PO*PO)*DR*DR)
      CALL GENST(PCR)
      QSTAR=TN*3.14*3.14*DS/(R*AL*AL)
      IF(QSTAR-2.*Q) 36,16,16
   36 X(2)=X(2)+.0001
      GO TO 1000
   16 EL=ELY-TR
      IF(AA-BBB) 5,6,7
    5 CC=Q*R/(8.*DD)
      DC=.5*SQRT(E*H/(DD*R*R))
      C=SQRT(-CC+DC)
      D=SQRT(CC+DC)
      V=C*EL/2.
      Y=D*EL/2.
      A1=V*V+Y*Y
      A2=SINH(V)*SINH(V)+SIN(Y)*SIN(Y)
      A3=V*SIN(Y)*COS(Y)+Y*SINH(V)*COSH(V)
      W=-16.*V*Y*A1*A2/(A3*EL**3)
      A4=V*COSH(V)*SIN(Y)-Y*SINH(V)*COS(Y)
      AJO=4.*A1*A4/(A3*EL*EL)
      A5=V*SIN(Y)*COS(Y)-Y*SINH(V)*COSH(V)
      AJL=4.*A1*A5/(A3*EL*EL)
      UO=-(Y*SINH(V)*COS(Y)+V*COSH(V)*SIN(Y))
      A6=V*SINH(V)*COS(Y)-Y*COSH(V)*SIN(Y)
      UL=A6*SINH(V)*SIN(Y)+UO*COSH(V)*COS(Y)
```

```
      HH=-A3
      GO TO 100
    6 V=SQRT(Q*R/(4.*DD))*EL/2.
      Y=V+SIN(V)*COS(V)
      W=-16.*V**3*SIN(V)*SIN(V)/(Y*EL**3)
      AJO=4.*V*V*(SIN(V)-V*COS(V))/(Y*EL*EL)
      AJL=4.*V*V*(SIN(V)*COS(V)-V)/(Y*EL*EL)
      UO=V*COS(V)+SIN(V)
      UL=UO*COS(V)+V*SIN(V)*SIN(V)
      HH=Y
      GO TO 100
    7 CC=Q*R/(8.*DD)
      DC=.5*SQRT(E*H/(DD*R*R))
      C=SQRT(CC-DC)
      D=SQRT(CC+DC)
      V=C*EL/2.
      Y=D*EL/2.
      B1=Y*Y-V*V
      B2=SIN(Y)**2-SIN(V)**2
      B3=V*SIN(Y)*COS(Y)+Y*SIN(V)*COS(V)
      W=-16.*V*Y*B1*B2/(B3*EL**3)
      B4=Y*SIN(V)*COS(Y)-V*COS(V)*SIN(Y)
      AJO=-4.*B1*B4/(B3*EL*EL)
      B6=Y*SIN(V)*COS(V)-V*SIN(Y)*COS(Y)
      AJL=-4.*B1*B6/(B3*EL*EL)
      UO=Y*SIN(V)*COS(Y)+V*COS(V)*SIN(Y)
      B7=(V*SIN(V)*COS(Y)+Y*COS(V)*SIN(Y))*SIN(V)*SIN(Y)
      UL=B7+UO*COS(V)*COS(Y)
      HH=B3
      GO TO 100
  100 AR=A+TR*H
      PP=R*R*W*H**3/(6.*(1.-PO*PO)*AR)
      TT=A/(AR*(1.-PP))
      QQR=Q*(TR-PP*(AR-PO*A/2.)/H)
      QQ=QQR/(1.-PP)
      SRY=QQ*R/AR
      IF(SRY*TR*2..GT.QCR)GO TO 222
      GO TO 333
  333 IF(SRY.GT.AYS) GO TO 8
      GO TO 9
    8 WRITE(6,10)SRY
```

```
   10 FORMAT(6X,'SRY = ',F10.2/)
    9 SKBO=Q*R*R*(1.-PO/2.)*TT*AJO/(2.*(1.-PO*PO))
      SKF1=-Q*R/H+Q*R*(1.-PO/2.)*TT*UO/(H*HH)
      SKF2=-Q*R/H+Q*R*(1.-PO/2.)*TT*UL/(H*HH)
      SKBL=Q*R*R*(1.-PO/2.)*TT*AJL/(2.*(1-PO*PO))
      SX1=SKBO-Q*R/(2.*H)
      SX2=-SKBO-Q*R/(2.*H)
      SY1=SKF1+PO*SKBO
      SY2=SKF1-PO*SKBO
      SX1L=SKBL-Q*R/(2.*H)
      SX2L=-SKBL-Q*R/(2.*H)
      SY1L=SKF2+PO*SKBL
      SY2L=SKF2-PO*SKBL
      SKY=SQRT(SX2**2+SY2**2-SX2*SY2)
      IF(SKY.GT.AYS) GO TO 51
      GO TO 13
   51 X(2)=X(2)+.0001
      GO TO 1000
   13 RO=R+H/2.
      RI=R-H/2.
      RWI=RI-DR
      RFI=RWI-TF
      VS=AL*(RO*RO-RI*RI)
      VW=TR*(RI*RI-RWI*RWI)*BK
      VF=WF*(RWI*RWI-RFI*RFI)*BK
      WUL=GS*3.14*(VS+VW+VF)/AL
      RYC=SRY/AYS
      RBC=SRY*TR*2./QCR
```

```
      PBC=2.*Q/PBCR
      SKYC=SKY/AYS
      GB=2.*Q/QSTAR
      WRITE(6,20)
   20 FORMAT(//25X,'D E S I G N     R E S U L T S '//)
      WRITE(6,21)DP
   21 FORMAT(6X,'OPERATING DEPTH = ',F8.0/)
      WRITE(6,22)ZZ,AL,R
   22 FORMAT(6X,'ZZ = ',F8.1,2X,'L = ',F7.1,2X,'R = ',F7.1/)
      WRITE(6,40)X(2),CY
   40 FORMAT(6X,'X(2) = ',F10.5,8X,'CY = ',F10.5/)
      WRITE(6,50)WUL
   50 FORMAT(6X,'WEIGHT PER INCH  = ',F10.2/)
      WRITE(6,23)H
   23 FORMAT(6X,'SKIN   THICKNESS = ',F10.5/)
      WRITE(6,24)DR
   24 FORMAT(6X,'DEPTH    OF    WEB = ',F10.5/)
      WRITE(6,25)TR
   25 FORMAT(6X,'WEB    THICKNESS = ',F10.5/)
      WRITE(6,26)WF
   26 FORMAT(6X,'FLANGE      WIDTH = ',F10.5/)
      WRITE(6,27)TF
   27 FORMAT(6X,'FLANGE THICKNESS = ',F10.5/)
      WRITE(6,28)ELY
   28 FORMAT(6X,'RING     SPACING = ',F10.5/)
      WRITE(6,41)TN,M,N
   41 FORMAT(6X,'CKYR=',F11.2,2X,'M = ',I5,2X,'N = ',I5/)
      WRITE(6,42)QSTAR
```

```
   42 FORMAT(6X,'QSTAR=',F15.2/)
      WRITE(6,30)QCR,SRY,PBCR
   30 FORMAT(6X,'QCR=',F10.2,2X,'SRY=',F10.2,2X,'PBCR=',F10.
2/)
      WRITE(6,111)SX2,SY2,QQ
  111 FORMAT(6X,'SX2 = ',F10.2,2X,'SY2 = ',F10.2,2X,'QQ = ',
F12.2/)
      WRITE(6,29)SKY
   29 FORMAT(6X,'SKY = ',F10.2/)
      WRITE(6,43)GB
   43 FORMAT(6X,'GB  = ',F10.5/)
      WRITE(6,31)PBC
   31 FORMAT(6X,'PBC = ',F10.5/)
      WRITE(6,32)RBC
   32 FORMAT(6X,'RBC = ',F10.5/)
      WRITE(6,34)RYC
   34 FORMAT(6X,'RYC = ',F10.5/)
      WRITE(6,35)SKYC
   35 FORMAT(6X,'SKYC= ',F10.5/)
      GO TO 55
  222 WRITE(6,3000)
 3000 FORMAT(6X,'RING BUCKLING FAILURE'/)
   55 GO TO 1
    4 CONTINUE
      END
```

BIBLIOGRAPHY

1. Gerard, G., "Optimum Structural Design Concepts for Aerospace Vehicles", Journal Spacecraft, Vol. 3, No. 1, 1966, pp. 5-18.

2. Niordson, F. I., and P. Pederson, "A Review of Optimal Structural Design", Paper Presented at the 13th International Congress of Theoretical and Applied Mechanics, Moscow, USSR, August 1972.

3. Schmit, L., "Structural Synthesis: 1959-1969: A Decade of Progress", Recent Advances in Matrix Methods of Structural Analysis and Design (Ed. R. H. Gallagher et al.), University of Alabama Press, 1971.

4. Schmit, L., "Structural Engineering Applications of Mathematical Programming Techniques", Symposium on Structural Optimization, AGARD Conference Proceedings No. 36 (Editor R. Gellatly), Advisory Group for Aeronautical Research and Development, NATO, October 1970.

5. Schmit, L., "Automated Design", International Science and Technology, June 1966.

6. Nickell, E. H., and R. F. Crawford, "Optimum Ring Stiffened Cylinders Subjected to a Uniform Hydrostatic Pressure", Society of Automotive Engineers, Reprint 578F, 1962.

7. Crawford, R. F., and A. B. Burns, "Minimum Weight Potentials for Stiffened Plates and Shells", AIAA Journal, Vol. 1, No. 4, April 1963, pp. 879-886.

8. Burns, A. B., and B. O. Almroth, "Structural Optimization of Axially Compressed Ring-Stringer Stiffened Cylinders", Journal Spacecraft, Vol. 3, No. 1, 1966, pp. 19-25.

9. Burns, A. B., and J. Skogh, "Combined Loads Minimum Weight Analysis of Stiffened Plates and Shells", Journal Spacecraft, Vol. 3, No. 2, February 1966, pp. 235-240.

10. Block, D. L., "Minimum Weight Design of Axially Compressed Ring Stringer Stiffened Cylindrical Shells", NASA CR-1766, July 1971.

11. Shideler, J. L., M. S. Anderson and L. R. Jackson, "Optimum Mass-Strength Analysis of Orthotropic Ring Stiffened Cylinders Under Axial Compression", NASA TND-6772, July 1972.

12. Spunt, L., Optimal Structural Design, Prentice Hall, 1971.

13. Thompson, J. M. T., and G. M. Lewis, "On the Optimum Design of Thin Walled Compression Members", Journal of Mechanics and Physics Solids, Vol. 20, 1972, pp. 101-109.

14. Thompson, J. M. T., J. D. Tulk, and A. C. Walker, "An Experimental Study of Imperfection-Sensitivity in the Interactive Buckling of Stiffened Plates". To be published.

15. Morrow, Wm. M., II, and L. A. Schmit, Jr., "Structural Synthesis of a Stiffened Cylinder", NASA CR-1217, December 1968.

16. Jones, R. T., and D. S. Hague, "Application of Multivariable Search Techniques to Structural Design Optimization", NASA CR-2039, June 1972.

17. Pappas, M., and A. Allentuch, "Structural Synthesis of Frame Reinforced Submersible, Circular, Cylindrical Hulls", NCE Report No. NV 5, Newark College of Engineering, Newark, New Jersey, May 1972.

18. Pappas, M., and A. Allentuch, "Optimal Design of Submersible Frame Stiffened Circular Cylindrical Hulls", NCE Report No. NV 6, Newark College of Engineering, Newark, New Jersey, July 1972.

19. Pappas, M., and A. Allentuch, "Mathematical Programming Procedures for Mixed Discrete-Continuous Design Problems", NCE Report No. NV 7, Newark College of Engineering, Newark, New Jersey, April 1973.

20. Pappas, M., and A. Allentuch, "Extended Structural Synthesis Capability of Automated Design of Frame-Stiffened, Submersible, Circular, Cylindrical Shells", NCE Report No. NV 8, Newark College of Engineering, Newark, New Jersey, June 1973.

21. Pappas, M., and A. Allentuch, "Pressure Hull Optimization Using General Instability Equations Admitting More than One Longitudinal Buckling Half-Wave", NCE Report No. NV 9, Newark College of Engineering, Newark, New Jersey, June 1973.

22. Pappas, M., and C. L. Amba-Rao, "A Direct Search Algorithm for Automated Structural Design", AIAA Journal, Vol. 9, No. 3, March 1971, pp. 387-393.

23. Simitses, G. J., and V. Ungbhakorn, "Minimum Weight Design of Stiffened Cylinders Under Axial Compression", AIAA Paper No. 74-101, Presented at 12th Aerospace Science Meeting, January-February 1974.

24. Ungbhakorn, V., "Minimum Weight Design of Fuselage Type Stiffened Circular Cylindrical Shells Subject to Uniform Axial Compression", Ph.D. Thesis, School of Engineering Science and Mechanics, Georgia Institute of Technology, June 1974.

25. Nelder, J. A., and R. Mead, "A Simplex Method of Function Minimization", Computer Journal, Vol. 7, 1964, pp. 308-313.

26. Courant, R., "Calculus of Variations and Supplementary Notes and Exercises", (Revised and amended by J. Moses) New York University Institute of Mathematical Sciences, New York, 1956-1957, pp. 270-276.

27. Wilde, D. and C. S. Beightler, Foundation of Optimization, 1967, pp. 242-245.

28. Pappas, M., and C. L. Amba-Rao, "A Discrete Search Procedure for the Minimization of Stiffened Cylindrical Shell Stability Equations", AIAA Journal, Vol. 9, No. 11, November 1970, pp. 2093-2094.

29. Spendley, W., G. R. Hext, and F. R. Himsworth, "Sequential Application of Simplex Designs in Optimization and Evolutionary Operations", Technometrics 4, 1962, pp. 441-461.

30. Aswani, M., "Design Charts for the Minimum Weight Design of Circular Cylindrical Shells Subject to Uniform Hydrostatic Pressure", School of Engineering Science and Mechanics, Georgia Institute of Technology, Atlanta, Georgia, 1974.

31. Singer, J., and M. Baruch, "Recent Studies on Optimization for Elastic Stability of Cylindrical and Conical Shells", Aerospace Proceedings, 1966.

32. Simitses, G. J., "A Note on the General Instability of Eccentrically Stiffened Cylinders", Journal Aircraft, Vol. 4, No. 5, 1967, pp. 473-475.

33. Timoshenko, S. P., and J. M. Gere, Theory of Elastic Stability, McGraw Hill Book Company, 1961.

34. Majumdar, S., "Buckling of a Thin Annular Plate Under Uniform Compression", AIAA Journal, Vol. 9, No. 9, September 1971, pp. 1701-1707.

35. Yamaki, N., "Buckling of Thin Annular Plate Under Uniform Compression", Journal of Applied Mechanics, June 1958, pp. 267-273.

36. Salereno, V. L., and J. G. Pulos, "Stress Distribution in Cylindrical Shells Under Hydrostatic Pressure Supported by

Equally Spaced Circular Ring Frames", PIBAL No. 171A, Polytechnic Institute of Brooklyn, Brooklyn, New York, June 1951.

VITA

Mohan Aswani was born on January 12, 1939 in Nawabshah (W. Pakistan). He received his Bachelor's degree in Civil Engineering from Birla College of Engineering, Pilani (India) in 1962. He was awarded, in 1962, a three year Senior Fellowship under the Technical Teachers Training Program, sponsored by the Government of India. Under this program he received his Master's degree in Structural Engineering from the College of Engineering, Poona (India) in 1965. Thereafter he was appointed as a lecturer in the Department of Civil Engineering, Delhi College of Engineering, Delhi (India) where he served from 1965 to 1970. He entered the Georgia Institute of Technology, Atlanta, Georgia as a doctoral student in the School of Engineering Science and Mechanics in September 1970.