MATERIAL DESIGN VS MATERIAL SELECTION: A TRADE-OFF BETWEEN DESIGN FREEDOM AND DESIGN SIMPLICITY

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MATERIAL DESIGN VS MATERIAL SELECTION: A TRADE-OFF BETWEEN DESIGN FREEDOM AND DESIGN SIMPLICITY

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NOMENCLATURE

B	in-plane spacing of webs of square honeycomb core
BRP(s)	blast resistant panel(s)
C_{dk}	Design capability index
CDSE	Cost of Design Space Expansion
d_i^+, d_i^-	deviation variables for goals in cDSP
DCI(s)	Design capability index (indices)
DSP(s)	Decision Support Problem(s)
cDSP	compromise Decision Support Problem
h_c, h_f, h_b	thickness of core webs and face sheets, respectively
δ, $\Delta\delta$	deflection of back face, variation in deflection of back face
$\bar{\varepsilon}_c$	average crushing strain of core
g_δ, Δg_δ	deflection constraint function, variation in deflection constraint
g_M, Δg_M	mass constraint function, variation in mass constraint
Γ, $\Delta\Gamma$	Gamma front face shear constraint, variation in gamma front face shear constraint
$H, \bar{H}$	height of un-deformed and deformed core, respectively
KE_I, KE_{II}	kinetic energy per unit area in panel after stages 1 and 2, respectively
L	width of plate
λ_c, λ_s	factors governing strength of core in crush and stretch
M, ΔM	total mass/area of plate, variation in total mass/area

μ, $\Delta\mu$	Mu front face shear constraint, variation in mu front face shear constraint
μ_p, Δ_p	mean and variation of peak pressure
μ_t, Δ_t	mean and variation of characteristic pulse time
p_0	peak pressure of free-field pulse
$p(t)$	Pressure of the blast as a function of time
ρ_f, ρ_b, ρ_c	density of front face sheet material, back face sheet material, and core base material
$\Delta\rho$	variation in density
R_c	relative density of core
$\sigma_{Y,f}, \sigma_{Y,b}, \sigma_{Y,c}$	yield strength of front face sheet material, back face sheet material, and core base material
$\Delta\sigma$	variation in yield strength
t_o	characteristic time of incident pressure pulse
T_δ, $T_{\Delta\delta}$	targets for deflection and variance of deflection goals
$VDSE$	Value of Design Space Expansion
W_i	weighting factors in Archimedean cDSP deviation function
W_{III}^{P}	plastic work per unit area dissipated in stage three
Z_A	Archimedean deviation function in the cDSP

SUMMARY

Materials have traditionally been selected for the design of a product; however, advances in the understanding of material processing along with simulation and computation techniques are now making it possible to systematically design materials by tailoring the properties of the material to achieve the desired product performance. Material design offers the potential to increase design freedom and enable improved product performance; however, this increase in design freedom brings with it significant complexity in predictive models used for design, as well as many new design variables to consider. Material selection, on the other hand, is a well-established method for identifying the best materials for a product and does not require the complex models needed for material design. But material selection inherently limits the design of products by only considering existing materials. To balance increasing design costs with potentially improved product performance, designers must have a method for assessing the value of material design in the context of product design.

In this thesis, the Design Space Expansion Strategy (DSES) and the Value of Design Space Expansion (VDSE) metric are proposed for supporting a designer's decision between material selection and material design in the context of product design. The strategy consists of formulating and solving two compromise Decision Support Problems (cDSP). The first cDSP is formulated and solved using a selected baseline material. The second cDSP is formulated and solved in an expanded material design space defined by material property variables in addition to other system variables. The two design solutions are then compared using the VDSE metric to quantify the value of expanding the material design space. This strategy is demonstrated in this thesis with an example of blast resistant panel design and is validated by application of the validation square, a framework for the validating design methods.

CHAPTER 1

MOTIVATION FOR DEVELOPING THE DESIGN SPACE EXPANSION STRATEGY

In this thesis, the first steps are taken towards a tool to support a designer's decision between material selection and material design in the context of product design. The Design Space Expansion Strategy is proposed here as a starting point to explore how to formulate such a tool. To elaborate on this decision, material design and material selection are discussed in Section 1.1. The decision between material design and selection represents a compromise between design freedom and design simplicity. These two concepts are defined and discussed in Section 1.2. The research questions for this thesis are introduced in Section 1.3. This chapter concludes with an overview of the validation strategy for this thesis in Section 1.4.

1.1 MATERIAL DESIGN VS. MATERIAL SELECTION

For every physically-realized system, there comes a point during the design process when designers must determine the material(s) that will give the product its substance while enabling the product to meet the design requirements. In practice, designers *select* a material from the set of existing materials either through designer expertise or by following a material *selection* procedure (Ashby, 1999); however, the achievement of design requirements may be limited by the selection of a material. Material *design* offers the potential to tailor materials to product-specific requirements, enabling improvement in product performance that is not possible with existing materials. To choose between material design and selection in product design, the potential performance benefits due to designing materials must be balanced by the increase in design timeline and expense of designing materials as compared to selecting materials.

To provide an analogy to explain a designer's choice between material design and selection, consider the design of a sandwich. A deli owner is interested in offering a new sandwich for his health-conscious customers. The sandwich will contain several layers of healthy vegetables and lean meats, so the bread in the sandwich must be strong enough to support the load. In addition, the bread itself must be nutritious, so it should be made with whole grains. And of course the sandwich must be tasty in order to be a success! When considering the bread for the sandwich, the deli owner has a choice to make: whether to purchase a ready-made loaf of bread from his baker or whether to design a new loaf of bread specifically for this sandwich. In this analogy the product in question is the sandwich, and the material which will either be selected or designed is the bread.

By designing the bread, the deli owner can achieve bread that meets the material requirements for the sandwich such that the sandwich as a whole meets the nutritional, textural, and taste requirements of his customers, but the deli owner will have to spend a lot of time and money working with his baker to try new recipes and ingredients and develop the procedure for baking the new bread. On the other hand, the customers may be just as satisfied with the sandwich if the deli owner uses bread that his baker has already developed, and by selecting rather than designing the bread, the new sandwich can be ready to sell much sooner and at less cost. How is the deli owner to decide between designing and selecting bread for new sandwiches? This is the same question that mechanical designers face when considering the selection or design of materials in product design. To explore this question further, material selection and material design are discussed in the next sections.

1.1.1 Material Design

Material design refers to the tailoring of material properties to meet product performance goals (McDowell, 1998; Seepersad, 2004). Existing materials can be adapted to produce new materials with desired properties and behavior. Olson identifies a logical structure for material design incorporating the four elements of material science: *processing*, *structure*, *properties*, and technological *performance* (Olson, 2000). A chain is formed by connecting adjacent elements of this structure, shown in Figure 1.1.

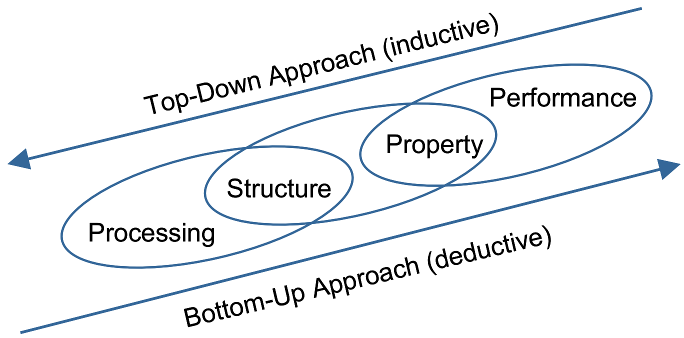


FIGURE 1.1. LOGICAL STRUCTURE FOR MATERIAL DESIGN (MODIFIED FROM OLSEN, 1997)

Processing refers to the manufacturing processes used to create the material, such as casting, machining, and heat-treating and determines the structure of the material. The *structure* of a material is identified by, for example, material constituents, grain size, morphology, and material defects. The *structure* in turn determines the material's *properties*. Material properties characterize the behavior of a material and can be found in many engineering material tables. Some examples of material properties are Young's modulus, thermal conductivity, and mass density. These properties are then used to determine the material performance. The *performance* of a material describes how a part constructed from the given material behaves under certain loading conditions or design requirements. Material properties are an indication of the performance of the material in use and thus link structure to material performance.

As shown in Figure 1.1, current material development processes are deductive in nature, i.e., a bottom-up approach. By changing the processing of a material, the structure of the material is affected, which results in a new material with new properties and performance. Although the material development process may begin with a goal or hypotheses in mind geared at achieving an improvement in some aspect of material performance, it is largely an educated trial-and-error method. From the systems perspective, it is preferred to take an inductive, or top-down, approach, in which the desired performance is the driving factor for identifying the processing path that achieves the structure and properties necessary for achieving the desired performance. To realize this inductive approach to material design, the implications of changes in processing and structure must be understood and modeled, which is not a trivial task.

As Smith has observed, people began using newly discovered materials long before understanding much about the new materials (Smith, 1988). This ad hoc, discovery-based tradition of material development has dominated from prehistory and continues to this day. Historically speaking, material scientists have only recently developed the theories which account for the material structure of old world products such as quench-hardened steel swords (Olson, 2000). Materials have a complex, multilevel structure in which strong interactions among levels and the inevitable interplay of perfection and imperfection at all levels determine the behavior of the material. Material *designers* must understand and model this complex multilevel structure in order to predict the behavior of a material before it is physically realized to achieve a top-down design process. The development and validation of multilevel material models are significant tasks requiring expertise that most mechanical designers do not have. In addition, these complex models are developed to describe the behavior of very specific material systems, so designers must have information about the requirements of the material before developing a predictive material model for design.

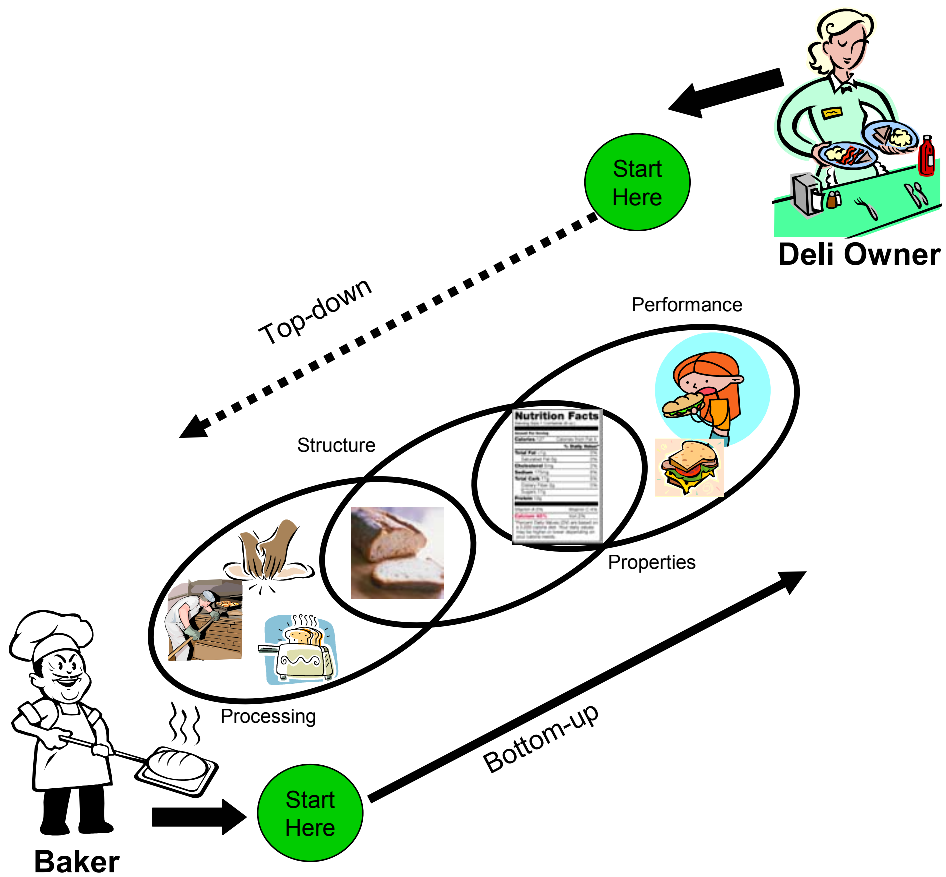


FIGURE 1.2. THE PROCESSING-STRUCTURE-PROPERTIES-PERFORMANCE CHAIN FOR SANDWICH DESIGN (MODIFIED FROM OLSEN, 1997)

To summarize, recall the sandwich design analogy. Bread, as a material, embodies the processing-structure-properties-performance chain introduced in Figure 1.1 and modified for sandwich design in Figure 1.2. The raw ingredients such as flour, yeast, water, oil, and whole grains are processed by mixing into dough, kneading the dough, letting the dough rise, shaping the loaves, and baking. These processing steps determine the structure of the bread. For example, kneading the dough encourages gluten formation, which traps the carbon dioxide expelled by the yeast, allowing the dough to rise. In the oven, the layers of gluten solidify, creating the spongy texture characteristic of yeast breads. Once baked, the loaves of bread are sliced to the desired thickness for the sandwich, and may be toasted for additional strength, similar to heat-treatment of metals.

The resulting structure can be characterized by properties. These properties may include nutritional information like the number of calories or grams of fat or structural properties. For example, if the deli owner is an engineer, he may wish to measure the compressive strength of the bread. The compressive strength of the bread can then be used to determine the performance of the material, such as whether or not the bread will fail under the load of the meats and vegetables.

The baker, similarly to the materials expert, utilizes a bottom-up approach to develop new breads. The baker makes changes to the ingredients and processing steps to realize new breads with new properties. He uses a trial-and-error process to develop new breads with better strength and nutrition. On the other hand, the deli owner, similarly to the product designer, prefers a top-down approach to develop new bread specifically for the new sandwich. The deli owner prefers to specify the requirements for the bread and use bread-baking models to determine the ingredients and processing steps necessary to create the new bread.

1.1.2 Material Selection

Material selection involves using a material database to select the best material for a product. By following materials selection techniques, the most appropriate material is selected from all known materials in order to satisfy product requirements and goals. The material selection method that is most widely used in practice is the method developed by Ashby (1999). This process begins with a database containing all known materials. Screening and ranking techniques reduce the number of feasible material based on product geometry and loading conditions. The resulting subset of feasible materials is further reduced by conducting research on these materials. At this stage in the material selection process, engineering expertise plays a role in eliminating materials that would be poor choices in the overall product design by including information about economics (cost and availability) and manufacturing needs. After the prime candidates have been selected, local load conditions combined with design requirements lead to the final material choice.

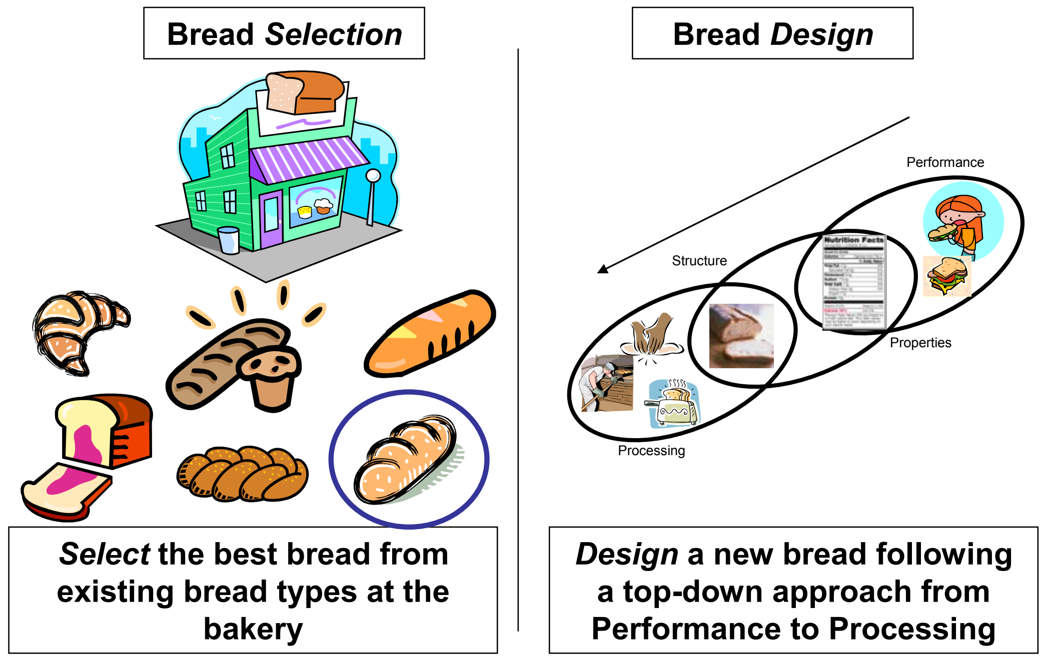


FIGURE 1.3. SELECTION VS. DESIGN IN THE CONTEXT OF THE SANDWICH EXAMPLE

In order to distinguish material selection from material design, reconsider the sandwich design example. A figure comparing material selection and material design in the context of the sandwich design example is shown in Figure 1.3. In material design, bread is tailor-made for the specific design problem. The resulting bread matches the material requirements exactly. In material selection, the process starts with a database of bread located at the bakery. A loaf of bread is chosen that best achieves the design requirements and is not tailored to meet the specific design requirements.

1.2 DESIGN FREEDOM AND DESIGN SIMPLICITY

As discussed in the previous sections, the potential improvement in product performance enabled by material design is tempered by an increase in the complexity of the design process, because sophisticated multilevel material models must be developed and employed to predict the behavior of designed materials. On the other hand, material selection is a well-established procedure for identifying the best material for a product while considering performance, processing and economic effects. However, material selection inherently limits the material design space by restricting the alternatives to existing materials, and this restriction may limit the performance of the product. A designer's choice between material selection and design involves a compromise between *design freedom* and *design simplicity*. These two terms are defined and discussed in the following sections.

1.2.1 Design Freedom

Simpson and co-authors define design freedom as the extent to which a system can be adjusted while still meeting its design requirements (Simpson et al., 1996). That is, the amount a design can be changed while still meeting the constraints and bounds on the system. Material design offers the potential for increased design freedom because the product design is no longer limited by the set of existing materials. Specifically, by introducing design variables related to the design of the *material*, the designer gains additional dimensions in which the design can be adjusted. Design freedom is determined by the range of performance of a design, not by the range of design variables directly; however, an increase in the range or the number of design variables has the potential to increase the performance range, thereby increasing the design freedom. Consequently, material design has the *potential* to increase design freedom relative to material selection but does not guarantee an increase in design freedom.

1.2.2 Design Simplicity

Design simplicity, or conversely, design complexity, refers to the ease or difficulty in identifying the best design alternative due to the number of design decisions and the degree of coupling between design decisions. A design process in which the material is selected is simpler than a design process in which the material is designed because there are fewer design decisions regarding the material, and there is less coupling between the decisions regarding the material. Design decisions in this case refer to determining the value of design variables, including both discrete and continuous variables. This definition for design simplicity is distinguished from the notion of design complexity in the Information Axiom developed by Suh (1990). The Information Axiom refers to the likelihood of a particular *design* to succeed given the simplicity of the design, whereas design simplicity in this thesis refers to the ease of *designing*.

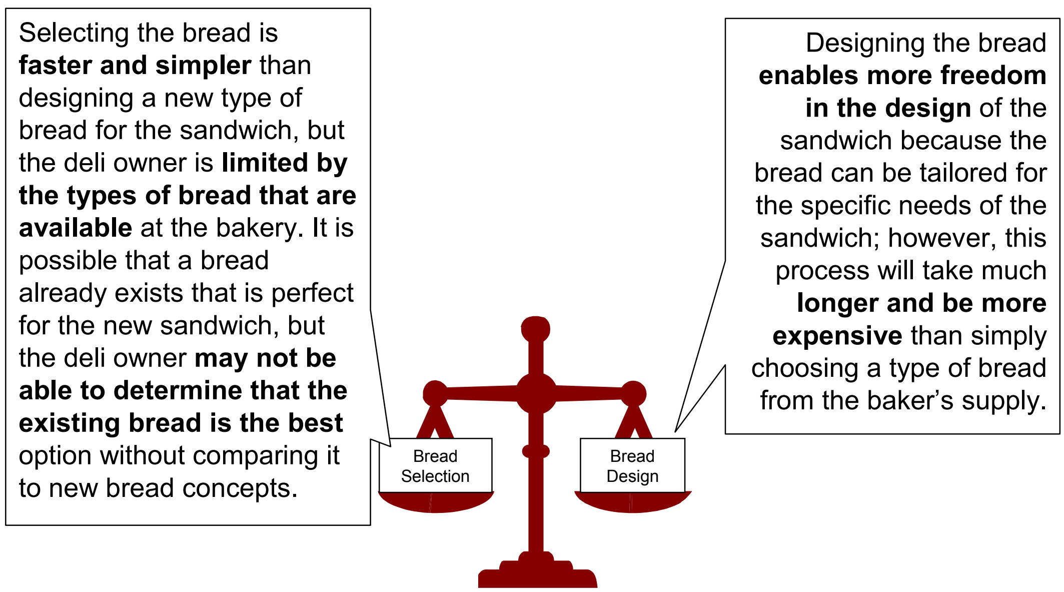


FIGURE 1.4. SELECTION VS DESIGN AS A COMPROMISE BETWEEN DESIGN FREEDOM AND DESIGN SIMPLICITY

The notions of design freedom and design simplicity can also be described in terms of the sandwich design example, shown graphically in Figure 1.4. Selecting a loaf of bread from the available bread at the bakery simplifies the design of the sandwich as compared to designing a new loaf of bread for the sandwich; however, the deli owner may increase the design freedom of the sandwich by designing the bread rather than selecting the bread. In making the decision between design and selection, the deli owner must make a compromise between design freedom and design simplicity.

1.3 REQUIREMENTS AND RESEARCH QUESTIONS

In the previous sections it is established that a choice between material selection and material design involves a compromise between design freedom and design simplicity. In this section, the requirements are identified for a strategy for choosing between material selection and material design in the context of product design. Also, requirements are identified for a metric that is used as a part of the strategy. The requirements are identified in Section 1.3.1. In Section 1.3.2 research questions and hypotheses are introduced and the mapping from requirements to research questions and hypotheses is established.

1.3.1 Requirements for a Method and a Metric

In this section, requirements are identified for a method and a metric to support a designer's decision between material selection and material design. To guide the identification of these requirements, recall the sandwich design example: *What does the deli owner need in order to help him decide between bread design and bread selection for his new sandwich?* It is apparent that the deli owner needs a systematic method or strategy to follow to guide his decision between selection and design, but what are the requirements for this method?

In Table 1.1 requirements for the method are listed in the context of the sandwich design example on the left and generalized for product design on the right. The first requirement is that the method be driven by sandwich requirements rather than bread requirements, following a top-down approach. That is, the deli owner prefers to make the decision between bread selection and design on the basis of meeting the sandwich requirements. Like the deli owner, designers also prefer to follow this top-down approach from a systems perspective.

TABLE 1.1. REQUIREMENTS FOR THE METHOD

	Requirements for the Method	
	Sandwich Design	**Product Design**
	Problem statement: Design a strategy that enables a *deli owner* to compare and choose between *bread* selection and *bread* design in the context of a *sandwich* design.	**Problem statement:** Design a strategy that enables a *designer* to compare and choose between *material* selection and *material* design in the context of a *product* design.
#	**Requirement**	**Requirement**
1	Top-Down: Driven by *sandwich* requirements rather than *bread* requirements	Top-Down: Driven by *product-level* requirements rather than *material-level* requirements
2	Computationally inexpensive as compared to designing a new *bread*	Computationally inexpensive as compared to designing a new *material*
3	Enables a quantitative comparison of the options of selection and design based on the achievement of the *sandwich* design objectives as well as process design objectives	Enables a quantitative comparison of the options of selection and design based on the achievement of the *product* design objectives as well as process design objectives
4	Enables the identification of solutions which provide guidance for subsequent phases of the *sandwich* design	Enables the identification of solutions which provide guidance for subsequent phases of the *product* design

Another requirement is that the method for deciding between bread selection and design should be computationally inexpensive as compared to bread design. This means that the strategy for choosing between the bread design and bread selection should be easier and faster to complete than designing a new type of bread. If it takes more time and effort to decide between selection and design than it takes to design bread in the first place, then the strategy is of no help to the deli owner. Again, this requirement holds true for designers; the strategy for choosing between material selection and material design should be computationally inexpensive as compared to the design of a new material.

The third requirement for the method is that it must enable a quantitative comparison of the options of bread selection and bread design on the basis of meeting sandwich requirements and design process requirements. An example of a design process requirement is staying within the design process timeline or budget. The deli owner wants to consider the sandwich requirements as well as the project timeline and budget when he decides between bread selection and design. Likewise, a product designer also wants to consider both product and process requirements when making the decision between material design and selection. An important part of this requirement for the method is that it provides a *quantitative* comparison of the options of selection and design. Therefore, the method should make use of a metric that allows the deli owner to quantitatively compare the options of bread selection and bread design. The requirements for the metric are shown separately in Table 1.2, again in the context of the sandwich design example on the left and generalized for product design on the right.

The last requirement for the method is that it should enable the identification of solutions which provide guidance for subsequent phases of sandwich design. Specifically, the method should not only help the deli owner to decide between bread selection and bread design, but it should also help the deli owner to determine the characteristics of the bread that is to be designed or selected. Similarly, a designer needs guidance on the characteristics of the material that is to be selected or designed.

TABLE 1.2. REQUIREMENTS FOR THE METRIC

	Requirements for the Metric	
	Sandwich Design	**Product Design**
	Problem Statement: Design a metric that quantifies the improvement in the achievement of *sandwich* design goals by expanding the *bread* design space.	**Problem Statement:** Design a metric that quantifies the improvement in the achievement of *product* design goals by expanding the *material* design space.
#	**Requirement**	**Requirement**
1	Enables comparison of an existing *bread* to an unrealized *bread concept*	Enables comparison an existing *material* to an unrealized *material concept*
2	Enables comparison on the basis of meeting *sandwich* and *sandwich design process* requirements	Enables comparison on the basis of meeting *product-level* and *product design process* requirements

The third requirement for the method is that it provides a *quantitative* comparison of the option of material selection and material design; therefore, a metric is needed. The first requirement of the metric is that it compares an existing type of bread to an unrealized bread concept. The existing bread represents the choice of bread selection while the unrealized bread concept represents the choice of bread design. Similarly, in product design, the metric should compare existing materials to unrealized material concepts in order to compare the options of material selection and material design.

The second requirement for the metric is that the comparison is made on the basis of meeting *sandwich* and *sandwich design process* requirements. This requirement comes from the method requirement on the comparison of selection and design. As before, the deli owner prefers to make the choice between bread selection and design by considering sandwich design requirements as well as sandwich design process requirements. The metric that enables this quantitative comparison must also, therefore, make the comparison on the basis of meeting both sandwich and sandwich design process requirements. Similarly, a product designer also prefers to make the decision between material design and selection on the basis of meeting both *product* requirements and design *process* requirements. Given these requirements for the method and metric, the research questions and hypotheses for this thesis are identified next.

1.3.2 Research Questions and Hypotheses

In this section, the research questions for this thesis are presented. The research questions follow from the identification of the method and metric requirements in the previous section. In Table 1.3 the mapping is shown between the requirements and the research questions and hypotheses. The method requirements map into all three secondary research questions and hypotheses, while the metric requirements map only to the second secondary research question and hypotheses. The research questions and hypotheses are then discussed in detail in this section.

TABLE 1.3. MAPPING REQUIREMENTS TO RESEARCH QUESTIONS AND HYPOTHESES

Requirements	**Research Questions**	**Hypotheses**
	Primary: How can designers choose between material selection and material design in the context of a product design?	Primary: Designers can use a value-of-information-based metric as part of a design space expansion strategy to assess the value of material design as compared to material selection.
Method Requirements: 1. Top-Down: Driven by product-level requirements rather than material-level requirements 2. Computationally inexpensive as compared to designing a new material	1. How can designers evaluate candidate material concepts without the use of complex material models?	1. Material properties are an abstraction of the structure of the material which simplifies the assessment of product concepts. A multidimensional material design space is created by assuming the material properties to be independent design variables that can be realized by adjusting the composition and processing path of existing materials.
3. Provides a quantitative comparison of the options of selection and design based on the achievement of the product design objectives as well as process design objectives 4. Provides guidance for subsequent phases of the product design	2. How can a value-of-information-based metric compare the options of material selection and material design?	2. A Value of Design Space Expansion metric can be calculated to support the designer's assessment of the improvement in the achievement of product goals by expanding the material design space around a baseline material point.
Metric Requirements: 1. Compares an existing material to an unrealized material concept 2. Compares on the basis of meeting product-level and product design process requirements	3. How can designers determine the material property targets for a product for subsequent phases of design?	3. By finding a design solution in the expanded material design space, designers gain insight into the material property targets for subsequent phases of design.

The primary research question comes from the initial question used to identify the method and metric requirements. In the context of the sandwich example, the question is "How can the deli owner decide between bread selection and bread design in the context of sandwich design?" To generalize this question, *deli owner* becomes *designer*, *bread* becomes *material*, and *sandwich* becomes *product*, as shown below.

<u>Primary Research Question:</u>

> *How can designers decide between material selection and material design in the context of a product design?*

<u>Primary Hypothesis:</u>

> *Designers can use a value-of-information-based metric as part of a design space expansion strategy to assess the value of material design as compared to material selection.*

The fundamental difference between material selection and material design is an expansion of the material design space. In material selection, the design space is limited to discrete points, whereas in material design, a multidimensional continuous design space is explored. The decision between material selection and design is similar to the decision whether or not to incorporate more information into the decision model prior to making the decision. Material design requires the use of a sophisticated material model to evaluate the performance of candidate designs. In material selection, the material models used to evaluate the performance of candidate design are essentially the properties of existing materials. By deciding to pursue material design, designers are implicitly making the decision to incorporate more information into the design decision model. Value of information metrics have been demonstrated for making these types of decisions (Panchal et al., 2007). In this thesis, a value of design space expansion metric is identified to support the decision between material design and material selection. To guide the development of the design space expansion strategy, three secondary research questions and hypotheses are identified.

Secondary Research Question 1:

> *How can designers evaluate candidate material concepts without the use of complex material models?*

Hypothesis 1:

> *Material properties are an abstraction of the structure of the material which simplifies the assessment of product concepts. A multidimensional* ***material design space*** *is created by assuming the material properties to be independent design variables that can be realized by adjusting the composition and processing path of existing materials.*

To compare the design process options of material selection and material design, designers must have a way to explore a material design space. Although complex material simulations provide a way to evaluate candidate material concepts, the generation of these models is too time consuming for efficient design space exploration in the early stages of design. Rather, the material properties can be assumed to be independent design variables, creating an n-dimensional material design space, where n is the number of independent material properties. In this material design space, existing materials are located at various discrete points throughout the space. It is assumed any design point in this continuous material design space can be realized by adjusting the composition or processing path of existing materials.

Secondary Research Question 2:

How can a value-of-information-based metric be used to compare the options of material selection and material design?

Hypothesis 2:

A Value of Design Space Expansion metric can be calculated to support a designer's assessment of the improvement in the achievement of product goals by expanding the material design space around a baseline material point.

Design solutions can be found for two scenarios, one in which the material design space is limited to an existing material, and one in which the material design space is expanded as described in Hypothesis 1. By comparing the achievement of design goals of the solutions to these two scenarios, the value of design space expansion metric indicates the value of expanding the material design space.

Secondary Research Question 3:

How can designers determine the material property targets for a product for subsequent phases of design?

Hypothesis 3:

By finding a design solution in the expanded material design space, designers gain insight into the material property targets for subsequent phases of design.

If material design is chosen over material selection, designers must have a means for identifying the material property targets for subsequent phases of design. The material property targets are determined from the product requirements by finding a design solution in the expanded material design space.

The design space expansion strategy is presented in this thesis as an embodiment of the hypotheses to these research questions. To validate these hypotheses, a validation strategy based on the Validation Square construct is presented in the next section.

1.4 AN OVERVIEW AND VALIDATION STRATEGY FOR THIS THESIS

The validation and verification strategy for this thesis is based on the validation square construct introduced by Pedersen and coauthors (Pedersen, et al., 2000) and extended by Seepersad and coauthors (Seepersad, et al., 2006). The overview of the validation square construct in Seepersad (2004) is presented here in Section 1.4.1 without major change. In Section 1.4.2, the strategy for validation and verification of this thesis is presented as an overview of the thesis.

1.4.1 The Validation Square Construct

Validation (justification of knowledge claims, in a modeling context) of engineering research has typically been anchored in formal, rigorous, quantitative validation based on logical induction and/or deduction. As long as engineering design is based primarily on mathematical modeling, this approach works well. Engineering design methods, however, rely on subjective statements as well as mathematical modeling; thus, validation solely by means of logical induction or deduction is problematic. Pedersen and coauthors and Seepersad and coauthors propose an alternative approach to validation of engineering design based on a relativistic notion of epistemology in which "knowledge validation becomes a process of building confidence in its usefulness with respect to a purpose" (Pedersen, et al., 2000; Seepersad, et al., 2006).

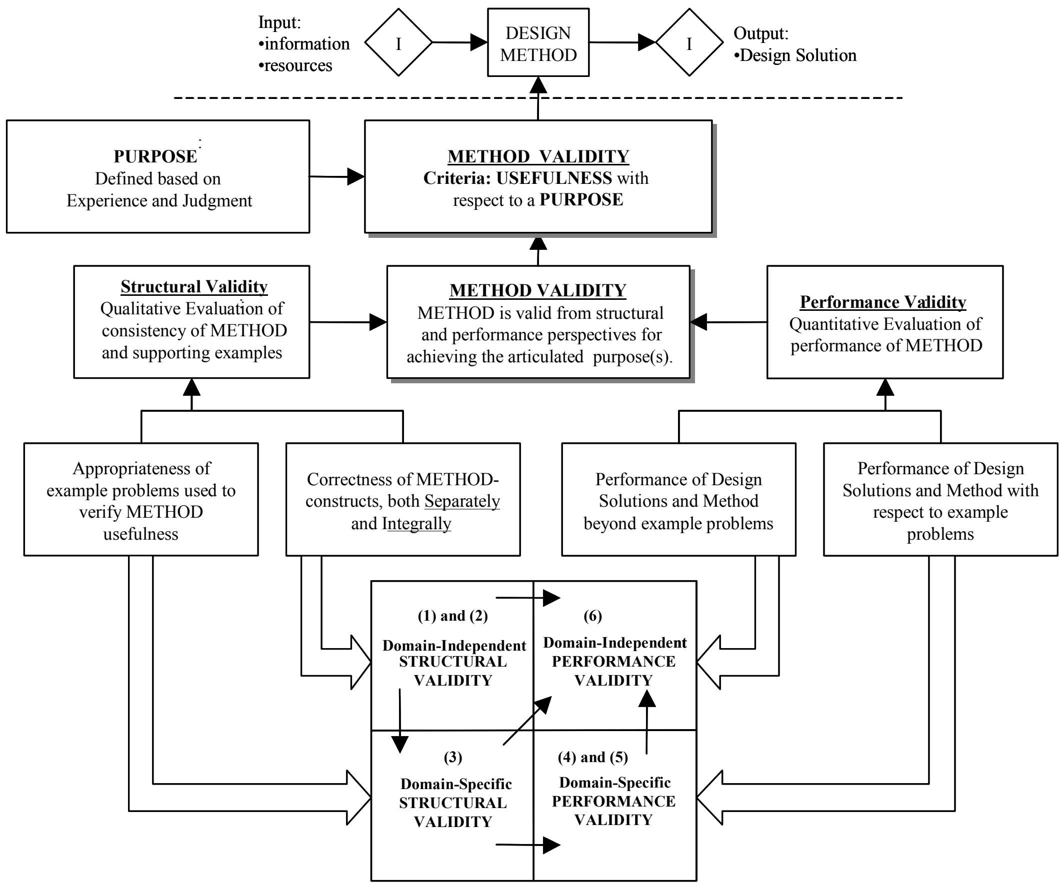


FIGURE 1.5. DESIGN METHOD VALIDATION: A PROCESS OF BUILDING CONFIDENCE IN USEFULNESS WITH RESPECT TO A PURPOSE (SEEPERSAD, ET. AL., 2006)

The Validation Square is a framework for validating design methods in which the 'usefulness' of a design method is associated with whether the method provides design solutions correctly (structure validity) and whether it provides correct design solutions (performance validity). This process of validation is represented graphically in Figure 1.5. With respect to the square, domain-independent structure validity involves accepting the individual constructs constituting a method as well as the internal consistency of the assembly of constructs to form an overall method. Domain-specific structure validity includes building confidence in the appropriateness of the example problems chosen for illustrating and verifying the performance of the design method. Domain-specific performance validity includes building confidence in the usefulness of a method using example problems. Domain-independent performance validity involves building confidence in the generality of the method and accepting that the method is useful beyond the example problems.

How can this validation framework be implemented in a thesis? Establishing domain-independent structure validity involves searching and referencing the literature related to each of the parent constructs utilized in the design method. In addition, flow charts are often useful for checking the internal consistency of the design method by verifying that there is adequate input for each step and that adequate output is provided for the next step. A list of criteria may be useful for establishing and comparing the domain-independent structure validity of methods and constructs with respect to a set of explicit, favorable properties.

Establishing domain-specific structure validity consists of documenting that the example problems are similar to the problems for which the methods/constructs are generally accepted, that the example problems represent actual problems for which the method is intended, and that the data associated with the example problems can be used to support a conclusion. Domain-specific performance validity can be established by using representative example problems to evaluate the outcome of the design method in terms of its usefulness. Metrics for usefulness should be related to the degree to which the method's purpose has been achieved (e.g., reduced cost, reduced time, improved quality). It is also important to establish that the resulting usefulness is, in fact, a result of applying the method. For example, solutions obtained with and without the construct/method can be compared and/or the contribution of each element of the method can be evaluated in turn.

An important part of domain-specific performance validity is empirical verification of data used to support domain-specific performance validation. Empirical verification can be established by demonstrating the accuracy and internal consistency of the data. For example, in optimization exercises, multiple starting points, active constraints and goals, and convergence can be documented to verify that the solution is stationary and robust. For any engineering model it is important to verify that data obtained from the model represents aspects of the real world that are relevant to the hypotheses in question. The model should react to inputs in an expected manner or in the same way that an actual system would react.

Finally, domain-independent performance validity can be established by showing that the method/construct is useful beyond the example problem(s). This may involve showing that the problems are representative of a general class of problems and that the method is useful for these problems; from this, the general usefulness of the method can be inferred.

In Figure 1.6, an outline of the validation strategy for this thesis is presented. It is arranged according to the quadrants in the validation square, and references are included for chapters and sections in which the validation is documented. A roadmap of this thesis is illustrated in Figure 1.7 with the purpose of implementing the validation strategy outlined in Figure 1.6.

1.4.2 The Validation Strategy for This Thesis

In Chapter 1, the foundations are established for the Design Space Expansion Strategy (DSES). The motivation and frame of reference are presented. The principal goal of the DSES is introduced along with the research questions and hypotheses. The expected contributions are summarized and a validation strategy is established for this thesis.

Domain-independent Structure Validity

• Critical review of literature that is foundational to the Design Space Expansion Strategy proposed in the research hypotheses. Topics include multi-objective decision-making with the cDSP and information economics. • *What are the advantages, limitations, and accepted domains of application for available approaches? What are the opportunities for future work? In light of this critical review, do the research tasks and hypotheses represent original, significant contributions?*	Ch. 2
• Presentation and discussion of the DSES, including the intellectual and methodological aspects of instantiating each associated hypothesis. • *What are the advantages, limitations, and accepted domains of application for the DSES? To what extent does it serve as a foundation for choosing between material selection and material design from a theoretical perspective?*	Ch. 3

Domain-Specific Structure Validity

• Identify the significance of the example problem from the perspective of identifying material property requirements and the need for the design space expansion strategy. • Discuss the appropriateness of the example problem o Document that the example problem is similar enough to problems for which the DSES is accepted theoretically. The characteristics of the proposed domain of application are enumerated in Ch. 3. o Document that the example is representative of actual problems for which the approach is intended. What are the key characteristics of examples? o Document that the data associated with the example problem can support a conclusion or conclusions with respect to the hypotheses	Section 3.3.2

Domain-Specific Performance Validity

• Build confidence in the utility of the DSES using the example. o Use the example problem to evaluate the utility of the DSES. ▪ Does the strategy facilitate exploration of the expanded design space? ▪ Does the value of design space expansion metric support the decision between material selection and material design? ▪ Does the strategy facilitate generation of material property targets? ▪ Does the method possess the advantages claimed in Chs 2 and 3? o Demonstrate that the observed usefulness is linked to applying the method. For example, compare results to those obtained with alternative or conventional methods or to benchmark products. o Verify the empirical data obtained in the experiments (e.g., compare to detailed computer simulations or analytical solutions). • Demonstrate material property identification significance and contributions.	Ch. 4

Domain-independent Performance Validity

• Build confidence in the generality and utility of the approach beyond the specific example problem. Argue that the approach is useful for the example problem and that the example problem is representative of general problems.	Ch. 5

FIGURE 1.6. VALIDATION STRATEGY FOR THIS THESIS

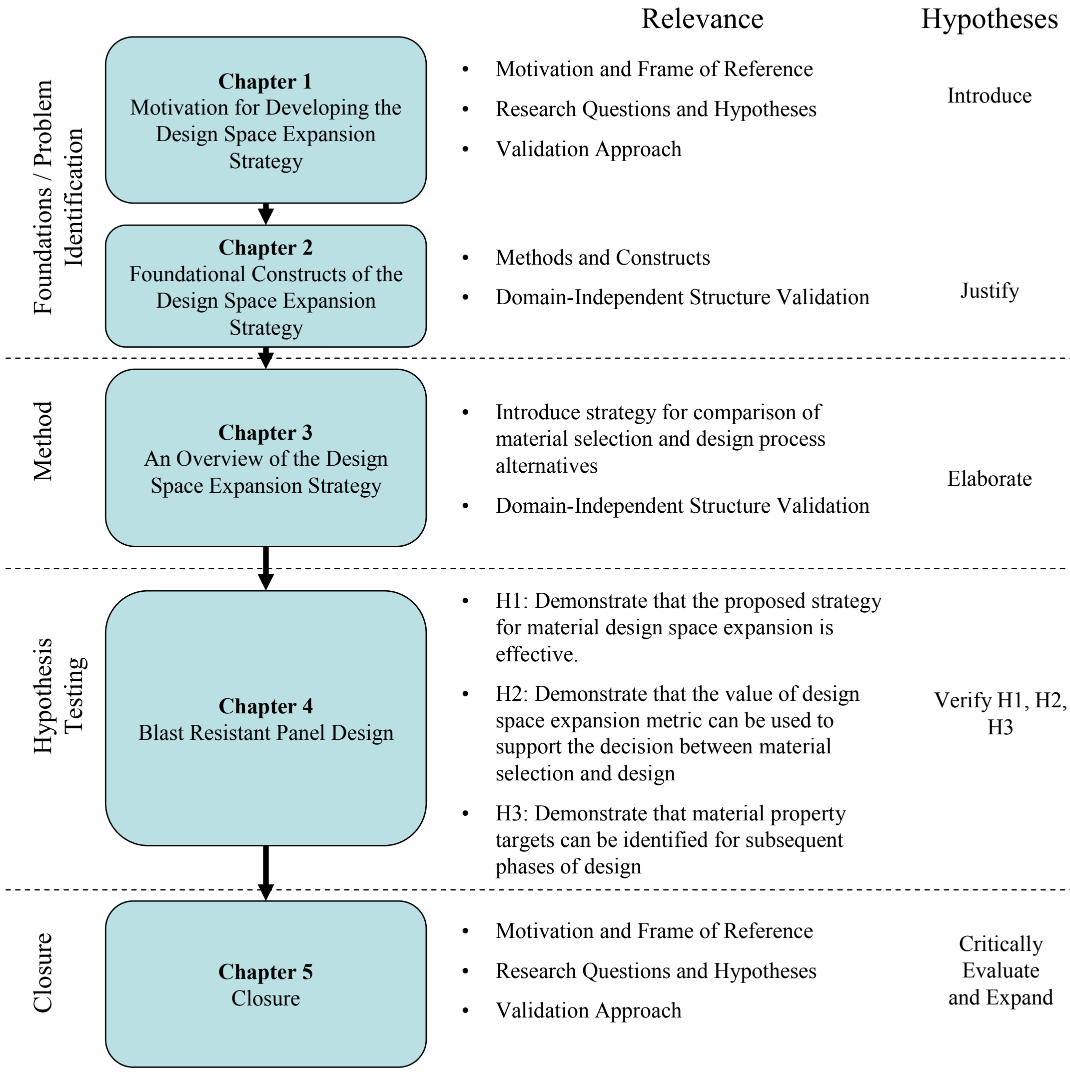


FIGURE 1.7. A THESIS OVERVIEW AND ROADMAP

In Chapter 2, the theoretical foundations of the DSES are introduced and discussed including multi-objective decision-making and information economics. For domain-independent structure validation, relevant literature in each of these research areas is referenced, discussed, and critically evaluated. The purpose is to discuss the availability, strengths, and limitations of methods or constructs that are foundational for the DSES and to identify research opportunities addressed in this thesis by the DSES.

In Chapter 3 an overview of the DSES is presented. The elements of the strategy are discussed in detail from the perspective of embodying the hypotheses presented in Chapter 1. For domain-independent structure validation, emphasis is placed on verifying the internal consistency of the method as well as its originality, advantages, limitations, and accepted domain of application. Advantages, limitations, and originality are discussed in relation to methods and constructs that are available in the literature. An example problem on blast resistant panel (BRP) design is introduced. For domain-specific structure validation, the appropriateness of the example for validating specific aspects of the DSES is discussed. Several BRP design scenarios are presented as an experimental plan to document how the example is used to generate information that can be used to test the hypotheses.

In Chapter 4 the example problem on the design of BRPs is discussed. A step-by-step implementation of the DSES is presented for the BRP example. The results are presented, verified, and critically discussed for the purpose of domain-specific validation of the hypotheses introduced in Chapter 1.

In Chapter 5 the thesis is summarized and critically reviewed and relevant contributions and avenues of future work are discussed. The advantages and domain of application are discussed for the DSES as presented in this thesis, and intellectual contributions are reviewed. For domain-independent performance validation, it is argued that the conclusions of this thesis are relevant beyond the example problem, and potential future applications are discussed. Conditions are identified under which the conclusions are valid, and limitations of the work are presented explicitly. Recommendations are proposed for future work that would make the DSES more effective for the example problem and extend it for a broader range of applications.

CHAPTER 2

FOUNDATIONAL CONSTRUCTS FOR THE DSES AND VDSE METRIC

In this chapter the foundational constructs of the Design Space Expansion Strategy (DSES) and the Value of Design Space Expansion (VDSE) metric are presented and critically reviewed as part of the validation strategy identified in Figure 1.6. In Section 1.3.1, the desirable characteristics of the DSES and the VDSE are identified and discussed. These are summarized in Tables 1.1 and 1.2. In Section 2.1 these characteristics are revisited in order to establish the context for a critical review of the literature in areas that are foundational to the DSES and the VDSE. These areas include multiobjective decision support and information economics, which are reviewed in Section 2.2 and 2.3, respectively. Research opportunities in these areas are identified in Section 2.4 by comparing the requirements for the DSES and VDSE metric discussed in Section 2.1 to the capabilities of existing methods and metrics presented in the literature and reviewed in Sections 2.2 and 2.3. The internal consistency of the parent constructs of the DSES is discussed in Section 2.5 to assess domain-independent structure validity. This chapter is concluded in Section 2.6 with a look back and a look ahead.

The work presented in this thesis is intended to illustrate the extension and implementation of existing concepts in engineering design. Therefore, topics such as multi-objective decision support and information economics are reviewed in order to present the foundation from which the work in this thesis begins. Significant portions of the following chapter are leveraged from the work of former and current graduate students in the Systems Realization Laboratory, Georgia Institute of Technology.

2.1 REQUIREMENTS FOR THE DSES AND THE VDSE METRIC

The requirements for the DSES and the VDSE metric are identified Tables 1.1 and 1.2 and are repeated here in Table 2.1 and Table 2.2. The purpose of this literature review is to determine how well these requirements are met by existing methods and metrics; therefore, it is important to review these requirements to set the context for this literature review.

TABLE 2.1. REQUIREMENTS FOR THE DSES

	Requirements for the Design Space Expansion Strategy
	Problem statement: Design a strategy that enables a designer to compare and choose between material selection and material design in the context of a product design.
#	**Requirement**
1	Top-Down: Driven by product-level requirements rather than material-level requirements
2	Computationally inexpensive as compared to designing a new material
3	Enables a quantitative comparison of the options of selection and design based on the achievement of the product design objectives as well as process design objectives
4	Enables the identification of solutions which provide guidance for subsequent phases of the product design

The first requirement of the DSES is that it be a top-down process that is driven by product (system) requirements rather than material requirements. That is, the method should facilitate a systems approach in which the preferred design alternatives are identified based on the system requirements. Therefore, the decision between material selection and material design for a component of a system must be made on the basis of meeting system-level requirements. To achieve this, multi-objective decision support tools are needed to identify whether material selection or material design is the preferred alternative in terms of multiple system goals. System variables such as the length or width of a component are often tightly linked to the material of the component. For example, the height of a cantilever beam may be calculated with knowledge of the material properties in order to choose a height that limits deflection or prohibits yielding of the beam. Because system variables are linked to the material properties, the decision between material selection and material design must be made in a manner that allows the system variables to vary if the material properties change. That is, the system variables must be allowed to have different values when the material properties change.

The second requirement of the DSES is that it must be computationally inexpensive as compared to designing a new material. Designing a new material requires complex material models that can predict the performance of candidate material concepts. The development and validation of these models is not trivial. With this knowledge, the point of the DSES is to give some indication of whether or not the development and validation of these predictive models is worth the effort. The DSES itself must not be nearly as cumbersome as the development of a new material model, or no designer will be willing to use the strategy to support this decision. Multi-objective decision support tools will again be useful for meeting this particular requirement, specifically decision support tools which enable an efficient search of the design space.

The third requirement of the DSES is that the options of material selection and material design must be compared on the basis of meeting product design objectives as well as process design objectives. Product design objectives are the objectives that pertain to the performance of the product itself, such as reducing deflection or lowering the mass of the system. The process design objective at hand in the choice between material selection and material design is to reduce the cost of design or the length of the design timeline. Because both product and process design objectives must be considered in the decision between material selection and material design, multiobjective decision support tools are needed to find solutions that simultaneously satisfy all the product and process objectives.

The last requirement of the DSES is that it must provide guidance for subsequent phases of design. Guidance must be provided for two different issues: whether to select or design a material, and which property values characterize the preferred material that is to be designed or selected. Multiobjective decision support tools will again be useful for meeting this requirement in order to find families of solutions for subsequent phases of design. These families of solutions indicate the properties of materials that meet the product and process design objectives. Furthermore, ranged sets of solutions can be identified that maintain design freedom for subsequent phases of design.

TABLE 2.2. REQUIREMENTS FOR THE VDSE METRIC

	Requirements for the VDSE metric
	Problem Statement: Design a metric that quantifies the improvement in the achievement of design goals by expanding the material design space.
#	**Requirement**
1	Enables comparison of an existing material to an unrealized material concept
2	Enables comparison on the basis of meeting product-level and product design process requirements

In addition to the requirements for the method, requirements for the metric are identified in Section 1.3.1 and are repeated here. The first requirement of the VDSE metric is that it enables the comparison of the use of an existing material to the use of an unrealized material concept. This is necessary because the VDSE is intended to be used to compare the design process options of material selection and material design, where an existing material is used in the material selection option and a new material is designed in the material design option. The decision between material selection and material design is one that involves a change in the extent of the material design space. It is the *value* of the expansion of the design space that must be quantified to provide support for the decision between material selection and design. This expansion of the material design space is similar to a source of new information that designers have the option of considering when making a design decision. Value of *information* metrics can be calculated to help a designer determine if gathering more information is worth the added cost and time needed to reduce the uncertainty in the problem. The second requirement of the VDSE metric is that it compares on the basis of meeting product and process requirements. The metric must therefore be compatible with multi-objective decision support tools. Value of information metrics exist that make use of multi-objective value or utility functions.

In the following sections, the research areas which most closely relate to these requirements are reviewed. These areas are multi-objective decision support and information economics, which are reviewed in Sections 2.2 and 2.3, respectively. The requirements discussed in this section are revisited in Section 2.4, and research opportunities in these areas are identified.

2.2 MULTIOBJECTIVE DECISION SUPPORT USING COMPROMISE DECISION SUPPORT PROBLEMS

The first requirement for a method is that it should facilitate exploration and generation of families of multiobjective or multifunctional compromise solutions. This requirement is driven by the fact that designers often must balance conflicting objectives in order to obtain viable solutions. For example, in the sandwich design example (see Chapter 1), the deli owner must balance conflicting objectives of reducing the fat content of the sandwich and maintaining a pleasing taste and texture. The challenge is to identify values of design parameters—which describe the structure or form of a design and possibly its environment—that yield preferred compromise solutions with respect to the set of objectives. The compromise Decision Support Problem has been established to identify values of design parameters that meet conflicting objectives.

In this section the compromise Decision Support Problem and its foundations in mathematical and goal programming are reviewed. The review of multiobjective decision support and the compromise Decision Support Problem in Seepersad (2004) is presented here without major change.

Mathematical programming is part of the foundation of the cDSP, which in turn is part of the foundation of the DSES proposed in this thesis; therefore, the review of multiobjective decision support begins with mathematical programming. In its most general form, a conventional mathematical programming problem is formulated as follows:

$$\text{Minimize } f(\mathbf{x}) \tag{2.1}$$

$$\text{Subject to } g(\mathbf{x}) < 0 \tag{2.2}$$

$$h(\mathbf{x}) = 0 \tag{2.3}$$

$$\boldsymbol{x_L} \leq \boldsymbol{x} \leq \boldsymbol{x_U} \tag{2.4}$$

where $f(\mathbf{x})$ is a function to be minimized, $g(\mathbf{x})$ and $h(\mathbf{x})$ are vectors of inequality and equality constraints, respectively, and $\boldsymbol{x_L}$ and $\boldsymbol{x_U}$ are vectors of lower and upper bounds for the vector of design variables, $\boldsymbol{x}$. When multiple objectives are considered, the objective function effectively becomes a vector, as well, and Equation 2.1 must be expressed as follows:

$$\text{Minimize } f = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_n(\mathbf{x})] \tag{2.5}$$

By placing different relative values or priorities on the individual objectives, it is possible to obtain many solutions to the multiobjective problem. The range of compromise solutions is often called a Pareto set, curve, or frontier. Individual solutions or members of the Pareto set are called Pareto solutions or points. A Pareto solution is one that is not dominated by any other solution in the feasible design space (defined by the set of constraints and bounds). A non-dominated or Pareto solution is one for which no other feasible solution yields preferred values for all objectives. In other words, it is impossible to locate another feasible solution that improves one or more objectives without worsening the values of other objectives. The concept of a Pareto solution and Pareto set is borrowed from economics and named for the economist Vilfredo Pareto who defined an allocation of resources as Pareto efficient if it is impossible to identify another allocation that makes some people better off without making others worse off (Pareto, 1909).

Design solutions are rarely judged on the basis of a single criterion; instead, their value is determined by how well they balance multiple criteria associated with cost, performance, environmental impact, robustness, and other categories. In a designer's choice between material selection and material design, both product and process design goals must be considered (see requirement 3 in Table 2.1). Therefore, it is reasonable to pursue a balance between these multiple criteria or objectives during the design process itself. Accordingly, many techniques have been proposed for generating Pareto sets of solutions and for determining the most preferable multiobjective solution. One of the most straightforward techniques is the weighted sum approach. A weighted sum formulation of an objective function, Z, is expressed as a linear, additive combination of the multiple objectives:

$$Z = \sum_{i=1}^{m} w_i f_i \tag{2.6}$$

where w_i is the weight for the i^{th} objective, f_i, and m is the number of objectives. The weighted sum formulation is straightforward and easy to implement. By varying the weights, it is possible to generate a family of Pareto solutions to the multiobjective design problem posed in Equations 2.2 through 2.5. However, it has been shown that many

Pareto solutions may be overlooked (i.e., it is not possible to identify all Pareto solutions) with a weighted sum formulation if the problem is non-convex (Koski, 1985). Also, if a single multiobjective solution is sought, it is difficult to determine a priori an appropriate set of weights that yield a preferable compromise solution that does not overemphasize one or more objectives relative to other objectives.

Messac (1996; 1996) has proposed a physical programming formulation to remedy the latter limitation. With the physical programming approach, a designer expresses his preferences for each objective through various degrees of desirability from unacceptable to ideal. Based on these preferences, sets of weights are determined automatically for each objective, with each weight valid over a specified range of objective function values, to form a convex, piecewise linear merit function for each objective. With physical programming formulations, solutions that achieve tolerable or desirable values for all criteria are preferred over solutions that achieve ideal values of some objectives at the expense of extremely poor values of other objectives. However, like the simple weighted sum approach, the physical programming formulation still suffers from inability to identify a full range of Pareto solutions (because it is based on a linear weighted sum formulation). Furthermore, many designers object to the use of semantic preference levels that are central to the physical programming formulation. The weighted sum approach is a special case of compromise programming (Yu and Leitmann, 1974; Zeleny, 1973) in which a multiobjective function is expressed as the distance between objective values, f(x), for a particular solution and a set of ideal or utopian objective values, f*, as follows:

$$Z = \left\{ \sum_{i=1}^{m} \left(w_i \left(f_i(\mathbf{x}) - f_i^* \right) \right)^p \right\}^{1/p} \tag{2.7}$$

where p is a positive integer. If p equals one and the ideal objective values have null values, the compromise programming formulation reduces to the weighted sum formulation. If p equals two, the Euclidean formulation is established. The Tchebycheff formulation is obtained by setting p equal to infinity:

$$Z = \min \max_{\mathrm{i=1, \ldots, m}} \left\{ w_i \left(f_i \left(\mathbf{x} \right) - f_i^* \right) \right\} \tag{2.8}$$

The Tchebycheff formulation has been shown to be much more effective for generating an entire Pareto set of options, even for non-convex problems, than the weighted sum formulation (c.f., (Bowman, 1976)) and has been used to generate a Pareto frontier for biobjective robust design problems that involve tradeoffs between nominal performance and robustness (Chen, et al., 1999b). The standard min-max formulation in engineering optimization is a special case of Equation 2.8 in which the ideal or utopia objective values are assigned null values and the weights are removed by assigning values of unity to all of them. Although compromise programming formulations have been shown to be effective for generating Pareto sets of solutions for multiobjective problems, they have the disadvantage of requiring ideal or utopian solutions within the problem formulation. In strictly keeping with the compromise programming approach, an ideal or utopian point must be identified separately for each objective by minimizing/maximizing the objective over the feasible solution space. This is an expensive requirement, and its cost grows with the number of objectives.

There are many other multiobjective formulations. For example, utility theory has been shown to be a mathematically rigorous, domain independent approach for multiobjective decision-making (Keeney and Raiffa, 1976; von Neumann and Morgenstern, 1947). A decision-maker's preferences are explicitly assessed and modeled as utility functions that are valid for conditions of risk and uncertainty as well as tradeoffs among multiple attributes. As long as a decision-maker's preferences obey a set of axioms, it can be proven mathematically that his/her preferred alternative—and therefore the rational choice—is the one with the highest expected utility. Although utility theory is a theoretically sound approach for identifying compromise solutions, especially when uncertainty is associated with the objectives, the associated informational demands on a decision-maker are very high (c.f., (Fernandez, 2002; Seepersad, 2001) for discussions). Among other demands, utility theory requires a decision-maker to assign probabilities to every possible outcome or set of objective function values and to know a priori exactly what his/her preferences are for combinations of multiple objectives. The latter requirement is particularly prohibitive in the early stages of design when a designer may be using multiobjective searches to discover or explore the potential range of compromise solutions for a specific problem; a designer may not know what he/she wants until he/she ascertains what is possible.

2.2.1 The compromise Decision Support Problem

Another mathematical construct for modeling multiple objectives in engineering design applications is the compromise Decision Support Problem (DSP) (c.f. (Mistree, et al., 1993a)). The compromise DSP is a hybrid formulation based on mathematical programming and goal programming. The focus of goal programming is to establish goals for each objective and to achieve each of the goals as closely as possible (Charnes and Cooper, 1961). The corresponding mathematical formulation is similar to compromise programming, but ideal or utopian objective function values are replaced with goals or targets established by a designer. For each objective, an achievement function, $A_i(\boldsymbol{x})$, represents the value of the objective as a function of a set of design variables, $\boldsymbol{x}$, and a goal or target value, G_i, is established for each objective. Deviation variables, d_i^- and d_i^+, represent the extent to which an objective underachieves or overachieves its target or goal, as follows:

$$A_i(\mathbf{x}) + d_i^- - d_i^+ = G_i \tag{2.9}$$

The overall objective function is expressed as a function of the deviation variables as follows:

$$Z = \underset{\mathrm{i=1,\,...,m}}{f}\left(d_i^-, d_i^+\right) \tag{2.10}$$

As expressed in Equation 2.10, the objective function in goal programming is exclusively a function of the deviation variables that measure the extent to which conflicting goals are achieved. The objective function could take many forms, the simplest of which is the weighted sum formulation:

$$Z = \sum_{i=1}^{m}\left(w_i^+ d_i^+ + w_i^- d_i^-\right) \tag{2.11}$$

Restrictions are placed on the deviation variables to limit them to positive values and ensure that only one deviation variable is positively valued at any specific point in the design space:

$$d_i^- \geq 0;\ d_i^+ \geq 0;\ d_i^- \cdot d_i^+ = 0 \tag{2.12}$$

Although strict formulations of goal programming do not support equality or inequality constraints, these constraints are supported in the compromise DSP with formulations borrowed from mathematical programming:

$$g_i(\mathbf{x}) \geq 0;\ i = 1,\ ...,\ p \tag{2.13}$$

$$h_i(\mathbf{x}) = 0;\ i = 1,\ ...,\ q \tag{2.14}$$

where p and q are the numbers of inequality and equality constraints, respectively. Bounds are also specified on the set of design variables that describe the form of potential solutions:

$$x_{i,L} \le x_i \le x_{i,U};\ i = 1, \ldots, n \tag{2.15}$$

where n is the number of design variables and $\boldsymbol{x}_{i,L}$ and $\boldsymbol{x}_{i,U}$ are the lower and upper bounds, respectively, for the i^{th} design variable.

Given

- An alternative to be improved through modification.
- Assumptions used to model the domain of interest.
- The system parameters (fixed variables).
- All other relevant information.

n	number of system variables
p + q	number of system constraints
p	equality constraints
q	inequality constraints
m	number of system goals
$g_i(\underline{X})$	system constraint function
W_i	weight for the Archimedean case

Find

- The values of the independent system variables
  $\underline{X} = X_1, \ldots, X_j$ $\quad$ j = 1, …, n
- The values of the deviation variable
  d_i^-, d_i^+ $\quad$ i = 1, …, m

Satisfy

- The system constraints that must be satisfied for the solution to be feasible.
  $g_r(\underline{X}) = 0$ $\quad$ r = 1, …, p
  $g_r(\underline{X}) \ge 0$ $\quad$ r = p+1, …, p+q
- The system goals that must achieve, to the extent possible, a specified target value.
  $A_i(\underline{X}) + d_i^- - d_i^+ = G_i;$ $\quad$ i = 1, …, m
- The lower and upper bounds on the system variables and bounds on the deviation variables.
  $X_j^{\min} \le X_j \le X_j^{\max}$ $\quad$ j = 1, …, n
  $d_i^-, d_i^+ \ge 0$ and $d_i^- \cdot d_i^+ = 0$

Minimize

The deviation function (a measure of the deviation of the system performance from that implied by the set of goals and their associated priority levels or relative weights):

$$Z = \sum_{i=1}^{m} W_i \left(d_i^- + d_i^+\right); \quad \sum W_i = 1; \quad W_i \ge 0 \qquad i = 1, \ldots, m$$

FIGURE 2.1. MATHEMATICAL FORMULATION OF THE COMPROMISE DSP (MISTREE, HUGHES, ET AL., 1993)

The objective function formulation and constraints borrowed from goal programming and mathematical programming, respectively, are unified with other constructs into a single decision support construct—the compromise DSP, as illustrated in Figure 2.1.

The compromise DSP is used to determine the values of design variables that satisfy a set of constraints and bounds and achieve a set of conflicting, multifunctional goals as closely as possible. As in goal programming formulations, the deviation function is formulated as a function of deviation variables that measure the extent to which multiple goals are achieved. The compromise DSP differs from goal programming, however, because it is tailored to handle common engineering design situations in which physical limitations are manifested as system constraints (mostly inequalities) and bounds on the system variables. In traditional mathematical programming, the objective function typically represents a single goal, by which the desirability of a design solution is measured. All other characteristics of a design are modeled as hard constraints. On the other hand, the compromise DSP is more flexible than traditional mathematical programming because it accommodates multiple constraints and objectives, as well as both quantitative information and information—such as bounds and assumptions—that may be based on a designer's judgment and experience (Marston, et al., 2000). In the compromise DSP, multiple goals have been considered conventionally by formulating the deviation function either with Archimedean weightings or preemptively (lexicographically) (Mistree, et al., 1993a). An Archimedean formulation is illustrated in Figure 2.1.

The conceptual basis of the compromise DSP is to minimize the difference between that which is desired (the goal, G_i) and that which can be achieved ($A_i(\mathbf{x})$) for multiple goals. The underlying philosophy of the compromise DSP and its goal programming foundations is similar to the concept of satisficing solutions and bounded rationality proposed by Simon (Simon, 1983; Simon, 1996). According to Simon's theory of bounded rationality, decision-makers are not omniscient and recognize that search is expensive. Consequently, they establish targets or thresholds and accept solutions that meet or exceed these targets as 'good enough' or satisficing. The thresholds are similar to the goal values specified in goal programming and the compromise DSP. Simon proposed a Nobel Prize-winning perspective with which to view human decision-making in a variety of contexts. It has very important consequences in an engineering design setting. For example, if the goal target values established by a designer for the compromise DSP are easily achieved, solution of the compromise DSP may produce solutions that are dominated (in the Pareto sense) by other feasible solutions.

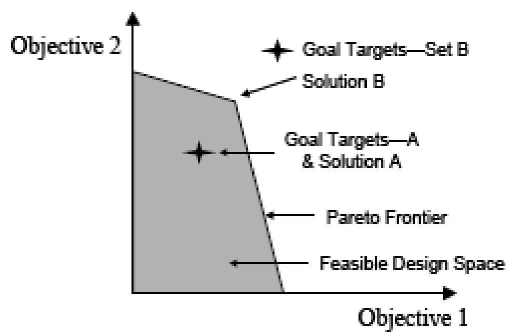


FIGURE 2.2. PARETO SOLUTIONS AND GOAL TARGETS IN THE COMPROMISE DSP (SEEPERSAD, 2004)

Suppose that two objectives are being balanced, as illustrated in Figure 2.2, and constraints limit achievement of the pair of objectives to the shaded feasible design space bounded by a Pareto frontier. If established goals are easily achieved (i.e., within the Pareto frontier) as with Solution A in Figure 2.2, then solution of the compromise DSP will satisfy the goals exactly, despite the fact that other feasible solutions dominate the targeted solution. Other solutions are feasible and offer preferred levels of all objectives. This is a common criticism of goal programming formulations—that they often deliver solutions that are inferior to other feasible Pareto solutions. However, this is not an inherent limitation of the compromise DSP formulation. Satisficing designs—such as Solution A in Figure 2.2—may actually be preferable to solutions on the Pareto frontier, especially in the early stages of design. As design parameters and conditions change, satisficing solutions are more likely to remain acceptable than Pareto solutions because satisficing solutions do not reside on the frontier of the feasible space and are therefore less likely to violate critical constraints as soon as design parameter values change. In the early stages of design, it is typical for assumptions and preliminary design parameter values to shift. As a result, 'optimal' designs may no longer be optimal; in fact, they may be infeasible. The flexibility built into satisficing solutions is particularly important for coupled, distributed design problems in which collaborating designers need this flexibility for adjusting design parameters without rendering the design unacceptable to other designers. The capabilities of the compromise DSP—coupled with design capability indices—for the generation of ranged solutions are discussed in further detail in Section 2.2.2. It is argued that flexible, satisficing solutions may be regarded as preferable rather than inferior to optimal solutions in some contexts. However, if Pareto solutions are sought, they are obtainable with the compromise DSP formulation. The enabling strategy is to set goal target values sufficiently high as with Solution B in Figure 2.2. In fact, it is easy to determine whether targets have been set sufficiently high because all of the deviation variables will have positive values.

In addition to facilitating the search for either flexible, satisficing solutions or Pareto solutions, the compromise DSP has additional capabilities that make it the construct of choice in this thesis for modeling multiobjective decisions in product design. For example, once a compromise DSP is formulated for a particular problem, it is possible to generate families of related designs by changing goal target values, weights, and/or design variable bounds without reformulating the problem. Unlike conventional single-objective optimization, a designer is not forced to choose a single objective and arbitrarily constrain other objectives. Instead, a designer can explore a range of tradeoffs between multiple conflicting objectives. Those objectives may include multiple measures of nominal performance (e.g., mass, heat transfer rates, effective stiffness) as well as measures of performance variation, induced by many sources of variation or uncertainty. Furthermore, the compromise DSP has been successfully utilized for designing many types of engineering systems, and its library of overall objective function formulations has been expanded to include physical programming (Hernandez, et al., 2001), Bayesian (Vadde, et al., 1994), fuzzy (Zhou, et al., 1992), and utility theory formulations (Seepersad, 2001) for specific contexts. Since the compromise DSP has been previously developed and utilized, the contribution in this thesis is in demonstrating that it can be used for comparing material design and material selection in the context of product design.

2.2.2 The cDSP with Design Capability Indices

There are some cases in the early stages of design when requirements are uncertain and are most appropriately expressed as a range (i.e., smaller than a lower limit, larger than an upper limit, or between lower and upper limits) rather than a target value, as shown in Figure 2.3. In these cases, it may be necessary to measure the extent to which a range or distribution of design performance (induced by a range of design specifications) satisfies a ranged set of design requirements. Chen and coauthors propose the design capability indices (DCIs) as a set of metrics for assessing the capability of a ranged set of design specifications for satisfying a ranged set of design requirements (Chen, Simpson, Allen, and Mistree, 1999). The design capability indices are incorporated as goals in the compromise DSP. In further work, Chen and Yuan (1999) introduce a design preference index that allows a designer to specify varying degrees of desirability for ranged sets of performance, rather than specifying precise target values or limits for a range of requirements beyond which designs are considered worthless.

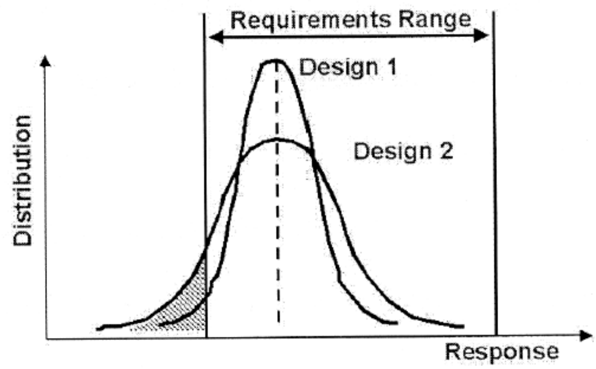


FIGURE 2.3. COMPARING TWO DESIGNS WITH RESPECT TO A RANGE OF REQUIREMENTS

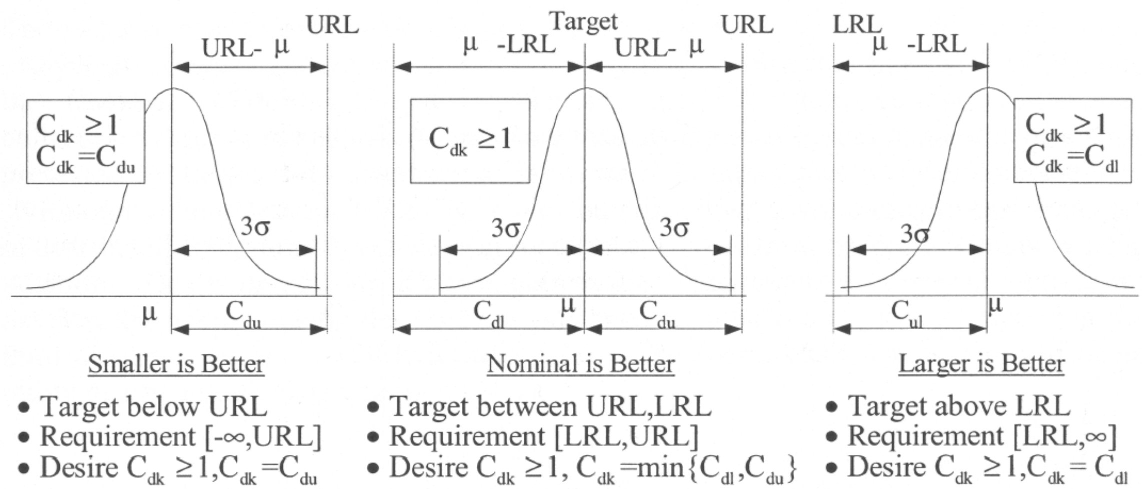


FIGURE 2.4. DESIGN CAPABILITY INDICES (CHEN, SIMPSON, ALLEN, AND MISTREE, 1999)

The DCIs are mathematical constructs for efficiently determining whether a ranged design specification is capable of satisfying a ranged set of design requirements. When the index is negative, the mean of the system performance is outside of the system requirement range. If the index is greater than one, the design will meet the requirement satisfactorily. Therefore, a designer's objective is to force the index to unity so that a larger portion of performance deviation falls into the range of design requirements. Forcing the index to unity is achieved by reducing performance deviation and/or locating the mean of performance deviation farther from requirement limits. The procedure to evaluate the index is illustrated in Figure 2.4.

Assuming that the design variable varies $\pm\Delta x$ around its nominal value x_0, then the corresponding deviation of the system response is $\pm\Delta y$ around its mean value μ, which is calculated by:

$$\mu = f\left(x_0\right) \tag{2.16}$$

C_{du}, C_{dl}, and C_{dk} in Figure 2.4 are calculated as follows:

$$C_{dl} = \frac{\mu - \mathrm{LRL}}{\Delta y}; \quad C_{du} = \frac{\mathrm{URL} - \mu}{\Delta y}; \quad C_{dk} = \min\left\{C_{dl}, C_{dk}\right\} \tag{2.17}$$

For cases in which the deviation of the input variables can be quantified by a statistical distribution, the amount of deviation in the system response is characterized by the number of standard deviations of the response distribution. The number of standard deviations indicates the percentage of the distribution that conforms to the design requirements. For three standard deviations, a C_{dk} of 1 indicates that 99.865% of the performance distribution conforms to the requirements (Chen, Simpson, Allen, and Mistree, 1999). Incorporating DCIs into the goals of the cDSP results in the particularized formulation of the cDSP shown in Figure 2.5. The DCIs can be used for both goals and constraints in the formulation of the particularized cDSP, depending on whether the ranged design requirement is a wish or a demand. Demands are requirements that must be met under all circumstances and are formulated as constraints under which the constraint DCIs must be greater than or equal to one. Wishes are requirements that should be taken into consideration whenever possible and are formulated as goals of maximizing the goal DCIs as close as possible to one.

Given

Functions $\boldsymbol{y}$ including those ranged design requirements which are constraints, $\mathbf{g}_i(\mathbf{X})$, and those which are objectives, $\mathbf{A}_j(\mathbf{X})$

Deviations of control variables $\sigma_{\mathbf{x}}$ or Δx

Target *ranges* for the design requirements, URL_i and LRL_i

Find

The location of the mean of the control variables $\mu_{\mathbf{x}}$

Satisfy

Constraints: $C_{\text{dk-constraints}} \geq 1$

Goals: $C_{\text{dk-objectives}} + d_i^- - d_i^+ = 1$

Bounds:

$d_i^-, d_i^+ \geq 0$; $i = 1,\ldots, m$

$d_i^- \cdot d_i^+ = 0$; $i = 1,\ldots, m$

Minimize: Deviation Function

Archimedean: $Z = \sum_{i=1}^{m} W_i \left(d_i^- + d_i^+\right)$, where $\sum_{i=1}^{m} W_i = 1$

Preemptive: $Z = \left[f_1\left(d_1^-, d_1^+\right), \ldots, f_m\left(d_m^-, d_m^+\right) \right]$

FIGURE 2.5. MATHEMATICAL FORMULATION OF THE COMPROMISE DSP PARTICULARIZED FOR DESIGN CAPABILITY INDICES (CHEN, SIMPSON, ALLEN, AND MISTREE, 1999; XIAO, 2003)

The compromise DSP formulation with DCIs provides several advantages. With DCIs, a designer can efficiently check whether a family of designs can satisfy design requirements while eliminating the tedious task of evaluating large numbers of discrete or continuous design specifications. In addition, a designer can consider multiple aspects of quality improvement by adjusting the location of the mean of the performance distribution as well as the variation. Finally, DCIs are easy for a designer to compute and understand using a simple index. The advantages of using DCIs for finding ranged sets of solutions in conjunction with the compromise DSP have been approved by applying them to sample engineering problems, such as the design of a solar powered irrigation system (Chen, Simpson, Allen, and Mistree, 1999), multidisciplinary decision making in the design of a circuit board (Xiao, et al., 2002), and the design of a Linear Cellular Alloy (Seepersad and Allen, 2003).

In the previous section it is established that the cDSP enables the search for solutions to multiobjective design problems, and that once formulated, the cDSP can be easily adapted to provide families of solutions and explore trades between the objectives. This capability is particularly important for the DSES in which it is desired to compare material design and selection because additional system variables can be easily added. Furthermore, the ranged cDSP enables the search for ranges of solutions that meet ranged performance specs, which is particularly useful in the early stages of design when requirements are uncertain. Therefore, it is argued that the cDSP with design capabilities is an appropriate foundation for finding ranged sets of design solutions in the DSES. Because DCIs are intended to be used to find ranges of design solutions that satisfy ranges of design requirements, DCIs are an appropriate choice for finding target ranges of material properties that meet ranged design requirements in the early stages of design. In the next section, the area of information economics and value of information metrics are reviewed.

2.3 INFORMATION ECONOMICS

One of the requirements for the DSES is that it must provide a *quantitative* comparison of the options of material selection and material design. This quantitative comparison is provided by the VDSE metric proposed in this thesis. The requirements for the metric are as follows: the comparison must be made on the basis of achieving product performance goals, and the metric enables comparison of an existing material and an unrealized material concept. Value of information metrics developed in the field of information economics provide the foundation for the VDSE metric and should therefore be reviewed. In particular, The VDSE metric is developed based on the Improvement Ratio developed by Panchal and coauthors (2005). In this section, a review of value of information for decision making is presented as background and for domain-independent structure validity. The review of Panchal (2005) is presented here without major change and is augmented by a review of the Improvement Ratio.

At any stage in the design process, designers possess some amount of information that can be used for selecting the best course of action. Designers have an option of either making a decision using the available information or gathering more information and then making a decision using the updated information. In this context of decision making, the value of this added information refers to the *improvement in designers' decision making capability*. The value of information metric is used by decision makers to make the meta-level decision involving the tradeoff between gathering more information to reduce uncertainty and reducing the associated cost of gather the additional information. The idea of using value of information for determining whether to consider additional information for decision making is not new. It was first introduced by Howard (1966). The expected value of information as defined by Howard is "*the difference between the expected value of the objective for the option selected with the benefit of the information less than without"*. Mathematically, the expected value of information is shown in Equation 2.18, where *x* is the state of the environment, EVI_{φ} is the expected value of information, φ is the available information, and $\underset{x}{E}\left[f\left(x\right)|\,\varphi\right]$ is the expected value of the function $f(x)$.

$$EVI_{\varphi} = \underset{\gamma}{E}\left[\max_{i}\left\{\underset{p}{E}\left[u\left(p, x^{(i)}\right)|\,\varphi\right]\right\}\right] - \max_{i}\left\{\underset{p}{E}\left[u\left(p, x^{(i)}\right)\right]\right\} \tag{2.18}$$

Bradley and Agogino use this value of information metric for a catalog selection problem, where a designer is faced with the task of choosing components from a catalog in order to satisfy some functional requirements (Bradley and Agogino, 1994). During the conceptual design phase, selection decisions are characterized by significant uncertainty due to limited understanding of requirements and constraints, inability to specify part dimensions, uncertainty in the environmental conditions, etc. However, before making the decision about the right component, a designer needs to make *another higher level decision* – whether to go ahead and make the decision using available information or to spend resources and gather more information before making the selection decision. This is a meta-level decision, for which Bradley and Agogino (Bradley and Agogino, 1994) utilize the value of information metric to quantify the expected benefit from additional information.

Poh and Horvitz use the value of information metric for refining decisions (Poh and Horvitz, 1993). The authors present three dimensions in which the decision models can be refined – quantitative, conceptual, and structural. Quantitative refinement of a decision model can be carried out by reducing the uncertainty in the decision problem or by refining the preference models. Conceptual refinement is carried out by refining the definition of alternatives and design variables, whereas structural refinement requires addition of dependencies in the simulation model. Poh and Horvitz use the value of information metric to determine which dimension is critical for refinement of the decision problem.

Lawrence provides a comprehensive overview of metrics for value of information (Lawrence, 1999). He argues that the value of information for decision making can be measured at different stages in the decision-making process. Accordingly, the value of information metrics are named differently based on the stage at which they are evaluated. Four different options for measuring the value of information are:

a) prior to consideration of incorporation of information,
b) Ex-ante value: after considering a message source but prior to receiving a message

c) Conditional value: after receiving additional information and making the decision, but before realization of the environmental state, or
d) Ex-post value: after addition of information and making a decision-based on acquired information.

Determination of value of information at different stages in the decision-making process results in different kinds of insight for meta-level decisions. The appropriateness of a stage for measuring the value of information depends on the problem at hand and the available information. Consider an example of a designer who has a simulation model for predicting the system behavior and is interested in making a decision using the model. Before making the decision, a designer has an option of increasing the fidelity of the model by considering additional physical phenomena in the model. For example, a structural analyst may improve the fidelity of a static model by adding dynamic behavior, creep, etc. Description of a physical phenomenon is equivalent to an information source that generates information about the system behavior. The output of the simulation i.e., system behavior is equivalent to the added information generated by the information source. Now, the decision maker can evaluate the expected value of information before even considering the incorporation of any additional physical phenomena. The second option (ex-ante value) is to decide which physical phenomena to model (i.e. information source) and evaluating the value of information before executing the simulation code. The third option (conditional value) is to evaluate the value after executing the simulation code and making decision about the system, but before manufacturing and testing the system. In this scenario, there is uncertainty in the actual system behavior that would be achieved due to factors such as manufacturing variability and changes in environmental conditions. The fourth option (ex-post value) is to evaluate the value of this additional information after making decision and also manufacturing and testing the system. In this scenario, designers know exactly how the system behaves.

Mathematically, the ex-post and ex-ante value of information are represented as follows:

1. Ex-post value: $v(x,y)=\pi(x,a_y)-\pi(x,a_0)$

Where a_0 and a_y represent the actions taken by the decision maker in the absence and presence of information y. $\pi(x,a)$ represents the payoff achieved by selecting action a, when the state realized by the environment is x.

2. Ex-ante value: $v(x,y)=E_{x|y}\pi(x,a_y)-E_x\pi(x,a_0)$

where $E_x f(x)$ is the expected value of $f(x)$ and $E_{x|y} f(x)$ is the expected value of $f(x)$ given y. It is important to realize that the key difference between ex-post and ex-ante value is that in ex-post value, the realization of state x is known. However, the realization of state x is not known in ex-ante value and the expected value of payoff is taken over the uncertain range of state x.

Ideally, designers are interested in the ex-post value of information because it truly reflects the value of information for a decision-based on the actual behavior of the system. There the system behavior is known deterministically. However, it is not possible to calculate the ex-post value of a decision before making the decision itself. Due to the ex-ante nature of decision making, the decisions about the information have to be made before the state actually occurs. Hence, the ex-ante value of information is generally used by designers. It captures the value of information by considering uncertainties in the system. In order to model uncertainty for evaluating value of information, it is assumed that the probability distributions are available. However, if these probability distributions are not available, they are generally generated through an educated guess that is based on the designers' prior knowledge. In order to address the problem of lack of knowledge about the probability distributions, Aughenbaugh and coauthors present an approach of measuring the value of information based on probability bounds (Aughenbaugh, Ling, et al., 2005). They assume that although the exact probability distributions are unavailable, the lower and upper bounds on these probabilities are available in terms of p-boxes. Using this p-box approach, they evaluate the value of added information that reduces the size of the interval for probability distribution (i.e., tightens the bounds on the p-box).

In most of the efforts, the value of information is based on the variability in the decision problem. This uncertainty is modeled using probability distributions; however, except for Aughenbaugh and co-authors, imprecision in the decision models which cannot be modeled in terms of probability distribution functions is generally not modeled (Aughenbaugh, Ling, et al. 2005). Imprecision relates to epistemic uncertainty (i.e., the lack of knowledge), whereas variability refers to aleatory uncertainty (i.e., inherent randomness in the system). The key difference between imprecision and uncertainty from a value of information standpoint is that imprecision can be reduced by the incorporation of more information but uncertainty cannot be reduced via incorporation of information. For example, consider a scenario where a designer has an option of making a decision using one of the two available simulation models. One of the simulation models has a higher fidelity representation of physics than the other. The meta-level decision that the designer has to make is – “Which simulation model should be used for making the decision?” This scenario is extremely common in design problems.

To account for imprecision in simulation models in addition to variability, Panchal and coauthors propose the Improvement Potential metric (Panchal et al., 2007). The Improvement Potential quantifies the maximum possible improvement in a designer’s decision that can be achieved by refining a simulation model. This metric is measured as the upper-bound on the increase in expected utility through model refinement, and the equation for the metric is shown in Equation 2.19, where $\max(U_{\max})$ is the maximum expected payoff that can be achieved by any point in the design space, and $(U_{\min})^*$ is the lowest expected payoff value achieved by the selected point in the design space (after making the decision without the added information).

$$P_I = \max\left(U_{\max}\right) - \left(U_{\min}\right)^* \tag{2.19}$$

With this metric, statistical variability is accounted for by using the expected utility and imprecision is accounted for by using the lower and upper bounds on expected utility. In addition to the metric, the authors put forth a method in which the metric is utilized for supporting model refinement decisions. The method and metric are demonstrated for the design of a pressure vessel and the design of a multifunctional material.

Pursuing the design of the material can be thought of as refining the material model that is used for the design of the product. In material selection the material model is defined by discrete points, whereas in material design the material model is continuous. Since the extension of the material design space is similar to the refinement of a decision model, it is argued that existing value of information metrics are a valid basis for the development of the VDSE metric. Specifically, the Improvement Potential proposed by Panchal and coauthors provides the foundation for the VDSE metric, introduced in Section 3.2.1

2.4 RESEARCH OPPORTUNITIES

The primary purpose of this literature review is to identify research opportunities relevant to the focus of this thesis. The identified research opportunities are organized in Tables 2.3 and 2.4 according to the requirements for the DSES and VDSE metric, where the requirements are listed in the second column, the support for these requirements in existing literature is listed in the third column and the research opportunities are listed in the fourth column. In the remainder of this section, these research opportunities are summarized with the goal of establishing the originality and significance of the primary

TABLE 2.3. IDENTIFYING RESEARCH OPPORTUNITIES, PART 1

		Requirement	Support in existing literature	What is needed	RQ
DSES	1	Driven by product-level requirements rather than material-level requirements	cDSP enables the search for design solutions that meet multiple conflicting product requirements	A way to determine how well material concepts meet product requirements	1
	2	Computationally inexpensive as compared to designing a new material	cDSP enables an efficient search of the design space as long as the performance models are quick to evaluate	A way to represent and evaluate the performance of new materials that is computationally efficient	1
	3	Provides a quantitative comparison of the options of selection and design based on the achievement of the product design objectives as well as process design objectives	- cDSP allows for both product and design process goals to be considered simultaneously - Value of Information metrics provide quantitative means to compare options based on overall performance	A Value of Information based metric that is compatible with the cDSP	2
			cDSP once formulated can be used to find families of design by changing bounds, targets, etc.	Show that the material design space can be expanded by adding material property variables to an existing cDSP	2
	4	Provides guidance for subsequent phases of design	ranged cDSP can be used to find ranges of design solutions that meet ranged performance specs	Use material property variables in cDSP solutions to set targets for material design	3

TABLE 2.4. IDENTIFYING RESEARCH OPPORTUNITIES, PART 2

		Requirement	Support in existing literature	What is needed	RQ
VDSE	1	Compares an existing material to an unrealized material concept	Improvement Potential measures the value of model refinement through the reduction of uncertainty	A metric that measures the value of the expansion of the design space rather than the value of the reduction of uncertainty	2
	2	Compares on the basis of meeting product-level and design process requirements	- Existing Value of Information metrics are based on the improvement in overall utility, which can include multiple goals or attributes - the cDSP provides a combined assessment of the value of the design in the Archimedean deviation function	A metric in which the value is measured by the reduction in overall deviation, which is compatible with the cDSP formulation	2

and secondary research questions and hypotheses. The research opportunities are thus organized by research question.

2.4.1 Research Question 1: Evaluation of Material Concepts

To compare the design process options of material selection and material design, designers must have a way to explore a material design space. Although multiscale material simulations provide a way to evaluate candidate material concepts, the generation of these models is too time consuming for efficient design space exploration in the early stages of design. The ranged cDSP with DCIs enables an efficient search of an expanded material design space. In this situation, multiobjective decision support is needed in order to determine families of compromise solutions that meet product and process objectives. By setting material properties as design variables in the ranged cDSP, the efficient search of an expanded material design space is enabled without the need for expensive and complex material models.

2.4.2 Research Question 2: A Value of Design Space Expansion Metric

To compare the options of material selection and material design, an ex-ante or conditional metric is desired, because the addition of information in this case refers to the inclusion of a multiscale material model in the decision model. The development of this complex simulation model or models is a difficult and time-consuming task, which is the very task designers wish to avoid by identifying the material property requirements. A new metric is needed that can be evaluated prior to the development of a complex material model. Metrics exist for measuring the value of information that reduces the uncertainty in a decision problem; however, in the choice between material selection and design, information to reduce the uncertainty in the material properties is not available. What is available in this choice is an expanded material design space. Therefore, a new metric is needed to quantify the value of *design space expansion*.

2.4.3 Research Question 3: Providing Guidance for Subsequent Phases of Design

The cDSP and the ranged cDSP with DCIs have been demonstrated in the literature for finding families of compromise, satisficing solutions to multiobjective design problems. The cDSP has also been used in conjunction with value of information metrics to determine when additional information is valuable. What is needed is the use of the ranged cDSP along with a new value of design space expansion metric to determine when expanding the design space is valuable. Furthermore, once ranged solution sets have been found and the metric has been calculated, a method for determining material property targets for subsequent phases of design is needed.

2.5 A LOOK BACK AND A LOOK AHEAD

In this chapter the foundational constructs of the DSES and VDSE metric are introduced and critically reviewed. The cDSP with DCIs and value of information metrics provide the foundation for the development of the DSES and the VDSE metric.

Next, in Chapter 3 the DSES and the VDSE metric are presented as embodiments of the research hypotheses. The domain-independent structure validity is assessed by critically reviewing the internal consistency of the DSES as an assemblage of the parent constructs. The domain-specific structure validity and domain-specific performance validity of the DSES are evaluated in Chapter 4 with the presentation of an example problem on the design of blast resistant panels. The thesis is concluded in Chapter 5 with a critical review of the thesis and the assessment of the domain-independent performance validity of the DSES. Limitations are identified and opportunities for future work are presented.

CHAPTER 3

AN OVERVIEW OF THE DESIGN SPACE EXPANSION STRATEGY AND THE VALUE OF DESIGN SPACE EXPANSION METRIC

In Chapter 2, the available design tools are reviewed and critically evaluated. In this chapter, the Design Space Expansion Strategy (DSES) and the Value of Design Space Expansion (VDSE) metric are introduced. An outline of the topics discussed in this chapter is shown in Table 3.1.

TABLE 3.1. A SUMMARY OF CHAPTER 3

Section	Heading	Information
3.1	The Design Space Expansion Strategy	
3.1.1	Introducing the Design Space Expansion Strategy	• Stage 1: Baseline • Stage 2: Expansion • Stage 3: Interpretation
3.1.2	Meeting the Requirements of the DSE Strategy	• Recap of the requirements for the DSE strategy • Analysis of whether or not the requirements are met by the proposed strategy
3.2	The Value of Design Space Expansion metric	
3.2.1	Introducing the VDSE metric	• An ex-ante metric for the value of design space expansion • Using the VDSE to support design process decision making
3.2.2	Meeting the Requirements of the VDSE metric	• Recap of the requirements for the VDSE metric • Analysis of whether or not the requirements are met by the proposed metric
3.3	Validation	
3.3.1	Domain-Independent Structure Validity	Does the strategy provide an internally consistent answer to the research questions?
3.3.2	Domain-Specific Structure Validity	Is the BRP example appropriate for testing the DSES?
3.4	A look back and a look ahead	

In Section 3.1, the DSES is introduced. The strategy itself is introduced in Section 3.1.1, and an assessment of how well the strategy meets the requirements is presented in Section 3.1.2. In Section 3.2 the metric for the VDSE is introduced and described. The metric is introduced in Section 3.2.1 and the requirements for the metric are discussed in Section 3.2.2. In Section 3.3 the structure validity of the design space expansion strategy is assessed. The domain-independent structure validity, or internal consistency, of the strategy is discussed in Section 3.3.1, and the domain-specific structure validity is explored in Section 3.3.2. In Section 3.4 the chapter is summarized in the context of the thesis with a look back and a look ahead.

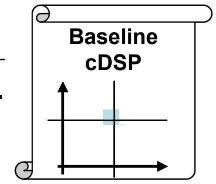
3.1 THE DESIGN SPACE EXPANSION STRATEGY

In Chapter 2, the available design tools are critically reviewed, and the research gap is identified. The systematic design of materials requires a top-down approach rather than the bottom-up approach taken by material scientists (see Figure 1.1). Inductive mappings must be identified from the desired performance down to the properties, structure, and processing of the material components. Mapping performance to properties is the first step in this top-down approach to material design. In this section, the DSES is proposed to identify the material properties that are needed to achieve the desired product performance.

3.1.1 Introducing the Design Space Expansion Strategy

To meet the requirements identified in previous chapters and to answer the research questions in this thesis, the DSES is proposed here. The strategy consists of three stages: baseline, expansion, and interpretation. A graphical overview of the strategy is presented in Figure 3.1.

Stage 1 – Baseline

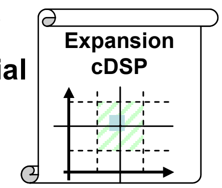


Find **the design solution using existing material(s).**

1.1 Baseline Material Design Space
- 1.1.1 Define the baseline material
- 1.1.2 Determine the variation in baseline material properties

1.2 The Baseline cDSP
- 1.2.1 The word formulation
- 1.2.2 The math formulation
- 1.2.3 Solving the baseline cDSP

Stage 2 – Expansion



Find **the design solution using independent material property design variables in place of existing material(s).**

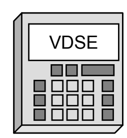
2.1 Expanded Material Design Space

Define upper and lower bounds for the material property variables.

2.2 The Expansion cDSP
- 2.2.1 The word formulation
- 2.2.2 The math formulation
- 2.2.3 Solving the expansion cDSP

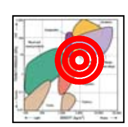
Stage 3 – Interpretation

Compare **the achievement of design objectives at the baseline and expansion solutions to assess the value of expanding the design space.**



3.1 Calculate the VDSE metric

If the value of expanding the design space is large enough, *identify* material property targets for subsequent phases of design.



3.2 Identify Material Property Targets
- 3.2.1 Search for existing materials that meet the targets
- 3.2.2 Identify the material property targets on material selection charts

FIGURE 3.1. GRAPHICAL OVERVIEW OF THE DSES

In the baseline stage, a design solution is found using existing material(s) by formulating and solving the baseline cDSP. Next, in the expansion stage, the materials are defined using independent material property variables, and a new solution is found in this expanded material design space. In this stage the expansion cDSP is formulated and solved. The main difference between the baseline and expansion stages is the definition of the material design space, which is shown in icon form at the right side of Figure 3.1. Lastly, in the interpretation stage the baseline and expansion design solutions are compared using the VDSE metric, which is introduced and discussed separately in Section 3.2. Also in the interpretation stage, designers gain insight into the material property targets for the product by analyzing the expansion stage solutions. All three stages are discussed in more detail in the following sections.

Stage 1: Baseline

In the baseline stage of the DSES the objective is to find a design solution using existing materials. The purpose of this stage is to determine the best product design that is achievable without engaging in the design of a new material. The baseline stage consists of two steps: defining the baseline material design space (Step 1.1) and formulating and solving the baseline cDSP (Step 1.2). In the baseline stage, the material design space is a point represented by the material properties of the baseline material; therefore, the first step is to identify the baseline material (Step 1.1.1). This is done by following a material selection procedure, solving a selection DSP, or, for variant or adaptive design, the baseline material may be the materials that have previously been used for the product. By assuming a baseline material, an approach is taken in which material property targets are sought in the vicinity of existing materials. In the author's opinion, this approach increases the likelihood that a material with the desired properties can be designed by adjusting the structure and processing path of an existing material.

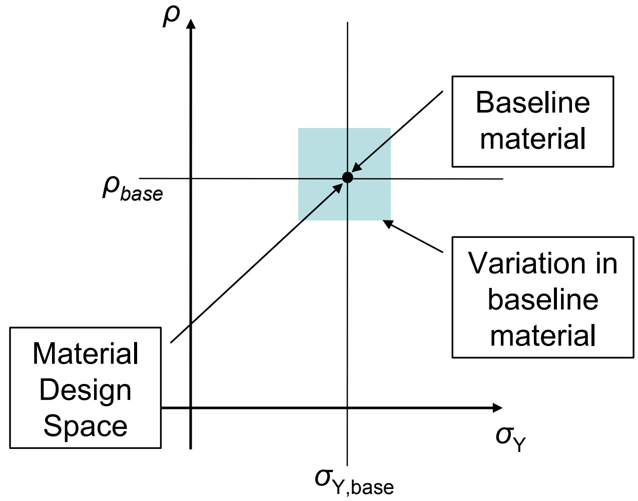


FIGURE 3.2. A SAMPLE BASELINE MATERIAL DESIGN SPACE

It is also necessary to characterize the uncertainty in the properties of the baseline material in Step 1.1.2. Due to factors such as variability in manufacturing and environmental conditions, the properties of the baseline material may not exactly match the values shown in handbooks or reported by the supplier; therefore, a designer must determine the amount of variation that is possible in these quantities so that the variation can be accounted for in design. The material design space in the baseline stage is thus an uncertain point, as shown in Figure 3.2.

The second step of the baseline stage is formulating and solving the baseline cDSP (Step 1.2). The baseline cDSP is a particularized form of the ranged cDSP in which DCIs are employed in the specification of system constraints and goals. A detailed explanation of the ranged cDSP as an extension of the original cDSP can be found in Section 2.2.2. The ranged cDSP is particularized here in the baseline cDSP to include given parameters relating to the baseline material(s) and a goal to reduce the cost of design space expansion (CDSE).

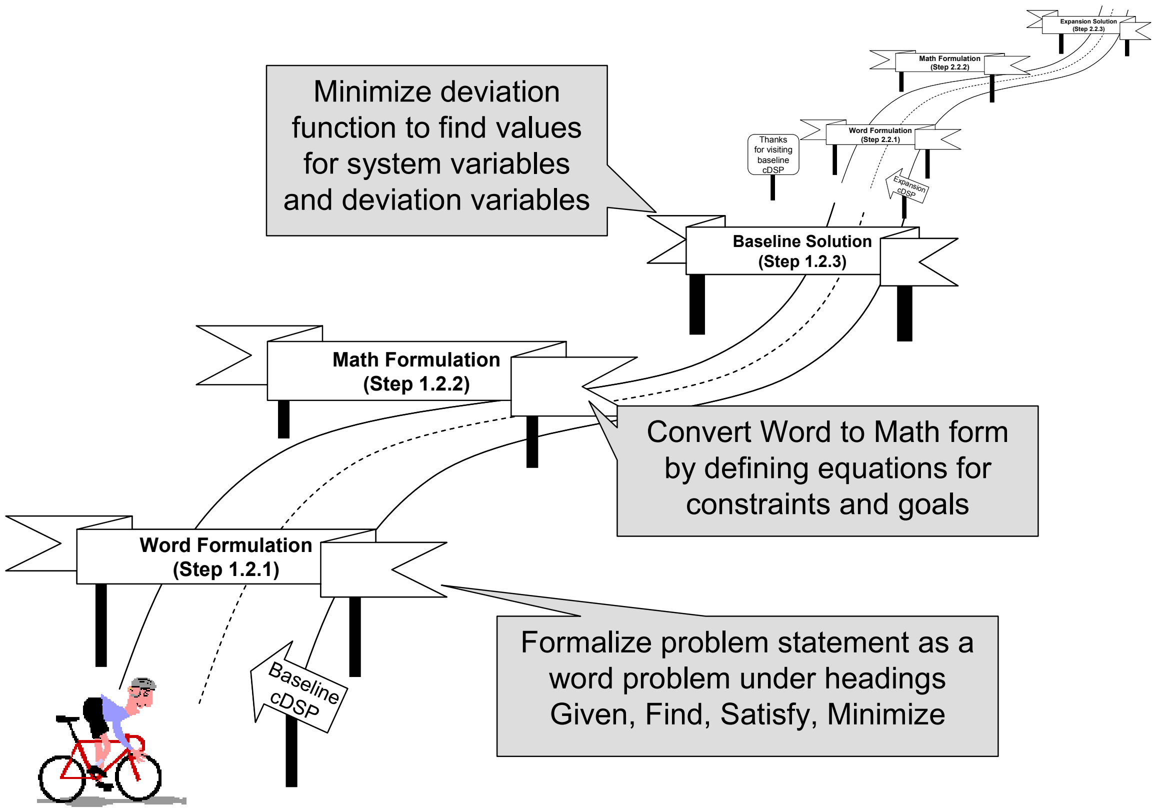


FIGURE 3.3. THE ROAD TO THE BASELINE SOLUTION

The process of formulating and solving the baseline cDSP is shown graphically in Figure 3.3. There are three milestones which a designer must reach to formulate and solve any cDSP: the word formulation (Step 1.2.1), the math formulation (Step 1.2.2), and the compromise solution (Step 1.2.3). The same process holds for the baseline cDSP as well. To reach the word formulation (Step 1.2.1), a designer formalizes the design problem statement under the headings *Given*, *Find*, *Satisfy*, and *Minimize*. The word formulation for the baseline cDSP is shown in Figure 3.4. The particularizations of the baseline cDSP from the ranged cDSP are shown in italics in the figure.

The particularizations of the baseline cDSP are located under the *Given* and *Satisfy* headings of the cDSP formulation. Under the *Given* heading, two types of system parameters are identified which must be included in both the baseline and expansion cDSPs. These system parameters are the properties of the baseline materials and the variation in these properties. These parameters are used in the models of system performance to predict the performance of the various design alternatives.

Given

- An alternative to be improved through modification.
- Assumptions used to model the domain of interest.
- Deviation of the design variables.
- Target ranges of performance measures.
- The system parameters (fixed variables).
    - *The properties of the baseline material(s).*
    - *The variation in the properties of the baseline material(s).*
- The constraints and goals for the design.

Find

- The values (locations) of the ranged system variables.
- The values of the deviation variables.

Satisfy

a. The system constraints that must be satisfied for the solution to be feasible, stated in terms of the design capability index, C_{dk}, which must be greater than or equal to one for the solution to be feasible.
b. The system goals that must achieve, to the extent possible, a specified target value. The system goals are stated in terms of the design capability index, C_{dk}, which must achieve, to the extent possible, the target value of one.
c. The CDSE goal that must achieve, to the extent possible, a specified target value. The CDSE goal is stated in terms of the design capability index, C_{dk}, which must achieve, to the extent possible, the target value of one.
d. The lower and upper bounds on the system variables and bounds on the deviation variables.

Minimize

The deviation function that is a measure of the deviation of the system performance from that implied by the set of goals and their associated priority levels or relative weights.

FIGURE 3.4. WORD FORMULATION OF THE BASELINE CDSP

Under the *Satisfy* heading, a new goal specific to the DSES is included to minimize the CDSE as closely as possible to zero. This goal relates to the design process objective of reducing design process complexity. By simultaneously seeking to minimize the CDSE while also achieving the system performance goals, the benefits of the design space expansion (improved system performance) are balanced by the cost of expanding the design space (increase in cost of design space expansion as solutions diverge from the baseline material). The choice of a model for CDSE and the implications of the CDSE model on the baseline cDSP are discussed in the next section regarding the math formulation of the baseline cDSP.

Having reached the word formulation, the next milestone is the math formulation of the cDSP (Step 1.2.2). The difference between the math and word forms of the cDSP is that in the math form, all the variables, constants, and equations are defined mathematically in order to obtain a deviation function that can be minimized. To do this, equations are defined or other models are identified to evaluate the constraints and goals. This portion of the problem formulation is highly problem-specific, as this is when the models of the system behavior are identified. The math formulation of the baseline cDSP is shown in Figure 3.5, with the particularizations for the DSES shown in boldface type.

The specification of a model for the CDSE is one aspect of the math formulation of the baseline cDSP that is particular to the DSES strategy. The CDSE quantifies the expected increase in product design cost due to deviations from known materials. It is assumed that the product design cost will increase as the candidate solutions move farther away from the baseline material properties. This assumption is based on the fact that complex material models will need to be developed and validated for use as predictive models to design a material that does not yet exist, and the cost of developing and validating these models must be considered when a designer is choosing between material selection and material design. In the early stages of design, explicit data on this cost is not likely to be available; however, a designer can define a function that reflects his beliefs about how the product design cost will be affected by choosing material properties that differ from the baseline materials.

Given

- An alternative to be improved through modification.
- Assumptions used to model the domain of interest.
- **Values of the baseline material properties, μMP_k. k = 1, …, s**
- **Variation in the material properties, ΔMP_k.**
- Variation of design variables, ΔX_i
- Target ranges of performance measures. URL_i and LRL_i i = 1, …, m
- The system parameters (fixed variables).

n	number of system variables
s	**number of baseline material properties**
p + q	number of system constraints
p	equality constraints
q	inequality constraints
m	number of system goals
$g_i(\underline{X})$	system constraint function
W_i	weight for the Archimedean case

Find

- The values (locations) of the ranged system variables
  $\underline{\mu}_X = \mu_{X1}, \ldots, \mu_{Xj}$ j = 1, …, n
- The values of the deviation variables
  d_i^-, d_i^+ i = 1, …, m

Satisfy

a. The system constraints that must be satisfied for the solution to be feasible.

$g_r(\underline{X}) = 0$ r = 1, …, p

$g_r(\underline{X}) \geq 0$ r = p+1, …, p+q

$C_{dk\text{-}constraints} \geq 1$

b. The system goals that must achieve, to the extent possible, a specified target value.

$A_i(\underline{X}) + d_i^- - d_i^+ = G_i;$ i = 1, …, m

$C_{dk\text{-}objectives} + d_i^- - d_i^+ = 1$

c. The CDSE goal that must achieve, to the extent possible, a specified target value.

$\mathbf{A_{CDSE}(\underline{X}) + d_{CDSE}^- - d_{CDSE}^+ = G_{CDSE};}$

$C_{dk\text{-}obj-CDSE} + d_i^- - d_i^+ = 1$

d. The lower and upper bounds on the system variables and bounds on the deviation variables.

$X_j^{min} \leq X_j \leq X_j^{max}$ j = 1, …, n

$d_i^-, d_i^+ \geq 0$ and $d_i^- \cdot d_i^+ = 0$

Minimize

The deviation function (a measure of the deviation of the system performance from that implied by the set of goals and their associated priority levels or relative weights):

$$Z = \sum_{i=1}^{m} W_i \left(d_i^- + d_i^+\right); \quad \sum W_i = 1; \quad W_i \geq 0 \qquad i = 1, \ldots, m$$

FIGURE 3.5. MATHEMATICAL FORMULATION OF THE BASELINE CDSP

A simple CDSE function is one that increases linearly with the Euclidean norm of the vector from the baseline material to the expansion solution, shown graphically in Figure 3.6. This linear model is easy to implement and reflects the assumption that it will be more difficult to design materials that have properties that are farther away from the baseline material properties, but a designer is free to choose other forms such as a quadratic or exponential model to represent his beliefs about how the product design cost will change as the expansion solutions diverge from the baseline material.

Since the material property design variables remain fixed at the baseline properties, the CDSE is constant throughout the baseline design space. One may wonder, then, why the CDSE goal is included in the formulation of the baseline cDSP. The CDSE is modeled in the baseline cDSP in order to reduce the changes in the formulation from the baseline to expansion cDSP and because the total deviation values in the baseline and expansion solutions must be compared in the VDSE metric. If there are a different number of goals in the two cDSPs, then the deviation function values at the two solutions cannot be compared. Since the CDSE is constant throughout the baseline design space, the baseline cDSP solution is not affected by the achievement of the CDSE goal.

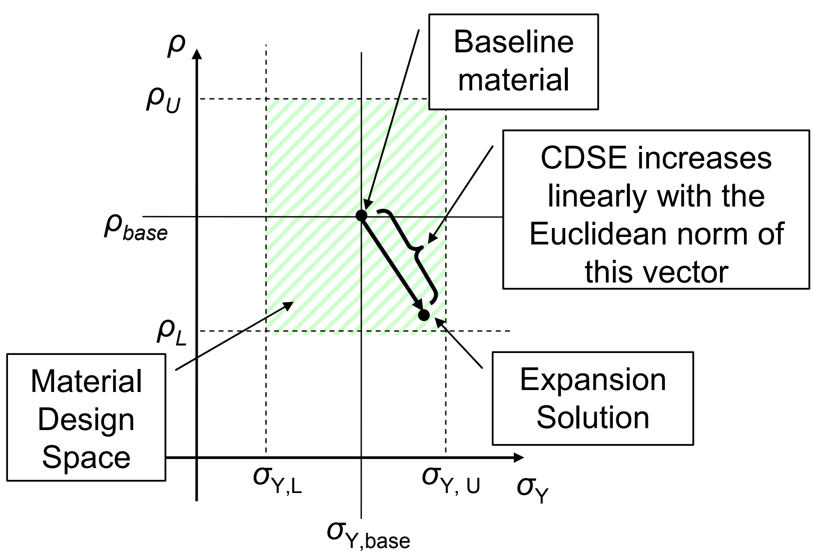


FIGURE 3.6. A GRAPHICAL REPRESENTATION OF A LINEAR CDSE FUNCTION

Having reached the math formulation of the baseline cDSP, the last remaining step (step 1.2.3) is to solve the baseline cDSP by minimizing the deviation function in order to reach the last milestone on the road to the baseline solution (see Figure 3.3). An algorithm is selected and applied based on the characteristics of the deviation function. The shape and behavior of the deviation function is dependent on the models of the constraints and goals, and is thus highly problem-specific. Once a baseline solution has been found, it must be verified, not only with respect to the correctness of the minimization results, but also with respect to the physical meaning of the solution and the resulting meaning in the context of the DSES. Specifically, a designer must verify that there is additional room for improvement in the system performance in order to proceed with the remainder of the DSES. A baseline solution with a corresponding deviation of zero, for example, indicates that the performance targets have all been met and there is no need to search for improved solutions in the expanded material design space. That is, the target product performance can exactly be achieved using existing materials, and there is no need to design a new material. If the deviation at the baseline solution has not been minimized to zero, then one proceeds with the next stage of the DSES: expansion.

Stage 2: Expansion

In the expansion stage of the DSES the objective is to find a design solution in an expanded design space in which the material properties are independent design variables (see Figure 3.1). The purpose of this stage is to determine the best product design that is achievable with a new material. Like the baseline stage, the expansion stage consists of two main steps: defining the expanded material design space (Step 2.1) and formulating and solving the expansion cDSP (Step 2.2).

Having defined the baseline material(s) previously in the baseline stage, the material design space is *expanded* in this stage by identifying bounds for the material property design variables (Step 2.1). A graphical representation of a sample expanded material design space in two material dimensions (density, ρ, and strength, σ_Y) is shown in Figure 3.7.

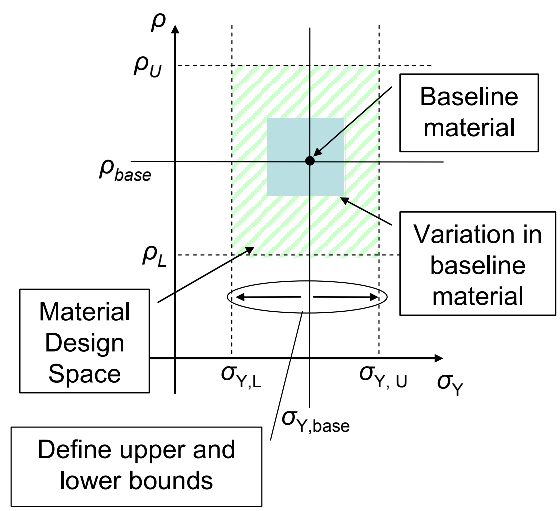


FIGURE 3.7. A SAMPLE EXPANDED MATERIAL DESIGN SPACE

The material property bounds are determined in the same manner as bounds for any other design variables; that is, upper and lower limits are selected for the design variables based on the limitations of performance models and designer knowledge of where the design solution is likely to occur. For example, if a designer is using a deflection model that is valid for metals, it is appropriate to select bounds for the material properties that represent the ranges of properties of existing metals. This is because materials represented by properties outside of these ranges are unlikely to exhibit the same stress-strain behavior as metals, and therefore the deflection model would not be accurate outside these ranges. By identifying upper and lower bounds for the material property design variables, the expanded material design space is fully defined. The next step in the expansion stage is to formulate and solve the expansion cDSP (Step 2.2).

Similar to the formulation of the baseline cDSP, there are three milestones in the formulation and solution of the expansion cDSP: the word formulation (Step 2.2.1), the math formulation (Step 2.2.2) and the expansion solution (Step 2.2.3). However, a designer can travel much faster on the road to the expansion cDSP because only a few changes are needed in the word and math formulations of the expansion cDSP relative to the baseline cDSP. This idea is shown graphically in Figure 3.8. In the figure, the designer has traded his bicycle in for a race car to reflect that he can travel more quickly along this road since much of the work involved in the formulation of the expansion cDSP has already been completed in the baseline stage.

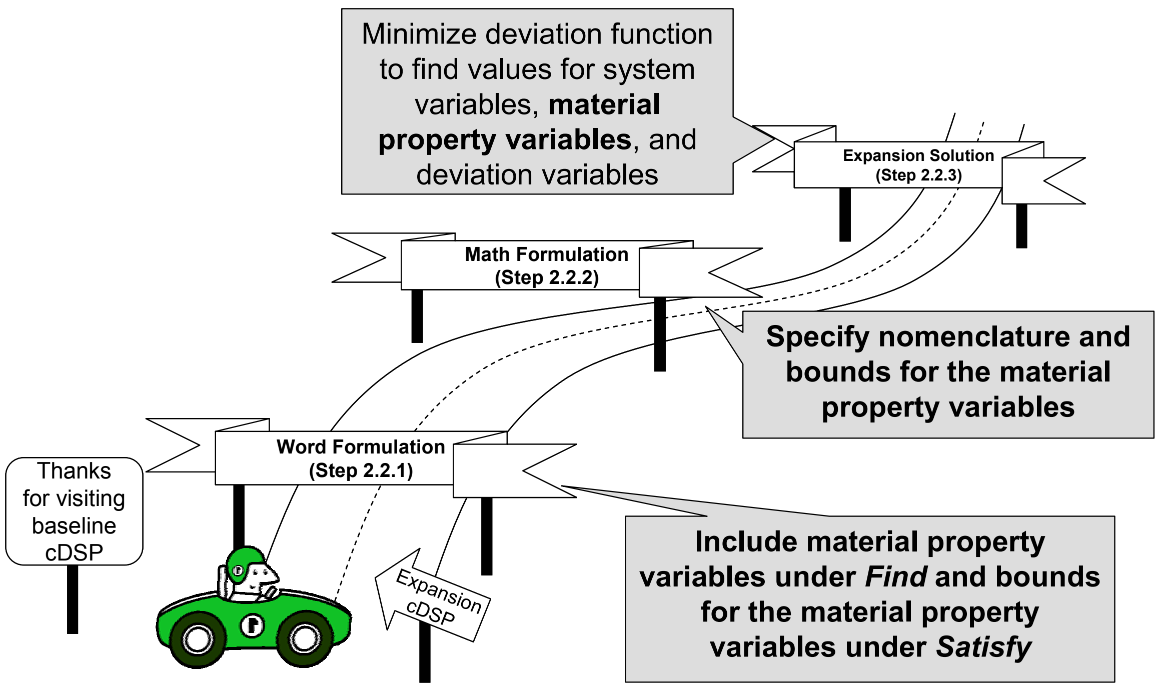


FIGURE 3.8. THE ROAD TO THE EXPANSION SOLUTION

The changes in the expansion cDSP relative to the baseline cDSP are located under the *Find* and *Satisfy* headings. Under the *Find* heading, the values of the material property variables are listed in addition to the system variables. Also, under the *Satisfy* heading, it is specified that the design solution must satisfy the material property bounds in addition to the system variable bounds and the bounds on the deviation variables. The word formulation for the expansion cDSP is shown in Figure 3.9, where the changes relative to the baseline cDSP are shown in boldface type.

Given

- An alternative to be improved through modification.
- Assumptions used to model the domain of interest.
- Deviation of the design variables.
- Target ranges of performance measures.
- The system parameters (fixed variables).
    - The properties of the baseline material(s).
    - The variation in the properties of the baseline material(s).
- The constraints and goals for the design.

Find

- The values (locations) of the ranged system variables.
- **The values (locations) of the material property variables.**
- The values of the deviation variables.

Satisfy

a. The system constraints that must be satisfied for the solution to be feasible, stated in terms of the design capability index, C_{dk}, which must be greater than or equal to one for the solution to be feasible.
b. The system goals that must achieve, to the extent possible, a specified target value. The system goals are stated in terms of the design capability index, C_{dk}, which must achieve, to the extent possible, the target value of one.
c. The CDSE goal that must achieve, to the extent possible, a specified target value. The CDSE goal is stated in terms of the design capability index, C_{dk}, which must achieve, to the extent possible, the target value of one.
d. The lower and upper bounds on the system variables **and material property variables** and bounds on the deviation variables.

Minimize

The deviation function that is a measure of the deviation of the system performance from that implied by the set of goals and their associated priority levels or relative weights.

FIGURE 3.9. WORD FORMULATION OF THE EXPANSION CDSP

The move to the math formulation from this word formulation in Step 2.2.2 requires the same additions under the *Find* and *Satisfy* headings, except that the values of the material property variables and the bounds for the material properties are given mathematical identifiers in the math form. The math formulation of the expansion cDSP is shown in Figure 3.10, where the boldface type indicates a change in the expansion cDSP relative to the baseline cDSP.

Given

- An alternative to be improved through modification.
- Assumptions used to model the domain of interest.
- Values of the baseline material properties, μMP_k. $k = 1, \ldots, s$
- Variation in the material properties, ΔMP_k.
- Variation of design variables, ΔX_i
- Target ranges of performance measures. URL_i and LRL_i $i = 1, \ldots, m$
- The system parameters (fixed variables).

n	number of system variables
s	number of baseline material properties
$p + q$	number of system constraints
p	equality constraints
q	inequality constraints
m	number of system goals
$g_i(\underline{X})$	system constraint function
W_i	weight for the Archimedean case

Find

- The values (locations) of the ranged system variables **and the material property variables**
    - $\boldsymbol{\underline{\mu}_X = \mu_{X1}, \ldots, \mu_{Xj}}$ **$j = 1, \ldots, n + s$**
- The values of the deviation variables
    - d_i^-, d_i^+ $i = 1, \ldots, m$

Satisfy

a. The system constraints that must be satisfied for the solution to be feasible.

$g_r(\underline{X}) = 0$ $r = 1, \ldots, p$

$g_r(\underline{X}) \geq 0$ $r = p+1, \ldots, p+q$

$C_{dk\text{-}constraints} \geq 1$

b. The system goals that must achieve, to the extent possible, a specified target value.

$A_i(\underline{X}) + d_i^- - d_i^+ = G_i;$ $i = 1, \ldots, m$

$C_{dk\text{-}objectives} + d_i^- - d_i^+ = 1$

c. The CDSE goal that must achieve, to the extent possible, a specified target value.

$A_{CDSE}(\underline{X}) + d_{CDSE}^- - d_{CDSE}^+ = G_{CDSE};$

$C_{dk\text{-}obj-CDSE} + d_i^- - d_i^+ = 1$

d. The lower and upper bounds on the system variables **and material property variables** and bounds on the deviation variables.

$\boldsymbol{X_j^{min} \leq X_j \leq X_j^{max}}$ **$j = 1, \ldots, n + s$**

$d_i^-, d_i^+ \geq 0$ and $d_i^- \cdot d_i^+ = 0$

Minimize

The deviation function (a measure of the deviation of the system performance from that implied by the set of goals and their associated priority levels or relative weights):

$$Z = \sum_{i=1}^{m} W_i \left(d_i^- + d_i^+ \right); \quad \sum W_i = 1; \quad W_i \geq 0 \qquad i = 1, \ldots, m$$

FIGURE 3.10. MATHEMATICAL FORMULATION OF THE EXPANSION CDSP

Having reached the second milestone on the road to the expansion solution, the next task (Step 2.2.3) is to find the expansion solution by minimizing the deviation function. It is possible that the same minimization algorithm may be used to find the expansion solution as was used to find the baseline solution; however, the addition of design variables may necessitate a change in solution strategy. For example, it may be possible to perform an exhaustive search of the baseline design space to find the baseline solution, but when additional variables are added in the expansion cDSP, the design space may be too large for an exhaustive search to be feasible. Once the expansion solution has been found, it must be verified as before in the baseline stage. That is, the solution must be verified with respect to the correctness of the minimization results as well as the physical meaning of the solution. After verifying the expansion solution, the next step is to proceed to the third stage of the DSES: Interpretation.

Stage 3: Interpretation

A designer has two objectives in the interpretation stage of the DSES (see Figure 3.1): to compare the baseline and expansion solutions using the VDSE (Step 3.1), and to gain insight into the material property targets of the product by analyzing the expansion solution (Step 3.2). The VDSE metric and the use of the metric are discussed in Section 3.2.1. The identification of material property targets is discussed here.

The material property targets of a product are the ranges of material properties exhibited in the expansion solution. Since DCIs are used in the formulation of the expansion cDSP, the values of the material property variables represent the location of the middle of the range of possible material properties and not a fixed point. Any value within this range is a feasible value for that particular property. These ranged targets can then be used to search material databases (in Step 3.2.1) to determine if there are existing materials that meet the targets, negating the need to design a new material. If an existing material is not found that meets the target range, then a new material must be designed that meets the target range.

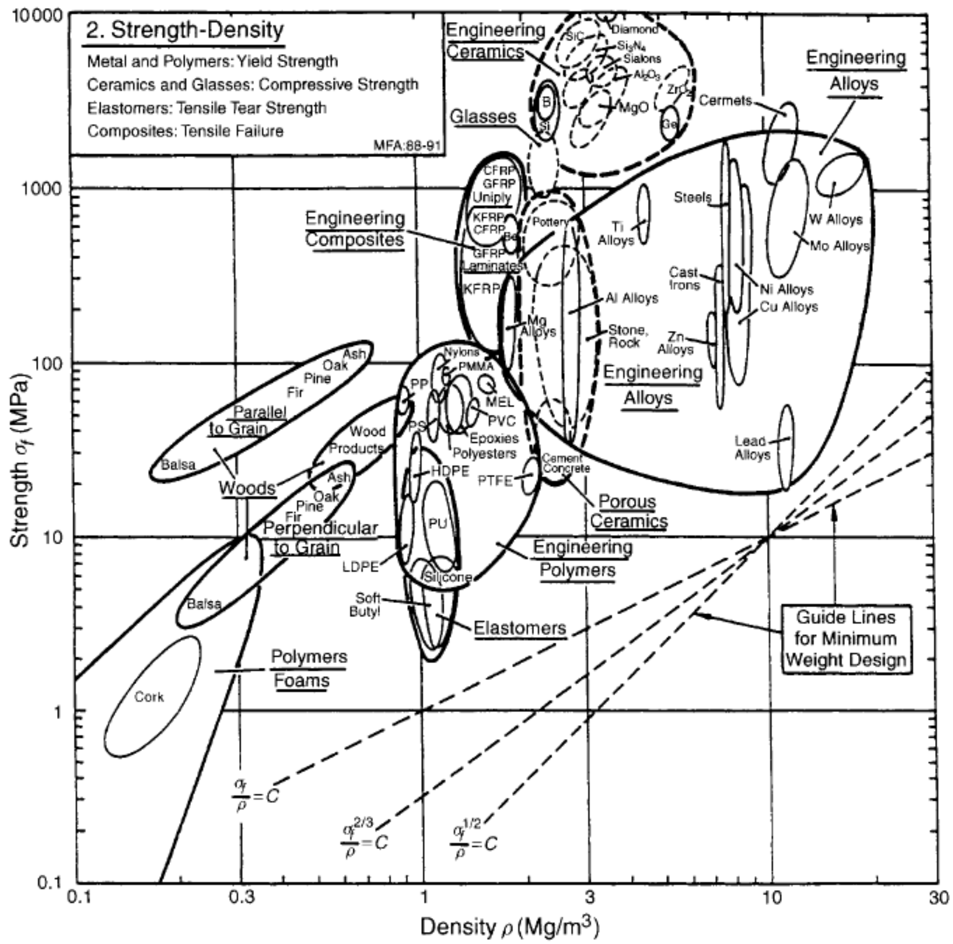


FIGURE 3.11. MATERIAL SELECTION CHART OF STRENGTH VERSUS DENSITY (ASHBY, 1999)

Designers can gain further insight into the targets (in Step 3.2.2) by locating the ranges on a material selection chart, such as the strength versus density chart shown in Figure 3.11. By locating the material property target ranges on a material selection chart, designers can determine what classes of materials meet the material property targets of the product. This knowledge of the classes of materials that meet the material property targets may help designers to identify materials to investigate and consider in subsequent phases of design. A designer may also determine that the material property targets that are identified fall outside of the accepted range of accuracy of the performance models of the product. In this case, the material property targets should not be used for subsequent phases of design unless they are verified using performance models that are accurate in that range. For example, if a designer is using a performance model that is accurate for metals but finds material property targets that fall in the range of ceramics, then the performance of the expansion solution must be verified using a model that is appropriate for ceramics before the material property targets can be used for design or selection. To summarize, material property targets are only valid if the performance model assumptions hold true for the classes of materials that the targets represent.

The calculation of the VDSE metric (Step 3.1) and the identification of material property targets (Step 3.2) are the last steps of the DSES. By calculating the VDSE, a designer is able to determine if the performance improvement realizable by expanding the material design space justifies the additional design cost of designing a material. Furthermore, by identifying material property targets, a designer is able to gain insight into the preferred material properties for the selection or design of a material in subsequent phases of the product design. In the next section, the requirements of the DSES are recalled and an assessment is made of whether or not the proposed strategy meets the requirements.

3.1.2 Meeting the Requirements of the DSES

In Section 1.3.1, requirements are identified for a strategy for choosing between material selection and material design. In the preceding section, the DSES is proposed to meet those requirements. Having proposed the strategy, it must now be determined if the DSES in fact meets the requirements.

The identification of the requirements for the DSES is a necessary step in determining the domain of application of the DSES for validation purposes. The requirements for the

DSES are identified in Section 1.3.1 in Table 1.1. A discussion of the strengths and limitations of the DSES in meeting these requirements follows.

The first requirement of the DSES is that it be a top-down process that is driven by product (system) requirements rather than material requirements. This requirement is met in the DSES because the baseline and expansion cDSPs are formulated based on the product requirements. The material property targets that are identified in the third stage of the DSES can be thought of as the material "requirements" for subsequent phases of design, and these targets are an output of this strategy rather than an input. Thus, the product requirements are the inputs to the DSES and the material requirements are the output of the strategy, which meets the top-down, product-driven nature of the first requirement.

The second requirement of the DSES is that it must be computationally inexpensive as compared to designing a new material. The requirement is met by expanding the material design space and treating the material properties as design variables. The material properties are viewed as quantities that a designer can control, rather than an output of a complex material model or simulation. By viewing the material properties as design variables, an efficient search of the material design space is enabled. This allows designers to determine if there is the potential for enough product performance improvement to justify the cost of developing, validating, and running the complex material models needed for design. Since the material is defined by independent material properties, no complex multiscale material models are needed for the comparison of material selection and design. This assumption undoubtedly ignores many complex material phenomena that may impact the design solution; however, this strategy is intended for use in the early stages of design and is not expected to reveal a "final answer". The complex multiscale material models necessary for material design as well as additional material behavior which are important in both material selection and design must be considered in subsequent stages of design, as more design knowledge is created.

The third requirement of the DSES is that the options of material selection and material design must be compared on the basis of meeting product design objectives as well as process design objectives. Product design objectives are the objectives that pertain to the performance of the product design, such as reducing deflection or lowering the mass of the system. The process design objective at hand in the choice between material selection and material design is to reduce the cost of design or the length of the design timeline. From the process design point of view, material selection is the preferred choice because it is much cheaper to select the best existing material than it is to develop predictive models and run complex simulations in order to design a new material. However, from the product design point of view, material design is the preferred choice because the material properties can be tailored such that the performance of the product is greatly improved. The DSES enables the consideration of both of these viewpoints by including the CDSE goal in addition to the system goals in the formulation of the baseline and expansion cDSPs. The baseline solution then represents the option of material selection, where the CDSE is at a minimum but the system goals may not be met, and the expansion solution represents the option of material design, where the CDSE is larger but the system goals are closer to being met. The deviation function values at the baseline and expansion solutions are thus representative of how well each option (material selection or material design) meets the combination of system and process objectives.

The last requirement of the DSES is that it must enable the identification of solutions which provide guidance for subsequent phases of design. This requirement is met by the calculation of the VDSE metric and the identification of material property targets in the interpretation stage of the DSES. Here guidance is provided for two different issues: whether to select or design a material, and which property values characterize the preferred material that is to be designed or selected. The VDSE metric indicates if there is an improvement in the achievement of product and process design goals that justifies the expense of designing a material. The value of the metric is used to decide whether to select or design. The identification of material property targets refers to the second issue. That is, by identifying material property targets, designers get a feel for the property values that the preferred material should exhibit. If a material does not already exist that meets the targets, then this information can be used as the targets or requirements to design a new material.

Having reviewed the requirements of the DSES, it is determined that the proposed strategy does in fact meet the requirements listed in Table 3.2; however, the strategy itself hinges upon the definition and application of the VDSE metric. In the next section, the VDSE metric is introduced and discussed.

3.2 THE VALUE OF DESIGN SPACE EXPANSION METRIC

As a critical part of the DSES, the VDSE metric is used to provide a quantitative comparison between material selection and material design. The metric is introduced and its use is discussed in Section 3.2.1. In Section 3.2.2 an assessment is made of how well the VDSE metric proposed here meets the requirements identified in Chapter 2.

3.2.1 Introducing the Value of Design Space Expansion Metric

The value of design space expansion (VDSE) is defined in words as *the improvement in the achievement of design goals at the expansion solution relative to the achievement of design goals at the baseline solution*. The mathematical definition of the VDSE metric is presented in equation 3.1 where $Z(\boldsymbol{x}_{base})$ is the value of the deviation function at the baseline solution and $Z(\boldsymbol{x}_{exp})$ is the value of the deviation function at the expansion solution.

$$VDSE = Z\left(\mathbf{x}_{base}\right) - Z\left(\mathbf{x}_{exp}\right) \tag{3.1}$$

Since smaller values of deviation are preferred, the deviation at the expansion solution is subtracted from the deviation at the baseline solution so that a positive VDSE indicates an improvement in the achievement of design goals in the expanded design space. Specifically, a positive value for VDSE indicates that the total deviation at the expansion solution is smaller than the total deviation at the baseline solution. This occurs when the improvement in the achievement of the system goals outweighs the deterioration in the achievement of the CDSE goal. Conversely, if the deterioration in the achievement of the CDSE goal outweighs the improvement in the achievement of the system goals, the total deviation at the expansion solution is larger than the total deviation at the baseline solution resulting in a negative VDSE.

The VDSE is implemented in the DSES in the interpretation stage (Step 3.1 in Figure 3.1). Having found both the baseline and expansion solutions in the first two stages, a designer can calculate the VDSE in the interpretation stage to quantify the improvement in the achievement of design goals by expanding the design space. A positive VDSE indicates that expanding the design space shows promise for improving the performance of the product with an acceptable increase in CDSE; hence, material design should be pursued. In this case material property targets should be identified to guide the material design and verify that a material does not already exist that meets the targets. A negative VDSE indicates that expanding the design space is too costly and material selection should be performed instead. In this case the baseline solution is used to determine the values of the system variables for subsequent phases of design.

A designer need not make the decision between selection and design solely on the value of the dimensionless VDSE metric. A designer may link the value of the dimensionless VDSE metric to the achievement of design goals by connecting the baseline and expansion solutions in the performance space. Based on the length and direction of the line connecting the baseline and expansion solutions, a designer can gain insight into the meaning of the VDSE metric in terms of the achievement of design goals. The length of the connecting line indicates the magnitude of the change in the achievement of design goals that is realized by expanding the design space. The direction of the connecting line indicates whether the change in achievement is an improvement or deterioration. A large magnitude change in which the expansion solution is closer to the targets than the baseline solution indicates that expanding the material design space enables significant improvements in the achievement of design goals. On the other hand, a small magnitude change or a change in the wrong direction indicates that expanding the material design space does not enable significant improvements in the achievement of design goals. Thus, by mapping the VDSE metric into the performance space, designers can gain insight into the meaning of the metric in practical terms.

3.2.2 Meeting the Requirements for the Value of Design Space Expansion Metric

Given the expanded material design space, and materials defined by independent material property variables, designers must have a metric with which to quantify the improvement in the achievement of design goals by expanding the design space. The value of design space expansion metric is proposed to meet this need. The requirements for the value of design space expansion metric are identified in Section 1.3.1 in Table 1.2. An assessment of how the proposed metric meets these needs is presented next.

The first requirement of the VDSE metric is that it enables the comparison of the use of an existing material to the use of an unrealized material concept. The VDSE metric meets this need by comparing the deviation at the baseline solution to the deviation at the expansion solution. The baseline solution represents the use of an existing material in the design of the product, while the expansion solution represents the use of an unrealized material concept. The second requirement of the VDSE metric is that it compares on the basis of meeting product-level requirements. The proposed metric meets this need because the total deviation represents how well the baseline and expansion solutions meet the overall product and process requirements and the VDSE metric provides a comparison of these two solutions on the basis of their total deviation values. Thus, the proposed metric does in fact meet the requirements identified in Section 1.3.1.

3.3 VALIDATION

This chapter fits into the validation strategy for this thesis in the assessment of structure validity. In Section 3.3.1, the domain-independent structure validity of the DSES and VDSE is discussed. The domain-specific structure validity of the DSES and the VDSE is then discussed in Section 3.3.2

3.3.1 Domain-Independent Structure Validity

Domain-independent structure validity refers to the internal consistency of the method and is determined by demonstrating the internal consistency of the parent constructs as well as the internal consistency of the integrated method as an assemblage of the parent constructs. This quadrant of the Validation Square is addressed partially in Chapter 2 via a critical review of the available literature to establish the internal consistency of the parent construct, the ranged cDSP. Given the internal consistency of the ranged cDSP (established in Section 2.2.2), the internal consistency of the DSES as an assemblage of two cDSPs is discussed in this section.

Is the integrated method internally consistent?

The DSES consists of two cDSPs and an interpretation stage which makes use of a value-of-information based metric. With the ranged cDSP as a parent construct, a design process objective and some given parameters are included in the baseline cDSP to particularize the formulation for the DSES. These two additions do not affect the internal consistency of the cDSP. With the ranged cDSP as a parent construct again, the number of design variables is increased in the expansion cDSP and the given parameters and design process objective are again included. An increase in the number of design variables also does not affect the internal consistency of the cDSP. Given two internally consistent cDSPs, the solutions are compared using the VDSE metric, which is based on the Improvement Potential, a value of information metric. Like the improvement potential metric, the VDSE compares design solutions from two design problem with slightly different formulations. In the case of the improvement potential the difference in the two problem formulations is the amount of uncertainty in some of the design parameters, whereas the difference in the two problem formulations in the VDSE metric is the number of design variables. This difference in the two metrics, although necessitating a new name for the type of metric (value of design space expansion rather than value of information), does not affect the structural validity of the metric.

Furthermore, the organization of the DSES into the three stages of baseline, expansion, and interpretation ensures that the information needed for each step of the strategy is available when needed. Specifically, the two cDSPs are solved prior to the calculation of the VDSE metric, which is a function of the total deviation at the baseline and expansion solutions. Also, the formulation of the expansion cDSP is built on the formulation of the baseline cDSP, requiring only minor changes; therefore, it is important to solve the baseline cDSP prior to solving the expansion cDSP. Considering its basis in the structurally valid cDSP and value of information metrics and the appropriate information flow through the strategy, it is argued that the DSES is internally consistent and structurally valid for the intended domain of application, which is for the support of decisions between material selection and material design in the context of product design.

3.3.2 Domain-Specific Structure Validity

Domain-specific structure validity refers to the appropriateness of the example problem for testing the proposed method. The example problems used to test the method must test the validity of the method in achieving all of the method requirements. In this section, Blast Resistant Panel (BRP) design scenarios are identified to test all aspects of the DSES.

<u>Introduction to BRPs</u>

BRPs are designed to absorb large amounts of energy per unit mass compared to solid plates. In one application of this design, BRPs can be attached on the outside of military vehicles to protect them from explosions. The BRPs considered in this example are sandwich structures that consist of two outer face sheets bonded to a square honeycomb core structure. Examples of blast resistant panels are shown in Figure 3.12.

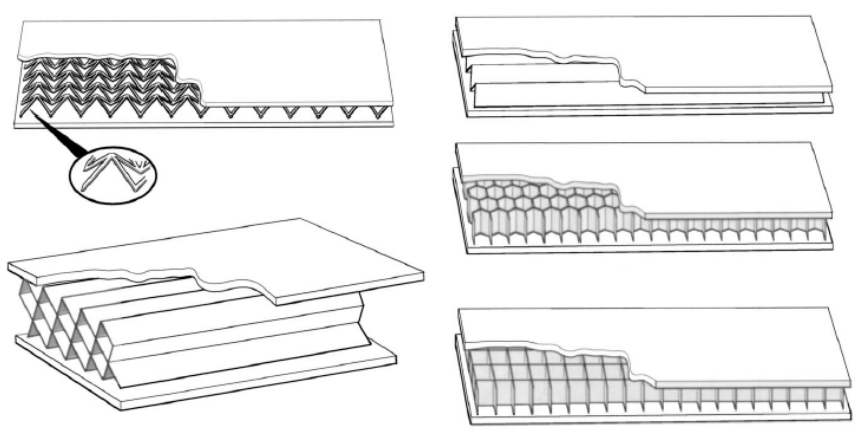


FIGURE 3.12. SAMPLE BLAST RESISTANT PANELS (FLECK AND DESHPANDE, 2004)

What are the requirements or expected capabilities of the DSES?

The DSES is intended to enable the comparison of material selection and material design as design process options in the context of a product design. The DSES is expected to be capable of handling the consideration of design alternatives incorporating multiple independent materials in addition to other system variables such as the geometry. The key assertion is that it is possible to simultaneously explore material solutions to the design problem along with solutions from another domain such as the structural solutions presented in the BRP example. The design decisions that are supported by the use of the DSES are ones that are made early in the design process, when the constraints and requirements of the design are likely to be uncertain and there is need to maintain design freedom in the solutions. Hence, the DSES must be capable of simultaneous exploration of solutions from multiple domains while generating ranged sets of solutions that are robust to variation in operating conditions, design variables and material properties.

Why is BRP design an appropriate example for testing these capabilities?

The design of BRPs is an appropriate example for the DSES because the BRP designer is not sure if the added cost of designing new materials is justified by the potential performance improvements. Specifically, design solutions may be possible with existing materials that meet the design objectives to some extent, but by designing new materials it may be possible to improve the achievement of the design objectives. To determine if material selection or material design is preferred in the design of BRPs, designers need a strategy for comparing the two design process options on the basis of how each option enables the achievement of design objectives. Furthermore, the conditions of early-stage BRP design conform closely to the intended domain of application of the DSES; i.e., simplified analysis models are available to predict BRP behavior, but the requirements of the BRP are uncertain and designers are unable to quantify the value of the design process options of material selection and material design. In addition, there are multiple objectives in the BRP example, and the trades between the achievements of these objectives are not immediately apparent.

What BRP design scenarios are necessary to test these capabilities?

The relationship between the BRP design scenarios and the design method capabilities they are intended to demonstrate is summarized in Table 3.2 along with a summary of the contributions of the examples to verification of the research hypotheses.

TABLE 3.2 BRP DESIGN SCENARIOS

Hyp.		Point	Ranged
	Comparison of material selection and design		
1	Expanding the material design space	x	x
2	Calculating the VDSE metric	x	x
3	Identifying material property targets	x	x
1,2,3	**Finding ranged sets of design solutions using DCIs**		
	Uncertainty in constraints and goals		x
	Uncertainty in material properties		x
	Uncertainty in noise factors		x

A Point Scenario and a Ranged Scenario are identified. In the Point Scenario, the constraints and goals in the cDSP are not formulated in terms of DCIs, and the variations in the material property variables and noise factors are ignored. In the Ranged Scenario the DSES is implemented precisely as described in this chapter, where the goals and constraints are formulated in terms of DCIs and ranged sets of solutions are sought to ranged specifications. The Point Scenario is not a precise application of the DSES, since the variations are ignored, but this scenario is investigated in order to gauge the impact of DCIs on the quantification of the value of design space expansion. Since all of the intended capabilities of the DSES are addressed by at least one of the design scenarios, the BRP example is appropriate for assessing the domain-specific performance validity of the DSES.

3.4 A LOOK BACK AND A LOOK AHEAD

In this chapter, the DSES and the VDSE metric are presented. The DSES and the VDSE metric are proposed to answer the research questions identified in Section 1.3.2. The DSES consists of three stages: baseline, expansion, and interpretation. In the baseline and expansion stages, cDSPs are formulated to find design solutions in two different design spaces. In the baseline cDSP a design solution is sought in the baseline design space in which the material properties are fixed. In the expansion cDSP a design solution is sought in the expanded design space in which the material properties are treated as variables. In the interpretation stage, the two design solutions are compared using the VDSE metric in order to gauge the value of the design space expansion. Material property targets are identified based on the expansion solution for guidance in subsequent phases of design. Because the DSES strategy is based on the cDSP and the VDSE metric is based on value of information metrics, both of which are internally consistent parent constructs, and because the augmentations to these constructs do not affect the internal consistency of the constructs, the DSES strategy is an internally consistent and thus structurally valid strategy. In Chapter 4, the DSES is implemented for the design of a blast resistant panel in order to assess the domain-specific validity of the strategy and to demonstrate the implementation of the method. Two scenarios are presented and the results are critically evaluated to test the capabilities of the DSES and the validity of the research hypotheses.

CHAPTER 4

BLAST RESISTANT PANEL DESIGN

In this chapter, the Design Space Expansion Strategy (DSES) is applied to an example problem on the design of blast resistant panels (BRPs) as part of the validation strategy for this thesis. The appropriateness of the BRP example for testing the design space expansion strategy is discussed in Section 3.3.2. The DSES consists of three stages: baseline, expansion, and interpretation. In the baseline stage a cDSP is formulated and solved to identify the best product design that is achievable with an existing material. In the baseline cDSP, the material design space is fixed at a point, and the system variables are allowed to vary between upper and lower bounds. In the expansion stage a cDSP is again formulated and solved; however, in this stage the aim is to determine the best product design that is achievable by designing a new material. In the interpretation stage, the value of design space expansion (VDSE) metric is calculated to determine the improvement of the expansion solution relative to the baseline solution. Depending on the value of the VDSE metric, a designer chooses either to pursue material design or material selection in the design of the product. Also in the interpretation stage, the values of material property design variables are studied to gain insight into the material property targets for subsequent phases of the product design.

A summary of this chapter is shown in Table 4.1. In Section 4.1, the baseline cDSP is discussed, including the formulation of the problem, the method of finding the solution, and the interpretation of the results. In Section 4.2, the expansion cDSP is discussed, also including the formulation of the problem, the method of finding the solution, and the interpretation of the results. In the discussion of the formulation of the expansion cDSP, an emphasis is placed on the differences between the baseline and expansion cDSPs.

TABLE 4.1. A SUMMARY OF CHAPTER 4

Section	Heading	Information
4.1	The Baseline cDSP	
4.1.1	Formulating the design problem	Baseline cDSP formulation • Defining the design space (Step 1.1) • Word formulation (Step 1.2.1) • Math formulation (Step 1.2.2)
4.1.2	Solving the design problem	Baseline cDSP solution (Step 1.2.3) • Solution method: exhaustive search • Presentation of solutions
4.1.3	Interpreting the design solution	Baseline cDSP interpretation • Verification of the optimization results • Discussion of the physical meaning of the solutions
4.2	The Expansion cDSP	
4.2.1	Formulating the design problem	Expansion cDSP formulation • Defining the design space (Step 2.1) • Word formulation (Step 2.2.1) • Math formulation (Step 2.2.2)
4.2.2	Solving the design problem	Expansion cDSP solution (Step 2.2.3) • Solution method: genetic algorithm • Conversion to classical unconstrained optimization problem • Application of the GA • Presentation of solutions
4.2.3	Interpreting the design solution	Expansion cDSP interpretation • Verification of the optimization results • Discussion of the physical meaning of the solutions
4.3	The VDSE metric and material property targets	
4.3.1	VDSE metric	Calculating and interpreting the VDSE metric (Step 3.1) • Calculation of the VDSE metric • Decision-making with the VDSE metric
4.3.2	Material property targets	Identifying material property targets for subsequent phases of design (Step 3.2)
4.4	Validation: domain-specific performance validity	
4.4.1	BRP Model Validity	Verification of the BRP analysis models
4.4.2	Assessing the usefulness of the DSES	The implications of the solutions to the BRP design example on the domain-specific performance validity of the DSES
4.5	A look back and a look ahead	

In Section 4.3, the VDSE metric and material property targets are discussed, including the calculation of the VDSE metric, the use of the VDSE metric in decision-making, and the identification of material property targets for subsequent phases of design. The implications of the results on the validity of the DSES are presented in Section 4.4. The chapter is then concluded in Section 4.5 with a look back and a look ahead.

4.1 THE BASELINE CDSP FOR BRP DESIGN

Formulating, solving, and interpreting the baseline cDSP is the first stage in the design space expansion strategy. The aim in solving the baseline cDSP is to identify the best possible performance of the system with no change in material properties. The formulation of the baseline cDSP is discussed in Section 4.1.1, including the BRP problem statement, the word formulation of the baseline cDSP, the derivation of the relevant equations, and the math formulation of the baseline cDSP. In Section 4.1.2, the method for finding the solution to the baseline cDSP is discussed, and the solution is presented. The solutions are analyzed in Section 4.1.3 and the physical meaning of the results is discussed.

4.1.1 Formulating the Baseline BRP Design Problem

The baseline cDSP is solved to determine the best design possible with an existing baseline material. In the following sections, the BRP problem statement is introduced and the word formulation of the cDSP is discussed. Then, the necessary equations are derived, and the mathematical formulation of the cDSP is presented.

BRP baseline problem statement

The goal in this design example is to design a BRP for minimum mass and deflection under an uncertain blast load and with uncertain material properties. The design variables include the thickness of each layer and the geometric parameters relating to the topology of the square honeycomb core. The panel must not exceed a maximum deflection and a maximum mass per unit area, and must not rupture or collapse under the blast loading. The panel is assumed to be clamped on all edges. There are two uncertain noise factors pertaining to the air blast received by the panel. These factors are the peak pressure of the incoming pulse and the characteristic time of the pulse.

The BRP consists of a front face sheet, cellular core, and back face sheet as shown in Figure 4.1. The front face sheet receives the pressure load from the blast. The topology of the core is designed to dissipate a majority of the impulse energy in crushing. The back face sheet provides additional protection from the blast as well as a means to confine the core collapse and absorb energy in stretching.

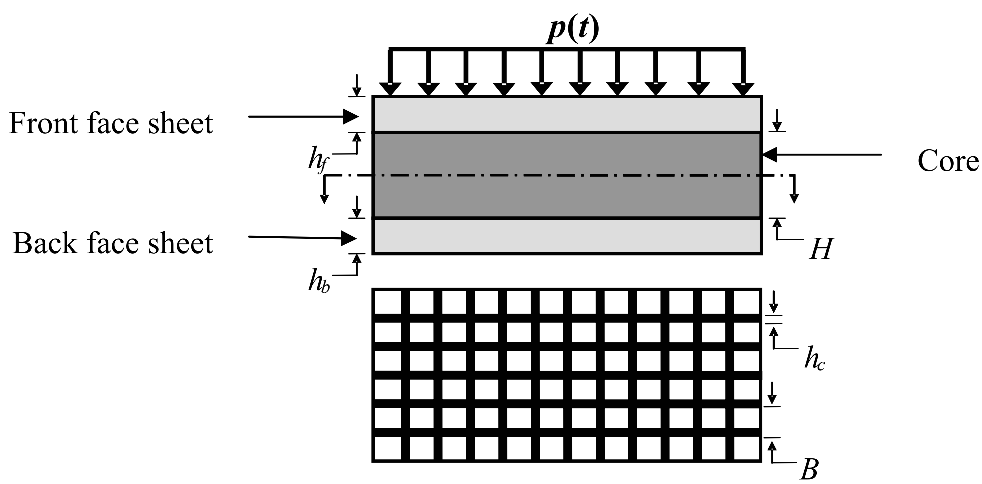


FIGURE 4.1. SCHEMATIC OF BRP STRUCTURE

Because blasts of different impulse amplitudes and duration can be expected, the design of the BRP should be robust to this uncertainty. In addition, due to variation in the materials of the BRP, the BRP should be robust to uncertainty in the material properties. The directionality of the blast, which may give rise to spatial gradients of pressure along the BRP surface, is not considered here.

In Section 3.3.2, two design scenarios are identified to test all aspects of the DSES. The two design scenarios differ in the consideration of the uncertainty in the design problem. In the Ranged Scenario, the uncertainty in the blast parameters and the material properties is considered, and the constraints and goals are formulated in terms of DCIs to find ranged sets of solutions that meet ranged performance requirements. In the Point Scenario, the uncertainty in blast parameters and material properties is ignored.

Defining the baseline material design space (Step 1.1 in Figure 3.1)

The first step of the baseline stage of the DSES is to define the baseline material design space. The first step in defining this design space is to identify baseline materials for each layer of the panel (Step 1.1.1). The baseline material is selected either through application of a material selection procedure or based on the expertise of the designer. The baseline materials for each layer of the panel are identified in Table 4.2. It is assumed that these materials are identified through application of material selection procedure; however, that process is not demonstrated here. These two materials are chosen for this example because steel is typically used in armor and magnesium has been applied in automotive applications to reduce weight.

TABLE 4.2. BASELINE MATERIALS FOR EACH LAYER OF THE BRP

Layer	**Baseline Material**
Front face sheet	Magnesium AZ31B-H24
Core	Steel AISI 1040
Back Face Sheet	Magnesium AZ31B-H24

The next step in defining the baseline material design space is to determine the amount of variability in the properties of the baseline materials due to differences in processing (Step 1.1.2). In the Point Scenario, all the uncertainty in the problem is ignored, and this step is omitted. A survey of the properties of available materials with similar material composition is conducted by consulting material databases and engineering handbooks (Avallone and Baumeister, 1996). A scatter plot of the variation in yield strength of the two baseline materials due to material processing is shown for each baseline material in (Shearouse, 1996; Paxton, 1996). From this plot, the amount of variation in yield strength is determined.

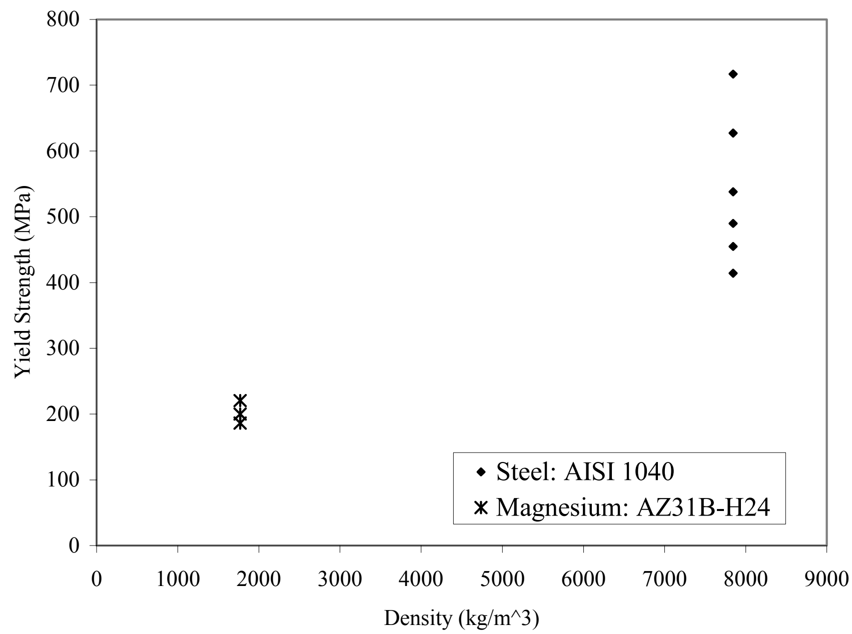


FIGURE 4.2. VARIATION IN DENSITY AND YIELD STRENGTH DUE TO MATERIAL PROCESSING (SHEAROUSE, 1996; PAXTON, 1996)

In Figure 4.3, a material selection chart of yield strength versus density is presented (Ashby, 1999). The amount of variation in density is estimated for both baseline materials by observing the variation in mass densities available in the magnesium and steel alloys on the strength versus density plot. The nominal values of density and yield strength and the amount of variation in density and yield strength for both baseline materials are summarized in Table 4.3.

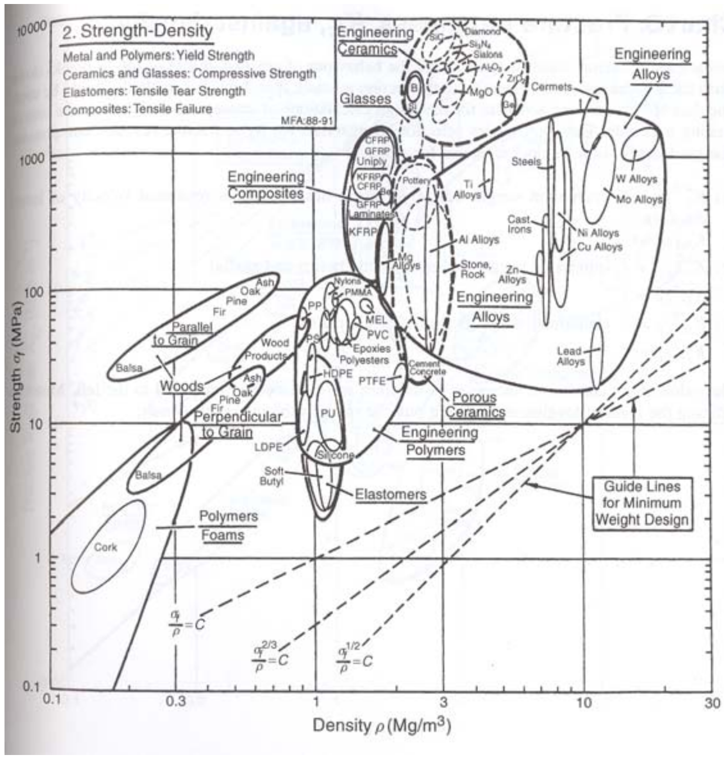


FIGURE 4.3. MATERIAL SELECTION CHART OF STRENGTH VERSUS DENSITY (ASHBY, 1999)

TABLE 4.3. SUMMARY OF MATERIAL PROPERTIES OF BASELINE MATERIALS (SHEAROUSE, 1996; PAXTON, 1996; ASHBY, 1999)

	Baseline		
Material Property	**Material Grade**	**Nominal Value**	**Variation**
Density (kg/m^3)	Steel AISI 1040	7845	±100
	Magnesium AZ31B-H24	1770	±170
Yield Strength (MPa)	Steel AISI 1040	538	±150
	Magnesium AZ31B-H24	200	±20

Word formulation of the baseline cDSP

The BRP problem statement is formalized as a word problem in the word formulation of the baseline cDSP (Step 1.2.1). Recall from Section 2.2.1 that the cDSP is formulated under the headings: given, find, satisfy, and minimize. *Given* a feasible alternative, *find* the values of the design variables and deviation variables which *satisfy* the constraints and bounds on the system and *minimize* the deviation from the system goals. The complete word formulation of the baseline cDSP is shown in the following tables. The formulation for the Ranged Scenario is shown in Table 4.4, and the formulation for the Point Scenario is shown in Table 4.5. An explanation of these word formulations is presented in the following sections.

The feasible alternative under the *given* heading is represented by the constant parameters and the models used to predict the performance of the design alternatives. The constant parameters include the properties of the baseline material, the peak pressure and characteristic time of the blast load, the variation of the uncertain load parameters, and the variation in the material properties. There are three objectives in the BRP example: minimize mass per area, minimize deflection and minimize the cost of design space expansion (CDSE). The first two objectives, relating to the mass and deflection of the panel, are the performance objectives in BRP design. The third objective, minimizing the CDSE, is included in this formulation as part of the DSES and relates to the design process objective of reducing design process complexity. By simultaneously seeking to minimize the CDSE while also achieving the system objectives of minimizing mass and deflection, the benefits of the design space expansion (smaller mass and deflection) are balanced by the cost of expanding the design space (increase in cost of design space expansion as solutions diverge from the baseline material).

TABLE 4.4. WORD FORMULATION OF THE BASELINE CDSP, RANGED SCENARIO

Given:
Baseline material properties An impulse load defined by peak pressure, p_0, and characteristic time, t_0 Mean and variation of loading parameters ($t_{0,}\, p_0$) Variation of material property design variables (ρ,σ_Y) Model for the deflection of the panel Model for the mass per area of the panel Model for the cost of design space expansion
Find:
Core geometry: h_c, B Height of each layer: h_f, h_b, H Deviation variables d_i^+, d_i^- (i = 1,2, 3)
Satisfy:
Constraints a) Mass/area of BRP must not exceed 150 kg/m^2 b) Deflection must not exceed 15 cm for specified boundary conditions c) Relative density must be greater than 0.07 to avoid buckling d) Front face shear-off parameter, μ , must not exceed 4/sqrt(3) e) Front face shear-off parameter, Γ, must not exceed 0.6 *Goals* a) The C_{dk} of mass per area is equal to or greater than the target value. b) The C_{dk} of deflection is equal to or greater than the target value. c) The C_{dk} of CDSE is equal to or greater than the target value. *Bounds* Upper and lower bounds for design variables
Minimize:
Archimedean deviation function

The objectives are converted into goals by specifying a target value. In the Point Scenario, the target values are all zero because the mass per area, deflection, and CDSE should all be minimized as close to zero as possible since none of these attributes can be negative. In the Ranged Scenario, the objectives are specified in terms of the design capability index C_{dk}, and the target for each of these objectives is one because a C_{dk} value of one indicates that the ranged goal is met by all of the designs represented by the ranged specification. A model is developed to predict the achievement of each of these goals.

TABLE 4.5. WORD FORMULATION OF THE BASELINE CDSP, POINT SCENARIO

Given:
Baseline material properties An impulse load defined by peak pressure, p_0, and characteristic time, t_0 Model for the deflection of the panel Model for the mass per area of the panel Model for the cost of design space expansion
Find:
Core geometry: h_c, B Height of each layer: h_f, h_b, H Deviation variables d_i^+, d_i^- (i = 1,2, 3)
Satisfy:
Constraints a) Mass/area of BRP must not exceed 150 kg/m^2 b) Deflection must not exceed 15 cm for specified boundary conditions c) Relative density must be greater than 0.07 to avoid buckling d) Front face shear-off parameter, μ , must not exceed 4/sqrt(3) e) Front face shear-off parameter, Γ, must not exceed 0.6 *Goals* a) The mass per area is equal to or less than the target value. b) The deflection is equal to or less than the target value. c) The CDSE is equal to or less than the target value. *Bounds* Upper and lower bounds for design variables
Minimize:
Archimedean deviation function

In the baseline cDSP, the material properties are fixed at the baseline material. The CDSE is determined by the distance of the material property variables away from the baseline material properties. Since the material properties are fixed in the baseline cDSP, the CDSE is also fixed. The objective of minimizing the cost of design space expansion is included in the baseline cDSP so that the design objectives will remain the same in both the baseline and expansion cDSP. It is necessary for the design objectives to remain the same in both cDSPs of the DSES so that the VDSE metric can be calculated, and by including the CDSE goal in the baseline cDSP, the changes in the formulation between the baseline and expansion cDSP are reduced. Also, the CDSE goal is included in the baseline cDSP for verification of the CDSE model. See Section 3.1.1 for more detail on the CDSE goal and its inclusion in the baseline cDSP.

The *find* heading of the cDSP refers to system variables and deviation variables. The design space of the BRP is defined by eleven variables which consist of six material property variables and five geometric variables. The five geometric system variables are used to define the height of each of the layers as well as the cell spacing and cell wall width of the square honeycomb core. Although these geometric variables may have associated uncertainty due to manufacturing variations, in this example these design variables are assumed to have no associated uncertainty, as this type of uncertainty is not pertinent to the demonstration and testing of the DSES. The *material* design space of the BRP is defined by the six material property variables consisting of independent yield strength and mass density variables for each of the three layers of the panel. The materials for each layer of the panel are assumed to have an elastic, perfectly-plastic stress-strain relationship. In the baseline cDSP, the properties for the material property variables are held constant, leaving only five independent system variables.

In addition to the five system variables in the baseline cDSP, there are six deviation variables, including two deviation variables for each goal, denoted by d^- and d^+, respectively. Depending on the goal, one or both of these deviation variables may represent *unwanted* deviation from the target value. For example, if the objective is to maximize a quantity to the target value, the underachievement variable, d^-, is the unwanted deviation, while the overachievement variable, d^+, is acceptable deviation. The unwanted deviation variables are used in the deviation function under the *minimize* heading, which is discussed later.

The design solution to the BRP problem must *satisfy* constraints, bounds, and goals. Constraints are imposed on the BRP design to limit mass and deflection, and prohibit failure. The maximum mass per unit area of the BRP is limited to 150 kg/m^2. In addition, the maximum allowable deflection of the back face sheet of the BRP is limited to 15 cm. A minimum relative density constraint is imposed to ensure crushing of the honeycomb core rather than buckling, which would not dissipate as much energy. Two constraints are imposed to avoid shear failure of the front face sheet (Hutchinson and Xue, 2005). The first criterion, denoted by Γ, prohibits shearing of the face sheet at the clamped ends of the plate, and the second criterion, denoted by μ, prohibits shearing of the face sheet at the core webs. In addition to the constraints, the design solution must satisfy the bounds on the design variables.

Although there are constraints on the mass and deflection of the BRP, it is desirable to achieve a panel that is as light as possible and as resistant to deflection as possible; therefore the objectives of minimizing the mass and deflection are included in the formulation of the design problem in addition to the constraints on these values. Constraints are demands, or design requirements that must be met at all costs, while goals are wishes, or design requirements that should be considered whenever possible. Accordingly, the maximum mass of 150 kg/m^2 and maximum deflection of 15 cm are demands, and are modeled as constraints, while the wishes of minimizing the mass and the deflection to zero are modeled as goals.

The final heading in the formulation of the cDSP is *minimize*, relating to the minimization of the deviation function. The deviation function is modeled in the

Archimedean form as a weighted sum of the unwanted deviation from each of the three goals. Next, the word formulation of the baseline cDSP is converted to the mathematical formulation of the baseline cDSP by deriving the equations for constraints and goals in the following sections.

Deriving the equations for the constraints and goals in the baseline BRP problem

To convert the word formulation of the baseline cDSP into the math formulation, equations must be derived for the constraints and goals. Also, the bounds on the system variables must be specified. In this section the equations for the constraints and goals are introduced. First, equations are derived for the BRP goals, including the deflection and mass per area of the panel, and the CDSE. Then, additional equations are derived for the BRP constraints, including equations for the relative density of the core and for the front face shear parameters μ and Γ.

Equations for the deflection of the BRP are developed based on the analysis by Hutchinson and Xue (2005). Their analysis considered sandwich plates with square honeycomb cores and blast loads in both water and air. For this example, only impulses in air are considered. Following the work of Taylor, the impulse load is considered to be exponential in form with a time dependence where p_0 is the peak pressure and t_0 is the characteristic pulse time (Taylor, 1963). The impulse load acts perpendicular to the surface of the BRP and is uniformly distributed over the area of the plate. For deflection calculations, the plate is assumed to be fully clamped at both ends, of width $L/2$, and of infinite extent in the y-direction (Hutchinson and Xue, 2005).

The equations for deflection of the back face sheet developed by Hutchinson and Xue are extended here to allow for independent adjustment of heights and material properties in each layer. Following the three stage deformation theory, the impulse of the blast is received by the front face sheet and momentum is transferred in stage one (Fleck and Deshpande, 2004). The equation for kinetic energy per unit area at the end of stage one is shown in Equation 4.1. In stage two, some of the kinetic energy is dissipated through crushing of the core layer. The equation for the amount of kinetic energy per unit area at the end of stage two is shown in Equation 4.2. The crushing strain is used to determine the crushed height of the core layer and is derived by equating the plastic dissipation per unit area in the core to the loss of kinetic energy per unit area in stage two (Hutchinson and Xue, 2005). The crushing strain is shown in Equation 4.3. The variables in these equations are defined in the Nomenclature section in the introductory pages of this thesis.

$$KE_I = \frac{2p_0^2 t_0^2}{\left(\rho_f h_f\right)} \tag{4.1}$$

$$KE_{II} = \frac{2p_0^2 t_0^2}{\left(\rho_f h_f + \rho_c R_c H + \rho_b h_b\right)} \tag{4.2}$$

$$\overline{\varepsilon}_c = \frac{2p_0^2 t_0^2 \left(\rho_b h_b + \rho_c R_c H\right)}{\lambda_c R_c \sigma_{Y,c} H \rho_f h_f \left(\rho_f h_f + \rho_b h_b + \rho_c R_c H\right)} \tag{4.3}$$

In stage three, the remaining kinetic energy must be dissipated through bending and stretching of the back face sheet. The equation for deflection is derived by equating the remaining kinetic energy per unit area to the plastic work per unit area dissipated through bending and stretching. The average plastic work per unit area dissipated in stage three is estimated by summing the dissipation from bending and stretching, following the work of Hutchinson and Xue (2005). The equation for this estimate is shown in Equation 4.4. The equation for deflection is shown in Equation 4.5.

$$W_{III}^P = \frac{2}{3}\left[\sigma_{Y,f} h_f + \sigma_{Y,c} R_c H \lambda_S + \sigma_{Y,b} h_b\right]\left(\frac{\delta}{L}\right)^2 + 4\sigma_{Y,b} h_b \frac{\overline{H}}{L}\left(\frac{\delta}{L}\right) \tag{4.4}$$

$$\delta = \frac{-3\left(\sigma_{Y,b} h_b \bar{H}\right) \pm \sqrt{9\sigma_{Y,b}^2 h_b^2 \bar{H}^2 + \left(\frac{3I_0^2 L^2\left[\sigma_{Y,f} h_f + \sigma_{Y,c} R_c H \lambda_S + \sigma_{Y,b} h_b\right]}{\left(\rho_f h_f + \rho_c R_c H + \rho_b h_b\right)}\right)}}{\left(\left[\sigma_{Y,f} h_f + \sigma_{Y,c} R_c H \lambda_S + \sigma_{Y,b} h_b\right]\right)} \tag{4.5}$$

where $\bar{H} = H\left(1-\bar{\varepsilon}_c\right) = \left(H - \frac{2I_0^2\left(\rho_b h_b + \rho_c R_c H\right)}{\lambda_c R_c \sigma_{Y,c} \rho_f h_f\left(\rho_f h_f + \rho_b h_b + \rho_c R_c H\right)}\right)$

Equations for the variation in deflection are also needed in order to determine the sensitivity of the panel to variation in noise factors and uncertain design variables. The equation for the variation in deflection is derived as a first-order Taylor series expansion and is shown in Equation 4.6.

$$\Delta\delta = \left|\frac{\partial\delta}{\partial\sigma_{Y,b}}\right|\Delta\sigma_{Y,b} + \left|\frac{\partial\delta}{\partial\sigma_{Y,c}}\right|\Delta\sigma_{Y,c} + \left|\frac{\partial\delta}{\partial\sigma_{Y,f}}\right|\Delta\sigma_{Y,f} + \left|\frac{\partial\delta}{\partial\rho_b}\right|\Delta\rho_b + \left|\frac{\partial\delta}{\partial\rho_c}\right|\Delta\rho_c + \left|\frac{\partial\delta}{\partial\rho_f}\right|\Delta\rho_f + \left|\frac{\partial\delta}{\partial p_0}\right|\Delta p_0 + \left|\frac{\partial\delta}{\partial t_0}\right|\Delta t_0 \tag{4.6}$$

In addition to minimizing the panel deflection, it is desired to minimize mass per area of the panel. Hence, a model is needed for the mass per area of the panel as a function of the system variables. The mass per area of the panel is a function of only the geometry and the mass density of the materials, shown in Equation 4.7. The variation in mass per area as a function of the variation in material properties is shown in Equation 4.8.

$$M = \rho_f h_f + \rho_c H R_c + \rho_b h_b \tag{4.7}$$

$$\Delta M = \Delta\rho_f h_f + \Delta\rho_c H R_c + \Delta\rho_b h_b \tag{4.8}$$

A model for the CDSE is needed to quantify the expected increase in design cost due to deviations from the baseline materials. It is assumed that the design cost will increase as the candidate solutions move farther away from the baseline solutions due to the increased complexity in the models of the candidate material behavior. In the early stages of design, explicit data on the cost of expected design process options is not likely to be available; however, a CDSE function can be defined that reflects the beliefs of the designer about how the design cost will be affected by choosing material properties that differ from the baseline materials. Here, the cost function is defined to increase linearly with the Euclidean norm of the vector from the baseline material to the expansion solution, shown graphically in Figure 4.4. The CDSE is chosen to increase linearly here for simplicity, but a designer is free to choose other forms such as a quadratic or exponential model to represent his beliefs about how the CDSE will change as the expansion solutions diverge from the baseline material.

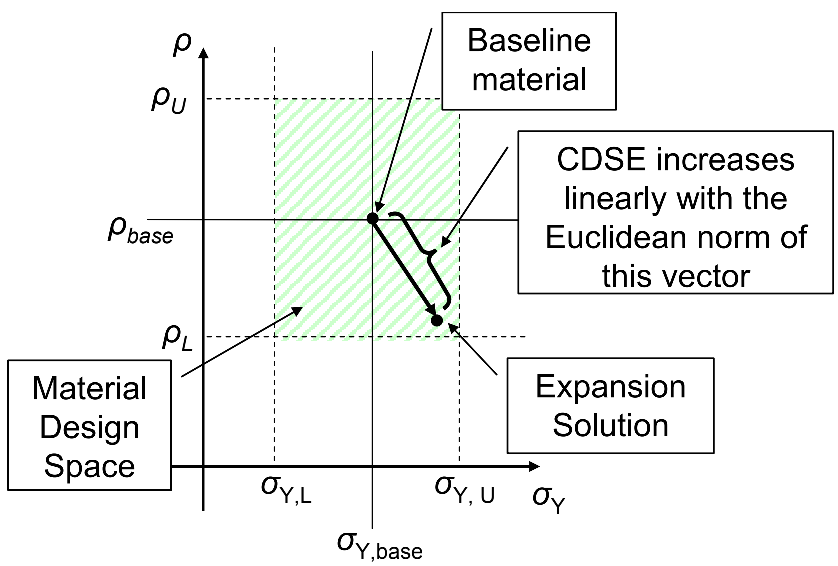


FIGURE 4.4. A GRAPHICAL REPRESENTATION OF THE CDSE FUNCTION

In addition, weighting coefficients are applied to certain material dimensions to represent the relative ease or difficulty in adjusting those properties, where a large coefficient indicates that it is difficult to achieve large variations in that material dimension. For instance, it is possible to achieve large variations in the yield strength of steel alloys through various heat treatments; however, it is not possible to achieve large variations in the density of steels because the constituent materials remain nearly the same for all steel alloys. Equation 4.9 is used to calculate a weighting coefficient for each material dimension as a function of the variation in each material dimension, where w_i is the weighting coefficient on material property design variable i, Δx_i is the variation in material property design variable i, and x_{bi} is the value of material property design variable i at the baseline material.

$$w_i = 1 - \frac{\Delta x_i}{x_{b,i}} \tag{4.9}$$

The equation for the weighting coefficient gives a value near one when the variation in the material dimension is a small fraction of the baseline material property, indicating that it is difficult to achieve large variations in that material dimension. Conversely, when the variation in the material dimension is a large fraction of the baseline value, the weighting coefficient approaches zero. This indicates that it is possible to achieve large variations in the material dimension.

An equation for the CDSE is shown in Equation 4.10 where $\boldsymbol{x}_{\mathrm{base}}$ is the vector of material property design variables at the baseline solution, $\boldsymbol{x}_{\mathrm{new}}$ is the vector of material property variables at the design point, and $\boldsymbol{w}$ is the vector of weighting factors (defined in Equation 4.9). In this example problem, CDSE is a dimensionless quantity intended to quantify the designer's expectation of the change in design costs as a result of diverging from the baseline materials. The change in CDSE due to variation in the uncertain factors is approximated with a 1st order Taylor series expansion, shown in equation 4.11. When the material property variables are equal to the baseline material properties, the CDSE is zero which represents no change in the cost of design because a new material does not need to be designed. The Taylor expansion of the CDSE is undefined when the CDSE is zero, so the change in CDSE at the baseline material properties is defined separately in equation 4.11.

$$CDSE\left(\mathbf{x}_{base},\mathbf{x}_{new},\mathbf{w}\right)=\sqrt{\sum_{i=1}^{n} w_i\left|\frac{x_{i,base}-x_{i,new}}{x_{i,base}}\right|} \tag{4.10}$$

$$\frac{dCDSE}{dx}=\begin{cases}\dfrac{1}{\sqrt{\text{CDSE}}}\displaystyle\sum_{i=1}^{n}\left|\frac{w_i\left(x_i-x_{b,i}\right)}{x_{b,i}}\right|\cdot\Delta\frac{x_i}{x_{b,i}} & \text{CDSE}\neq 0\\ \displaystyle\sum_{i=1}^{n}\frac{w_i\Delta x_i}{x_{b,i}} & \text{CDSE}=0\end{cases} \tag{4.11}$$

Having established equations for the BRP objectives, the equations for the BRP constraints are presented next, including equations for the relative density of the core and for the front face shear parameters μ and Γ.

The equation for the calculation of the relative density constraint is shown in Equation 4.12. The relative density of the core is not a function of uncertain factors, so no equation for the variation in this quantity due to variation in uncertain factors is needed.

$$\begin{aligned}&R_c=\left(2Bh_c-h_c^2\right)/B^2\\&R_c\geq 0.07\end{aligned} \tag{4.12}$$

The equations for Γ and μ are shown in Equations 4.13 and 4.15, respectively. The quantities are functions of uncertain factors, so equations are needed for the variation in these quantities due to the variation in the uncertain factors. The equations for the variations in Γ and μ are shown in Equations 4.14 and 4.16, respectively.

$$\Gamma=\left(2p_0t_0\Big/h_f\sqrt{\sigma_{Y,f}\rho_f}\right)\leq 0.6 \tag{4.13}$$

$$\Delta\Gamma=\left|\frac{2t_0}{h_f\sqrt{\sigma_{Y,f}\rho_f}}\right|\Delta p_0+\left|\frac{2p_0}{h_f\sqrt{\sigma_{Y,f}\rho_f}}\right|\Delta t_0+\left|\frac{-p_0t_0}{h_f\sqrt{\sigma_{Y,f}^3\rho_f}}\right|\Delta\sigma_{Y,f}+\left|\frac{-p_0t_0}{h_f\sqrt{\sigma_{Y,f}\rho_f^3}}\right|\Delta\rho_f \tag{4.14}$$

$$\mu=\frac{\rho_cHR_c}{\rho_fh_f}\leq\frac{4}{\sqrt{3}} \tag{4.15}$$

$$\Delta\mu = \left|\frac{-\rho_c H R_c}{\rho_f^2 h_f}\right| \Delta\rho_f + \left|\frac{H R_c}{\rho_f h_f}\right| \Delta\rho_c \tag{4.16}$$

For the Point Scenario of the BRP problem, the constraints are stated such that each of the constraints is expressed as a quantity which must be less than or equal to zero. The constraint equations for mass, deflection, relative density, Γ, and μ are shown in Equations 4.17-21, respectively.

$$\rho_f h_f + \rho_c H R_c + \rho_b h_b - 150 \text{ kg/m}^2 \leq 0 \tag{4.17}$$

$$\delta - 0.15 \text{ m} \leq 0 \tag{4.18}$$

$$0.07 - R_c \leq 0 \tag{4.19}$$

$$\left(2 p_0 t_0 \big/ h_f \sqrt{\sigma_{Y,f} \rho_f}\right) - 0.6 \leq 0 \tag{4.20}$$

$$\frac{\rho_c H R_c}{\rho_f h_f} - \frac{4}{\sqrt{3}} \leq 0 \tag{4.21}$$

For the Ranged Scenario of the BRP problem, the goals and constraints are expressed in terms of design capability indices (DCIs). A DCI value of one indicates that all the possible designs in the uncertain range of design variables meet the goal or constraint. The use of DCIs is reviewed in Section 2.3.3. The relative density of the core is a function of the geometric design variables B and h_c. Because there is assumed to be no uncertainty in these variables, the relative density is also deterministic. Therefore, it is not necessary or possible to convert the relative density constraint into DCI form.

With the exception of the relative density constraint, each of the constraints are specified as less than or equal to some maximum allowable value. The DCI for a less-than-or-equal-to constraint is calculated by the difference of the upper requirement limit (URL) and the mean value divided by the variation. Equations for the DCIs of the mass, deflection, Γ, and μ constraints are given in Equations 4.22, 4.23, 4.24, and 4.25.

$$C_{\text{dk-mass-constraint}} = \left(\frac{150 - \mu_{mass}}{\Delta M} \right) \tag{4.22}$$

$$C_{\text{dk-deflection-constraint}} = \left(\frac{0.15 - \mu_{deflection}}{\Delta \delta} \right) \tag{4.23}$$

$$C_{\text{dk-gamma-constraint}} = \left(\frac{0.6 - \mu_{gamma}}{\Delta \Gamma} \right) \tag{4.24}$$

$$C_{\text{dk-mu-constraint}} = \left(\frac{4/\sqrt{3} - \mu_{mu}}{\Delta \mu} \right) \tag{4.25}$$

The constraints for the ranged scenario are summarized in Table 4.6 along with the transformation into C_{dk}. The design capability index for each of the constraints must be greater than or equal to one to satisfy the design requirements.

TABLE 4.6. CONSTRAINTS ON THE BRP SYSTEM, RANGED SCENARIO

Constraint	≤, ≥	Value	C_{dk}	Eq#
Mass	≤	150 kg/m^2	$C_{\text{dk-mass-constraint}} = \left(\frac{150 - \mu_{mass}}{\Delta M} \right)$	(4.22)
Deflection	≤	15 cm	$C_{\text{dk-deflection-constraint}} = \left(\frac{0.15 - \mu_{deflection}}{\Delta \delta} \right)$	(4.23)
Relative Density	≥	0.07	$0.07 - R_c \le 0$	(4.19)
Front Face Shear - Γ	≤	0.6	$C_{\text{dk-gamma-constraint}} = \left(\frac{0.6 - \mu_{gamma}}{\Delta \Gamma} \right)$	(4.24)
Front Face Shear – μ	≤	$\frac{4}{\sqrt{3}}$	$C_{\text{dk-mu-constraint}} = \left(\frac{4/\sqrt{3} - \mu_{mu}}{\Delta \mu} \right)$	(4.25)

There are three objectives for the BRP system: minimize mass per area, minimize deflection and minimize the CDSE. These objectives are converted into goals by specifying target values and deviation variables. The specification of goal equations in the cDSP is reviewed in detail in Section 2.2.1. For the Point Scenario, the target value for each of the goals is zero. For non-zero minimization targets, the achievement of the objective is normalized by the target value in the goal equation. For a target of zero, this normalization procedure results in a division by zero. Instead of normalizing by the target value, the achievement of the objective is normalized by the estimated maximum possible value of the objective, resulting in deviation variables that vary between zero and one. These maximum values are estimated by discretizing the design space into a grid with three points for each design variable, and evaluating the objective at each point. These values are only estimates of the true maximum value of the objectives because the actual maximum value may occur at a point in the design space that is not captured by the coarse grid. The goals for the point scenario are shown in Equations 4.26-4.28. The estimates of the maximum values for each objective are shown in Table 4.7. Since the CDSE is constant at zero throughout the baseline design space, which would result in division by zero, the CDSE goal is normalized by one rather than the maximum value. The unwanted deviation for each of these goals is d_i^+, the overachievement of the objective.

$$\frac{M}{568.4} + d_{mass}^{-} - d_{mass}^{+} = 0 \tag{4.26}$$

$$\frac{\delta}{7.6} + d_{defl}^{-} - d_{defl}^{+} = 0 \tag{4.27}$$

$$\frac{CDSE}{1} + d_{CDSE}^{-} - d_{CDSE}^{+} = 0 \tag{4.28}$$

TABLE 4.7. ESTIMATED MAXIMUM VALUES OF OBJECTIVES IN THE BASELINE CDSP

	Max	Units
Mass	568.4	kg/m2
Deflection	7.6	m
CDSE	0	none

In the Ranged Scenario, the design objectives are specified in terms of the DCIs for each objective. The objectives for the Ranged Scenario are converted to DCI form in a similar manner to the constraints and are shown in Equations 4.29-4.31. To meet the target values specified in the table, the C_{dk} for each of the objectives must be maximized to be as close as possible to one. The goal equations for the ranged scenario are shown in Equations 4.32-4.34. The unwanted deviation for each of these goals is d_i^-, the underachievement of the objective. The objectives for the Ranged Scenario are summarized in Table 4.8 along with the transformation into C_{dk} and the formulation of the goal equation.

$$C_{\text{dk-mass-objective}} = \left(\frac{0 - \mu_{mass}}{\Delta M} \right) \tag{4.29}$$

$$C_{\text{dk-deflection-objective}} = \left(\frac{0 - \mu_{deflection}}{\Delta \delta} \right) \tag{4.30}$$

$$C_{\text{dk-CDSE-objective}} = \left(\frac{0 - \mu_{\text{CDSE}}}{\Delta \text{CDSE}} \right) \tag{4.31}$$

$$C_{\text{dk-mass-objective}} + d_{\text{mass}}^{-} - d_{\text{mass}}^{+} = 1 \tag{4.32}$$

$$C_{\text{dk-deflection-objective}} + d_{\text{deflection}}^{-} - d_{\text{deflection}}^{+} = 1 \tag{4.33}$$

$$C_{\text{dk-CDSE-objective}} + d_{\text{CDSE}}^{-} - d_{\text{CDSE}}^{+} = 1 \tag{4.34}$$

TABLE 4.8. OBJECTIVES CONVERTED TO GOALS IN TERMS OF DESIGN CAPABILITY INDICES

<table>
<tr><th colspan="2">Objective</th><th>Maximize Cdk of mass as close as possible to 1</th><th></th></tr>
<tr><td rowspan="2">Ranged</td><td>C_{dk}</td><td>$C_{\text{dk-mass-objective}} = \left(\frac{0-\mu_{mass}}{\Delta M}\right)$</td><td>(4.29)</td></tr>
<tr><td>Goal</td><td>$C_{\text{dk-mass-objective}} + d^{-}_{\text{mass}} - d^{+}_{\text{mass}} = 1$</td><td>(4.32)</td></tr>
<tr><th colspan="2">Objective</th><th>Maximize Cdk of deflection as close as possible to 1</th><th></th></tr>
<tr><td rowspan="2">Ranged</td><td>C_{dk}</td><td>$C_{\text{dk-deflection-objective}} = \left(\frac{0-\mu_{deflection}}{\Delta \delta}\right)$</td><td>(4.30)</td></tr>
<tr><td>Goal</td><td>$C_{\text{dk-deflection-objective}} + d^{-}_{\text{deflection}} - d^{+}_{\text{deflection}} = 1$</td><td>(4.33)</td></tr>
<tr><th colspan="2">Objective</th><th>Maximize Cdk of CDSE as close as possible to 1</th><th></th></tr>
<tr><td rowspan="2">Ranged</td><td>C_{dk}</td><td>$C_{\text{dk-CDSE-objective}} = \left(\frac{0-\mu_{\text{CDSE}}}{\Delta \text{CDSE}}\right)$</td><td>(4.31)</td></tr>
<tr><td>Goal</td><td>$C_{\text{dk-CDSE-objective}} + d^{-}_{\text{CDSE}} - d^{+}_{\text{CDSE}} = 1$</td><td>(4.34)</td></tr>
</table>

The final task in formulating the baseline cDSP is to model the preferences of the designer by the specification of a deviation function. The deviation function is specified in terms of a weighted sum of the deviation variables, which is termed an Archimedean formulation. Eleven Archimedean weighting schemes are identified in order to create plots of Pareto curves to assess the trades between goals. The Archimedean weightings are shown in Table 4.9. The Archimedean deviation function Z_A is shown for the point and ranged scenarios in Equations 4.35 and 4.36, respectively, where W_m, W_d and W_c are the weighting factors for the mass, deflection, and CDSE goals.

$$Z_{A,point}(\mathbf{x}) = W_{\text{m}}\left(d^{+}_{\text{mass}}\right) + W_{\text{d}}\left(d^{+}_{\text{defl}}\right) + W_{\text{c}}\left(d^{+}_{\text{CDSE}}\right) \tag{4.35}$$

$$Z_{A,ranged}(\mathbf{x}) = W_{\text{m}}\left(d^{-}_{\text{mass}}\right) + W_{\text{d}}\left(d^{-}_{\text{defl}}\right) + W_{\text{c}}\left(d^{-}_{\text{CDSE}}\right) \tag{4.36}$$

TABLE 4.9. ELEVEN ARCHIMEDEAN WEIGHTING SCHEMES

Scheme	Mass	Deflection	CDSE
1	1.00	0.00	0.00
2	0.00	1.00	0.00
3	0.00	0.00	1.00
4	0.60	0.20	0.20
5	0.20	0.60	0.20
6	0.20	0.20	0.60
7	0.20	0.40	0.40
8	0.40	0.20	0.40
9	0.40	0.40	0.20
10	0.33	0.33	0.33
11	0.50	0.50	0.00

Mathematical formulation of the baseline cDSP

The word formulation of the cDSP is converted into the mathematical formulation of the cDSP by the inclusion of the parameters, bounds, and equations specified in the preceding sections. The math formulation of the baseline cDSP for the Ranged Scenario is shown in Table 4.10, while the math formulation of the baseline cDSP for the Point Scenario is shown in Table 4.11. The next step is to minimize the deviation function to solve the baseline cDSP. This procedure is discussed in the following section.

TABLE 4.10. MATH FORMULATION OF THE BASELINE CDSP FOR THE RANGED SCENARIO

Given:		Eq#
A impulse load defined by peak pressure, p_0, and characteristic time, t_0	$\mu_t = 10^{-4}$ seconds, $\Delta t = 0.15 \cdot \mu_t$ $\mu_p = 25$ MPa, $\Delta p = 0.15 \cdot \mu_p$	
Material properties of the baseline materials	Faces: $\rho_{mg} = 1770$ kg/m^3, $\sigma_{Y,mg} = 200$ MPa Core: $\rho_{steel} = 7845$ kg/m^3, $\sigma_{Y,steel} = 538$ MPa	
Variation in material properties (ρ,σ_Y)	Faces: $\Delta\rho_{mg} = 170$ kg/m^3, $\Delta\sigma_{Y,mg} = 20$ MPa Core: $\Delta\rho_{steel} = 100$ kg/m^3, $\Delta\sigma_{Y,steel} = 150$ MPa	
BRP Deflection Model $\left[\delta, \Delta\delta, M, \Delta M\right] = f\left(\sigma_{Y,b}, \sigma_{Y,c}, \sigma_{Y,f}, \rho_b, \rho_c, \rho_f, p_0, t_0, B, H, h_c, h_f, h_b, L\right)$		(4.5)
Find:		
Core geometry:	h_c, B	
Height of each layer:	h_f, h_b, H	
Value of deviation variables	d_i^+, d_i^- (i = mass, defl, CDSE)	
Satisfy:		
Constraints	C_{dk}'s defined in Tables 4.6 and 4.8	
a) Mass/area of BRP must not exceed 150 kg/m^3	$C_{\text{dk-mass-constraint}} \geq 1$	(4.22)
b) Deflection must not exceed 15% of span for specified boundary conditions	$C_{\text{dk-deflection-constraint}} \geq 1$	(4.23)
c) Relative Density must be greater than 0.07 to avoid buckling	$R_c - 0.07 \geq 0$	(4.19)
Front face shear-off constraints		
d) Gamma must not exceed 0.6	$C_{\text{dk-gamma-constraint}} \geq 1$	(4.24)
e) Mu must not exceed 4/sqrt(3)	$C_{\text{dk-mu-constraint}} \geq 1$	(4.25)
Deviation variables must be greater than or equal to zero and multiply to zero	$d_i^+ \cdot d_i^- = 0,\; d_i^+, d_i^- \geq 0$ $\left(i = \text{mass, defl, CDSE}\right)$	
Goals		
a) The C_{dk} of mass per area is greater than or equal to the target of 1.	$C_{\text{dk-mass-objective}} + d_{\text{mass}}^- - d_{\text{mass}}^+ = 1$	(4.32)
b) The C_{dk} of deflection is greater than or equal to the target of 1.	$C_{\text{dk-deflection-objective}} + d_{\text{defl}}^- - d_{\text{defl}}^+ = 1$	(4.33)
c) The C_{dk} of CDSE is greater than or equal to the target of 1.	$C_{\text{dk-CDSE-objective}} + d_{\text{CDSE}}^- - d_{\text{CDSE}}^+ = 1$	(4.34)
Bounds (0.1 mm $\leq h_c \leq$ 1 cm) (5 mm $\leq H \leq$ 5 cm) (1 mm $\leq B \leq$ 2 cm) (1 mm $\leq h_f \leq$ 5 cm) (1 mm $\leq h_b \leq$ 5 cm)		
Minimize:		
Deviation Function: Archimedean $Z_{A,ranged}\left(\mathbf{x}\right) = W_m\left(d_{\text{mass}}^-\right) + W_d\left(d_{\text{defl}}^-\right) + W_c\left(d_{\text{CDSE}}^-\right);\; \sum W_i = 1$		(4.36)

TABLE 4.11. MATH FORMULATION OF THE BASELINE CDSP FOR THE POINT SCENARIO

		Eq#
Given:		
A impulse load defined by peak pressure, p_0, and characteristic time, t_0	$t_0 = 10^{-4}$ seconds, $p_0 = 25$ MPa	
Material properties of the baseline materials	Faces: $\rho_{mg} = 1770$ kg/m^3, $\sigma_{Y,mg} = 200$ MPa Core: $\rho_{steel} = 7845$ kg/m^3, $\sigma_{Y,steel} = 538$ MPa	
BRP Deflection Model	$[\delta, \Delta\delta, M, \Delta M] = f\left(\sigma_{Y,b}, \sigma_{Y,c}, \sigma_{Y,f}, \rho_b, \rho_c, \rho_f, p_0, t_0, B, H, h_c, h_f, h_b, L\right)$	(4.5)
Find:		
Core geometry:	h_c, B	
Height of each layer:	h_f, h_b, H	
Value of deviation variables	d_i^+, d_i^- (i = mass, defl, CDSE)	
Satisfy:		
Constraints		
a) Mass/area of BRP must not exceed 150 kg/m^3	$\rho_f h_f + \rho_c H R_c + \rho_b h_b - 150 \text{ kg/m}^2 \leq 0$	(4.17)
b) Deflection must not exceed 15% of span for specified boundary conditions	$\delta \leq 0.15$ m	(4.18)
c) Relative Density must be greater than 0.07 to avoid buckling	$0.07 - R_c \leq 0$	(4.19)
Front face shear-off constraints		
d) Gamma must not exceed 0.6	$\left(2 p_0 t_0 \big/ h_f \sqrt{\sigma_{Y,f} \rho_f}\right) - 0.6 \leq 0$	(4.20)
e) Mu must not exceed 4/sqrt(3)	$\frac{\rho_c H R_c}{\rho_f h_f} - \frac{4}{\sqrt{3}} \leq 0$	(4.21)
Deviation variables must be greater than or equal to zero and multiply to zero	$d_i^+ \cdot d_i^- = 0,\ d_i^+, d_i^- \geq 0$ $(i = \text{mass, defl, CDSE})$	
Goals		
a) The mass per area is less than or equal to the target value of zero.	$\frac{M}{568.4} + d_{mass}^- - d_{mass}^+ = 0$	(4.26)
b) The deflection is less than or equal to the target value of zero.	$\frac{\delta}{7.6} + d_{defl}^- - d_{defl}^+ = 0$	(4.27)
c) The CDSE is less than or equal to the target value of zero.	$\frac{CDSE}{1} + d_{CDSE}^- - d_{CDSE}^+ = 0$	(4.28)
Bounds (0.1 mm $\leq h_c \leq$ 1 cm) (5 mm $\leq H \leq$ 5 cm) (1 mm $\leq B \leq$ 2 cm) (1 mm $\leq h_f \leq$ 5 cm) (1 mm $\leq h_b \leq$ 5 cm)		
Minimize:		
Deviation Function: Archimedean	$Z_{A,point}(\mathbf{x}) = W_m\left(d_{mass}^+\right) + W_d\left(d_{defl}^+\right) + W_c\left(d_{CDSE}^+\right);\ \sum W_i = 1$	(4.35)

4.1.2 Solving the Baseline cDSP

With the problems formulated, the baseline solutions can now be found by minimizing the deviation function. Since there are only five system variables for the baseline cDSPs, an exhaustive search is performed. A grid of design points is created with sixteen points in each dimension, yielding 16^5 or 1,048,576 points to evaluate. The feasible points are first gleaned from the group by evaluating the constraints and eliminating the points which violate any of the constraints. Then the deviation variables are found for each point. This process is detailed in Appendix A. The deviation variables for the three goals are combined using the weighting factors for each scheme identified in Table 4.9 to obtain the Archimedean deviation function value. The design point with the lowest deviation function value is the solution. The system variables for the baseline solutions are shown in Table 4.12 and Table 4.13. The bold entries indicate that the particular design variable is at one of the bounds of that variable. The deviation variables for the baseline solutions are presented in Table 4.14 and Table 4.15. The solutions for weighting Scheme 10 are shown on the BRP schematic in Figure 4.5 and Figure 4.6. These solutions are discussed in the following section.

TABLE 4.12. BASELINE SYSTEM VARIABLES, POINT SCENARIO

Scheme	B	H	h_c	h_f	h_b
1	0.0187	0.0350	0.00076	0.0141	0.0173
2	**0.0200**	**0.0500**	0.00076	0.0173	**0.0500**
3	**0.0010**	**0.0050**	**0.00010**	0.0141	0.0337
4	**0.0200**	0.0470	0.00076	0.0141	0.0141
5	**0.0200**	0.0470	0.00076	0.0141	0.0141
6	**0.0200**	0.0470	0.00076	0.0141	0.0141
7	**0.0200**	0.0470	0.00076	0.0141	0.0141
8	**0.0200**	0.0470	0.00076	0.0141	0.0141
9	**0.0200**	0.0470	0.00076	0.0141	0.0141
10	**0.0200**	0.0470	0.00076	0.0141	0.0141
11	**0.0200**	0.0470	0.00076	0.0141	0.0141

TABLE 4.13. BASELINE SYSTEM VARIABLES, RANGED SCENARIO

Scheme	B	H	h_c	h_f	h_b
1	0.0023	**0.0050**	**0.00010**	0.0239	**0.0500**
2	**0.0200**	**0.0500**	0.00076	0.0206	0.0402
3	**0.0010**	**0.0050**	**0.00010**	0.0206	0.0467
4	**0.0200**	**0.0050**	0.00076	0.0239	**0.0500**
5	**0.0200**	0.0200	0.00076	0.0206	**0.0500**
6	**0.0200**	0.0080	0.00076	0.0206	**0.0500**
7	**0.0200**	0.0200	0.00076	0.0206	**0.0500**
8	**0.0200**	**0.0050**	0.00076	0.0239	**0.0500**
9	**0.0200**	0.0080	0.00076	0.0206	**0.0500**
10	**0.0200**	0.0080	0.00076	0.0206	**0.0500**
11	**0.0200**	0.0080	0.00076	0.0206	**0.0500**

TABLE 4.14. BASELINE DEVIATION VARIABLES, POINT SCENARIO

Scheme	d^-_{mass}	d^+_{mass}	$d^-_{\text{deflection}}$	$d^+_{\text{deflection}}$	d^-_{CDSE}	d^+_{CDSE}	Z_A
1	0	0.136	0	0.020	0	0	0.136
2	0	0.261	0	0.005	0	0	0.005
3	0	0.162	0	0.019	0	0	0.000
4	0	0.136	0	0.020	0	0	0.086
5	0	0.136	0	0.020	0	0	0.039
6	0	0.136	0	0.020	0	0	0.031
7	0	0.136	0	0.020	0	0	0.035
8	0	0.136	0	0.020	0	0	0.058
9	0	0.136	0	0.020	0	0	0.062
10	0	0.136	0	0.020	0	0	0.052
11	0	0.136	0	0.020	0	0	0.078

TABLE 4.15. BASELINE DEVIATION VARIABLES, RANGED SCENARIO

Scheme	d^-_{mass}	d^+_{mass}	$d^-_{\text{deflection}}$	$d^+_{\text{deflection}}$	d^-_{CDSE}	d^+_{CDSE}	Z_A
1	11.6	0	3.03	0	1	0	11.60
2	13.8	0	2.39	0	1	0	2.39
3	12.0	0	3.16	0	1	0	1.00
4	11.6	0	3.00	0	1	0	7.76
5	12.2	0	2.58	0	1	0	4.19
6	11.7	0	2.86	0	1	0	3.51
7	12.2	0	2.58	0	1	0	3.87
8	11.6	0	3.00	0	1	0	5.64
9	11.7	0	2.86	0	1	0	6.02
10	11.7	0	2.86	0	1	0	5.19
11	11.7	0	2.86	0	1	0	7.28

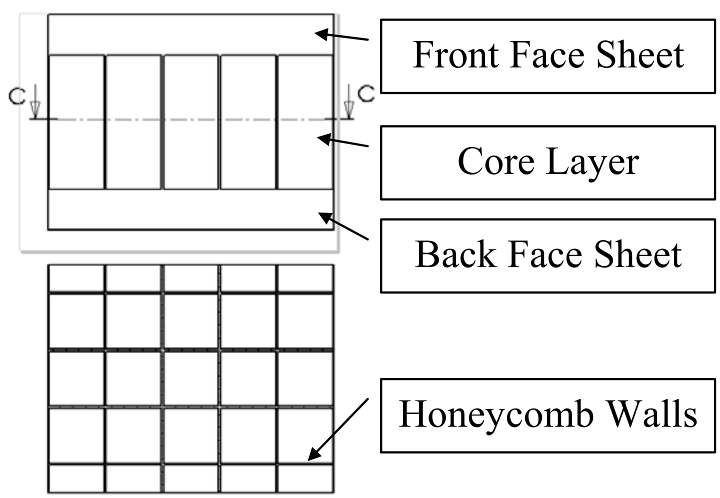


FIGURE 4.5. BRP SCHEMATIC OF THE SOLUTION TO THE POINT SCENARIO OF THE BASELINE CDSP, SCHEME 10

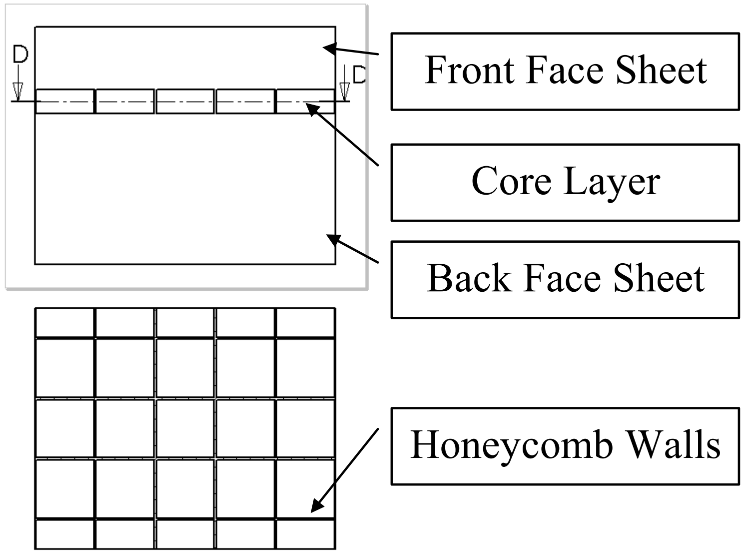


FIGURE 4.6. BRP SCHEMATIC OF THE SOLUTION TO THE RANGED SCENARIO OF THE BASELINE CDSP, SCHEME 10

4.1.3 Interpreting the Baseline Solution

The baseline cDSP is solved to identify the best design possible with the existing baseline material. A designer must consider the validity of the minimization results and the physical meaning of the results when interpreting the solutions. These two topics are discussed in the next two sections.

The minimization results for the baseline solution

A valid solution to the baseline cDSP is the global minimum of the design space. With an exhaustive search of the design space, the global minimum is guaranteed to be found, because each possible design is evaluated. There is the possibility that a better solution could be found by using a finer grid of points, but for the purpose of this example, it is assumed that these solutions are adequate. To build confidence in the solutions, the active constraints of each solution are investigated.

There are two types of constraints that can be active on the design solution: the design space bounds and the system constraints. In Table 4.12 and Table 4.13, the active bounds are denoted by the bold entries in the design variable tables. The constraints are shown for each solution to the Ranged Scenario of the baseline cDSP in Table 4.16. In Table 4.17 the constraints are shown for each solution to the Point Scenario of the baseline cDSP. Values less than or equal to zero indicate that the constraints are satisfied and the solutions are feasible. A value of zero for the constraint indicates that the particular constraint is active.

TABLE 4.16. CONSTRAINTS FOR THE BASELINE SOLUTION TO THE RANGED SCENARIO

Scheme	mass	deflection	R_c	Γ	μ
1	-0.27	-0.157	-0.0163	-0.775	-255.085
2	-0.21	-1.812	-0.0046	-0.183	-16.259
3	-1.00	-0.123	-0.1200	-0.183	-93.663
4	-0.27	-0.086	-0.0046	-0.775	-296.690
5	-0.07	-0.676	-0.0046	-0.183	-55.935
6	-0.65	-0.021	-0.0046	-0.183	-155.126
7	-0.07	-0.676	-0.0046	-0.183	-55.935
8	-0.27	-0.086	-0.0046	-0.775	-296.690
9	-0.65	-0.021	-0.0046	-0.183	-155.126
10	-0.65	-0.021	-0.0046	-0.183	-155.126
11	-0.65	-0.021	-0.0046	-0.183	-155.126

TABLE 4.17. CONSTRAINTS FOR THE BASELINE SOLUTION OF THE POINT SCENARIO

Scheme	mass	deflection	R_c	Γ	μ
1	-72.60	**0.000**	-0.0095	-0.004	-1.435
2	-2.00	-0.109	-0.0046	-0.114	-1.354
3	-58.10	-0.009	-0.1200	-0.004	-2.011
4	-72.70	-0.001	-0.0046	-0.004	-1.207
5	-72.70	-0.001	-0.0046	-0.004	-1.207
6	-72.70	-0.001	-0.0046	-0.004	-1.207
7	-72.70	-0.001	-0.0046	-0.004	-1.207
8	-72.70	-0.001	-0.0046	-0.004	-1.207
9	-72.70	-0.001	-0.0046	-0.004	-1.207
10	-72.70	-0.001	-0.0046	-0.004	-1.207
11	-72.70	-0.001	-0.0046	-0.004	-1.207

It can be seen that the constraints are in fact less than or equal to zero for all solutions; however, the relative density, deflection, mass and gamma constraints approach zero for many of the solutions. Also, the deflection constraint is equal to zero, and therefore active, in the solution to Scheme 1 in the Point Scenario. In both scenarios, the μ constraint has little effect on the solutions and does not approach zero. In the Ranged Scenario, the relative density constraint approaches zero for all schemes except Scheme 3. In Scheme 3 all the weight is placed on the CDSE goal, the achievement of which is the same for all feasible solutions to the baseline cDSP. The baseline solution to Scheme 3 is thus arbitrary, but is included here for the comparison to the expansion solution and to verify that the CDSE function is behaving as expected. In the Point Scenario, the relative density constraint again approaches zero for all schemes except Scheme 3. The magnitude of the mass constraint is very large for all schemes in the Point Scenario except Scheme 2, in which all the weight is placed on the minimizing deflection objective. This makes sense because a BRP with thicker face sheets would be more massive and more resistant to deflection, meaning that the deflection constraint should have a large magnitude and the mass constraint should have a small magnitude for Scheme 2, which is in fact the case.

The physical meaning of the baseline solution

To analyze the baseline design solutions, hypotheses are identified for each of the preference schemes and each scenario. These hypotheses and the corresponding results are displayed in Tables 4.18 through 4.22. By analyzing the results, it is confirmed that the CDSE goal has no effect on the baseline cDSP as is expected, since the CDSE is constant throughout the baseline design space. It is seen that the solutions for Schemes 4-11 of the Point Scenario are identical. This means that the baseline solution is insensitive to small changes in the weighting factors on the mass and deflection goals. Thus, the achievement of either the mass or deflection goal is dominating the deviation function. Referring to the values of the constraints in Table 4.17, it is seen that in Schemes 4-11 the deflection, Γ, and relative density constraints are nearly active, while the mass is more than 72 kg from the limit. Therefore, it can be concluded that the achievement of the minimum mass goal is dominating the deviation function in the Point Scenario.

In the solutions to the Ranged Scenario, there are distinct solutions for the three possible mass-deflection weighting combinations: mass weight greater than deflection weight, mass weight less than deflection weight, and mass weight equal to deflection weight. Thus, in the Ranged Scenario, neither the mass goal nor the deflection goal consistently dominates the deviation function.

TABLE 4.18. BASELINE SOLUTION ANALYSIS, SCHEMES 1-2

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
1	1.00	0.00	0.00	In this scheme the only objective is to minimize mass; therefore the layer heights and cell wall thicknesses are likely to be small, the cell spacing is likely to be large, and the deflection constraint is likely to be active.	Point	The front and back face sheet and cell wall thicknesses tend towards the lower bounds. The core layer height is closer to the upper bound, but is lower in this scheme than in the solutions to the other schemes. The deflection constraint is active.
					Ranged	The cell wall thickness and the core layer are at the lower bounds, but the front face sheet thickness is closer to the upper bound and the back face sheet thickness is at the upper bound. The deflection constraint is not active as it is expected to be.
2	0.00	1.00	0.00	In this scheme the only objective is to minimize deflection; therefore the layer and cell wall thicknesses are likely to be large and the mass constraint is likely to be active.	Point	The cell spacing, core layer height, and back face sheet thickness are at their upper bounds; however, the front face sheet thickness and the cell wall thickness tend toward the lower bounds. The mass constraint is not active, but is within 2 kg of the constraint.
					Ranged	The cell spacing and core layer height are at the upper bounds, and the back face sheet thickness is near the upper bound. The cell wall thickness is very near the lower bound, while the front face sheet thickness is just less than the midpoint. The mass constraint is not active.

TABLE 4.19. BASELINE SOLUTION ANALYSIS, SCHEMES 3-4

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
3	0.00	0.00	1.00	In this scheme the only objective is to minimize the CDSE. In the baseline cDSP, the CDSE goal is mathematically independent of the system variables; therefore the baseline solution for this scheme is arbitrary. This scheme is included for demonstration purposes in the expansion cDSP.	Point	No constraints are active, but the cell spacing and cell wall thickness variables are at their lower bounds while the core height variable is at its upper bound. This point is chosen because it is the first feasible point in the design space of the Point Scenario, according to the solution-finding method used in the baseline cDSP.
					Ranged	No constraints are active, but the cell spacing and cell wall thickness variables are at their lower bounds while the core height variable is at its upper bound. This point is chosen because it is the first feasible point in the design space of the Ranged Scenario, according to the solution-finding method used in the baseline cDSP.
4	0.60	0.20	0.20	In the baseline cDSP, the CDSE goal is mathematically independent of the system variables; therefore the solution is the same in this scheme as for Scheme 8 in which more weight is placed on the mass goal.	Point	The solutions for Schemes 4 and 8 are indeed the same; however the solutions for Schemes 4-11 are identical in the Point Scenario.
					Ranged	The solutions for Schemes 4 and 8 are indeed the same.

TABLE 4.20. BASELINE SOLUTION ANALYSIS, SCHEMES 5-7

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
5	0.20	0.60	0.20	In the baseline cDSP, the CDSE goal is mathematically independent of the system variables; therefore the solution is the same as for Scheme 7 in which more weight is placed on the deflection goal.	Point	The solutions for Scheme 5 and 7 are indeed the same; however the solutions for Schemes 4-11 are identical in the Point Scenario.
					Ranged	The solutions for Schemes 5 and 7 are indeed the same.
6	0.20	0.20	0.60	In the baseline cDSP, the CDSE goal is mathematically independent of the system variables; therefore the solution is the same as for Schemes 9, 10, and 11 in which equal weight is placed on the mass and deflection goals.	Point	The solutions for Schemes 6, 9, 10, and 11 are indeed the same; however the solutions for Schemes 4-11 are identical in the Point Scenario.
					Ranged	The solutions for Schemes 6, 9, 10, and 11 are indeed the same.
7	0.20	0.40	0.40	In the baseline cDSP, the CDSE goal is mathematically independent of the system variables; therefore the solution is the same as for Scheme 5 in which more weight is placed on the deflection goal.	Point	The solutions for Schemes 5 and 7 are indeed the same; however the solutions for Schemes 4-11 are identical in the Point Scenario.
					Ranged	The solutions for Schemes 5 and 7 are indeed the same.

TABLE 4.21. BASELINE SOLUTION ANALYSIS, SCHEMES 8-10

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
8	0.40	0.20	0.40	In the baseline cDSP, the CDSE goal is mathematically independent of the system variables; therefore the solution is the same as for Scheme 4 in which more weight is placed on the mass goal.	Point	The solutions for Schemes 4 and 8 are indeed the same; however the solutions for Schemes 4-11 are identical in the Point Scenario.
					Ranged	The solutions for Schemes 4 and 8 are indeed the same.
9	0.40	0.40	0.20	In the baseline cDSP, the CDSE goal is mathematically independent of the system variables; therefore the solution is the same as for Schemes 6, 10, and 11 in which equal weight is placed on the mass and deflection goals.	Point	The solutions for Schemes 6, 9, 10, and 11 are indeed the same; however the solutions for Schemes 4-11 are identical in the Point Scenario.
					Ranged	The solutions for Schemes 6, 9, 10, and 11 are indeed the same.
10	0.33	0.33	0.33	In the baseline cDSP, the CDSE goal is mathematically independent of the system variables; therefore the solution is the same as for Schemes 6, 9, and 11 in which equal weight is placed on the mass and deflection goals.	Point	The solutions for Schemes 6, 9, 10, and 11 are indeed the same; however the solutions for Schemes 4-11 are identical in the Point Scenario.
					Ranged	The solutions for Schemes 6, 9, 10, and 11 are indeed the same.

TABLE 4.22. BASELINE SOLUTION ANALYSIS, SCHEME 11

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
11	0.50	0.50	0.00	In the baseline cDSP, the CDSE goal is mathematically independent of the system variables; therefore the solution is the same as for Schemes 6, 9, and 10 in which equal weight is placed on the mass and deflection goals.	Point	The solutions for Schemes 6, 9, 10, and 11 are indeed the same; however the solutions for Schemes 4-11 are identical in the Point Scenario.
					Ranged	The solutions for Schemes 6, 9, 10, and 11 are indeed the same.

In the plot of a Pareto surface the achievements of the design objectives are plotted against each other in the criterion space to assess trade-offs between the objectives. Approximations of the Pareto surface can be created using a weighted sum approach by varying the Archimedean weights on the three design objectives and plotting achievement of each objective versus each other. Eleven weighting schemes are identified in Table 4.9 to generate multiple points along this surface. Pareto plots for the two BRP design scenarios are presented in Figure 4.7 and Figure 4.8. These plots are created by plotting the deviation variable of one goal in one weighting scheme versus the deviation variable for another goal in the same weighting scheme. Thus, the ideal solution is represented by the origin of these plots, which represents a deviation of zero from all goals. Alternatively, the values of mass, deflection, and CDSE can be plotted directly instead of the deviation variables for these objectives.

After plotting the deviation variables versus each other for all the solutions, conclusions can be made about the effects of the achievement of one goal versus another. It is expected that for the baseline solutions, the CDSE goal will have no effect on the achievement of the other goals, because the CDSE function is constant throughout the baseline cDSP. The CDSE is modeled in the baseline cDSP in order to reduce the amount of changes in the formulation from the baseline to expansion cDSP, to verify the proper behavior of the CDSE function, and because the total deviation values in the baseline and expansion solutions must be compared in the VDSE metric. If there are only two goals in the baseline cDSP and three goals in the expansion cDSP, then it is not valid to compare the deviation values at the two solutions. For both the Point and Ranged Scenarios, this expectation holds true because there is no correlation between the deviation variables for the mass or deflection goals and the deviation variables for the CDSE goal. There does appear to be a correlation between the achievements of the deflection goal and mass goal, as evidenced by the declining trend in the deviation from the deflection goal as the deviation from the mass goal increases. This trend is evident in both the point and ranged scenario; however, the effect is stronger in the point scenario due to the steeper slope in the trendline. The negative correlation in the deviation variables for deflection and mass indicate that there is a conflict between the achievement of these two goals, i.e., an improvement in the achievement of the deflection goal is only possible with an accompanying deterioration in the achievement of the mass goal and vice versa. Physically, this conflict in the mass and deflection goals is expected, because it is known that the BRP can be made stronger by increasing the thickness of the face sheets, which will increase the mass of the panel. Conversely, if the mass is decreased by taking material away from the face sheets, it is expected that the panel is less resistant to deflection.

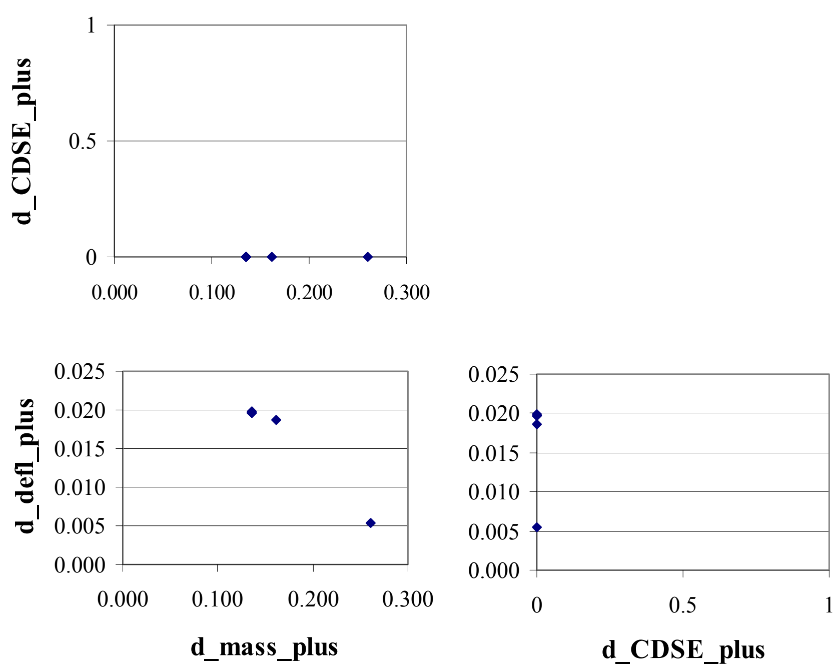


FIGURE 4.7. PARETO PLOTS FOR THE POINT SCENARIO

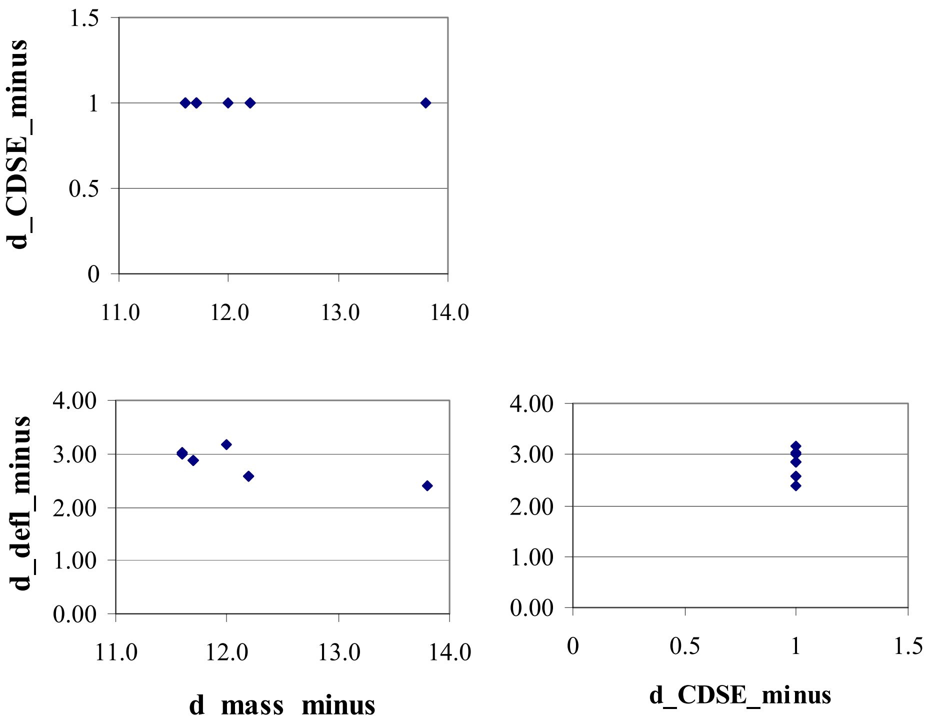


FIGURE 4.8. PARETO PLOTS FOR THE RANGED SCENARIO

A designer must consider two issues when interpreting the solutions to the baseline cDSP as part of the design space expansion strategy. The CDSE function should behave as expected, and there must be room for improvement in the achievement of design goals in order to proceed with the design space expansion strategy. Both of these issues can be assessed by looking at the deviation variables for the design solutions.

As stated previously in Section 4.1.2, the CDSE represents the belief of the designer about the increase in the design cost as a result of moving away from the baseline material. The value of the CDSE at the baseline solution is the minimum value in the design space because the least costly design process option is to select an existing material. It is expected that the achievement of the CDSE goal will not affect the search for design solutions to the baseline cDSP because the material property design variables are all fixed at the minimum point of the CDSE. For example, in Scheme 4 the mass is preferred to deflection and CDSE equally. If the CDSE has no effect on the solutions, the solution should be the same for all scenarios in which mass is preferred to deflection, regardless of the weight on the CDSE goal. In fact, this effect is seen in the results. In Scheme 8 also mass is preferred to deflection, and the solutions are the same for both of those schemes in both the Ranged and Point Scenarios. Similarly, deflection is preferred to mass in both Schemes 5 and 7, with different weights on the CDSE goal. Again, the solutions for the two schemes are the same in both the Ranged and Point Scenarios. Finally, in Schemes 6, 9, and 10 an equal weight is placed on mass and deflection. Once again, the solutions are the same for all three schemes in both the Ranged and Point Scenarios. These results support the conclusion that the CDSE goal has no effect on the solutions to the baseline cDSP as expected.

A designer must also examine the deviation function values for each solution to make sure that the design targets have not already been met in the baseline cDSP. If the targets are met in the baseline solution, there is not room for additional improvement by expanding the material design space. It is also possible that the deviation function may be at the absolute minimum in the design space at the baseline solution without meeting the design targets. In the case that the targets are met in the baseline solution, a deviation function value greater than zero indicates that there is room for improvement in the achievement of design goals. It can be seen that the solution to Scheme 3 in the point scenario has a deviation function value of zero, indicating no further improvement is possible. This is because all the weight in Scheme 3 is placed on the CDSE goal, and the CDSE goal is at its minimum value of zero in the baseline cDSP. The solution to Scheme 3 in the Ranged Scenario has a deviation function value of one rather than zero. This is because the C_{dk} of the CDSE goal is equal to zero when the CDSE is equal to zero, and the goal in the ranged scenario is to maximize the C_{dk} to one. Since zero is the minimum of the CDSE function, zero is the maximum value of C_{dk} for the CDSE function, and one is the minimum possible value of the deviation function for Scheme 3 in the Ranged Scenario. Therefore, there is no room for improvement in the minimization of the deviation function in Scheme 3 by expanding the material design space. If Scheme 3 were the only weighting scheme under consideration, the DSES would end here.

A designer must critically evaluate the baseline solutions to verify that a valid solution has been found, and that there is still room for improvement in the achievement of design goals beyond the achievement of the baseline solution. Although it is known that the deviation variable for the CDSE goal is at its minimum point for all scenarios, the mass per area and deflection goals have room for improvement, indicated by the nonzero deviation function values for the solution to each scheme except Scheme 3. Since there *is* additional room for improvement, the next step is to revise the formulation of the cDSP by expanding the material design space in the expansion stage of the DSES.

4.2 THE EXPANSION CDSP

In the expansion cDSP, the design space is expanded from a point to a multidimensional material design space. The intention in the expansion cDSP is to identify the design that represents the best goal achievement in the expanded design space. After solving the expansion cDSP, the VDSE metric is calculated to support the decision between material design and material selection. Also, the material property targets for subsequent phases of design are identified by analyzing the values of the material property variables in the solution to the expansion cDSP. The revision of the cDSP formulation and the solution of the expansion cDSP are discussed in the following sections.

4.2.1 Formulating the Expansion BRP Design Problem

In the following sections, the word and math formulations of the expansion cDSP are presented. Rather than starting from scratch, the baseline cDSP is revised to reflect the changes in the problem formulation. These changes are highlighted in the following sections.

Defining the expanded design space

The material design space is expanded in Step 2.1 of the DSES to allow for adjustment of the material properties of the layers of the BRP. In the expansion stage, the expanded material design space is defined by identifying upper and lower bounds for the material property variables. The upper and lower bounds for the BRP material property variables are selected based on the region of engineering alloys on the material selection chart shown in Figure 4.3, giving a rectangular design space that encompasses the properties of most engineering metals. The bounds on the material property design variables are summarized in Table 4.23.

TABLE 4.23. BOUNDS ON MATERIAL PROPERTY DESIGN VARIABLES (ASHBY, 1999)

	Design Variable	
Material Property	**Lower Bound**	**Upper Bound**
Density (kg/m^3)	1600	20000
Yield Strength (MPa)	20	1200

The primary changes in the expansion cDSP relate to the material design space and the goals. The first change from the baseline formulation involves the material design space. In the expansion cDSP, the material design space is expanded from the single design point of the baseline cDSP to a multidimensional design space by allowing the material properties to vary between upper and lower bounds. The material properties are included in the design variables listed under the *find* heading of the cDSP formulation. In the BRP design problem there are six material property variables, because there is both a mass density and a yield strength variable for each layer of the panel.

The second change from the baseline cDSP to the expansion cDSP is the formulation of the goals. Because the design space has expanded, the normalization factors in the point scenario may have changed. The estimates of the maximum values of the objectives must be reevaluated to find the new estimate in the expanded design space.

Word formulation of the expansion cDSP (Step 2.2.1 of Figure 3.1)

Having identified the changes from the baseline cDSP to the expansion cDSP, the word formulations of the expansion cDSP are presented in the following tables. The formulation for the Point Scenario is shown in Table 4.25 and the formulation for the Ranged Scenario is shown in Table 4.26. The underlined items indicate the new information that is added or changed to convert from the baseline to the expansion cDSP.

Deriving the equations for the expansion cDSP

The BRP analysis and constraint equations derived in Section 4.1.1 remain the same for the expansion cDSP. The goals for the Ranged Scenario also remain the same. The goal equations for the Point Scenario are revised to reflect the new estimates of the maximum values of each of the objectives. These maximum value estimates are found by discretizing the expanded design space into a grid with three points for each geometric design variable and two points for each material property variable. The revised goal equations reflecting the new maximum value estimates are shown in Equations 4.37-4.39. The new estimates of the maximum objective values in the expanded design space are listed in Table 4.24.

$$\frac{M}{2997.7} + d_{mass}^{-} - d_{mass}^{+} = 0 \tag{4.37}$$

$$\frac{\delta}{234.5} + d_{defl}^{-} - d_{defl}^{+} = 0 \tag{4.38}$$

$$\frac{CDSE}{5.35} + d_{CDSE}^{-} - d_{CDSE}^{+} = 0 \tag{4.39}$$

TABLE 4.24. ESTIMATED MAXIMUM VALUES OF OBJECTIVES IN THE EXPANSION CDSP

	Max	Units
Mass	2997.7	kg/m2
Deflection	234.5	m
CDSE	5.35	none

TABLE 4.25. WORD FORMULATION OF THE EXPANSION CDSP, POINT SCENARIO

Given:
Baseline material properties An impulse load defined by peak pressure, p_0, and characteristic time, t_0 Model for the deflection of the panel Model for the mass per area of the panel Model for the cost of design space expansion
Find:
Material properties: $\sigma_{Y,f}, \rho_f, \sigma_{Y,b}, \rho_b, \sigma_{Y,c}, \rho_c$ Core geometry: h_c, B Height of each layer: h_f, h_b, H Deviation variables d_i^+, d_i^- (i = 1, 2, 3)
Satisfy:
Constraints a) Mass/area of BRP must not exceed 150 kg/m^3 b) Deflection must not exceed 15 cm for specified boundary conditions c) Relative density must be greater than 0.07 to avoid buckling d) Front face shear-off parameter, μ , must not exceed 4/sqrt(3) e) Front face shear-off parameter, Γ, must not exceed 0.6 *Goals* a) The mass per area is equal to or less than the target value. b) The deflection is equal to or less than the target value. c) The CDSE is equal to or less than the target value. *Bounds* Upper and lower bounds for system variables
Minimize:
Archimedean deviation function

TABLE 4.26. WORD FORMULATION OF THE EXPANSION CDSP, RANGED SCENARIO

Given:
Baseline material properties An impulse load defined by peak pressure, p_0, and characteristic time, t_0 Mean and variance of loading parameters (t_0, p_0) Variance of material property design variables (ρ,σ_Y) Model for the deflection of the panel Model for the mass per area of the panel Model for the cost of design space expansion
Find:
Material properties: $\sigma_{Y,f}, \rho_f, \sigma_{Y,b}, \rho_b, \sigma_{Y,c}, \rho_c$ Core geometry: h_c, B Height of each layer: h_f, h_b, H Deviation variables d_i^+, d_i^- (i = 1, 2, 3)
Satisfy:
Constraints a) Mass/area of BRP must not exceed 150 kg/m^3 b) Deflection must not exceed 15 cm for specified boundary conditions c) Relative density must be greater than 0.07 to avoid buckling d) Front face shear-off parameter, μ , must not exceed 4/sqrt(3) e) Front face shear-off parameter, Γ, must not exceed 0.6 *Goals* a) The C_{dk} of mass per area is equal to or greater than the target value. b) The C_{dk} of deflection is equal to or greater than the target value. c) The C_{dk} of CDSE is equal to or greater than the target value. *Bounds* Upper and lower bounds for system variables
Minimize:
Archimedean deviation function

Mathematical formulation of the expansion cDSP (Step 2.2.2)

The word form of the expansion cDSP is converted into the math form of the expansion cDSP by the inclusion of the information specified in the preceding section. The math formulation of the expansion cDSP for the Ranged Scenario is shown in Table 4.27, while the math form of the expansion cDSP for the Point Scenario is shown in Table 4.28. The next step is to apply an algorithm to minimize the deviation function.

TABLE 4.27. MATH FORM OF THE EXPANSION CDSP, RANGED SCENARIO

Given:		Eq#
A impulse load defined by peak pressure, p_0, and characteristic time, t_0	$\mu_t = 10^{-4}$ seconds, $\Delta t = 0.15\cdot \mu_t$ $\mu_p = 25$ MPa, $\Delta p = 0.15\cdot \mu_p$	
Material properties of the baseline materials	Faces: $\rho_{mg} = 1770$ kg/m^3, $\sigma_{Y,mg} = 200$ MPa Core: $\rho_{steel} = 7845$ kg/m^3, $\sigma_{Y,steel} = 538$ MPa	
Variation in material properties (ρ,σ_Y)	Faces: $\Delta\rho_{mg} = 170$ kg/m^3, $\Delta\sigma_{Y,mg} = 20$ MPa Core: $\Delta\rho_{steel} = 100$ kg/m^3, $\Delta\sigma_{Y,steel} = 150$ MPa	
BRP Deflection Model $\left[\delta,\Delta\delta,M,\Delta M\right] = f\left(\sigma_{Y,b},\sigma_{Y,c},\sigma_{Y,f},\rho_b,\rho_c,\rho_f,p_0,t_0,B,H,h_c,h_f,h_b,L\right)$		(4.5)
Find:		
Material properties:	$\sigma_{Y,f}$, ρ_f, $\sigma_{Y,b}$, ρ_b, $\sigma_{Y,c}$, ρ_c	
Core geometry:	h_c, B	
Height of each layer:	h_f, h_b, H	
Value of deviation variables	d_i^+, d_i^- (i = mass, defl, CDSE)	
Satisfy:		
Constraints	C_{dk}'s defined in Tables 4.6 and 4.8	
a) Mass/area of BRP must not exceed 150 kg/m^3	$C_{\text{dk-mass-constraint}} \geq 1$	(4.22)
b) Deflection must not exceed 15% of span for specified boundary conditions	$C_{\text{dk-deflection-constraint}} \geq 1$	(4.23)
c) Relative Density must be greater than 0.07 to avoid buckling	$R_c - 0.07 \geq 0$	(4.19)
Front face shear-off constraints		
d) Gamma must not exceed 0.6	$C_{\text{dk-gamma-constraint}} \geq 1$	(4.24)
e) Mu must not exceed 4/sqrt(3)	$C_{\text{dk-mu-constraint}} \geq 1$	(4.25)
Deviation variables must be greater than or equal to zero and multiply to zero	$d_i^+ \cdot d_i^- = 0,\ d_i^+, d_i^- \geq 0$ $\left(i = \text{mass, defl, CDSE}\right)$	
Goals		
a) The C_{dk} of mass per area is greater than or equal to the target of one.	$C_{\text{dk-mass-objective}} + d_{\text{mass}}^- - d_{\text{mass}}^+ = 1$	(4.32)
b) The C_{dk} of deflection is greater than or equal to the target of one.	$C_{\text{dk-deflection-objective}} + d_{\text{defl}}^- - d_{\text{defl}}^+ = 1$	(4.33)
c) The C_{dk} of CDSE is greater than or equal to the target of one.	$C_{\text{dk-CDSE-objective}} + d_{\text{CDSE}}^- - d_{\text{CDSE}}^+ = 1$	(4.34)
Bounds (0.1 mm $\leq h_c \leq$ 1 cm) (20 MPa $\leq \sigma_{Y,b} \leq$ 1200 MPa) (1600 $\leq \rho_b \leq$ 20,000 kg/m^3) (5 mm $\leq H \leq$ 5 cm) (20 MPa $\leq \sigma_{Y,f} \leq$ 1200 MPa) (1600 $\leq \rho_c \leq$ 20,000 kg/m^3) (1 mm $\leq B \leq$ 2 cm) (20 MPa $\leq \sigma_{Y,c} \leq$ 1200 MPa) (1600 $\leq \rho_f \leq$ 20,000 kg/m^3) (1 mm $\leq h_f \leq$ 5 cm) (1 mm $\leq h_b \leq$ 5 cm)		
Minimize:		
Deviation Function: Archimedean $Z_{A,ranged}\left(\mathbf{x}\right) = W_{\text{m}}\left(d_{\text{mass}}^-\right) + W_{\text{d}}\left(d_{\text{defl}}^-\right) + W_{\text{c}}\left(d_{\text{CDSE}}^-\right);\ \sum W_i = 1$		(4.36)

TABLE 4.28. MATH FORM OF THE EXPANSION CDSP, POINT SCENARIO

Given:		Eq#
A impulse load defined by peak pressure, p_0, and characteristic time, t_0	$t_0 = 10^{-4}$ seconds, $p_0 = 25$ MPa	
Material properties of the baseline materials	Faces: $\rho_{mg} = 1770$ kg/m^3, $\sigma_{Y,mg} = 200$ MPa Core: $\rho_{steel} = 7845$ kg/m^3, $\sigma_{Y,steel} = 538$ MPa	
BRP Deflection Model	$[\delta, \Delta\delta, M, \Delta M] = f(\sigma_{Y,b}, \sigma_{Y,c}, \sigma_{Y,f}, \rho_b, \rho_c, \rho_f, p_0, t_0, B, H, h_c, h_f, h_b, L)$	(4.5)
Find:		
Material properties:	$\sigma_{Y,f}, \rho_f, \sigma_{Y,b}, \rho_b, \sigma_{Y,c}, \rho_c$	
Core geometry:	h_c, B	
Height of each layer:	h_f, h_b, H	
Value of deviation variables	d_i^+, d_i^- (i = mass, defl, CDSE, VDSE)	
Satisfy:		
Constraints		
a) Mass/area of BRP must not exceed 150 kg/m^3	$\rho_f h_f + \rho_c H R_c + \rho_b h_b - 150 \text{ kg/m}^2 \le 0$	(4.17)
b) Deflection must not exceed 15% of span for specified boundary conditions	$\delta \le 0.15$ m	(4.18)
c) Relative Density must be greater than 0.07 to avoid buckling	$0.07 - R_c \le 0$	(4.19)
Front face shear-off constraints		
d) Gamma must not exceed 0.6	$\left(2 p_0 t_0 \big/ h_f \sqrt{\sigma_{Y,f} \rho_f}\right) - 0.6 \le 0$	(4.20)
e) Mu must not exceed 4/sqrt(3)	$\dfrac{\rho_c H R_c}{\rho_f h_f} - \dfrac{4}{\sqrt{3}} \le 0$	(4.21)
Deviation variables must be greater than or equal to zero and multiply to zero	$d_i^+ \cdot d_i^- = 0,\ d_i^+, d_i^- \ge 0$ $(i = \text{mass, defl, CDSE})$	
Goals		
The mass per area is less than or equal to the target value of zero.	$\dfrac{M}{2997.7} + d_{mass}^- - d_{mass}^+ = 0$	(4.37)
The deflection is less than or equal to the target value of zero.	$\dfrac{\delta}{234.5} + d_{defl}^- - d_{defl}^+ = 0$	(4.38)
The CDSE is less than or equal to the target value of zero.	$\dfrac{CDSE}{5.35} + d_{CDSE}^- - d_{CDSE}^+ = 0$	(4.39)
Bounds		
(0.1 mm $\le h_c \le$ 1 cm) (20 MPa $\le \sigma_{Y,b} \le$ 1200 MPa) (1600 $\le \rho_b \le$ 20,000 kg/m^3) (5 mm $\le H \le$ 5 cm) (20 MPa $\le \sigma_{Y,f} \le$ 1200 MPa) (1600 $\le \rho_c \le$ 20,000 kg/m^3) (1 mm $\le B \le$ 2 cm) (20 MPa $\le \sigma_{Y,c} \le$ 1200 MPa) (1600 $\le \rho_f \le$ 20,000 kg/m^3) (1 mm $\le h_f \le$ 5 cm) (1 mm $\le h_b \le$ 5 cm)		
Minimize:		
Deviation Function: Archimedean	$Z_{A,point}(\mathbf{x}) = W_m(d_{mass}^+) + W_d(d_{defl}^+) + W_c(d_{CDSE}^+);\ \sum W_i = 1$	(4.35)

4.2.2 Solving the Expanded BRP Design Problem

With the problems formulated, the solutions to the expansion cDSP can now be found by minimizing the deviation function (Step 2.2.3). An exhaustive search of the expanded design space using the same resolution as the baseline cDSP would require evaluating 16^{11} or 17,592,186,044,416 design points. This is 16,777,216 times the number of design points evaluated for the baseline solution. The evaluation of feasibility for the design points in the baseline cDSP takes approximately 15 hours of computation on a computer with dual 3.1 GHz processors and 2 GB of RAM. Assuming that the amount of computational time for the expanded design space is 16,777,216 times the amount of computation time for the baseline design space, the evaluation of feasibility for the expanded design space would take approximately 215,658,240 hours or 10,485,760 days to complete (roughly 29,000 years). Since an exhaustive search of the design space is not feasible, a genetic algorithm (GA) and a pattern search are applied to solve the expansion cDSP in this work.

A GA is a Monte-Carlo-type method in which solutions are found by the "survival of the fittest" concept of evolution. Rules are specified for selection of mating pairs, crossover of genetic material, and mutation of genetic material. The genetic material is a bit string which represents a vector of design variables. Based on the selection, crossover, and mutation rules, the population evolves and converges to a solution. Due to the large size of the design space in this example and the complexity of the deflection model, it is unlikely that unimodality is an appropriate assumption, meaning that it is probable that there are local minima in the design space. The solution finding method presented here is for the general case where unimodality is not an appropriate assumption. If the goals are unimodal, then the solution finding method can be simpler. For this solution finding method genetic algorithm is selected because although it is not guaranteed that the global minimum will be found, a genetic algorithm is less likely to settle in a local minimum than gradient-based search methods such as Newton's method or the method of steepest descent. In this work, the MATLAB built-in genetic algorithm is used to find the solutions.

The MATLAB GA is an unconstrained optimization algorithm, meaning that the algorithm searches for the minimum of the fitness function without regard to any constraints on the system. However, there are constraints on the BRP system, and these constraints must be taken into account in the search for the design solutions. To apply an unconstrained optimization algorithm to a problem with constraints, penalty functions are employed. A penalty function is a function that is added to the fitness function to apply a penalty to the fitness when a constraint is violated. When all the constraints are met, the penalty function is inactive. To apply an unconstrained genetic algorithm to solve the cDSPs formulated above, the problems are converted in two steps. This process is discussed in the following sections.

The first step towards applying the unconstrained genetic algorithm is to convert the cDSP formulation into a constrained minimization formulation. This is accomplished by stating the Archimedean deviation function, Z_A, as an objective function which must be minimized subject to constraints. The two constrained minimization problems for the Ranged and Point Scenarios are shown in Figure 4.9 and Figure 4.10. In the objective function Z_A, W_m, W_d, and W_c are the Archimedean weighting factors for the mass, deflection, and CDSE goals specified in the eleven weighting schemes (Table 4.9), and $d_{VDSE,mass}$, $d_{VDSE,defl}$, and $d_{VDSE,CDSE}$ are the deviation variables of the VDSE goals for mass, deflection and CDSE, respectively. The constraint functions are shown as functions g_1, g_2, g_3, g_4, and g_5, and correspond to the constraints on mass, deflection, relative density, Γ, and μ. Prior to solving with an unconstrained minimization algorithm, the problem formulations must again be adapted. In this step the constrained minimization problems are converted into unconstrained minimization problems using penalty functions. The unconstrained objective functions are shown in Equations 4.38 and 4.39 where P is a constant penalty coefficient acting on the summation of all five constraint functions. The Ranged Scenario is shown in Equation 4.40 using DCIs for the constraint functions, and the Point Scenario is shown in Equation 4.41.

$$
\begin{array}{ll}
\min & Z_{A,ranged}(\mathbf{x}) = W_{\mathrm{m}}\left(d^{-}_{\mathrm{mass}}\right) + W_{\mathrm{d}}\left(d^{-}_{\mathrm{defl}}\right) + W_{\mathrm{c}}\left(d^{-}_{\mathrm{CDSE}}\right) \\
 & \\
s.t. & g_1 = 1 - \dfrac{150 - M}{\Delta M} \le 0 \\
 & g_2 = 1 - \dfrac{0.15 - \delta}{\Delta \delta} \le 0 \\
 & g_3 = 0.07 - R_c \le 0 \\
 & g_4 = 1 - \dfrac{0.6 - \Gamma}{\Delta \Gamma} \le 0 \\
 & g_5 = 1 - \dfrac{4/\sqrt{3} - \mu}{\Delta \mu} \le 0
\end{array}
$$

FIGURE 4.9. CONSTRAINED MINIMIZATION, RANGED SCENARIO

$$
\begin{array}{ll}
\min & Z_{A,point}(\mathbf{x}) = W_{\mathrm{m}}\left(d^{+}_{\mathrm{mass}}\right) + W_{\mathrm{d}}\left(d^{+}_{\mathrm{defl}}\right) + W_{\mathrm{c}}\left(d^{+}_{\mathrm{CDSE}}\right) \\
 & \\
s.t. & g_1 = M - 150 \le 0 \\
 & g_2 = \delta - 0.15 \le 0 \\
 & g_3 = 0.07 - R_c \le 0 \\
 & g_4 = \Gamma - 0.6 \le 0 \\
 & g_5 = \mu - 4/\sqrt{3} \le 0
\end{array}
$$

FIGURE 4.10. CONSTRAINED MINIMIZATION, POINT SCENARIO

$$
\begin{aligned}
\min Z_{A,ranged}(\mathbf{x}) = \; & W_{\mathrm{m}}\left(d^{-}_{\mathrm{mass}}\right) + W_{\mathrm{d}}\left(d^{-}_{\mathrm{defl}}\right) + W_{\mathrm{c}}\left(d^{-}_{\mathrm{CDSE}}\right) + \\
& P\left(\max\left\{ 0, \left[\begin{array}{l} \left(1 - \dfrac{150 - M}{\Delta M}\right) + \\ \left(1 - \dfrac{0.15 - \delta}{\Delta \delta}\right) + \left(0.07 - R_c\right) + \\ \left(1 - \dfrac{0.6 - \Gamma}{\Delta \Gamma}\right) + \left(1 - \dfrac{4/\sqrt{3} - \mu}{\Delta \mu}\right) \end{array} \right] \right\} \right)
\end{aligned}
\tag{4.40}
$$

$$\min Z_{A,point}(\mathbf{x}) = W_{\mathrm{m}}\left(d^{+}_{\mathrm{mass}}\right) + W_{\mathrm{d}}\left(d^{+}_{\mathrm{defl}}\right) + W_{\mathrm{c}}\left(d^{+}_{\mathrm{CDSE}}\right) + P\left(\max\left\{0, \left[\begin{array}{l}(M-150)+ \\ (\delta-0.15)+(0.07-R_c)+ \\ (\Gamma-0.6)+\left(\mu-4/\sqrt{3}\right)\end{array}\right]\right\}\right) \tag{4.41}$$

Prior to running the GA, fitness and constraint functions are defined in MATLAB for each problem formulation. The fitness functions are the same as the objective functions specified in the unconstrained problem formulations above; however, the fitness function must convert a binary bit string into the numerical values of the design variables. Each design variable is represented by five bits, which enables the discretization of the design space into thirty-two points for each design variable between the upper and lower bounds for the variables. The default settings of the MATLAB GA are used to find the solutions, with the exception of the settings for the population. The individuals are defined using a bit string and the population size is set at three times the length of the bit string. The MATLAB code for each of the fitness functions and constraint functions is included in Appendix A.

The GA can get close to the global optimum, but since the design space is discretized rather than continuous, the GA is not guaranteed to reach the true minimum of the deviation function. To remedy this problem, a pattern search is used to find the minimum using the GA solution as the starting point. On its own, a pattern search is likely to fall into a local minimum, but by starting from the GA solution, which is close to the global minimum, the pattern search is much less likely to fall into a minimum other than the global minimum. The *patternsearch* function in MATLAB is used to implement the pattern search in this example.

Presentation of the expansion solutions

The design variables for the expansion cDSP solutions are shown in Tables 4.29 through 4.32. The bold entries indicate that the particular design variable is at one of the bounds of that variable. The deviation variables for the expansion solutions are presented in Tables 4.33 and 4.34. The solutions for weighting Scheme 10 are shown on the BRP schematic in Figures 4.11 and 4.12. The interpretation of these solutions is discussed in the following section.

TABLE 4.29. EXPANSION SYSTEM VARIABLES, POINT SCENARIO

Scheme	B	H	h_c	h_f	h_b
1	**0.0200**	0.0056	0.0007	0.0194	**0.0010**
2	0.0187	**0.0500**	0.0049	0.0121	**0.0500**
3	0.0188	0.0253	0.0027	0.0247	0.0121
4	**0.0200**	0.0390	0.0007	0.0228	0.0106
5	0.0118	0.0270	0.0004	0.0212	0.0152
6	**0.0200**	0.0119	0.0007	0.0264	0.0168
7	**0.0200**	0.0251	0.0012	0.0184	0.0152
8	**0.0200**	**0.0050**	0.0050	0.0186	0.0219
9	0.0118	0.0160	0.0004	0.0262	0.0152
10	**0.0200**	0.0254	0.0011	0.0168	0.0168
11	0.0111	0.0093	0.0011	0.0184	**0.0010**

TABLE 4.30. EXPANSION MATERIAL PROPERTY VARIABLES, POINT SCENARIO

	Yield Strength			Density		
Scheme	Back	Core	Front	Back	Core	Front
1	**1200.0**	**1200.0**	**1200.0**	**1600.0**	**1600.0**	**1600.0**
2	**1200.0**	**1200.0**	**1200.0**	**1600.0**	**1600.0**	2787.1
3	200.0	538.0	200.0	1770.0	7845.0	1770.0
4	200.0	538.1	200.1	1770.7	7844.8	1770.7
5	200.0	538.0	200.0	1770.1	7844.9	1770.1
6	200.0	538.0	200.0	1770.1	7844.9	1770.1
7	200.0	538.0	200.0	1770.1	7845.4	1770.1
8	200.0	538.0	200.0	1770.7	7846.1	1770.7
9	200.0	538.1	200.0	1770.7	7845.4	1770.7
10	200.1	538.1	200.0	1770.7	7845.4	1770.7
11	**1200.0**	**1200.0**	**1200.0**	**1600.0**	**1600.0**	**1600.0**

TABLE 4.31. EXPANSION SYSTEM VARIABLES, RANGED SCENARIO

Scheme	B	H	h_c	h_f	h_b
1	**0.0200**	**0.0050**	0.0007	**0.0500**	0.0270
2	**0.0200**	0.0294	0.0011	0.0121	0.0468
3	0.0126	0.0322	0.0011	0.0437	0.0057
4	**0.0200**	**0.0050**	0.0007	0.0230	**0.0500**
5	**0.0200**	0.0182	0.0007	0.0291	0.0433
6	**0.0200**	**0.0050**	0.0008	0.0218	**0.0500**
7	0.0118	0.0107	0.0004	0.0495	0.0247
8	**0.0200**	**0.0050**	0.0007	0.0434	0.0325
9	**0.0200**	**0.0050**	0.0007	0.0230	**0.0500**
10	**0.0200**	**0.0050**	0.0007	0.0230	**0.0500**
11	**0.0200**	0.0383	0.0007	0.0121	**0.0500**

TABLE 4.32. MATERIAL PROPERTY VARIABLES IN THE SOLUTION TO THE EXPANSION CDSP, RANGED SCENARIO

	Yield Strength			Density		
Scheme	Back	Core	Front	Back	Core	Front
1	738.7	879.7	701.9	**1770.0**	**1700.0**	**1770.0**
2	**1180.0**	**170.0**	504.3	2352.6	2509.5	**1770.0**
3	200.0	538.0	200.0	**1770.0**	7845.0	**1770.0**
4	200.0	538.0	200.0	**1770.0**	7845.0	**1770.0**
5	257.0	**170.0**	175.3	**1770.0**	7150.9	**1770.0**
6	200.0	538.0	200.0	**1770.0**	7845.1	**1770.0**
7	200.0	538.0	200.0	**1770.0**	7845.0	**1770.0**
8	200.0	538.2	200.0	**1770.0**	7845.3	**1770.0**
9	200.0	538.0	200.0	**1770.0**	7844.7	**1770.0**
10	200.0	538.0	200.0	**1770.0**	7844.9	**1770.0**
11	**1180.0**	198.4	504.3	**1770.0**	**1700.0**	**1770.0**

TABLE 4.33. EXPANSION DEVIATION VARIABLES, POINT SCENARIO

Scheme	d^-_{mass}	d^+_{mass}	$d^-_{deflection}$	$d^+_{deflection}$	d^-_{CDSE}	d^+_{CDSE}	Z_A
1	0	0.0111	0	0.0006	0	1.2102	0.0111
2	0	0.0500	0	0.0000	0	1.2140	0.0000
3	0	0.0391	0	0.0005	0	0.0000	0.0000
4	0	0.0268	0	0.0006	0	0.0001	0.0163
5	0	0.0264	0	0.0006	0	0.0000	0.0057
6	0	0.0277	0	0.0006	0	0.0000	0.0057
7	0	0.0275	0	0.0006	0	0.0000	0.0058
8	0	0.0296	0	0.0006	0	0.0001	0.0120
9	0	0.0274	0	0.0006	0	0.0001	0.0112
10	0	0.0271	0	0.0006	0	0.0001	0.0093
11	0	0.0112	0	0.0006	0	1.2102	0.0059

TABLE 4.34. EXPANSION DEVIATION VARIABLES, RANGED SCENARIO

Scheme	d^-_{mass}	d^+_{mass}	$d^-_{deflection}$	$d^+_{deflection}$	d^-_{CDSE}	d^+_{CDSE}	Z_A
1	11.4293	0.0000	3.4384	0.0000	11.4995	0.0000	11.4293
2	14.5109	0.0000	1.3581	0.0000	14.546	0.0000	1.3581
3	15.3505	0.0000	3.3295	0.0000	1.0132	0.0000	1.0132
4	11.603	0.0000	2.9743	0.0000	1.0094	0.0000	7.7585
5	12.0614	0.0000	2.1445	0.0000	3.4773	0.0000	4.3944
6	11.6413	0.0000	2.9975	0.0000	1.0226	0.0000	3.5414
7	11.8114	0.0000	3.227	0.0000	1.0116	0.0000	4.0577
8	11.5959	0.0000	3.276	0.0000	1.0587	0.0000	5.7170
9	11.6049	0.0000	2.975	0.0000	1.0296	0.0000	6.0379
10	11.6049	0.0000	2.9721	0.0000	1.0194	0.0000	5.1988
11	11.5808	0.0000	1.3379	0.0000	15.275	0.0000	6.4594

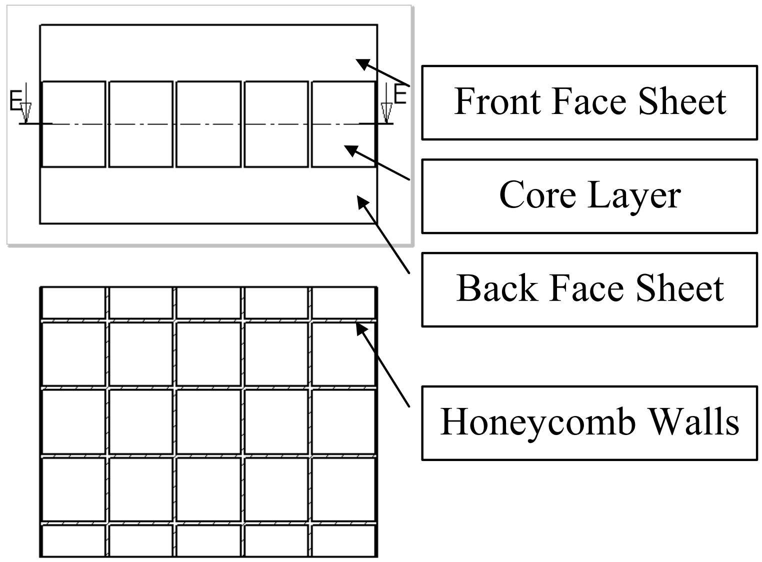


FIGURE 4.11. BRP SCHEMATIC OF THE SOLUTION TO THE POINT SCENARIO OF THE EXPANSION CDSP, SCHEME 10

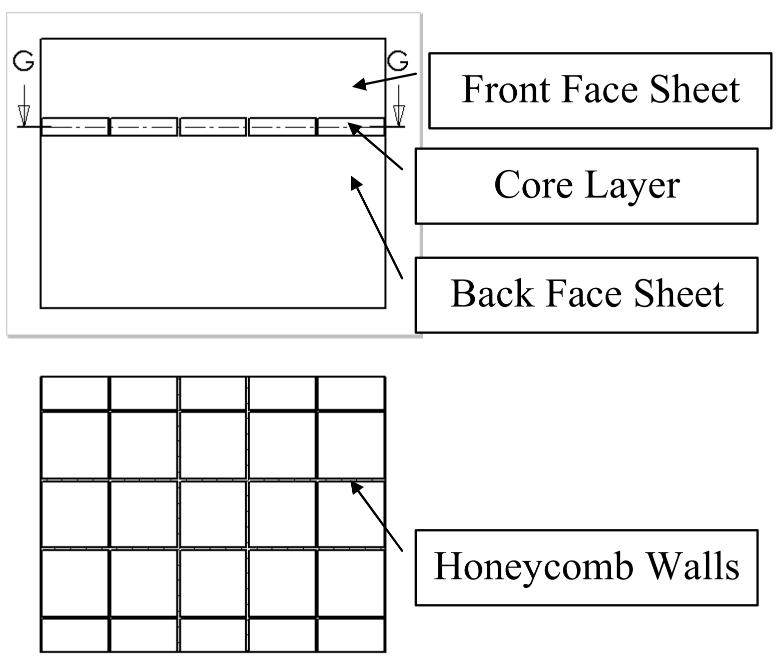


FIGURE 4.12. BRP SCHEMATIC OF THE SOLUTION TO THE RANGED SCENARIO OF THE EXPANSION CDSP, SCHEME 10

4.2.3 Interpreting the Expansion Solution

As before with the baseline cDSP solution, the minimization results must be evaluated for validity in addition to the solutions themselves. These two topics are addressed in the next sections.

The minimization results for the expansion solutions

To build confidence in the validity of the design solution, convergence plots of the genetic algorithm and pattern search are consulted along with an analysis of the active constraints. As shown in the sample convergence plots in Figures 4.13 and 4.14, the algorithm converges smoothly to a solution. This behavior is observed for all solutions, and the convergence plots for each solution are included in Appendix B. The active constraints are also investigated. Identifying the active constraints helps to determine if the solutions make sense theoretically. The active constraints for the solutions are shown by the bold entries in Tables 4.35 and 4.36. Design variable bounds also act as constraints and can be active for a particular solution. These active bounds are denoted by the bold entries in Tables 4.29 through 4.32. When the bounds on the design variables are acting as active constraints, this indicates that it may be beneficial to expand these bounds if possible. The active constraints identified in the solution tables build confidence in the solutions to the expansion cDSPs. In particular, it is noted that there are no active constraints for Scheme 3 in both scenarios. This is expected because it is known that the minimum of the CDSE function is in the interior of the design space, therefore the solution to that particular scheme should not have any active constraints because it is an interior solution.

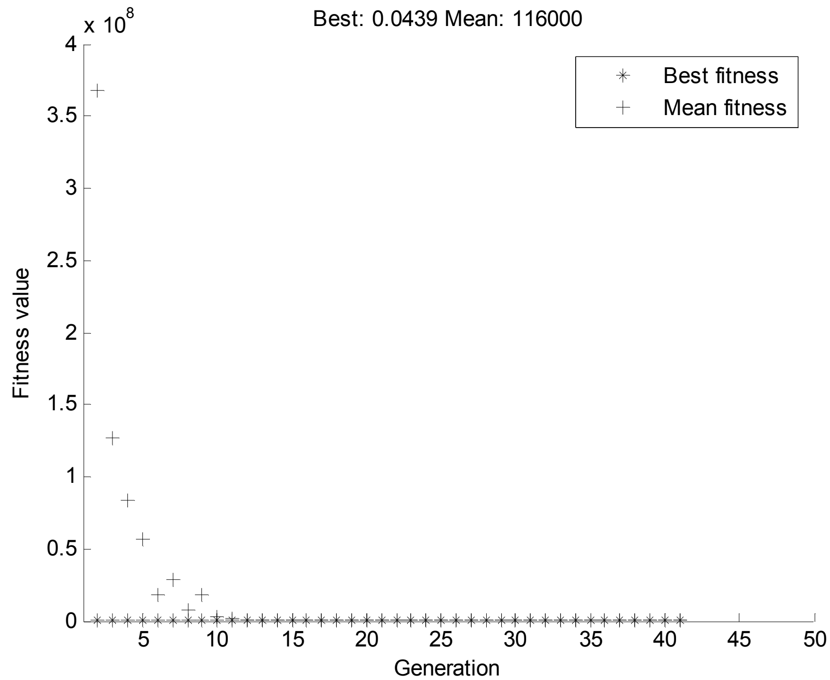


FIGURE 4.13. SAMPLE CONVERGENCE PLOT FROM THE GENETIC ALGORITHM–POINT SCENARIO, SCHEME 10

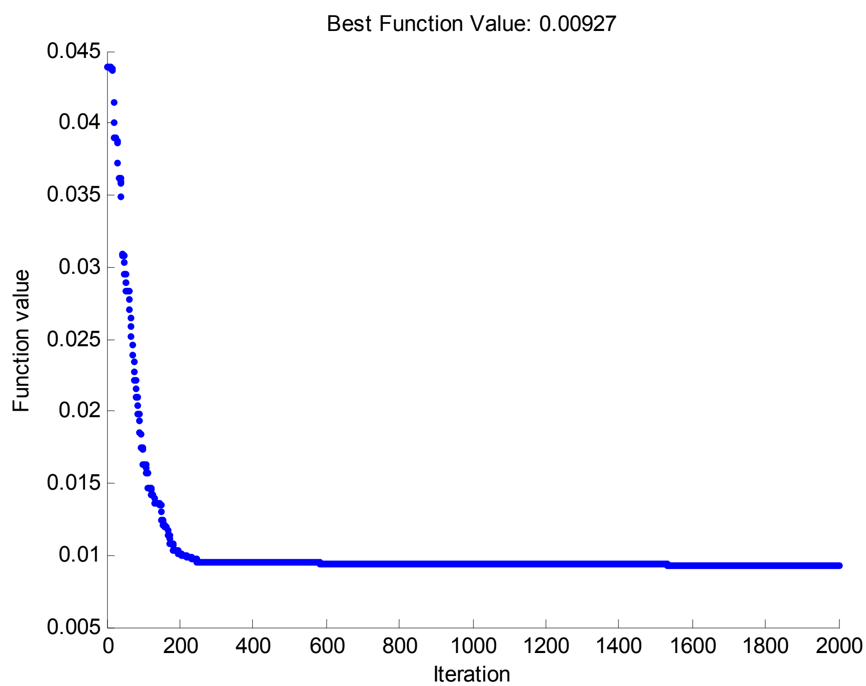


FIGURE 4.14. SAMPLE CONVERGENCE PLOT FROM THE PATTERN SEARCH - POINT SCENARIO, SCHEME 10

TABLE 4.35. CONSTRAINTS FOR THE SOLUTIONS TO THE RANGED SCENARIO OF THE EXPANSION CDSP

Scheme	mass	deflection	R_c	Γ	μ
1	**0.0000**	-4.588	**0.000**	-14.701	-2210.545
2	**0.0000**	**0.000**	-0.040	-0.707	-50.686
3	-1.4703	-0.004	-0.090	-12.375	-123.990
4	-0.4435	**0.000**	**0.000**	-4.732	-1012.824
5	**0.0000**	-0.001	-0.002	-6.984	-334.951
6	-0.6052	**0.000**	-0.013	-4.289	-812.076
7	**0.0000**	-0.632	**0.000**	-14.516	-1018.628
8	-0.0008	-0.484	**0.000**	-12.264	-1917.769
9	-0.4519	**0.000**	**0.000**	-4.732	-1005.588
10	-0.4473	**0.000**	**0.000**	-4.732	-1012.824
11	-2.2712	**0.000**	-0.003	-0.707	-60.199

TABLE 4.36. CONSTRAINTS FOR THE SOLUTIONS TO THE POINT SCENARIO OF THE EXPANSION CDSP

Scheme	mass	deflection	R_c	Γ	μ
1	-116.69	**0.000**	**0.000**	-0.414	-2.289
2	**0.000**	-0.143	-0.385	-0.302	-0.430
3	-32.699	-0.036	-0.193	-0.454	-2.040
4	-69.554	**0.000**	**0.000**	-0.442	-2.190
5	-70.718	**0.000**	**0.000**	-0.430	-2.220
6	-67.036	**0.000**	**0.000**	-0.463	-2.278
7	-67.628	**0.000**	-0.046	-0.404	-2.151
8	-61.230	**0.000**	-0.365	-0.406	-2.193
9	-67.913	**0.000**	**0.000**	-0.462	-2.267
10	-68.895	**0.000**	-0.038	-0.385	-2.146
11	-116.28	**0.000**	-0.112	-0.404	-2.217

The physical meaning of the solution

To analyze the expansion design solutions, hypotheses are again identified for each of the preference schemes and each scenario. These hypotheses and the corresponding results are displayed in Tables 37 through 47. For each of the schemes in which a non-zero weight is placed on the CDSE goal, the material property variables are nearly identical to the baseline material properties, which indicate that the minimization of the CDSE goal is a significant factor in the expansion solutions. The only exception is the solution to Scheme 5 in the Ranged Scenario, in which the strength and density variables are distinct from the baseline material properties. The weight on the CDSE goal in Scheme 5 is the smallest non-zero weight placed on the CDSE in any scheme. Also, the solutions to many of the schemes in the Point Scenario indicate that the achievement of the mass goal continues to dominate the achievement of the deflection goal in the expansion CDSP.

TABLE 4.37. EXPANSION SOLUTION ANALYSIS, SCHEME 1

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
1	1.00	0.00	0.00	In this scheme the only objective is to minimize mass; therefore the layer and cell wall thicknesses are likely to be small, the cell spacing is likely to be large, the strength variables are likely to be large, the density variables are likely to be small, and the deflection constraint is likely to be active.	Point	The cell spacing is at the upper bound, the back face sheet thickness is at the lower bound and the core layer height and cell wall thickness each tend towards the lower bounds. The front face sheet thickness tends slightly toward the lower bound. All three density variables are at the lower bound and all three yield strength variables are at the upper bound. Thus, all of the variables behave as expected in this solution. The deflection constraint and the relative density constraint are both active.
					Ranged	All three density variables are at the lower bound, while the strength variables tend slightly toward the upper bound. The cell spacing is at the upper bound, the core height is at the lower bound, the cell wall thickness is nearly at the lower bound, the front face sheet thickness is at the upper bound, and the back face sheet thickness is near the midpoint. Unexpectedly, the mass constraint is active, along with the relative density constraint.

TABLE 4.38. EXPANSION SOLUTION ANALYSIS, SCHEME 2

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
2	0.00	1.00	0.00	In this scheme the only objective is to minimize deflection; therefore the layer and cell wall thicknesses are likely to be large, the strength variables are likely to be large, the density variables are likely to be small, and the mass constraint is likely to be active.	Point	The core layer height and back face sheet are at their upper bounds as expected, and the cell spacing tends toward its upper bound. The cell wall thickness is near the midpoint of its range, and the front face sheet tends towards the lower bound. All three yield strength variables are at the upper bound as expected, and the core and back face sheet densities are at the lower bound as expected. The front face sheet density tends toward the lower bound. The mass constraint is active as expected.
					Ranged	The strength variable for the back face sheet is at the upper bound, while the strength variable for the core is at the lower bound, and the strength variable for the front face sheet is near the midpoint. The density variables for the back face sheet and core are near the lower bound, and the density of the front face sheet is at the lower bound. The cell spacing is at the upper bound, the core height is near the midpoint, the cell wall thickness tends toward the lower bound, the front face sheet thickness is less than the midpoint, and the back face sheet thickness is near the upper bound. The mass is active as expected; however the deflection constraint is also active, which is not expected..

TABLE 4.39. EXPANSION SOLUTION ANALYSIS, SCHEME 3

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
3	0.00	0.00	1.00	In this scheme, the only objective is to minimize CDSE; therefore, the material property variables are expected to be the same as the baseline material properties. The remaining system variables are arbitrary other than for constraint satisfaction.	Point	No geometric variables are at their bounds, and the material property variables are identical to the baseline material properties as expected. No constraints are active, but the deflection is within 4mm of the limit.
					Ranged	The material property variables are identical to the baseline material properties as expected, but the face sheet densities are at their lower bounds at the baseline properties. No constraints are active, but the deflection constraint is nearly active.

TABLE 4.40. EXPANSION SOLUTION ANALYSIS, SCHEME 4

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
4	0.60	0.20	0.20	The majority of the weight is placed on the mass goal with equal weight placed on the deflection and CDSE goals; therefore the material property variables are likely to be similar to the baseline material properties but tending towards lower densities.	Point	The material property variables are nearly identical to the baseline material properties, differing at most by one tenth of one MPa in the case of the yield strength variables and seven tenths of one kg/m^3 in the case of the density variables. The cell spacing is at the upper bound, the core height tends toward the upper bound, the cell wall thickness is nearly at the lower bound, the front face sheet thickness is near the midpoint, and the back face sheet thickness tends toward the lower bound. These dimensions are consistent with minimizing the mass of the panel. Both the deflection and relative density constraints are active, similarly to the solution for Scheme 1 in which minimizing the mass is the only goal.
					Ranged	The material property variables are identical to the baseline properties. The cell spacing is at the upper bound, the core height is at the lower bound, the cell wall thickness is near the lower bound, the front face sheet thickness is near the midpoint, and the back face sheet thickness is at the upper bound. The deflection and relative density constraints are active.

TABLE 4.41. EXPANSION SOLUTION ANALYSIS, SCHEME 5

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
5	0.20	0.60	0.20	The majority of the weight is placed on the deflection goal with equal weight placed on the mass and CDSE goals; therefore the material property variables are likely to be similar to the baseline materials but tending towards higher strengths and lower densities. The deflection constraint is not likely to be active but the mass constraint is likely to be active.	Point	The material property variables are nearly identical to the baseline material properties, differing at most by one tenth of one kg/m^3 in the case of the density variables and not at all in the case of the strength variables. The cell spacing tends toward the upper bound, the core height is near the midpoint, the cell wall thickness is nearly at the lower bound, the front face sheet thickness is slightly less than the midpoint, and the back face sheet thickness tends toward the lower bound. The deflection and relative density constraints are both active, and the mass is more than 70 kg away from the limit. On the contrary, the mass constraint is expected to be active, not the deflection constraint. This indicates that the mass goal is dominating the solution.
					Ranged	The yield strength variables all tend toward the lower bounds, while the core strength is at the lower bound. The face sheet densities are at the lower bound, which is the density of the face sheet baseline material. The density of the core layer is less than the density of the core baseline material by about 700 kg/m^3. The cell spacing is at the upper bound, the core height is less than the midpoint, the cell wall thickness is near the lower bound, the front face sheet thickness is somewhat larger than the midpoint, and the back face sheet thickness tends toward the upper bound. The mass constraint is active as expected, and the deflection and relative density constraints are nearly active.

TABLE 4.42. EXPANSION SOLUTION ANALYSIS, SCHEME 6

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
6	0.20	0.20	0.60	The majority of the weight is placed on the CDSE goal with equal weight placed on the mass and deflection goals; therefore, the material property variables are likely to be the same as the baseline material properties.	Point	The material property variables are nearly identical to the baseline material properties, differing at most by one tenth of one kg/m^3 in the case of the density variables and not at all in the case of the strength variables. The cell spacing is at the upper bound, the core height is near the lower bound, the cell wall thickness is nearly at the lower bound, the front face sheet thickness is near the midpoint, and the back face sheet thickness tends toward the lower bound. The deflection and relative density constraints are both active.
					Ranged	The material property variables are all identical to the baseline material properties, except for the core density which differs by one tenth of one kg/m^3. The cell spacing is at the upper bound, the core height is at the lower bound, the cell wall thickness is near the lower bound, the front face sheet thickness is near the midpoint, and the back face sheet thickness is at the upper bound. The deflection constraint is active.

TABLE 4.43. EXPANSION SOLUTION ANALYSIS, SCHEME 7

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
7	0.20	0.40	0.40	The majority of the weight is split equally between the CDSE and deflection goals, with a smaller weight placed on the mass goal; therefore, the mass constraint is likely to be active, and the material properties variables are likely to be similar to the baseline properties but tending towards higher strength and lower density.	Point	The material property variables are nearly identical to the baseline material properties, differing at most by four tenths of one kg/m^3 in the case of the density variables and not at all in the case of the strength variables. The cell spacing is at the upper bound, the core height is near the midpoint, the cell wall thickness tends toward the lower bound, and the face sheet thicknesses are somewhat less than the midpoint. The deflection constraint is active, but the mass constraint is not active as expected. This indicates that the mass goal is dominating the solution..
					Ranged	The material property variables are identical to the baseline material properties. The cell spacing is near the upper bound, the core height is near the lower bound, the cell wall thickness is near the lower bound, the front face sheet thickness is near the upper bound, and the back face sheet thickness is near the midpoint. The mass and relative density constraints are active.

TABLE 4.44. EXPANSION SOLUTION ANALYSIS, SCHEME 8

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
8	0.40	0.20	0.40	The majority of the weight is split equally between the mass and CDSE goals, with a smaller weight placed on the deflection goal; therefore the material property variables are likely to be similar to the baseline material properties but tending towards smaller densities. Also, the layer and wall thicknesses are likely to be smaller and the deflection constraint is likely to be active.	Point	The material property variables are nearly identical to the baseline material properties, differing at most by 1.1 kg/m^3 in the case of the density variables and not at all in the case of the strength variables. The cell spacing is at the upper bound, the core height is at the lower bound, the cell wall thickness is near the midpoint, and the face sheet thicknesses are somewhat less than the midpoint. The deflection constraint is active as expected.
					Ranged	The material property variables are all identical to the baseline material properties, except for the core density which differs by three tenths of one kg/m^3 and the strength of the core layer which differs by two tenths of one MPa. The cell spacing is at the upper bound, the core height is at the lower bound, the cell wall thickness is near the lower bound, the front face sheet thickness is near the upper bound and the back face sheet thickness is slightly larger than the midpoint. The relative density constraint is active and the mass constraint is nearly active. The deflection constraint is not active.

TABLE 4.45. EXPANSION SOLUTION ANALYSIS, SCHEME 9

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
9	0.40	0.40	0.20	The majority of the weight is equally divided between the mass and deflection goals with the remaining weight placed on the CDSE goal; therefore, the mass and deflection constraints are not likely to be active, but the material property variables are likely to tend towards the baseline material properties.	Point	The material property variables are nearly identical to the baseline material properties, differing at most by one tenth of one MPa in the case of the yield strength variables and seven tenths of one kg/m^3 in the case of the density variables. The cell spacing is near the upper bound, the core height is less than the midpoint, the cell wall thickness is nearly at the lower bound, the front face sheet thickness is near the midpoint, and the back face sheet thickness tends toward the lower bound. Both the deflection and relative density constraints are active, similarly to the solution for Scheme 1 in which minimizing the mass is the only goal. This again indicates that the mass goal dominates the solution.
					Ranged	The material property variables are all identical to the baseline material properties, except for the core density which differs by three tenths of one kg/m^3. The cell spacing is at the upper bound, the core height is at the lower bound, the cell wall thickness is near the lower bound, the front face sheet thickness is near the midpoint, and the back face sheet thickness is at the upper bound. The deflection and relative density constraints are active.

TABLE 4.46. EXPANSION SOLUTION ANALYSIS, SCHEME 10

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
10	0.33	0.33	0.33	In this scheme an equal weight is placed on all three goals; therefore, it is likely that the material properties will tend towards the baseline material properties and the mass and deflection constraints will not be active.	Point	The material property variables are nearly identical to the baseline material properties, differing at most by one tenth of one MPa in the case of the yield strength variables and seven tenths of one kg/m^3 in the case of the density variables. The cell spacing is at the upper bound, the core height is near the midpoint, the cell wall thickness tends toward the lower bound, and the face sheet thicknesses are equal at a value slightly less than the midpoint. The deflection constraint is active, which is not expected.
					Ranged	The material property variables are all identical to the baseline material properties, except for the core density which differs by one tenth of one kg/m^3. The cell spacing is at the upper bound, the core height is at the lower bound, the cell wall thickness is near the lower bound, the front face sheet thickness is near the midpoint, and the back face sheet thickness is at the upper bound. The geometric variables in this solution are identical to the solution to Scheme 9, which also has equal weights on the mass and deflection goals. The deflection and relative density constraints are active, which is also the same as in the solution to Scheme 9.

TABLE 4.47. EXPANSION SOLUTION ANALYSIS, SCHEME 11

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
11	0.50	0.50	0.00	In this scheme an equal weight is placed on the mass and deflection goals with no weight placed on the CDSE goal; therefore, it is expected that the density variables will tend towards the lower bound and the strength variables will tend toward the upper bound. Also, it is unlikely for the mass and deflection constraints to be active.	Point	As expected, all density variables are at their lower bounds and the strength variables are at their upper bounds. The cell spacing is near the midpoint, the core layer height is near the lower bound, the cell wall thickness tends toward the lower bound, the front face sheet thickness is slightly less than the midpoint, and the back face sheet thickness is at the lower bound. The deflection constraint is active, which is not expected..
					Ranged	The back face sheet strength is at the upper bound, while the core layer and front face sheet strength variables tend toward the lower bounds. All three density variables are at their lower bounds. The cell spacing is at the upper bound, the core height is greater than the midpoint, the cell wall thickness is near the lower bound, the front face sheet thickness is near the lower bound, and the back face sheet thickness is at the upper bound. The deflection constraint is active and the relative density constraint is nearly active, which is not expected.

To investigate the trades between the three objectives of minimizing mass, deflection and CDSE, the solutions for each problem are summarized by the Pareto front of the three design goals, shown in Figures 4.15 and 4.16. As shown in the Pareto front for the Point Scenario, there appears to be a conflict between minimizing mass and minimizing cost, indicated by the convex relationship between the mass and deflection deviation variables. There does not appear to be a strong relationship between mass and CDSE or deflection and CDSE. In the Pareto front for the Ranged Scenario, there are weak relationships between mass and deflection and between CDSE and deflection, but no apparent relationship between mass and CDSE.

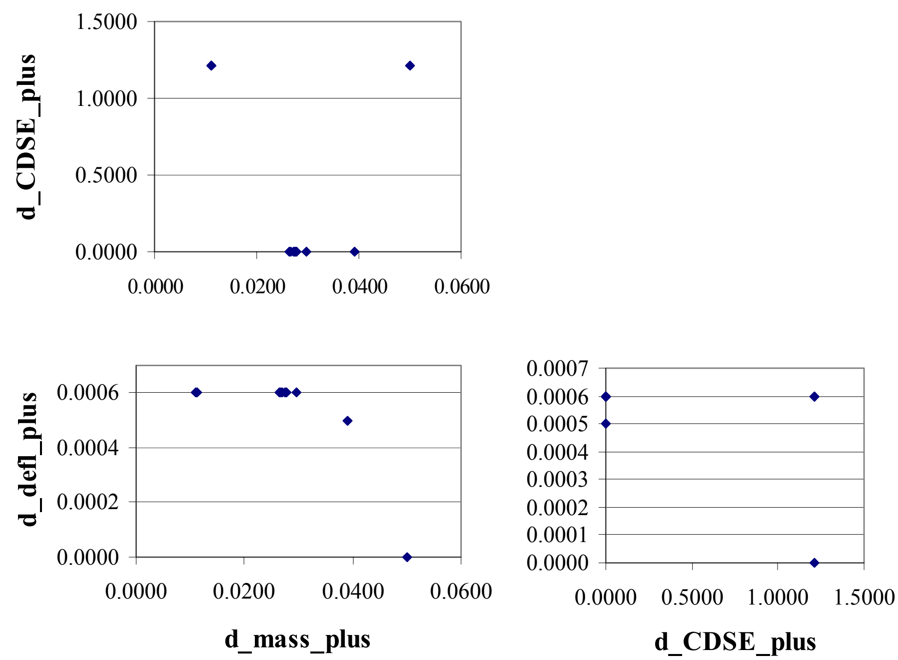


FIGURE 4.15. PARETO FRONT FOR THE POINT SCENARIO

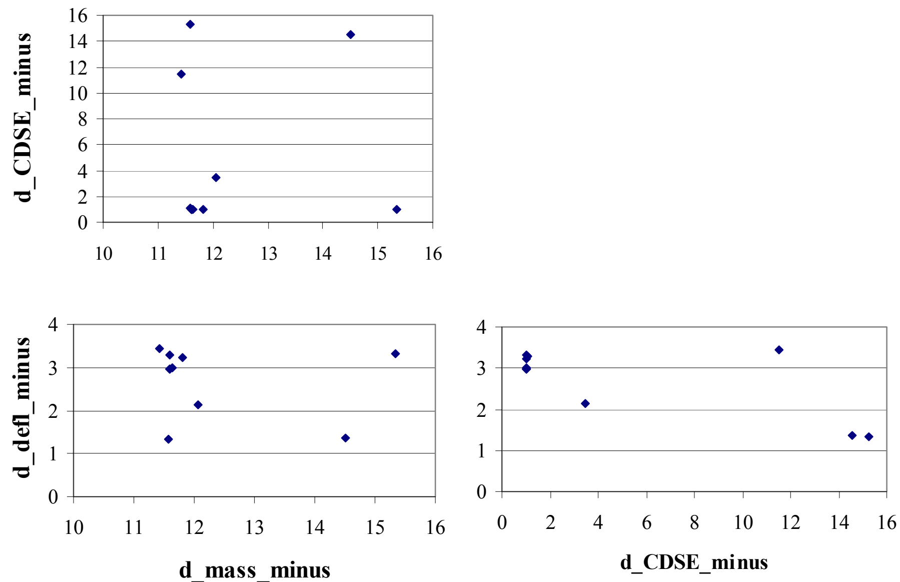


FIGURE 4.16. PARETO FRONT FOR THE RANGED SCENARIO

4.3 THE VDSE METRIC AND MATERIAL PROPERTY TARGETS

With the solutions to the expansion cDSP found, the VDSE metric can be calculated, and the material property targets can be specified for subsequent phases of design. In Section 4.3.1 the calculation of the VDSE metric is discussed and the impact of the values of the VDSE metric on the decision between material design and material selection is discussed. In Section 4.3.2, the material property targets are identified for a sample solution.

4.3.1 The VDSE Metric

Recall from Section 3.2.1 the equation for the VDSE metric, repeated here in Equation 4.32.

$$VDSE = Z_{A,base} - Z_{A,expanded} \tag{4.32}$$

The VDSE metric is calculated for each preference scheme of both the Point and Ranged Scenarios (Step 3.1 of the DSES). These values are shown in Tables 4.48 and 4.49.

TABLE 4.48. VDSE METRIC FOR EACH SOLUTION TO THE POINT SCENARIO

Scheme	Baseline Deviation	Expansion Deviation	VDSE
1	0.136	0.0111	**0.125**
2	**0.005**	0.0000	**0.005**
3	0.000	0.0000	0.000
4	0.086	0.0163	**0.069**
5	0.039	0.0057	**0.033**
6	0.031	0.0057	**0.025**
7	0.035	0.0058	**0.029**
8	0.058	0.0120	**0.046**
9	0.062	0.0112	**0.051**
10	0.052	0.0093	**0.043**
11	0.078	0.0059	**0.072**

TABLE 4.49. VDSE METRIC FOR EACH SOLUTION TO THE RANGED SCENARIO

Scheme	Baseline Deviation	Expansion Deviation	VDSE
1	11.60	11.429	**0.015**
2	2.39	1.358	**0.432**
3	1.00	1.013	*-0.013*
4	7.76	7.759	0.000
5	4.19	4.394	*-0.049*
6	3.51	3.541	*-0.009*
7	3.87	4.058	*-0.049*
8	5.64	5.717	*-0.014*
9	6.02	6.038	*-0.003*
10	5.19	5.199	*-0.002*
11	7.280	6.459	**0.821**

To analyze the VDSE values, hypotheses are again identified for each of the preference schemes and each scenario. These hypotheses and the corresponding results are displayed in Tables 50 through 55. For each of the schemes in which no weight is placed on the CDSE goal, it is expected that the VDSE is positive, because there is no penalty for drastic changes in the material property variables. It is seen that this is indeed the case for both the Point and Ranged Scenarios (see Schemes 1, 2, and 11). It is also seen that increasing the weight on the CDSE goal, while keeping the weights on the mass and deflection goals equal to each other results in a lower VDSE value. This is the expected result.

A positive value for the VDSE indicates that the expanded material design space enabled an improvement in performance relative to the baseline solution without too much of an increase in CDSE. Conversely, a negative value for the VDSE indicates that the improvement in performance in the solution in the expanded material design space is outweighed by the increase in the cost of expanding the design space. Excluding Scheme 3, in which no improvement is possible by expanding the material design space, the solutions to all schemes of the Point Scenario in the expansion cDSP represent an improvement in the achievement of design goals as compared to the baseline solution. Furthermore, in Scheme 2 of the Point Scenario, the VDSE metric is equal to the deviation at the baseline solution, indicating that expanding the material design space enabled the deviation to be minimized to zero in that scheme. For the Ranged Scenario, only Schemes 1, 2, and 11 show an improvement due to expanding the design space, and in each of these schemes no weight is placed on the CDSE goal. In Scheme 4 of the Ranged Scenario there is no change in the achievement of design goals, but in the remaining schemes, the performance improvements in the expansion solutions are outweighed by the cost of expanding the design space.

TABLE 4.50. VDSE ANALYSIS, SCHEMES 1-3

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
1	1.00	0.00	0.00	In this scheme the only objective is to minimize mass with no weight on the CDSE goal; therefore the VDSE is likely to be positive because there is no penalty for large changes in the material property variables.	Point	The VDSE is positive and equal to 0.125.
					Ranged	The VDSE is positive and equal to 0.015.
2	0.00	1.00	0.00	In this scheme the only objective is to minimize deflection with no weight placed on the CDSE goal; therefore the VDSE is likely to be positive because there is no penalty for large changes in the material property variables.	Point	The VDSE is positive and equal to 0.005.
					Ranged	The VDSE is positive and equal to 0.432.
3	0.00	0.00	1.00	In this scheme the only objective is to minimize CDSE; therefore, the VDSE is expected to be equal to zero because the it is not possible to improve in the achievement of the CDSE goal in the expanded design space.	Point	The VDSE is indeed equal to zero.
					Ranged	The VDSE is slightly less than zero due to small variations in the material properties in the expansion solution.

TABLE 4.51. VDSE ANALYSIS, SCHEMES 4-5

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
4	0.60	0.20	0.20	The majority of the weight is placed on the mass goal with equal but less weight placed on the deflection and CDSE goals; therefore, due to the non-zero weight on the CDSE goal the VDSE is likely to be smaller than the VDSE for Scheme 1, but the VDSE may or may not be positive.	Point	The VDSE is equal to 0.069, which is indeed smaller than the VDSE for Scheme 1.
					Ranged	The VDSE is equal to zero because there is no change in the deviation from the baseline to the expansion solution. This value is indeed smaller than the VDSE for Scheme 1.
5	0.20	0.60	0.20	The majority of the weight is placed on the defleciton goal with equal but smaller weight placed on the mass and CDSE goals; therefore, due to the non-zero weight placed on the CDSE goal, the VDSE is likely to be smaller than the VDSE for Scheme 2, but the VDSE may or may not be positive.	Point	The VDSE is equal to 0.033, which is larger than the VDSE in Scheme 2, but smaller than the VDSE in Scheme 1. This is consistent with the finding that the mass goal dominates the deflection goal in the Point Scenario.
					Ranged	The VDSE is equal to -0.049, which is indeed smaller than the VDSE in Scheme 2.

TABLE 4.52. VSDE ANALYSIS, SCHEMES 6-7

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
6	0.20	0.20	0.60	The majority of the weight is placed on the CDSE goal with equal weight placed on the mass and deflection goals. Compared to other schemes in which an equal weight is placed on the mass and deflection goals (Schemes 6, 9, 10, and 11), this scheme has a larger weight placed on the CDSE goal than in Schemes 9, 10, and 11. Therefore, the VDSE for this scheme is expected to be smaller than the VDSEs for Schemes 9, 10, and 11.	Point	The VDSE is equal to 0.025, which is indeed smaller than the VDSE for Schemes 9, 10, and 11.
					Ranged	The VDSE is equal to -0.009, which is indeed smaller than the VDSE for Schemes 9, 10, and 11.
7	0.20	0.40	0.40	The majority of the weight is split equally between the CDSE and deflection goals, with a smaller weight placed on the mass goal; therefore, due to the large weight placed on the CDSE goal the VDSE for this scheme is likely to be small.	Point	The VDSE is equal to 0.029.
					Ranged	The VDSE is equal to -0.049.

TABLE 4.53. VDSE ANALYSIS, SCHEMES 8-9

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
8	0.40	0.20	0.40	The majority of the weight is split equally between the mass and CDSE goals, with a smaller weight placed on the deflection goal; therefore due to the large weight placed on the CDSE goal the VDSE is likely to be small.	Point	The VDSE is equal to 0.046.
					Ranged	The VDSE is equal to -0.014.
9	0.40	0.40	0.20	The majority of the weight is equally divided between the mass and deflection goals with the remaining weight placed on the CDSE goal. Compared to other schemes in which an equal weight is placed on the mass and deflection goals (Schemes 6, 9, 10, and 11), this scheme has a smaller weight placed on the CDSE goal than in Schemes 6 and 10 and a larger weight placed on the CDSE goal than in Scheme 11. Therefore, the VDSE for this scheme is expected to be smaller than the VDSE for Scheme 11 but larger than the VDSE's for Schemes 6 and 10.	Point	The VDSE is equal to 0.051, which is indeed smaller than the VDSE for Scheme 11 but larger than the VDSE's for Schemes 6 and 10.
					Ranged	The VDSE is equal to -0.003, which is indeed smaller than the VDSE for Scheme 11 and larger than the VDSE for Scheme 6; however the VDSE in this scheme is not larger than the VDSE in Scheme 10, as expected, but is within one one-thousandth of the VDSE for Scheme 10.

TABLE 4.54. VDSE ANALYSIS, SCHEME 10

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
10	0.33	0.33	0.33	In this scheme an equal weight is placed on all three goals. Compared to other schemes in which an equal weight is placed on the mass and deflection goals (Schemes 6, 9, 10, and 11), this scheme has a smaller weight placed on the CDSE goal than in Scheme 6 and a larger weight placed on the CDSE goal than in Schemes 9 and 11. Therefore, the VDSE for this scheme is expected to be smaller than the VDSEs for Schemes 9 and 11 but larger than the VDSE for Scheme 6.	Point	The VDSE is equal to 0.043, which is indeed smaller than the VDSEs for Schemes 9 and 11 but larger than the VDSE for Scheme 6.
					Ranged	The VDSE is equal to -0.002, which is indeed smaller than the VDSE for Scheme 11 and larger than the VDSE for Scheme 6; however the VDSE in this scheme is not smaller than the VDSE in Scheme 9, as expected, but is within one one-thousandth of the VDSE for Scheme 9.

TABLE 4.55. VDSE ANALYSIS, SCHEME 11

Scheme	Mass	Deflection	CDSE	Hypothesis		Result
11	0.50	0.50	0.00	In this scheme an equal weight is placed on the mass and deflection goals with no weight placed on the CDSE goal. Compared to other schemes in which an equal weight is placed on the mass and deflection goals (Schemes 6, 9, 10, and 11), this scheme has a smaller weight placed on the CDSE goal than in Schemes 6, 9, and 10. Therefore, the VDSE for this scheme is expected to be larger than the VDSEs for Schemes 6, 9, and 10. Also, since there is zero weight placed on the CDSE goal, the VDSE is expected to be positive.	Point	The VDSE is positive and equal to 0.072, which is indeed larger than the VDSE for Schemes 6, 9, and 10.
					Ranged	The VDSE is positive and equal to 0.821, which is indeed larger than the VDSE in Schemes 6, 9, and 10, as expected.

Depending on the targets for each objective and the weighting coefficients on each of the deviation variables, it is possible to achieve a deviation function value of zero at the baseline solution, indicating that the targets have been met. Another possibility is that the absolute minimum of the goal may be realized at the baseline solution. In those situations, no improvement is possible in the deviation function by expanding into the material design space. Accordingly, the VDSE metric will be negative for all design points in the expanded design space except at the baseline solution. This is the case in Scheme 3 of both scenarios. If the deviation function is at a minimum in the baseline solution, the VDSE will be negative, indicating that moving away from the baseline solution results in a worse design.

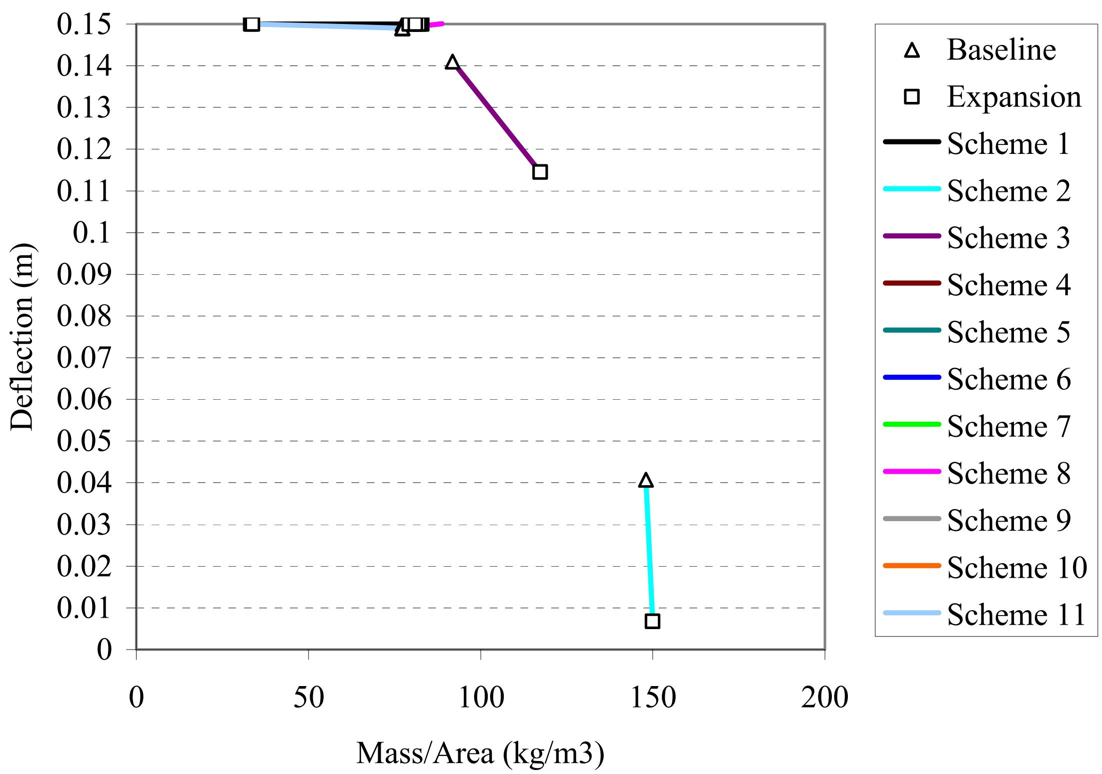


FIGURE 4.17. MAPPING THE VDSE METRIC IN THE PERFORMANCE SPACE, POINT SCENARIO

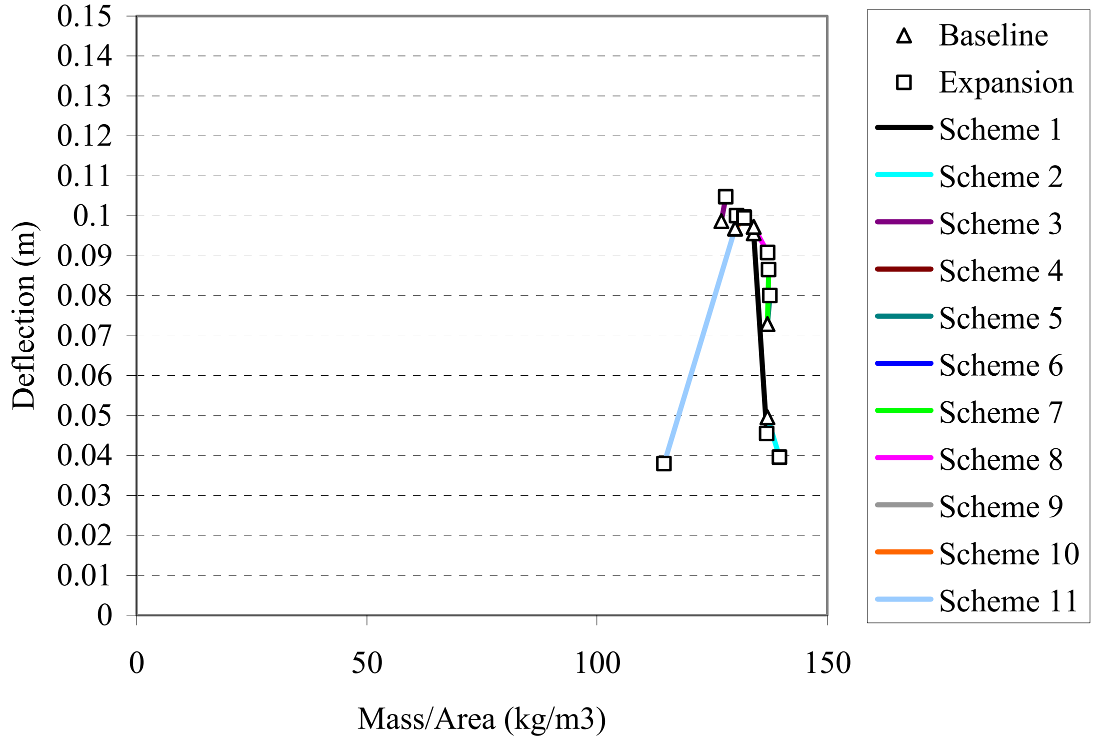


FIGURE 4.18. MAPPING THE VDSE IN THE PERFORMANCE SPACE, RANGED SCENARIO

To understand the meaning of the VDSE in terms of the achievement of design goals, the baseline and expansion solutions for each scenario are compared in the performance space. In Figures 4.17 and 4.18 the VDSE is indicated by lines connecting the baseline and expansion solutions in the performance space (i.e. on a plot of deflection versus mass per area). The lengths of the connecting lines indicate the magnitude of the change in the achievement of the design goals. The solutions for the Point Scenario are shown in Figure 4.17. Recall that the VDSE metric is positive for all preference schemes in the Point Scenario; however, the length of the lines connecting the baseline solutions to the expansion solutions indicates that the amount of improvement in the achievement of the deflection and mass goals is very small. Therefore, a designer may choose not to pursue material design when the VDSE metric is positive if the improvement in the achievement in the design goals is small. The schemes which show large improvements (Schemes 1, 2, 3, and 11) have at least one goal with zero weight, meaning that at least one of the three goals is ignored in these schemes. Similarly, the lengths of the connecting lines in the Ranged Scenario (shown in Figure 4.18) show large changes for Schemes 1 and 11, both of which have zero weight placed on the CDSE goal.

The directions of the connecting lines are also important. An overall improvement in the achievement of design goals occurs when the expansion solution (shown as a square point) is closer to the origin than the baseline solution (shown as a triangular point). This can be seen in the connecting line for Scheme 11 in the Ranged Scenario (Figure 4.18).

4.3.2 Material Property Targets

If the VDSE metric is positive and the magnitude of the improvement in design goals is large enough, a designer may decide to pursue material design. In that case, material property targets can be identified based on the values of material property variables in the solution to the expansion cDSP. To demonstrate the identification of material property targets, the process is discussed for three schemes in the BRP example. For the Point Scenario, the material property targets are identified for Scheme 10. For the Ranged Scenario, the material property targets are identified for Scheme 11, which had the largest value of VDSE in the previous section, and for Scheme 2, which also has a positive value of VDSE. For the Point Scenario, the targets are a single point; however, for the Ranged Scenario, the material property targets are a range of values. The material property targets for all three schemes are shown in Table 4.56. These targets are compared to databases of material properties to determine if an existing material meets the targets (Step 3.2.1). If an existing material does not meet the targets, then the targets are used to guide the design of the material.

TABLE 4.56. MATERIAL PROPERTY TARGETS FOR THREE SCHEMES

	Units	Mean Value	+/-	Targets	
Point Scenario, Scheme 10					
Yield Strength, Back	MPa	200.06	n/a	200.06	
Yield Strength, Core	MPa	538.06	n/a	538.06	
Yield Strength, Front	MPa	200.02	n/a	200.02	
Density, Back	kg/m3	1770.70	n/a	1770.70	
Density, Core	kg/m3	7845.43	n/a	7845.43	
Density, Front	kg/m3	1770.70	n/a	1770.70	
Ranged Scenario, Scheme 2				lower	upper
Yield Strength, Back	MPa	1180.00	20	1160.00	1200.00
Yield Strength, Core	MPa	170.00	150	20.00	320.00
Yield Strength, Front	MPa	504.32	20	484.32	524.32
Density, Back	kg/m3	2352.58	170	2182.58	2522.58
Density, Core	kg/m3	2509.47	100	2409.47	2609.47
Density, Front	kg/m3	1770.00	170	1600.00	1940.00
Ranged Scenario, Scheme 11				lower	upper
Yield Strength, Back	MPa	1180.00	20	1160.00	1200.00
Yield Strength, Core	MPa	198.39	150	48.39	348.39
Yield Strength, Front	MPa	504.32	20	484.32	524.32
Density, Back	kg/m3	1770.00	170	1600.00	1940.00
Density, Core	kg/m3	1700.00	100	1600.00	1800.00
Density, Front	kg/m3	1770.00	170	1600.00	1940.00

To determine if an existing material meets the material property targets, material databases are consulted (Step 3.2.1). In this example the online database maintained by Automation Creations is used for a quick search of available materials (Automation Creations, 2007). For the Point Scenario, the targets are single numbers rather than a range of acceptable properties, and as such, it is unlikely that an exact match will be found in a search of existing materials. However, the targets that are identified for this particular scheme are very similar to the properties of the baseline materials. Therefore, the baseline materials could be selected for this scheme.

In the case of the Ranged Scenario, it is much more likely to find an existing material that meets the targets, because the targets are a range of material properties rather than a point solution. For the front face sheet in Scheme 2, a carbon composite material is found that meets the range of material properties (Industrial Laminates/Norplex NP545 Carbon Fiber). For the core layer, several aluminum alloys are found that meet the property targets. For the back face sheet, no materials are found in the database that meet the property targets. For Scheme 11, no materials are found in the database that meet the property targets for the back face sheet. Many magnesium alloys are found that meet the property targets for the core layer. And for the front face sheet, the material property targets are the same for Scheme 11 as for Scheme 2, so the carbon composite material found for Scheme 2 also meets the targets for Scheme 11.

To gain further insight, the targets are highlighted on the material selection chart shown in Figure 4.19 in Step 3.2.2 of the DSES. For the Point Scenario, the targets are actually points, and the points fall into the expected points on the plot for steel and magnesium. In the Ranged Scenario, the targets for the back face sheet fall into a range common to glasses. The front face sheet, on the other hand, falls into material property ranges that are common to engineered composites. The targets for the core cover an area in the plot populated by aluminum alloys. Composites and glasses have very different material behavior as compared to metals, and the analysis models used in the BRP design assume a stress-strain behavior of metals. Therefore, the differences in the behavior of these types of materials must be investigated before using them in the BRP design. For the cases in which no material is found that meets the targets, a material must be designed to meet these targets. For Scheme 2 of the Ranged Scenario, a designer could select materials that meet the front face sheet and core layer material property targets, but a material must be designed to meet the material property targets for the back face sheet.

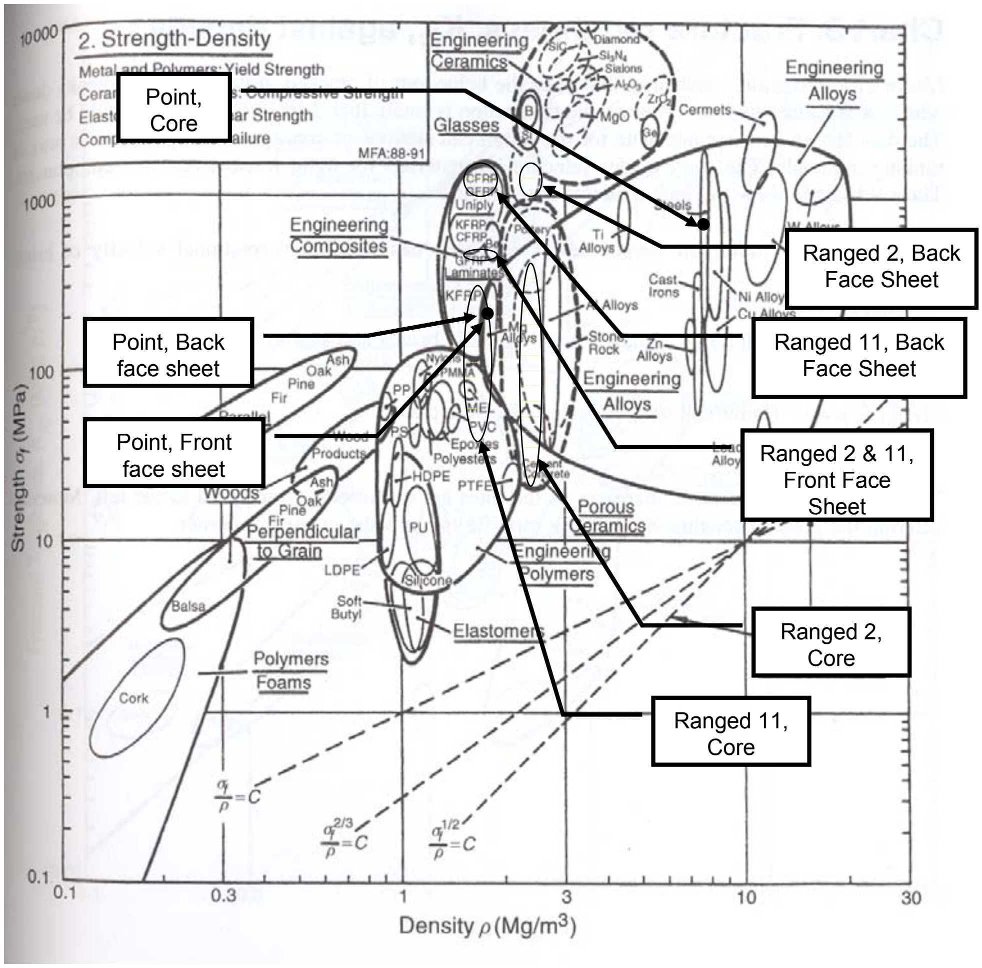


FIGURE 4.19. MATERIAL SELECTION CHART WITH TARGETS HIGHLIGHTED (ASHBY, 1999)

4.4 VALIDATION: DOMAIN-SPECIFIC PERFORMANCE VALIDITY

Domain-specific performance validity refers to the usefulness of the proposed method in achieving the intended purpose of the method, as introduced in Section 1.4. The usefulness must be attributed to the application of the proposed method. In addition, the accuracy and internal consistency of the empirical data used in the example must be verified. The intended purpose of the design space expansion strategy is to identify the value of expanding the material design space in the context of a product design. The empirical validity of the models is discussed in Section 4.4.1. The domain-specific performance validity of the DSES is discussed as well as the implications of the design solutions on the usefulness of the method in Section 4.4.2.

4.4.1 BRP Model Validity

In the following section, a look at the validity of the models used in the BRP design example is presented. The BRP analysis models are discussed in addition to the models for the expected cost of design space expansion. The following two sections were developed in collaboration with Hannah Muchnick and also appear in her thesis (Muchnick, 2007). The finite element analysis reported here was conducted by Jin Song.

<u>*BRP Analysis Models*</u>

The equations used in generating BRP computational models are based on the work of Hutchinson and Xue (2005) and are detailed in Section 4.1.3. A cost function is also defined in that section to reflect the designer's beliefs about how the cost of the design process will be affected by diverging from the baseline material. The validation of the cost model is addressed later in this section.

Confidence is built in the validity of the BRP models due to the fact that they are based on existing performance calculations of BRPs found in the literature (Hutchinson and Xue, 2005). Although the equations for BRP deflection are taken directly from the work of Hutchinson and Xue, BRP deflection solutions calculated as part of the research in this thesis do not match the deflection results published by Hutchinson and Xue (2005). Deflection calculations based on the work in this thesis disagree with published deflection calculations by approximately an order of 2. After rigorous examination for possible errors, researchers in the Systems Realization Lab (SRL) at Georgia Tech are confident that the BRP performance equations have been implemented correctly. Due to this inconsistency, researchers in the SRL at Georgia Tech have contacted Hutchinson and Xue in an effort to resolve this in consistency. Since this issue is not yet resolved, it is left as future work in the BRP design project.

Finite Element Analysis

To provide further validation to the computational design tools used to predict BRP performance, finite element analysis (FEA) of BRP performance is currently being conducted. Once BRP a design solution is obtained, a 3D model of the designed BRP is imported to the commercial FEA software, ABAQUS for further deformation analysis. An example of a BRP analyzed in ABAQUS is shown in Figure 4.20.

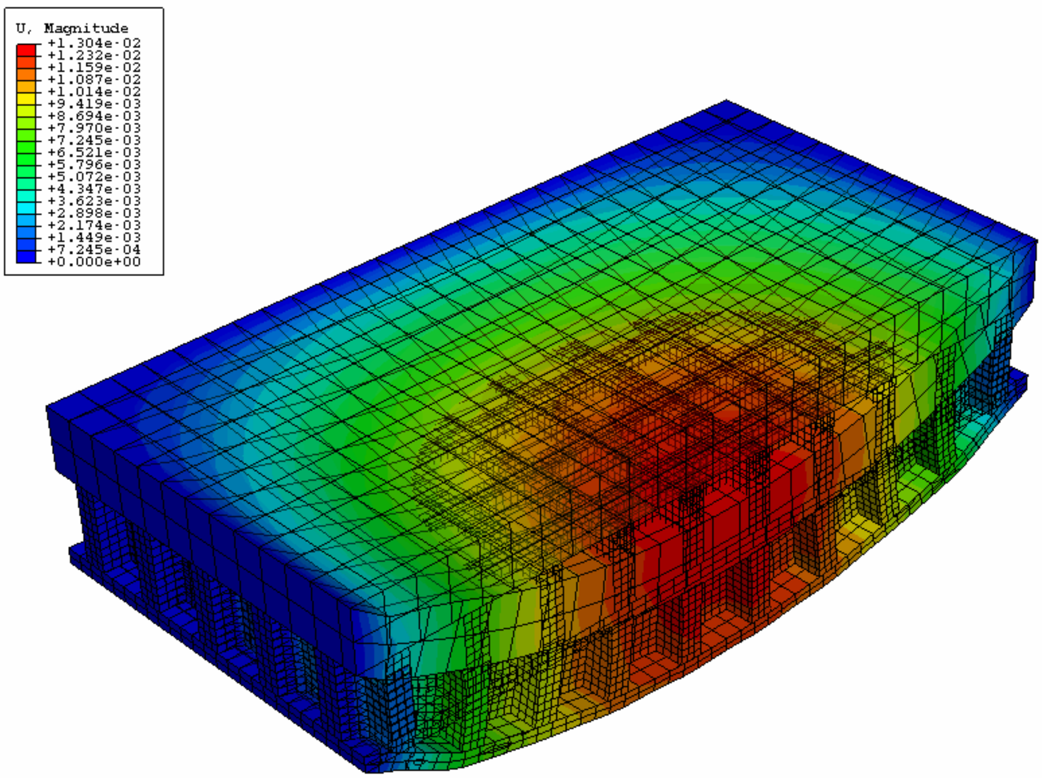


FIGURE 4.20. BRP DEFLECTION ANALYZED IN ABAQUS

Current efforts to validate the mathematical models used to predict BRP performance using FEA are in progress. Due to the complexity of parametrizing FEA simulations and the time required to simulate the plastic behavior of the panel, collecting enough data to validate the models via FEA simulations is very time-consuming. As such, the number of

FEA simulations needed to validate these empirical models are currently unavailable, and this portion of model validation will be addressed in the future work of BRP design.

Analyzing BRP performance using FEA will become extremely valuable in the future when BRP design continues to increase in complexity. For example, there is a research interest in filling various cells in the BRP core layer with a ceramic particle powder. It is assumed that this will further decrease BRP deflection without significantly increasing panel mass. Analyzing complex phenomena such as energy dissipation in ceramic particles will most likely be completed using FEA software.

Cost of Design Space Expansion

To verify that the CDSE function performs as desired, contour plots are generated for the two baseline materials identified in Section 4.3.1. Plots are generated for two sets of weighting coefficients to see the effect of the weighting coefficients on the CDSE. The contour plots are shown in Figures 4.21 through 4.24. As expected, the minimum of the CDSE lies at the point in the material design space embodied by the corresponding baseline material. In addition, the CDSE monotonically increases as the solutions diverge from the baseline material. By comparing the plots with the nominal weights to the plots with the doubled weights, it is evident that increasing the weighting factor indeed results in an increase in the slope of the CDSE. Furthermore, by the shape of the contour lines, it can be seen that the weights chosen for the yield strength and density dimensions result in a CDSE that reflects the beliefs that it is easier (i.e., less costly) to vary in the yield strength dimension than it is to vary in the density dimension. This is the expected behavior because the yield strength of alloys can be varied through heat treatment, but the density is difficult to change because it is dependent on the constituent materials of the particular alloy.

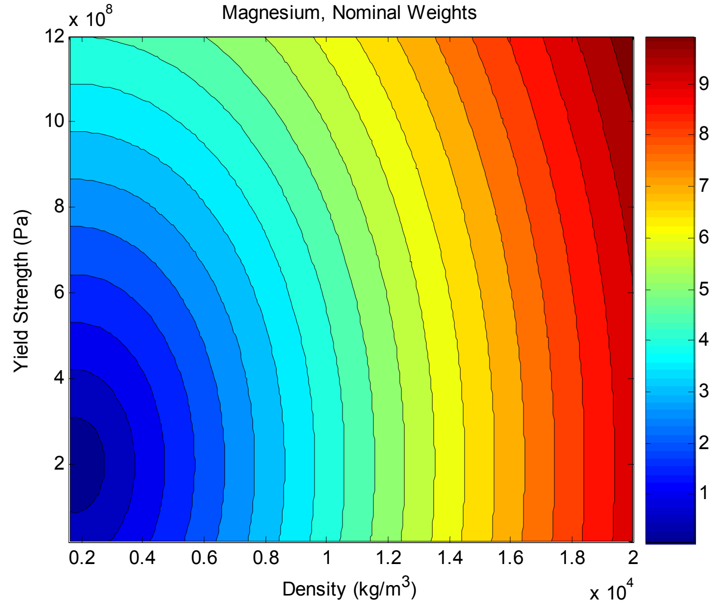


FIGURE 4.21. MAGNESIUM BASED CDSE FUNCTION WITH NOMINAL WEIGHTS

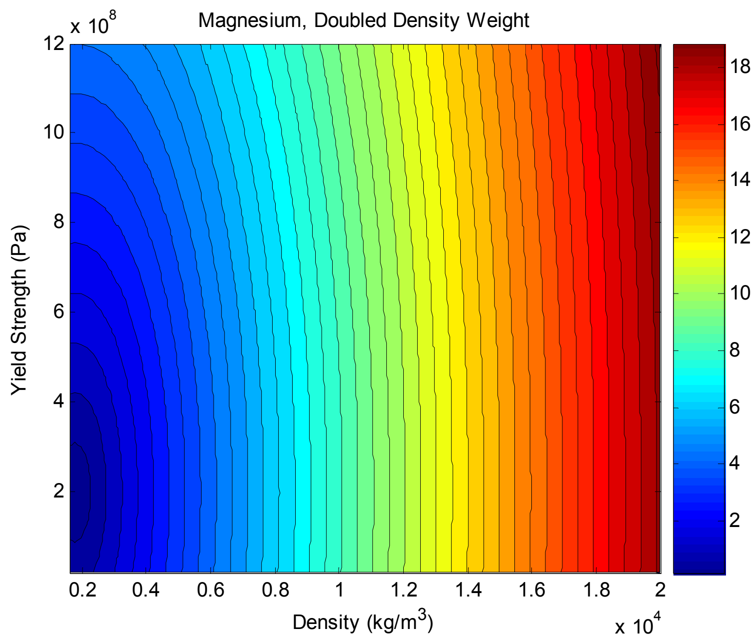


FIGURE 4.22. MAGNESIUM BASED CDSE FUNCTION WITH DOUBLED DENSITY WEIGHT

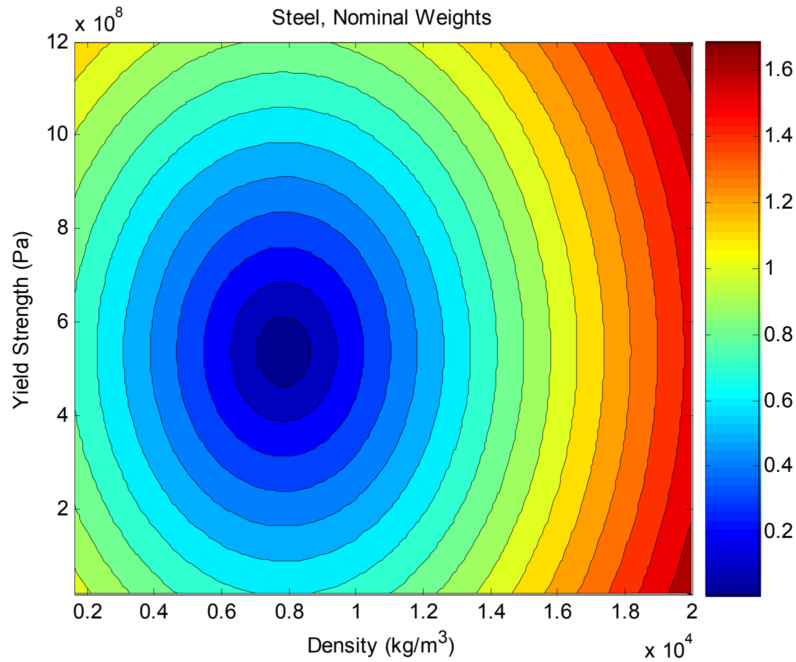


FIGURE 4.23. STEEL BASED CDSE FUNCTION WITH NOMINAL WEIGHTS

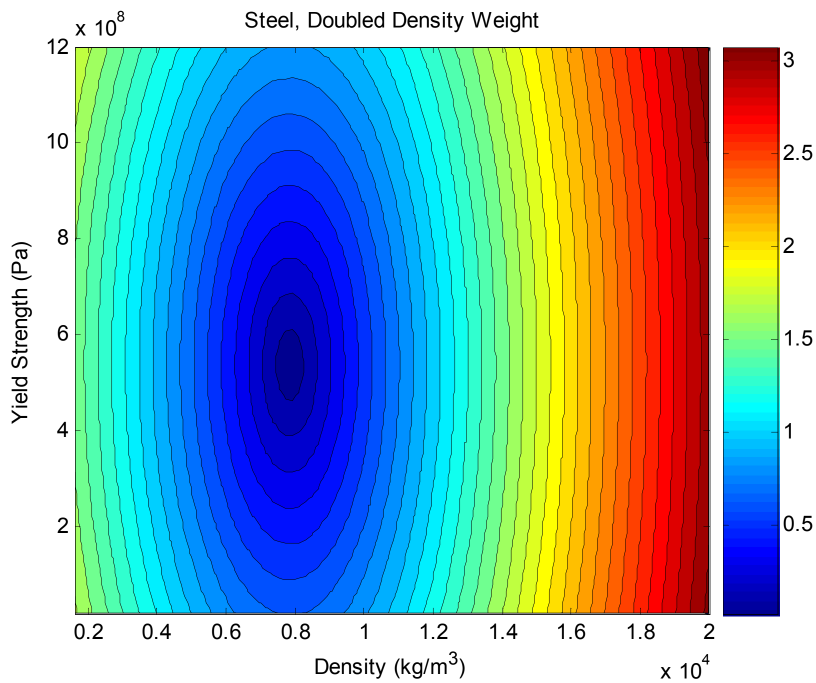


FIGURE 4.24. STEEL BASED CDSE FUNCTION WITH DOUBLED DENSITY WEIGHT

4.4.2 Assessing the Usefulness of the DSES

An assessment must be made of the usefulness of the design space expansion strategy with respect to its intended purpose: assessing the value of expanding the material design space for supporting the decision between material selection and material design. In demonstrating the strategy for the design of BRPs, it is shown that useful results can be found. By using DCIs in the ranged BRP scenario, ranged sets of design solutions are found that include values for material property variables as well as values for system variables important for BRP design. These ranged sets of solutions meet ranged constraints and goals and thus preserve design freedom for subsequent phases of design. By defining the material properties as independent design variables, the freedom of material design is embodied without the development of a complex material model, which is a difficult task in itself.

How do the VDSE metric values calculated in this example support the decision between material design and selection?

For every scenario in which the VDSE is positive, the solutions that are presented represent an improvement in the achievement of system goals that outweighs the increase in the CDSE. By comparing the expansion solutions to the baseline solutions, it is apparent that improvement in design goals is facilitated by tailoring the material properties to meet the identified material property requirements. The actual choice between material design and selection is inherently linked to the expected cost of design space expansion and the expression of preferences in the deviation function. Designers must have confidence in these preference models in order to use this strategy to make the decision between design and selection. With confidence in the models and preferences, the designer can use the VDSE metric to determine when to choose design over selection.

The conclusion in this example is that material selection is the preferred course of action in BRP design; however, the validity of this conclusion rests on the assumptions made in the BRP performance models, the target values for the design goals, the CDSE function, and the weights in the deviation function. A change in any of these assumptions will likely change the results. Specifically, if a different designer formulates this same BRP problem, she is likely to have different preferences and beliefs, and these preferences and beliefs are manifested in the formulation of the baseline and expansion cDSPs. As such, the baseline and expansion solutions are likely to change and the VDSE metric and resulting decision between material selection and design are also likely to change

4.5 A LOOK BACK AND A LOOK AHEAD

In this chapter, the DSES is demonstrated in a BRP design example. This demonstration is provided both as an explanation of the implementation of the strategy and for the assessment of domain-specific structure and performance validity of the strategy. In Chapter 5, the thesis is summarized and critically reviewed. The validation of the research hypotheses is concluded with the assessment of domain-independent performance validity. In addition, opportunities for future work are identified.

CHAPTER 5

CLOSING REMARKS

In this chapter, the thesis is summarized and critically reviewed. The intention in this thesis is to develop a strategy for supporting a designer's choice between material selection and material design in the context of product design. The motivation for establishing the Design Space Expansion Strategy and the VDSE metric, the details of the strategy itself, and the results obtained by applying the strategy to a blast resistant panel design problem are summarized in Section 5.1. In Section 5.2, the research questions and hypotheses posed in Section 1.3.2 are revisited and critically evaluated with emphasis on the validity of the research hypotheses beyond the example problem described in this thesis. Based on the summary and critical review, the achievements and contributions reported in this thesis are presented in Section 5.3, followed in Section 5.4 by opportunities for future work.

5.1 A SUMMARY OF THIS THESIS

A paradigm shift is underway in engineering design in which the classical approach of material *selection* is being replaced by the systematic *design* of materials in the context of product designs. In this thesis, the focus is on developing a strategy for supporting a designer's decision between material selection and material design in the context of product design. Material property targets are determined from the product requirements and detail a range of acceptable material properties to most closely achieve the product requirements. A range of material property targets is preferred to a deterministic set of requirements to maintain design freedom for subsequent phases of design.

In this thesis, the Design Space Expansion Strategy (DSES) is presented as a strategy for assessing the value of expanding the material design space by designing relative to material selection. A material design space is defined by assuming the material properties to be independent design variables which vary continuously between an upper and lower bound. It is assumed that points in this material design space can be realized by modifying existing materials in a multiscale material design process. This material design space is *expanded* relative to the point design space of a baseline material. The expanded design space embodies an increase in the design freedom because it allows more freedom for the designer to tailor the material properties to the product. This expansion of the material design space also has the potential to significantly increase the complexity of the product design process by the inclusion of material design; therefore, a cost function is defined by the designer to quantify the designer's beliefs about the design process impact of diverging from the existing baseline material. This function is called the Cost of Design Space Expansion (CDSE). By including a designer-specified cost or penalty function in the multi-objective formulation of the DSES, a designer is able to weigh the trade-off between the design freedom and complexity of material design with the simplicity and possible design limitations of material selection.

A key component of the DSES is the Value of Design Space Expansion (VDSE) metric. Based on existing value of information metrics, the VDSE metric quantifies the improvement in the achievement of design goals by expanding the material design space. By calculating the VDSE metric, designers are able to measure the value of expanding the material design space in terms of how an expansion of the design space enables an improvement in the achievement of product and design process goals. Positive values of the VDSE metric indicate that the solution is an improvement in the achievement of product goals over the baseline solution and that this improvement outweighs the increase in design complexity associated with expanding the material design space. Conversely, the VDSE metric is negative when the increase in the design complexity due to expanding the material design space outweighs the improvement in product goals in the expansion solution relative to the baseline solution.

The DSES is demonstrated in Chapter 4 for the design of blast resistant panels. The design of blast resistant panels is an appropriate example problem because there is a trade-off between material and structural solutions. At the outset of BRP design it is unclear if the freedom of material design is necessary for achieving product goals, given that material design increases the complexity of the design process. Two BRP design scenarios are presented, and several preference schemes are considered. The results from this example problem are used to build confidence in the validity of the DSES.

5.2 ANSWERING THE RESEARCH QUESTIONS AND VALIDATING THE HYPOTHESES

The DSES and VDSE metric are established to answer the research questions posed in Section 1.3.2. The DSES and the VDSE metric are an embodiment of the three hypotheses proposed in Section 1.3.2 for answering the research questions. In this section, each hypothesis is revisited and the validation of each hypothesis according to the validation plan identified in Figure 1.6 is discussed. In Sections 5.2.1 through 5.2.3 summaries are provided of the arguments made throughout the thesis regarding the domain-independent and domain-specific validity of each hypothesis. In Section 5.2.4, attention is focused on the domain-independent performance validity of the DSES, which involves building confidence that the strategy is valid in a general sense beyond the scope of the example problem presented in this thesis.

5.2.1 Question 1: Evaluating Material Concepts

In Hypothesis 1, an expansion of the material design space is proposed by assuming that material properties are independent design variables rather than fixed, discrete solutions. A designer models the material by assuming that the material property variables vary continuously between upper and lower bounds, and that a material defined by any point in the material design space can be realized by modifying the composition and processing path of existing materials using a systematic material design process. The material model is then embedded in the performance models commonly used by mechanical designers which use tabulated material properties to represent an abstraction of the actual material. Agile material design space exploration is enabled by the formulation of the expansion cDSP and the simplicity of the performance models. By adjusting the weighting factors of an Archimedean deviation function, designers are able to explore the trade-offs between several goals while leaving the doors open to material design or material selection.

The domain-independent structure validity of the cDSP is established in Chapter 2 with a review of the literature and a critical discussion of the domain of application of the construct. The validity of the cDSP for multi-objective decision making in engineering design has been well established in the literature. Archimedean formulations of the deviation function allow designers to specify preferences on the achievement of multiple goals using simple weights. The cDSP has also been extended in the utility-based cDSP to model non-linear designer preferences in the form of utility functions. This widely applicable multi-objective decision support construct is appropriate for a foundation of the DSES.

By using the cDSP as the multi-objective decision support foundation of the DSES, multiple product and design process goals can be included. Moreover, by defining the material design space using independent material property variables, the models used to predict product performance embody the freedom of material design without an increase in complexity which would negatively impact the ease of design space exploration.

A ranged form of the cDSP using design capability indices (DCIs) is used to find ranged sets of material property targets which meet ranged design requirements. Design capability indices are used for both constraints and goals in the formulation of the ranged cDSP, where constraints represent requirements that must be met and goals represent requirements that should be taken into consideration whenever possible.

The structure validity of the ranged form of the cDSP with design capability indices is established in Chapters 2 and 3 with a review of the literature in Section 2.2.2 and a discussion of the internal consistency of the DSES in Section 3.3.1. DCIs are appropriate for finding ranged sets of design solutions which meet ranged sets of design requirements and have been demonstrated by several authors (see Section 2.2.2). In the DSES, the DCIs are used to find ranged sets of material property targets that satisfy ranged goals and constraints. To apply DCIs, the designer must have a way to quantify the sensitivity of the product performance as a result of variation in some uncertain input factors. In the DSES, the uncertain input factors are the material property variables. The amount of uncertainty in the material property variables is defined by the designer by surveying the variability in the material properties of the baseline material due to differences in processing. This gives an indication of the degree to which the material properties can be tailored to product-specific requirements. The sensitivity of the product performance due to variations in the material properties is approximated by a 1st order Taylor series expansion, which has been shown to be an appropriate method for quantifying the sensitivity of the response when an analytical expression for the response is available.

In Chapter 4, the DSES is demonstrated for the design of blast resistant panels, and the domain-specific structure and performance validity of the DSES are discussed. Two scenarios are identified for testing all aspects of the DSES with respect to the domain-specific validity. Variation in the material property variables as well as uncertainty in noise factors is considered in a Ranged Scenario. By applying constraints and goals in the DSES in the form of DCIs, ranges of material property targets are found which are robust to both sources of uncertainty and meet the ranges of design requirements modeled by the constraints and goals.

5.2.2 Question 2: The Value of Design Space Expansion Metric

In Section 3.2, the VDSE metric is proposed as the metric for supporting the decision between material selection and material design. The VDSE quantifies the improvement in the achievement of design goals in the expanded design space relative to the achievement of design goals at a baseline solution. The VDSE is based on the Improvement Potential metric which has been used in engineering design to make the meta-level of decision of when to stop gathering information for simulation model refinement. In this thesis, the meta-level decision is whether or not to gather more information by developing a complex multiscale material model. Since the cost of development of this model is the very cost that designers are hoping to avoid, the ex-post value of information metrics used previously in engineering design are not appropriate, and an ex-ante or conditional value of information metric is desired. Rather than directly quantifying the value of the additional information of the material model, the VDSE metric quantifies the value of the additional design freedom afforded by expanding the material design space from a discrete point to a continuous design space.

The VDSE is applied in the example problem on BRP design in Section 4.3.1. By calculating the VDSE metric for the BRP design problem, the value of expanding the material design space for the BRP is assessed. Based on the calculated VDSE values, it is found that the improvements in the system goals are outweighed by the increase in the cost of expanding the design space, and as such, material selection is the preferred option in the BRP example.

5.2.3 Question 3: Providing Guidance for Subsequent Phases of Design

In the third hypothesis, it is proposed that the ranged sets of solutions found using the ranged cDSP can be analyzed in order gain to insight into the material property targets of the product. Furthermore, by calculating the VDSE metric, guidance is provided to support the decision between material selection and material design. The structural validity of the ranged cDSP is established in Chapters 2 and 3 and is discussed previously in Section 5.2.1. Also, the structural validity of value of information metrics is established in Chapters 2 and 3 and is discussed previously in Section 5.2.2. The DSES is applied in Chapter 4 to an example problem on BRP design in order to assess the performance validity. In Section 4.3.2, material property targets are identified from the expansion solutions, and the targets are located both in material databases and on material selection charts. By locating the material property targets both in databases of existing materials and on material selection charts, designers gain insight into the types of materials that may meet the targets.

5.2.4 Domain-Independent Performance Validation of Hypotheses 1, 2 and 3

As introduced in Section 1.4.1, domain-independent performance validity involves establishing that the proposed methods are useful beyond the example problems. This involves determining the characteristics of the example problems that make them representative of general classes of problems. Based on the usefulness of the method for the example problem, its usefulness for general classes of problems is inferred.

For domain-specific structure validation, it is argued in Section 3.3.2 that the example problem on BRP design is representative of a general class of problems, defined by the following characteristics:

- Multiple-conflicting objectives must be balanced in order to achieve families of compromise solutions.
- Motivation exists to tailor the material to the product through material design, but the value of this material design freedom relative to the increase in complexity of the design process is unknown. It is assumed that the designers beliefs about the extent of the material design freedom is embodied in the bounds, constraints, and the cost of design space expansion function in the formulation of the problem.
- Variations in the material properties cause significant performance variation, the nature and/or magnitude of which is influenced by the levels of all design variables. This provides a rationale for modeling the sensitivity of the product performance to the variability in the material properties and seeking ranged sets of solutions which meet ranged design requirements.
- Analytical models are available that relate material properties and other design variables to the performance of the product, and these models are relatively precise, accurate, and fast enough to permit agile design space exploration.

This is intended to be a list of the characteristics of problems for which the effectiveness of the DSES has been demonstrated. In Section 4.4.2 it is demonstrated that the DSES is effective for an example problem on BRP design which shares these characteristics. Therefore, there is reason to believe that the DSES is effective for a *general* class of problems with these characteristics. The capabilities, advantages and limitations of the

DSES for the general class of problems represented by the example problem in BRP design are summarized in Section 5.2.1 through 5.2.3 and are not repeated here.

The human designer is essential for the successful application of the DSES as an attention directing tool in the early stages of design. That is, the preferences, beliefs, and expertise of the designer have significant impacts on the usefulness of the strategy and the validity of the conclusions, and the design solutions and conclusions are likely to be different for different designers. By completing the validation square, it is shown that the DSES can be a useful tool for supporting the decision between material selection and material design for the general class of problems with the characteristics listed above; however, the usefulness of the DSES is dependent upon the meaningfulness of the assumptions made by the human designer. With meaningless assumptions the DSES is rendered useless because no confidence can be placed on the resulting conclusions.

In the next section, the achievements and contributions to the field of engineering design that have been established by answering the research questions and demonstrating and validating the research hypotheses are highlighted.

5.3 ACHIEVEMENTS AND CONTRIBUTIONS

To identify the achievements and contributions in this thesis, the research opportunities identified in Section 2.4 are revisited in this section. Contributions are identified here both in the field of engineering design as well as in BRP design. First, the research opportunities are revisited to identify the achievements and contributions in the field of engineering design.

The first two research opportunities identified in Table 2.3 are embodied in Research Question 1. These are to identify a way to determine how well material concepts meet product requirements and a computationally efficient way to represent and evaluate the performance of new materials. This gap is addressed by the identification of the continuous material design space represented by independent material property variables. As design variables and used in conjunction with engineering equations, the material property variables provide both a way to represent material concepts and a way to evaluate their performance.

Several of the research opportunities identified in Tables 2.3 and 2.4 refer to the establishment of the Value of Design Space Expansion metric, which is embodied in Research Question 2. In these tables it is identified that a metric is needed that is similar to value of information metrics but quantifies the value of expanding the design space. Furthermore, this metric must be compatible with the cDSP, because the cDSP enables the simultaneous consideration of both product performance goals and the cost of design space expansion goal, a design process goal. Thus the metric must determine the value of design space expansion based on the reduction of overall deviation in the expansion solution relative to the baseline solution. These needs are met in the VDSE metric proposed in Section 3.2. This metric enables the quantitative comparison of material selection and material design on the basis of meeting product and design process objectives. By calculating the VDSE metric and identifying material property targets, designers can decide whether or not material design is warranted in the context of the product design, given the increase in design complexity. Furthermore, this decision can be made prior to the development of complex material models for material design.

Finally, a need is established in Table 2.3 to identify material property targets from the cDSP solutions, which is embodied in Research Question 3. In Section 4.3.2 a process is established for identifying material property targets from the expansion cDSP solutions and to locate these targets in material databases and on material selection charts. This process allows designers to gain insight into the *material* requirements of the product.

The primary contribution in this thesis is the establishment of a strategy to support a designer's choice between material selection and material design in the context of product design. The DSES enables the identification of material property targets through agile design space exploration throughout an expanded material design space defined by independent material property variables. The flexibility of the cDSP construct is leveraged by including additional system variables in the expansion cDSP to enable design space expansion. Also, the multiobjective capabilities of the cDSP are leveraged to find solutions to product and design process objectives simultaneously. Specifically, the cost of design space expansion objective is sought simultaneously with the product performance objectives. The strategy is applicable in a more general sense in the context of scoping decisions in engineering design, not just for the decision between material selection and material design. The extension of this strategy towards a more comprehensive strategy for generic design scoping decisions is discussed in the next section.

In addition to the contributions listed above in the field of design, contributions are made in this thesis in the field of BRP design (see Chapter 4). First, by implementing the DSES strategy for BRP design, it is found that the VDSE is positive for all preference schemes in the Point Scenario while the VDSE is negative for all preference schemes in the Ranged Scenario which have a nonzero weight on the CDSE goal (see Section 4.3). Also, it is seen that the material property variables tend to stay very close to the baseline properties when the weight on the CDSE goal is nonzero. Thus, the increase in complexity due to the design of new materials outweighs the improvements in mass and deflection of the panel when uncertainty is considered in the design. When the uncertainty is ignored and a point solution is sought, the improvements in mass and deflection outweigh the increase in complexity due to material design; however, it is unlikely that the material properties can be precisely achieved, and the variation in the performance of the panel as a result of the variation in these properties can be significant.

In addition, some interesting trends are identified. It is found that in the Ranged Scenario, there is a tendency towards thicker face sheets and a thin core layer. Conversely, in the Point Scenario, a tendency towards a thicker core layer with nearly equal face sheet thicknesses is found. In both scenarios, a tendency towards smaller cell wall thicknesses and larger cell spacing is found. Therefore, it seems that the ranged solutions rely on thicker back face sheets while the point solutions take advantage of the crushing of the core layer. These results indicate that the opportunities to leverage the crushing layer are localized minima in the design space that cannot be reached when seeking ranged solutions. This conclusion is further supported by the tendency of the mass goal to dominate the deflection goal in the Point Scenario (see Sections 4.1.3 and 4.2.3). This is because reducing the cell wall thickness, increasing the cell spacing, and reducing the mass density of the material in the core layer facilitates a reduction in mass of the panel but does not necessarily help to reduce the deflection of the panel. The impact of changes in these variables on the deflection of the panel depends on the values of the other system variables as well, thus it is easier to improve the deviation function by reducing the panel mass because it is easier to achieve a reduction in mass. In future work it is prudent to expand the bounds on cell spacing and cell wall thickness if possible because these variables tended toward their bounds in all cases. Also, because the mass goal dominates the achievement of the deflection goal in the Point Scenario, it may be more appropriate to constrain rather than minimize the panel deflection.

5.4 LIMITATIONS AND OPPORTUNITIES FOR FUTURE WORK

The Design Space Expansion Strategy is proposed in this thesis as a starting point to explore how to formulate a tool to support a designer's decision between material selection and material design in the context of product design. However, there are limitations to the breadth and extent of this work, and these limitations offer opportunities for future work. In this section the simplifying assumptions made in this work are identified and the resulting opportunities for future work are outlined. First, the assumptions in the BRP example are discussed, and are summarized in Table 5.1.

TABLE 5.1. SIMPLIFYING ASSUMPTIONS AND ASSOCIATED FUTURE WORK IN THE BRP EXAMPLE

	Assumptions	Future Work
1	The baseline materials used in the example are the preferred materials for selection.	Conduct a material selection procedure to determine the most preferred existing materials for the BRP.
2	The BRP analysis models used in the example provide results that are accurate enough to support the decision between selection and design.	Determine the error in the BRP performance models and incorporate this model uncertainty into the decision-making process.
3	The mass and deflection targets are appropriate, and the CDSE function appropriately models the designer's preferences in this example.	• Conduct a sensitivity analysis of the mass and deflection targets and the weighting coefficients in the CDSE function. • Consult industry experts to verify the targets and CDSE function.

In the BRP example, assumptions are made in the problem formulation as well as in the equations used to predict the BRP performance. Many simplifying assumptions are made by Hutchinson and Xue to arrive at the equations for panel deflection, and these assumptions are thus made here as well (Hutchinson and Xue, 2005). In future work, the error in these equations should be quantified so that it can be modeled in the decision making process as a type of uncertainty. In the formulation of the BRP design problem in this thesis, assumptions are made concerning the baseline materials, the mass and deflection targets, and the CDSE function. It is assumed that the set of baseline materials used in the example comes as a result of undertaking a material selection procedure; however, that procedure is not demonstrated here. To verify or correct this assumption, a material selection procedure should be conducted to determine the most preferred *existing* materials for the BRP.

Assumptions are also made in the selection of mass and deflection targets in the BRP problem formulation. These targets can significantly impact the solution, and as a result, a sensitivity analysis should be performed to determine the sensitivity of the solutions to these targets. Industry experts can be consulted to determine more appropriate targets if necessary. These same assumptions apply to the specification of the CDSE function in the BRP example. A linear function with weighting coefficients is used in the example to reflect a designer's belief that the complexity of the design will increase as the solutions diverge from the baseline materials. A sensitivity analysis should be performed to determine the sensitivity of the solutions to the weighting coefficients on the CDSE functions as well as to the form of the CDSE function itself. Again, industry experts can be consulted to identify a CDSE function that more accurately reflects their beliefs.

The assumptions made in the DSES itself and the VDSE metric are summarized in Table 5.2. The first assumption in the DSES strategy is that the continuous material design space combined with the CDSE goal captures the complexity of material design sufficiently for making the decision between material design and selection. To improve on this assumption, a more thorough survey of existing materials should be conducted to determine the distribution of existing materials throughout this design space supports this assumption. Additionally, other forms of the CDSE function should be investigated including value and/or utility functions.

To balance the costs of material design with the potential improvement in performance, a cost of design space expansion (CDSE) is defined by the designer in the formulation of the DSES. This cost function is intended to reflect the beliefs of the designer about how the cost or complexity of the design process will be impacted by choosing material properties that diverge from the baseline material. This cost function is critical for balancing the potential benefits of material design with the increases in design complexity that are expected if a material must be designed concurrently with the product. Because minimizing the cost or complexity of the design process is only one of multiple design objectives, it is included in the formulation of the DSES as a design goal. The CDSE function used in the BRP design example in this thesis is only one option for a cost function, and some designers may prefer a more rigorous definition of the CDSE. Another option for a cost function is a utility-based cost function. Both utility and value functions should be investigated as potential forms for the CDSE function.

The monotonically increasing CDSE function that is used in the BRP example is appropriate for incremental improvements in material properties in the neighborhood of existing materials. Outside of the neighborhood of an existing material, it is unlikely that a monotonically increasing CDSE function is an appropriate assumption. This is because there are other existing materials which populate the material design space, and the CDSE should therefore decrease as solutions approach existing materials. Because of this, the DSES using a monotonically increasing CDSE function is only appropriate for supporting the decision between material selection and *incremental* material design. In *revolutionary* material design new materials are sought which have properties that are drastically different from properties of existing materials. To support a designer's decision between material selection and *revolutionary* material design, a different non-monotonically increasing CDSE function should be employed.

By selecting a set of baseline materials in the DSES strategy, two assumptions are made. First, it is assumed that a set of baseline materials can be identified and that there is a feasible solution using these baseline materials. The need for a baseline material limits the applicability of this strategy to cases in which feasible design solutions can be found using existing materials. Some critics may argue that if a feasible solution (one that meets design constraints) can be found using an existing material, there is no value in *designing* a material. In the author's opinion, this argument ignores the potential improvement in the achievement of design goals by tailoring a material to the particular requirements of the product; this potential improvement may outweigh the increase in complexity of the design due to the design of the material.

If there is no feasible design solution with existing materials, then it is possible that there is no need for the DSES because the only option is to design a new material. However, it is also possible that a revision of the problem formulation may allow for a feasible solution using existing materials. There is a trade-off here between the lost performance of the product due to the revision of the problem formulation and the expense of material design. This is a similar decision to the decision between material design and material selection that is addressed in this thesis, but this decision cannot be made using the DSES as it is proposed here, and significant changes to the DSES are needed to address this question.

Another assumption relating to the baseline materials is that only one baseline material set can be considered at a time in the DSES strategy. To address this assumption, a more sophisticated CDSE function can be identified that incorporates multiple minima representing multiple baseline solutions. In addition, a systematic method for comparing the expansion solution to the multiple baseline solutions is needed. This new method may require changes to the VDSE metric.

TABLE 5.2. SIMPLIFYING ASSUMPTIONS AND ASSOCIATED FUTURE WORK IN THE DSES

	Assumptions	**Future Work**
	DSES	
1	A continuous material design space combined with the CDSE goal appropriately models the complexities of material design for the purpose of making the decision between material selection and material design.	• Conduct a more thorough survey of the material design space to determine the distribution of existing materials throughout the design space. • Investigate other forms for the CDSE function including utility and/or value functions.
2	Baseline material properties can be identified at this stage in the design process, and there is a feasible design solution with baseline properties.	Identify a method to determine if material design is the only option when there is no feasible baseline solution, or if it is preferred to revise the problem formulation in order to find a feasible baseline solution.
3	Only one set of baseline material properties is considered at a time.	• Identify a CDSE function that reflection multiple baseline material options. • Identify a systematic method for comparing the expansion solution to multiple baseline solutions.
4	The goal targets and Archimedean deviation function of the cDSP effectively model the designer's preferences.	Adapt the DSES to be compatible with other preference models including utility and/or value functions.
5	The ranged form of the cDSP using DCIs is sufficient for characterizing the uncertainties in the choice between selection and design.	Identify methods for incorporating model uncertainty in addition to uncertainty in requirements, noise factors and system variables
	VDSE	
6	The cDSP deviation function is sufficient for calculating the VDSE and effectively models the designer's preferences concerning trade-offs and risks.	Identify forms of the VDSE metric that are compatible with other forms of preference modeling including value and/or utility functions.

An assumption is made in the DSES that the target values for the goals and the Archimedean deviation function in the cDSP effectively model the preferences of a designer for the purposes of making the decision between material selection and design. Other preference models such as utility and value functions exist that may offer a better representation of a designer's preferences. The DSES should be adapted to be compatible with these other preference models. These assumptions relating to the preference models also extend to the VDSE metric. The metric proposed here is compatible with the cDSP deviation function, but is not directly compatible with utility and/or value functions. This incompatibility is due to the fact that the objective with a deviation function is to minimize the deviation and the objective with a utility or value function is to maximize. Therefore, the VDSE metric will have to be revised to be compatible with utility and value functions.

In addition to assumptions about preference models, assumptions are made in the DSES that the ranged form of the cDSP using DCI's is sufficient for characterizing the uncertainties in the decision between material selection and design. However; model uncertainty can also be a significant factor. Therefore the DSES should be adapted to account for this type of uncertainty as well.

* * *

I believe that decisions in design should be made consciously, not implicitly; however, many design alternatives are discarded early in the design process due to lack of knowledge or inexperience with the technologies. Therefore, I believe there is a need for decision support constructs which help designers to determine when it is beneficial to take the risk to develop new technologies. Strategies such as the DSES can be used to quickly determine the value of expanding the design space to include new technologies. I also believe that it is prudent to adopt a set-based approach to design in which efforts are directed at eliminating poor solutions rather than identifying the one best solution.

I believe that the DSES proposed in this thesis is an effective strategy for supporting a designer's decision between material selection and material design in the context of product design, but the decision between material selection and material design is only one type of design scoping decision that may arise in a design process. In my PhD research I hope to develop a more comprehensive strategy in which the notion of information economics is infused in a set-based design approach. The opportunities for future work in extending the DSES are summarized in proposal form in Figure 5.1 by identifying the problem and the primary research question.

<table>
<tr><td>Problem</td><td colspan="2">The generation of alternatives is an important step in any design process, one which may ultimately limit the success of the final product if the best alternative is not identified. Designers often make implicit decisions to rule out design alternatives that increase the complexity or cost of the design without analyzing the potential benefits of the complicating technology. Material design is one such example; designers are quick to rule out designing a new material for a product in favor of material selection because material design is costly and complicated, although designing the material may enable product improvements that are not possible with existing materials.

The Design Space Expansion Strategy has been established to support a designer's decision between material selection and material design in the context of product design. This strategy can be generalized into a more comprehensive set-based design approach.</td></tr>
<tr><td rowspan="2">Research Questions</td><td rowspan="2">Primary</td><td>Research Question
How can the notion of information economics be infused into design scoping decisions to balance the cost of exploring additional alternatives with the potential for improving product performance?</td></tr>
<tr><td>Hypothesis
By generalizing the DSES into a more comprehensive set-based design approach, information economics can be infused into design scoping decisions.</td></tr>
</table>

FIGURE 5.1. PROBLEM AND RESEARCH QUESTIONS FOR PHD RESEARCH

The DSES is the first step towards infusing information economics into a comprehensive, systematic approach for set-based design. I believe that a special class of value of information metrics can be defined to assess the value of *design space expansion* in a more general sense. These new metrics can then be used to support all types of scoping decisions in a set-based approach to design.

* * *

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APPENDIX A: MATLAB CODE

In Chapter 4, baseline and expansion cDSPs are solved as part of the example problem on BRP design (see Sections 4.1 and 4.2). In this appendix, the MATLAB m-files that are used for finding the baseline and expansion solutions are presented. In addition, algorithms for finding the solutions using these m-files are presented. The algorithm for finding the baseline solutions is presented in Table A.1, and the algorithm for finding the expansion solutions is presented in Table A.2. In both the baseline and expansion cDSPs, separate m-files are used for the Point and Ranged Scenarios. The Point Scenario is denoted by "_nr" in the filename, while the Ranged Scenario is denoted by "_r" in the filename. For example, *BRPfitness_r.m* is the GA fitness function for the Ranged Scenario, while *BRPfitness_nr.m* is the GA fitness function for the Point Scenario. The m-files are listed after the algorithms in alphabetical order. For convenience, the page numbers for each of the m-files are listed in Table A.3.

In Table A.1 the algorithm for finding the baseline solutions is presented. There are four tasks, which are described in the second column. The functions that are called by the scripts for each task are listed in the third column, and the names of the files in which the results are stored are listed in the fourth column. This procedure must be carried out for each of the two design scenarios, but the scripts are written such that the solutions for all eleven preference schemes are found at the same time. The resulting text files should be copied outside of the MATLAB current directory once all the tasks have been performed for one of the scenarios. Otherwise, the files will be overwritten when running the scripts for the next scenario. The only exception is the first task in which the text file of baseline points is created. This task can be performed only once as the same file can be used for both scenarios.

TABLE A.1. ALGORITHM FOR FINDING BASELINE SOLUTIONS

Task	Description	Function Calls (from current directory)	Stores (in current directory)
1	**Generate points** by running "get_points_baseline.m"	• get_points_baseline.m	• baseline_points.txt
2	**Find the feasible points** • Point Scenario: run "baseline_nr_ getfeasible.m" • Ranged Scenario: run "baseline_r_ getfeasible.m"	• baseline_nr/r_ getfeasible.m • BRPdefl.m • ineq_constraints_ BRP_nr/r.m • baseline_points.txt	• feasible_A2.txt • infeasible_A2_ gamma.txt • infeasible_A2_mu.txt • infeasible_A2_R_c.txt • infeasible_A2_defl.txt • infeasible_A2_mass.txt • infeasible_A2_hcB.txt • timelog.txt
3	**Calculate deviation variables** • Point Scenario: run "baseline_nr.m" • Ranged Scenario: run "baseline_r.m"	• baseline_nr/r.m • BRPdefl.m • DSEcost.m • deviation.m • feasible_A2.txt	• a2_devvars.txt • a2_cost.txt
4	**Find and report the minimum** by running "baseline_getsolutions.m"	• baseline_ getsolutions_nr/r.m • a2_devvars.txt • feasible_a2.txt • a2_cost.txt	a2_solutions_arch.txt

The algorithm for finding the expansion cDSP solutions is shown in Table A.2. The tasks are described in the second column and the functions that are called during each task are listed in the third column. This procedure must be carried out for both design scenarios and for each preference scheme separately. The results of each step in this algorithm are not stored in text files as in the algorithm for the baseline solutions. Rather, the solutions must be exported to the MATLAB workspace from the GA and pattern search tools. At the end of the procedure the output from the *solution_check_pattern_nr/r.m* scripts should be copied from the command window to a separate file to save the data.

TABLE A.2. ALGORITHM FOR FINDING EXPANSION SOLUTIONS

Task	Description	function calls (in current directory)
1	**Set the scheme number** in the appropriate m-files - BRPfitness_nr/r.m (line 33) - BRPpattern_nr/r.m (line 33) - solution_check_pattern_nr/r.m (line 30)	
2	**Run the GA** • open GA tool by typing "gatool" at the command line • enter the fitness function @BRPfitness_nr/r • Number of variables = 55 • Select "Best Fitness" and "Mean Fitness" for the plots • Population type = "bit string" • Population size = 200 • Click on "Run Solver"	• BRPfitness_ nr/r.m • BRPdefl.m • norm01.m • unnorm01.m • ineq_constraints_ BRP_nr/r.m • DSEcost.m • deviation.m
3	**Export the GA solution** to the workspace and save plot	
4	**Convert the bit string** into a vector of normalized design variables using the following commands, where 'gasol' is the name of the exported structure containing the GA solution ➢ xga = gasol.x, [xps] = xga2xps(xga)	xga2xps.m
5	**Run the pattern search** • Open pattern search tool by typing "psearchtool" at the command line • Enter the objective function @BRPpattern_nr/r • Start point = xps • Set the bounds: LB = zeros(1,11) UB = ones(1,11) • Select "Best Fitness" to plot • Click on "Start"	• BRPpattern_ nr/r.m • unnorm01.m • ineq_constraints_ BRP_nr/r.m • BRPdefl.m • DSEcost.m • deviation.m
6	**Export the pattern search solution** to the workspace and save plot	
7	**Check the pattern search solution** by entering the following command where "pssol" is the pattern search solution vector and save the output solution_check_pattern_nr/r(pssol)	solution_check_ pattern_nr/r.m
	Repeat for remaining schemes	

TABLE A.3. INVENTORY OF M-FILES INCLUDED IN THIS APPENDIX

baseline_getsolutions_nr.m

```
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%
% PURPOSE: to solve the baseline cDSP for the Point Scenario BRP
%
% ASSUMPTIONS:
%        - The file 'a2_devvars.txt' is in the working directory and
%          contains the deviation variables for the feasible points
%        - The file 'feasible_a2.txt' is in the working directory and
%          contains the design variables and performance values for the
%          feasible points
%        - The file 'a2_cost.txt' is in the working directory and
%          contains the Design Space Expansion cost of each of the
%          feasible points
```

```
% -----------------------------------------------------------------------
% Problem formulation:
% -----------------------------------------------------------------------

% Define weighting factors
W = [1 0 0; 0 1 0; 0 0 1; ...
    .6 .2 .2; .2 .6 .2; .2 .2 .6; ...
    .2 .4 .4; .4 .2 .4; .4 .4 .2; ...
    1/3 1/3 1/3; 0.5 0.5 0];

% Load deviation variables, feasible points and cost values
a2_devvars = load('a2_devvars.txt');
dplus = [a2_devvars(:,2) a2_devvars(:,4) a2_devvars(:,6)];

% -----------------------------------------------------------------------
% Problem solution:
% -----------------------------------------------------------------------

% Deviation Function: Archimedian formulation
% Compute the value of the deviation function
devfunc = dplus*W';

% recall design variables, etc. and concatenate matrices
feasible_points = load('feasible_a2.txt');
cost = load('a2_cost.txt');
points = [feasible_points, cost, a2_devvars];

% Find the minimum and report the solution
[min_val_ArchDev1, index_ArchDev1]= min(devfunc(:,1));
sol_ArchDev1 = [points(index_ArchDev1,:) min_val_ArchDev1];
[min_val_ArchDev2, index_ArchDev2]= min(devfunc(:,2));
sol_ArchDev2 = [points(index_ArchDev2,:) min_val_ArchDev2];
[min_val_ArchDev3, index_ArchDev3]= min(devfunc(:,3));
sol_ArchDev3 = [points(index_ArchDev3,:) min_val_ArchDev3];
[min_val_ArchDev4, index_ArchDev4]= min(devfunc(:,4));
sol_ArchDev4 = [points(index_ArchDev4,:) min_val_ArchDev4];
[min_val_ArchDev5, index_ArchDev5]= min(devfunc(:,5));
sol_ArchDev5 = [points(index_ArchDev5,:) min_val_ArchDev5];
[min_val_ArchDev6, index_ArchDev6]= min(devfunc(:,6));
sol_ArchDev6 = [points(index_ArchDev6,:) min_val_ArchDev6];
[min_val_ArchDev7, index_ArchDev7]= min(devfunc(:,7));
sol_ArchDev7 = [points(index_ArchDev7,:) min_val_ArchDev7];
[min_val_ArchDev8, index_ArchDev8]= min(devfunc(:,8));
sol_ArchDev8 = [points(index_ArchDev8,:) min_val_ArchDev8];
[min_val_ArchDev9, index_ArchDev9]= min(devfunc(:,9));
sol_ArchDev9 = [points(index_ArchDev9,:) min_val_ArchDev9];
[min_val_ArchDev10, index_ArchDev10]= min(devfunc(:,10));
sol_ArchDev10 = [points(index_ArchDev10,:) min_val_ArchDev10];
[min_val_ArchDev11, index_ArchDev11]= min(devfunc(:,11));
sol_ArchDev11 = [points(index_ArchDev11,:) min_val_ArchDev11];

solutions_ArchDev = [sol_ArchDev1; sol_ArchDev2; sol_ArchDev3; ...
                     sol_ArchDev4; sol_ArchDev5; sol_ArchDev6; ...
                     sol_ArchDev7; sol_ArchDev8; sol_ArchDev9; ...
                     sol_ArchDev10; sol_ArchDev11];
```

```
% ---------------------------------------------------------------------
% Problem interpretation:
% ---------------------------------------------------------------------

dlmwrite('a2_solutions_arch.txt', solutions_ArchDev , ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
% ---------------------------------------------------------------------
```

baseline_getsolutions_r.m

```
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%
% PURPOSE: to solve the baseline cDSP for the Ranged Scenario BRP
%
% ASSUMPTIONS:
%        - The file 'a2_devvars.txt' is in the working directory and
%          contains the deviation variables for the feasible points
%        - The file 'feasible_a2.txt' is in the working directory and
%          contains the design variables and performance values for the
%          feasible points
%        - The file 'a2_cost.txt' is in the working directory and
%          contains the Design Space Expansion cost of each of the
%          feasible points

% ---------------------------------------------------------------------
% Problem formulation:
% ---------------------------------------------------------------------
% Define weighting factors
W = [1 0 0; 0 1 0; 0 0 1; ...
    .6 .2 .2; .2 .6 .2; .2 .2 .6; ...
    .2 .4 .4; .4 .2 .4; .4 .4 .2; ...
    1/3 1/3 1/3; 0.5 0.5 0];

% Load deviation variables, feasible points and cost values
a2_devvars = load('a2_devvars.txt');
dminus = [a2_devvars(:,1) a2_devvars(:,3) a2_devvars(:,5)];

% ---------------------------------------------------------------------
% Problem solution:
% ---------------------------------------------------------------------

% Deviation Function: Archimedian formulation
% Compute the value of the deviation function
devfunc = dminus*W';

% recall design variables, etc. and concatenate matrices
feasible_points = load('feasible_a2.txt');
cost = load('a2_cost.txt');
points = [feasible_points, cost, a2_devvars];
```

```
% Find the minimum and report the solution
[min_val_ArchDev1, index_ArchDev1]= min(devfunc(:,1));
sol_ArchDev1 = [points(index_ArchDev1,:) min_val_ArchDev1];
[min_val_ArchDev2, index_ArchDev2]= min(devfunc(:,2));
sol_ArchDev2 = [points(index_ArchDev2,:) min_val_ArchDev2];
[min_val_ArchDev3, index_ArchDev3]= min(devfunc(:,3));
sol_ArchDev3 = [points(index_ArchDev3,:) min_val_ArchDev3];
[min_val_ArchDev4, index_ArchDev4]= min(devfunc(:,4));
sol_ArchDev4 = [points(index_ArchDev4,:) min_val_ArchDev4];
[min_val_ArchDev5, index_ArchDev5]= min(devfunc(:,5));
sol_ArchDev5 = [points(index_ArchDev5,:) min_val_ArchDev5];
[min_val_ArchDev6, index_ArchDev6]= min(devfunc(:,6));
sol_ArchDev6 = [points(index_ArchDev6,:) min_val_ArchDev6];
[min_val_ArchDev7, index_ArchDev7]= min(devfunc(:,7));
sol_ArchDev7 = [points(index_ArchDev7,:) min_val_ArchDev7];
[min_val_ArchDev8, index_ArchDev8]= min(devfunc(:,8));
sol_ArchDev8 = [points(index_ArchDev8,:) min_val_ArchDev8];
[min_val_ArchDev9, index_ArchDev9]= min(devfunc(:,9));
sol_ArchDev9 = [points(index_ArchDev9,:) min_val_ArchDev9];
[min_val_ArchDev10, index_ArchDev10]= min(devfunc(:,10));
sol_ArchDev10 = [points(index_ArchDev10,:) min_val_ArchDev10];
[min_val_ArchDev11, index_ArchDev11]= min(devfunc(:,11));
sol_ArchDev11 = [points(index_ArchDev11,:) min_val_ArchDev11];

solutions_ArchDev = [sol_ArchDev1; sol_ArchDev2; sol_ArchDev3; ...
                     sol_ArchDev4; sol_ArchDev5; sol_ArchDev6; ...
                     sol_ArchDev7; sol_ArchDev8; sol_ArchDev9; ...
                     sol_ArchDev10; sol_ArchDev11];

% ---------------------------------------------------------------------
% Problem interpretation:
% ---------------------------------------------------------------------

dlmwrite('a2_solutions_arch.txt', solutions_ArchDev , ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
% ---------------------------------------------------------------------
```

baseline_nr.m

```
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%
% PURPOSE: to calculate the deviation variables for the Point Scenario
%          of the baseline cDSP for BRP design
%
% ASSUMPTIONS:
%       -   The file 'feasible_a2.txt' is in the working directory and
%           contains the feasible points for this scenario
%


% ---------------------------------------------------------------------
% Problem formulation:
% ---------------------------------------------------------------------
```

```
% CONSTANTS
% Geometry
L = 1; % 1 meter

% Noise factors (blast loading)
t_0 = 0.0001; % 10^-4 seconds
delta_t_0 = 0.15*t_0; % 0.000015 seconds
p_0 = 25 * 10^6; % 25 MPa
delta_p_0 = 0.15*p_0; % 0.15*25 MPa

% Baseline Material Properties
rho_mg = 1770; % 1770 kg/m3
rho_st = 7845; % 7845 kg/m3
sigma_y_mg = 200 * 10^6; % 200 MPa
sigma_y_st = 538 * 10^6; % 538 MPa

% Variation in material properties
delta_rho_mg = 170; % 170 kg/m3
delta_rho_st = 100; % 100 kg/m3
delta_sigma_y_mg = 20 * 10^6; % 20 MPa
delta_sigma_y_st = 150 * 10^6; % 150 MPa

x_base = [sigma_y_mg, sigma_y_st, sigma_y_mg, rho_mg, rho_st, rho_mg];
dx = [delta_sigma_y_mg, delta_sigma_y_st, delta_sigma_y_mg, ...
    delta_rho_mg, delta_rho_st, delta_rho_mg, ...
    delta_p_0, delta_t_0];
weights = 1-dx(1:6)./x_base;
dx = zeros(1,8);

tic
dmass_minus = [];
dmass_plus = [];
ddefl_minus = [];
ddefl_plus = [];
dcost_minus = [];
dcost_plus = [];
cost = [];
d = [];

% load feasible points
feasible_points = load('feasible_a2.txt');
% Note: columns of feasible_points are as follows:
% (B, H, h_c, h_f, h_b, defl, delta_defl, M, delta_M, R_c, m_c, m_f)

% Goals
% Minimize mass as close as possible to 0 kg/m2 with a maximum value
% of 568.4
M_max = 568.4;

% Minimize deflection as close as possible to 0 cm with a maximum
% value of 7.553 m
defl_max = 7.553;
```

```
% Minimize cost as close as possible to 0, normalized by 1
cost_max = 1;

% Get cost
[cost, delta_cost] = DSEcost(x_base, x_base, weights, dx(1:6));

% -------------------------------------------------------------------
% Problem solution:
% -------------------------------------------------------------------
% Get values of deviation variables for all feasible points
for i = [1:size(feasible_points,1)]
    % Evaluate BRP deflection code
    [defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
        feasible_points(i,1), ...
        feasible_points(i,2), ...
        feasible_points(i,3), ...
        feasible_points(i,4), ...
        feasible_points(i,5), ...
        sigma_y_mg, sigma_y_st, sigma_y_mg,...
        rho_mg, rho_st, rho_mg, ...
        L, p_0, t_0, dx);

    % calculate deviation vector
    [dmass_minus,dmass_plus] = deviation(M,M_max,'m20');
    [ddefl_minus,ddefl_plus] = deviation(defl,defl_max,'m20');
    [dcost_minus,dcost_plus] = deviation(cost,cost_max,'m20');
    % save deviation variables in a text file using 3 sig digits
dlmwrite('a2_devvars.txt', [dmass_minus, dmass_plus, ...
                            ddefl_minus, ddefl_plus, ...
                            dcost_minus, dcost_plus], ...
                            'delimiter', '\t', 'precision', 3, ...
                            'newline', 'pc', '-append')
end

% -------------------------------------------------------------------
% Problem interpretation:
% -------------------------------------------------------------------
% save cost information in a text file using 3 sig figs
dlmwrite('a2_cost.txt', [cost*ones(size(feasible_points,1),1), ...
    delta_cost*ones(size(feasible_points,1),1)] , ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
% -------------------------------------------------------------------
```

baseline_nr_getfeasible.m

```
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%
% PURPOSE: to separate feasible points from infeasible points in the
%          Point Scenario of the baseline cDSP for BRP design
%
% ASSUMPTIONS:
%       -  The file 'baseline_points.txt' is in the working directory
```

```
%

%  WARNING: This script creats several .txt files in the current
%           directory and takes some time to execute.

% ------------------------------------------------------------------
% Problem formulation:
% ------------------------------------------------------------------

% CONSTANTS
% Geometry
L = 1; % 1 meter

% Noise factors (blast loading)
t_0 = 0.0001; % 10^-4 seconds
delta_t_0 = 0.15*t_0; % 0.000015 seconds
p_0 = 25 * 10^6; % 25 MPa
delta_p_0 = 0.15*p_0; % 0.15*25 MPa

% Baseline Material Properties
rho_mg = 1770; % 1770 kg/m3
rho_st = 7845; % 7845 kg/m3
sigma_y_mg = 200 * 10^6; % 200 MPa
sigma_y_st = 538 * 10^6; % 538 MPa

% Variation in material properties
delta_rho_mg = 170; % 170 kg/m3
delta_rho_st = 100; % 100 kg/m3
delta_sigma_y_mg = 20 * 10^6; % 20 MPa
delta_sigma_y_st = 150 * 10^6; % 150 MPa

x_base = [sigma_y_mg, sigma_y_st, sigma_y_mg, ...
          rho_mg, rho_st, rho_mg];
dx = [delta_sigma_y_mg, delta_sigma_y_st, delta_sigma_y_mg, ...
    delta_rho_mg, delta_rho_st, delta_rho_mg, ...
    delta_p_0, delta_t_0];
weights = 1-dx(1:6)./x_base;
dx = zeros(1,8);

% Initialize vectors of points
points = load('baseline_points.txt');


tic

feasible_points_3 = [];
infeasible_hcB_3 = [];
infeasible_mass_3 = [];
infeasible_defl_3 = [];
infeasible_R_c_3 = [];
infeasible_mu_3 = [];
infeasible_gamma_3 = [];

% store begintime
```

```
dlmwrite('timelog.txt', 'Beginning feasible points', ...
    'newline', 'pc', '-append')
dlmwrite('timelog.txt', clock, 'delimiter', '\t', ...
    'newline', 'pc', '-append')

% ----------------------------------------------------------------------
% Problem solution:
% ----------------------------------------------------------------------
% create vector of feasible points for the three layer panel
for i = 1:length(points)
    if points(i,3)<points(i,1)

    % evaluate BRPdefl
    [defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
        points(i,1), ...
        points(i,2), ...
        points(i,3), ...
        points(i,4), ...
        points(i,5), ...
        sigma_y_mg, sigma_y_st, sigma_y_mg, ...
        rho_mg, rho_st, rho_mg, ...
        L, p_0, t_0, dx);

    % evaluate constraints
    gx = ineq_constraints_BRP_nr([points(i,:), ...
            sigma_y_mg, sigma_y_st, sigma_y_mg, ...
            rho_mg, rho_st, rho_mg], dx);
        if gx(1) == 0
            if  gx(2) ==0
                if gx(3) == 0
                    if gx(4) ==0
                        if gx(5) == 0
                            feasible_points_3 = [feasible_points_3;...
                                                 points(i,1), ...
                                                 points(i,2), ...
                                                 points(i,3), ...
                                                 points(i,4), ...
                                                 points(i,5), ...
                                                 defl, delta_defl, ...
                                                 M, delta_M, ...
                                                 R_c, m_c, m_f];
                        else
                            infeasible_gamma_3 = [...
                                                 infeasible_gamma_3;...
                                                 points(i,1), ...
                                                 points(i,2), ...
                                                 points(i,3), ...
                                                 points(i,4), ...
                                                 points(i,5), ...
                                                 defl, delta_defl, ...
                                                 M, delta_M, ...
                                                 R_c, m_c, m_f];
                        end
                    else
                        infeasible_mu_3 = [infeasible_mu_3; ...
                                           points(i,1), ...
```

```
                                            points(i,2), ...
                                            points(i,3), ...
                                            points(i,4), ...
                                            points(i,5), ...
                                            defl, delta_defl, ...
                                            M, delta_M, ...
                                            R_c, m_c, m_f];
                    end
                else
                    infeasible_R_c_3 = [infeasible_R_c_3; ...
                                        points(i,1), ...
                                        points(i,2), ...
                                        points(i,3), ...
                                        points(i,4), ...
                                        points(i,5), ...
                                        defl, delta_defl, ...
                                        M, delta_M, R_c, m_c, m_f];
                end
            else
                infeasible_defl_3 = [infeasible_defl_3; ...
                                    points(i,1), ...
                                    points(i,2), ...
                                    points(i,3), ...
                                    points(i,4), ...
                                    points(i,5), ...
                                    defl, delta_defl, ...
                                    M, delta_M, R_c, m_c, m_f];
            end
        else
            infeasible_mass_3 = [infeasible_mass_3; ...
                                points(i,1), ...
                                points(i,2), ...
                                points(i,3), ...
                                points(i,4), ...
                                points(i,5), ...
                                defl, delta_defl, ...
                                M, delta_M, R_c, m_c, m_f];
        end
    else
        infeasible_hcB_3 = [infeasible_hcB_3; ...
                            points(i,1), ...
                            points(i,2), ...
                            points(i,3), ...
                            points(i,4), ...
                            points(i,5)];
    end
end
toc

% ----------------------------------------------------------------------
% Problem interpretation:
% ----------------------------------------------------------------------
dlmwrite('timelog.txt', 'Beginning storing points', ...
    'newline', 'pc', '-append')
dlmwrite('timelog.txt', clock, 'delimiter', '\t', ...
    'newline', 'pc', '-append')
```

```
tic
dlmwrite('feasible_a2.txt', feasible_points_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
dlmwrite('infeasible_a2_gamma.txt', infeasible_gamma_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
dlmwrite('infeasible_a2_mu.txt', infeasible_mu_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
dlmwrite('infeasible_a2_R_c.txt', infeasible_R_c_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
dlmwrite('infeasible_a2_defl.txt', infeasible_defl_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
dlmwrite('infeasible_a2_mass.txt', infeasible_mass_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
dlmwrite('infeasible_a2_hcB.txt', infeasible_hcB_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
toc
% -------------------------------------------------------------------
```

baseline_r.m

```
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%
% PURPOSE: to calculate the deviation variables for the Ranged
%          Scenario of the baseline cDSP for BRP design
%
% ASSUMPTIONS:
%       -   The file 'feasible_a2.txt' is in the working directory and
%           contains the feasible points for this scenario

% -------------------------------------------------------------------
% Problem formulation:
% -------------------------------------------------------------------
% CONSTANTS
% Geometry
L = 1; % 1 meter

% Noise factors (blast loading)
t_0 = 0.0001; % 10^-4 seconds
delta_t_0 = 0.15*t_0; % 0.000015 seconds
p_0 = 25 * 10^6; % 25 MPa
delta_p_0 = 0.15*p_0; % 0.15*25 MPa

% Baseline Material Properties
rho_mg = 1770; % 1770 kg/m3
rho_st = 7845; % 7845 kg/m3
sigma_y_mg = 200 * 10^6; % 200 MPa
sigma_y_st = 538 * 10^6; % 538 MPa

% Variation in material properties
delta_rho_mg = 170; % 170 kg/m3
```

```
delta_rho_st = 100; % 100 kg/m3
delta_sigma_y_mg = 20 * 10^6; % 20 MPa
delta_sigma_y_st = 150 * 10^6; % 150 MPa

x_base = [sigma_y_mg, sigma_y_st, sigma_y_mg, rho_mg, rho_st, rho_mg];
dx = [delta_sigma_y_mg, delta_sigma_y_st, delta_sigma_y_mg, ...
    delta_rho_mg, delta_rho_st, delta_rho_mg, ...
    delta_p_0, delta_t_0];
weights = 1-dx(1:6)./x_base;

tic
dmass_minus = [];
dmass_plus = [];
ddefl_minus = [];
ddefl_plus = [];
dcost_minus = [];
dcost_plus = [];
cost = [];
d = [];

% load feasible points
feasible_points = load('feasible_a2.txt');
% Note: columns of feasible_points are as follows:
% (B, H, h_c, h_f, h_b, defl, delta_defl, M, delta_M, R_c, m_c, m_f)

% Goals
% Minimize mass as close as possible to 0 kg/m2
M_target = 0;

% Minimize deflection as close as possible to 0 cm
defl_target = 0;

% Minimize cost as close as possible to 0
cost_target = 0;

% Get cost
[cost, delta_cost] = DSEcost(x_base, x_base, weights, dx(1:6));

% ----------------------------------------------------------------------
% Problem solution:
% ----------------------------------------------------------------------
% Get values of deviation variables for all feasible points
for i = [1:size(feasible_points,1)]
   % Evaluate BRP deflection code
    [defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
        feasible_points(i,1), ...
        feasible_points(i,2), ...
        feasible_points(i,3), ...
        feasible_points(i,4), ...
        feasible_points(i,5), ...
        sigma_y_mg, sigma_y_st, sigma_y_mg,...
        rho_mg, rho_st, rho_mg, ...
        L, p_0, t_0, dx);

    % Get Cdk's
```

```
    Cdk_M = (M_target - M)/delta_M;
    Cdk_defl = (defl_target - defl)/delta_defl;
    Cdk_cost = (cost_target - cost)/delta_cost;

    % calculate deviation vector
    [dmass_minus,dmass_plus] = deviation(Cdk_M,1,'max');
    [ddefl_minus,ddefl_plus] = deviation(Cdk_defl,1,'max');
    [dcost_minus,dcost_plus] = deviation(Cdk_cost,1,'max');
    d = [d; [dmass_minus, dmass_plus, ...
             ddefl_minus, ddefl_plus, ...
             dcost_minus, dcost_plus]];
    dmass_minus = [];
    dmass_plus = [];
    ddefl_minus = [];
    ddefl_plus = [];
    dcost_minus = [];
    dcost_plus = [];
end

% --------------------------------------------------------------------
% Problem interpretation:
% --------------------------------------------------------------------
% save deviation variables in a text file using 3 significant digits
dlmwrite('a2_devvars.txt', [d], ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')

% save cost information in a text file using 3 sig figs
dlmwrite('a2_cost.txt', [cost*ones(size(feasible_points,1),1), ...
    delta_cost*ones(size(feasible_points,1),1)] , ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')

toc
% --------------------------------------------------------------------
```

baseline_r_getfeasible.m

```
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%
% PURPOSE: to separate feasible points from infeasible points in the
%          Ranged Scenario of the baseline cDSP for BRP design
%
% ASSUMPTIONS:
%       -  The file 'baseline_points.txt' is in the working directory
%

%  WARNING: This script creats several .txt files in the current
%           directory and takes some time to execute.

% --------------------------------------------------------------------
% Problem formulation:
% --------------------------------------------------------------------
```

```
% CONSTANTS
% Geometry
L = 1; % 1 meter

% Noise factors (blast loading)
t_0 = 0.0001; % 10^-4 seconds
delta_t_0 = 0.15*t_0; % 0.000015 seconds
p_0 = 25 * 10^6; % 25 MPa
delta_p_0 = 0.15*p_0; % 0.15*25 MPa

% Baseline Material Properties
rho_mg = 1770; % 1770 kg/m3
rho_st = 7845; % 7845 kg/m3
sigma_y_mg = 200 * 10^6; % 200 MPa
sigma_y_st = 538 * 10^6; % 538 MPa

% Variation in material properties
delta_rho_mg = 170; % 170 kg/m3
delta_rho_st = 100; % 100 kg/m3
delta_sigma_y_mg = 20 * 10^6; % 20 MPa
delta_sigma_y_st = 150 * 10^6; % 150 MPa

x_base = [sigma_y_mg, sigma_y_st, sigma_y_mg, rho_mg, rho_st, rho_mg];
dx = [delta_sigma_y_mg, delta_sigma_y_st, delta_sigma_y_mg, ...
    delta_rho_mg, delta_rho_st, delta_rho_mg, ...
    delta_p_0, delta_t_0];
weights = 1-dx(1:6)./x_base;

% Initialize vectors of points
points = load('baseline_points.txt');

% create vector of feasible points for the three layer panel
tic

feasible_points_3 = [];
infeasible_hcB_3 = [];
infeasible_mass_3 = [];
infeasible_defl_3 = [];
infeasible_R_c_3 = [];
infeasible_mu_3 = [];
infeasible_gamma_3 = [];

% -------------------------------------------------------------------
% Problem solution:
% -------------------------------------------------------------------
% Create a vector of feasible points:
for i = 1:length(points)
    if points(i,3)<points(i,1)

    % evaluate BRPdefl
    [defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
        points(i,1), ...
        points(i,2), ...
        points(i,3), ...
        points(i,4), ...
```

```
        points(i,5), ...
        sigma_y_mg, sigma_y_st, sigma_y_mg, ...
        rho_mg, rho_st, rho_mg, ...
        L, p_0, t_0,dx);

    % evaluate constraints
    gx = ineq_constraints_BRP_r([points(i,:), ...
        sigma_y_mg, sigma_y_st, sigma_y_mg, ...
            rho_mg, rho_st, rho_mg], dx);
        if gx(1) == 0
            if  gx(2) ==0
                if gx(3) == 0
                    if gx(4) ==0
                        if gx(5) == 0
                            feasible_points_3 = [feasible_points_3;...
                                                    points(i,1), ...
                                                    points(i,2), ...
                                                    points(i,3), ...
                                                    points(i,4), ...
                                                    points(i,5), ...
                                                    defl, delta_defl, ...
                                                    M, delta_M, ...
                                                    R_c, m_c, m_f];
                        else
                            infeasible_gamma_3 = [...
                                                    infeasible_gamma_3; ...
                                                    points(i,1), ...
                                                    points(i,2), ...
                                                    points(i,3), ...
                                                    points(i,4), ...
                                                    points(i,5), ...
                                                    defl, delta_defl, ...
                                                    M, delta_M, ...
                                                    R_c, m_c, m_f];
                        end
                    else
                        infeasible_mu_3 = [infeasible_mu_3; ...
                                                points(i,1), ...
                                                points(i,2), ...
                                                points(i,3), ...
                                                points(i,4), ...
                                                points(i,5), ...
                                                defl, delta_defl, ...
                                                M, delta_M, ...
                                                R_c, m_c, m_f];
                    end
                else
                    infeasible_R_c_3 = [infeasible_R_c_3; ...
                                            points(i,1), ...
                                            points(i,2), ...
                                            points(i,3), ...
                                            points(i,4), ...
                                            points(i,5), ...
                                            defl, delta_defl, ...
                                            M, delta_M, R_c, m_c, m_f];
                end
```

```
            else
                infeasible_defl_3 = [infeasible_defl_3; ...
                                    points(i,1), ...
                                    points(i,2), ...
                                    points(i,3), ...
                                    points(i,4), ...
                                    points(i,5), ...
                                    defl, delta_defl, ...
                                    M, delta_M, R_c, m_c, m_f];
            end
        else
            infeasible_mass_3 = [infeasible_mass_3; ...
                                points(i,1), ...
                                points(i,2), ...
                                points(i,3), ...
                                points(i,4), ...
                                points(i,5), ...
                                defl, delta_defl, ...
                                M, delta_M, R_c, m_c, m_f];
        end
    else
        infeasible_hcB_3 = [infeasible_hcB_3; ...
                            points(i,1), ...
                            points(i,2), ...
                            points(i,3), ...
                            points(i,4), ...
                            points(i,5)];
    end
end
toc

% ---------------------------------------------------------------------
% Problem interpretation:
% ---------------------------------------------------------------------
tic
dlmwrite('feasible_a2.txt', feasible_points_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
dlmwrite('infeasible_a2_gamma.txt', infeasible_gamma_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
dlmwrite('infeasible_a2_mu.txt', infeasible_mu_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
dlmwrite('infeasible_a2_R_c.txt', infeasible_R_c_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
dlmwrite('infeasible_a2_defl.txt', infeasible_defl_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
dlmwrite('infeasible_a2_mass.txt', infeasible_mass_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
dlmwrite('infeasible_a2_hcB.txt', infeasible_hcB_3, ...
    'delimiter', '\t', 'precision', 3, 'newline', 'pc')
toc
% ---------------------------------------------------------------------
```

BRPdefl.m

```
function [defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
    B, H, h_c, h_f, h_b, ...
    sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
    rho_b, rho_c, rho_f, ...
    L, p_0, t_0, uncert_vars)

%   BRPdefl Calculates the mass per area and deflection per
%       length of a metal sandwich plate subjected to an impulse load
%       defined by pressure, p_0, and characteristic time, t_0.  The
%       variation of mass and deflection as a result of 8 uncertain
%       factors is also calculated.
%
%   Inputs:
%                  B: cell spacing in [m]
%                  H: core layer height in [m]
%                h_c: cell wall thickness in [m]
%                h_f: front face sheet thickness in [m]
%                h_b: back face sheet thickness in [m]
%      sigma_yield_b: the back face sheet yield strength in [Pa]
%      sigma_yield_c: the core material yield strength in [Pa]
%      sigma_yield_f: the front face sheet yield strength in [Pa]
%              rho_b: the back face sheet density in [kg/m^3]
%              rho_c: the core material density in [kg/m^3]
%              rho_f: the front face sheet density in [kg/m^3]
%                  L: the length of the panel in [m]
%                p_0: the peak pressure in [Pa]
%                t_0: the characteristic time of the blast in [s]
%        uncert_vars: a vector of uncertain parameters
%
%   Outputs:
%             defl: the panel deflection in [m]
%       delta_defl: the variation in the panel deflection in [m]
%                M: the panel mass per area in [kg/m^2]
%          delta_M: the variation in the panel deflection in [kg/m^2]
%              R_c: the relative density of the core layer [no units]
%              m_c: the mass/area of the core layer in [kg/m^2]
%              m_f: the mass/area of the front face sheet in [kg/m^2]

%   Assumptions:
%          - the panel is clamped on all edges
%          - the pressure blast is uniform
%          - elastic, perfectly plastic materials

% Created by Stephanie Thompson and Hannah Muchnick
% Summer 2006

%----------------------------------------------------------------------
% Define uncertain parameters
%----------------------------------------------------------------------
delta_sigma_yield_b = uncert_vars(1);
delta_sigma_yield_c = uncert_vars(2);
delta_sigma_yield_f = uncert_vars(3);
delta_rho_b = uncert_vars(4);
```

```
delta_rho_c = uncert_vars(5);
delta_rho_f = uncert_vars(6);
delta_p_0 = uncert_vars(7);
delta_t_0 = uncert_vars(8);
%----------------------------------------------------------------------
% End uncertain parameter definition
%----------------------------------------------------------------------


%----------------------------------------------------------------------
% Mass Calculation
%----------------------------------------------------------------------
% Relative Density of Square Core
R_c = (2*B*h_c-h_c^2)/B^2;

% mass/area of the core
m_c = rho_c*R_c*H;

% mass/area of one front sheet
m_f = rho_f*h_f;

% mass/area of one back sheet
m_b = rho_b*h_b;

% mass/area of total sandwich plate
M = m_c + m_f + m_b;
%----------------------------------------------------------------------
% End Mass Calculation
%----------------------------------------------------------------------


%----------------------------------------------------------------------
% Change in Mass Calculation
%----------------------------------------------------------------------
% change in mass is the sum of the partial derivatives of the mass
% function with respect to each uncertain parameter multiplied by the
% change of the uncertain parameter

delta_M = (R_c*H)*delta_rho_c + (h_f)*delta_rho_f + (h_b)*delta_rho_b;
%----------------------------------------------------------------------
% End Change in Mass Calculation
%----------------------------------------------------------------------


%----------------------------------------------------------------------
% Deflection Calculation
%----------------------------------------------------------------------
% Define lambda_c, "core compression strength factor", approximately
% equal to one for all extruded honeycomb cores
lambda_c = 1; % for square honeycomb core

% Define lambda_s, "core stretching strength factor", depends on core
% geometry
lambda_s = 0.5; % for square honeycomb core



% the deflection equation is the sum of two functions, f1 and f2
```

```
% defl = f1 + f2;  % must define f1 and f2 first

% f1 is divided into g1N and g1D
% f1 = g1N/g1D; % must define g1N and g1D

% f2 is also divided into two function g2N and g2D
% f2 = g2N/g2D; % must define g2N and g2D

% g2N is divided into j2N and j2D
% g2N = sqrt(j2N/j2D); % must define j2N and j2D

% j2N is the sum of j2Na through j2Nf
% j2N = j2Na + j2Nb + j2Nc + j2Nd + j2Ne + j2Nf;
% must define j2Na through j2Nf

j2Na = (9*h_b^2*H^2*lambda_c^2*R_c^2*h_f^2*...
    sigma_yield_b^2*sigma_yield_c^2*...
    (rho_f^4*h_f^2 + rho_f^2*rho_b^2*h_b^2 + ...
    rho_f^2*rho_c^2*R_c^2*H^2 + ...
    2*rho_f^3*h_f*rho_b*h_b + ...
    2*rho_f^3*h_f*rho_c*R_c*H + 2*rho_f^2*rho_b*h_b*rho_c*R_c*H));

j2Nb = (-36*h_b^2*H*lambda_c*R_c*h_f*p_0^2*t_0^2*...
    sigma_yield_b^2*sigma_yield_c*...
    (rho_b*h_b*rho_f^2*h_f + rho_f*rho_b^2*h_b^2 + ...
    2*rho_f*rho_b*h_b*rho_c*R_c*H + ...
    rho_f^2*h_f*rho_c*R_c*H + rho_f*rho_c^2*R_c^2*H^2));

j2Nc = (36*h_b^2*p_0^4*t_0^4*sigma_yield_b^2*...
    (rho_b^2*h_b^2 + 2*rho_b*h_b*rho_c*R_c*H + ...
    rho_c^2*R_c^2*H^2));

j2Nd = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*rho_f^3*...
    (sigma_yield_f*sigma_yield_c^2*h_f^2 + ...
    sigma_yield_c^3*R_c*H*h_f*lambda_s + ...
    sigma_yield_c^2*sigma_yield_b*h_f*h_b);

j2Ne = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*rho_f^2*rho_b*...
    (sigma_yield_f*sigma_yield_c^2*h_f*h_b + ...
    sigma_yield_c^3*R_c*H*h_b*lambda_s +...
    sigma_yield_c^2*sigma_yield_b*h_b^2);

j2Nf = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*rho_f^2*rho_c*...
    (sigma_yield_f*sigma_yield_c^2*R_c*H*h_f + ...
    sigma_yield_c^3*R_c^2*H^2*lambda_s + ...
    sigma_yield_c^2*sigma_yield_b*R_c*H*h_b);

j2N = j2Na + j2Nb + j2Nc + j2Nd + j2Ne + j2Nf;

j2D = L^2*lambda_c^2*R_c^2*h_f^2*sigma_yield_c^2 * ...
    (rho_f^4*h_f^2 + ...
     rho_f^2*rho_b^2*h_b^2 + ...
     rho_f^2*rho_c^2*R_c^2*H^2 + ...
     2*rho_f^3*h_f*rho_b*h_b + ...
```

```
     2*rho_f^3*h_f*rho_c*R_c*H + ...
     2*rho_f^2*rho_b*h_b*rho_c*R_c*H);

g2N = sqrt(j2N/j2D);

g2D = (sigma_yield_f*h_f + ...
        sigma_yield_c*R_c*H*lambda_s + ...
        sigma_yield_b*h_b);

g1N = (-3*H*lambda_c*R_c*h_f*h_b*sigma_yield_c*sigma_yield_b*...
    (rho_f^2*h_f + rho_f*rho_b*h_b + rho_f*rho_c*R_c*H) + ...
    6*p_0^2*t_0^2*sigma_yield_b*h_b * (rho_b*h_b + rho_c*R_c*H));

g1D = lambda_c*R_c*h_f * ...
    (rho_f^2*h_f + rho_f*rho_b*h_b + rho_f*rho_c*R_c*H)*...
    (sigma_yield_c*sigma_yield_f*h_f + ...
    sigma_yield_c^2*R_c*H*lambda_s +...
    sigma_yield_c*sigma_yield_b*h_b);

f2 = g2N/g2D;

f1 = g1N/g1D;

defl = f1 + L*f2;
%----------------------------------------------------------------------
% End Deflection Calculation
%----------------------------------------------------------------------


%----------------------------------------------------------------------
% Change in Deflection Calculation
%----------------------------------------------------------------------
% change in deflection is the sum of the partial derivatives of the
% deflection function with respect to each uncertain parameter
% multiplied by the change of the uncertain parameter

% delta_defl = ddefl_dsigma_yield_b*delta_sigma_yield_b + ...
%     ddefl_dsigma_yield_c*delta_sigma_yield_c + ...
%     ddefl_dsigma_yield_f*delta_sigma_yield_f + ...
%     ddefl_drho_b*delta_rho_b + ...
%     ddefl_drho_c*delta_rho_c + ...
%     ddefl_drho_f*delta_rho_f + ...
%     ddefl_dp_0*delta_p_0 + ...
%     ddefl_dt_0*delta_t_0

% General partial derivative of defl for referece:
% ddefl_dX = (g1D*dg1N_dX - g1N*dg1D_dX)/g1D^2 + ...
%     (((sqrt(j2D)/(2*sqrt(j2N)) * dj2N_dX) -...
%     (sqrt(j2N)/(2*sqrt(j2D)) * dj2D_dX))/(j2D*g2D) - ...
%     (sqrt(j2N/j2D)*dg2D_dX)/(g2D^2))

%%-- Partial Derivatives with respect to sigma_yield_b ---------------
dg1N_dsigma_yield_b = (-3*H*lambda_c*R_c*h_f*h_b*sigma_yield_c*...
    (rho_f^2*h_f + rho_f*rho_b*h_b + rho_f*rho_c*R_c*H) + ...
    6*p_0^2*t_0^2*h_b * (rho_b*h_b + rho_c*R_c*H));
```

```
dg1D_dsigma_yield_b = lambda_c*R_c*h_f * ...
    (rho_f^2*h_f + rho_f*rho_b*h_b + rho_f*rho_c*R_c*H)*...
    (sigma_yield_c*h_b);

dj2Na_dsigma_yield_b = (18*h_b^2*H^2*lambda_c^2*R_c^2*h_f^2*...
                        sigma_yield_b*sigma_yield_c^2*...
    (rho_f^4*h_f^2 + rho_f^2*rho_b^2*h_b^2 + ...
    rho_f^2*rho_c^2*R_c^2*H^2 + ...
    2*rho_f^3*h_f*rho_b*h_b + ...
    2*rho_f^3*h_f*rho_c*R_c*H + ...
    2*rho_f^2*rho_b*h_b*rho_c*R_c*H));
dj2Nb_dsigma_yield_b = (-72*h_b^2*H*lambda_c*R_c*h_f*p_0^2*t_0^2*...
    sigma_yield_b*sigma_yield_c*...
    (rho_b*h_b*rho_f^2*h_f + ...
    rho_f*rho_b^2*h_b^2 + ...
    2*rho_f*rho_b*h_b*rho_c*R_c*H + ...
    rho_f^2*h_f*rho_c*R_c*H + ...
    rho_f*rho_c^2*R_c^2*H^2));
dj2Nc_dsigma_yield_b = (72*h_b^2*p_0^4*t_0^4*sigma_yield_b*...
    (rho_b^2*h_b^2 + ...
    2*rho_b*h_b*rho_c*R_c*H + ...
    rho_c^2*R_c^2*H^2));
dj2Nd_dsigma_yield_b = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*...
    rho_f^3*(sigma_yield_c^2*h_f*h_b);
dj2Ne_dsigma_yield_b = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*...
    rho_f^2*rho_b*...
    (sigma_yield_c^2*h_b^2);
dj2Nf_dsigma_yield_b = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*...
    rho_f^2*rho_c*...
    (sigma_yield_c^2*R_c*H*h_b);

dj2N_dsigma_yield_b = dj2Na_dsigma_yield_b + dj2Nb_dsigma_yield_b +...
                      dj2Nc_dsigma_yield_b + dj2Nd_dsigma_yield_b + ...
                      dj2Ne_dsigma_yield_b + dj2Nf_dsigma_yield_b;

dj2D_dsigma_yield_b = 0;

dg2D_dsigma_yield_b = (h_b);

ddefl_dsigma_yield_b = (g1D*dg1N_dsigma_yield_b - ...
                        g1N*dg1D_dsigma_yield_b)/g1D^2 + ...
    (((sqrt(j2D)/(2*sqrt(j2N)) * dj2N_dsigma_yield_b) -...
    (sqrt(j2N)/(2*sqrt(j2D)) * dj2D_dsigma_yield_b))/(j2D*g2D) - ...
    (sqrt(j2N/j2D)*dg2D_dsigma_yield_b)/(g2D^2));
%%-- End Partial Derivatives with respect to sigma_yield_b -----

%%-- Partial Derivatives with respect to sigma_yield_c ---------------
dg1N_dsigma_yield_c = (-3*H*lambda_c*R_c*h_f*h_b*sigma_yield_b*...
    (rho_f^2*h_f + rho_f*rho_b*h_b + rho_f*rho_c*R_c*H));

dg1D_dsigma_yield_c = lambda_c*R_c*h_f * ...
    (rho_f^2*h_f + rho_f*rho_b*h_b + rho_f*rho_c*R_c*H)*...
    (sigma_yield_f*h_f + 2*sigma_yield_c*R_c*H*lambda_s +...
    sigma_yield_b*h_b);
```

```
dj2Na_dsigma_yield_c = (18*h_b^2*H^2*lambda_c^2*R_c^2*h_f^2*...
    sigma_yield_b^2*sigma_yield_c*...
    (rho_f^4*h_f^2 + rho_f^2*rho_b^2*h_b^2 + ...
    rho_f^2*rho_c^2*R_c^2*H^2 + ...
    2*rho_f^3*h_f*rho_b*h_b + ...
    2*rho_f^3*h_f*rho_c*R_c*H + ...
    2*rho_f^2*rho_b*h_b*rho_c*R_c*H));
dj2Nb_dsigma_yield_c = (-36*h_b^2*H*lambda_c*R_c*h_f*p_0^2*t_0^2*...
    sigma_yield_b^2*...
    (rho_b*h_b*rho_f^2*h_f + ...
    rho_f*rho_b^2*h_b^2 + ...
    2*rho_f*rho_b*h_b*rho_c*R_c*H + ...
    rho_f^2*h_f*rho_c*R_c*H + ...
    rho_f*rho_c^2*R_c^2*H^2));
dj2Nc_dsigma_yield_c = 0;
dj2Nd_dsigma_yield_c = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*...
    rho_f^3*(2*sigma_yield_f*sigma_yield_c*h_f^2 + ...
    3*sigma_yield_c^2*R_c*H*h_f*lambda_s + ...
    2*sigma_yield_c*sigma_yield_b*h_f*h_b);
dj2Ne_dsigma_yield_c = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*...
    rho_f^2*rho_b*...
    (2*sigma_yield_f*sigma_yield_c*h_f*h_b + ...
    3*sigma_yield_c^2*R_c*H*h_b*lambda_s +...
    2*sigma_yield_c*sigma_yield_b*h_b^2);
dj2Nf_dsigma_yield_c = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*...
    rho_f^2*rho_c*...
    (2*sigma_yield_f*sigma_yield_c*R_c*H*h_f + ...
    3*sigma_yield_c^2*R_c^2*H^2*lambda_s + ...
    2*sigma_yield_c*sigma_yield_b*R_c*H*h_b);

dj2N_dsigma_yield_c = dj2Na_dsigma_yield_c + dj2Nb_dsigma_yield_c +...
    dj2Nc_dsigma_yield_c + dj2Nd_dsigma_yield_c + ...
    dj2Ne_dsigma_yield_c + dj2Nf_dsigma_yield_c;

dj2D_dsigma_yield_c = 2*L^2*lambda_c^2*R_c^2*h_f^2*sigma_yield_c * ...
    (rho_f^4*h_f^2 + ...
    rho_f^2*rho_b^2*h_b^2 + ...
    rho_f^2*rho_c^2*R_c^2*H^2 + ...
    2*rho_f^3*h_f*rho_b*h_b + ...
    2*rho_f^3*h_f*rho_c*R_c*H + ...
    2*rho_f^2*rho_b*h_b*rho_c*R_c*H);

dg2D_dsigma_yield_c = (R_c*H*lambda_s);

ddefl_dsigma_yield_c = (g1D*dg1N_dsigma_yield_c - ...
                        g1N*dg1D_dsigma_yield_c)/g1D^2 + ...
    L*(((sqrt(j2D)/(2*sqrt(j2N)) * dj2N_dsigma_yield_c) -...
    (sqrt(j2N)/(2*sqrt(j2D)) * dj2D_dsigma_yield_c))/(j2D*g2D) - ...
    (sqrt(j2N/j2D)*dg2D_dsigma_yield_c)/(g2D^2));
%%-- End Partial Derivatives with respect to sigma_yield_c -----------

%%-- Partial Derivatives with respect to sigma_yield_f ---------------
dg1N_dsigma_yield_f = 0;
```

```
dg1D_dsigma_yield_f = lambda_c*R_c*h_f * ...
    (rho_f^2*h_f + rho_f*rho_b*h_b + rho_f*rho_c*R_c*H)*...
    (sigma_yield_c*h_f);


dj2Na_dsigma_yield_f = 0;
dj2Nb_dsigma_yield_f = 0;
dj2Nc_dsigma_yield_f = 0;
dj2Nd_dsigma_yield_f = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*...
    rho_f^3*(sigma_yield_c^2*h_f^2);
dj2Ne_dsigma_yield_f = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*...
    rho_f^2*rho_b*...
    (sigma_yield_c^2*h_f*h_b);
dj2Nf_dsigma_yield_f = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*...
    rho_f^2*rho_c*...
    (sigma_yield_c^2*R_c*H*h_f);

dj2N_dsigma_yield_f = dj2Na_dsigma_yield_f + dj2Nb_dsigma_yield_f +...
    dj2Nc_dsigma_yield_f + dj2Nd_dsigma_yield_f + ...
    dj2Ne_dsigma_yield_f + dj2Nf_dsigma_yield_f;

dj2D_dsigma_yield_f = 0;

dg2D_dsigma_yield_f = (h_f);

ddefl_dsigma_yield_f = (g1D*dg1N_dsigma_yield_f - ...
                        g1N*dg1D_dsigma_yield_f)/g1D^2 + ...
    L*(((sqrt(j2D)/(2*sqrt(j2N)) * dj2N_dsigma_yield_f) -...
    (sqrt(j2N)/(2*sqrt(j2D)) * dj2D_dsigma_yield_f))/(j2D*g2D) - ...
    (sqrt(j2N/j2D)*dg2D_dsigma_yield_f)/(g2D^2));
%%-- End Partial Derivatives with respect to sigma_yield_f -----------

%%-- Partial Derivatives with respect to rho_b -----------------------
dg1N_drho_b = (-3*H*lambda_c*R_c*h_f*h_b*...
    sigma_yield_c*sigma_yield_b*...
    (rho_f*h_b) + 6*p_0^2*t_0^2*sigma_yield_b*h_b * (h_b));

dg1D_drho_b = lambda_c*R_c*h_f * ...
    (rho_f*h_b)*(sigma_yield_c*sigma_yield_f*h_f + ...
    sigma_yield_c^2*R_c*H*lambda_s +...
    sigma_yield_c*sigma_yield_b*h_b);

dj2Na_drho_b = (9*h_b^2*H^2*lambda_c^2*R_c^2*h_f^2*...
    sigma_yield_b^2*sigma_yield_c^2*...
    (2*rho_f^2*rho_b*h_b^2 + ...
    2*rho_f^3*h_f*h_b + ...
    2*rho_f^2*h_b*rho_c*R_c*H));
dj2Nb_drho_b = (-36*h_b^2*H*lambda_c*R_c*h_f*p_0^2*t_0^2*...
    sigma_yield_b^2*sigma_yield_c*...
    (h_b*rho_f^2*h_f + ...
    2*rho_f*rho_b*h_b^2 + ...
    2*rho_f*h_b*rho_c*R_c*H));
dj2Nc_drho_b = (36*h_b^2*p_0^4*t_0^4*sigma_yield_b^2*...
    (2*rho_b*h_b^2 + 2*h_b*rho_c*R_c*H));
dj2Nd_drho_b = 0;
dj2Ne_drho_b = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*rho_f^2*...
```

```
    (sigma_yield_f*sigma_yield_c^2*h_f*h_b + ...
    sigma_yield_c^3*R_c*H*h_b*lambda_s +...
    sigma_yield_c^2*sigma_yield_b*h_b^2);
dj2Nf_drho_b = 0;

dj2N_drho_b = dj2Na_drho_b + dj2Nb_drho_b + dj2Nc_drho_b +...
    dj2Nd_drho_b + dj2Ne_drho_b + dj2Nf_drho_b;

dj2D_drho_b = L^2*lambda_c^2*R_c^2*h_f^2*sigma_yield_c^2 * ...
    (2*rho_f^2*rho_b*h_b^2 + ...
    2*rho_f^3*h_f*h_b + ...
    2*rho_f^2*h_b*rho_c*R_c*H);

dg2D_drho_b = 0;

ddefl_drho_b = (g1D*dg1N_drho_b - g1N*dg1D_drho_b)/g1D^2 + ...
    L*(((sqrt(j2D)/(2*sqrt(j2N)) * dj2N_drho_b) -...
    (sqrt(j2N)/(2*sqrt(j2D)) * dj2D_drho_b))/(j2D*g2D) - ...
    (sqrt(j2N/j2D)*dg2D_drho_b)/(g2D^2));
%%-- End Partial Derivatives with respect to rho_b -------------------

%%-- Partial Derivatives with respect to rho_c -----------------------
dg1N_drho_c = (-3*H*lambda_c*R_c*h_f*h_b*...
    sigma_yield_c*sigma_yield_b*...
    (rho_f*R_c*H) + ...
    6*p_0^2*t_0^2*sigma_yield_b*h_b * (R_c*H));

dg1D_drho_c = lambda_c*R_c*h_f * (rho_f*R_c*H)*...
    (sigma_yield_c*sigma_yield_f*h_f + ...
    sigma_yield_c^2*R_c*H*lambda_s +...
    sigma_yield_c*sigma_yield_b*h_b);

dj2Na_drho_c = (9*h_b^2*H^2*lambda_c^2*R_c^2*h_f^2*...
    sigma_yield_b^2*sigma_yield_c^2*...
    (2*rho_f^2*rho_c*R_c^2*H^2 + ...
    2*rho_f^3*h_f*R_c*H + ...
    2*rho_f^2*rho_b*h_b*R_c*H));
dj2Nb_drho_c = (-36*h_b^2*H*lambda_c*R_c*h_f*p_0^2*t_0^2*...
    sigma_yield_b^2*sigma_yield_c*...
    (2*rho_f*rho_b*h_b*R_c*H + ...
    rho_f^2*h_f*R_c*H + ...
    2*rho_f*rho_c*R_c^2*H^2));
dj2Nc_drho_c = (36*h_b^2*p_0^4*t_0^4*sigma_yield_b^2*...
    (2*rho_b*h_b*R_c*H + 2*rho_c*R_c^2*H^2));
dj2Nd_drho_c = 0;
dj2Ne_drho_c = 0;
dj2Nf_drho_c = 3*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*rho_f^2*...
    (sigma_yield_f*sigma_yield_c^2*R_c*H*h_f + ...
    sigma_yield_c^3*R_c^2*H^2*lambda_s + ...
    sigma_yield_c^2*sigma_yield_b*R_c*H*h_b);

dj2N_drho_c = dj2Na_drho_c + dj2Nb_drho_c + dj2Nc_drho_c +...
    dj2Nd_drho_c + dj2Ne_drho_c + dj2Nf_drho_c;

dj2D_drho_c = L^2*lambda_c^2*R_c^2*h_f^2*sigma_yield_c^2 * ...
```

```
    (2*rho_f^2*rho_c*R_c^2*H^2 + ...
    2*rho_f^3*h_f*R_c*H + ...
    2*rho_f^2*rho_b*h_b*R_c*H);

dg2D_drho_c =0;

ddefl_drho_c = (g1D*dg1N_drho_c - g1N*dg1D_drho_c)/g1D^2 + ...
    L*(((sqrt(j2D)/(2*sqrt(j2N)) * dj2N_drho_c) -...
    (sqrt(j2N)/(2*sqrt(j2D)) * dj2D_drho_c))/(j2D*g2D) - ...
    (sqrt(j2N/j2D)*dg2D_drho_c)/(g2D^2));
%%-- End Partial Derivatives with respect to rho_c -------------------

%%-- Partial Derivatives with respect to rho_f -----------------------
dg1N_drho_f = (-3*H*lambda_c*R_c*h_f*h_b*...
    sigma_yield_c*sigma_yield_b*...
    (2*rho_f*h_f + rho_b*h_b + rho_c*R_c*H));

dg1D_drho_f = lambda_c*R_c*h_f * ...
    (2*rho_f*h_f + rho_b*h_b + rho_c*R_c*H)*...
    (sigma_yield_c*sigma_yield_f*h_f + ...
    sigma_yield_c^2*R_c*H*lambda_s +...
    sigma_yield_c*sigma_yield_b*h_b);

dj2Na_drho_f = (9*h_b^2*H^2*lambda_c^2*R_c^2*h_f^2*...
    sigma_yield_b^2*sigma_yield_c^2*...
    (4*rho_f^3*h_f^2 + ...
    2*rho_f*rho_b^2*h_b^2 + ...
    2*rho_f*rho_c^2*R_c^2*H^2 + ...
    6*rho_f^2*h_f*rho_b*h_b + ...
    6*rho_f^2*h_f*rho_c*R_c*H + ...
    4*rho_f*rho_b*h_b*rho_c*R_c*H));
dj2Nb_drho_f = (-36*h_b^2*H*lambda_c*R_c*h_f*p_0^2*t_0^2*...
    sigma_yield_b^2*sigma_yield_c*...
    (2*rho_b*h_b*rho_f*h_f + ...
    rho_b^2*h_b^2 + ...
    2*rho_b*h_b*rho_c*R_c*H + ...
    2*rho_f*h_f*rho_c*R_c*H + ...
    rho_c^2*R_c^2*H^2));
dj2Nc_drho_f = 0;
dj2Nd_drho_f = 9*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*rho_f^2*...
    (sigma_yield_f*sigma_yield_c^2*h_f^2 + ...
    sigma_yield_c^3*R_c*H*h_f*lambda_s + ...
    sigma_yield_c^2*sigma_yield_b*h_f*h_b);
dj2Ne_drho_f = 6*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*...
    rho_f*rho_b*...
    (sigma_yield_f*sigma_yield_c^2*h_f*h_b + ...
    sigma_yield_c^3*R_c*H*h_b*lambda_s +...
    sigma_yield_c^2*sigma_yield_b*h_b^2);
dj2Nf_drho_f = 6*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0^2*...
    rho_f*rho_c*...
    (sigma_yield_f*sigma_yield_c^2*R_c*H*h_f + ...
    sigma_yield_c^3*R_c^2*H^2*lambda_s + ...
    sigma_yield_c^2*sigma_yield_b*R_c*H*h_b);

dj2N_drho_f = dj2Na_drho_f + dj2Nb_drho_f + dj2Nc_drho_f +...
```

```matlab
    dj2Nd_drho_f + dj2Ne_drho_f + dj2Nf_drho_f;

dj2D_drho_f = L^2*lambda_c^2*R_c^2*h_f^2*sigma_yield_c^2 * ...
    (4*rho_f^3*h_f^2 + ...
    2*rho_f*rho_b^2*h_b^2 + ...
    2*rho_f*rho_c^2*R_c^2*H^2 + ...
    6*rho_f^2*h_f*rho_b*h_b + ...
    6*rho_f^2*h_f*rho_c*R_c*H + ...
    4*rho_f*rho_b*h_b*rho_c*R_c*H);

dg2D_drho_f = 0;

ddefl_drho_f = (g1D*dg1N_drho_f - g1N*dg1D_drho_f)/g1D^2 + ...
    L*(((sqrt(j2D)/(2*sqrt(j2N)) * dj2N_drho_f) -...
    (sqrt(j2N)/(2*sqrt(j2D)) * dj2D_drho_f))/(j2D*g2D) - ...
    (sqrt(j2N/j2D)*dg2D_drho_f)/(g2D^2));
%%-- End Partial Derivatives with respect to rho_f -------------------

%%-- Partial Derivatives with respect to p_0 -------------------------
dg1N_dp_0 = (12*p_0*t_0^2*sigma_yield_b*h_b * ...
    (rho_b*h_b + rho_c*R_c*H));

dg1D_dp_0 = 0;

dj2Na_dp_0 = 0;
dj2Nb_dp_0 = (-72*h_b^2*H*lambda_c*R_c*h_f*p_0*t_0^2*...
    sigma_yield_b^2*sigma_yield_c*...
    (rho_b*h_b*rho_f^2*h_f + ...
    rho_f*rho_b^2*h_b^2 + ...
    2*rho_f*rho_b*h_b*rho_c*R_c*H + ...
    rho_f^2*h_f*rho_c*R_c*H + ...
    rho_f*rho_c^2*R_c^2*H^2));
dj2Nc_dp_0 = (144*h_b^2*p_0^3*t_0^4*sigma_yield_b^2*...
    (rho_b^2*h_b^2 + 2*rho_b*h_b*rho_c*R_c*H + ...
    rho_c^2*R_c^2*H^2));
dj2Nd_dp_0 = 6*L^2*lambda_c^2*R_c^2*h_f^2*p_0*t_0^2*rho_f^3*...
    (sigma_yield_f*sigma_yield_c^2*h_f^2 + ...
    sigma_yield_c^3*R_c*H*h_f*lambda_s + ...
    sigma_yield_c^2*sigma_yield_b*h_f*h_b);
dj2Ne_dp_0 = 6*L^2*lambda_c^2*R_c^2*h_f^2*p_0*t_0^2*rho_f^2*rho_b*...
    (sigma_yield_f*sigma_yield_c^2*h_f*h_b + ...
    sigma_yield_c^3*R_c*H*h_b*lambda_s +...
    sigma_yield_c^2*sigma_yield_b*h_b^2);
dj2Nf_dp_0 = 6*L^2*lambda_c^2*R_c^2*h_f^2*p_0*t_0^2*rho_f^2*rho_c*...
    (sigma_yield_f*sigma_yield_c^2*R_c*H*h_f + ...
    sigma_yield_c^3*R_c^2*H^2*lambda_s + ...
    sigma_yield_c^2*sigma_yield_b*R_c*H*h_b);

dj2N_dp_0 = dj2Na_dp_0 + dj2Nb_dp_0 + dj2Nc_dp_0 + ...
            dj2Nd_dp_0 + dj2Ne_dp_0 + dj2Nf_dp_0;

dj2D_dp_0 = 0;

dg2D_dp_0 = 0;
```

```
ddefl_dp_0 = (g1D*dg1N_dp_0 - g1N*dg1D_dp_0)/g1D^2 + ...
    L*(((sqrt(j2D)/(2*sqrt(j2N)) * dj2N_dp_0) -...
    (sqrt(j2N)/(2*sqrt(j2D)) * dj2D_dp_0))/(j2D*g2D) - ...
    (sqrt(j2N/j2D)*dg2D_dp_0)/(g2D^2));
%%-- End Partial Derivatives with respect to p_0 ---------------------

%%-- Partial Derivatives with respect to t_0 -------------------------
dg1N_dt_0 = (12*p_0^2*t_0*sigma_yield_b*h_b * ...
    (rho_b*h_b + rho_c*R_c*H));

dg1D_dt_0 = 0;

dj2Na_dt_0 = 0;
dj2Nb_dt_0 = (-72*h_b^2*H*lambda_c*R_c*h_f*p_0^2*t_0*...
    sigma_yield_b^2*sigma_yield_c*...
    (rho_b*h_b*rho_f^2*h_f + ...
    rho_f*rho_b^2*h_b^2 + ...
    2*rho_f*rho_b*h_b*rho_c*R_c*H + ...
    rho_f^2*h_f*rho_c*R_c*H + ...
    rho_f*rho_c^2*R_c^2*H^2));
dj2Nc_dt_0 = (144*h_b^2*p_0^4*t_0^3*sigma_yield_b^2*...
    (rho_b^2*h_b^2 + 2*rho_b*h_b*rho_c*R_c*H + ...
    rho_c^2*R_c^2*H^2));
dj2Nd_dt_0 = 6*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0*rho_f^3*...
    (sigma_yield_f*sigma_yield_c^2*h_f^2 + ...
    sigma_yield_c^3*R_c*H*h_f*lambda_s + ...
    sigma_yield_c^2*sigma_yield_b*h_f*h_b);
dj2Ne_dt_0 = 6*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0*rho_f^2*rho_b*...
    (sigma_yield_f*sigma_yield_c^2*h_f*h_b + ...
    sigma_yield_c^3*R_c*H*h_b*lambda_s +...
    sigma_yield_c^2*sigma_yield_b*h_b^2);
dj2Nf_dt_0 = 6*L^2*lambda_c^2*R_c^2*h_f^2*p_0^2*t_0*rho_f^2*rho_c*...
    (sigma_yield_f*sigma_yield_c^2*R_c*H*h_f + ...
    sigma_yield_c^3*R_c^2*H^2*lambda_s + ...
    sigma_yield_c^2*sigma_yield_b*R_c*H*h_b);

dj2N_dt_0 = dj2Na_dt_0 + dj2Nb_dt_0 + dj2Nc_dt_0 + ...
            dj2Nd_dt_0 + dj2Ne_dt_0 + dj2Nf_dt_0;

dj2D_dt_0 = 0;

dg2D_dt_0 = 0;

ddefl_dt_0 = (g1D*dg1N_dt_0 - g1N*dg1D_dt_0)/g1D^2 + ...
    L*(((sqrt(j2D)/(2*sqrt(j2N)) * dj2N_dt_0) -...
    (sqrt(j2N)/(2*sqrt(j2D)) * dj2D_dt_0))/(j2D*g2D) - ...
    (sqrt(j2N/j2D)*dg2D_dt_0)/(g2D^2));
%%-- End Partial Derivatives with respect to t_0 ---------------------
% Now that all the partial derivatives have been defined, the change
% in deflection can be calculated.
delta_defl = abs(ddefl_dsigma_yield_b)*delta_sigma_yield_b + ...
    abs(ddefl_dsigma_yield_c)*delta_sigma_yield_c + ...
    abs(ddefl_dsigma_yield_f)*delta_sigma_yield_f + ...
    abs(ddefl_drho_b)*delta_rho_b + ...
```

```
    abs(ddefl_drho_c)*delta_rho_c + ...
    abs(ddefl_drho_f)*delta_rho_f + ...
    abs(ddefl_dp_0)*delta_p_0 + ...
    abs(ddefl_dt_0)*delta_t_0;

%-----------------------------------------------------------------------
% End Change in Deflection Calculation
%-----------------------------------------------------------------------
```

BRPfitness_nr.m

```
function [Z] = BRPfitness_nr(x)

%   BRPfitness_nr Determines the value of the fitness function Z
%       given a vector of design variables, x. This fitness function
%       is used to solve the expansion cDSP for the Point Scenario of
%       the BRP example
%
%   Inputs:
%      X:   a vector of design variables represented by a vector of
%           55 bits
%   Outputs:
%      Z:   the value of the fitness function at design point X
%
%   Assumptions:
%         - design variable vector in the order of:
%           X = [B, H, h_c, h_f, h_b, ...
%                 sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f]
%         - uncertain parameter vector in the order of:
%           DX = [sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f, ...
%                 p_0, t_0]
%         - constraint values vector in the order of:
%           GX = [mass_con, defl_con, Rc_con, gamma_con, mu_con]

%
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%

% Set scheme
scheme = 11;

% -----------------------------------------------------------------------
% Convert x from bit string to normalized design variables
% -----------------------------------------------------------------------
% There are eleven design variables, each discretized into 32 points
% between zero and one.  The 32 points are represented by a binary
% string of 5 bits.

% separate bit string x into 11 design variables:
```

```
x = [x(1:5); x(6:10); x(11:15); x(16:20); x(21:25); x(26:30);...
     x(31:35); x(36:40); x(41:45); x(46:50); x(51:55)];

% convert binary to base ten:
x = bin2dec(strcat(num2str(x)))';

% normalize from 0 to 1:
x = norm01(x,zeros(1,11),31*ones(1,11));

% ----------------------------------------------------------------------
% GIVEN
% ----------------------------------------------------------------------
% Constant parameters
L = 1;
p_0 = 25*10^6;
t_0 = 0.0001;
P = 1000; % Penalty parameter
delta_p_0 = 0.15*p_0;
delta_t_0 = 0.15*t_0;

% Baseline Material Properties
rho_mg = 1770; % 1770 kg/m3
rho_st = 7845; % 7845 kg/m3
sigma_y_mg = 200 * 10^6; % 200 MPa
sigma_y_st = 538 * 10^6; % 538 MPa

% Variation in material properties
delta_rho_mg = 170; % 170 kg/m3
delta_rho_st = 100; % 100 kg/m3
delta_sigma_y_mg = 20 * 10^6; % 20 MPa
delta_sigma_y_st = 150 * 10^6; % 150 MPa

x_base = [sigma_y_mg, sigma_y_st, sigma_y_mg, rho_mg, rho_st, rho_mg];
dx = [delta_sigma_y_mg, delta_sigma_y_st, delta_sigma_y_mg, ...
    delta_rho_mg, delta_rho_st, delta_rho_mg, ...
    delta_p_0, delta_t_0];
weights = 1-dx(1:6)./x_base;
dx = zeros(1,8);



% Archimedean weights
% W = [mass, defl, cost];
W = [1 0 0; 0 1 0; 0 0 1; ...
    .6 .2 .2; .2 .6 .2; .2 .2 .6; ...
    .2 .4 .4; .4 .2 .4; .4 .4 .2; ...
    1/3 1/3 1/3; 0.5 0.5 0];

% [defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
%     x(1), x(2), x(3), x(4), x(5), ...
%     x(6), x(7), x(8), ...
%     x(9), x(10), x(11), ...
%     L, p_0, t_0, uncert_vars);

% ----------------------------------------------------------------------
```

```
% FIND
% -------------------------------------------------------------------
% Design Variables
% xabs = [B, H, h_c, h_f, h_b, ...
%     sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%     rho_b, rho_c, rho_f];

% Deviation Variables
% d = [sum(deviation(M,M_target,'min')),...
%      sum(deviation(defl,defl_target,'min')),...
%      sum(deviation(cost,cost_target,'min'))];
% -------------------------------------------------------------------
% SATISFY
% -------------------------------------------------------------------
% Bounds
ub = [2/100, 5/100, 1/100, 5/100, 5/100, ...
    1200e6, 1200e6, 1200e6, ...
    20000, 20000, 20000];
lb = [1/1000, 5/1000, 0.1/1000, 1/1000, 1/1000, ...
    20e6, 20e6, 20e6 ...
    1600, 1600, 1600];
xabs = unnorm01(x, lb, ub);

% Constraints
% constraints are defined and evaluated in function
% ineq_constraints_BRP_nr
gx = ineq_constraints_BRP_nr(xabs, dx);

% Goals
% Minimize mass as close as possible to 0 kg/m2 with a maximum value
% of 2997.7
M_max = 2997.7;

% Minimize deflection as close as possible to 0 cm with a maximum
% value of 234.5 m
defl_max = 234.5;

% Minimize cost as close as possible to 0, with a maximum value of
% 5.35
cost_max = 5.35;

% Evaluate BRP deflection code
[defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
    xabs(1), xabs(2), xabs(3), xabs(4), xabs(5), ...
    xabs(6), xabs(7), xabs(8), ...
    xabs(9), xabs(10), xabs(11), ...
    L, p_0, t_0, dx);

% Get cost
[cost, delta_cost] = DSEcost(xabs(6:11), x_base, weights, dx(1:6));



    [dmass_minus,dmass_plus] = deviation(M,M_max,'m20');
    [ddefl_minus,ddefl_plus] = deviation(defl,defl_max,'m20');
    [dcost_minus,dcost_plus] = deviation(cost,cost_max,'m20');
```

```
    d = [(dmass_minus + dmass_plus), ...
        (ddefl_minus+ddefl_plus), ...
        (dcost_minus+ dcost_plus)];
    dmass_minus = [];
    dmass_plus = [];
    ddefl_minus = [];
    ddefl_plus = [];
    dcost_minus = [];
    dcost_plus = [];

% ---------------------------------------------------------------------
% MINIMIZE
% ---------------------------------------------------------------------
% Archimedean formulation
Z = W(scheme,:)*(d)' + P*sum(gx.^2);
% ---------------------------------------------------------------------
```

BRPfitness_r.m

```
function [Z] = BRPfitness_r(x)

%   BRPfitness_r Determines the value of the fitness function Z
%       given a vector of design variables, x. This fitness function
%       is used to solve the expansion cDSP for the Ranged Scenario of
%       the BRP example
%
%   Inputs:
%      X:   a vector of design variables represented by a vector of
%           55 bits
%   Outputs:
%      Z:   the value of the fitness function at design point X
%
%   Assumptions:
%         - design variable vector in the order of:
%           X = [B, H, h_c, h_f, h_b, ...
%                 sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f]
%         - uncertain parameter vector in the order of:
%           DX = [sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f, ...
%                 p_0, t_0]
%         - constraint values vector in the order of:
%           GX = [mass_con, defl_con, Rc_con, gamma_con, mu_con]


%
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%


%Set Scheme
scheme = 11;
```

```
% -------------------------------------------------------------------
% Convert x from bit string to normalized design variables
% -------------------------------------------------------------------
% There are eleven design variables, each discretized into 32 points
% between zero and one.  The 32 points are represented by a binary
% string of 5 bits.

% separate bit string x into 11 design variables:
x = [x(1:5); x(6:10); x(11:15); x(16:20); x(21:25); x(26:30);...
     x(31:35); x(36:40); x(41:45); x(46:50); x(51:55)];

% convert binary to base ten:
x = bin2dec(strcat(num2str(x)))';

% normalize from 0 to 1:
x = norm01(x,zeros(1,11),31*ones(1,11));

% -------------------------------------------------------------------
% GIVEN
% -------------------------------------------------------------------
% Constant parameters
L = 1;
p_0 = 25*10^6;
t_0 = 0.0001;
P = 1000; % Penalty parameter
delta_p_0 = 0.15*p_0;
delta_t_0 = 0.15*t_0;

% Baseline Material Properties
rho_mg = 1770; % 1770 kg/m3
rho_st = 7845; % 7845 kg/m3
sigma_y_mg = 200 * 10^6; % 200 MPa
sigma_y_st = 538 * 10^6; % 538 MPa

% Variation in material properties
delta_rho_mg = 170; % 170 kg/m3
delta_rho_st = 100; % 100 kg/m3
delta_sigma_y_mg = 20 * 10^6; % 20 MPa
delta_sigma_y_st = 150 * 10^6; % 150 MPa

x_base = [sigma_y_mg, sigma_y_st, sigma_y_mg, rho_mg, rho_st, rho_mg];
dx = [delta_sigma_y_mg, delta_sigma_y_st, delta_sigma_y_mg, ...
    delta_rho_mg, delta_rho_st, delta_rho_mg, ...
    delta_p_0, delta_t_0];
weights = 1-dx(1:6)./x_base;


% Archimedean weights
% W = [mass, defl, cost];
W = [1 0 0; 0 1 0; 0 0 1; ...
    .6 .2 .2; .2 .6 .2; .2 .2 .6; ...
    .2 .4 .4; .4 .2 .4; .4 .4 .2; ...
    1/3 1/3 1/3; 0.5 0.5 0];
```

```
% [defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
%     x(1), x(2), x(3), x(4), x(5), ...
%     x(6), x(7), x(8), ...
%     x(9), x(10), x(11), ...
%     L, p_0, t_0, uncert_vars);

% --------------------------------------------------------------------
% FIND
% --------------------------------------------------------------------
% Design Variables
% xabs = [B, H, h_c, h_f, h_b, ...
%     sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%     rho_b, rho_c, rho_f];

% Deviation Variables
% d = [sum(deviation(M,M_target,'min')),...
%      sum(deviation(defl,defl_target,'min')),...
%      sum(deviation(cost,cost_target,'min'))];
% --------------------------------------------------------------------
% SATISFY
% --------------------------------------------------------------------
% Bounds
ub = [2/100, 5/100, 1/100, 5/100, 5/100, ...
    1200e6-delta_sigma_y_mg, ...
    1200e6-delta_sigma_y_st, ...
    1200e6-delta_sigma_y_mg, ...
    20000-delta_rho_mg, 20000-delta_rho_st, 20000-delta_rho_mg];
lb = [1/1000, 5/1000, 0.1/1000, 1/1000, 1/1000, ...
    20e6+delta_sigma_y_mg, ...
    20e6+delta_sigma_y_st, ...
    20e6+delta_sigma_y_mg, ...
    1600+delta_rho_mg, 1600+delta_rho_st, 1600+delta_rho_mg];
xabs = unnorm01(x, lb, ub);

% Constraints
% constraints are defined and evaluated in function
% ineq_constraints_BRP_r
gx = ineq_constraints_BRP_r(xabs, dx);

% Goals
% Minimize mass as close as possible to 0 kg/m2
M_target = 0;

% Minimize deflection as close as possible to 0 cm
defl_target = 0;

% Minimize cost as close as possible to 0
cost_target = 0;

% Evaluate BRP deflection code
[defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
    xabs(1), xabs(2), xabs(3), xabs(4), xabs(5), ...
    xabs(6), xabs(7), xabs(8), ...
    xabs(9), xabs(10), xabs(11), ...
    L, p_0, t_0, dx);
```

```
% Get Cdk's
Cdk_M = (M_target - M)/delta_M;
Cdk_defl = (defl_target - defl)/delta_defl;
[cost, delta_cost] = DSEcost(xabs(6:11), x_base, weights, dx(1:6));
Cdk_cost = (cost_target - cost)/delta_cost;

    % calculate deviation vector
    [dmass_minus,dmass_plus] = deviation(Cdk_M,1,'max');
    [ddefl_minus,ddefl_plus] = deviation(Cdk_defl,1,'max');
    [dcost_minus,dcost_plus] = deviation(Cdk_cost,1,'max');
    d = [(dmass_minus), (ddefl_minus), (dcost_minus)];
    dmass_minus = [];
    dmass_plus = [];
    ddefl_minus = [];
    ddefl_plus = [];
    dcost_minus = [];
    dcost_plus = [];

% -----------------------------------------------------------------------
% MINIMIZE
% -----------------------------------------------------------------------
% Archimedean formulation
Z = W(scheme,:)*(d)' + P*sum(gx.^2);
% -----------------------------------------------------------------------
```

BRPpattern_nr.m

```
function [Z] = BRPpattern_nr(x)

%   BRPpattern_nr Determines the value of the objective function Z
%       given a vector of design variables, x. This objective function
%       is used to solve the expansion cDSP for the Point Scenario of
%       the BRP example
%
%   Inputs:
%      X:   a vector of design variables normalized between 0 and 1
%
%   Outputs:
%      Z:   the value of the objective function at design point X
%
%   Assumptions:
%         - design variable vector in the order of:
%           X = [B, H, h_c, h_f, h_b, ...
%                 sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f]
%         - uncertain parameter vector in the order of:
%           DX = [sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f, ...
%                 p_0, t_0]
%         - constraint values vector in the order of:
%           GX = [mass_con, defl_con, Rc_con, gamma_con, mu_con]
```

```
%
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%

% Set Scheme
scheme = 11;

% -------------------------------------------------------------------
% GIVEN
% -------------------------------------------------------------------
% Constant parameters
L = 1;
p_0 = 25*10^6;
t_0 = 0.0001;
P = 1000; % Penalty parameter
delta_p_0 = 0.15*p_0;
delta_t_0 = 0.15*t_0;

% Baseline Material Properties
rho_mg = 1770; % 1770 kg/m3
rho_st = 7845; % 7845 kg/m3
sigma_y_mg = 200 * 10^6; % 200 MPa
sigma_y_st = 538 * 10^6; % 538 MPa

% Variation in material properties
delta_rho_mg = 170; % 170 kg/m3
delta_rho_st = 100; % 100 kg/m3
delta_sigma_y_mg = 20 * 10^6; % 20 MPa
delta_sigma_y_st = 150 * 10^6; % 150 MPa

x_base = [sigma_y_mg, sigma_y_st, sigma_y_mg, rho_mg, rho_st, rho_mg];
dx = [delta_sigma_y_mg, delta_sigma_y_st, delta_sigma_y_mg, ...
    delta_rho_mg, delta_rho_st, delta_rho_mg, ...
    delta_p_0, delta_t_0];
weights = 1-dx(1:6)./x_base;
dx = zeros(1,8);



% Archimedean weights
% W = [mass, defl, cost];
W = [1 0 0; 0 1 0; 0 0 1; ...
    .6 .2 .2; .2 .6 .2; .2 .2 .6; ...
    .2 .4 .4; .4 .2 .4; .4 .4 .2; ...
    1/3 1/3 1/3; 0.5 0.5 0];

% [defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
%     x(1), x(2), x(3), x(4), x(5), ...
%     x(6), x(7), x(8), ...
%     x(9), x(10), x(11), ...
%     L, p_0, t_0, uncert_vars);

% -------------------------------------------------------------------
% FIND
```

```
% ------------------------------------------------------------------
% Design Variables
% xabs = [B, H, h_c, h_f, h_b, ...
%     sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%     rho_b, rho_c, rho_f];

% Deviation Variables
% d = [sum(deviation(M,M_target,'min')),...
%      sum(deviation(defl,defl_target,'min')),...
%      sum(deviation(cost,cost_target,'min'))];
% ------------------------------------------------------------------
% SATISFY
% ------------------------------------------------------------------
% Bounds
ub = [2/100, 5/100, 1/100, 5/100, 5/100, ...
    1200e6, 1200e6, 1200e6, ...
    20000, 20000, 20000];
lb = [1/1000, 5/1000, 0.1/1000, 1/1000, 1/1000, ...
    20e6, 20e6, 20e6 ...
    1600, 1600, 1600];
xabs = unnorm01(x, lb, ub);

% Constraints
% constraints are defined and evaluated in function
% ineq_constraints_BRP_nr
gx = ineq_constraints_BRP_nr(xabs, dx);

% Goals
% Minimize mass as close as possible to 0 kg/m2 with a maximum value
% of 2997.7
M_max = 2997.7;

% Minimize deflection as close as possible to 0 cm with a maximum
% value of 234.5 m
defl_max = 234.5;

% Minimize cost as close as possible to 0, with a maximum value of
% 5.35
cost_max = 5.35;

% Evaluate BRP deflection code
[defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
    xabs(1), xabs(2), xabs(3), xabs(4), xabs(5), ...
    xabs(6), xabs(7), xabs(8), ...
    xabs(9), xabs(10), xabs(11), ...
    L, p_0, t_0, dx);

% Get cost
[cost, delta_cost] = DSEcost(xabs(6:11), x_base, weights, dx(1:6));



    [dmass_minus,dmass_plus] = deviation(M,M_max,'m20');
    [ddefl_minus,ddefl_plus] = deviation(defl,defl_max,'m20');
    [dcost_minus,dcost_plus] = deviation(cost,cost_max,'m20');
    d = [(dmass_minus + dmass_plus), ...
```

```
        (ddefl_minus+ddefl_plus), ...
        (dcost_minus+ dcost_plus)];
    dmass_minus = [];
    dmass_plus = [];
    ddefl_minus = [];
    ddefl_plus = [];
    dcost_minus = [];
    dcost_plus = [];

% ----------------------------------------------------------------------
% MINIMIZE
% ----------------------------------------------------------------------
% Archimedean formulation
Z = W(scheme,:)*(d)' + P*sum(gx.^2);
% ----------------------------------------------------------------------
```

BRPpattern_r.m

```
function [Z] = BRPpattern_r(x)

%   BRPpattern_r Determines the value of the objective function Z
%       given a vector of design variables, x. This objective function
%       is used to solve the expansion cDSP for the Ranged Scenario of
%       the BRP example
%
%   Inputs:
%      X:   a vector of design variables normalized between 0 and 1
%
%   Outputs:
%      Z:   the value of the objective function at design point X
%
%   Assumptions:
%         - design variable vector in the order of:
%           X = [B, H, h_c, h_f, h_b, ...
%                 sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f]
%         - uncertain parameter vector in the order of:
%           DX = [sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f, ...
%                 p_0, t_0]
%         - constraint values vector in the order of:
%           GX = [mass_con, defl_con, Rc_con, gamma_con, mu_con]

%
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%

% Set Scheme
scheme = 11;

% ----------------------------------------------------------------------
```

```
% GIVEN
% -------------------------------------------------------------------
% Constant parameters
L = 1;
p_0 = 25*10^6;
t_0 = 0.0001;
P = 1000; % Penalty parameter
delta_p_0 = 0.15*p_0;
delta_t_0 = 0.15*t_0;

% Baseline Material Properties
rho_mg = 1770; % 1770 kg/m3
rho_st = 7845; % 7845 kg/m3
sigma_y_mg = 200 * 10^6; % 200 MPa
sigma_y_st = 538 * 10^6; % 538 MPa

% Variation in material properties
delta_rho_mg = 170; % 170 kg/m3
delta_rho_st = 100; % 100 kg/m3
delta_sigma_y_mg = 20 * 10^6; % 20 MPa
delta_sigma_y_st = 150 * 10^6; % 150 MPa

x_base = [sigma_y_mg, sigma_y_st, sigma_y_mg, rho_mg, rho_st, rho_mg];
dx = [delta_sigma_y_mg, delta_sigma_y_st, delta_sigma_y_mg, ...
    delta_rho_mg, delta_rho_st, delta_rho_mg, ...
    delta_p_0, delta_t_0];
weights = 1-dx(1:6)./x_base;


% Archimedean weights
% W = [mass, defl, cost];
W = [1 0 0; 0 1 0; 0 0 1; ...
    .6 .2 .2; .2 .6 .2; .2 .2 .6; ...
    .2 .4 .4; .4 .2 .4; .4 .4 .2; ...
    1/3 1/3 1/3; 0.5 0.5 0];

% [defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
%     x(1), x(2), x(3), x(4), x(5), ...
%     x(6), x(7), x(8), ...
%     x(9), x(10), x(11), ...
%     L, p_0, t_0, uncert_vars);

% -------------------------------------------------------------------
% FIND
% -------------------------------------------------------------------
% Design Variables
% xabs = [B, H, h_c, h_f, h_b, ...
%     sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%     rho_b, rho_c, rho_f];

% Deviation Variables
% d = [sum(deviation(M,M_target,'min')),...
%      sum(deviation(defl,defl_target,'min')),...
%      sum(deviation(cost,cost_target,'min'))];
% -------------------------------------------------------------------
```

```
% SATISFY
% ---------------------------------------------------------------------
% Bounds
ub = [2/100, 5/100, 1/100, 5/100, 5/100, ...
    1200e6-delta_sigma_y_mg, ...
    1200e6-delta_sigma_y_st, ...
    1200e6-delta_sigma_y_mg, ...
    20000-delta_rho_mg, 20000-delta_rho_st, 20000-delta_rho_mg];
lb = [1/1000, 5/1000, 0.1/1000, 1/1000, 1/1000, ...
    20e6+delta_sigma_y_mg, ...
    20e6+delta_sigma_y_st, ...
    20e6+delta_sigma_y_mg, ...
    1600+delta_rho_mg, 1600+delta_rho_st, 1600+delta_rho_mg];
xabs = unnorm01(x, lb, ub);

% Constraints
% constraints are defined and evaluated in function
% ineq_constraints_BRP_r
gx = ineq_constraints_BRP_r(xabs, dx);

% Goals
% Minimize mass as close as possible to 0 kg/m2
M_target = 0;

% Minimize deflection as close as possible to 0 cm
defl_target = 0;

% Minimize cost as close as possible to 0
cost_target = 0;

% Evaluate BRP deflection code
[defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
    xabs(1), xabs(2), xabs(3), xabs(4), xabs(5), ...
    xabs(6), xabs(7), xabs(8), ...
    xabs(9), xabs(10), xabs(11), ...
    L, p_0, t_0, dx);

% Get Cdk's
Cdk_M = (M_target - M)/delta_M;
Cdk_defl = (defl_target - defl)/delta_defl;
[cost, delta_cost] = DSEcost(xabs(6:11), x_base, weights, dx(1:6));
Cdk_cost = (cost_target - cost)/delta_cost;

    % calculate deviation vector
    [dmass_minus,dmass_plus] = deviation(Cdk_M,1,'max');
    [ddefl_minus,ddefl_plus] = deviation(Cdk_defl,1,'max');
    [dcost_minus,dcost_plus] = deviation(Cdk_cost,1,'max');
    d = [(dmass_minus), (ddefl_minus), (dcost_minus)];
    dmass_minus = [];
    dmass_plus = [];
    ddefl_minus = [];
    ddefl_plus = [];
    dcost_minus = [];
    dcost_plus = [];
```

```
% -------------------------------------------------------------------
% MINIMIZE
% -------------------------------------------------------------------
% Archimedean formulation
Z = W(scheme,:)*(d)' + P*sum(gx.^2);
% -------------------------------------------------------------------
```

deviation.m

```
function [di_minus, di_plus] = deviation(A, G, minmax)

%   DEVIATION Determines the values of the deviation variables
%       DI_MINUS and DI_PLUS given the achievement A, the target
%       (goal) G, and the direction MINMAX which can have values of
%       'min', 'max', or 'm20'. This function is used in the context
%       of a compromise Decision Support problem.
%
%   Inputs:
%      A:           the achievment of the objective
%      G:           the target for the objective, or the maximum value
%                   in the design space for 'm20' case
%      MINMAX:      minimize 'min' or maximize 'max' or
%                   minimize to zero 'm20'
%
%   Outputs:
%      DI_MINUS:    the underachievement of the objective
%      DI_PLUS:     the overachievement of the objective
%


%
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%
%--------------------------------------------------------------------
% initialize deviation variables
di_minus = [];
di_plus = [];

% Input error checking

if G == 0
    error('Target value cannot be zero'), return
end

if minmax == 'min'

    % Minimize :
    % G + di_minus - di_plus = A
    di_minus = 1 - G/A;
        if di_minus <= 0
            di_plus = -1*di_minus;
            di_minus = 0;
```

```
        else
            di_plus = 0;
        end
else
    if minmax == 'max'
        % Maximize :
        % A + di_minus - di_plus = G
        di_minus = 1 - A/G;
            if di_minus <= 0
                di_plus = -1*di_minus;
                di_minus = 0;
            else
                di_plus = 0;
            end
    else
            if minmax == 'm20'
            % Minimize :
            % A/Aimax + di_minus - di_plus = 0
            di_minus = 0 - A/G;
                if di_minus <= 0
                    di_plus = -1*di_minus;
                    di_minus = 0;
                else
                    di_plus = 0;
                end
            else
                error('minmax must be either min, max, or min2zero')
            end
    end
end
%----------------------------------------------------------------------
```

DSEcost.m.m

```
function [cost, delta_cost] = DSEcost(x_sol, x_base, weights, delta_x)

%   DSEcost Determines the design space expansion cost at design point
%       X_SOL given a baseline point X_BASE, directional weights
%       WEIGHTS, and variation due to uncertainty in the design
%       variables defined in DELTA_X. The function defines cost as the
%       vector length of the difference in the evaluation point and
%       the baseline material properties multiplied by a weighting
%       factor.
%
%   Inputs:
%      X_SOL:           a vector of material property design variables
%      X_BASE:          a vector of baseline material properties
%      WEIGHTS:         a vector of material property weights
%      DELTA_X:         a vector of variation in material property
%                       design variables
%
%   Outputs:
%      COST:            (scalar) the cost of design space expansion at
%                       design point X_SOL
```

```
%      DELTA_COST:      (scalar) the variation in the cost of design
%                       space expansion at design point X_SOL
%
%   Assumptions:
%         - goals are evaluated in terms of design capability indices
%         - material property design variable vectors in the order of:
%           X_SOL = [sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                    rho_b, rho_c, rho_f]
%           X_BASE = [sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                     rho_b, rho_c, rho_f]
%         - variation in material property design variable vector in
%           the order of:
%           DELTA_X = [sigma_yield_b, sigma_yield_c, sigma_yield_f,...
%                 rho_b, rho_c, rho_f]
%         - material property weights in the order of:
%           WEIGHTS = [sigma_yield_b, sigma_yield_c, sigma_yield_f,...
%                      rho_b, rho_c, rho_f]

%
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%
%----------------------------------------------------------------------
cost = norm((weights).*abs(x_sol - x_base)./x_base);

delta_cost = 0;

for i = 1:length(x_sol)
    if cost == 0
        delta_cost = norm(weights.*delta_x./x_base);
    else
        delta_cost = delta_cost + ...
        (weights(i)/(x_base(i)))*...
        (abs((x_sol(i) - x_base(i))/sqrt(cost)))* ...
        (delta_x(i)/x_base(i));
    end
end
%----------------------------------------------------------------------
```

get_points_baseline.m

```
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%
% PURPOSE: create a grid of points in the baseline BRP design space
%
% ASSUMPTIONS:
%       -   The grid is defined by 16 grid points
%       -   Bounds listed below
%       -   Five variables/dimensions (B, H, h_c, h_f, h_b)
%
```

```
%  WARNING: This script creats a .txt file that is approx 40 MB and
%           takes some time to execute.

% -------------------------------------------------------------------
% Problem formulation:
% -------------------------------------------------------------------

tic % start timer to determine the time to create and store points
points = [];

grid_pts = 16;

% Bounds
h_c_lb = 0.1/1000; % 0.1 mm
h_c_ub = 1/100; % 1 cm
H_lb = 5/1000; % 5 mm
H_ub = 5/100; % 5 cm
B_lb = 1/1000; % 1 mm
B_ub = 2/100; % 2 cm
h_f_lb = 1/1000; % 1 mm
h_f_ub = 5/100; % 5 cm
h_b_lb = 1/1000; % 1 mm
h_b_ub = 5/100; % 5 cm

% -------------------------------------------------------------------
% Problem solution:
% -------------------------------------------------------------------
for h_c = [h_c_lb:((h_c_ub - h_c_lb)/(grid_pts-1)):h_c_ub]
    for H = [H_lb:((H_ub - H_lb)/(grid_pts-1)):H_ub]
        for B = [B_lb:((B_ub - B_lb)/(grid_pts-1)):B_ub]
            for h_f = [h_f_lb:((h_f_ub - h_f_lb)/(grid_pts-1)):h_f_ub]
                for h_b = ...
                        [h_b_lb:((h_b_ub-h_b_lb)/(grid_pts-1)):h_b_ub]

                    points = [points; B, H, h_c, h_f, h_b];

                end
            end
        end
    end
end
toc
% -------------------------------------------------------------------
% Problem interpretation:
% -------------------------------------------------------------------
tic
dlmwrite('baseline_points.txt', points, ...
    'delimiter', '\t', 'newline', 'pc')
toc
% -------------------------------------------------------------------
```

ineq_constraints_BRP_nr.m

```
function [gx, varargin] = ineq_constraints_BRP_nr(x, dx)

%   INEQ_CONSTRAINTS_BRP_NR Determines the extent to which a design
%       point X violates the constraints on the BRP problem in the
%       Point Scenario. GX is equal to zero if the constraint is not
%       violated and is equal to the value of the constraint if the
%       constraint is violated.
%
%   Inputs:
%      X:   a vector of design variables
%      DX:  a vector of uncertain parameters
%
%   Outputs:
%      GX:       a vector of constraint values
%      VARARGIN: a matrix containing the evaluated constraints and
%                their limits.
%
%   Assumptions:
%         - all constraints are inequality constraints <= 0
%         - design variable vector in the order of:
%           X = [B, H, h_c, h_f, h_b, ...
%                 sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f]
%         - uncertain parameter vector in the order of:
%           DX = [sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f, ...
%                 p_0, t_0]
%         - constraint values vector in the order of:
%           GX = [mass_con, defl_con, Rc_con, gamma_con, mu_con]

%
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%

% Define limits
mass_max = 150;
defl_max = 0.15;
Rc_min = 0.07;
gamma_max = 0.6;
mu_max = 4/sqrt(3);

% Constant parameters
L = 1;
p_0 = 25*10^6;
t_0 = 0.0001;
uncert_vars = dx;

% ---------------------------------------------------------------------
% Several constraint evaluations require the evaluation of the BRPdefl
% function.
```

```
[defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
    x(1), x(2), x(3), x(4), x(5), ...
    x(6), x(7), x(8), ...
    x(9), x(10), x(11), ...
    L, p_0, t_0, uncert_vars);

gamma = (2*p_0*t_0/(x(4)*sqrt(x(8)*(x(11)))));

mu = m_c/m_f;

% -------------------------------------------------------------------
% Constraints
% -------------------------------------------------------------------
mass_con = min([0,(mass_max-M)]);
defl_con = min([0,(defl_max-defl)]);
Rc_con = min([0,(R_c-Rc_min)]);
gamma_con = min([0,(gamma_max-gamma)]);
mu_con = min([0,(mu_max-mu)]);

gx = [mass_con, defl_con, 100*Rc_con, gamma_con, mu_con];
varargin = [M mass_max;...
            defl defl_max;...
            R_c Rc_min;...
            gamma gamma_max;...
            mu mu_max];
% -------------------------------------------------------------------
```

ineq_constraints_BRP_r.m

```
function [gx, varargin] = ineq_constraints_BRP_r(x, dx)

%   INEQ_CONSTRAINTS_BRP_R Determines the extent to which a design
%       point X violates the constraints on the BRP problem in the
%       Ranged Scenario. GX is equal to zero if the constraint is not
%       violated and is equal to the value of the constraint if the
%       constraint is violated.
%
%   Inputs:
%      X:   a vector of design variables
%      DX:  a vector of uncertain parameters
%
%   Outputs:
%      GX:  a vector of constraint values
%      VARARGIN: a matrix containing the evaluated constraints and
%                their limits.
%
%   Assumptions:
%         - all constraints are inequality constraints <= 0
%         - robust constraints are evaluated in terms of design
%           capability indices
%         - design variable vector in the order of:
%           X = [B, H, h_c, h_f, h_b, ...
%                 sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
```

```
%                  rho_b, rho_c, rho_f]
%          - uncertain parameter vector in the order of:
%            DX = [sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                  rho_b, rho_c, rho_f, ...
%                  p_0, t_0]
%          - constraint values vector in the order of:
%            GX = [mass_con, defl_con, Rc_con, gamma_con, mu_con]

%
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%

% Define limits
mass_max = 150;
defl_max = 0.15;
Rc_min = 0.07;
gamma_max = 0.6;
mu_max = 4/sqrt(3);

% Constant parameters
L = 1;
p_0 = 25*10^6;
t_0 = 0.0001;
uncert_vars = dx;

% ------------------------------------------------------------------
% Several constraint evaluations require the evaluation of the BRPdefl
% function.

[defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
    x(1), x(2), x(3), x(4), x(5), ...
    x(6), x(7), x(8), ...
    x(9), x(10), x(11), ...
    L, p_0, t_0, uncert_vars);

gamma = (2*p_0*t_0/(x(4)*sqrt(x(8)*(x(11)))));

delta_gamma = (abs((2*t_0/(x(4)*sqrt(x(8)*x(11))))*dx(7))+...
    (abs(2*p_0/(x(4)*sqrt(x(8)*x(11))))*dx(8))+...
    (abs(-t_0*p_0/(x(4)*sqrt(x(8)^3*x(11))))*dx(3))+...
    (abs(-t_0*p_0/(x(4)*sqrt(x(8)*x(11)^3)))*dx(6)));

mu = m_c/m_f;

delta_mu = (abs((-x(10).*x(2).*R_c)./(x(11).^2.*x(4))).*dx(6) + ...
    (abs((x(2).*R_c)./(x(11).*x(4))).*dx(5)));

% ------------------------------------------------------------------
% Constraints
% ------------------------------------------------------------------
mass_con = max(0,(1-((mass_max-M)/delta_M)));
defl_con = max(0,(1-((defl_max-defl)/delta_defl)));
```

```
Rc_con = max(0,(Rc_min-R_c));
gamma_con = max(0,(1-((gamma_max-gamma)/delta_gamma)));
mu_con = max(0,(1-((mu_max-mu)/delta_mu)));

gx = [mass_con, defl_con, 100*Rc_con, gamma_con, mu_con];
varargin = [((mass_max-M)/delta_M) 1;...
            ((defl_max-defl)/delta_defl) 1;...
            R_c Rc_min;...
            ((gamma_max-gamma)/delta_gamma) 1;...
            ((mu_max-mu)/delta_mu) 1];
% --------------------------------------------------------------------
```

norm01.m

```
function [x_norm] = norm01(x, lb, ub)

%   NORM01 Returns a vector X_NORM that is normalized to be between
%       zero and one using an input vector X, a lower bound vector LB,
%       and an upper bound vector UB such that the lower bound is
%       equal to zero and the upper bound is equal to one. The inverse
%       operation which restores the absolute x values is performed by
%       the function unnorm01.m
%
%   Inputs:
%      X:          a vector of absolute design variables
%      LB:         a vector of lower bounds
%      UB:         a vector of upperbounds
%
%   Outputs:
%      X_NORM:     a normalized vector
%


%
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%
%---------------------------------------------------------------------
x_norm = (x - lb)./(ub - lb);
%---------------------------------------------------------------------
```

solution_check_pattern_nr.m

```
function [d] = solution_check_pattern_nr(x)

%   solution_check_pattern_nr Evaluates the design point x and prints
%       an analysis of the performance of BRP at that point.  This
%       version is for the Point Scenario of the expansion cDSP
%
%   Inputs:
%      X:   a vector of design variables (normalized between 0 & 1)
%
```

```
%   Outputs:
%
%   Assumptions:
%         - design variable vector in the order of:
%           X = [B, H, h_c, h_f, h_b, ...
%                 sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f]
%         - uncertain parameter vector in the order of:
%           DX = [sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f, ...
%                 p_0, t_0]
%         - constraint values vector in the order of:
%           GX = [mass_con, defl_con, Rc_con, gamma_con, mu_con]
%
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%

% Set Scheme
scheme = 11;

% --------------------------------------------------------------------
% GIVEN
% --------------------------------------------------------------------
% Constant parameters
L = 1;
p_0 = 25*10^6;
t_0 = 0.0001;
P = 1000; % Penalty parameter
delta_p_0 = 0.15*p_0;
delta_t_0 = 0.15*t_0;

% Baseline Material Properties
rho_mg = 1770; % 1770 kg/m3
rho_st = 7845; % 7845 kg/m3
sigma_y_mg = 200 * 10^6; % 200 MPa
sigma_y_st = 538 * 10^6; % 538 MPa

% Variation in material properties
delta_rho_mg = 170; % 170 kg/m3
delta_rho_st = 100; % 100 kg/m3
delta_sigma_y_mg = 20 * 10^6; % 20 MPa
delta_sigma_y_st = 150 * 10^6; % 150 MPa

x_base = [sigma_y_mg, sigma_y_st, sigma_y_mg, rho_mg, rho_st, rho_mg];
dx = [delta_sigma_y_mg, delta_sigma_y_st, delta_sigma_y_mg, ...
    delta_rho_mg, delta_rho_st, delta_rho_mg, ...
    delta_p_0, delta_t_0];
weights = 1-dx(1:6)./x_base;
dx = zeros(1,8);

% Archimedean weights
% W = [mass, defl, cost];
W = [1 0 0; 0 1 0; 0 0 1; ...
```

```
    .6 .2 .2; .2 .6 .2; .2 .2 .6; ...
    .2 .4 .4; .4 .2 .4; .4 .4 .2; ...
    1/3 1/3 1/3; 0.5 0.5 0];

% [defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
%     x(1), x(2), x(3), x(4), x(5), ...
%     x(6), x(7), x(8), ...
%     x(9), x(10), x(11), ...
%     L, p_0, t_0, uncert_vars);


% --------------------------------------------------------------------
% FIND
% --------------------------------------------------------------------
% Design Variables
% xabs = [B, H, h_c, h_f, h_b, ...
%     sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%     rho_b, rho_c, rho_f];

% Deviation Variables
% d = [sum(deviation(M,M_target,'min')),...
%      sum(deviation(defl,defl_target,'min')),...
%      sum(deviation(cost,cost_target,'min'))];
% --------------------------------------------------------------------
% SATISFY
% --------------------------------------------------------------------
% Bounds
ub = [2/100, 5/100, 1/100, 5/100, 5/100, ...
    1200e6, 1200e6, 1200e6, ...
    20000, 20000, 20000];
lb = [1/1000, 5/1000, 0.1/1000, 1/1000, 1/1000, ...
    20e6, 20e6, 20e6 ...
    1600, 1600, 1600];
xabs = unnorm01(x, lb, ub);

% Constraints
% constraints are defined and evaluated in function
% ineq_constraints_BRP_nr
[gx, cons] = ineq_constraints_BRP_nr(xabs, dx);

% Goals
% Minimize mass as close as possible to 0 kg/m2 with a maximum value
% of 2997.7
M_max = 2997.7;

% Minimize deflection as close as possible to 0 cm with a maximum
% value of 234.5 m
defl_max = 234.5;

% Minimize cost as close as possible to 0, with a maximum value of
% 5.35
cost_max = 5.35;

% Evaluate BRP deflection code
[defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
    xabs(1), xabs(2), xabs(3), xabs(4), xabs(5), ...
```

```
    xabs(6), xabs(7), xabs(8), ...
    xabs(9), xabs(10), xabs(11), ...
    L, p_0, t_0, dx);

% Get cost
[cost, delta_cost] = DSEcost(xabs(6:11), x_base, weights, dx(1:6));


    [dmass_minus,dmass_plus] = deviation(M,M_max,'m20');
    [ddefl_minus,ddefl_plus] = deviation(defl,defl_max,'m20');
    [dcost_minus,dcost_plus] = deviation(cost,cost_max,'m20');
    d = [(dmass_minus + dmass_plus), ...
        (ddefl_minus+ddefl_plus), ...
        (dcost_minus+ dcost_plus)];
    dmass_minus = [];
    dmass_plus = [];
    ddefl_minus = [];
    ddefl_plus = [];
    dcost_minus = [];
    dcost_plus = [];

% ---------------------------------------------------------------------
% MINIMIZE
% ---------------------------------------------------------------------
% Archimedean formulation
Z = W(scheme,:)*(d)' + P*sum(gx.^2);
% ---------------------------------------------------------------------

fid = 1;

% Design Scenario
fprintf(fid, ...
    '| Design Scenario and Deviation Function Value--------------|\n')
fprintf(fid, '| Mass Weight                              | % 5.4f |\n',...
    W(scheme,1) )
fprintf(fid, '| Deflection Weight                        | % 5.4f |\n',...
    W(scheme,2) )
fprintf(fid, '| Cost Weight                              | % 5.4f |\n',...
    W(scheme,3) )
fprintf(fid, '| Deviation Function Value                 | % 5.4f |\n',...
    Z )
fprintf(fid, '| Deviation - Mass                         | % 5.4f |\n',...
    d(1) )
fprintf(fid, '| Deviation - Deflection                   | % 5.4f |\n',...
    d(2) )
fprintf(fid, '| Deviation - Cost                         | % 5.4f |\n',...
    d(3) )

fprintf(fid, '| Material Properties------------------------------|\n')
fprintf(fid,...
    '| Yield Strength, Back |%6.2f|MPa|between|%6.2f|and|%6.2f|\n',...
    xabs(6)/1e6, lb(6)/1e6,ub(6)/1e6)
fprintf(fid,...
    '| Yield Strength, Core |%6.2f|MPa|between|%6.2f|and|%6.2f|\n',...
    xabs(7)/1e6, lb(7)/1e6,ub(7)/1e6)
```

```
fprintf(fid,...
    '| Yield Strength, Front|%6.2f|MPa|between|%6.2f|and|%6.2f|\n',...
    xabs(8)/1e6, lb(8)/1e6,ub(8)/1e6)
fprintf(fid,...
    '| Density, Back | % 6.2f|kg/m^3|between|% 6.2f|and|% 6.2f|\n',...
    xabs(9), lb(9),ub(9))
fprintf(fid,...
    '| Density, Core | % 6.2f|kg/m^3|between|% 6.2f|and|% 6.2f|\n',...
    xabs(10), lb(10),ub(10))
fprintf(fid,...
    '| Density, Front| % 6.2f|kg/m^3|between|% 6.2f|and|% 6.2f|\n',...
    xabs(11), lb(11),ub(11))

fprintf(fid, '| BRP Dimensions ----------------------------------|\n')
fprintf(fid,...
    '|Cell Spacing, B | %6.4f |m| between |% 6.4f|and|% 6.4f |\n',...
    xabs(1), lb(1), ub(1))
fprintf(fid,...
    '|Core Height, H | %6.4f |m| between |% 6.4f |and|% 6.4f |\n',...
    xabs(2), lb(2), ub(2))
fprintf(fid,...
    '|Cell Wall Thickness, h_c|%6.4f|m|between|%6.4f|and|%6.4f|\n',...
    xabs(3), lb(3), ub(3))
fprintf(fid,...
    '|Front FS Thickness, h_f|%6.4f|m|between|%6.4f|and|%6.4f|\n',...
    xabs(4), lb(4), ub(4))
fprintf(fid,...
    '|Back FS Thickness, h_b|%6.4f|m|between|%6.4f|and|%6.4f|\n',...
    xabs(5), lb(5), ub(5))

fprintf(fid, '| Performance -------------------------------------|\n')
fprintf(fid,...
    '| Deflection                          | % 5.4f | m       |\n',...
    defl )
fprintf(fid,...
    '| Variation in Deflection             | % 5.4f | m       |\n',...
    delta_defl )
fprintf(fid,...
    '| Mass per Area                       | %6.4f | kg/m^2  |\n',...
    M )
fprintf(fid,...
    '| Variation in Mass per Area          | % 6.4f | kg/m^2  |\n',...
    delta_M )
fprintf(fid,...
    '| Cost                                | % 6.4f | no unit |\n',...
    cost )
fprintf(fid,...
    '| Variation in Cost                   | % 6.4f | no unit |\n',...
    delta_cost )

fprintf(fid, '| Constraint Analysis -----------------------------|\n')
fprintf(fid,...
    '| Mass              | % 5.4f |  at most | %6.4f |kg/m^2  |\n',...
    cons(1,1:2))
fprintf(fid,...
    '| Deflection        | % 5.4f |  at most | % 5.4f |m       |\n',...
```

```
    cons(2,1:2))
fprintf(fid,...
    '| Relative Density | % 5.4f | at least | % 5.4f |no unit |\n',...
    cons(3,1:2) )
fprintf(fid,...
    '| Mu               | %5.4f |  at most   | % 5.4f |no unit |\n',...
    cons(4,1:2))
fprintf(fid,...
    '| Gamma            | %6.4f |  at most   | % 5.4f |no unit |\n',...
    cons(5,1:2))

%---------------------------------------------------------------------
% End Print Solution to Command Window
%---------------------------------------------------------------------
```

solution_check_pattern_r.m

```
function [d] = solution_check_pattern_r(x)

%   solution_check_pattern_r Evaluates the design point X and prints
%       an analysis of the performance of BRP at that point.  This
%       version is for the Ranged Scenario of the expansion cDSP
%
%   Inputs:
%      X:   a vector of design variables (normalized between 0 & 1)
%
%   Outputs:
%
%   Assumptions:
%         - design variable vector in the order of:
%           X = [B, H, h_c, h_f, h_b, ...
%                 sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f]
%         - uncertain parameter vector in the order of:
%           DX = [sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%                 rho_b, rho_c, rho_f, ...
%                 p_0, t_0]
%         - constraint values vector in the order of:
%           GX = [mass_con, defl_con, Rc_con, gamma_con, mu_con]
%
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%


% Set Scheme
scheme = 11;

% --------------------------------------------------------------------
% GIVEN
% --------------------------------------------------------------------
% Constant parameters
L = 1;
p_0 = 25*10^6;
```

```
t_0 = 0.0001;
P = 1000; % Penalty parameter
delta_p_0 = 0.15*p_0;
delta_t_0 = 0.15*t_0;

% Baseline Material Properties
rho_mg = 1770; % 1770 kg/m3
rho_st = 7845; % 7845 kg/m3
sigma_y_mg = 200 * 10^6; % 200 MPa
sigma_y_st = 538 * 10^6; % 538 MPa

% Variation in material properties
delta_rho_mg = 170; % 170 kg/m3
delta_rho_st = 100; % 100 kg/m3
delta_sigma_y_mg = 20 * 10^6; % 20 MPa
delta_sigma_y_st = 150 * 10^6; % 150 MPa

x_base = [sigma_y_mg, sigma_y_st, sigma_y_mg, rho_mg, rho_st, rho_mg];
dx = [delta_sigma_y_mg, delta_sigma_y_st, delta_sigma_y_mg, ...
    delta_rho_mg, delta_rho_st, delta_rho_mg, ...
    delta_p_0, delta_t_0];
weights = 1-dx(1:6)./x_base;



% Archimedean weights
% W = [mass, defl, cost];
W = [1 0 0; 0 1 0; 0 0 1; ...
    .6 .2 .2; .2 .6 .2; .2 .2 .6; ...
    .2 .4 .4; .4 .2 .4; .4 .4 .2; ...
    1/3 1/3 1/3; 0.5 0.5 0];

% [defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
%     x(1), x(2), x(3), x(4), x(5), ...
%     x(6), x(7), x(8), ...
%     x(9), x(10), x(11), ...
%     L, p_0, t_0, uncert_vars);

% -------------------------------------------------------------------
% FIND
% -------------------------------------------------------------------
% Design Variables
% xabs = [B, H, h_c, h_f, h_b, ...
%     sigma_yield_b, sigma_yield_c, sigma_yield_f, ...
%     rho_b, rho_c, rho_f];

% Deviation Variables
% d = [sum(deviation(M,M_target,'min')),...
%      sum(deviation(defl,defl_target,'min')),...
%      sum(deviation(cost,cost_target,'min'))];
% -------------------------------------------------------------------
% SATISFY
% -------------------------------------------------------------------
% Bounds
ub = [2/100, 5/100, 1/100, 5/100, 5/100, ...
    1200e6-delta_sigma_y_mg, ...
```

```
    1200e6-delta_sigma_y_st, ...
    1200e6-delta_sigma_y_mg, ...
    20000-delta_rho_mg, 20000-delta_rho_st, 20000-delta_rho_mg];
lb = [1/1000, 5/1000, 0.1/1000, 1/1000, 1/1000, ...
    20e6+delta_sigma_y_mg, ...
    20e6+delta_sigma_y_st, ...
    20e6+delta_sigma_y_mg, ...
    1600+delta_rho_mg, 1600+delta_rho_st, 1600+delta_rho_mg];
xabs = unnorm01(x, lb, ub);

% Constraints
% constraints are defined and evaluated in function
% ineq_constraints_BRP_r
[gx, cons] = ineq_constraints_BRP_r(xabs, dx);

% Goals
% Minimize mass as close as possible to 50 kg/m2
M_target = 0;

% Minimize deflection as close as possible to 5 cm
defl_target = 0;

% Minimize cost as close as possible to 1
cost_target = 0;

% Evaluate BRP deflection code
[defl, delta_defl, M, delta_M, R_c, m_c, m_f] = BRPdefl(...
    xabs(1), xabs(2), xabs(3), xabs(4), xabs(5), ...
    xabs(6), xabs(7), xabs(8), ...
    xabs(9), xabs(10), xabs(11), ...
    L, p_0, t_0, dx);

% Get Cdk's
Cdk_M = (M_target - M)/delta_M;
Cdk_defl = (defl_target - defl)/delta_defl;
[cost, delta_cost] = DSEcost(xabs(6:11), x_base, weights, dx(1:6));
Cdk_cost = (cost_target - cost)/delta_cost;

% calculate deviation vector
[dmass_minus,dmass_plus] = deviation(Cdk_M,1,'max');
[ddefl_minus,ddefl_plus] = deviation(Cdk_defl,1,'max');
[dcost_minus,dcost_plus] = deviation(Cdk_cost,1,'max');
d = [(dmass_minus), (ddefl_minus), (dcost_minus)];

% ---------------------------------------------------------------------
% MINIMIZE
% ---------------------------------------------------------------------
% Archimedean formulation
Z = W(scheme,:)*(d)' + P*sum(gx.^2);
% ---------------------------------------------------------------------

fid = 1;

% Design Scenario
```

```
fprintf(fid, '| Design Scenario and Deviation Function Value-----|\n')
fprintf(fid, '| Mass Weight                               | % 5.4f |\n',...
    W(scheme,1) )
fprintf(fid, '| Deflection Weight                         | % 5.4f |\n',...
    W(scheme,2) )
fprintf(fid, '| Cost Weight                               | % 5.4f |\n',...
    W(scheme,3) )
fprintf(fid, '| Deviation Function Value                  | % 5.4f |\n',...
    Z )
fprintf(fid, '| Deviation - Mass                          | % 5.4f |\n',...
    d(1) )
fprintf(fid, '| Deviation - Deflection                    | % 5.4f |\n',...
    d(2) )
fprintf(fid, '| Deviation - Cost                          | % 5.4f |\n',...
    d(3) )

fprintf(fid, '| Material Properties------------------------------|\n')
fprintf(fid,...
    '| Yield Strength, Back |%6.2f|MPa|between|%6.2f|and|%6.2f|\n',...
    xabs(6)/1e6, lb(6)/1e6,ub(6)/1e6)
fprintf(fid,...
    '| Yield Strength, Core |%6.2f|MPa|between|%6.2f|and|%6.2f|\n',...
    xabs(7)/1e6, lb(7)/1e6,ub(7)/1e6)
fprintf(fid,...
    '| Yield Strength, Front|%6.2f|MPa|between|%6.2f|and|%6.2f|\n',...
    xabs(8)/1e6, lb(8)/1e6,ub(8)/1e6)
fprintf(fid,...
    '| Density, Back | % 6.2f|kg/m^3|between|% 6.2f|and|% 6.2f|\n',...
    xabs(9), lb(9),ub(9))
fprintf(fid,...
    '| Density, Core | % 6.2f|kg/m^3|between|% 6.2f|and|% 6.2f|\n',...
    xabs(10), lb(10),ub(10))
fprintf(fid,...
    '| Density, Front| % 6.2f|kg/m^3|between|% 6.2f|and|% 6.2f|\n',...
    xabs(11), lb(11),ub(11))

fprintf(fid, '| BRP Dimensions ----------------------------------|\n')
fprintf(fid,...
    '| Cell Spacing, B| %6.4f | m |between|% 6.4f |and|% 6.4f |\n',...
    xabs(1), lb(1), ub(1))
fprintf(fid,...
    '| Core Height, H | %6.4f | m |between|% 6.4f |and|% 6.4f |\n',...
    xabs(2), lb(2), ub(2))
fprintf(fid,...
    '|Cell Wall Thickness, h_c|%6.4f|m|between|%6.4f|and|%6.4f|\n',...
    xabs(3), lb(3), ub(3))
fprintf(fid,...
    '|Front FS Thickness, h_f|%6.4f|m|between|%6.4f|and|%6.4f|\n',...
    xabs(4), lb(4), ub(4))
fprintf(fid,...
    '|Back FS Thickness, h_b|%6.4f|m|between|%6.4f|and|%6.4f|\n',...
    xabs(5), lb(5), ub(5))

fprintf(fid, '| Performance -------------------------------------|\n')
fprintf(fid, '| Deflection            | % 5.4f | m       |\n',...
    defl )
```

```
fprintf(fid, '| Variation in Deflection | % 5.4f | m        |\n',...
    delta_defl )
fprintf(fid, '| Mass per Area           | %6.4f | kg/m^2  |\n',...
    M )
fprintf(fid, '| Variation in Mass per Area | % 6.4f | kg/m^2  |\n',...
    delta_M )
fprintf(fid, '| Cost                       | % 6.4f | no unit |\n',...
    cost )
fprintf(fid, '| Variation in Cost          | % 6.4f | no unit |\n',...
    delta_cost )

fprintf(fid, '| Constraint Analysis -----------------------------|\n')
fprintf(fid,...
    '| Mass, Cdk        | % 5.4f |  at least | %6.4f |kg/m^2  |\n',...
    cons(1,1:2))
fprintf(fid,...
    '| Deflection, Cdk  | % 5.4f | at least | % 5.4f |m       |\n',...
    cons(2,1:2))
fprintf(fid,...
    '| Relative Density | % 5.4f | at least | % 5.4f |no unit |\n',...
    cons(3,1:2) )
fprintf(fid,...
    '| Mu, Cdk          | % 5.4f |  at least | % 5.4f |no unit |\n',..
cons(4,1:2))
fprintf(fid,...
    '| Gamma, Cdk       | %6.4f |  at least | % 5.4f |no unit |\n',...
    cons(5,1:2))

%----------------------------------------------------------------------
% End Print Solution to Command Window
%----------------------------------------------------------------------
```

unnorm01.m

```
function [x] = unnorm01(x_norm, lb, ub)

%   UNNORM01 Reverses the normalization of a vector of design
%       variables X_NORM to restore the absolute values of the design
%       variables in the vector X using a lower bound vector LB, and
%       an upper bound vector UB. The inverse operation which
%       normalizes the design variables is performed by the function
%       norm01.m
%
%   Inputs:
%      X_NORM:     a vector of absolute design variables
%      LB:         a vector of lower bounds
%      UB:         a vector of upperbounds
%
%   Outputs:
%      X:          a normalized vector
%


%
% Fall 2006
```

```
%
% AUTHOR:  Stephanie Thompson
%
%----------------------------------------------------------------------
x = (ub - lb).*x_norm + lb;
%----------------------------------------------------------------------
```

xga2xps.m

```
function [xps] = xga2xps(xga)

%   XGA2XPS converts a 55 bit string into a vector of 11 design
%       variables, normalized between zero and one.
%
%   Inputs:
%      XGA: a bit string of 55 bits
%
%   Outputs:
%      XPS: a vector of 11 design variables normalized between zero
%           and one
%
%   Assumptions:
%         - XGA is a vector of length 55 containing only zeros and
%           ones

%
% Fall 2006
%
% AUTHOR:  Stephanie Thompson
%
%----------------------------------------------------------------------
% separate bit string x into 11 design variables:
x = [xga(1:5); xga(6:10); xga(11:15); xga(16:20); xga(21:25); ...
     xga(26:30); xga(31:35); xga(36:40); xga(41:45); xga(46:50); ...
     xga(51:55)];

% convert binary to base ten:
x = bin2dec(strcat(num2str(x)))';

% normalize from 0 to 1:
xps = norm01(x,zeros(1,11),31*ones(1,11));
%----------------------------------------------------------------------
```

APPENDIX B: CONVERGENCE PLOTS

In this appendix, convergence plots are presented for each solution to the expansion cDSP for BRP design (see Section 4.2). A hybrid solution-finding approach is used in which a genetic algorithm (GA) is first used to find the general area of the global minimum in a discretized design space and then a pattern search algorithm is used to pinpoint the location of the minimum. As such, two convergence plots are shown for each solution: one GA convergence plot and one pattern search convergence plot. The plots are shown below in Figures B.1 through B.44. In both cases, the plots are expected to exhibit smooth convergence to the solution. On the GA convergence plots, both the mean fitness and the best fitness are shown on the plot. All of the plots show the desired smooth convergence to the solution. Although the deviation function is still decreasing in Figure B.44, the pattern search is terminated after 2000 iterations because the reduction in the deviation function is occurring in the sixth decimal place, which is an insignificant digit. These convergence plots confirm the smooth convergence of the solution-finding algorithms to the design solutions reported in Section 4.2.2, and this smooth convergence contributes to the confidence in the validity of these design solutions.

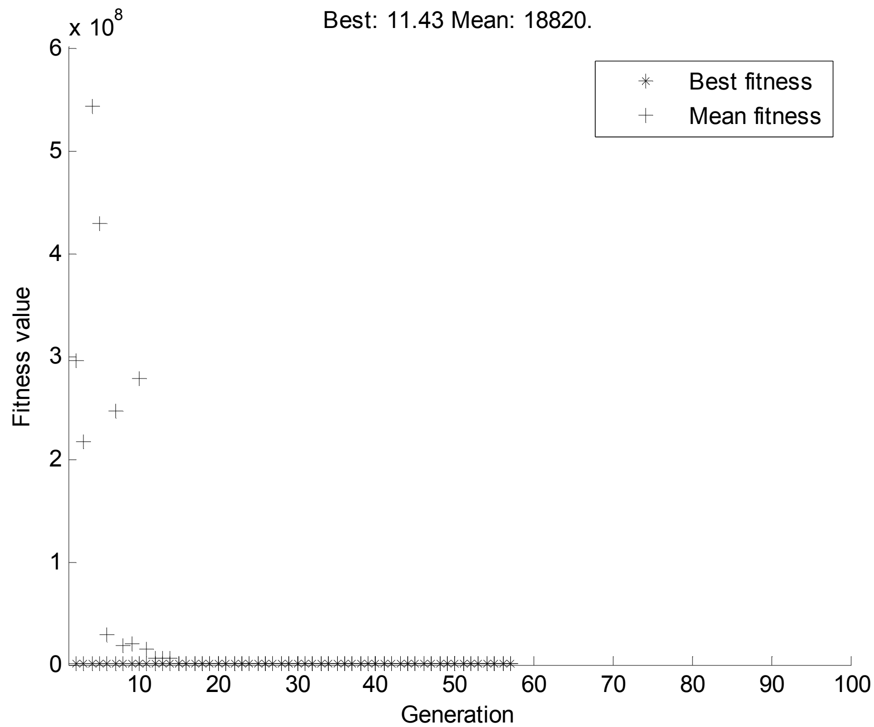

FIGURE B.1. SCHEME 1, RANGED SCENARIO, GA CONVERGENCE

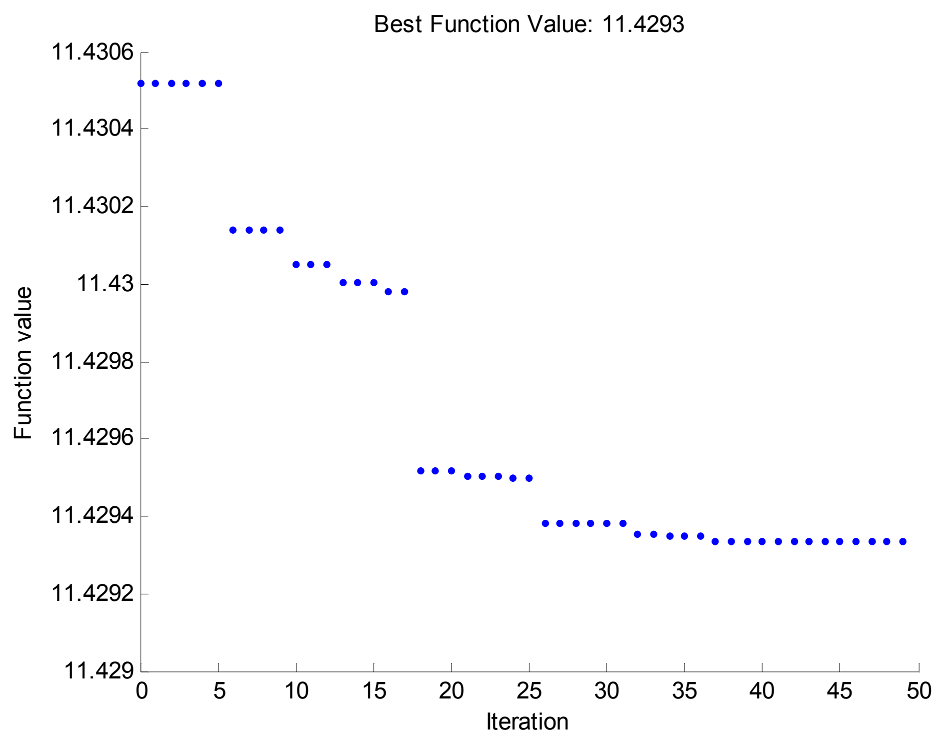

FIGURE B.2. SCHEME 1, RANGED, PATTERN SEARCH CONVERGENCE

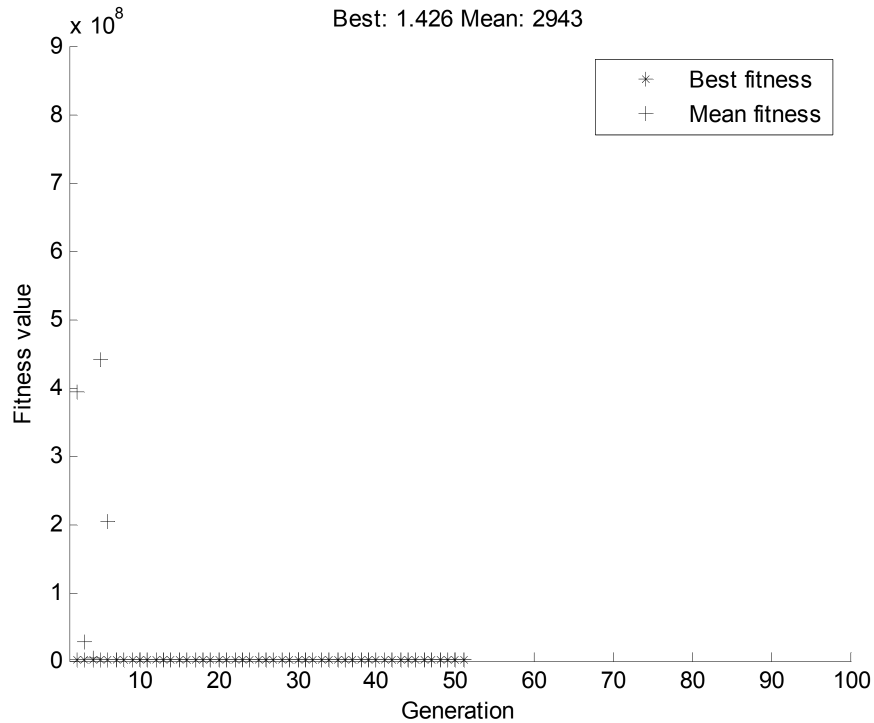

FIGURE B.3. SCHEME 2, RANGED, GA CONVERGENCE

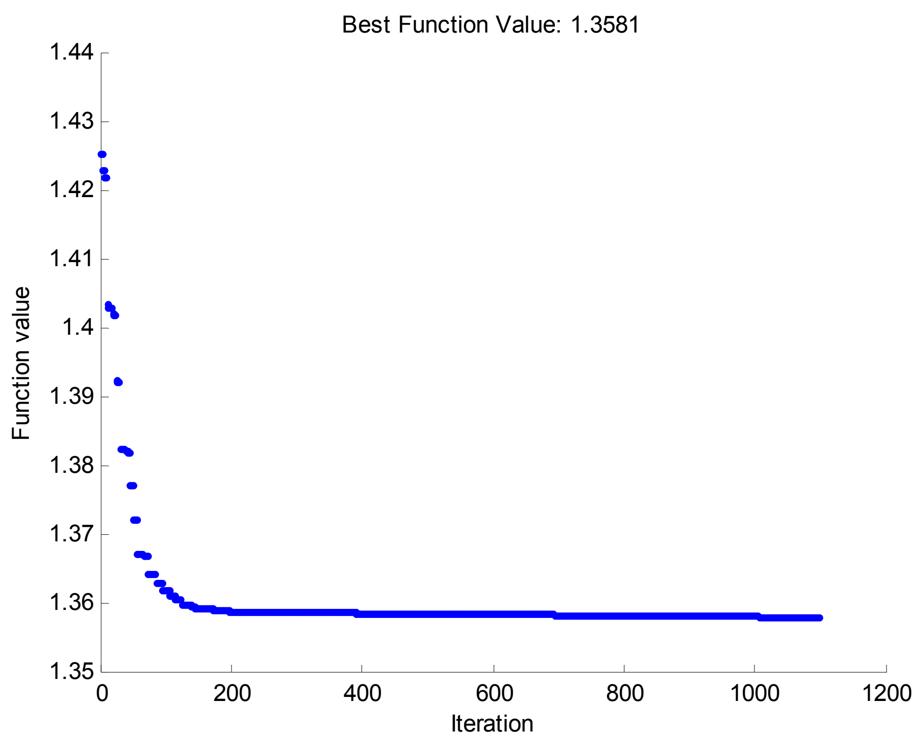

FIGURE B.4. SCHEME 2, RANGED, PATTERN SEARCH CONVERGENCE

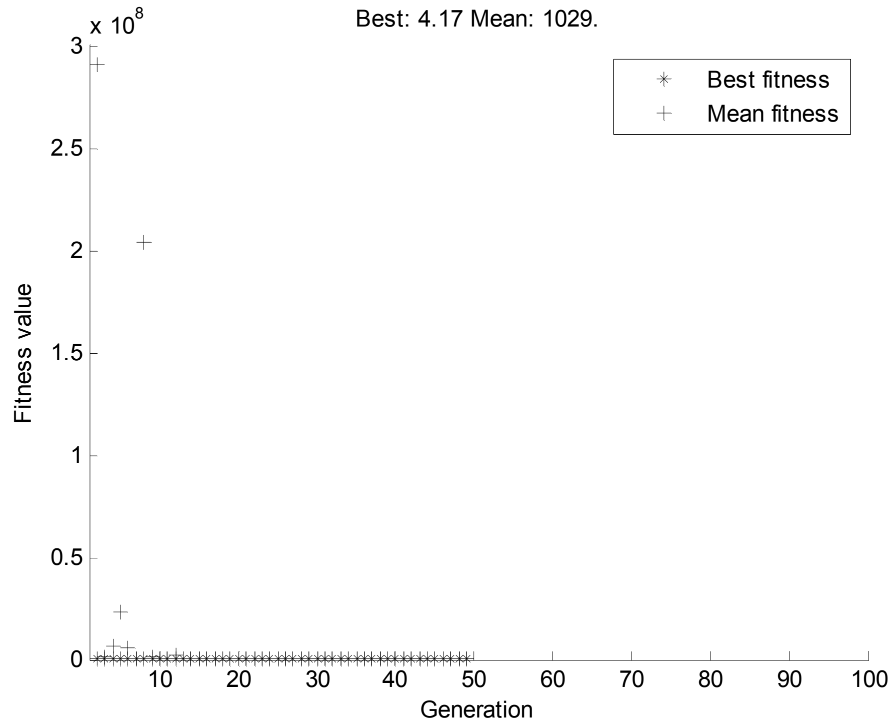

FIGURE B.5. SCHEME 3, RANGED, GA CONVERGENCE

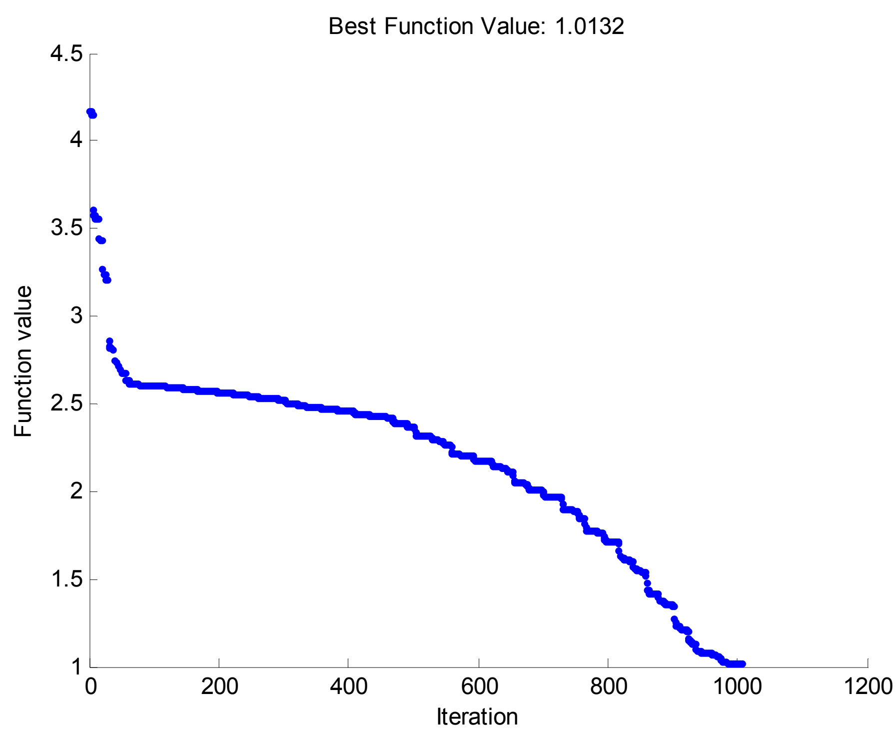

FIGURE B.6. SCHEME 3, RANGED, PATTERN SEARCH CONVERGENCE

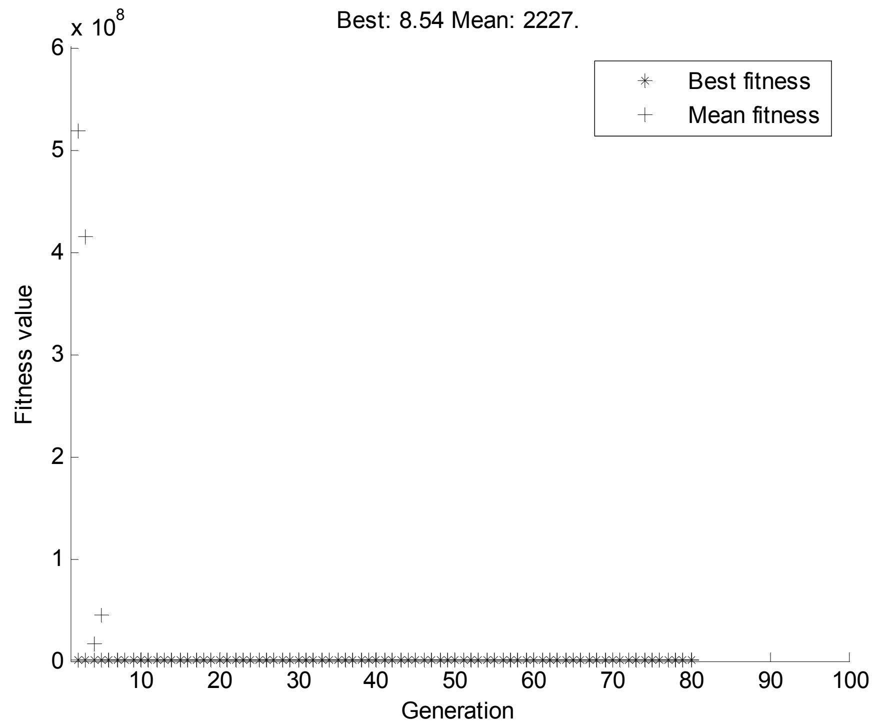

FIGURE B.7. SCHEME 4, RANGED, GA CONVERGENCE

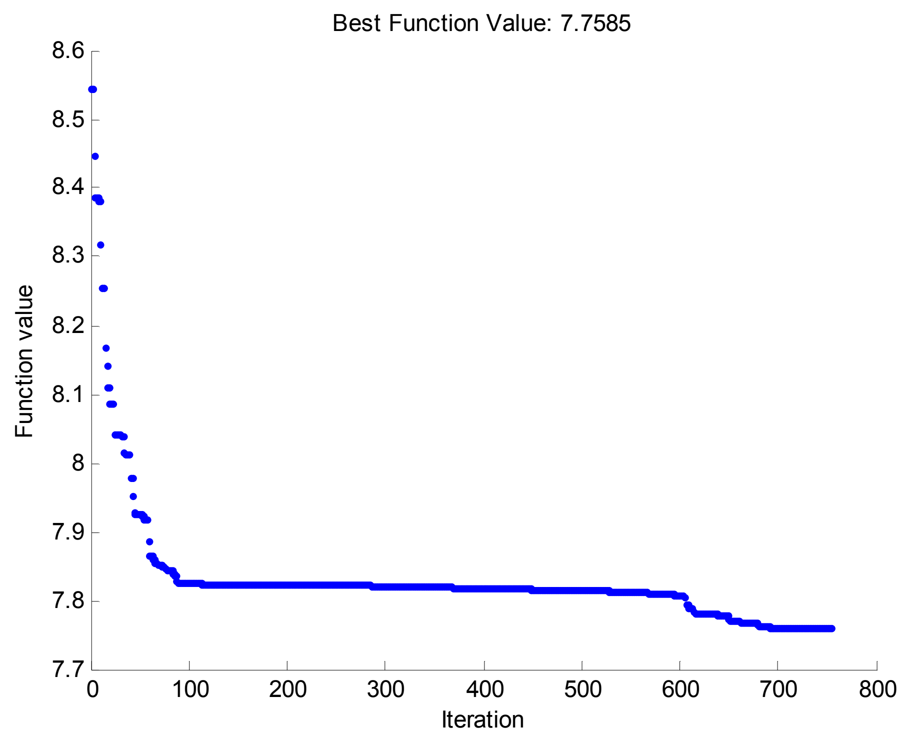

FIGURE B.8. SCHEME 4, RANGED, PATTERN SEARCH CONVERGENCE

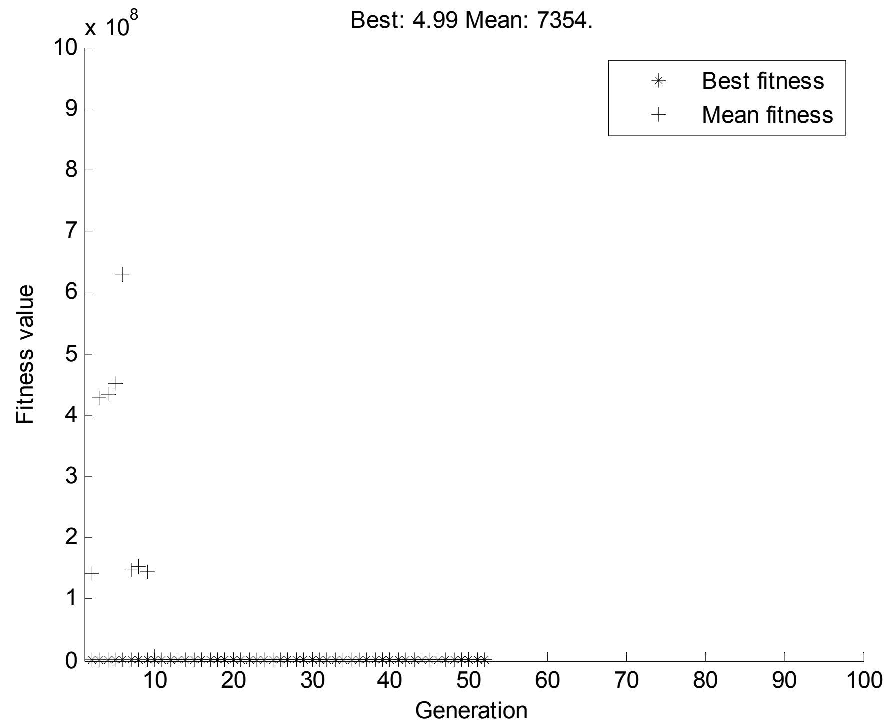

FIGURE B.9. SCHEME 5, RANGED, GA CONVERGENCE

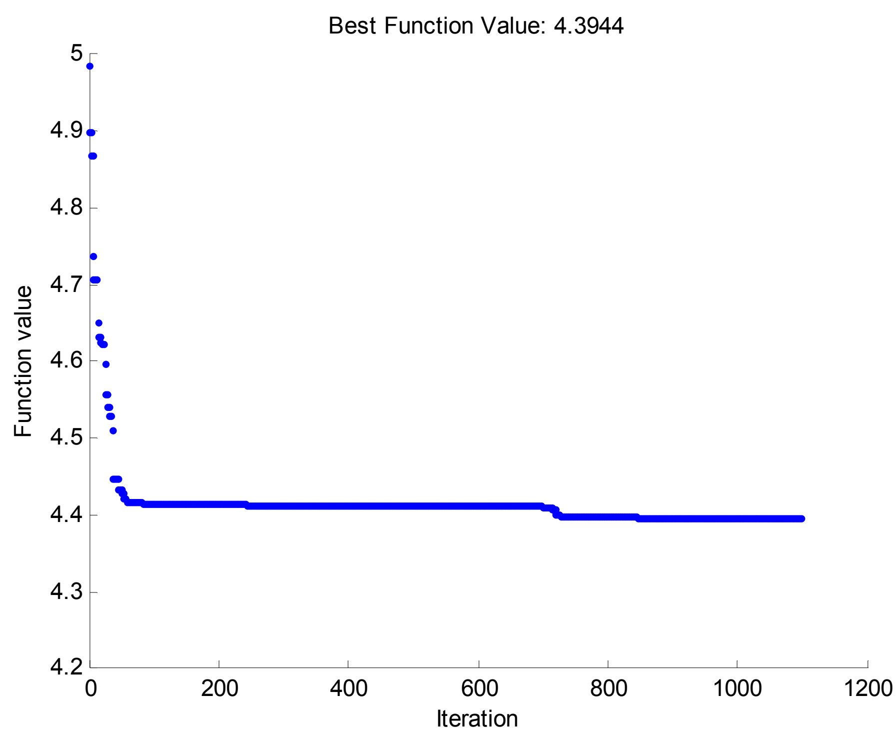

FIGURE B.10. SCHEME 5, RANGED, PATTERN SEARCH CONVERGENCE

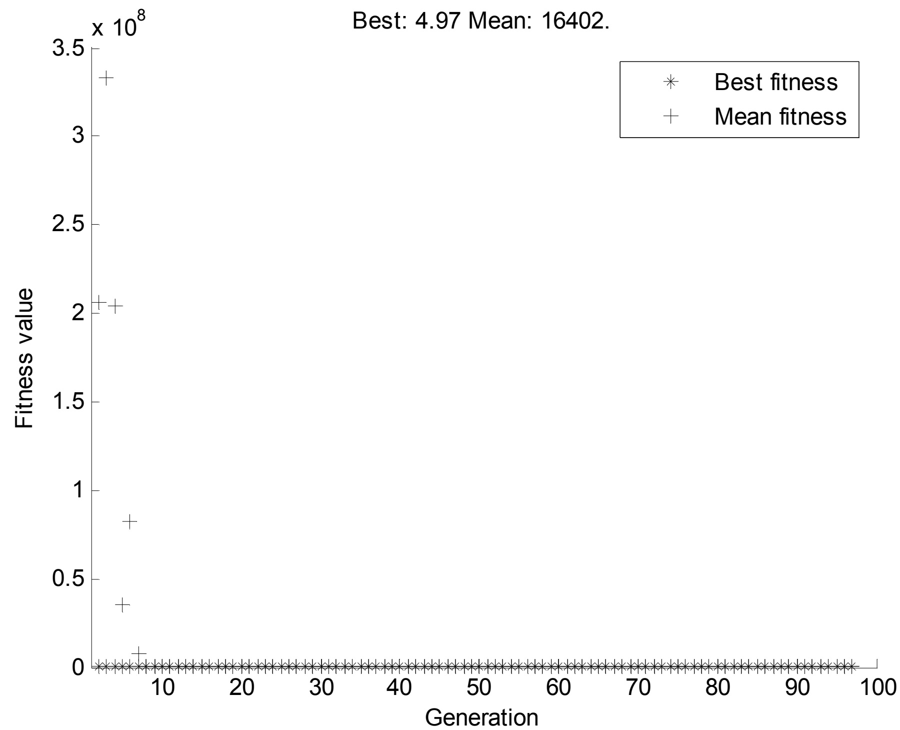

FIGURE B.11. SCHEME 6, RANGED, GA CONVERGENCE

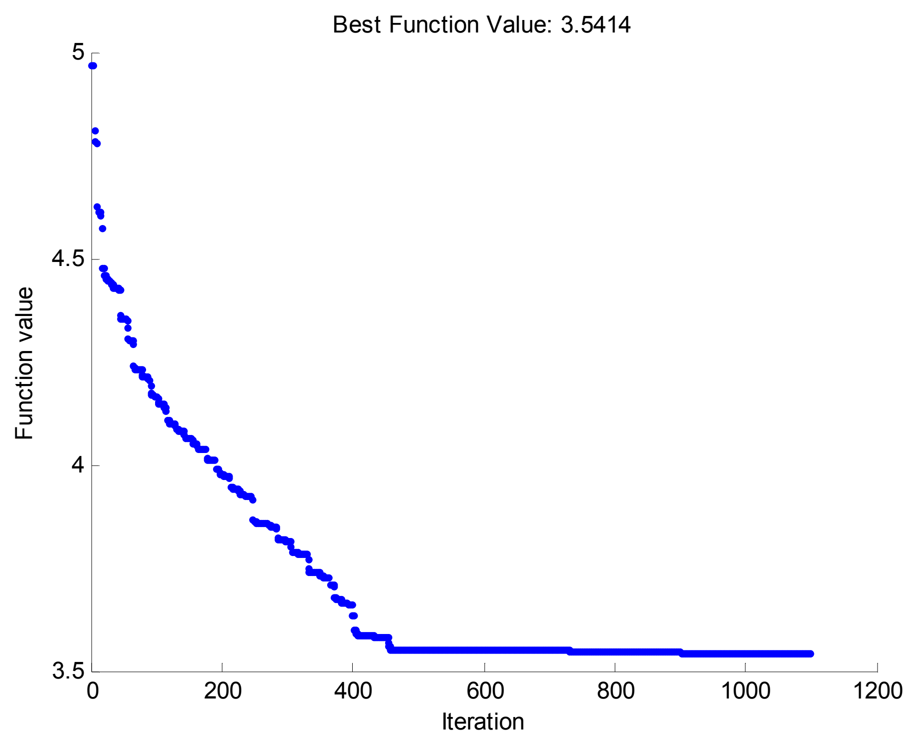

FIGURE B.12. SCHEME 6, RANGED, PATTERN SEARCH CONVERGENCE

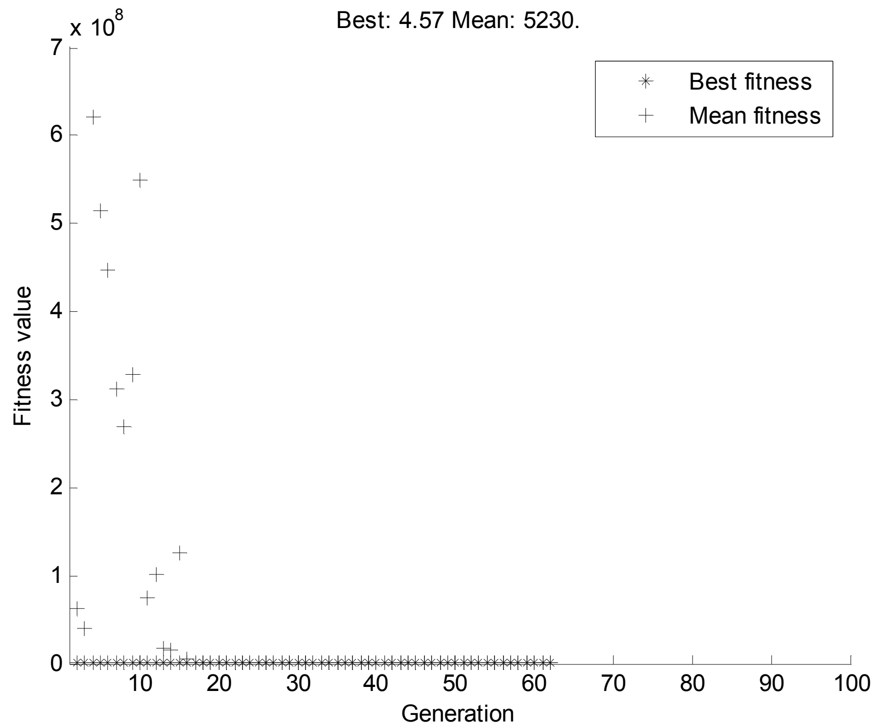

FIGURE B.13. SCHEME 7, RANGED, GA CONVERGENCE

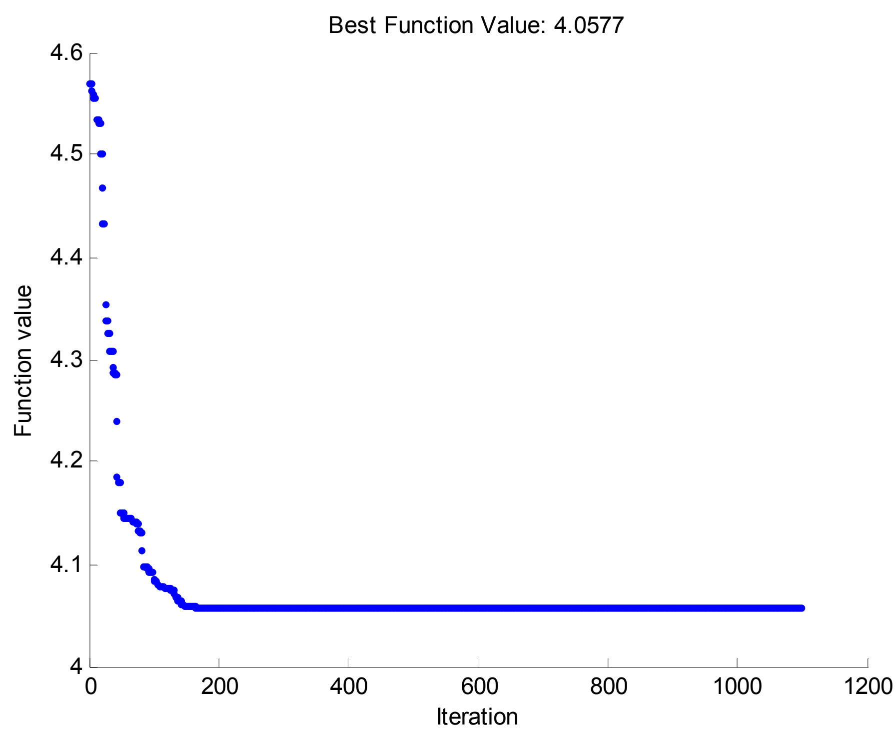

FIGURE B.14. SCHEME 7, RANGED, PATTERN SEARCH CONVERGENCE

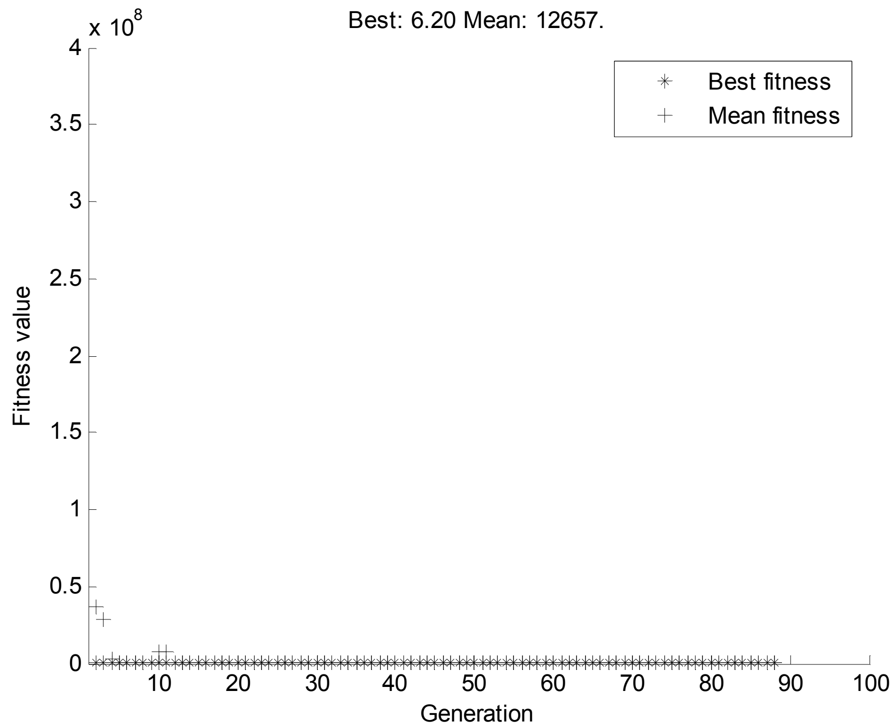

FIGURE B.15. SCHEME 8, RANGED, GA CONVERGENCE

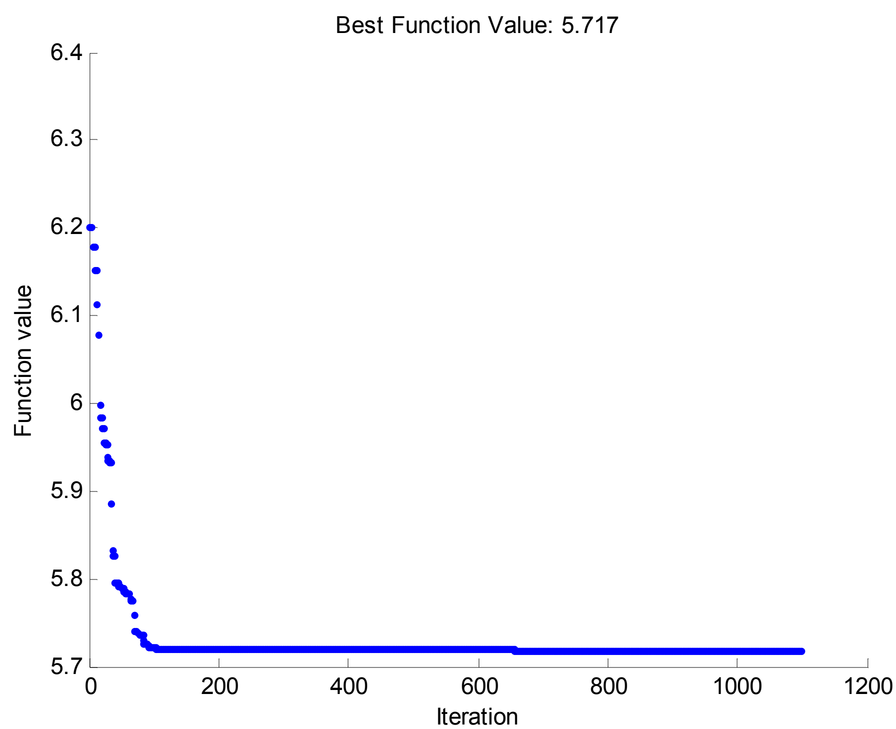

FIGURE B.16. SCHEME 8, RANGED, PATTERN SEARCH CONVERGENCE

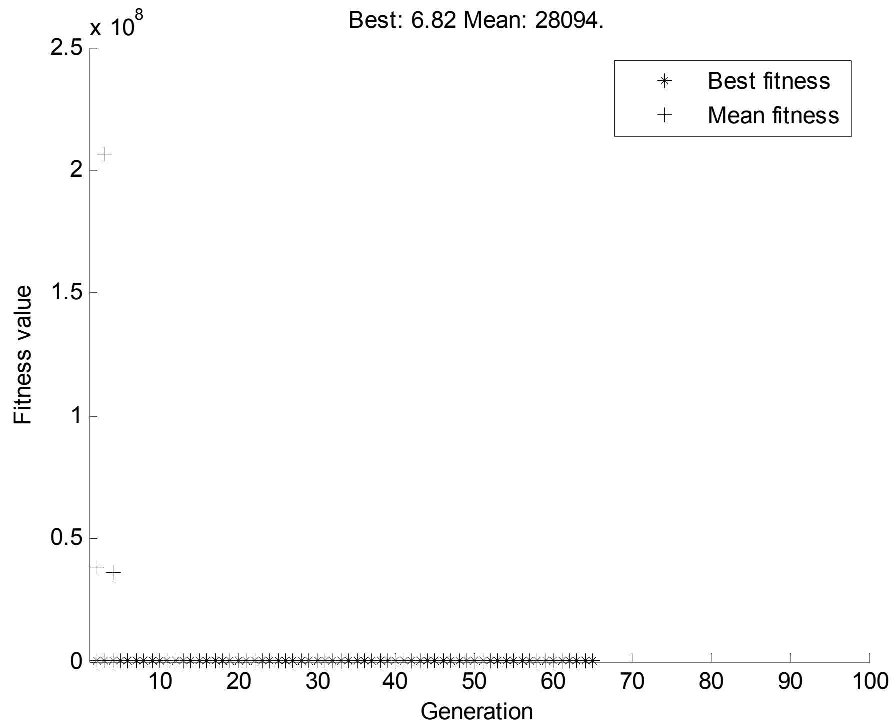

FIGURE B.17. SCHEME 9, RANGED, GA CONVERGENCE

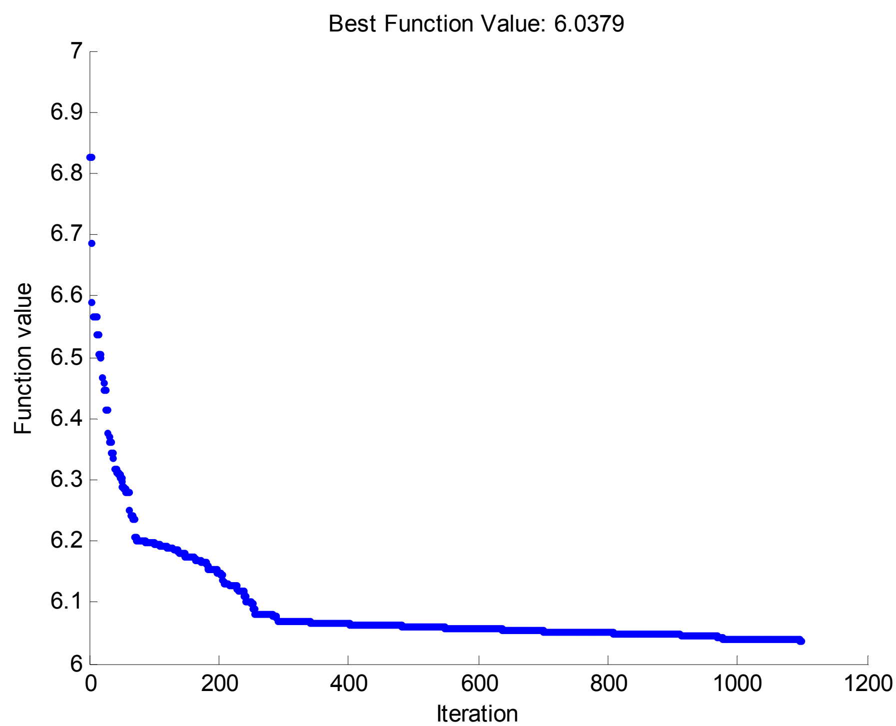

FIGURE B.18. SCHEME 9, RANGED, PATTERN SEARCH CONVERGENCE

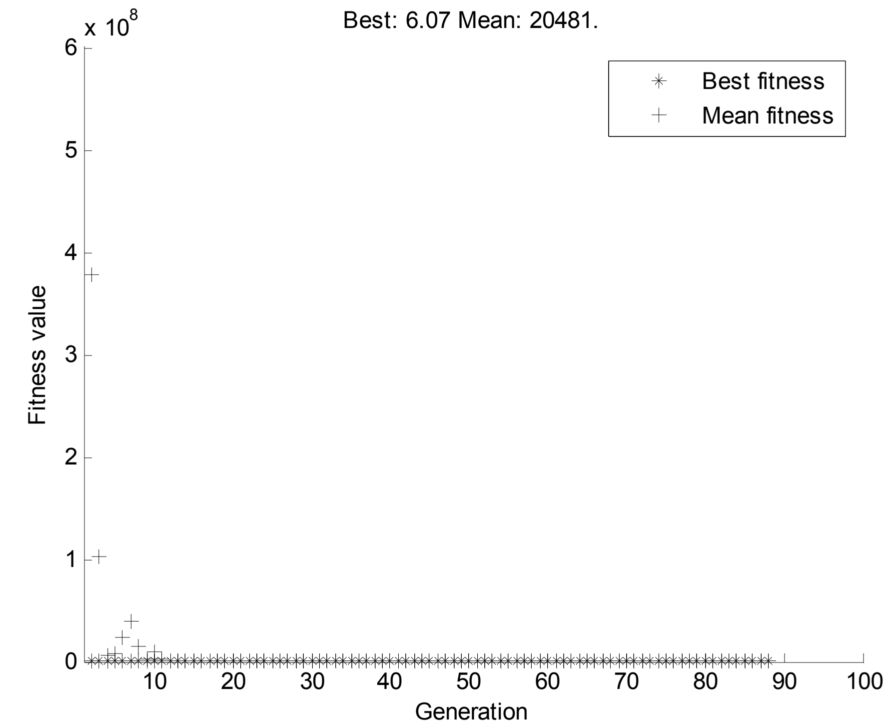

FIGURE B.19. SCHEME 10, RANGED, GA CONVERGENCE

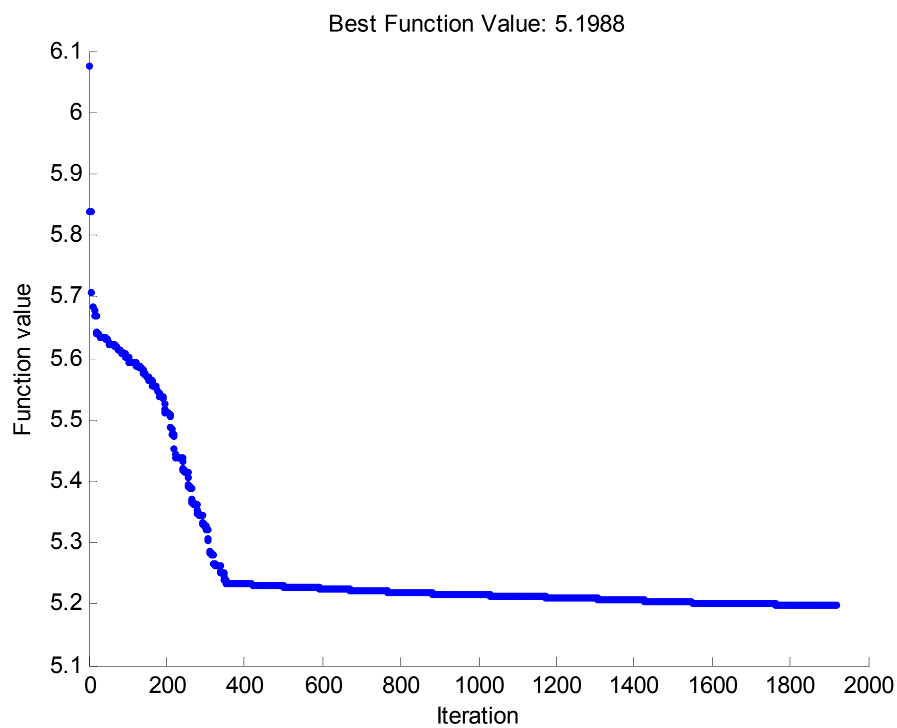

FIGURE B.20. SCHEME 10, RANGED, PATTERN SEARCH CONVERGENCE

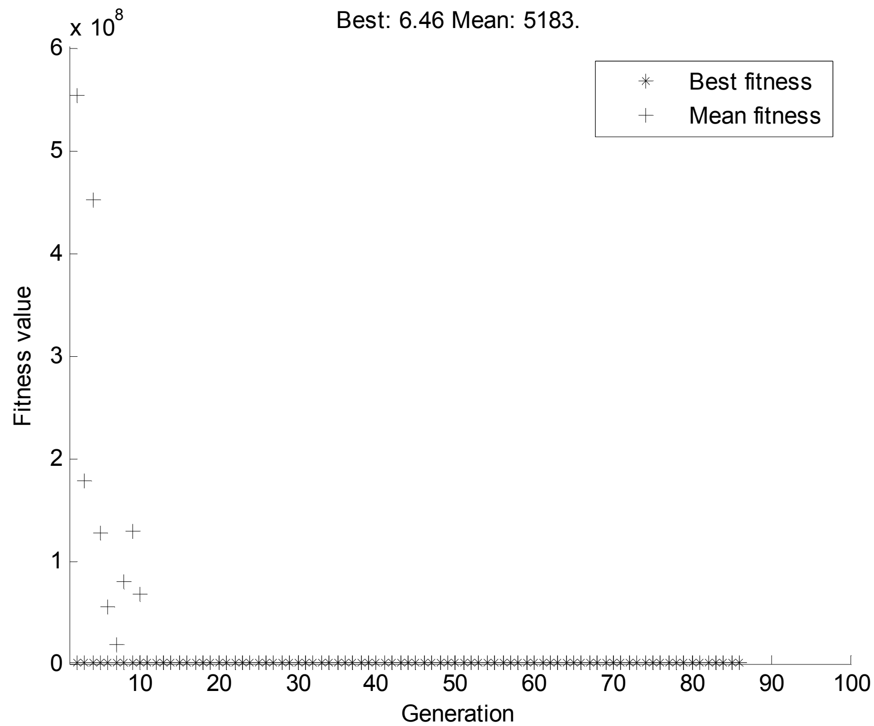

FIGURE B.21. SCHEME 11, RANGED, GA CONVERGENCE

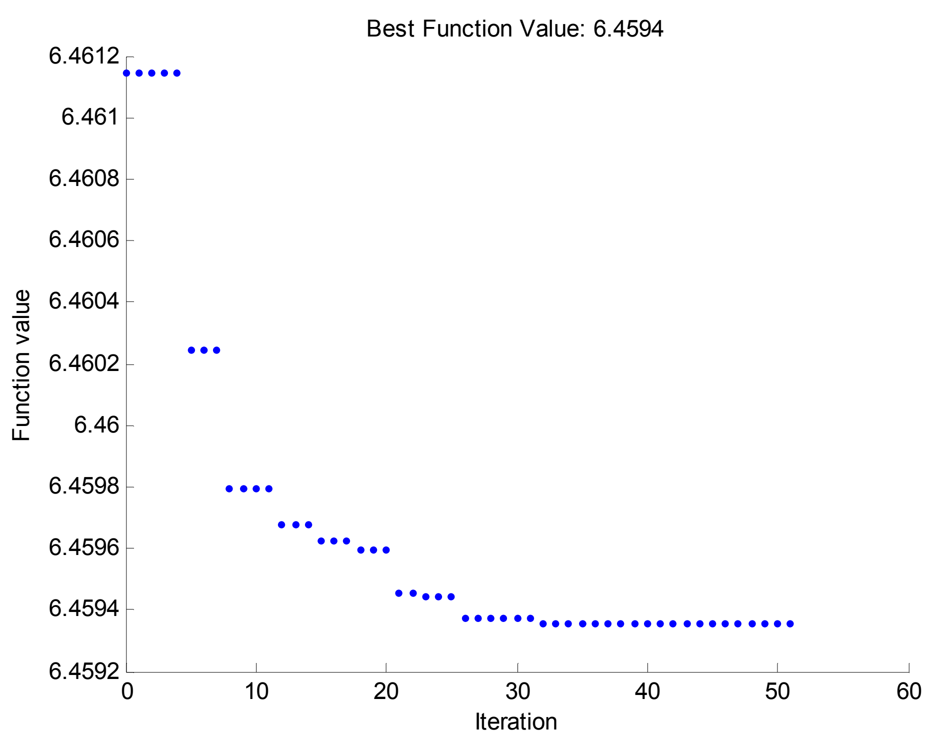

FIGURE B.22. SCHEME 11, RANGED, PATTERN SEARCH CONVERGENCE

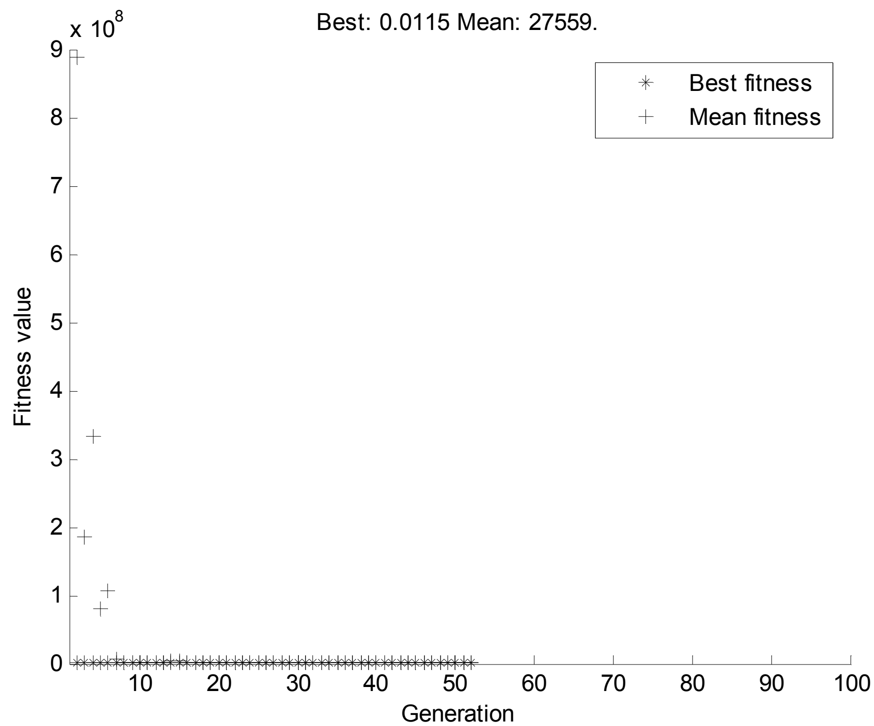

FIGURE B.23. SCHEME 1, POINT SCENARIO, GA CONVERGENCE

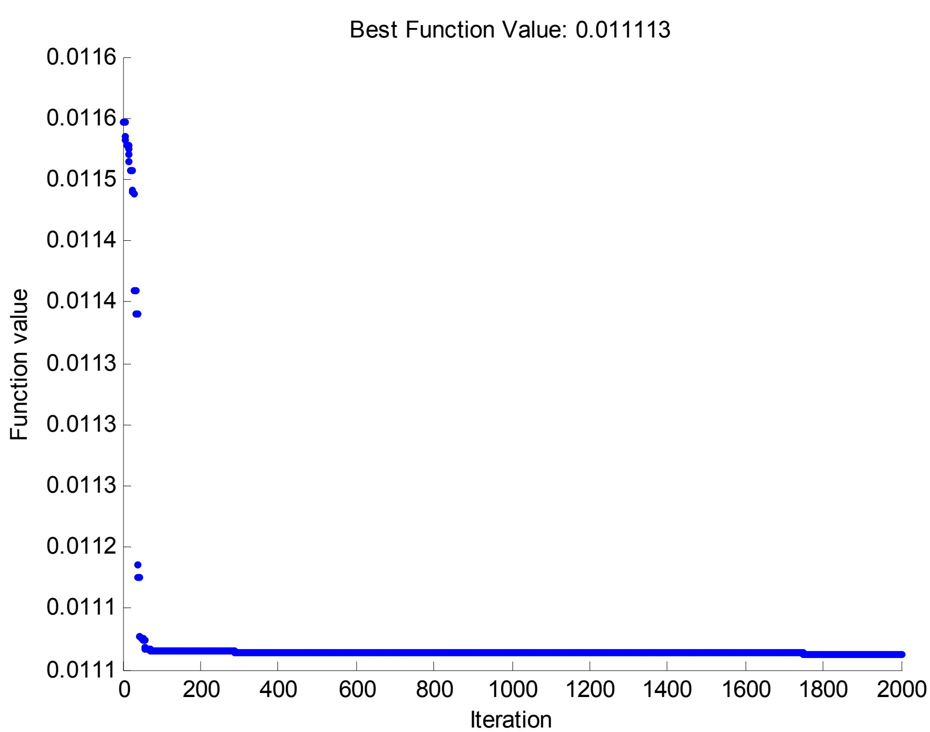

FIGURE B.24. SCHEME 1, POINT, PATTERN SEARCH CONVERGENCE

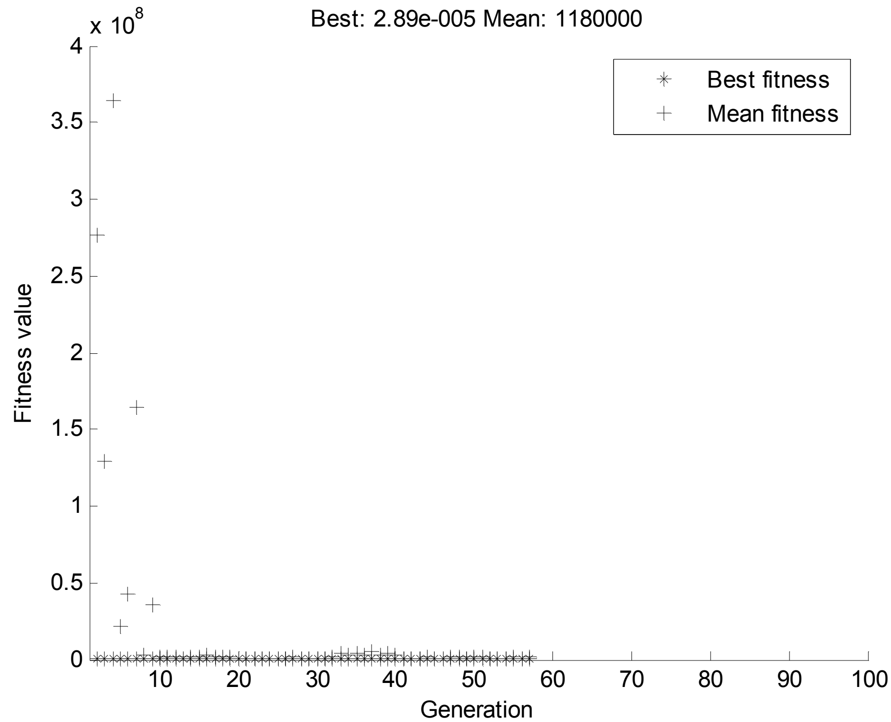

FIGURE B.25. SCHEME 2, POINT, GA CONVERGENCE

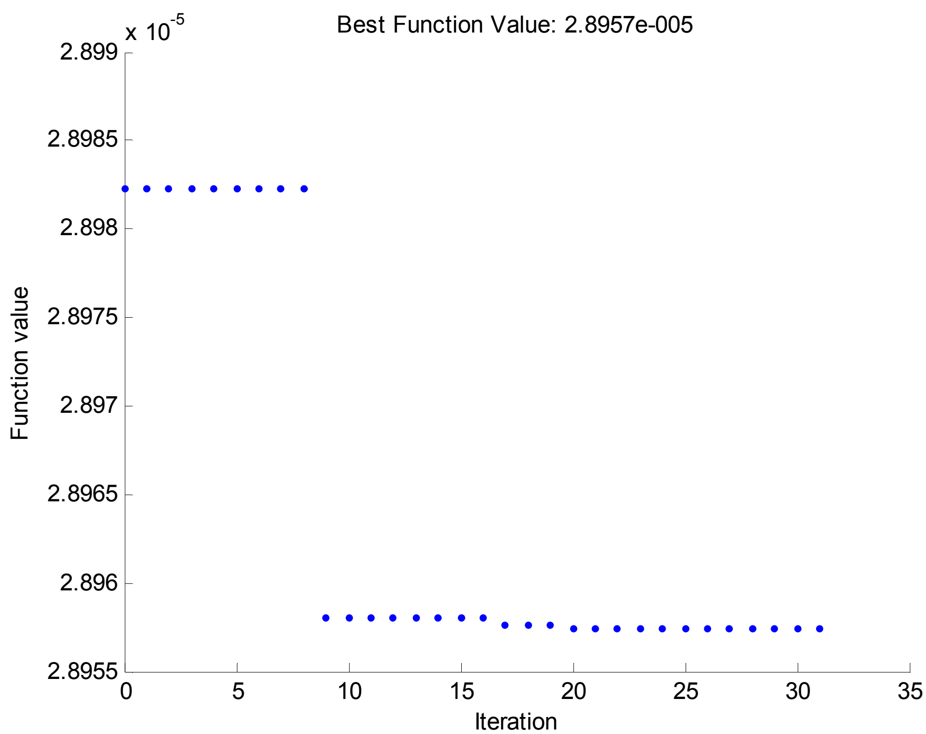

FIGURE B.26. SCHEME 2, POINT, PATTERN SEARCH CONVERGENCE

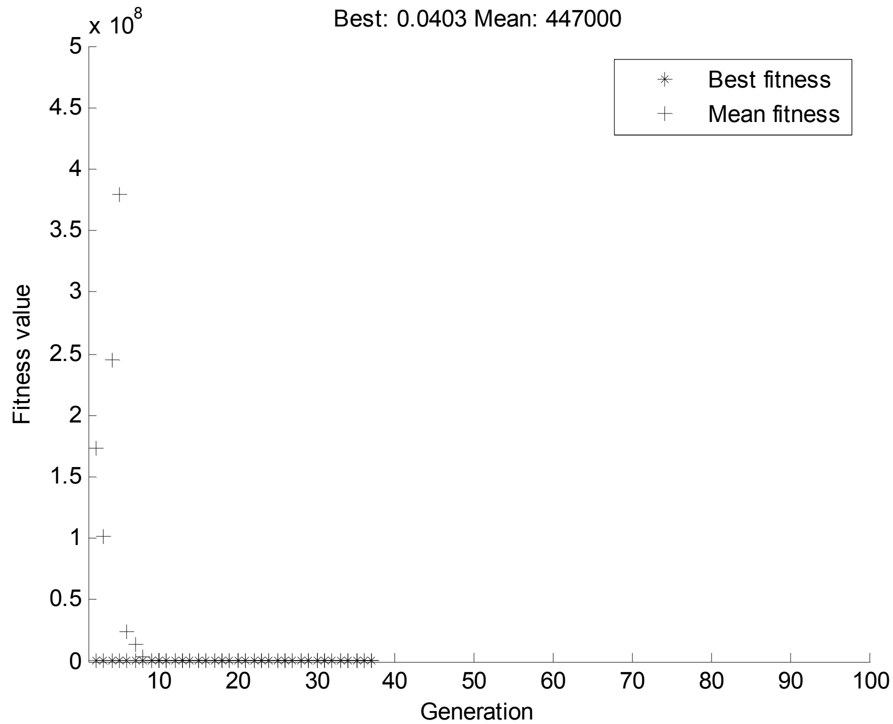

FIGURE B.27. SCHEME 3, POINT, GA CONVERGENCE

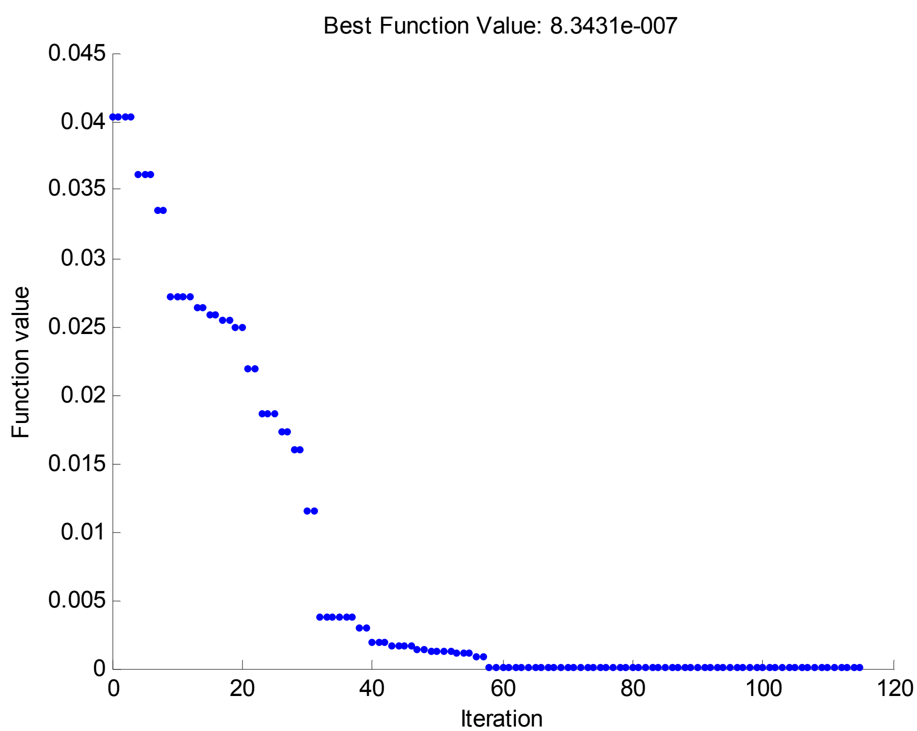

FIGURE B.28. SCHEME 3, POINT, PATTERN SEARCH CONVERGENCE

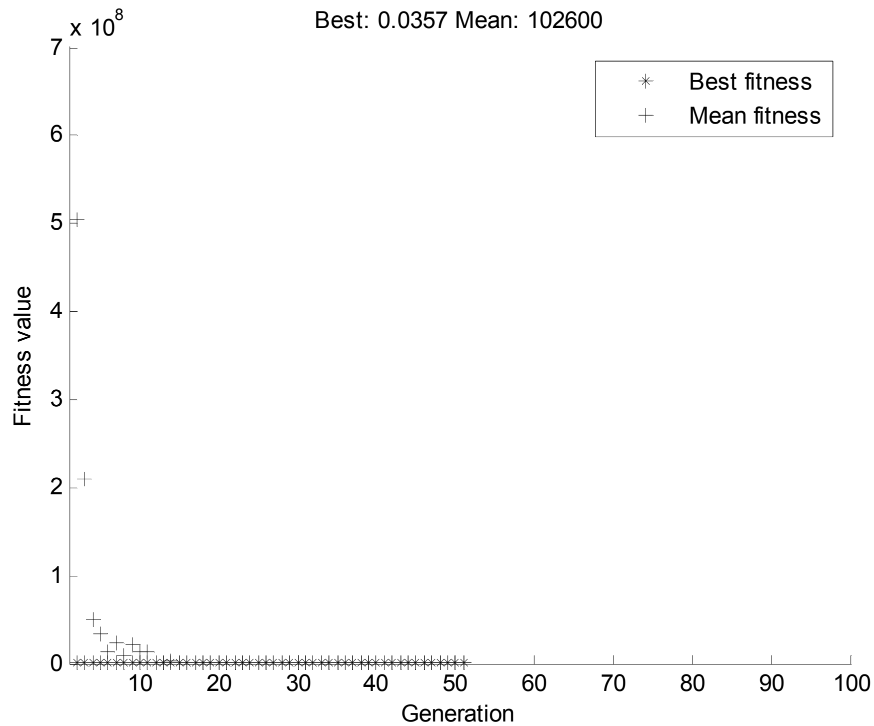

FIGURE B.29. SCHEME 4, POINT, GA CONVERGENCE

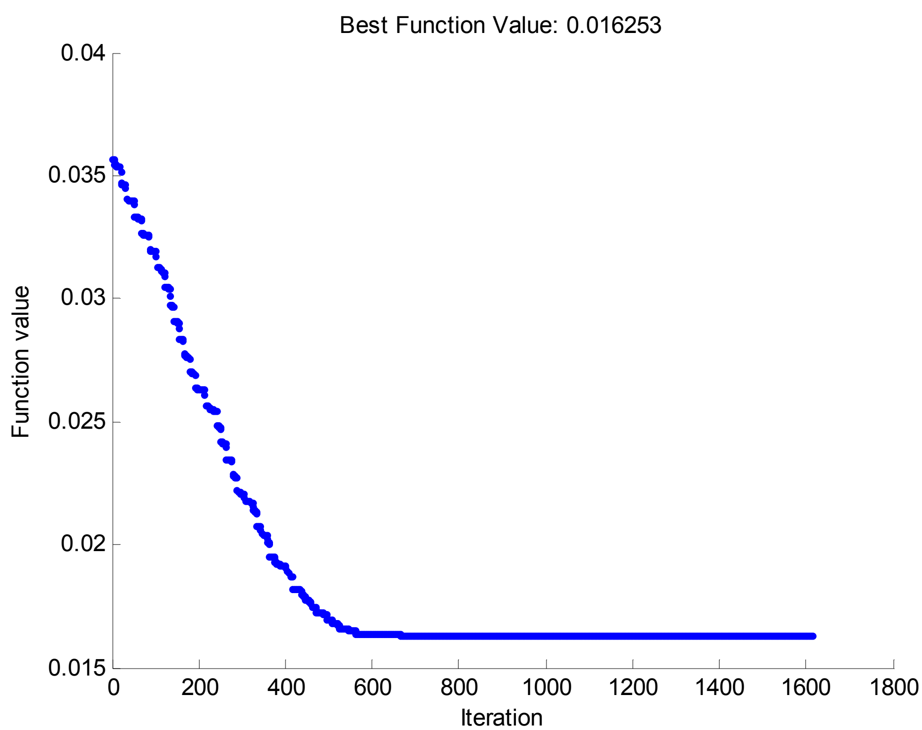

FIGURE B.30. SCHEME 4, POINT, PATTERN SEARCH CONVERGENCE

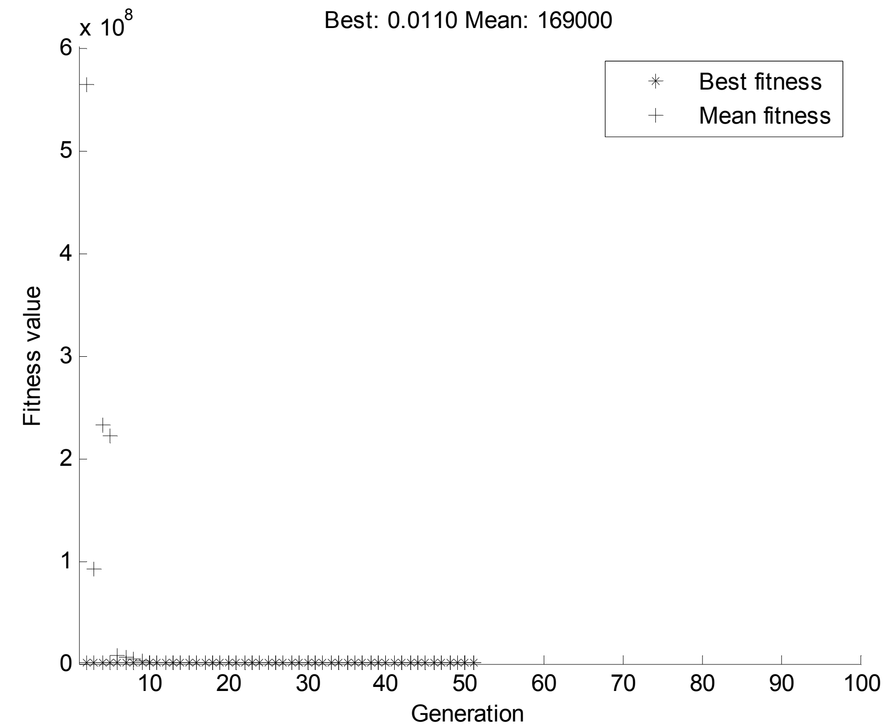

FIGURE B.31. SCHEME 5, POINT, GA CONVERGENCE

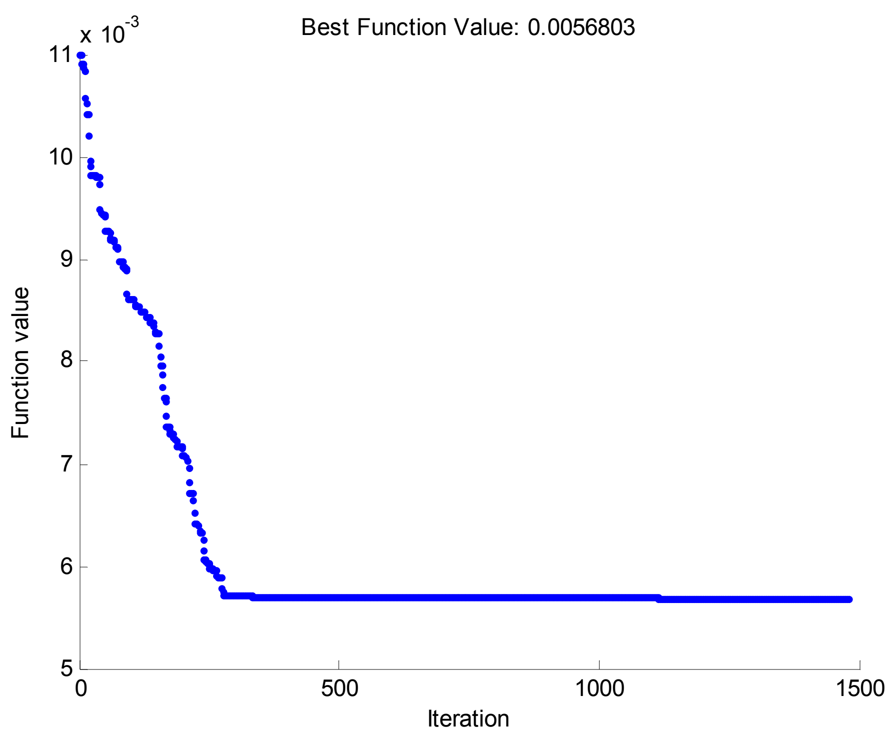

FIGURE B.32. SCHEME 5, POINT, PATTERN SEARCH CONVERGENCE

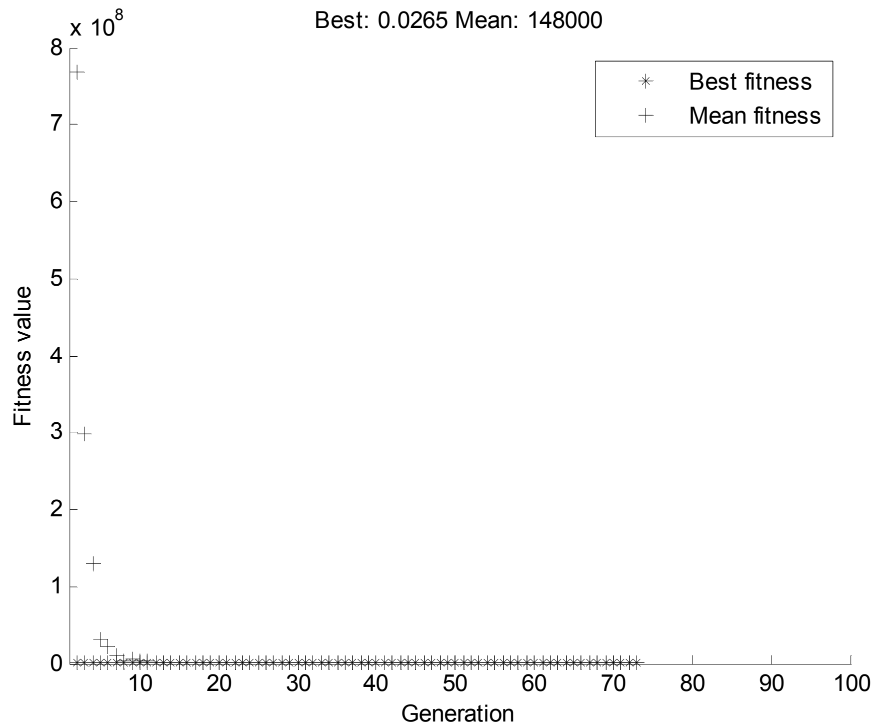

FIGURE B.33. SCHEME 6, POINT, GA CONVERGENCE

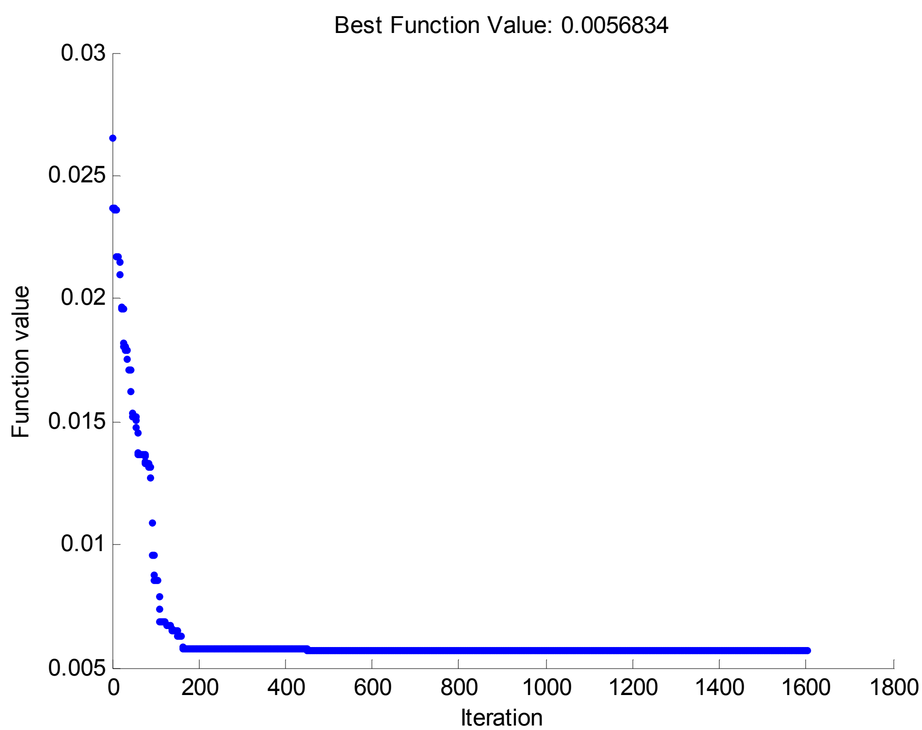

FIGURE B.34. SCHEME 6, POINT, PATTERN SEARCH CONVERGENCE

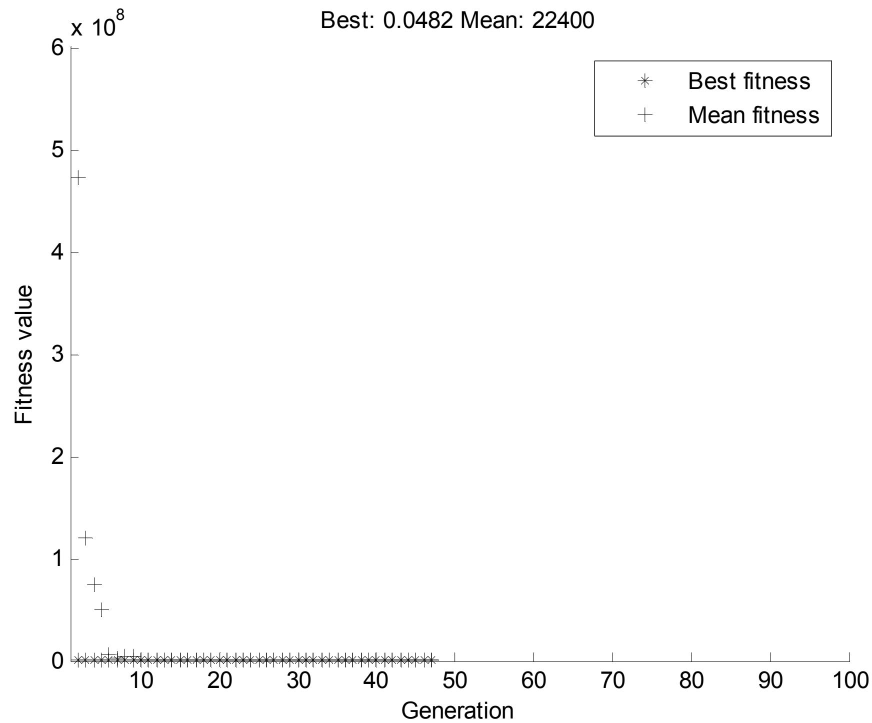

FIGURE B.35. SCHEME 7, POINT, GA CONVERGENCE

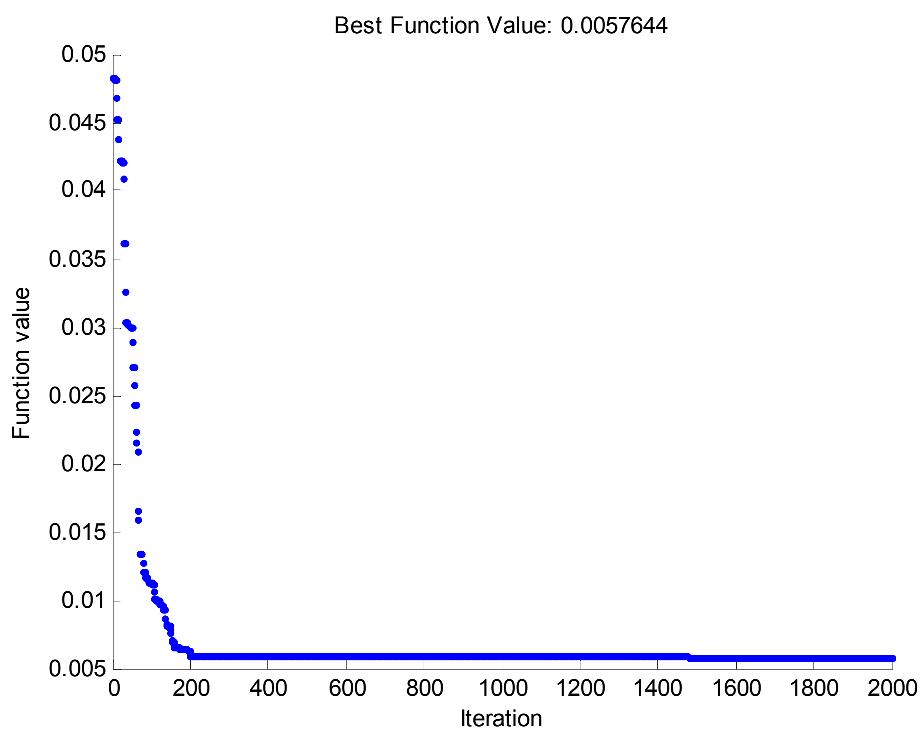

FIGURE B.36. SCHEME 7, POINT, PATTERN SEARCH CONVERGENCE

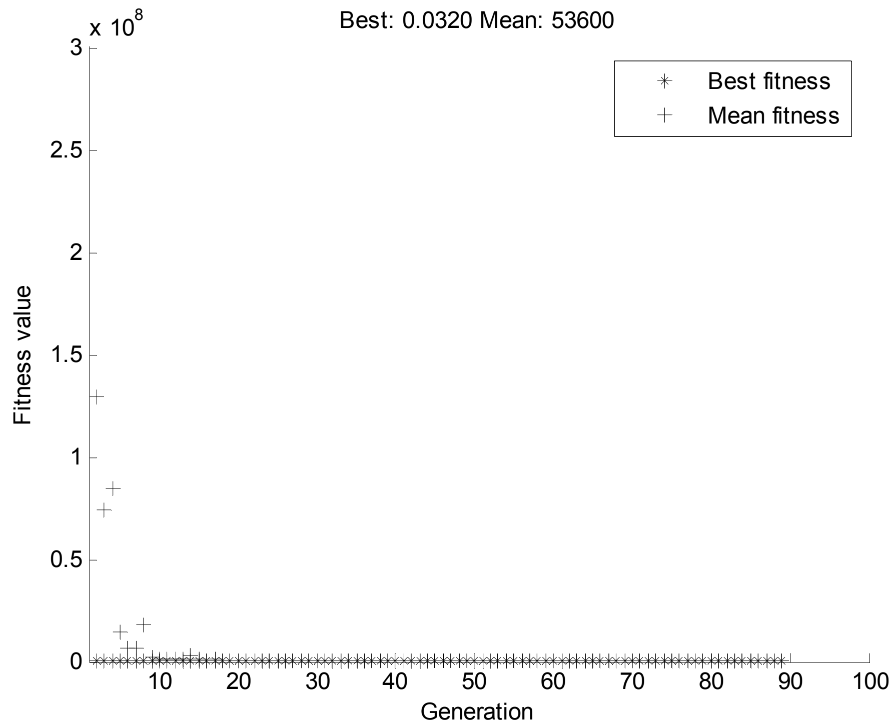

FIGURE B.37. SCHEME 8, POINT, GA CONVERGENCE

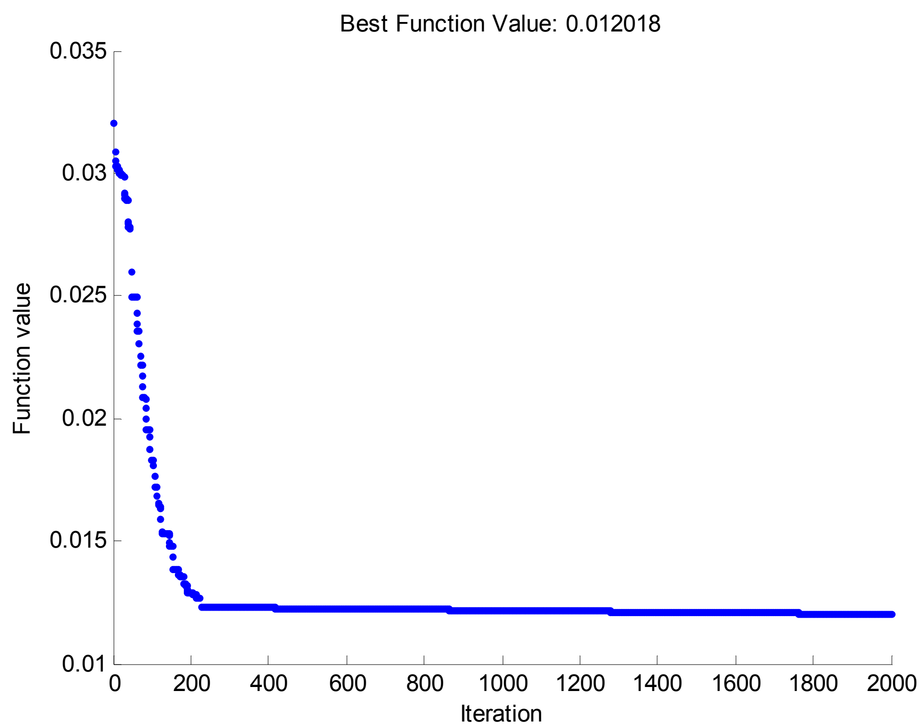

FIGURE B.38. SCHEME 8, POINT, PATTERN SEARCH CONVERGENCE

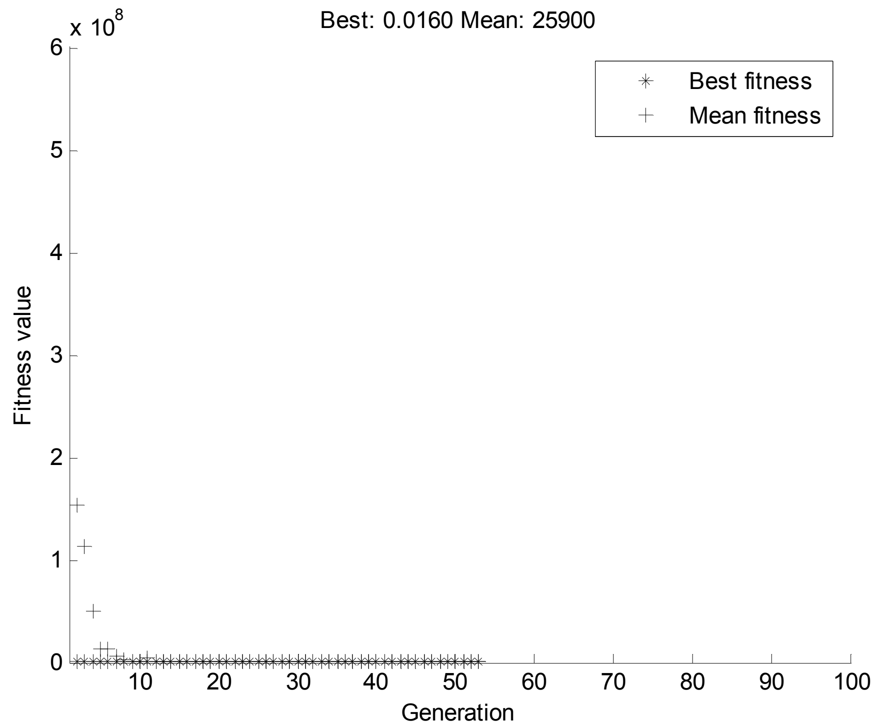

FIGURE B.39. SCHEME 9, POINT, GA CONVERGENCE

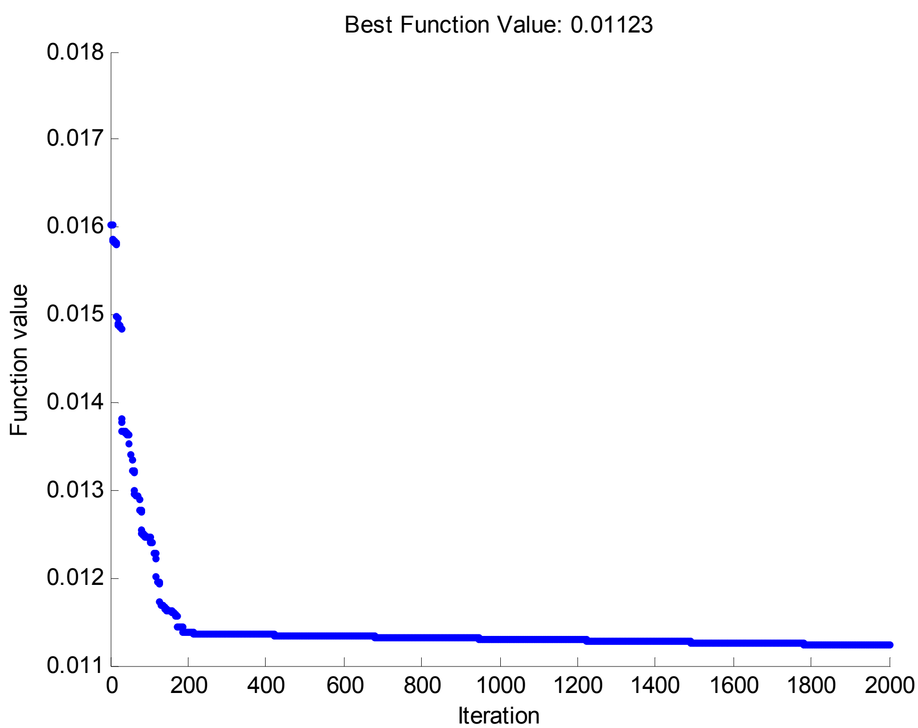

FIGURE B.40. SCHEME 9, POINT, PATTERN SEARCH CONVERGENCE

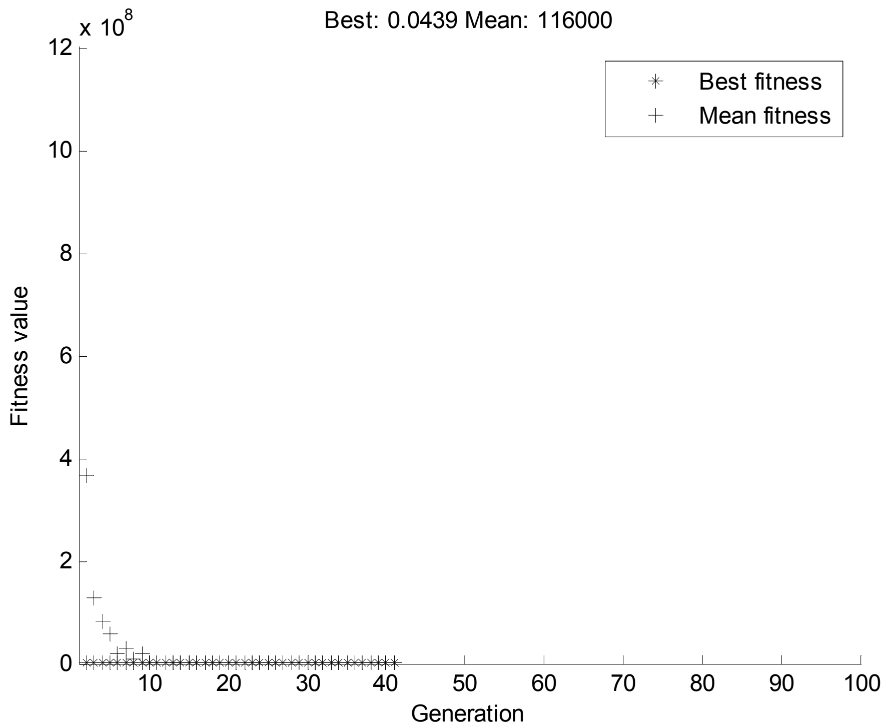

FIGURE B.41. SCHEME 10, POINT, GA CONVERGENCE

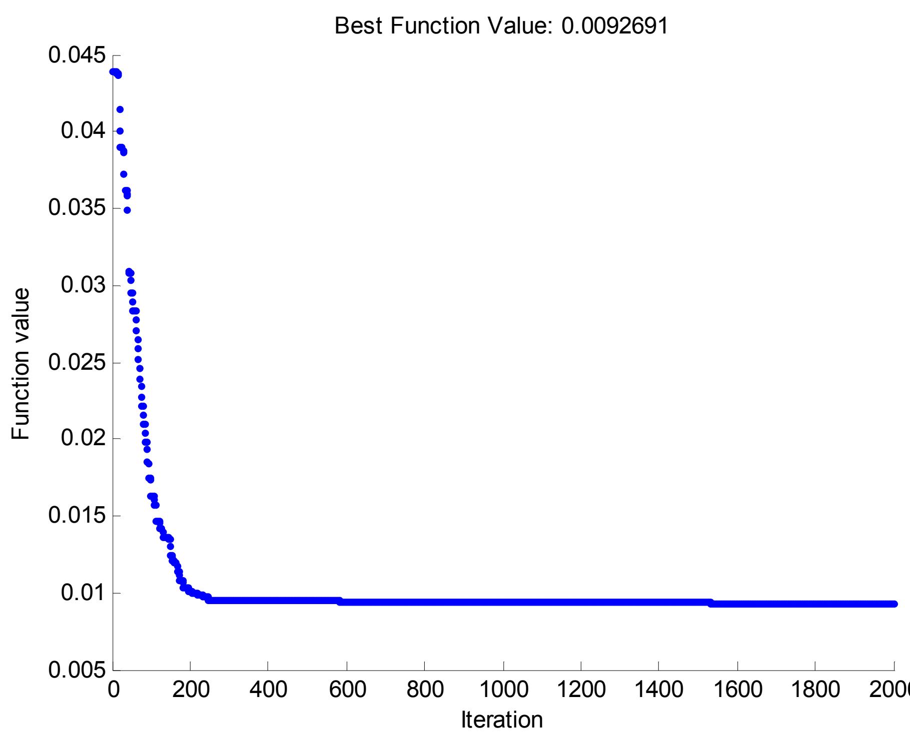

FIGURE B.42. SCHEME 10, POINT, PATTERN SEARCH CONVERGENCE

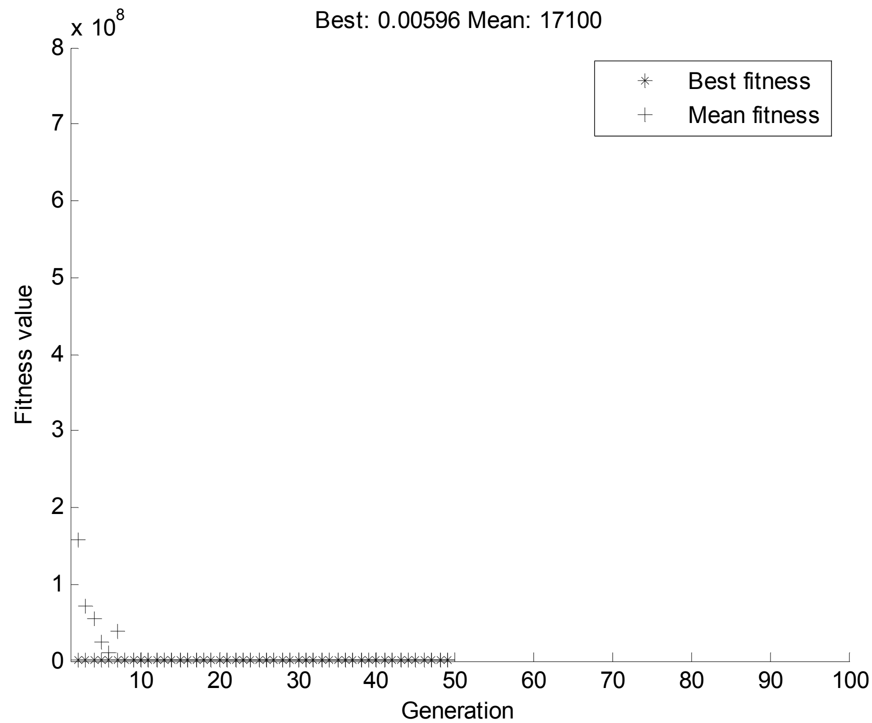

FIGURE B.43. SCHEME 11, POINT, GA CONVERGENCE

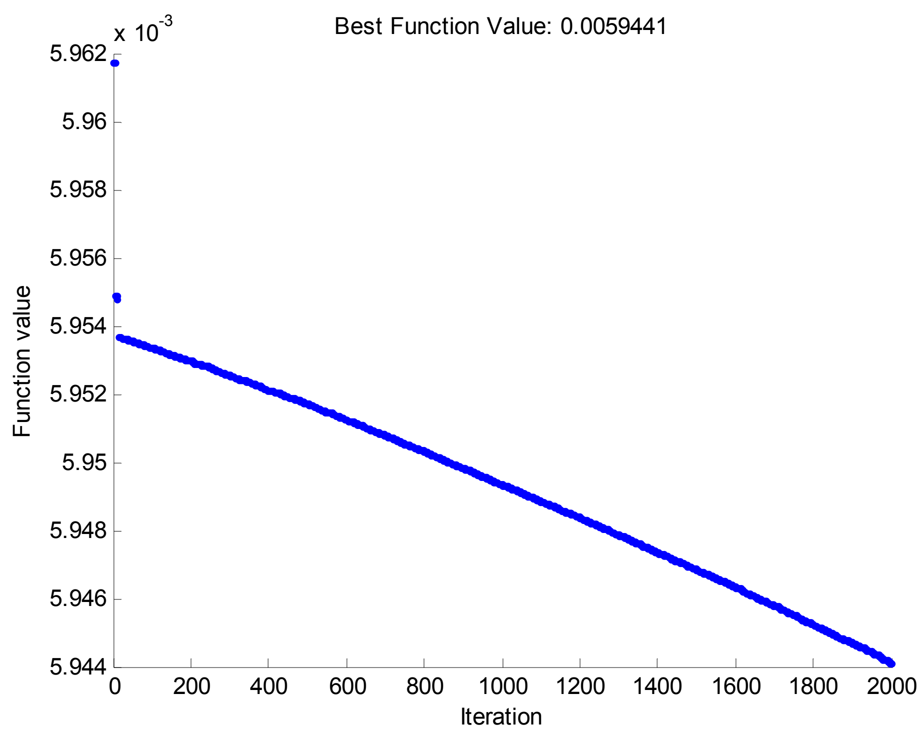

FIGURE B.44. SCHEME 11, POINT, PATTERN SEARCH CONVERGENCE