

ECONOMICAL COMPARISON OF A REINFORCED CONCRETE
HIGHWAY ARCH BRIDGE AND REINFORCED CONCRETE
HIGHWAY BEAM BRIDGE.

A Thesis

Submitted in partial fulfillment of the
requirements for the Degree of Master of
Science in Civil Engineering .

by

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PREFACE

The purpose of this thesis is to give the designs of a reinforced concrete highway beam bridge and reinforced concrete highway arch bridge. Also to give an economical discussion about both types of concrete highway bridges. The two bridges are to have the same spans as they are designed for the same crossing. Throughout the thesis it is aimed to put the material in such a form and layout as to help a student or an experienceless engineer to be able to follow it with ease.

The most used reference books in order to obtain necessary information for writing the thesis have been the following: " Principles of Highway Engineering, " by Willey; " Reinforced Concrete Construction, " by Hool, Volume III; " Reinforced Concrete Design, " by Sutherland and Clifford; " Handbook of Reinforced Concrete " by Arthur R. Lord; " Economics of Highway Bridge Types, " by Mo. Cullough; " Class Notes on Reinforced Concrete Arch Design, " by Prof. F. C. Snow, of the Georgia School of Technology.

Sincere appreciation is wished to express to Prof. F. C. Snow, of Georgia School of Technology, for his kind assistance and willing guidance given throughout the preparation of this thesis.

Most of the design computations are given as slide-rule results, which are believed to be accurate enough for all practical purposes.

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INTRODUCTION

The two bridges which are designed in this thesis do not refer to any particular crossing, but rather to design such types of bridges which are mostly used in the country where I come from, and where probably my whole engineering career is going to take place. Thus the major object of this thesis was to get a valuable experience in the design of such types of bridges which will be mostly benefactory in my future engineering work.

There are no wide rivers in Turkey where I live, so that the demand for the construction of long span bridges is very little. This was particularly the chief reason for adopting a short span of 60 feet for both types of bridges designed in the thesis. The concrete bridges are preferred over steel bridges in Turkey, for the simple reason that there are more concrete factories than steel mills, and thus concrete is much more abundant and relatively cheaper than steel, as compared to some other countries. Due to these facts, at least for the coming few years most of the bridges will be made of reinforced-concrete. At this stage since the two bridges were not built for a particular crossing, it was impossible to state without further investigation which of the two is the more economic bridge, and would prove to be the most suitable.

The economics of the two types of bridges is fully discussed at the end of the thesis giving all the important elements which enter to such a discussion. The following items were mostly stressed upon such as

the floor system of the two types comparing the slab, beam and girder of one type with the other, and showing how they differ in size and amount of tension and compression steel bars. Columns and an arch, which are not necessary in the beam bridge, are required in the arch bridge, whereas in the beam bridge heavier girders are used .

Important considerations are also entered in this discussion concerning the economy in this design , and construction of such types of concrete bridges are generally discussed, which concludes the thesis.

PART I

DESIGN OF THE REINFORCED-CONCRETE
BEAM BRIDGE.

SPAN = 60 FT.

DESIGN OF REINFORCED CONCRETE HIGHWAY BEAM BRIDGE

Highway bridges of reinforced concrete are built as arches, as cantilevers and as continuous or non-continuous beams. The type to be designed in the following pages is the most common of these forms namely the beam bridge. These type of bridges are considered more economical over other type of bridges up to 60 feet spans. In a beam bridge the load of the road slab is carried by beams which rest on the abutments.

The clear span of this beam bridge is taken as 60 feet, while the width of the roadway is 18 feet. Materials used are 2000 lbs. of concrete with maximum aggregate of 1 inch. Curbs are dimensioned by 9 inches wide by 12 inches above surface of paving, 2 inches of bituminous wearing surface being used for paving. The railing is to be constructed by concrete 3 feet 6 inches above road surface. H-15 loading is used in this design which gives a total load on each traffic lane composed of a uniform load of 450 lbs. per linear foot plus a single concentrated load of 21,000 lbs.

SLAB DESIGN

The slab is treated as a rectangular beam, continuous over the several longitudinal supporting beams, and with a width which is equal to the length of the bridge. For simplicity a simple strip one foot wide is used in the computations.

In this beam bridge beam are used at 7 feet 6 inches apart. As shown in Fig. 1 the width of the slab in feet is found by using the following formula:

$$W = \frac{4x}{3} + T$$

in which the axles of the wheel of the truck are parallel to the supports,

in which W = Width of strip in feet

T = Width of tire in feet, taken as one inch for each 1000 lbs. of wheel load.

x = Distance in feet from center of the nearer support to the middle point of the line of contact of the tire with the slab. For shear x will be taken as $2\frac{1}{2}$ times the effective depth of the slab.

The total moment is found by adding live load, impact and dead load moments. In order to find the dead load moment, the thickness of the slab is necessary to be assumed, and if it comes out to be too big, corrections are made. Then the total shear is found by adding the live load, dead load and impact shears. The depth of the slab may then be found by using both total shear and total moment in their respective formulas and selection is given to the larger depth found which shows whether shear or moment governs the design. Then using the corrected depth the whole design

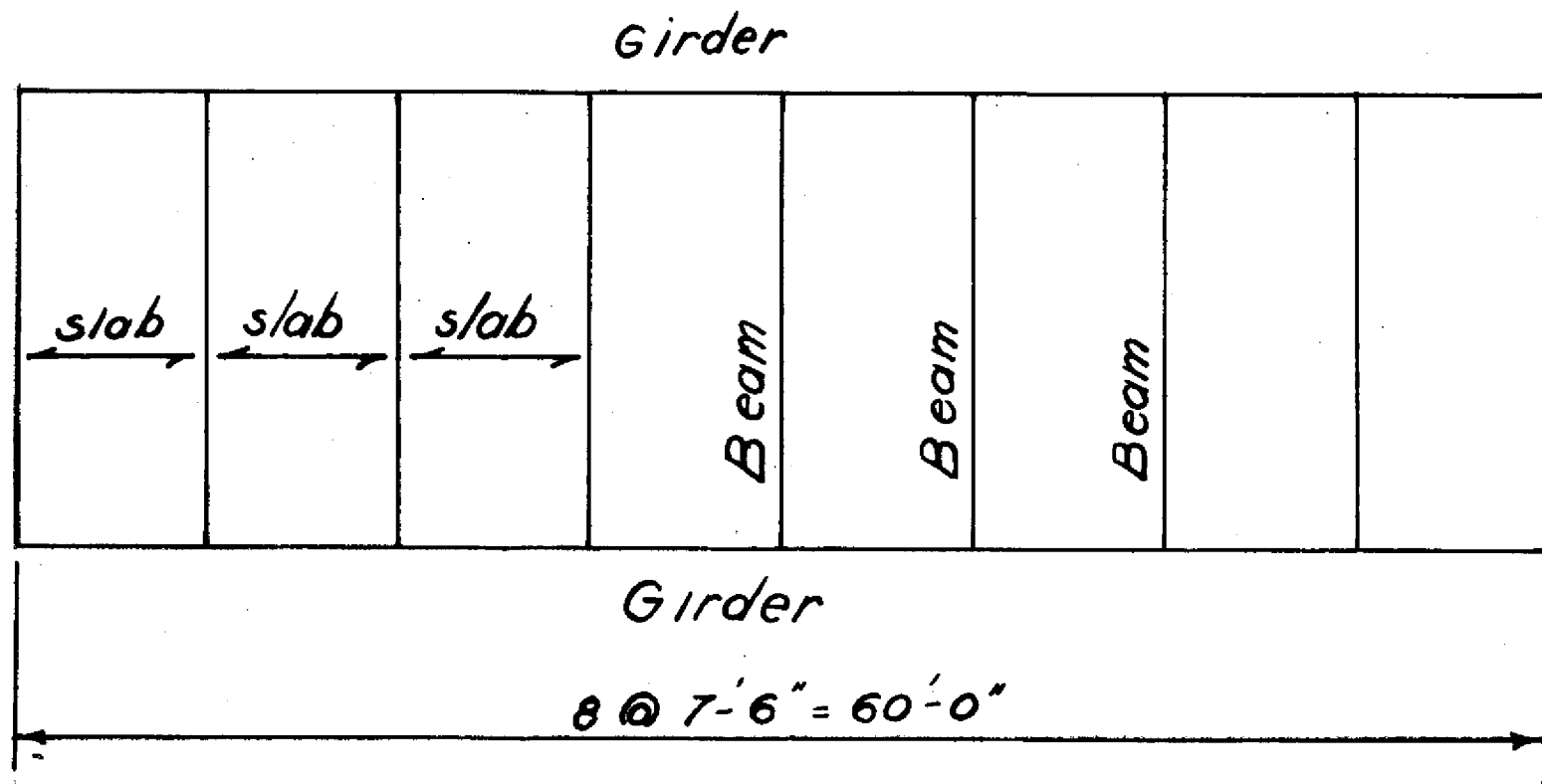


Fig. 1

*Floor system
of beam bridge*

is repeated until the results check with the assumed dimensions.

The area of the steel required is obtained by the following formula

$$A_s = \frac{M}{f_s j d} \text{ in which}$$

A_s = Area of steel in square inches.

M = Total moment.

f_s = 20,000, fiber stress for tension.

j = 7/8;

d = Depth of the slab.

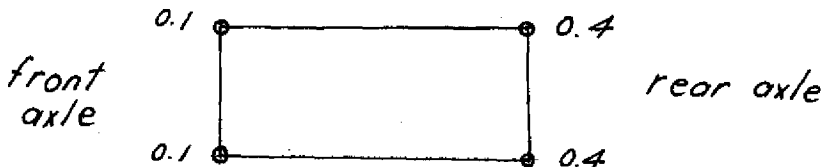
After the area of steel required is found, the number, size and spacing of the steel bars are determined.

Computations

$$X = L/2 = 7.5/2 = 3.75 \text{ ft.}$$

where L is the distance from beam to beam.

Distribution of Loads for Truck (H-15)



It is assumed that the front axle takes .2 of the total of the truck while the rear axle takes .8 of the load. Thus each front wheel carries only .1 of the load of the truck, which is

Load Carried by Each Front Wheel

$$.1 \times 15 = 1.5 \text{ tons.}$$

Load Carried by Each Rear Wheel

$$.4 \times 15 = 6.0 \text{ tons.} = 12,000 \text{ lbs.}$$

Width of Tire

$$T = \frac{12,000}{1000} = 12 \text{ in.} = 1 \text{ ft.}$$

$$W = \frac{4\pi}{3} \cdot T = \frac{4}{3} (3.75) \cdot 1 = 6 \text{ ft.}$$

Moment

Assume 15" Beams

Clear Span of Slab

$$7.50 - 1.25 = 6.25 \text{ ft.}$$

Live Load

$$P = 12,000/6 \text{ lbs. per ft. of span}$$

Live Load Moment

$$1/4 PL = 1/4 \times 12,000/6 \times \frac{6.25}{8/16} = 2,500 \text{ 'lbs.}$$

Impact Moment

$$30\% \text{ Live Load Moment} = .30 \times 2500 = 750 \text{ 'lbs.}$$

Dead Load

Bituminous wearing surface of 2" of thickness is taken as

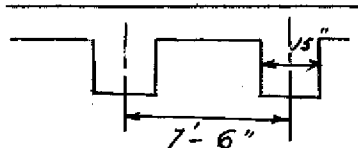
$$25 \text{ lbs./sq.ft.}$$

Assume 8 in. of slab

Concrete Weight = 150 lbs./cu.ft.

Weight of Slab = $8/12 \times 150 = 100 \text{ lbs./sq.ft.}$

Total Dead Load = $100 \cdot 25 = 125 \text{ lbs./sq.ft.}$



Dead Load Moment

$$125 \times (6.25)^2 \times \frac{1}{8} \times \frac{8}{16} = 490 \text{ 'lbs.}$$

Total Moment

$$2500 \cdot 750 \cdot 490 = 3,740 \text{ 'lbs.}$$

Shear

W for Shear = W for Moment = 6 ft.

Live Load Shear

$$12,000/6 \times \frac{1}{2} = 1000 \text{ lbs.}$$

Impact Shear

$$.30 \times 1000 = 300 \text{ lbs.}$$

Dead Load Shear

$$125 \times 6.25 \times \frac{1}{2} = 390 \text{ lbs.}$$

Total Shear

$$= 1690 \text{ lbs.}$$

Depth (d)

For Moment

$$d = \sqrt{\frac{3,740 \times 12}{12 \times 131}} = 5.35 \text{ in.}$$

For Shear

$$d = \frac{1690}{12 \times 7/8 \times 40} = 4.00 \text{ in.}$$

Use 7" Slab (d = 5.5")

Correction

Dead Load Moment

$$2" \text{ Surface} : 25 \text{ lbs./sq.ft.}$$

$$7" \text{ Slab} : 87.5 \text{ \# /sq.ft.}$$

$$112.5 \text{ \# /sq.ft.} \times (6.25)^2 \times 1/8 \times 8/10 = 440 \text{ \#}$$

Total Moment

$$2,500 \text{ \#} / 750 \text{ \#} / 110.0 = 3,690 \text{ 'lbs.}$$

Dead Load Shear

$$112.5 \times 6.25 \times \frac{1}{2} = 350 \text{ lbs.}$$

Total Shear

$$1000 \div 300 \div 350 = 1,650 \text{ lbs.}$$

Depth (d)

For Moment

$$d = \sqrt{\frac{3,690 \times 12}{12 \times 131}} = 5.31 \text{ in.}$$

For Shear

$$d = \frac{1650}{12 \times 7/8 \times 40} = 3.93 \text{ in.}$$

Use 7" Slab (d = 5.5")

Steel

Area of Steel

$$A_s = \frac{M}{f_s j d} = \frac{3,690 \times 12}{20,000 \times 7/8 \times 5.5} = .46 \text{ sq.in./ft. width.}$$

Try $\frac{1}{2}$ " Round Bars

$$A = .196 \text{ sq.in.}$$

$$\text{Spacing} = \frac{.196}{.0383} = 5.1" \text{ use } 5"$$

$$v = V/bjd = \frac{1650}{12 \times 7/8 \times 5.5} = 28.6 \text{ lbs./in}^2.$$

$$u = vb/\zeta_o = \frac{28.6 \times 5}{1.57} = 91 \text{ lbs./in}^2. \quad (\text{less than } 100) \\ \text{O.K.}$$

Use $\frac{1}{2}$ " Round Bars @ 5" (Alternate Bars Bent)

BEAM DESIGN (INTERIOR BEAM)

In designing the beam, loads on it must be put in such a position as to give maximum moment and maximum shear. This loading is shown in the Fig. 2, where two rear wheels of each of two trucks fall on the beam. In maximum moment, computation is carried both for maximum positive and maximum negative moment. It is also found out whether maximum moment or maximum shear governs the design. In finding the dead load of the beam assumption is necessary for its size and is corrected later on, if necessary. The number and size of bars are determined by the area of steel required; checking for bond stress is done by the following formula:

$$u = \frac{V}{\sum_0 j d} \text{ in which,}$$

$$u = \text{Unit bond, and should not exceed } 100 \text{ lbs./in.}^2$$

$$V = \text{Maximum shear.}$$

$$\sum_0 = \text{Total perimeter of bars in inches.}$$

$$j = 7/8;$$

$$d = \text{Depth of beam in inches.}$$

It will be found that choice was given to two layers of bars, 6 on the top and 7 on the bottom layer. As trial calculations show, the use of more number of bars will make it necessary to use three layers of rods, which is not desirable. The requirements as to bar clearances are illustrated by the figure used later in the design. The minimum center to center distance between parallel bars being $2\frac{1}{2}$ times the diameter for round bars, or three times the side dimension for square bars; and is in no case the clear spacing between bars is taken less than 1 inch. The bending up or the bending down of the rods are calculated by the following formula:

$$d_1^2 / d_2^2 = a/b$$

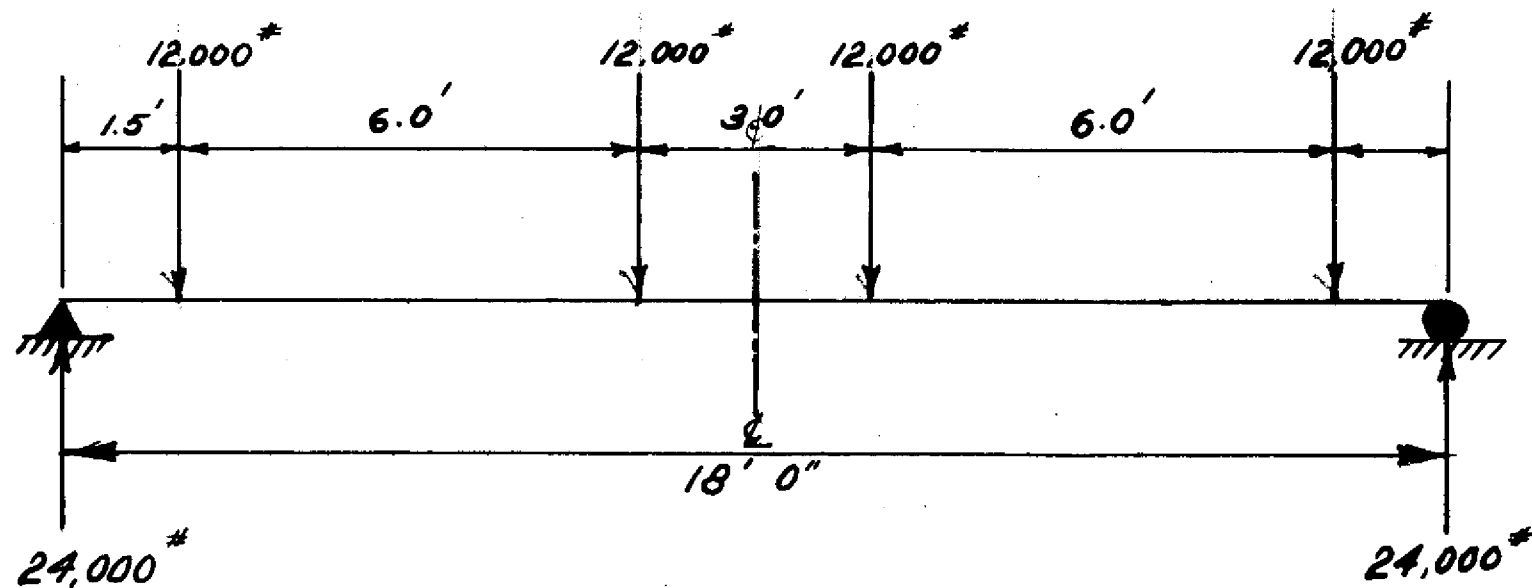


Fig. 2

Interior beam

The use of this formula is readily seen in Fig. 4 and Fig. 5 and thus needs no further explanation.

In finding out the stirrup requirements the maximum shear at the center line and the maximum shear at the support are calculated and a straight line variation is drawn between the two, because any possible shear curve due to any loading will fall within the shear curve thus drawn and accordingly the reinforcement is proportioned by its use. The total stress in the stirrup is computed by the following formula:

$$S = A_s f_s$$

in which S = Strength of stirrup.

$$f_s = 16,000 \text{ lbs./sq.in.}$$

The spacing between the stirrups are found by the following formula:

$$s = S/v'b$$

in which s = Spacing in inches.

$$v' = \text{Total unit shear at the section - ultimate unit shear of concrete which is taken as } 40 \text{ lbs./sq.in.}$$

The size of stirrups are taken as 7/8 inches in diameter. The spacing and the number of stirrups used are shown clearly in Fig. 6.

Computations

Moment

Live Load Moment

$$24,000 \times 9 - 12,000 \times 7.5 - 12,000 \times 1.5 = 1,300,000 \text{ "}\#$$

Dead Load

$$2" \text{ Surface} = 25 \text{ lbs./ft}^2$$

$$7" \text{ Slab} = 87.5 \text{ "}$$

$$= 112.5 \text{ lbs./ft}^2$$

$$112.5 \times 7.5 = 845 \text{ lbs./ft.}$$

$$\text{Assumed Stem} = 255 \text{ lbs./ft.}$$

$$w = 1,100 \text{ lbs./ft.}$$

Dead Load Moment (Positive)

$$M = 1/8 w l^2 = 1/8 \times 1100 \times (18)^2 \times 12 = 535,000 \text{ "lbs.}$$

Dead Load Moment (Negative)

$$M = 1/20 w l^2 = 1/20 \times 1100 \times (18)^2 \times 12 = 214,000 \text{ "lbs.}$$

Impact Moment

$$.30 \times 1,300,000 = 390,000 \text{ "lbs.}$$

Total Moment (Positive)

$$1,300,000 + 390,000 - 535,000 = 2,225,000 \text{ "lbs.}$$

Total Moment (Negative)

$$8/20 \times 2,225,000 = 890,000 \text{ "lbs.}$$

Shear

$$\text{Live Load Shear} = 24,000 \text{ lbs.}$$

$$\text{Impact Shear} = 7,200 \text{ lbs.}$$

Dead Load Shear

$$\frac{1}{2} \times 1100 \times 18 = 9,900 \text{ lbs.}$$

$$\text{Total Maximum Shear} = 41,100 \text{ lbs.}$$

Depth (d)

For Positive Moment

$$d = \sqrt{\frac{2,225,000}{131 \times 64}} = 17.8 \text{ in.}$$

$$b = \frac{1}{4} \times 12 \times 18 = 54"$$

For Negative Moment

$$d = \sqrt{\frac{890,000}{157 \times 16}} = 18.8 \text{ in.}$$

Assume 16" Beam

For Shear

$$d = \frac{41,100}{120 \times 7/8 \times 18} = 24.5 \text{ in.} \quad \text{Shear Governs.}$$

Try $b = 14"$

$$d_v = \frac{41,100}{120 \times 7/8 \times 14} = 28 \text{ in.}$$

Use $14" \times 30"$ Beam ($d = 28"$)

Correction

Dead Load Moment

$$\text{Surface } \nearrow \text{ Slab} = 845 \text{ lbs./ft.}$$

$$\text{Stem} = \frac{30 - 7 \times 14/12 \times 160}{12} = 335 \text{ lbs./ft.}$$

$$w = 1180 \text{ lbs./ft.}$$

Maximum Positive Moment

$$1,360,000 \nearrow 390,000 \nearrow 1/8 \times 1180 \times (18)^2 \times 12 = 2,265,000 \text{ "lb.}$$

Maximum Negative Moment

$$8/20 \times 2,265,000 = 905,000 \text{ "lbs.}$$

Maximum Dead Load Shear

$$\frac{1}{2} \times 1180 \times 18 = 10,600 \text{ lbs.}$$

Maximum Total Shear

$$10,600 \nearrow 7,200 \nearrow 24,000 = 41,800 \text{ lbs.}$$

$$d_v = \frac{41,800}{120 \times 7/8 \times 14} = 28.4 \text{ in.}$$

Steel

Area for Positive Moment

$$A_s = \frac{2,265,000}{20,000 \times 7/8 \times 28} = 4.61 \text{ sq.in.}$$

Area for Negative Moment

$$A_s = \frac{905,000}{20,000 \times 7/8 \times 28} = 1.85 \text{ sq.in.}$$

Use $7 - 3/4"$ Round Bar Steel $A_s = 3.1 \text{ sq.in.}$

$$\text{Total } A_s = 4.94 \text{ sq.in.}$$

Bond

$$u = \frac{41,800 \times 8}{16.5 \times 28 \times 7} = 103 \text{ lbs./in.}^2 \quad \text{O.K.}$$

Bent Up

$$d_1^2 / (108)^2 = .61 / 4.94$$

$$d_1 = 38 \text{ in.}$$

$$= 108 - 38 = 70 \text{ in. from end of beam.}$$

$$d_2 = \sqrt{\frac{1.23 \times (108)^2}{4.94}} = 54 \text{ in.}$$

$$= 108 - 54 = 54 \text{ in. from end of beam.}$$

$$d_3 = \sqrt{\frac{1.84 \times (108)^2}{4.94}} = 66 \text{ in.}$$

$$= 108 - 66 = 42 \text{ in. from end of beam.}$$

Bend up 2 bars at a distance of 70 in. from end of beam.

" " 2 " " " " " 54 in. " " " "

" " 2 " " " " " 42 in. " " " "

Bent Down

$$d_1 = \sqrt{\frac{13 \times (108)^2}{15}} = 100.5 \text{ in.}$$

$$= 108 - 100.5 = 7.5 \text{ in. from end.}$$

$$d_2 = \sqrt{\frac{11 \times (108)^2}{15}} = 92.5 \text{ in.}$$

$$= 108 - 92.5 = 15.5 \text{ in. from end.}$$

$$d_3 = \sqrt{\frac{9 \times (108)^2}{15}} = 83.5 \text{ in.}$$

$$= 108 - 83.5 = 24.5 \text{ in. from end.}$$

Bend down 2 bars at a distance of 24.5 in. from end of beam.

" " 2 " " " " " 15.5 " " " "

" " 2 " " " " " 7.5 " " " "

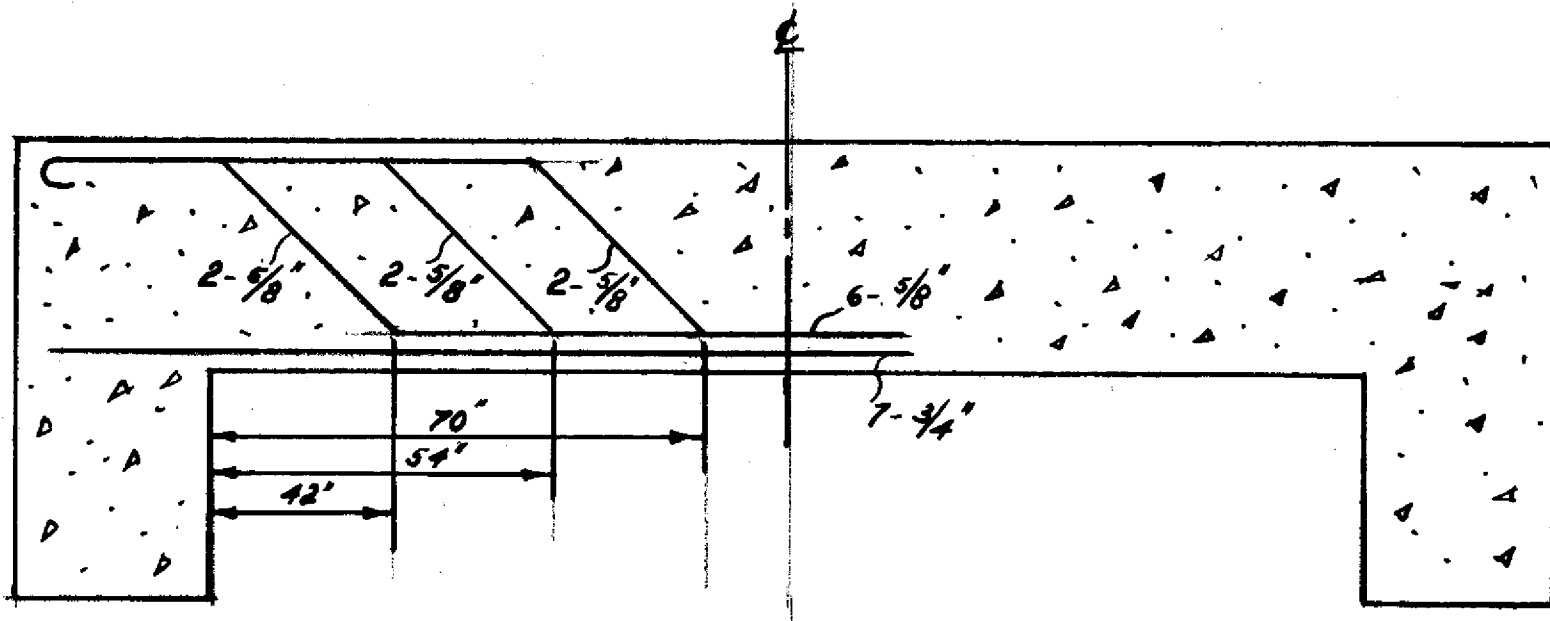


Fig 3(0)

Steel in beam

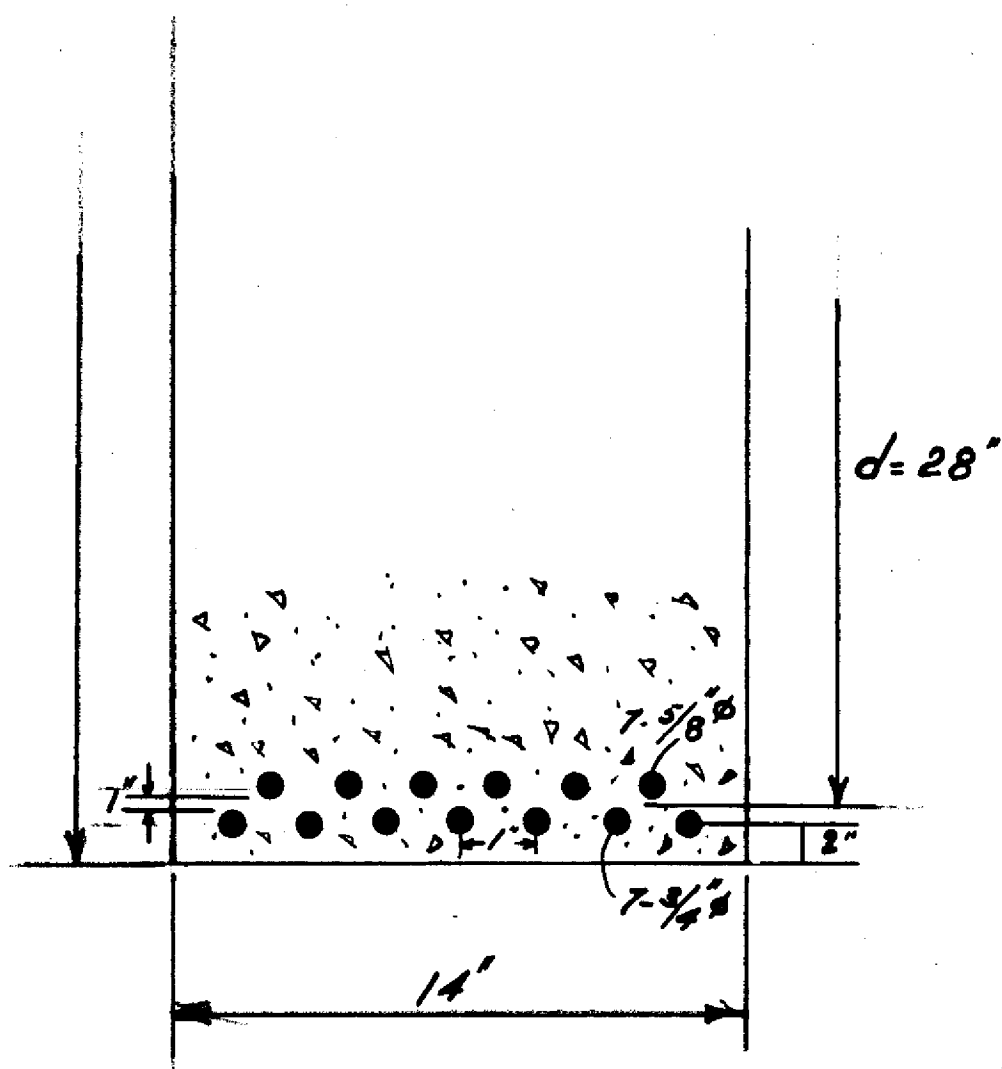


Fig 3 (b)

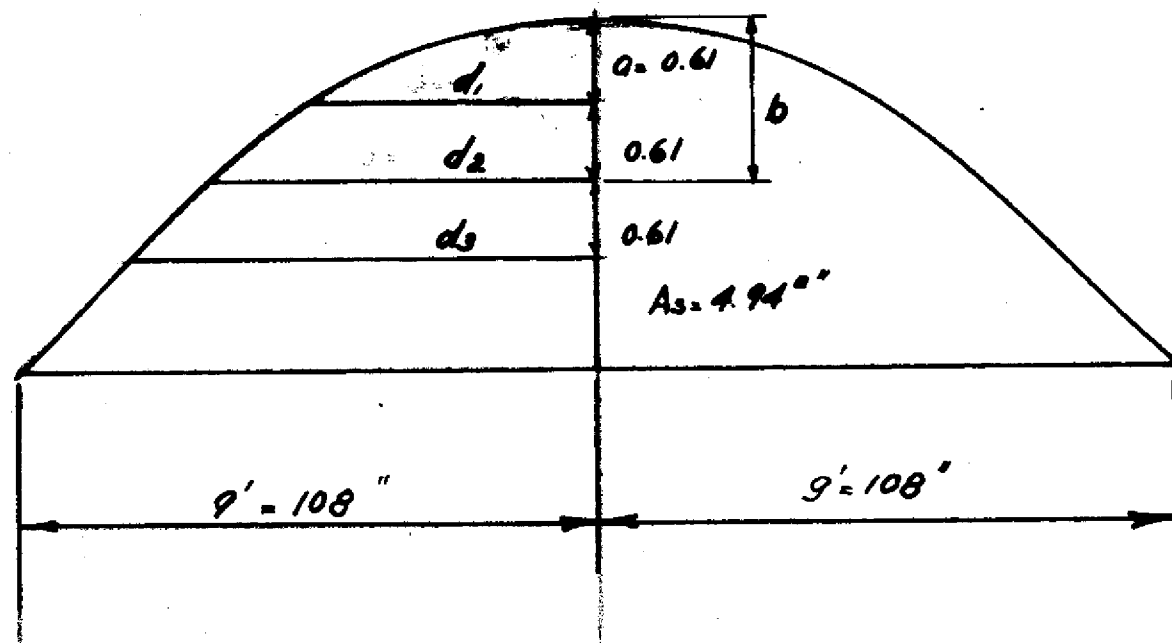


Fig 4

Bent up in beam

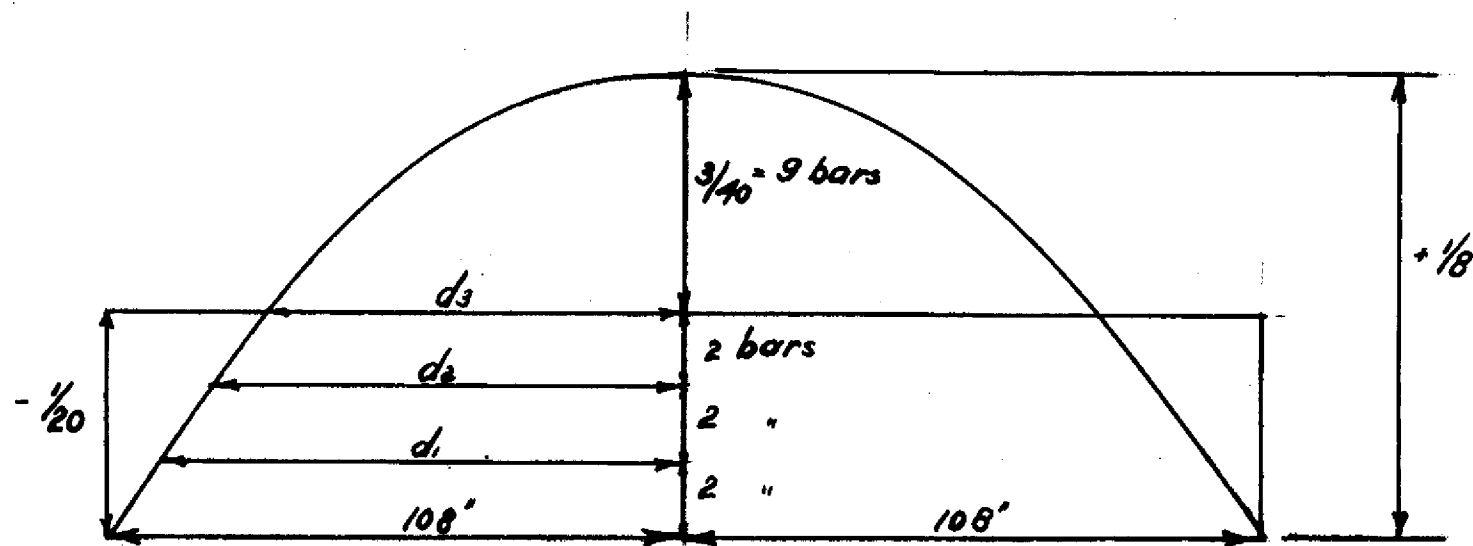


Fig 5

Bent down in beam

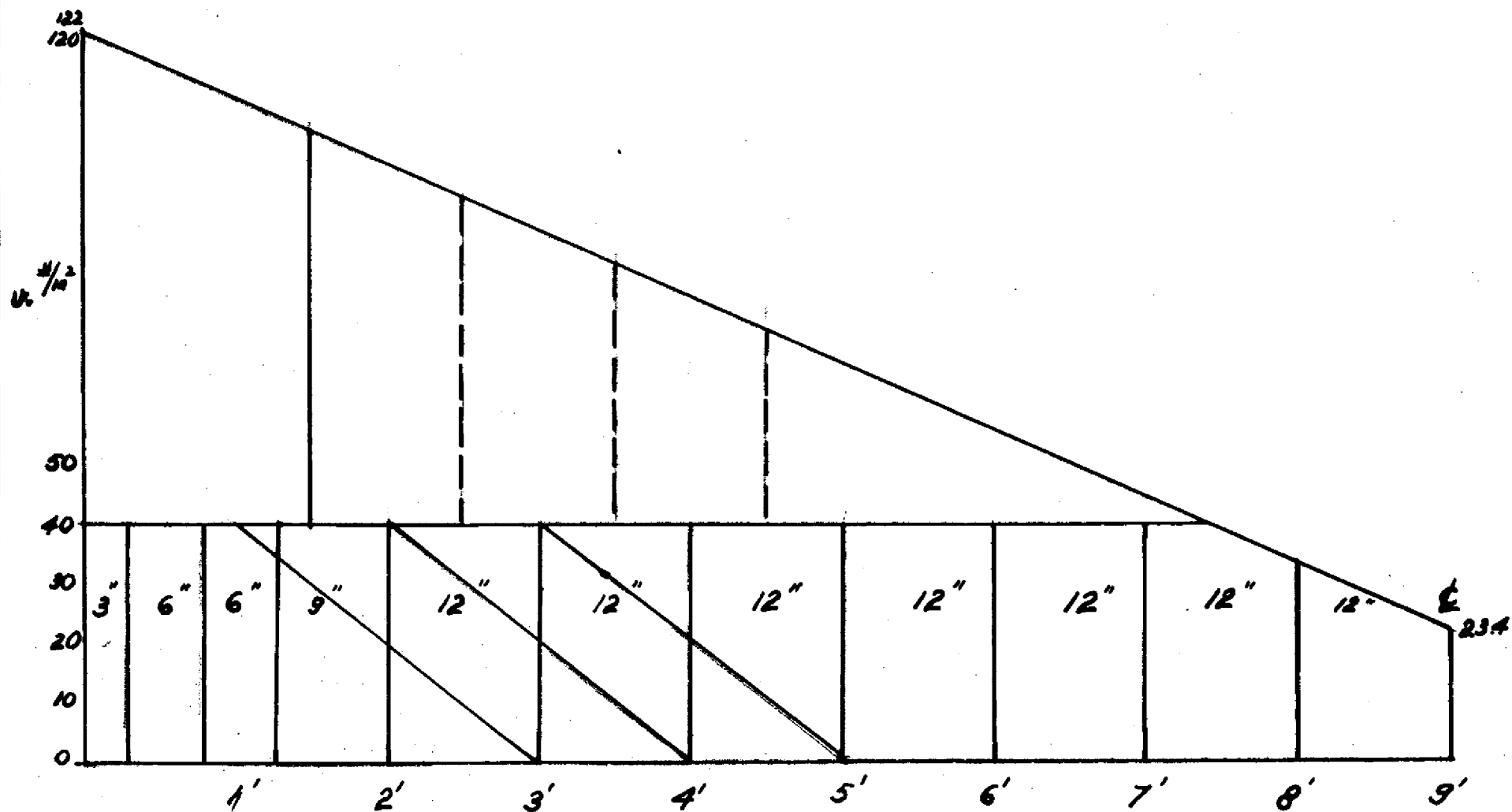


Fig 6

Stirrup spacing
in beam

END BEAM DESIGN

In the design of the end beam, the same method is followed as used in the design of the interior beam, the only difference between the two being that the interior beam is a T beam while the end beam is an L beam.

Computations

Assume 14" Beam

Moment

$$b = 7.5/2 \times 12 \div 7 = 52"$$

Live Load Moment

$$108,000 \text{ 'lbs.} \times 12 = 1,300,000 \text{ "lbs.}$$

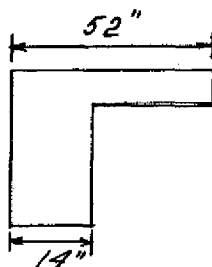
Impact Moment

$$60\% \times 1,300,000 = 780,000 \text{ "lbs.}$$

Dead Load

$$2" \text{ Surface} = 25 \text{ lbs./ft.}^2$$

$$7" \text{ Slab} = 87.5 \text{ lbs./ft.}^2$$



$$112.5 \text{ lbs./ft.}^2 \times 52/12 = 490 \text{ lbs./ft.}$$

Assume Stem

$$= 310 \text{ lbs./ft.}$$

w =

$$800 \text{ lbs./ft.}$$

Dead Load Moment

$$1/8 \times 800 \times (18)^2 \times 12 = 390,000 \text{ "lbs.}$$

Total Moment (Positive)

$$1,300,000 \div 780,000 \div 390,000 = 2,470,000 \text{ "lbs.}$$

Total Moment (Negative)

$$2,470,000 \times 8/20 = 987,000 \text{ "lbs.}$$

Shear

$$\text{Live Load Shear} = 24,000 \text{ lbs.}$$

$$\text{Impact (60\%)} = 14,400 \text{ lbs.}$$

Dead Load Shear

$$\frac{1}{2} \times 870 \times 18 = 7,850 \text{ lbs.}$$

Maximum Shear

$$7,850 \div 24,000 \div 14,400 = 45,600 \text{ lbs.}$$

Depth

For Positive Moment

$$d = \sqrt{\frac{2,470,000}{131 \times 52}} = 19.1 \text{ in.}$$

For Negative Moment

$$d = \sqrt{\frac{987,000}{157 \times 14}} = 21.2 \text{ in.}$$

For Shear

$$d = \frac{45,600}{120 \times 7/8 \times 14} = 31 \text{ in.}$$

Try 14" x 33" Beam

Correction

Moment for Dead Load

$$\text{Surface } \div \text{ Slab} = 490 \text{ lbs./ft.}$$

$$\text{Stem} = \frac{33 - 7}{12} \times \frac{14}{12} \times 150 = 380 \text{ lbs./ft.}$$

$$w = 870 \text{ lbs./ft.}$$

$$M = 1/8 \times 870 (18)^2 \times 12 = 423,000 \text{ "lbs.}$$

Total Moment (Positive)

$$423,000 \div 1,300,000 \div 780,000 = 2,503,000 \text{ "lbs.}$$

Total Negative Moment

$$2,503,000 \times 8/20 = 1,000,000 \text{ "lbs.}$$

Dead Load Shear

$$\frac{1}{2} \times 870 \times 18 = 7,850 \text{ lbs.}$$

Total Shear

$$24,000 \div 14,400 \div 7,850 = 46,250 \text{ lbs.}$$

$$d_v = \frac{46,250}{120 \times 7/8 \times 14} = 31.5 \text{ in.}$$

Use 14" x 35" Beam. (d = 32")

Steel

Area of Steel for Positive Moment

$$A_s = \frac{2,503,000}{20,000 \times 7/8 \times 32} = 4.48 \text{ sq.in.}$$

Area of Steel for Negative Moment

$$A_s = \frac{1,000,000}{20,000 \times 7/8 \times 32} = 1.79 \text{ sq.in.}$$

<u>Use</u>	7 - 3/4" Round Bars (Stirrup)	3.1 sq.in.
	6 - 5/8" Round Bars (Bent)	1.84 sq.in.
	Total A_s	<u>4.94 sq.in.</u>

Bond

$$u = \frac{46,250}{16.5 \times 32 \times 7/8} = 100 \text{ lbs./in.}^2 \quad \text{O.K.}$$

For the Bend Up and Bend Down of bars for the end beam use same as the interior beam.

Stirrups

$$v = \frac{46,250}{14 \times 7/8 \times 32} = 118 \text{ lbs./in.}^2$$

$$v' = 118 - 40 = 78 \text{ lbs./in.}^2$$

Maximum Shear at Center Line of Beam = 8,000 lbs.

$$v = \frac{8,000}{14 \times 7/8 \times 32} = 20.4 \text{ lbs./in.}^2 \text{ at C.L.}$$

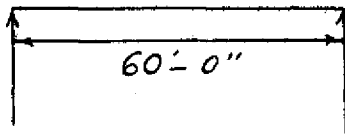
Use the same detailing as for interior beams, because maximum unit stresses at the support and the center line are just a little less than the ones used in the beam, thus the same detailing makes the design on the safe side.

GIRDER DESIGN

The design of the girder is very similar to the design of the end beam both being in L shape. The maximum moment and shear are found by the usual methods already explained for beam design. Assumption for the dead weight of the girder is first made and corrected later on. In the total dead load calculations, dead loads of the beam are transformed from concentrated loads to uniform load by dividing half the weight of a beam by the clear span between beams which simplifies the work a great deal and is also accurate enough for practical purposes. Curb and railing loads are also included in the totalling of dead loads.

Steel and stirrup designs are similar to the ones used in beam design. Fig. 7 and 8 show clearly the detailing of the bars and stirrups.

Computations



Moment

Dead Load

$$2'' \text{ Surface} = 25 \text{ lbs./ft.}^2$$

$$7'' \text{ Slab} = 87.5 \text{ "}$$

$$= 112.5 \text{ "}$$

$$112.5 \times 9 = 1,010 \text{ lbs./ft.}$$

$$\text{Beam : } 24/12 \times 14/12 \times 9 \times 150 \times 1/7.5 = 420 \text{ "}$$

$$\text{Curb : } 9/12 \times 12/12 \times 150 = 110 \text{ "}$$

$$\text{Railing : } 3.5 \times 150 = 530 \text{ "}$$

$$\text{Girder : Assume} = 1,530 \text{ "}$$

$$w = 3,600 \text{ lbs./ft.}$$

Dead Load Moment

$$1/8 \times 3600 \times (60)^2 \times 12 = 19,500,000 \text{ "lbs.}$$

Live Load Moment

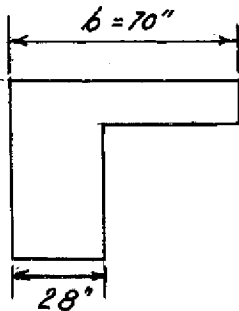
$$1/8 \times 450 \times (60)^2 \times 12 \quad \neq \quad \frac{1}{4} \times 21,000 \times 60 \times 12 = 6,200,000 \text{ "lbs.}$$

Impact Moment

$$30\% \times 6,200,000 = 1,860,000 \text{ "lbs.}$$

Total Moment

$$19,500,000 \neq 6,200,000 \neq 1,860,000 = 27,560,000 \text{ "lbs.}$$



Assume $b' = 28''$

$$b = 6 \times 7 \neq 28 = 70''$$

$$= 1/12 \times 60 \times 12 = 88''$$

Shear

Dead Load Shear

$$\frac{1}{2} \times 3600 \times 60 = 108,000 \text{ lbs.}$$

Live Load Shear

$$\frac{1}{2} \times 450 \times 60 \neq 21,000 = 34,500 \text{ lbs.}$$

Impact Shear

$$30\% \times 34,500 = 10,300 \text{ lbs.}$$

Total Shear

$$= 152,800 \text{ lbs.}$$

Depth (d)

For Moment

$$d = \frac{27,560,000}{131 \times 70} = 54.8 \text{ in.}$$

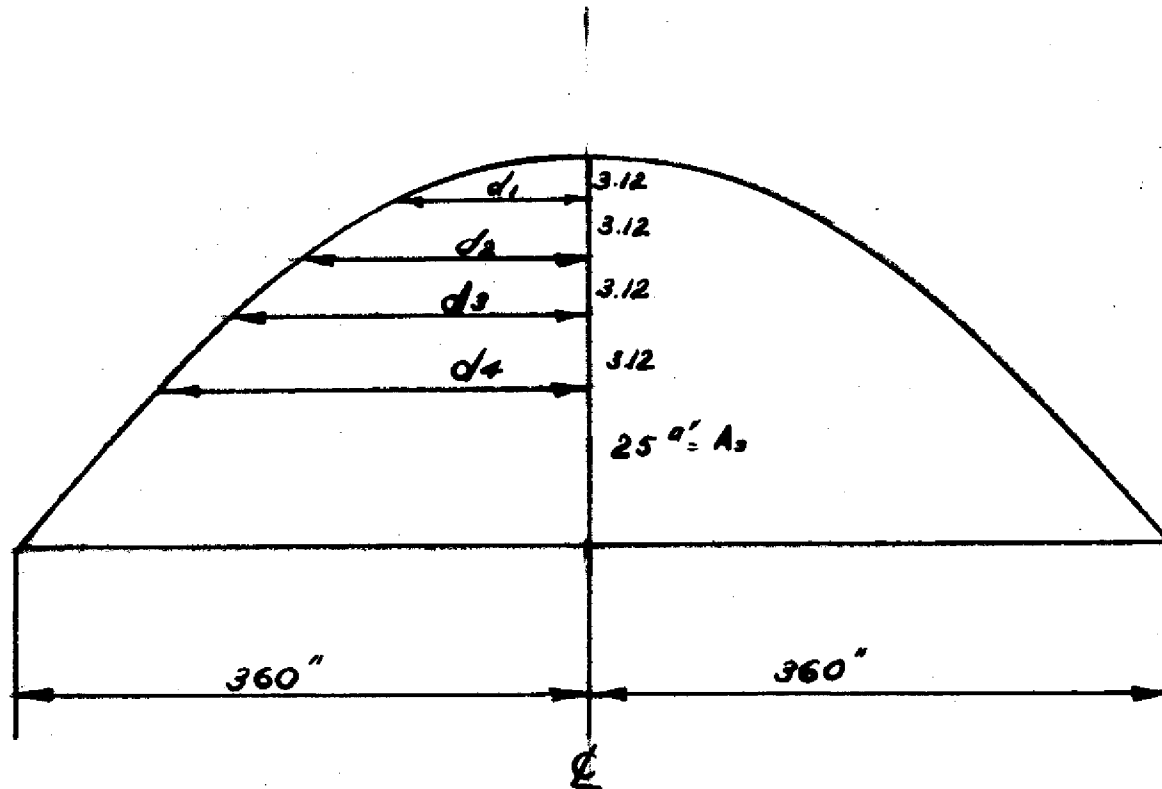


Fig 7

Bent up in Girder

For Shear

$$d = \frac{152,800}{120 \times 7/8 \times 28} = 52 \text{ in.}$$

Moment Governs

Try 28" x 59" Girder (d = 55")

True Dead Load of Girder

$$28/12 \times 59/12 \times 150 = 1720 \text{ lbs./ft.}$$

$$w = 3600 - 1530 / 1720 = 3790 \text{ lbs./ft.}$$

Use 28" x 66" Girder (d = 63")

Steel

$$A_s = \frac{27,560,000}{20,000 \times 7/8 \times 63} = 25 \text{ sq.in.}$$

Use 8 - 1 $\frac{1}{4}$ " Square Bars St. (Lower Layer).

8 - 1 $\frac{1}{4}$ " Square Bars Bt. (Upper Layer).

Bond

$$u = \frac{152,800}{40 \times 7/8 \times 63} = 69.5 \quad \begin{matrix} \text{(less than 100)} \\ \text{O. K.} \end{matrix}$$

Bend Up

$$d_1 = \sqrt{\frac{2 \times (360)^2}{16}} = 128" \text{ from C. L.}$$

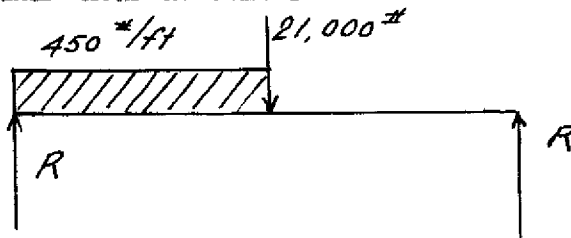
$$d_2 = \sqrt{\frac{4 \times (360)^2}{16}} = 180" \quad " \quad "$$

$$d_3 = \sqrt{\frac{6 \times (360)^2}{16}} = 220" \quad " \quad "$$

$$d_4 = \sqrt{\frac{8 \times (360)^2}{16}} = 256" \quad " \quad "$$

Maximum Shear

Maximum Shear at Center Line



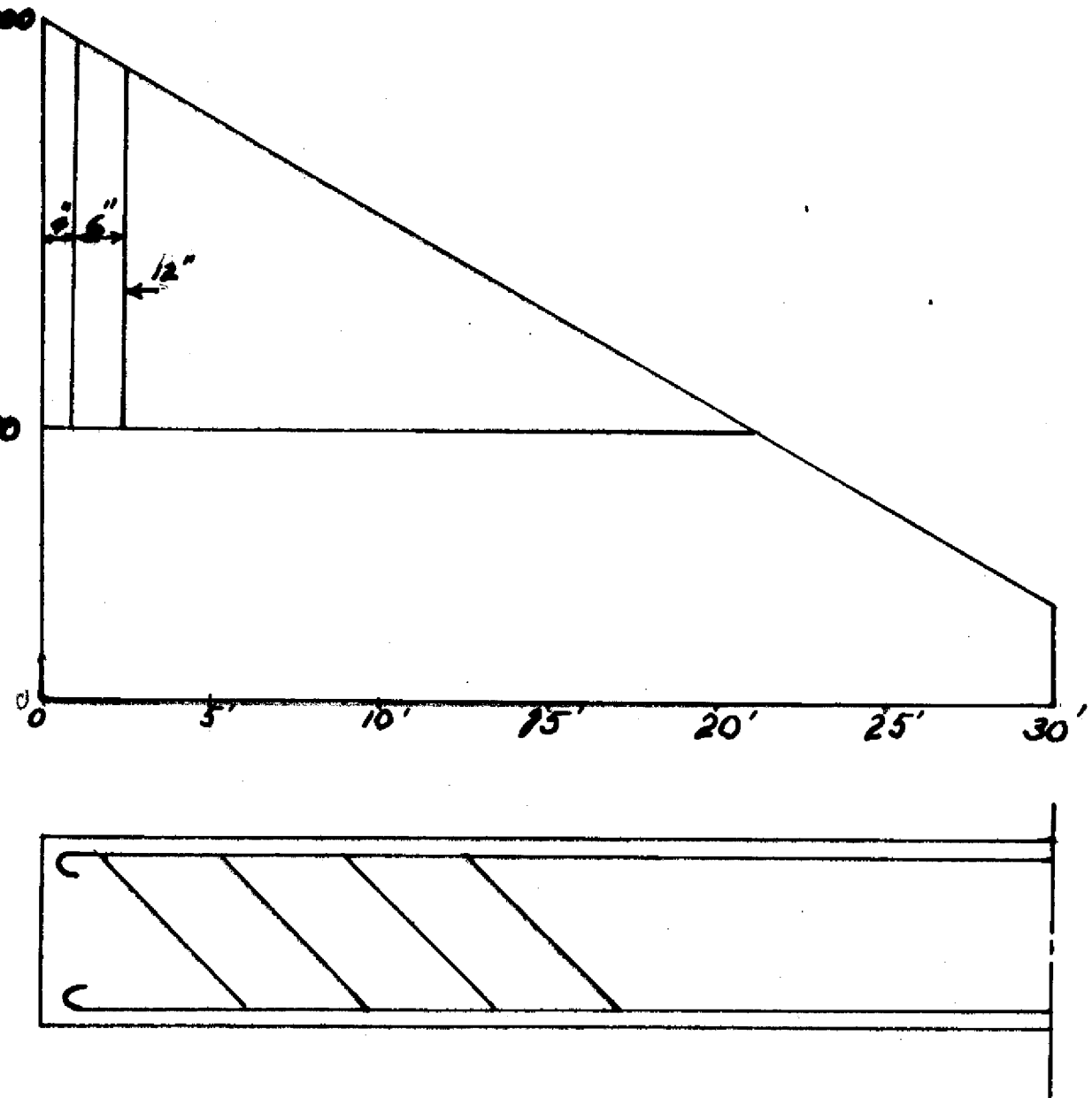


Fig 8

Stirrup
in Girder

$$R = \frac{21,000}{2} \times (450 \times 30) \times 1/60 \times 30/2 = 13,880 \text{ lbs.}$$

$$v = \frac{13,880}{28 \times 7/8 \times 63} = 9 \text{ lbs./in.}^2$$

$$v' = \frac{152,800}{28 \times 7/8 \times 63} - 40 = 59 \text{ lbs./in.}^2$$

Bent Bars

$$s = \frac{16,000 \times 3.12 \times 2^{\frac{1}{2}}}{54 \times 28} = 46.5 \text{ in.} \quad \text{Use } 3\frac{1}{2}' \text{ for all Bent Bars}$$

End Spacing

$$s = \frac{16,000 \times .39}{56 \times 28} = 4 \text{ in.}$$

Maximum Spacing

$$.6 \times 63 = 37.8 \text{ in.}$$

$$s = \frac{16,000 \times .39}{15 \times 28} = 14.8 \text{ in.} \quad \text{Use } 12''.$$

END BEARING DESIGN

End bearings are necessary at one end of the bridge for preventing high temperature stresses and undesirable crackings, thus provision is made for free expansion and contraction of the structure with temperature changes. A rocker is designed and shown in Fig. 9 placed between steel bearings plates,, proportioned to bring the bearing stresses on the concrete within the given limit of 500 lbs. per square inch. The bearing of the cast iron rocker on steel is limited to $300 \frac{d}{\text{in.}}$ lbs. per inch of length, where d is the diameter of the rounded surface of contact.

Computation

$$\text{Fixed End : } f_b = 500 \text{ lbs./in.}^2$$

$$A = \frac{152,800}{500} = 305 \text{ sq.in.}$$

$$28" \times 36" = 1010 \text{ sq.in.}$$

Use 3' Abutments

Expansion End : C. I. Rocker

Allowable bearing of Rocker on Steel Plate

$$f_b = 300 \text{ D per inch length}$$

$$\text{L. D.} = \frac{152,800}{300} = 510$$

$$D = 24 \text{ in.}$$

$$L = 22 \text{ in.}$$

Expansion (Assuming 80° F. in difference of temperature)

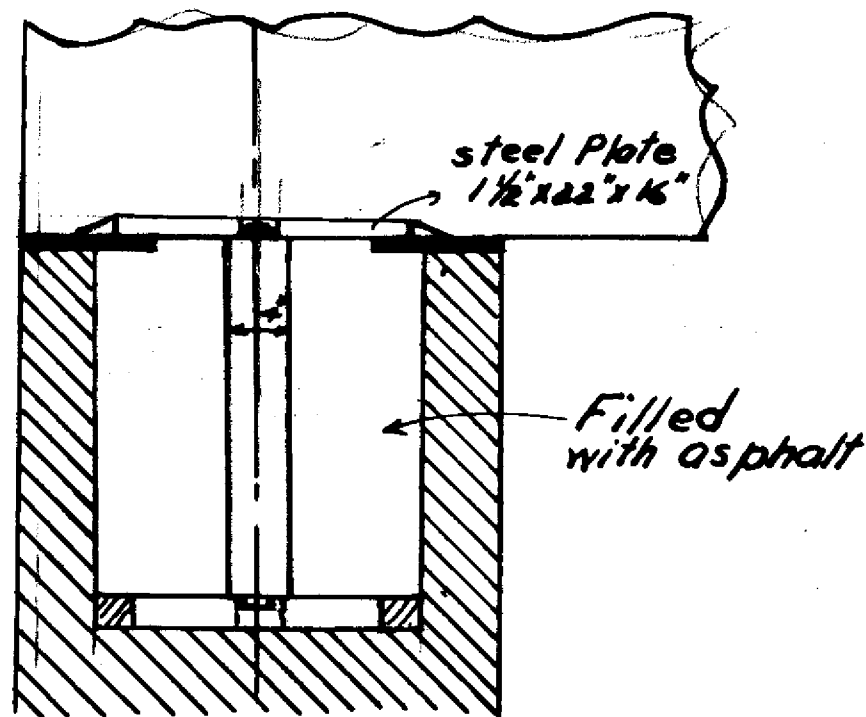
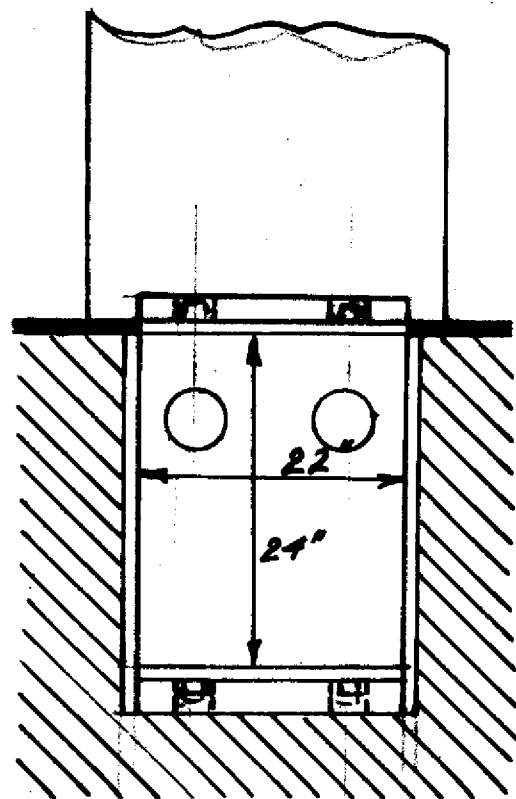
$$80 \times .0000065 \times 63 \times 12 = .394 \text{ in.}$$

Plates $22" \times 16" = 352$ (greater than 305)

Rocker Thickness

$$f_b = \frac{152,800}{22 \times 4} = 1720 \text{ lbs./in.}^2 \quad \text{O.K.}$$

See Fig. 9 for Rocker.



Rocker
Fig 9

PART II

DESIGN OF THE REINFORCED-CONCRETE
ARCH BRIDGE.

SPAN = 60 FT.

RISE = 10 FT.

DESIGN OF CONCRETE ARCH BRIDGE

Ordinary way of construction of a concrete arch with fixed ends is done by the elastic theory, because it is statically indeterminate. The arch ring is nothing but a curved beam and is so considered.

In arches made of stone, it is essential that the line of pressure of any possible loading should pass through the middle third of each joint of the arch ring, in order to avoid a tendency for any joint to open. But in arches made of concrete, the structure considered may be made monolithic and thus be capable of withstanding tension, which means that the line of pressure may pass outside of the middle third without endangering the structure. An arch being mostly in compression, reinforcing it with steel plays very little part in its strength. However it is customary to use reinforcement for some distance at least, near both upper and lower surfaces and carry both rows of steel throughout the entire span, thus eliminating any possible failure due to inadequate provision for tensile stresses. In order to prevent buckling, the upper and lower reinforcement is tied together. There are several methods to design a hinge-less arch. The method used here is taken from the "Bureau of Public Roads" publications "Public Road, Vol VIII Nos. 4 & 5 for sale at 10 cents a copy by the Supt. of Documents, U.S. Printing Office, Washington D.C."

The design of the arch bridge is carried in steps, starting with the floor system design which includes designs of slabs, beam and girder; then the columns are designed. In designing the girder the Theorem of Three Moments is used. The span for the arch is taken as 60 feet. The width of roadway used is 18 feet, and loading H-15.

Also the following data and information is used in the design of the bridges

2000 lbs. Concrete. (1 : 2 : 4) $6\frac{1}{2}$ " slump, Maximum Aggregate \leq 1 in

Stresses: $f_c = 800 \text{ lbs./in.}^2$

$f_s = 20,000 \text{ lbs./in.}^2$

$n = 15.$

$v = 40 \text{ lbs./in.}^2$ or 120 lbs./in.^2

$u = 100 \text{ lbs./in.}^2$

Curbs are 9" wide by 12" above Surface of Paving.

Paving is done by 2" of Bituminous Wearing.

Railing is constructed by Concrete 3'-6" above Road Surface.

SLAB DESIGN

Beams are placed 7 feet 6 inches apart, center to center with a total number of 9 beams, two being the end beams as shown in Fig. 10. In finding the thickness of the slab, the Ketchum's formula has been used, the explanation of which is already given in the first part of this thesis in the diagram of the Beam Bridge. The design is carried out in the following order: First the maximum moment is found by taking the sum of the dead load, live load and impact moments; Then the total shear is calculated. The depth of the slab is next calculated by the use of depth formulas for moment and shear and choice is given to the bigger depth to take care of both. Then the number, size and spacing of bars are determined by calculating the required area of steel.

Beams are assumed 16 inches wide for obtaining the clear span of the slab.

Computations (Alternate Design for Slab)

Method Used: From Ketchum's Specifications, in " Reinforced Concrete Design ", by Sutherland and Clifford.

$$W = 2/3 S \sqrt{T}$$

S = Spacing of floorbeams in feet.

$$W = 2/3 \times 7.5 \sqrt{1} = 6 \text{ ft.}$$

Moment

$$\text{Live Load Moment} = 2500 \text{ 'lbs.}$$

$$\text{Impact Moment} = 750 \text{ 'lbs.}$$

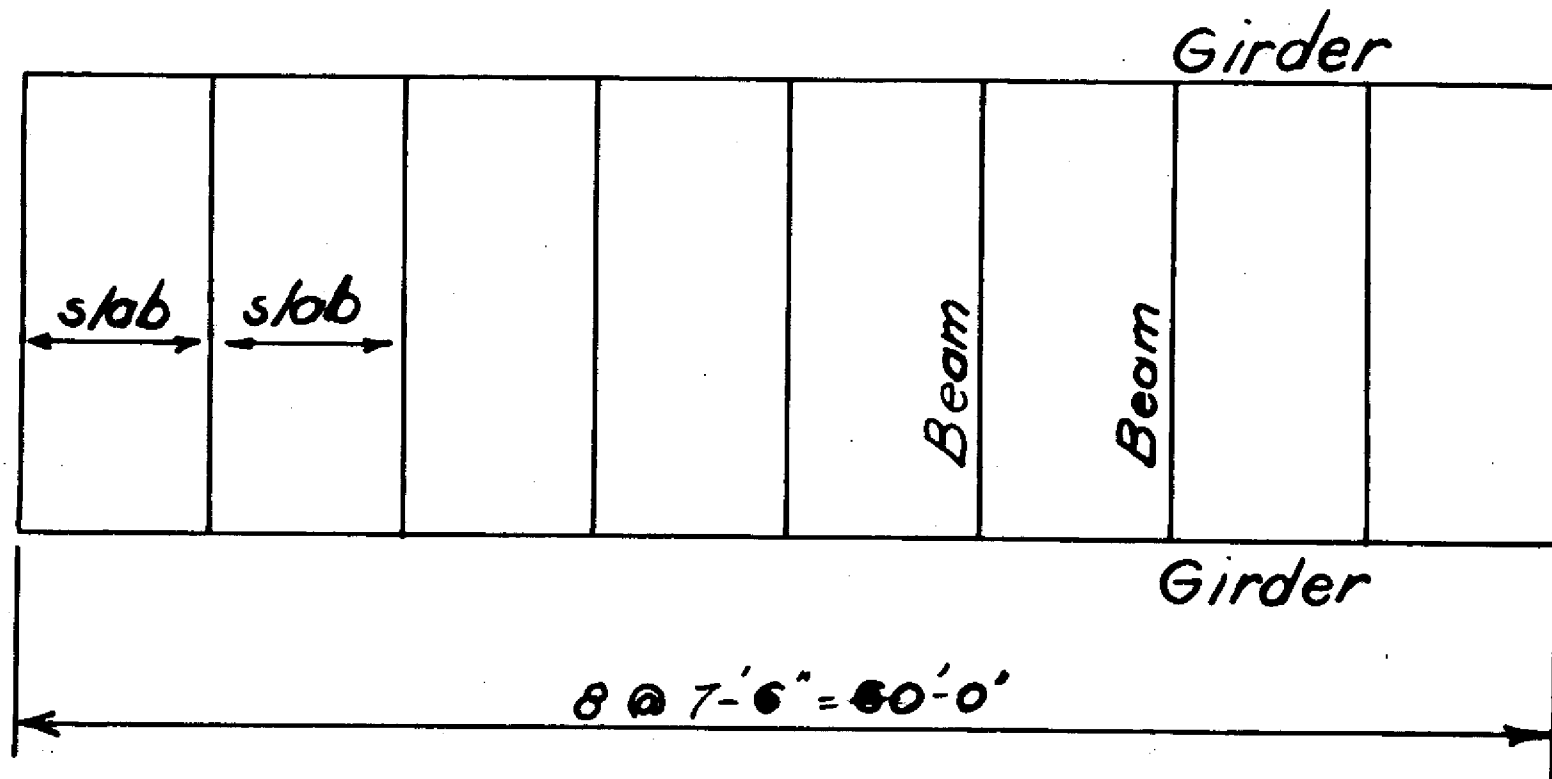


Fig 10

Floor system of
Arch bridge

Dead Load Moment

$$2" \text{ Surface} : 25 \text{ lbs./sq.ft.}$$

$$7" \text{ Slab} : 87.5 \text{ \# /sq.ft.} = 7/12 \times 150$$

$$112.5 \text{ \# /sq.ft.}$$

$$112.5 \times (6.25)^2 \times 1/8 \times 8/10 = 440 \text{ 'lbs.}$$

Total Moment

$$2500 \text{ \#} / 750 \text{ \#} / 440 = 3,690 \text{ 'lbs.}$$

Shear

Live Load Shear

$$12,000/6 \times \frac{1}{2} = 1,000 \text{ lbs.}$$

Impact Load Shear

$$30\% \times 1000 = 300 \text{ lbs.}$$

Dead Load Shear

$$112.5 \times 6.25 \times \frac{1}{2} = 350 \text{ lbs.}$$

Total Shear

$$= 1,650 \text{ lbs.}$$

Depth

For Moment

$$d = \sqrt{\frac{3,690 \times 12}{12 \times 131}} = 5.31 \text{ in.}$$

For Shear

$$d = \frac{1650}{12 \times 7/8 \times 40} = 3.93 \text{ in.}$$

Use Slab ($d = 5.5"$)

Steel

Area of Steel

$$A_s = M/f_sjd = \frac{3,690 \times 12}{20,000 \times 7/8 \times 5.5} = .46 \text{ sq.in./ft.width}$$

Try $\frac{1}{2}$ " round Bars

$$A = .196 \text{ sq.in.}$$

$$\text{Spacing} = \frac{.196}{.0383} = 5.1" \text{ use } 5" \text{ in.}$$

$$v = V/bjd = \frac{1,650}{12 \times 7/8 \times 5.5} = 28.6 \text{ lbs./in}^2$$

$$u = vb/z_o = \frac{28.6 \times 5}{1.57} = 91 \text{ lbs./in}^2 \quad (\text{less than } 100) \\ \text{O. K.}$$

Use $\frac{1}{2}$ " Round Bars @ 5" (Alternate Bars Bent)

BEAM DESIGN (Fig. 11, 12, 13, 14, 15.)

Interior beams carry the load coming from the slab. The beam is designed very similar to the slab design. Formulas used in the beam design are the same as in the slab design, except for a few additional formulas which are used in order to find the area of tension steel and for bent bars which are as follows:

$$A_s = M/f_s j d$$

in which,

M = Maximum total moment either positive or negative.

f_s = Unit tension stress.

d = Depth of slab from top up to the center of the steel bars used for reinforcement.

$$j = 7/8.$$

To find the places where the bars are to be bent either up or down, we use the following formula:

$$d_1^2/d_2^2 = a/b$$

where the position of the maximum moment can be obtained and shown in Fig. 11.

The size of the beam used is first assumed to be corrected later on, after corrected depth is calculated.

The procedure for the design of the end beam is exactly the same as in the interior beam and the same formulas are used.

Computations (Interior Beam)

Moment

Live Load Moment

$$24,000 \times 9 - 12,000 \times 7.5 - 12,000 \times 1.5 = 1,300,000 \text{ #}$$

Dead Load

$$2" \text{ Surface} : 25 \text{ lbs. ft}^2$$

$$7" \text{ Slab} : 87.5^{\#} / \text{ft}^2$$

$$112.5^{\#} / \text{ft}^2$$

$$112.5 \times 7.5 = 845 \text{ lbs. / ft.}$$

$$\text{Assumed Stem} = 255 \text{ lbs. / ft.}$$

$$w = 1,100 \text{ lbs. / ft.}$$

Dead Load Moment (Positive)

$$M = 1/8 w l^2 = 1/8 \times 1100 \times (18)^2 \times 12 = 535,000 \text{ "lbs.}$$

Dead Load Moment (Negative)

$$M = 1/20 w l^2 = 1/20 \times 1100 \times (18)^2 \times 12 = 214,000 \text{ "lbs.}$$

Impact Moment

$$M = .30 \times 1,300,000 = 390,000 \text{ "lbs.}$$

Total Moment (Positive)

$$M = 1,300,000 + 390,000 + 535,000 = 2,225,000 \text{ "lbs.}$$

Total Moment (Negative)

$$M = 8/20 \times 2,225,000 = 890,000 \text{ "lbs.}$$

Shear

$$\text{Live Load Shear} = 24,000 \text{ lbs.}$$

$$\text{Impact Shear} = 7,200 \text{ lbs.}$$

Dead Load Shear

$$\frac{1}{2} \times 1100 \times 18 = 9,900 \text{ lbs.}$$

$$\text{Total Maximum Shear} = 41,100 \text{ lbs.}$$

Depth (d)

For Negative Moment

$$d = \sqrt{\frac{890,000}{157 \times 16}} = 18.8 \text{ in.} \quad \text{Assume 16" Beam}$$

For Positive Moment

$$d = \sqrt{\frac{2,225,000}{131 \times 54}} = 17.8 \text{ in.} \quad b = \frac{1}{4} \times 18 = 4.5' = 54"$$

For Shear

$$d = \frac{41,100}{120 \times 7/8 \times 16} = 24.5 \text{ in.} \quad \text{Shear Governs.}$$

Try b = 14"

$$d_v = \frac{41,100}{120 \times 7/8 \times 14} = 28 \text{ in.}$$

Use 14" x 30" Beam (d = 28")

Correction

Maximum Positive Moment

$$1,300,000 \text{ } \cancel{\text{ } } 390,000 \text{ } \cancel{\text{ } } \text{ Corrected Dead Load Moment.}$$

Dead Load Moment

$$\text{Surface } \cancel{\text{ } } \text{ Slab} = 845 \text{ lbs.ft.}$$

$$\text{Stem} = \frac{30 - 7}{12} \times 14/12 \times 160 = 335 \text{ lbs.ft.}$$

$$w = 1180 \text{ lbs./ft.}$$

$$M = 1/8 \times 1180 \times (18)^2 \times 12 = 575,000 \text{ "lbs.}$$

Total Maximum Positive Moment

$$1,300,000 \text{ } \cancel{\text{ } } 390,000 \text{ } \cancel{\text{ } } 575,000 = 2,265,000 \text{ "lbs.}$$

Maximum Negative Moment

$$8/20 \times 2,265,000 = 905,000 \text{ "lbs.}$$

Maximum Total Shear = Maximum Corrected Dead Load Shear

Impact Shear + Live Load Shear

Maximum Dead Load Shear

$$\frac{1}{2} \times 1180 \times 18 = 10,600 \text{ lbs.}$$

Maximum Total Shear

$$10,600 + 7,200 + 24,000 = 41,800 \text{ lbs.}$$

$$d_v = \frac{41,800}{120 \times 7/8 \times 14} = 28.4 \text{ in.}$$

Steel

Area for Positive Moment

$$A_s = \frac{2,265,000}{20,000 \times 7/8 \times 28} = 4.61 \text{ sq.in.}$$

Area for Negative Moment

$$A_s = \frac{905,000}{20,000 \times 7/8 \times 28} = 1.85 \text{ sq.in.}$$

Use

7 - 3/4" Round Bar Steel

$$A_s = 3.1 \text{ sq.in.}$$

6 - 5/8" Round Bar Steel

$$A_s = 1.84 \text{ in.sq.}$$

$$\text{Total } A_s = 4.94 \text{ sq.in.}$$

$$u = \frac{41,800 \times 8}{18.5 \times 28 \times 7} = 103 \text{ lbs./in.}^2 \quad \text{O. K.}$$

Bent Up

$$d_2 = \sqrt{\frac{1.23 \times (108)^2}{4.94}} = 54 \text{ in.}$$

$$= 108 - 54 = 54 \text{ in from end of beam.}$$

$$d_3 = \sqrt{\frac{1.84 \times (108)^2}{4.94}} = 66 \text{ in.}$$

$$= 108 - 66 = 42 \text{ in. from end.}$$

$$d_1 = \sqrt{.61 \times (108)^2} = 80 \text{ in.}$$

$$= 108 - 38 = 70 \text{ in.}$$

Bend up 2 bars at a distance of 70 in. from end of beam.

" " 2 " " " " " 54 " " " " "

" " 2 " " " " " 42 " " " " "

Bent Down

$$d_1 = \sqrt{\frac{13 \times (108)^2}{16}} = 100.5 \text{ in.}$$

$$= 108 - 100.5 = 7.5 \text{ in. from end.}$$

$$d_2 = \sqrt{\frac{11 \times (108)^2}{16}} = 92.5 \text{ in.}$$

$$= 108 - 92.5 = 15.5 \text{ in. from end.}$$

$$d_3 = \sqrt{\frac{9 \times (108)^2}{16}} = 83.5 \text{ in.}$$

$$= 108 - 83.5 = 24.5 \text{ in. from end.}$$

Bend down 2bars at a distance of 24.5 in. from end of beam.

" " 2 " " " " " 15.5 in. " " " "

" " 2 " " " " " 7.5 in. " " " "

$$v = \frac{41,800}{14 \times 7/8 \times 28} = 122 \text{ lbs./in.}^2$$

$$v' = 122 - 40 = 82 \text{ lbs./in.}^2$$

Maximum Shear at Center Line

$$R_2 = 12,000 \times 3 \text{ } \nearrow \text{ } 12,000 \times 9 = 8,000 \text{ lbs.}$$

$$v = \frac{8,000}{14 \times 7/8 \times 28} = 23.4 \text{ lbs./in.}^2 \text{ at C.L.}$$

Stirrups

Use $\frac{1}{2}$ " Round Bars

$$S = 2A_s f_s = .39 \times 16,000 = 6,240 \text{ lbs.}$$

$$s = \frac{16,000 \times .39}{28 \times 14} = 16 \text{ in. Use 12"}$$

Maximum Spacing

$$.6 d = .6 \times 28 = 16.8 \text{ in.}$$

Use 12" spacing for all stirrups after the bent bars.

$$s_1 = \frac{16,000 \times .39}{73 \times 14} = 6 \text{ in.}$$

$$s_2 = \frac{16,000 \times .39}{82 \times 14} = 5.5 \text{ in. Min. end spacing.}$$

Use First Stirrup 3 in. From End.

Second " 9 in. " "

Third " 12 in. " "

All other Stirrups 12 in. Apart.

Spacing for Bent Bars

True Spacing

$$s = S/\sqrt{v'}(\sin a) = \frac{16,000 \times .61 \times 2^{\frac{1}{2}}}{65 \times 14} = 15.2 \text{ in. Use } 12".$$

Use 12" Spacing For the other Bars also.

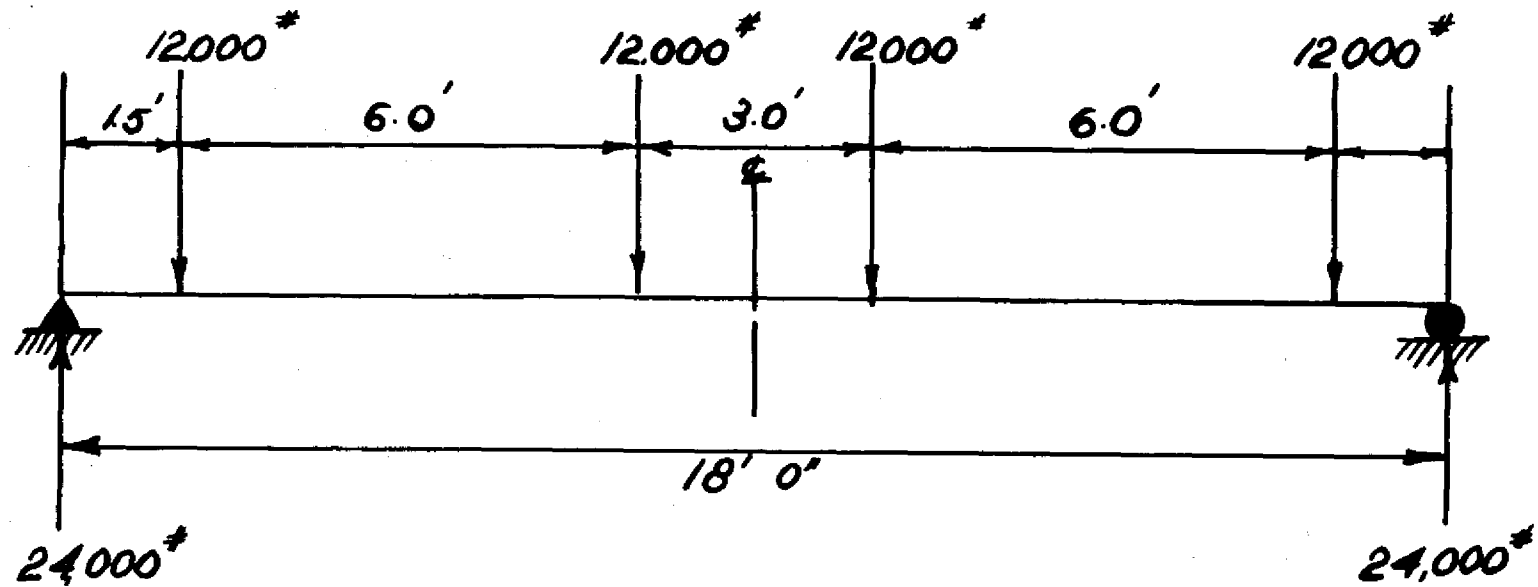


Fig 11

*Interior beam of
Arch bridge*

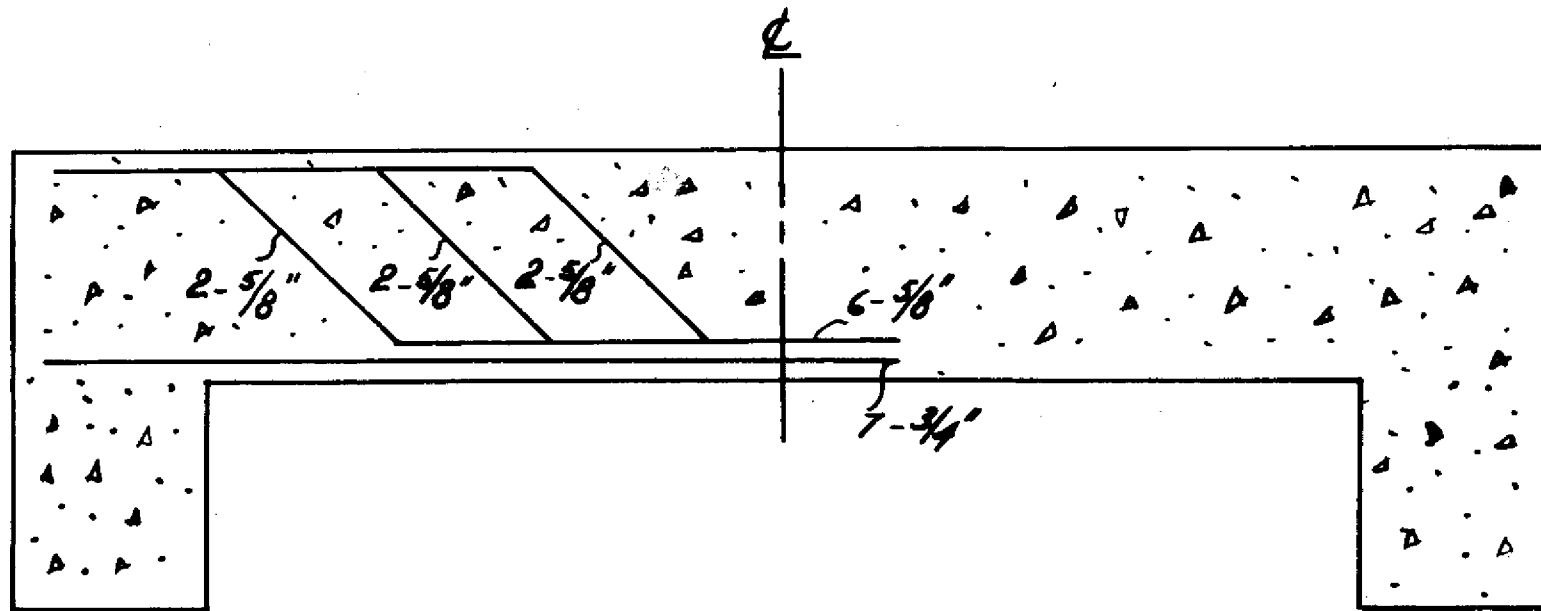


Fig 12

*Steel in beam
of Arch bridge*

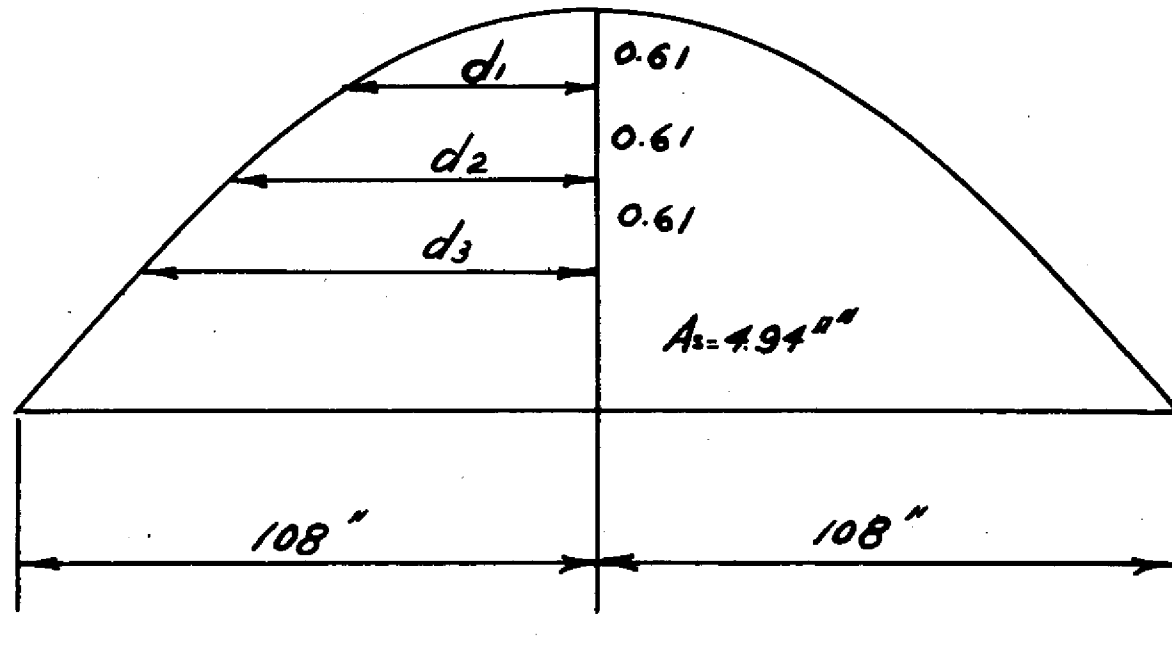


Fig 13

Bent up in beam
of Arch bridge

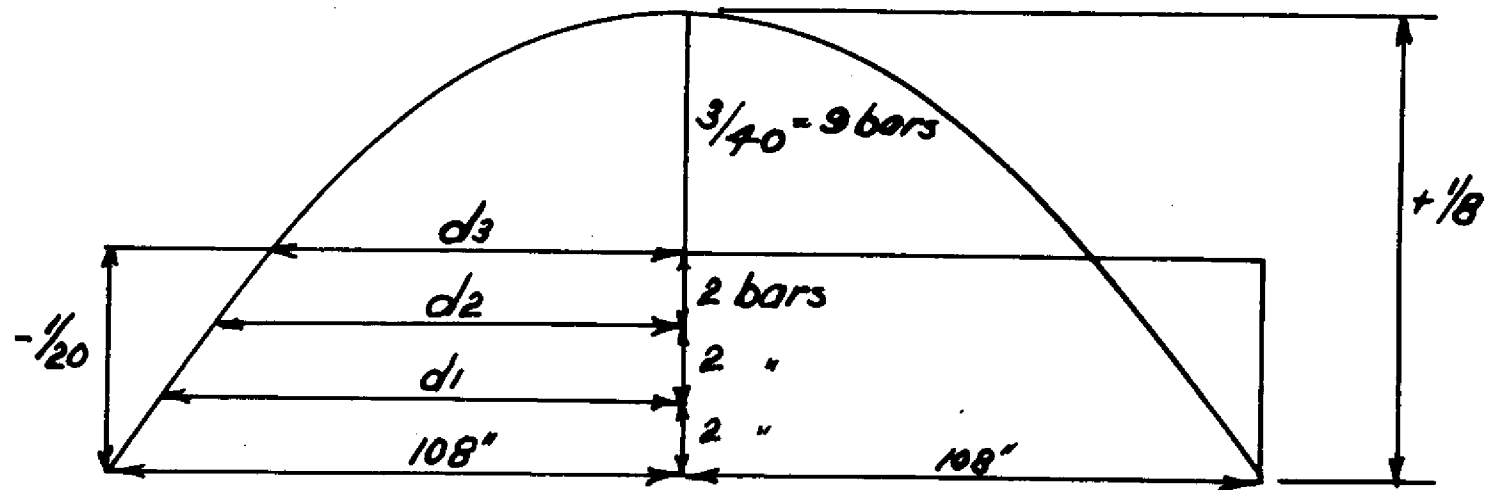


Fig 14

Bent down in beam
of Arch bridge

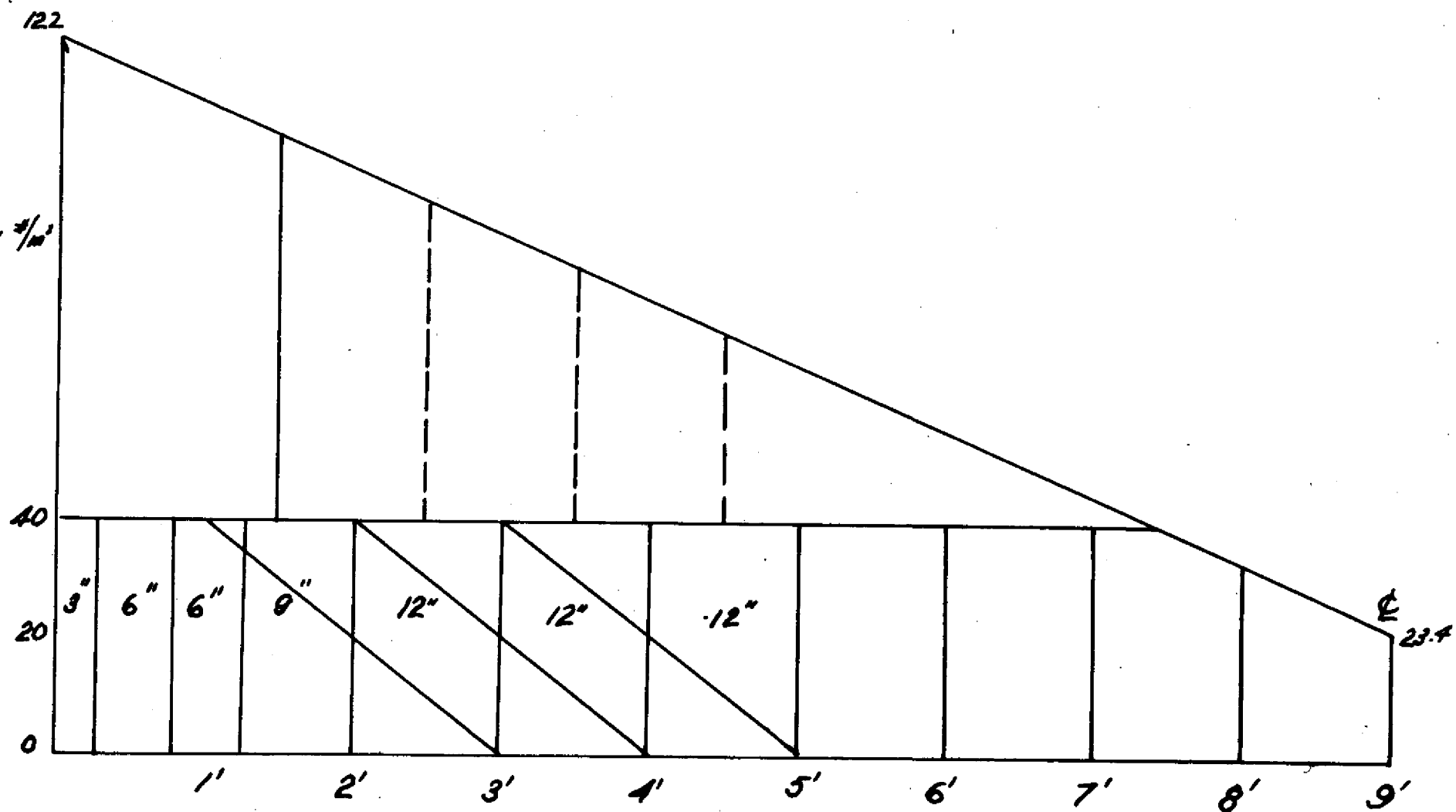


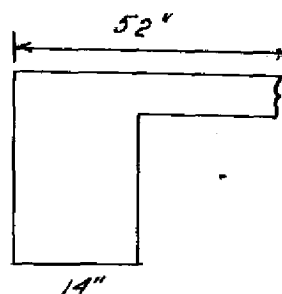
Fig 15

Stirrup spacing
in beam of Arch bridge

Computations (End Beam)

Assume 14" Beam

$$b = 7.5/2 \times 12 \neq 7 = 52"$$



Moment

Live Load Moment

$$108,000 \times 12 = 1,300,000 \text{ "lbs.}$$

Impact Moment

$$60\% \times 1,300,000 = 780,000 \text{ "lbs.}$$

Dead Load

$$2" \text{ Surface} = 25.0 \text{ lbs./ft.}^2$$

$$7" \text{ Slab} = 87.5 \text{ lbs./ft.}^2$$

$$= 112.5 \text{ lbs./ft.}^2 \times 52/12 = 490 \text{ lbs./ft.}$$

$$\text{Assume Stem} = 390 \text{ lbs./ft.}$$

$$w = 800 \text{ lbs./ft.}$$

Dead Load Moment

$$1/8 \times 800 \times (18)^2 \times 12 = 390,000 \text{ "lbs.}$$

Total Moment (Positive)

$$1,300,000 \neq 780,000 \neq 390,000 = 2,470,000 \text{ "lbs.}$$

Total Moment (Negative)

$$2,470,000 \times 8/20 = 987,000 \text{ "lbs.}$$

Shear

$$\text{Live Load Shear} = 24,000 \text{ lbs.}$$

$$\text{Dead Load Shear} = 7,850 \text{ lbs.} = \frac{1}{2} \times 18 \times 870$$

$$\text{Impact Shear (30\%)} = 14,400 \text{ lbs.}$$

$$\text{Total Shear} = 45,600 \text{ lbs.}$$

Depth

For Positive Moment

$$d = \frac{2,470,000}{131 \times 52} = 19.1 \text{ in.}$$

For Negative Moment

$$d = \frac{987,000}{157 \times 14} = 21.2 \text{ in.}$$

For Shear

$$d = \frac{45,600}{120 \times 7/8 \times 14} = 31 \text{ in.}$$

Try 14" x 33" Beam

Corrections

Dead Load Moment

$$\text{Surface } \nearrow \text{ Slab} \quad 490 \text{ lbs/ft.}$$

$$\text{Stem} = \frac{33 - 7}{12} \times \frac{14}{12} \times 150 = 380 \text{ lbs/ft.}$$

$$w = \quad 870 \text{ lbs/ft.}$$

$$M = 1/8 \times 870 \times (18)^2 \times 12 = 423,000 \text{ "lbs.}$$

Total Moment (Positive)

$$423,000 \nearrow 1,300,000 \nearrow 780,000 = 2,503,000 \text{ "lbs.}$$

Total Moment (Negative)

$$2,503,000 \times 8/20 = 1,000,000 \text{ "lbs.}$$

Dead Load Shear

$$\frac{1}{2} \times 870 \times 18 = 7,850 \text{ lbs.}$$

Total Shear

$$24,000 \nearrow 14,400 \nearrow 7,850 = 46,250 \text{ lbs.}$$

$$d_v = \frac{46,250}{120 \times 7/8 \times 14} = 31.5 \text{ in.}$$

Use 14" x 35" Beam (d = 32")

GIRDER DESIGN

The girder to be designed in this case is one which is continuous and is supported by six columns. That is 60 feet of the span of the bridge is divided into 5 spans at 12 feet. It is evident that the moments and the shears in such a girder that which is rigid over the columns, cannot be determined by principle of statics alone, since there are two unknown end moments, two unknown vertical reactions and possibly two unknown horizontal reactions which makes a possible total of six unknowns while there are only three equations in statics for bodies in equilibrium. There are several methods for finding moments for such continuous girders such as the Theorem of Three Moments, Least Work Method and Slope Deflection Method. The three moments method was used in our thesis.

By the use of the first (three moment) method, moment factors are found for beams and girder carrying uniform load and continuous over several spans. A table of these coefficients for maximum moment in continuous beams is given by Sutherland and Clifford in their book of Reinforced Concrete Design on page 174. In finding the maximum moments in the above girder design the use of this table will be made which is believed to give accurate enough results for practical purposes. The formula to be used to get the moment factor is,

$$M = xwL^2$$

where

M is the maximum moment.

x is the moment factor.

w is the uniform load (dead or live).

L is the span length.

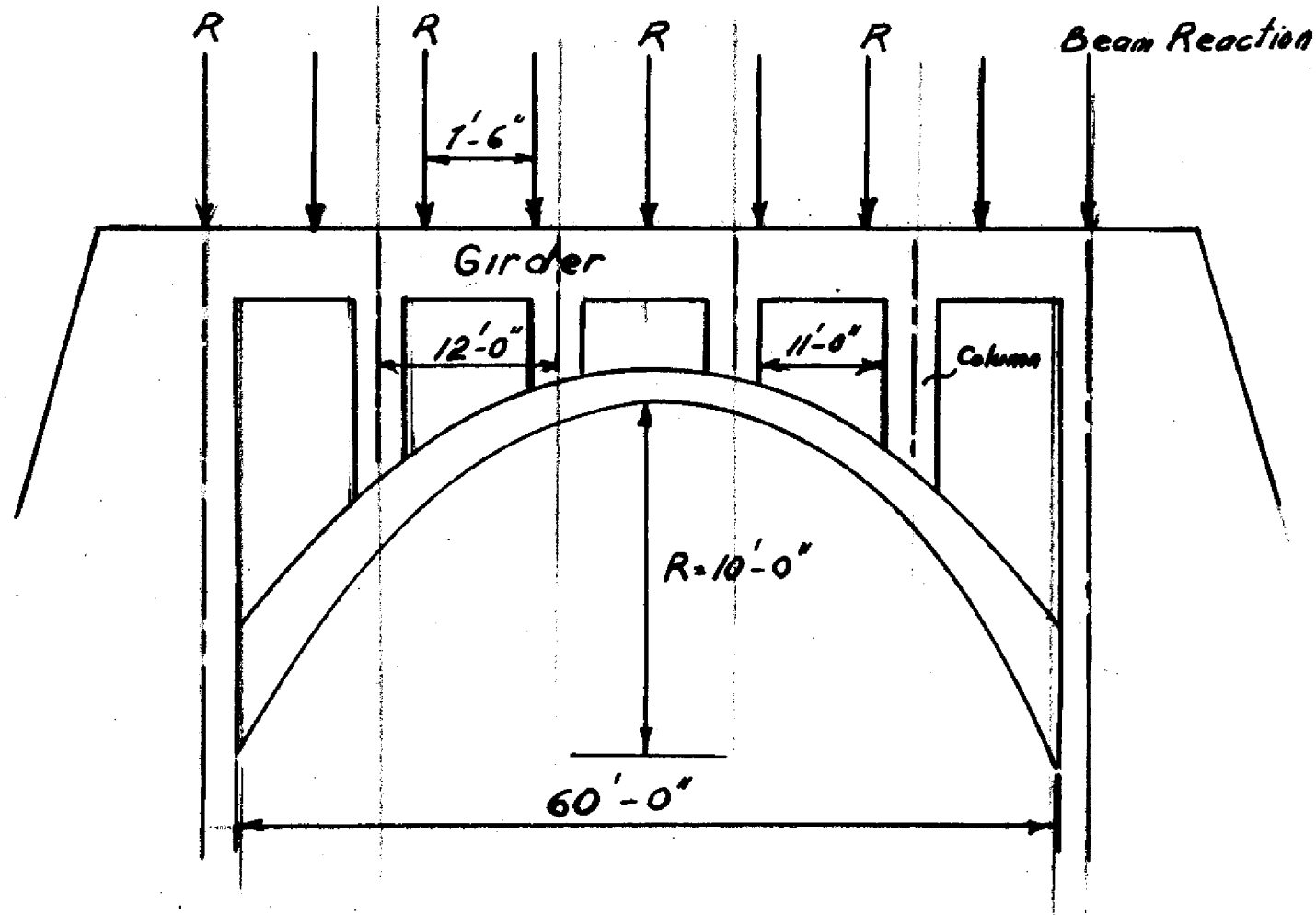


Fig. 16

Position of beams
and columns on Arch

The equivalent uniform load for H-15 loading is found to be 480 lbs. per linear foot of lane and a concentrated load of 13,000 lbs. for moment and 19,500 lbs. for shear these being found in Design of Steel Structure by Urquhart and O'Rourke on page 422. The maximum shear is obtained by the use of the following formula:

$$V = cWL$$

where

c is the coefficient and is .6 for end beam and .5 for interior beam.

The depth of the girder and the steel reinforcement used are found in similar processes as in the case of the beams.

The size of the columns are assumed to be 18" x 12" in order to obtain approximately the clear span of the girder between the columns. This is obtained to be 11 feet.

Computations

Dead Concentrated Load Carried to Girder

Dead Load of Slab (carried by $\frac{1}{2}$ of each beam)

$$7/12 \times 9 \times 7.5 \times 150 = 5,900 \text{ lbs.}$$

Dead Load of $\frac{1}{2}$ Beam

$$14/12 \times 30/12 \times 9 \times 150 = 3,930 \text{ lbs.}$$

2" Bituminous Surface

$$25 \times 9 \times 7.5 = 1,690 \text{ lbs.}$$

$$\text{Dead Concentrated Load} = 10,520 \text{ lbs.}$$

Live Concentrated Load Carried to Girder

Live Load

$$480 \times 7.5 = 3,600 \text{ lbs.}$$

Impact Load

$$60\% \times 3600 = 2,160 \text{ lbs.}$$

$$5,760 \text{ lbs.}$$

Total Concentrated Load Carried to Girder

$$10,520 \neq 5,760 = 16,280 \text{ lbs.}$$

Uniform Dead Load

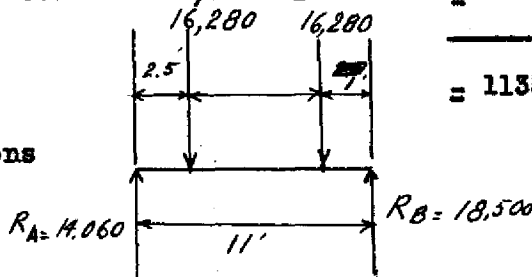
Assume Dead Load of Girder = 500 lbs./ft.

Railing $3.5 \times 1 \times 150 = 525 \text{ lbs./ft.}$

Curbs $9/12 \times 1 \times 150 = 110 \text{ lbs./ft.}$

$$= 1135 \text{ lbs./ft.}$$

Reactions



$$R_B = 2.5/11 \times 16,280 \neq 10/11 \times 16,280 = 18,500 \text{ lbs.}$$

$$R_A = 16,280 \neq 16,280 - 18,500 = 14,060 \text{ lbs.}$$

Shear

R_B (Beam Reaction) = 18,500 lbs.

Live Load Shear (from Table) = 19,500 lbs.

Impact Shear (60%) = 11,700 lbs.

Uniform Dead Load Shear

$$.6 \times 1135 \times 11 = 7,500 \text{ lbs.}$$

Total Shear = 57,200 lbs.

Moment

Uniform Dead Load Moment

$$\text{(Positive)} \quad .072 \times 1135 \times (11)^2 = 9,900 \text{ 'lbs. at center.}$$

$$\text{(Negative)} \quad .105 \times 1135 \times (11)^2 = 14,400 \text{ ' lbs.}$$

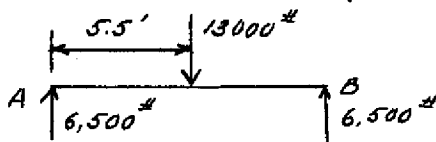
Moment at the Reactions

$$\begin{aligned} \text{(Negative)} \quad M_A &= \frac{16,280 \times 2.5 \times (8.5)^2}{(11)^2} \div \frac{16,280 \times 10 \times (1)^2}{(11)^2} \\ &= 20,340 \text{ ft.lbs.} \end{aligned}$$

$$\begin{aligned} \text{(Negative)} \quad M_B &= \frac{16,280 \times 8.5 \times (2.5)^2}{(11)^2} \div \frac{16,280 \times 1 \times (10)^2}{(11)^2} \\ &= 20,500 \text{ 'lbs.} \end{aligned}$$

$$\begin{aligned} \text{(Positive)} \quad M_{C.L.} &= -20,500 \div 18,500 \times 5.5 - 16,280 \times 4.5 \\ &= 8000 \text{ 'lbs.} \end{aligned}$$

Concentrated Live Load Moment (from table)



$$\text{(Negative)} \quad M_{A \& B} = \frac{13,000 \times 5.5 \times (5.5)^2}{(11)^2} = 17,900 \text{ 'lbs.}$$

$$\text{(Positive)} \quad M_{\text{Center}} = 6,500 \times 5.5 - 17,900 = 17,900 \text{ 'lbs.}$$

Impact Moment

$$\text{Negative Moment} \quad 30\% \times 17,900 = 5,370 \text{ 'lbs.}$$

$$\text{Positive Moment} \quad 30\% \times 17,900 = 5,370 \text{ 'lbs.}$$

Total Negative Moment

$$14,400 \div 20,500 \div 17,900 \div 5,370 = 58,170 \text{ 'lbs.}$$

Total Positive Moment

$$8,000 \div 9,900 \div 17,900 \div 5,370 = 41,170 \text{ 'lbs.}$$

Depth

For Positive Moment

$$d = \sqrt{\frac{41,170 \times 12}{131 \times 33}} = 11 \text{ in.}$$

$$\begin{aligned} b &= \frac{1}{4} \text{span} = \frac{1}{4}(11)(12) \\ &= 33'' \end{aligned}$$

For Negative Moment

$$d = \sqrt{\frac{58,170 \times 12}{157 \times 18}} = 16 \text{ in.}$$

Trying $b = 18"$

For Shear

$$d = \frac{57,200}{120 \times 7/8 \times 18} = 33 \text{ in.}$$

Shear Governs

Use 18" x 36" (d = 33")

Steel

Area of Steel For Positive Moment

$$A_s = \frac{41,170 \times 12}{20,000 \times 7/8 \times 33} = 8.55 \text{ sq.in.}$$

Use 9 - 1" Square Bars

Bond

$$u = \frac{57,200}{36 \times 7/8 \times 33} = 55 \quad \begin{matrix} \text{(less than 100)} \\ \text{O. K.} \end{matrix}$$

Area of Steel for Negative Moment

$$A_s = \frac{58,170 \times 12}{20,000 \times 7/8 \times 33} = 10.90 \text{ sq.in.}$$

Use 11 - 1" Square Bars

Bond

$$u = \frac{57,200}{44 \times 7/8 \times 33} = 67 \quad \begin{matrix} \text{(less than 100)} \\ \text{O. K.} \end{matrix}$$

Use 9 - 1" Square Bars
11 - 1" Square Bars

Bend Down

$$d_1 = \sqrt{\frac{5 \times (5.5)^2}{13}} = 3.4 \text{ in.}$$

$$d_2 = \sqrt{\frac{6 \times 5.5 \times 5.5}{13}} = 3.75 \text{ in.}$$

Bend Down 1 Bar 2'-4" from End.

" " 2 " 1'-10" " " .

Bend Up

$$d_1 = \sqrt{\frac{2 \times (5.5)^2}{10.7}} = 2.38 \text{ in.}$$

$$d_2 = \sqrt{\frac{4 \times (5.5)^2}{10.7}} = 3.35 \text{ in.}$$

Bend 2 Bars Up at 3' from End.

" 2 " " " 2' " "

Use

1 - 1" Square Bars. Straight at Top.

3 - 1" Square Bars. Bent Down.

(Fig. 9)

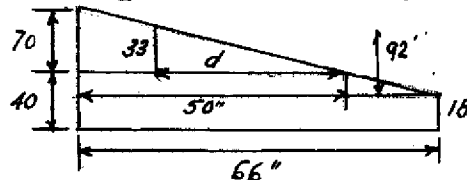
4 - 1" Square Bars. Bent Up.

Stirrups

$$v = \frac{57,200}{18 \times 7/8 \times 33} = 110 \text{ lbs./in.}^2$$

$$v' = 110 - 40 = 70 \text{ lbs./in.}^2$$

$$v \text{ at C. L.} = 70 \times \frac{1}{4} = 18 \text{ lbs./in.}^2$$



End Spacing

$$s = S/v'b = \frac{3540}{70 \times 18} = 2.8 \text{ in.}$$

$$v' = \frac{3,540}{6 \times 18} = 32.8$$

$$x = \frac{70 \times 66}{92} = 50$$

$$d = \frac{33 \times 50}{70} = 23.5 \text{ in. or } 26.5 \text{ in. from End.}$$

Use

3/8 Round Stirrups 1 @ 2" (2")

5 @ 6" (30")

3 @ 8" (24")

(62")

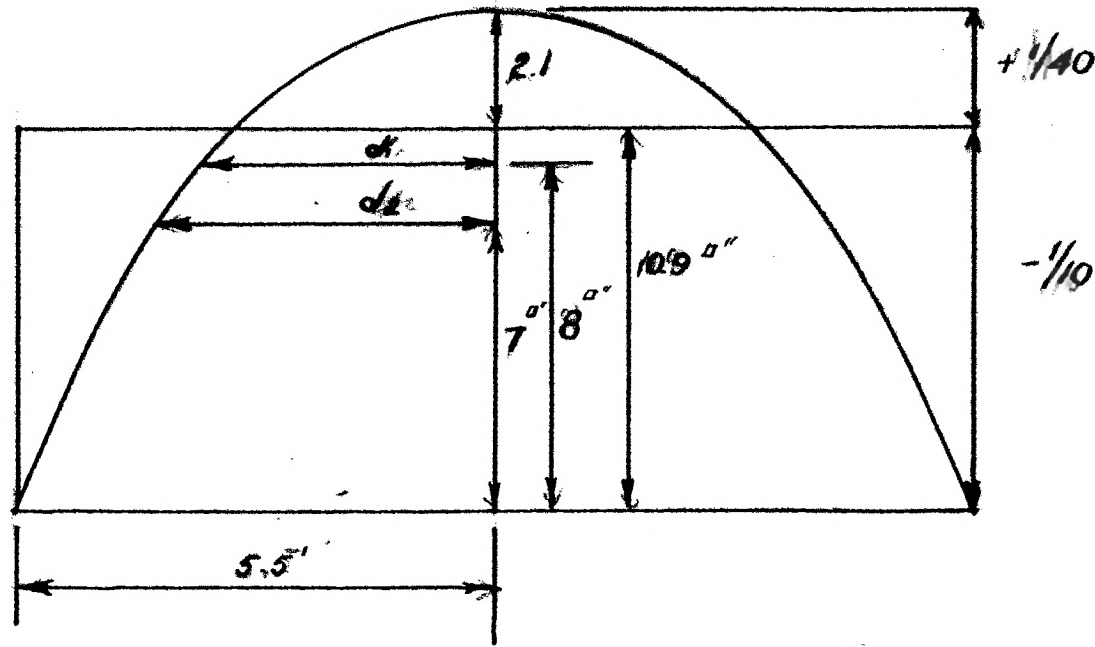


Fig 17

Bent down of bars
in Girder

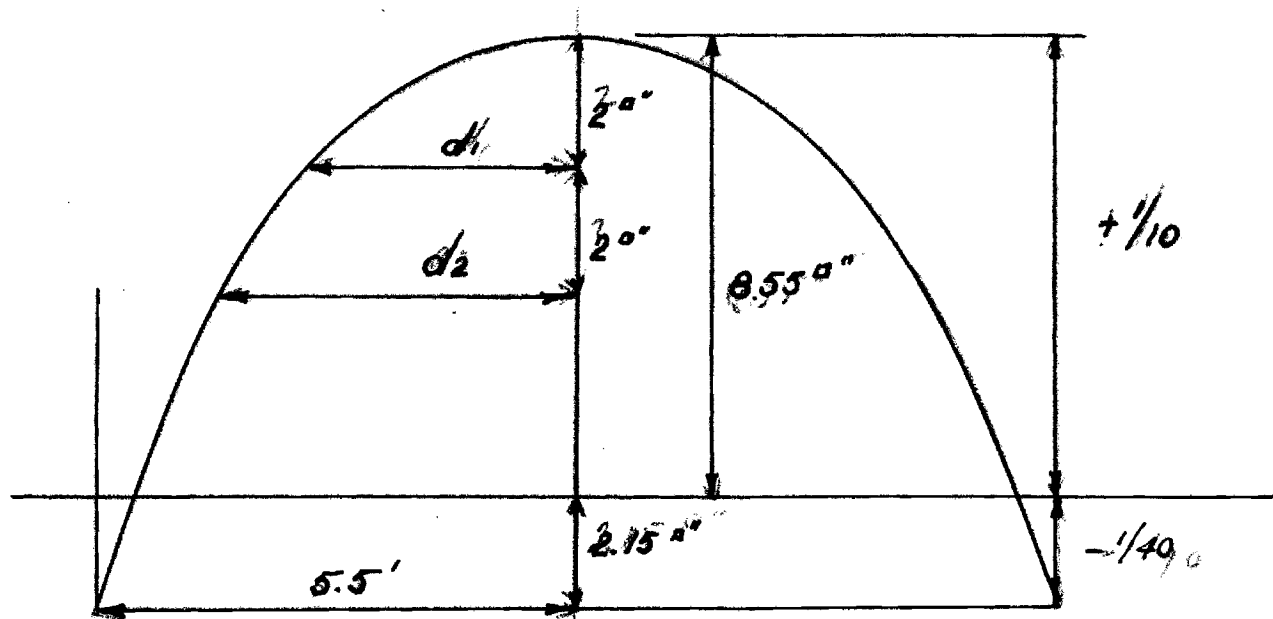


Fig 18

Bent up of bars
in Girder

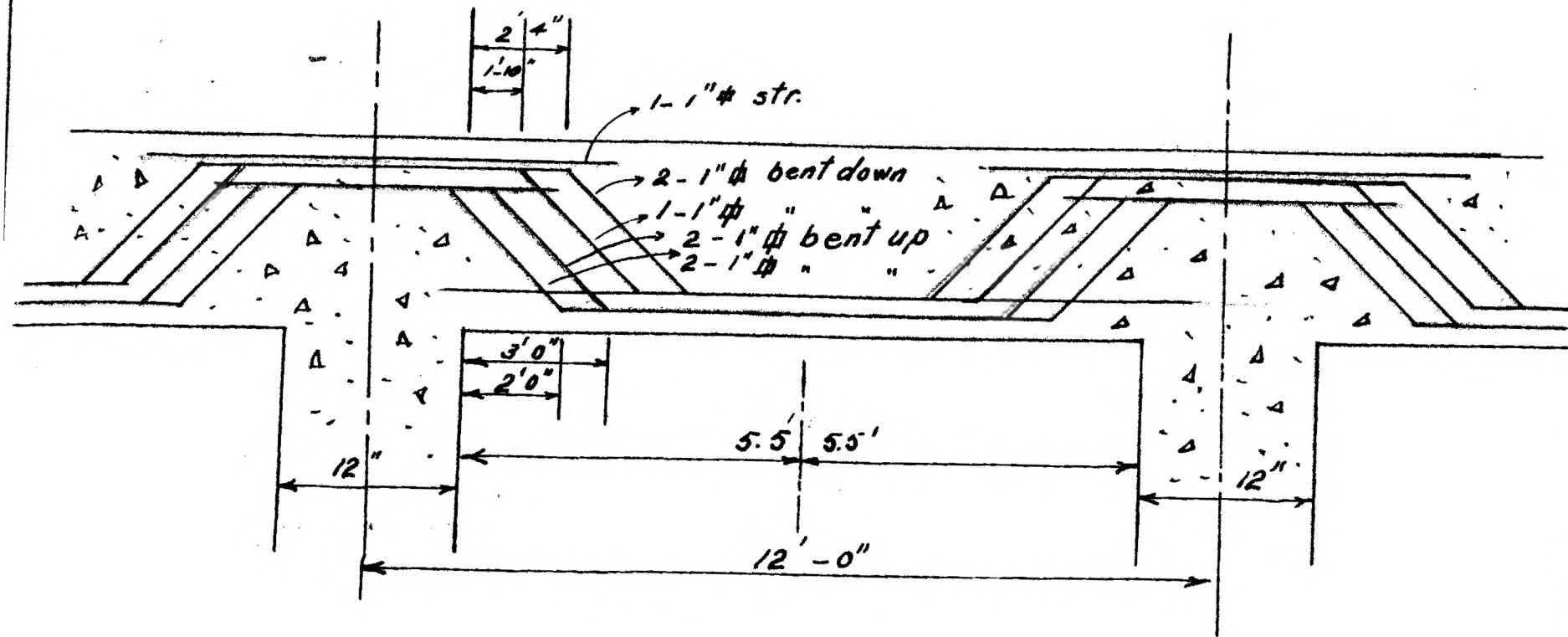


Fig 19

Position of steel
in Girder.

COLUMN DESIGN

Columns carry the loads coming from girders to the arch. Six columns are used in the design, three on each half of the bridge. The cross-sectional area of the columns is found simply by using the formula:

$$A_g = \text{Load}/f_c$$

where A_g is the cross-sectional area and f_c equals 450 lbs./in²

Computations

Total Load on Column

Girder

$$150 \times \frac{18 \times 36}{144} \times 12 = 8,100 \text{ lbs.}$$

Slab

$$7/12 \times 52.5 \times 9/4 \times 150 = 10,400 \text{ lbs.}$$

Beam

$$7 \times 9 \times \frac{14 \times 30 \times 150}{4 \times 144} = 6,900 \text{ lbs.}$$

2" Bituminous Surface

$$\frac{25 \times 52.5 \times 9}{4} = 3,000 \text{ lbs.}$$

Railing

$$3.5 \times 150 \times 1 \times 12 = 6,300 \text{ lbs.}$$

Curbs

$$9/12 \times 1 \times 150 \times 12 = 1,350 \text{ lbs.}$$

Live Load

$$480 \times 12 = 5,760 \text{ lbs.}$$

Impact

$$30\% \times 5,760 = 1,728 \text{ lbs.}$$

Concentrated Load

$$= 19,500 \text{ lbs.}$$

Impact for Concentrated Load

$$30\% \times 19,500 = 5,850 \text{ lbs.}$$

Total Load

$$= 60,275 \text{ lbs.}$$

$$f_c = 450 \text{ #/in}^2$$

Cross-sectional Area

$$A_g = \frac{60,275}{450} = 134 \text{ sq.in.}$$

Use 12" x 12" Column Area = 144 sq.in.

Steel Required

$$1\% A_g = .01 \times 144 = 1.44 \text{ sq.in.}$$

Use 4 - 3/4" Round

1/4" Round Ties @ 12"

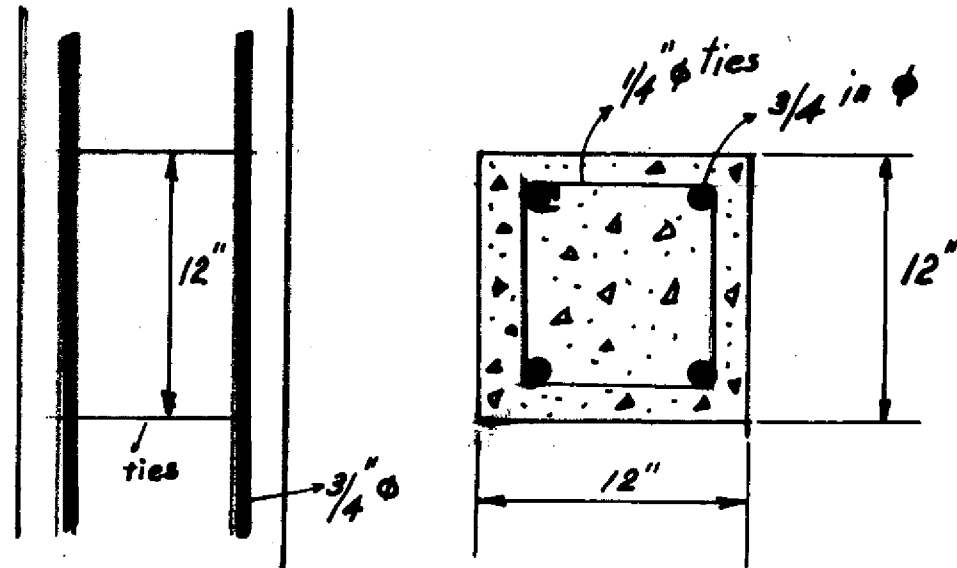


Fig 20

Cross Section
of Column

DESIGN OF ARCH

The hingeless arch is statically indeterminate as there are only three equations for six unknowns; thus three additional equations are required. These are derived from the elastic property of the arch; the derivation can be found in the book "Analysis of Concrete Arches" which is a reprint from the Bureau of Public Road Publications. Arch computations are best handled on tabulated forms as shown in the prints Tables 1-9.

The thickness of the arch at various points is found by a trial arch as shown in Fig. 19 where the graphical method is used. To find the shape of the center line of the arch, half of the arch is divided into ten equal parts and midpoints of these are drawn vertically. The respective "y" s of these are found by the use of the formula:

$$y = b - \frac{8rL}{6 \sqrt{5r}} (3c^2 \sqrt{10c^4 r})$$

in which

b = the rise of the arch and is taken as 10 feet.

r = b/L

$$c = \frac{.5 - x}{L}$$

x = horizontal distance from the point o to various points as shown in Fig. 19.

After the 'y' s of each point are found the shape of the arch is drawn to scale. The thickness of the crown is found by the following formula:

$$h = .0001 (11,000 \sqrt{l^2}) = \text{Crown thickness}$$

where l is the span.

Thickness of springing line is obtained by a general rule, by taking 2.5 times the thickness of the crown.

The various thicknesses of the arch is found by stretching the arch axis into a straight line as shown in fig. 19 and drawing in the thickness of the crown and the springing line. There we obtain the thickness of the arch at various points and the d's of points from one to ten. Then these informations being used as finding the stresses of all the arches completely designed. The stresses are found on the tabulated forms, tables 1-9. Explanation of these tables are found below.

The value of h_x for each of the ten points between 1 and 10, are computed by the above formula and entered in Col. 2 of table 1, Prints.

Col. 3 to 5, follow the table.

Col. 6 $d' = \text{Fireproofing or steel cover} = 2" = .17 \text{ Ft.}$

Col. 8 $n = 10, 12 \text{ or } 15$ according to the value of f_c' used.

$A_s = \text{Total area of steel per Ft. of arch ring in Sq. Ft.}$

Usually $1"$, $1\frac{1}{8}"$ or $1\frac{1}{4}"$ Sq. bars are used, one per Foot at both top and bottom of arch ring.

Col. 9 Add values in Col. 4 and in Col. 8.

Col. 10 $d_s = \text{length of arch axis for each arch division, Scaled from drawing. } d_s \text{ is not constant as } dx \text{ is. See Pt. 3, Fig. 1,}$

Col. 11 Sum of this Col. = $(\frac{1}{2})$ as in this column only one half of the arch ring is used.

Col. 12 $dx = 0.05L$. The values in this Col. are the same for all arches.

Col. 13 Multiply values in Col. 12 by those in Col. 11. Add column.

Col. 14 Start at Pt. 1 with value as in Col. 13, with the sign changed.

(89.5) Prints. Add 89.3 and 171.7 = 261.0 for value at Pt. 2.

261.0 and 243.0 = 504.0 for value at Pt. 3. Continue this method and add column.

Col. 15 Value at Pt. 10 = Sum of Col. 14. Value at Pt. 9 = value at Pt. 10 in Col. 15 plus value at Pt. 9 in Col. 14. Value at Pt. 8 = Value at Pt. 9 in Col. 15 plus value at Pt. 8 in Col. 14. Continue this process to top of Col..

Since Pt. 1 is not far from 0 and since V_0 should be very nearly 1 for a unit load at Pt. 1, the value of F from the formula should be very nearly to the value at Pt. 1 in Col. 15. In computed arch it is equal.

Col. 16 Follow the table of the computed arch. Same for all arches.

Col. 17 Products of values in Col. 16 by those in Col. 11.

Col. 18 Compute F by formula. Divide values in Col. 15 by F_0 .

Col. 20 Values of y are computed by formula for y.

Col. 21 Same as Col. 11.

Col. 22 Products of values in Col. 20 by those in Col. 21.

Add column to obtain $(1/2) \sum y \Delta$

$\sum y \Delta / \sum \Delta$ = Sum of Col. 22 divided by Sum of Col. 11.

Col. 23 Subtract this value from each value of y in Col. 20.

Col. 24 Products of values in Col. 21 by those in Col. 23.

Col. 25 Obtained from Col. 24 in this same manner as in Col. 14 from Col. 13. Add the column.

Col. 26 At Pt. 1 value is 0. At Pt. 2 value is the value of Pt. 1 in Col. 26 plus the value in Col. 26 at Pt. 2 plus the value at Pt. 2 in Col. 25. Continue this method. The signs are all negative.

Col. 27 Values are the products of values in Col. 20 by those in 24.

$$\text{Add for } (1^{1/2}) \sum y \Delta / y - (\sum y \Delta) / \sum \Delta$$

Col. 28 Compute by formula for Tan- ϕ or scale from drawing. ϕ is the angle between the horizontal and a tangent to the arch axis at the point. Look up Cos. ϕ .

Col. 29 Divide values in Col. 28 by those in Col. 2. A = cross sectional area of the arch ring. Since the ring that is being designed is 1 Ft. wide $h = A$. Add Col. for $\frac{(1^{1/2}) \text{ Cos. } \phi}{A}$.

Col. 30 Compute C by formula.

$C = (1/0.05L)$ times twice the sum of Col. 27 plus twice the sum of Col. 29. $H_o =$ Values in Col. 26 divided by C. Compute H_t by $H_t = 34560t/C$. See print.

$$34560 = 20 Ee = 20 (144) (2000000) (0.000006).$$

Col. 32 To obtain Z, subtract 0.5 from the point and multiply by 2. As at Pt. 10.2 = $(10-0.5)/2 = 19$. Also for Pt. 10'.

Col. 33 Same as Col. 32 for Pts. 1 to 10 and Pts. 1' to 10'.

Col. 34 Same as Col. 18 for Pts. 1 to 10. Values between 10' and 1' are found by subtracting the corresponding values for Pts. between 1 and 10 from 1. At Pt. 7' value = 1-79915.

Col. 35 Same as Col. 11 for Pts. between 1 and 10 and 10' and 1'.

Col. 36 Value at Pt. 1' is 0. Value at Pt. 2' is the sum of values at Pt. 1' in Col. 35 and Pt. 1' in Col. 36. Value at Pt. 3' is the sum of values at Pt. 2' in Col. 35 and 2' in Col. 36. Continue this process.

Col. 37 Value at Pt. 1' is 0. Value at Pt. 2' = Value at Pt. 1' in Col. 37 plus value at Pt. 2' in Col. 36. Value at Pt. 3' = Value at Pt. 2' in Col. 37 plus value at Pt. 3' in Col. 36. Continue this method.

Col. 38

Compute $dx/\Sigma \Delta \cdot \Sigma \Delta$ - twice the sum of Col. 11. Multiply each value in column 37 by this constant.

Col. 39 Multiply the values in Col. 33 by $\Sigma y \Delta / \Sigma \Delta$ as computed for use in Col. 23.

Col. 40 Compute $20dx/2$ and multiply values in Col. 34 by it.

Col. 41 Value for each point in this Col. = Some of the values for the same point in Cols. 38, 39, 40.

Sum the Cols. 38, 39, 40, 41 and apply the following check: Sum of Col. 38 \neq Sum of Col. 39, \neq Sum of Col. 40 \neq Sum of Col. 41.

Cols. 43, 44, 45. Same as columns 33, 34, 35.

Col. 46 $Z_3 = 2x_3/dx = 2(7.5)/3 = 5$. $y_3 = 3.51$. Compute $V_0 Z_3$.

Col. 47 $a_1 = x_1$, $a_2 = x_2$ etc. For Pt. 1 $Z_3 - 2a/dx = 5 - 2(1.5)/3 = 4$. For Pt. 2, $Z_3 - 2a/dx = 5 - 2(4.5)/3 = 2$ etc.

Col. 48 For Pts. up to but not including No. 3 multiply the values in Col. 47 by $dx/2 = 3/2 = 1.5$. For Pt. 3 and beyond, multiply the values of v_0 in Col. 44 by $Z_3 dx/2 = 5(3/2) = 7.5$.

Col. 49. Multiply values of H_0 in Col. 43 by $-y_3$.

Col. 50 Value at any point = Sum of values for same points in Col. 45, 48 and 49. See formula on print.

Cols. 51, 52, 53, 54, and 55. are computed in like manner to Cols. 46, 47, 48, 49, and 50. Carry 51 and 52 to Pt. 7 inclusive.

Col. 56, 57, 58, 59, 60. are computed in same manner as cols. 46, 47, 48, 49, and 50. Carry to Pt. 10 inclusive Cols. 56 and 57. For checks see computations and equations on page or print 4.

Cols. 61 to 67 (inclusive). Copy Cols. 43, 44, 45, 50, 55, 60.

Col. 68. D.L. Dead Loads are really the weights of the arch ring in Open

Spandril Arches and the weights of the arch ring plus the weights of the fill for Filled Spandril Arches. The dead load at each point is the weight of the section of arch ring and fill which is ds long. In the dead load table on print 3 only Cols. 1, 2, and 3 are needed for open spandril arches.

Col. 69 are Cols. where the values are obtained by multiplying the values to 74 in Cols. 62 to 67 by values in Col. 68. Sum of Col. 69 \times H for D. L. and is entered on table 9 Col. 77 as shown. Sum of Col. 70 $=$ V for dead load at Pt. 0 and is entered on table 9 at Pt. 0 as shown. Sums of Cols. 71, 72, 73, and 74 are the dead load moments for Pts. 0, 3, 8, and 10.5 and are entered on table 9 in Col. 78 as shown.

Computations

Sample to find the y for the respective x .

$$b = 10 \text{ ft.}$$

$$r = 10/60 = 1/6$$

$$c = .5 - x/L \quad \text{for } x_1 = 1.5$$

$$= .5 - 1.5/60 = .475.$$

$$\begin{aligned} y_1 &= b - \frac{8rL}{6 \sqrt{5r} (3c^2 \sqrt{10c^4 r})} = \\ &= 10 - \frac{8(1/6)60}{6 \sqrt{5(1/6)}} \frac{(\sqrt{3} \times (4.75)^2 \sqrt{10(.475)^4} 1/\sqrt{6})}{1} = 3.05 \end{aligned}$$

Thus for the respective values of x values for y are found in this manner and the curve of the arch can be plotted. The values of y for the different values of x are ~~is~~ tabulated below, the calculation being performed for each one as above.

$x_1 = 1.5$	$y_1 = 1.12$
$x_2 = 4.5$	$y_2 = 3.05$
$x_3 = 7.5$	$y_3 = 4.70$
$x_4 = 10.5$	$y_4 = 6.10$
$x_5 = 13.5$	$y_5 = 7.26$
$x_6 = 16.5$	$y_6 = 8.18$
$x_7 = 19.5$	$y_7 = 8.90$
$x_8 = 22.5$	$y_8 = 9.45$
$x_9 = 25.5$	$y_9 = 9.803$
$x_{10} = 28.5$	$y_{10} = 9.98$
$x_{10.5} = 30.0$	$y_{10.5} = 10.00$

Crown Thickness (Sample)

$$h = .001 (11,000 / 12) = .001 (11,000 / 36) = 1.46 \text{ ft.}$$

Thickness of Springing

$$h = 1.46 \times 2.5 = 3.65 \text{ ft.}$$

Values Obtained from Drawing Number 1

$h_0 = 3.65'$	$ds_1 = 4.00$	$\theta_1 = 41.5^\circ$
$h_2 = 3.50'$	$ds_2 = 3.70$	$\theta_2 = 37.00$
$h_3 = 3.30'$	$ds_3 = 3.57$	$\theta_3 = 33.00$
$h_4 = 3.00'$	$ds_4 = 3.45$	$\theta_4 = 28.50$
$h_5 = 2.80'$	$ds_5 = 3.25$	$\theta_5 = 23.50$
$h_6 = 2.50'$	$ds_6 = 3.10$	$\theta_6 = 19.00$
$h_7 = 2.35'$	$ds_7 = 3.05$	$\theta_7 = 15.00$
$h_8 = 2.10'$	$ds_8 = 3.02$	$\theta_8 = 11.50$
$h_9 = 1.90'$	$ds_9 = 3.00$	$\theta_9 = 7.00$
$h_{10} = 1.50'$	$ds_{10} = 3.00$	$\theta_{10} = 3.00$
$h_{10.5} = 1.46'$	$ds_{10.5} = 3.00$	

All the remaining results of the arch are shown in tables 1-9 which are in the thesis in blueprinted form.

Table 1 Computations of Δ

1	2	3	4	5	6	7	8	9	10	11
Points	$h = A$	h^3	$I_c = \frac{h^3}{12}$	$\frac{h}{2}$	$d = .17'$ $\frac{h}{2} - d'$	$[\frac{h}{2} - d']^2$	$I_s = \frac{h^3}{12} - h A d'$ $[\frac{h}{2} - d']^2$	$I = I_c + I_s$	d_s	$\Delta = \frac{d_s}{I}$
0	3.65	48.5	4.04	1.82	1.65	2.72	1.135	5.175		
1	3.50	42.7	3.56	1.75	1.58	2.50	1.042	4.602	4.00	.870
2	3.30	36.0	3.00	1.65	1.48	2.18	.825	3.825	3.700	.967
3	3.00	27.0	2.25	1.50	1.33	1.78	.742	2.992	3.57	1.193
4	2.80	22.0	1.83	1.40	1.23	1.51	.629	2.459	3.45	1.400
5	2.50	16.0	1.33	1.25	1.08	1.17	.487	1.817	3.25	1.790
6	2.35	13.0	1.08	1.175	1.005	1.05	.437	1.517	3.10	2.044
7	2.10	9.3	.775	1.05	.88	.773	.322	1.097	3.05	2.780
8	1.90	6.84	.570	.95	.78	.608	.253	.823	3.02	3.680
9	1.70	4.90	.408	.85	.68	.462	.192	.600	3.00	5.000
10	1.50	3.35	.280	.75	.58	.336	.140	.420	3.00	7.150
									$1/2 \Sigma \Delta$	26.874

-46

Table 2 Computations of V_0

	12	13	14	15	16	17	18
Points	$Z-20$ where $Z = \frac{2x}{\Delta x}$	$Q = (Z-20)\Delta$	$-\sum_0^L (Z-20)\Delta$	$-\frac{1}{2} \sum_0^L (Z-\frac{20}{\Delta x})Q$	$Z^2 + (40-Z)^2$	$[Z^2 + (40+Z)^2]/\Delta$	V_0
0							
1	-19	-16.52	16.52	1755.93	1522	1325	1.000
2	-17	-16.42	32.94	1739.41	1378	1330	.983
3	-15	-17.90	50.84	1706.47	1250	1490	.965
4	-13	-18.20	69.04	1655.63	1138	1590	.935
5	-11	-19.70	88.74	1586.59	1042	1865	.895
6	-9	-18.40	107.14	1497.85	962	1970	.845
7	-7	-19.48	126.62	1390.71	898	2500	.786
8	-5	-18.40	144.02	1264.09	850	3130	.715
9	-3	-15.00	159.02	1120.07	818	4100	.634
10	-1	-7.15	166.17	961.05	802	5740	.544
		-167.17	961.05		$\sum Z^2 \Delta = 25040$		

$$V_0 = \frac{-\frac{1}{2} \sum_0^L (Z - \frac{20}{\Delta x})(Z-20)\Delta}{F}$$

$$F = \frac{1}{2} \sum Z^2 \Delta - 200 \sum \Delta = 1770.4$$

$$H_T = 20 \text{ cfs } E = \frac{345.60 \text{ ft}}{100.78}$$

$$= +10300 \text{ for } t = +30^\circ$$

$$-13810 \text{ for } t = -40^\circ$$

$$\text{Span} = L = 60'$$

$$dx = \frac{L}{20} = 3.00'$$

$$\text{Rise} = 10.0'$$

$$Z = \frac{2x}{dx} \quad \frac{\sum Y \Delta}{\sum \Delta} = \frac{226.725}{26.874} = 8.44$$

Table 3 Computation of H_0

19	20	21	22	23	24	25	26	27	28	29	30
Points	Y	Δ	$Y\Delta$	$Y - \frac{\sum Y\Delta}{\sum \Delta}$	$R = \Delta(Y - \frac{\sum Y\Delta}{\sum \Delta})$	$\sum_a^L \Delta(Y - \frac{\sum Y\Delta}{\sum \Delta})$	$\frac{1}{2} \sum_a^L (Z - \frac{\sum Z\Delta}{\sum \Delta}) R$	$Y\Delta(Y - \frac{\sum Y\Delta}{\sum \Delta})$	$\cos \phi$	$\frac{\cos \phi}{A}$	H_0
0	0			-8.440					.700		
1	1.12	.870	.975	-7.320	-6.37	6.37	0	-7.13	.732	.209	0
2	3.05	.967	2.950	-5.390	-5.21	11.58	-4.40	-15.90	.798	.242	.044
3	4.70	1.193	5.600	-3.740	-4.46	15.98	-15.98	-20.95	.838	.279	.158
4	6.10	1.400	8.540	-2.340	-3.28	18.32	-31.02	-20.00	.878	.314	.310
5	7.26	1.790	13.000	-1.180	-2.11	20.43	-49.34	-15.32	.916	.367	.491
6	8.18	2.044	16.740	-.26	-.53	20.96	-69.77	-4.35	.945	.402	.696
7	8.90	2.780	23.720	.460	1.28	19.66	-90.73	10.90	.965	.460	.905
8	9.45	3.680	34.800	1.010	3.72	15.94	-110.39	35.10	.984	.518	1.100
9	9.80	5.000	49.000	1.363	6.82	9.12	-126.33	66.70	.990	.582	1.252
10	9.98	7.150	71.400	1.540	11.02	-1.90	-135.45	110.00	1.000	.666	1.345
10 1/2	10.0	1/2 $\sum Y\Delta$	226.725	1.560	0	135.45	1/2 $\sum Z =$	139.05	1/2 $\sum Z =$	4.039	

$$H_0 = \frac{-\frac{1}{2} \sum_a^L (Z - \frac{\sum Z\Delta}{\sum \Delta})(Y - \frac{\sum Y\Delta}{\sum \Delta}) \Delta}{C}$$

$$C = \frac{1}{dx} \sum_a^L Y\Delta(Y - \frac{\sum Y\Delta}{\sum \Delta}) + \sum \frac{\cos \phi}{A}$$

$$= 100.78$$

Computation of Dead L.

Pl.	hds	150 hds	D.L.
1	14.0	2100	2100
2	12.2	1830	1830
3	10.7	1605	1605
4	9.65	1450	1450
5	8.12	1220	1220
6	7.27	1090	1090
7	6.40	960	960
8	5.74	860	860
9	5.1	765	765
10	4.5	675	675

Table 4 Computations for M_0

31	32	33	34	35	36	37	38	39	40	41
	Z	H ₀	V ₀	Δ	Σ ₀ Δ	$\frac{1}{2} \sum_0^L (Z \frac{dZ}{dx}) \Delta$	$\frac{dx}{\sum \Delta} (\text{col 37})$	$\frac{\sum Y \Delta H_0}{\sum \Delta}$	$-20 \frac{d}{2} V_0$	M_0
0	0									
1	1	0	1.000	.870	52.778	509.906	29.05	0	-30.000	-.950
2	3	.044	.983	.967	51.811	457.128	26.06	.372	-29.500	-3.068
3	5	.158	.965	1.193	50.618	405.317	23.10	1.330	-28.950	-4.520
4	7	.310	.935	1.400	49.218	354.699	20.20	2.620	-28.050	-5.230
5	9	.491	.895	1.790	47.428	305.481	17.38	4.140	-26.850	-5.330
6	11	.696	.845	2.044	45.384	258.053	14.70	5.870	-25.250	-4.680
7	13	.905	.786	2.780	42.604	212.669	12.10	7.640	-23.600	-3.860
8	15	1.100	.715	3.680	39.024	170.065	9.70	9.280	-21.450	-2.470
9	17	1.252	.634	5.000	34.024	131.041	7.46	10.580	-19.000	-.960
10	19	1.345	.544	7.150	26.874	97.017	5.52	11.350	-16.300	.570
10'	19	1.345	.456	7.150	19.724	70.143	4.00	11.350	-13.700	1.650
9'	17	1.252	.366	5.000	14.724	50.419	2.88	10.580	-11.000	2.460
8'	15	1.100	.285	3.680	11.044	35.695	2.03	9.280	-8.550	2.760
7'	13	.905	.214	2.780	8.264	24.651	1.40	7.640	-6.420	2.620
6'	11	.696	.155	2.044	6.220	16.387	.93	5.870	-4.650	2.150
5'	9	.491	.105	1.790	4.430	10.167	.58	4.140	-3.150	1.470
4'	7	.310	.065	1.400	3.030	5.737	.326	2.620	-1.950	.996
3'	5	.158	.035	1.193	1.837	2.707	.154	1.330	-1.050	.434
2'	3	.044	.017	.967	.870	.870	.050	.372	-.510	.088
1'	1	0	0	.870	0	0	0	0	0	0
0'	0									
		12.602	10.00				177.620	106.364	-299.930	-16.046

$V_0 = 1 - V_7 = 1 - .786 = .214$
 $M_0 = \frac{dx}{\sum \Delta} \frac{1}{2} \sum_0^L (Z \frac{dZ}{dx}) \Delta + H_0 \frac{\sum Y \Delta}{\sum \Delta} - 20 \frac{d}{2} V_0$
 $\frac{dx}{\sum \Delta} = \frac{3}{52.748} = .057 \quad -20 \frac{d}{2} = -20(\frac{3}{2}) = -30$
 $\sum 38 + \sum 39 + \sum 40 = \sum 41$
 $177.620 + 106.364 - 299.930 = -16.046$
 $-20 \frac{d}{2} (\sum 34) = \sum 40 = -300$
 $226.725 \times 12.602 = 106.36$
 $\frac{26.874}{12 \text{ sum of 11}}$
 For check: $\frac{1}{2} \sum 0' \text{ to } 22 \text{ sum } 33 - \sum 39$

Table 5 Computation of M_3

42	43	44	45	46	47	48	49	50
				Point 3 $Z_3 = 5$ $Y_3 = 4.7$				
	H_0	V_0	M_0	$V_0 Z_3$	$V_0 Z_3 - (Z_3 - \frac{20}{2})$	m_3	$-H_0 Y_3$	M_3
0								
1				5.00	1.0	1.500	0	.550
2				4.92	1.92	2.880	-.207	-.395
3						7.240	-.741	1.979
4						7.020	-1.460	.330
5	33	34	41			6.700	-2.310	-.940
6	same as column 5					6.340	-3.270	-1.610
7						5.900	-4.250	-2.210
8		"	"			5.360	-5.160	-2.270
9						4.750	-5.880	-2.090
10						4.080	-6.330	-1.680
10'						3.420	-6.330	-1.270
9'		"	"			2.740	-5.880	-.680
8'						2.140	-5.160	-.260
7'						1.600	-4.250	-.030
6'		"	"			1.160	-3.270	.040
5'						.786	-2.310	-.050
4'						.487	-1.460	.020
3'						.263	-.741	-.044
2'						.127	-.207	0
1'						0	0	0
0'								
	12.602	10.0	-16.046			64.493	-59.202	-10.61

$$M_x = M_0 + m_x - H_0 Y$$

$$m_x = \left[V_0 Z_x - \left(Z_x - \frac{20}{2} \right) \right] \frac{dx}{2} \quad \text{when } Z > \frac{20}{dx}$$

$$m_x = V_0 Z_x \frac{dx}{2} \quad \text{when } Z \leq \frac{20}{dx}$$

Check.

$$\begin{aligned} \text{Sum } 45 + 48 + 49 - 50 \\ -16.046 - 59.202 + 64.493 \\ = 10.7 \quad \text{O.K.} \end{aligned}$$

Table 6 Computation of M_8

	51	52	53	54	55
	Point 8. $Z_8 = 15$ $Y_8 = 9.45$				
	$V_0 Z_8$	$V_0 Z_8 - (Z_8 - \frac{20}{24})$	m_8	$-H_0 Y_8$	M_8
0					
1	15.00	1.0	1.50	0	.550
2	14.70	2.7	4.05	- .415	.567
3	14.47	4.47	6.70	- 1.492	.688
4	14.00	6.00	9.00	- 2.930	.840
5	13.40	7.40	11.10	- 4.640	1.130
6	12.68	8.68	13.00	- 6.560	1.760
7	11.80	7.80	11.70	- 8.550	-.710
8			16.10	-10.400	3.230
9			14.25	-11.830	1.460
10			12.20	-12.700	.070
10'			10.25	-12.700	-.800
9'			8.24	-11.830	-1.130
8'			6.18	-10.400	-1.460
7'			4.81	- 8.550	-1.120
6'			3.48	- 6.560	-.930
5'			2.36	- 4.640	-.810
4'			1.46	- 2.930	-.474
3'			.78	- 1.492	-.278
2'			.38	- .415	-.123
1'			0	0	0
0'					
			143.11	-119.034	7.030

Check: Sums 45 + 53 + 54 = Sum of 55
 - 16.046 + 143.11 - 119.034 = 7.03 O.K.

Table 7 Computation of $M_{10\frac{1}{2}}$

	56	57	58	59	60
	Point $10\frac{1}{2}$ $Z_{10\frac{1}{2}} = 20$ $Y_{10\frac{1}{2}} = 10$				
	$V_0 Z_{10\frac{1}{2}}$	$V_0 Z_{10\frac{1}{2}} - (Z_{10\frac{1}{2}} - 20)$	$m_{10\frac{1}{2}}$	$-H_0 Y_{10\frac{1}{2}}$	$M_{10\frac{1}{2}}$
0					
1	20.000	1.000	1.500	0	.550
2	19.680	2.680	4.020	- .440	.512
3	19.300	4.300	6.450	- 1.580	.350
4	18.700	5.700	8.550	- 3.100	.220
5	17.900	6.900	10.320	- 4.910	.080
6	16.900	7.900	11.840	- 6.960	.200
7	15.720	8.720	13.090	- 9.050	.180
8	14.300	9.300	13.950	- 11.000	.480
9	12.680	9.680	14.510	- 12.520	1.030
10	10.890	9.890	14.830	- 13.450	1.950
10'			13.680	- 13.450	1.880
9'			10.980	- 12.520	.920
8'			7.550	- 11.000	-.690
7'			6.420	- 9.050	-.010
6'			4.650	- 6.960	-.160
5'			3.150	- 4.910	-.290
4'			1.950	- 3.100	-.154
3'			1.050	- 1.580	-.090
2'			.510	- .440	-.018
1'			0	0	0
0'					
			149.000	- 126.020	+ 7.010

Check: $\text{Sum}_5 + 58 + 59 = \text{Sum } 60$
 $- 16.046 + 149.000 - 126.020 = 6.94 \text{ O.K.}$

Table 8 Computations of H, V, and M for D.L.

61	62	63	64	65	66	67	68	69	70	71	72	73	74
	Unit Loads						Dead	Dead Loads					
	H ₀	V ₀	M ₀	M ₃	M ₈	M _{10%}	Load	H ₀	V ₀	M ₀	M ₃	M ₈	M _{10%}
1							2100	0	2100	-1995	1154	1154	1154
2							1830	80.5	1800	-5620	-723	1040	936
3							1605	253.0	1550	-7250	3180	1105	562
4							1450	450.0	1355	-7570	478	1220	319
5	33	4	1	5	5	0	1220	600.0	1090	-6500	-1150	1380	98
6	33	3	4	5	5	6	1090	758.0	920	-5100	-1750	1920	218
7	column						960	870.0	755	-3710	-2120	-680	173
8							860	945.0	615	-2130	-1950	2780	413
9		"	"	"	"	"	765	957.0	485	-735	-1600	1120	787
10							675	907.0	367	385	-1130	47	1320
10'							675	907.0	308	1114	-855	-540	1270
9'	0.5	"	"	"	"	"	765	957.0	280	1880	-520	-864	705
8'	same						860	945.0	245	2380	-224	-1260	-595
7'							960	870.0	206	2520	-29	-1075	-10
6'		"	"	"	"	"	1090	758.0	169	2340	44	-1020	-175
5'	5						1220	600.0	128	1790	-61	-987	-354
4'							1450	450.0	94	1445	29	-687	-224
3'							1605	253.0	56	696	-70	-446	-145
2'							1830	80.5	31	-161	0	-225	-33
1'							2100	0	0	0	0	0	0
	12.602	10.0	-31.156	-13.529	+14.865	8.352	25710	11641.0	12554	-40610	-12182	11766	7955
			15.11	2.919	-7.835	-1342				+14550	+4885	-7784	-1536
			-16.016	-10.610	7.030	7.010				-26221	-7297	3982	6419

Table 9 Computation of Maximum Stresses

75	76	77	78	79	80	81	82	83
		H	M	V	Hcos φ	Vsin φ	N	Computation of Unit Stresses, f_c and f_s
Point 0 $\sin \phi = .668$ $\cos \phi = .700$ $h = 3.65$	D.L.	11641.0	- 26221	12554	8150	8550	16700	$-M$ shows stress is max. in Extrados $x_0 = \text{eccentricity} = \frac{342720}{41100} = 8.5' = 102''$ $\frac{h}{x_0} = \frac{3.65}{8.5} = .43$ $p = \frac{1.20}{144 \times 3.65} = .0022$ Use Diag. 5 $\frac{N x_0}{f_c b h^2} = .2$ $\frac{t}{x_0} = \frac{h}{x_0} = .43$ $f_c = 980$ in Extrados $f_s = n f_c \left(\frac{h-d''}{k h} - 1 \right) = 18000$ $K = .03$
	+C.L.L.	21000	58000	5500	14700	3680	18380	
	+C.D.L.	99200		69300	69300	46000	115000	
	-C.L.L.	21000	- 92000		14700		14700	
	-C.D.L.		-101000					
	+T	+10300	86800	0	7200	0	7200	
	-T	-13810	-116500	0	-9700	0	-9700	
	-M		-342720				-41100	
	+M		144800				140580	
Point 8 $\sin \phi = .20$ $\cos \phi = .984$ $h = 1.90$	D.L.	11641	3982	1440	11400	288	11688	$-M$ $x_0 = \frac{20800}{155648} = .134'$ $\frac{h}{x_0} = \frac{1.9}{.134} = 14.2$ $p = \frac{1.20}{144 \times 1.9} = .0044$ $15 \times .0022 = .033$ $\frac{N x_0}{f_c b h^2} = .09$ $f_c = 445$ $K = 1$ $f_s = 0$ Extrados
	+C.L.L.	20900	46400	-15800	20500	-3160	17340	
	+C.D.L.	111600	95300	35000	109500	7000	116500	
	-C.L.L.	20900	-10400	5550	20500	1110	21610	
	-C.D.L.							
	+T	+10300	+13950	0	10120	0	10120	
	-T	-13810	-10400	0	-13600	0	-13600	
	-M		-20800				155648	
	+M		+159632				80100	
								$+M$ $x_0 = \frac{159632}{8010} = 1.9$ $\frac{h}{x_0} = \frac{1.9}{1.9} = 1$ $\frac{N x_0}{f_c b h^2} = .06$ $f_c = 490$ $K = .02$ $f_s = 3700$ Intrados

$$N = H \cos \phi + V \sin \phi$$

Temperature Mom.

$$M_t = -H_t \left(Y - \frac{\sum Y \Delta}{\sum \Delta} \right)$$

For H_t see table 2

$$Y - \frac{\sum Y \Delta}{\sum \Delta} \text{ from col 23}$$

Point

$$0 \quad M_t = - \left\{ \begin{matrix} 10300 \\ -13810 \end{matrix} \right\} \times (-8.44) = \left\{ \begin{matrix} +86800 \\ -116500 \end{matrix} \right\}$$

$$8 \quad M_t = - \left\{ \begin{matrix} 10300 \\ -13810 \end{matrix} \right\} \times (1.01) = \left\{ \begin{matrix} -10400 \\ +13950 \end{matrix} \right\}$$

PART III

ECONOMICAL DISCUSSION

REINFORCED-CONCRETE BEAM & ARCH
BRIDGES.

ECONOMIC ANALYSIS AND TYPE SELECTION

Type selection is unquestionably the highest, most difficult and most important feature of bridge engineering from start to finish. Millions of dollars can be and have been wasted through improper type adaptation resulting in unwarranted first costs, or needless maintenance expenses. Correct type of selection is the very corner stone of economy. A failure to recognize the principles involved, or to evaluate correctly the factors entering into the problem may frequently result in a waste many times greater than any saving which may result from refinements in stress analysis and design. It is the truth that type selection calls for the exercise of the rarest judgement, tempered by long experience in the design, construction, maintenance and operation of bridges under a wide variety of conditions and it is also true that as a general rule, nothing but time will give the bridge engineer the maturity of judgement needed. It is quite possible however, to analyze this problem, to separate it, as it were, into its component parts, to state certain fundamental fundamentals and submit certain data which may aid in forming judgements as to probable first costs maintenance costs, renewal costs, etc., for the various construction types commonly employed. After the preliminary surveys have been computed, for any bridge structure and before any work can be done on the detailed design or preparation of plans, it becomes necessary to make at least a tentative selection of the type of construction, best suited to the particular needs involved. The question of economy in first cost, maintenance, and renewals is naturally a major controlling consideration and one which in order of importance should possibly, receive first mention. The economics of any bridge

problem however, are generally investigated after certain other controlling conditions have received consideration. The major controlling factors are conveniently grouped as follows:

- A- Stream behavior.
- B- Requirements of navigation.
- C- Traffic considerations.
- D- Architectural features and scenic considerations.
- E- Condition of available funds.

The term "stream behavior" is here used to signify the peculiar characteristics of the waterway during periods of high water as regards erosion of bed and banks, lateral shifting of channels, carriage of drift, ice and debris, etc. Such characteristics many times operate to place certain limits upon type selection entirely independent of considerations of economy and it is with such tendencies that this article has to deal.

The second factor, namely, requirements of navigation, will generally affect type selection; as regards both vertical and horizontal clearances for the main channel span. Where movable spans are used, the type of design for the moving leaves may also be controlled by considerations of water traffic.

Factor D in the above list, which is the architectural feature and scenic considerations, has in certain cases a very important role in dictating type selection. Grouped in order of their architectural possibilities bridge types may be classified, as follows:

- a) Masonry arch construction.
- b) Reinforced concrete deck construction.
- c) Deck truss or plate girder construction with concrete deck and railing.

- e) Through truss or girder construction
- f) Timber construction

The last factor listed as controlling the economics of any bridge problem was the condition of available funds. In certain instances the choice between types may hinge upon the amount of money actually available for construction purposes and thus render of no avail any theoretical consideration of the economics of first cost, maintenance, renewals, etc. Perhaps the selection of a cheaper construction type even, in this case, may be false economy, but it is apparent that if there are no additional funds available and no legal machinery provided for the borrowing of the same, the garment must, to a certain extent, be cut in accordance with the cloth.

Having disposed of the general features controlling type selection for highway bridges, the question of economic analysis may now be considered.

FUNDAMENTALS OF ECONOMIC ANALYSIS

Source of funds for highway bridge improvements

Funds for the construction of highway bridges are in general derived from two main sources, as follows:

- a) From direct revenue (property tax, license fees, gasoline tax, etc.)
- b) By borrowing (issuing bonds).

In either case a fund is created termed the "Capital Account" out of which the money necessary for construction can be drawn.

Capital costs

Let us consider the case of a bridge costing C dollars, built of absolutely permanent construction and upon which there is no need for maintenance expenditures. The capital account is charged with C dollars,

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withdrawn but credited with a bridge worth C dollars, so that the balance remains unchanged. In other words, the total wealth of the state is in no way changed by exchanging C dollars in money to C dollars in bridge construction. However, the state has by no means received a big free, owing to the fact that the C dollars in liquid funds had an earning of rC capacity per annum, r being the interest rate, while the C dollars in the bridge has no earning capacity. The net result of the state therefore, is the gain of a bridge but the loss of rC dollars interest money per year. The capital cost of any bridge structure therefore, can be represented by an annual charge representing the interest on the amount expended; thus for permanent construction without maintenance and costing C dollars, the annual expense is equal to rC dollars.

Maintenance and Renewal Costs

It is apparent that no bridge can be built satisfying the above requirements. Regardless of its excellence, some money for maintenance is required and at some future period the structure is bound to be worn out and need complete renewal.

If we assume that the average maintenance cost per year is estimated M dollars and that the profitable life of structure is equal to n years, the amount of money R , which must be deposited at the end of each year to accumulate with the compound interest at $r\%$ per year, an amount equal to C dollars in n yearst time is given by the expression

$$R = \frac{Cr}{(1 + r)^n - 1}$$

The term R obviously represents the measures of the renewal charges against the structure. The total annual cost for capital, maintenance and renewal is therefore represented by the expression

$$E = rC \div M \div R$$

Gillette's Handbook of Cost Data and other handbooks contain tables which aid in the calculation of annual expenses of renewal funds, giving the amount accumulated when one dollar is deposited annually in a fund drawing compound interest at the rates from three to ten percent and for time periods one to fifty years.

Insurance Costs

Fire insurance in highway bridges is only necessary when the bridge is of timber construction. In certain cases the danger from flood loss is so great as to warrant the consideration, an annual item for flood insurance. In general this item is not considered except when it becomes necessary to determine the relative economy of two type of constructions which differ in regard to their respective liabilities for flood loss. In such a case the proper annual charge for flood insurance can be determined from the probable frequency of flood losses in connection with the probable service life of the structure by a consideration of the theory of probabilities.

Operation Costs

In addition to the foregoing the last item of annual expense to be considered is that of operation. Operation expenses may be divided into two main classes as follows:

- a) Operation of the bridge.
- b) Traffic operation.

The first operating cost mentioned above is simply an annual charge occurring in the case of movable bridges or in connection with the operation of crossing gates, the employment of watchmen, etc. The second operating cost is the cost to the traffic operating over the bridge.

This cost can be determined from the length of the bridge, the traffic density and the unit traffic cost, which latter value may be expressed in terms of either vehicle mile or ton-mile units. The latter is the more accurate but the former lends itself more readily to this type of economic analysis since the use of the ton-mile requires that the traffic units be segregated. The vehicle mile unit will therefore, be used here although the ton mile may be used if desired.

As an illustration of the method of arriving at traffic operation costs consider a bridge 1000 feet in length carrying an average traffic of 1660 vehicles per day. At an average cost of 8 cents per vehicle mile the total annual operation costs

$$\frac{(1000)}{(5280)} (\$0.08) (1000) (365) = \$553.30$$

It is true that the cost of traffic operation is paid for out of a different fund than that from which other annual costs are paid, it is however, none the less a legitimate charge against the structure and should be considered in any economic comparison.

The cost per vehicle mile varies with a large number of factors and data at hand are not sufficient to place anything more than a very rough estimate of evaluation upon the same. The following discussion of transportation costs, as affected by bridge design may prove enlightening, in so far as general principles are concerned, even though no definite or accurate quantitative conclusions are reached. Among the factors involved in bridge construction which affect transportation costs may be mentioned the following:

- a) Character of roadway surface.
- b) Width of roadway.

c) Horizontal alignment of structure.

d) Grade line treatments.

The character of the roadway surface (a) adopted for the bridge affects transportation costs in a variety of ways. First of all, a rough roadway surface increases the rolling resistance of the vehicle and in this measure operates to increase the fuel consumption. The presence of a rough surface also increases the tire expense and in addition, introduces certain impact and vibration strains in the vehicle itself, which result in a faster depreciation of the equipment. These vibrations are not altogether a function of the bridge floor surface, but may result from a vibration set up throughout the entire superstructure during the passage of a load.

The effect of an unduly restricted roadway is a slackening in the average speed of traffic movement and a consequent increase in time element transportation costs. Narrow roadways also operate to introduce greater liability of accident, not only major accidents, but also injury to fenders, bumpers and other like minor damages growing out of the closely restricted clearance between vehicle and rail.

The effect of sharp curvature is to increase the liability of collision, to slacken the average of safe traffic speed and to introduce an element of added wear on tires due to longitudinal and lateral slippage which results. A general stress upon the entire vehicular mechanism also results from travel over an alignment having undue curvature.

The introduction of grades operates to increase the rolling resistance of the vehicle and therefore, the fuel consumption. Sharp changes in the direction of grade, unless modified by long vertical curves, introduce an added impact, both to the structure and to the vehicle. Conditions of this kind are generally observed either at grade vertices or at the end of the structure

where the same joins the roadway approach,

In general it may be stated that any condition which will impair the riding qualities of the bridge roadway surface, operates to increase transportation costs.

ECONOMICS OF REINFORCED--CONCRETE BRIDGES

Reinforced concrete structures being the most modern of all the general types of bridge construction, the economics of their designing is not so highly developed as compared with the older types. In studying their economy, it is convenient to divide it to different topics and discuss them one by one.

REINFORCED STEEL

One of the important points in the designing specifications for reinforced concrete bridges is the proper intensity of working stress for the reinforcing concrete bars. It is generally conceded that it should not exceed one-half of the elastic limit of the material, and in consequence, the practice in engineering in the past has limited it to 18,000 lbs./in.² but using a higher carbon-steel many manufacturer's are trying to raise it to 20,000 lbs./in.². The higher intensity saves in the quantity steel, but generally increases the amount of concrete used. That is due to the fact that the higher stress in the steel reduces the moment of resistance of the concrete about 6 percent. If the amount of concrete is increased, the net saving is about 2 percent in slabs and 3 percent in beams; but if the section of the concrete is determined by shear or other considerations, so

that no increase is necessary, those percentages will be increased by unity. There are two great objections to use higher steel. The first one is that when bent cold it is liable to crack on account of its increased hardness. The second it opens up cracks in the concrete.

INTENSITY OF WORKING STRESS IN CONCRETE

For many years the practice has been to stress the concrete in compression only 600 lbs./in²., but lately the Joint Committee of Technical Societies has reported in favor of adopting 650 lbs./in². When a good aggregate is procurable, there is no objection to this increase of 8 percent. The actual reduction in the amount of concrete of a beam due to this difference of intensity of working stress is about 6 percent, but this is partially offset by the increase in the amount of steel required.

PAVINGS

With a concrete base any desired type of paving can be employed as wood blocks, brick, asphalt, or any other kind of bituminous paving, plain concrete or granitoid. Wood block is the most expensive as far as first cost is concerned, but it makes a much better showing in the comparison when maintenance and renewals are considered. Brick per se is less expensive, but it is heavy and in consequence, requires more metal to carry it. This is not a serious drawback in short-span bridges, but on long-span ones it is almost prohibitory.

Asphalt and bituminous bridges in general are good; and usually they are not heavier than the wooden-block ones. Unfortunately they require an extensive plant to lay them; and as the total area of paved surface on most bridges is comparatively small, the charge per square yard for use of plant will be excessive, unless there be a nearby plant available. To adopt an asphalt or bithulitic paving on a bridge in a small town is,

for that reason rarely economic practice. This difficulty however, can be overcome by adopting an asphalt block pavement, which requires no plant for its construction.

A concrete wearing-surface in many cases is both satisfactory and comparatively inexpensive, for it requires no special plant to lay it; nevertheless an extra hard and durable aggregate is obligatory, and the concrete must be carefully mixed, placed, and finished, and must be kept properly wet while curing, especially in hot, dry weather. Unless these precautions be observed, the concrete pavement will not prove economic because of short life and the expense of repairs and replacements. It will be found advisable to design with an allowance in dead load for an extra two inches of concrete, so that a thicker wearing surface may be put on, if ever desired, without overloading the floor-system or the trusses.

DESIGNS

The economics of design are rather difficult to determine, as the quantities involved are influenced quite largely by the individual tastes of the designer. The problem is also complicated by the facts that the unit costs of the various portions of a structure may be more or less different, and that the unit costs of different types of construction may be decidedly unlike. In general, it may be said that the unit costs are lower for those structures which have the simplest form-work; and a reduction will also be effected by decreasing the area of form-surface per cubic yard of concrete. For instance, in the case of a wall or slab, the form-cost per cubic yard will vary practically inversely as the thickness of the said wall or slab. Evidently therefore, it is desirable to concentrate the concrete into a few large members, rather than to employ a great number of small ones. The economics of the designing of the different parts of reinforced concrete struct-

ures are discussed in logical order below.

SLABS

A primary economic problem in slab designing is that of two-way versus one-way reinforcing. Two-way reinforcing involves less concrete but more steel than does one-way reinforcing; hence it has but little advantage unless the reduction of dead-load to a minimum be of prime importance.

Barring most of those in railway bridges, slabs are usually continuous over panel points, excepting at the expansion points. There is but a little difference in the actual costs of continuous and non-continuous slabs but continuity is desirable from the standpoint of paving and drainage; also with continuous slabs T-beams construction can be employed. The continuity of slabs and girders complicates construction problems sometimes very seriously. The various processes of the construction of a proposed design should be studied through completely in order to make certain that no impracticable or unnecessarily expensive work is involved.

GIRDERS

Girders are of two main types, single or continuous; and there is no great difference in their costs, there being more concrete but less steel in their spans, of the single-span type. The two-span continuous type is nearly always more expensive than the single-span type. Comparing simple girders and continuous ones having three or more spans, the following observations may be made:

If there is no T-beam action, the simple spans will be the more expensive; but if the bottoms of all girders are curved, the continuous girders will be cheaper there being decidedly much less concrete required for them.

The character of the foundations should be duly considered in deciding between simple and the continuous girders; for if there is a danger of settlement, the simple-girder is the preferable type - in fact it is obligatory.

COLUMNS

Columns are generally square or rectangular in cross-section for constructive or architectural reasons. A round or octagonal column is in reality a better structural member; and if the lines of the bridge are also worked in accordance with it, there should seldom be any difficulty in the matter of appearance. A round column can be hooped or banded better than any other type. Frequently for the sake of appearance, the size of the column must be made greater than that necessitated by theoretical requirements.

FOOTINGS

Footings may be either plain or reinforced, and the question as to which style to adopt is one solely of economics, because as they are buried out of sight, the consideration of appearance will not apply. If the area of the footing is a little larger than that of the column supported, plain concrete will be cheaper; while for a spread foundation reinforced concrete will be always found to be more economical. If a footing has to be poured under water, plain concrete should invariably employed.

Plain footings are made of 1 : 3 : 5 concrete or sometimes 1 : 3 : 6 but the latter, is considered too weak. The use of 1 : 2 : 4 concrete permits thinner footings, but this is not of much importance when plain concrete bases are used.

In respect to the economics of girder bridges resting on columns, the following points must be considered: 1. The panel length; 2. The number and spacing of the longitudinal girders; 3. The number of columns per

bent; 4. The span length.

ARCH BRIDGES

The economics of arch bridges are much more complicated than those of girder bridges. The important factors to be considered are the costs of arch ribs and those of the abutments; and the main economic point to determine is the ratio of rise to span-length. For any fixed span length, the greater the rise the smaller will be the cost of both arch ribs and abutments. By increasing further, the cost of the rib will be but little augmented whereas the cost of the abutments above the springing will be increased while the part below will be decreased. If the increase in rise is secured by lowering the springings, the greater the rise the greater the economy of material and cost; but if the increase must be secured by raising the grade, the springing remaining at a fixed elevation, it will rarely be economical to increase the rise above the limit of one-third of the opening.

The principle economic problem in the arch bridges is the determination of the best span-length. The principle factors to be considered are the following:

- A. The rise of the arch.
- B. The distance from springing to bottom of base.
- C. The character of the substructure work.
- D. The massiveness or lightness of the piers, determined from the aesthetic viewpoint.

The rise of the arch is evidently of greatest importance, because the greater it is, the greater will be the economic length of the span. The distance from springing to bottom of base is another very important factor. In general, for ribbed arches, when the adjoining spans are of

equal length and when the springings are a short distance above the bottom of the base, a ratio of rise to span length of one-third or even less will be quite economic; while if the springings are a considerable distance above the bottom, a ratio of one-half will be better. Generally speaking, low ratios of rise to span are more pleasing to the eye than higher ones, so that the use of longer spans is preferable from the view of appearance. Also longer spans involve larger members, and consequently lower unit costs.

Difficult foundations favor long spans, not only because in the reduction of the number of piers but also because the unit costs of small piers is much higher than those for large ones. On the other hand, if the foundations are very deep, the effect of unbalanced thrust becomes of great importance, and this favors shorter spans.

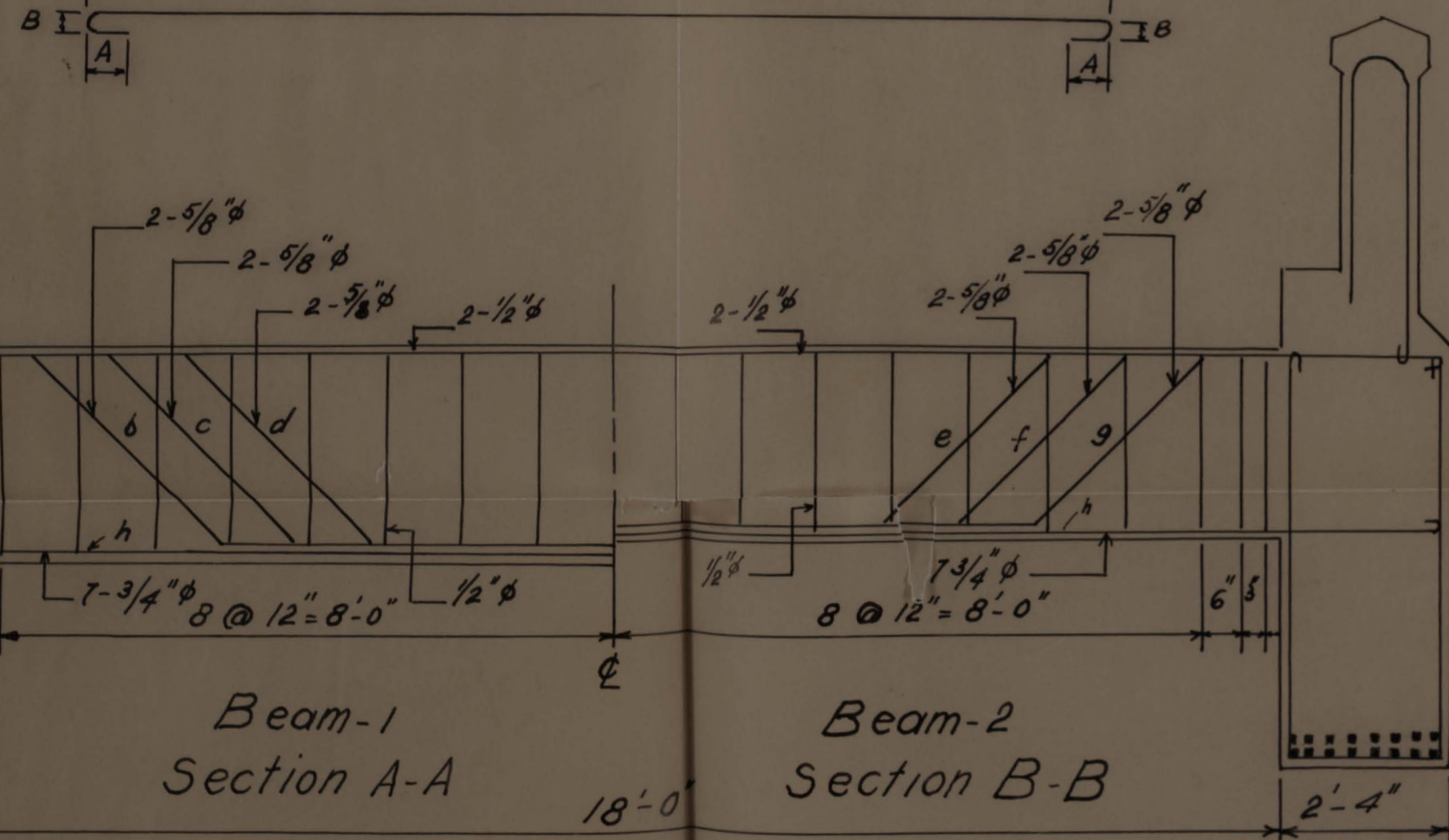
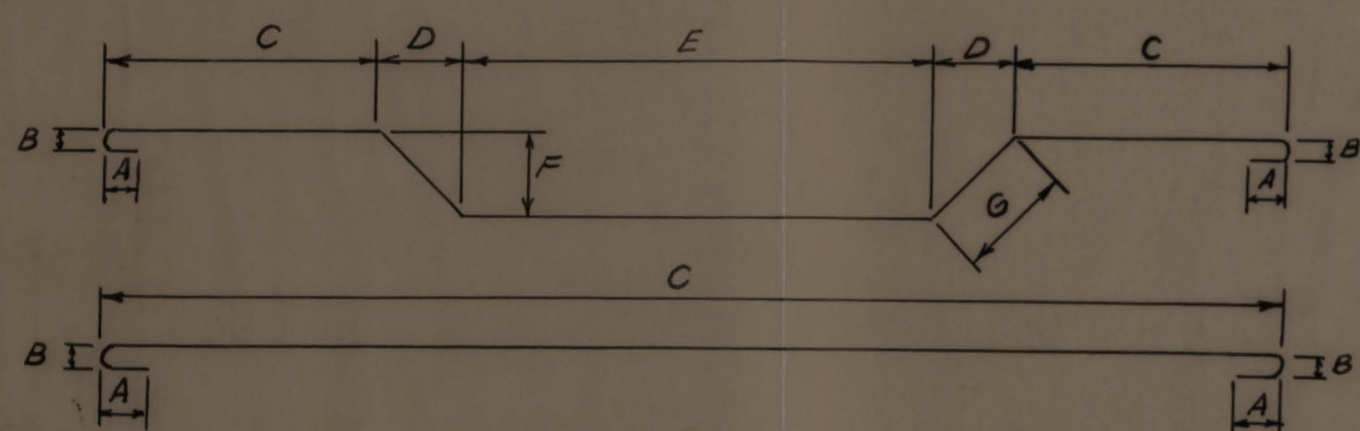
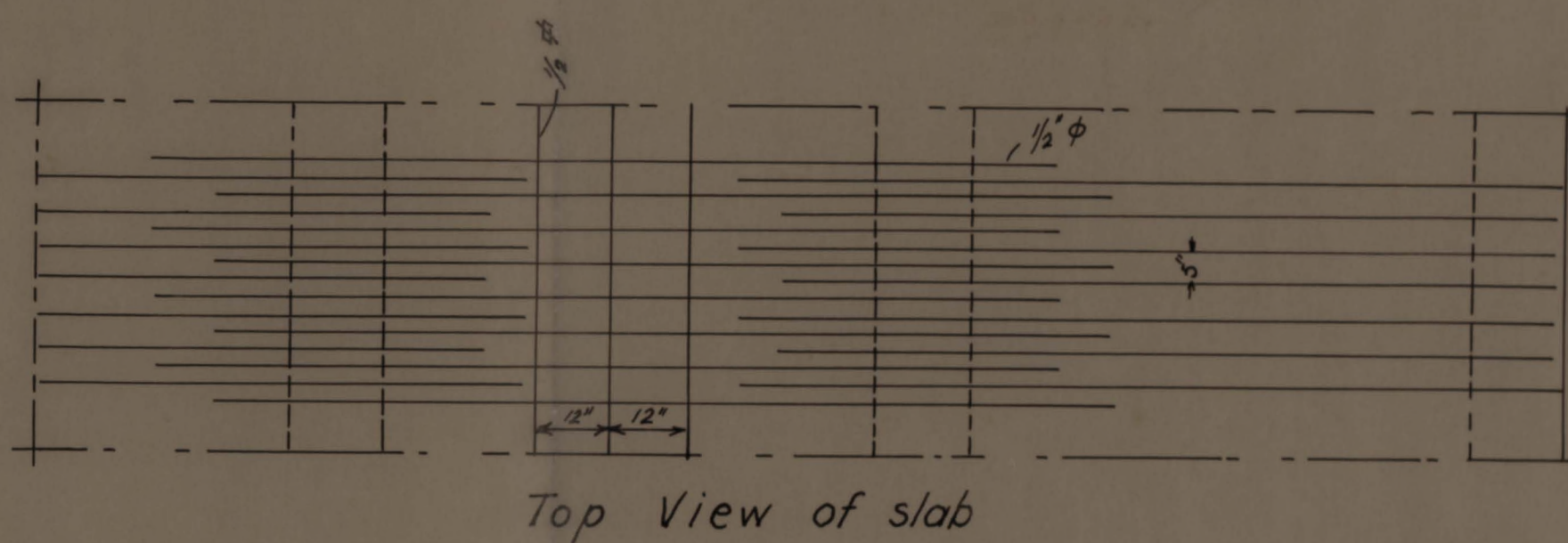
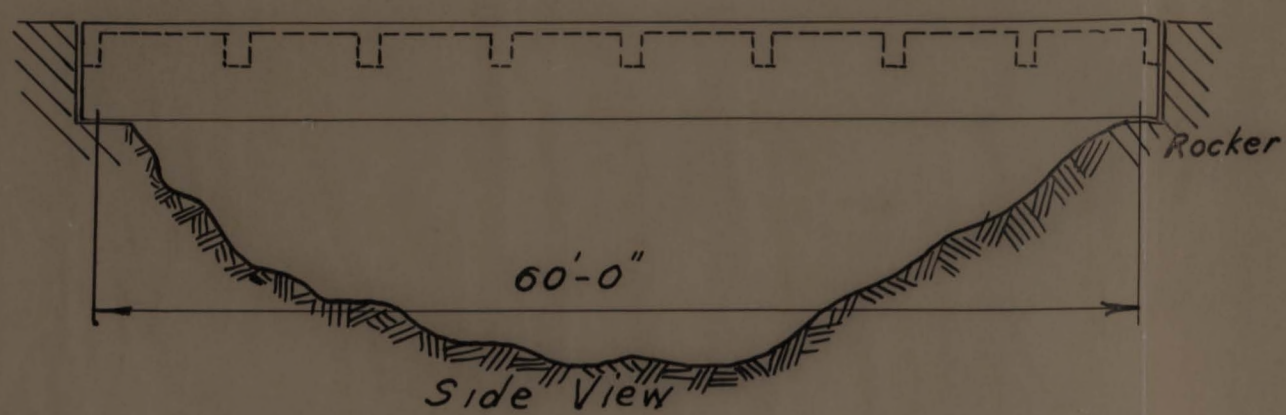
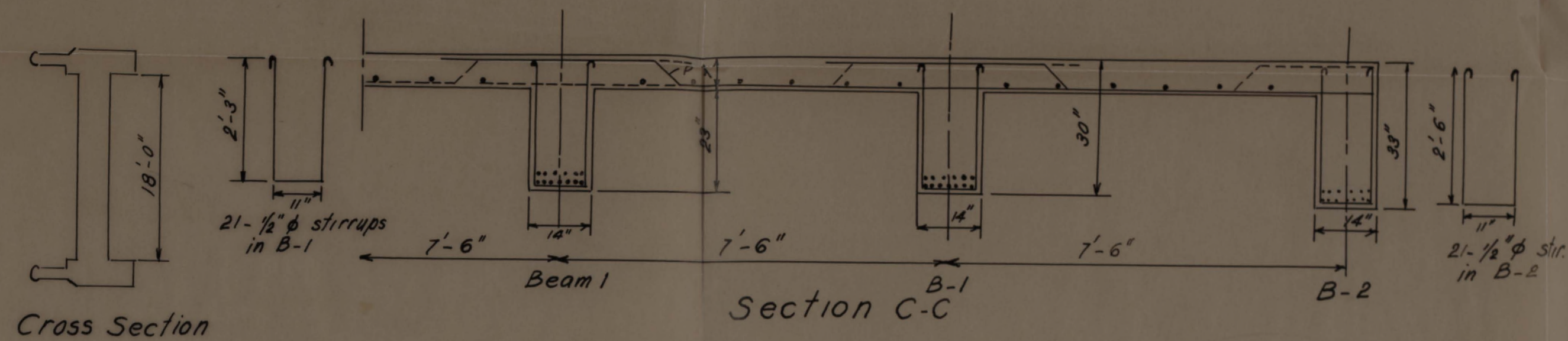
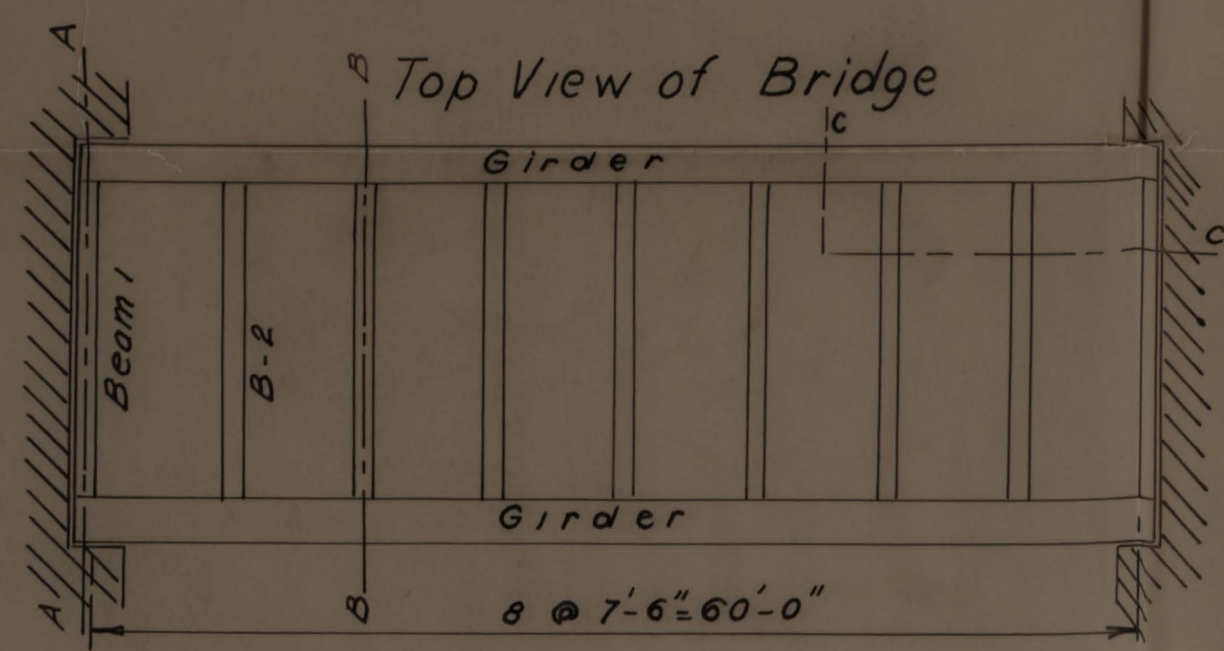
If it be decided for the sake of appearance to make the piers heavy and massive, this will tend towards greater span length; because an increase in span length will augment the size of each individual pier but little, if any. It will rarely pay to reduce the span length, if such reduction will not decrease the size of the pier or piers.

COMPARISON OF SOLID-SPRANDEL WITH OPEN-SPRANDEL

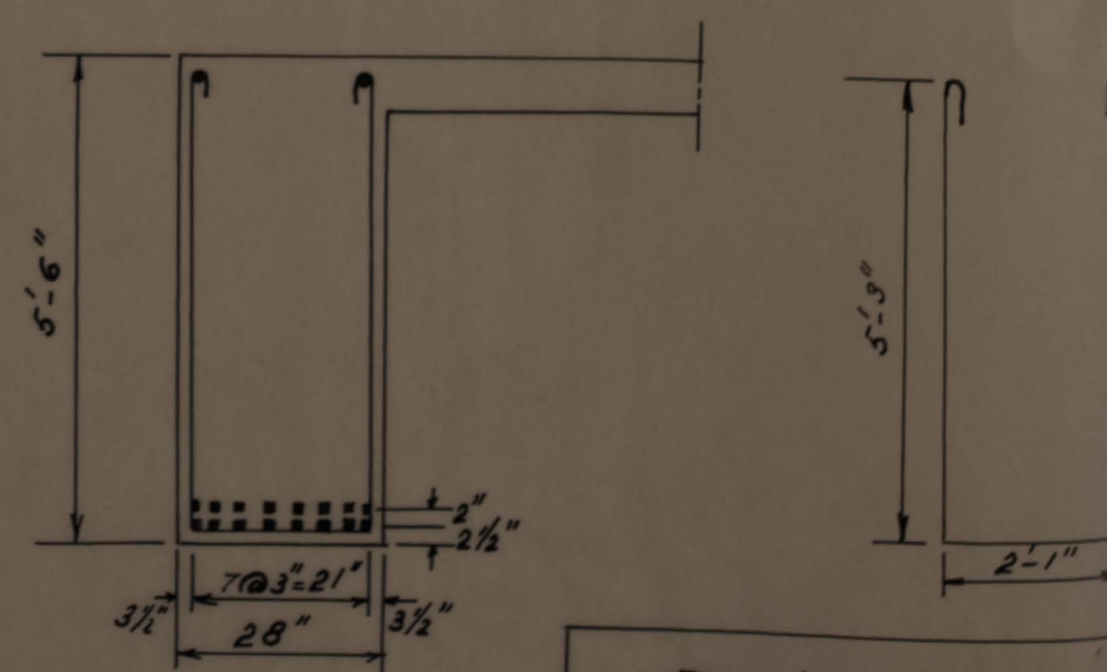
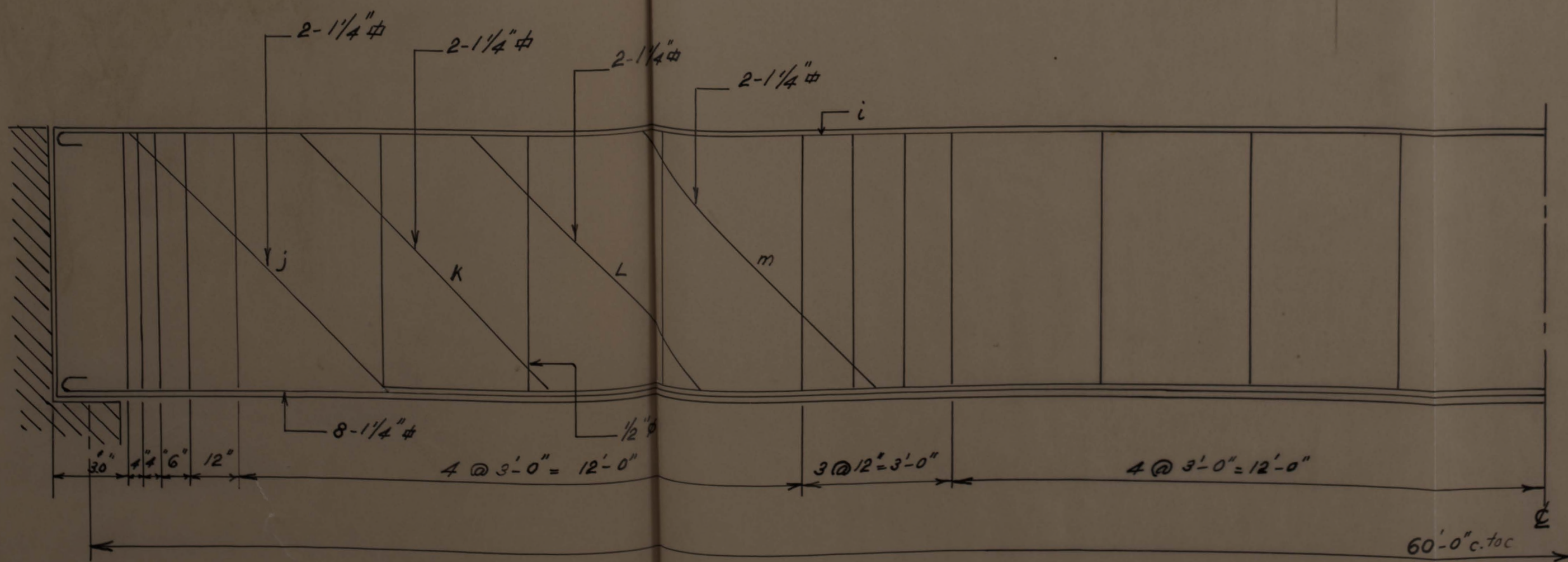
In highway structures, the open-sprandel type is generally preferred to the filled type, for various good and economic reasons, as it is a fact that with the latter type, it will be found desirable for the sake of appearance, to make the ring the full width of the deck; whereas for the former type it will be satisfactory to carry a part of the deck on cantilevers. The consequent narrowing of the arch-rings and shortening of the piers involve quite a saving in cost.

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DRAWINGS .

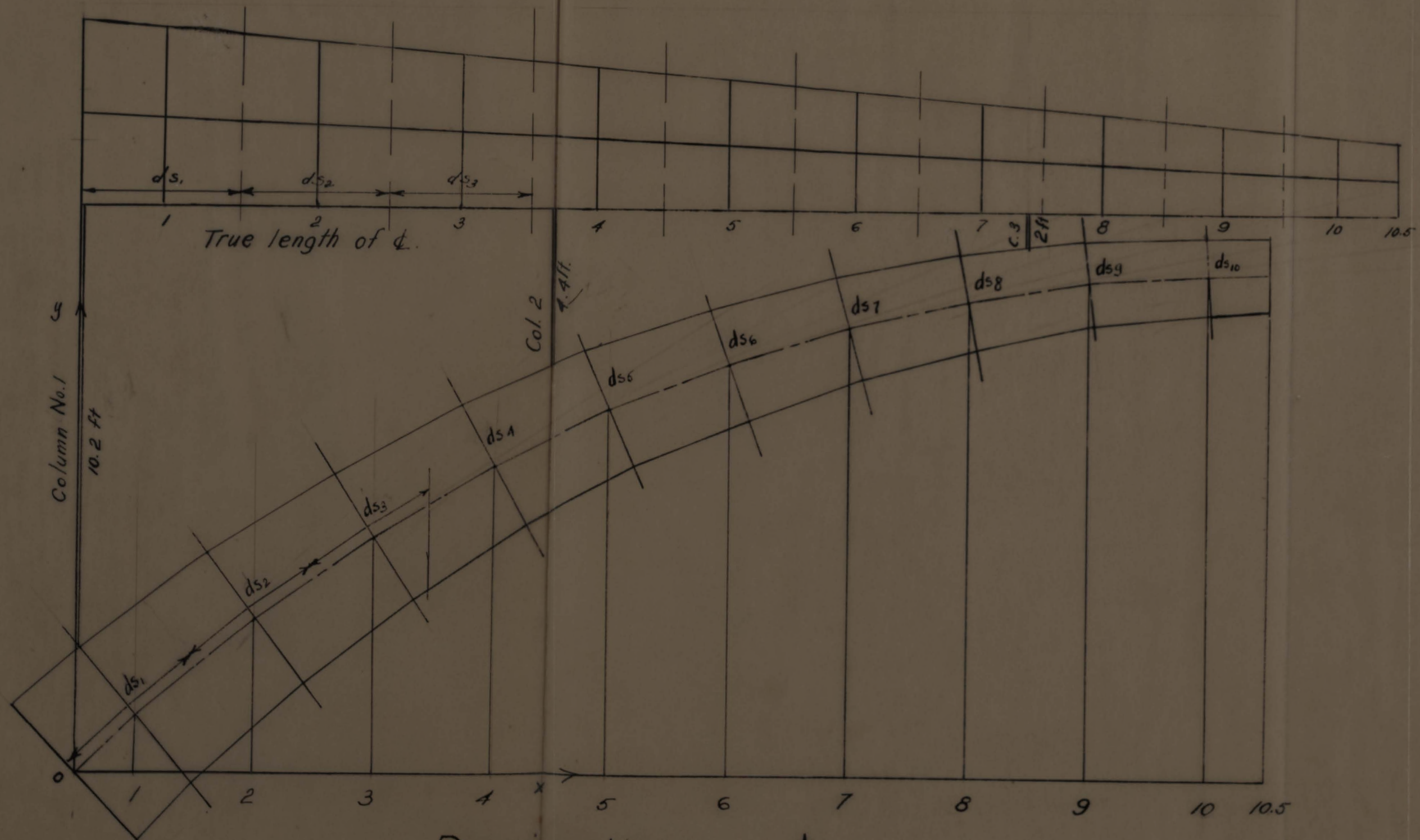


Mark	In Beam or Girder	No. of bars in each Beam or Girder	Total No. of Bars	Size	A	B	C	D	E	F	G	Total Length
a	B ₁ B ₂	2	16	1/2"φ	4"	4"	22'-4"	—	—	—	—	23'-8"
b	B ₁	2	4	5/8"φ	6"	6"	3'-8"	2'-3"	10'-6"	2'-3"	3'-2"	26'-0"
c	B ₁	2	4	5/8"φ	"	"	4'-8"	"	8'-6"	"	"	"
d	B ₁	2	4	"	"	"	5'-8"	"	6'-6"	"	"	"
e	B ₂	2	14	"	"	"	3'-8"	2'-0"	11'-0"	2'-0"	2'-10"	"
f	B ₂	2	14	"	"	"	4'-8"	"	9'-0"	"	"	"
g	B ₂	2	14	"	"	"	5'-8"	"	7'-0"	"	"	"
h	B ₁ B ₂	7	63	3/4"φ	"	"	22'-4"	—	—	—	—	24'-4"
i	Girder	2	4	1"φ	10"	10"	62'-8"	—	—	—	—	66'-0"
j	"	2	4	1 1/4"φ	"	"	1'-6"	5'-0"	49'-8"	5'-0"	7'-0"	70'-0"
k	"	2	4	"	"	"	5'-0"	"	42'-8"	"	"	"
l	"	2	4	"	"	"	8'-6"	"	36'-8"	"	"	"
m	"	2	4	"	"	"	12'-0"	"	28'-8"	"	"	"
n	"	8	16	"	"	"	62'-8"	—	—	—	—	66'-0"
P	Slab	5 spacing	150	1/2"φ	5"	5"	2'-0"	5"	7'-0"	5"	7"	3'-7"



Design of Reinforced Concrete Highway Bridge

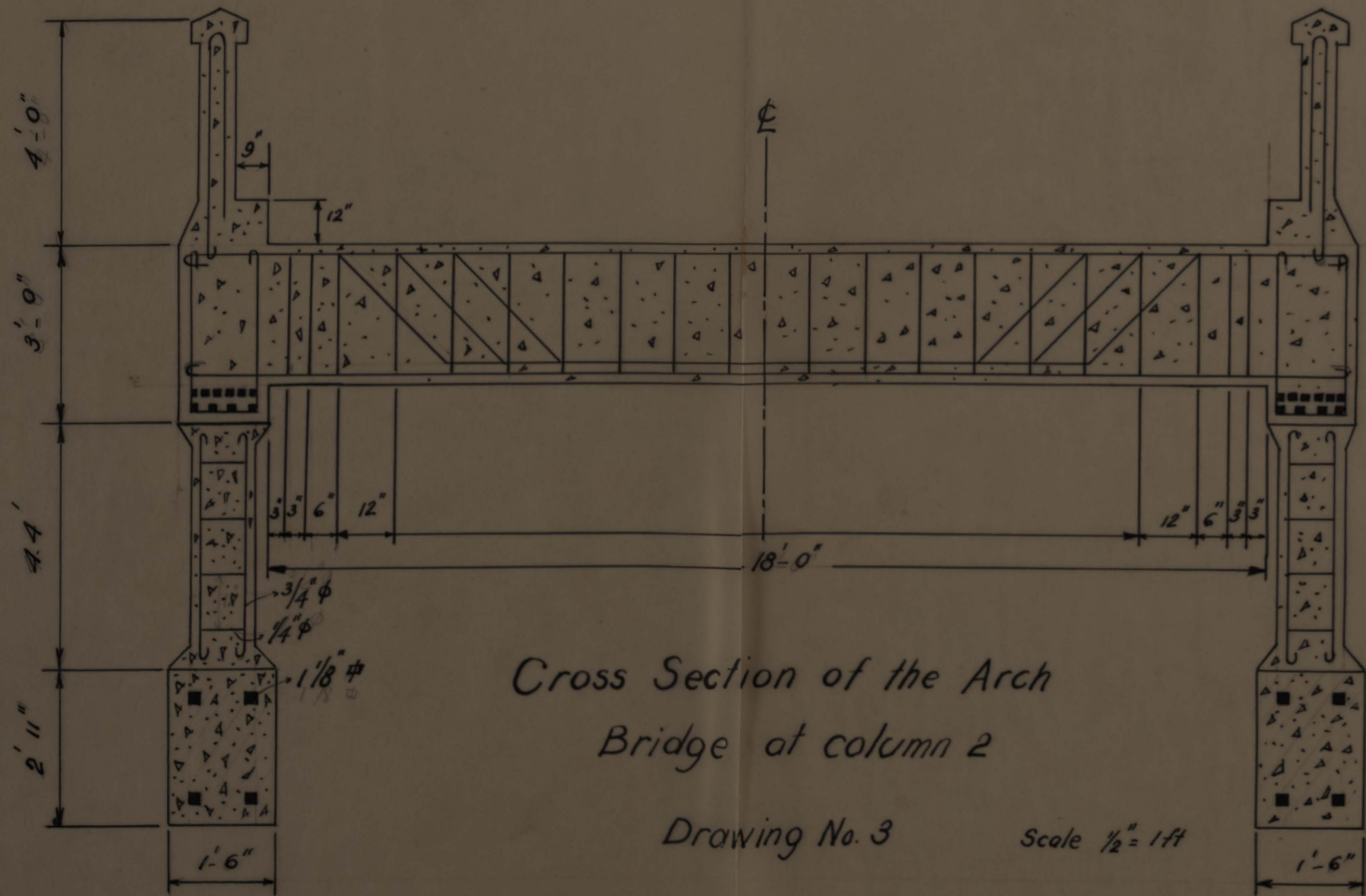
Drawing No. I Span = 60 ft
scale = 1 ft = 1/2 inch



Drawing No. 2

$1'' = 2'$
 scale
 $1'' = 2.5'$

Arch Thickness at different
 Points. (Grafical Method)
 Drawn by Leon Bolamutoglu



*Cross Section of the Arch
Bridge at column 2*

Drawing No. 3

Scale 1/2" = 1 ft