A METHOD FOR PROBABILISTIC SENSITIVITY ANALYSIS OF COMMERCIAL AIRCRAFT ENGINES

Prof. Dimitri Mavris,* Mr. Bryce Roth,† and Mr. Noel Macsotai†
Aerospace Systems Design Laboratory
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150

Abstract

The objective of this paper is to illustrate how probabilistic methods can be utilized to rationally and analytically make design decisions in the presence of uncertainty, with emphasis on the use of probabilistic sensitivities in the aircraft gas turbine engine preliminary design process. A brief review of risk and uncertainty in the engine design process is given, and the role of probabilistic methods is discussed. Probabilistic sensitivity analysis, used in conjunction with response surface methods, is proposed as a computationally-efficient method to address defined sources of uncertainty and risk in engine design from a system level perspective. The method outlined is then applied to the analysis of engine component performance uncertainty impact on the performance of a notional four-engine wide-body commercial transport. More specifically, uncertainty in engine design parameters is shown to have a direct and quantifiable impact on aircraft system figures of merit such as design range and fuel burn. The methods developed are then used to create a set of contour plots showing the behavior of vehicle performance uncertainty over the design space of interest.

Background

The business of design, test, and production of aerospace systems has historically, and is continuing to increase in complexity and cost for the foreseeable future.[1] The days of the "tin bender" aircraft industry have been replaced by the high precision machinist's mentality demanded by the performance and complexity of modern aerospace systems. This growing complexity, in conjunction with increases in capital cost of design and development, shareholder demand for stock value, industrial competition, and a volatile political environment are the underlying elements driving the current need for more sophisticated and accurate tools to manage development risk in a technical setting.

One potential answer to the need for risk management tools is to use probabilistic design methods. The objective of probabilistic design is to analytically quantify the impact of uncertainty in terms of probabilities by describing design performance in terms of *distributions* instead of point values. The result is an analytical estimate of uncertainty that can be used as a tool to aid the decision maker in selecting alternatives that have a level of risk consistent with program objectives and risk tolerance level. Probabilistic techniques have strong potential to assist the designer in finding solutions to risk management problems, and for this reason, have undergone a great deal of development over the past several years.

The goal of this paper is to illustrate how probabilistic methods can be used to provide the reader with a rational framework or methodology from within which a designer can consider uncertainty in an analytical and a self-consistent fashion. The discussion begins with a fairly broad overview of risk, uncertainty, and probabilistics in the engine design process, and then focuses on one aspect of probabilistic design, that being probabilistic sensitivity techniques. The proposed method, probabilistic sensitivity analysis, is discussed in detail and then applied to preliminary cycle analysis of a large commercial transport engine in order to illustrate the process and show ways to visualize results. The objective of this exercise is to show how uncertainty in system figures of merit (FoMs) such as design range and fuel burn changes as component performance uncertainty changes. In addition, this paper will also illustrate a technique for using a normalized probabilistic sensitivity in conjunction with response surface methods to create contour plots showing how the probabilistic sensitivities change over the cycle design space.

Uncertainty in the Context of Engine Design

It was pointed out earlier that the utility of probabilistic methods is to provide a uniform framework for *quantification of uncertainty*. In an engineering sense, uncertainty in the design process can come from a variety of sources, several of the most prominent being: uncertainty in mission requirements, infusion of new technologies, and analysis model fidelity (or lack thereof). Mission uncertainty is typically important during the very early stages of program development when the vehicle design requirements are not yet "set in stone". For these scenarios, selection of an appropriate engine size and cycle to begin preliminary studies is ambiguous because the ultimate design requirements are unknown.[2]

*Senior Member, AIAA
†Student Member, AIAA

For the technology selection problem, the objective is to select from amongst a set of new technologies, those that will yield the best possible product with minimal risk. Uncertainty in this scenario takes the form of: 1) uncertainty as to the performance of technology concepts when used outside of the laboratory environment, and 2) uncertainty that the technologies can meet established program goals. Finally, the third scenario involves uncertainty in analytical model fidelity and the ability to build hardware that meets performance predictions.[3] For instance, uncertainty in the various engine component performance predictions must be 'rolled up' into an uncertainty on overall propulsion system performance, weight, cost, etc. It is this last type of uncertainty that will serve as a demonstration of the approach described in this paper.[‡] There are three missions of interest for this analysis: a design range mission, a 6,000 nmi mission, and a 3,000 nmi mission.

Deterministic vis 〈 vis Probabilistic Sensitivities

The fundamental difference between probabilistic sensitivities and their deterministic counterparts is that the former provides information regarding the impact of *uncertainty* on design performance whereas the latter provides information as to the change in average performance. To understand these differences consider first the deterministic sensitivity. Deterministic sensitivities are essentially a first order Taylor series approximation at a point. Thus, they are the partial derivative of a change in response mean with respect to a change in variable mean. For example, one could use a parametric deck in conjunction with a mission analysis code to calculate sensitivity of design range and mission fuel burn with respect to engine component efficiencies, pressure losses, etc. Based on this information, the designer can make decisions as to which components are important drivers on system performance and place appropriate emphasis in these areas. The advantage of sensitivities is that they are easy to calculate, intuitive to use, and are an excellent tool for decomposing a highly complex problem into manageable pieces.

When uncertainty becomes a first-order effect,[§] the deterministic sensitivities are no longer sufficient to completely describe the problem. Specifically, the deterministic sensitivity says nothing about the variances or the change in distribution width of one parameter with respect to another. To capture this information, one must create an auxiliary set of sensitivities that quantify the change in variance of one parameter with respect to another. In effect, the input and output parameters in the sensitivity Taylor series now have a mean and a variance. Therefore, one now needs a deterministic sensitivity in addition to sensitivities describing the change in response variance with respect to *design variable mean*, and change in response variance with change in *design variable variance*. As a result, the number of sensitivities for potential consideration has doubled, and if mean-variance interactions are significant, the number of sensitivities increases by a factor of four, which means the complexity of the problem has increased markedly.[4]

The advantage of this approach is that it provides a more complete picture of the impact that uncertainty has on overall system performance. In effect, it allows one to gauge not only mean sensitivities, but also sensitivity of variance or spread. This is useful information because it indicates which component-level uncertainties are the biggest contributors to uncertainty in overall system performance.

Another advantage is that it facilitates direct quantification of design margin required to meet program goals. This concept is most easily explained through an example, in this case an aircraft with a 7,500 nmi design range target. Assume for the sake of argument that the performance of each component in an engine has some uncertainty associated with it. If this uncertainty can be described as a distribution, the resultant design range must also be a distribution. Given this range distribution, one can select a desired confidence level and estimate the corresponding design range.

Alternatively, given a 7,500 nmi design range target, one could estimate the probability (confidence) of meeting the design range target. This is contrary to the usual approach wherein one would typically add a margin to the target to obtain a nominal range, and design for this increased range with the margin as a buffer against unknowns. In the design world, the margin adder is typically based on past experience or a statistical analysis if sufficient production data is available.

Figure 1 illustrates the probabilistic point of view, where instead of having a point value for design range, it is now a distribution. The abscissa is design range, and the ordinate shows probability of failing to meet the 7,500 nmi design range target. Given this distribution and a design range target, it is possible to estimate the probability of failure. In this figure, the distribution has a 20% probability of failure (or alternatively, 80% confidence). The design margin in this case would be the distance between the target range and the 50% probability (nominal) range. Thus, the probabilistic approach works in terms of confidence levels instead of design margins. For a given confidence level, the margin is a fall-out.

[‡] In addition, manufacturing process capability is an important source of uncertainty in the operations area, but this discussion will focus on the design aspects of uncertainty only.

§"First order effect" meaning that the uncertainty (or variance) of system performance is on the same order of magnitude as changes in the mean value of performance.

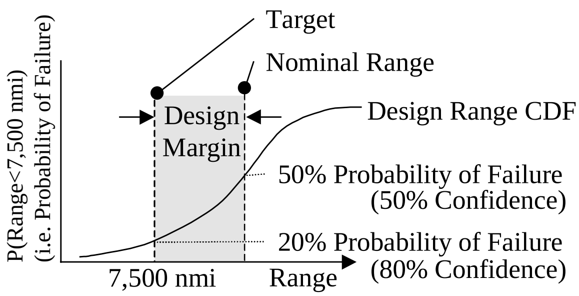

Figure 1: Comparison Between Deterministic and Probabilistic Approaches to Design Margin

Probabilistic sensitivities are useful in this setting because they quantify the relative change in probability of success per unit change in design range. In turn, this facilitates trades that take into account all considerations impacting engine performance in a design environment where uncertainty exists. Traditionally, the lack of knowledge about uncertainty in design makes it difficult to take advantage of available design margin because it is not known exactly how much margin is *really* available. Probabilistic sensitivities facilitate *direct trades* of margin against performance.

To summarize, the main innovation in a probabilistic approach is that a probabilistic sensitivity is formulated in terms of a change in *probability of meeting the target* over change in an FoM or probability of meeting another target. Examples of typical probabilistic sensitivities and their deterministic analogues are given in Table 1. The left column gives the change of confidence in meeting a target per unit change in FoM, while the right column gives the deterministic analogue sensitivity in terms of the value of one FoM relative to another. Note that the probabilistic sensitivities could also be formulated in terms of FoM margin over FoM margin (as, for example, probability of meeting design range target over probability of meeting *acoustic noise* target). However, it should be noted that the price is paid for this analytical capability is further complication of the sensitivity analysis process and additional computational effort.

Probabilistic Sensitivity Analysis Method

The basic idea behind probabilistic sensitivities is very similar to ordinary sensitivities where the designer calculates point sensitivities for various figures of merit at a design point. This information is then used to make trades of one FoM against others until a solution is found which is satisfactory (i.e. good performance, meets requirements, satisfactory margins, etc.). The objective of this section is to detail the probabilistic sensitivity analysis methods used herein, and discuss details of implementation of probabilistic sensitivity methods.

In a mathematical sense, deterministic sensitivities can be thought of as a subset of a more comprehensive set of probabilistic sensitivities, as illustrated in Figure 2. The element in the upper left-hand corner of the top matrix corresponds to the classic deterministic sensitivity, and is the partial derivative of response mean i with respect to the mean of variable j. However, for the probabilistic case, the inputs and outputs must be described in terms of a distribution, or, at least a variance. Thus, there must be an additional set of sensitivities that relates the change in probability of meeting a target with respect to a change in the mean of an FoM or with respect to the change in probability of meeting other targets. These are the three remaining elements in Figure 2.

An example illustrating the fundamental idea is given in the matrix of Figure 3 for the case where the objective is to find the sensitivity of design range with respect to change in 3,000 nmi mission (3K) fuel burn. The sensitivity at the upper left is the deterministic case which quantifies the relative worth of design range in terms of 3K fuel burn. The sensitivity in the upper right quantifies the change in probability of meeting design range target per unit change in 3K mission fuel burn. The lower right is the same sensitivity except quantified in terms of probability of meeting range and fuel burn targets. Finally, the lower left is the change in design range mean value per unit change in probability of meeting 3K fuel burn target. Note that since there are no 3K fuel burn targets defined for the example to be used later in this problem, these last two sensitivities are not used in this paper. These sensitivities can be calculated using an approximate calculation method, to be described later.

<u>Probabilistic Sensitivity Factors</u>

The previous example discussed the sensitivity between two response parameters, which can be useful when making system-level trades between competing objectives. However, another scenario is to inquire as to how *system performance uncertainty* changes with *component performance uncertainty*. A typical example would be the sensitivity of design range uncertainty with respect to changes in HPT efficiency uncertainty. This

<u>Probabilistic Sensitivity</u>	*<u>Deterministic Sensitivity</u>*
$P_{success}$(Range)/ lb Engine Weight	nmi Range / lb Engine Weight
$P_{success}$(Range)/ lb 3K Fuel Burn	nmi Range / lb 3K Fuel Burn
$P_{success}$(Range)/ \$ Shop Cost	nmi Range / \$ Shop Cost
$P_{success}$(Range)/ dB Margin	nmi Range / dB Margin

Table 1 : Probabilistic Sensitivities and Their Deterministic Analogues

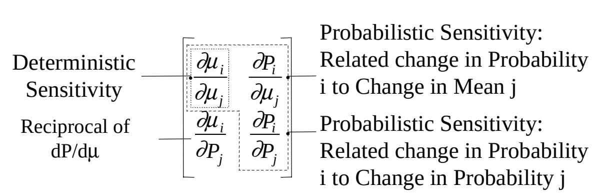

Figure 2: Relationship between Probabilistic and Deterministic Sensitivities

$$\begin{bmatrix} \dfrac{\Delta \text{ NMi Design Range}}{\Delta \text{ lb 3K Fuel Burn}} & \dfrac{\Delta \text{ Prob. of Meeting DR Target}}{\Delta \text{ lb 3K Fuel Burn}} \\[2ex] \dfrac{\Delta \text{ NMi Design Range}}{\Delta \text{ Prob. Meeting 3K FB Target}} & \dfrac{\Delta \text{ Prob. of Meeting DR Target}}{\Delta \text{ Prob. Meeting 3K FB Target}} \end{bmatrix}$$

Figure 3: Example Probabilistic Sensitivities for Design Range and Fuel Burn

type of information can be presented using the "probabilistic sensitivity factor" concept developed by Wu et al,[5] which is particularly useful in weighing the relative importance of a set of input random variables on response uncertainty.

Probabilistic sensitivity factors are essentially normalized probabilistic sensitivities wherein the vector norm[**] of a set of principal sensitivity factors at a point must be one. They are a first-order accurate estimate of probabilistic sensitivities, and can be thought of in geometric terms as the directional cosines of the random variables. They must therefore satisfy the equation:

$$\sum_n \alpha_i^2 = 1$$

where α_i = Probabilistic sensitivity factor for random variable i
n = Number of random variables in analysis

Additionally, the probabilistic sensitivity factors satisfy the proportionality:

$$|\alpha_i| \propto \left(\frac{\partial g}{\partial x_i}\right)\sigma_i$$

where: g = Response parameter of interest
x_i = Input variable i
σ_i = Standard deviation of random variable i

In other words, the probabilistic sensitivity factor is proportional to the deterministic sensitivity times the standard deviation of the random variable. Thus, the probabilistic sensitivity factor reflects dispersion in the response due to uncertainty in the input random variable as well as the scaling effect of the deterministic sensitivity.

For example, suppose that it is desired to know the sensitivity of design range uncertainty with respect to uncertainty in a set of seven component efficiency parameters. Using the probabilistic sensitivity factor method, one would obtain a set of seven probabilistic sensitivity factors, one for each input distribution. The vector norm of the seven components must be one, and the magnitude of each component would be proportional to its overall influence.

Probabilistic Sensitivity Contours

From a design point of view, a fundamental shortcoming of sensitivity methods as they are ordinarily applied is that the sensitivities are valid at a single point only, and do not give information as to how the sensitivities vary throughout the entire design space. This is reflected in the way that sensitivities are typically used: sensitivities are considered *after* the design point has been selected, rather than playing an active role *during* the design point selection. Using sensitivities in this manner limits their usefulness in the design process.

It is therefore very desirable to find a means of obtaining a broad picture of how sensitivities vary throughout the design space. This was a driving factor in the development of the analysis methods used here. The objective was to develop techniques that would broaden the application of sensitivities from being strictly an analysis technique to being a design technique.

In order to get a global view of how sensitivities change in the design space, one needs a plot of sensitivity contours that depict regions of high and low sensitivity over the *entire* design space. However, the data required to generate such a plot would ordinarily require an exhaustive matrix of points at which sensitivity analyses must be run. This is tedious and not very practical if the design space has more than three dimensions (three variables). The solution to this problem is to use response surface methodology (RSM) in conjunction with design of experiments (DoE) techniques to drastically reduce the computational workload required to obtain the desired sensitivity contours.

Both RSM and DoE are fairly standard analysis techniques that have been in use in other fields for some time. RSM is essentially a formalized method for multivariate regression, and is a tool commonly used in process engineering.[6] The objective of RSM is to create an approximate analytical model of a given data set using general regression equations to model data behavior. DoE is a scientific method for maximizing the power of experiments.[7] It is well known in the scientific community, and was first developed in the 1920's. DoE methods offer a much more efficient means of obtaining information than the exhaustive grid search and matrix methods. These two techniques, in conjunction with the probabilistic sensitivities described earlier, enable probabilistic sensitivities to be applied in a more general, design-oriented way than would otherwise be the case.

Analysis Method

The first element required for the analysis is a means of generating propulsion system performance data and calculating performance of the engine/aircraft combination. In this case, a standard parametric deck was used to generate all cycle and engine weight/geometry data required for the analysis. This is then used in the mission analysis code to estimate vehicle performance with the desired engine installation. For the results shown in this paper, the parametric deck and mission analysis are linked together using an automated script. This script takes cycle parameters and component efficiency

[**]Recall that the mathematical definition of a vector norm is the square root of the sum of the squares for all principal components of a vector.

modifiers as input, runs the analysis, and returns outputs for engine and vehicle performance. The creation and validation of this script is no mean task, but it greatly reduces the run time and the opportunity for human error in the analysis process.

Once the basic engine/aircraft analysis script is in place, the next step is to link this to a probabilistic analysis package. Recall that the basic engine and aircraft analysis codes are deterministic in nature in that they must have point values for all inputs and return point values as outputs. Therefore, if probabilistic results are desired, it is necessary to "wrap" a probabilistic analysis package around the engine/aircraft script. This allows it to obtain probabilistic results by repeated interrogation of the script analysis. In this case, the probabilistic analysis package chosen for use is the fast probability integrator (FPI) described in references 5 and 8. Linking the probabilistic package to the script analysis allows FPI to automatically generate result distributions given a set of input distributions, without the need for user intervention.

The basic architecture resulting from the analysis code set-up previously described is depicted in Figure 4. First, a design of experiments approach is used to define a set of cases varying the cycle parameters over a prescribed range. This enables maximal information to be extracted from a minimal number of cases, and reduces the number of analysis cases required to generate the contour plots mentioned earlier. Meanwhile, the component uncertainty parameters are assigned a distribution that reflects the dispersion of preliminary component performance estimates, and does not change from case to case. Thus, the input uncertainty is assumed to be constant over the entire design space.

This set of cases is the run using the FPI probabilistic analysis package in conjunction with a parametric deck and a mission analysis code. This analysis produces two types of output data. First, a set of raw output distributions for design range, mission fuel burn, and fan diameter are created, one for each case. In addition, FPI also gives a set of probabilistic sensitivity factors for each case.

These sensitivity factor data sets can be visualized using response surface methods in order to make contour plots showing the variation of sensitivities throughout the cycle design space. For the case examined in this paper, there are seven input distributions on component performance uncertainties. There will therefore be seven probabilistic sensitivity factors for each case run, and there are a total of 15 unique cases for a three factor central composite DoE. In total, there are seven data sets with 15 cases each. These data sets can be used to create seven response surface equations, one for each of the seven sensitivity factors. These response surface equations are quadratic approximations of the actual data

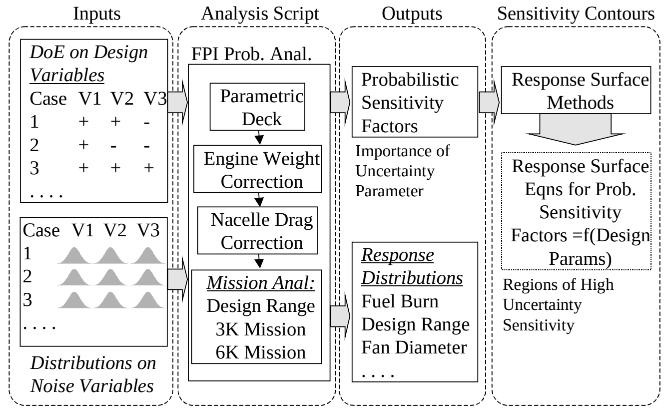

Figure 4: Probabilistic Sensitivity Analysis Methodology

behavior, and can therefore be plotted in the form of contour plots.

Application to Commercial Engine Cycle Design

In order to illustrate the ideas presented herein, the probabilistic methods discussed in the previous section are applied to the analysis of uncertainty in a large commercial aircraft engine. Specifically, the problem analyzed here focuses on the impact of component performance uncertainty (meaning component efficiencies, parasitic cooling flows, and pressure losses) on vehicle performance. This analysis is complementary to that described in reference 3, and the interested reader is referred there for more details on the analytical setup and probability distribution results obtained. Whereas reference 3 deals with the analysis of the probability distributions themselves, this work deals in-depth with the analysis of probabilistic sensitivities.

Specifically, this paper is concerned with uncertainty in the cycle selection process. Cycle analysis is a process of application of conservation of energy, momentum, and mass to achieve a thermodynamic balance amongst all components in the engine. The output is engine performance as a function of flight condition and throttle setting. Sir Frank Whittle himself was one of the first persons to develop and apply cycle analysis to the turbojet engine in the 1930s in order to estimate the performance of his original designs.[9] Later, Whittle's pioneering work was furthered by men such as Sir Stanley Hooker, who contributed to this area by developing more formalized and accurate methods for estimating the performance of compressors, turbines, and turbojet engines.[10] Since then, the field has matured considerably, and the gas property models, nomenclature, and results presentation are now standardized.[11,12] Furthermore, there are now numerous texts on the subject such as reference 13, and therefore, the theoretical background of engine cycle design is not discussed in detail here.

Instead, it is simply stated that the engine cycle selection process typically involves a myriad of considerations such as cost, noise, emissions, technology capability, customer desires, etc. In addition, cycle selection must be accomplished concurrently with other analyses, such as mechanical design, weight and flowpath design, etc. It is impractical to attempt to address all of the elements relevant to cycle selection in the confines of this paper. Therefore, the focus here is on a portion of the larger problem, this being the classic trade between design range and mission fuel burn. Emphasis is placed on using probabilistic sensitivity contours as a means of describing the impact of uncertainty over the entire design space, and assisting in the selection of an appropriate cycle based on this information.

What is meant by the previous reference to trades between design range and mission fuel burn is fundamentally a trade on propulsive efficiency and engine weight. In order to have less fuel burn for short-range missions, one must improve the propulsive efficiency of the engine by selecting a lower fan pressure ratio. This, in turn, increases fan diameter for the same thrust, and also increases engine weight. Ultimately, this results in a lower design range due to: 1) the increased nacelle drag of the larger fan and 2) increased empty weight (decreased fuel fraction) of the aircraft. Thus, short range missions demand lower fan pressure ratio and higher extraction ratio cycles with larger (and more expensive) engines, while an engine designed for maximum range will tend to have a higher fan pressure ratio and lower extraction ratio. The best engine cycle is always a balance between the two competing requirements. Although this example is a simplification of the larger problem, it is sufficient to convey the basic concepts, and extension of these methods to other portions of the problem is a relatively straightforward process.

The basic tools used for the present analysis consist of a parametric deck to generate engine performance and weight data, a mission analysis code, and the FPI probabilistic analysis package with the AMV method described in reference 8. The analytical setup also includes a correction on nacelle drag as a function of fan diameter, and a correction on aircraft empty weight as a function of engine weight. The model inputs are three cycle parameters and seven component performance

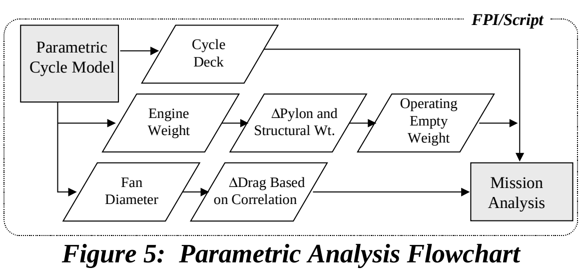

Figure 5: Parametric Analysis Flowchart

distributions. The outputs are *distributions* on: design range, 3,000 nmi mission (3K) fuel burn, and 6,000 nmi mission (6K) fuel burn. In addition, point values for fan diameter are tracked for each analysis case.[††]

Baseline Engine and Aircraft

The baseline aircraft is a notional 420 passenger commercial transport in the 800-900,000 lb gross weight class. The vehicle has a design range target of 7,500 nmi, and is limited to a 100 inch fan diameter to ensure adequate ground clearance. The mission analysis model assumes a fixed operational empty weight minus propulsion system weight, and includes adjustments for increased pylon structural weight as a function of engine weight, as well as a correction for nacelle drag as a function of fan diameter. The three missions considered assume typical mission rules, weights, and profiles, as well as standard day flight conditions. Schedules for power extraction and customer bleed air are based on typical requirements for a commercial aircraft of this size class.

The baseline engine used for this study is a dual spool high bypass separate flow turbofan engine. The core is photographically scaleable, while the low pressure spool is fully variable. The engine model assumes a constant (fixed) technology level and a two stage booster configuration.

The design parameters of interest for this study are extraction ratio,[‡‡] fan pressure ratio (FPR), and maximum turbine inlet temperature (T4). The normalized ranges used for these parameters are shown in Table 2. Note that since the core and booster configurations are fixed, the overall pressure ratio of the machine varies with fan pressure ratio. Also note that the range selected for fan pressure ratio is much narrower than that used for T4 or extraction ratio. The reason for this is that large changes in FPR would force a change in booster configuration in order to maintain the same overall pressure ratio, which was not desired for this study.

The uncertainty (or noise) parameters of interest for this study are given at the bottom of Table 2. These seven parameters were selected out of a total of 15 parameters via a screening test that showed them to have the greatest impact on uncertainty in vehicle performance. The distribution shape used for these parameters is the normal distribution, which is intended to be a first approximation since detailed statistical data was not available. The standard deviation (denoted as σ) was selected based on

[††]Engine fan diameter is not treated as a distribution in this analysis because it is not a strong function of the noise parameters and ranges selected for this study. See reference 3 for a more detailed explanation.

[‡‡]Extraction ratio is defined as the ratio of bypass duct discharge pressure to core stream discharge pressure.

Control Parameter	Upper	Nominal	Lower
Fan Pressure Ratio	+0.065	Base	-0.065
Extraction Ratio	+0.15	Base	-0.15
Max. Turbine Inlet Temp.	+200°F	Base	-200°F

Noise Parameter	Upper	Nominal	Lower
Fan Thrust Coefficient	+2σ	μ	-2σ
HPT Efficiency	+2σ	μ	-2σ
Compressor Efficiency	+2σ	μ	-2σ
LPT Efficiency	+2σ	μ	-2σ
HPT Chargeable Cooling	+2σ	μ	-2σ
Mid-Frame ΔP/P	+2σ	μ	-2σ
Booster Efficiency	+2σ	μ	-2σ

Table 2: Engine Cycle Parameter Range Specification (from ref. 3)

design experience to be representative of the typical spread of actual versus predicted component performance that has historically been observed at the preliminary level of design detail. Note that the actual values of the baseline cycle, the means (denoted as μ) and standard deviations are not given here due to the proprietary nature of the data.

Results

Before discussing the probabilistic sensitivity results in detail, it is instructive to first reinforce the previous discussion on the differences between the probabilistic and deterministic points of view. Consider Figure 6, which shows contour plots for vehicle design range as a function of FPR and extraction ratio with a T4 of base+122°F selected to give optimal design range. The plot on the left is the classical deterministic point of view showing contours of vehicle design range assuming mean values for the seven noise parameters. The constraint on fan diameter cuts the design space at the lower right of the figure, and the cycle for best design range is shown at top center of the plot. The best design range cycle is biased towards a high FPR, an extraction ratio near the baseline value, and a high turbine inlet temperature, yielding a *optimal* design range of 7,528 nmi.

The panel to the right shows the same problem from the probabilistic point of view. The difference in this case is that the contours express the probability that the vehicle will meet or exceed the 7,500 nmi design range target given the seven noise distributions defined previously. Thus, for example, all points inside the 60% probability of success contour have at least a 60% chance of meeting the design range target, with the optimum being 63%. Note that the fan diameter constraint could have been treated probabilistically, but in this case, fan diameter is only a weak function of the uncertainty parameters. As a result, the distance from tail to tail of the fan diameter distribution is very small, and the fan diameter probability contours effectively collapse into a single line for this plot.

The contour plot in the right panel shows the absolute probability level, which is effectively an aggregate view of the contribution from all seven uncertainty distributions to overall vehicle performance. However, it is also desirable to know what the individual contributions from each source of uncertainty are in order to understand how to improve the design. The most convenient means of doing this is by examining the probabilistic sensitivity factors for each of the seven uncertainty parameters.

Recall that the probabilistic sensitivity factor is proportional to the deterministic sensitivity times the standard deviation of the noise parameter of interest. The results for the probabilistic sensitivity factors at the center-point in the design space are shown for design range, 3K, and 6K fuel burn in Figure 7. This center point refers to a design in which all the control parameters are set at their base values and is not necessarily the

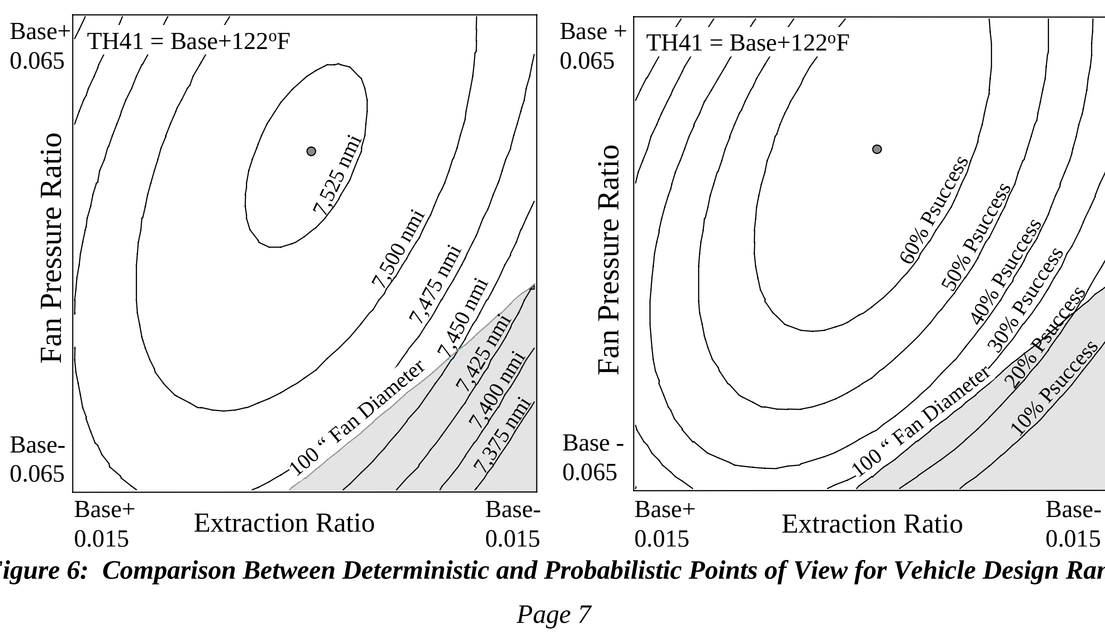

Figure 6: Comparison Between Deterministic and Probabilistic Points of View for Vehicle Design Range

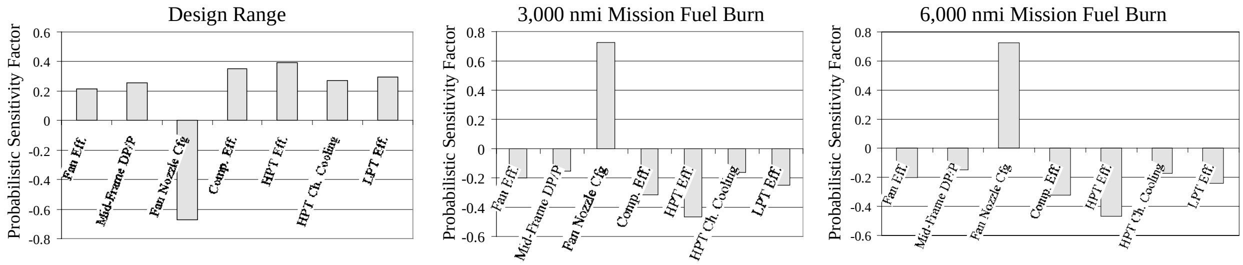

Figure 7: Probabilistic Sensitivity Factors for Vehicle Design Range and Fuel Burn

"best" design. The results show that uncertainty in engine performance is highly sensitive to uncertainty in fan nozzle thrust coefficient. Uncertainty in high pressure turbine (HPT) and compressor efficiencies is also moderately influential on uncertainty in engine performance. Note that the vector norm of the seven components for each response is one.

The drawback of these results is that they are valid for the center point only. Their value in active selection of a cycle design point is thus limited to an after-the-fact analysis on a single point. However, the objective here is to characterize the sensitivity throughout the entire design space.

It was pointed out earlier that a good way to convey sensitivity information throughout the entire design space is to use contour plots by using a DoE of analysis cases such as shown in Figure 7 and using RSEs to generate contour equations, as shown in Figures 8 and 9. These figures are probabilistic sensitivity factor *contours* for vehicle design range at a fixed T4 of base+122°F, with each plot showing how one of the seven sensitivity factors varies as a function of extraction ratio and fan pressure ratio. It is evident from these plots that fan nozzle thrust coefficient uncertainty (panel A) is the dominant parameter, exhibiting wide changes over the design space. Note that as the cycle design point approaches the fan diameter limit, the fan nozzle thrust coefficient uncertainty becomes increasingly dominant while the others generally become less important. Clearly, *the lower the design point fan pressure ratio, the more*

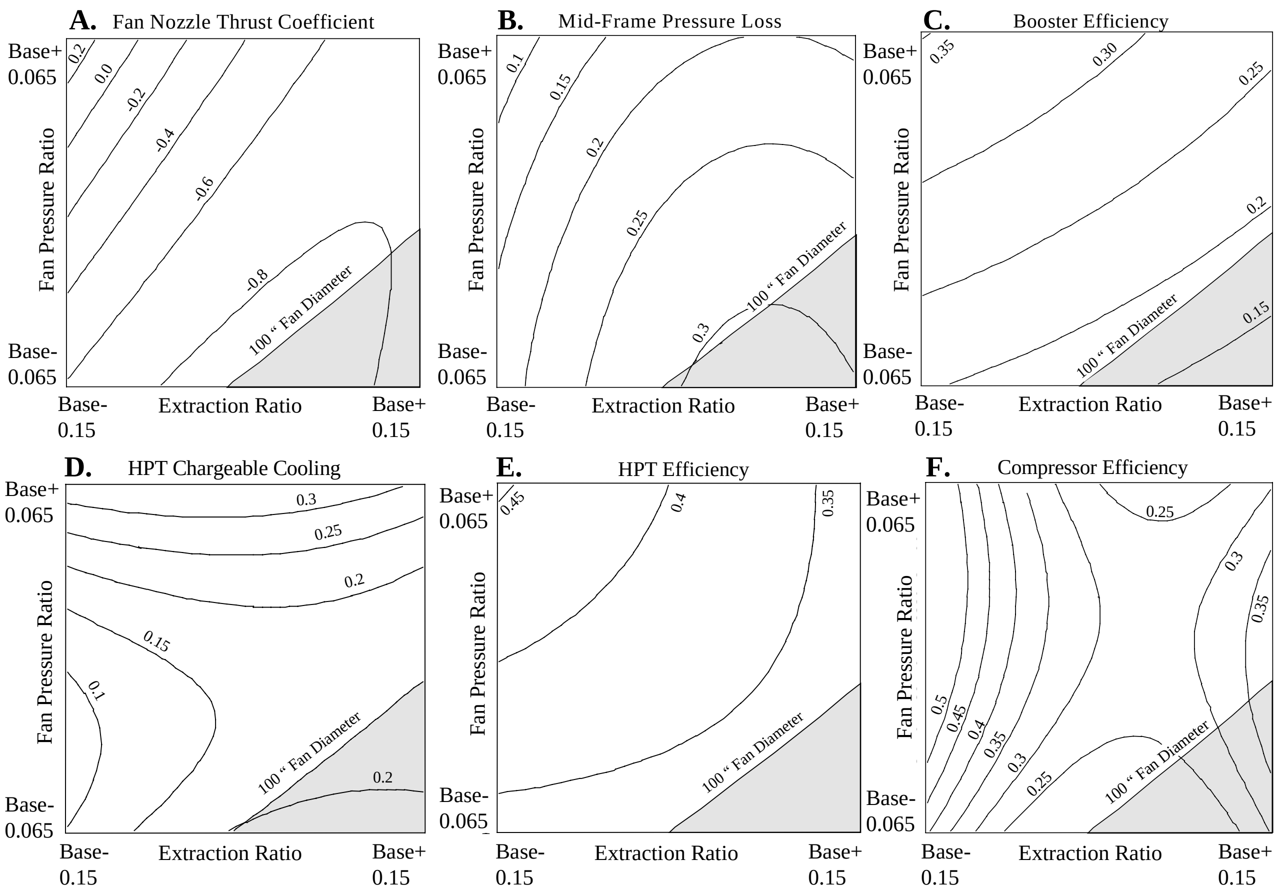

Figure 8: Probabilistic Sensitivity Factor Contour Plots for Design Range (at T4 = +122°F)

important it will become to accurately predict fan nozzle thrust coefficient.

It therefore becomes apparent that if there is some reason to move the design into these regions, the nozzle designer must strive to control and minimize uncertainty in the nozzle performance estimates, due to the strength of its impact. Conversely, by knowing that these regions exist, controlling the impact of uncertainty can be as simple as avoiding them (if possible). The last two statements, reducing the impact or avoiding uncertainty, are different approaches to achieving a robust design.

Note that HPT efficiency uncertainty (panel E) makes a moderately strong contribution to design range uncertainty throughout the entire design space. Moreover, the uncertainties associated with core components, namely HPT chargeable cooling, HPT efficiency, and, to a lesser extent, compressor efficiency (panels D, E, and F) are relatively insensitive to changes in fan pressure ratio or extraction ratio. This is not surprising given that these cycle parameters are not strong drivers on core engine performance.

On the other hand, the remaining four uncertainty sensitivities (shown in panels A, B, C, and Figure 9) are strongly influenced by fan pressure ratio and extraction ratio. Since these component uncertainties are associated with the low pressure spool, it is not surprising that they show such a strong correlation. Another noteworthy feature of the contour plots is that most of the contours are oriented at a diagonal to the axes, indicating that none of the uncertainty factors examined here is driven purely by fan pressure ratio or extraction ratio individually. Rather, the sensitivities are all strong functions of both design parameters.

Probabilistic sensitivity factor contours for 3,000 nmi mission fuel burn are shown in Figures 10 and 11. Contours for 6,000 nmi mission fuel burn look very similar, and are therefore excluded here in the interest of brevity. Note that the contours for mission fuel burn are generally shaped much different than that shown for the design range case, with the exception of compressor efficiency sensitivity (panel F). Also, note that HPT efficiency uncertainty (panel E) is a moderately strong driver on 3,000 nmi mission fuel burn.

Another notable feature of the 3K fuel burn contours is that fan nozzle thrust coefficient uncertainty (panel A) exerts a strong influence throughout the *entire* design space. This is different from the design range case, where it exhibits large swings going from being very strong at low FPR/high extraction ratio to relatively weak at high FPR/low extraction ratio. This general observation applies to all panels shown for the 3K fuel burn case, in that the range of extremes is much narrower than it was for the design range case. The conclusion that can be drawn based on this is that the relative importance of 3K fuel burn uncertainty sensitivities is not strongly impacted by cycle selection. As a result, there is little the designer can do if it is desired to tailor the cycle with regards to 3K fuel burn uncertainty.

This was not the case for the design range contours, which showed large swings in sensitivity values through the design space. As a result, it is possible to influence the relative importance of design range sensitivities through appropriate choice of cycle design point. If a desirable cycle design region can be identified based on the design requirements, the probabilistic sensitivity contour information can be used to bias the design point towards a portion of the design space with a desirable uncertainty sensitivity. For example, it may be desirable to bias the design towards having fan nozzle thrust coefficient be the dominant factor, as it is far cheaper to rectify uncertainty in this component, should it become necessary, than it is to modify turbomachinery.

Before continuing with this analysis, it is important to point out that these probabilistic sensitivity factor contours were created by using a response surface fit of the same 15 case, 3 factor central composite DoE data set

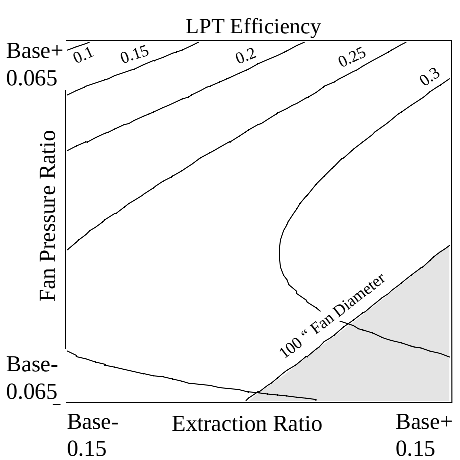

Figure 10: Probabilistic Sensitivity Factor Contours for Design Range (at T4 = base+122°F)

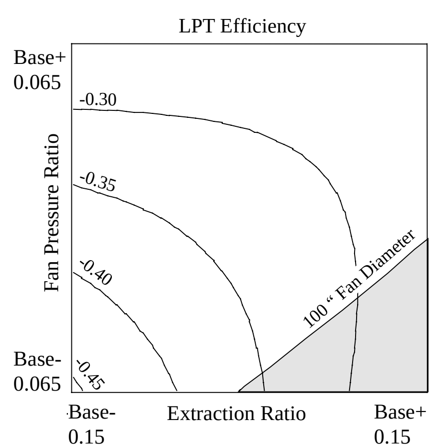

Figure 9: Probabilistic Sensitivity Factor Contour Plots for 3,000 nmi Fuel Burn (at T4 = base+122°F)

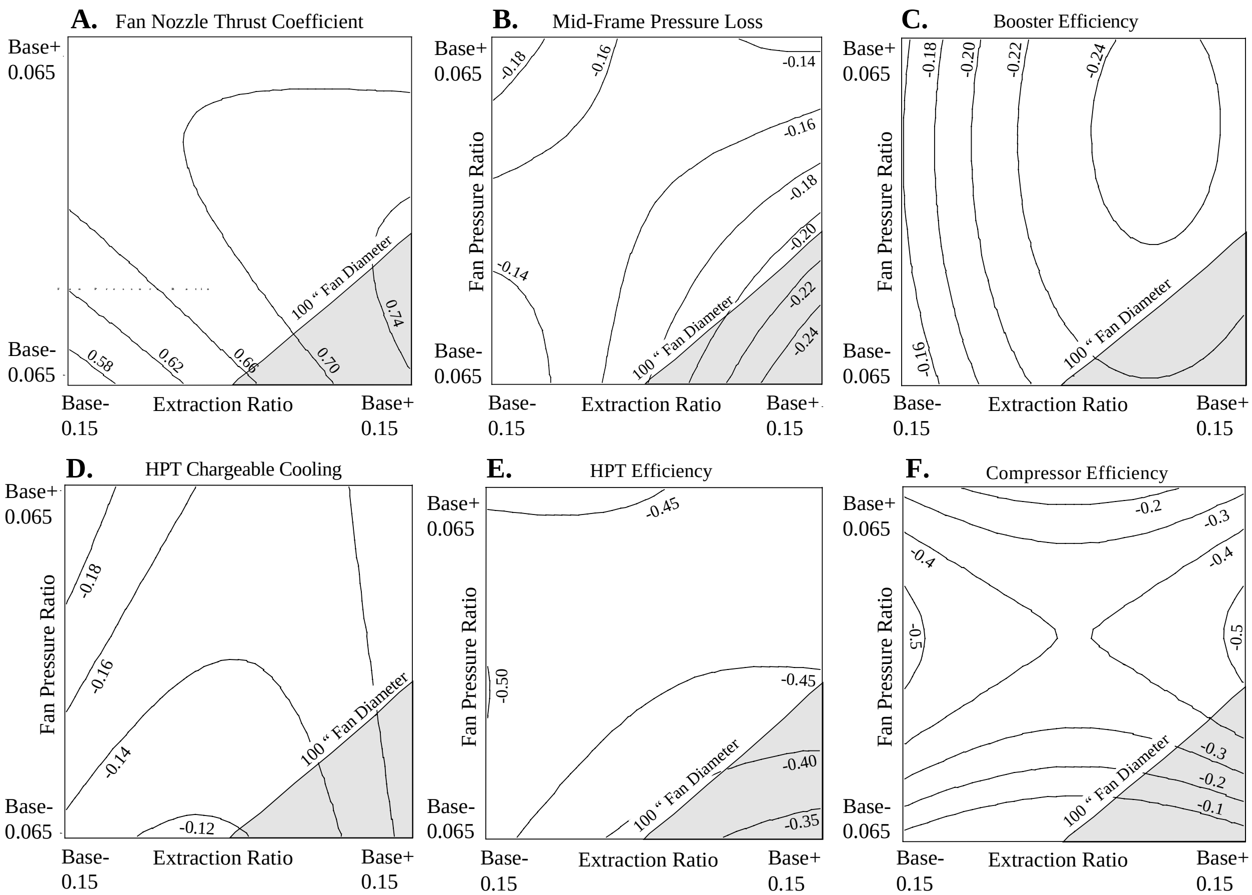

Figure 11 : Probabilistic Sensitivity Factor Contours for 3,000 nmi Mission Fuel Burn (at T4 = base+122°F)

described previously in the analysis method section. Therefore, the contour plots of sensitivities shown in this section are quadratic *approximations* only, and are not exact representations of the actual contours. This explains why the contour shapes shown in the plots all have a conic section behavior. The contour plots used here are a good visualization tool for the entire design space, are useful to find regions of desired performance, and can be used to "zero-in" on a smaller region of interest. However, if exact probabilistic sensitivity estimates are desired at point in the design space, one *must* run the probabilistic analysis at that point.

Up to now, the focus here has been to examine how uncertainty in component performance impacts uncertainty in engine performance. However, at the system level, it is also useful to know the sensitivity of one figure of merit to another such that trades on one against the other can be evaluated. This is especially useful when the two objectives are conflicting (as the best range and fuel burn cycles are in this case).

One such probabilistic sensitivity of interest is change in probability of success with respect to design range and fuel burn. These sensitivities can be estimated by superimposing contour plots of different FoMs. For example, probability contours for design range target as well as contours for constant 3K fuel burn are shown in Figure 12, with T4 optimized for the best design range scenario. Note that the region for best design range has a higher FPR and extraction ratio than that for the best fuel burn (lowest SFC). Note also that the best fuel burn design is estimated to have only a 30% chance of meeting or exceeding the design range target, whilst the best design range cycle has in excess of 60% probability of success.

It is fairly simple to estimate the probabilistic sensitivities of FoMs relative to one another by simply estimating contour gradients based on the figure or by numerically calculating contour gradients using the underlying response surface equations. In this case, the cycle for best design range yields probabilistic sensitivities of 5% probability of success per 8.8 nmi range and 5% probability of success per 350 lb of 3K fuel burn.

Based on the results presented up to now, the situation for the simple problem discussed here is as follows: the best design range cycle occurs for an FPR of base+0.035, a extraction ratio of base+0.0, and a T4 of base+122°F. The probabilistic sensitivity analysis indicates that it is desirable to bias this towards the best 3K fuel burn cycle, as a reduction of 5% in probability of meeting design range will enable a significant decrease in the 3K mission fuel burn. Moreover, the contours for

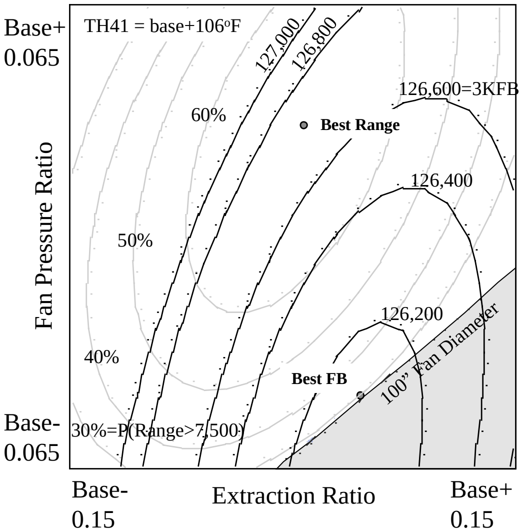

Figure 12: Design Range and 3,000 nmi Mission Fuel Burn Contours for Constant T4

probabilistic sensitivity factors indicate that this will tend to make fan nozzle thrust coefficient the dominant uncertainty parameter, which is probably more desirable than the alternative of having turbomachinery uncertainties dominate. The cycle design point should therefore lie between the best design range and best fuel burn cycle design points. The final choice as to where along this locus of points the design will finally settle depends on the relative importance of mission fuel burn and design range missions (as well as engine cost, nacelle drag, etc.).

Conclusions

This paper has shown that the probabilistic method is useful for formulating direct trades of design margin against performance or other FoMs such as mission fuel burn, thus enabling the existing design margin to be capitalized upon in the interest of obtaining better system performance. In order to leverage the available design margin, a probabilistic sensitivity analysis method was developed and executed. Two types of sensitivities were used: a probabilistic sensitivity useful for making trades of performance against design confidence (margin), and probabilistic sensitivity factors which are useful for weighing the relative importance of uncertainty parameters. Moreover, these were visualized using contours created via response surface methods, which allow computationally efficient estimation of approximate contour shapes. It was shown that use of sensitivities in combination with the response surface contours enabled sensitivity results to be actively used in the cycle selection process rather than after-the-fact, as is ordinarily the case.

Based on the discussion and results shown here, one can conclude that the probabilistic approach is inherently more computationally intensive that the deterministic approach. It therefore behooves the designer to choose wisely when setting up the problem in order to avoid unnecessary work. However, a properly formulated probabilistic method provides a much clearer picture of how the various system trades "stack up" against one another and enables the ultimate cycle selection to be made based on a more complete picture of the problem behavior than is possible using deterministic methods.

The results for a notional four engine wide-body commercial transport show that a 5% reduction in the probability of meeting a 7,500 nmi design range target is worth roughly: 8.8 nmi range, or 350 lb of 3,000 nmi mission fuel burn for the best design range cycle. Based on the results of the probabilistic sensitivity factors for design range and fuel burn, it is clear that fan nozzle thrust coefficient and HPT efficiency are dominant factors on design range and fuel burn uncertainty throughout the entire design space.

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