THE APPLICATION OF THYRITE TO A NONLINEAR SERVO-CONTROLLER

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SUMMARY

In the design of a servomechanism it is necessary to compromise between conflicting performance requirements. As the relative importance of these requirements varies with the magnitudes of signals within the system, the response characteristics of the system would be improved if the compromise could be made dependent on these signal magnitudes. The problem examined here is the application of thyrite (a nonlinear resistor) to a servo-controller to permit such a compromise. The conflicting requirements considered are those of low rise time, low velocity lag error, and small overshoot.

The experimental system used a synchro error-detector system and an amplidyne for the major part of the power amplification. Function generators based on the nonlinear characteristics of thyrite were used to make the system gain and the tachometric feedback nonlinear functions of the magnitude of the error. The general types of nonlinear functions used were chosen on the basis of examination of loci for the M point on the B_0 versus B_1 curves of Mitrovic's method, and consideration of the characteristics of the linear portion of the system.

Two nonlinear systems were examined: (1) a system with constant gain and tachometric feedback decreasing with increasing error, and (2) a system with saturation on gain and tachometric feedback decreasing with increasing error.

It was concluded that the inclusion of thyrite resistors in a control system permitted a more favorable compromise to be made between

conflicting requirements. The two systems examined both had lower maximum velocity lag errors than a linear system with similar overshoot for large step inputs. The response of the constant gain nonlinear system to small step inputs was overdamped, however. The use of a saturation characteristic on the gain permitted high gain for small error while preventing overload for large errors. The higher gain for small error improved the response of the system to small input signals and improved the steady-state accuracy and resistance to load torques.

The experimental system used thyrite resistors only to modify the magnitudes of system quantities. The use of thyrite resistors in resistance-capacitance compensation networks would permit time constants to be functions of signal magnitudes and might be useful in some applications.

CHAPTER I

INTRODUCTION

Classification of control systems.--Most automatic control systems now in use can be divided into two main groups: (1) systems which apply no correction unless the error exceeds a predetermined value and maximum correction if this value is exceeded, and (2) systems which apply a corrective effort whose magnitude is a continuous function of the error. Some major systems not fitting into either classification are those which apply fixed values of correction for errors between predetermined limits, and dual-mode systems (which operate as a system of the first group for error larger than a predetermined value and as a system of the second group for error less than this value).

Linear and nonlinear systems.--All systems of the first group are nonlinear. Most systems of the second group now in use are nearly linear in their operating ranges. The principle of superposition holds for linear systems, and they can be described by linear differential equations with coefficients that are not functions of system variables (1).[1] As the characteristics of a linear system are independent of the input signal, such a system is useful in applications where the input signal varies widely or is unknown. A nonlinear system is more specialized than a linear system and can in most cases give faster, more accurate

[1] Reference symbols refer to the Bibliography.

response than a linear system of the same degree of complexity for a specific type of input signal.

Statement of the problem.--The problem investigated here is the application of a nonlinear resistor to the improvement of the rise time, overshoot, and velocity lag error characteristics of a basic control system of the second group. Circuits containing thyrite[2] resistors are used to cause the system gain and the tachometric feedback to vary as chosen nonlinear functions of the magnitude of the error.

Rise time.--The rise time of a system is a measure of the time required for the system to respond to a sudden change in the input and is inversely proportional to the system bandwidth. As the bandwidth of a control system is limited by the inertia and viscous friction of the load and by time lags in the system components, attempts to reduce the rise time by the use of high gains and compensation networks result in increased energy storage in the system. This stored energy causes larger overshoot. Rise time is defined in terms of a step input of position for most control system applications and is the time required for the output to increase from ten per cent to ninety per cent of its final value in response to the instant change of input position caused by the step input.

Step inputs for the experimental system correspond to an instant change of the control transformer shaft position by an angle equal to the value of the step.

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[2]"Thyrite" is a trademark of the General Electric Company.

<u>Overshoot</u>.--The overshoot of a control system is the amount by which the output of the system exceeds the input while responding to a sudden change in the input. The overshoot of a system can be reduced to any desired value by increasing the damping. The increased damping results in larger rise times and velocity lag errors. Overshoot is usually defined in terms of a percentage of the input change or in terms of the magnitude of the overshoot for a stated input.

<u>Velocity lag error</u>.--The velocity lag error of a system is the difference which exists between the output and input positions when the system is subjected to a constant velocity input signal. The velocity lag error may be reduced by decreasing the coefficient of the "s" term in the denominator of the open loop transfer function or by increasing the gain. Either of these changes will reduce the damping and increase the overshoot.

CHAPTER II

THE NONLINEAR FUNCTION GENERATOR

Basic requirements.--To facilitate the modification of the nonlinear transfer functions used in the experimental system, it was considered desirable to isolate the nonlinear function generator as a separate circuit. However, a control system designed for a specific application would in most cases incorporate the circuits used to generate the nonlinear functions into the amplifiers in the controller. To make the experimental results more readily applicable to specific systems, the use of high gain operational amplifiers has been avoided.

Characteristics of thyrite (2).--Thyrite is a nonlinear resistance material consisting of silicon carbide crystals in a ceramic binder. The current in a thyrite resistor is related to the applied voltage by

$$i = Kv^n. \tag{1}$$

The constant, K, is determined by the size and shape of the resistor and by the characteristics of the crystals used. The constant, n, is determined by the characteristics of the crystals and is available in values between 2 and 7 in commercial units. Although thyrite has a negative temperature coefficient of approximately one per cent per degree centigrade, equation (1) is a direct relationship with no thermal lag involved.

Characteristics of functions generated with thyrite.--The nonlinear variation of the current in a thyrite resistor makes possible the generation of a wide range of nonlinear transfer functions by the use of resistive networks containing thyrite resistors. The circuits to be considered here will consist of networks in which only one resistor is a thyrite resistor as variations in commercial units make pairs with closely related characteristics difficult to obtain.

The circuit equations for a resistive network containing a thyrite resistor are algebraic equations having non-integral exponents. This type of equation is not adapted to analytic solution and a point by point solution is required. A number of general characteristics of networks containing thyrite elements may be noted, however, to assist in the determination of the possible functions obtainable.

As the characteristics of a thyrite resistor are not dependent on the sign of the applied voltage, a plot of network output as a function of input will be symmetrical about the origin.

The current in a thyrite resistor increases at a greater rate than the terminal voltage for all values of terminal voltage. As the circuit is linear with the exception of the thyrite resistor, the voltage at one of the two nodes across which the output voltage is measured will increase at a greater rate than that at the other node for all values of input voltage. Hence, the output voltage can be zero for at most one nonzero magnitude of input voltage.

Previous use of nonlinear functions obtained with thyrite.--Most of the previous use of thyrite as a nonlinear element of specified characteristics has been in the analog computer field where there is a continuing search for means by which a square law characteristic can be provided for use in quarter square analog multipliers. Several attempts which have been made to approximate a square law characteristic by the use of a linear resistive network and thyrite resistors are of interest here because of the insight which they give into methods of producing nonlinear functions with thyrite resistors.

Kovach and Comley (3) describe an analog multiplier in which thyrite resistors are used to provide square law characteristics. A thyrite resistor and a linear resistor in series are driven by an operational amplifier and the output voltage is measured across the linear resistor. Values of the thyrite resistor at two voltages are used in calculations to determine the correct value of the linear resistor. The circuit equations are kept in linear form by the use of logarithms of the current and voltage as variables. In a subsequent paper Kovach and Comley (4) describe a more general nonlinear function generator for analog computer use which uses an operational amplifier with networks containing thyrite resistors which have been corrected to square law or cubic characteristics by resistive padding. An experimental method of adjusting the corrective resistors is described, and circuits are given for some of the basic functions used in analog computer work.

A more complex method of approximating a square law characteristic is given by Maslov (5). In this case, a circuit consisting of the parallel combination of a linear resistor with the series combination of a second linear resistor and the thyrite resistor is driven by the input voltage. The output voltage is measured across the thyrite resistor. The characteristics of the circuit are made to match the square law characteristic at three points by choice of the values for the linear resistors. Design calculations were made on the basis of solution of the three simultaneous equations obtained by the substitution of measured values of the resistance of the thyrite resistor at three voltages into the transfer function.

Difficulties encountered in the use of passive circuits.--In most cases, the input and output quantities of the nonlinear network are voltages. Two major exceptions are the use of a nonlinear network as a coupling network between a pentode and a following stage, or between a cathode follower with low output impedance and a load with low input impedance such as the field winding of a tachometer or motor. As the nonlinearity of a thyrite resistor is in the relationship between current and voltage, it is necessary for the input voltage to control the current in the circuit. The output voltage is then measured across an element or elements of the circuit through which a portion of the current flows. If the average steady-state impedance of the elements which determine the relationship between the output voltage and the current is large enough to partially determine the relationship between the current and the input voltage, a degenerative effect will be present which will tend to linearize the transfer function. As the output voltage is proportional to this impedance a network with an appreciably nonlinear transfer function will also have an unavoidably high attenuation. More amplification would be required in a servo controller because of this attenuation. The advantages of increased flexibility and isolation of the input from loading effects of the network can be obtained by incorporating the additional gain into the nonlinear network to make it an active network.

An active network to generate a nonlinear voltage transfer function.--A block diagram of the nonlinear function generator is shown in Figure 1. The comparator adjusts the current, i, to make the voltage, $V_{reference}$, equal to the input voltage. If the reference voltage is related to the current by

$$V_{reference} = f_1(i) \tag{2}$$

and if the output voltage is related to the current by

$$V_{output} = f_2(i), \tag{3}$$

then the output voltage will be related to the reference voltage by

$$V_{output} = f_2 \quad f_3(V_{reference}) , \tag{4}$$

where $f_3(V_{reference})$ is the inverse of $f_1(i)$ and relates the current to the reference voltage.

As the circuit from the input to the reference voltage terminals is basically a unity gain amplifier, the linearizing effect of the large amount of negative feedback will cause the reference voltage to closely approximate the form

$$K_1 V_{in} + K_2 \tag{5}$$

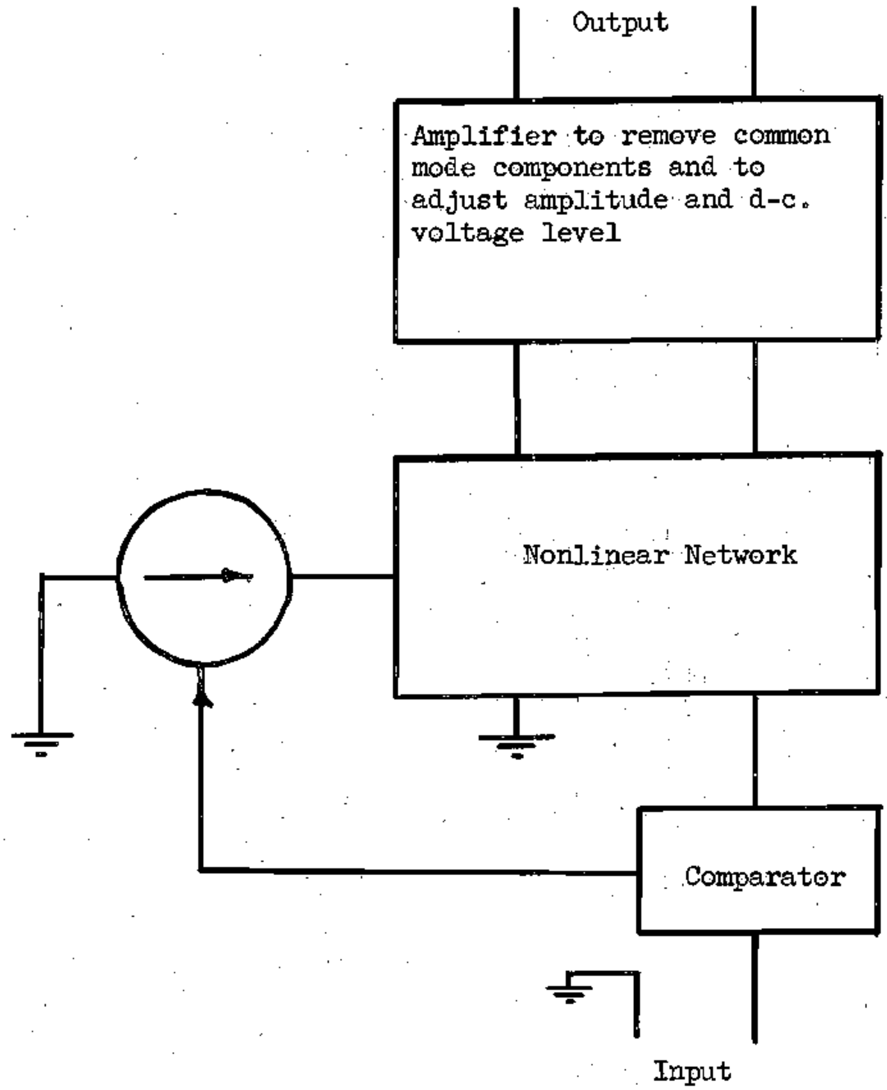


Figure 1. Block Diagram of Nonlinear Function Generator

where K_1 is nearly unity and K_2 is very small. Provision of an adjustment to set the output voltage to zero for zero input voltage should make the accuracy of the circuit adequate for control system use even if the comparator has relatively low gain.

Circuits used to implement active portion of the function generator.--A circuit diagram of the comparator and the source of Figure 1 is given in Figure 2a. In this circuit one terminal of the source is common to one terminal of the reference voltage. To permit the reversal of the output voltage while preserving a single-ended form of circuit the d-c. voltage level at point "2" is maintained at approximately 125 volts. The direction of current flow in the circuit then depends on whether the voltage at point "1" is less than or greater than 125 volts. The voltage dividers in the grid circuits of the 6SL7 balanced amplifier are needed to permit the grids of the tube to operate at lower potentials than the plates. The variable resistance in the voltage divider connected to point "3" permits adjustment for differences in the resistance ratios of the two dividers.

There are two possible sources of error: (1) the loading effect of the voltage divider in the reference voltage circuit, and (2) the d-c. voltage level introduced in the input voltage by current flow from the 125 volt source through the potentiometer used to adjust the input voltage level. Two means are used to minimize the loading effect of the voltage divider: (1) making its resistance high with respect to the resistance levels in the network, and (2) providing an adjustment to set the direct-current voltage level at point "3" (which prevents current flow to ground through the nonlinear network due to the constant 125 volt

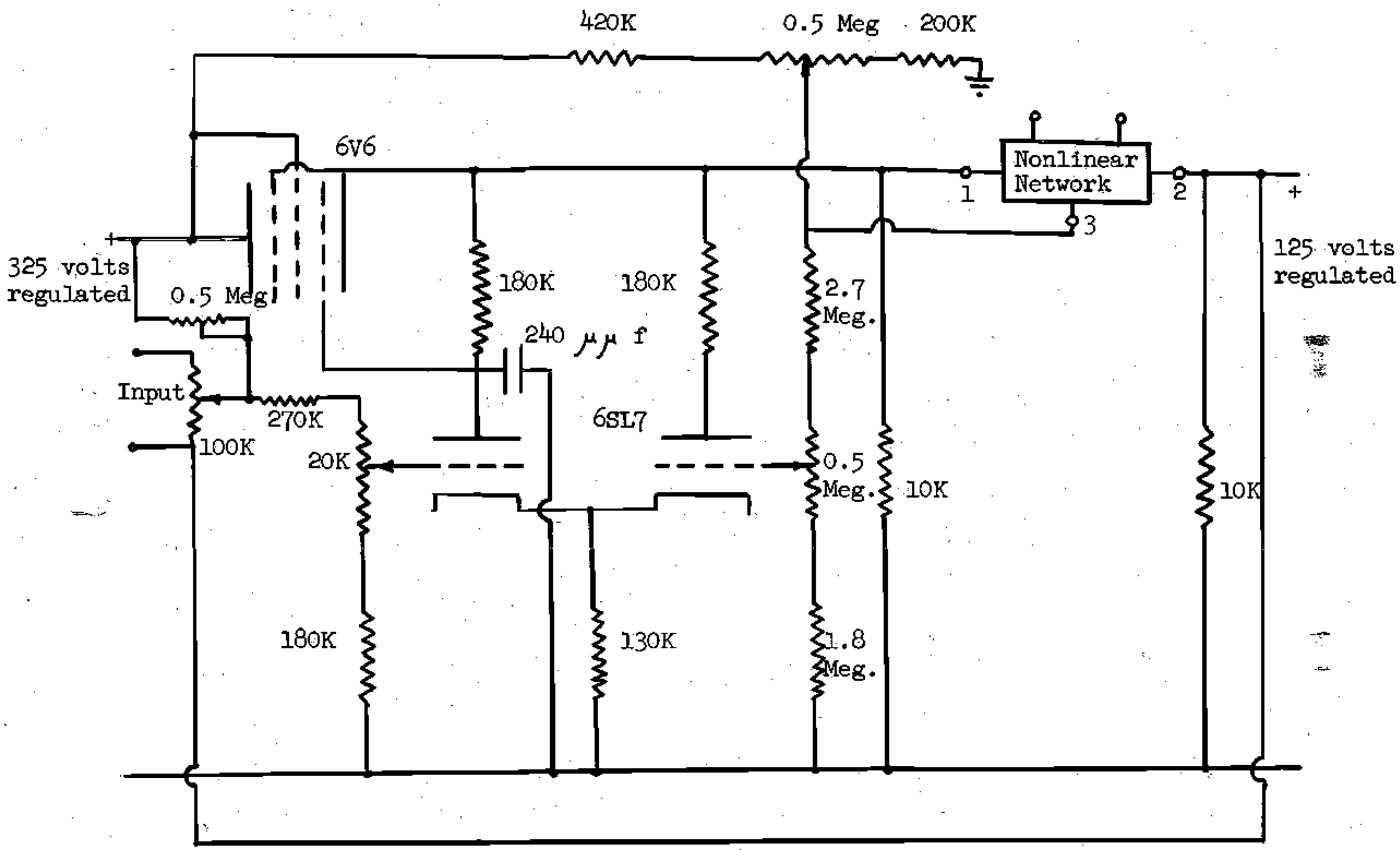


Figure 2a. Circuit used to Implement Comparator and Source of Figure 1

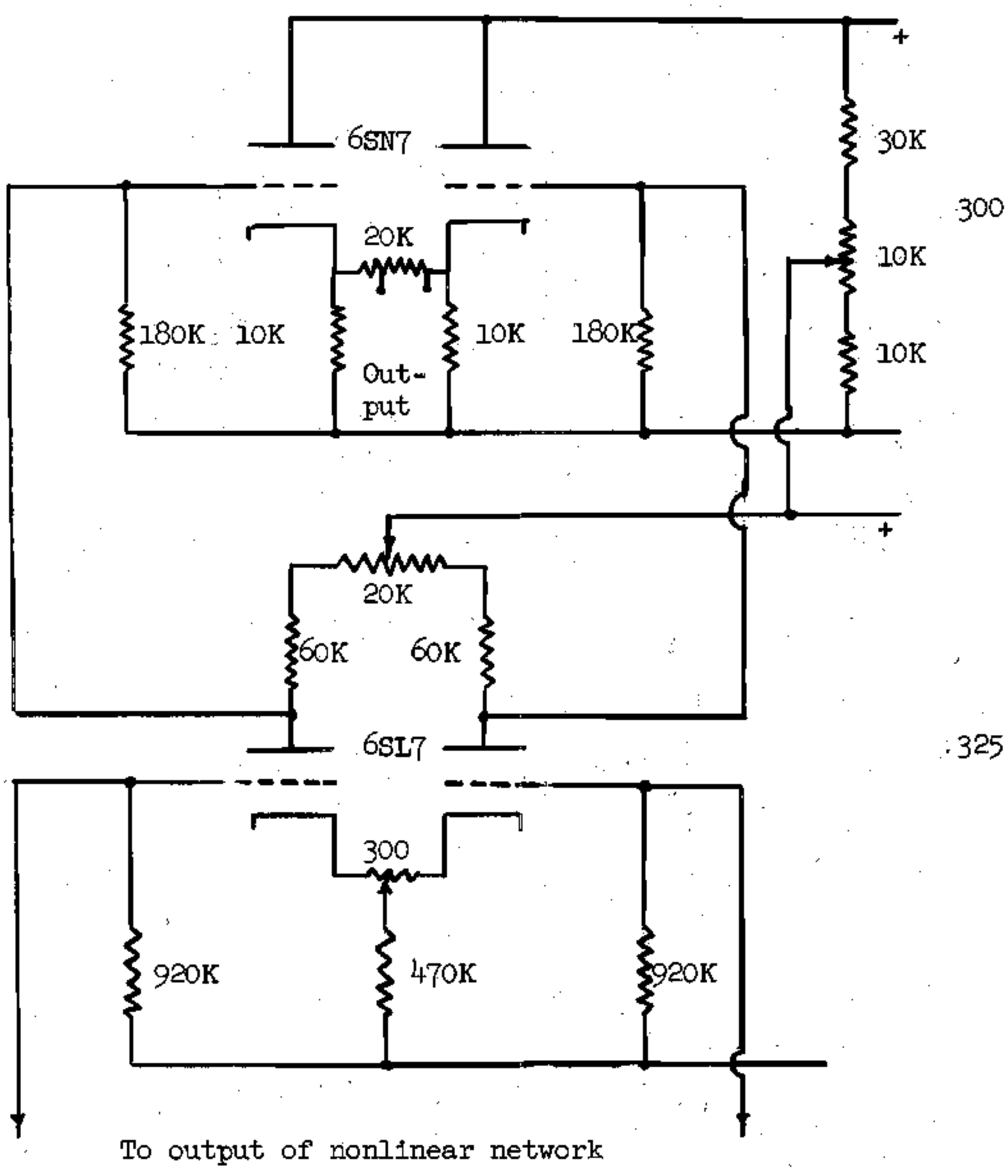
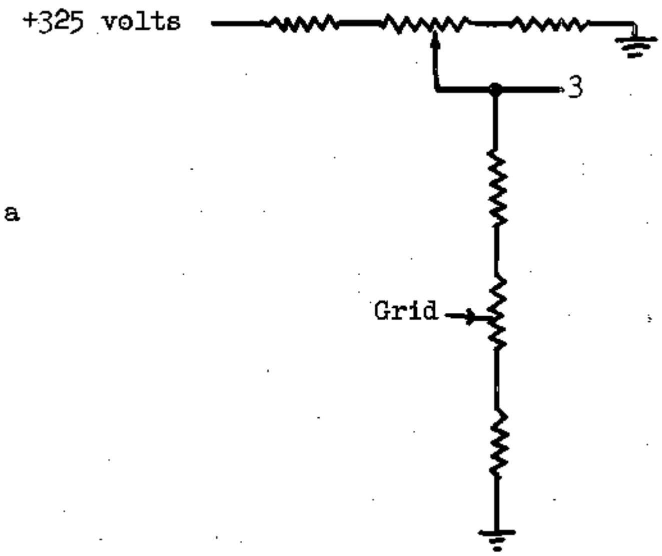


Figure 2b. Circuit of Output Amplifier for Nonlinear Function Generator

potential). Figure 3a shows the circuit used to provide this adjustment. The circuit used to eliminate the undesired d-c. voltage level in the input voltage is quite similar, as is shown in Figure 3b. Current flow from the 325 volt source is used to cancel that from the 125 volt source so that there is no voltage between points "a" and "b" except that caused by the input signal.

Characteristics of the function generator.--Figure 4 and Figure 5 show the two basic functions produced by the function generator. A wide range of functions can be obtained by the addition or subtraction of a linear component at the output of the function generator and by the use of series and shunt resistances to modify the characteristics of the thyrite at large and small signals, respectively. The linear term of approximately 0.44 volts per volt evident in the functions of Figures 4 and 5 is due to the loading of the nonlinear networks by the circuits of the function generator. A linear term within the function generator is desirable as the circuit will be unstable if $f_1(i)$ of equation (2) has zero or negative slope at any point.

The possible functions that could be obtained from the two basic functions of Figures 4 and 5 by adjustment of the linear term may be examined by rotating the axes of the figures by the angle of the linear term.



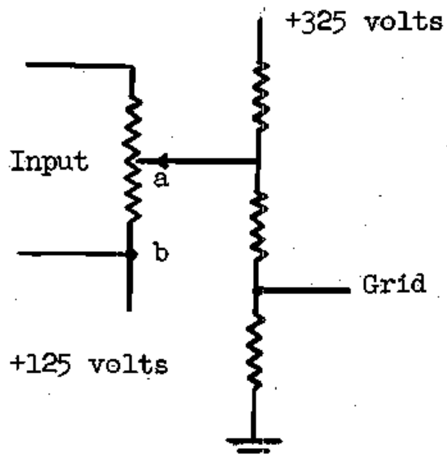


Figure 3. Output and Input d-c. Level Adjustments

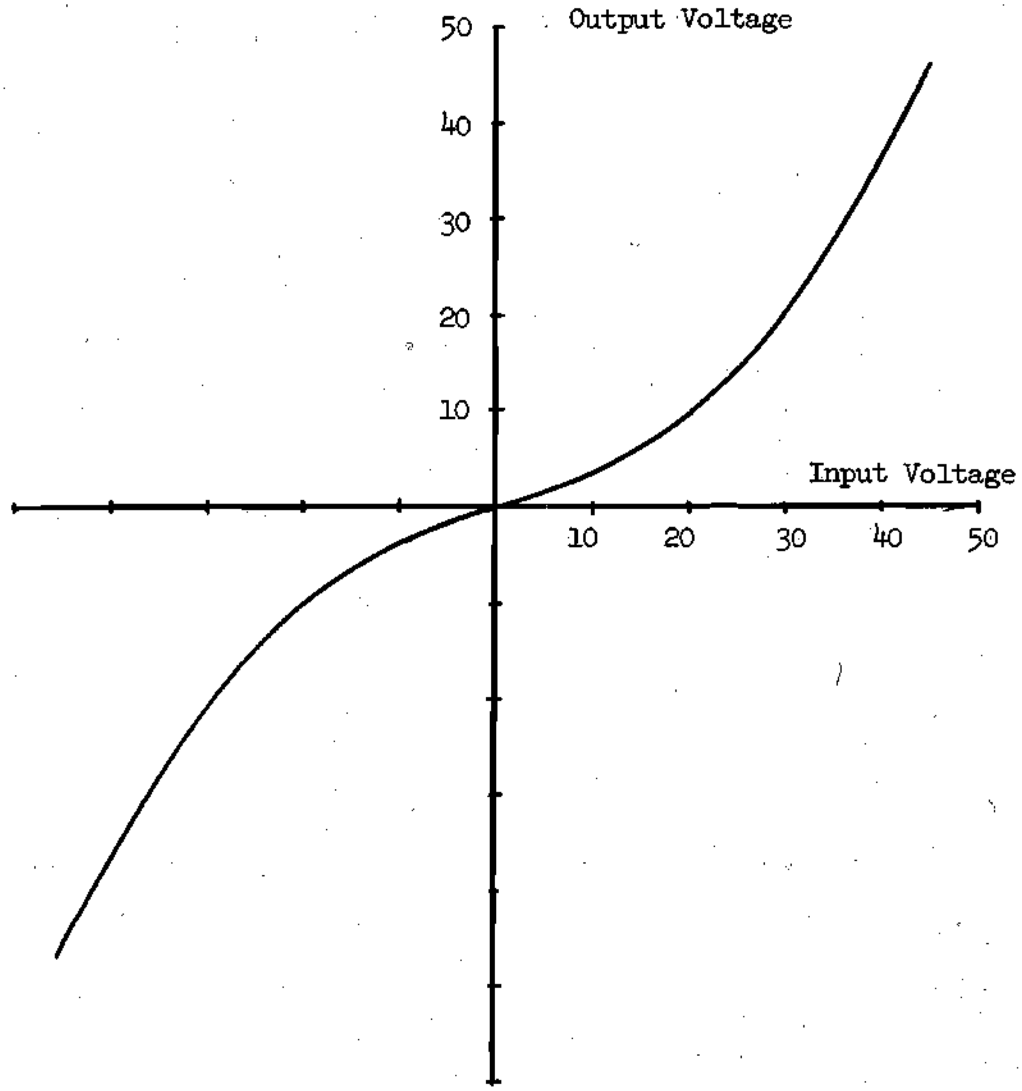


Figure 4. Output of Function Generator-Increasing Function

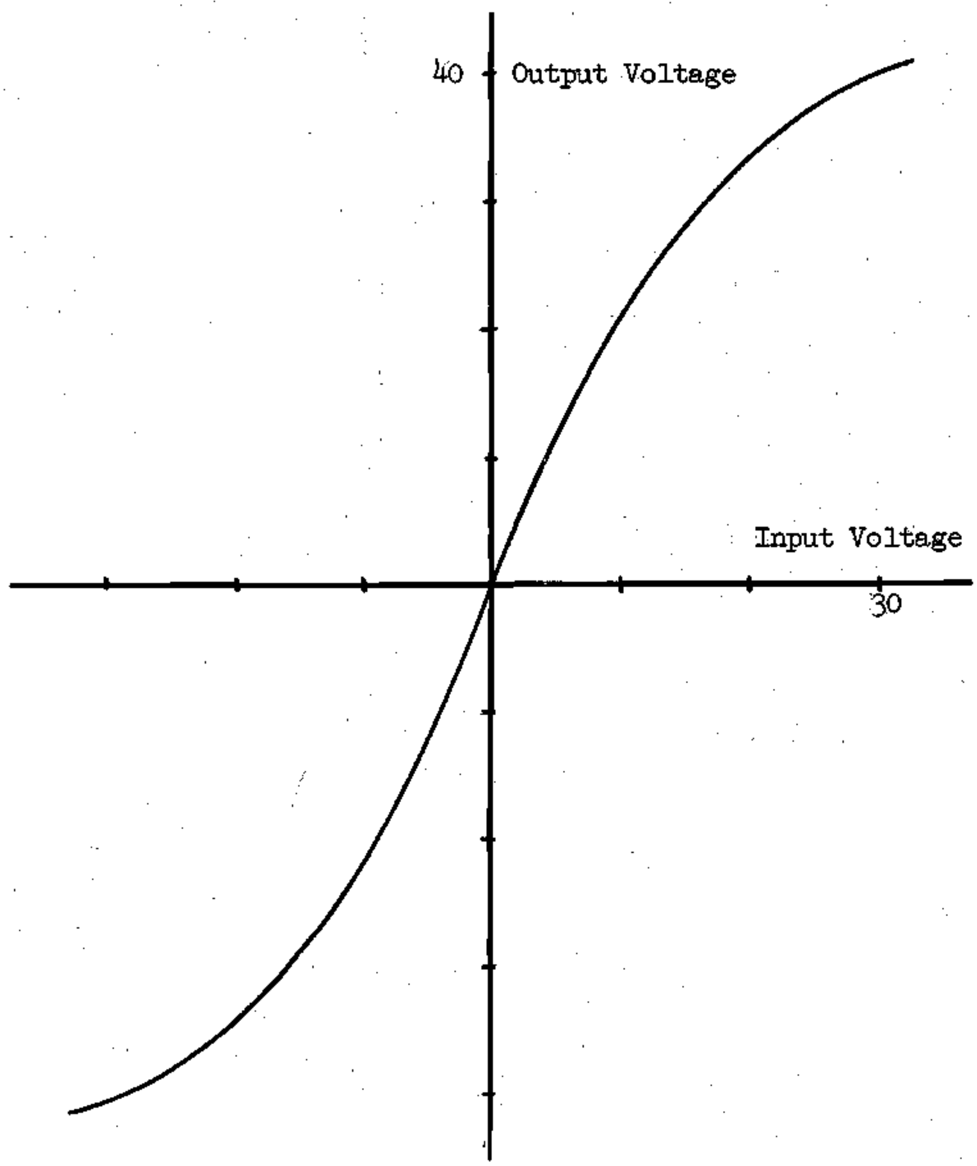


Figure 5. Output of Function Generator-Saturation Function

CHAPTER III

EXPERIMENTAL RESULTS

Basic system construction.--The experimental system is shown in Figure 6. Standard components have been used wherever possible. In the usual system of this type employing synchros as error detectors and an amplidyne for the major part of the power amplification the signal would be kept in a-c. form until the stage proceeding the amplidyne to avoid the need for several stages of direct-coupled amplification. In the system used here, however, the phase sensitive detector directly follows the control transformer to provide a d-c. signal to the nonlinear function generator. Although the linear portions of the system are characterized by the roots obtained from open loop frequency response measurements, the characteristics of the individual components must be examined to determine the types of signals in each part of the system. Characteristics of the basic system components are listed in Appendix I.

Experimental procedure.--The circuits used to measure the characteristics of the experimental system are shown in Figure 7. The error voltage and the permanent-magnet tachometer output voltage were recorded on parallel channels of a Sanborn recorder for all measurements. A paper speed of one hundred millimeters per second was used for all step response tests, and a speed of ten millimeters per second was used for all velocity lag error measurements. The error channel was calibrated in terms of the error angle to remove the effect of the sinusoidal variation of error voltage with error angle introduced by the synchros.

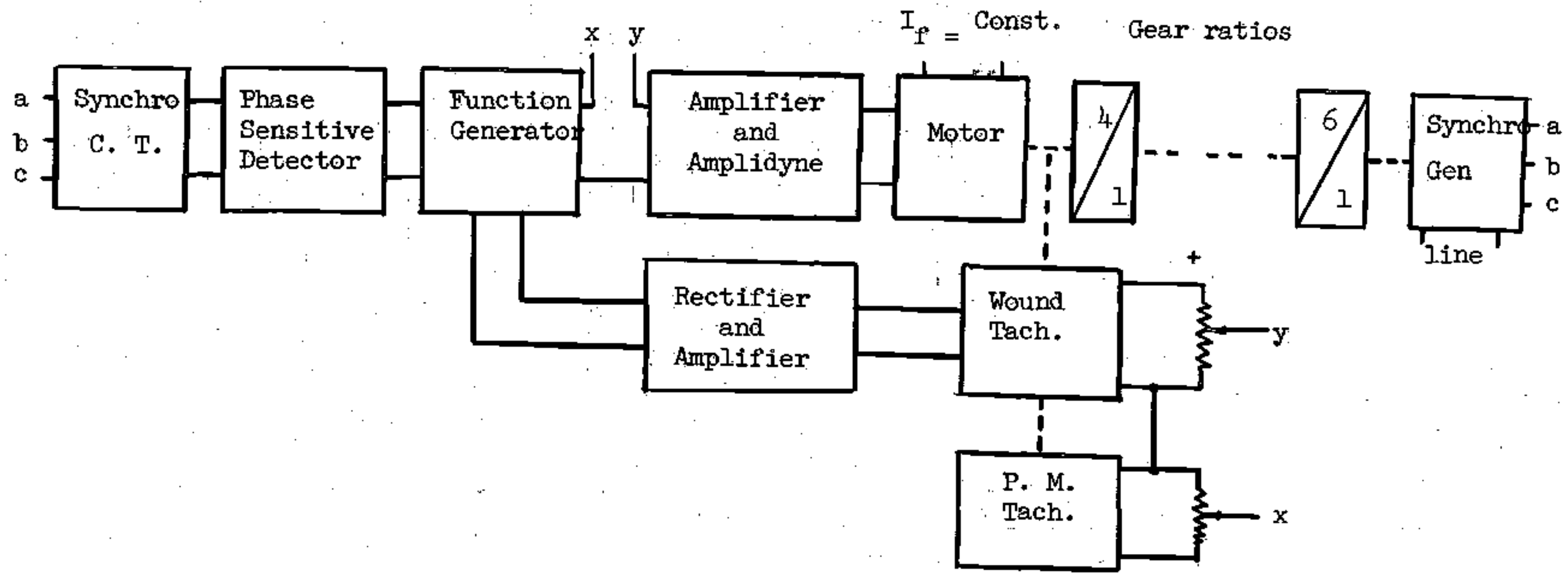


Figure 6. System with Nonlinear Gain and Nonlinear Tachometric Feedback

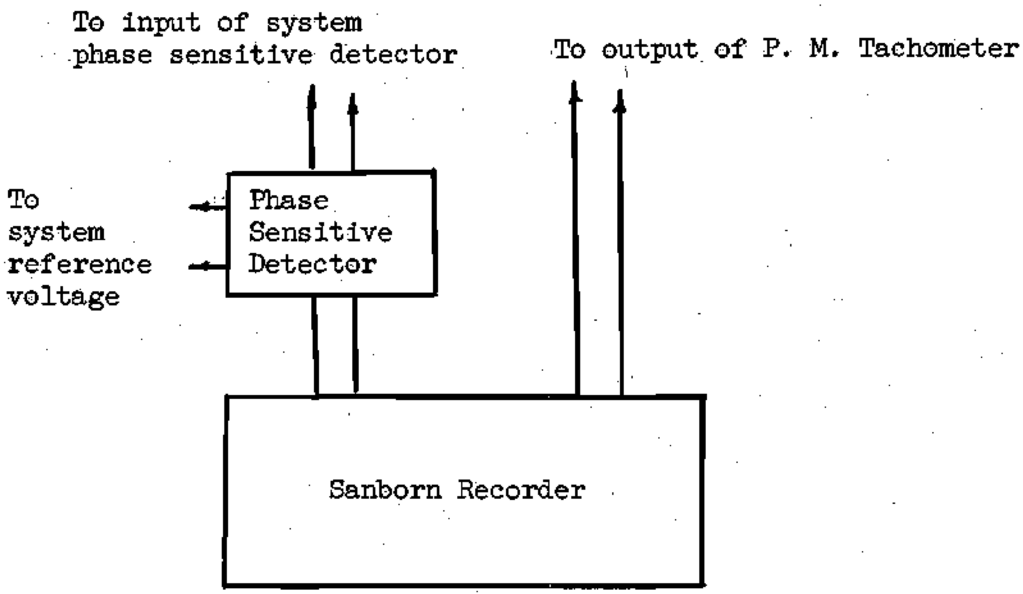


Figure 7. Measurement Circuit for Nonlinear Systems

Step inputs were simulated by closing the loop with an initial error present. As the input position was fixed, the permanent magnet tachometer output voltage was proportional to the derivative of error.

For steady velocity input signals the system input control transformer was replaced by a control transformer driven by an adjustable-speed drive.

<u>Nonlinear system with constant gain and tachometric feedback decreasing with increasing error</u>.--Figure 8 shows the locus on the B_o versus B_1 chart which describes this system. The downward motion of the locus for large error is caused by the sinusoidal variation of the synchro output voltage with angle. For comparison, the locus of a nearly linear system having the same M point for zero error is also shown. The points on the loci are spaced at intervals of five degrees of error. The error voltage used to drive the wound tachometer was weighted by the nonlinear function of Figure 4 to cause the tachometric feedback to decrease more rapidly with large error than with small error.

The velocity lag error of the system with, and without, the nonlinear tachometric feedback is shown in Figure 9. The increase in slope of the curve for the nearly linear system for values of error greater than 25 degrees is due to the effective reduction of gain at large angles caused by the sinusoidal characteristic of the synchros. The lower curve shows the effect of the nonlinear tachometric feedback. The nonlinear tachometer has no detectable effect at velocities of 1.5 radians per second or less but reduces the velocity lag error for all velocities above 1.5 radians per second. At a velocity of 7.0 radians

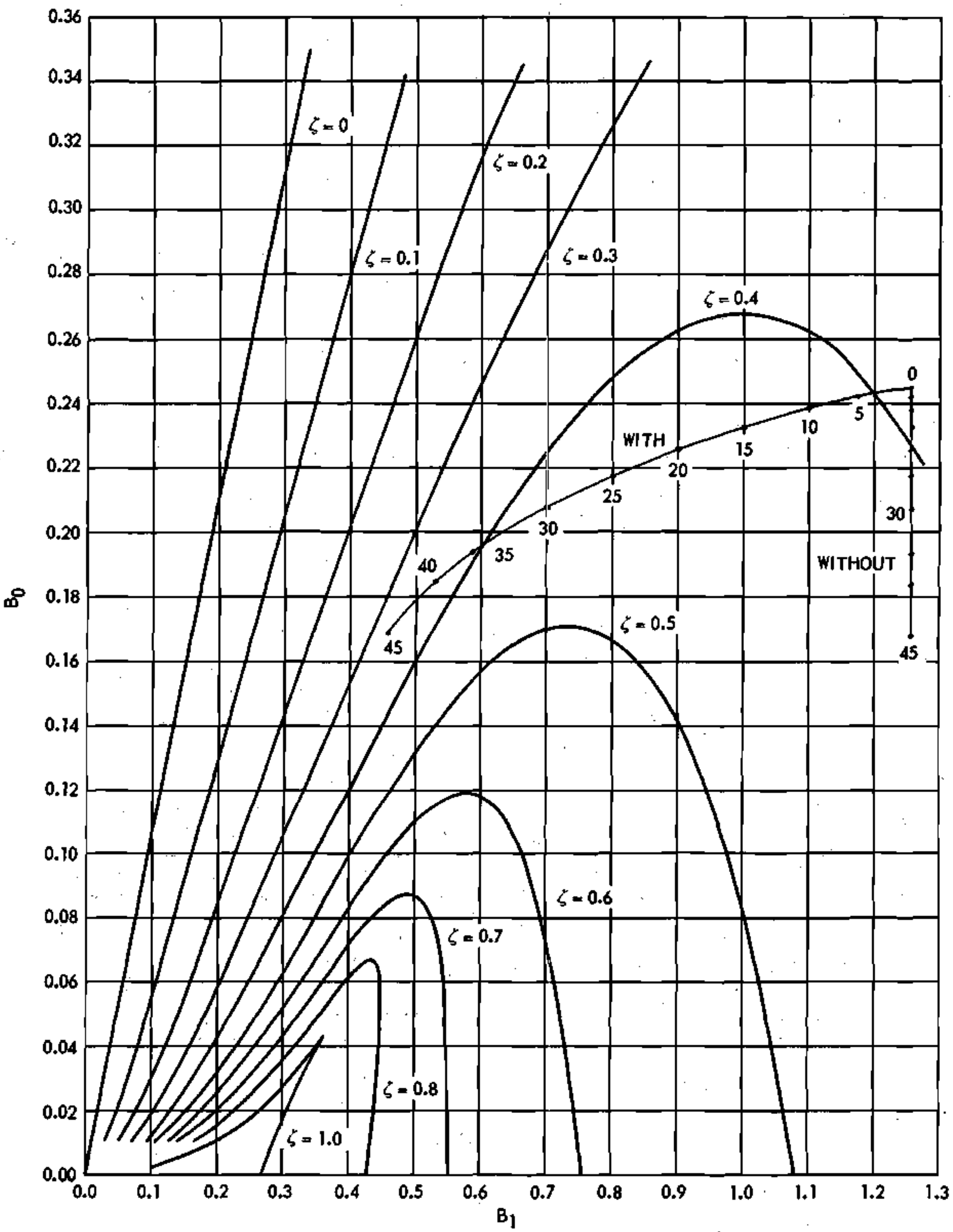


Figure 8. Loci on B_o Versus B_1 Chart for Constant Gain System With, and Without, Nonlinear Tachometric Feedback.

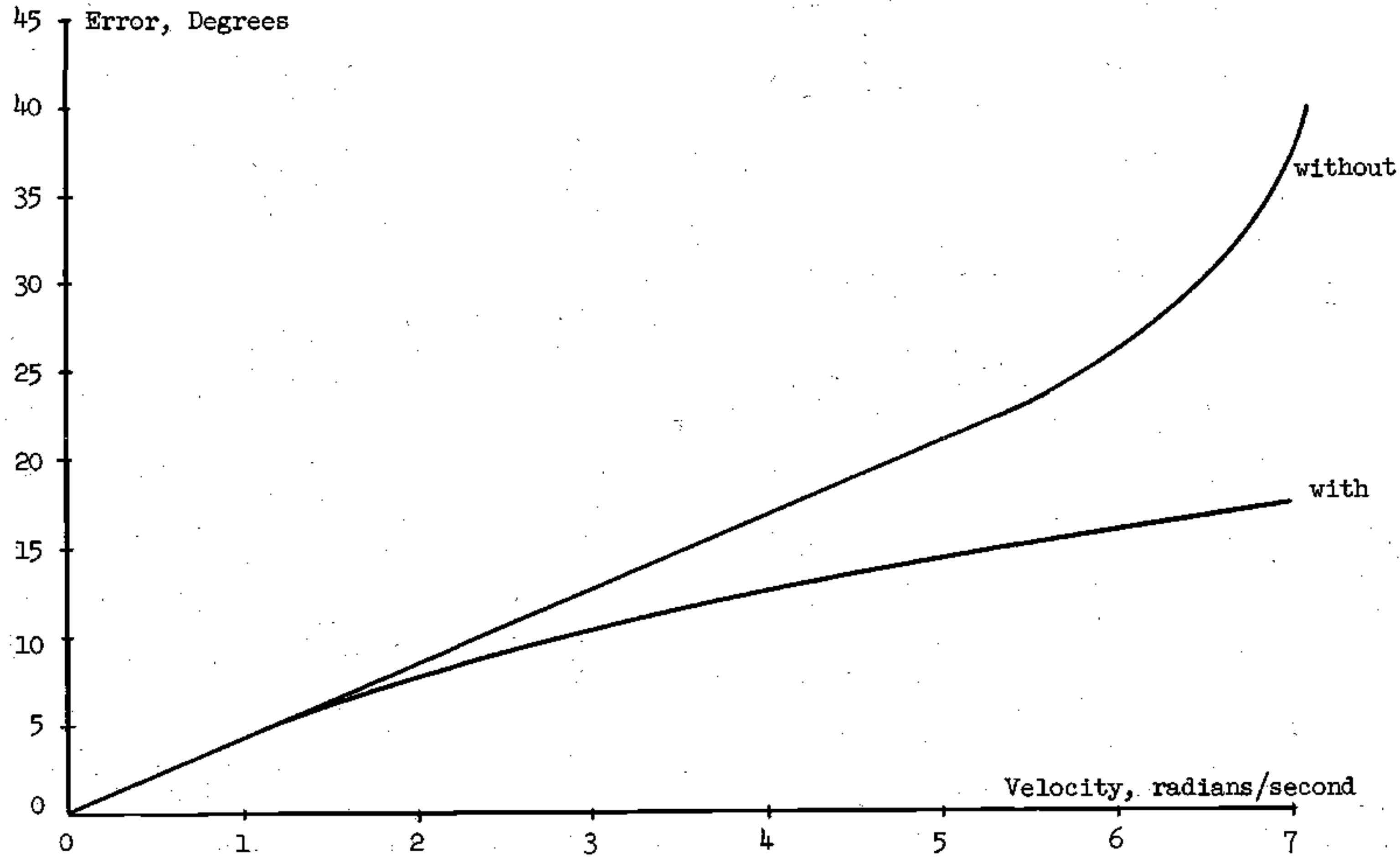


Figure 9. Velocity Lag Error of Constant Gain System with, and without Nonlinear Tachometric Feedback

per second the velocity lag error is only 48 per cent of that of the nearly linear system. At high velocities the error is large enough to cause the locus to cross the $\zeta = 0$ curve and the system oscillates about the input velocity with an amplitude that increases with velocity.

The step response of the system with, and without, the nonlinear tachometric feedback is shown in Figure 10. A step of 45 degrees is used for the comparison as the effects of the nonlinear tachometer are greater for large errors. The nearly linear system is overdamped. The nonlinear tachometric feedback allows a higher velocity to be attained and results in an initial overshoot of 14 degrees. Figure 11 shows the output waveforms for step inputs. The nonlinear tachometer causes the system to respond more rapidly to large inputs. The rise time is limited to a maximum of 100 milliseconds for step inputs of 45 degrees or less while the initial overshoot increases with the size of the step to 14 degrees for a 45 degree step.

Nonlinear system with saturation on gain and tachometric feedback decreasing with increasing error.--Figure 12 shows the locus on the B_o versus B_1 chart which describes this system. Figure 5 shows the transfer function of the nonlinear gain in the error channel and Figure 4 shows the weighting function used to drive the wound tachometer from the error voltage. These two nonlinear characteristics are evident in the vertical and horizontal spacing of the points on the locus.

The velocity lag error as a function of velocity is shown in Figure 13. The error is very nearly a linear function of velocity as the increase in error due to the saturation of gain is canceled by the decrease

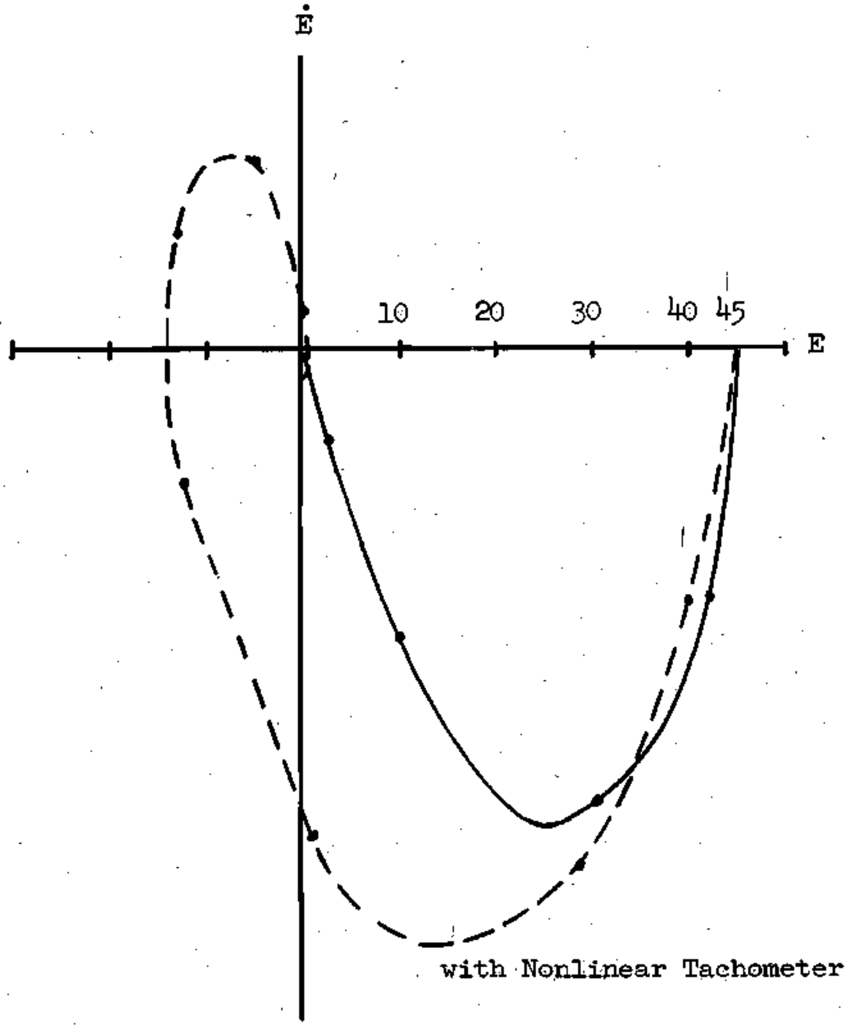


Figure 10. Effect of Nonlinear Tachometric Feedback on Response of Constant Gain System to a 45 Degree Step Input

(The points are at 50 millisecond intervals)

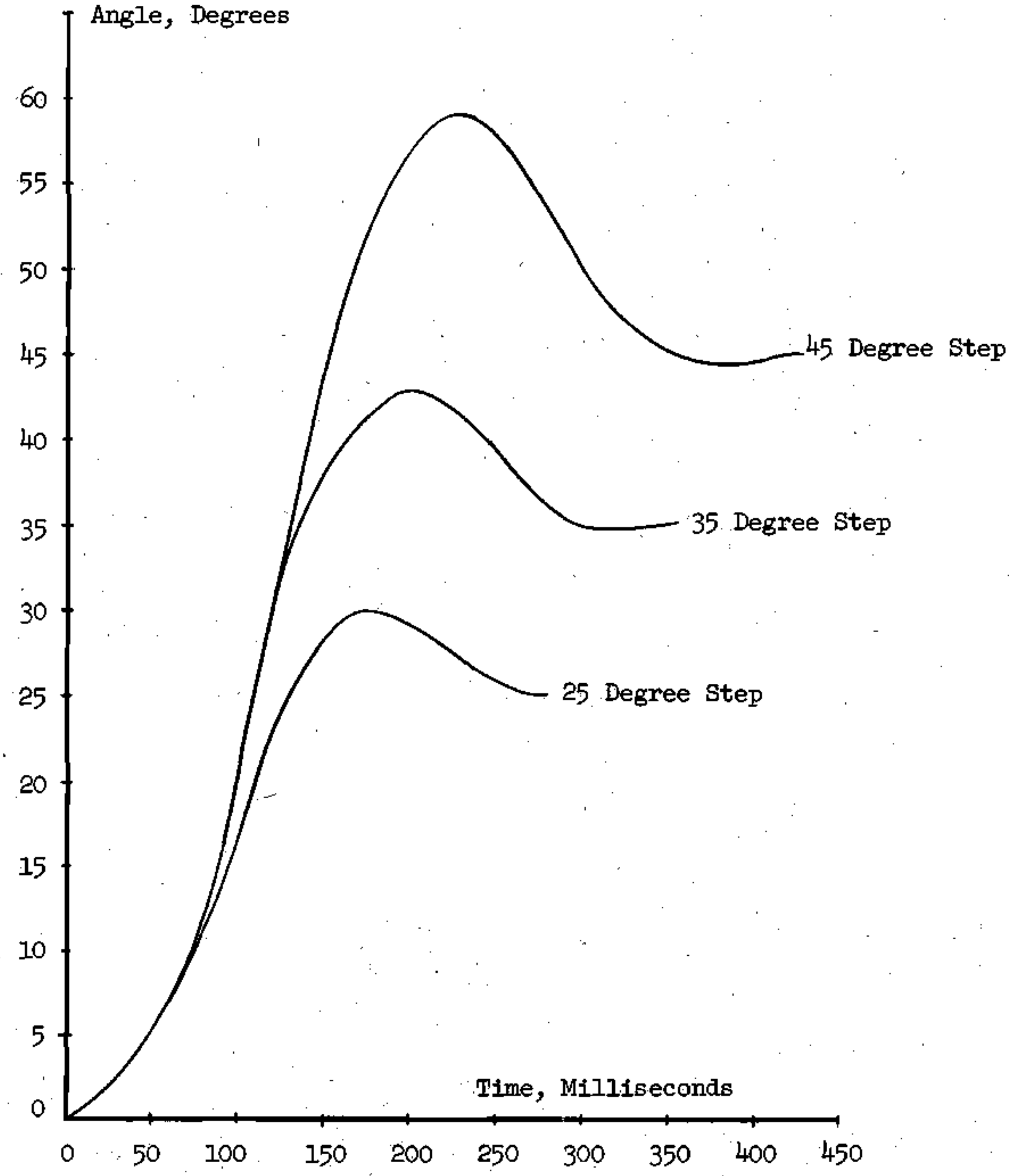


Figure 11. Step Response of Constant Gain Nonlinear System

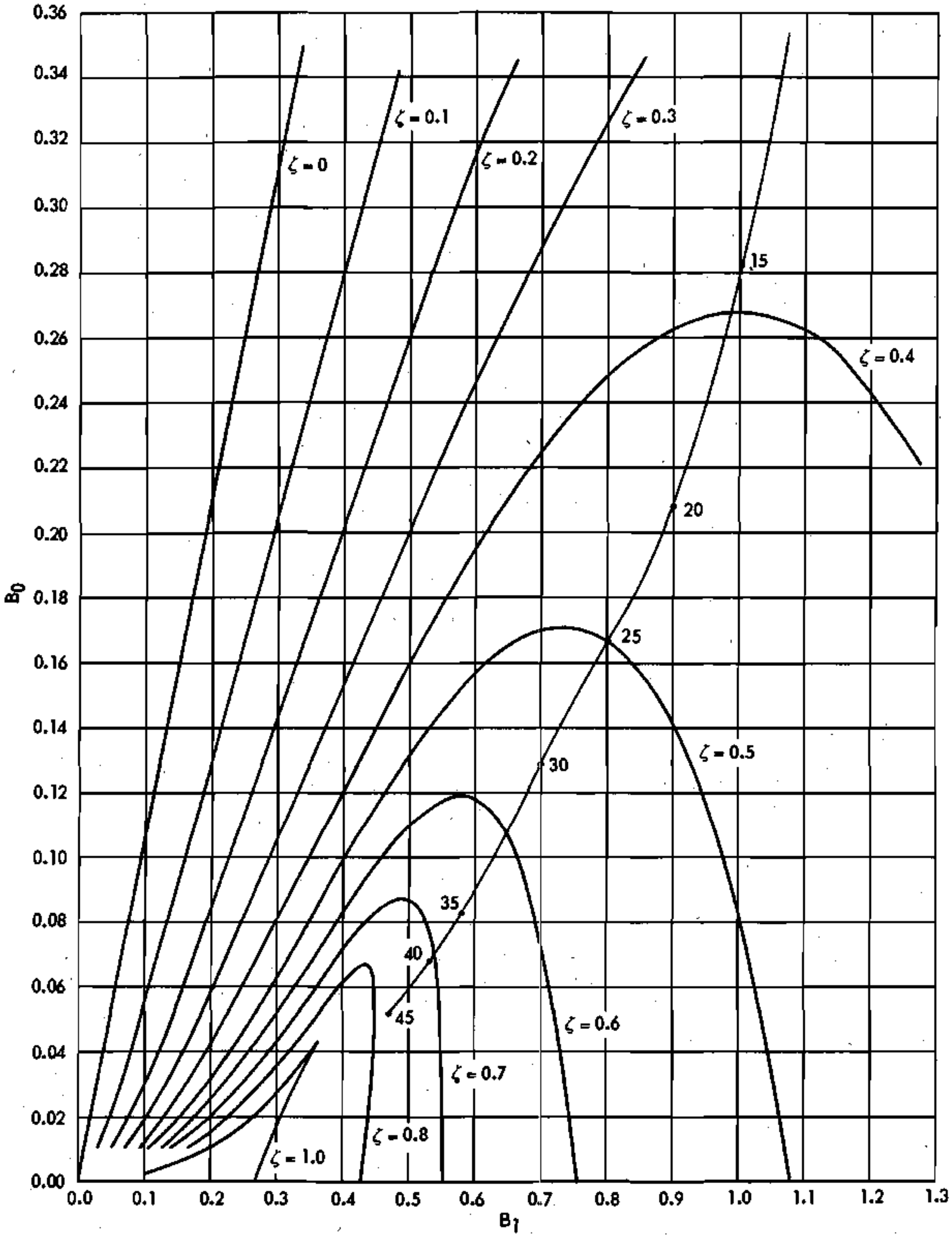


Figure 12. Locus on B_o Versus B_1 Chart for System with Saturation on Gain and Nonlinear Tachometric Feedback.

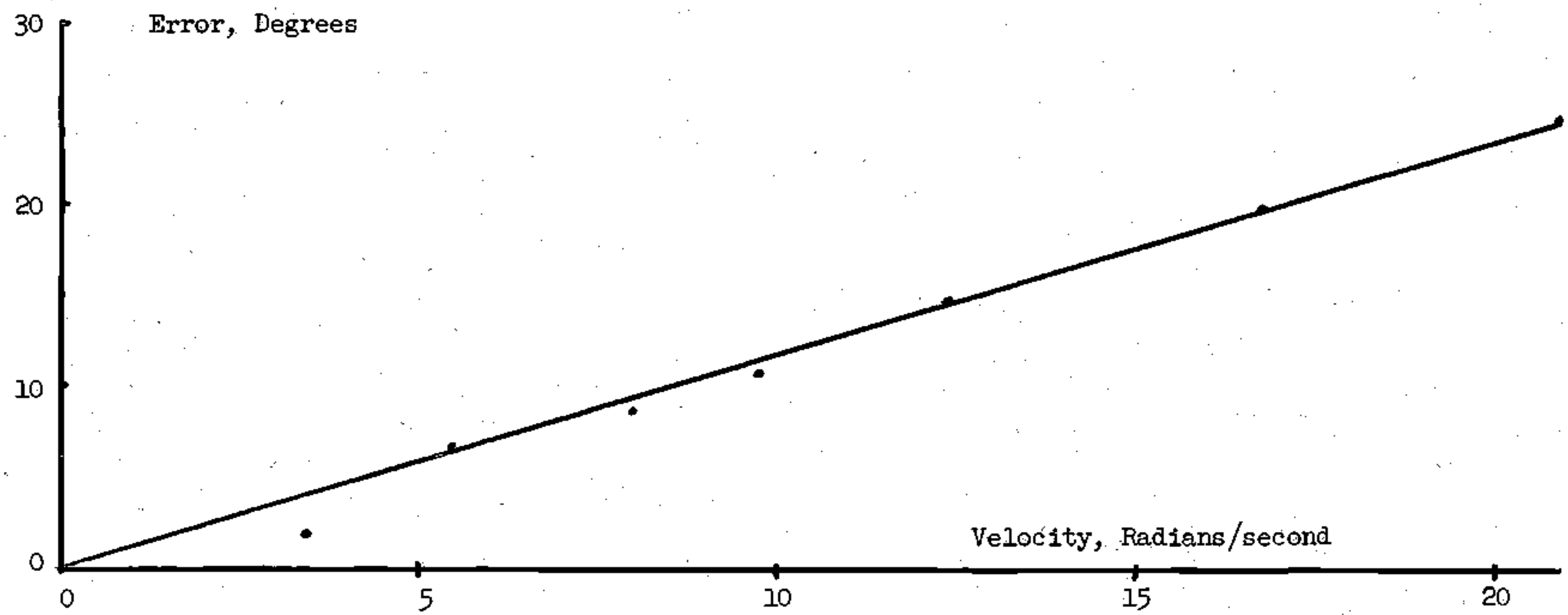


Figure 13. Velocity Lag Error of System with Saturation on Gain and Nonlinear Tachometric Feedback

in error caused by the increase in the positive tachometric feedback at large errors. The system is stable over the full range of velocities which the motor can provide within its ratings because the downward motion of the M point due to the saturation characteristic prevents the decreased tachometric feedback from causing the locus to cross the $\zeta = 0$ curve.

Figure 14 shows the step response of the system for a 45-degree step. The response of the system to smaller steps is similar. Figure 15 shows the output waveforms for step inputs. The overshoot increases from 28 per cent for a 5.0 degree step to 36 per cent for a 25 degree step but then remains less than 37 per cent for steps between 25 and 45 degrees being 34 per cent for a 35 degree step.

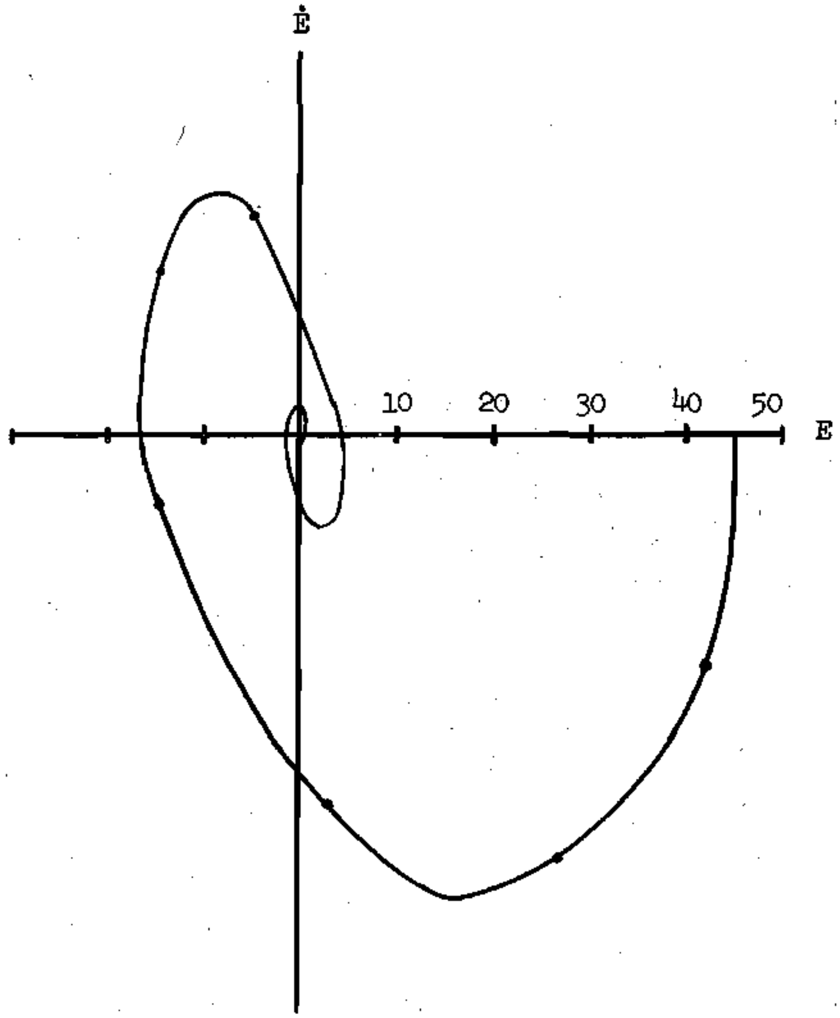


Figure 14. Response of Nonlinear System to 45 Degree Step Input

(The points are at 50 millisecond intervals)

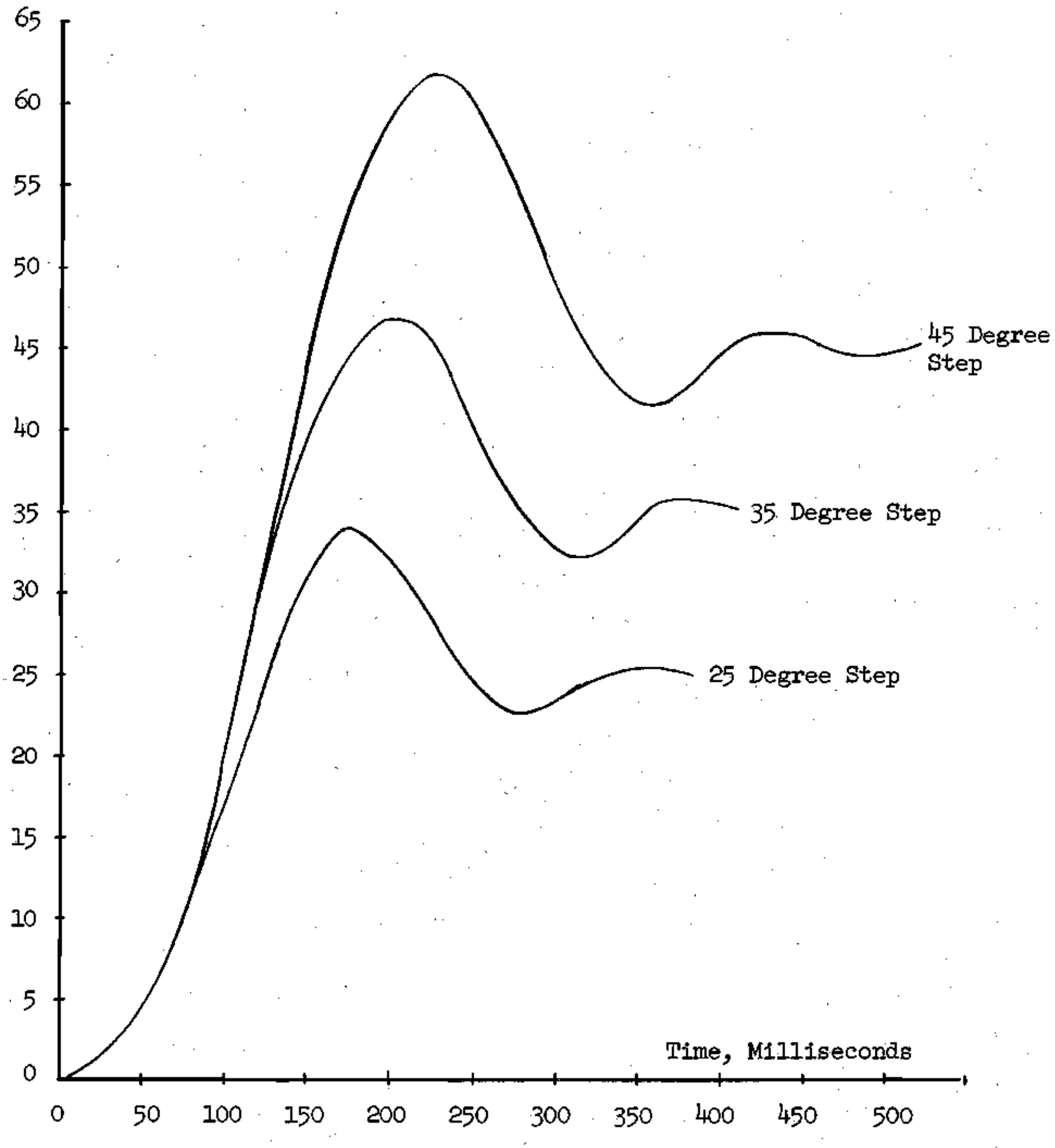


Figure 15. Step Response of Nonlinear System

CHAPTER IV

CONCLUSIONS

The inclusion of nonlinear function generators in an automatic control system permits system characteristics to be made dependent on signal magnitudes within the system. A function generator based on the nonlinear characteristics of thyrite can generate functions of use in control systems.

The use of a nonlinear function obtained by the use of a thyrite resistor to reduce the tachometric feedback at an increasing rate with increasing error permitted the reduction of the maximum velocity lag error. At the same time this caused less increase in the overshoot and settling time than would be caused by fixed adjustment of the tachometric feedback for the same maximum velocity lag error.

The use of nonlinear functions obtained by the use of thyrite resistors to cause saturation of the gain for large values of error and reduction of the tachometric feedback at an increasing rate with increasing error permitted the experimental system to have high gain for low error to provide good steady-state accuracy and resistance to load torques. At the same time it prevented the large maximum velocity lag errors usually caused by saturation characteristics.

CHAPTER V

RECOMMENDATIONS

The system examined in this paper used thyrite resistors only to modify the magnitudes of system quantities. The use of thyrite resistors in resistance capacitance compensation networks would permit time constants to be functions of signal magnitudes and might be of use in some applications.

APPENDIX I

CHARACTERISTICS OF BASIC LINEAR PORTION OF SYSTEM

Error detector.--The pair of synchros used as an error detector had an output of 52 volts for an error of 90 degrees, and an output of approximately one volt per degree for small error when the input voltage was 115 volts. The carrier frequency was 60 cycles per second.

Phase sensitive detector.--The circuit diagram of the phase sensitive detector is shown in Figure 16. The output voltage was one volt direct current per rms volt input for a reference voltage of 60 volts rms.

Direct current amplifier.--The circuit diagram of the direct current amplifier used to drive the amplidyne is shown in Figure 17. As the units which drive this amplifier are all electrically isolated, series inputs rather than a parallel summing network were used. Comparison of the amplifier transfer function in Figure 18 with that of the amplidyne in Figure 19 shows the amplifier to be linear over the range required to drive the amplidyne.

Amplidyne.--A General Electric Amplidyne Motor Generator was used for the major portion of the power amplification. The transfer function curve of the machine is shown in Figure 19. The curve shown is reduced in scale from one drawn by hand over several curves made by an X-Y-recorder, as noise voltages caused ragged variations in individual runs.

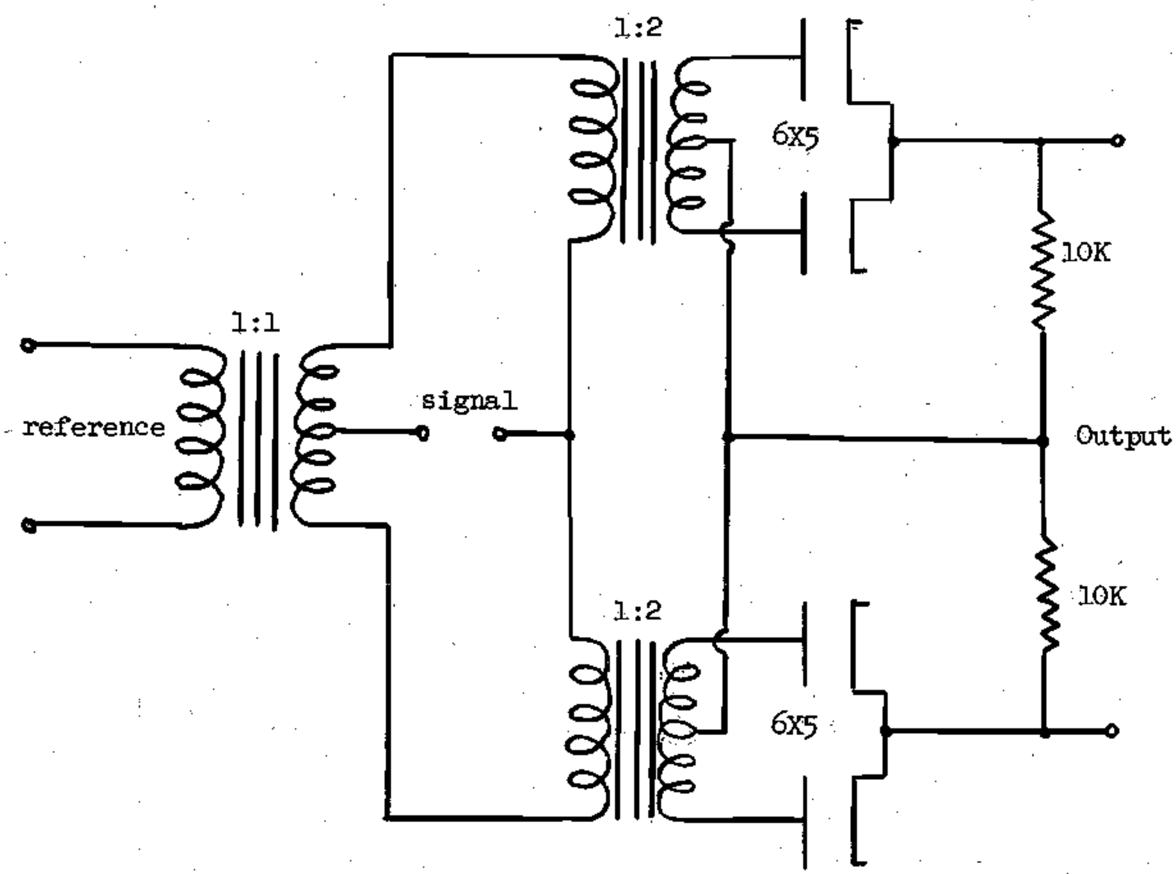


Figure 16. Circuit Diagram of Phase Sensitive Detector

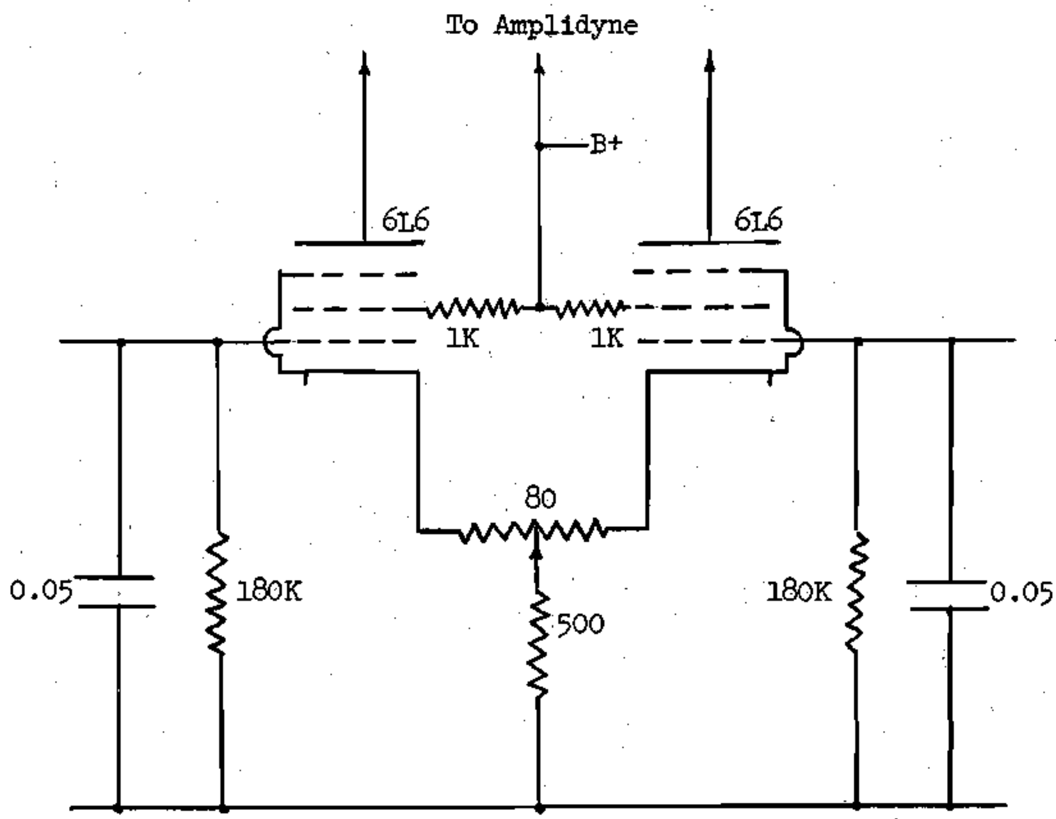


Figure 17. System Amplifier

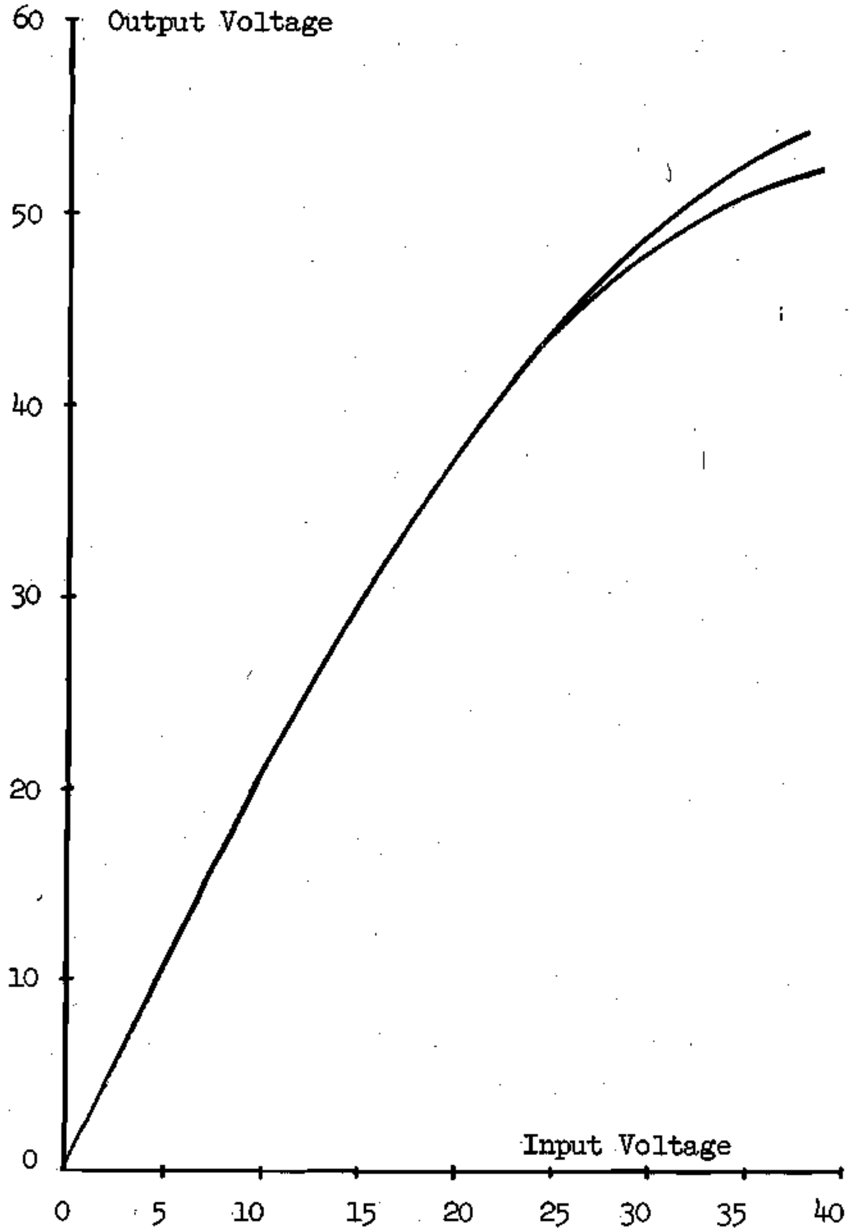


Figure 18. Transfer Function of System Amplifier

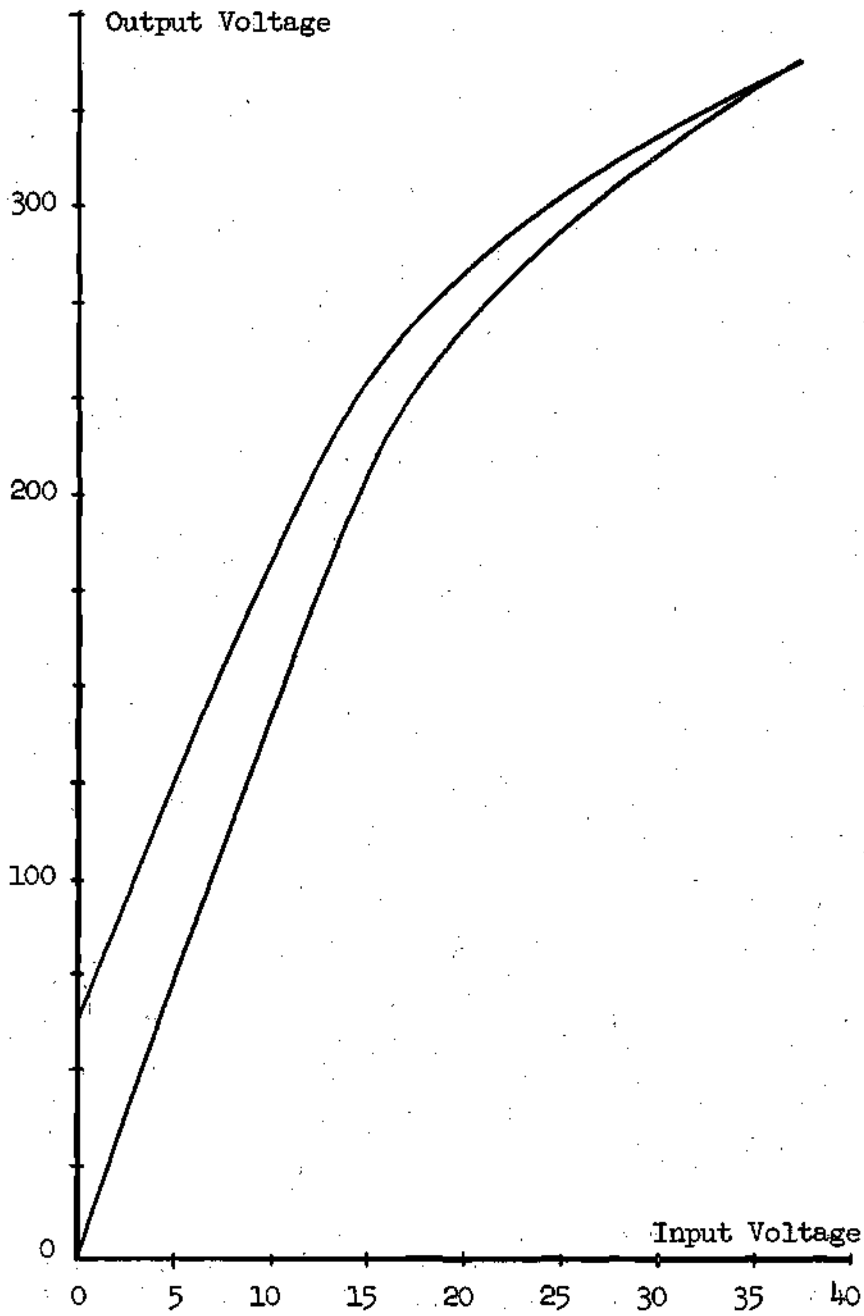


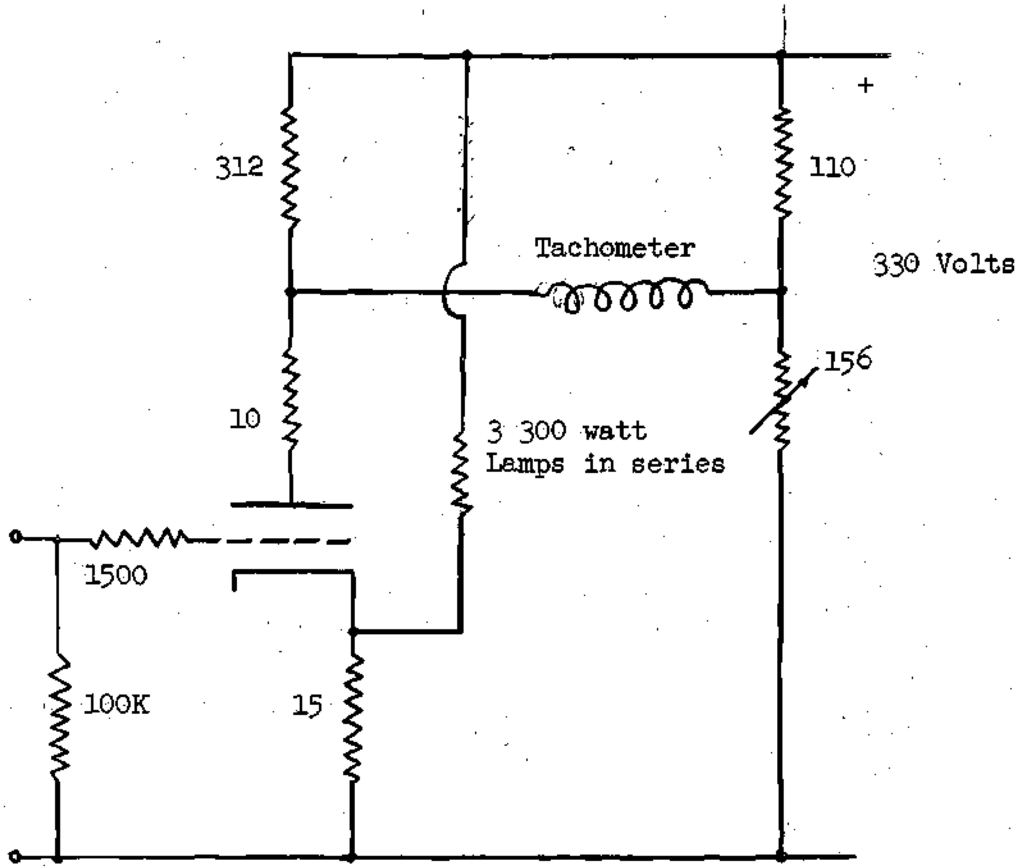
Figure 19. Transfer Function of Amplidyne

Name plate data for the machine is reproduced below.

Input voltage	78/110 dc.
Amperage	9/6.5
r. p. m.	3800/5000
Output	250 volts, 1 ampere.

Tachometers.--Two tachometer generators were used in the experimental system. A permanent magnet tachometer having an output of 0.0735 volts per radian per minute referred to the synchro shaft was used to produce a voltage proportional only to velocity. A tachometer with a wound field was used to produce an output porportional to the product of velocity and the quantity used to control the field current. Figure 21 shows the combined transfer function of the tachometer and the amplifier used to drive its field. The increase in gain of the amplifier for large signals tends to cancel the effect of saturation in the tachometer. The circuit diagram of the amplifier used to drive the tachometer is shown in Figure 20.

Frequency response measurements.--To determine the transfer function of the linear components of the system, a sinusoidal test signal was used to measure the open loop gain as a function of frequency. The circuit usually used for this type of measurement is shown in Figure 22. A diagram of the experimental set-up showing the modifications required to overcome experimental difficulties is shown in Figure 23. As slight variations in component characteristics caused a drift of the output zero position, a small amount of negative feedback was required to



(The vacuum tube is six parallel 6AS7 G tubes with individual ten ohm plate and 1500 ohm grid resistors.)

Figure 20. Circuit of Amplifier used to Drive Wound Tachometer

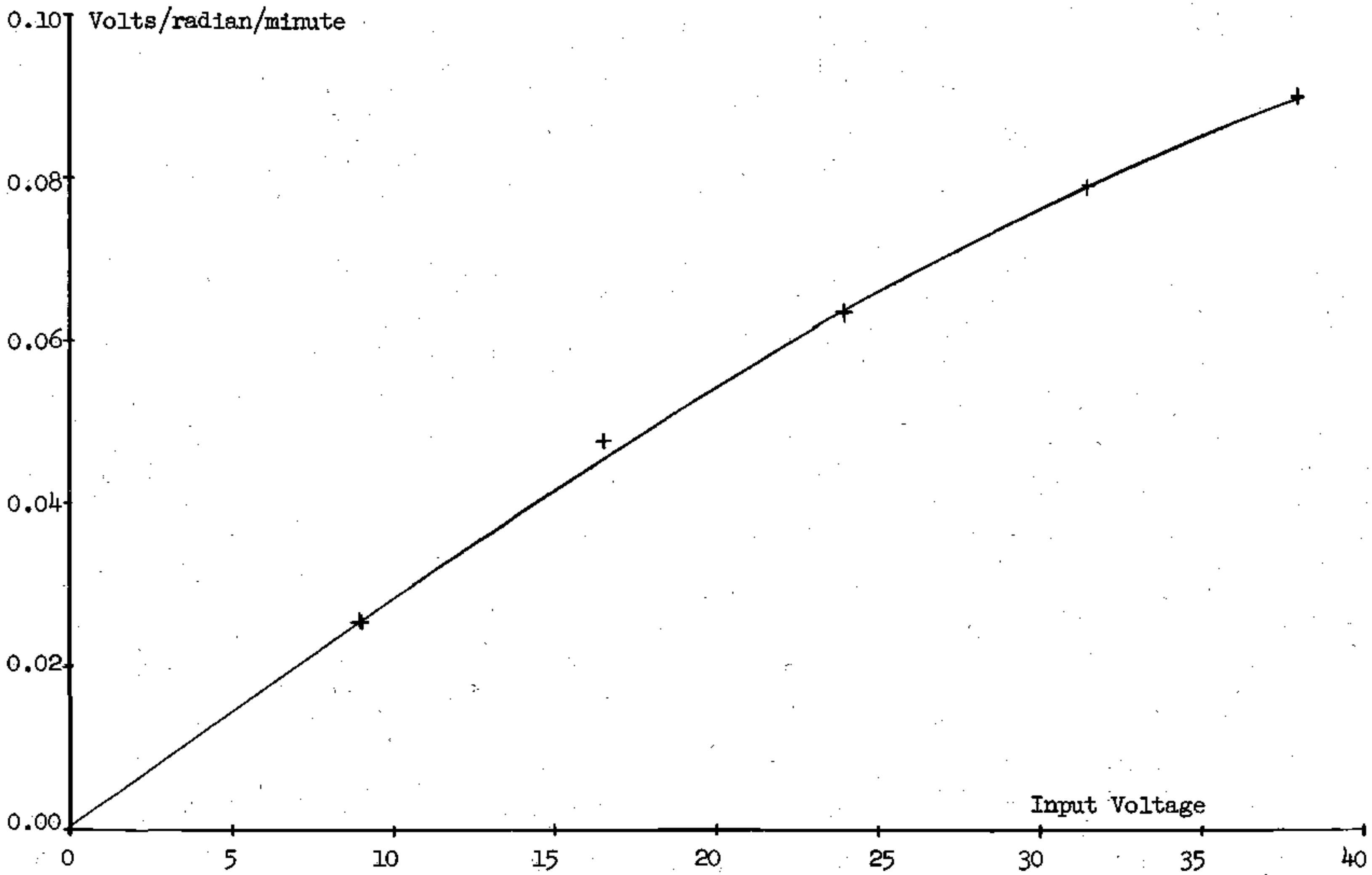


Figure 21. Combined Transfer Function of Wound Tachometer and Driving Amplifier

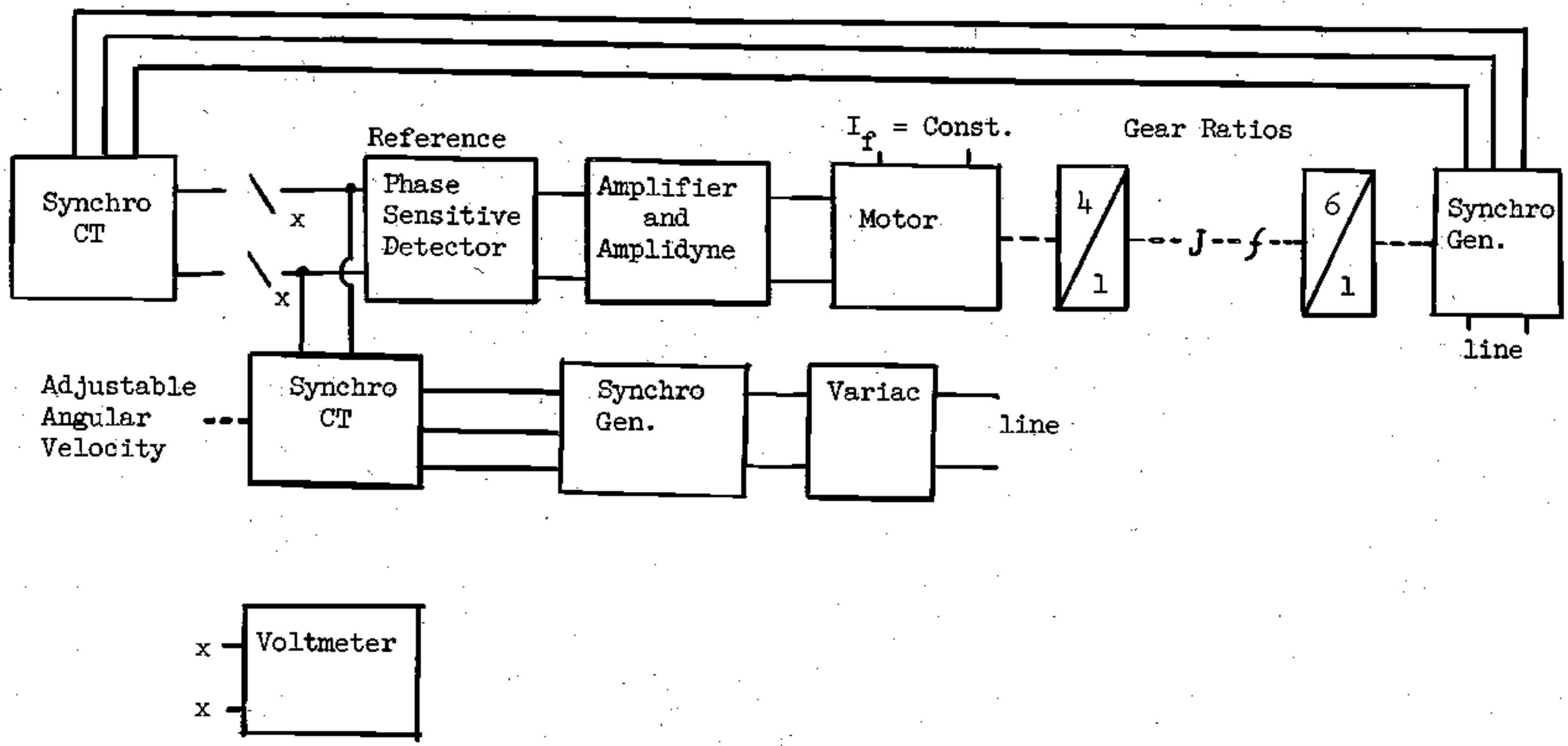


Figure 22. Standard Frequency Response Test Set-Up

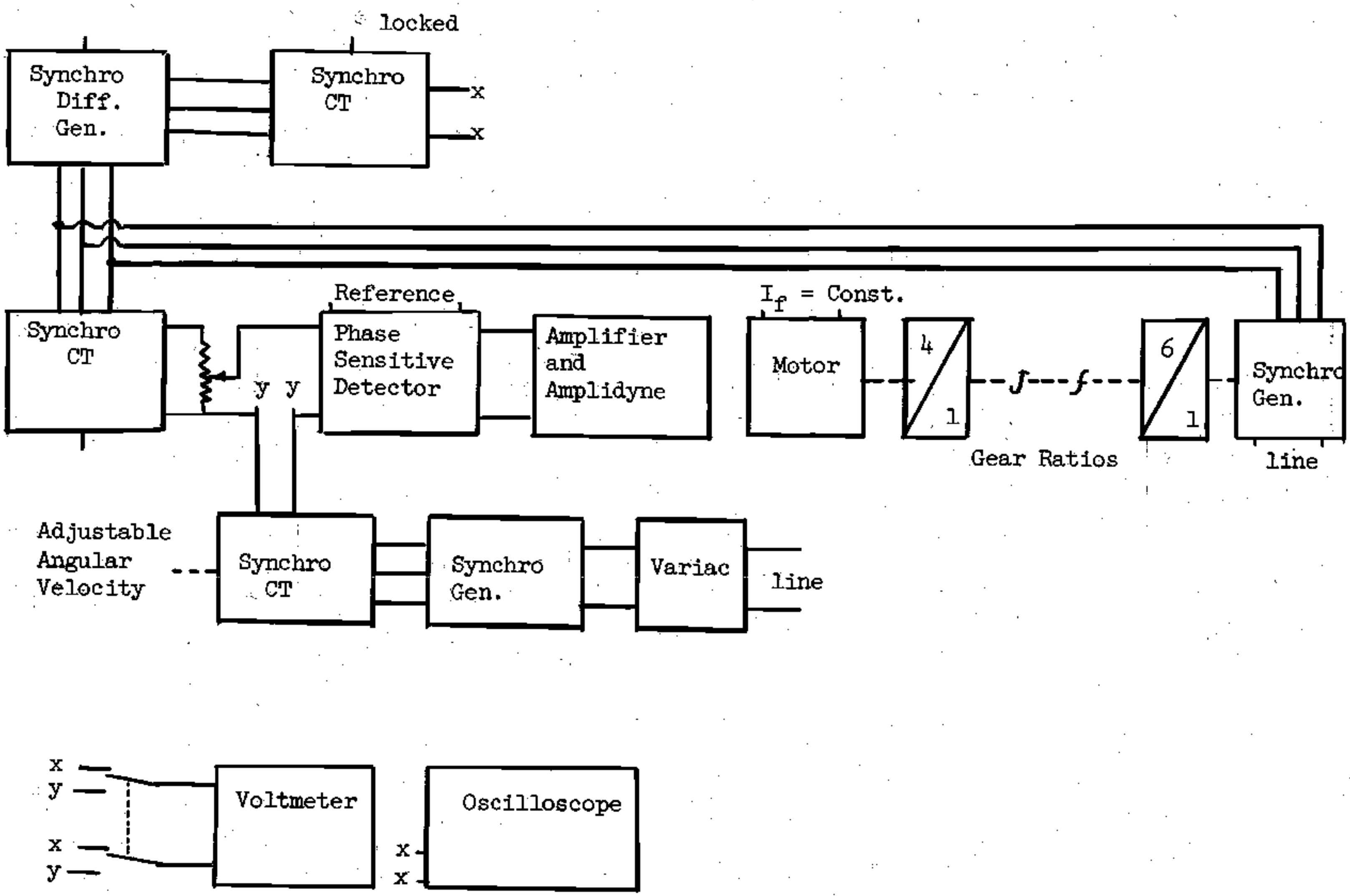
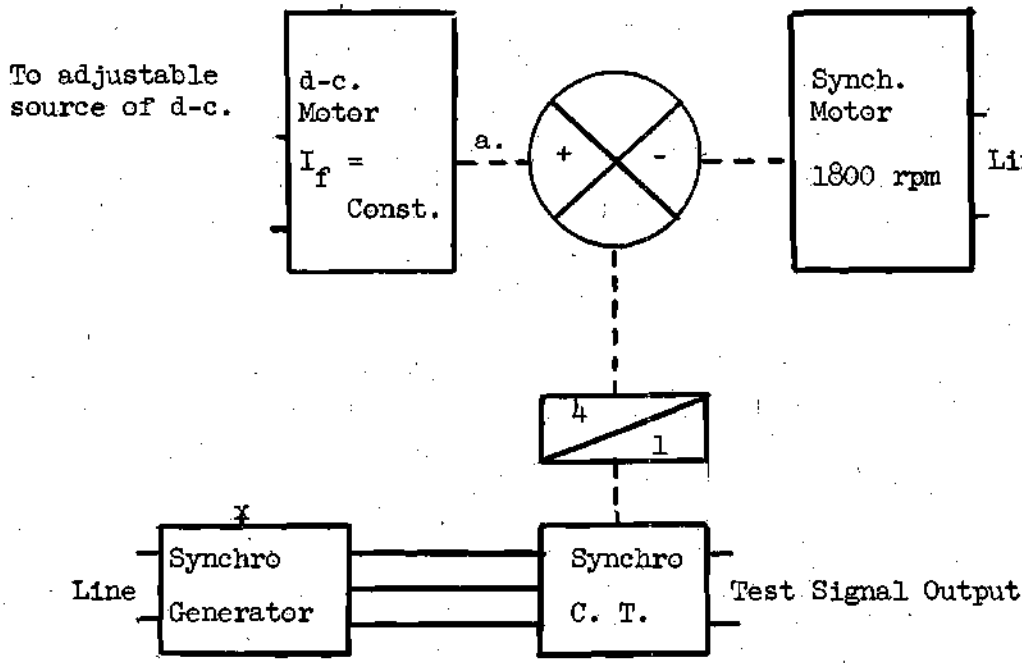


Figure 23. Experimental Frequency Response Test Set-Up

provide stability. The resulting system was then a closed loop system whose characteristics were an approximation to the desired open loop characteristics. As the carrier frequency for the synchro error detectors was sixty cycles per second, some difficulty was caused by stray voltages picked up from the power lines and other equipment. These stray voltages caused a shift in the system reference position. The effect of this shift of reference position was compensated for by the use of a separate control transformer for measurement of the output signal to allow adjustment of the output reference position despite the effect of the feedback used to provide stability. Although the adjustment of reference position could have been made by adjustment of the shaft position of the output control transformer, it was found more convenient to lock this shaft and obtain the necessary displacement by the use of a differential generator which had a geared-down dial to permit smooth adjustment.

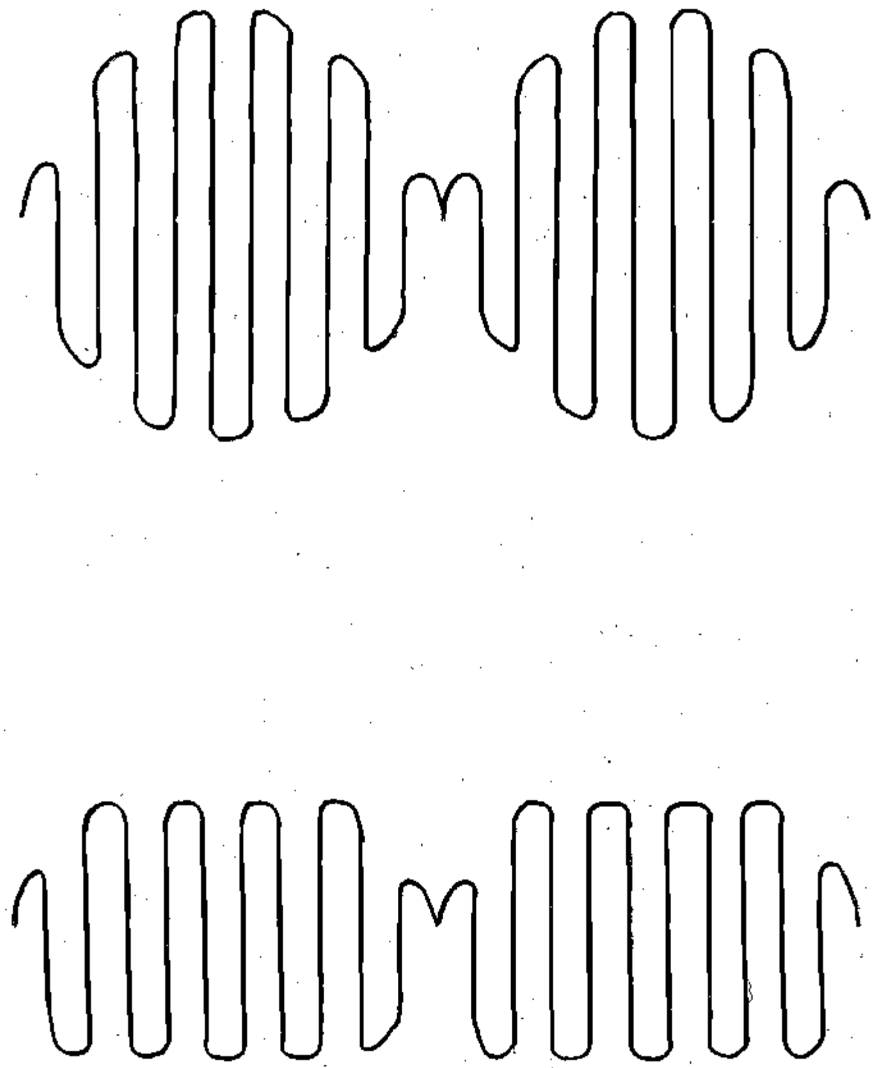
Measurement procedure.--To measure the response of the system to a sinusoidal variation of input position, it was necessary to supply a signal to the phase sensitive detector of the same type as would be generated by a sinusoidal variation of the input shaft. The required signal is a double-sideband suppressed-carrier-modulated signal. This test signal was produced by the circuit shown in Figure 24. The output signal is shown in Figure 25. The modulation frequency was determined for each measurement by measurement of the angular velocity at point "a" with a Jaquets Indicator. Input and output voltages were measured with a Hewlett-Packard Model 400-D vacuum tube voltmeter. The voltages



The 1800 rpm speed of the synchronous motor is subtracted from the speed of the d-c. motor so that the d-c. motor will not be required to operate at extremely low speeds.

The modulation frequency is equal to the speed in rpm at point "a" minus 1800 divided by $8\pi \times 60$.

Figure 24. Circuit to Generate Sinusoidal Displacement Test Signal



a

b

Figure 25. Modulation Patterns

recorded are rms values for the modulated wave except at extremely low frequencies. The rms value of the carrier at the peak of the modulation waveform was recorded for these frequencies. In each case, the synchro differential generator was adjusted to place the output zero reference position at the center of the sinusoidal variation. The correct reference position was determined by the appearance of a double sideband suppressed carrier modulation pattern on the oscilloscope used to monitor the output. Figure 25a shows the correct modulation pattern.

The frequency response curve.--Figure 26 shows the gain of the system as a function of frequency. This curve is plotted from the data in Table 2 and applies to the system shown in Figure 23 with test conditions as given in Table 1. As the amount of negative feedback used was only two per cent, the measured frequency response is a close approximation to the true open loop characteristics of the system. At frequencies below 0.09 radians per second, the velocity of the system was low enough to permit static friction to arrest the motion of the output shaft until the error increased to a value sufficient to produce enough torque to overcome the friction. These nonlinear friction effects limited the motion of the output shaft at low velocities and caused a flattening of the frequency response curve at the low frequencies. Figure 25b shows the effect of this limiting on the modulation pattern.

Open loop gain function.--The open loop gain function was obtained from the frequency response data by the asymptotic procedures given in references (6) and (7). The low frequency asymptote was started at a slope of minus twenty decibels per decade as it was known that the

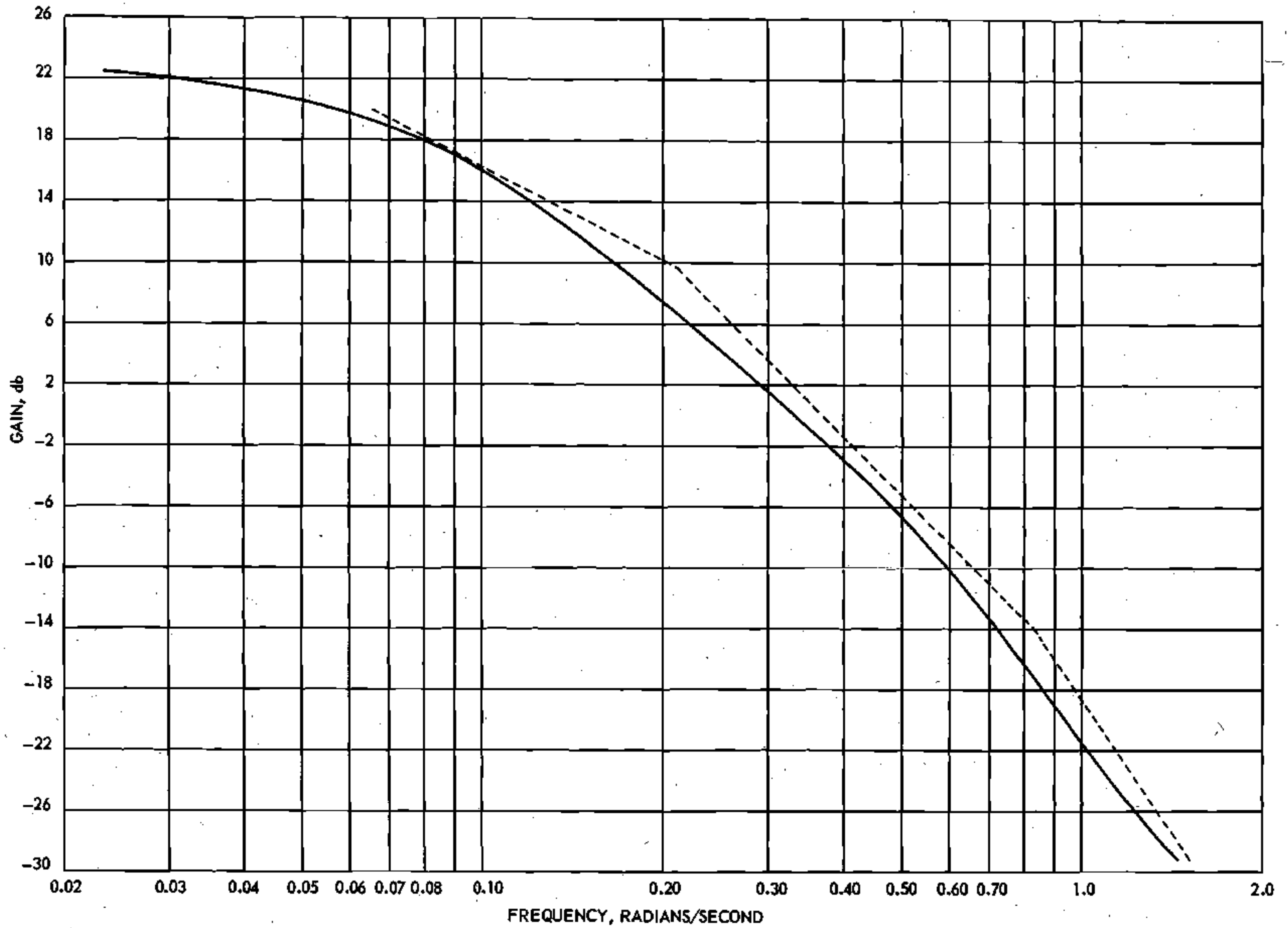


Figure 26. Frequency Response Plot.

Table 1. Test Conditions for Frequency Response Measurements

Negative feedback:	1.0 volts rms for an error of 90 degrees
Output voltage:	26 volts rms for an error of 90 degrees
Signal voltage:	1.0 volts rms for an angle of 90 degrees between the signal control transformer and the generator which drives it.

direct-current shunt motor should cause a pole at the origin and that the flattening of the frequency response curve at low frequencies was due to nonlinear friction effects. The function obtained,

$$G(s) = \frac{K_{ab}}{s(s + 0.215)(s + 0.83)} \quad (6)$$

should accurately represent the variation of the gain with frequency as both of the two higher frequency poles are above the region of uncertainty caused by the nonlinear friction. As K_{ab} should be a constant, the range of variation of the values obtained by substituting the measured values of gain into the gain function gives a measure of the accuracy of the approximation. Figure 26 shows the variation of K_{ab} with frequency for the frequency range from 0.1 radian per second to 1.5 radian per second.

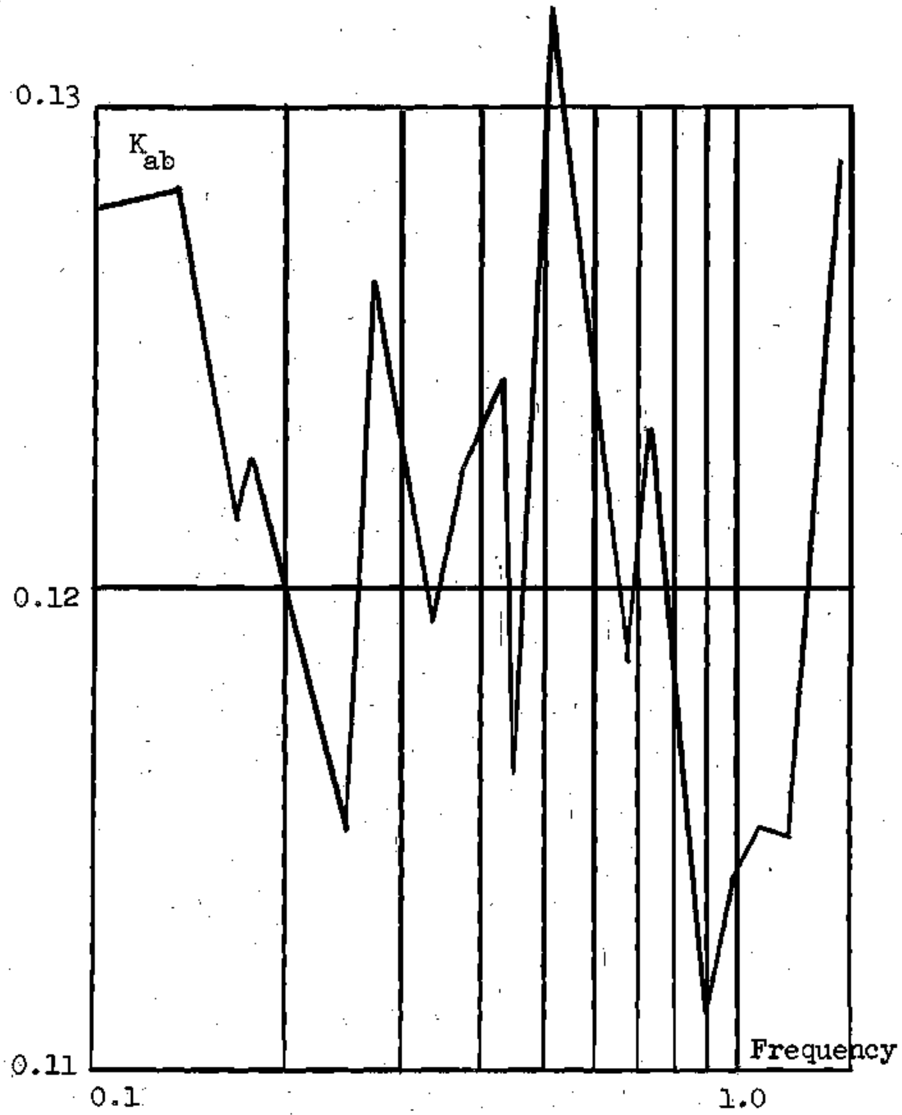


Figure 27. Variation of K_{ab} with Frequency in Equation (6)

(This variation includes experimental error and the inaccuracy of the approximation.)

APPENDIX II

DESIGN CONSIDERATIONS FOR NONLINEAR CHARACTERISTICS OF SYSTEM

Choice of design methods.--In the experimental system nonlinear function generators are used to cause the gain and the tachometric feedback to vary as functions of the error magnitude. A method of design or analysis must show the effects of continuous variation of these quantities in a form which may readily be related to the response characteristics of the system. Mitrovic's method (8) (9) is useful in this case as it defines the characteristics of the system in terms of the location of a point, the M point, with respect to a set of curves, the B_o versus B_1 curves. For the case in which the open loop transfer function has no zeros, the B_o versus B_1 curves are independent of the gain and the tachometric feedback, and the motions of the M point in the B_o and B_1 directions are determined independently by the gain and the tachometric feedback respectively. One zero in the transfer function will not affect the B_o versus B_1 curves but will cause the gain to affect both the B_o and the B_1 coordinates of the M point. If the transfer function has more than one zero, the B_o versus B_1 curves would be altered by changes in the gain, and the use of Mitrovic's method for a system with varying gain and tachometric feedback would require excessive calculations.

The accuracy with which Mitrovic's method can be applied to a nonlinear system is dependent upon the accuracy with which consideration of only the fundamental component of a nonsinusoidal waveform describes the system (10). The frequency response characteristics of the linear components of the experimental system strongly attenuate high frequency components, and the symmetry of the nonlinear functions about the origin insures that only odd harmonics will be generated. As there are no sharp breaks in the functions used, the higher frequency components should be of small magnitude. It may be assumed from these considerations that Mitrovic's method will accurately describe the characteristics of the experimental system.

Design considerations.--As all of the poles of the open loop transfer function are on the negative real axis, the most practical linear means of modifying the system characteristics would be the use of resistance-capacitance compensation networks (11). The transfer function of the uncompensated system has been taken as the starting point here, however, to evaluate the effect of modification of the system by the inclusion of nonlinear elements. In addition to the examination of loci for the M point on the B_o versus B_1 curves to determine the variation of system characteristics with error magnitude, consideration of the characteristics of the uncompensated system is of value in the choice of functions to determine the motion of the M point along the loci.

Although the pole at the origin in the open loop gain function indicates that the system should have zero steady state error, nonlinear friction effects at low velocities cause error in the form of a dead zone which can be reduced by an increase in the gain.

Physical characteristics of the components limit the signal levels which can effectively be used. The signal level at the input of a component should not greatly exceed the value required to drive it to full output to avoid the possibility of a temporary loss of effectiveness of the component due to overload.

The requirements of high gain for small error and limited signal magnitude can be satisfied by the use of a saturation characteristic in the error channel. A saturation characteristic would result in a velocity lag error which would increase at a greater than linear rate with velocity and would require the use of separate compensation to prevent the velocity lag error from becoming excessive.

Velocity lag error can be reduced by the use of positive tachometric feedback to provide part of the voltage required to drive the system at the input velocity. To reduce the effect on the overshoot, the tachometric feedback may be made a function of the error so that the damping is reduced only for large error. For any function, however, there will be some increase in the overshoot because of the higher velocity attained by the system while operating in the lightly damped region.

Loci on the B_o versus B_1 chart.--The B_o versus B_1 curves determined from the open loop gain function are shown in Figure 28. The effects of making the gain a function of error can be determined by examination of the motion of the M point along a locus of constant B_1. Such a locus would be a vertical line on the B_o versus B_1 chart in Figure 28. If the tachometric feedback is made a function of error the resulting locus will be a line of constant B_o. This locus would be a horizontal line on the chart in Figure 28. More general loci may be obtained by making both

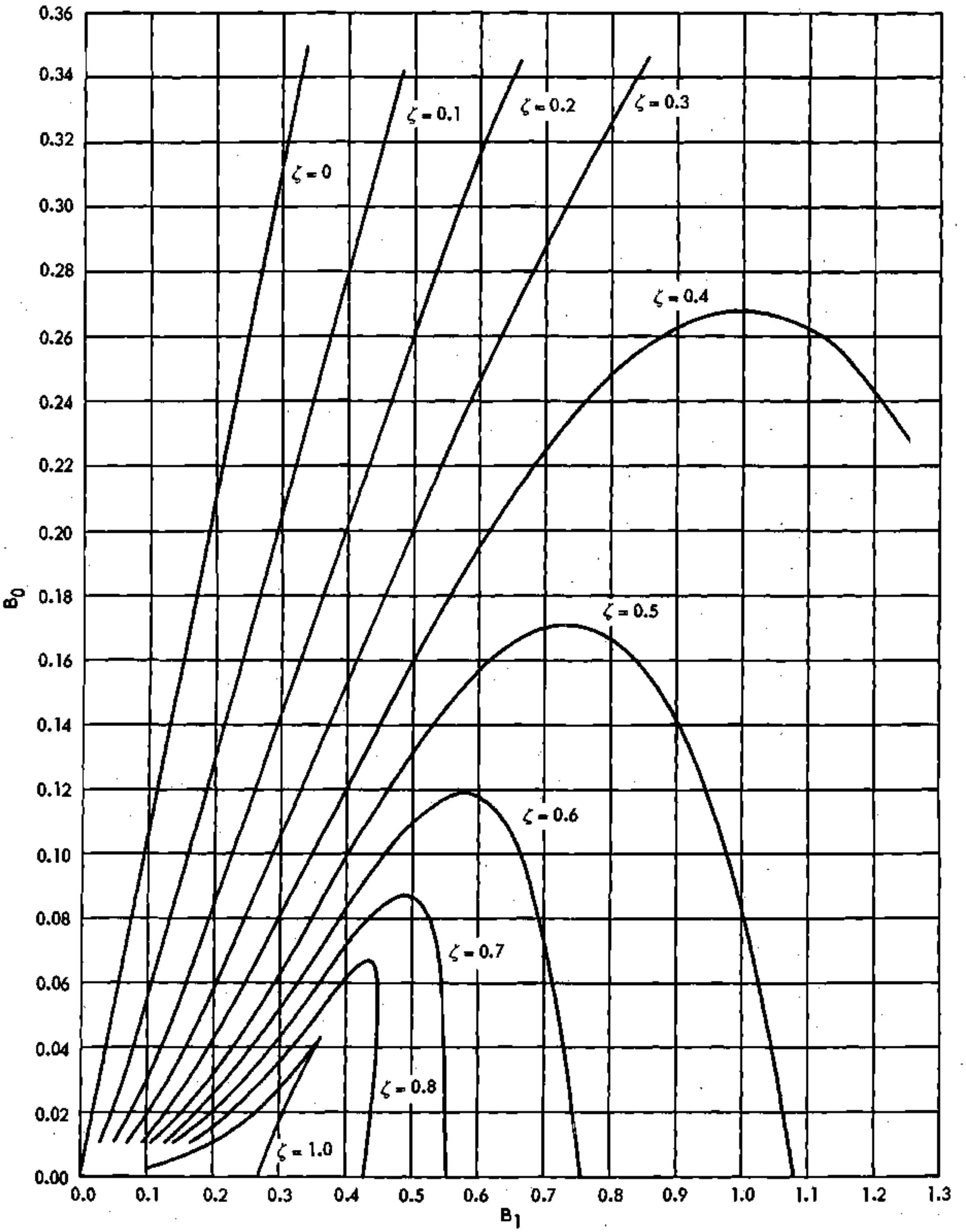


Figure 28. B_o Versus B_1 Curves.

the gain and the tachometric feedback functions of error. The possible loci are examined separately to determine their value for the experimental system.

Vertical loci.--Decreased damping for large error may be obtained by causing the gain of the system to increase with error. The function chosen to relate the gain to the error would determine the rate at which the M point moved along the locus. Requirements of low steady-state error would make such a locus undesirable for a type zero control system or systems of type one or higher which have excessive static friction. The reduced gain for small error would also permit load torques to cause larger errors. Although a system with this gain characteristic would have a larger velocity lag error at low velocities than a system with a fixed M point and similar overshoot for large step inputs, the error would increase more slowly with velocity.

Horizontal locus.--As the lines of constant damping on the B_o versus B_1 chart have a large slope for most values of B_1, a horizontal locus will provide a greater change of damping than a vertical locus unless extremely low gains are used and B_1 is less than 0.3. For high gains, a horizontal locus might permit the M point to move to the left of the $\zeta = 0$ curve and cause instability.

General loci.--For applications where the slight increase in complexity is permissible, adjustment of both the gain and the tachometric feedback with error will permit a wider range of response characteristics to be realized. A greater variation in the location of the M point is possible as the combination of vertical and horizontal position may vary over a wider range in either coordinate if a corresponding shift is made in the remaining coordinate to preserve stability.

<u>Effect of tachometric feedback on the B_1 coordinate</u>.--The B_1 coordinate of the M point is the coefficient of the S term in the characteristic equation of the system.

$$G(S) = \frac{K_{ab}}{S(S + 0.215)(S + 0.83)} = \frac{V_b}{V_a}$$

$$S^3 + 1.045\ S^2 + 0.1785\ S \quad V_b = K_{ab}\ V_a$$

At constant velocity,

$$0.1785 \quad S\ V_b = K_{ab}\ V_a.$$

With tachometric feedback a new voltage,

$$V_a' = V_a + V_{tach}$$

will be required to maintain a given S V_b.

$$0.1785 \quad S\ V_b = K_{ab}\ (V_a' - V_{tach})$$

$$K_f \quad S\ V_b = K_{ab}\ V_a'$$

$$V_{tach} = \frac{K_{ab} \quad K_t\ V_a}{0.1785}$$

$$K_f = 0.1785 \frac{\dfrac{K_{ab}\ K_t\ V_a}{0.1785} + V_a}{V_a}$$

$$K_f = 0.1785 + K_t\ K_{ab}$$

K_t = Tachometer output voltage per radian per second velocity referred to the synchro shaft.

K_{ab} = System gain past point where tachometer is introduced.

APPENDIX III

TABULATED POINTS FOR FREQUENCY RESPONSE CURVE AND B_o VERSUS B_1 CURVES

Table 2. Tabulated Points for Frequency Response Measurements

ω radians per second	Gain decibels
0.023	22.4
0.047	21.1
0.063	19.5
0.080	17.7
0.086	17.5
0.095	16.7
0.136	12.9
0.165	10.1
0.172	9.7
0.207	7.0
0.245	4.4
0.272	3.5
0.335	0.0
0.374	-3.6
0.418	-3.3
0.455	-5.4
0.487	-6.0
0.536	-7.3
0.614	-10.5
0.673	-12.2
0.703	-13.2
0.732	-14.0
0.782	-15.7
0.871	-18.6
0.975	-20.8
1.07	-22.8
1.28	-26.8
1.45	-28.8

Table 3. Calculated Points for B_o versus B_1 Curves*

(continued)

$\zeta = 0.1$ $\phi_1 = -1.0$ $\phi_2 = 0.2$ $\phi_3 = 0.96$

$$B_o = 1.045\,\omega_n^2 - 0.2\,\omega_n^3$$

$$B_1 = 0.209\,\omega_n + 0.96\,\omega_n^2$$

ω_n	B_o	B_1
0.00	0.0 x 10^{-3}	0.0
0.05	2.6	0.013
0.10	10.3	0.031
0.15	22.9	0.053
0.20	40.4	0.080
0.25	62.5	0.112
0.30	89.1	0.149
0.35	121.	0.191
0.40	155.	0.237
0.45	195.	0.289
0.50	238.	0.345
0.55	285.	0.406
0.60	335.	0.471
0.65	389.	0.542
0.70	452.	0.617
0.75	517.	0.697

Table 3. Calculated Points for B_o versus B_1 Curves*

(continued)

$\zeta = 0.2$ $\phi_1 = -1.0$ $\phi_2 = 0.4$ $\phi_3 = 0.84$

$$B_o = 1.045\,\omega_n^2 - 0.4\,\omega_n^3$$

$$B_1 = 0.418\,\omega_n + 0.84\omega_n^2$$

ω_n	B_o	B_1
0.00	0.0 x 10^{-3}	0.0
0.05	2.6	0.023
0.10	10.1	0.050
0.15	22.3	0.082
0.20	38.8	0.117
0.25	59.4	0.157
0.30	83.7	0.201
0.35	112.	0.250
0.40	142.	0.302
0.45	177.	0.359
0.50	213.	0.419
0.55	252.	0.484
0.60	292.	0.553
0.65	330.	0.627
0.70	378.	0.705
0.75	422.	0.787
0.80	467.	0.872
0.85	483.	0.962
0.90	558.	1.06

(continued)

Table 3. Calculated Points for B_o versus B_1 Curves*

(continued)

$\zeta = 0.3$ $\phi_1 = -1.0$ $\phi_2 = 0.6$ $\phi_3 = 0.64$

$$B_o = 1.045\,\omega_n^2 - 0.6\,\omega_n^3$$

$$B_1 = 0.627\,\omega_n^2 + 0.64\,\omega_n^2$$

ω_n	B_o	B_1
0.00	0.0×10^{-3}	0.0
0.05	2.5	0.032
0.10	9.9	0.069
0.15	21.6	0.108
0.20	37.2	0.151
0.25	56.2	0.197
0.30	78.3	0.246
0.35	103.	0.298
0.40	130.	0.353
0.45	158.	0.412
0.50	187.	0.474
0.55	218.	0.539
0.60	248.	0.606
0.65	279.	0.678
0.70	309.	0.753
0.75	338.	0.831
0.80	365.	0.911
0.85	391.	0.996
0.90	423.	1.08
0.95	433.	1.17
1.00	445.	1.27
1.05	464.	1.36

(continued)

Table 3. Calculated Points for B_o versus B_1 Curves*

(continued)

$\zeta = 0.4$ $\phi_1 = -1.0$ $\phi_2 = 0.2$ $\phi_3 = 0.36$

$$B_o = 1.045\,\omega_n^2 - 0.8\,\omega_n^3$$

$$B_1 = 0.836\,\omega_n + 0.36\,\omega_n^2$$

ω_n	B_o	B_1
0.00	0.0 x 10^{-3}	0.0
0.05	2.5	0.043
0.10	8.7	0.087
0.15	20.9	0.134
0.20	35.6	0.182
0.25	53.1	0.232
0.30	72.9	0.283
0.35	94.7	0.337
0.40	117.	0.392
0.45	140.	0.449
0.50	163.	0.508
0.55	185.	0.569
0.60	205.	0.631
0.65	224.	0.696
0.70	241.	0.762
0.75	254.	0.830
0.80	262.	0.900
0.85	268.	0.971
0.90	267.	1.04
0.95	262.	1.12
1.00	245.	1.20
1.05	222.	1.27

(continued)

Table 3. Calculated Points for B_o versus B_1 Curves*

(continued)

$\zeta = 0.5$ $\phi_1 = -1.0$ $\phi_2 = 1.0$ $\phi_3 = 0$

$$B_o = 1.045\,\omega_n^2 - \omega_n^3$$

$$B_1 = 1.045\,\omega_n$$

ω_n	B_o	B_1
0.00	0.0 x 10^{-3}	0.0
0.05	2.5	0.052
0.10	9.5	0.105
0.15	20.3	0.157
0.20	34.0	0.210
0.25	49.1	0.262
0.30	67.5	0.314
0.35	85.1	0.367
0.40	104.	0.420
0.45	122.	0.470
0.50	138.	0.523
0.55	152.	0.577
0.60	162.	0.630
0.65	169.	0.680
0.70	171.	0.730
0.75	169.	0.787
0.80	160.	0.840
0.85	145.	0.890
0.90	121.	0.940
0.95	89.6	0.997
1.00	45.0	1.05
1.05	-8.0	1.10

(continued)

Table 3. Calculated Points for B_o versus B_1 Curves*

(continued)

$\zeta = 0.6$ $\phi_1 = -1.0$ $\phi_2 = 1.2$ $\phi_3 = -0.44$

$$B_o = 1.045\,\omega_n^2 - 1.2\,\omega_n^3$$

$$B_1 = 1.25\,\omega_n - 0.44\,\omega_n^2$$

ω_n	B_o	B_1
0.00	0.0 x 10^{-3}	0.0
0.05	2.5	0.062
0.10	9.3	0.121
0.15	19.6	0.178
0.20	32.4	0.232
0.25	46.9	0.285
0.30	62.1	0.335
0.35	77.6	0.384
0.40	91.2	0.430
0.45	104.	0.473
0.50	113.	0.515
0.55	118.	0.555
0.60	119.	0.592
0.65	114.	0.627
0.70	103.	0.659
0.75	85.0	0.690
0.80	57.6	0.718
0.85	22.2	0.745
0.90	-24.8	0.768

(continued)

Table 3. Calculated Points for B_o versus B_1 Curves*

(continued)

$\zeta = 0.7$ $\phi_1 = 1.0$ $\phi_2 = 1.4$ $\phi_3 = -0.96$

$$B_o = 1.045\,\omega_n^2 - 1.4\,\omega_n^3$$

$$B_1 = 1.46\,\omega_n - 0.96\,\omega_n^2$$

ω_n	B_o	B_1
0.00	0.0	0.0
0.05	2.5	0.071
0.10	9.1	0.136
0.15	18.9	0.187
0.20	30.8	0.254
0.25	43.8	0.305
0.30	56.7	0.352
0.35	69.0	0.393
0.40	78.4	0.430
0.45	85.5	0.462
0.50	87.5	0.490
0.55	85.5	0.512
0.60	75.6	0.531
0.65	69.0	0.543
0.70	34.3	0.552
0.75	0.2	0.555
0.80	-44.8	0.553

(continued)

Table 3. Calculated Points for B_o versus B_1 Curves*

(continued)

$\zeta = 0.8$ $\phi_1 = 1.0$ $\phi_2 = 1.6$ $\phi_3 = -1.56$

$$B_o = 1.045\,\omega_n^2 - 1.6\,\omega_n^3$$

$$B_1 = 1.68\,\omega_n^2 - 1.56\,\omega_n^2$$

ω_n	B_o	B_1
0.00	0.0	0.0
0.05	2.4	0.080
0.10	8.9	0.152
0.15	18.2	0.217
0.20	29.2	0.274
0.25	40.6	0.323
0.30	51.3	0.364
0.35	60.4	0.397
0.40	65.6	0.422
0.45	67.2	0.440
0.50	62.5	0.450
0.55	51.0	0.452
0.60	32.0	0.446
0.65	4.0	0.433
0.70	-34.5	0.412

(continued)

Table 3. Calculated Points for B_o versus B_1 Curves*

$\xi = 1.0$ $\phi_1 = 1.0$ $\phi_2 = 2.0$ $\phi_3 = -3$

$$B_o = 1.045\,\omega_n^2 - 2.0\,\omega_n^3$$

$$B_1 = 2.09\,\omega_n^2 - 3.0\,\omega_n^2$$

ω_n	B_o	B_1
0.00	0.0	0.0
0.05	2.4	0.097
0.10	8.5	0.179
0.15	16.9	0.246
0.20	26.0	0.298
0.25	34.5	0.335
0.30	40.5	0.357
0.35	43.2	0.364
0.40	40.0	0.356
0.45	30.8	0.333
0.50	12.5	0.295
0.55	-14.0	0.242

*These values were calculated by the procedure given in reference (9).

(continued)

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