

CASCADE SYNTHESIS OF THREE-TERMINAL
RC TRANSFER FUNCTIONS WITH
RIGHT-HALF-PLANE TRANSMISSION ZEROS

A THESIS

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Theodore Dean Lindgren

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Approved: _____

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SUMMARY

In this investigation, a cascade synthesis procedure is developed to realize RC transfer functions with transmission zeros in part of the right half of the complex-frequency plane. The procedure is similar to the Dasher procedure for realizing transmission zeros in the left half of the complex-frequency plane and is to be used in conjunction with that procedure. A brief description of the procedure is contained in the following paragraph:

Given an RC immittance function of suitable degree, $Y_1(s)$ or $Z_1(s)$, the immittance is developed by removing a three-terminal network which produces a pair of conjugate transmission zeros in the region of the complex-frequency plane given by

$$60^\circ < |\arg s| < 90^\circ ,$$

and which is terminated in an RC immittance function, $Y_2(s)$ or $Z_2(s)$, whose degree is less than $Y_1(s)$ or $Z_1(s)$.

The cascade procedure may then be repeated using $Y_2(s)$ or $Z_2(s)$. Several repetitions of this procedure result in a three-terminal network made up of cascaded networks, each of which produces a transmission zero or pair of conjugate transmission zeros and is called a "coupling circuit." The procedure may be used to realize the transfer functions E_2/I_1 , E_2/E_1 , I_2/I_1 and I_2/E_1 . For these cases, a driving-point immittance function, $Y_1(s)$ or $Z_1(s)$, is derived which is compatible with the given transfer function. The immittance function is then developed using the cascade

procedure to realize the transmission zeros of the transfer function.

The investigation is divided into four interrelated areas.

These are:

1. Determination of the effect of surplus factors.
2. Determination of a set of coupling circuits for producing right-half-plane transmission zeros.
3. Determination of a set of preparatory-step conditions which will result in an RC termination.
4. Determination of a procedure for choosing the most suitable coupling circuit for use with a given driving-point immittance and a pair of transmission zeros.

The procedure for realizing right-half-plane zeros is shown to be more difficult than the Dasher procedure for producing left-half-plane zeros. The increased complexity is caused by the necessity of the use of augmenting factors which result in non-negative numerator coefficients of the numerator of the transfer function before cancellation of any common factors. The augmenting factors are restricted to specific ranges of values depending on the location of the transmission zero. The factors cause additional preparatory-step conditions and restrict the location of the compact poles of the coupling circuits. However, the factors are shown to cancel in the overall transfer function and therefore are not transmission zeros. Furthermore, it is shown that the augmenting factors need not be present in the driving-point immittance. The augmenting factors are referred to as surplus factors, because of the similarity to surplus factors used in other synthesis procedures.

The procedure developed during the investigation is confined to

the one-surplus factor case, which limits the location of the transmission zeros to the first 30° -sector of the right half of the complex-frequency plane. Many of the results of the investigation are shown to apply to the use of multiple surplus factors and therefore to an extended region of the right-half plane.

Eight coupling circuits are listed for use in the new cascade synthesis procedure. The list of circuits is not necessarily complete. Each circuit is described in terms of the allowable pole locations of the short-circuit admittance functions, y_{11} , $-y_{12}$, and y_{22} or of the open-circuit impedance functions, z_{11} , z_{12} , and z_{22} . The pole locations are described as functions of the surplus-factor zero, which is restricted to a specific range of values. The eight circuits are shown to cover a wide range of possible pole locations. The circuits are z-compact and y-compact.

As in the Dasher synthesis procedure, the given driving-point immittance, $Y_1(s)$ or $Z_1(s)$, must be prepared for removal of the coupling circuit by removing series or shunt elements. It is shown that in certain cases the new procedure requires removal of both shunt and series elements. The preparatory step is shown to place requirements on the function $Y_1 - y_{11}$ and its derivative at both the transmission zero and at the surplus-factor zero. The derivation of the preparatory-step conditions shows that satisfaction of the preparatory-step conditions will not only result in an RC termination, but will also cause cancellation of the surplus factor in the overall transfer functions of circuit.

A method is derived for choosing the most suitable coupling circuit to use for a given synthesis problem. The circuit is chosen during

the first step of the procedure. It is shown that the circuit may be chosen on the basis of the relative pole-locations of Y_1 and y_{11} , or the relative pole-locations of Z_1 and z_{11} .

A discussion of the compactness property of the coupling circuits is included. A simple procedure that can be used to determine whether a given set of z - or y - functions are y - and z - compact is developed. Use of non-compact circuits for the cascade synthesis procedure is discussed and shown to be feasible.

A special technique is demonstrated for the use of the procedure in realizing a given transfer function with a specific termination.

CHAPTER I

INTRODUCTION

Statement of the Problem

The cascade synthesis problem discussed in this thesis is an intended extension of the Dasher¹ method of cascade synthesis of RC networks to include transmission zeros in part of the right half of the complex-frequency plane. The problem differs from the Dasher problem in certain aspects because of a surplus factor requirement. The problem is stated in the following paragraph:

Given an RC immittance function of suitable degree, $Y_1(s)$ or $Z_1(s)$, develop that immittance by removing a three-terminal network which (a) produces a pair of conjugate transmission zeros in the region of the s -plane given by

$$60^\circ < |\arg s| < 90^\circ \quad (1)$$

and (b) is terminated in an RC immittance function, $Y_2(s)$ or $Z_2(s)$, the degree of which is less than $Y_1(s)$ or $Z_1(s)$.

Although the procedure developed in solution to the problem may be used to produce left-half-plane transmission zeros as an alternative to the Dasher method of synthesis, it offers no particular advantage. Therefore, the discussion in this thesis concentrates on the development of the procedure for producing right-half-plane transmission zeros.

The procedure to be developed in this thesis may be repeated using $Y_2(s)$ or $Z_2(s)$. Several repetitions of the procedure result in a three

terminal network made up of cascaded networks, each of which produces a pair of conjugate transmission zeros and is called a "coupling circuit." The process may be used in conjunction with other cascade synthesis methods. For example, negative real transmission zeros may be produced by an extension of the Cauer ladder development and conjugate pairs of left-half-plane transmission zeros may be produced using the Dasher procedure.

The procedures may be used to realize transfer functions, such as E_2/I_1 , E_2/E_1 , I_2/I_1 and I_2/E_1 . First, a driving-point function must be derived which is compatible with the given transfer function. The driving-point function may be either an impedance, Z_1 , or an admittance, Y_1 . Second, the driving-point immittance is developed using cascade synthesis methods to produce transmission zeros or pairs of transmission zeros of the given transfer function in the step-by-step manner described previously.

Origin and History of the Problem

The study of two-terminal-pair RC transfer functions is of prime interest in servomechanism and other low-frequency applications of electrical networks. The results of the study of RC network synthesis may be applied to LC and RL network synthesis.

Existing methods for RC synthesis of right-half-plane transmission zeros using three-terminal networks are the Guillemin parallel-ladder procedure² and the Fialkow-Gerst polynomial-partitioning procedure³. The Guillemin procedure realizes only Y_{12} transfer functions or voltage transfer functions. The Fialkow-Gerst procedure may be used to produce Z_{12} , Y_{12} , or voltage transfer functions. Both methods require an excessively large number of elements. No single element or group of elements in

either procedure controls the location of each transmission zero. Instead, each element affects the performance of the network at all frequencies and therefore no independent adjustments are possible in the physical network.

The Dasher procedure overcomes all of the disadvantages listed in the preceding paragraph when compared to those procedures as they are used to produce left-half-plane transmission zeros. In addition, the Dasher procedure may yield a larger gain constant than the Guillemin procedure. The primary disadvantage of the Dasher procedure is the difficult preparatory step, which often results in a small gain constant. Seshu and Hakimi⁴ studied the problems of simplifying the preparatory step and extending the procedure to include right-half-plane transmission zeros. They were not successful in simplifying the preparatory step. However, they did conclude that the procedure could be extended to the right-half-plane and they suggested several circuits which might be used in the extension.

Meadows⁵ observed that surplus factors could be made to cancel from the overall transfer function for cascade synthesis.

Properties of RC Functions

For subsequent reference, a brief discussion of certain properties of RC functions is given here.

The Foster expansion of an RC driving-point admittance may be written as

$$Y_1(s) = k_{\infty} s + k_0 + \sum_{v=1}^n \frac{k_v s}{s_v + \sigma_v} \quad (2)$$

In this expression the k 's must be real and positive and the σ 's must be real and positive. The expression may be obtained by expanding Y_1/s in partial fractions and multiplying the result by s . The k_v 's represent the residues of the corresponding poles. The values of k_∞ and k_0 may be zero. The poles and zeros of an RC driving point admittance alternate along the negative real axis of the s -plane. The first critical point (zero or pole) starting from the origin is a zero. The origin may or may not be a zero. The last critical point is a pole. A typical graph of an RC admittance function along the negative real axis is indicated in Figure 1.

The Foster expansion for an RC impedance function may be written as

$$Z_1(s) = k_\infty + \frac{k_0}{s} + \sum_{v=1}^n \frac{k_v}{s + \sigma_v} \quad (3)$$

As for the admittance function, the k 's and the σ 's must be real and positive. The k_v 's represent the residues of corresponding poles and the values of k_∞ and k_0 may be zero. The poles and zeros also alternate along the negative real axis. The first critical point starting from the origin is a pole. The origin may or may not be a pole. The last critical frequency is a zero. A typical plot of an RC impedance function is indicated in Figure 2.

RC transfer functions, $Y_{12}(s)$ and $Z_{12}(s)$, for three-terminal networks have the following properties:

1. Transmission zeros may lie anywhere in the complex-frequency plane except along the positive real axis.

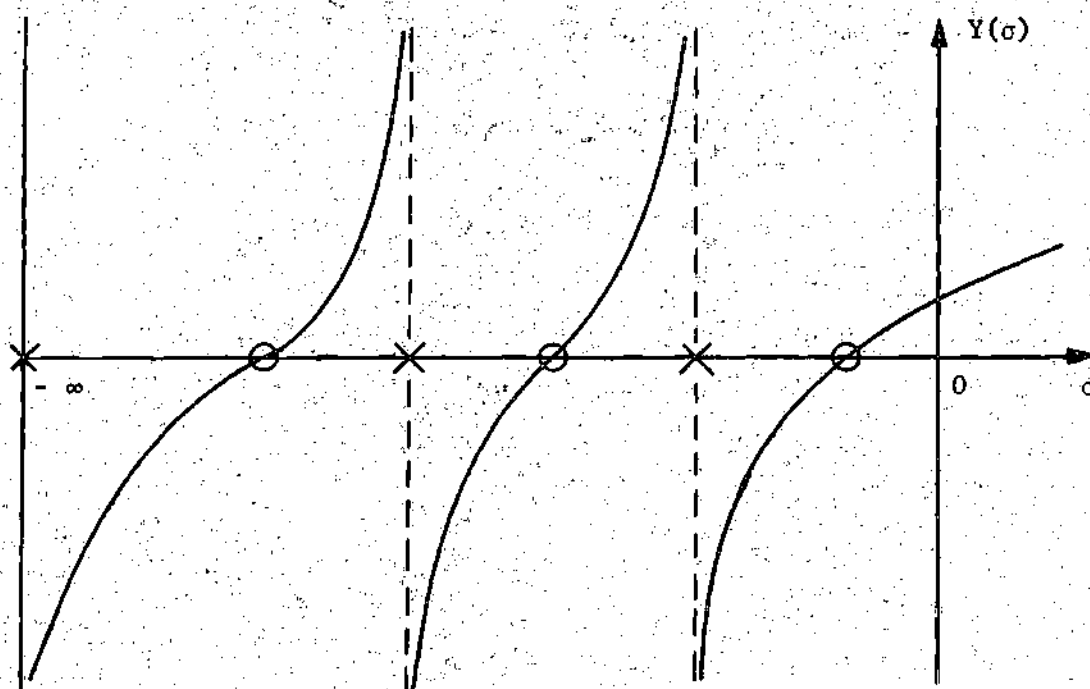


Figure 1. Typical RC Admittance Function.

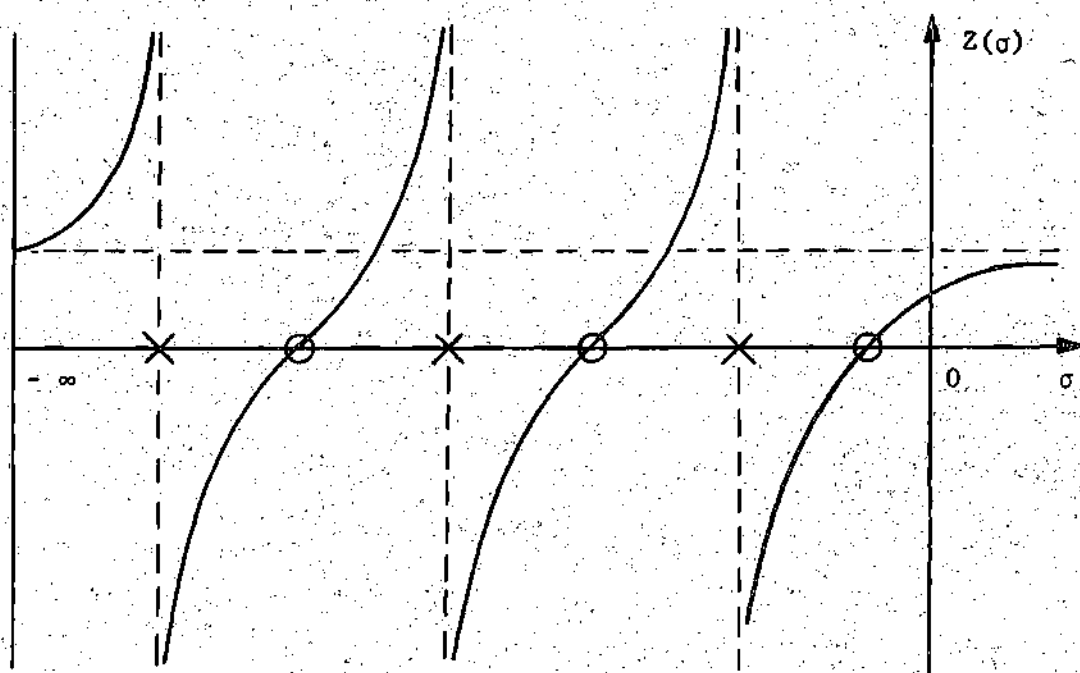


Figure 2. Typical RC Impedance Function.

2. Poles of Z_{12} must also be poles of Z_1 , the associated driving point impedance. Poles of Y_{12} must also be poles of Y_1 , the associated driving-point admittance.

3. The transfer functions, when related to a given circuit, must have non-negative numerator coefficients before cancellation of factors common to both numerator and denominator. This property is of particular importance for right-half-plane transmission zeros.

Notation

The notation used throughout this thesis is given by the following scheme:

1. Lower-case letters refer to quantities related directly to the coupling circuits. Lower-case letters having double subscripts always refer to the coupling-circuit parameters z_{11} , z_{12} , z_{22} , y_{11} , y_{12} , and y_{22} .
2. Capital letters refer to the over-all network impedance or admittance functions.
3. Transmission zeros are defined as $s_o = \alpha_o + j\beta_o = \omega_o \angle \theta$ and $\bar{s}_o = \alpha_o - j\beta_o = \omega_o \angle -\theta$ where $|s_o| = \omega_o$. For left-half-plane transmission zeros, α_o represents a negative number; for right-half-plane transmission zeros, α_o represents a positive number.
4. The current and voltage reference directions and the pi and tee representations of the coupling circuits are indicated in Figure 3.

Example of the Dasher Procedure

The sequence of steps in the Dasher procedure, assuming that an RC admittance Y_1 is given and that one of the desired conjugate transmission zeros is at $s_o = \alpha_o + j\beta_o$, where α_o represents a negative number, is as follows:

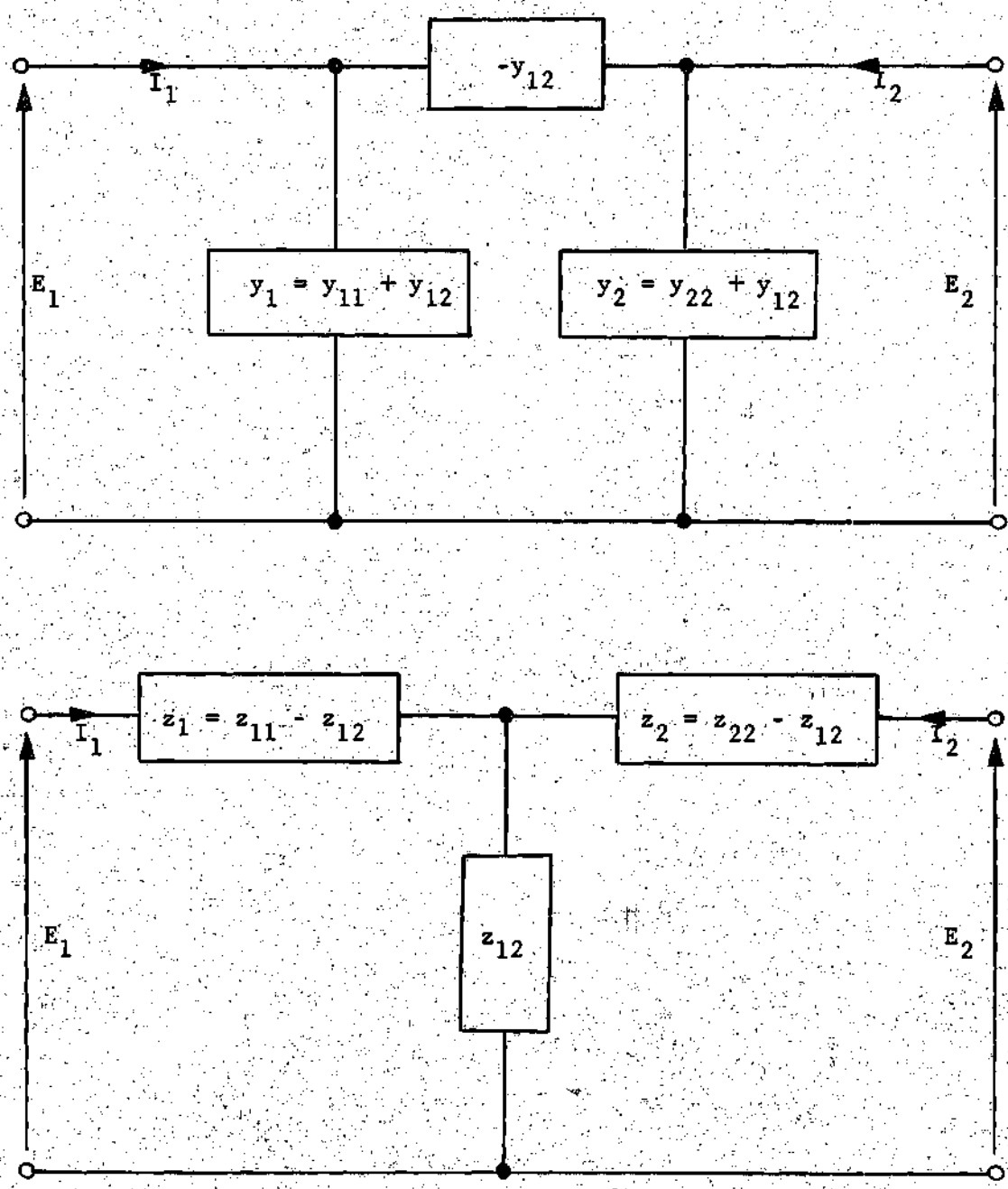


Figure 3. Pi and Tee Representations of Coupling Circuits.

Step 1. Remove an admittance, Y_p , which is part of Y_1 such that the remainder, Y_{1p} , will not only be RC, but will satisfy the requirements:

$$Y_{1p}(s_o) = y_{11}(s_o) \quad (4)$$

$$Y_{1p}'(s_o) = y_{11}'(s_o), \quad (5)$$

where the prime indicates differentiation with respect to s . Satisfaction of the requirements assures that the terminating function, $Y_2(s)$, will be RC.

Step 2. Remove a second admittance y_1 , where

$$y_1 = y_{11} + y_{12}, \quad (6)$$

to satisfy the relation, $Y_{1p}(s_o) = y_1(s_o)$.

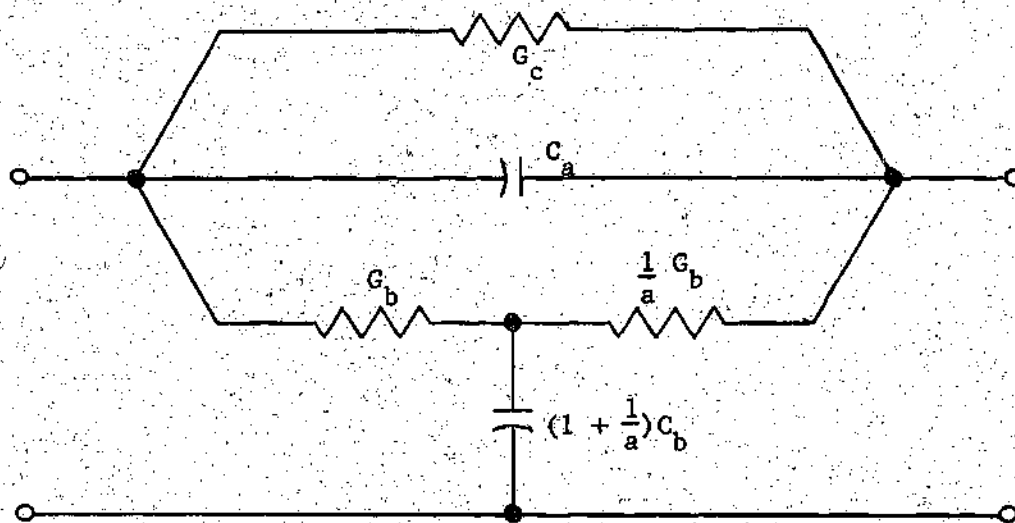
Step 3. The remainder in Step 2, Y_a , has a pair of conjugate zeros at $s = \alpha_o \pm j\beta_o$. Y_a is inverted and $1/-y_{12}$ is removed. The constant multiplier of $1/-y_{12}$ is chosen to remove the pair of conjugate poles from $1/Y_a$.

Step 4. The reciprocal of the impedance remaining in Step 3, Z_p , has the same pole on the negative-real axis as y_1 . This pole is removed as an admittance y_2 where

$$y_2 = y_{22} + y_{12}, \quad (7)$$

to complete the cycle. The final remainder, Y_2 , is an RC admittance.

Step 5. The three admittances, y_1 , y_2 , and $-y_{12}$ are completely specified by the preceding steps and the associated coupling circuit is chosen from one of the three circuits of Figures 4, 5, and 6.



$$C_a = k_y$$

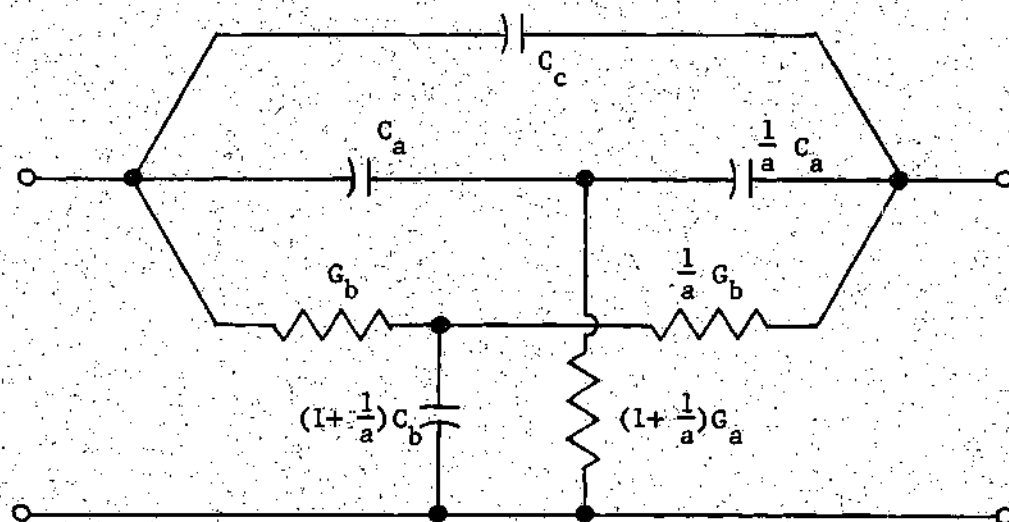
$$C_b = \frac{k_y (1+a) (\omega_o^2 + 2\alpha_o \sigma_1 + \sigma_1^2)}{\sigma_1^2}$$

$$G_b = \sigma_1 C_b$$

$$G_c = k_y (-2\alpha_o - \sigma_1)$$

$$\text{Restriction: } -2\alpha_o - \sigma_1 \geq 0$$

Figure 4. Dasher Coupling Circuit A.



$$C_a = k_y (1+a) \frac{(\sigma_1 + 2\alpha_o)}{\sigma_1}$$

$$C_b = k_y (1+a) \frac{\omega_o^2}{2\sigma_1}$$

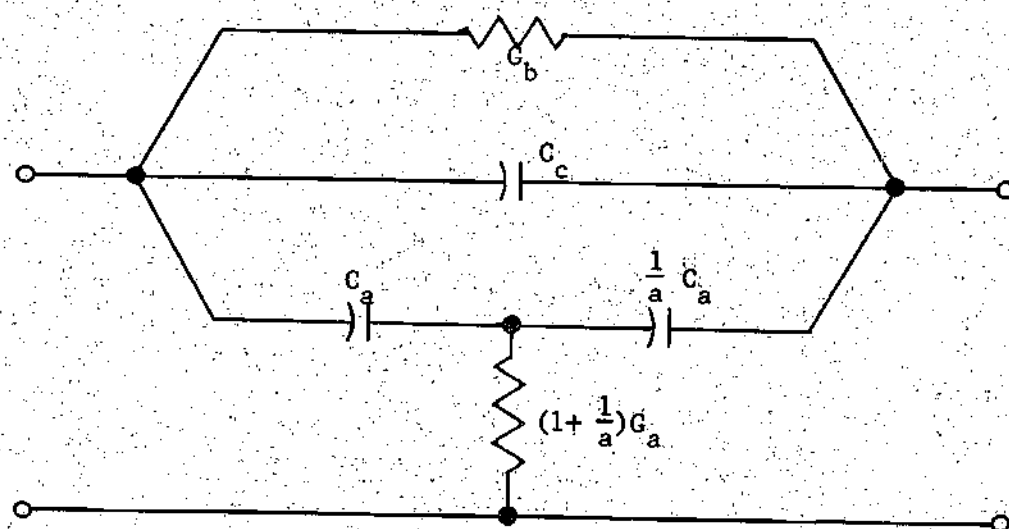
$$C_c = k_y \frac{-2\alpha_o}{\sigma_1}$$

$$G_a = \sigma_1 C_a$$

$$G_b = \sigma_1 C_b$$

$$\text{Restriction: } \sigma_1 + 2\alpha_o \geq 0$$

Figure 5. Dasher Coupling Circuit B.



$$C_a = \frac{k_y (1+a) (\omega_o^2 + 2\alpha_o \sigma_1 + \sigma_1^2)}{\sigma_1^2}$$

$$C_c = k_y \left(-\frac{2\alpha_o}{\sigma_1} - \frac{\omega_o^2}{\sigma_1^2} \right)$$

$$G_a = \sigma_1 C_a$$

$$G_b = k_y \frac{\omega_o^2}{\sigma_1^2}$$

$$\text{Restriction: } \sigma_1 + \frac{\omega_o^2}{2\alpha_o} \geq 0$$

Figure 6. Dasher Coupling Circuit C.

The steps are illustrated in Figure 7.

The y-parameters for the Dasher coupling circuits are given by the following equations:

$$-y_{12} = k_y \frac{s^2 - 2\alpha_o s + \omega_o^2}{s + \sigma_1} \quad (8)$$

or

$$-y_{12} = k_y \left[s + \frac{\omega_o^2}{\sigma_1} + (\sigma_1^2 + 2\alpha_o \sigma_1 + \omega_o^2) \frac{s}{\sigma_1(s + \sigma_1)} \right] \quad (9)$$

$$y_{11} = k_y \left[s + \frac{\omega_o^2}{\sigma_1} + a(\sigma_1^2 + 2\alpha_o \sigma_1 + \omega_o^2) \frac{s}{\sigma_1(s + \sigma_1)} \right] \quad (10)$$

$$y_{22} = k_y \left[s + \frac{\omega_o^2}{\sigma_1} + \frac{1}{2}(\sigma_1^2 + 2\alpha_o \sigma_1 + \omega_o^2) \frac{s}{\sigma_1(s + \sigma_1)} \right] \quad (11)$$

The conditions of Step 1 are equivalent to satisfying the equation

$$\frac{g}{b} = \frac{1 - \frac{\beta_o}{\alpha_o} \frac{b - g'\beta_o}{b'\beta_o}}{\frac{b - g'\beta_o}{b'\beta_o} + \frac{\beta_o}{\alpha_o}} \quad (12)$$

where

$$Y_{1p}(s_o) = g + jb \quad (13)$$

$$Y_{1p}'(s_o) = g' + jb' \quad (14)$$

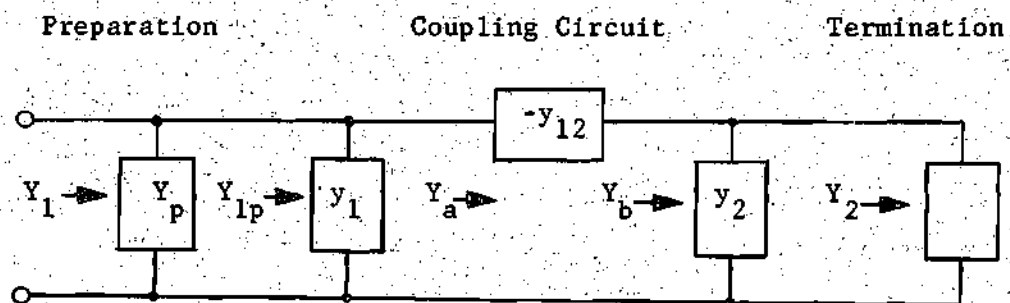


Figure 7. One Cycle of the Dasher Procedure.

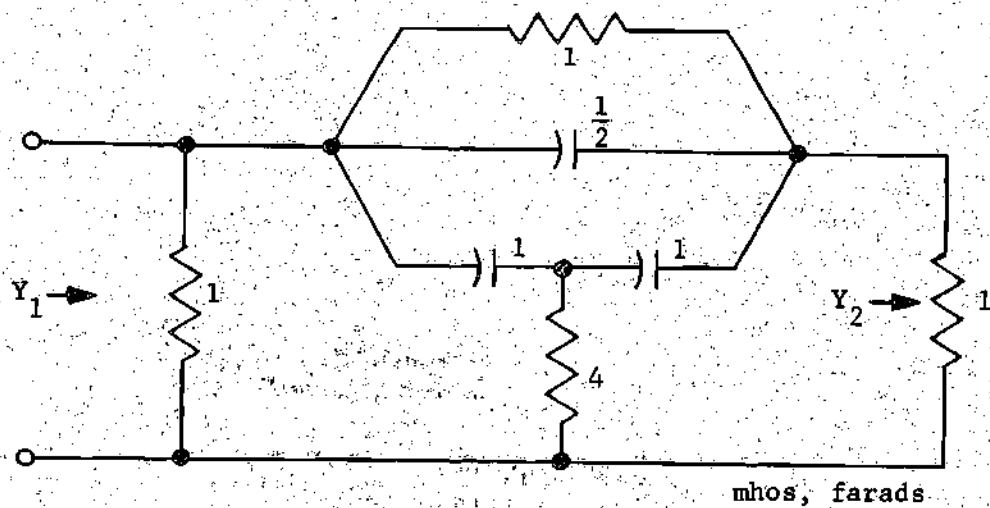


Figure 8. Example of the Dasher Procedure

Dasher also developed (12) in another form involving summation of the residues of the poles of Y_{lp} . If the conditional equation cannot be satisfied by removing a shunt conductance or shunt capacitance or partially removing a pole from Y_1 , then the equation is satisfied by removing series elements. Y_{lp} is replaced in (13) and (14) by $s/Y_{lp} = sZ_{lp}$ which also has the form of an RC admittance. The Dasher procedure thus allows one to work with admittance functions only.

For example, given a particular driving point admittance,

$$Y_1 = \frac{(3s + 2)(2s + 3)}{(s + 1)(s + 4)}, \quad (15)$$

it is desired to realize a pair of transmission zeros at $s = -1 \pm j1$.

Equation (12) may be used to satisfy the conditions of Step 1 of the procedure while removing a shunt conductance, G_p . The example is chosen for its simplicity. One should expect that, in general, Y_p will be more complex than indicated here. Equations (13) and (14) become

$$Y_{lp}(s_o) = g + jb = 1 - G_p + j2 \quad (16)$$

$$Y_{lp}'(s_o) = g' + jb' = 1 - j2. \quad (17)$$

Substituting these values in (12) along with

$$\alpha_o = -1 \quad (18)$$

and

$$\beta_o = 1, \quad (19)$$

one obtains the equation

$$\frac{1 - G_p}{2} = \frac{1 + \frac{2 - 1}{-1}}{\frac{2 - 1}{-1} - 1} \quad (20)$$

The equation is satisfied and Step 1 is performed by the removal of a shunt conductance of value

$$G_p = 1, \quad (21)$$

leaving the remainder RC function

$$y_{lp} = \frac{5s^2 + 8s + 2}{s^2 + 5s + 4} \quad (22)$$

For Step 2, the substitution $s = -1 + j1$ is made in (22) and in the equation for y_1 . Equating the two results, one obtains

$$j2 = k_y(1 + a)\left[\frac{2 - \sigma_1}{\sigma_1} + j1\right] \quad (23)$$

Equating the real and imaginary parts of (23) yields

$$\sigma_1 = 2 \quad (24)$$

and

$$k_y(1 + a) = 2. \quad (25)$$

Therefore

$$y_1 = \frac{2s}{s + 2} \quad (26)$$

Now y_1 is removed from Y_{lp} , obtaining

$$Y_a = Y_{lp} - y_1 = \frac{(3s + 2)(s^2 + 2s + 2)}{(s + 2)(s^2 + 5s + 4)} \quad (27)$$

Because σ_1 is known, $-y_{12}$ is known except for k_y . When Step 3 is performed, the value of k_y will be chosen to remove the conjugate pair of poles from $1/Y_a$.

$$Z_b = \frac{1}{Y_a} - \frac{1}{-y_{12}} = \frac{(s + 2)(s^2 + 5s + 4)}{(3s + 2)(s^2 + 2s + 2)} - \frac{s + 2}{k_y(s^2 + 2s + 2)} \quad (28)$$

Setting $s = -1 + j1$ in (28) and equating to zero yield

$$k_y = 1 \quad (29)$$

Completing the operation indicated in (28) using the value for k_y obtained in (29) gives

$$Z_b = \frac{s + 2}{3s + 2} \quad (30)$$

Substituting (29) in (25) yields

$$a = 1 \quad (31)$$

Therefore

$$y_2 = \frac{2s}{s + 2} \quad (32)$$

and the remainder calculated in Step 4 is

$$Z_2 = \frac{1}{Z_b} - y_2 = \frac{3s + 2}{s + 2} - \frac{2s}{s + 2} = 1 \quad (33)$$

Circuit C, Figure 6, is chosen for use in the circuit. The complete circuit is indicated in Figure 8.

Introduction to the New Procedure

Extension of the cascade synthesis procedure to include right-half-plane transmission zeros may be divided into four inter-related areas for discussion. These are:

1. Determination of the effect of surplus factors.
2. Determination of a set of coupling circuits for producing right-half-plane transmission zeros.
3. Determination of a set of preparatory conditions which will ensure RC termination.
4. Determination of the most suitable coupling circuit for use with a given driving-point immittance and pair of transmission zeros.

The preceding topics are discussed briefly and, where possible, the results are compared to the Dasher procedure here.

Surplus Factors

The numerator coefficients of the transfer function, when calculated for any three-terminal network with no mutual coupling, must be non-negative. This fact was first established by Fialkow and Gerst.⁶

(If the circuit is constructed in a manner such that a common factor is present in both the numerator and the denominator of the transfer function, the negative coefficients may appear.) Thus, for a pair of right-half-plane transmission zeros at $s = \alpha_0 \pm j\beta_0$, at least one surplus factor $(s + \sigma_0)$ is required, and the expression for $-y_{12}$ becomes

$$-y_{12} = \frac{(s + \sigma_0)(s^2 - 2\alpha_0 s + \omega_0^2)}{(s + \sigma_1)(s + \sigma_2)} \quad (34)$$

or

$$-y_{12} = \frac{s^3 + (\sigma_0 - 2\alpha_0)s^2 + (\omega_0^2 - 2\alpha_0\sigma_0)s + \sigma_0\omega_0^2}{(s + \sigma_1)(s + \sigma_2)} \quad (35)$$

For non-negative numerator coefficients in (35), the range of the surplus factor is limited to

$$2\alpha_0 \leq \sigma_0 \leq \frac{\omega_0^2}{2\alpha_0} \quad (36)$$

Multiplying condition (36) by $2\alpha_0$, it is apparent that

$$\omega_0 \geq 2\alpha_0, \quad (37)$$

and that one surplus factor will be sufficient only if the right-half-plane transmission zeros are limited to the region of the s -plane given by

$$60^\circ \leq |\arg s| \leq 90^\circ. \quad (38)$$

The solution to the cascade synthesis problem discussed in this thesis is limited to transmission zeros in the region indicated by (38). The problems involved in the extension of the procedure for use with two or more surplus factors, and therefore use with transmission zeros located in an extended region of the right-half-plane, will be discussed in Chapter V.

Although the surplus factor $(s + \sigma_0)$ is a transmission zero of the coupling-circuit transfer function, $-y_{12}$, the discussion of Chapter

III will show that it is not a transmission zero of the overall transfer function, Y_{12} , nor does it appear in the driving-point admittance, Y_1 . In fact, it will be shown that the factor, $(s + \sigma_0)$, augments both the numerator and the denominator of both Y_{12} and Y_1 , and therefore is a surplus factor according to the conventional definition.

Although the discussion of this section has used admittance functions, the same arguments hold for the synthesis procedure if impedance functions are used.

Coupling Circuits

As in the Dasher procedure, the coupling circuits used in the new procedure are both z - and y - compact. That is, the residue condition $k_{11}k_{22} - k_{12}^2 \geq 0$ at each pole is satisfied with an equality sign for both the z - and y - functions.

In the Dasher procedure, the three listed coupling circuits will realize any transmission zero in the left-half plane using any given driving-point immittance function of sufficient degree.

For synthesis of right-half-plane transmission zeros, eight coupling circuits are listed in Chapter II. In certain cases, the given driving-point immittance function, Y_1 , may require extensive alteration for use in realizing a desired transmission zero using any of the circuits listed.

The element values for the coupling circuits of the Dasher procedure, listed in Figures 4, 5, and 6, are derived from the variable parameters of the admittance functions, y_{11} , $-y_{12}$, and y_{22} . It is not always possible to use admittance parameters in the new procedure because of the more complex preparatory-step requirement. Therefore, circuits are derived for use with either admittance parameters or impedance

parameters. Most circuits are limited to use with one or the other.

The one finite pole of the Dasher circuits can be located anywhere on the negative real axis, and therefore σ_1 can be used as a variable in satisfying the preparatory-step conditions. The locations of the poles of the coupling circuits for the new procedure are restricted because, in general, the pole locations are functions of the surplus-factor zero, $s = -\sigma_0$, the range of which is limited as discussed in the preceding section.

The Dasher circuits are derived from symmetrical circuits. The circuits of the new procedure are, in general, not derivable from symmetrical circuits.

Preparatory Step

In the Dasher procedure, the given driving-point function, Y_1 , can be altered to meet the preparatory-step requirements,

$$y_{11}(s_0) = Y_{1p}(s_0) \quad (39)$$

and

$$y_{11}'(s_0) = Y_{1p}'(s_0), \quad (40)$$

by removal of either series or shunt elements. It is not necessary to remove both. The conditions (39) and (40) can be satisfied by using equation (12) or a residue condition derived from (12). Equating the real and imaginary parts of (39) and (40) yields four equations. However, y_{11} has only three variable parameters in equation (10). These are k_y , a , and σ_1 . Equation (12) is derived by eliminating these variables from the four equations. After satisfying (12) or its equivalent in terms of impedance, the Dasher procedure is performed using admittance functions.

In the procedure for right-half-plane transmission zeros, there

are two requirements in addition to (39) and (40). These are

$$y_{11}(-\sigma_0) = Y_{1p}(-\sigma_0) \quad (41)$$

$$y_{11}'(-\sigma_0) > Y_{1p}'(-\sigma_0). \quad (42)$$

In this case,

$$y_{11} = k_y \left[s + \frac{\sigma_0 \omega_0^2}{\sigma_1 \sigma_2} + \frac{a_1 B_2 s}{s + \sigma_1} + \frac{a_2 B_2 s}{s + \sigma_2} \right] \quad (43)$$

where

$$B_1 = \left| \frac{(\sigma_0 - \sigma_1)(\sigma_1^2 + 2\alpha_0 \sigma_1 + \omega_0^2)}{\sigma_1(\sigma_2 - \sigma_1)} \right| \quad (44)$$

$$B_2 = \left| \frac{(\sigma_0 - \sigma_2)(\sigma_2^2 + 2\alpha_0 \sigma_2 + \omega_0^2)}{\sigma_2(\sigma_2 - \sigma_1)} \right|$$

The variable parameters of y_{11} are k_y , a_1 , and a_2 . In addition, the parameters σ_0 , σ_1 , and σ_2 are variable over certain ranges. The new procedure has no equation similar to Dasher equation (12). The additional condition (41), plus the fact that there are still essentially only three variable parameters of y_{11} , makes the existence of such an equation highly improbable. Even if such an equation does exist, it may be shown that both series and shunt elements must be removed, in general, from Y_1 in order to satisfy the equation. Unfortunately, the lack of such an equation makes the procedure more complex and necessitates the use of both admittance and impedance functions in the procedure. The impedance conditions equivalent to the previously listed admittance conditions are:

$$z_{11}(s_o) = z_{1p}(s_o) \quad (46)$$

$$z_{11}'(s_o) = z_{1p}'(s_o) \quad (47)$$

$$z_{11}(-\sigma_o) = z_{1p}(-\sigma_o) \quad (48)$$

$$z_{11}'(-\sigma_o) < z_{1p}'(-\sigma_o). \quad (49)$$

The derivation of the preceding preparatory-step conditions is contained in Chapter III.

To demonstrate the method used in the new procedure to satisfy the conditional equations, another approach to the solution of the previously discussed example of the Dasher procedure may be used. Equations (50) and (51) represent the real and imaginary parts of (39). The assumption is made that a shunt conductance may be used to satisfy the conditions. Equations (52) and (53) represent the real and imaginary parts of (40). Y_1 and y_{11} are given by (15) and (10).

$$\frac{k(1+a)}{\sigma_1} (2 - \sigma_1) = 1 - G_p \quad (50)$$

$$k_y (1 + a) = 2 \quad (51)$$

$$k(1+a) - \frac{2k_y a}{\sigma_1^2 - 2\sigma_1 + 2} = 1 \quad (52)$$

$$\frac{-2k_y a(\sigma_1 - 1)}{\sigma_1^2 - 2\sigma_1 + 2} = 1. \quad (53)$$

Substitution of (51) in (52), then (52) in (53) yields

$$\sigma_1 = 2. \quad (54)$$

Solving (53) for $k_y a$ using this value for σ_1 gives

$$k_y a = 1. \quad (55)$$

Substituting (55) in (51) yields

$$k_y = 1, \quad (56)$$

and therefore from (55)

$$a = 1. \quad (57)$$

Solving for G_p in equation (50) yields

$$G_p = 1. \quad (58)$$

This method of solution gives information necessary to calculate all circuit element values as well as values of elements removed in the preparatory step. The remaining information to be obtained is the expression for Y_2 , the terminating function. The method is demonstrated again in Chapter IV.

The disadvantage of using the preceding procedure in the Dasher method is that there is no way of knowing beforehand whether removal of a shunt conductance or a shunt capacitance or perhaps partial removal of a pole of Y_1 , or even removal of all of these will satisfy the preparatory conditions. Unfortunately, this disadvantage extends to the new procedure which has an additional equality to satisfy, namely, equation (41).

Choice of Coupling Circuit

In the Dasher procedure, the choice of coupling circuit depends upon the pole location of the y -parameters at $s = -\sigma_1$, and the value of σ_1 is determined by the conditional equations. There is no apparent restriction on the location of the pole, so long as it lies on the negative real axis. Regardless of the value of σ_1 , one of the three circuits will be acceptable for use in the procedure.

In the new procedure, the poles of the coupling circuits are restricted to certain ranges of values. For each circuit there is a complex relationship between the pole locations, σ_1 and σ_2 , and the surplus-factor zero, σ_0 . The poles of the y -functions must interlace with the poles of the given admittance function, Y_1 , in a prescribed manner. At the same time, the poles of the z -functions for the same circuit must interlace with the zeros of the Y_{1p} -function in a similarly prescribed manner. For this reason and because the poles of the y - and z -functions have restricted ranges of values, a circuit is chosen at the beginning of the procedure which will satisfy the pole-interlacing property. The values of σ_0 , σ_1 , and σ_2 are chosen during this step and these values used when satisfying the preparatory-step conditions. The pole-interlacing requirement and the method of choosing the proper circuit are explained in greater detail in Chapter III.

CHAPTER II

COUPLING CIRCUITS

Properties of the Coupling Circuits

A two-terminal-pair network for which the residue condition, $k_{11}k_{22} - k_{12}^2 \geq 0$, is satisfied with an equality sign for the z-functions is by definition z-compact. Another manner of defining a z-compact network is to specify that the function $z_{11}z_{22} - z_{12}^2$ must contain no second-order poles after cancellation of common factors.

A two-terminal-pair network for which the residue condition is satisfied with an equality sign for the y-functions is by definition y-compact. Similarly, a second definition for the y-compact property requires the absence of any second-order poles in the function $y_{11}y_{22} - y_{12}^2$.

A circuit which is y-compact is not necessarily z-compact and a circuit which is z-compact is not necessarily y-compact. However, given the y-functions for a two-terminal-pair network (which may or may not be y-compact), a simple check will determine whether the network is or is not z-compact without actually calculating the z-functions. Similarly, given the z-functions, a simple check will determine whether the network is or is not y-compact. For example, suppose the y-functions are given. The y-functions are related to the z-functions by the following equations:

$$y_{11} = \frac{z_{22}}{z_{11}z_{22} - z_{12}^2} \quad (59)$$

$$-y_{12} = \frac{z_{12}}{z_{11} z_{22} - z_{12}^2} \quad (60)$$

$$y_{22} = \frac{z_{11}}{z_{11} z_{22} - z_{12}^2} \quad (61)$$

If the circuit is assumed to be z-compact, an examination of the right side of each equation above reveals that the poles of the numerator function cancel with the poles of the denominator function. If the circuit is not z-compact at a pole, that pole will appear as a second-order factor in the denominator function, and therefore as a numerator factor of each of the functions y_{11} , y_{12} , and y_{22} .

Thus a two-terminal-pair network is z-compact unless the same numerator factor appears in all three of the functions z_{11} , z_{12} , and z_{22} .

For example, suppose that the functions

$$-y_{12} = \frac{s^2 + 4}{s + 2} = s + 2 - \frac{4s}{s + 2} \quad (62)$$

$$y_{11} = \frac{s^2 + 9s + 4}{s + 2} = s + 2 + \frac{5s}{s + 2} \quad (63)$$

$$y_{22} = \frac{s^2 + 9s + 4}{s + 2} = s + 2 + \frac{5s}{s + 2} \quad (64)$$

are given. The z-functions associated with the preceding y-functions must be z-compact because of the absence of a common numerator factor in (62), (63), and (64). In this example, the y-functions do not happen to be y-compact. The associated z-functions are:

$$z_{12} = \frac{(s+2)(s^2+4)}{9s(2s^2+9s+6)} \quad (65)$$

$$z_{11} = \frac{(s+2)(s^2+9s+4)}{9s(2s^2+9s+6)} \quad (66)$$

$$z_{22} = \frac{(s+2)(s^2+9s+4)}{9s(2s^2+9s+6)} \quad (67)$$

These functions have a common numerator factor corresponding to the non-compact pole of the y-functions.

For a second example, one may note that the Guillemin parallel-ladder networks are not, in general, y-compact but they are z-compact. The sufficient condition is that Y_{12} and Y_{11} have no common numerator factor.

Now, to show the effect of a non-compact network imbedded in a set of cascaded networks, straightforward calculation of the transfer functions for the simple cascade connection of Figure 9 reveals that

$$-Y_{12} = \frac{I_2}{E_1} = \frac{-y_{12}Y_2}{y_{22} + Y_2} = \frac{-z_{12}Y_2}{z_{11} + (z_{11}z_{22} - z_{12}^2)Y_2} \quad (68)$$

and that

$$z_{12} = \frac{E_2}{I_1} = \frac{z_{12}Z_2}{z_{22} + Z_2} = \frac{-y_{12}Z_2}{y_{11} + (y_{11}y_{22} - y_{12}^2)Z_2} \quad (69)$$

If the network to be used in the cascade synthesis procedure is not z-compact at a pole, the right side of equation (68) shows that $-Y_{12}$ will have a transmission zero at the non-compact pole. If the network to be

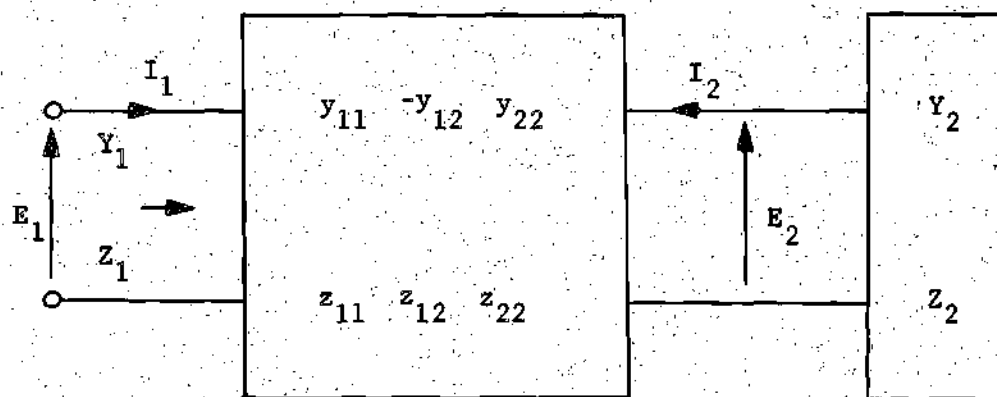


Figure 9. Cascade Connection of Networks

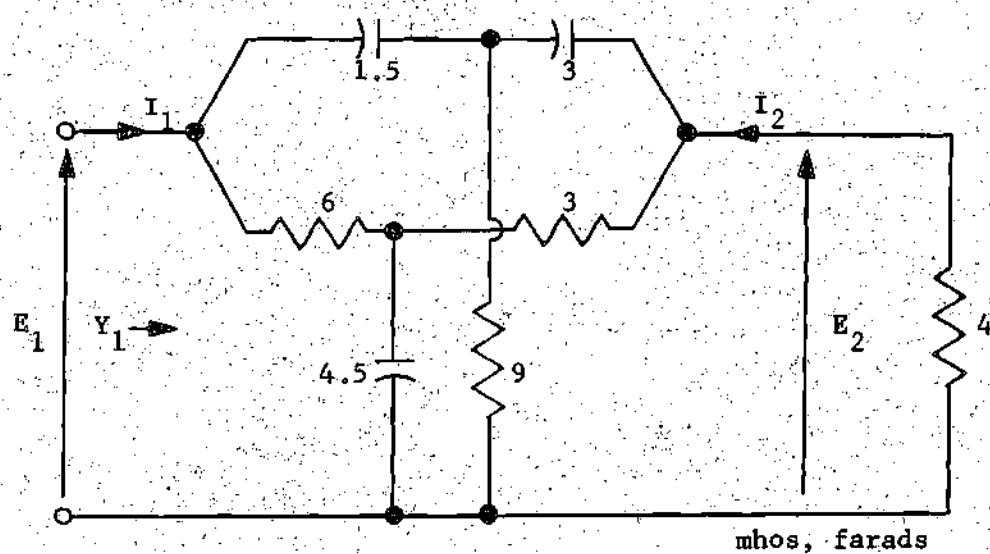


Figure 10. Example of a Non-Compact Circuit.

used in the cascade synthesis procedure is not y-compact at a pole, the right side of equation (69) shows that Z_{12} will have a transmission zero at the non-compact pole.

For example, if the circuit associated with the y- and z-functions of equations (62) through (67) is cascaded with a 4-mho conductance as indicated in Figure 10, the overall transfer functions are

$$-Y_{12} = \frac{4(s^2 + 4)}{(s + 1)(s + 12)} \quad (70)$$

and

$$Z_{12} = \frac{(s + 2)(s^2 + 4)}{22s^3 + 125s^2 + 160s + 32} \quad (71)$$

The $(s + 2)$ factor in the numerator of the Z_{12} function is a transmission zero caused by the use of a circuit which is not y-compact at the corresponding pole.

Consequently, a coupling circuit intended for use in cascade synthesis should be both z-compact and y-compact for general use in the synthesis of either Z_{12} or Y_{12} -functions. However, Y_{12} transfer functions may be formed by cascade synthesis of circuits which are z-compact but not y-compact and Z_{12} transfer functions may be formed by cascade synthesis of circuits which are y-compact but not z-compact. A preliminary investigation of the use of non-compact circuits for cascade synthesis has indicated that the disadvantages of such circuits are many when compared to the advantages. Coupling circuits which are y-compact but not z-compact are difficult to derive. Circuits which are z-compact but not y-compact may be derived as parallel ladders, but these require more

elements than circuits which are both z- and y-compact. In addition, the non-compact poles of the coupling circuit are poles or zeros of the given driving-point immittance.

The coupling circuits used for cascade synthesis of transmission zeros in the part of the right-half plane under consideration have compact y- and z-functions given by the following equations:

$$-y_{12} = k_y \frac{(s + \sigma_0)(s^2 - 2\alpha_0 s + \omega_0^2)}{(s + \sigma_1)(s + \sigma_2)} \quad (72)$$

or

$$-y_{12} = k_y \left(s + \frac{\sigma_0 \omega_0^2}{\sigma_1 \sigma_2} + \frac{A_1 s}{s + \sigma_1} + \frac{A_2 s}{s + \sigma_2} \right) \quad (73)$$

and

$$y_{11} = k_y \left(s + \frac{\sigma_0 \omega_0^2}{\sigma_1 \sigma_2} + a_1 \frac{|A_1| s}{s + \sigma_1} + a_2 \frac{|A_2| s}{s + \sigma_2} \right) \quad (74)$$

$$y_{22} = k_y \left(s + \frac{\sigma_0 \omega_0^2}{\sigma_1 \sigma_2} + \frac{1}{a_1} \frac{|A_1| s}{s + \sigma_1} + \frac{1}{a_2} \frac{|A_2| s}{s + \sigma_2} \right) \quad (75)$$

where

$$A_1 = \frac{(\sigma_0 - \sigma_1)(\sigma_1^2 + 2\alpha_0 \sigma_1 + \omega_0^2)}{-\sigma_1(\sigma_2 - \sigma_1)} \quad (76)$$

and

$$A_2 = \frac{(\sigma_0 - \sigma_2)(\sigma_2^2 + 2\alpha_0 \sigma_2 + \omega_0^2)}{\sigma_2(\sigma_2 - \sigma_1)}, \quad (77)$$

$$z_{12} = k_z \frac{(s + \sigma_o)(s^2 - 2\alpha_o s + \omega_o^2)}{s(s + \sigma_3)(s + \sigma_4)} \quad (78)$$

or

$$z_{12} = k_z \left(1 + \frac{\sigma_o \omega_o^2}{\sigma_3 \sigma_4 s} + \frac{A_3}{s + \sigma_3} + \frac{A_4}{s + \sigma_4} \right) \quad (79)$$

and

$$z_{11} = k_z \left(1 + \frac{\sigma_o \omega_o^2}{\sigma_3 \sigma_4 s} + a_3 \frac{|A_3|}{s + \sigma_3} + a_4 \frac{|A_4|}{s + \sigma_4} \right) \quad (80)$$

$$z_{22} = k_z \left(1 + \frac{\sigma_o \omega_o^2}{\sigma_3 \sigma_4 s} + \frac{1}{a_3} \frac{|A_3|}{s + \sigma_3} + \frac{1}{a_4} \frac{|A_4|}{s + \sigma_4} \right) \quad (81)$$

where

$$A_3 = \frac{(\sigma_3 - \sigma_o)(\sigma_3^2 + 2\alpha_o \sigma_3 + \omega_o^2)}{-\sigma_3(\sigma_4 - \sigma_3)} \quad (82)$$

and

$$A_4 = \frac{(\sigma_4 - \sigma_o)(\sigma_4^2 + 2\alpha_o \sigma_4 + \omega_o^2)}{-\sigma_4(\sigma_4 - \sigma_3)} \quad (83)$$

Circuits which have the y- and z-functions of equations (72) through (83) have capacitive feed-through paths for very high-frequency voltage and current and have resistive feed-through paths for very low-frequency voltage and current. Coupling circuits with these properties may be used for cascade synthesis of the most general transfer functions possible. In certain cases, however, elements removed in the preparatory step may have an adverse affect on the performance of the circuit at the extreme frequencies. This is one of the disadvantages of the cascade synthesis procedure.

Other properties which the coupling circuits should have include

a wide range of compact-pole locations and also ease in deriving circuit element values from the variable parameters of y_{11} or z_{11} .

For compact coupling circuits, the surplus factor $(s + \sigma_0)$ must not cancel from the $-y_{12}$ function. If such a cancellation did take place, $-y_{12}$ would have zero residue at the pole at $s = -\sigma_0$; however, y_{11} and y_{22} would have non-zero residues. The same rule applies for the z -functions.

Coupling Circuits for Use with Admittance Functions

If a driving-point admittance has a pole at $s = \infty$ (or if the driving-point impedance has a zero at $s = \infty$), the preparatory step of the synthesis procedure should, in general, be performed using admittance functions. The use of admittance functions allows removal of a shunt capacitance and perhaps a shunt conductance, which will aid in satisfying an impedance pole-interlacing requirement which in turn is related to the preparatory-step conditions. A detailed explanation is contained in the next chapter.

The coupling circuits of Figures 11, 13, 15, 17, and 19 are for use with admittance functions. The element values for each circuit are listed in terms of the variable parameters of the y_{11} -function, as given in (74), (76), and (77).

For each of the circuits, the compact poles at $s = -\sigma_2$ and $s = -\sigma_1$ are restricted to certain ranges of values. For uniformity, it is assumed that σ_2 is larger than σ_1 in all cases. Graphs of the variation of σ_2 and σ_1 as the value of σ_0 is varied over its allowable range are sketched for each circuit in Figures 12, 14, 16, 18, and 20. The graphs are sketched with the assumption that the transmission zeros are close to the 60-degree line of the right-half plane. For transmission zeros close to the $j\omega$

axis, the range of σ_0 is increased and therefore the ranges of σ_2 and σ_1 are increased.

None of these circuits can be used for cascade synthesis of transmission zeros which lie on the 60-degree line in the right-half plane. For transmission zeros close to the 60-degree line, the circuit elements approach extremely large or extremely small values.

This list of coupling circuits is not necessarily complete. However, a wide range of allowable values for σ_1 and σ_2 is provided. Many other circuits were analyzed for use in the procedure. However, none offered any particular advantage over those listed.

The z-functions of the admittance coupling circuits are difficult to calculate. For example, the poles of the z-functions of Coupling Circuit A, Figure 11, are the factors of the polynomial

$$s^3 + B_1 s^2 + B_2 s \quad (84)$$

where

$$B_1 = \frac{\frac{(a_1 + 1)^2}{a_1} (A^2 + \sigma_2 A) + \frac{(a_2 + 1)^2}{a_2} \sigma_2 (A + \sigma_1) + \frac{(a_1 - a_2)}{a_1 a_2} \sigma_2 A}{\frac{(a_1 + 1)^2}{a_1} A + \frac{(a_2 + 1)^2}{a_2} \sigma_2} \quad (85)$$

$$B_2 = \frac{\frac{(a_1 + 1)^2}{a_1} \sigma_2 A^2 + \frac{(a_2 + 1)^2}{a_2} \sigma_2 \sigma_1 A}{\frac{(a_1 + 1)^2}{a_1} A + \frac{(a_2 + 1)^2}{a_2} \sigma_2} \quad (86)$$

$$A = \frac{\sigma_0 \omega_0^2}{\sigma_1 \sigma_2} \quad (87)$$

The values of a_1 and a_2 and therefore the pole locations are not known until after the completion of the preparatory step, which will be discussed in Chapter III.

The admittance circuits are y-compact by construction. However, because of the difficulty in calculating the z-functions, one cannot ascertain whether the circuits are z-compact until after the completion of the preparatory step. The circuits will be z-compact if y_{11} , y_{12} , and y_{22} have no common numerator factor. The only possible common numerator factor is $(s + \sigma_0)$. If $(s + \sigma_0)$ is a common numerator factor of the y-functions, then the z-functions must have a non-compact pole at $s = -\sigma_0$. A non-compact pole of the z-functions must be a zero of Y_{lp} , the driving point admittance after the preparatory step has been performed. In the synthesis procedure, the assumption is made initially that the circuits are z-compact. The assumption is checked for validity after completion of the preparatory step by examining the zeros of y_{11} and y_{22} or by examining the zeros of Y_{lp} . It is highly improbable that the functions y_{11} and y_{22} will have a common zero and that the common zero will be located at $s = -\sigma_0$. It is also improbable that Y_{lp} will have a zero at $s = -\sigma_0$. If the circuit should be non-compact, the preparatory step must be recalculated using another choice of variables.

The conditions under which the numerators of y_{11} and y_{22} will contain the factor $(s + \sigma_0)$ may be determined by evaluating (74) and (75) at $s = -\sigma_0$ and equating the result to zero. The conditions are:

$$-\sigma_0 + \frac{\sigma_0 \omega_0^2}{\sigma_1 \sigma_2} - a_1 \frac{|A_1| \sigma_0}{\sigma_1 - \sigma_0} - a_2 \frac{|A_2| \sigma_0}{\sigma_2 - \sigma_0} = 0 \quad (88)$$

$$-\sigma_0 + \frac{\sigma_0 \omega_0^2}{\sigma_1 \sigma_2} - \frac{1}{a_1} \frac{|A_1| \sigma_0}{\sigma_1 - \sigma_0} - \frac{1}{a_2} \frac{|A_2| \sigma_0}{\sigma_2 - \sigma_0} = 0. \quad (89)$$

Both of the conditions must be satisfied in order for the procedure to fail. The circumstances under which the conditions will be satisfied, if they exist, are unknown. Several calculations were performed using randomly chosen values for the parameters a_1 and a_2 with a particular circuit and transmission zero. The conditions were not satisfied.

The significance of the graphs of pole locations is related to the selection of the proper circuit for use in the synthesis procedure and is discussed in the next chapter.

Coupling Circuits for Use with Impedance Functions

If the driving-point impedance has a finite, non-zero value at $s = \infty$ (or if the driving-point admittance has a finite, non-zero value at $s = \infty$), the preparatory step of the synthesis procedure should, in general, be performed using impedance functions. The use of impedance functions will allow removal of a series resistance and perhaps a series capacitance, which may satisfy the preparatory-step conditions, while at the same time, satisfy the admittance pole-interlacing requirement. The preparatory-step conditions and the related pole-interlacing property are discussed in detail in the next chapter.

The coupling circuits of Figures 21, 23, and 25 are for use with impedance functions. The element values for each circuit are listed in terms of the variable parameters of the z_{11} -function, as given in (80), (82), and (83).

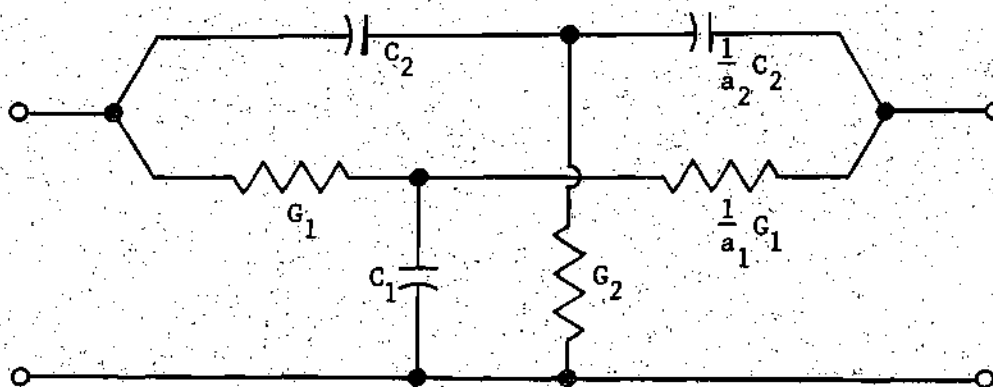
Graphs of the variation of the compact poles, σ_3 and σ_4 , along the negative real axis are indicated in Figures 22, 24, and 26.

None of the circuits may be used to realize transmission zeros on the 60-degree line of the right half of the s -plane.

As in the case of the admittance circuits, the list of circuits is not necessarily complete. Many other circuits were analyzed; however, none were adaptable to the procedure.

Coupling Circuits G and H are z -compact by construction. They are not necessarily y -compact. If they are not y -compact, the z -functions and Z_{lp} will have a zero at $s = -\sigma_0$. As for the admittance circuits, the circuits are assumed to be y -compact until after completion of the preparatory step.

Coupling Circuit F has the same twin-tee structure as Coupling Circuit A. The restriction $a_4 = 1$ is the result of requiring that $a_1 = a_2$ in Coupling Circuit A. Circuit F is both y - and z -compact and the pole locations of both the y - and z -functions may be calculated before performance of the preparatory step. When using the circuit in the synthesis procedure, the number of variable parameters is one less because of the restriction on a_4 . The solution of the preparatory-step equations in this case is demonstrated by the impedance example of Chapter IV.



$$C_1 = \frac{k_y (1+a_1)^2 \sigma_o \omega_o^2}{a_1 \sigma_1 \sigma_2}$$

$$C_2 = k_y(1+a_2)$$

$$G_1 = \frac{k_y(1+a_1)\sigma_o\omega_o^2}{\sigma_1\sigma_2}$$

$$G_2 = \frac{k_y (1+a_2)^2 \sigma_2}{a_2}$$

$$\sigma_2 = \frac{\sigma_0^2 \omega^2}{\omega_0^2 - 2\alpha_0 \sigma_0}$$

$$\sigma_1 = \sigma_0 - 2\alpha_0$$

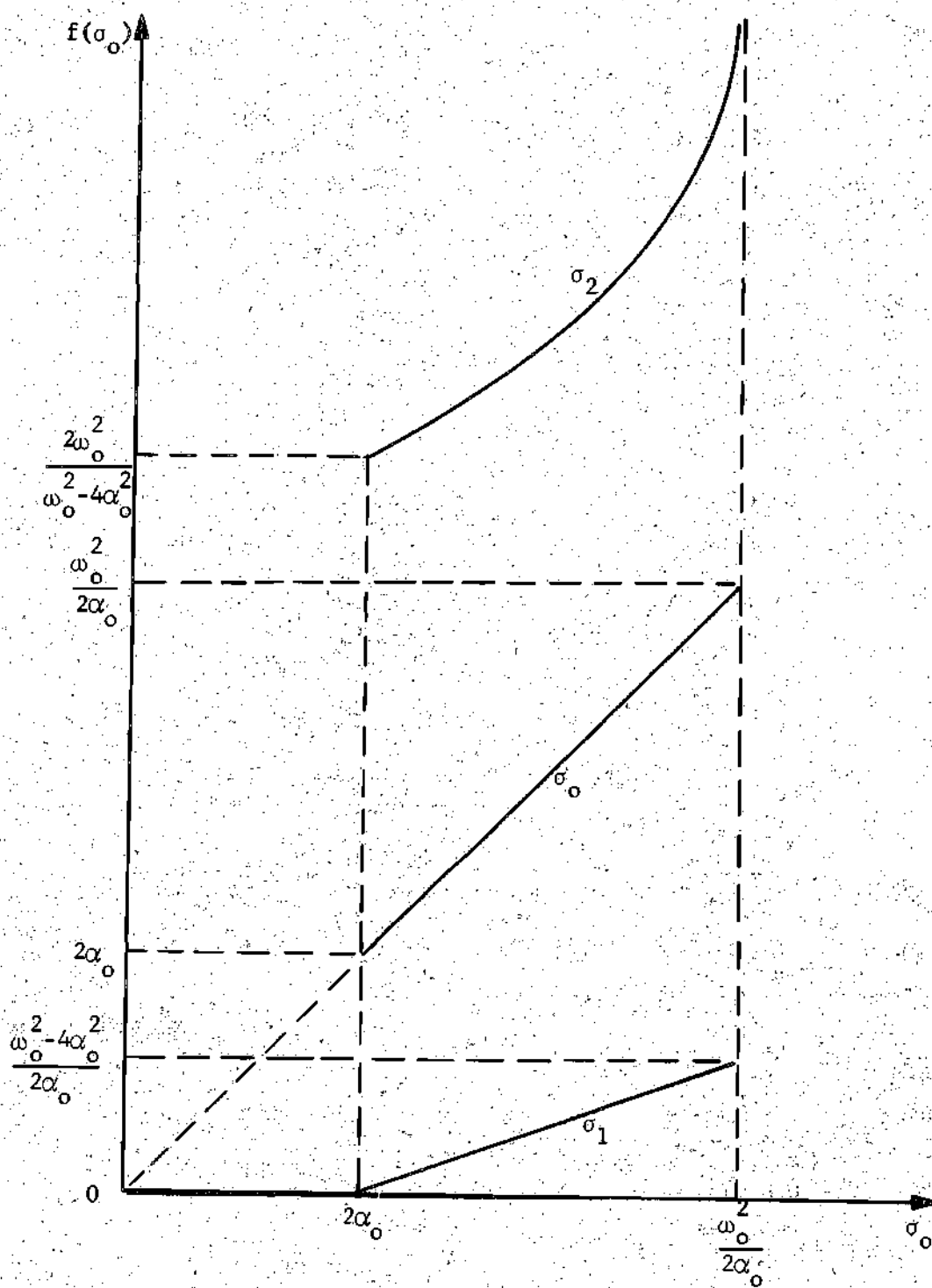
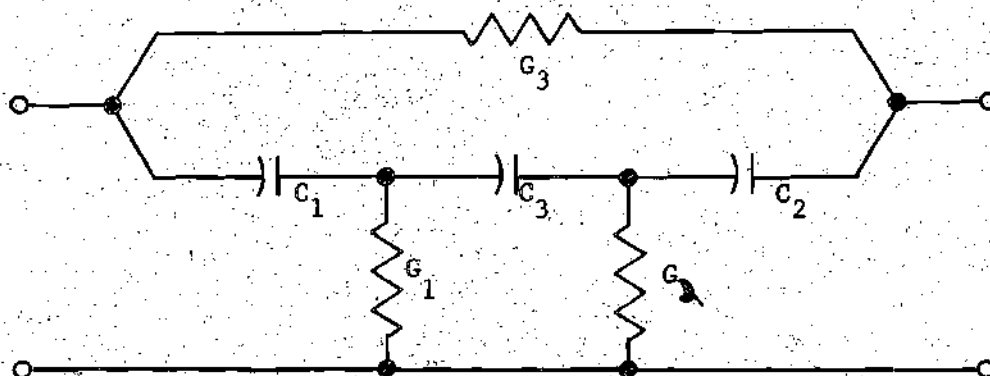


Figure 12. Possible Pole Locations for Coupling Circuit A.



$$C_1 = \frac{k_y [(a_1 - 1)\sigma_1 + (a_2 + 1)\sigma_2]}{\sigma_2 - \sigma_1}$$

$$C_2 = \frac{k_y [a_1(a_2 + 1)\sigma_2 - a_2(a_1 - 1)\sigma_1]}{a_1 a_2 (\sigma_2 - \sigma_1)}$$

$$C_3 = \frac{k_y [(a_1 - 1)\sigma_1 + (a_2 + 1)\sigma_2] [a_1(a_2 + 1)\sigma_2] a_2(a_1 - 1)\sigma_1]}{(a_1 + a_2)^2 \sigma_1 \sigma_2}$$

$$G_1 = \frac{k_y [(a_1 - 1)\sigma_1 + (a_2 + 1)\sigma_2]^2}{(a_1 + a_2)(\sigma_2 - \sigma_1)}$$

$$G_2 = \frac{k_y [a_1(a_2 + 1)\sigma_2] a_2(a_1 - 1)\sigma_1]^2}{a_1 a_2 (a_1 + a_2)(\sigma_2 - \sigma_1)}$$

$$G_3 = \frac{k_y \sigma_1^2 \sigma_2^2}{\sigma_1 \sigma_2}$$

$$\sigma_2, \sigma_1 = \frac{\omega_o^2 - 2\alpha_o \sigma_o + [(\omega_o^2 + 2\alpha_o \sigma_o)^2 - 4\sigma_o^2 \omega_o^2]^{\frac{1}{2}}}{2(\sigma_o - 2\alpha_o)}$$

Figure 13. Coupling Circuit B.

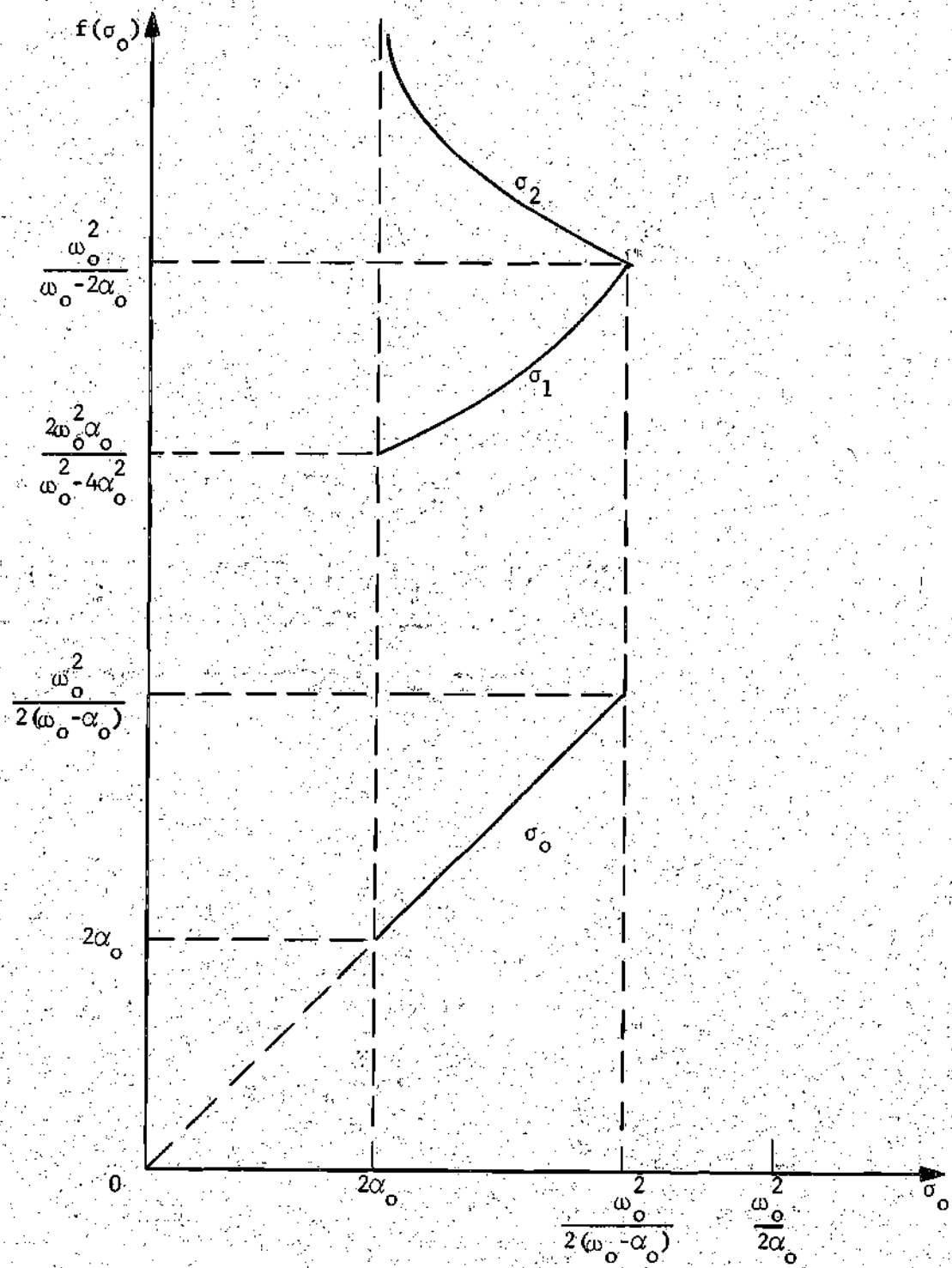
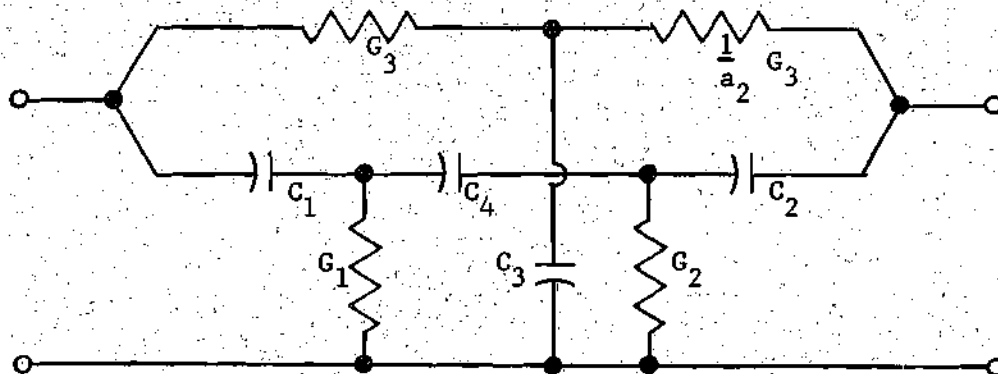


Figure 14. Possible Pole Locations for Coupling Circuit B.



$$C_1 = \frac{k_y [(a_1 - 1)\sigma_1 + (a_2 + 1)\sigma_2]}{\sigma_2 - \sigma_1}$$

$$G_1 = \frac{k_y [(a_1 - 1)\sigma_1 + (a_2 + 1)\sigma_2]^2}{(a_1 + a_2)(\sigma_2 - \sigma_1)}$$

$$C_2 = \frac{k_y [a_1(a_2 + 1)\sigma_2 - a_2(a_1 - 1)\sigma_1]}{a_1 a_2 (\sigma_2 - \sigma_1)}$$

$$G_2 = \frac{k_y [a_1(a_2 + 1)\sigma_2 - a_2(a_1 - 1)\sigma_1]^2}{a_1 a_2 (a_1 + a_2)(\sigma_2 - \sigma_1)}$$

$$C_3 = \frac{k_y (1 + a_2)^2 \sigma_o \omega_o^2}{a_2 \sigma_1 \sigma_2^2}$$

$$G_3 = \frac{k_y (1 + a_2) \sigma_o \omega_o^2}{\sigma_1 \sigma_2}$$

$$C_4 = \frac{k_y [(a_1 - 1)\sigma_1 + (a_2 + 1)\sigma_2] [a_1(a_2 + 1)\sigma_2 - a_2(a_1 - 1)\sigma_1]}{(a_1 + a_2)^2 \sigma_1 \sigma_2}$$

$$\sigma_o = 2\alpha_o$$

$$\sigma_1 = \frac{2\omega_o^2 \alpha_o}{\omega_o^2 - 4\alpha_o^2}$$

The pole at $s = -\sigma_2$ is restricted to any value within the range,

$$-\infty < -\sigma_2 < -\frac{2\omega_o^2 \alpha_o}{\omega_o^2 - 4\alpha_o^2}$$

Figure 15. Coupling Circuit C.

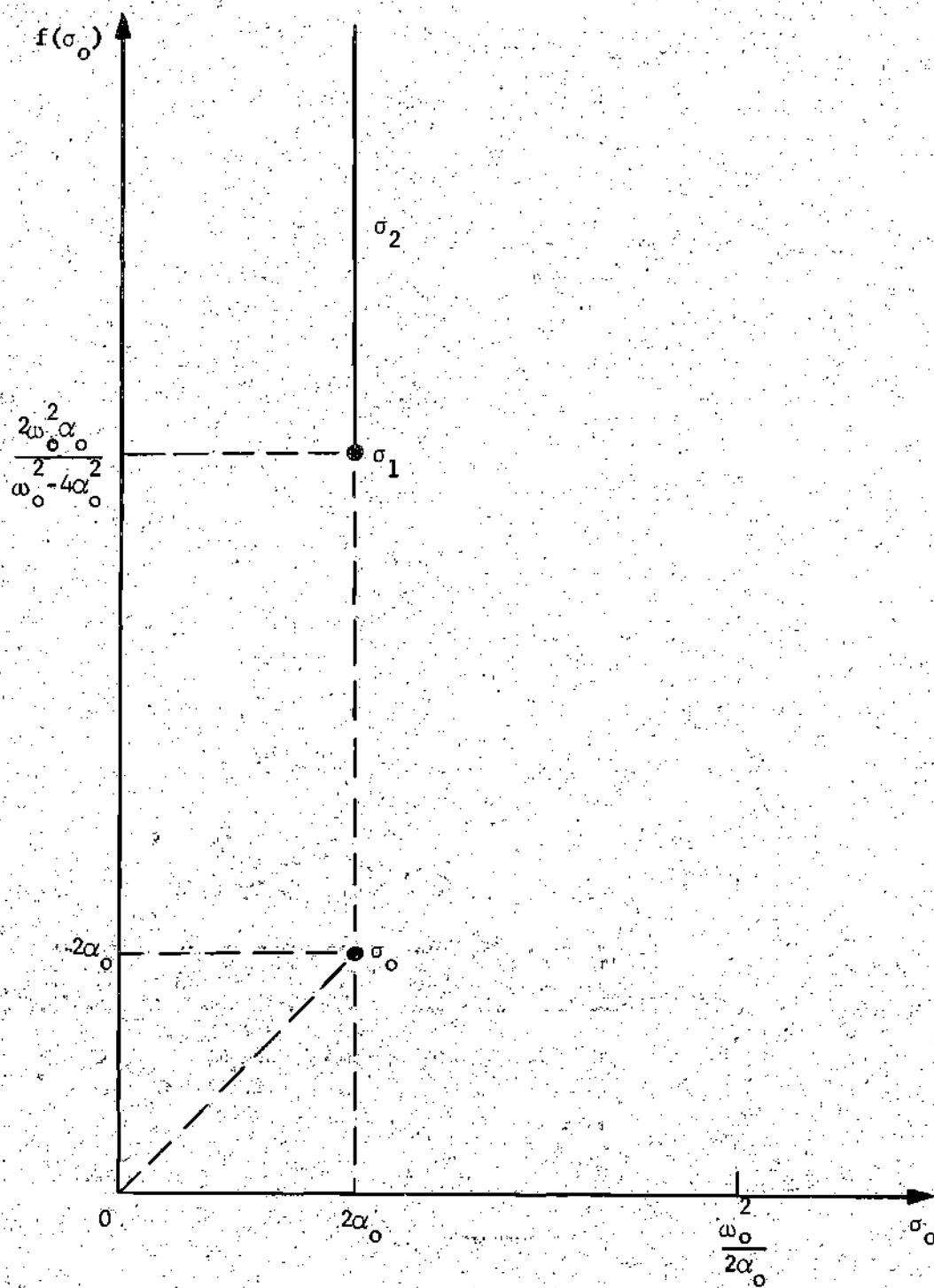
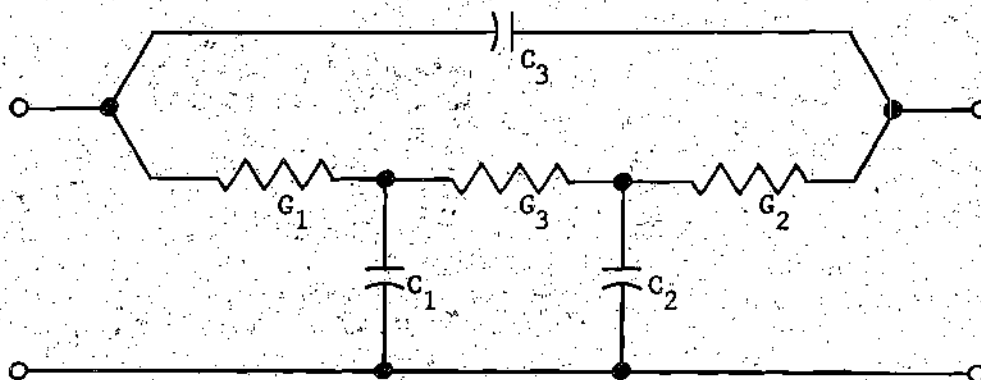


Figure 16. Possible Pole Locations for Coupling Circuit C.



$$C_1 = \frac{k_y \sigma_o \omega_o^2 [(a_2 - 1)\sigma_1 + (a_1 + 1)\sigma_2]^2}{\sigma_1^2 \sigma_2^2 (a_1 + a_2) (\sigma_2 - \sigma_1)}$$

$$C_2 = \frac{k_y \sigma_o \omega_o^2 [a_2 (a_1 + 1)\sigma_2 - a_1 (a_2 - 1)\sigma_1]^2}{\sigma_1^2 \sigma_2^2 a_1 a_2 (a_1 + a_2) (\sigma_2 - \sigma_1)}$$

$$C_3 = k_y$$

$$G_1 = \frac{k_y \sigma_o \omega_o^2 [(a_2 - 1)\sigma_1 + (a_1 + 1)\sigma_2]}{\sigma_1 \sigma_2 (\sigma_2 - \sigma_1)}$$

$$G_2 = \frac{k_y \sigma_o \omega_o^2 [a_2 (a_1 + 1)\sigma_2 - a_1 (a_2 - 1)\sigma_1]}{\sigma_1 \sigma_2 a_1 a_2 (\sigma_2 - \sigma_1)}$$

$$G_3 = \frac{k_y \sigma_o \omega_o^2 [(a_1 + 1)\sigma_2 + (a_2 - 1)\sigma_1] [a_2 (a_1 + 1)\sigma_2 - a_1 (a_2 - 1)\sigma_1]}{\sigma_1^2 \sigma_2^2 (a_1 + a_2)^2}$$

$$\sigma_2, \sigma_1 = \frac{\sigma_o - 2\alpha_o \pm [(\sigma_o + 2\alpha_o)^2 - 4\omega_o^2]^{\frac{1}{2}}}{2}$$

Figure 17. Coupling Circuit D.

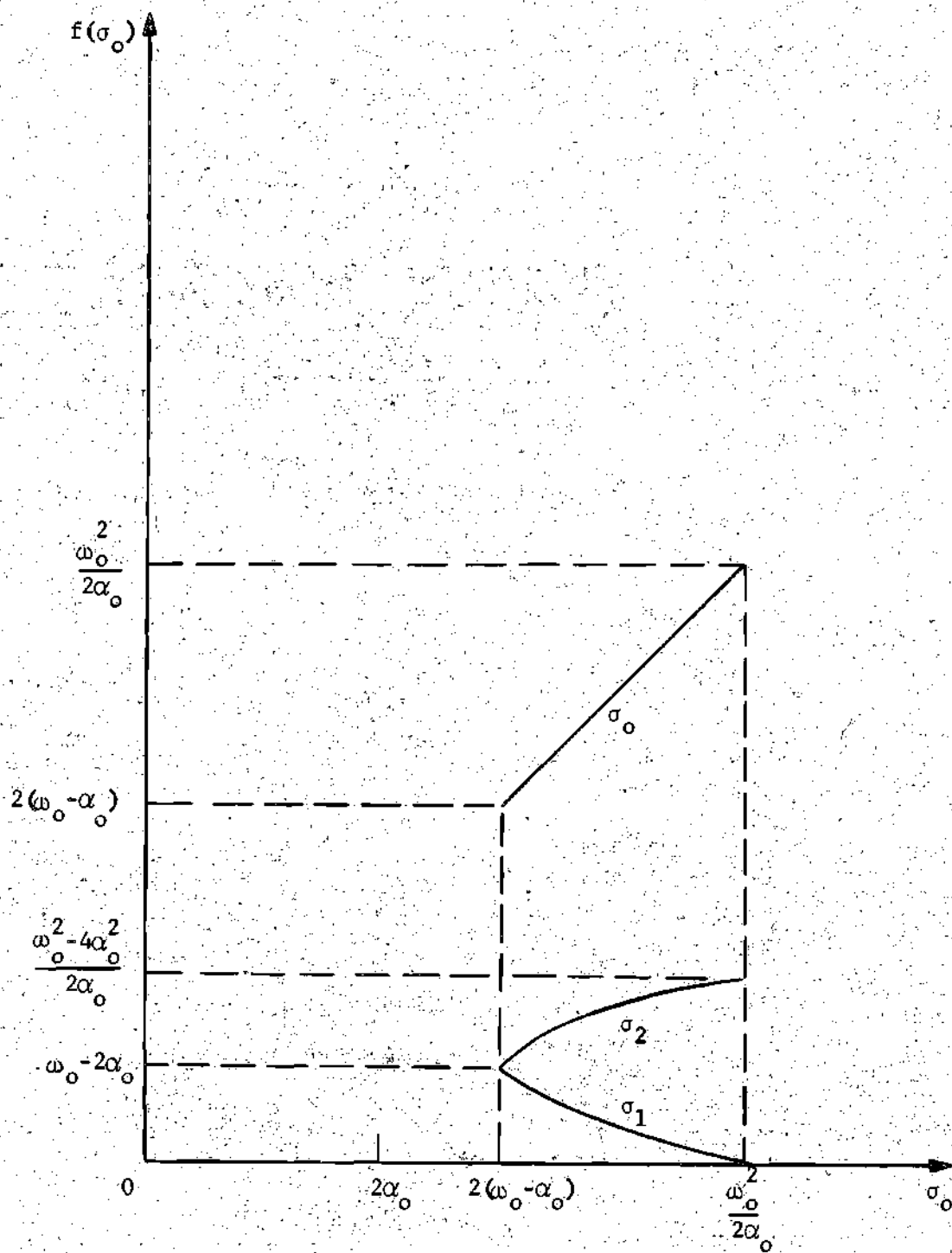
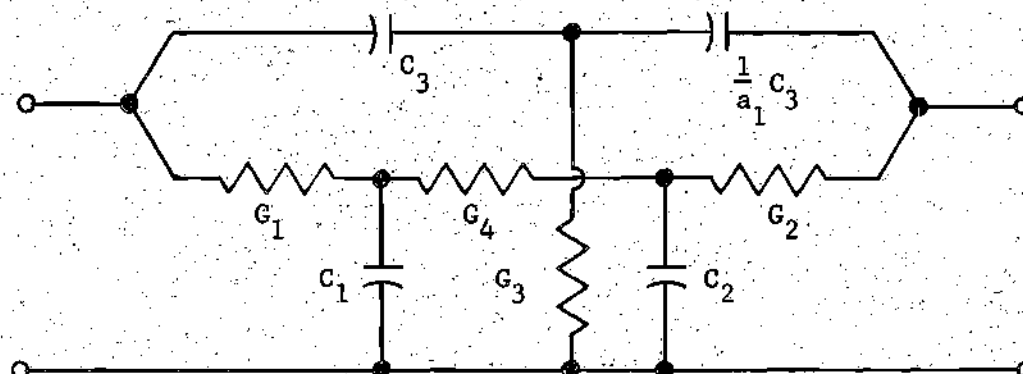


Figure 18. Possible Pole Locations for Coupling Circuit D.



$$C_1 = \frac{k \sigma \omega_o^2 [(a_2 - 1)\sigma_1 + (a_1 + 1)\sigma_2]^2}{\sigma_1^2 \sigma_2^2 (a_1 + a_2) (\sigma_2 - \sigma_1)}$$

$$G_1 = \frac{k \sigma \omega_o^2 [(a_2 - 1)\sigma_1 + (a_1 + 1)\sigma_2]}{\sigma_1 \sigma_2 (\sigma_2 - \sigma_1)}$$

$$C_2 = \frac{k \sigma \omega_o^2 [a_2(a_1 + 1)\sigma_2 - a_1(a_2 - 1)\sigma_1]^2}{\sigma_1^2 \sigma_2^2 a_1 a_2 (a_1 + a_2) (\sigma_2 - \sigma_1)}$$

$$G_2 = \frac{k \sigma \omega_o^2 [a_2(a_1 + 1)\sigma_2 - a_1(a_2 - 1)\sigma_1]}{\sigma_1 \sigma_2 a_1 a_2 (\sigma_2 - \sigma_1)}$$

$$C_3 = k_y (1 + a_1)$$

$$G_3 = \frac{k_y (1 + a_1)^2 \sigma_1}{a_1}$$

$$G_4 = \frac{k_y \sigma \omega_o^2 [(a_1 + 1)\sigma_2 + (a_2 - 1)\sigma_1] [a_2(a_1 + 1)\sigma_2 - a_1(a_2 - 1)\sigma_1]}{\sigma_1^2 \sigma_2^2 (a_1 + a_2)^2}$$

$$\sigma_o = \frac{\omega_o^2}{2\alpha_o}$$

$$\sigma_2 = \frac{\omega_o^2 - 4\alpha_o^2}{2\alpha_o}$$

The pole at $s = -\sigma_1$ is restricted to any value within the range,

$$-\frac{\omega_o^2 - 4\alpha_o^2}{2\alpha_o} < -\sigma_1 < 0.$$

Figure 19. Coupling Circuit E.

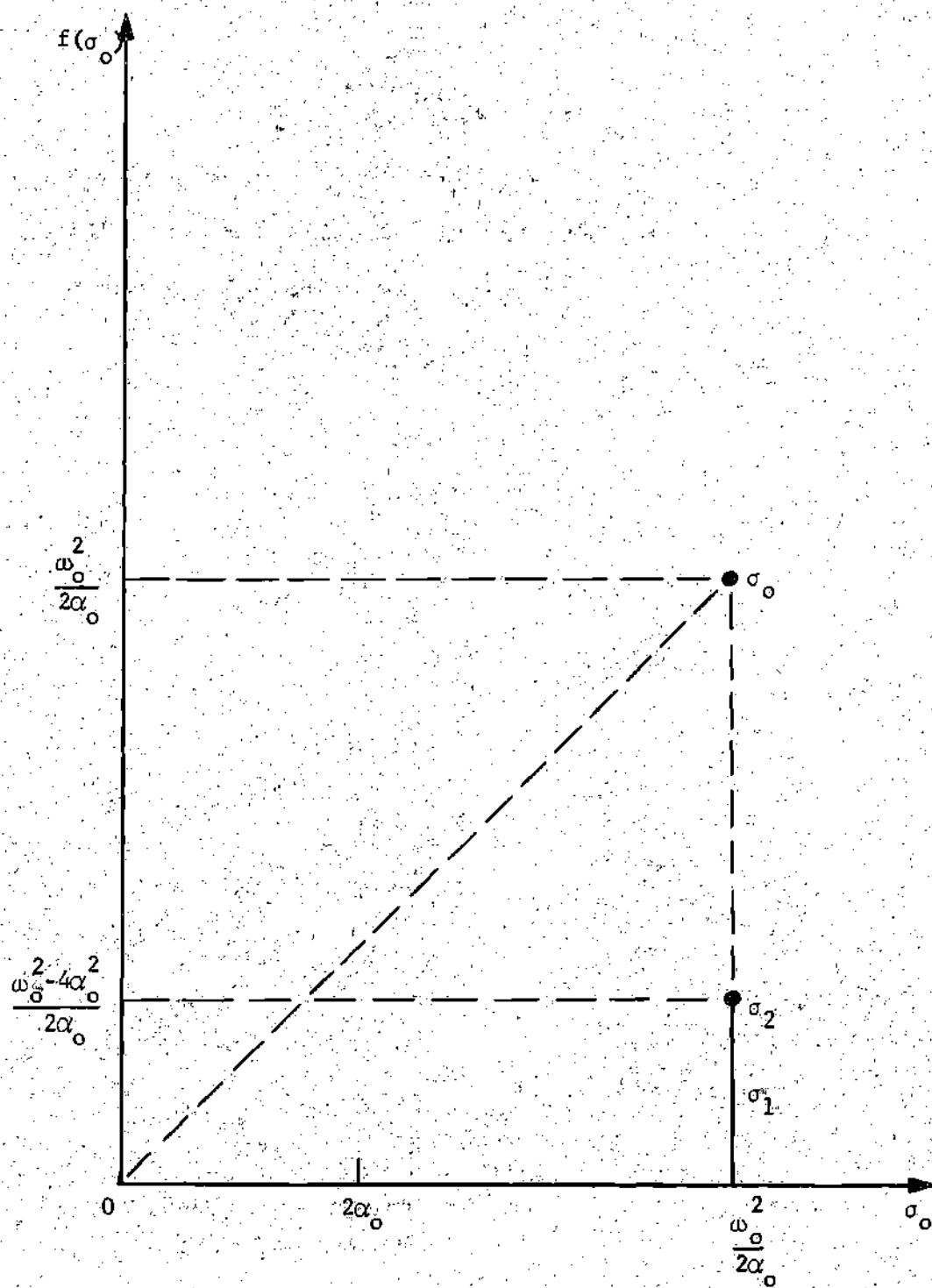
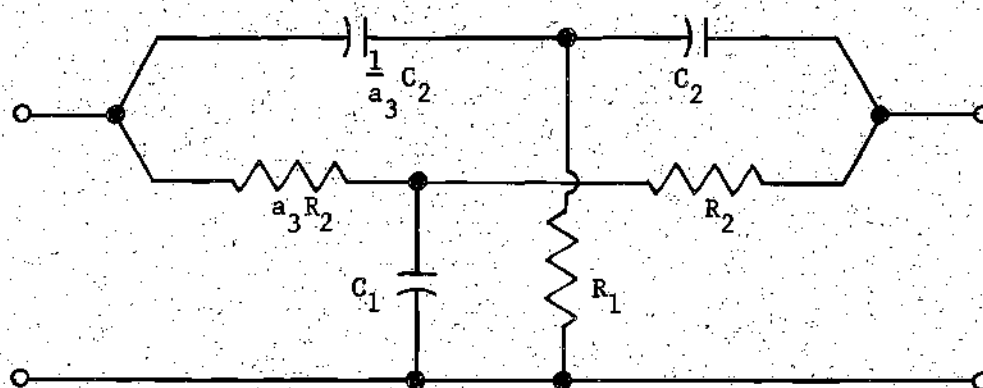


Figure 20. Possible Pole Locations for Coupling Circuit E.



$$c_1 = \frac{\sigma_3^2}{\omega_o^2 \sigma_o R_1}$$

$$c_2 = \frac{1}{\sigma_3 R_2}$$

$$R_1 = k_z \left[1 + \frac{(\omega_o^2 - 2\alpha_o \sigma_o) \sigma_3}{\omega_o^2 \sigma_o} \right]$$

$$R_2 = \frac{k_z (1 + a_3)}{a_3} \left[1 + \frac{\omega_o^2 \sigma_o}{(\omega_o^2 - 2\alpha_o \sigma_o) \sigma_3} \right]$$

$$\sigma_4 = \frac{\omega_o^2 [(\sigma_o - 2\alpha_o)^2 + \omega_o^2 - 2\alpha_o \sigma_o] \sigma_o}{\omega_o^2 \sigma_o (\sigma_o - 2\alpha_o) + (\omega_o^2 - 2\alpha_o \sigma_o)^2}$$

$$\sigma_3 = \frac{\omega_o^2 - 2\alpha_o \sigma_o}{\sigma_o - 2\alpha_o}$$

$$a_4 = 1$$

Figure 21.. Coupling Circuit F.

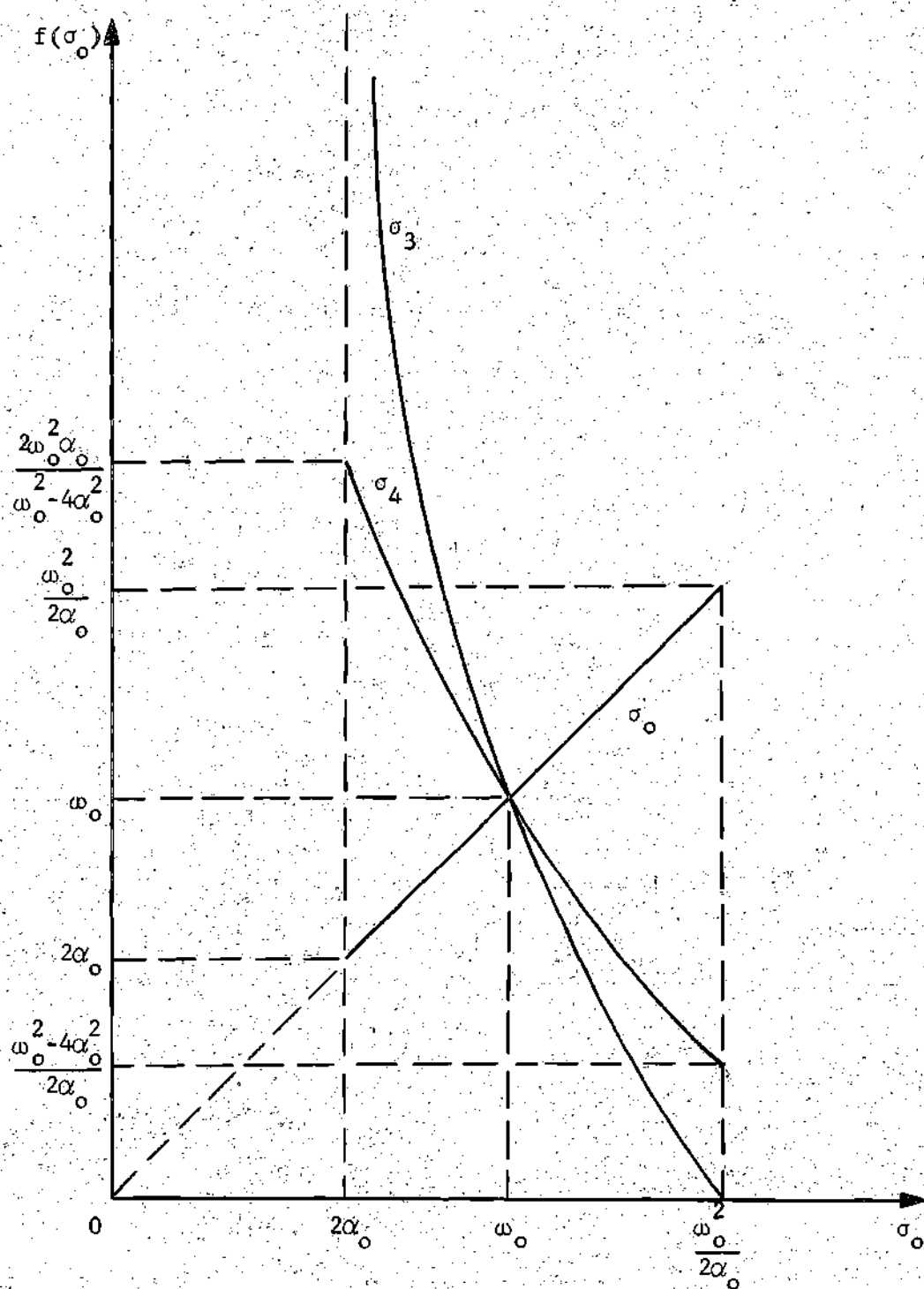
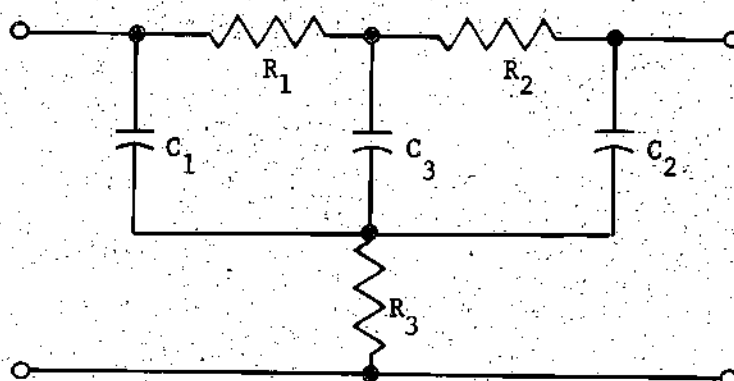


Figure 22. Possible Pole Locations for Coupling Circuit F.



$$C_1 = \frac{\sigma_3 \sigma_4 (\sigma_4 - \sigma_3)}{k_z \sigma_o \omega_o^2 [(a_4 - 1)\sigma_3 + (a_3 + 1)\sigma_4]}$$

$$C_2 = \frac{\sigma_3 \sigma_4 a_3 a_4 (\sigma_4 - \sigma_3)}{k_z \sigma_o \omega_o^2 [a_4 (a_3 + 1)\sigma_4 - a_3 (a_4 - 1)\sigma_3]}$$

$$C_3 = \frac{\sigma_3^2 \sigma_4^2 (a_3 + a_4)^2}{k_z \sigma_o \omega_o^2 [(a_3 + 1)\sigma_4 + (a_4 - 1)\sigma_3] [a_4 (a_3 + 1)\sigma_4 - a_3 (a_4 - 1)\sigma_3]}$$

$$R_1 = \frac{k_z \sigma_o \omega_o^2 [(a_4 - 1)\sigma_3 + (a_3 + 1)\sigma_4]^2}{\sigma_3^2 \sigma_4^2 (a_3 + a_4) (\sigma_4 - \sigma_3)}$$

$$R_2 = \frac{k_z \sigma_o \omega_o^2 [a_4 (a_3 + 1)\sigma_4 - a_3 (a_4 - 1)\sigma_3]^2}{\sigma_3^2 \sigma_4^2 a_3 a_4 (a_3 + a_4) (\sigma_4 - \sigma_3)}$$

$$R_3 = k_z$$

$$\sigma_4, \sigma_3 = \frac{\sigma_o - 2\alpha_o + [(\sigma_o + 2\alpha_o)^2 - 4\omega_o^2]^{\frac{1}{2}}}{2}$$

Figure 23. Coupling Circuit G.

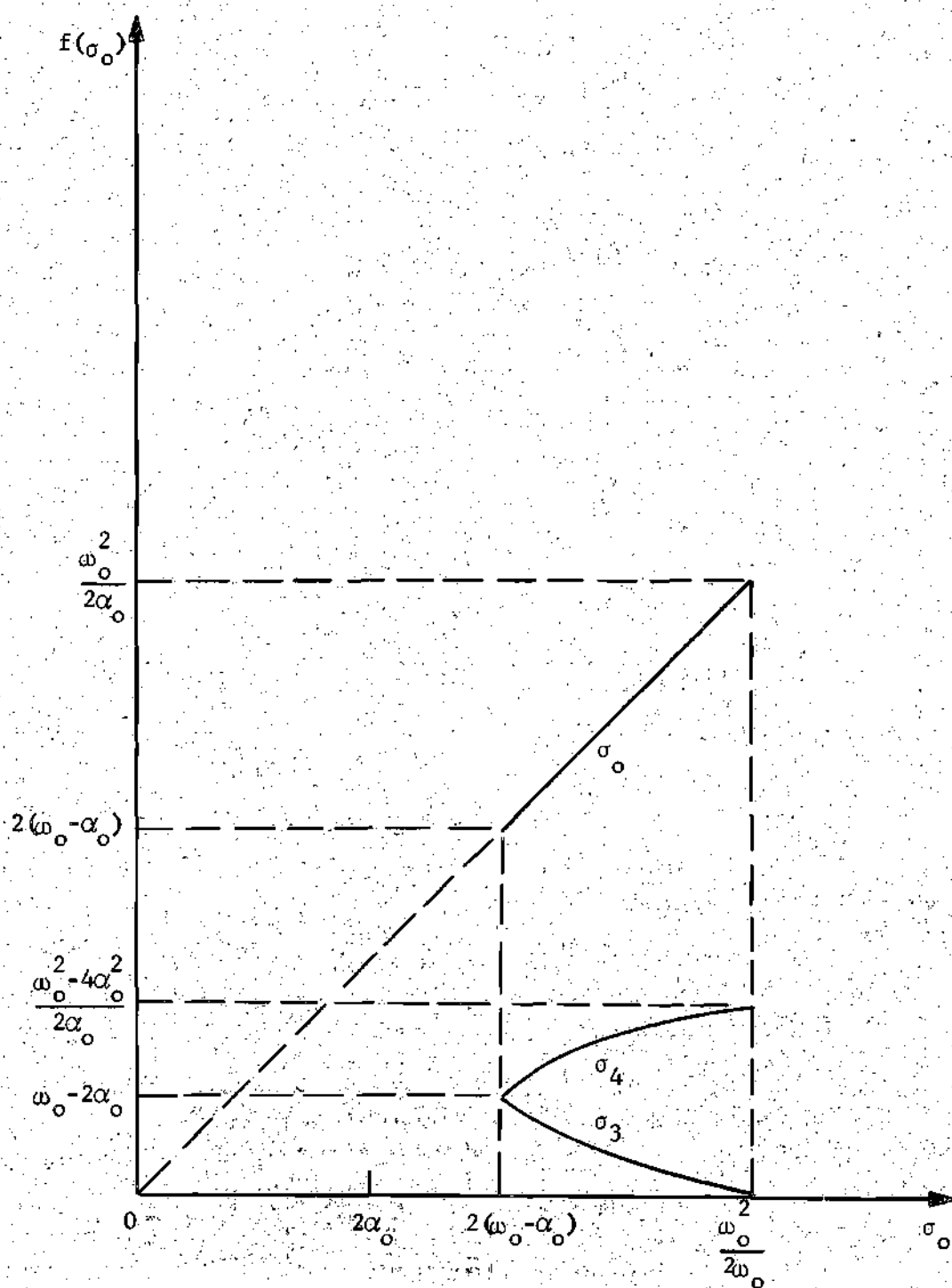
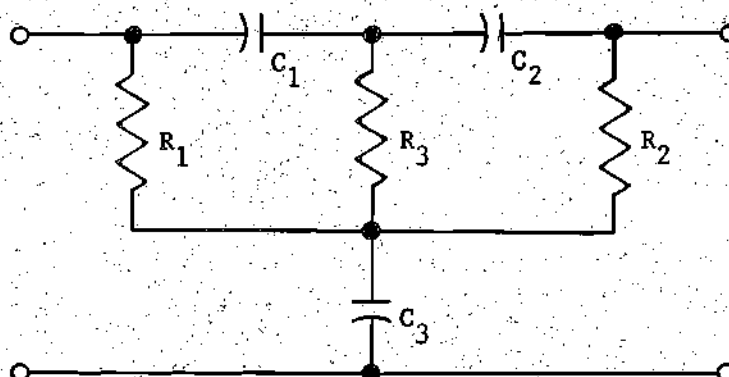


Figure 24. Possible Pole Locations for Coupling Circuit G.



$$C_1 = \frac{(a_3 + a_4)(\sigma_4 - \sigma_3)}{k_z [(a_3 - 1)\sigma_3 + (a_4 + 1)\sigma_4]^2}$$

$$C_2 = \frac{a_3 a_4 (a_3 + a_4)(\sigma_4 - \sigma_3)}{k_z [a_3(a_4 + 1)\sigma_4 - a_4(a_3 - 1)\sigma_3]^2}$$

$$C_3 = \frac{\sigma_3 \sigma_4}{k_z \sigma_0 \omega_0^2}$$

$$R_1 = \frac{k_z [(a_3 - 1)\sigma_3 + (a_4 + 1)\sigma_4]}{\sigma_4 - \sigma_3}$$

$$R_2 = \frac{k_z [a_3(a_4 + 1)\sigma_4 - a_4(a_3 - 1)\sigma_3]}{a_4 a_3 (\sigma_4 - \sigma_3)}$$

$$R_3 = \frac{k_z [(a_3 - 1)\sigma_3 + (a_4 + 1)\sigma_4] [a_3(a_4 + 1)\sigma_4 - a_4(a_3 - 1)\sigma_3]}{(a_3 + a_4)^2 \sigma_3 \sigma_4}$$

$$\sigma_4 - \sigma_3 = \frac{\omega_0^2 - 2\alpha_0 \sigma_0 + [(\omega_0^2 + 2\alpha_0 \sigma_0)^2 - 4\sigma_0^2 \omega_0^2]^{\frac{1}{2}}}{2(\sigma_0 - 2\alpha_0)}$$

Figure 25. Coupling Circuit H.

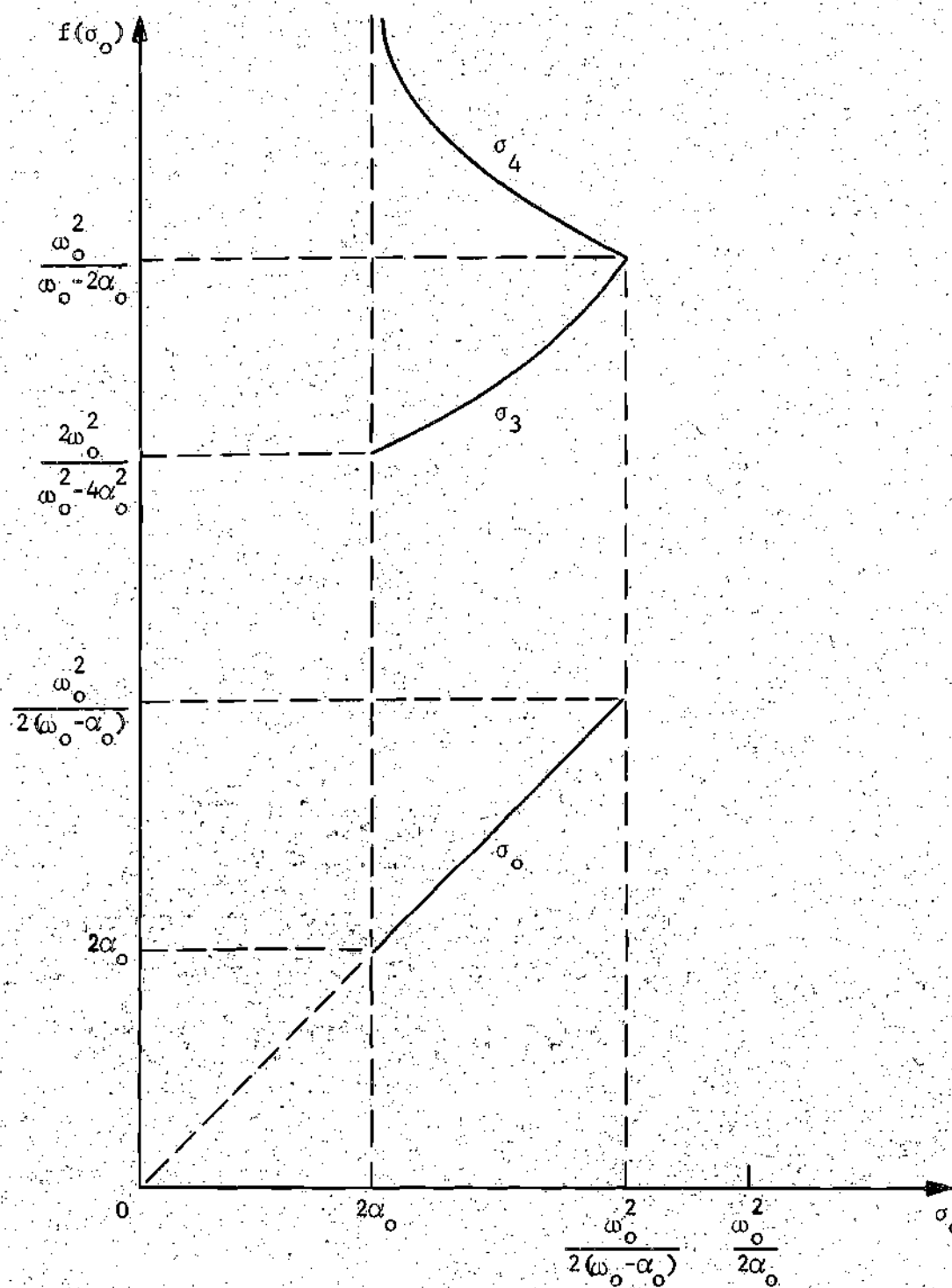


Figure 26. Possible Pole Locations for Coupling Circuit H.

CHAPTER III

THEORY OF THE PROCEDURE

The Preparatory Step Conditions

The problem of satisfying the preparatory-step conditions, (39) through (42) or (46) through (49), is the major problem associated with the cascade synthesis procedure. The problem involves satisfying five equations and therefore at least five variables are required. In general, three of the variables are present in the coupling circuit functions, y_{11} or z_{11} . The other variables must be obtained by removing series or shunt elements from the driving-point immittance, Y_1 or Z_1 .

If the removal of these elements, which are represented by Y_p or Z_p , consists of partially removing one or more of the finite poles of Y_1 or Z_1 , no new transmission zeros are introduced and the remainder Y_{1p} or Z_{1p} is RC. Partial removal of finite poles in the form of a shunt removal of an RC series branch or a series removal of an RC parallel branch maintains the capacitive feed-through path and the resistive feed-through path. These paths are also present in the coupling circuits and thus optimum gain for the overall transfer function is ensured.

Partial removal of a pole at infinity or the origin as a shunt or series capacitance along with the removal of a series or shunt resistor is often necessary in order to satisfy the preparatory-step conditions. In some cases, these will adversely affect the gain of the overall transfer function. At each realization of a pair of transmission zeros, several options are available to satisfy the preparatory-step conditions.

In many cases, one of the options will lead to the most satisfactory solution of the given problem. Synthesis of right-half-plane transmission zeros should take place in the realization before other transmission zeros because of their larger number of restrictions and because more avenues are open for satisfactory solutions.

The preparatory-step conditions may be obtained by first writing the cascade-synthesis equation

$$(y_{11} - Y_{1p})(y_{22} + Y_2) = y_{12}^2 \quad (90)$$

in the form

$$\frac{1}{y_{22} + Y_2} = \frac{y_{11} - Y_{1p}}{y_{12}^2} \quad (91)$$

The problem is to determine the relationship between y_{11} and Y_{1p} which will ensure that the function

$$\frac{1}{y_{22} + Y_2} \quad (92)$$

will have the properties of an RC driving-point impedance and then finally to determine the conditions under which Y_2 will be RC admittance.

Function (92) may contain only real poles and real zeros if it is to be an RC impedance. Therefore, the second-order complex pole present in y_{12}^2 on the right side of (91) must cancel. The numerator function, $y_{11} - Y_{1p}$, then must contain the factor $(s^2 - 2\alpha_0 s + \omega_0^2)^2$ which is equivalent to requiring

$$y_{11}(s_0) = Y_{1p}(s_0) \quad (93)$$

and

$$y_{11}'(s_o) = Y_{1p}'(s_o). \quad (94)$$

Because function (92) must contain only simple poles, the right side of (91) must have only simple poles. Therefore, at least one cancellation of the surplus factor present in y_{12}^2 must take place with the numerator function, $y_{11} - Y_{1p}$. If $y_{11} - Y_{1p}$ is to contain the factor $(s + \sigma_o)$, then it is required that

$$y_{11}(-\sigma_o) = Y_{1p}(-\sigma_o). \quad (95)$$

Under the requirements given thus far, function (92) has real poles and real zeros of first order. There is one pole at $s = -\sigma_o$ and all of the other poles are also the poles of Y_{1p} . If the residues of all of the poles of the impedance function (92) are positive, then the function will be RC.

Now let the residue of the pole of Y_{1p} at $s = -s_v$ be given by $-k_{11v}$. (The residues in an expansion of an RC driving-point admittance are negative for all finite poles.) The residue of function (92) which also has a pole at $s = -s_v$, is given by

$$\frac{\left[(s + s_v) y_{11} - (s + s_v) Y_{1p} \right]_{s = -s_v}}{\left[y_{12}(s_v) \right]^2}, \quad (96)$$

except in the case where y_{11} and Y_{1p} have a common pole. This case will be discussed later. The first term in the numerator of (96) is zero if

y_{11} does not also have a pole at $s = -s_v$. The second term of the numerator of (96) becomes

$$[-(s + s_v) y_{1p}]_s = -s_v = +k_{1lv}. \quad (97)$$

The denominator term of (96) is always positive. The conclusion is that the residues of (92) are always positive for all such poles.

The residue of the pole of (92) at $s = -\sigma_0$ is given by

$$\left[\frac{(s + \sigma_0)(y_{11} - y_{1p})}{y_{12}^2} \right]_{s = -\sigma_0}. \quad (98)$$

The denominator of (98) may be replaced by

$$y_{12}^2 = k_y^2 \frac{(s + \sigma_0)^2 (s^2 - 2\alpha_0 s + \omega_0^2)^2}{(s + \sigma_1)^2 (s + \sigma_2)^2}, \quad (99)$$

The residue (98) then becomes

$$\left[\frac{(s + \sigma_1)^2 (s + \sigma_2)^2 (y_{11} - y_{1p})}{k_y^2 (s + \sigma_0) (s^2 - 2\alpha_0 s + \omega_0^2)^2} \right]_{s = -\sigma_0}. \quad (100)$$

The quantity in (100) is indeterminate because $y_{11} - y_{1p}$ contains the factor $(s + \sigma_0)$. Applying l'Hospital's rule to (100), the residue becomes

$$\frac{(\sigma_1 - \sigma_0)^2 (\sigma_2 - \sigma_0)^2 [y_{11}'(-\sigma_0) - y_{1p}'(-\sigma_0)]}{k_y^2 (\sigma_0^2 + 2\alpha_0 \sigma_0 + \omega_0^2)^2}. \quad (101)$$

The residue will be positive, if

$$y_{11}'(-\sigma_0) > Y_{1p}'(-\sigma_0). \quad (102)$$

Consequently, if Y_{1p} is RC and if y_{11} and Y_{1p} have no pole in common, it has been shown that satisfying (93), (94), (95), and (102) will cause function (92) to be RC.

The equivalent set of preparatory-step conditions for the impedance case may be obtained by examining the equation

$$\frac{1}{z_{22} + z_2} = \frac{z_{11} - z_{1p}}{z_{12}^2}. \quad (103)$$

The function

$$\frac{1}{z_{22} + z_2} \quad (104)$$

will be RC if z_{1p} is RC, if z_{1p} and z_{11} have no poles in common, and if

$$z_{11}(s_0) = z_{1p}(s_0) \quad (105)$$

$$z_{11}'(s_0) = z_{1p}'(s_0) \quad (106)$$

$$z_{11}(-\sigma_0) = z_{1p}(-\sigma_0) \quad (107)$$

$$z_{11}'(-\sigma_0) < z_{1p}'(-\sigma_0). \quad (108)$$

Either set of conditions, admittance or impedance, may be applied during the preparatory step. If one set of conditions is satisfied, the

other is also satisfied.

If Y_{1p} and y_{11} have a pole in common, that pole will also be a pole of Y_2 . This may be determined from the second term of the right side of the equation

$$Y_2 = \frac{y_{11}y_{22} - y_{12}^2}{Y_{1p} - y_{11}} + \frac{y_{22}Y_{1p}}{y_{11} - Y_{1p}}, \quad (109)$$

after noting that the pole is also a pole of y_{22} by construction. The pole is canceled in the first term because of the compact nature of the coupling circuit. Similarly, if Z_{1p} and z_{11} have a pole in common, that pole is also a pole of Z_2 and z_{22} .

The conditions under which $1/(y_{22} + Y_2)$ and $1/(z_{22} + Z_2)$ are RC functions are the same. That is, if $1/(y_{22} + Y_2)$ is RC, then $1/(z_{22} + Z_2)$ is RC, and vice-versa. The only possibility that Y_2 is not RC is that the Foster residue of Y_2 , at a pole common to y_{22} , is negative and less in magnitude than the corresponding Foster residue of y_{22} . However, this would mean that the reciprocal of Y_2 , Z_2 , must have at least one pole in common with z_{22} and that its negative residue would be less in magnitude than that of the z_{22} -pole in order that $1/(z_{22} + Z_2)$ may be RC. But if z_{22} and Z_2 have a pole in common, that pole must also be a zero of Y_{1p} . Thus, for the case of common poles, one cannot be completely sure of an RC termination until after completion of the preparatory step.

In the synthesis procedure, the assumption is made initially that the satisfaction of the preparatory-step conditions (93), (94), (95), and (102) or conditions (105), (106), (107), and (108) will ensure an RC termination. If satisfaction of the conditions does not result in an RC

termination, the cause will be that both the admittance and the impedance functions have common poles and that the residues exceed the allowable values in both cases. It is highly improbable that these phenomena will ever occur. In the case of occurrence, the preparatory-step equations may be recalculated using a different pole of the driving point immittance as a variable parameter.

As a matter of interest, the condition for common poles may be derived for the admittance case from equation (109). For a common pole at $s = -s_c$, the residue of Y_2 at the pole will be positive if

$$\left[(s + s_c) \frac{y_{11}}{s} \right]_{s = -s_c} > \left[(s + s_c) \frac{y_{1p}}{s} \right]_{s = -s_c} . \quad (110)$$

It is highly probable that condition (110) will be satisfied if the impedance or admittance conditions listed previously are satisfied, and therefore the condition is not used in the procedure. The condition applies for poles at infinity and also for the zero-frequency values of the admittance functions.

Surplus Factor Cancellation

The numerator of the function $y_{22} + Y_2$ will contain the factor $(s + \sigma_0)$ assuming that the preparatory-step conditions of the preceding section are satisfied. The transfer function Y_{12} then contains the factor $(s + \sigma_0)$ once in the numerator and once in the denominator as indicated by the equation,

$$Y_{12} = \frac{y_{12} Y_2}{y_{22} + Y_2} . \quad (111)$$

Therefore the factor $(s + \sigma_0)$ is not a transmission zero but is a surplus factor of the same type as that used, for example, in the Guillemin parallel-ladder procedure for realizing right-half-plane transmission zeros. The factor is also cancelled from the transfer impedance function,

$$Z_{12} = \frac{z_{12}z_2}{z_{22} + z_2} \quad (112)$$

The factor also augments both numerator and denominator of the driving-point immittances, Y_1 and Z_1 , as indicated by the equations

$$Y_1 = \frac{(y_{22} + Y_2)(y_{11} + Y_p) - y_{12}^2}{y_{22} + Y_2} \quad (113)$$

and

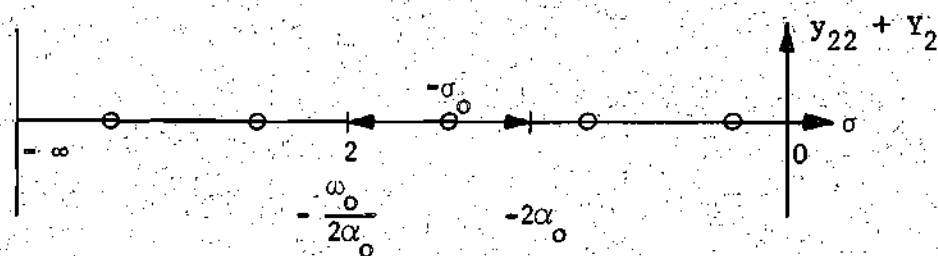
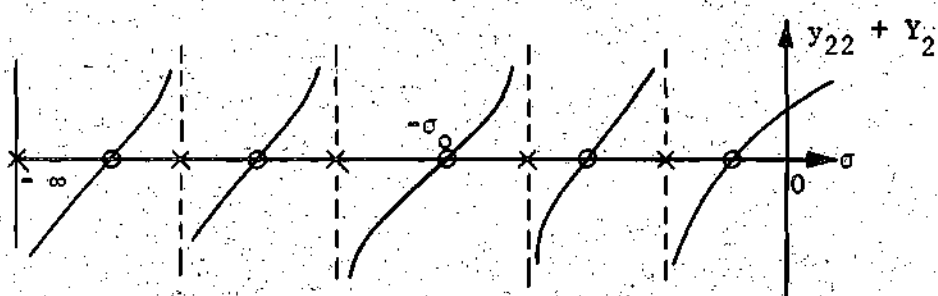
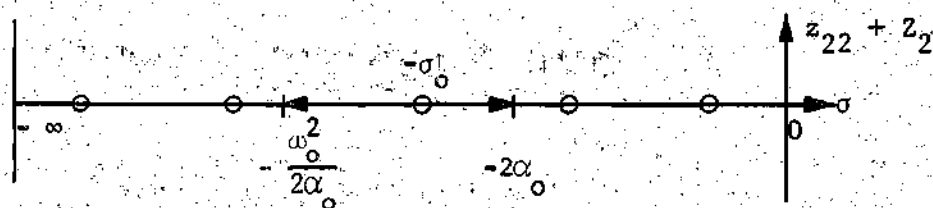
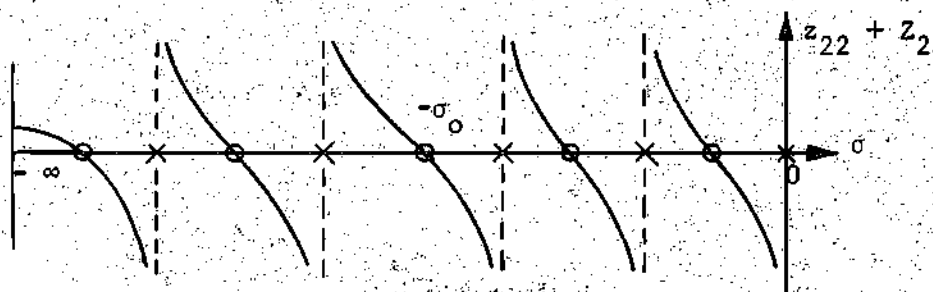
$$Z_1 = \frac{(z_{22} + Z_2)(z_{11} + Z_p) - z_{12}^2}{z_{22} + Z_2} \quad (114)$$

where the factor $(s + \sigma_0)$ is present in $y_{22} + Y_2$, y_{12}^2 , $z_{22} + Z_2$, and z_{12}^2 .

The Choice of Coupling Circuit

The zeros of $y_{22} + Y_2$ are located at $s = -\sigma_0$ and at the poles of the driving-point function Y_1 , if the assumptions are made that y_1 and Y_1 have no common poles and that the preparatory conditions may be satisfied by removal of shunt elements. The choice of coupling circuit for use in satisfying the preparatory conditions may be made by first plotting the zeros of $y_{22} + Y_2$ on the negative σ -axis. A typical plot is indicated in Figure 27. The zero at $s = -\sigma_0$ is located in the range,

$$-2\alpha_0 > -\sigma_0 > -\frac{\omega_0^2}{2\alpha_0} \quad (115)$$

Figure 27. Zeros of $y_{22} + Y_2$.Figure 28. Sketch of $y_{22} + Y_2$.Figure 29. Zeros of $z_{22} + Z_2$.Figure 30. Sketch of $z_{22} + Z_2$.

The next step is to sketch the RC function $y_{22} + Y_2$ which is similar to the sketch of Figure 1, using the known zeros to determine the ranges of the pole locations. The sketched function should have a pole at $s = -\infty$ because the y_{22} function for each coupling circuit has a pole at $s = -\infty$. A typical sketch is indicated in Figure 28. Two of the poles on the finite part of the axis are the poles of y_{22} , located at $s = -\sigma_2$ and $s = -\sigma_1$. The remaining poles are poles of Y_2 . The last step is to determine which, if any, of the coupling circuits of Chapter II have ranges of σ_2 and σ_1 that will each coincide at some point with the ranges of any two of the finite poles in the sketch of $y_{22} + Y_2$. Trial-and-error calculation using different values for σ_0 to calculate σ_2 and σ_1 is required at this point. If a circuit cannot be found which will satisfy the alternating-pole-zero requirement, the poles of the driving-point immittance may be shifted to acceptable ranges by removing series elements or a combination of series and shunt elements. Once the values of σ_0 , σ_1 , and σ_2 are specified, they are used in the equation for y_{11} when satisfying the preparatory-step requirements.

If it is desired to make one of the poles of y_{11} coincide with a pole of Y_1 , the sketch of $Y_2 + y_{22}$ must be redrawn, this time with a pole replacing the zero which corresponded to the pole of Y_1 . This pole will be a pole of both y_{22} and Y_2 . As in the previous case, Y_2 will have two less poles than Y_1 . If both σ_1 and σ_2 coincide with poles of Y_1 , Y_2 will also have two poles less than Y_1 .

For the impedance case, the zeros of $z_{22} + Z_2$ are located at $s = -\sigma_0$ and at the poles of Z_1 , if the assumptions are made that Z_1 and z_{11} have no pole in common and that the preparatory-step conditions may

be satisfied by removing series elements only. The choice of a coupling circuit is made by first plotting the zeros of $z_{22} + Z_2$ on the negative σ -axis, as in Figure 29, then sketching an impedance function similar to Figure 2 using the zeros. The sketch will always have a pole at the origin and will have a finite, positive value at $s = -\infty$ because of the properties of z_{22} . A typical sketch is indicated in Figure 30. Two of the finite poles are the z_{22} poles at $s = -\sigma_3$ and $s = -\sigma_4$. Hopefully, at least one of the coupling circuits of Chapter II will have pole locations which will coincide with two of the allowable pole locations of the sketch. Again, trial-and-error calculation is required and if no circuit fits the requirement, shunt elements must be removed in an attempt to shift the Z_1 poles to more suitable locations.

As in the admittance case, the sketch must be redrawn if the poles of z_{11} coincide with those of Z_1 . The common poles are also poles of z_{22} and of Z_2 . Z_2 will always have two poles less than Z_1 . Z_2 was shown to have two zeros less than Z_1 in the preceding discussion for the admittance case. The degree of the terminating immittance is two less than that of the driving-point immittance in both numerator and denominator, as in the Dasher synthesis procedure.

After choosing the circuit, either admittance or impedance, the y_{11} -function or z_{11} -function is used to attempt to satisfy the preparatory-step conditions. A solution to the preparatory-step conditions is not possible unless the alternating-pole-zero requirement is met on both the impedance and the admittance basis for a given circuit. The procedure for choosing a circuit caused the alternating-pole-zero requirement for either the impedance or the admittance functions to be satisfied, but not

both. Except in the case of Coupling Circuit F, the compact pole locations of the circuits are not known for both the z - and y -functions until after solution of the preparatory-step equations.

However, the success of the procedure is almost certain if the poles of either Z_1 or Y_1 can be spread over as wide a range of the finite negative σ -axis as possible. For example, if $Z_1(s)$ has a positive and finite value at $s = \infty$ and has a pole at $s = 0$, removal of a series resistor and a series capacitor will move one zero in the direction of $s = -\infty$ and one zero in the direction of $s = 0$. The result is a Y_1 -function with widely spaced poles. The same argument holds for Y_1 -functions, which should have a pole at $s = \infty$. Removal of a shunt capacitance and a shunt conductance while satisfying the preparatory step will spread the poles of Z_1 , making a solution more certain.

It is always possible to spread the poles of the Y_1 or Z_1 function in order to satisfy the pole-interlacing requirement for one of the coupling circuits. The method used to spread the poles of a Y_1 function with a pole at $s = \infty$ is indicated in Figure 31. First, a shunt conductance and a shunt capacitance are removed to shift the extreme zeros outward. The admittance function, Y_a , is then inverted and two RC-parallel branches are removed in series, partially removing the extreme poles and shifting the original poles of the function outward. The final result is an admittance function, Y_b , with widely spaced poles. A similar procedure is illustrated in Figure 32 for a Z_1 function which does not have a zero at $s = \infty$. In this case, a series resistance is removed in the first step to shift a zero near to the origin. The remainder, Z_a , is inverted and an RC series branch removed in shunt to partially remove the pole

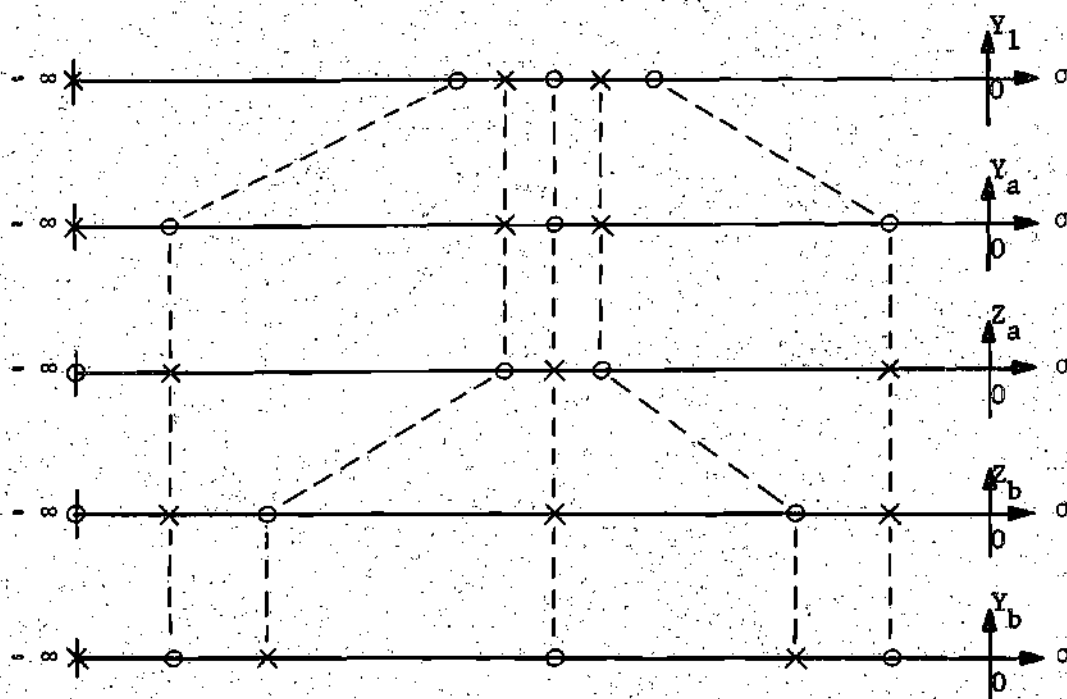


Figure 31. Spreading the Poles of Y_1 .

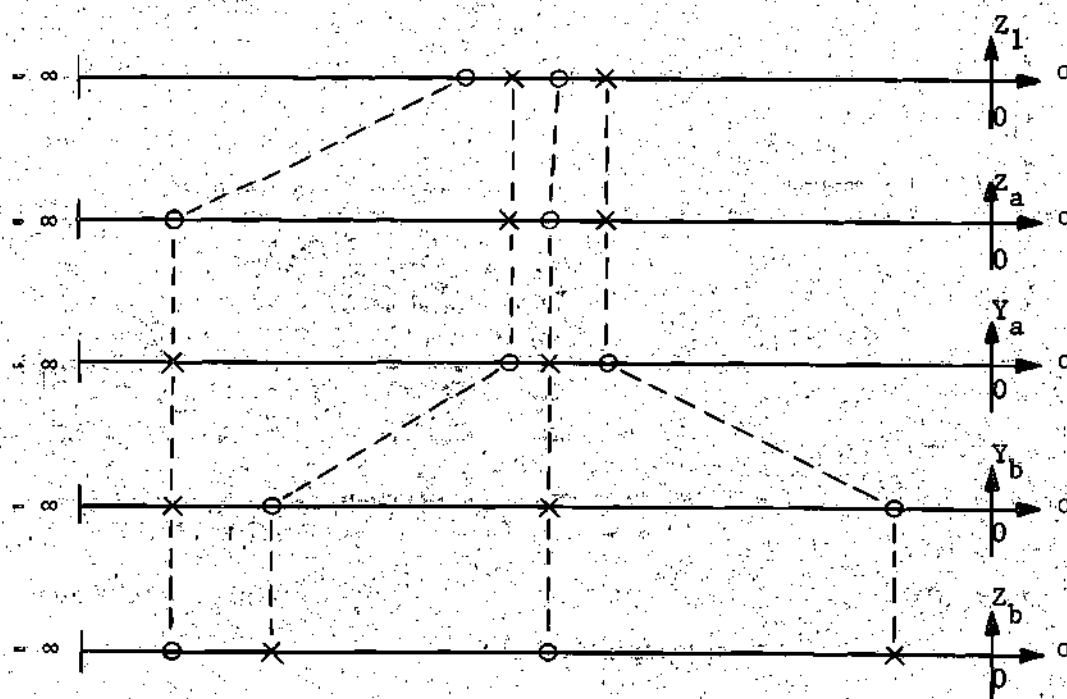


Figure 32. Spreading the Poles of Z_1 .

nearest to $s = -\infty$. A shunt conductance is also removed and the original poles have been shifted outward. The final result is an impedance function, Z_b , with widely spaced poles.

CHAPTER IV

EXAMPLES OF THE PROCEDURE

Example Using Admittance Functions

As an example illustrating the procedure, assume

$$Y_1 = \frac{(s + 0.5)(s + 1.5)(s + 2.5)(s + 3.5)}{(s + 1)(s + 2)(s + 3)} \quad (116)$$

and that it is desired to realize a pair of transmission zeros at

$$s = 3 \pm j9. \quad (117)$$

The first step is to choose a circuit. The admittance circuits are to be used because Y_1 has a pole at infinity. (If Y_1 did not have a pole at infinity, impedance circuits would be used.) The range of allowable values for σ_0 is calculated from (36) to be

$$6 \leq \sigma_0 \leq 15. \quad (118)$$

The numerator factors of the function $y_{22} + Y_2$ are

$$(s + \sigma_0)(s + 1)(s + 2)(s + 3). \quad (119)$$

These are plotted on the negative σ -axis in Figure 33. A sketch of an RC driving-point admittance function with a pole at $s = \infty$ is made using these zeros. The sketch is indicated in Figure 34. Because all of the finite poles are to the right of $-\sigma_0$ on the sketch, it is evident that the circuit chosen must be such that

$$\sigma_0 > \sigma_2 > \sigma_1 \quad (120)$$

Coupling Circuits D and E of Chapter II meet requirement (120). Coupling Circuit D is chosen arbitrarily and by trial-and-error calculation, the values

$$\sigma_0 = 13.5, \quad (121)$$

$$\sigma_2 = 6, \quad (122)$$

and

$$\sigma_1 = 1.5, \quad (123)$$

are found to be compatible with the sketch of $y_{22} + Y_2$ in Figure 34. Removal of series elements to shift the poles of Y_1 is not necessary for this example.

The next step is to see if it is possible to satisfy the preparatory-step conditions using the values of σ_0 , σ_2 , and σ_1 of the previous step in the y_{11} -function. The real and imaginary parts of the following equations must be equal:

$$y_{11}(-13.5) = Y_{1p}(-13.5) \quad (124)$$

$$y_{11}(3 + j9) = Y_{1p}(3 + j9) \quad (125)$$

$$y_{11}'(3 + j9) = Y_{1p}'(3 + j9) \quad (126)$$

where

$$Y_{1p} = Y_1 - G - Cs - \frac{As}{s+1} - \frac{Bs}{s+2} - \frac{Ds}{s+3} \quad (127)$$

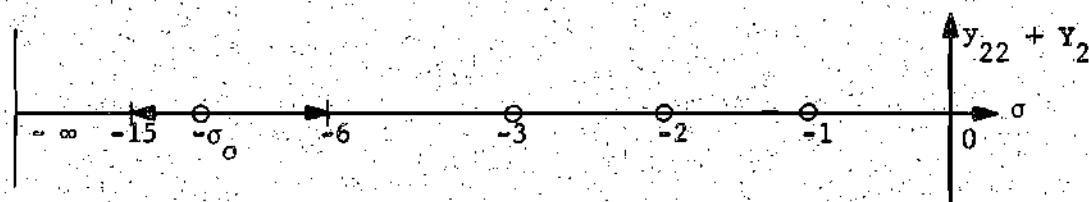
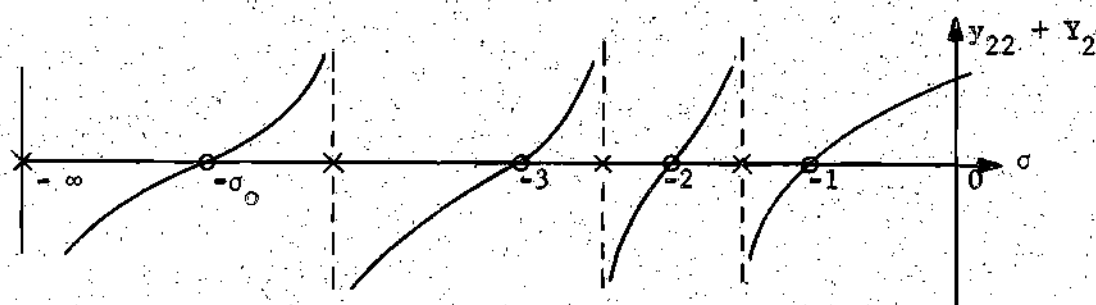
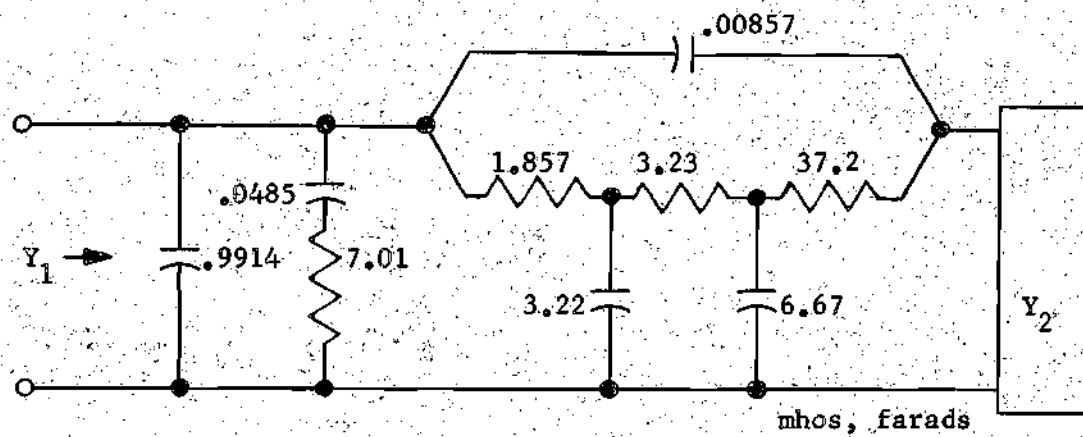
Figure 33. Zeros of $y_{22} + Y_2$.Figure 34. Sketch of $y_{22} + Y_2$.

Figure 35. Final Circuit of Admittance Example.

and

$$y_{11} = k_y \left\{ s + 135 + \frac{180a_1s}{s + 1.5} + \frac{45a_2}{s + 6} \right\} \quad (128)$$

In (127) it is assumed that

$$Y_p = G + C_s + \frac{A_s}{s + 1} + \frac{B_s}{s + 2} + \frac{D_s}{s + 3} \quad (129)$$

where the values of G, C, A, B, and D must be less than the corresponding values in the Foster expansion of Y_1 . At least two of the variables will be required to complete the five unknowns necessary to solve the five equations represented by (124), (125), and (126). There is no way of knowing at the outset which of the variables should be used to satisfy the equations. The five equations become

$$\begin{aligned} 202.5k_y a_1 + 81k_y a_2 + 121.5k_y &= \\ -11.37 - G + 13.5C - 1.08A - 1.174B - 1.286D \end{aligned} \quad (130)$$

$$\begin{aligned} 168.0k_y a_1 + 30k_y a_2 + 138.0k_y &= \\ 4.93 - G - 3C - .959A - .906B - .846D \end{aligned} \quad (131)$$

$$\begin{aligned} 24.0k_y a_1 + 15k_y a_2 + 9.0k_y &= \\ 9.13 - 9C - .093A - .170B - .231D \end{aligned} \quad (132)$$

$$-1.6k_y a_1 + k_y = .992 - C + .007A + .010B + .010D \quad (133)$$

$$-2.1k_y a_1 - 1.7k_y a_2 = -.0118 + .008A + .016B + .024D \quad (134)$$

Truncated numbers are used in the preceding equations. The actual calculations were carried out using ten significant figures for each term of

the equations. The Gauss-Jordan reduction method may be used to eliminate the variables in the order: k_{y1} , k_{y2} , k_y , and C . A solution which satisfies the equations but does not over-remove the poles of Y_1 is

$$k_y = .00857 \quad (135)$$

$$a_1 = .451 \quad (136)$$

$$a_2 = .0114 \quad (137)$$

$$A = 0 \quad (138)$$

$$B = 0 \quad (139)$$

$$C = .9914 \quad (140)$$

$$D = .143. \quad (141)$$

Next, the inequality given by (102),

$$y_{11}'(-13.5) = .016286 > y_{1p}'(-13.5) = .016175 \quad (142)$$

is checked using the solution of the equations. The inequality is satisfied, and therefore the coupling circuit may be removed.

The admittance functions of the ladder-circuit representation for the coupling circuit and the preparatory step are calculated from the solution of the preparatory-step equations. The resulting functions are

$$Y_p = .9914s + \frac{.1428s}{s + 3} \quad (143)$$

$$y_1 = \frac{2.238s}{s + 1.5} - \frac{.381s}{s + 6} = \frac{1.857s(s + 6.924)}{s^2 + 7.5s + 9} \quad (144)$$

$$-y_{12} = \frac{.00857(s + 13.5)(s^2 - 6s + 90)}{(s + 1.5)(s + 6)} \quad (145)$$

$$y_2 = \frac{4.9616s}{s + 1.5} + \frac{33.44s}{s + 6} \quad (146)$$

The y_1 -function does not have a zero at $s = -\sigma_0$ and therefore the coupling circuit is z -compact.

Removing the elements in a step-by-step manner results in the RC terminating admittance

$$Y_2 = \frac{6.532(s + 1.99)(s + 4.28)}{s + 2.77} \quad (147)$$

which may be used to realize other transmission zeros. The function Y_2 could have been calculated directly from the equation

$$Y_2 = \frac{y_{12}^2}{y_{11} - y_{1p}} - y_{22} \quad (148)$$

for the solution of the preparatory step leaves Y_2 as the only unknown parameter. All element values of the coupling circuit and of the preparatory shunt branches may be calculated using the solution to the equations.

It is interesting to note that Y_2 has a pole at $s = 2.77$, which is within the range of values predicted from the sketch of $y_{22} + Y_2$.

The element values are calculated with the aid of Figure 17 and the final circuit is given in Figure 35.

Example Using Impedance Functions

As a second example, assume

$$Z_1 = \frac{(s + 0.5)(s + 1.5)(s + 2.5)(s + 3.5)}{s(s + 1)(s + 2)(s + 3)} \quad (149)$$

and that the cascade synthesis procedure is to be used to realize a pair of transmission zeros at

$$s = 1 \pm j2. \quad (150)$$

The range of σ_0 calculated from (36) is

$$2 \leq \sigma_0 \leq 2.5. \quad (151)$$

Impedance functions and circuits are used because Z_1 does not have a zero at $s = -\infty$.

The functions Z_1 and z_{11} have a common pole at $s = 0$. The pole will also be common to Z_2 and z_{22} . Therefore the zeros of the function, $z_{22} + Z_2$, are given by the factors

$$(s + \sigma_0)(s + 1)(s + 2)(s + 3). \quad (152)$$

These zeros are plotted on the negative σ -axis, along with the known pole at $s = 0$, in Figure 36. A sketch of an RC driving point impedance with a finite positive value at $s = \infty$ is made using the zeros and pole of Figure 36. The sketch is indicated in Figure 37.

An examination of the impedance circuit listed in Chapter II reveals that Coupling Circuit F has pole locations compatible with Figure 37. By trial-and-error calculation, the value

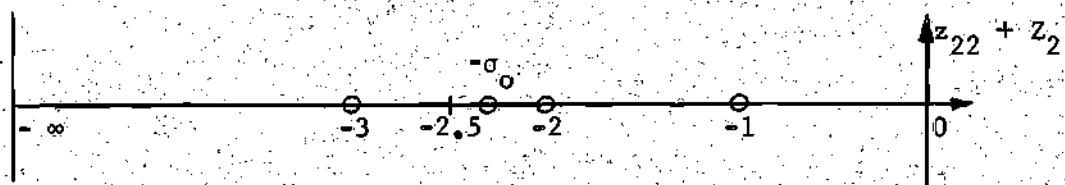
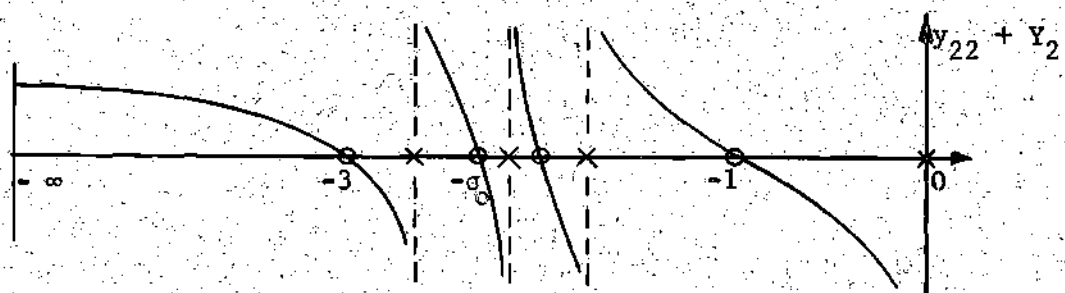
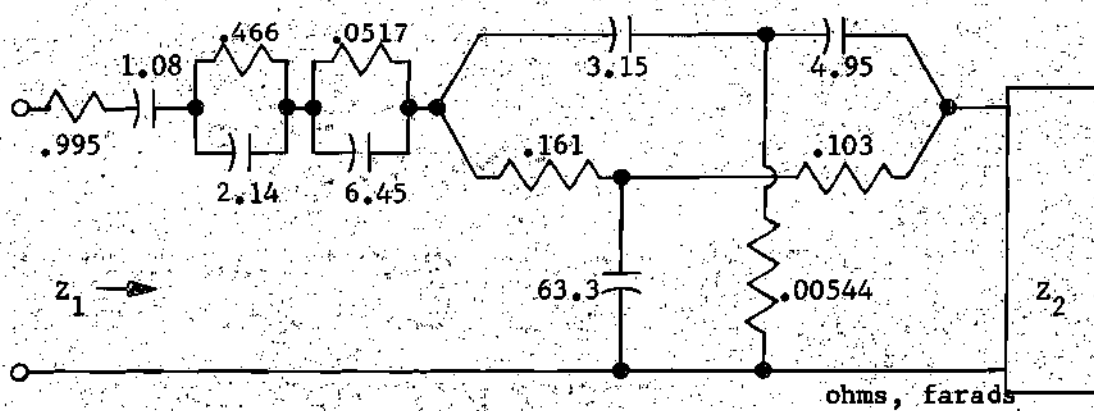
Figure 36. Zeros of $z_{22} + z_2$.Figure 37. Sketch of $z_{22} + z_2$.

Figure 38. Final Circuit of Impedance Example.

$$\sigma_0 = 2.252 \quad (153)$$

is found to place poles at $s = -\sigma_4$ and $s = -\sigma_3$ on either side of the zero at $s = -2$. These values are

$$\sigma_4 = 2.0431138277 \quad (154)$$

and

$$\sigma_3 = 1.9682539682. \quad (155)$$

The preceding values are then used to calculate z_{11} when satisfying the preparatory-step equations,

$$z_{11}(1 + j2) = z_{1p}(1 + j2) \quad (156)$$

$$z_{11}'(1 + j2) = z_{1p}'(1 + j2) \quad (157)$$

$$z_{11}(-2.252) = z_{1p}(-2.252), \quad (158)$$

where

$$z_{1p} = z_1 - R - \frac{1}{Cs} - \frac{A}{s+1} - \frac{B}{s+2} - \frac{D}{s+3} \quad (159)$$

and the values of R , C , A , B , and D are less than the corresponding values of the partial fraction expansion of z_1 .

The z_{11} -function, using truncated numbers and using the circuit restriction that $a_4 = 1$, is

$$z_{11} = k_z \left(1 + \frac{2.80}{s} + \frac{18.11}{s+2.04} + \frac{24.67a_3}{s+1.97} \right). \quad (160)$$

Equations (156), (157), and (158), using all possible variables and equating the real and imaginary parts, become

$$-86.94k_z(1 + a_3) = -.767 - R + .444 \frac{1}{C} + .798A + 3.97B - 1.33D \quad (161)$$

$$5.71k_z(1 + a_3) = 1.432 - R - .200 \frac{1}{C} - .250A - .230B - .20D \quad (162)$$

$$-3.85k_z(1 + a_3) = -.613 + .400 \frac{1}{C} + .250A + .154B - .1D \quad (163)$$

$$-.517k_z a_3 - .205k_z(1 + a_3) = .118 - .120 \frac{1}{C} + .030B + .03D \quad (164)$$

$$.083k_z a_3 + 1.701k_z(1 + a_3) = .259 - .160 \frac{1}{C} + .125A - .071B - .04D. \quad (165)$$

Again, truncated numbers are used in the preceding equations.

One procedure for solving the equations is to eliminate the variables in the order: $k_z(1 + a_3)$, R , $k_z a_3$, and $1/C$. One solution which satisfies the restriction that Z_{1p} must be RC is

$$k_z = .003686 \quad (166)$$

$$a_3 = 2.474 \quad (167)$$

$$R = .996455 \quad (168)$$

$$\frac{1}{C} = 1.0841 \quad (169)$$

$$A = .4624 \quad (170)$$

$$B = 0 \quad (171)$$

$$D = .1500. \quad (172)$$

However, the preceding solution does not meet the preparatory-step inequality, condition (108),

$$z_{11}'(-2.252) < z_{1p}'(-2.252). \quad (173)$$

Therefore a new solution must be found. By trial and error, the following solution which does satisfy the inequality is determined.

$$k_z = .005001 \quad (174)$$

$$a_3 = 1.56979 \quad (175)$$

$$R = .995041 \quad (176)$$

$$\frac{1}{C} = 1.07998 \quad (177)$$

$$A = .46633 \quad (178)$$

$$B = 0 \quad (179)$$

$$D = .15500. \quad (180)$$

Using the latter solution, the z-functions of the tee representation of the coupling circuit are

$$z_1 = \frac{.3171}{s + 1.97} \quad (181)$$

$$z_{12} = \frac{.005001(s + 2.252)(s^2 - 2s + 5)}{(s + 1.97)(s + 2.04)} \quad (182)$$

$$z_2 = \frac{.2020}{s + 1.97} \quad (183)$$

and the RC terminating impedance is

$$Z_2 = \frac{.5791(s + 1.228)(s + 2.712)}{s(s + 2.422)} \quad (184)$$

The final circuit is indicated in Figure 38.

CHAPTER V

DISCUSSION OF THE PROCEDURE

A Special Technique

The procedure developed in this thesis may be applied in many cases to the problem where the transfer function and termination are given. As an illustration, the given transfer function is

$$z_{12} = \frac{s^2 - 6s + 90}{(s + 1)(s + 2)} \quad (185)$$

and the given terminating function is

$$z_2 = 1. \quad (186)$$

The zeros of the function $z_{22} + z_2$ are given by the factors

$$(s + \sigma_0)(s + 1)(s + 2).$$

The alternating-pole-zero requirement is satisfied in this case by Coupling Circuit G with

$$\sigma_0 = 13.5 \quad (188)$$

$$\sigma_4 = 6 \quad (189)$$

$$\sigma_3 = 1.5. \quad (190)$$

The values of σ_0 , σ_4 , and σ_3 are used to calculate z_{22} and the following evaluations are made:

$$[z_{22} + z_2]_{s=-1} = 0 = -134k_z + 360k_z a_3 + 9k_z a_4 + 1 \quad (191)$$

$$[z_{22} + z_2]_{s=-2} = 0 = -66.5k_z - 360k_z a_3 + 7.5k_z a_4 + 1 \quad (192)$$

$$[z_{22} + z_2]_{s=-13.5} = 0 = -9k_z - 15k_z a_3 - 6k_z a_4 + 1 \quad (193)$$

Solving the preceding equations results in

$$k_z = .02078100023 \quad (194)$$

$$a_3 = .08058608067 \quad (195)$$

$$a_4 = 6.318681315. \quad (196)$$

These values are used to calculate the circuit-element values and the final circuit is indicated in Figure 39.

There is no assurance that a coupling circuit may be found which will match the alternating-pole-zero requirement. The terminating function should not contain any poles not present in z_{22} , for if such were the case, $z_{22} + z_2$ would have more than three zeros. More than three equations similar to (191), (192), and (193) would result, although only three variable parameters are available to solve them.

The procedure may be used with non-RC terminations, such as a simple inductance or a series RL branch.

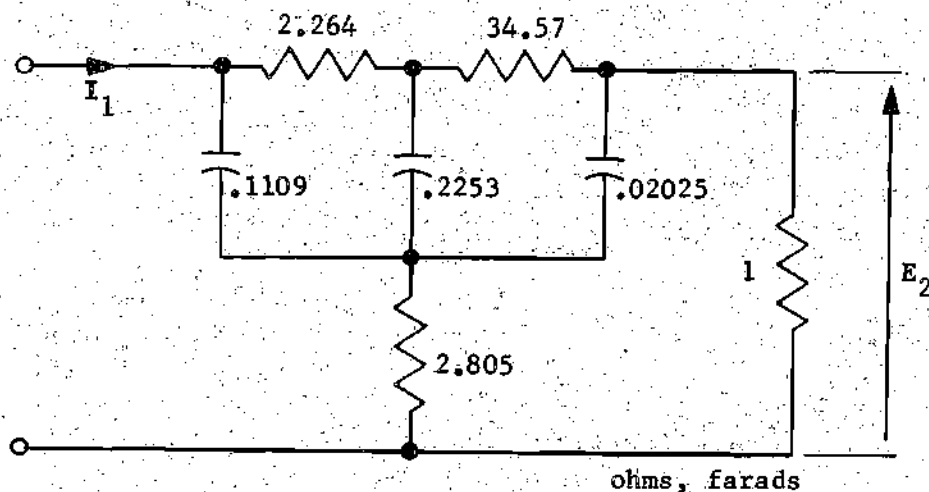


Figure 39. Circuit of Special Technique Example.

Discussion of Future Work

The procedure developed in this thesis is limited to realization of transmission zeros in the first 30° -sector of the right-half plane. In that limited region, realization is not ensured in all possible cases because of the restrictions of the coupling circuits. The problems involved in removing the limitations are discussed in the following sections.

Additional Surplus Factors

Use of more than one surplus factor extends the permissible region for right-half-plane transmission zeros. The surplus-factor zeros, $\sigma_{o1}, \sigma_{o2}, \dots, \sigma_{on}$ must be real and positive for RC circuits.

For example, the function

$$(s + \sigma_{o1})(s + \sigma_{o2})(s^2 - 2\alpha_o s + \omega_o)^2 \quad (197)$$

will have non-negative coefficients if the following conditions are satisfied:

$$\sigma_{o1} + \sigma_{o2} \geq 2\alpha_o \quad (198)$$

$$\omega_o^2 + \sigma_{o1}\sigma_{o2} \geq 2\alpha_o(\sigma_{o1} + \sigma_{o2}) \quad (199)$$

$$\omega_o^2(\sigma_{o1} + \sigma_{o2}) \geq 2\alpha_o\sigma_{o1}\sigma_{o2} \quad (200)$$

The additional requirement that the quadratic polynomial,

$$s^2 + (\sigma_{o1} + \sigma_{o2})s + \sigma_{o1}\sigma_{o2} \quad (201)$$

must have real zeros will be satisfied if

$$(\sigma_{o1} + \sigma_{o2})^2 \geq 4\sigma_{o1}\sigma_{o2} \quad (202)$$

A straightforward procedure for determining the conditions under which all of the inequalities are satisfied is unknown. Condition (202) is satisfied with the restricted condition that

$$\sigma_{o1} = \sigma_{o2} \quad (203)$$

Substituting (203) in (198) and using the equality sign, the surplus factor values become

$$\sigma_{o1} = \sigma_{o2} = \alpha_o \quad (204)$$

Restriction (204), when substituted in condition (199), results in

$$\omega_0^2 \geq 3\alpha_0^2. \quad (205)$$

The same condition, (205), is obtained by substituting (203) in (200). The resulting value of σ_{01} using the equality sign of (200) is then substituted in (199).

Condition (205) limits the right-half-plane transmission zero to the region of the s -plane given by

$$54.8^\circ < |\arg s| \leq 90^\circ, \quad (206)$$

an extension of approximately five degrees over the one-surplus-factor region. The actual extension is less than five degrees because the surplus factors cannot be equal.

As an example of the preceding results, the zeros of the polynomial

$$s^2 - 2s + 3 \quad (207)$$

lie on the 54.8° -lines of the s -plane. When multiplied by $(s + 1)^2$ the polynomial has non-negative coefficients given by

$$s^4 + 4s + 3. \quad (208)$$

When multiplied by $(s + 3)^2$, the non-negative coefficients are given by

$$s^4 + 4s^3 + 27. \quad (209)$$

If the surplus factors are made slightly unequal in either case, negative coefficients appear in the result of the multiplication.

Similarly, for three equal surplus factors, the transmission-zero restriction is

$$\omega_o^2 \geq \frac{8}{3} \alpha_o^2 \quad (210)$$

or

$$52.3^\circ < |\arg s| \leq 90^\circ \quad (211)$$

The preparatory-step conditions for additional surplus factors become:

$$y_{11}(s_o) = y_{1p}(s_o) \quad (212)$$

$$y_{11}'(s_o) = y_{1p}'(s_o) \quad (213)$$

$$y_{11}(-\sigma_{o1}) = y_{1p}(-\sigma_{o1}) \quad (214)$$

$$\vdots$$

$$y_{11}(-\sigma_{on}) = y_{1p}(-\sigma_{on}) \quad (215)$$

$$y_{11}'(-\sigma_{o1}) > y_{1p}'(-\sigma_{o1}) \quad (216)$$

$$\vdots$$

$$y_{11}'(-\sigma_{on}) > y_{1p}'(-\sigma_{on}) \quad (217)$$

or

$$z_{11}(s_o) = z_{1p}(s_o) \quad (218)$$

$$z_{11}'(s_o) = z_{1p}'(s_o) \quad (219)$$

$$z_{11}(-\sigma_{o1}) = z_{1p}(-\sigma_{o1}) \quad (220)$$

$$\vdots$$

$$z_{11}(-\sigma_{on}) = z_{1p}(-\sigma_{on}) \quad (221)$$

$$z_{11}'(-\sigma_{01}) < z_{1p}'(-\sigma_{01}) \quad (222)$$

$$\begin{array}{c} \vdots \\ \vdots \\ \vdots \\ z_{11}'(-\sigma_{on}) < z_{1p}'(-\sigma_{on}) \end{array} \quad (223)$$

where Y_{1p} and Z_{1p} are RC functions and $\sigma_{01}, \dots, \sigma_{on}$ represent the surplus-factor zeros. The procedure for proving these conditions is a straightforward extension of the proof given in Chapter III for one surplus factor, and therefore is not repeated here.

Coupling circuits for the procedure must be derived. The family of circuits for use with two surplus factors and parameters will include a triple-tee circuit, the circuits of Figures 15 and 19, the circuits of Figures 13 and 17 with extended ladder sections, and other circuits.

The terminating immittance must be two degrees less in the numerator and in the denominator than the driving-point immittance, regardless of the number of surplus factors.

Non-Compact Coupling Circuits

The list of coupling circuits given in Chapter II is not complete enough to include all possible combinations of given driving-point functions and given transmission zeros without the removal of both series and shunt elements. The procedure would be improved with the addition of other compact three-terminal networks.

An alternative to finding other compact networks would be to develop two sets of non-compact networks - one set for use in Z_{12} synthesis and the other for use in Y_{12} synthesis. The necessity for two sets of coupling circuits was discussed in Chapter II.

The non-compact networks appear to have fewer restrictions on pole locations. For example, the Guillemin parallel-ladder networks

have no restrictions on pole locations. However, the non-compact poles of the coupling circuit must also be poles of the driving-point immittance. In addition, the alternating-pole-zero requirement for the $y_{22} + Y_2$ or $z_{22} + Z_2$ functions must be satisfied. The functions have the numerator factor $(s + \sigma_0)$ as in the compact case. The preparatory-step conditions appear to be the same for non-compact and compact coupling circuits.

The family of non-compact coupling circuits for use with one surplus factor may contain circuits with one or two non-compact poles. For example, the coupling circuits of Figures 15 and 19 are non-compact at the σ_2 -pole if σ_2 is chosen to be less in magnitude than σ_1 . The Guillemin parallel-ladder networks are, in general, non-compact at both poles. In certain cases, the non-compact pole may be made to coincide with the surplus-factor zero σ_0 . If this is done, the $(s + \sigma_0)$ factor will cancel in $-y_{12}$, simplifying the preparatory-step conditions. An interesting sub-case to consider here, is the effect of causing $(s + \sigma_0)$ to cancel in y_{11} or y_{22} as well as in $-y_{12}$. The cancellation may be realized using the methods of this thesis.

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VITA

Theodore Dean Lindgren was born in Forest City, Iowa, on February 5, 1933. He is the son of Gilmore R. and Hazel Hill Lindgren. He was married to Henrietta Pannell of Montgomery, Alabama, in June of 1960 and they have two children.

He attended public school in Lanyon, Iowa, where he graduated from high school in 1950. In 1955 he received a B.S.E.E. degree and in 1959 a M.S.E.E. degree from Iowa State University.

From June of 1955 to March of 1956, he held the position of Electrical Engineer with Sandia Corporation, Albuquerque, New Mexico. From March of 1956 to March of 1958, he was a project officer in the U.S. Army. From March of 1958 to September of 1958, he was a field engineer for Vitro Laboratories, assigned to the Bahama Islands. He was a graduate assistant at Iowa State University in 1959 and joined the staff of the Georgia Institute of Technology as an Instructor in September of that year. He held a Ford Foundation Fellowship at the Georgia Institute of Technology from September of 1962 to September of 1964.