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# DIGITAL SIMULATION OF THUNDERSTORM RAINFALL 

## A THESIS <br> Presented to <br> The Faculty of the Graduate Division by

Unal Ali Sorman

In Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the School of Civil Engineering

# DIGITAL SIMULATION OF THUNDERSTORM RAINFALL 

Approved:



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## LIST OF SYMBOLS

$A A=\left(D_{M}\right)_{\text {max }} / 2$. One-half of the maximum major cell axis $a_{i}, i=1,2,3,--n$ Coefficients of regression equation $\overline{\mathrm{b}}_{1}=$ average distribution parameter along minor axis of cell (mile ${ }^{-2}$ ) $b_{i}, i=1,2=$ Distribution coefficients along minor and major axis (mile $e^{-2}$ ) $C C=\left(D_{M}\right)_{\text {max }} / 1.2$ One and two-tenths of the maximum major cell axis (mile ${ }^{-2}$ ) DMMAJ $=\left(D_{M}\right)_{\text {max }}=$ Maximum major cell axis (mile) DMMIN $=\left(D_{M}\right)_{\text {max }}=$ Maximum minor cell axis (mile) $E=$ Eccentricity of the elliptical isohyets defined in Reference (3) $\mathrm{F}=$ Triangular distribution density function
$\left(I_{o}\right)_{\max }=\begin{aligned} & \text { Maximum rainfall intensity at the center of the cell during } \\ & \text { the cell 1ife (inches/ten-minute) }\end{aligned}$
$\left(I_{o}\right)_{t}=$ Maximum rainfall intensity at time $t$ (inches/ten-minute)
$I_{t}=$ Rainfall intensity at time $t$ (inches/ten-minute)
$\mathrm{N}=$ Number of observations
$\mathrm{N}_{\mathrm{T}}=$ Total number of raingages
$r=$ Distance along the axis from the cell center (mile)
$R=$ Correlation coefficient
$R_{o}=$ Maximum rainfall intensity defined in Reference (3)
RN = Uniform random number
RNN $=$ Random normal number
$S^{2}=$ Geometric mean of the semi axes of isohyets
$\mathrm{t}=\mathrm{time}$ (minutes)
$t_{D}=$ Duration of cell at a raingage (minutes)

```
tmax}= Cell life (minutes
T = t/t max = cumulative percent of cell life
THETAD = Deviation of cell direction from the wind direction = 的 - - 餄
THETAW = Wind direction = 旗 (degree)
V
\overline{V}
V
VELD = V 
(X,Y) = Coordinates of raingages with respect to cell center (miles)
Y = Dependent variable in regression equation
XI = tmax = Cell life (minutes)
XMAXI = (I_ ) max = Maximum rainfall intensity (inches/ten-minute)
Z = 2/CC-AA
\varepsilon
\sigma = standard deviation of dependent variable
0c
0 w = Wind direction (degree)
0
```


## ABSTRACT

The purpose of this study was to obtain a better understanding of the temporal and spatial variability of thunderstorm rainfall and to develop a digital model for the stochastic simulation of thunderstorm rainfall for the Southeast Coastal Plain areas.

In the present study rainfall cells were thoroughly analyzed from rainfall data made available by the Agricultural Research Service, U.S. Department of Agriculture. The data source was a dense network of raingages located over the Little River Experimental watershed near Tifton, Georgia. The study area is monitored by a network of 55 digitaltype raingages covering a 250 square mile area. Statistical properties and frequency distributions of cell characteristics, such as cell duration, size, spatial and temporal distribution of rainfall intensity, cell movement, and number and orientation of cells, were analyzed.

A conceptual model of thunderstorm rainfall was formulated from the observed storm cell characteristics and the parameters of the model were evaluated from isolated thunderstorms which occurred in the summer of 1967. The model was coded for a digital computer and a number of rainfall events were generated by the dynamic model, which is based on the stochastic generation of rainfall patterns from thunderstorm cells. These simulated precipitation sequences preserved certain of the fundamental statistics of the historical thunderstorm rainfall records. The model was validated by comparing synthetic precipitation events with events observed on Little River watershed during the summers of 1968 ,

1969, and 1970. Rainfall characteristics which are considered to be representative of the most important features of thunderstorm rainfall were analyzed during the model validation. For the validation of the model a number of statistical rainfall parameters determined from simulated rainfall, such as frequency distribution of the maximum amount of rainfall, maximum accumulated rainfall versus duration of rainfall at the maximum rainfall raingage, and maximum ten-minute rainfall intensity, were compared with the 1968, 1969, and 1970 historical data. In addition, relationships between correlation coefficients and spacing between the first and the second maximum rainfall gages, as well as time lag of rainfall between them, were derived graphically and a comparison was made between the simulated and observed results. The performance of the model was considered to be successful on the basis of comparisons made between the observed and simulated rainfall characteristics.

This study has lead to the development of a body of knowledge on the characteristics of summer thunderstorm rainfall in the Coastal Plain of Georgia. The size, movement, and intensity of rainfall thunderstorm cells has been measured, and a stochastic model has been developed which will generate precipitation patterns like those observed by the raingage network. The dependence of the characteristics of individual cells on the location, movement, and size of other cells already existing in the same general area needs additional study. Such a study will require a network of gages covering an area larger than that available for the current study or a study using radar measurements in combination with a raingage network.

It is suggested that the simulation model can be used in conjunction with watershed models for generation of synthetic streamflows and that the knowledge gained through this study will aid in the efficient use of water resources throughout the Coastal Plain areas.

## CHAPTER I

## INTRODUCTION

## Objectives

There are two basic objectives of this research. The first is to gain insight into the temporal and spatial variability of thunderstorm rainfall from analysis of rainfall data in Georgia. The second is to develop a computer model for the stochastic simulation of thunderstorm rainfall.

The development of the thunderstorm model is seen to play a dual role: the first role is to provide a framework for the evaluation of quantitative relationships among a large number of parameters useful in describing thunderstorm rainfall. The second role of the model is to generate surface rainfall patterns that may be useful in studies which trace rainfall through the land phase of the hydrologic cycle.

It has become clearly evident in recent years that information on the temporal and spatial variation of precipitation is needed on a small scale (micro-scale). There are several reasons why such information is needed, but most of these can be related, in one way or another, to the various fields of study dealing with the transport of matter through the biosphere. Regardless of whether one is concerned with the transport of particles from the atmosphere to the earth or with the movement of chemicals in solution or particles in suspension in stream channels and in soil moisture, or whether one is concerned with only the movement of water
itself a knowledge of high intensity rainfall and its microscale variation is needed. This is particularly true in the field of urban hydrology and urban storm drainage. National studies by the American Society of Civil Engineers(1), the U.S. Geological Survey(39), and the Office of Water Resources Research(49) have continually placed the need for additional knowledge of high intensity, short duration rainfall in a position of high priority on lists of needed research. The primary use of such information is expected to be in the development of water drainage systems, watershed models, and also in the application of models for synthesizing streamflow. Many different hydrologic models now in use could employ a rainfall model to generate temporal and spatial varying input data.

## Justification

There is a need for detailed knowledge of thunderstorm rainfall. For example, the temporal and spatial distribution of rainfall has a very strong influence on the accuracy of the computation of storm runoff, the rate of which may be strongly affected by high intensity and short duration rainfall. The areal distribution of rainfall over long periods of time (months, years) and the time distributions of point rainfall data have been studied by many investigators in the past. However, the areal distribution of precipitation for short time intervals have not been investigated in these studies. The need for such investigations has been recognized in recent years. This need has been stated by Eagleson(23), Huff $(32,33)$, and Amorocho (2).

More recently, the recommendations (35) made at the Engineering Foundation Research Conference in 1968 indicated that research is needed

on the characteristics of thunderstorm rainfall. These recommendations are partially quoted below:

Research is particularly needed in developing stochastic models of precipitation, both in time and space; including thunderstorm and cyclonic models, and perhaps others; and method of use should be stressed, either in rainfall-runoff models or in design method for small urban basins.

The need for information on precipitation patterns and for simulation models for the study of rainfall patterns has been placed in a position of high priority by recent national studies. $(1,35,39)$

The following quotes from these studies convey the need for data collection and modeling of small scale precipitation patterns.

Analysis of time and space variations of rainfall in metro-politan-scale storms, particularly thunderstorms, should be initiated very soon, using all available pertinent data. $(1,39)$

The storm pattern will most likely be probabilistic, synthetic storm patterns founded on the characteristics of actual storm histories. $(35,39)$

The input for the water cycle is precipitation, and little is known about it in the urban context...a form of historical storm data is needed as input for planning, development and management. $(1,39)$

For large urban areas, spatial and temporal rainfall variations may be very important. Hence, a simulation model must be able to utilize all available precipitation data to simulate the effect of these variations. (39)

Storm totality in time and space is required, not merely a measure of, say, the maximum intensity at a point for a given storm. (39)

The above recommendations were influential in the development of the research presented in this study.

## Methodology

Convective circulation units, called rainfall cells*, are the fundamental components of the model developed in this investigation. The model is based on the stochastic generation of rainfall patterns associated with these convective units. The purpose of the model is to simulate precipitation sequences which preserve certain of the fundamental statistics of the available historical thunderstorm rainfall records.

The model simulates, in a Lagrangian reference frame, the cell size, the distribution of precipitation throughout the cell, the cell life duration, the direction and speed of cell movement and the number and location of cells in a storm. The statistical characteristics of the model parameters were studied to develop a representation of the parameters that best fit observation on the Little River Experimental watershed at Tifton, Georgia. In the operation of the mode1, a sequence of input parameters is stochastically selected such that certain of the fundamental statistics of the available historical record are preserved. Parameter evaluation is based on data from thunderstorms which occured over the Little River watershed in the summer of 1967. The model is validated with the rainfall data from the Tifton network which was collected during the summers of 1968,1969 , and 1970 . Four years of data were collected by the Agricultural Research Service of the U.S. Department of Agriculture.

[^0]The results of the study should provide an important advance in the understanding of the pattern of thunderstorm rainfall in the Coastal Plain area, particularly in view of the scarcity of knowledge currently available.

## CHAPTER II

## LITERATURE SURVEY

## Introduction

A survey of literature was conducted to determine the relation between previous studies and the current study. The relavent previous works are given in the References. Only a small number of studies concerned with temporal and spatial simulation of rainfall were found. In the following discussion only a brief summary is presented, because in most cases the investigations were quite involved and inclusion of many details would obscure the relationship to the current work.

The first detailed study of thunderstorm activity may be attributed to the Thunderstorm Project (12). During the period July, 1945 through May, 1949, four government agencies - U.S. Air Force, Navy, National Advisory Committee for Aeronautics, and Weather Bureau - cooperated in a project to study the internal structure and behavior of thunderstorms. Following the cessation of World War II, the sponsoring agencies were able to mount a project that remains to this day as one of the important investigations in meteorological research. Much of the information summarized in the following paragraphs was developed as part of the Thunderstorm Project.

Definition of Thunderstorm Cell

Convective overturning results as the atmosphere becomes unstable,
and may lead to the development of units of convective circulation. These convective circulation units usually form a regular pattern within a thunderstorm. The convective units, which are sometimes called cells by meteorologists, can be detected on a radar scope and the precipitation resulting from these units can be observed as isohyets on the ground surface. This leads to the fundamental concept that in the thunderstorm there are a number of convective units having similar properties and characteristics which are capable of analysis as a class of convective phenomena.

Use of the term cell as applied to these individual convection units is not new; many previous investigators have indicated that there are subdivisions or regions of localized convective activity within a thunderstorm. However, there is some confusion concerning the meaning of this term. Various definitions of cells refer to different meteorological units which range in size from a single cumulus cloud to a large thunderstorm.

During a storm period there may be more than one cell unit, each of which may be dependent or independent of surrounding cells in the same storm. The number of cells depends on the type of thunderstorm and also upon the physiographic characteristics of the region. It has been found that there is a sequence of irregular motions of the air, or turbulence, within the storm area. This motion can be separated into two classes, drafts and gusts. Drafts are by far the more important as far as thunderstorms are concerned, since they make possible the principal energy releases of strong convection within the storm area.

Each cell unit during the period of storm activity may be in different stages of development at any one instant. The boundaries of the cell are identified as narrow zones of inactive or nonturbulent cloudy air. The direction of the air motion largely depends upon the stages of development of the cell. In early stages of development air motion is upward and during the later stages it is predominantly downward, particularly where rain develops.

## Stages of Development of Thunderstorm Cell

The detailed and comprehensive observations made by the Thunderstorm Project have permitted the identification and study of cells and measurement of the duration of cell life. Through this and other similar studies it has been found that there are three stages in the life of a cell. The stage is determined by the magnitude and direction of the predominating vertical motions during the life cycle of a cell. These stages are:
(1) The cumulus stage - updraft air throughout the cell
(2) The mature stage - presence of both updrafts and downdrafts and heavy rain at the surface
(3) The dissipating stage - weak downdrafts throughout the cell

As a summary, it can be concluded that during the cumulus stage of cell development updraft causes the cell cloud to extend in height, air flows in through the sides and mixes with the updrafts. It is difficult . to give a definite time duration for this stage but if the duration is recorded from the time of the initial detection of the radar echo, it may be ten to fifteen minutes. When rain begins heat is given up by the air
as the falling water evaporates, and the density of air increases. This initiates a downdraft in part of the cell region which was previously updraft and is the start of the mature stage of development. The occurance of precipitation on the ground surface is a signal for the beginning of this stage. As the rainfall continues throughout the mature stage, the downdraft area increases in size until it extends over the entire cell. This is considered to be the beginning of the dissipating stage or the end of the mature stage. The mature stage exists for a period of fifteen to thirty minutes and the cell in this stage reaches its greatest height, normally about 40,000 feet.

## Cell Characteristics

$\because: A$ first appearance of a cloud cell on radar is quite sudden. A thunderstorm first appears as a small and isolated cell and then develops rapidly. G. R. Hilst and G. P. MacDowell (31) studied the rate of growth of precipitation cells and observed that the rate of horizontal growth was remarkably uniform and the vertical growth was rapid. Various other studies of cell growth, such as those by E. J. Workman and S. E. Reynolds (54), have indicated different growth rates. The study of a large number of observations made during the Thunderstorm Project has shown that, at least statistically, the maximum horizontal diameter of a convective cell has the same order of magnitude as the vertical extent. A somewhat similar investigation which was carried out by D. R. Mather (38) and by H. B. Brooks (10) indicated that as a convective cell increases in horizontal dimension it also increases in vertical extent. The mean precipitation echo diameter ranged from about one to thirteen
miles in Mather's study and from one-half mile to forty miles in H. B. Brooks' investigation. Two miles was the most frequent size, while four and one-half miles was the average. On the basis of radar investigations made by Byers (13), Newton and Frankhauser (41), Braham (8), and Clark (19), individual raincells of medium size thunderstorms ranged in size from one to twenty miles with an average diameter of four and one-half miles.

It was found that the rainfall pattern over a ground surface follows closely the arrangement of the cells in the storm and reflects, to a considerable extent, the various stages of development. Correlation of the observations aloft and at the surface shows that the rain at the surface is in the downdraft area of the cell. Figure 1 shows a typical relation between radar echo and surface rainfall. The first rain reaching the ground is limited in area to a few square miles. Later, as the cell develops, the rain area expands with the increase of the downdraft with which the rain is associated.

The number of cells developed within a thunderstorm depends on the region under study. In some regions, such as the New Mexico area studied by Workman and co-workers (53), single isolated thunderstorm ce11s are common. In some humid areas, such as the eastern and southern U.S., a single celled thunderstorm is comparatively rare and when it occurs it is generally weak. Usually a thunderstorm consists of a group of three or more cells adjacent to each other. Several studies $(17,46)$ have shown that the number of active cells per thunderstorm may vary from one to 22.

Another parameter of interest is the duration of rainfall. In the R. R. Braham (8) study, the rainfall duration from a cell was determined
from an analysis of the isohyetal patterns for the accumulated rainfall during five minute periods. The average duration of cell rainfall was found to be 23 minutes. It was found that the duration of moderate or heavy rain from a single cell may vary from a few minutes for short lived cells to an hour in a large active cell. The average duration of cells observed by radar was found to be 20 to 30 minutes with a maximum life of about 90 minutes.

At a fixed point on the ground the duration of rain depends on several factors such as number of cells, size of the cells, rate of storm movement, and the direction of cell movement. According to the Thunderstorm Project report, the average duration of rain cells for 16 Florida storms was 27 minutes. For 11 storms in Ohio (18), it was 24 minutes. The average duration of rain from single cells of Ohio storms was 23 minutes.

A problem which frequently arises in dealing with the duration of a convective rain cloud is the fact that the convective cells tend to merge as they move along and develop. L. J. Battan (5) investigated the duration of individual radar cells which did not appear to merge with any other cell during their lifetime. The data indicated that maximum duration of radar cells was equal to about 40 to 45 minutes. However, the duration of an extremely large cell which forms part of larger thunderstorms may exceed 45 minutes. These results were confirmed by work of Workman and Reynolds (54) and M. Satman (46).

The variation of rainfall with the life of the cell was studied by R. R. Braham (8). In his study, the cumulative percentage of total rainfall was plotted against the cumulative percentage of cell duration.

From the plot, it was observed that the rate of accumulation of rain from a cell is greatest during the interval from 20 percent to 40 percent of storm cell duration. This may be compared with the results of a similar analysis made by the Thunderstorm Project, which showed that the maximum rate of rain at a station as a thunderstorm cell passes overhead occured in the first two five-minute periods of rainfall. The most intense rain occurred under the center of the cell within a few minuts after the first measurable rain from the cell reached the ground. This corresponds to the beginning of the mature stage, and the rain remains heavy for a period of five to fifteen minutes. Then the rainfall rate decreases, but much more slowly than it first increased. The Thunderstorm Project publication has reported the same conclusion and noted that the major portion of the rainfall occurs in the early part ( 10 to 15 minutes) of the rain period.
R. R. Braham (8) also investigated the relation between the maximum area covered by rainfall at any one time from a single cell against the total rainfall. It was stated that there is much greater variation in the total amount of rain than in the maximum area covered, since the accumulated rainfall depends on duration of the cell as well as cell size. From these data, the average cell was found to cover a maximum area of eight square miles at any one time with values ranging from less than one up to 30 square miles.

In the above discussion, most of the findings were for a single, isolated cell for any one storm. However, new cells frequently form adjacent to those which have already developed. As a consequence of this, the passage of several cells over a given station, or the tendency of cells to
merge as they move along, causes a great variation in the rainfall at a station and also in the rainfall duration. It has been stated by many authors that new cell developments take place around the initial cells.

Many studies have been made in the past and recent years to find the relation between the movement of thunderstorm echoes and the winds. The conclusion of the various studies differs in detail because of the loose terminology which is used to describe thunderstorm movement. The full discussion of the detailed findings by several authors is omitted at this point in the study because of the divergence in the results. However, some general results of previous storm movement studies on the speed and direction of cell trajectories and on similarities and differences in movement are explained in the last part of Chapter III. The reader may refer to Brooks (10), the Thunderstorm Project (12), Newton and Fankhauser (41), J. Charba, and Y. Sasaki (18), J. C. Frankhauser (23), M. G. Ligda (36) for detailed analyses of thunderstorm movement. The above literature review summarizes the analysis of weather radar and surface rainfall data made by several investigators. The conclusions derived from the above studies played an important role in the formulation and development of the rainfall model described in subsequent chapters of this study.

## Related Current Research

An intensive literature review has also been conducted to determine the relation between recent studies and the present study. Only a small number of current studies concerned with temporal and spatial simulation of rainfall were found.

The meteorologists of the Travelers Research Center undertook a brief, crash study of the subject, with partial support of the Office of Water Resources Research (OWRR). A report on this study (48) written by A. Thomase11 and entitled "Rainfall Variability Research for Urban Drainage Problems" was based on the work of the Travelers Research Corporation for the American Society of Civil Engineers. The Travelers Research Corporation utilized an "Objective Analysis Technique"* to develop a runoff model that leads to an assessment of the temporal and spatial variability of precipitation. The investigators suggested that the model could be useful both for the daily operation of urban water drainage systems and for the design of new systems. Emphasis is placed on transferability of the results to as many locations as possible. In the report, a research program is outlined, but how one goes about defining the probability of various configurations of the precipitation pattern as a function of time throughout a drainage area was not mentioned.

Amorocho and Shack developed a simulation of a cyclonic storm and presented it at the 51st Annual Meeting of the AGU in Washington, D.C. Since this paper has not been published, it could not be used for this study. In another thunderstorm research project D. Amorocho and D. Morgan (3) investigated convective storm cells, the frequency distributions of cell parameters, and rainfall intensities in the state of Arizona. The modeling process adopted in this project involved three steps which are stated as:

[^1]1. Simulation of a sequence of thunderstorm occurances
2. Simulation of total storm precipitation, duration and surface area from the Weinstein-Davis convective model
3. Simulation of storm fields by producing isohyetal maps which were plotted by computer.

The results of the first part of the study showed that the number of thunderstorms which occur in a region is equal at all the sites in the region over a period of time, and that the frequency distributions of thunderstorm hits for the stations within an area of 7100 square miles have similar parameters.

In the third modeling process adopted by Amorocho, a new procedure was developed for simulation. A relationships was found between the maximum intensity in the cell and the intensities at surrounding points. From the analysis of individual cells, a bivariate Gaussian distribution was found to represent the general pattern of storm precipitation. The isohyetal maps were drawn for each five minute interval during storms by computer on the basis of the Walnut Gulch Watershed data. A similar approach has been discussed by Court (1961) for the storm events in eastern and southern United States. The equation used by Amorocho to represent the rainfall intensity at the ground surface referred to the point of maximum rainfall intensity ( $R_{0}$ ) as well as the eccentricity of the elliptical isohyets (E), and the geometric mean of the major and minor semi-axes of the isohyets $\left(S^{2}\right)$. These parameters, $E, S$ and $R_{o}$, define the surface of a bivariate Gaussian distribution and were estimated from the isohyetal map values from a least square fit of the intensity surfaces. A set of these parameters was obtained for each time interval. An attempt was made to
represent the entire life cycle of a cell by three functions $E(t), S(t)$, and $R_{o}(t)$. These were assumed to be random functions for the universe of cells. Because an insufficient number of cells were analyzed in that study, the frequency distributions of the functions with respect to time were not established. Only a parabolic fit for the maximum intensity of rainfall at the cell center for all the intensity values was included in the simulation process, and $E(t)$, and $S(t)$ were assumed to be constant for the life of a given cell. The values of these last two quantities were adjusted until the simulated volume of precipitation equaled the value given by the Weinstein-Davis convection model, a meteorological model based on atmospheric thermodynamics and an equation of vertical motion.

It should be stated that the frequency distributions of ce11 parameters such as eccentricity, geometric mean of the major and minor axes, and maximum cell center intensity are actually not constant for the life span of a cell and that the rainfall intensity of a cell cannot be approximated by a single parabola over all the range of intensities. Parameters similar to the ones mentioned in the above paragraph are studied in more detail in Chapter III of the present study. It should be noted that the development of new cells, the orientation of multiple cells with respect to existing ones, and the motion (direction and speed) of the cells and their dependance on the prevailing winds aloft were not investigated in the Amorocho study.

The stochastic rainfall generating model developed by D. D. Franz (25) is limited to a three station network of hourly rainfall data in northern California where the major rainfall type is orographic frontal.

Another study on spatial and temporal distribution of thunderstorm rainfall for southeastern Arizona was reported by H. B. Osborn and L. J. Lane (42). An oral presentation of part of their work was given at the Symposium on Statistical Hydrology in Tucson, Arizona, 1971, but the details were not available to the writer.

In a recent report published by W. M. Grayman and Peter S. Eagleson (28), meteorological events were classified by the scale or level of the event. one of their models simulates the two major sections of a cyclone (prewarm frontal and cold front). A second model is used to simulate a squall line. Storm types such as air mass thunderstorms and orographic storms were not modeled in that study due to the lack of required data for the Boston area. Their models were based on data collected by radar in which the storm characteristics were viewed in a Lagrangian frame of reference. The statistical characteristics of these storms, such as size, intensity, duration, average number of cells present at one time and the direction and speed of activity movement, were determined for the climatic, macro-scale, and mesoscale and microscale levels of activity. The largest scale was the climatic scale. Progressing downward in size, the synoptic or macro-scale came next. The next level was the mesoscale which Austin defines as having an area between 25 and 5,000 square miles. This scale is almost of the same order of magnitude in size as most catchment areas of interest to engineers. The next smaller scale is the microscale or cellular. The term "meso-meteorological" has been used to designate storm structure having space and time scales somewhat between the microscale and the mesoscale. This definition would include thunderstorms, squall lines and tornados.


Rainfall on the macro and meso scales has been studied extensively for many years. A national network of precipitation gages has been established, and the body of knowledge has been an invaluable asset to the nation in terms of the large scale variations in precipitation. However, the information gathered in the past has been regional in character and on a scale of many hundreds or thousands of square miles. It has become evident in recent years that information on the temporal and spatial variation of precipitation is needed on a much smaller scale. Although, the microscale characteristics of storms have been studied within the last 20 years, in all these studies generally only one characteristic of rainfall, either temporal or spatial distribution, has been analyzed.

## CHAPTER III

## CHARACTERISTICS OF THUNDERSTORM RAINFALL IN LITTLE RIVER WATERSHED

## Characteristics of Precipitation in Georgia

On the basis of causative factors precipitation may be divided into three types, namely, convective, orographic, and advective. Thunderstorm rainfall, with associated thunder and lightening, is one type of precipitation associated with strong convective air mass movement. Several types of thunderstorms have been recognized by meteorologists. The distinction between types has been based on the actions of the air masses* in which the storms occur. These types include:

1) Local convection (also know as "air mass thunderstorm")
2) Frontal type
a) Those associated with a warm front
b) Those associated with a cold front
3) Squall 1ine thunderstorm

The properties of thunderstorms are such that they may be distinguished on radarscopes. On a plan-position indicator scope, on which the return pulse from a radar signal is presented, thunderstorms are characterized by the tendency toward oval shape, high intensity, and high intensity gradients of the radar signals. On a range-height indicator scope they are characterized by their vertical extent and fairly uniform,

[^2]high intensity radar signal over a relatively narrow vertical column. Since, by definition, a convective cloud does not become a thunderstorm until thunder is heard or lightning is seen one cannot be certain that a given radar echo from a convective cell is a "thunderstorm". Nevertheless, it is safe to assume that an echo whose top has grown rapidly to over 25,000 or 30,000 feet is a thunderstorm.

In summer, two different thunderstorm distribution patterns can be recognized on long range radar scopes. One consists of an irregular spacing of air mass storms, and the other type consists of a line of thunderstorms which usually runs parallel to the low level wind. In low latitudes and in the tropics, the air mass thunderstorm is predominant. In middle latitudes, lines of thunderstorms are most frequent. Frontal thunderstorms develop as warm air advances over cold air (warm front), or they develop in front of a cold air mass (cold front).

In a discussion of thunderstorms in Georgia, H.A. Scott (47) reported, "Apart from the tropical disturbance, the main rainfall producer during summer months in Georgia is the thunderstorm. Convective action is at its highest during the warm season and the cyclonic movement is weak." He states further, "the thunderstorm, being the result often of purely local convective action, affects only a limited area, and we have therefore at times single, isolated heavy downpours and at others a series of locally heavy showers."

Personal communication with meteorologists at the U.S. Weather Bureau in Atlanta and study of reports on thunderstorm activity in Georgia (44) have confirmed that summer thunderstorms in this region are
typically air mass thunderstorms. These storms characteristically generate intense rainfall within a short period of time and the rainfall has a highly variable areal distribution. Convective rains are frequent in southern Georgia during the summer months, especially in July and August. The monthly precipitation distribution for June, July and August at Tifton, Georgia, is shown in Figure 2. Monthly precipitation for these three months as recorded during 1967, 1968, 1969, and 1970 and the normal monthly precipitation (1931-1960) for each month of the year are tabulated in Table 1. From observation of the table, it can be concluded that, on an average, one-third of the annual average rainfall comes during the months of June, July, and August. Thiry years of records at the Georgia Coastal Plain Experimental Station at Tifton showed that average annual rainfall is 45.71 inches and that the month of greatest rainfall is July with 6.30 inches. There is a very pronounced maximum and minimum in the seasonal distribution. Maximum rain occurs in mid summer. The average monthly rainfall for July is more than three times that for either October or November. The differences in rainfall during these three months were sporadic for the years 1967 to 1970. See Table 1.

No data is available on the average number of thunderstorm days at Tifton. However, such data is available for Atlanta, which is 183 miles north of Tifton. According to the U.S. Weather Bureau an average of about 59 thunderstorm days per year has been recorded in the vicinity of Atlanta, Georgia. During the period 1904-1943 there were 2348 thunderstorm days, and 534 of these occurred in July, the month with the highest number of thunderstorms. (See Table 1, pp. 8 in the Thunderstorm Project


Figure 2. Monthly Precipitation Distribution
at Tifton, Georgia


Table 1. Monthly Precipitation Averages at Tifton, Georgia (in inches)

*Deviation from the normal precipitation

Report.) The summer thunderstorms were most frequently late afternoon storms with high rainfall intensity.

Moisture, one of the primary requirements for the occurance of thunderstorms, is drawn into the study area from the Gulf of Mexico and the Atlantic Ocean. The Tropical Gulf (Tg) and the Tropical Atlantic Air Masses (Ta) are of primary importance to this region, especially during the summer months. In spite of different life histories and movement, the two tropical air masses which are predominant are so similar that they can scarcely be distinguished from each other. The symbol " Tm " is used to represent the tropical maritime air mass from either source region. Two factors make the development and movements of Ta and Tg air masses much more favorable in summer. First, there is the development of a low pressure area over the interior of North America; second, there is the development of an area of high pressure over the western Atlantic Ocean. These two factors combine to produce a pressure gradient from the Ocean toward the southeastern United States. Such a condition is illustrated in Figure 3, a surface synoptic map for the date of July 7, 1967. The meteorological conditions depicted on this map are typical of those which occur during the summer months when thunderstorm activity is highest.

## Data Collection

The Agricultural Research Service (ARS) of the U.S. Department of Agriculture is conducting an intensive hydrologic investigation (30) on the Little River watershed near Tifton, Georgia. The watershed is located in the Southern Coastal Plain approximately 120 miles from the Atlantic Ocean and 90 miles from the Gulf of Mexico. The ARS has been collecting


Figure 3. Synoptic Surface Map for
the Date of July 7, 1967
rainfall data from the Little River Experimental watershed since 1966, and four years of data were available for the current thunderstorm study. Fifty-two raingages were put into operation in 1967, six more gages were installed during the summer of 1967 in an adjacent urban watershed, and three gages were terminated in June of 1969. The raingages are spaced one and one-ha1f miles apart in intensive study areas and three miles in other areas. The gages monitor the rainfall on the 150 square mile watershed in addition to some area in a buffer strip beyond the watershed boundary. In all, rainfall is measured over 250 square miles. See Figure 4.

Precipitation caught by gages in the network is automatically recorded at five-minute intervals in digital form on four-channel paper tape. Amounts are recorded to the nearest tenth of an inch, and a code is punched to indicate trace rainfall. The data is edited, translated to cards for computer input and finally recorded on magnetic tape for analysis and storage.

For processing the Little River precipitation data, two computer programs entitled Preprocessor and Processor have been developed by the ARS. Figure 5 is a system schematic showing inputs and outputs for the programs. Data from all raingages for a month constitute the input file for one run of the program. The preprocessor edits the data until a complete run is made. Then it changes the form of the rainfall data from cumulative amounts to five minute rainfall increments with the trace codes* which

[^3]
19

Legend




Figure 4. Raingage Network on Little River Experimental Watershed at Tifton, Ga.


Figure 5. System Schematic of Computer Programs for Little River Precipitation Data
indicate rain occurance. An example is given in Appendix A to show how the rainfall data and the trace codes are printed on the computer printout. (See Table A.1) The processor computer program uses the monthly precipitation files as input and builds an annual precipitation file, which is stored on nine-track magnetic tapes. The tapes include a summary and the five-minute rainfall increments and trace codes.

Tapes for 1968,1969 and 1970 are operational and were used for validation of the model. The data was not complete and trace codes for some of the raingage stations were not reliable during 1967, the year the gages were put into operation. For that reason, the 1967 data was not recorded on magnetic tape. A complete manual search was done on the 1967 data by the writer to isolate thunderstorm rainfall. From the computer printouts of the 1967 rainfall data, the rainfall events during the months of June, July and August were analyzed. Events covered by a large number of raingages and which also contained rainfall of high intensity (. 10 inch per five-minute or more) and short duration (2 to 3 hour storm duration at most) were selected for analysis. Seven storms which had these characteristics were selected. These seven storms included 25 individual rainfall cells, the spatial and temporal characteristics of which were thoroughly studied. These data were used as the basis for the formulation and parameter evaluation of a digital model of thunderstorm rainfall. Most of these storms occurred in mid-afternoon with a relatively large cumulative rainfall amount recorded by most of the raingages. The mean time of occurance of the thunderstorms which were studied over the watershed was found to be $4: 20 \mathrm{p} . \mathrm{m}$.

Printouts of five-minute rainfall increments were obtained for June, July and August of 1968, 1969 and 1970 for each raingage for each rainy day. Thunderstorm events were isolated from this printout information and data derived from these printouts formed the basis for the validation of the simulation model which will be discussed in Chapter $V$.

## Cell Characteristics

The seven storms that were selected for study were analyzed by utilizing raingage records, weather data from daily weather maps, and synoptic storm characteristics from Weather Bureau publications. The analysis was concentrated on June, July, and August data because individual thunderstorm cells could be more easily identified (had a higher maximum intensity) during the summer months. The dates and general description of each selected storm appear in Table 2 with a summary of the total number of cells, number of cells studied, i.e., followed throughout a cell life, and cell life durations. As can be seen from this table, most of the selected events are air mass thunderstorms which occurred during the midafternoon. According to the surface weather maps, two storms are classified as stationary fronts, and one is classified as a squall line. As previously mentioned, the number of cells studied in the selected storms is usually less than the number observed. The reason is that cells which originated or terminated outside the gage network and for which data on the entire life of the cell was not available are not included in this study.

The various scales of atmospheric phenomena involved in the research are shown on Figure 6. The synoptic scale is represented by the circles of 100 mile radius and the mesoscale is represented by the

Table 2. Storm Characteristics for the Year of 1967



Figure 6. Synoptic and Mesoscale Storm Activity on August 20, 1967.
smaller elliptical patterns. The mesoscale activity is composed of an agglomeration of cells. There may be a number of mesoscale phenomena present at any time in a region of synoptic size. In Figure 6, mesoscale areas are plotted for each one hour interval on August 20, 1967. From these consecutive hourly diagrams, the movement of mesoscale structures were observed. The smallest scale of storm is called cell and is observed most clearly in air mass thunderstorm structures. It is this smallest scale that is modeled in the current study.

The cell parameters are grouped according to spatial and temporal variability of precipitation. Each of the cell characteristics is explained in more detail in the following subsection. The information on size, speed, and direction and maximum rainfall intensity at the center of cell as well as the wind speed and the wind direction are summarized in Table B. 1 given in Appendix B.

The cell parameters included in the present study are:

1. Cell shape
2. Cell size
3. Cell duration
4. i - temporal variation of cell diameters
ii - temporal variation of rainfall intensity at the center of the cell
5. Maximum rainfall intensity at the cell center during the life of the cell
6. Spatial variation of rainfall intensity along minor and major axes
7. Direction and speed of cell movement
8. Number and orientation of cells
9. Other aspects of cell characteristics
10. Cell Shape

The shape of the cells observed on the ground surface appeared as elliptical isohyets which were plotted at ten-minute increments. The plots provided the information to define temporal and spatial characteristics of the cells. The determination of elliptical cell boundaries involved a large degree of judgment since the cell boundaries are not clearly defined by the raingage network. In order to determine the total number of cells passing over a raingage and to determine the cell shape, the accumulated five-minute rainfall intensity values were plotted on a mass diagram. Each sharp increase in the slope of the curve was due to a single cell passing over a point and plateaus indicate periods of low, or no, rain. This type of study was done for all the storms analyzed and for all the gages recording rainfall. Figure 7 shows one of the mass diagrams plotted for raingage number 32 on August 20 , 1967. It can be seen on this figure that two cells passed over this gage. Ce11 1 (See Figure 8) had a duration of 40 minutes and cell 2 had a 20 minute duration.

- During the plotting of isohyetal maps, the question was raised as to whether they should be plotted at five or ten minute time intervals. In this study, the objective was to get the best cell definition and a clear picture of cell movement. In order to obtain a graphical representation of the rainfall over the experimental watershed, a series of hand plots of isohyets for both ten and five-minute time increments were prepared. These plots are given in Figure 8 and 9. Figure 8 shows the


Figure 7. Mass Diagram Between Accumulated Rainfall vs. Rainfall Duration at a Raingage


Figure 8. Rainfall Cell Isohyetal Patterns at Successive Ten-minute Intervals on August 20, 1967.


Figure 9. Rainfall Cell Isohyetal Patterns at Successive Fiveminute Intervals on August 20, 1967.
isohyetal patterns at successive ten-minute periods for the storm on August 20, 1967 from 4 p.m. to 6 p.m. On Figure 9 the same storm is studied with five-minute periods from $4: 10$ p.m. to $5: 30$ p.m. The shape of cells in Figure 8 were defined as clearly as the cells which were drawn with five-minute time periods. However, during the study of some other storms on a five-minute basis, it was practically impossible to illustrate cell movement. In general, it was found that a ten-minute period provides for adequate definition of cell shape and movement, while five-minute intervals did not. Therefore, the decision was made to use ten-minute periods as the basic time interval.

Due to the large areal extent of storm activity (macroscale) during which the complete movement of cells could not be observed over the watershed throughout the cell life. Therefore, the cells for which the trajectories could not be detected on the ground surface were excluded from the study. As it can be seen in Figure 8 and 9 , the origins of some of the cells during the storm on August 20 , 1967 occurred outside of the watershed boundary and could not be detected at the early part of the growing stage.
2. Cell Size

Figures B. 1 - B.7, which are attached in Appendix B, show the series of thunderstorm cells which occurred over the experimental watershed for each storm event in 1967. By measuring the width and length of the ten-minute isohyetal patterns the variation in size of thunderstorm cells was determined. The values are tabulated in Table B. 1 and the frequency distributions of maximum minor and major diameter of cells are plotted in Figure 10 and 11. The parameters of the proposed probability density



Figure 11. Frequency Distribution of Maximum Major Cell Axis

functions in these and subsequent figures were estimated by the method of moments and the estimated probability density functions for the cell characteristics were plotted with the histograms of the observed data. No exhaustive studies were conducted to select the specific form of the probability density functions used to fit the data. More complete studies may later be conducted if the sensitivity of the model described in Chapter IV proves such studies are needed. The parameters of the various distributions utilized will be discussed in more detail in a subsequent section of this study.

In the present study, it was decided to consider the cell confined to the area bounded by the 0.10 -inch isohyet on each ten-minute map. Ce11 diameter measurements were limited in some cases because of interaction, or merging of cells, and because cells could remain undetected at the beginning or at the end of their lifetime.

It is felt that the selection of the ten-minute time scale for analysis provided accurate measurement of the cell size. On an average, the maximum values of the minor and major axes of the isohyetal cells were about 4.70 and 7.16 miles, respectively. The maximum value of the minor cell axes ranged from three to seven miles and the maximum major cell axes ranged from four to 12 miles.

As can be seen in Figure 12, the maximum major and minor cell axes are related. On an average, the maximum major cell axis was found to be 1.5 times greater than the maximum average minor cell axis. The values of the ratio of maximum major axis to maximum minor axis ranged between one and two.

Figure 12. Relation Between Maximum Major and Minor Cell Axes

## 3. Cell Duration

There was no general agreement in the literature on the definition of thunderstorm cell duration. In this paper cell duration is defined as the difference in time between the first and last recorded continuous rainfall increment. The frequency distribution of cell duration is shown in Figure 13. The duration of the cells varied between 30 minutes and 100 minutes with a mean of 53 minutes.
4. Temporal Variation of Cell Diameters and Rainfall Intensity at the Center of the Cell

The variation of rainfall intensity at the center of the cell and the variation of cell size with respect to time was studied from the series of isohyetal maps drawn at ten-minute intervals. The intensity and cell size versus time graphs from the beginning of cell growth until cell decay were plotted. The rainfall intensity and the length of the major and minor axes were measured and changed into dimensionless ratios by dividing them by the maximum values observed during the life of the cell.
i. Temporal Variation of Cell Axes. The dimensionless values of minor and major cell axes ratios were grouped with respect to the maximum cell sizes. Two categories of these ratios were defined. The dimensionless distribution of minor cell axes with maximum minor cell dimension ranging between four to five miles was studied and then the maximum minor cell dimension of five to eight miles was considered. Similarly, two categories were used in the analysis of major cell axes. The values in the first group ranged between six and eight miles, and in the second group they ranged from eight to 12 miles. From


Figure 13. Frequency Distribution of Cell Duration
these studies, it was found that the distribution of dimensionless cell size ratio versus cumulative percent of cell duration is not dependent on maximum cell diameter. The maximum cell size occurs in the range 0.20 to $0.50 \mathrm{t} / \mathrm{tmax}^{\text {where }} \mathrm{t}_{\max }$ is the duration of cell life.

A fourth order polynomial was fitted to the dimensionless distributions of the major and minor cell axes to facilitate the use of this parameter in the simulation model. Figure 14 shows the observed and fitted curves for both axes.
ii. Temporal Variation of Rainfall Intensity at the Cell Center. The dimensionless intensity values were grouped with respect to the maximum intensity and plotted with respect to cumulative percent of cell duration. Three different categories were defined. The intensity distribution curves for three categories were grouped as follows: less than .40 , between .40 and .60 , and greater than .60 inches per ten-minutes. The corresponding relationships are shown in Figure 15.

In the first intensity range, a fourth order polynomial was fitted. The other two intensity curves could not be fitted by a fourth order polynomial. This was due to the greater skewness and steepness of the curves. A method of linear interpolation was applied in the simulation studies to model the relationships that could not be fit by a low degree polynomial. This is described in more detail in the chapter on model description.
5. Maximum Rainfall Intensity at the Cell Center During the Life of the Cell

Precipitation intensity at the center of a cell reaches a maximum when the cell size is a maximum. In addition, a particular trend was


Figure 14. Dimensionless Distributions of Major and Minor Axes


Figure 15. Dimensionless Rainfall Intensity H. .w. Distribution Curves
observed between the maximum rainfall intensity at the center of the cell and cell duration. The relationship indicated that rainfall intensity increased with an increase in cell duration. Figure 16 shows such a trend observed from the tabulated values on Table B.1. Each point shown on this figure corresponds to the average maximum-rainfall intensity at the cell duration range from 30 minutes to 100 minutes.

The frequency distribution of rainfall intensity at the cell center obtained from an analysis of 23 thunderstorm cells was fitted to a log normal probability distribution (See Figure 17) with the method of moments. The maximum rainfall intensities range from 0.10 to 1.20 inches per ten minutes. The most frequent intensity values range from 0.40 to 0.50 inches per tenminutes. Intensities less than 0.10 inches per ten minutes were not recorded as continuous rainfall by the raingages, and the cells which had a duration of less than 20 minutes were not considered because they apparently were not observed through the complete life cycle.

## 6. Spatial Variation of Rainfall Intensity Along Minor and Major Cell Axes

Rainfall intensity at the center of cells and the variation of precipitation along both axes was measured at each ten-minute interval from the historical data. A consistent relationship was observed between the rainfall intensity at the cell center and the intensities at the surrounding gages. The spatial variation of rainfall intensity along the minor and major axis was fitted by a function of the type (as shown in Figure 18)

$$
\begin{equation*}
I_{t}=\left(I_{o}\right)_{t} e^{-b} i^{r^{2}} \tag{1}
\end{equation*}
$$



Figure 16. Relation Between Maximum Rainfall Intensity at the Cell Center vs. Cell Duration


Figure 17. Frequency Distribution of Maximum Rainfall Intensity at the Cell Center
where $I_{t}=$ the rainfall intensity (inches $/ 10$ minutes) at a distance $r$ miles along an axis from the center of the cell
$\left(I_{o}\right)_{t}=$ the maximum rainfall intensity at the center at time $t$ $\mathrm{b}_{\mathrm{i}}=$ distribution coefficients $\mathrm{C}_{1}=$ coefficient for rainfall distribution along minor axis, $b_{2}=$ coefficient for rainfall distribution along major axis.)

4

Figure 18. Spatial Variation of Rainfall Intensity along Cell Axes. Accordingly, the relationship between $\log \frac{I_{t}}{\left(I_{o}\right)_{t}}$ and $r^{2}$ can be derived as

$$
\begin{align*}
& \frac{I_{t}}{\left(I_{o}\right)_{t}}=e^{-b_{i} r^{2}}  \tag{2}\\
& b_{i}=\frac{\log \left(I_{t} /\left(I_{o}\right)_{t}\right)}{r^{2}} \tag{3}
\end{align*}
$$

The average values of $b_{i}$ were determined from the slopes of the lines drawn between $\log \left(I_{t} /\left(I_{o}\right)_{t}\right)$ and $r^{2}$. The mean values of $b_{i}$ are tabulated with respect to the maximum rainfall intensities in Table 3. No correlation was observed between the values of $b_{i}$ and the maximum rainfall intensity. The frequency distributions of $b_{i}$ were studied and they are plotted in Figures 19 and 20. The mean value of $b_{1}$ was substituted into equation (1) to illustrate the spatial distribution of cell intensity along the minor axis. The plot is shown in Figure 21.

The parameters $b_{1}$ and $b_{2}$ were studied and a relationship between these parameters was discovered. The details of this relationships are presented in Chapter IV.

In order to specify the precipitation intensity at any location within a cell boundary, a bivariate distribution function, based on equation 1, was employed. The relationship was expressed as

$$
\begin{equation*}
I_{t}(X, Y)=\left(I_{o}\right)_{t} \exp -\left[b_{2} X^{2}+b_{1} Y^{2}\right] \tag{4}
\end{equation*}
$$

where $X, Y$ are the coordinates of raingage stations with respect to a cell center and $\left(I_{o}\right)_{t}$ is the maximum precipitation intensity at time $t$. The parameter $\left(I_{o}\right)_{t}$ is a function of cell duration.

## 7. Direction and Speed of Cell Movement

The ten-minute isohyetal patterns for each storm in 1967 were drawn in order to make a comparison of the movement of cells with the wind speed and direction. As an example, the cell paths of the storm on June 22, 1967 were plotted and shown on Figure 22 to indicate trajectories of cells at each ten-minute interval. The cell trajectories of

Table 3. Relation Between Distribution Coefficients and Maximum Cell Center Intensity

| Max. | Distribution Coefficients |  | Max. | Distribution |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Cell |  |  | Cell | Coeff | cients |
| Center |  |  | Center |  |  |
| Intensity | Minor | Major | Intensity | Minor | Major |
| Io(in/l0 min) | $\mathrm{b}_{1}$ | $\mathrm{b}_{2}$ | Io | $\mathrm{b}_{1}$ | $\mathrm{b}_{2}$ |
| . 11 | . 385 | . 138 | . 41 | . 300 | . 109 |
| . 13 | . 225 | - | . 42 | . 131 | . 075 |
| . 21 | . 190 | - | . 44 | . 130 | . 080 |
| . 22 | . 242 | . 089 | . 44 | . 403 | . 195 |
| . 23 | . 244 | . 077 | . 45 | . 290 | . 094 |
| . 24 | . 260 | . 121 | . 46 | . 180 | . 060 |
| . 25 | . 260 | . 100 | . 47 | . 280 | . 150 |
| . 25 | . 210 | . 093 | . 47 | . 385 | - |
| . 25 | . 285 | . 045 | . 48 | . 415 | . 130 |
| . 25 | . 222 | . 123 | . 50 | - | . 131 |
| . 25 | . 230 | . 087 | . 50 | . 167 | . 067 |
| . 26 | . 166 | . 099 | . 50 | . 260 | . 135 |
| . 28 | . 350 | . 210 | . 50 | . 340 | . 229 |
| . 30 | . 270 | . 064 | . 50 | . 150 | . 093 |
| . 32 | . 195 | . 130 | . 52 | . 262 | . 102 |
| . 32 | . 260 | . 121 | . 53 | . 143 | . 064 |
| . 34 | . 154 | . 088 | . 56 | . 292 | . 097 |
| . 34 | . 213 | . 130 | . 56 | . 317 | . 193 |
| . 35 | . 310 | . 143 | . 57 | . 273 | . 160 |
| . 35 | . 253 | . 075 | . 60 | . 260 | . 121 |
| . 35 | . 230 | . 077 | . 60 | . 245 | . 054 |
| . 36 | . 415 | . 127 | . 62 | . 115 | - |
| . 37 | . 260 | . 136 | . 68 | . 209 | . 116 |
| . 38 | - | . 186 | . 70 | . 267 | . 156 |
| . 40 | . 135 | . 067 | . 70 | . 316 | . 109 |
| . 40 | . 360 | . 096 | . 70 | . 175 | . 112 |
| . 40 | . 317 | . 227 | . 76 | . 780 | . 144 |
|  |  |  | . 80 | . 212 | . 142 |
|  |  |  | . 80 | . 241 | . 115 |
|  |  |  | . 96 | . 135 | . 074 |



Figure 19. Frequency Distribution of the Coefficient (bl)

> O



Figure 20. Frequency Distribution of the Coefficient $\left(b_{2}\right)$



85

Figure 22. Motions of Rain Cell Centers for the Storm of June 22, 1967
seven storms were thoroughly studied. The cell speed at each time increment and the direction of motion were compared with the wind speed and direction at the 500 mb leve1. The 7:00 p.m. E.S.T. wind direction and speed at 500 mb level were obtained from the daily weather maps. The orientations of major cell axes were generally in the direction of the 500 mb wind. However, the individual cells showed varying speeds and directions around the mean wind movement.

From observations of daily weather maps, it was noted that the wind velocity at the 500 mb level averaged about 16 mph and it varied from about six to 32 mph . The frequency distribution of 500 mb velocites observed on each of the thunderstorm days used in this study is shown on Figure 23.

The quantity $\Sigma\left(\mathrm{V}_{\mathrm{c}}-\mathrm{V}_{\mathrm{w}}\right) / \mathrm{N}$, where $\mathrm{V}_{\mathrm{c}}$ is the cell speed, $\mathrm{V}_{\mathrm{W}}$ is the wind speed and $N$ is the number of observations, represents the average deviation of wind speed from the cell speed. The mean deviation between the cell speed and wind speed was -4 mph . This leads to the conclusion that the cell speed is usually less than the mean wind speed at 500 mb level. In most of the cases, movement of cells was within $\pm 15 \mathrm{mph}$ in wind speed. The mean deviation of the cell direction from the wind was $+11.0^{\circ}$. That is, when looking down wind, the cells mostly moved to the left and at a slower rate than the wind. Frequency distributions of deviations of wind speed and direction from the cell movement are plotted in Figures 24 and 25 .

Some inconsistencies exist between the cell direction and speed and wind direction and speed. In some cases, the cells had no particular direction and they seemed to develop and grow against the wind. This


Figure 23. Frequency Distribution of 500 mb Wine. Velocity



Figure 24. Frequency Distribution of Deviations of Cell Speed from the Wind Speed

$$
\begin{aligned}
& 8 \\
& 4
\end{aligned}
$$



Figure 25. Frequency Distribution of Deviations of
Cell Direction from the Wind Direction
Figure 25. Frequency Distribution of Deviations of
?
might be due to a difference between the movement of the rainfall pattern and the cloud cell. That is, the rainfall pattern observed on the ground does not necessarily coincide with the cloud cell. The Thunderstorm Project Report indicated that thunderstorm movement has a very complex nature. It has been shown by other investigators $(12,13)$ that cloud cells usually move with a speed near the mean wind as computed by integrating between the cloud base and cloud top. As a result, some of the observed surface isohyets moved very little, or were even stationary, and some others showed very erratic movement.

The deviation of cell speed from the wind was correlated with respect to cell area in square miles. Figure 26 shows that a cell of average area moves slower than the wind speed and as the cell area increases in size the deviation in the velocity becomes greater and the cell moves much slower. The dotted lines shown in Figure 26 are boundaries of the data points.
8. Number and Orientation of Cells

Of the seven storms analyzed, the number of cells within each storm varied from two to nine with an average of five. A total of 25 cells out of 35 were analyzed in full detail to provide the numerical basis for the thunderstorm model.

The isohyetal patterns clearly demonstrate that new cells have a tendency to form adjacent to the ones which have already developed. The position of a new cell at the time of initial appearence was studied and its location coordinates were determined in a polar coordinate system. Figure 27 shows that new cells were most likely to form within four


Figure 26. Deviation of Cell Speed from Wind Speed vs. Cell Area
to eight mile distances from the existing cell center. The mean value was five miles. The most frequently occurring time lapse from the start of a cell to generation of a new cell was 20 minutes, with values ranging from zero to 40 minutes. All the new cells were formed within the third and fourth quadrant of the cartesian coordinate system (see Figure 27) where the system origin was selected as the center of the existing cell.

The distance between the center of the new cell and the existing cell was analyzed in relation to the speed and the diameter of the existing ce11. No particular trend was observed between the distance and these other parameters. On some occasions, cells which were separate and distinct at the time of formation grew into or merged with adjacent cells. When this happened, the newly developed cells moved and grew faster than the existing cells, so that the isolation of individual cells became more difficult.

The cell centers occurred anywhere in the raingage network area with equal probability and, on the average, the number of cells per thunderstorm was about five.

It should be noted that the generation of cell patterns, i.e., the spatial and temporal distribution of cells, is very complex in nature. Under certain conditions, thunderstorms develop in groups or families of cells, thus increasing the area subject to rainfall. In order to understand the relationships that govern these patterns, it will be necessary to discover the interaction of the cells with each other, and also whether there is any preferred location for the origin of individual cells within a family. The data available for the present study are not sufficient to completely define the spatial relationships of cells within a family.


Figure 27. Position of New Cell with Respect to

This needs to be studied in more detail on the basis of information gathered from a macroscale level of activity.

## 9. Other Aspects of Cell Characteristics

Various other aspects of cell characteristics, such as maximum cell area, relation between the percent of cell duration and accumulated rainfall, and the amount of rain accumulating on the ground from single cells, were studied. The relationship between the total accumulated rainfall during the cell life as a function of the maximum cell area is shown in Figure 28. The lines shown in Figure 28 are drawn by eye to indicate the boundary and the mean of data points. The maximum area covered by a cell ranged from 20 to 55 square miles. The variation in the volume of rain from a single cell during its life time was much greater than the variation in the maximum area covered by the same cell. The reason for this is that the volume of rain depends on cell duration as well as the cell size. The study conducted by R. R. Braham (8) of thunderstorm cells over Ohio showed that the cell area ranged from one to 30 square miles and the cloud cell had the average minor diameter of 4.5 miles.

The variation of rainfall during the life of a cell was also studied and the cumulative percent of total rainfall versus the cumulative percent of cell duration was plotted in Figure 29. The rate of accumulation of rain from a cell was greatest during the interval from 20 to 40 per- $N$ cent of elapsed cell duration. This was in close agreement with the findings of other studies $(8,12)$.

The duration of rain from a cell on a surface station depends upon a number of factors such as the size of a cell, position of the rain station


Figure 28. Relation Between the Amount of Rain vs. the Maximum Cell Area


Figure 29. Cumulative Percent of Accumulated Rainfall Vs. Cumulative Percent of Cell Duration
with respect to passing cell, and also the rate of cell movement. Without any attempt to separate any of the above factors, the rain duration period at a station as a cell passes over the station was studied. The ratio of each five minute rainfall depth to the average rainfall depth at each rain station was calculated for all the storms. This ratio was called the rainfall depth ratio. Figure 30 shows the plot of average rainfall depth ratios against time. The rainfall rate at a station normally reaches its maximum within the first ten-minute rainfall period and the average rainfall duration at a station was found to be about 30 minutes.

An estimate of the average cell size can be obtained with the above given information by making use of the information that the velocity of the surface cell is 4 mph less than the average wind speed of 16 mph at 500 mb level. Then the average cell size is calculated by multiplying the average cell speed by the duration of the rain at the station.
$\overline{\mathrm{V}}_{\mathrm{c}}=12 \mathrm{mph}$
$t_{D}=30$ minutes at the station
Average cell size diameter $=12 \mathrm{mph} \times 1 / 2 \mathrm{hr}=6$ miles which is within the acceptable range of cell diameter observed on the ground surface.

The relationship between the maximum ten-minute point intensity and the average amount of rain per each ten minute interval at the station was studied. On an average, the maximum rainfall intensity at a point was found to be almost three times as great as the average rain at the same recording gage during the entire period of the storm. (See Figure 31)

## Comparison with Previous Studies

The cell characteristics measured on the Little River network were

5
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3
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3



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Figure 30. Average Time Duration of Cell at a Station t

compared with results found by previous investigators. In general, most of the cell characteristics were in close agreement with the findings of other investigators, but a few results were inconsistent. One should be aware of the fact that each researcher investigates and attempts to solve problems from his own point of interest and his own background. This is part of the reason why some of the present results confirm the findings of previous authors and some of the other do not. One of the problems lies in defining the elements which are measured by the investigators. For example, during the present study, several definitions were found in the literature for "thunderstorm cell". Some authors defined the cell as one or more individual radar cells at a particular time and space, but some others defined it as small precipitation areas. In G. H. Ligda's paper (36), the term "small precipitation area" was applied to the radar echo which are commonly called "cells" by the radar observer. In the present study, the analysis of motion of convective cell units is studied by the rainfall patterns produced by these precipitation units, and it was not possible to measure the spatial distribution characteristics of the cells covering an area larger than the raingage networks.

The results of several studies differ because of the methods used in data collection and data analysis. Data collected by surface raingages may not produce the same results as data collected by meteorological radars. The degree of discrepancy depends on the angle to which the radar beam is elevated, the beam width, the range of the radar, and also on the accuracy and density of the raingages. The way that the data is handled and analyzed has a significant influence on the thunderstorm
studies. First of all, the cell characteristics found from an analysis of a Eulerian frame of reference, such as in Franz's model, will naturally be different from the studies of moving cells, as in the present study, in W. M. Grayman and P. S. Eagleson's study and also in J. Amorocho's study. Another obvious reason for discrepancies is the fact that observation methods, whether by radar or by surface raingages, puts some physical limitations on the recorded data. The range of the radar and the number and spacing of raingages will determine whether an area of several thousand miles or only a few square miles will be studied. Limitations on other available recources also influence the investigation. For example, resource limitations may impose constraints which will not allow investigation of the larger areas or smaller scales of activity. Last but not least, the cell parameters and the nature of the statistical distributions of these parameters depend upon the type of thunderstorm which prevails over the region and also upon the physiographic and atmospheric characteristics of the region. Keeping these facts in mind, the following comparisons were made.

Cell Size
Surface raincells, when they first become apparent on raingages in the present study, were small and isolated and were two to four miles in diameter and showed a minimum of 0.10 inches per five-minute interval. The cell developed rapidly by extending its major and minor axes. At the same time the rainfall intensity at the cell center increased. The cell axes were at a maximum when the cell center intensity reached its maximum. Studies made by previous investigators showed that the dimensions
of individual rain cells of medium size thunderstorms ranged from one to 20 miles with an average of 4.5 miles. From the studies of historical records, the mean maximum cell axes at the mature stage of development were found to be 4.70 and 7.20 miles for minor and major axis respectively. Temporal Variation of Cell Intensity and Size

The maximum rainfall intensity in the present study was reached at 25 to 60 percent of cell duration and the maximum cell size occurred on the average at 40 percent of the cell duration with the time of occurrace ranging from 20 percent to 50 percent of duration. From the R. R. Braham study, rainfall intensity was observed to be greatest during the interval from 20 percent to 40 percent of total duration. The result has also been confirmed by the Thunderstorm Project Report. Furthermore, it was stated in the Thunderstorm Project Report that the maximum size of the cell at the mature stage of development reached 6.5 miles in diameter as the cloud reached its highest vertical extent.

## Cell Center Intensity

By the use of the surface rainfall records from the Thunderstorm Project, a study was made by R. R. Braham for 53 thunderstorm cells over Ohio. The rainfall intensity ranged from 0.01 to .40 inches per five minutes. In the present study, the Coastal Plain thunderstorm cell intensities were ranging from 0.20 to 1.20 inches per ten minutes. Cells with an intensity of 0.10 inches or less could not be identified. The most frequent maximum intensity at the cell center was found to be about 0.40 to 0.50 inches per ten minutes duration.

## Cell Duration

From the radar studies made by several authors, the moving cells had an average life of 20 to 30 minutes with a maximum life of 90 minutes. The Satyam (46) and Aiya (17) studies showed that the cell life of thunderstorms over Bangalone ranged from ten minutes to 80 minutes with a mean of 35 minutes. L. J. Battan (5) also investigated the duration of individual radar cells which did not merge with other cells. The maximum radar cell duration was about 40 to 45 minutes. In the present study, the duration of 25 surface thunderstorm cells over the experimental watershed varied between 20 to 90 minutes with a mean of 53 minutes. The ones which had a duration of less than 20 minutes dropped out of the study. In other words, the thunderstorms selected from the summer months of 1967 had large intensive cells.

Relation Between Maximum Rainfall Intensity and Cell Duration
The logarithms of maximum intensities were found to be related to the cell duration. R. A. House also noted a relationship between cell duration and rainfall intensity but the range of cell duration was small as compared to the values in the present study. (See reference 28)

## Number of Ce11s

From the M. Satyam and Aiya paper, it was found that the number of active cells per thunderstorm changes from one to 22 with an average number of five cells per storm. Given that a thunderstorm was in observable range, only one storm was found to be active within a range of 12.5 miles at any given time. In the present study, a total of 35 cells were ob$4 \times$ served within the seven storms. The number of active cells ranged from


## $H$

two to nine per storm, and the most recurrent number of active cells per ten-minute period was two and the average number of cells per storm was five. Thus, the present results confirm the findings of other investigators. Location and Formation of Cells

In the present study, 40 percent of the total number of cells were generated as individual cells. An additional 40 percent had a tendency to form near the existing cells. Those that formed near existing cells appeared to be "primary baby cells" spawned in the wake of the existing "mother cells". The remaining 20 percent are "secondary baby ce11s" developed from the primary baby cells. The time lag between the initiation of a primary baby ce11 and a secondary baby cell was, on the average, 20 minutes. The initial location of the primary baby cell varied from four to eight miles behind the mother cell, and a lag of from ten to forty minutes occurred between the birth of the mother and that of the baby cell. Of course, these figures are approximations to be real phenomenon occurring in nature. Usually, the cell development activity studied during this project occurred within an area larger than 250 square miles. Therefore more accurate and reliable estimates of the cell patterns and the time lag for the cell generation have to be collected from the macro-level of storm events for better definition of these characteristics.

It is found from the present studies that during a thunderstorm event, on the average, 50 percent of the raingages on the Little River watershed recorded rainfall. The distribution of rainfall in space during each storm event was random. Amorocho (3) noted that the thunderstorm activity may be regarded as random over the region.

## Direction and Speed of Cell Movement

Most of the studies indicated that radar echoes, which may be composed of more than one cell at any instant, may move in a direction from 40 degrees to the left of the wind at 700 mb level to 60 degrees to the right and that large groups of cloud cells tend to move to the right. The reason for this is attributed to the cell growth and new cell development adjacent to the previous cell. Frequency distributions of the direction of cell motion studied by Brooks (10), Huff (32), and Ligda (36) indicated that speed is more constant than direction for the individual small rainfall area. In the Ligda paper, fifty percent of the storm directions were found to be within $\pm 2.0$ degrees of the mean storm direction, and fifty percent were found to be within $\pm 1.4 \mathrm{mph}$ of the mean storm speed.

Based on studies by J. C. Fankhauser on the angular difference between the direction of mean wind and the direction of generating system (echo) movement, 70 percent of the echoes moved from a direction that was within $\pm 10$ degrees of the mean wind with a standard deviation of 9 degree. The deviation of the mean wind direction from thunderstorm cell direction increased for greater storm intensity. The study made by J. Charba and Y. Sasaki (18) also showed an average deviation of 37 degrees for the left moving storms and 5 to 25 degrees to the right of the mean wind direction for the right moving storms.

In the present study, individual cell speed determination were made by measuring the displacement of the rainfall pattern for each tenminute interval. The direction of each cell was also recorded. The speed and direction measurements observed during the entire cell duration were
compared with the speed and direction of the wind at 500 mb level. Most of the individual cell speed and direction variations were within $\pm 90$ degrees in wind direction and $\pm 15 \mathrm{mph}$ in speed. Due to lack of accurate data on winds aloft, it was impossible to relate the surface cell speed and direction with several levels of wind motion. It is suggested that the main reason for the large variations in the cell motion can be attributed to the fact that speed and direction were determined on tenminute intervals during the storm, while wind speed was based on the 7:00 p.m. 500 mb wind velocity. Use of average values for speed and direction, taken over time and space during the life of the cell, and better wind data may serve to reduce the large variations.

Previous investigators have sought causal relationships between radar detected cell speed and winds aloft at various levels. Investigations conducted by Brooks (10), Mather (38), Ligda(36), and the Thunderstorm Project (12) have found fair to excellent correlations between direction and speed of cell and wind. One of the earliest studies of cell movements made by Brooks showed that small radar cells moved with winds at 5000 feet and that the larger ones moved with the winds at 11,000 feet. According to the J. R. Mather study, 70 percent of the radar cloud cells fall in the range of 6000 to 14,000 feet in vertical thickness. Later the thunderstorm investigators correlated the speed of mean wind between 5000 and 20,000 feet with radar echo movement. Poor correlation was noted. It is of interest that in Ligda's study, geostrophic* wind speed and direction

[^4]i was correlated with the direction and speed of precipitation area. High correlation was found between direction and speed at 700 mb level. In the same paper, the velocity of precipitation areas is also correlated very well with atmosphere flow as obtained from upper level pressure charts, especially near the 700 mb ( 10,000 feet) level.

Cell characteristics, as determined in the present study, have been compared with previously published results. It can be concluded that there is a general agreement with the previous investigators, although it is difficult to compare the present findings with some results of previous investigators.


## CHAPTER IV

## DESCRIPTION OF RAINFALL SIMULATION MODEL

## Introduction

A simulation of a system or an organism is the operation of a model or simulator which is a representative of the system or organism. (48)

We therefore define system simulation as the technique of solving problems by following the changes over time of a dynamic model of the system. (26)

Simulation is a numerical technique for conducting experiments on a digital computer, which involves certain types of mathematical and logical models that describe the behavior of a ... [hydrologic]... system (or some component thereof) over extended periods of time. (40)

Each of the three definitions given above is appropriate in the present study. Each definition contains the implication that simulation involves operations with a model, and more specifically, with a dynamic model, a model in which the values of the variables change in time. The purpose of this chapter is to describe the formulation of a dynamic, digital model to be used to simulate thunderstorm rainfall, and to present data and methods used for evaluating parameters and functional relationships contained within the model.

The role of the model in the simulation study is to provide a representation of the real world system that is under study, while at the same time providing a simplification of the real world system. Rosenblueth and Wiener (45) have stated, "No substantial part of the universe is so simple that it can be grasped and controlled without abstraction.

Abstraction consists in replacing the part of the universe under consideration by a model of similar but simpler structure. Models ... are thus a central necessity of scientific procedure."

Obviously computer models are an approximation of the natural phenomenon which occur in the real world. The degree of approximation depends on a number of considerations, including the applications for which the model will be used. The computer model which is presented in this paper will not reproduce many of the meteorological aspects of rainfall. On the other hand, the computer model can be used by the hydrologists to generate thunderstorm rainfall patterns satisfactory for use as a synthetic sequence of rainfall data for input to rainfall-runoff models.

Development of a simulation model involves a number of steps. The steps included in the present model development were:

1. Formulation of the problem or objectives
2. Collection and processing real world data
3. Formulation of conceptual model
4. Estimation of model parameters
5. Evaluation of parameter estimates
6. Formulation of computer program
7. Validation of simulation model

The objectives of this simulation study have already been discussed in Chapter I, but these objectives will be reiterated here in slightly different terms. Two general reasons can be given for developing a model to simulate thunderstorm rainfall. First, simulated rainfall can provide a test of certain assumptions and hypotheses on which the mode1 is based.

Failure of the model of reproduce observed rainfall patterns would imply that some, or all, of the hypotheses on which the model is founded must be rejected, while successful reproduction of historical rainfall patterns can be interpreted as a validation of the relationships incorporated in the model. A second reason for simulating thunderstorm precipitation is to provide a data sequence that can be used in hydrologic studies which require precipitation input. Hydrologists have devised causal models for streamflow generation which convert precipitation over a watershed into streamflow. In calibrating such watershed model historical sequences of rainfall and the associated streamflow are required. However, if the watershed model is to be used to predict the statistical characteristics of streamflow expected in the future, simulated precipitation values may be used, as long as the simulated precipitation sequences manifest the characteristics expected, in a statistical sense, from the actual rainfall.

Hydrologists have long used statistical descriptions of point precipitation to predict the runoff to be expected with a given frequency from a given watershed. These predictions have, in the past, been based on very simple rainfall-runoff relations which were not capable of incorporating temporal and spatial variation in rainfall. Presently the more advanced watershed models are able to take into account such variation in precipitation. Thus, the development of a model for simulating thunderstorm rainfall will provide input data for these watershed models and will allow increased accuracy in the prediction of storm runoff.

The second major step in model development, collection and processing of real world data, was described in detail in Chapter III. Chapter III
also described parameter estimation (step 4) and parameter evaluation through graphical comparison of historical and fitted relationships (step 5). Formulation of a conceptual model (step 3), and formulation of a computer program (step 6), are described in the following paragraphs. Model validation, the final step in model development, is presented in the following chapter, Chapter V.

## The Conceptual Mode1

The basic element in the construction of the model is a thunderstorm cell which is manifested as an isohyetal pattern on the ground, moving in a certain direction along a certain path. The direction of the cell path, the velocity of the cell and the size of the cell are considered random cell parameters. The number and the initial positions of cells are also considered as random. Estimates of the probabilities which describe the operation of the model were obtained from an analysis of rainfall data as described in Chapter III. A basic assumption is that the particular probability density functions which describe the population parameters are know from Chapter III.

The concept of the model given in the previous paragraph emphasizes that the model is fundamentally stochastic. Three types of stochastic elements were envisioned to operate within the system being modeled. First of all, a number of parameters were considered to be independent random variables. For examples, the initial location of a cell and the number of cells within a storm system were treated as independent random variables and were generated by the computer by random sampling techniques. The reader is referred to Naylor et al (40) for a detailed explanation of these
techniques. Other techniques were needed to handle stochastic parameters that were related to some other element of the system. For example, in Chapter III it was found that the logarithm of the maximum rainfall intensity is linearly related to the cell duration. In cases like this where there was a clear relationship between variables, the values of the dependent variable was generated by adding a random component to the value predicted from the current value of the independent variable. Such techniques are frequently used in hydrologic simulation. See references (24, 55). In the case of generating cell directions with respect to the wind direction there was no apparent functional relation between the sequential cell direction values, but the value of cell direction at time $\mathrm{t}+1$ was influenced by the cell direction at time t . In order to generate the direction of cell movement, a matrix of conditional or marginal probabilities was employed. For additional background on marginal probability matrices, see reference (6). The various statistical characteristics of the model parameters are summarized in Table 4.

## Generation of Stochastic Elements

The elements of the simulation model are formulated and the stochastic characteristics of model parameters are described in the following paragraphs.

Maximum Cell Size
The maximum dimension of the minor cell axis was generated from the relation derived in Chapter III between the maximum major and minor cell axes. A graph of the density function relating the maximum major and minor axes is given in Figure 32. The method followed during the

Table 4. Statistical Characteristics of Model

## a. Parameters of Probability Density Functions



Table 4. Statistical Characteristics of Model
b. Relationships Between Model Parameters

Variable

Temporal Variation
of Cell Axes

Temporal Variation
of Rainfall Intensity at the Cell Center

Along Minor Axis
Spatial Var. of Rainfall $I_{t}$ Int. Along the Axes

Maximum Rainfall Int. XMAXI Lin. reg. (3) $=0.0667+0.0068$ XI+RNNX $\sigma\left(1-R^{2}\right)^{1 / 2}$
Distribution Parameter $\quad b_{1} \quad$ Reg. eq. $\quad=0.112+1.187 \mathrm{~b}_{2}+\mathrm{RNNx} \mathrm{\sigma}_{1}\left(1-\mathrm{R}^{2}\right)^{1 / 2}$
Symbol Type of Relation

DMAJ Reg. eq. (1) $=.39-1.06 \mathrm{~T}+15.04 \mathrm{~T}^{2}-27.8 \mathrm{~T}^{3}+$ DMIN $13.8 \mathrm{~T}^{4}+\varepsilon$

$$
\begin{aligned}
& =37+0.57 \mathrm{~T}+8.50 \mathrm{~T}^{2}-19.9 \mathrm{~T}^{3}+ \\
& 11.0 \mathrm{~T}^{4}+\varepsilon
\end{aligned}
$$

XMAX Reg. eq. $=.14+1.91 T+4.95 T^{2}-14.8 T^{3}+8.18 T^{4}+\varepsilon$
Lin. int. (2) 0.4-0.6
Lin. int. < 0.6

Bivariate $=I_{o} \exp -\left(b_{2} X^{2}+b_{1} Y^{2}\right)$
(1) Reg. eq. = Regression equation
(2) Lin. int. = Linear interpolation
(3) Lin. reg. = Linear regression
$\mathrm{T}=$ Percent of cell duration
$\varepsilon=$ Random element


Figure 32. Probability Density Function for Maximum Minor Cell Axis

- generation process of the maximum minor cell axis is called a rejection method (40).

1. The maximum major cell axis (DMMAJ) was drawn from a normal probability density function with a mean of 7.16 miles, and a standard deviation of 1.47 miles. (Samples from the normal density function with mean zero and standard deviation of unity were generated by summing twelve random numbers and subtracting six fromthe total.)
2. A pair of random numbers* $\mathrm{RN}_{1}$ and $R N_{2}$ were generated by a function subroutine.
3. By using the information obtained in parts 1 and 2 , the following relation was established to generate a random value for the maximum minor cell axis in the range (AA, CC). (See Figure 32)

DMMIN $=A A+\mathrm{RN}_{1}(\mathrm{CC}-\mathrm{AA})$
4. By the use of second random number $\left(\mathrm{RN}_{2}\right)$, a value for function $Z=R N_{2}(2 /(C C-A A))$ was found. If the density function $F$ was less than or equal to the function $Z$, then the value of DMMIN was accepted; otherwise, a new pair of random numbers were generated and the method was repeated starting from step 2.

## Cell Duration

The cell duration was selected from a normal density function with a mean of 53.3 minutes and standard deviation of 15.9 minutes.

[^5]Temporal Variation of Cell Axes and Rainfall Intensity at the Cell Center
i. Temporal Variation of Cell Axes. As was mentioned in Chapter III, the relationship between the dependent variable $Y$, which in this case is the value of the dimensionless cell axis, and the independent variable $T$, which is the cumulative percentage of cell life, was in the power series form. The first four terms of the polynomial were retained and fitted to data by the least squares method. The coefficients of the polynomial were chosen so that the sum of the squares of deviations from the line was minimum. The following relationship was used to generate random values of $Y$ by adding a random error component as shown in equation (6)

$$
\begin{equation*}
Y_{j}=a_{0}+a_{1} T+a_{2} T^{2}+a_{3} T^{3}+a_{4} T_{4}+\varepsilon_{j} \tag{6}
\end{equation*}
$$

where $\varepsilon_{j}$ is the random component associated with the $j^{\text {th }}$ values of $Y$. The random component was generated from a normal distribution function and $a_{0}, a_{1}, a_{2}, a_{3}$, and $a_{4}$ are the values of the polynomial coefficients. The random error component was obtained by multiplying the standard error of estimate by a random normal number with zero mean and unit variance. The values of the coefficients for major and minor axes and the standard errors of estimae ( $\mathrm{S}_{\mathrm{e}}$ 's) are

ii. Temporal Variation of Rainfall Intensity at the Center of
the Cell. The variation of intensity was grouped into three categories. In the first category, the rainfall intensities at the cell centers were
less than 0.40 inches per ten minutes and were represented by a polynomial equation of the form shown in equation (6). The coefficients of the equation were $a_{0}=0.14, a_{1}=1.90, a_{2}=4.95, a_{3}=14.82$, and $a_{4}=8.18$. The other two categories, which were grouped in two ranges, 0.40 to 0.60 , and greater than 0.60 inches per ten minutes, were not fitted by a polynomial because of greater skewness and steepness of the distribution curves. A method of linear interpolation was used in the simulation runs. A series of 50 T 's (dimensionless cell duration) and Y's (ordinates of the rainfall intensity curves) were read into the : computer; the rainfall intensity at intermediate values of $T$ were found by linear interpolation.

Maximum Rainfall Intensity at the Center of the Cell
A relationship between the cell duration ( t ) and the logarithm of maximum rainfall intensity $\left(I_{o}\right)_{\text {max }}$ at the center of the cell was linear. The linear regression relation defined by a straight line in Figure 16 of Chapter III provided the basis of a procedure for generating the maximum rainfall intensity which occurs during the life of a cell. A random component was added as shown in the following equation during the generation of maximum rainfall intensities.

$$
\begin{equation*}
Y=a_{0}+a_{1} t+R N N \sigma_{y}\left(1-R^{2}\right)^{1 / 2} \tag{7}
\end{equation*}
$$

In equation (7) $a_{0}=-0.667, a_{1}=0.0068, t$ is the cell duration in minutes, RNN is a normal random variate with zero mean and unit standard deviation, $R$ is the coefficient of correlation between cell duration and maximum rainfall intensity, $\sigma_{y}$ is the and $Y$ is a logarithm of the maximum rainfall intensity.

The distribution of the maximum rainfall intensities at the cell centers generated by equation (7) showed a distribution shifted to the right as compared to the historical data. The reason for this was that in the simulated model the maximum rainfall intensities were assumed to be at the center of elliptical isohyets, but in the observed storm cells the maximum intensities were always assumed to be at a raingage. The simulated model study showed that the distribution of the maximum rainfall intensities could be improved if 0.065 inches of rainfall, as shown in equation (8), is added to an antilogarithm of a simulated maximum rainfall intensity. Therefore the following relation was used to make this adjustment.

$$
\begin{equation*}
\left(I_{o}\right)_{\max }=\exp (Y)+0.065 \tag{8}
\end{equation*}
$$

where $\left(I_{o}\right)_{\max }$ is the maximum rainfall intensity at the center of the cell during its life.

Spatial Variation of Rainfall Intensity Along the Cell Axes
The point rainfall intensities at the ground surface were generated by a bivariate distribution equation. Equation (4) given in Chapter III was used to describe the distribution of rainfall intensities along the cell axes and the following procedure was used for determining the point rainfall intensities $\left(I_{t}\right)$ at any raingage location which was within a rainfall pattern:

1. The distribution coefficient $b_{2}$ along the major axis was generated from a normal density function given in Figure 20. A linear relation between the distribution coefficients $b_{1}$ and $b_{2}$ was established in Chapter III.
2. Simulation required the addition of a random component to the relationship developed between $b_{1}$ and $b_{2}$. To generate the distribution coefficient $b_{1}$ along the minor axis, equation (9) was used.
$b_{1}=a_{0}+a_{1} b_{2}+R N N \sigma_{1}\left(1-R^{2}\right)^{1 / 2}$
where $a_{0}=0.112, a_{1}=1.187, \sigma_{1}$ is the standard deviation of distribution coefficient along minor axis, RNN is a random normal number and $R$ is the correlation coefficient.
3. Simulation proceeded by generating a maximum rainfall intensity $\left(I_{o}\right)_{\text {max }}$ at the cell center (see equations 7 and 8 ) and by using dimensionless rainfall intensity curves to find the maximum cell center intensity at time $t$.
4. In order to compute the point rainfall intensity at any location within a cell boundary, a bivariate distribution function expressed by the following equation was used

$$
\begin{equation*}
I_{t}(X, Y)=\left(I_{o}\right)_{t} \exp \left(-\left(b_{2} x^{2}+b_{1} Y^{2}\right)\right) \tag{10}
\end{equation*}
$$

where $X$ and $Y$ are the coordinates of a raingage location with respect to the cell center.

Direction and Speed and Cell Movement
The first step in generating direction and speed of cell movement was to simulate two values for the wind speed and direction at the 500 mb level.

During the simulation of the cell speed deviation from the wind speed, two normal probability density functions were used. During the
time when the rainfall intensity at the cell center was increasing, the cell speed deviation was generated from a normal function with a mean of -1.94 mph . When the rainfall intensity was decreasing, the mean cell speed deviation from the wind was -4.92 mph . The mean and the standard deviation for both increasing and decreasing rainfall intensity were -4.0 mph and 8.0 mph , respectively. During the middle period of cell life (35 to 50 percent of cell duration), rainfall intensity and cell size generally reached a maximum value, and the cells were observed to move with a velocity equal to or less than the wind velocity. In order to simulate the cell movement during this period, cell speed deviations from the wind were limited to zero or negative values.

The mean wind direction was found to lie along a line East 24.5 degrees North. The distribution of the wind direction was represented by a normal density function. The cell direction during period $t$ was related to the cell direction during period $t-1$. This approach required only the first cell direction to be generated from a relationship in which the wind direction was a factor. The other cell direction deviations were generated by utilizing a marginal probability table. The use of marginal probabilities was necessary in order to preserve the serial correlation between the successive cell directions. In Table 5, the number of the observed cell direction deviations during period $t$ with respect to the period $t-1$ is tabulated. The cell direction deviations are grouped into eight categories, each of which covers a 30 degree interval. The observed number of cell direction deviations grouped into the middle six categories, between 0.0 and $\pm 90$ degrees, showed that the data can be fitted by normal distribution functions. The statistics of these distri-

Table 5. Marginal Probability Distribution
$\theta_{2}$ (cell direction deviation during period $t$ )

butions are given in Table 4 a , Due to the lesser number of observed cell direction deviations in the range outside of $\pm 90$ degrees, the probability functions are represented by continuous uniform distributions. During the simulation process of cell direction deviations from the wind the following steps were followed:

1. The cell direction deviation from the wind during the first period of cell life was generated from a normal density function presented in Figure 25. (Refer to Chapter III)
$\therefore$ 2. The other cell direction deviations were generated by the use of Table 5 in which the marginal probability distribution of cell direction deviation during period $t$ was tabulated with respect to the period $t-1$. As an example, if the deviation of ce11 direction from the wind at the first five minute period is in the range of $0^{\circ}$ to $-30^{\circ}$, then the cell direction deviation at 15 minutes of cell life would be generated from a normal distribution function whose mean and standard deviation was -5.6 and $40^{\circ}$, respectively. (Refer to Table 4a)
2. The procedure reverts to step 2 for the other periods of cell life until the cell duration is completed.

Number and Orientation of Cells
. $\cdot$.. The total number of cells simulated within a storm period* ranged between two and nine. The number of cells was selected from a discrete uniform distribution. The number of independent cells active at one time

[^6]
was also generated from a discrete uniform distribution on the interval one to three. Seventy-five percent of the time, a dependent, or a baby cell, was generated at a mean distance of five miles from the center of the independent cell. The origin of the baby cell was within the third or fourth quadrant of a cartesian coordinate system formed by the major and minor axes. The time lag between generation of the two cells was 10 to 40 minutes. In addition to the formation of primary baby cells, there was a 50 percent chance of forming secondary baby cells from primary cells. The origin of secondary baby cells was related to the temporal and spatial characteristics of primary baby cells in the same way as the primary baby cells were related to the independent cell. The same method of generating baby cells was followed for all the independent number of cells generated at one time until the total number was completed. Then, if the total number of cells within a storm was not complete, a new set of independent cells was generated with a time lag of 10 to 80 minutes from the previous set of independent cells and the procedure was repeated. When the duration of the existing cells was equal to the time lag between generation of baby cell and existing cell, then two primary baby cells were generated from the existing cell at the downwind (first and second quadrant) side of the existing cell major axis direction.

When simulated cells did not pass over any raingage in the network, the cells were dropped from consideration and additional cells were generated to take the place of those dropped.

## Simulation Program

A computer program was written in FORTRAN language which has
the following structure. The main program statements handle input-output
as well as the sequence of the simulation procedure. The first part of the program includes instructions to control the value of the model parameters as well as to read the raingage locations. Part two pertains to the dynamic part of the program in which the cell is moved forward through tenminute time increments, starting from the first five minute period. Rainfall intensity values are generated throughout the life of the cell and are printed at $5,15,25, \ldots$ etc. minutes of cell durection, the midpoints of the ten-minute periods. Subroutines are provided to generate primary and secondary baby cells and to store the necessary values for printing and plotting of the computer outputs. The raingage locations and the isohyetals cells patterns were plotted by a Calcomp plotter. During the process of simulation, subprograms are called for generating new values of the cell parameters whenever they are needed.

To simulate stochastic processes, uniformly distributed and statistically independent numbers are needed. For the purpose of this study, a multiplicative congruential method was used to generate random numbers. This method has been found to behave statistically quite well and, furthermore, by selecting an appropriate multiplier and random number, a maximum period in the generated sequences is insured.

The origins of the independent cells were generated within a rectangular area, 25.0 miles by 7.5 miles, with the major axis of the rectangle placed along the major axis of the watershed, which was at an angle of 30 degrees counterclockwise with respect to north. Two rectangular coordinate systems were used in the computer program; one had a fixed origin and fixed axis orientation to which the raingage
coordinates were referred. The origin of this system was selected as the middle raingage station. In the second coordinate system, the origin moved along the trajectory of the storm cell and the perpendicular axes were oriented along the minor and major axes or the elliptical cell. The independent thunderstorm cells, which were initially generated at random within the rectangular area, could easily be followed. The primary and secondary baby cells were expressed in terms of the Lagrangian coordinate system and the cell boundaries were established so that the location of raingages could be expressed with respect to the isohyetal cell center.

In order to describe the simulation procedure followed in the model, a step by step sequence is presented and a flow chart is drawn to show the sequences of model operation in a schematic manner. The schematic flow chart is shown in Figure 33. A detailed flow chart is attached in Appendix C, in Figure C.2.

## Step 1

The means and variances of the probability density functions used in the model are specified as input data. Polynomial coefficients for the fourth order equations and the coordinates of the raingages are also input data.

## Step 2

The wind speed is selected from a triangular distribution which covers the range between zero and 32 mph and shows a peak wind velocity of 12 mph . The direction of the wind is generated from a normal density function with a mean of E24.5N degrees and standard deviation of E74.ON degrees.


Figure 33. Schematic Flow Chart of the Computer Program

## Step 3

The total number of cells simulated within a storm period is generated from a uniform distribution on the interval two to nine. The number of independent cells generated at one time ranges from one to three, with a time lag of 10 to 80 minutes from the previous set of independent cells.

## Step 4

The origin of an independent cell is generated at random within the target area. By a random process, a decision is made as to whether or not a baby cell will be allowed to originate from an existing cell. Seventy-five percent of the time, a primary baby cell originates from an existing cell.

## Step 5

The cell duration, the maximum rainfall intensity at the center of the cell during the life of the cell, the maximum major cell axis at the time of maximum cell growth, and the distribution coefficients of rainfall intensity along the major axis $\left(b_{2}\right)$ are generated from normal frequency curves and the distribution coefficient $b_{1}$ and the maximum minor cell axis are determined from the relations established among $b_{1}$ and $b_{2}$ and maximum major and minor axes. The values are stored for use in later steps.

The following steps keep track of an existing cell trajectory and cell characteristics at times of $5,15,25, \ldots$ etc. minutes of cell duration to insure the use of proper dimensions, and also to provide
the necessary information for locating the origin or primary and secondary baby cells.

Step 6
In the first step, the program simulates the major and minor cell axes and the rainfall intensity at the cell center for the first five minute period. Then the cell is moved forward through ten minute time increments. The cell speed deviation and cell direction deviation from the wind are generated to determine the position of the cell with respect to the first cartesian coordinate system. The cell coordinates are expressed in terms of the Lagrangian frame of reference.

## Step 7

After the transformation of the raingage coordinates and the cell isohyet boundaries are completed, the point rainfall intensities at the raingages within the cell boundary are calculated.

The sixth and seventh steps are repeated as many times as needed until the cell duration is completed, or the cell center coordinates fall outside of the target area.

## Step 8

Depending upon the decision made in Step 4, a primary baby cell originates from an existing cell provided that the existing cell
r has dropped rainfall on at least one raingage. In addition to the formation of the primary baby cell, 50 percent of the time the secondary cell is generated from the primary cell within a distance of two to eight miles from the center of the primary cell.

The size, rainfall intensity, and the duration of a baby cell are generated as the program sequence is returned to Step 5.

## Step 9

After simulation is completed for the total number of independent cells generated at a given time, a check is made to determine if the total number of cells generated equals that selected in Step 3 . If it does, the sequence of the program begins again at Step 3. Otherwise, one to three additional cells are generated and the process described in Step 4 through 9 is continued.
;



DATA ANALYSIS AND MODEL TESTING

This chapter is devoted to data analysis and comparison of
historical and simulated thunderstorm rainfall. The stochastic thunderstorm model is designed to reproduce the cell characteristics of the historical storms through simulation of surface rainfall patterns. The testing of the model is based on a comparison of thunderstorm rainfall data generated by a computer model with data from three years of historical records.

The procedure for testing the model was, in principle, straightforward. Basically, the process was one of comparing simulated data with historical data. However, difficulties arose because the simulated and historic data are probabilistic in nature and because a large amount of effort is required to define the details of isohyets generated over short time intervals. Therefore, a method of testing based on data other than that given by short-time interval isohyets was used. The rainfall parameters which were used for model testing were selected because they (1) were throught to be representative of many of the most important features of thunderstorm rainfall, (2) were relatively easy to obtain, and (3) had not been directly built into the model.*

[^7]Most of the model testing involved analysis of the rainfall which occurred at the gage with maximum rainfall. This is the gage which recorded the maximum total depth within a closed isohyetal pattern during the life of a storm. Thus, more than one gage could be used, since distinct isohyetal patterns created by separate cells could occur over the gage network. This definition of the maximum rainfall gage applies to both historical and simulated data. The number of distinct patterns usually ranged from one to four with two being the most commonly observed number. Rainfall at the gage with the second highest rainfall was also used to test the model, as were statistics on the number of cells active at ten-minute intervals over the gage network. Hence, the testing of the model was based on rainfall characteristics as seen from an Eulerian frame of reference, while the generation of the rainfall took place in a Lagrangian reference frame.

It should be pointed out that the historical data used to test the model was not the same as that used to set the parameter values of the model. The model formulation and parameter evaluation was based on data from 1967 while data from 1968, 1969, and 1970 were used for model testing. The rainfall characteristics used to test the model are listed below and described in subsequent paragraphs.

1. The frequency distribution of rainfall duration at the raingage recording the maximum amount of accumulated rainfall.
2. The frequency distribution of the maximum amount of accumulated rainfall at the gage with maximum rainfall.
3. The frequency distribution of maximum ten-minute rainfall intensity at the maximum-rainfall gage.
4. The relationship between the duration of rainfall and the maximum amount of accumulated rainfall at the maximumrainfall gage.
5. The relation between auto correlation coefficients and the time lag of rainfalls between the maximum-rainfall gages and second maximum-rainfall gage.
6. The relationship between correlation coefficients of tenminute rainfall and distance between the gages recording the two highest accumulated amounts of rainfall.
7. The frequency distribution of the number of active cells present during ten-minute periods.

Initial runs with the computer model indicated that some modifications were needed to bring the performance of the model into line with the observed storms. Some of the modifications were simply changes in the parameters of the frequency distributions of the cell characteristics. Some other changes were required in the interior structure of the model. These changes are included in the model description in Chapter IV. On the basis of knowledge gained from these initial runs, further computer runs were performed to test the effect of the improvements made on the model parameters as well as to get a more complete understanding of the capability of the model.

One hundred and twelve thunderstorms were studied during the summer months of 1968,1969 , and 1970. To facilitate the model testing, and to make the comparison of both historical and simulated data easier, the total storm isohyetals for observed and simulated storms were drawn and the maximum rainfall gages were selected. From the total rainfall plots, 231 gages with maximum rainfall were selected and the rainfall data from these gages were compared with similar data obtained from the generated thunderstorms.

The observed data on the temporal distribution of rainfall
intensities recorded at the first and second maximum-rainfall gages were studied. Peculiar intensity characteristics were observed at the end of each rainfall period. The intensities with less than one tenth of an inch of rainfall during the dissipating stage of the cell were not recorded as a continuous sequence of rainfall intensities. Instead, onetenth increments were recorded within varying time intervals which ranged between ten minutes and approximately two hours. It was also observed that the time lag between the initiation of rainfall at the maximum and second maximum rainfall gages depended upon the rate of rainfall and also the number of cells passing over the raingages. The temporal distribution of rainfall at the maximum and at the second maximum-rainfall gages were similar. The initiation of rain, the time of occurance of rainfall of highest intensity, and the cessation of rainfall generally occurred with a zero to ten-minute lag between the maximum and second maximum rainfall gages. The shorter time lags were measured on the more densely gaged areas of the watershed. The second maximum rainfall was recorded at a raingage along the major axis of the total storm isohyet. The average distance between the two maximum-rainfall gages was three or one and one-half miles depending upon the density of raingage network.

In Table D.1, the temporal distributions of the maximum rainfalls at the gages are tabulated. The analysis of data indicated, in general, three types of cell movements over maximum rainfall gages. The following list of cell movements were most commonly observed:

1. Eighty percent of the time a single cell passed over the maximum-rainfall gage.
2.a.Ten percent of the time, two cells passed over the maximum rainfall gage within a time interval of 10 to 40 minutes. b.Three to four percent of the time two independent cells passed
over the maximum-rainfall gage within a time period of two hours or more.
2. About six or seven percent of the data showed three or more cells within a varying time interval (from 10 minutes up to two hours) passing over the same maximum-rainfall gage.

In each of the three cases stated above, the following assumptions and corresponding adjustments were made on the rainfall data for the maximum-rainfall gage and second maximum-rainfall gage.

1. When one-tenth of an inch of rainfall was recorded within a time interval of 20 minutes or less, the rainfall rate was assumed to be continuous.
2. If an increase of one-tenth of an inch in the amount of rainfall required more than 20 minutes to accumulate, the rainfall rate was assumed to be zero during this interval. This assumption was made even though the rain code indicated that rain fell on the raingage within that time period.
3. If two consecutive continuous sequences of rainfalls were observed within a time interval of two hours at the maximum rainfall gage, then each of the rainfall sequences was considered to be a separate record.

The above assumptions were needed because the model was not programmed to generate low intensity continuous rainfall sequences at the end of a rainfall duration from a single thunderstorm cell traveling over the raingage. The model was also not programmed to generate multicells within a time interval of two hours or more. The rainfall intensities grouped in categories 1 and 2 a were generated reasonably well by the model.

For model testing, one hundred and seventeen storms were generated.
The total storm isohyetals were drawn to select the synthetic maximumrainfall gages. Three hundred and eight maximum-rainfall gages were identified and the synthetic data at the raingages was stored to be used in the model testing. A greater number of isolated rainfall patterns were generated in the simulated storms because:

1. The parameters (distance, location, speed, and direction) which controlled the formation and movement of baby cells in the simulated rainfall with respect to an existing cell were not capable of producing well organized storm isohyetal patterns such as appeared in the observed storms.
2. The generation of baby cells from the existing cells was not allowed in the case of no rainfall records at the raingages from the movement of existing cells. Instead, an additional number of independent cells were generated in place of the discorded cells. Consequently, the cells were more scattered over the watershed.

Some other discrepencies were noted between the model results and the historical rainfall. Some of the differences could be attributed to the complexity of the cell characteristics and to the high probability that no gage was present at the actual cell centers where the maximum rainfall activity occurred. Some other differences were due to generation of overlapping cells by the model. In the case of overlapping cells, the amount of precipitation was overestimated. When no gage was present near the center of the cell, the values of maximum accumulated rainfalls were underestimated.

In Figures 34 to 40 the results of the simulated studies are compared with the historical values. The rainfall parameters utilized for the model testing will be discussed in more detail in subsequent sections of this chapter.

1. The Frequency Distribution of Rainfall Duration at the Gage with

Maximum Rainfall
In order to determine the duration of rainfall at the maximum rainfall gage, the centers of the storm isohyets were identified from the storm maps. Depending upon the temporal distribution characteristics of
rainfall generated at the maximum and second maximum gages*, the three types of adjustments which were already mentioned were made on the rainfall data. The duration of rainfall at the gage recording the maximumrainfall in the historical storms ranged from ten minutes for a single cell passing over the raingage to 150 minutes for a storm during which continuing cells, with small time intervals between cells (ten minutes to 40 minutes), were observed. The frequency distributions of rainfall duration for the observed and the simulated rainfall data are plotted in Figure 34. The type of trend represented by the frequency bars was similar for both historical and generated storm studies. The most frequent duration of rainfall at the maximum rainfall gage was between 30 and 40 minutes. At the 0.01 level of significance, a t-test, showed that the mean values of the observed and simulated storms were not different. 2. The Frequency Distribution of the Maximum Accumulated Rainfall at the Gage with Maximum Rainfall

The statistic for the maximum amount of rainfall for the historical storms was obtained from the data presented in Table D.1. The frequency distributions of the historical and simulated data are drawn in Figure 35. The most frequent maximum amount of rainfall was found to be in the range of 0.5 to 1.5 inches. The generated data showed comparatively higher frequencies in the range 0.5 to 1.5 inches, and lower frequencies outside this range. The reason for this difference between generated and observed

[^8]

Figure 34. Rainfall Duration at the Maximum Rainfall Recording Raingage

Figure 35. Frequency Distribution of Maximum Amount of Rainfall

Cutate
data was related to the categories of thunderstorm rainfall included in the model. Categories 1 and $2 a$, the categories of observed precipitation included in the model, did not include some of the multicell events observed historically. It was these multicell events which were responsible for a portion of the high historical rainfalls, and hence the generated data showed a lower frequency of large events. The hypothesis that there was no difference between the means of the observed values and the mean of the generated values could not be rejected at the 0.005 level of significance with the " $t_{0}$ " test of statistics.
3. The Frequency Distribution of Maximum Ten-minute Rainfall Intensity at the Maximum Recording Gage

The analysis of observed and simulated rainfall data indicated that the maximum rainfall intensity occurred in the first two ten-minute time periods. The frequency distributions of the rainfall intensities observed and generated by the model at the maximum-rainfall gage are plotted in Figure 36. The maximum ten-minute rainfall intensities ranged from 0.10 inches for short duration cells to 1.20 inches per ten minutes for highly intensive and long duration cells. The most frequent maximum rainfall intensity was about 0.30 inches per ten-minute period. The mean values of the generated and the observed distributions were 0.41 and 0.44 inches respectively. The hypothesis that there was no difference between the mean of the observed values and the mean of the generated values could not be rejected at the 0.05 level of significance.
4. The Relationship Between the Duration of Rainfall and the Maximum Amount of Accumulated Rainfall

The average duration of rainfall was computed for each total maxi-


Figure 36. Frequency Distribution of Maximum TenMinute Rainfall Intensity
mum rainfall depth, e.g., all storms in which the maximum total rainfall depth was 0.4 inches were grouped and the average duration computed. This was done for all recorded depths where there were at least four storms producing the same maximum total depth. The maximum amount of rainfall found in both the historical and generated data at the maximum-rainfall gage during a total storm period was plotted against the rainfall duration averaged for each total depth of rainfall. The relationship between the rainfall duration and the maximum amount of rainfall at the maximum rainfall gage showed very high correlation. The points shown in Figure 37 for observed and simulated data indicated a similar trend between the maximum amount of rainfall from 0.2 inches to 2.0 inches. Generally, less than four storm records for the same maximum total depth were available for the accumulated rainfalls greater than 2.0 inches. Therefore, the study for both simulated and observed data was limited up to 2.0 inches of maximum rainfall.
5. and 6. The Relation Between the Correlation Coefficients versus Time

Lag and Distance
For additional comparisons of rainfall characteristics in the historic and synthetic data, the following relations were derived:
$\because$ a. The correlation coefficient computed from ten-minute rainfall values for the two raingages recording the maximum and second maximum-rainfall was analyzed in relation to the distance between the gages.
b. The correlation coefficient versus the time lag between the rainfalls recorded at the gages with maximum-rainfall and the second highest rainfall.

The resulting correlation functions for each of the above relations were plotted in Figures 38 and 39 for the observed and simulated values. The

[^9]


Figure 38. Distribution of Correlation Coefficients vs. Time Lag



Figure 39. Distribution of Correlation Coefficients Vs. Distance between Raingages
following conclusions were drawn from each of these plots:
a. The correlation coefficients found for each 0,10 or 20 minute time intervals at the two raingages were about ten percent lower in the simulated storms than those observed from the historical values.
b. For the zero time lag between the rainfalls, the highest correlation coefficient was 0.63 and it dropped to about
? 0.40 when the time difference between the initiation of rainfalls was increased to 20 minutes.
c. An asymptotically decreasing type of relation was apparently observed between the spacing of the two raingages and the correlation coefficients.
d. Larger correlation coefficients were obtained in the denser part of the raingage network where the raingages were spaced
. 4 st one and one-half miles apart.
e. The curves appeared to be asymptotic to the 0.40 line of space correlation coefficient when the spacing between the two gages was increased to seven or eight miles.
7. The Frequency Distribution of the Number of Active Cells Present at

## Ten-minute Periods

The frequencies of the number of active cells over the gage network at ten-minute time intervals for the observed and generated data are plotted in Figure 40. The plot indicated that two was the most frequent number of cells present with a ten-minute period. The frequency diagram obtained from the model gave a similar distribution pattern to that observed historically, but the ordinates of the bars showing the relative frequencies were slightly different. In this study, the highest number of cells observed or simulated within ten-minute period was seven. The hypothesis that there was no difference between the mean of the observed values and the mean of the generated values could not be rejected at the 0.05 leve1 of significance.


Figure 40. Frequency Distribution of Number of Active Cells

Summary
It can be concluded from the results discussed in the above paragraphs that the model was capable of adequately reproducing the selected parameters. The frequency distributions of the parameters and the correlation coefficients were about the same for both historical and generated data. Some variability in frequencies, such as in Figures 34 , 35, 36 , and 40 , for the observed and simulated data should be expected because the presence of active cells over the gage network is a random phenomena. : $\because \because^{\prime \rightarrow}$

## CHAPTER VI

## CONCLUSIONS AND RECOMMENDATIONS

A simple thunderstorm rainfall model has been developed based on observed storm cell characteristics. The present model permits the simulation of thunderstorm rainfall over a network of gages in the Southeastern Coastal P1ain area.

The performance of the thunderstorm model can be judged successful on the basis of comparisons made in Chapter V between the historical and simulated rainfall characteristics. The rainfall characteristics which are considered to be representative of the most important features of thunderstorm rainfall were produced satisfactorily by the model.

Some modifications may result from the study of additional historical data. For example, to get good agreement between simulated and observed data, it was, in a few cases, necessary to make parameter adjustments. Therefore, any conclusions drawn from this study should recognize these limitations and restrictions. However, the overall performance of the model is considered quite good.

The following conclusions and recommendations derived from this study should have an important bearing in future thunderstorm rainfall studies.

1. The results obtained from the plots of successive isohyetal cell patterns at ten-minute intervals indicated that further improvements in
understanding thunderstorm rainfall will require studies on two spatial scales:
a. An intensive investigation of the cell characteristics should be done on a microscale level of storm activity. The reason for this is that the sizes of convective thunderstorm cells are small and characterized by high temporal and spatial intensity gradients.
b. The occurrence of cells with respect to each other must be viewed on a larger scale of storm activity, the macroscale.

Because of these two levels of storm activity, it is recommended that both radar observations and surface rainfall measurement are needed. The movement of surface cells observed by raingages could then be compared with storm echo patterns. The raingages would provide accurate precipitation measurement and the radar would measure the larger spatial variability of rainfall.
2. It was found that a ten-minute period for plotting isohyetal maps provides for adequate definition of cell shape and movement. Analyses based on five-minute intervals showed complex fluctuations (noise) in some of the time varying characteristics of the rainfall. Use of tenminute intervals tended to smooth the data and to make it more meaningful by eliminating these higher frequency variations.
3. Quantitative knowledge was gained about many of the characteristics of thunderstorm rainfall. The following list includes some of the information which can be readily summarized.
a. On an average, the maximum values of the minor and major axes of rainfall cells were about 4.7 and 7.2 miles, respectively.
b. The cell life varied between 30 minutes and 100 minutes with a mean of 53 minutes.
c. The maximum cell size was found to occur in the range from 20 to 50 percent of cell 1ife.
d. The maximum rainfall intensity rainged from 0.10 to 1.20 inches per ten minutes.
e. The most frequent maximum intensity values were between 0.40 and 0.50 inches per ten minutes.
f. The average number of cells per storm was found to be five, with the number ranging from two to nine.
4. It can be concluded that representative rainfall characteristics for model testing may be determined from an Eulerian frame of reference, although the model operates in a Lagrangian reference frame. This method of model validation is relatively easy and simple compared to methods which require large amounts of effort to analyze the rainfall data.
5. In order to study the motion of surface thunderstorm cells, more accurate measurement of the upper wind data at different pressure levels ( 850,700 , and 500 mb level) is needed. This would allow more accurate measurement of the deviations of cell direction and speed from the upper wind data.
6. Due to the large amount of manual work needed to process and analyze the rainfall data, simulation of the rainfall process with digital computers is a better way to predict the surface storm patterns assocaited
with cell units. This information can readily serve as input data to a runoff model or a watershed model for predicting discharge from small drainage areas.
7. The computer program developed in this investigation, was relatively simple. A more detailed computer program may be needed in future studies to describe a variety of cases in which multicells might be generated within a time interval of two hours or more.
8. A sensitivity test of the model should be conducted to determine which cell elements have the greatest effect on the model outputs. Such a study could lead to a deeper understanding of the model and indicate improvements which might be made in future studies.
9. It is suggested that operation and design application of the model should be kept in mind during future studies of the rainfall model. The information provided by the model can serve as input to watershed models for estimates of flow. This is of fundamental importance to hydrologists who are basically interested in rainfall-runoff relationships.
10. The model described by this study should be tested for prediction of surface rainfall patterns in those areas of the country with meteorological and hydrological characteristics similar to the experimental watershed at Georgia Coastal Plain.
11. Transferability of the model to other regions and locations should also be studied. The model parameters could be generalized in terms of physical and meteorological characteristics of the regions. In simulating watershed inputs, some of the model parameters might be controlled by particular physical characteristics of the watershed. The continuation
of this study will be conducted in the School of Civil Engineering, Georgia Institute of Technology to test the generality and the applicability of the model in other areas of the United States as more data becomes available from different climatological areas.


Table A.1. Accumulated Rainfall Data at Five-minute Increments With the Trace Codes.



Table B.l. Cell Characteristics

*Positive sign means angle measured in counterclockwise direction from east. Negative sign means angle measured in clockwise direction from east.


*Positive sign means angle measured in counterclockwise direction from east.
Negative sign means angle measured in clockwise direction from east.

Motion Diameter Remarks
Date Time (mph) (degree) (mile) (mile) inch/ten-minute (mph) (degree) Speed Direction* Minor Major Core Int. Wind Wind Speed Direction


[^10]

| Date | Motion | Diameter |  | Remarks |
| :--- | :--- | :--- | :--- | :--- |
| $\because$ | Time | (mph) (degree) (mile) (mile) | inch/ten-minute (mph) (degree) |  |
|  |  | Speed Direction* Minor Major | Core Int. | Wind Wind |
|  |  |  |  | Speed Direction |


*Positive sign means angle measured in counterclockwise direction from east Negative sign means angle measured in clockwise direction from east.

Motion
(degree) (mile) (mile) inch/ten-minute (mph) (degree) Speed Direction* Minor Major Core Int. Wind Wind Speed Direction
$18{ }^{00}-_{19} 00$
8/20/67 $16^{00}{ }_{-16}{ }^{10}$

| $16^{10}$ | 16.8 | 25 | 6.5 | 8.8 | .64 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $17^{20}$ | 25.2 | 34 | 6.5 | 7.8 | .40 |
| $16^{30}$ | 15.0 | 17 | 3.5 | 3.5 | .14 |
| $16^{40}-16^{50}$ | 10.2 | -30 | 3.0 | 3.4 | .10 |

$16^{4}-16^{50}{ }_{20.4}$
17
2.5
3.5
. 14
$16^{50} \quad 20.4$
$17^{00} \quad 9.6$
$17^{10}-17^{20}$
$16^{20}-16^{30}$
$16^{30}$
$16^{40}$
$16^{50}{ }_{17} 00$
18.0
13.2
28.2

51
6.0
9.2
.45
66
6.5
8.5
.40
4.5
6.0
. 30
$-10$
2.3
2.3
. 10
18
6.6
8.6
.60
25
6.6
8.6
. 50
4.0
4.0
. 24
$16^{50}-17^{00}$
$17^{00}$
$17^{10}$
21.6
9.6
6.6
8.4
$17^{30}$
10.8
$17^{50}-1800$
120
3.0
5.5
. 34
155
3.0
5.5
.40
-55
5.0
7.0
.42
$17^{20}$
47
5.5
8.0
. 75
$17^{40}$
47
3.5
7.0
.44
$-12$
2.5
4.0
.22
2.0
2.5
. 10
*Positive sign means angle measured in counterclockwise direction from east.
Negative sign means angle measured in clockwise direction from east.


| $17^{00}-17^{10}$ | 16.8 | 170 | 3.5 | 6.0 |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $17^{10}$ | 4.8 | -40 | 4.6 | 6.5 | $\ddots$ |
| $17^{20}$ | 5.4 | 13 | 6.0 | 8.5 | .32 |
| $17^{30}$ | 10.8 |  | 82 | 5.0 | 6.0 |
| $17^{40}-17^{50}$ |  | $\therefore$ | 4.4 | 6.0 | .50 |
|  |  |  |  |  | .30 |



| $16^{40}-16^{50}$ | 16.8 | 25 | 2.0 | 2.0 |  |
| :--- | ---: | ---: | ---: | :--- | ---: |
| $16^{50}$ | 26.4 | 110 | 6.0 | 8.5 | .10 |
| $17^{00}$ | 11.4 | 30 | 6.0 | 8.0 | .50 |
| $17^{10}$ | 27.0 | 45 | 5.0 | 8.0 | .50 |
| $17^{20}$ | 3.6 | 30 | 2.0 | 2.0 | .20 |
| $17^{30}-17^{40}$ |  |  |  |  |  |


| $17^{00} \underline{-17 ~}^{10}$ |  | + |  | 5.0 | 9.0 | $\because$ | . 70 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $17^{10}$ | 6.6 |  | 77 | 7.0 | 12.0 |  | 1.20 |
| $17^{20}$ | 16.2 | $\cdots$ | 87 | $8.0{ }^{\circ}$ | 11.5 |  | 1.00 |
| $17^{30}$ | 18.0 |  | 10 | 6.0 | 11.0 | : | . 60 |
| $17^{40}$ | 7.8 |  | - 20 | 4.8 | 8.8 |  | . 40 |
| $17^{50} \underline{-18}^{00}$ | 15.0 |  | -45 | 3.0 | 4.5 |  | . 22 |

[^11]

Figure B.1. Thunderstorm Isohyetal Patterns on June 21, 1967.

|  |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Figure B.2. Thunderstorm Isohyetal Patterns on June 22, 1967.


Figure B.3. Thunderstorm Isohyetal Patterns on June 28, 1967.

|  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

Figure B.4. Thunderstorm Isohyetal Patterns on July 7, 1967.



Figure B.6. Thunderstorm Isohyetal Patterns on August 2, 1967.
$3+\quad$ "


Figure B.7. Thunderstorm Isohyetal Patterns on August 20, 1967.


A computer program has been written in FORTRAN language to simulate thunderstorm rainfall patterns as they are described in Chapter III. The computer program listing is given in Table C.1. The program as a whole is composed of one main program and seven subroutine subprograms and one subroutine subfunction. During the simulation of temporally and spatially distributed cell characteristics, the related subroutines are called a number of times.

The following variable names are used in the main and subroutine subprograms.

B1,B2 - Distribution coefficients along the minor and major axes
COEF(,) - Polynomial coefficients
CTIME - Time lag between the two independent cell groups
DMAJ ( ) - Temporal variation of major axis
DMIN( ) - Temporal variation of minor axis
DMMAJ - Maximum major cell axis
DMMIN - Maximum minor cell axis
DTIME - Time lag between the existing cell and the baby ce11
DT( ),DI( ) - Pairs of dimensionless values between percent of cell duration and rainfall intensity at the cell center (between 0.40 and 0.60 in./ten-min.)

DTT( ),DII( ) - Pairs of dimensionless values between percent of cell duration and rainfall intensity at the cell center (greater than 0.6 in./ten-min.)

```
DX( ),DY( ) - Raingage coordinates with respect to Lagrangian frame of reference
DIST1,POST1 - Location of primary or secondary baby cell at the upwind side of the existing cell major axis
DIST2,POST2 - Location of new baby cells at the downwind (northwest DIST3, POST3 -
through northeast) side of the existing cell
EX1, STDX1 - Mean and standard deviation of ce11 duration
EX2,STDX2 - Mean and standard deviation of ce11 speed deviation from the wind speed during the increasing rainfall intensity period
EX3, STDX3 - Mean and standard deviation of cell direction deviation from the wind direction
EX5, STDX5 - Mean and standard deviation of maximum major cell axis
EX7, STDX7 - Mean and standard deviation of wind direction
EX8, STDX8 - Mean and standard deviation of distribution parameter of rainfall along the major axis
EX9,STDX9 - Mean and standard deviation of conditional probability distribution of cell direction deviation from the previous ce11 direction
EX15, STDX15 - Mean and standard deviation of spacing between the baby cell origin and the existing cell core
EX16,STDX16 - Mean and standard deviation of cell speed deviation from the wind speed during the descending cell intensity period
II - Set to 1 and increment by 1 around the DO loop
IX - Initial random number
```

M - Number of cells in a storm
MM - Storm number
MK - Nonzero value in column 21 to indicate the last card of data deck MKK to interpolate rainfall intensities linearly with respect to percent of cell life

NN - Number of independent cell
PT(,1),PT(,2) - Coordinates of cell centers at $5,15,25$,etc., minutes of cell duration

PTI(,) - Rainfall intensity at a raingage
RN - Uniform random number
STAT(,) - Raingage coordinates with respect to Eulerian reference frame THETAD - Cell direction deviation from the wind direction

THETAW - Wind direction
VELD - Deviation of cell speed from the wind speed
VELW - Wind speed
XI - Ce11 duration
XK - Cell generation control number
XMAX ( ) - Temporal variation of rainfall intensity
XMAXI - Maximum ten-minute rainfall intensity during the cell life
ZL( ) - Set to zero to start and increment by 1 each time when raingage records rainfall

The names of the subroutines are
MAXINT - Generates the maximum rainfall intensity at the center of the cell

NORM - Generates cell direction deviation from the wind direction at the first period of cell duration

NORMAL - Generates normal random numbers
POLYN - Determines the temporal variation of cell size and cell intensity for less than 0.40 in./ten-minute ,

RANDU - Generates uniform random number between ( 0,1 )
REGION - Determines intermediate cell center coordinates
TRANS - Determines raingage coordinates with respect to the Lagrangian frame of reference

TRANSF - Transforms the coordinates of the cell boundaries with respect to the new cell centers

The program uses the Calcomp Plotter for indicating the isohyetal cell patterns at each ten-minute time increment. A portion of the Computer and Calcomp Plotter outputs are given in Table C. 2 and in Figure C.1. Figure C. 2 shows the detail flow chart of the computer program.

Table C.l. Computer Program Listing.



Table C.l (Continued).


Table C. 1 (Continued)


Table C.l (Continued)


Table C. 1 (Continued).


Table C. 1 (Continued)


Table C.l (Continued)


Table C.1 (Continued).


Table C. 1 (Continued).


Table C. 1 (Continued).


Table C.l (Concluded).


Table C.2. Portion of Program Output.


Table C. 2 (Continued).


Table C. 2 (Concluded).



Figure C.1. Calcomp Plotter Output.



Figure C. 2 (Continued)


Figure C. 2 (Continued)

$$
\begin{aligned}
& \begin{array}{r}
167
\end{array} \\
& \text { A4-A } \\
& \text { APPENDIX D } \\
& \because A^{*}+ \\
& \text { Ant: }
\end{aligned}
$$ s.

$-1$
$\square$
$\qquad$

Table D.1. Tabulation of the highest historical maximum accumulated rainfalls at the maximum-rainfall gages


$$
\begin{aligned}
& 35+64
\end{aligned}
$$

Total Acc

Rainfall

$$
\begin{array}{llllll}
5 & 15 & 25 \quad 35 \quad 45 \quad 55 \quad 65 \quad 75 \\
\hline
\end{array}
$$

. 2 . 2 . 1
. 1 . 3.1
. 1 . 2 . 1 . 1
$.1 .3-$ - 1
. 1 . 3.1
. 3 . 1 . 1
. 2.3
.4 .1
. 1 . 2 . 1 . 1
$\begin{array}{ll}.1 .^{4} \\ .1 & . \\ . & .2\end{array}$
. 2 . 2 . 1
. 3 . 2
. 3 . 2
.6
. 1 . 4 . 1
. 2 . 2 . 1 . 1
. 2 . 3 . 1
. 3.3
. 3 . 2 . 1
\%
$\begin{array}{lll}.1 & .4 & .1 \\ . & \text {. } 2 & .1\end{array}$
. 1 . 3 . 2
. 2 . 3 . 1
.7 . 1 . 4 . 1 . 1
. 2 . 4 . 1
. 3.4
. 1 . 1 . 1 . 2 . 1 . 1
.1 - -3 . 2 . 1
,
. 2 . 3 . 1 . 1

|  | .2 | .3 | .2 |
| :--- | :--- | :--- | :--- |
|  | .2 | .2 | .2 |

. 2 . 4 . 1
. 3 . 2 . 1 . 1
. 1 . 1 . 3 . 2
. 1 . 3 . 1 . 1 . 1

$86 x^{2}$

Total Acc.
Duration(min.)
Rainfall
$\begin{array}{llllllll}5 & 15 & 25 & 35 & 45 & 55 & 65 & 75\end{array}$

1.2
.1 .1 - - - . 4.5 .1
. 3 . 1 . 1 . 2 . 2 . 3
.1 . 8 . 2 . 1
$.2-.1 .4 .3 .2$
.2 . 7 . 3
.1 . 1 . 1 . 1 . 2 . 3 . 2 . 1
$.1 .2 .2 .3 .3-1$
$.5 .6-\quad-.1$
.1 . 5 . 5 . 1
.1 .5 .3 .2 .1

$.1 .4 \cdot 1.1(70 \mathrm{~min}) .2 .2 \cdot 1$

1.4
.1 . 8 . 3 . 1 . 1
.1 .3 . 2.1 . 1 (110 min) . 1.3 .1 . 1
.1 . 4 . 5 . 2 . 2


.2 . 5 . 5

Total Acc. Duration(min.)
$\begin{array}{llllllll}\text { Rainfall } & 5 & 15 & 25 & 35 \quad 45 \quad 55 \quad 65 \quad 75\end{array}$

. 11.1 . 1 . 3 . 1 . 1

1.8 . $\quad .2$. 3 . 4 . 3 . 2 . 1 . 1 . 1 . 1
. 21.1 . 5 . 1
. 2 . 9 . 4 . 2 . 1
. 2 . 5 . 6 . 4 . 1
.4 . 7 . 6 . 1
. 5 . 6 . 3 . 1 . 1 . 11
1.9
. 3 . 5 ( 110 min ). 1 . 1 . 3 . 6
. 2.2 - - . 4 . 5 . 3 . 2 . 1 .
. 1.1 . 3 - - -6 . 3 . 4 . 1
. 1 . 6 . 3 - . 1 . 3 . 3 . 1 . 1
. 2 . 8 . 1 . 1 ( 250 min ). 1 . 1 . 4 . 1
. 1 . 2 . 3 . 3 . 1 . 1 - . 1 - $-\quad-\quad-.1$. 2 . 2 . 1
2.0
$\begin{array}{llllllllll}.1 & .6 & .5 & .3 & .2 & .2 & .1 & & & \\ .1 & .1 & .4 & .2 & .4 & .4 & .1 & .1 & .1 & .1\end{array}$
. 2 . 8 . 3 . 3 . 2 . 2
. 1 . 3 . 7 . 1 . 1 . 1 . 1 . 3 . 2
. 2 . 9 . 6 . 2 . 1
. 3 . 4 . 7 . 3 . 1 . 1 . 1

Total Acc. Duration(min.)

| Rainfall | 5 | 15 | 25 | 35 | 45 | 55 | 65 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |




$\begin{array}{lllllllll} & .2 & . & .7 & .8 & .4 & .2 & .1 & .1\end{array}$
2.6
. 2 . 7 . 6 . 7 . $3-.1$
. 1 . 3 . 9 . 5 . 3 . 3 . 2
$\begin{array}{llllllllll}.3 & .6 & .8 & .7 & .1 & .1 & & & \\ . & . & .4 & .1 & . & . & .1 & .3 & .8 & .2\end{array}$
$\begin{array}{llllllllllllllllll}.3 & .2 & .2 & .4 & .3 & .2 & .2 & .1 & .1 & .1 & .1 & - & - & .2 & .1 & .1\end{array}$
2.7

3.0
3.1
$\begin{array}{lllllllll}.1 & .6 & .6 & .6 & .6 & .3 & .1 & .1 \\ .2 & .9 & .7 & .5 & .3 & .2 & - & .2\end{array}$
3.2 . $24-.3 .2--.2 .8 .7 .4$
3.3
3.5
3.6
.31 .5 . 8 . 5 . 2
. 3 . 6 . 7 . 4 . 2 . 5 . 3 . 1
3.7

. 4 . 7 . 5 . 6 . 6 . 3 . 2 . 1 . 1
3.7

$$
\begin{array}{lllllllllll}
.2 & .5 & .7 & .3 & .4 & .5 & .4 & .1- & -. \\
.2 & .1 & .5 & .8 & .31 .1 & .7
\end{array}
$$

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VITA

Unal Ali Sorman was born on May 31, 1943, in Ankara, Turkey. Elementary, junior high school and high school education was received in Ankara. The author received a B.S. in Civil Engineering at the Middle East Technical University (M.E.T.U.), Ankara, Turkey, in June, 1965. In 1966, he graduated with a M.S. degree in Civil Engineering from M.E.T.U. After graduation, the author entered the University as an Assistant and, one year later was promoted to the position of Instructor. During his two year teaching career from 1966 to 1968 , he was enrolled in a Ph.D. program at the Middle East Technical University.

During the period from September 1966 to November 1966, he attended two symposiums. The first one was held in Bursa, Turkey on Agricultural Development, sponsored by UNESCO, and the second one was on Hydrology and Ground Water, sponsored by CENTO in Baghdad, Iraq. From April 1968 to August 1968, he was enrolled in the International PortGraduate Course on Hydrologic Data for Water Resources Planning, sponsored by UNESCO, in Prague, Czechoslovakia.

From the Fall quarter of 1968 through the summer quarter of 1972 the author was a Ph.D. candidate and Graduate Research Assistant in the School of Civil Engineering, Georgia Institute of Technology.

Professional organizations in which the author is currently a member of include the American Society of Civil Engineers, and the American Geophysical Union.

The author will go back to the Middle East Technical University, from which he is currently on leave, to continue his academic career in the School of Civil Engineering




[^0]:    *Small precipitation areas characterized by elliptical isohyets and high temporal and spatial intensity gradients on the ground surface are defined as cells.

[^1]:    *An objective analysis technique is a method for interpolating at regular gridpoints from observations made at random locations in space and/or time. See reference (49).

[^2]:    *The term "air mass" is applied to a portion of the atmosphere that has remained nearly stationary over an extensive area of comparatively uniform characteristics until it has acquired an approximate horizontal homogeneity of such properties as temperature, moisture, and their vertical gradients.

[^3]:    *Trace number 8 is an indication of rainfall and trace number 9 corresponds to a non-working raingage or missing data. " 0 " is an indication of no rain or non-working tracing.

[^4]:    *The time and space average of actual atmospheric currents measured from the pressure contour spacing and orientation.

[^5]:    *The term "random number" refers to a random variate selected from a uniform probability density function over the interval 0.0 to 1.0 .

[^6]:    *Storm period is a time difference between the initiation and cessation of precipitation during which the number of cell generated ranged between two and nine.

[^7]:    *The statistical characteristics built into the model were, of course, reproduced by the model. This was affirmed by comparing the characteristics of the generated data with the historical data on which the model was based.

[^8]:    *If the time interval between the two consecutive cells passing over the maximum rainfall gage is equal to greater than one hour, then the duration of rainfall for each cell was considered to be a separate record. If the time interval was less than an hour, the duration of rainfall was assumed to be a continuous record.

[^9]:    AV, Figure 37. Relation Between Maximum Amount of Rainfall Versus Average Rainfall Duration

[^10]:    *Positive sign means angle measured in counterclockwise direction from east.
    Negative sign means angle measured in clockwise direction from east.

[^11]:    *Positive sign means angle measured in counterclockwise direction from east.
    Negative sign means angle measured in clockwise direction from east.

