GEORGIA INSTITUTE OF TECHNOLOGY

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PROJECT ADMINISTRATION DATA SHEET

[x] ORIGINAL [ ] REVISION NO. ______

Project No. G-37-608 GTRI/~~GIT~~ DATE 7 /12 / 84

Project Director: R.P. Kertz and T.P Hill School/~~Lab~~ Math

Sponsor: National Science Foundation

Type Agreement: Grant No. DMS-8401604

Award Period: From 7/1/84 To 12/31/85* (Performance) 3/31/86 (Reports)

Sponsor Amount:

	This Change	Total to Date
Estimated:	$	$ 28,400
Funded:	$	$ 28,400

Cost Sharing Amount: $ 7,909 Cost Sharing No: G-37-316

Title: "Mathematical Sciences: Optimization and Conditioning Principles for Discrete Parameter Stochastic Processes"

ADMINISTRATIVE DATA OCA Contact Lynn Boyd x4820

1) Sponsor Technical Contact:

Dr. Jerome Sacks
Statistics and Probability Program
Division of Mathematical and Computer Sciences
National Science Foundation
Washington, DC 20550
(202) 357-9764

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Grants Official
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(202) 357-9671

Defense Priority Rating: n/a

Military Security Classification: n/a

(or) Company/Industrial Proprietary: n/a

RESTRICTIONS

See Attached NSF Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval – Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of $500 or 125% of approved proposal budget category.

Equipment: Title vests with GIT.

COMMENTS:

*Includes usual 6 month unfunded flexibility period.

This is a continuing grant.

JUL 1984 RECEIVED Research Reports Office

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Accounting	Research Communications (2)	Other Newton

GEORGIA INSTITUTE OF TECHNOLOGY

OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date 4-15-87

Project No. G-37-608

School/~~Lab~~ Math

Includes Subproject No.(s) N/A

Project Director(s) T.P. Hill

GTRC / ~~GIT~~

Sponsor National Science Foundation

Title "Mathematical Sciences: Optimization and Conditioning Principles for Discrete Parameter Stochastic Processes"

Effective Completion Date: 12/31/86 (Performance) 3/31/87 (Reports)

Grant/Contract Closeout Actions Remaining:

- [X] None
- [ ] Final Invoice or Final Fiscal Report
- [ ] Closing Documents
- [ ] Final Report of Inventions
- [ ] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other

Continues Project No.

Continued by Project No.

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Project File
Other Duane H.
Angela DuBose
Russ Embry

FORM OCA 69.285

FINAL REPORT

MATHEMATICAL SCIENCES: OPTIMIZATION AND CONDITIONING PRINCIPLES FOR DISCRETE PARAMETER STOCHASTIC PROCESSES

Theodore P. Hill
Robert P. Kertz

Award Period
July 1, 1984 to December 31, 1986

Under

NSF Award Number DMS-8401604

March 1987

GEORGIA INSTITUTE OF TECHNOLOGY
A UNIT OF THE UNIVERSITY SYSTEM OF GEORGIA
SCHOOL OF MATHEMATICS
ATLANTA, GEORGIA 30332

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

FINAL PROJECT REPORT
NSF FORM 98A

PLEASE READ INSTRUCTIONS ON REVERSE BEFORE COMPLETING

PART I–PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
Georgia Tech Research Corp. Georgia Institute of Technology Atlanta, GA 30332	Statistics and Probability	DMS-8401604
	4. Award Period From 7/1/84 To 12/31/86	5. Cumulative Award Amount $59,400

6. Project Title
Mathematical Sciences: Optimization and Conditioning Principles for Discrete Parameter Stochastic Processes

PART II–SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

Research was carried out in the mathematical theory of probability, focusing chiefly on problems in sequential decision theory and on optimal-partitioning (or "fair-division") problems. The primary objectives were to determine: (i) for various classes of sequences of random variables and information structures, how much better a person with specified information concerning the process can do than one without that information; and (ii) quantitative bounds for the optimal-partitioning of a set or probability space based on various parameters of the underlying distributions.

The project involved two faculty researchers over a two-year period, with input from various collaborators and two beginning graduate students; nearly all of the work was purely theoretical and did not involve computers.

The primary findings included universal bounds for comparisons of various classes of discrete-parameter stochastic processes including averages of independent random variables, powers of martingales, exchangeable random variables and order statistics, and universal bounds for the optimal-partitioning of various classes of value measures such as general nonatomic measures and probability measures with atoms. Applications of those results were given to applied areas such as operations research, control theory and statistics, as well as to theoretical areas such as classical probability theory, function theory and measure theory.

PART III–TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM: Check (✓)	TO BE FURNISHED SEPARATELY TO PROGRAM: Approx. Date
a. Abstracts of Theses	X				
b. Publication Citations		X			
c. Data on Scientific Collaborators		X			
d. Information on Inventions	X				
e. Technical Description of Project and Results					
f. Other *(specify)*					

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
Theodore P. Hill/Robert P. Kertz		3/11/87

b. Publication Citations

List of papers by T. P. Hill which were partially supported by NSF Grant DMS 84-01604.

1. "Equipartitioning Common Domains of Non-Atomic Measures," Math. Z. 189, 415-419 (1985).

2. "Optimal-Partitioning Inequalities for Non-Atomic Probability Measures," Trans. Amer. Math. Soc. 296, 703-725 (1986) (with J. Elton and R. Kertz).

3. "Prophet Inequalities for Averages of Independent Non-Negative Random Variables," Math. Z. 192, 427-436 (1986).

4. "A Generalization of Lyapounov's Convexity Theorem to Measures with Atoms," Proc. Amer. Math. Soc. 99, 297-304 (1987) (with J. Elton).

5. "Monotonically Improving Limit-Optimal Strategies in Finite-State Decision Processes," to appear in Math. of O.R. (with J. Van der Wal).

6. "Partitioning General Probability Measures," to appear in Ann. Prob.

7. "The Existence of Good Markov Strategies in Decision Processes with General Payoffs," to appear in Stoch. Proc. Appl. (with V. Pestien).

8. "Common Hyperplane Medians for Random Vectors," to appear in Amer. Math. Monthly.

9. "A Sharp Partitioning-Inequality for Non-Atomic Probability Measures Based on the Mass of the Infimum of the Measures," to appear in Prob. Th. Rel. Topics (Z. Wahrscheinlichkeitsth.)

10. "Expectation Inequalities Associated with Prophet Problems," to appear in Stoch. Anal. Appl.

11. "Equitable Distribution of Indivisible Objects," (with S. Demko), submitted for publication.

12. "Random-Number Guessing and the First-Digit Phenomenon," submitted for publication.

13. "Prophet Inequalities for Parallel Processes," (with D. Kennedy), submitted for publication.

14. "Minimax-Risk Inequalities in the Classification Problem," (with Y. Tong) in preparation.

List of papers by Robert P. Kertz which were partially supported by NSF Grant DMS 84-01604.

1. "Common Strict Character of Some Sharp Infinite-Sequence Martingale Inequalities," co-authored with David C. Cox (1985) Stochastic Processes and Their Applications, 20, 169-179.

2. "Prophet Regions and Sharp Inequalities for pth Absolute Moments of Martingales," co-authored with David C. Cox (1986) Journal of Multivariate Analysis, 18, 242-273.

3. "Comparison of Optimal Value and Constrained Maxima Expectations for Independent Random Variables," (1986) Advances in Applied Probability, 18, 311-340.

4. "Optimal-Partitioning Inequalities for Non-Atomic Probability Measures," co-authored with J. Elton and T. P. Hill (1986) Transactions of the American Mathematical Society, 296, 703-725.

5. "Comparison of Stop Rule and Maximum Expectations for Finite Sequences of Exchangeable Random Variables," co-authored with J. Elton (1986). Submitted for publication.

6. "Decision Processes Under Total Expected Concomitant Constraints, with Applications to Bandit Processes," (1986). Submitted for publication.

7. "Leaving an Interval in Limited Playing Time," co-authored with David Heath (1986). Submitted for publication.

8. "Prophet Problems in Optimal Stopping: Results, Techniques, and Variations," (1986) Technical Report, School of Mathematics, Georgia Institute of Technology, Atlanta.

9. "Simulated Annealing, Diffusions, and Large Deviations for Nonlinear Programming," with Craig Tovey (1987), in preparation.

10. "Measures of Perfect Information in Bernoulli Bandit Processes," with Donald Berry (1987), in preparation.

11. "Comparisons Between Multi-Choice Gamblers and Prophets, with Application to Optimal Job Assignment Procedures," with Douglas P. Kennedy (1987), in preparation.

12. "Limit Theorems for Threshold-Stopped Random Variables," with Douglas P. Kennedy (1987), in preparation.

c. Scientific Collaborators

Principal Investigators for NSF Grant DMS 84-01604 were Theodore P. Hill and Robert P. Kertz, both associate professors in the School of Mathematics, Georgia Institute of Technology.

e. Technical Summary of Activities and Results

Results of research by T. P. Hill and R. P. Kertz which was supported in part by NSF Grant DMS 84-01604 are summarized below. Abstracts of pertinent papers are given, followed by conferences, workshops, meetings, and universities at which talks were given and names of three professors who were brought in for consultation.

Abstracts of papers by T. P. Hill which were partially supported by NSF Grant DMS 84-01604.

1. "Equipartitioning common domains of non-atomic measures," Math. Z. 189 415-419 (1985).

Abstract. If $\mu_1,\mu_2,\ldots,\mu_n$ are non-atomic finite measures on a measure space $(S,\mathcal{F})$, then there is a measurable partition $A_1,\ldots,A_n$ of S satisfying $\mu_i(A_i) \geq n^{-1}h(\|\mu_1\|,\ldots,\|\mu_n\|)$ for all i, where $h(\cdot)$ is the harmonic mean, and $\|\mu\|$ is the total mass of μ. Moreover, this bound is sharp and attained and strict inequality is possible if not all the measures are proportional. A probabilistic corollary is the following: If $X_1,\ldots,X_n$ are continuous non-negative L_1 random variables on $(\Omega,\mathcal{F},P)$, then there is a measurable partition $A_1,\ldots,A_n$ of Ω such that $\int_{A_i} X_i dP \geq ((EX_1)^{-1} + \ldots + (EX_n)^{-1})^{-1}$ for all i and this bound is best possible.

2. "Optimal-partitioning inequalities for non-atomic probability measures," Trans. Amer. Math. Soc. 296, 703-725 (1986) (with J. Elton and R. Kertz).

Abstract. Suppose $\mu_1,\ldots,\mu_n$ are nonatomic probability measures on the same measurable space $(S,\mathcal{B})$. Then there exists a measurable partition $\{S_i\}_{i=1}^n$ of S such that $\mu_i(S_i) \geq (n+1-M)^{-1}$ for all $i = 1,\ldots,n$, where M is the total mass of $\vee_{i=1}^n \mu_i$ (the smallest measure majorizing each μ_i). This inequality is the best possible for the functional M, and sharpens and quantifies a well-known cake cutting theorem of Urbanik and of Dubins and Spanier. Applications are made to L_1-functions, discrete allocation problems, statistical decision theory, and a dual problem.

3. "Prophet inequalities for averages of independent non-negative random variables," Math. Z. 192, 427-436 (1986).

Abstract. If $X_1,\ldots,X_n$ are independent non-negative random variables, then $E\{\max_{j\leq n}(X_1+\ldots+X_j)/j\} \leq (2-n^{-1})\sup\{E[(X_1+\ldots+X_t/t]: t$ is a stop rule for $X_1,\ldots,X_n\}$, and this bound is best possible for all $n \geq 1$. This result sharpens results of Krengel and Sucheston, in which the weaker bound $2(1+\sqrt{3})$ was obtained, and yields the $k = 2$ prophet inequality for independent non-negative random variables.

4. "A generalization of Lyapounov's convexity theorem to measures with atoms," Proc. Amer. Math. Soc. 99, 297-304 (1987) (with J. Elton).

Abstract. The distance from the convex hull of the range of an n-dimensional vector-valued measure to the range of that measure is no more than $\alpha n/2$, where α is the largest (one-dimensional) mass of the atoms of the measure. The case $\alpha = 0$ yields Lyapounov's Convexity Theorem; applications are given to the bisection problem and to the bang-bang principle of optimal control theory.

5. "Monotonically improving limit-optimal strategies in finite-state decision processes," accepted for publication in Math. of O.R. (with J. Van der Wal).

Abstract. In every finite state leavable gambling problem and in every finite-state Markov decision process with discounted, negative or positive reward criteria there exists a Markov strategy which is monotonically improving and optimal in the limit along every history. An example is given to show that for the positive and gambling cases such strategies cannot be constructed by simply switching to a "better" action or gamble at each successive return to a state.

6. "Partitioning general probability measures," accepted for publication in Ann. Prob.

Abstract. Suppose $\mu_1,\ldots,\mu_n$ are probability measures on the same measurable space $(\Omega,\mathcal{F})$. Then if all atoms of each μ_i have mass α or less, there is a measurable partition $A_1,\ldots,A_n$ of Ω so that $\mu_i(A_i) \geq V_n(\alpha)$ for all $i = 1,\ldots,n$, where $V_n(\cdot)$: $[0,1] \to [0,n^{-1}]$ is the unique non-increasing function satisfying $V_n(\alpha) = 1-k(n-1)\alpha$ for $\alpha \in [(k+1)k^{-1}((k+1)n-1)^{-1},(kn-1)^{-1}]$ for all $k \geq 1$. Moreover, the bound $V_n(\alpha)$ is attained for all n and all α. Applications are given in L_1 spaces, to statistical decision theory, and to the classical non-atomic case.

7. "The existence of good Markov strategies in decision processes with general payoffs," accepted for publication in Stoch. Proc. Appl. (with V. Pestien).

Abstract. For countable-state decision processes (dynamic programming problems), a general class of objective functions is identified for which it is shown that good Markov strategies always exist. This class includes product and lim inf rewards, as well as practically all the classical dynamic programming expected payoff functions.

8. "Common hyperplane medians for random vectors," accepted for publication in Amer. Math. Monthly.

Abstract. Every collection of $m \leq n$ random n-dimensional vectors has a common hyperplane median, that is, there is a single hyperplane which simultaneously bisects each of the distributions in the sense that at least half the mass of each distribution lies on one side of the hyperplane (including the hyperplane) and at least half lies on the other side (again including the hyperplane). This result generalizes the well-known Ham Sandwich Theorem; the proof is based on first principles using an application of the Borsuk-Ulam Theorem to a "midpoint median" function.

9. "A sharp partitioning-inequality for non-atomic probability measures based on the mass of the infimum of the measures," accepted for publication in Prob. Theory (Z. Wahrscheinlichkeitsth.).

Abstract. If $\mu_1,\ldots,\mu_n$ are non-atomic probability measures on the same measurable space (S,F), then there is an F-measurable partition $\{A_i\}_{i=1}^n$ of S so that $\mu_i(A_i) \geq (n-1+m)^{-1}$, where $m = \|\bigwedge_{i=1}^{n} \mu_i\|$ is the total mass of the largest measure dominated by each of the μ_is; moreover, this bound is attained for all $n \geq 1$ and all m in $[0,1]$. This result is an analog of the bound $(n+1-M)^{-1}$ of Elton, Hill, and Kertz based on the mass M of the supremum of the measures; each gives a quantative generalization of a well-known cake cutting inequality of Urbanik and of Dubins and Spanier.

10. "Expectation inequalities associated with prophet problems," accepted for publication in Stoch. Anal. Appl.

Abstract. Applications of the original prophet inequalities of Krengel and Sucheston are made to problems of order selection, non-measurable stop rules, look-ahead stop rules, and iterated maps of random variables. Also, proofs are given of two results of Hill and Hordijk concerning optimal orderings of uniform and exponential distributions.

11. "Equitable distribution of indivisible objects," (with S. Demko), submitted for publication.

Abstract. Deterministic and randomized solutions are developed for the problem of equitably distributing m indivisible objects to n people (whose values may differ), without the use of outside judges or side-payments. Several general bounds for the minimal share are found; a practical method is given for determining an optimal lottery and the largest minimal share; and the case of repeated allocations is analyzed.

12. "Random-number guessing and the first digit phenomenon," submitted for publication.

Abstract. A widely-accepted empirical observation called the First Digit Phenomenon or Benford's Law says that in collections of random tables of data (such as physical constants, almanacs, newspaper articles, etc.), the first significant digit is more likely to be a low number than a high number. In this study, an analysis of the frequencies of the first and second digits of "random" six-digit numbers guessed by people suggests that people's responses share some of the properties of Benford's Law: first digit 1 occurs much more frequently than expected; first digit 8 or 9 occurs much less frequently; and the second digits are much more uniformly distributed than the first.

13. "Prophet inequalities for parallel processes," (with D. Kennedy), submitted for publication.

Abstract. Generalizations of prophet inequalities for single sequences are obtained for optimal stopping of several parallel sequences of independent random variables. For example, if $\{X_{i,j}\}_{j=1,j=1}^{n \quad \infty}$ are independent non-negative random variables, then $E(\sup_{i,j} X_{ij}) \leq (n+1)\max_i \sup\{E(X_{i,t}): t$ is a stop rule for $X_{i,1}, X_{i,2}, \ldots\}$, and this bound is best possible. Applications are made to comparisons of the optimal expected returns of various alternative methods of stopping of parallel processes.

14. "Minimax-risk inequalities in the classification problem," (with Y. Tong) in preparation.

Abstracts of papers by Robert P. Kertz which were partially supported by NSF Grant DMS 84-01604.

1. "Common strict character of some sharp infinite-sequence martingale inequalities," co-authored with David C. Cox (1985) Stochastic Processes and Their Applications, 20, 169-179.

Abstract. A procedure is given for proving strictness of some sharp, infinite-sequence martingale inequalities, which arise from sharp, finite-sequence martingale inequalities attained by degenerating extremal distributions. The procedure is applied to obtain strictness of the sharp inequalities of Cox and Kemperman $P(|X_i| \geq 1 \text{ for some } i = 1,2,\ldots) \leq (\ln 2)^{-1} \sup_n E|\sum_{i=0}^{n} X_i|$ and of Cox (sharp form of Burkholder's inequality) $P(\sum_{i=0}^{\infty} X_i^2 \geq 1) \leq e^{\frac{1}{2}} \sup_n E|\sum_{i=0}^{n} X_i|$ for all nontrivial martingale difference sequences $X_0, X_1, \ldots$.

2. "Prophet regions and sharp inequalities for pth absolute moments of martingales," co-authored with David C. Cox (1986) Journal of Multivariate Analysis, 18, 242-273.

Abstract. Exact comparisons are made relating $E|Y_0|^p$, $E|Y_{n-1}|^p$, and $E(\max_{j\leq n-1}|Y_j|^p)$, valid for all martingales $Y_0,\ldots,Y_{n-1}$, for each $p \geq 1$. Specifically, for $p > 1$, the set of ordered triples $\{(x,y,z): x = E|Y_0|^p$, $y = E|Y_{n-1}|^p$, and $z = E(\max_{j\leq n-1}|Y_j|^p)$ for some martingale $Y_0,\ldots,Y_{n-1}\}$ is precisely the set $\{(x,y,z): 0 \leq x \leq y \leq z \leq \Psi_{n,p}(x,y)\}$, where $\Psi_{n,p}(x,y) = x\psi_{n,p}(y/x)$ if $x > 0$, and $= a_{n-1,p}y$ if $x = 0$; here $\psi_{n,p}$ is a specific recursively defined function. The result yields families of sharp inequalities, such as $E(\max_{j\leq n-1}|Y_j|^p) + \psi^*_{n,p}(a)E|Y_0|^p \leq aE|Y_{n-1}|^p$, valid for all martingales $Y_0,\ldots,Y_{n-1}$, where $\psi^*_{n,p}$ is the concave conjugate function of $\psi_{n,p}$. Both the finite sequence and infinite sequence cases are developed. Proofs utilize moment theory, induction, conjugate function theory, and functional equation analysis.

3. "Comparison of optimal value and constrained maxima expectations for independent random variables," (1986) Advances in Applied Probability, 18, 311-340.

Abstract. For all uniformly bounded sequences of independent random variables $X_1,X_2,\ldots$, a complete comparison is made between the optimal value $V(X_1,X_2,\ldots) = \sup\{EX_t: t \text{ is an (a.e.) finite stop rule for } X_1,X_2,\ldots\}$ and $E(k^{-1}\Sigma_{i=1}^{k} M_i(X_1,X_2,\ldots))$, where $M_i(X_1,X_2,\ldots)$ is the ith largest order statistic for $X_1,X_2,\ldots$. In particular, for $k > 1$, the set of ordered pairs $\{(x,y): x = V(X_1,X_2,\ldots)$ and $y = E(k^{-1}\Sigma_{i=1}^{k} M_i(X_1,X_2,\ldots))$ for some independent

random variables $X_1, X_2, \ldots$ taking values in [0,1]} is precisely the set $\{(x,y)\colon k^{-1}x \leqq y < B_k(x);\ 0 < x \leqq 1\} \cup \{(0,0),(1,1)\}$, where $B_k(0) = 0$, $B_k(1) = 1$, and for $0 < x < 1$, $B_k(x) = 1 - k^{-1}\Sigma_{i=0}^{k-1} \Sigma_{j=0}^{i} (1-x)^{2^k}(-\ell n(1-x))^j/j!$. The result yields sharp, universal inequalities for independent random variables comparing two choice mechanisms, the mortal's value of the game $V(X_1, X_2, \ldots)$ and the prophet's constrained maxima expectation of the game $E(k^{-1}\Sigma_{i=1}^{k} M_i(X_1, X_2, \ldots))$. Techniques of proof include probability- and convexity-based reductions; calculus-based, multivariate, extremal problem analysis; and limit theorems of Poisson-approximation type. Precise results are also given for finite sequences of independent random variables.

4. "Optimal-partitioning inequalities for non-atomic probability measures." co-authored with J. Elton and T. P. Hill (1986) Transactions of the American Mathematical Society, 296, 703-725.

Abstract. Suppose $\mu_1, \ldots, \mu_n$ are nonatomic probability measures on the same measurable space $(S, \mathcal{B})$. Then there exists a measurable partition $\{S_i\}_{i=1}^n$ of S such that $\mu_i(S_i) \geq (n+1-M)^{-1}$ for all $i = 1, \ldots, n$, where M is the total mass of $\vee_{i=1}^n \mu_i$ (the smallest measure majorizing each μ_i). This inequality is the best possible for the functional M, and sharpens and quantifies a well-known cake-cutting theorem of Urbanik and of Dubins and Spanier. Applications are made to L_1-functions, discrete allocation problems, statistical decision theory, and a dual problem.

5. "Comparison of stop rule and maximum expectations for finite sequences of exchangeable random variables," co-authored with J. Elton (1986). Submitted for publication.

Abstract. For sequences of exchangeable random variables $X_1, \ldots, X_n$ taking values in [0,1], results are given which compare $E(\max_{1 \leq i \leq n} X_i)$ and $V(X_1, \ldots, X_n) = \sup\{EX_t\colon t \text{ is a stop rule for } X_1, \ldots, X_n\}$. Complete results are given for the $n = 2$ and $n = \infty$ cases; results are also given for the $2 < n < \infty$ case. A constructive approach is taken, with extremal distributions given where applicable. For the $n = 2$ case, conjugate function theory is used in an essential way. The transformations and constructions of finite sequences of exchangeable random variables which are given should be of independent interest to those working with this class of processes.

6. "Decision processes under total expected concomitant constraints, with applications to bandit processes," (1986). Submitted for publication.

Abstract. A problem of optimization under constraints is stated for general nonstationary stochastic decision models, and studied using convexity theory. For constrained multi-armed bandit processes with geometric

discounting, optimal plans are characterized through dynamic allocation indices, and value functions are expressed in terms of single-arm value functions. Techniques of proof include randomizations, conjugate duality, and characterizations for the unconstrained problem. Explicit closed-form representations are given for some Bernoulli bandit processes.

7. "Leaving an interval in limited playing time," co-authored with David Heath (1986). Submitted for publication.

Abstract. A player starts at x in (-G,G) and attempts to leave the interval in a limited playing time. In the discrete time problem, G is a positive integer and the position is described by a random walk starting at integer x, with mean increments zero, and variance increment chosen by the player from [0,1] at each integer playing time. In the continuous time problem, the player's position is described by an Ito diffusion process with infinitesimal mean parameter zero and infinitesimal diffusion parameter chosen by the player from [0,1] at each time instant of play. To maximize the probability of leaving the interval (-G,G) in a limited playing time, the player should play boldly by always choosing largest possible variance increment in the discrete-time setting and largest possible diffusion parameter in the continuous-time setting, until the player leaves the interval. In the discrete time setting, this result affirms a conjecture of Spencer. In the continuous time setting, the value function of play is also identified.

8. "Prophet problems in optimal stopping: results, techniques, and variations," (1986) Technical Report, School of Mathematics, Georgia Institute of Technology, Atlanta.

Abstract. This paper presents background, statements, techniques, and variations of prophet problems in optimal stopping. The main formulation of the prophet problem and equivalent formulations are given in sections one and two. Techniques of proof are given in sections three and four; these include reduction procedures, use of verification lemmas, use of moment theory, and other techniques. Variations of the prophet problem in optimal stopping are given in section five. Prophet-type stochastic control problems for transforms of processes are discussed in section six. Examples of prophet inequalities and prophet regions for many classes of processes are given. Proofs in section 5b and in the appendix illustrate the techniques used to prove these results.

9. "Simulated annealing, diffusions, and large deviations for nonlinear programming," with Craig Tovey (1987), in preparation.

10. "Measures of perfect information in Bernoulli bandit processes," with Donald Berry (1987), in preparation.

11. "Comparisons between multi-choice gamblers and prophets, with application to optimal job assignment procedures," with Douglas P. Kennedy (1987), in preparation.

12. "Limit theorems for threshold-stopped random variables," with Douglas P. Kennedy (1987), in preparation.

Talks were presented at the following meetings, conferences, and universities by T. P. Hill.

1.	Amer. Math. Society Annual Meeting, Anaheim	Jan 1985
2.	Mathematics Dept., University of Miami	Apr 1986
3.	Mathematics and Statistics Departments, Ohio State	Apr 1986
4.	Conference on optimal stopping and gambling theory, Oberwolfach	Jun 1986
5.	Mathematical Stochastics Department, Univ. of Göttingen	Jun 1986
6.	Applied Mathematics Department, Univ. of Bonn	Jun 1986
7.	International Congress of Mathematicians, Berkeley	Aug 1986
8.	Mathematics Dept., U.S. Military Academy	Oct 1986

Talks were presented at the following meetings, conference, and universities by Robert P. Kertz.

1.	Amer. Math. Society 91st Annual Meeting, Anaheim	Jan 1985
2.	Southeastern Meeting on Diff. Eqns., Atlanta	Oct 1985
3.	ORSA/TIMS 20th Joint National Meeting, Atlanta	Nov 1985
4.	Inst. of Math. Statist. 195th Meeting, Atlanta	Mar 1986
5.	Conference on Optimal Stopping and Gambling, Oberwolfach	Jun 1986
6.	Workshop on Probabilistic Problems Concerning Strategies, Göttingen	Jun 1986
7.	Statistical Laboratory, Cambridge Univ., Cambridge	Jul 1986

The following professors were brought to Georgia Tech for research consultations with T. P. Hill and R. P. Kertz with partial assistance from NSF Grant DMS 84-01604.

1. Prof. Dr. Ulrich Krengel, Director, Institu für Mathematische Stochastik, Georg-August-Universität, Göttingen (August 1985).

2. Prof. Donald Geman, Department of Mathematics and Statistics, University of Massachusetts, Amherst (March 1986).

3. Prof. Douglas Kennedy, Statistical Laboratory and Trinity College, University of Cambridge (September 1986).